DIGITAL COMPUTER SIMULATION MODEL OF THE ENGLISHTOWN AQUIFER IN THE NORTHERN COASTAL PLAIN OF NEW JERSEY

U.S. GEOLOGICAL SURVEY

Water-Resources Investigations

Open-File Report 77-73

Prepared in cooperation with the

NEW JERSEY DEPARTMENT OF ENVIRONMENTAL PROTECTION, DIVISION OF WATER RESOURCES

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NOTATION

[Number in parentheses refers to the page, where the symbol first appears or where additional clarification may be obtained. Symbols are also defined in the text. There is some duplication of symbols because of the desire to preserve the notation used in the original papers.]

A	area (58) $[(L^2)]$
a_v	coefficient of compressibility (43)
b	thickness of aquifer (25) [L]
e	void ratio at time t (43)
e_o	initial void ratio (43)
Δe	change in void ratio (43)
H_o	stepwide change in head in the aquifer (46) [L]
h'	hydraulic head differential (42) [L]
h'	excess head (46) [L]
$h_{i,j,o}$	initial head in the aquifer (70) [L]
$\underline{h}_{i,j,o}$	hydraulic head on the other side of the confining bed (71) [L]
I, I_{min}, I_{max}	iteration parameters (71)
i,j	index in the y and x direction (66)
K	hydraulic conductivity (25) $[LT^{-1}]$
$K'K'',K'''$	vertical hydraulic conductivity of confining layers (22) (55) (63) $[LT^{-1}]$
k	time index (66)
ℓ'	thickness of the aquifer (46) [L]
ℓ',ℓ'',ℓ'''	thickness of the confining layers (46) (55) (63) [L]
m_v	coefficient of volume compressibility (43)
N_x, N_y	number of nodes in the x and y direction (71)
n	porosity (43)
n	index indicating the cycle of iteration (70)
p	pressure (43) $[ML^{-1}T^{-2}]$
Q,Q',Q''	flow out of confining layer per unit area (50) (70) (57) $[LT^{-1}]$
Q_D	dimensionless total flow from confining layer per unit area (54)
Q_r	steady leakage function (68)
Q'_{ST}	total volume of steady leakage (61)

NOTATION--Continued

Q''_T	total volume of flow (57) $[L^3]$
Q_w	well discharge or recharge (68) $[LT^{-1}]$
q'	specific discharge (44)
q'_ℓ	specific flux at aquifer-confining layer interface per unit area (48) $[LT^{-1}]$
q'_s	rate of steady leakage per square foot of confining layer-aquifer interface (61) $[LT^{-1}]$
q_w	average rate of well discharge or recharge (68) $[LT^{-1}]$
r	radial distance from pumped well [L]
S	storage coefficient (22) [dimensionless]
S_s	specific storage of aquifer (45) $[L^{-1}]$
S_s', S_s'', S_s'''	specific storage of confining beds (22) (55) (63) $[L^{-1}]$
T	transmissivity (22) $[L^2T^{-1}]$
T_D	dimensionless time (54)
T_{xx}, T_{yy}	components of the transmissivity tensor (65) $[L^2t^{-1}]$
t	time (T)
t_k	total elapsed time since pumping began
u	excess hydrostatic pressure (42)
u	r^2S/Tt (22)
W	volume flux per unit area (65) $[LT^{-1}]$
$\Delta x, \Delta y$	space increment in the x and y direction (66)
Z,Z',Z"	vertical dimension (44) (46) [L]
α	compressibility of sediment (45) $[ML^{-1}T^{-2}]$
β	compressibility of water (45) $[ML^{-1}T^{-2}]$
β	$1/4\ r\ \lambda$ (22)
γ_w	unit weight of water (42) $[M/L^3]$
λ	$\sqrt{\frac{K'S'_s}{TS} + \frac{K''S''_s}{TS}}$ (22)

CONVERSION FACTORS

Factors for converting English units to the International System of Units (SI) are given below to four significant figures. However, in the text the metric equivalents are shown only to the number of significant figures consistent with the values for the English units.

English	Multiply by	Metric
ft (feet)	0.3048	m (meters)
ft/mi (feet per mile)	0.1894	m/km (meters per kilometer)
ft^2/d (square feet per day)	0.0929	m^2/d (square meters per day)
gal (gallons)	0.003785	m^3 (cubic meters)
mi (miles)	1.609	km (kilometers)
mi^2 (square miles)	2.590	km^2 (square kilometers)
Mgal/d (million gallons per day)	0.04381	m^3/s (cubic meters per second)

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By W. D. Nichols

ABSTRACT

Continued decline of water levels in the Englishtown aquifer has caused considerable concern regarding the ability of the aquifer to meet future yield demands. A detailed study of the capability of the aquifer to yield water entailed the use of a digital computer simulation model as a quantitative experimental tool to evaluate aquifer and confining layer coefficients and to test alternative concepts of the hydrodynamics of the flow system. The modeled area of principle concern includes about 750 square miles of the northern Coastal Plain of New Jersey and encompasses all the major centers of pumping from the Englishtown aquifer. The simulation model was calibrated by matching computed declines with historical water-level declines over the 12-year period, 1959-70.

The Englishtown Formation of Late Cretaceous age is exposed in the western part of the New Jersey Coastal Plain along a northeast-southwest zone extending from Raritan Bay to Delaware Bay. The formation, which typically consists of a series of thin, cross-stratified, fine- to medium-grained lignitic quartzose sand beds intercalated with thin beds of silty clay and clay silt, dips gently toward the southeast. The confined part of the aquifer in the Englishtown Formation is utilized as a source of water in the New Jersey Coastal Plain and is an important source of supply in Monmouth and northern Ocean Counties. The annual average rate of withdrawal from the aquifer in the two-county area increased from 5.5 million gallons per day in 1959 to 10 million gallons per day in 1970. Water levels in parts of this area are declining 8 to 12 feet per year and declined as much as 140 feet between 1959 and 1970 near pumping centers. The aquifer transmissivity ranges from 2,400 square feet per day to 650 square feet per day, and the estimated hydraulic conductivity ranges from about 11 feet per day to 20 feet per day; the storage coefficient ranges from 8×10^{-5} to 3×10^{-4}. The underlying and overlying confining beds, which have an average thickness of 200 feet and 40 feet, respectively, have vertical hydraulic conductivities of approximately 1×10^{-5} feet per day and specific storage of approximately 8×10^{-5} ft^{-1}.

The volume of transient and steady leakage into the Englishtown aquifer from and through the adjacent confining layers equaled more than 90 percent of the total volume of water withdrawn from the aquifer between 1959 and 1970. The analytical estimate of transient leakage indicates that about 60 percent of the water withdrawn from the Englishtown between 1959 and 1970 was replaced by water released from storage in the

adjacent confining beds. An additional 34 percent of the withdrawal over this time period was supported by steady leakage through the overlying confining bed from the Mount Laurel aquifer. Water lost by the Mount Laurel through steady leakage is, in turn, replaced by induced recharge from the outcrop area of the Mount Laurel Sand and by water released from storage in the thick confining layer sequence overlying this aquifer.

The initial, or prototype, simulation model was constructed early in the study, using only those data that were immediately available. The results obtained from the prototype model clearly indicated the additional data required for improved simulation capabilities. The second stage of model development involved extensive revisions of and additions to the original computer program to provide for more accurate simulation of a confined leaky aquifer. The results obtained from this model led to a detailed analytical evaluation of the significance of regional transient and steady leakage and to further changes in the conceptual model of the hydrodynamics of the Englishtown aquifer. The third version of the Englishtown aquifer simulation model includes transient leakage from the two adjacent confining beds and controlled steady leakage through the overlying confining beds from the Mount Laurel aquifer. The volumes of transient and steady leakage calculated by the model agree closely with those determined analytically. This version of the simulation model computes water-level declines that satisfactorily approximate historical data. The values of aquifer and confining-layer coefficients used in the model are nearly the same as the average values obtained from field and laboratory data.

INTRODUCTION

The aquifer in the Englishtown Formation is one of the more important sources of ground water in the northern part of the New Jersey Coastal Plain. Total withdrawals for public supply from the Englishtown in Monmouth and northern Ocean Counties increased from about 5.5 Mgal/d (million gallons per day) in 1959 to about 10.0 Mgal/d in 1970. Although these withdrawals may seem small in comparison with withdrawals from ground-water sources in other areas, they have placed a considerable stress on the yield capabilities of the Englishtown aquifer. Water levels in the aquifer have been declining 8 to 12 ft (feet) per year over large areas since 1959. The rate of decline has accelerated during the past several years near some centers of pumping in response to increased withdrawals.

Recognizing the need to provide quantitative solutions to the questions arising from the demands for continued increased development of the ground-water resources of the New Jersey Coastal Plain, the U.S. Geological Survey in cooperation with the Division of Water Resources of the New Jersey Department of Environmental Protection began a program of regional geohydrologic studies of the aquifers in the Coastal Plain aquifer system. These studies, in addition to utilizing the applicable analytical techniques needed to define system parameters, will include simulation modeling of individual aquifers. The models in most, if not all, cases will be digital computer simulation models and will serve three specific purposes. First, the modeling effort will identify those data that are essential to a quantitative study of the hydraulics of the aquifer and the areas where such data are most needed. Second, the model provides a tool for the investigator to test and evaluate alternative concepts of the functioning of the ground-water system. Third, when completed and calibrated, the model is a tool that can be used by the water-resource planner and manager to predict the effects of alternative schemes of resource development.

Digital computer simulation of the functioning of aquifers uses hydrodynamic theory, geohydrologic data, and computer programming logic to symbolically replicate the pertinent elements of a ground-water flow system. The interactions among the various elements of the system are described by mathematical formulations, which are finite-difference approximations of the differential equations of flow. By themselves, the equations and the computer program that embodies them provide a general mathematical model that is applicable to a variety of complex flow problems encountered in field situations. Specific problems are investigated by providing the general model with the geohydrologic data and boundary conditions needed to define the system under consideration. With the completely assembled model it is possible to examine the validity of any number of assumptions concerning the behavior of the real-world aquifer. The ultimate goal, of course, is to create a simulation model, using geohydrologic parameters that are accurate within the limits determined by the investigator for the problem to be resolved, that will imitate the response of the aquifer to hydrologic stress. Once the behavioral imitation is achieved, the calibrated model can be used to explore the effects on the ground-water system of manipulating and changing controllable variables and thus develop reliable predictions of aquifer response to a variety of ground-water development and management schemes.

Purpose and Scope

This report describes the geohydrologic parameters required for a quantitative analysis of the aquifer system in Monmouth County and northern Ocean and Burlington Counties, where the aquifer constitutes a major source of water supply, and documents the development and calibration of the Englishtown aquifer simulation model. The geologic framework of the Englishtown Formation is briefly discussed. The lithologic character of the formation is described, and the geometry of the aquifer and adjacent confining beds is defined. The hydraulic parameters that control the movement of water in the aquifer system have been computed from field and laboratory test data. The hydrologic significance of confining beds in a confined aquifer system is discussed, and an analytical regional evaluation is made of transient and steady leakage in the Englishtown aquifer system. These data and the concepts of the response of the aquifer system to hydrologic stress provide the basis for the construction and development of a digital computer simulation model of the Englishtown aquifer system.

The mathematical model and the method used to solve the finite-difference equations are summarized. The model assumes two-dimensional flow in the plane of the aquifer and includes both transient and steady leakage from two confining layers as a one-dimensional flux at each confining layer-aquifer interface. Although additional refinements can and will be made as new data are obtained, the model in its present stage of development can be used by planners to provide reliable estimates of the effects of proposed management schemes.

An aquifer simulation model is commonly developed with the objective that it will reliably predict aquifer response to hydrologic stress. This is the use to which it will be put by the water-resource planner and manager. From the standpoint of the investigator, a simulation model is a powerful experimental tool with which alternative concepts of the hydrodynamics of a given ground-water system can be tested. Aquifer simulation models are not produced as the culmination of an aquifer study. Rather, model development should begin at the inception of the investigation or as soon after as possible. During the initial stage of model development it may be necessary to estimate the values of several pertinent parameters or even to exclude temporarily some aspects of the problem from consideration. As new data are obtained and various concepts tested the model will be refined. Additionally, the significant aspects of the flow system, those that will require more accurate representation in the model, will become obvious; and the investigator can concentrate his efforts in the most productive direction.

Previous Investigations

The geology of the Coastal Plain of New Jersey and of the Englishtown Formation has been discussed by several authors. Among the more recent studies are those by Owens and others (1968; 1970), Minard (1969), Minard and others (1969), Owens and Sohl (1969), and Owens and Minard (1970). A more extensive list of references to the geologic literature of the New Jersey Coastal Plain and the Englishtown and adjacent formations can be found in the articles cited above.

The regional hydrology of the Englishtown Formation is discussed briefly by Barksdale and others (1958) and by Parker and others (1964). Reports by Jablonski (1959, 1960, and 1968) on Monmouth County; Rush (1962 and 1968) on Burlington County; Anderson and Appel (1969) on Ocean County; Donsky (1963) and Farlekas and others (1976) on Camden County; Vecchioli and Palmer (1962) on Mercer County; Hardt (1963) and Hardt and Hilton (1969) on Gloucester County; and Rosenau and others (1969) on Salem County contain well records, logs, chemical analyses, and brief descriptions of the geology, hydrology, and water quality of the Englishtown Formation in these counties. Seaber (1965) discussed the geology, geohydrology, and chemical quality of the water in the Englishtown Formation and their interrelationships throughout much of the formation's occurrence in the New Jersey Coastal Plain. Nichols (1977) defined the geohydrologic parameters of the Englishtown Formation required for a quantitative analysis of the aquifer in the northern part of the coastal plain of New Jersey.

The computer program that was eventually developed for the Englishtown simulation model is based on programs developed and written by Pinder (1970), Pinder and Bredehoeft (1968), and Bredehoeft and Pinder (1970). The program used during the initial stage of model development was written by Pinder (1970) and modified only slightly for application to the present study. G. F. Pinder (written commun., 1970) subsequently revised the original version of the program to include an improved technique for solving the finite-difference equation. This version was extensively revised, rewritten, and expanded by this author for simulation of the Englishtown aquifer.

Area Modeled

The geohydrologic investigation of the Englishtown Formation covers an area of about 1,450 mi^2 of the northern Coastal Plain of New Jersey. The simulation model of the Englishtown aquifer covers about 1,900 mi^2 and includes the approximately 750 mi^2 area of principle concern in Monmouth County, northern Ocean County, and northeastern Burlington County (fig. 1). The modeled area is bounded on the north by Raritan Bay and on the northwest by the outcrop area of the Englishtown Formation. The southern boundary of the modeled area was somewhat arbitrarily located at about lat 39°45' N. or about 22 mi (miles) south of the southernmost well field developed in the aquifer in Ocean County. The eastern border of the model extends from 18 to 20 mi offshore. The region encompassed by the model includes the principle areas of ground-water development in the aquifer in southeastern Monmouth and northeastern Ocean Counties. The eastern and southern model boundaries were extended considerably beyond the area of main interest in order to avoid model response interference by improperly posed boundary conditions. This is more fully discussed in the section that considers assumptions made in the simulation model.

Acknowledgments

The author is especially indebted to several U.S. Geological Survey personnel: George F. Pinder, for making available the several versions of his simulation model computer program and for his explanations and guidance in using the program; John D. Bredehoeft, for his explanations and discussions of the mathematical equations of the hydrodynamics of fluid flow in low permeability sediments, and for providing the author

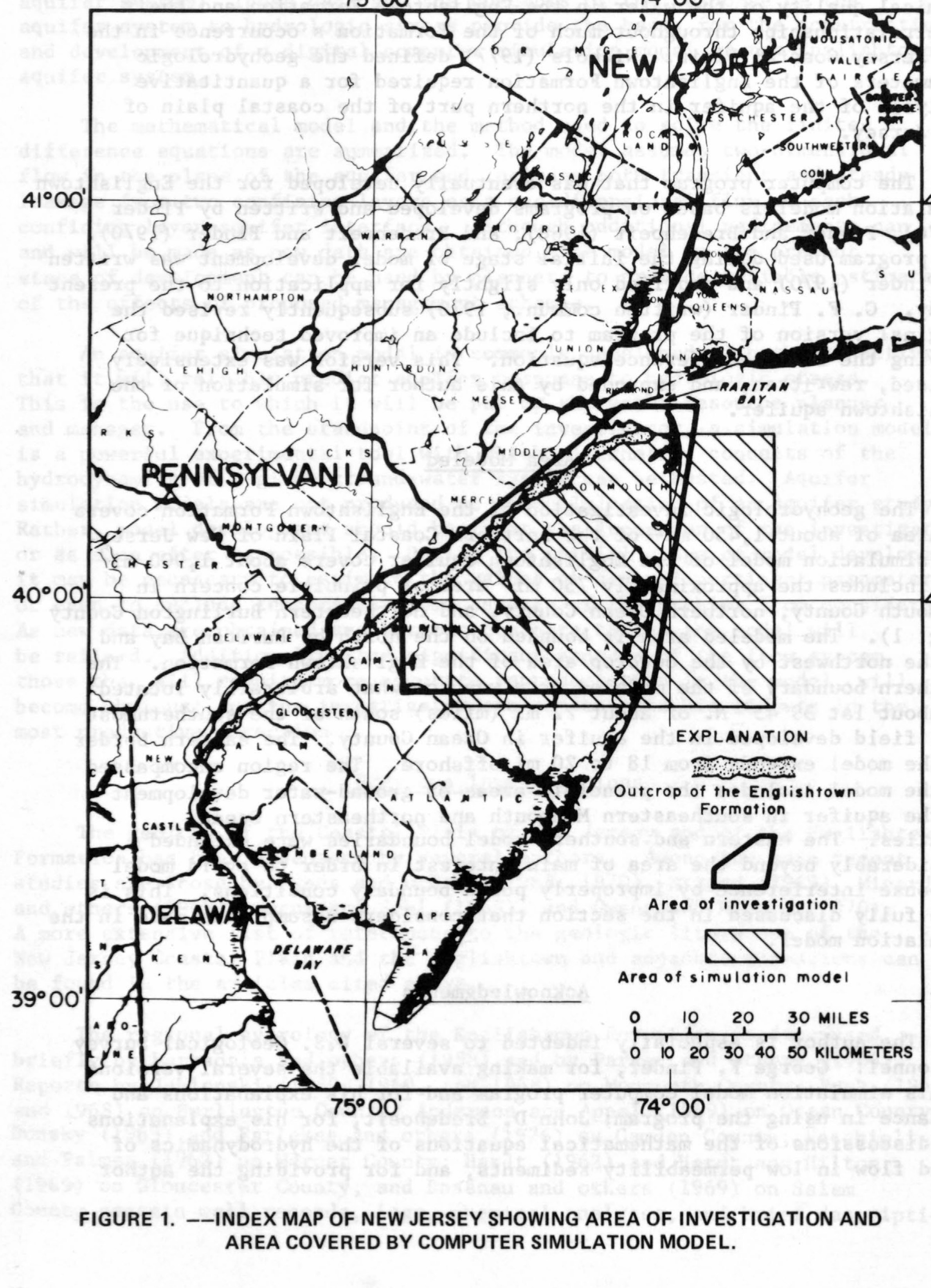


FIGURE 1. —INDEX MAP OF NEW JERSEY SHOWING AREA OF INVESTIGATION AND AREA COVERED BY COMPUTER SIMULATION MODEL.

with the initial version of the computer subroutine for calculating transient flow from a confining bed; Frances S. Riley, for his helpful discussions of the compaction of clayey sediments and of the application of the modified theory of leaky aquifers to aquifer test data; and Roger G. Wolff, for his assistance in interpreting soil consolidation test data.

GEOLOGIC FRAMEWORK OF THE ENGLISHTOWN AQUIFER SYSTEM

The New Jersey Coastal Plain is underlain by a wedge-shaped mass of unconsolidated and partly consolidated marine, marginal marine, and nonmarine deposits of clay, silt, sand, and gravel. The sediments range in age from Cretaceous to Holocene (table 1) and lie unconformably on the pre-Cretaceous basement. The total thickness of the sedimentary sequence in the outcrop area ranges from 500 to 1,000 ft; the sequence thickens downdip, toward the southeast, and attains a maximum aggregate thickness of about 6,500 ft at the extreme southern tip of New Jersey.

The geology of the pre-Quaternary Coastal Plain formations of New Jersey has been discussed by several investigators and was recently summarized by Owens and Minard (1970) and Owens and Sohl (1969). These studies, especially the more recent ones, have shown that there are significant differences in the character and occurrence of these formations between Raritan Bay in the northeast and Delaware Bay to the southwest. The differences reflect the varying patterns and environments of sedimentation that existed throughout the region, especially during Late Cretaceous time. Depositional patterns in the northern Coastal Plain of New Jersey seem to have been controlled largely by two basement tectonic elements; a trough or small basin centering in the vicinity of Raritan Bay and a northwest-southeast trending high in southern New Jersey.

The stratigraphic units that are included in this study as components of the Englishtown aquifer system all belong to the Matawan Group of Late Cretaceous age and include the Merchantville Formation, Woodbury Clay, Englishtown Formation, Marshalltown Formation, and Wenonah Formation. The Matawan Group is underlain by the Upper Cretaceous Raritan and Magothy Formations and is overlain by formations of the Monmouth Group, also of Late Cretaceous age (table 1).

Merchantville Formation and Woodbury Clay

The Merchantville Formation disconformably overlies the Magothy throughout the New Jersey Coastal Plain. It consists primarily of an interbedded marine sequence of dark-colored thin micaceous clayey silt and very fine silty sand with massive thick beds of silty glauconite sand in the Raritan Bay area. The formation becomes thicker-bedded toward the southwest.

Table 1.--Stratigraphic units of the northern Atlantic Coastal Plain of New Jersey[1]

System	Series	Group	Formation	Lithology
Quaternary	Holocene		Alluvium	Sand, silt, and black mud.
			Beach sand and gravel	Sand, quartz, light-colored, medium-grained, pebbly.
	Pleistocene		Cape May Formation Pensauken Formation[2] Bridgeton Formation	Sand, quartz, light-colored, heterogeneous, clayey, pebbly, glauconitic.
Tertiary	Pliocene(?)		Beacon Hill Gravel	Gravel, quartz, light-colored, sandy.
	Pliocene(?) and Miocene(?)		Cohansey Sand	Sand, quartz, light-colored, medium- to coarse-grained, pebbly; local clay beds.
	Miocene		Kirkwood Formation	Sand, quartz, gray to tan, very fine to medium-grained, micaceous, and dark-colored diatomaceous clay.
	Eocene		Shark River Marl	Sand, quartz and glauconite, gray, brown, and green, fine- to coarse-grained, clayey, and green silty and sandy clay.
		Rancocas Group	Manasquan Formation	
	Paleocene	Rancocas Group	Vincentown Formation	Sand, quartz, gray and green, fine- to coarse-grained, glauconitic, and brown clayey, very fossiliferous, glauconite and quartz calcarenite.
		Rancocas Group	Hornerstown Sand	Sand, glauconite, green, medium- to coarse-grained, clayey.
Cretaceous	Upper Cretaceous	Monmouth Group	Tinton Sand and Red Bank Sand undivided	Sand, quartz and glauconite, brown and gray, fine- to coarse-grained, clayey, micaceous
		Monmouth Group	Navesink Formation	Sand, glauconite and quartz, green, black, and brown, medium- to coarse-grained, clayey.
		Monmouth Group	Mount Laurel Sand	Sand, quartz, brown and gray, fine- to coarse-grained, glauconitic.
		Matawan Group	Wenonah Formation	Sand, quartz, gray and brown, very fine to fine-grained, glauconitic, micaceous.
		Matawan Group	Marshalltown Formation	Sand, quartz and glauconite, gray and black, very fine to medium-grained, very clayey.
		Matawan Group	Englishtown Formation	Sand, quartz, tan and gray, fine- to medium-grained; local clay beds.
		Matawan Group	Woodbury Clay	Clay, gray and black, micaceous.
		Matawan Group	Merchantville Formation	Clay, gray and black, micaceous, glauconitic, silty; locally very fine grained quartz and glauconite sand.
			Magothy Formation	Sand, quartz, light-gray, fine-grained, and dark-gray lignitic clay.
			Raritan Formation	Sand, quartz, light-colored, fine- to coarse-grained, pebbly, arkosic, and red, white, and variegated clay.
Pre-Cretaceous				Precambrian and early Paleozoic crystalline rocks - metamorphic schist and gneiss; locally Triassic basalt, sandstone, and shale.

1/ Modified after Seaber, 1965, table 3.

2/ Age of Pensauken Formation now considered late Miocene.

The Woodbury Clay overlies the Merchantville Formation in the northern and central part of the New Jersey Coastal Plain. It pinches out in the area just north of Swedesboro in southwestern New Jersey and is not seen in outcrop south of that point. The contact between the two formations is gradational. The Woodbury is a massive-bedded dark gray to grayish-black lignitic sandy to clayey silt and silty clay of marine origin.

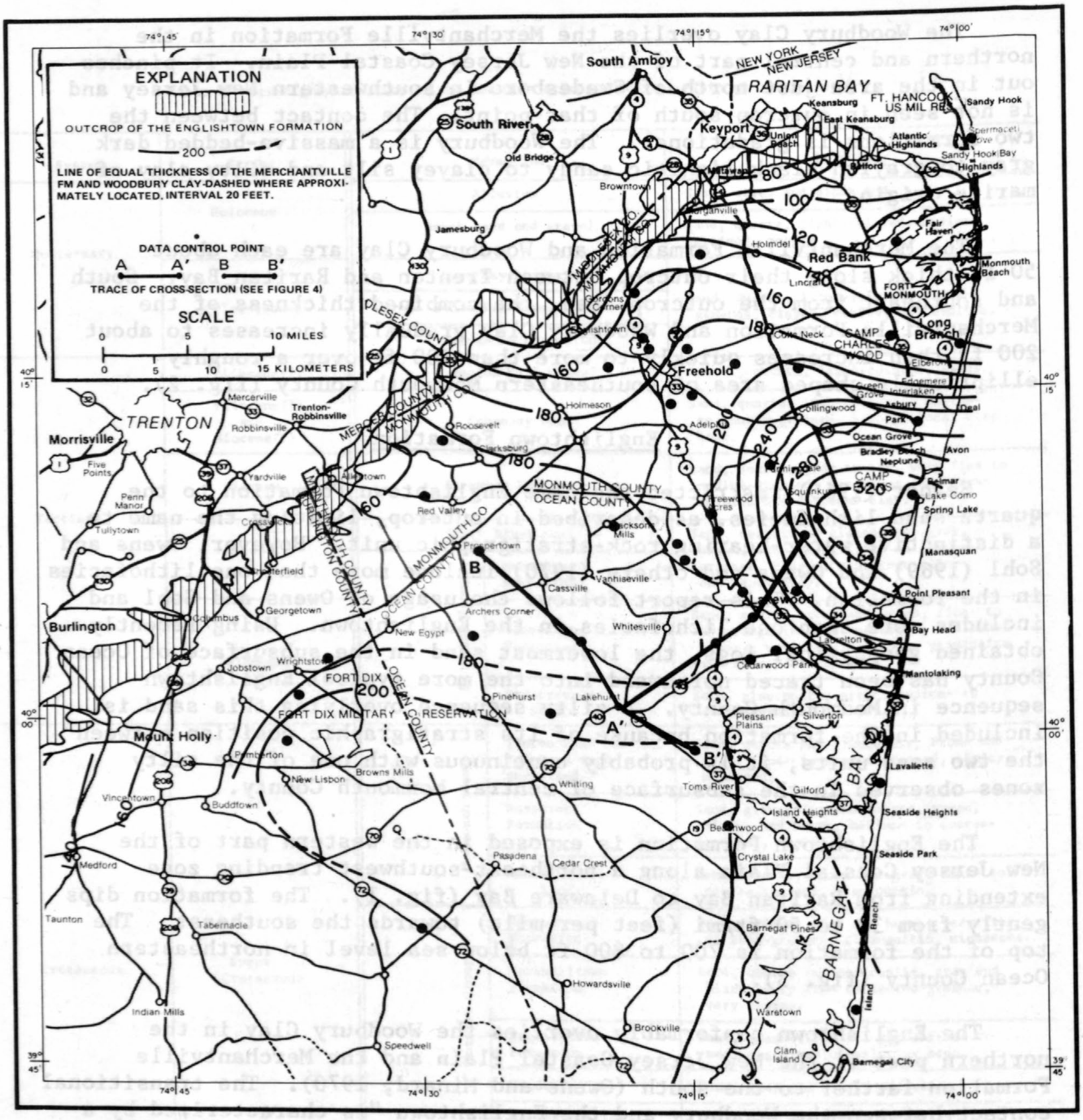
The Merchantville Formation and Woodbury Clay are each about 50 ft thick along their outcrop between Trenton and Raritan Bay. South and southeast from the outcrop area, the combined thickness of the Merchantville Formation and Woodbury Clay gradually increases to about 200 ft then increases quickly to more than 300 ft over a roughly elliptically shaped area of southeastern Monmouth County (fig. 2).

Englishtown Formation

Seaber (1965) restricted the name Englishtown Formation to the quartz sand lithofacies, as described in outcrop, limiting the name to a distinctive water-bearing rock-stratigraphic unit. However, Owens and Sohl (1969) and Owens and others (1970) include more than one lithofacies in the formation. This report follows the usage of Owens and Sohl and includes more than one lithofacies in the Englishtown. Using recently obtained geophysical logs, the lowermost sand in the subsurface of Ocean County has been traced northward into the more typical Englishtown sequence in Monmouth County. A silty sequence overlying this sand is included in the formation because of its stratigraphic position between the two sand units; it is probably continuous with one of the silty zones observed in the subsurface of central Monmouth County.

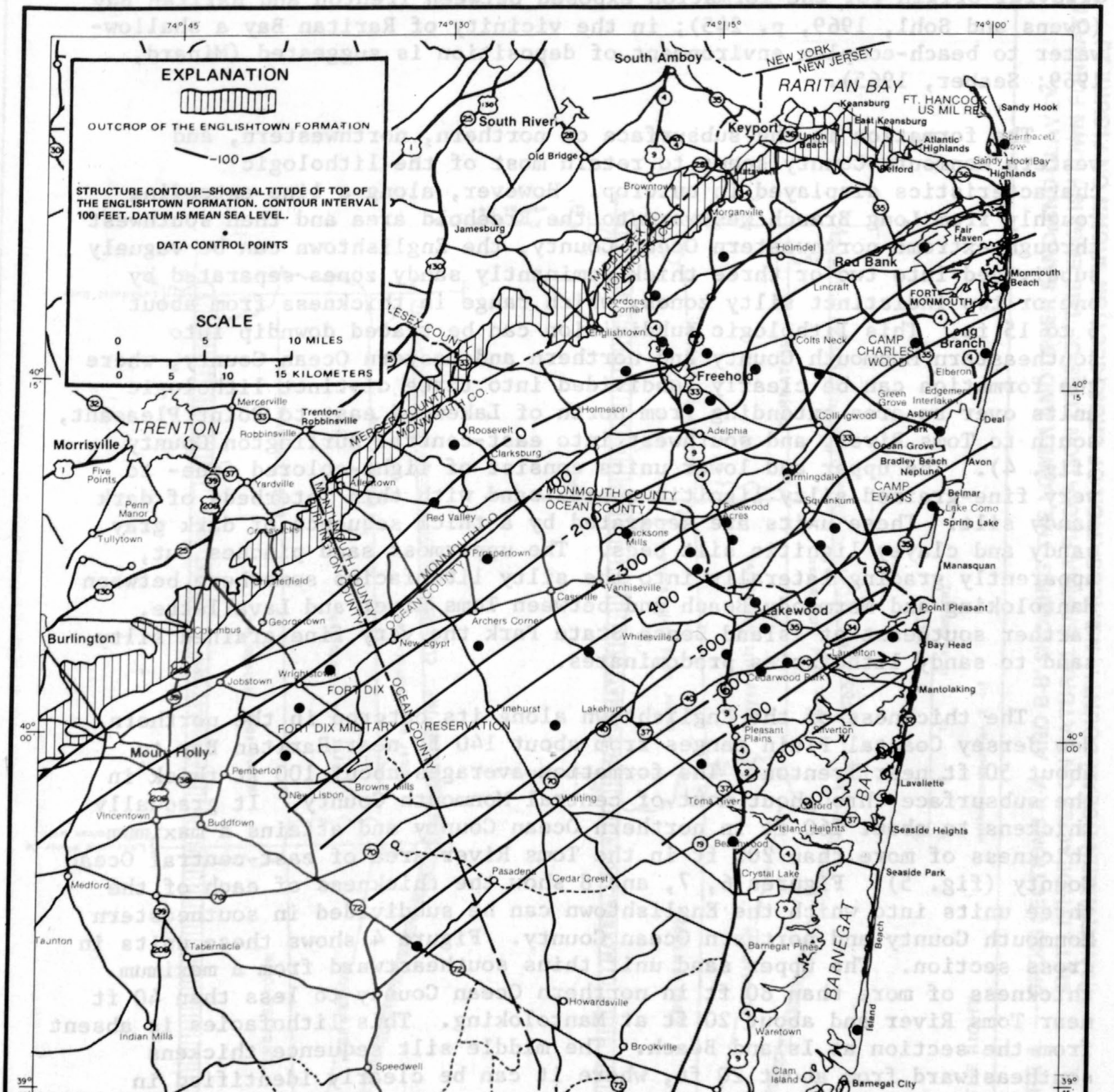
The Englishtown Formation is exposed in the western part of the New Jersey Coastal Plain along a northeast-southwest trending zone extending from Raritan Bay to Delaware Bay (fig. 1). The formation dips gently from 30 to 50 ft/mi (feet per mile) towards the southeast. The top of the formation is 700 to 800 ft below sea level in northeastern Ocean County (fig. 3).

The Englishtown conformably overlies the Woodbury Clay in the northern part of the New Jersey Coastal Plain and the Merchantville Formation farther to the south (Owens and Minard, 1970). The transitional contact between the Woodbury and the Englishtown "is characterized by a gradual increase in sand-sized quartz and a decrease in silt and clay" (Owens and Sohl, 1969, p. 244). The Englishtown Formation along its outcrop in the northern part of the Coastal Plain is typically a series of light gray to white thin cross-stratified fine- to medium-grained lignitic quartz sand beds intercalated with thin beds of dark gray sandy silty clay and clayey silt. A marine and marginal marine origin has been established for parts of the Englishtown, especially along its outcrop south of Trenton. The internal structure of the Englishtown indicates an



Base from Army Map Services, Newark, N.J., 1964 and Wilmington, Del., NJ-PA-MD 1966, 1:250000.
Edited and published by U.S. Geological Survey

FIGURE 2. —COMBINED THICKNESS OF THE MERCHANTVILLE FORMATION AND WOODBURY CLAY IN MONMOUTH AND NORTHERN OCEAN AND BURLINGTON COUNTIES, N.J.



Base from Army Map Services, Newark, N.J., 1964 and Wilmington, Del., NJ-PA-MD 1966, 1:250000.
Edited and published by U.S. Geological Survey

FIGURE 3. —STRUCTURE CONTOURS OF THE TOP OF THE ENGLISHTOWN FORMATION IN MONMOUTH AND NORTHERN OCEAN COUNTIES, N.J.

alluvial origin for the formation exposed between Trenton and Raritan Bay (Owens and Sohl, 1969, p. 245); in the vicinity of Raritan Bay a shallow-water to beach-complex environment of deposition is suggested (Minard, 1969; Seaber, 1965).

The formation in the subsurface of northern, northwestern, and western Monmouth County seems to retain most of the lithologic characteristics displayed in outcrop. However, along a line extending roughly from Long Branch westward to the Freehold area and then southwest through extreme northwestern Ocean County, the Englishtown can be vaguely subdivided into two or three thick dominantly sandy zones separated by one or two indistinct silty zones, which range in thickness from about 5 to 15 ft. This lithologic subdivision can be traced downdip into southeastern Monmouth County and northern and eastern Ocean County, where the formation can be clearly subdivided into three distinct lithologic units over an area extending from north of Lakewood east to Point Pleasant, south to Toms River, and southwest into east-central Burlington County (fig. 4). The upper and lower units consist of light-colored fine- to very fine-grained silty lignitic quartz sand with thin interbeds of dark sandy silt. These units are separated by a thick sequence of dark gray sandy and clayey lignitic silt beds. The uppermost sand pinches out, apparently grading laterally into the silty lithofacies somewhere between Mantoloking and Normandy Beach and between Toms River and Lavallette. Farther southeast at Island Beach State Park the very fine-grained silty sand to sandy lithofacies predominates.

The thickness of the Englishtown along its outcrop in the northern New Jersey Coastal Plain ranges from about 140 ft near Raritan Bay to about 50 ft near Trenton. The formation averages about 100 ft thick in the subsurface throughout most of central Monmouth County. It gradually thickens to about 160 ft in northern Ocean County and attains a maximum thickness of more than 200 ft in the Toms River area of east-central Ocean County (fig. 5). Figures 6, 7, and 8 show the thickness of each of the three units into which the Englishtown can be subdivided in southeastern Monmouth County and northern Ocean County. Figure 4 shows these units in cross section. The upper sand unit thins southeastward from a maximum thickness of more than 80 ft in northern Ocean County to less than 40 ft near Toms River and about 20 ft at Mantoloking. This lithofacies is absent from the section at Island Beach. The middle silt sequence thickens southeastward from about 20 ft, where it can be clearly identified in northern Ocean County and southeastern Monmouth County, to more than 130 ft in the Toms River-Lavallette area. The lower sand has an average thickness of between 30 and 40 ft in north-central Ocean County. The unit thickens slightly toward the southeast to a maximum of about 50 ft at Toms River and Lavallette. It cannot readily be recognized in the section at Island Beach State Park.

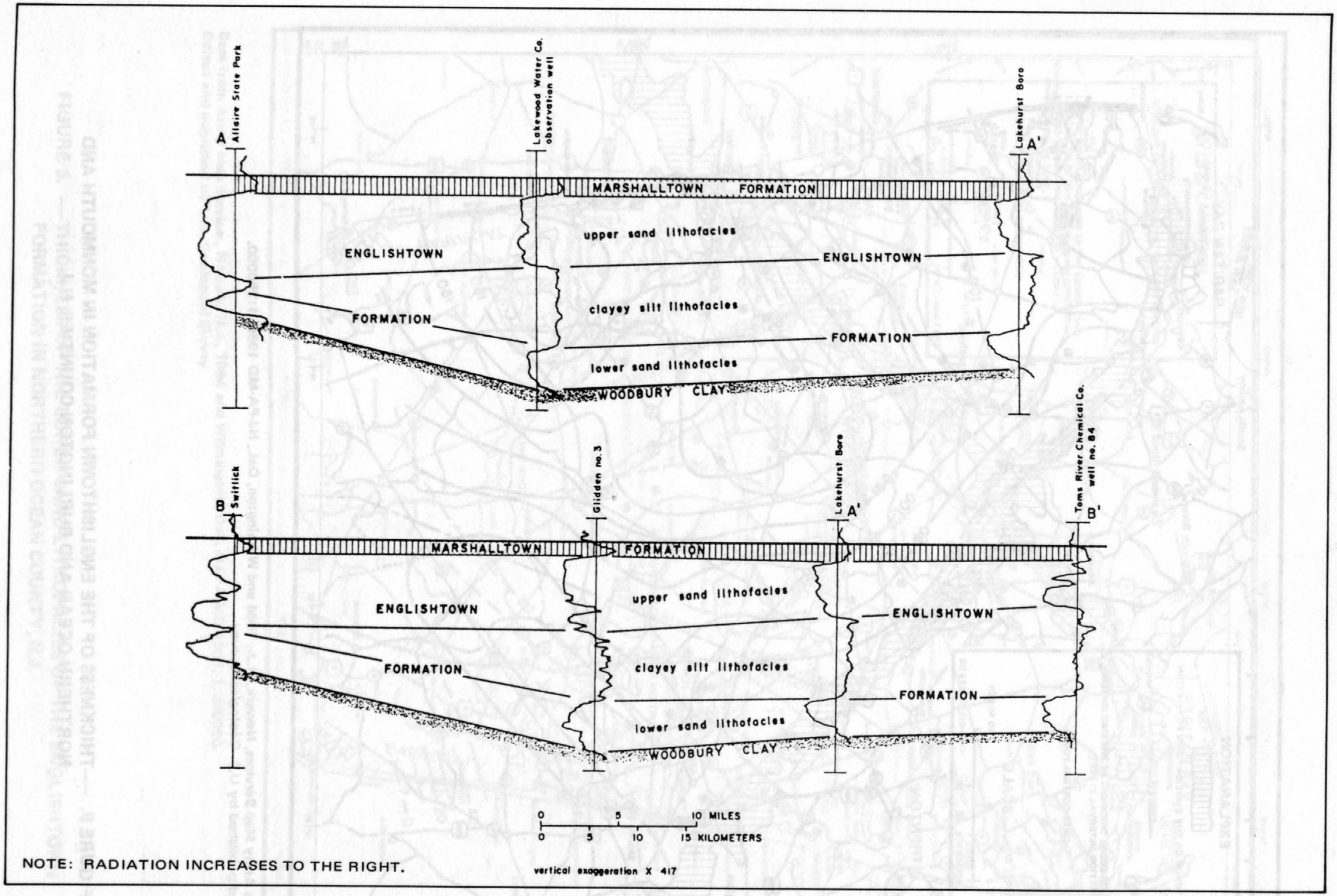
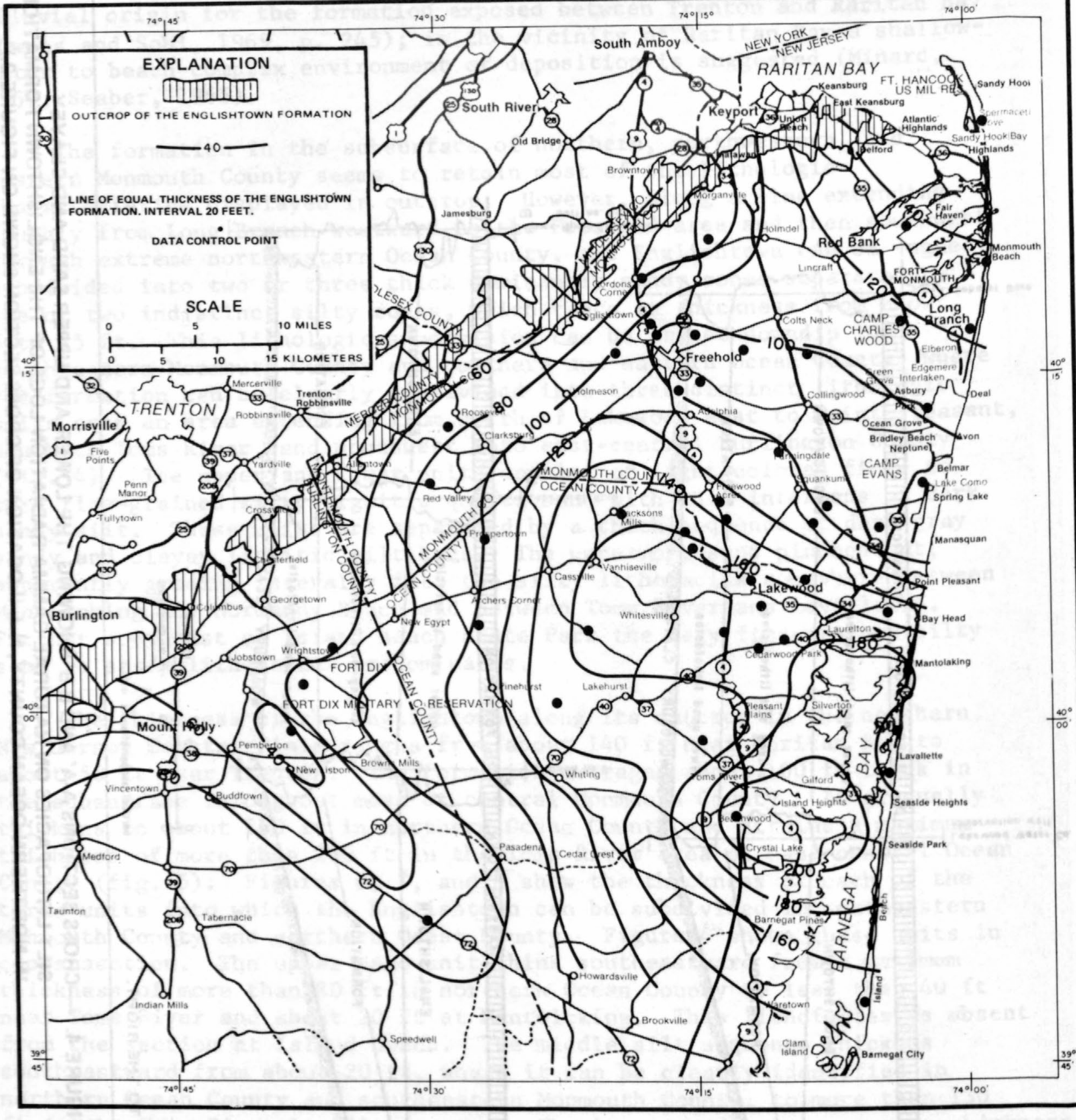


FIGURE 4. —CROSS SECTIONS A-A' AND B-B' SHOWING THE LOWER AND UPPER SAND AND CLAYEY SILT LITHOFACIES ON NATURAL GAMMA RADIATION LOGS OF THE ENGLISHTOWN FORMATION IN NORTHERN OCEAN COUNTY, N.J. (LOCATION OF CROSS-SECTION SHOWN ON FIGURE 2.)



Base from Army Map Services, Newark, N.J., 1964 and Wilmington, Del., NJ-PA-MD 1966, 1:250000.
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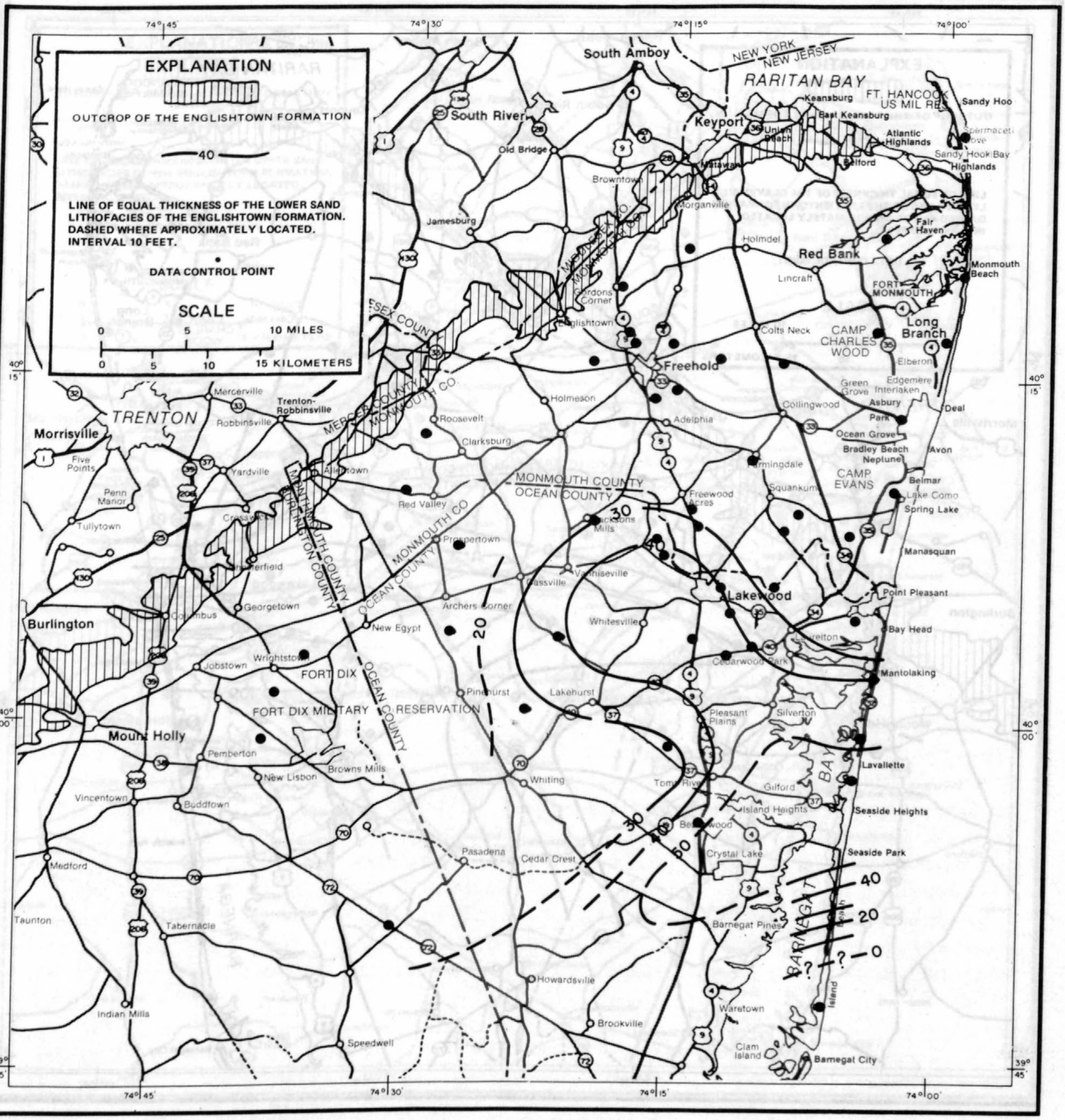
FIGURE 5. —THICKNESS OF THE ENGLISHTOWN FORMATION IN MONMOUTH AND NORTHERN OCEAN AND BURLINGTON COUNTIES, N.J.



Base from Army Map Services, Newark, N.J., 1964 and Wilmington, Del., NJ-PA-MD 1966, 1:250000.
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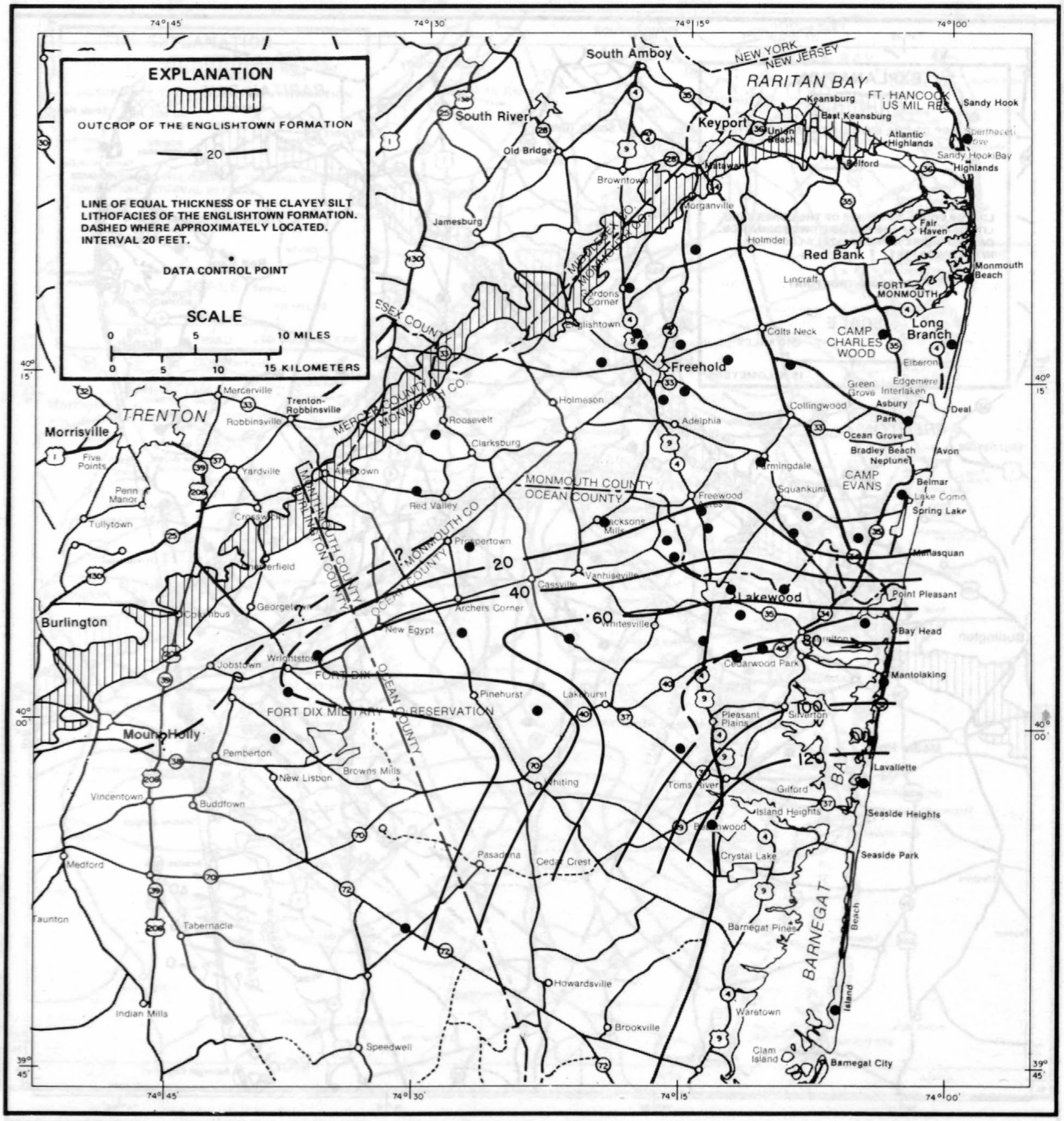
FIGURE 6. —THICKNESS OF THE LOWER SAND LITHOFACIES OF THE ENGLISHTOWN FORMATION IN NORTHERN OCEAN COUNTY, N.J.



Base from Army Map Services, Newark, N.J., 1964 and Wilmington, Del., NJ-PA-MD 1966, 1:250000.
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FIGURE 7. —THICKNESS OF THE CLAYEY SILT LITHOFACIES OF THE ENGLISHTOWN FORMATION IN NORTHERN OCEAN COUNTY, N.J.

Base from Army Map Services, Newark, N.J., 1964 and Wilmington, Del., NJ-PA-MD 1966, 1:250000. Edited and published by U.S. Geological Survey

FIGURE 8. —THICKNESS OF THE UPPER SAND LITHOFACIES OF THE ENGLISHTOWN FORMATION IN NORTHERN OCEAN COUNTY, N.J.

Marshalltown and Wenonah Formations

The Marshalltown Formation, which overlies the Englishtown throughout the New Jersey Coastal Plain, is a massive dark-greenish-black fine- to very fine-grained clayey glauconite-quartz sand and silt. The abrupt contact between the Marshalltown and Englishtown and the presence of reworked Englishtown material in the basal few feet of the Marshalltown suggest a disconformable relationship (Owens and Sohl, 1969). The formation is remarkably uniform in thickness along its entire outcrop in New Jersey, ranging from 10 to 20 ft. The same uniformity is observed everywhere in the subsurface of the northern Coastal Plain of New Jersey.

The overlying Wenonah Formation is a poorly sorted dark silty very fine to fine sand. The contact with the underlying Marshalltown is gradational. The Wenonah obtains a maximum thickness of about 70 ft in outcrop in the vicinity of Mount Holly, N.J. (Minard and others, 1964). It thins along the outcrop both to the northeast and southwest. Minard (1965, 1969) reports a thickness of 25 to 30 ft near Raritan Bay and 15 to 25 ft in extreme southwestern New Jersey. A similar range in thickness is observed in the subsurface of the northern Coastal Plain of New Jersey, although accurate identification of the formation on geophysical logs is difficult.

GEOHYDROLOGY OF THE ENGLISHTOWN AQUIFER SYSTEM

The sand lithofacies of the Englishtown Formation, part of which is shown in figure 4, is utilized as a source of water throughout much of the northern and western Coastal Plain of New Jersey. It is an important source of water supply in the northeastern Coastal Plain in Monmouth County and northern Ocean County. The aquifer is less developed in Burlington, Camden, Gloucester, and Salem Counties to the southwest, largely because of a decrease in thickness and water-yield capabilities and the presence of other more productive aquifers. Total ground-water withdrawals for public supply in Monmouth and northern Ocean Counties increased from 5.5 Mgal/d in 1959 to 10.0 Mgal/d in 1969. Water levels in some parts of this area are declining 8 to 12 ft per year and have declined more than 100 ft near centers of pumping between 1959 and 1970 (fig. 9).

The Englishtown aquifer in Monmouth County including the upper sand lithofacies, together with the underlying and overlying confining beds in the Matawan Group, constitute the Englishtown aquifer system for this study. This system is, in turn, a subsystem of the larger Coastal Plain aquifer system of New Jersey. The overlying confining bed, which includes the Marshalltown Formation and part of the Wenonah Formation, provides the interconnection between the Englishtown aquifer system and the overlying part of the larger Coastal Plain aquifer system. The Merchantville Formation and Woodbury Clay, which together constitute the lower confining bed, perform the same function with respect to that part of the larger Coastal Plain aquifer system underlying the Englishtown aquifer system.

Base from Army Map Services, Newark, N.J., 1964 and Wilmington, Del., NJ-PA-MD 1966, 1:250000.
Edited and published by U.S. Geological Survey

FIGURE 9. —DECLINE IN WATER LEVEL BETWEEN JANUARY 1959 AND NOVEMBER 1970 IN THE ENGLISHTOWN AQUIFER IN MONMOUTH AND NORTHERN OCEAN COUNTIES, N.J.

Aquifer Geometry and Boundaries

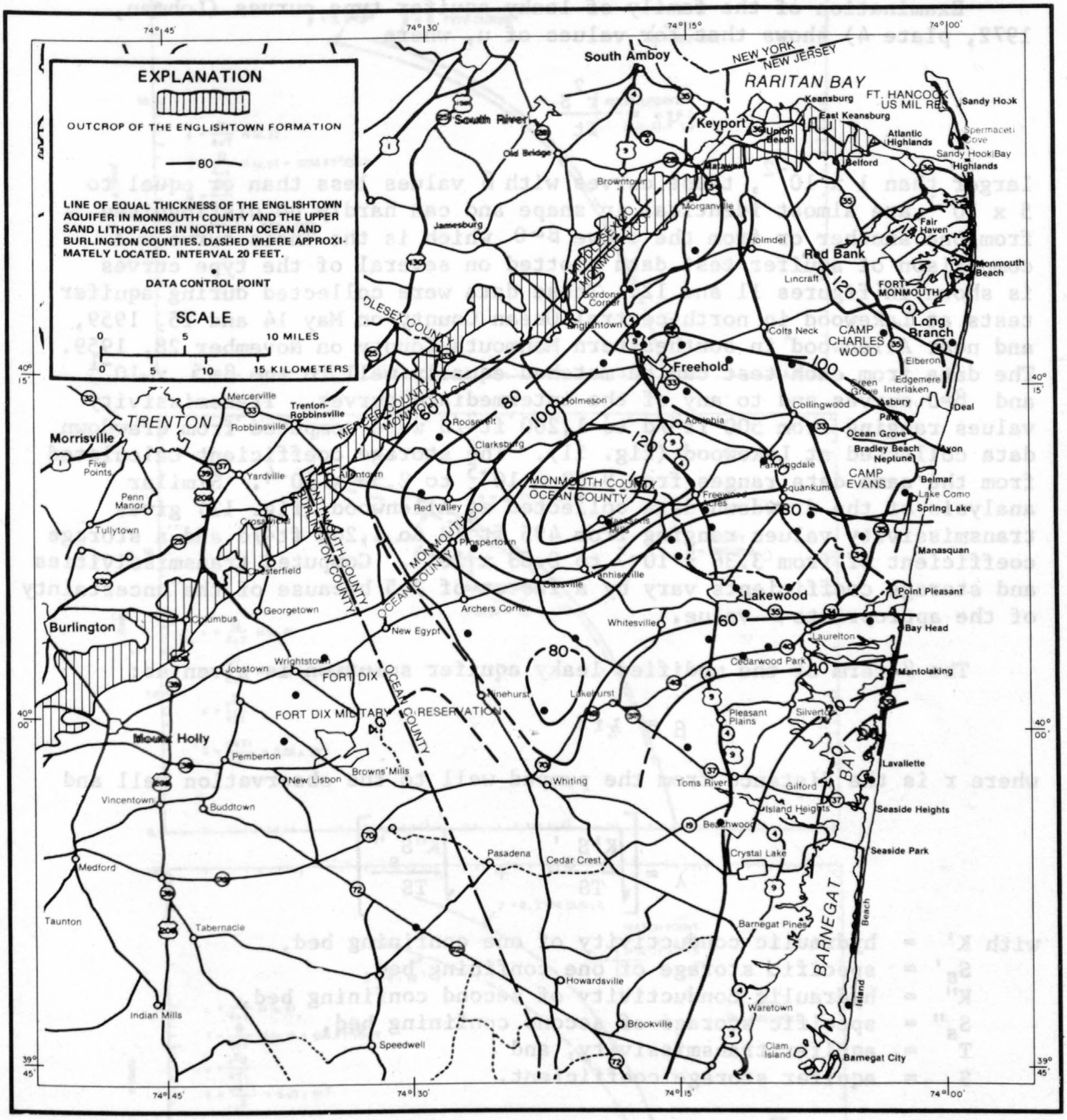
The thickness of the Englishtown aquifer in Monmouth County and the upper sand lithofacies of the aquifer in northern Ocean and Burlington Counties is shown in figure 10. Throughout most of Monmouth County the aquifer includes the entire thickness of the Englishtown Formation. In southeastern Monmouth County and northern Ocean and Burlington Counties, however, only the upper sand lithofacies (fig. 8) is included as part of the aquifer. The lower sand lithofacies (fig. 6) is not included in the aquifer for the purposes of this study because of a lack of water-level data and because of limitations in the capability for simulation modeling of three-dimensional cases, even though the lower sand lithofacies is lithologically and hydrologically continuous with the aquifer over some undefined area across southern Monmouth County and is, in fact, part of the Englishtown aquifer.

The outcrop of the upper contact of the Englishtown Formation marks the northwestern boundary of the confined part of the aquifer. The southward extent of the aquifer into southern Ocean County cannot be determined because of a lack of data. The eastward extent and thickness trends of the aquifer, beyond the New Jersey coast, are also unknown. The thickest parts of the aquifer are in northeastern Monmouth County and south-central Monmouth and north-central Ocean Counties, where it is more than 120 ft thick. The aquifer gradually thins toward the south into central and eastern Ocean County.

Hydraulic Coefficients of the Englishtown Aquifer

Estimated values of transmissivity were calculated using specific capacity data from selected wells. The technique used (Hurr, 1966) to make these estimates is based on a manipulation of the Theis equation. The estimates range from 650 ft^2/d (square feet per day) to 2,400 ft^2/d. The arithmetic mean of 15 estimated transmissivity values is 1,200 ft^2. Hydraulic conductivity values computed from estimated transmissivities range from 11 ft/d (feet per day) to 20 ft/d; the mean and median of 15 values of hydraulic conductivity is 15 ft/d.

Data collected during two aquifer tests are available from which to compute aquifer transmissivity and storage coefficient. The leaky nature of the Englishtown aquifer (which will be discussed subsequently) strongly suggests that these data should be evaluated using modified leaky aquifer theory (Hantush, 1960). However, a reliable match of aquifer test data from one test to type curves of the modified leaky aquifer theory requires data from at least two observation wells at different distances from the pumped well. Analysis of data from the single observation well used in each test in the Englishtown aquifer has been shown to be generally unreliable where significant leakage may be encountered (F. S. Riley and E. J. McClelland, written commun., 1972).



Base from Army Map Services, Newark, N.J., 1964 and Wilmington, Del., NJ-PA-MD 1966, 1:250000.
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FIGURE 10. —THICKNESS OF THE ENGLISHTOWN AQUIFER IN MONMOUTH COUNTY AND OF THE UPPER SAND LITHOFACIES OF THE ENGLISHTOWN AQUIFER IN NORTHERN OCEAN AND BURLINGTON COUNTIES, N.J.

Examination of the family of leaky aquifer type curves (Lohman, 1972, plate 4) shows that for values of u, where

$$u = \frac{r^2S}{Tt}$$

larger than 1×10^{-2}, those curves with β values less than or equal to 5×10^{-1} are almost identical in shape and can hardly be distinguised from one another or from the curve $\beta=0$ which is the Theis curve. A comparison of aquifer test data plotted on several of the type curves is shown in figures 11 and 12. These data were collected during aquifer tests at Lakewood in north-central Ocean County on May 14 and 15, 1959, and near Allenwood in southeastern Monmouth County on November 28, 1959. The data from each test can be matched equally well to the $\beta=5 \times 10^{-1}$ and $\beta=0$ curves and to any of the intermediate curves. Transmissivity values ranging from 500 ft^2/d to 1,260 ft^2/d were computed from drawdown data collected at Lakewood (fig. 11). The storage coefficient calculated from the same data ranges from 9.63×10^{-5} to 2.53×10^{-4}. Similar analysis of the drawdown data collected at Allenwood (fig. 12) gives transmissivity values ranging from 495 ft^2/d to 1,240 ft^2/d and a storage coefficient of from 3.36×10^{-5} to 8.83×10^{-5}. Computed transmissivities and storage coefficients vary by a factor of 2.5 because of the uncertainty of the appropriate β value.

The β term of the modified leaky aquifer solution is given as:

$$\beta = \tfrac{1}{4} r\lambda$$

where r is the distance from the pumped well to the observation well and

$$\lambda = \left[\sqrt{\frac{K'S_s'}{TS}} + \sqrt{\frac{K''S_s''}{TS}}\right]$$

with K' = hydraulic conductivity of one confining bed,
S_s' = specific storage of one confining bed,
K'' = hydraulic conductivity of second confining bed,
S_s'' = specific storage of second confining bed,
T = aquifer transmissivity, and
S = aquifer storage coefficient.

An average value, $\overline{\lambda}$, can be computed for the aquifer using average values for hydraulic conductivity and specific storage for each of the confining beds (table 6), the average value of aquifer hydraulic conductivity obtained from estimated transmissivity values, an average aquifer storage coefficient of 1×10^{-4}, and assuming an average aquifer thickness of 80 ft. The actual aquifer thickness, as determined from geophysical logs, is 70 to 75 ft at Lakewood and 90 ft at Allenwood. Using the following data:

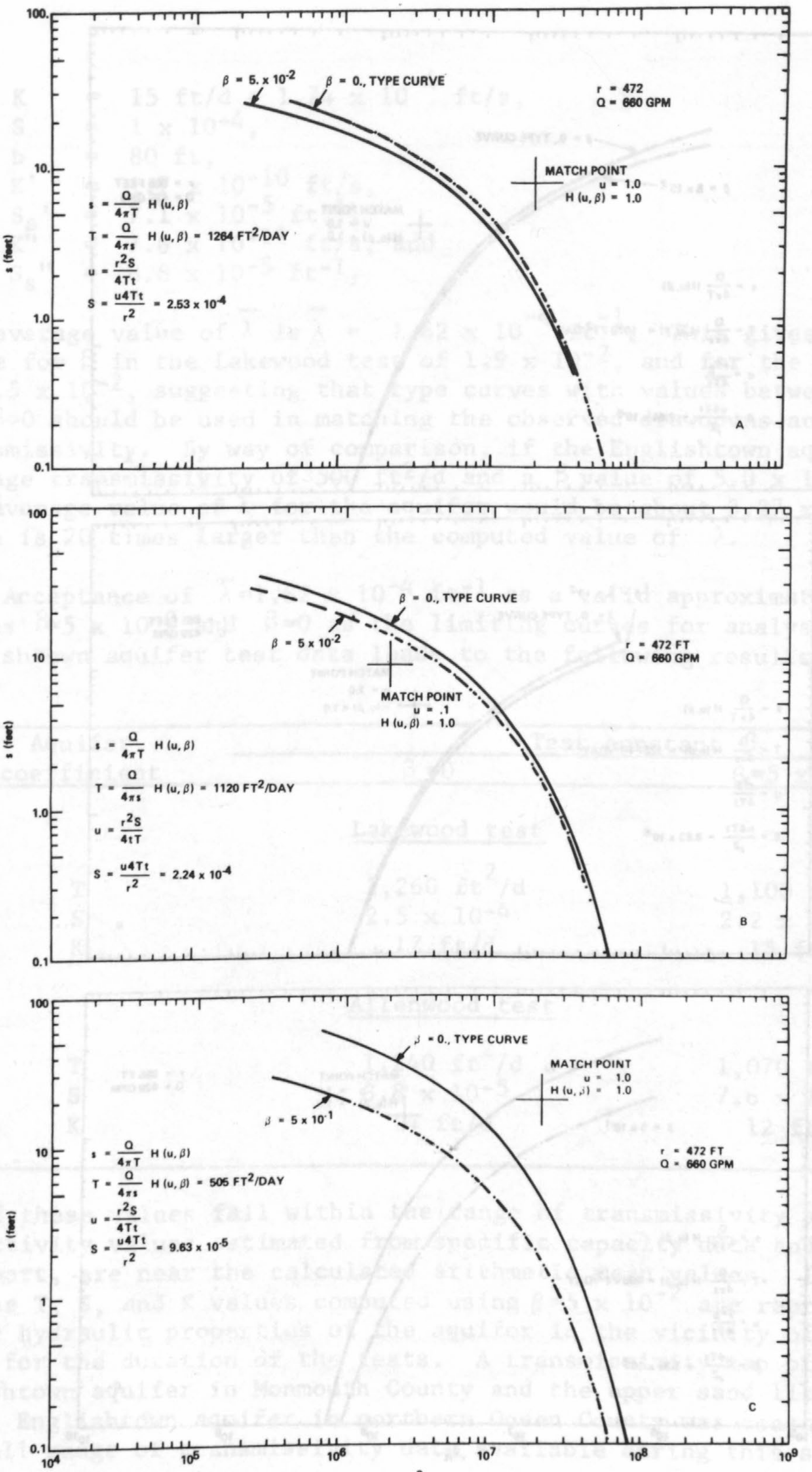


FIGURE 11. —LOGARITHMIC GRAPH OF DRAWDOWN, s, AS A FUNCTION OF r^2/t FOR AQUIFER TEST DATA COLLECTED AT LAKEWOOD, N.J., MAY 14-15, 1959, COMPARED WITH GRAPH OF H(u, β) VS. u FOR A) $\beta = 0.$, B) $\beta = 5 \times 10^{-2}$, AND C) $\beta = 5 \times 10^{-1}$, FOR MODIFIED LEAKY AQUIFIER THEORY.

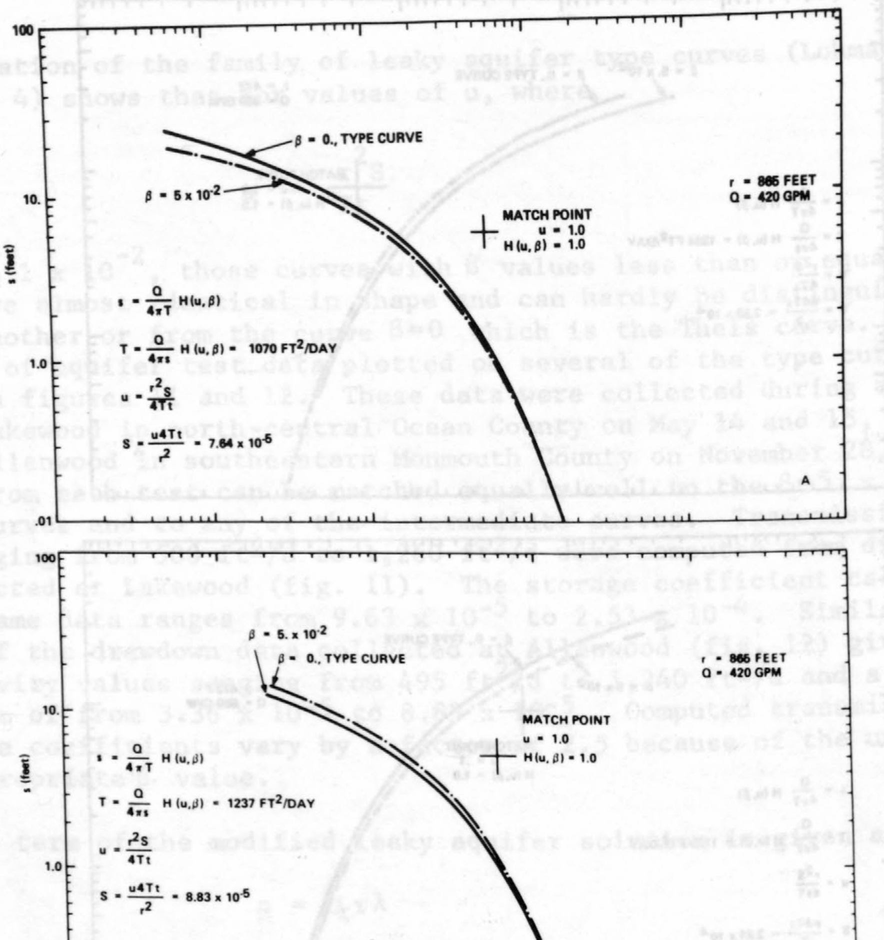


FIGURE 12. —LOGARITHMIC GRAPH OF DRAWDOWN, s, AS A FUNCTION OF r^2/t FOR AQUIFER TEST DATA COLLECTED AT ALLENWOOD, N.J., NOV. 28, 1959, COMPARED WITH GRAPH OF H(u, β) VS. u FOR A) $\beta = 0.$, B) $\beta = 5 \times 10^{-2}$ AND C) $\beta = 5 \times 10^{-1}$, FROM MODIFIED LEAKY AQUIFER THEORY.

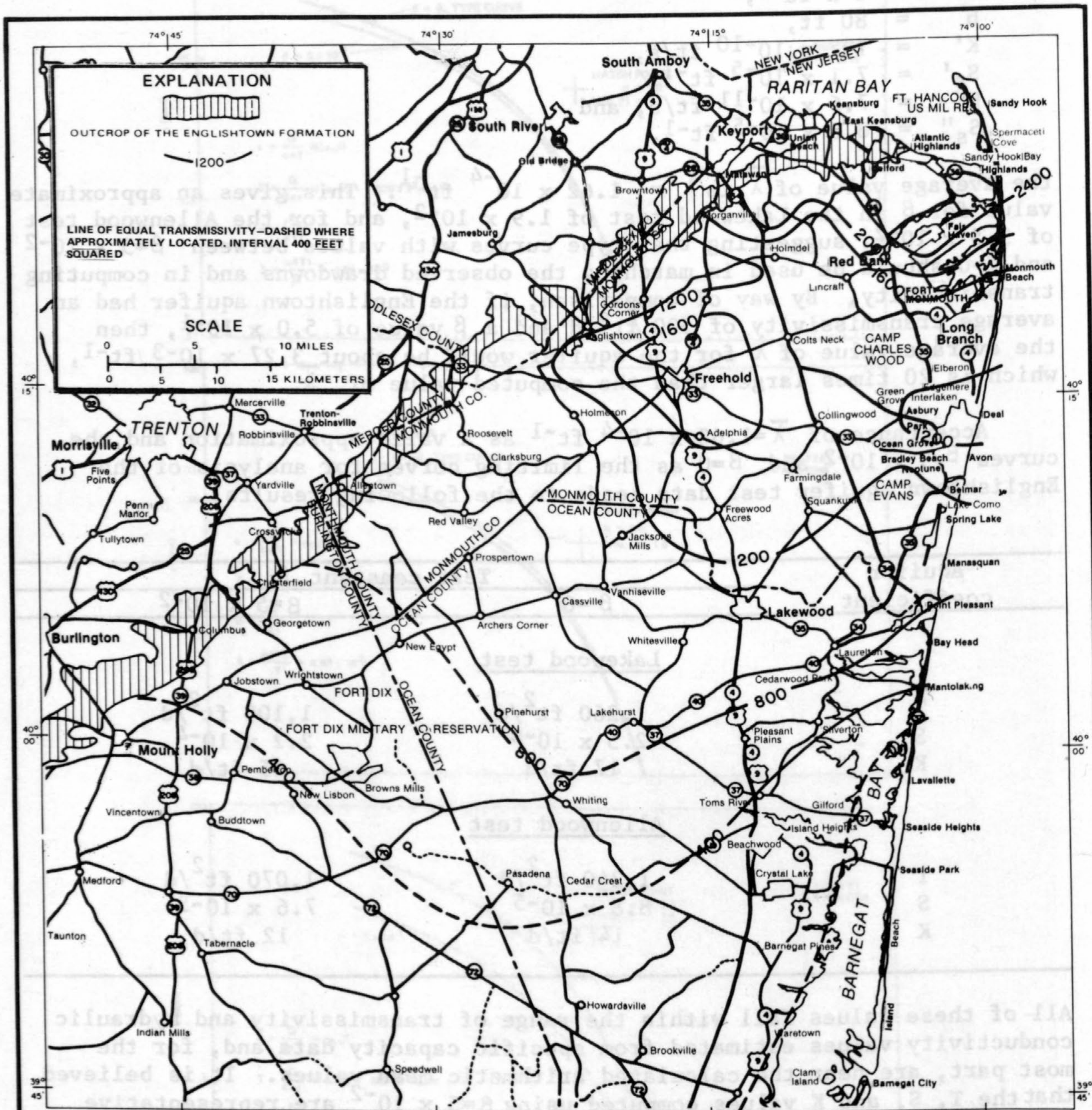
$K = 15$ ft/d $= 1.74 \times 10^{-4}$ ft/s,
$S = 1 \times 10^{-4}$,
$b = 80$ ft,
$K' = 1.7 \times 10^{-10}$ ft/s,
$S_s' = 7.1 \times 10^{-5}$ ft^{-1},
$K'' = 7.6 \times 10^{-11}$ ft/s, and
$S_s'' = 8.8 \times 10^{-5}$ ft^{-1},

the average value of $\bar{\lambda}$ is $\bar{\lambda} = 1.62 \times 10^{-4}$ ft^{-1}. This gives an approximate value for β in the Lakewood test of 1.9×10^{-2}, and for the Allenwood test of 3.5×10^{-2}, suggesting that type curves with values between $\beta=5 \times 10^{-2}$ and $\beta=0$ should be used in matching the observed drawdowns and in computing transmissivity. By way of comparison, if the Englishtown aquifer had an average transmissivity of 500 ft^2/d and a β value of 5.0×10^{-1}, then the average value of λ for the aquifer would be about 3.27×10^{-3} ft^{-1}, which is 20 times larger than the computed value of $\bar{\lambda}$.

Acceptance of $\bar{\lambda}=1.62 \times 10^{-4}$ ft^{-1} as a valid approximation and the curves $\beta=5 \times 10^{-2}$ and $\beta=0$ as the limiting curves for analysis of the Englishtown aquifer test data leads to the following results:

Aquifer coefficient	Test constant β: $\beta=0$	Test constant β: $\beta=5 \times 10^{-2}$
	Lakewood test	
T	1,260 ft^2/d	1,100 ft^2/d
S	2.5×10^{-4}	2.2×10^{-4}
K	17 ft/d	15 ft/d
	Allenwood test	
T	1,240 ft^2/d	1,070 ft^2/d
S	8.8×10^{-5}	7.6×10^{-5}
K	14 ft/d	12 ft/d

All of these values fall within the range of transmissivity and hydraulic conductivity values estimated from specific capacity data and, for the most part, are near the calculated arithmetic mean values. It is believed that the T, S, and K values computed using $\beta=5 \times 10^{-2}$ are representative of the hydraulic properties of the aquifer in the vicinity of the test sites for the duration of the tests. A transmissivity map of the Englishtown aquifer in Monmouth County and the upper sand lithofacies of the Englishtown aquifer in northern Ocean County was constructed using the full range of transmissivity data available during this study (fig. 13).



Base from Army Map Services, Newark, N.J., 1964 and Wilmington, Del., NJ-PA-MD 1966, 1:250000.
Edited and published by U.S. Geological Survey

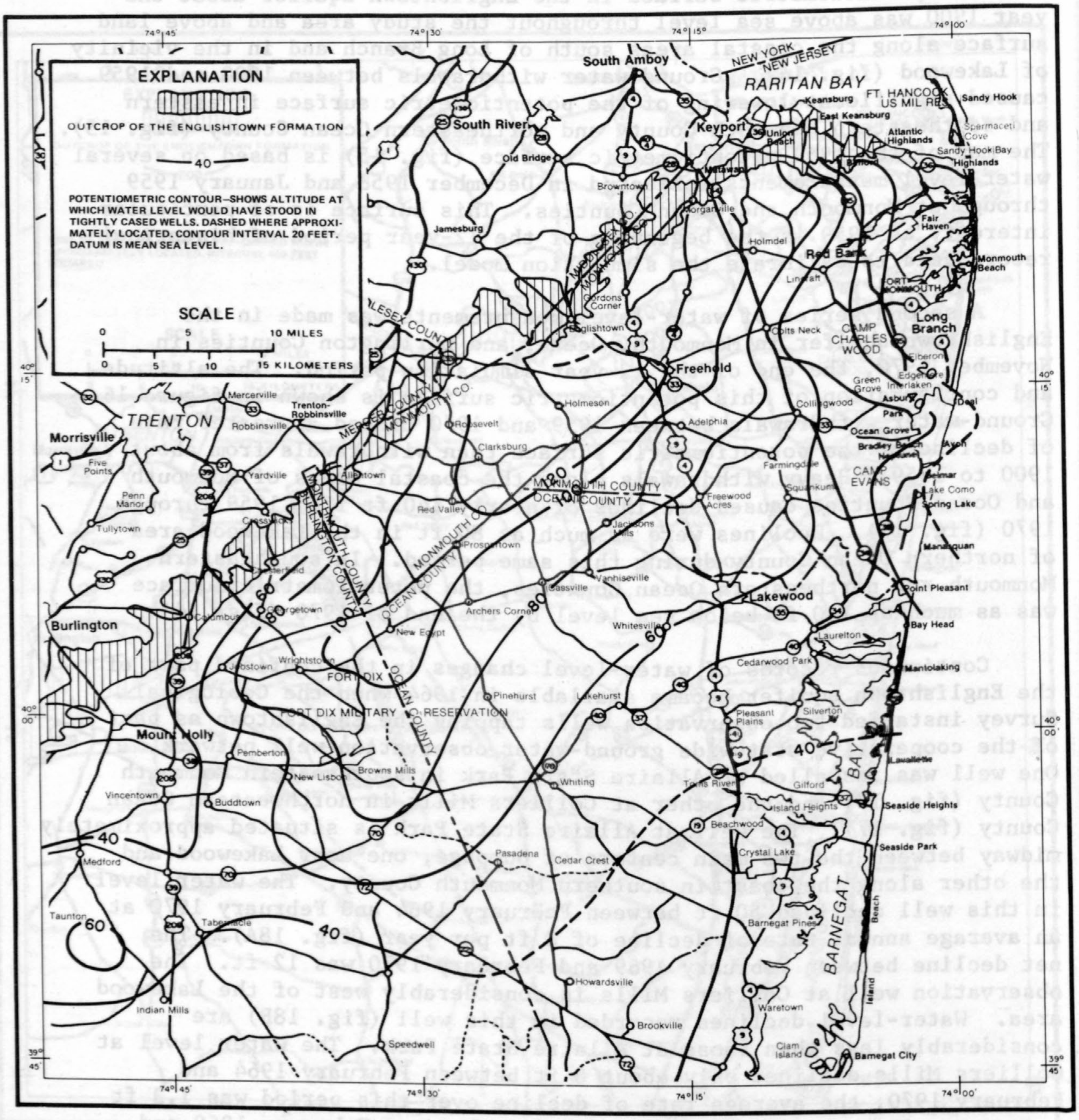
FIGURE 13. —TRANSMISSIVITY OF THE ENGLISHTOWN AQUIFER IN MONMOUTH AND NORTHERN OCEAN COUNTIES, N.J.

The Potentiometric Surface and Water-level Trends

The potentiometric surface in the Englishtown aquifer about the year 1900 was above sea level throughout the study area and above land surface along the coastal areas south of Long Branch and in the vicinity of Lakewood (fig. 14). Ground-water withdrawals between 1900 and 1959 caused significant lowering of the potentiometric surface in eastern and southeastern Monmouth County and northeastern Ocean County (fig. 15). The map of the 1959 potentiometric surface (fig. 15) is based on several water-level measurements collected in December 1958 and January 1959 throughout Monmouth and Ocean Counties. This surface is of particular interest, as 1959 is the beginning of the 12-year period of historical record used to calibrate the simulation model.

A second series of water-level measurements was made in the Englishtown aquifer in Monmouth, Ocean, and Burlington Counties in November 1970, the end of the 12-year simulation period. The altitude and configuration of this potentiometric surface is shown in figure 16. Ground-water withdrawals between 1959 and 1970 caused a greater rate of decline of the potentiometric surface than withdrawals from about 1900 to 1959. Heavy withdrawals along the coastal areas of Monmouth and Ocean Counties caused declines of nearly 140 ft from 1959 through 1970 (fig. 17). Declines were as much as 80 ft in the Lakewood area of northern Ocean County during this same period. In southeastern Monmouth and northeastern Ocean Counties, the potentiometric surface was as much as 160 ft below sea level by the end of 1970 (fig. 16).

Continuous records of water-level changes in the confined part of the Englishtown aquifer became available in 1964 when the Geological Survey installed two observation wells tapping the Englishtown as part of the cooperative statewide ground-water observation well network. One well was installed at Allaire State Park in southeastern Monmouth County (fig. 17) and the other at Colliers Mills in northwestern Ocean County (fig. 17). The well at Allaire State Park is situated approximately midway between the two main centers of pumpage, one near Lakewood and the other along the coast in southern Monmouth County. The water level in this well declined 50 ft between February 1964 and February 1970 at an average annual rate of decline of 8 ft per year (fig. 18A). The net decline between February 1969 and February 1970 was 12 ft. The observation well at Colliers Mills is considerably west of the Lakewood area. Water-level declines recorded in this well (fig. 18B) are considerably less than those at Allaire State Park. The water level at Colliers Mills declined only about 8 ft between February 1964 and February 1970; the average rate of decline over this period was 1.3 ft per year. The water level declined 1.2 ft between February 1969 and February 1970.



Base from Army Map Services, Newark, N.J., 1964 and Wilmington, Del., NJ-PA-MD 1966, 1:250000.
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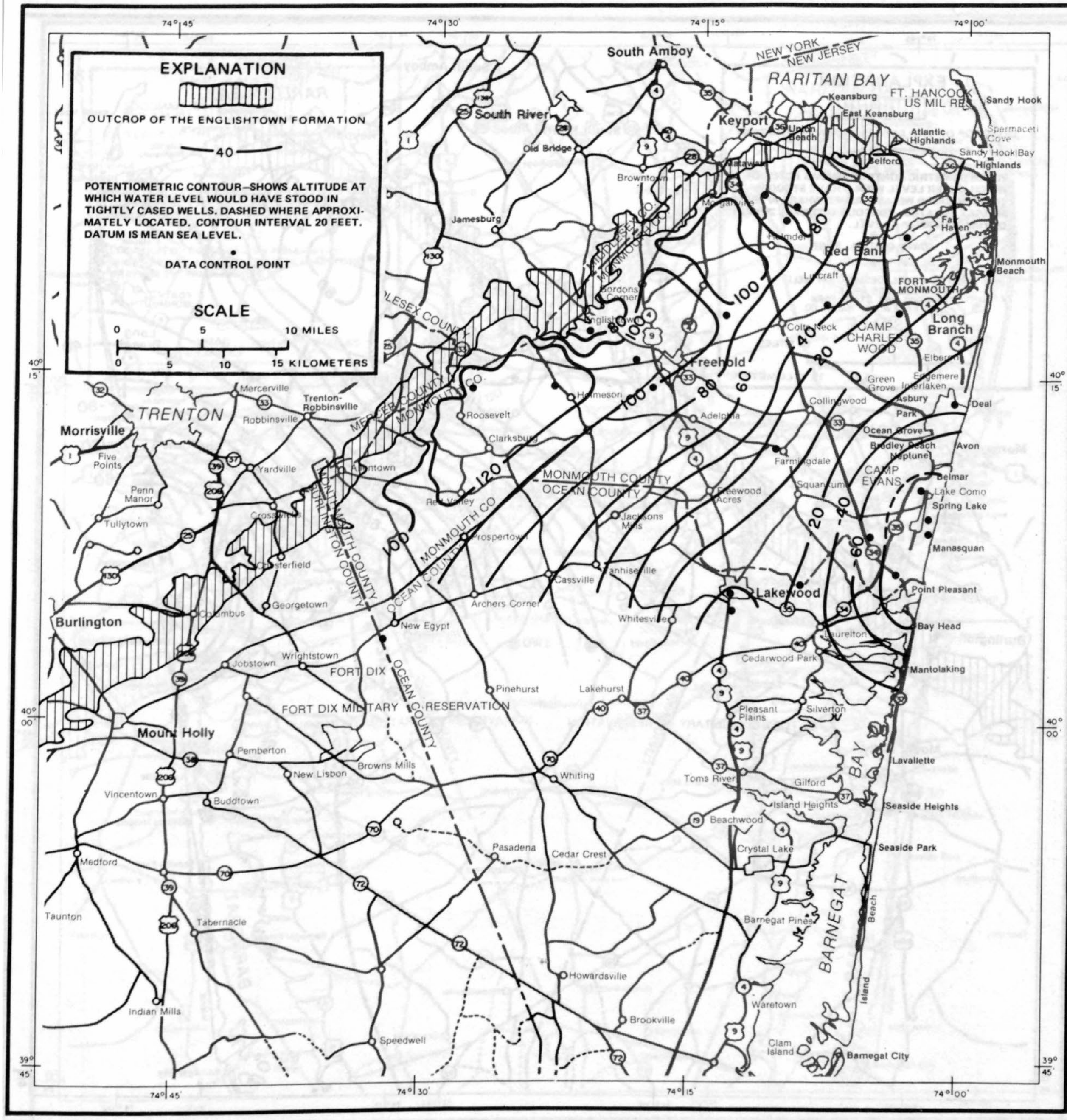
FIGURE 14. --ALTITUDE OF THE POTENTIOMETRIC SURFACE IN THE ENGLISHTOWN AQUIFER ABOUT 1900 IN MONMOUTH AND NORTHERN OCEAN AND BURLINGTON COUNTIES, N.J.



Base from Army Map Services, Newark, N.J., 1964 and Wilmington, Del., NJ-PA-MD 1966, 1:250000.
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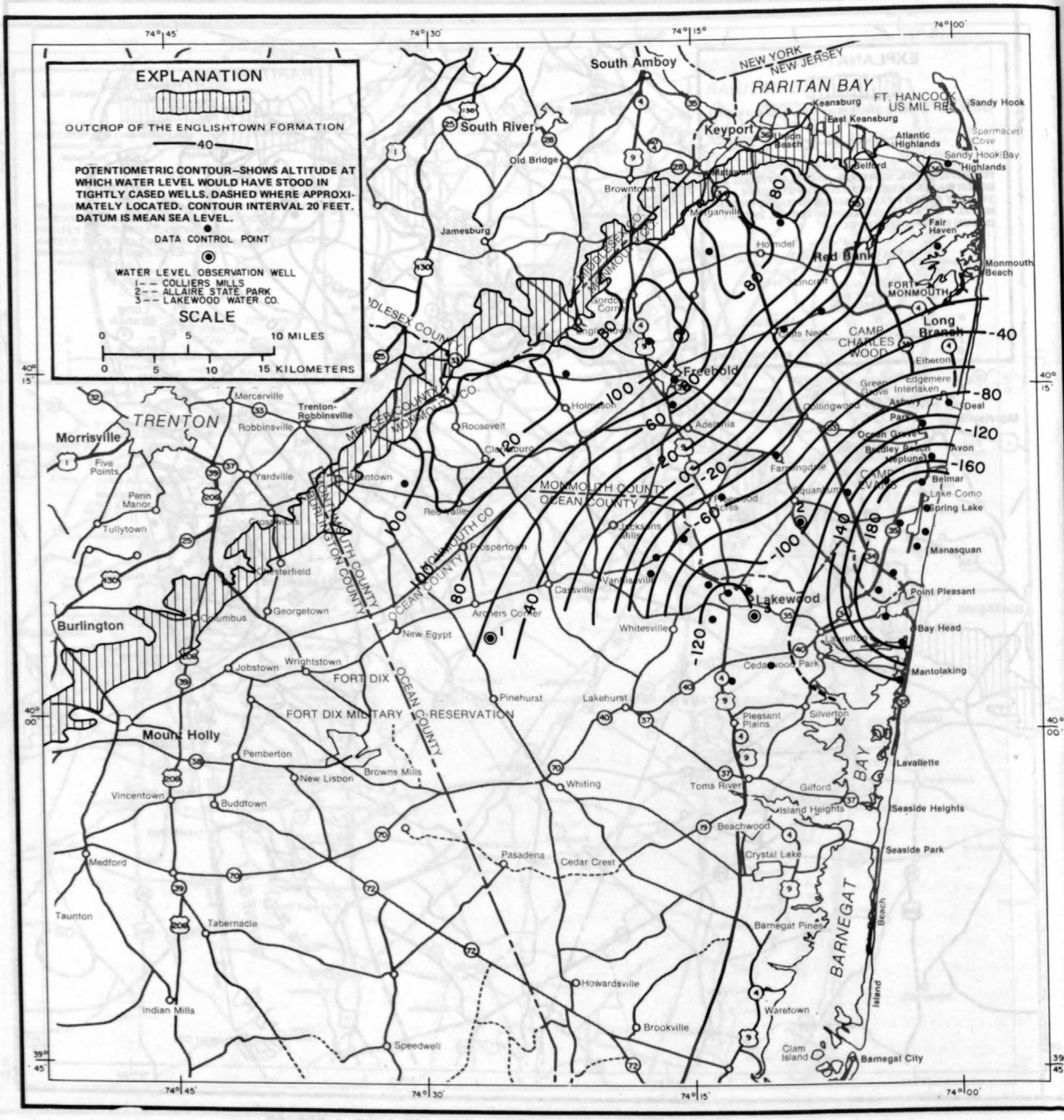
FIGURE 15. —ALTITUDE OF THE POTENTIOMETRIC SURFACE IN THE ENGLISHTOWN AQUIFER IN JANUARY 1959 IN MONMOUTH AND NORTHERN OCEAN COUNTIES, N.J.



Base from Army Map Services, Newark, N.J., 1964 and Wilmington, Del., NJ-PA-MD 1966, 1:250000.
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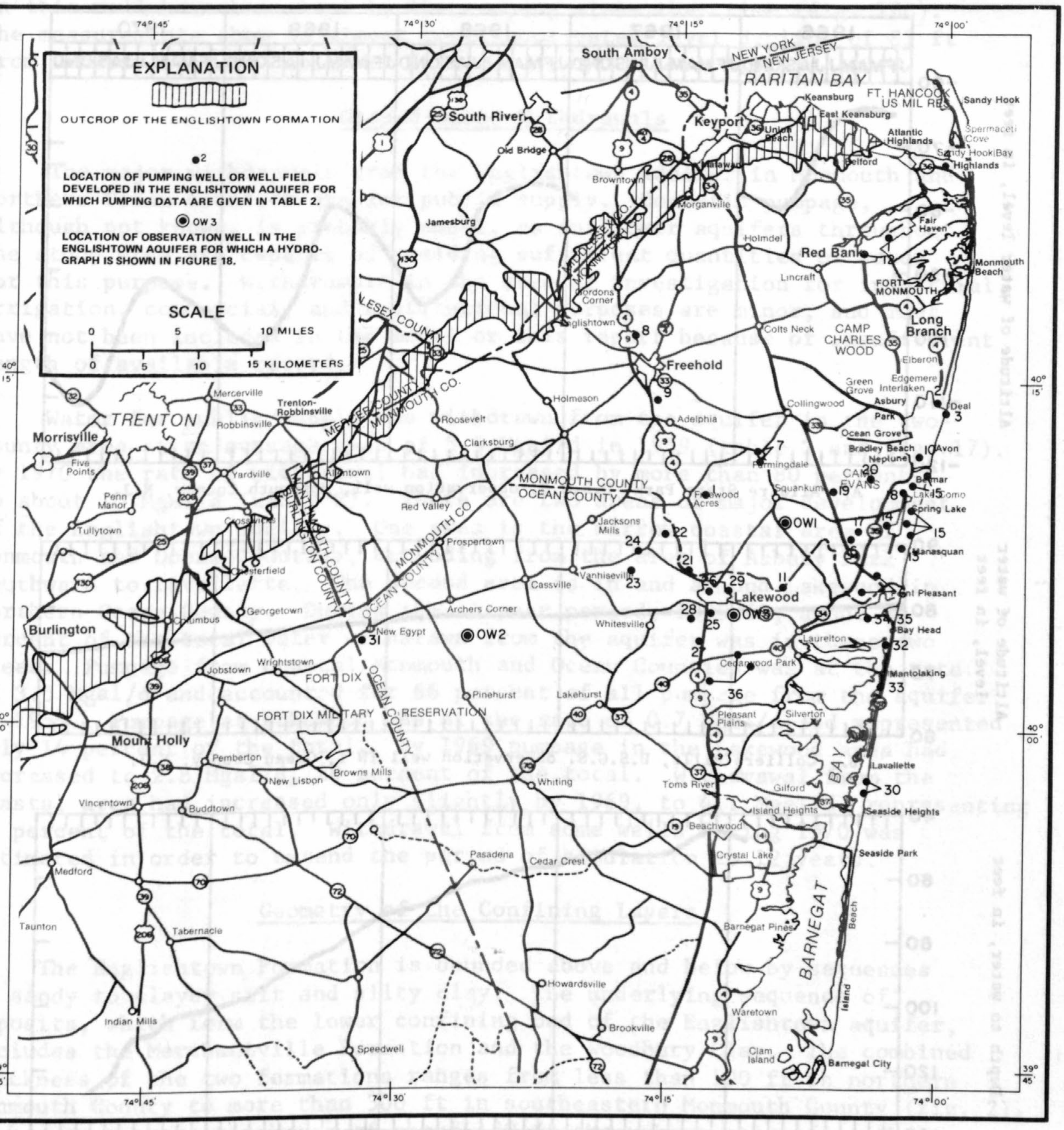
FIGURE 16. —ALTITUDE OF THE POTENTIOMETRIC SURFACE IN THE ENGLISHTOWN AQUIFER IN NOVEMBER 1970 IN MONMOUTH AND NORTHERN OCEAN COUNTIES, N.J.



Base from Army Map Services, Newark, N.J., 1964 and Wilmington, Del., NJ-PA-MD 1966, 1:250000.
Edited and published by U.S. Geological Survey

FIGURE 17. —LOCATION OF PUMPING WELLS LISTED IN TABLE 2 AND OBSERVATION WELLS DESCRIBED IN FIGURE 18.

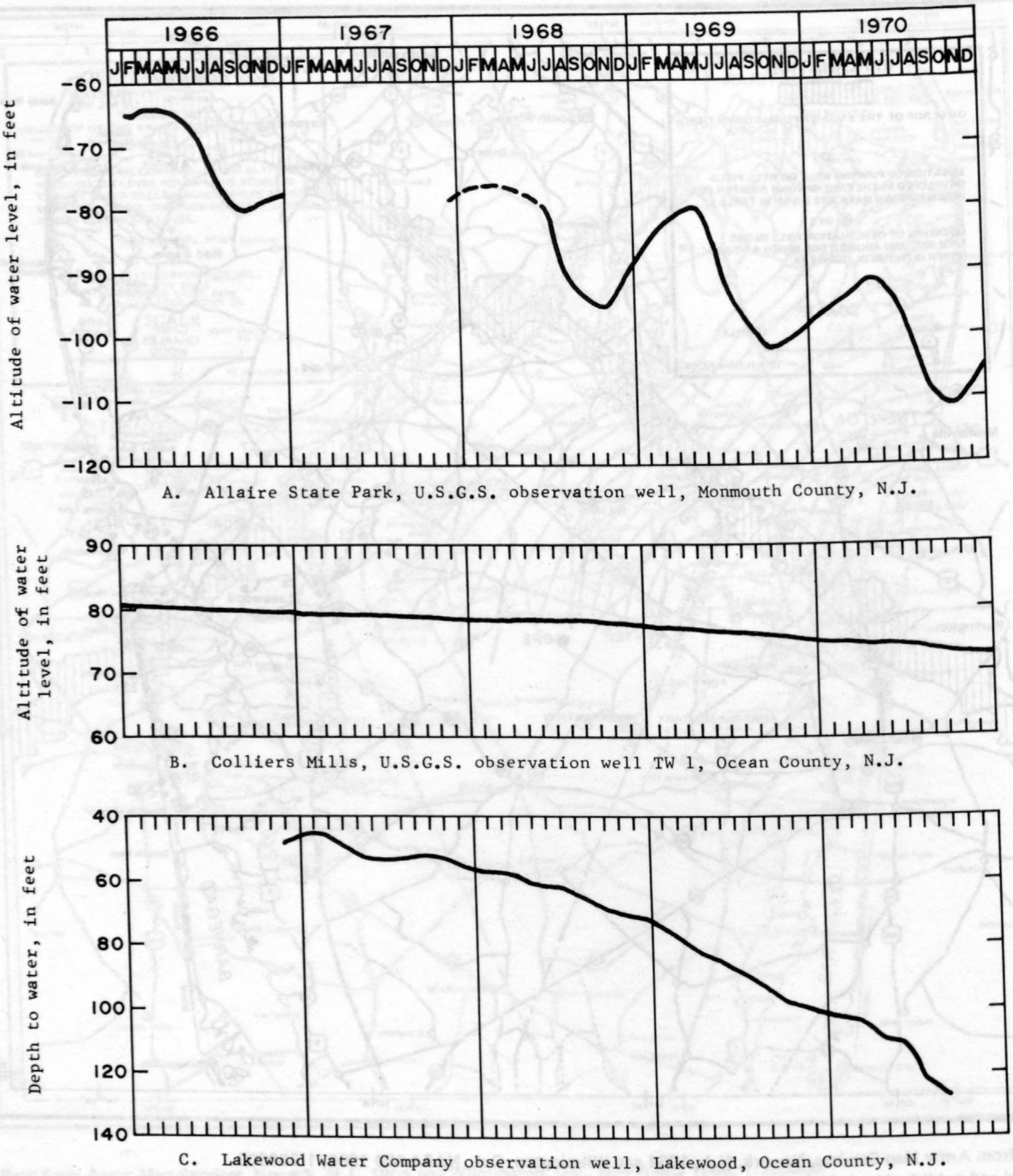


FIGURE 18. —HYDROGRAPHS OF OBSERVATION WELLS TAPPING THE ENGLISHTOWN AQUIFER IN MONMOUTH AND NORTHERN OCEAN COUNTIES, N.J.

The Lakewood Water Company installed an observation well southeast of Lakewood in late 1966 (fig. 17). Continuous records of water levels in this well have been kept by the company since that time (fig. 18C). The measurements show an almost continuous water-level decline of 81 ft from January 1967 to September 1970.

Ground-water Withdrawals

The major withdrawals from the Englishtown aquifer in Monmouth and northern Ocean Counties are for public supply. Domestic pumpage, although not known, is probably small, as shallower aquifers throughout the study area are capable of yielding sufficient quantities of water for this purpose. Withdrawals in the area of investigation for industrial, irrigation, commercial, and institutional purposes are minor, and most have not been included in the model or this report because of insufficient length of available record.

Water for public supply was withdrawn from the aquifer in the two-county area at an average rate of 5.5 Mgal/d in 1959 (table 2 and fig. 17). By 1970 the rate of withdrawal had increased by more than 80 percent, to about 10 Mgal/d (table 2). There are two areas of major development of the Englishtown aquifer. One area is the narrow coastal area of Monmouth and Ocean Counties, extending from the area of Asbury Park southward to Lavallette. The second area is in and around Lakewood in northern Ocean County. During the 11-year period, 1959-69, about 80 percent of the total water withdrawn from the aquifer was in these two areas. Pumpage from coastal Monmouth and Ocean Counties was at the rate of 3.5 Mgal/d and accounted for 66 percent of all pumpage from the aquifer in 1959. Pumpage at Lakewood was at the rate of 0.7 Mgal/d and represented only 14 percent of the total. By 1969 pumpage in the Lakewood area had increased to 2.8 Mgal/d, 29 percent of the total. Withdrawal from the coastal area had increased only slightly by 1969, to 4.1 Mgal/d, representing 42 percent of the total. Withdrawal from some wells during 1970 was estimated in order to extend the period of simulation to 12 years.

Geometry of the Confining Layers

The Englishtown Formation is bounded above and below by sequences of sandy to clayey silt and silty clay. The underlying sequence of deposits, which form the lower confining bed of the Englishtown aquifer, includes the Merchantville Formation and the Woodbury Clay. The combined thickness of the two formations ranges from less than 100 ft in northern Monmouth County to more than 300 ft in southeastern Monmouth County (fig. 2). It has an average thickness of about 180 ft throughout most of northern Ocean County.

The sequence of fine-grained sediments above the Englishtown Formation in the study area include the Marshalltown Formation and the finer-grained lower part of the Wenonah Formation. These sediments act as the upper confining bed of the aquifer. The thickness of the Marshalltown is relatively uniform throughout the area of investigation, ranging from 10 to 20 ft. The Marshalltown grades upward into the Wenonah Formation, a

Table 2.--Rate of withdrawal for public supply from the Englishtown aquifer in Monmouth and Ocean Counties

Map No.	Water department or company	Owner well number (see fig. 17 for locations)	Average annual rate of withdrawal (Mgal/d) Year 1959	1960	1961	1962	1963	1964	1965	1966	1967	1968	1969	1970
1	Aldrich Water Co.	#4	--	--	--	--	--	--	--	--	--	.04	.11	.17
2	Allenhurst Water Co.	#4	.13	.14	.14	.14	.15	.14	.14	.15	.15	.17	.15	.15
3	Asbury Park Water Dept.	#4	.10	.10	.10	.10	.10	.10	.10	10	.10	.10	.10	.10*
4	Atlantic Highlands Water Dept.	#2	.23	.25	.27	.29	.30	.20	.13	.15	.13	.17	.19	.19
5	Belmar Water Dept.	10 wells	.93	.70	.68	.72	.83	.83	.79	.80	.84	.93	.92	.91
6	Brielle Water Dept.	#2-3	.17	.18	.15	.16	.16	.16	.16	.21	.20	.32	.34	.32
7	Farmingdale Water Dept.	All wells	.04	.04	.05	.05	.06	.07	.09	.14	.15	.09	.10	.09
8	Freehold Water & Util.	Point Ivy #1	--	--	--	--	--	--	--	--	--	--	.02	.04*
9	Freehold Twp. Water Dept.	Koenig Lane #2	--	--	--	--	--	--	--	--	--	--	--	.60
10	Monmouth Consolidated Water Co.	Ocean Grove Sta. #20	.33	.07	.60	.71	.04	.05	.02	.08	--	--	--	.01
11	Parkway Water Co.		.02	.03	.04	.07	.07	.10	.08	.08	.08	.09	.09	.10
12	Red Bank Water Dept.	#2-3	.64	.68	.69	.67	.67	.67	.65	.67	.64	.77	.72	.62*
13	Sea Girt Water Dept.	#4-5	.19	.19	.19	.20	.23	.21	.21	.21	.18	.19	.19	.05
14	Spring Lake Heights Water Dept.	#2-3	.20	.23	.24	.23	.30	.33	.28	.29	.32	.42	.44	.53*
15	Spring Lake Water Dept.	#1-4	.48	.52	.45	.47	.49	.52	.45	.49	.46	.53	.55	.63
16	Wall Twp. Water Dept.	Allenwood #1	--	--	--	.11	.13	.16	.17	.19	.19	.18	.15	.18
17	Wall Twp. Water Dept.	Allenwood #2	--	--	--	.10	.12	.14	.16	.18	.20	.20	.19	.20
18	Wall Twp. Water Dept.	West Belmar	--	--	--	.09	.10	.11	.12	.14	.13	.13	.13	.15
19	Wall Twp. Water Dept.	Route 34 well	--	--	--	--	--	--	--	--	--	.08	.16	.18
20	Wall Twp. Water Dept.	Imperial Park #2	--	--	--	--	--	--	--	--	--	--	--	.06

Table 2.--Rate of withdrawal for public supply from the Englishtown aquifer in Monmouth and Ocean Counties--Continued

Map No.	Water department or company	Owner well number (see fig. 17 for locations)	Average annual rate of withdrawal (Mgal/d) Year 1959	1960	1961	1962	1963	1964	1965	1966	1967	1968	1969	1970
21	Jackson Twp. MUA	#1	--	--	--	.09	.19	.17	.17	.13	.13	.11	.13	.19
22	Jackson Twp. MUA	#2	--	--	--	--	.07	.21	.17	.17	.15	.18	.19	.28
23	Jackson Twp. MUA	#3	--	--	--	--	.01	.02	.01	.11	.12	.14	.17	.17
24	Jackson Twp. MUA	#4	--	--	--	--	--	--	--	.16	.16	.18	.14	.11
25	Lakewood Water Co.	#5	.75	.77	.78	.75	.65	.50	.22	.20	.17	.10	.08	.10*
26	Lakewood Water Co.	#6	--	--	--	.12	.27	.42	.56	.53	.47	.47	.32	.33*
27	Lakewood Water Co.	#7	--	--	--	--	--	.24	.44	.53	.45	.63	.60	.63*
28	Lakewood Water Co.	#8	--	--	--	--	--	--	--	.06	.29	.30	.32	.29*
29	Lakewood Water Co.	#9	--	--	--	--	--	--	--	--	--	.01	.25	.40*
30	Lavallette Water Dept.	#2-3	.32	.31	.30	.29	.30	.32	.33	.25	.19	.37	.40	.35*
31	New Egypt Water Co.		.10	.11	.11	.10	.12	.11	.12	.14	.11	.11	.11	.12*
32	Ocean County Water Co.	Bay Head #5-6	.38	.39	.41	.44	.45	.47	.47	.47	.51	.63	.60	.65
33	Ocean County Water Co.	Mantoloking #6	--	--	--	--	--	--	--	--	.01	.00	.03	.04
34	Point Pleasant Water Dept.	#3	.20	.09	.12	.25	.26	.45	.40	.21	.03	.05	--	--
35	Point Pleasant Water Dept.	#6	--	--	--	--	--	--	.33	.72	.32	.54	.47	.39
36	South Lakewood Water Co.	#1-3 (#1 drilled 69)	--	--	--	--	--	--	.05	.06	.18	.38	.56	.73

*Estimated

poorly sorted dark silty very fine- to fine-grained sand. As much as 80 ft of the Wenonah is included in the thickness of the upper confining bed. The total thickness of this confining layer ranges from about 12 ft in northern Monmouth County to 100 ft in central Ocean County (fig. 19).

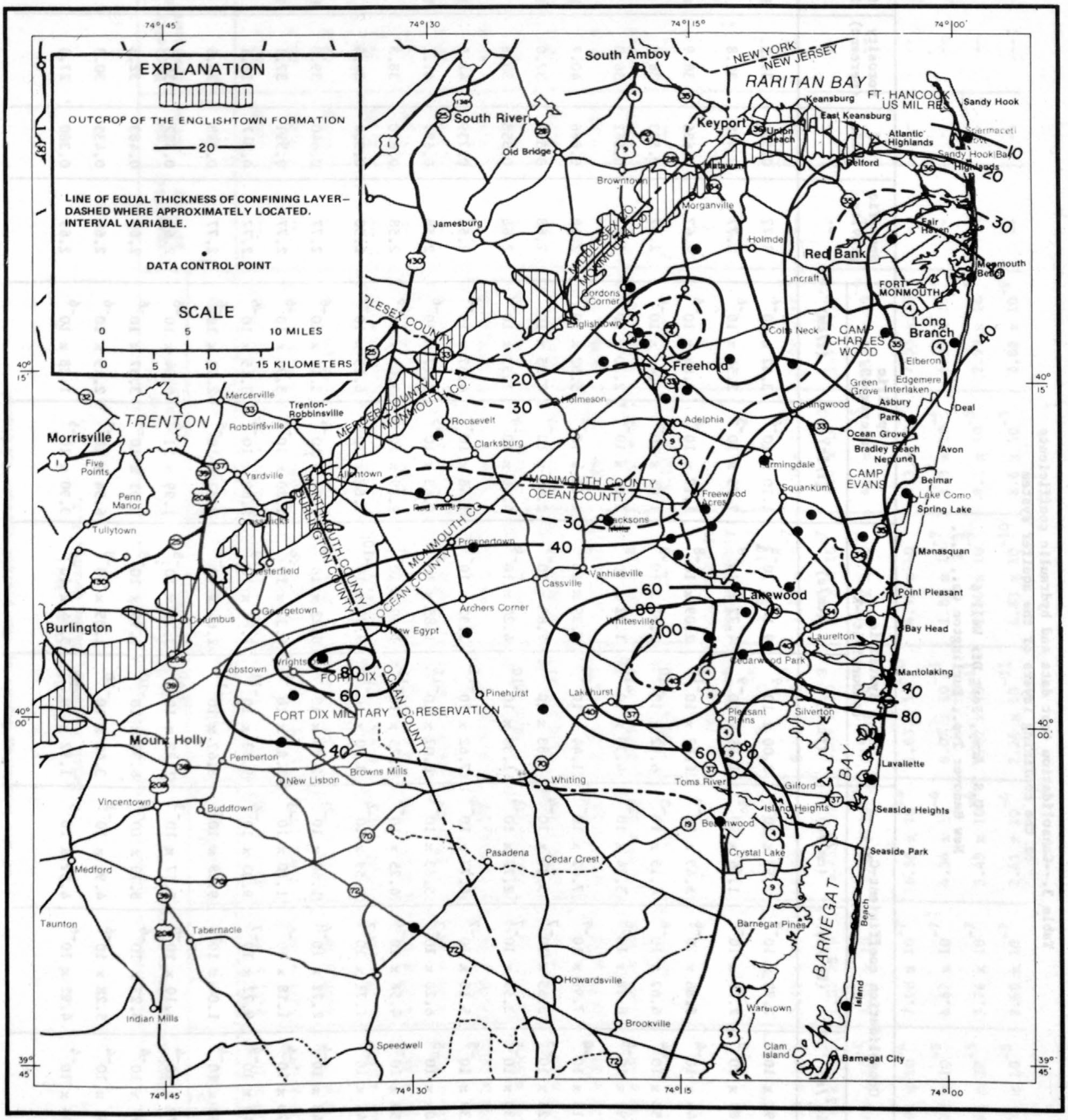
Hydraulic Coefficients of the Confining Layers

Undisturbed core samples of the confining layer sediments were obtained at three sites. Laboratory soil consolidation tests were made to obtain values of hydraulic conductivity and specific storage (Nichols, 1977). Hydraulic conductivity values for the lower confining bed computed from test data range from 3.6×10^{-6} ft/d to 5.9×10^{-5} ft/d and average (harmonic mean) about 6.6×10^{-6} ft/d; the specific storage ranges from 6.5×10^{-5} ft^{-1} to 4.6×10^{-4} ft^{-1} and average (harmonic mean) 8.8×10^{-5} ft^{-1}. Computed values of hydraulic conductivity of the upper confining layer range from 5.6 to 10^{-6} ft/d to 4.9×10^{-4} ft/d and average (harmonic mean) about 1.4×10^{-5} ft/d; computed specific storage values range from 5.1×10^{-6} ft^{-1} to 9.2×10^{-5} ft^{-1} and average (harmonic mean) about 7.1×10^{-5} ft^{-1} All values of hydraulic conductivity and specific storage were calculated from consolidation test data (Lambe, 1951; Domenico and Mifflin, 1965) for the load increment nearest the approximate computed overburden pressure (table 3).

HYDROLOGIC SIGNIFICANCE OF CONFINING LAYERS

A confined aquifer that receives water from adjacent confining layers is called a "leaky" aquifer, and the water received has been termed "leakage." Most analyses of leaky aquifers have been concerned primarily with the time-drawdown effects of pumpage in the aquifer rather than the rate and volume of leakage. While recognizing that the flow in the pumped aquifer is augmented by water derived from beds lying above or below the aquifer, most analyses of leakage have been concerned with its effect on time-drawdown relationships in the pumped aquifer. In reality, however, most leaky aquifers are but one part of a more complex multiaquifer system, wherein two or more aquifers are separated by layers of compressible and relatively low-permeability sediments. As flow in such a system is not restricted to the pumped aquifer, the removal of water from a leaky aquifer must eventually affect the flow behavior of the entire system. Thus, the effects of pumping in a leaky aquifer can be fully evaluated only by considering the hydrodynamics of all parts of the system. This may require a simulation model analysis of the entire system.

The focusing of attention on flow in individual aquifers of complex ground-water systems and the lack of mathematical techniques for evaluating transient (nonsteady) flow in low-permeability sediments has obscured the hydrologic significance of confining layers. In the analysis of leakage the assumption is commonly made that confining layers are incompressible and, consequently, do not release water from storage in response to a change in head, a serious oversimplification if the confining layers contain significant amounts of clay. The compressibility of aquifer systems was recognized by several early investigators of confined fluid flow. Meinzer (1928) summarized the findings of these earlier studies and extended the concept of compressibility to confined aquifers. Jacob



Base from Army Map Services, Newark, N.J., 1964 and Wilmington, Del., NJ-PA-MD 1966, 1:250000.
Edited and published by U.S. Geological Survey

FIGURE 19. —THICKNESS OF THE CONFINING LAYER OVERLYING THE ENGLISHTOWN AQUIFER IN MONMOUTH AND NORTHERN OCEAN AND BURLINGTON COUNTIES, N.J.

Table 3.--Consolidation test data and hydraulic coefficients of the confining layers of the aquifer system

U.S. Army, Fort Dix Well 6
New Hanover Twp., Burlington Co., N.J.

Sample depth	Consolidation load		Consolidation coefficient-C_v			Hydraulic conductivity-K		Specific storage-S_s		Specific gravity	Void ratio	Porosity (percent)	Geol. Fm.*
(ft)	(psi)	(g/cm^2)	(in^2/s)	(ft^2/s)	(cm^2/s)	(ft/s)	(cm/s)	(1/ft)	(1/cm)				
390.0	222.2	1.56×10^{4}	3.92×10^{-3}	2.72×10^{-5}	2.53×10^{-2}	3.00×10^{-9}	9.14×10^{-8}	1.10×10^{-4}	3.62×10^{-6}	2.77	0.551	35.5	Kmt
	444.4	3.12×10^{4}	2.88×10^{-3}	2.00×10^{-5}	1.86×10^{-2}	1.55×10^{-9}	4.72×10^{-8}	7.75×10^{-5}	2.54×10^{-6}	2.77	0.488	32.8	
456.0	222.2	1.56×10^{4}	8.44×10^{-4}	5.86×10^{-6}	5.45×10^{-3}	6.87×10^{-10}	2.09×10^{-8}	1.17×10^{-4}	3.84×10^{-6}	2.67	0.636	38.9	Kw
	361.1	2.54×10^{4}	9.53×10^{-4}	6.62×10^{-6}	6.15×10^{-3}	6.92×10^{-10}	2.11×10^{-8}	1.05×10^{-4}	3.43×10^{-6}	2.67	0.583	36.8	
	444.4	3.12×10^{4}	8.90×10^{-4}	6.18×10^{-6}	5.74×10^{-3}	4.33×10^{-10}	1.32×10^{-8}	7.01×10^{-5}	2.30×10^{-6}	2.67	0.562	36.0	
523.0	111.1	7.81×10^{3}	1.15×10^{-4}	7.99×10^{-7}	7.42×10^{-4}	1.44×10^{-10}	4.39×10^{-9}	1.81×10^{-4}	5.92×10^{-6}	2.68	0.686	40.7	Kwb-
	222.2	1.56×10^{4}	5.25×10^{-5}	3.65×10^{-7}	3.39×10^{-4}	8.83×10^{-11}	2.69×10^{-9}	2.42×10^{-4}	7.95×10^{-6}	2.68	0.588	37.0	Kmv
	402.8	2.83×10^{4}	4.30×10^{-5}	2.99×10^{-7}	2.77×10^{-4}	1.39×10^{-10}	4.24×10^{-9}	4.66×10^{-4}	1.53×10^{-5}	2.68	0.558	35.8	
546.0	111.1	7.81×10^{3}	7.39×10^{-5}	5.13×10^{-7}	4.77×10^{-4}	7.62×10^{-11}	2.32×10^{-9}	1.48×10^{-4}	4.87×10^{-6}	2.58	0.739	42.5	Kwb-
	222.2	1.56×10^{4}	9.04×10^{-5}	6.28×10^{-7}	5.83×10^{-4}	6.07×10^{-11}	1.85×10^{-9}	9.68×10^{-5}	3.17×10^{-6}	2.58	0.697	41.1	Kmv
	444.4	3.12×10^{4}	6.59×10^{-5}	4.58×10^{-7}	4.25×10^{-4}	4.25×10^{-11}	1.30×10^{-9}	9.29×10^{-5}	3.05×10^{-6}	2.58	0.620	38.3	
	888.9	6.25×10^{4}	2.46×10^{-5}	1.71×10^{-7}	1.59×10^{-4}	3.20×10^{-11}	9.75×10^{-10}	1.87×10^{-4}	6.15×10^{-6}	2.58	0.360	26.5	
569.0	111.1	7.81×10^{3}	2.48×10^{-4}	1.72×10^{-6}	1.60×10^{-3}	2.70×10^{-10}	8.23×10^{-9}	1.57×10^{-4}	5.14×10^{-6}	2.77	0.640	39.0	Kwb-
	222.2	1.56×10^{4}	1.70×10^{-4}	1.18×10^{-6}	1.10×10^{-3}	1.42×10^{-10}	4.32×10^{-9}	1.20×10^{-4}	3.94×10^{-6}	2.77	0.591	37.1	Kmv
	444.4	3.12×10^{4}	1.40×10^{-4}	9.72×10^{-7}	9.03×10^{-4}	9.33×10^{-11}	2.84×10^{-9}	9.60×10^{-5}	3.15×10^{-6}	2.77	0.517	34.1	
	888.9	6.25×10^{4}	1.50×10^{-4}	1.04×10^{-6}	9.68×10^{-4}	8.67×10^{-11}	2.64×10^{-9}	8.32×10^{-5}	2.73×10^{-6}	2.77	0.398	28.5	
625.0	111.1	7.81×10^{3}	8.79×10^{-4}	6.10×10^{-6}	5.67×10^{-3}	1.22×10^{-9}	3.71×10^{-8}	1.99×11^{-4}	6.54×10^{-6}	2.69	0.522	34.3	Kmv
	222.2	1.56×10^{4}	8.97×10^{-4}	6.23×10^{-6}	5.79×10^{-3}	6.39×10^{-10}	1.95×10^{-8}	1.03×10^{-4}	3.37×10^{-6}	2.69	0.483	32.6	
	444.4	3.12×10^{4}	7.61×10^{-4}	5.28×10^{-6}	4.91×10^{-3}	3.46×10^{-10}	1.05×10^{-8}	6.54×10^{-5}	2.15×10^{-6}	2.69	0.435	30.3	
	888.9	6.25×10^{4}	7.08×10^{-4}	4.92×10^{-6}	4.57×10^{-3}	1.92×10^{-10}	5.84×10^{-9}	3.90×10^{-5}	1.28×10^{-6}	2.69	0.380	27.5	

Bradlees Corp. Well
Brick Township, Ocean County, N.J.

Sample depth (ft)	Consolidation load (psi)	(g/cm^2)	Consolidation coefficient-C_v (in^2/s)	(ft^2/s)	(cm^2/s)	Hydraulic conductivity-K (ft/s)	(cm/s)	Specific storage-S_s (1/ft)	(1/cm)	Specific gravity	Void ratio	Porosity (percent)	Geol. Fm.*
605.0	138.0	9.70×10^{3}	8.39×10^{-4}	5.83×10^{-6}	5.41×10^{-3}	1.97×10^{-9}	5.99×10^{-8}	3.38×10^{-4}	1.11×10^{-5}	--	0.920	47.9	Kw
	276.0	1.94×10^{4}	5.36×10^{-4}	3.72×10^{-6}	3.46×10^{-3}	6.88×10^{-10}	2.10×10^{-8}	1.85×10^{-4}	6.07×10^{-6}	--	0.860	46.2	
	551.0	3.87×10^{4}	3.30×10^{-4}	2.29×10^{-6}	2.13×10^{-3}	2.10×10^{-10}	6.40×10^{-9}	9.16×10^{-5}	3.01×10^{-6}	--	0.780	43.8	
625.0	141.0	9.91×10^{3}	5.40×10^{-4}	3.75×10^{-6}	3.48×10^{-3}	6.88×10^{-10}	2.10×10^{-8}	1.84×10^{-4}	6.02×10^{-6}	--	1.130	53.1	Kw
	282.0	1.98×10^{4}	6.00×10^{-4}	4.17×10^{-6}	3.87×10^{-3}	4.27×10^{-10}	1.30×10^{-8}	1.02×10^{-4}	3.36×10^{-6}	--	1.100	52.4	
	564.0	3.97×10^{4}	1.84×10^{-4}	1.28×10^{-6}	1.19×10^{-3}	6.57×10^{-11}	2.00×10^{-9}	5.14×10^{-5}	1.69×10^{-6}	--	1.020	50.5	
635.0	133.0	9.35×10^{3}	3.01×10^{-4}	2.09×10^{-6}	1.94×10^{-3}	6.57×10^{-10}	2.00×10^{-8}	3.14×10^{-4}	1.03×10^{-5}	--	0.920	47.9	Kmt
	265.0	1.86×10^{4}	2.46×10^{-4}	1.71×10^{-6}	1.59×10^{-3}	2.76×10^{-10}	8.41×10^{-9}	1.61×10^{-4}	5.30×10^{-6}	--	0.860	46.2	
	531.0	3.73×10^{4}	2.23×10^{-4}	1.55×10^{-6}	1.44×10^{-3}	1.28×10^{-10}	3.91×10^{-9}	8.29×10^{-5}	2.72×10^{-6}	--	0.790	44.1	
645.0	151.0	1.06×10^{4}	6.75×10^{-4}	4.69×10^{-6}	4.35×10^{-3}	1.28×10^{-9}	3.91×10^{-8}	2.74×10^{-4}	8.98×10^{-6}	--	0.870	46.5	Kmt
	302.0	2.12×10^{4}	2.94×10^{-4}	2.04×10^{-6}	1.90×10^{-3}	2.79×10^{-10}	8.51×10^{-9}	1.37×10^{-4}	4.49×10^{-6}	--	0.860	46.2	
	500.0	--	--	--	--	1.00×10^{-10}	--	8.00×10^{-5}	--	--	--	--	
New Jersey Water Co. Well 10 Lakewood Township, Ocean County, N.J.													
566.0	100.0	7.03×10^{3}	---	--	--	1.29×10^{-8}	3.93×10^{-7}	--	--	--	--	--	Kmt
	275.0	1.93×10^{4}	1.33×10^{-2}	9.21×10^{-5}	8.56×10^{-2}	6.45×10^{-9}	1.96×10^{-7}	7.0×10^{-5}	2.29×10^{-6}	--	--	--	
	435.0	3.06×10^{4}	1.51×10^{-2}	1.05×10^{-4}	9.75×10^{-2}	5.69×10^{-9}	1.73×10^{-7}	5.4×10^{-5}	1.77×10^{-6}	--	--	--	
	685.0	4.82×10^{4}	1.48×10^{-2}	1.03×10^{-4}	9.57×10^{-2}	4.55×10^{-9}	1.39×10^{-7}	4.4×10^{-5}	1.44×10^{-6}	--	--	--	
692.0	100.0	7.03×10^{3}	1.02×10^{-4}	7.08×10^{-7}	6.58×10^{-4}	1.63×10^{-10}	4.96×10^{-9}	2.3×10^{-4}	7.55×10^{-6}	--	--	--	Ket
	275.0	1.93×10^{4}	6.72×10^{-5}	4.67×10^{-7}	4.34×10^{-4}	6.07×10^{-11}	1.85×10^{-9}	1.3×10^{-4}	4.27×10^{-6}	--	--	--	
	435.0	3.06×10^{4}	5.41×10^{-5}	3.76×10^{-7}	3.49×10^{-4}	3.42×10^{-11}	1.04×10^{-9}	9.1×10^{-5}	2.99×10^{-6}	--	--	--	
	530.0	3.73×10^{4}	3.74×10^{-5}	2.60×10^{-7}	2.41×10^{-4}	2.24×10^{-11}	6.83×10^{-10}	8.6×10^{-5}	2.82×10^{-6}	--	--	--	

Table 3.--Consolidation test data and hydraulic coefficients of the confining layers of the aquifer system--Continued

New Jersey Water Co. Well 10--Continued
Lakewood Township, Ocean County, N.J.

Sample depth	Consolidation load		Consolidation coefficient-C_v			Hydraulic conductivity-K		Specific storage-S_s		Specific gravity	Void ratio	Porosity (percent)	Geol. Fm.*
(ft)	(psi)	(g/cm^2)	(in^2/s)	(ft^2/s)	(cm^2/s)	(ft/s)	(cm/s)	(1/ft)	(1/cm)				
795.0	100.0	7.03×10^{3}	7.89×10^{-4}	5.48×10^{-6}	5.09×10^{-3}	4.93×10^{-10}	1.50×10^{-9}	9.0×10^{-5}	2.95×10^{-6}	--	--	--	Kwb
	610.0	4.29×10^{4}	8.37×10^{-5}	5.81×10^{-7}	5.40×10^{-4}	4.18×10^{-11}	1.27×10^{-9}	7.2×10^{-5}	2.36×10^{-6}	--	--	--	
	1150.0	8.09×10^{4}	4.00×10^{-5}	2.78×10^{-7}	2.58×10^{-4}	1.67×10^{-11}	5.09×10^{-10}	6.0×10^{-5}	1.97×10^{-6}	--	--	--	
845.0	115.0	8.09×10^{4}	3.61×10^{-3}	2.51×10^{-5}	2.33×10^{-2}	1.63×10^{-9}	4.97×10^{-8}	6.5×10^{-5}	2.13×10^{-6}	--	--	--	Kwb
	645.0	4.53×10^{4}	1.23×10^{-4}	8.58×10^{-7}	7.97×10^{-4}	5.32×10^{-11}	1.62×10^{-9}	6.2×10^{-5}	2.03×10^{-6}	--	--	--	
	1190.0	8.37×10^{4}	2.10×10^{-5}	1.46×10^{-7}	1.35×10^{-4}	8.35×10^{-12}	2.54×10^{-10}	5.7×10^{-5}	1.87×10^{-6}	--	--	--	
896.0	145.0	1.02×10^{4}	6.47×10^{-4}	4.49×10^{-6}	4.17×10^{-3}	4.94×10^{-10}	1.51×10^{-9}	1.1×10^{-4}	3.61×10^{-6}	--	--	--	Kmv
	690.0	4.85×10^{4}	1.32×10^{-4}	9.18×10^{-7}	8.53×10^{-4}	5.69×10^{-11}	1.73×10^{-9}	6.2×10^{-5}	2.03×10^{-6}	--	--	--	
	1225.0	8.61×10^{4}	5.05×10^{-5}	3.51×10^{-7}	3.26×10^{-4}	1.44×10^{-11}	4.39×10^{-10}	4.1×10^{-5}	1.35×10^{-6}	--	--	--	
928.0	180.0	1.27×10^{4}	--	--	--	5.69×10^{-10}	1.73×10^{-8}	--	--	--	--	--	Kmv
	715.0	5.03×10^{4}	--	--	--	1.63×10^{-10}	4.97×10^{-9}	--	--	--	--	--	
	1245.0	8.75×10^{4}	--	--	--	1.18×10^{-10}	3.59×10^{-9}	--	--	--	--	--	

*Ket - Englishtown Formation, clayey silt lithofacies
Kmt - Marshalltown Formation
Kmv - Merchantville Formation
Kw - Wenonah Formation
Kwb - Woodbury Clay

(1940, p. 574) recognized the compressibility of low-permeability sediments and the significance of the water they contain in storage by defining the aquifer storage coefficient in terms of all sources of stored water, including that water stored in intercalated clay and adjacent clay beds (Jacob, 1940, p. 577). However, the complications of mathematically handling the time-lag effects of the drainage of stored water from the clay limited his more practical definition of the aquifer storage coefficient to the properties of the aquifer and contained water (Jacob, 1940, p. 576).

Hantush's (1960) modified theory of leaky aquifers provided a solution for the pumped aquifer that included, for the first time, the effects of water released from storage in overlying and underlying confining beds. In effect, Hantush was able to extend mathematically the concept of compressibility to the adjacent confining layers. Again, however, the main emphasis was on the description of head in the pumped aquifer, although several expressions were included for the quantitative evaluation of the rate and volume of flow from storage in the confining layers at various times after pumping began (Hantush, 1960, p. 3716). These equations are given in terms of the pumping rate of a well, but are difficult to apply to large areas involving multiple pumping centers and confining beds with complex geometry and variable hydraulic parameters.

The significance of leakage on a regional scale can be realized by evaluating flow within the confining layer and computing the rate and volume of transient and steady leakage into the pumped aquifer. This approach to the regional evaluation of leakage utilizes equations that describe the head distribution in the confining layers as a function of the head distribution in the pumped aquifer. Once head distribution in a confining bed is known, leakage into the aquifer can be computed by applying Darcy's law at the aquifer-confining layer interface.

Equations of One-Dimensional Vertical Flow in Low-Permeability Sediments

Until recently, the equations of one-dimensional vertical transient flow in low-permeability sediments have not been generally used by hydrologists. Domenico and Mifflin (1965) discussed the relationships between the recovery of water from storage in compressible low-permeability sediments and land subsidence and have given several expressions that quantitatively describe the release of stored water. Hanshaw and Bredehoeft (1968) and Bredehoeft and Hanshaw (1968), in their analysis of anomalous fluid pressures at depth, discussed vertical transient flow in low-permeability sediments. The usual ground-water equations for transient flow can be shown to be identical to, and can be developed from, the equations of soil mechanics.

The equations of the hydrodynamic theory of consolidation (Terzaghi, 1943) were developed to calculate the overall change in thickness, or consolidation, caused by an increase in effective pressure and the subsequent drainage of stored water from compressible clay. According to the Terzaghi theory, the volume consolidation of a clayey sediment layer in response to a change in stress is a measure of the reduction in that drains from the layer. The water released during consolidation of a confining layer is the transient component of leakage.

The applicability of the principles of consolidation theory to thick sedimentary sequences has been demonstrated by several studies of compaction and land subsidence in areas of heavy ground-water withdrawals and large water-level declines (Poland, 1961, 1970a, and 1970b; Poland and Davis, 1959; Miller, 1961; Lofgren and Klausing, 1969; Lofgren, 1970; Riley, 1970). These studies have used consolidation equations to compute reasonably good estimates of compaction as a function of changes in hydraulic head in complex ground-water systems. The stress-strain relationships between the lowering of water levels in confined aquifer systems and the process of consolidation that have been applied successfully in these analyses are discussed in detail by Lofgren and Klausing (1969, p. B65-B69). As the volume compaction of a clayey sediment is, according to Terzaghi, a function of the volume of water removed from storage in the sediment, these same equations, when solved for the appropriate initial and boundary conditions, can be used to compute the rate and volume of transient leakage from compressible confining layers as a function of head changes in the adjacent aquifer. Furthermore, by demonstrating that the differential equation of vertical flow given by Hanshaw and Bredehoeft (1968) is equivalent to the differential equation of Terzaghi consolidation theory, it will then be possible to use the solutions of Hanshaw and Bredehoeft to make quantitative estimates of the rate and volume of flow from storage in a confining bed as a result of an increase in stress.

The differential equation of consolidation theory describes the change in excess hydrostatic pressure as a function of time and the vertical dimension (thickness). The excess hydrostatic pressure, u, is given by

$$u = \gamma_w h'$$

where γ_w = unit weight of water $[M/L^3]$
h' = hydraulic head differential $[L]$

As the excess hydrostatic pressure, u, decreases at any point in the clay layer, there is a corresponding increase in the effective stress, $\Delta\overline{p}$, transmitted from grain to grain of the sediment at that point. The total consolidation pressure, Δp, at any point and at any time is then the sum of excess hydrostatic pressure and effective pressure and can be expressed as

$$\Delta p = u + \Delta\overline{p}$$

The decrease in excess hydrostatic pressure and corresponding increase in effective stress with respect to time is given by

$$\frac{\partial u}{\partial t} = - \frac{\partial (\Delta\overline{p})}{\partial t} \qquad (1)$$

An increase in effective pressure, $\Delta\overline{p}$, results in a corresponding decrease in porosity, n, where the change in porosity, Δn, is a function of the change in void ratio, Δe. The change in void ratio in response to an increase in effective pressure is given by

$$\Delta e = e_o - e = a_v(\Delta\overline{p}) \qquad (2)$$

where e_o = initial void ratio

e = subsequent void ratio

a_v = coefficient of compressibility

As the porosity is defined as $n = \frac{e}{1+e_o}$

the change in void ratio given by equation 5 may be expressed as

$$\Delta n = \frac{\Delta e}{1+e_o} = \frac{a_v}{1+e_o} = (\Delta\overline{p}) = m_v(\Delta\overline{p})$$

where m_v = coefficient of volume compressibility and the change in porosity with respect to time as a function of the change in effective pressure with respect to time is

$$m_v \frac{\partial(\Delta\overline{p})}{\partial t} = -\frac{\partial n}{\partial t} \qquad (3)$$

From equations 1 and 3

$$-\frac{\partial n}{\partial t} = -m_v \frac{\partial u}{\partial t} \qquad (4)$$

which expresses the change in porosity with respect to time as a function of the compressibility of the clayey sediment and of the change in excess hydrostatic pressure with respect to time.

Consolidation theory assumes that the clay is completely saturated and that the contained water is incompressible so that the rate at which porosity is changing, represented by

$$-\frac{\partial n}{\partial t}$$

within a volume element of the clay must represent the net change in outward flux of water from the volume element per unit of time (Terzaghi, 1943). This, in essence, is a verbal statement of the principle of continuity commonly expressed in the form of an equation based on the familiar law of conservation of mass. The flux out of the unit volume at any time in terms of excess hydrostatic pressure is given by Darcy's law

$$q' = -\frac{K'}{\gamma_w}\frac{\partial u}{\partial Z} \tag{5}$$

where K' is the vertical hydraulic conductivity of the clay and q' is the specific discharge. The change in specific discharge per unit time is then given by

$$\frac{\partial q'}{\partial Z} = -\frac{\partial n}{\partial t} \tag{6}$$

where $\partial q'/\partial Z$ is the change in specific discharge and is equal to the change in the volume of void space, $-\partial n/\partial t$. The change in the volume of voids is equal to the quantity of water expelled from the volume element per unit of time.

The rate of change in the specific discharge as a function of excess hydrostatic pressure is given by

$$\frac{\partial q'}{\partial Z} = \frac{\partial}{\partial Z}\left(-\frac{K'}{\gamma_w}\frac{\partial u}{\partial Z}\right) = -\frac{K'}{\gamma_w}\frac{\partial^2 u}{\partial Z^2}$$

Substituting this term into equation 5, the change in porosity per unit time can be expressed as

$$-\frac{K'}{\gamma_w}\frac{\partial^2 u}{\partial Z^2} = -\frac{\partial n}{\partial t}$$

Finally, combining this equation with equation 4 leads to

$$\frac{K'}{\gamma_w}\frac{\partial^2 u}{\partial Z^2} = m_v\frac{\partial u}{\partial t} \tag{7}$$

which is the differential equation of consolidation (Terzaghi, 1943).

Equation 7 can be solved for appropriate initial and boundary conditions to provide equations that can be used to compute the rate and magnitude of consolidation of a clayey sediment at any time as the result of an increase in stress. As the rate and magnitude of compression is a function of the drainage of the clay layer, the same equation, with minor modification, can be utilized to calculate the rate and volume of flow of water from storage as a result of a change in effective stress.

When used to compute the volume of water released from storage, however, the consolidation equation must be modified to take into account the compressibility of water. An expression for the storativity of a volume element of sediment that includes the compressibility of water can be obtained from the storage coefficient, as given by Jacob (1940). Hantush (1960) introduced the term specific storage for this expression which he defined as "the volume of water which a unit volume. . . releases from storage because of expansion of water and compression of . . . [the unit volume] under a unit decline in the average head within the unit volume . . ." (Hantush, 1964). The specific storage, S_s, is given by

$$S_s = \gamma_w (\alpha + n\beta) \tag{8}$$

where S_s = specific storage

α = compressibility of sediment

β = compressibility of water

This expression has been rigorously verified by Cooper (1966).

Terzaghi's equation of consolidation (equation 7) can be expressed as

$$\frac{K'}{\gamma_w} \frac{\partial^2 u}{\partial Z^2} = (m_v + n\beta) \frac{\partial u}{\partial t} \tag{9}$$

The differential equation of consolidation can also be expressed in terms of excess head rather than excess hydrostatic pressure. Recalling that excess hydrostatic pressure is given by

$$u = \gamma_w h'$$

the quantity $\gamma_w h'$ can be substituted for u in equation 9. This substitution leads to

$$K' \frac{\partial^2 h'}{\partial Z^2} = \gamma_w (m_v + n\beta) \frac{\partial h'}{\partial t} = S_s' \frac{\partial h'}{\partial t} \tag{10}$$

which is the equation used by Hanshaw and Bredehoeft (1968).

A head decline caused by pumpage in a confined leaky aquifer creates, in effect, a change in the loading, or stress, conditions on the adjacent confining beds (Lofgren and Klausing, 1969). The resulting hydraulic gradient and stress changes within the confining layer initiate the gradual drainage and compaction described mathematically by equation 7. The rate and volume of flow from storage in the confining bed in response to the change in hydraulic gradient can be computed by solving the equivalent equation (equation 10) describing vertical nonsteady flow for the appropriate boundary and initial conditions. It is useful to consider the drawdown in the aquifer as a stepwise change in head; this boundary condition is approximated when the head declines rapidly and then maintains a relatively constant level. Wolff (1970) has described several situations in which this stepwise approximation gives reasonably good results.

Following Hanshaw and Bredehoeft (1968) the stepwise boundary-value problem is described by

$$K' \frac{\partial^2 h'}{\partial Z^2} = S_s' \frac{\partial h'}{\partial t} \quad \text{where } 0 < Z \leq \ell \tag{11}$$

The initial condition is

$$h'(Z,0) = 0 \text{ at } t=0$$

and the boundary conditions are

$$(1)\ h'(o,t) = 0 \text{ at } t>0$$

$$(2)\ h'(\ell',t) = H_o \text{ at } t>0$$

where H_o = stepwise change in head in the aquifer

ℓ' = thickness of confining layer

The coordinate system for the several cases of interest are shown in figure 20. It is important to take care in establishing the origin of these systems. Figure 20 is the coordinate system for the case of one or two confining layers draining in a single direction into one pumped aquifer. The origin for the vertical dimension, Z' and Z", is at the top of the overlying confining bed and at the base of the underlying confining bed, respectively. At the aquifer-confining bed interface, $Z' = \ell'$ and $Z'' = \ell''$.

Carslaw and Jaeger (1959) give two solutions for the boundary-value problem described by equation 11. One solution is for small values of the dimensionless time variable, $K't/S_s'\ell^2$, and one solution is for large values of the variable. The excess head distribution for a layer of finite thickness at small values of dimensionless time is given by (Carslaw and Jaeger, 1959, p. 310 and Hanshaw and Bredehoeft, 1968, p. 1109)

$$h' = H_o \sum_{n=o}^{\infty} \left\{ \text{erf}\left[\frac{(2n+1) + Z/\ell}{\sqrt{4K't/S_s'\ell^2}}\right] - \text{erf}\left[\frac{(2n+1) - Z/\ell}{\sqrt{4K't/S_s'\ell^2}}\right] \right\} \tag{12}$$

where h' is the excess head at any point, Z, and erf(x) is the error function, which is tabulated in several standard mathematical tables. For large values of dimensionless time the solution is (Carslaw and Jaeger, 1959, p. 313)

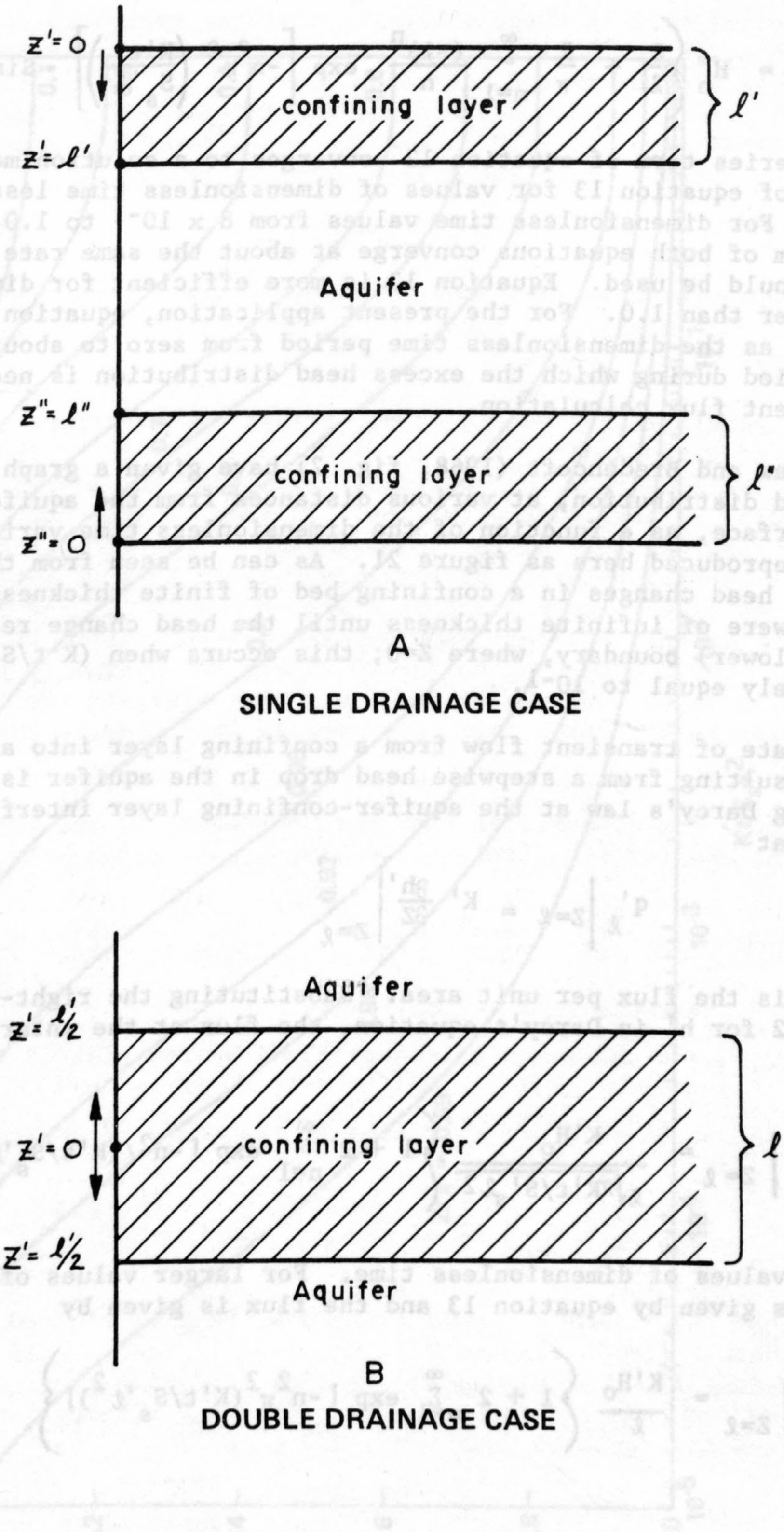


FIGURE 20. —COORDINATE SYSTEM USED FOR SOLVING THE EQUATIONS OF TRANSIENT FLOW.

$$h' = H_o \left\{ \frac{Z}{\ell} + \frac{2}{\pi} \sum_{n=1}^{\infty} \frac{(-1)^n}{n} \exp\left[-n^2\pi^2 \left(\frac{K't}{S_s'\ell^2} \right) \right] \cdot \operatorname{Sin}\left(\frac{n\pi Z}{\ell} \right) \right\} \tag{13}$$

The series term of equation 12 converges to a solution more rapidly than that of equation 13 for values of dimensionless time less than 8×10^{-1}. For dimensionless time values from 8×10^{-1} to 1.0, the series term of both equations converge at about the same rate, and either equation could be used. Equation 13 is more efficient for dimensionless time greater than 1.0. For the present application, equation 12 is more efficient, as the dimensionless time period from zero to about 2×10^{-1} is the period during which the excess head distribution is needed for the transient flux calculation.

Hanshaw and Bredehoeft (1968, fig. 2) have given a graph of the excess head distribution, at various distances from the aquifer-confining layer interface, as a function of the dimensionless time variable. Their graph is reproduced here as figure 21. As can be seen from this figure, the excess head changes in a confining bed of finite thickness as though the layer were of infinite thickness until the head change reaches the upper (or lower) boundary, where Z=0; this occurs when $(K't/S_s\ell^2)$ is approximately equal to 10^{-1}.

The rate of transient flow from a confining layer into a pumped aquifer resulting from a stepwise head drop in the aquifer is computed by applying Darcy's law at the aquifer-confining layer interface, where $Z=\ell$, so that

$$q'_\ell \Big|_{Z=\ell} = K' \frac{\partial h'}{\partial Z} \Big|_{Z=\ell}$$

where q'_ℓ is the flux per unit area. Substituting the right-hand side of equation 12 for h' in Darcy's equation, the flux at the interface is given by

$$q'_\ell \Big|_{Z=\ell} = \frac{K'H_o}{\ell\sqrt{\pi K't/S'_s\ell^2}} \left\{ 1 + 2 \sum_{n=1}^{\infty} \exp\left[-n^2/(K't/S_s'\ell^2) \right] \right\} \tag{14}$$

for small values of dimensionless time. For larger values of dimensionless time, h' is given by equation 13 and the flux is given by

$$q'_\ell \Big|_{Z=\ell} = \frac{K'H_o}{\ell} \left\{ 1 + 2 \sum_{n=1}^{\infty} \exp\left[-n^2\pi^2(K't/S_s'\ell^2) \right] \right\} \tag{15}$$

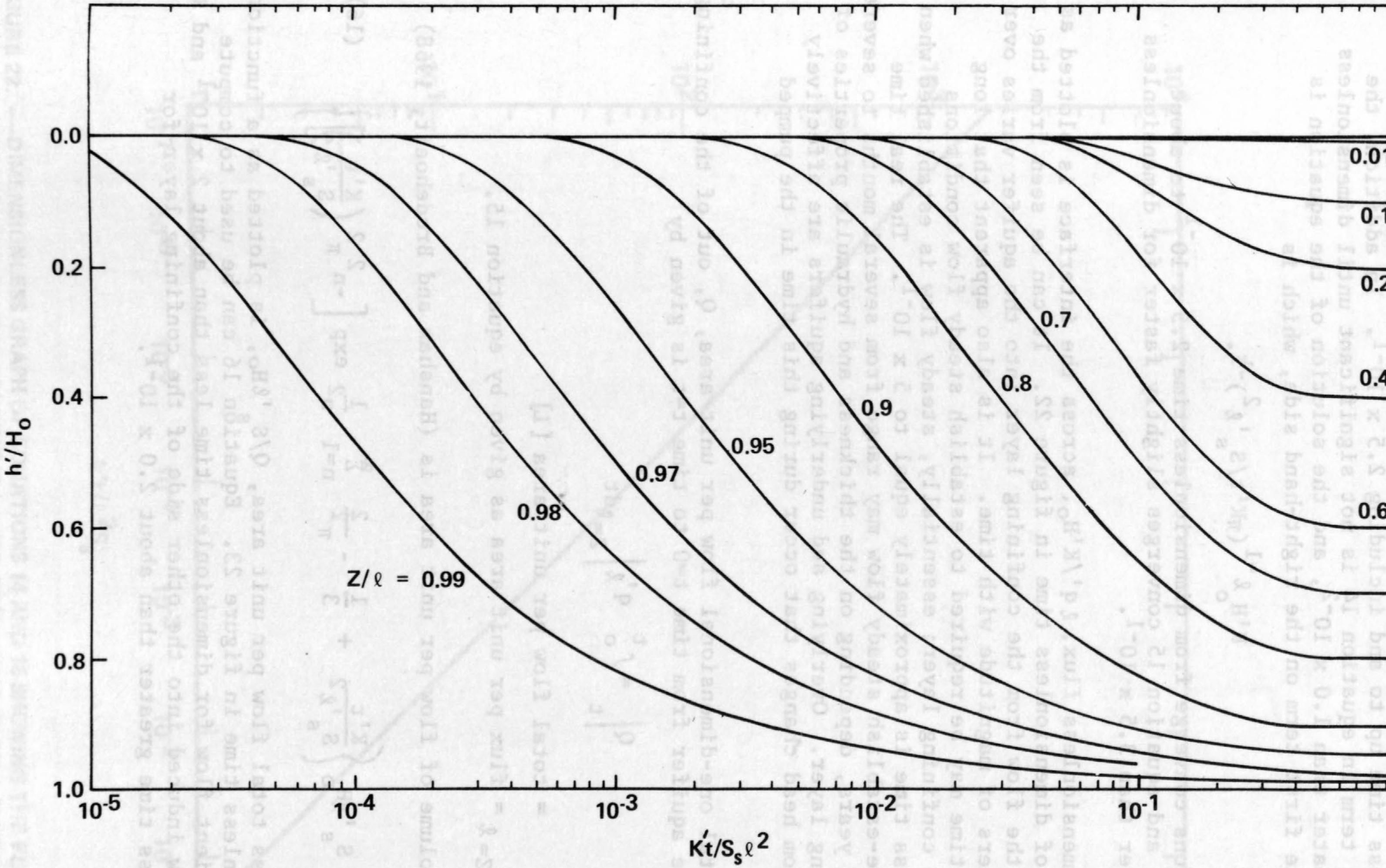


FIGURE 21. —DIMENSIONLESS GRAPH OF SOLUTIONS TO EQUATIONS 12 AND 13 ILLUSTRATING THE EXCESS-HEAD DISTRIBUTION IN A FINITE LAYER WITH A STEPWISE HEAD CHANGE AT ONE BOUNDARY (MODIFIED AFTER HANSHAW AND BREDEHOEFT, 1968, FIG. 2).

The series term in these equations also was evaluated for convergence. Equation 14 converges to a solution more rapidly than equation 15 for dimensionless time up to and including 2.5×10^{-1}. In addition, the exponential term in equation 14 is not significant until dimensionless time is greater than 1.0×10^{-2}, and the solution of the equation is given by the first term on the right-hand side, which is

$$K'H_o \ell^{-1}(\pi K't/S_s'\ell^2)^{-\frac{1}{2}}.$$

Both equations converge from dimensionless time 2.5×10^{-1} to about 3.5×10^{-1}, and equation 15 converges slightly faster for dimensionless times greater than 3.5×10^{-1}.

The dimensionless flux, $\ell q'/K'H_o$, across the interface is plotted as a function of dimensionless time in figure 22. It can be seen from the graph that the flow from the confining layer into the aquifer varies over several orders of magnitude with time. It is also apparent that long periods of time may be required to establish steady flow conditions through the confining layer; essentially, steady flow is established when dimensionless time is approximately equal to 5×10^{-1}. The real time needed to re-establish steady flow may range from several months to several hundreds of years, depending on the thickness and hydraulic properties of the confining layer. Overlying and underlying aquifers are effectively isolated from head changes that occur during this time in the pumped aquifer.

The total one-dimensional flow per unit area, Q, out of the confining bed into the aquifer from time t=0 to time t=t is given by

$$Q\Big|_t = \int_o^t q'_\ell\Big|_{Z=\ell} dt$$

where $Q\big|_t$ = total flow per unit area [L]

$q'_\ell\big|_{Z=\ell}$ = flux per unit area as given by equation 15.

The total volume of flow per unit area is (Hanshaw and Bredehoeft, 1968)

$$Q\Big|_t = S_s'\ell H_o \left\{\frac{K't}{S_s'\ell^2} + \frac{1}{3} - \frac{2}{\pi^2} \sum_{n=1}^{\infty} \frac{1}{n^2} \exp\left[-n^2\pi^2\left(\frac{K't}{S_s'\ell^2}\right)\right]\right\} \qquad (16)$$

Dimensionless total flow per unit area, $Q/S_s'\ell H_o$, is plotted as a function of dimensionless time in figure 23. Equation 16 can be used to compute total transient flow for dimensionless time less than about 2×10^{-1} and to compute flow induced into the other side of the confining layer for dimensionless time greater than about 2.0×10^{-1}.

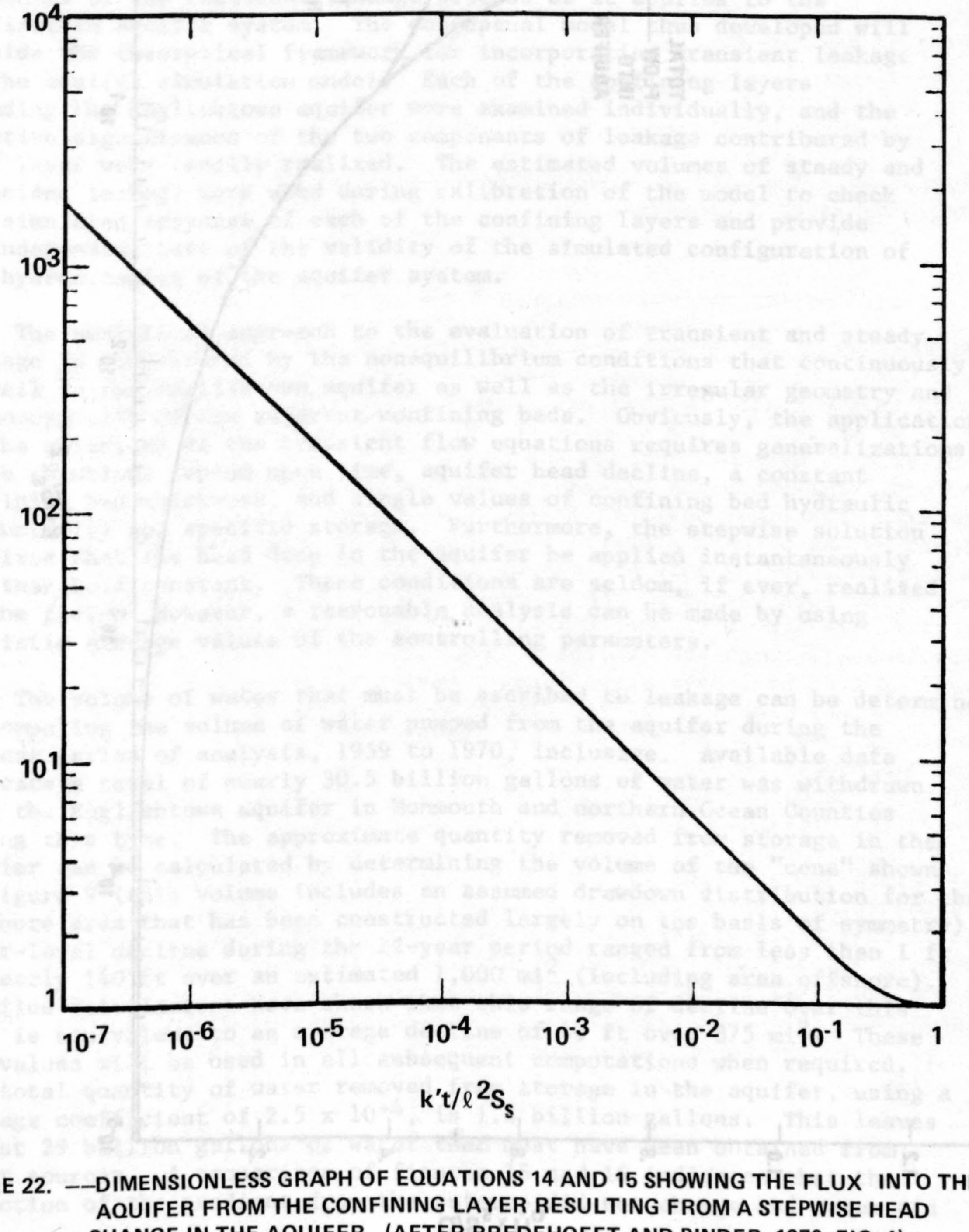


FIGURE 22. —DIMENSIONLESS GRAPH OF EQUATIONS 14 AND 15 SHOWING THE FLUX INTO THE AQUIFER FROM THE CONFINING LAYER RESULTING FROM A STEPWISE HEAD CHANGE IN THE AQUIFER. (AFTER BREDEHOEFT AND PINDER, 1970, FIG. 4).

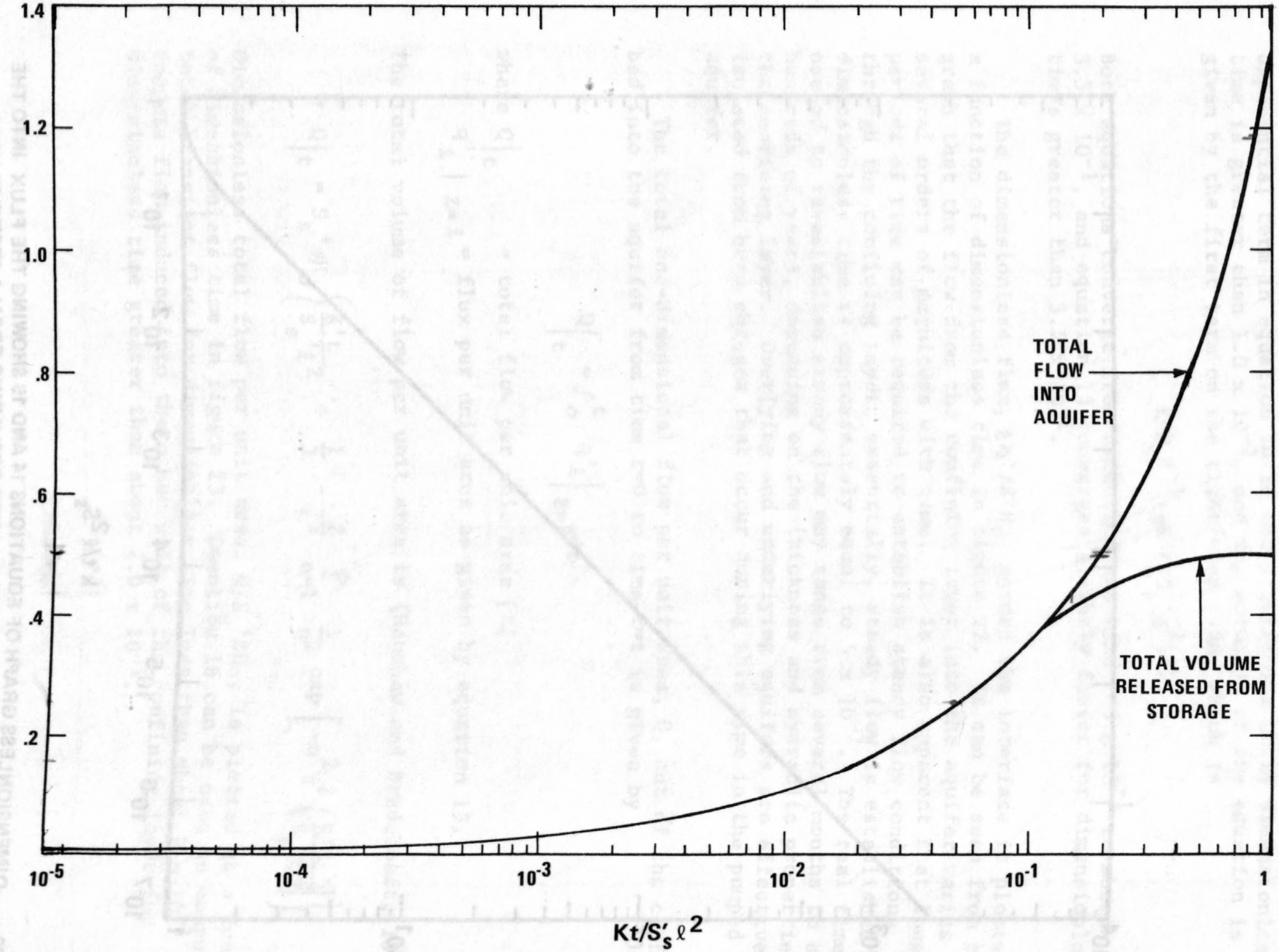


FIGURE 23. ––DIMENSIONLESS GRAPH OF SOLUTION TO EQUATION 16 OF THE TOTAL FLOW INTO THE AQUIFER FROM STORAGE IN THE CONFINING LAYER RESULTING FROM A STEPWISE CHANGE IN HEAD IN THE AQUIFER (MODIFIED AFTER HANSHAW AND BREDEHOEFT, 1968, FIG. 3).

Regional Evaluation of Transient and Steady Leakage in the Englishtown Aquifer System

A reasonable quantitative estimate of transient and steady leakage in the Englishtown aquifer system can be made with the aid of the foregoing equations and their graphical solutions. The analytical evaluation is used to explore and define the broader aspects of the mechanics of the transient leakage process as it applies to the Englishtown aquifer system. The conceptual model thus developed will provide the theoretical framework for incorporating transient leakage in the aquifer simulation model. Each of the confining layers bounding the Englishtown aquifer were examined individually, and the relative significance of the two components of leakage contributed by each layer were readily realized. The estimated volumes of steady and transient leakage were used during calibration of the model to check the simulated response of each of the confining layers and provide an independent test of the validity of the simulated configuration of the hydrodynamics of the aquifer system.

The analytical approach to the evaluation of transient and steady leakage is complicated by the nonequilibrium conditions that continuously prevail in the Englishtown aquifer as well as the irregular geometry and nonhomogeneity of the adjacent confining beds. Obviously, the application of the solutions of the transient flow equations requires generalizations. These solutions depend upon time, aquifer head decline, a constant confining bed thickness, and single values of confining bed hydraulic conductivity and specific storage. Furthermore, the stepwise solution requires that the head drop in the aquifer be applied instantaneously and then held constant. These conditions are seldom, if ever, realized in the field. However, a reasonable analysis can be made by using realistic average values of the controlling parameters.

The volume of water that must be ascribed to leakage can be determined by computing the volume of water pumped from the aquifer during the 12-year period of analysis, 1959 to 1970, inclusive. Available data indicate a total of nearly 30.5 billion gallons of water was withdrawn from the Englishtown aquifer in Monmouth and northern Ocean Counties during this time. The approximate quantity removed from storage in the aquifer can be calculated by determining the volume of the "cone" shown on figure 9 (this volume includes an assumed drawdown distribution for the offshore area that has been constructed largely on the basis of symmetry). Water-level decline during the 12-year period ranged from less than 1 ft to nearly 140 ft over an estimated 1,000 mi^2 (including area offshore). Detailed calculations have shown that this range of decline over this area is equivalent to an average decline of 40 ft over 875 mi^2. These two values will be used in all subsequent computations when required. The total quantity of water removed from storage in the aquifer, using a storage coefficient of 2.5×10^{-4}, is 1.8 billion gallons. This leaves almost 29 billion gallons of water that must have been obtained from other sources. A comparison of figures 15 and 16 indicates that the direction of the gradient from the outcrop did not change and water did not enter the aquifer from the outcrop area of the Englishtown Formation. The only remaining "source" from which this volume of water could be obtained is leakage from and through the adjacent confining layers and from captured discharge.

The volume of water released from storage in each of the confining layers can be calculated with equation 16 or can be determined graphically from figure 23. This graph shows dimensionless total flow per unit area, which is defined as

$$Q_D = \frac{Q}{S_{s\ell}' H_o}$$

plotted as a function of dimensionless time defined, in terms of the confining layer parameters, as

$$T_D = \frac{K't}{S_s' \ell^2} \tag{17}$$

The response of the system to a head drop in the pumped aquifer is divided into three periods. The first period, in which storage in the confining layer is important, extends until dimensionless time is approximately 10^{-1}, by which time the effects of the stepwise head drop in the pumped aquifer have extended through the confining layer (fig. 21). From time t=0 until dimensionless time 10^{-1}, the pumped aquifer is effectively isolated from other aquifers in the system, and the analysis of leakage caused by the assumed stepwise head drop in the pumped aquifer can be made by considering only the pumped aquifer and the adjacent confining beds. The last period of the analysis extends from dimensionless time greater than 5×10^{-1}, when essentially steady flow conditions are established through the confining layer (fig. 21); storage in the confining layer is then no longer significant. During the intermediate period from dimensionless time 1×10^{-1} to dimensionless time 5×10^{-1}, flow in the confining layer is coming both from storage in the confining layer and from the overlying or underlying aquifers of the system. Thus, the value of the dimensionless time parameter is critical in determining the volume of leakage derived from storage in the confining layer.

Dimensionless time is a function of the confining-layer parameters and the total elapsed time since the stepwise head change in the aquifer. The extreme and average values of thickness, hydraulic conductivity, and specific storage for each confining layer of the Englishtown aquifer system are given in table 4. These data can be used to compute the approximate real time required for the effects of the stepwise head ·· to extend through the confining bed, since from equation 17

$$t = \frac{(1 \times 10^{-1}) S_s' \ell^2}{K'}$$

where $T_D = 1 \times 10^{-1}$.

They also can be used to determine the approximate real time when essentially steady flow conditions are established in the confining layer, which is given by

$$t = \frac{(5 \times 10^{-1}) S_s' \ell^2}{K'}$$

Both of these solutions assume that head has dropped in only one of the aquifers bounding the confining layer and that flow in the confining layer is in one direction only. The approximate real times needed for the effects of a stepwise head change to extend through each confining bed and to establish essentially steady flow conditions, using the extreme and average values of confining bed thickness, are given in table 4. All the calculations of real time were made using average values of hydraulic conductivity and specific storage.

The lower confining layer of the Englishtown aquifer system, the Merchantville and Woodbury Formations, presents fewer computational problems than the thinner overlying confining bed and, therefore, will be considered first. The dimensionless time parameter for the lower confining layer at the end of the 12-year period can be determined with the following average data (table 4):

$$K'' = 7.6 \times 10^{-11} \text{ ft/s}$$
$$S_s'' = 8.8 \times 10^{-5} \text{ ft}^{-1}$$
$$\ell'' = 200 \text{ ft}$$
$$t = 3.78 \times 10^{8} \text{ s}$$

These give a dimensionless time of

$$T_D'' = \frac{(7.6 \times 10^{-11})(3.78 \times 10^{8})}{(8.8 \times 10^{-5})(4.0 \times 10^{4})} = 8.16 \times 10^{-3}$$

which is considerably less than the limiting value of 1×10^{-1} and indicates that nearly all new leakage during the 12-year period from the lower confining bed has come from storage within the layer. The estimated real times required for the effects of the head change in the Englishtown aquifer to extend through the lower confining layer (table 4) are considerably greater than the 12 years covered in the analysis.

The use of dimensionless time 8.16×10^{-3} to compute the total quantity of water released from storage implies that the total head change measured between 1959 and 1970 was applied instantaneously at t=0 (1959) and then remained constant throughout the 12-year period. This obviously was not the case. Available continuous water-level measurements (figs. 18A and 18C) indicate that the head in the Englishtown has been declining

Table 4.--Average and extreme values of confining-layer parameters that control transient leakage

	Upper confining layer			Lower confining layer		
	Average[1/]	Smallest	Largest	Average[1/]	Smallest	Largest
Thickness	40	10	100	200	80	320
Hydraulic conductivity (ft/s)	1.7×10^{-10}	6.5×10^{-11}	5.7×10^{-9}	7.6×10^{-11}	4.2×10^{-11}	6.9×10^{-10}
Specific storage (ft^{-1})	7.1×10^{-5}	5.1×10^{-5}	9.2×10^{-5}	8.8×10^{-5}	6.5×10^{-5}	4.6×10^{-4}
	Time required for effects of head change to extend through confining layer (years)[2/]					
	2.1	.13	13.2	146	23.4	376
	Time required to establish steady flow (years)[2/]					
	10.5	.66	66.2	734	117	1,880

[1/] Harmonic mean values of hydraulic conductivity and specific storage.

[2/] Using average values of hydraulic conductivity and specific storage.

at fairly constant rates, ranging from 8 to 20 ft per year near the centers of pumping. This is analogous to problems in soil mechanics when a load (head change) is applied continuously over a long period of time. Soil mechanics studies have shown that this condition produces the same compaction as would the same load applied instantaneously at the midpoint of the period. Consequently, the transient leakage from the underlying confining bed can be approximated better by assuming that a stepwise head drop occurred at $t/2 = 6$ years. The dimensionless time is then 4.08×10^{-3}

Using figure 23, dimensionless total transient flow per unit area from the lower confining bed at dimensionless time 4.08×10^{-3} is about

$$\frac{Q''}{S_s''\ell''H_o} = 6.5 \times 10^{-2}$$

The total transient flow per unit area, Q, is given by

$$Q'' = (6.5 \times 10^{-2})\, S_s''\ell''H_o$$

Using the same values of specific storage and thickness as before

$$Q'' = H_o\,(1.14 \times 10^{-3})$$

It now remains to determine the value of excess head to be used in calculating the quantity of flow per unit area. Several different approaches were taken, and all yielded substantially the same result. The simplest approach is to compute a weighted average head decline for the entire area. The average head decline is about 40 ft. Substituting this value into the above equation gives

$$Q'' = (1.14 \times 10^{-3})(40)$$

$$= 4.5 \times 10^{-2}\ \text{ft}$$

which is the average quantity of water per square foot released from storage from the lower confining bed. The total quantity released from storage during the 12-year period is obtained by multiplying the quantity per unit area by the total area

$$Q_T'' = (4.5 \times 10^{-2}\ \text{ft})(2.79 \times 10^{7}\ \text{ft}^2/\text{mi}^2)(8.75 \times 10^{2}\ \text{mi}^2)$$

$$= 1.12 \times 10^{9}\ \text{ft}^3$$

$$= 8.4 \times 10^{9}\ \text{gallons}$$

$$= (8.4\ \text{billion gallons})$$

where Q_T'' is the total volume from the lower confining layer.

The calculation of transient flow from the upper confining bed of the Englishtown aquifer system is complicated by a combination of geologic and hydrologic conditions. The overlying confining layer, which includes the Marshalltown Formation and part or all of the Wenonah Formation, varies in thickness from less than 20 ft to just over 100 ft in the area of the analysis, but has an average thickness of only 40 ft. The average time for the effects of a head drop in the Englishtown to extend through this layer is about 2 years (table 4); in some places, the effects of a head decline in the aquifer could reach the top of the confining layer in as little as 0.13 year (47 days). Steady flow conditions could be established in the confining layer in an average of about 10 years (table 4) or in as little as 0.66 year (240 days), but only if the head in the overlying Mount Laurel aquifer remains at a constant level. However, the water level in the Mount Laurel has declined continuously since 1959 and at rates similar to those in the underlying Englishtown aquifer.

The declining head on the upper boundary of the overlying confining layer, believed to be in response to pumping in the Englishtown aquifer, violates the first boundary condition on which the solution of equation 11 is based. This condition, together with the relatively short time needed to establish steady flow in the confining bed, makes it difficult to apply the transient flow equation solution (equation 16, fig. 23) in calculating the total volume of stored water released into the Englishtown aquifer. The analysis of release of confining bed storage based on the equation

$$Q' = H_o \ell' S_s' A$$

is more appropriate.

This method of computing the volume of stored water releases requires careful determination of the applicable confining-layer thickness that should be used in the equation. The head decline in the Englishtown will be transmitted through the confining layer in a fairly short time, 47 days, in some places, but requires a significantly longer time, 13 years, in other places (table 4). About half of the total drainable volume would be released from the average confining-layer thickness of 40 ft in about 10 years, nearly the length of the period of analysis. Consequently, it seems that a reasonable estimate of the total volume of water released from storage can be computed by the above method using a confining-layer thickness of 20 ft, the thickness from which the total drainable volume would be obtained in a 10- to 12-year period of time.

The following data are used to calculate the total volume of water released from storage in the overlying confining layer:

$$H_o = 40 \text{ ft}$$

$$\ell' = 20 \text{ ft}$$

$$S_s' = 7.1 \times 10^{-5} \text{ ft}^{-1}$$

$$A = 875 \text{ mi}^2$$

The total volume released from storage is

$$Q' = 1.387 \times 10^9 \text{ ft}^3$$
$$= 1.037 \times 10^{10} \text{ gal}$$
$$(10.4 \text{ billion gallons}).$$

Combining this quantity of 10.4 billion gallons from storage in the upper confining layer with the 1.8 billion gallons from storage in the aquifer and 8.4 billion gallons from storage in the underlying confining layer gives a total of about 20.6 billion gallons of water obtained from immediate sources of storage. This amounts to about 65 percent of the total volume withdrawn from the aquifer from 1959 to 1970. It could be assumed that the remaining 35 percent of the total pumpage was obtained from flow out of the Mount Laurel through the intervening confining layer and from captured discharge. Hereafter, steady leakage from the Mount Laurel refers to the flux across the Mount Laurel-Wenonah contact that supports the movement of water through the Marshalltown into the Englishtown. Steady leakage from the Magothy-Raritan aquifer is similarly defined with respect to the intervening Merchantville-Woodbury confining layer.

Available data indicate major predevelopment natural discharge by interaquifer leakage from the Englishtown into the underlying Magothy-Raritan aquifer. The head in the Englishtown aquifer was higher than that in the Magothy-Raritan aquifer system in Monmouth and northern Ocean County in 1900. Head differences ranged from as much as 80 ft at places 4 to 5 mi downdip from the Englishtown Formation outcrop area in Monmouth County to 20 ft or less near the Monmouth County coast. By 1959, the head in the Englishtown aquifer downdip from the outcrop area was still about 30 ft higher than that in the Magothy-Raritan aquifer, but was as much as 70 ft lower than that in the Magothy-Raritan aquifer in southeastern Monmouth County. Between 1900 and 1959 much of the discharge from the Englishtown aquifer to the Magothy-Raritan aquifer had been captured and in some areas head differences were reversed, providing the potential for recharge to the Englishtown aquifer by leakage from the Magothy-Raritan aquifer across the Merchantville-Woodbury confining layer. It is questionable that this condition was realized because of the long average time (table 4) needed to transmit head changes across the confining layer separating the two aquifers.

Although by 1959, discharge from the Englishtown (perhaps substantially reduced) to the underlying Magothy-Raritan aquifer, in the area 4 to 5 mi downdip from the outcrop, is supported by leakage from the overlying Mount Laurel aquifer through the intervening Marshalltown-Wenonah confining layer; the rate of flux through the Englishtown aquifer from the Mount Laurel and into the Magothy-Raritan aquifer probably changed insignificantly from 1959 to 1970. This is because head differences between each of the aquifers remained nearly the same during this time interval. For the purposes of this analysis, this volume of leakage need not be considered. All captured discharge from the Englishtown aquifer to the Magothy-Raritan aquifer is assumed to have been accounted for before 1959. It is further assumed, based on all available data, that there is no significant steady leakage from the Magothy-Raritan into the Englishtown during the period of analysis.

Other historical data suggest that there were two additional areas of significant natural discharge from the Englishtown aquifer in Monmouth and northern Ocean County. One area was along the coast where water was discharged upward through the overlying confining layer into the Mount Laurel aquifer. The aquifer also discharged into the outcrop area. Water-level data available for the early 1950's strongly suggest that all natural discharge from the Englishtown to the Mount Laurel had been captured by that time. The outcrop area was still a discharge area in 1970 although the ground-water divide downdip from the outcrop area (fig. 16) appears to have shifted slightly toward the west (toward the outcrop area) between 1959 and 1970. This suggests a decrease in the rate and volume of natural discharge to the outcrop area. However, the discharge is supported by leakage from the overlying Mount Laurel aquifer and any capture of this discharge by the Englishtown aquifer is, in effect, an increase in the volume of leakage into the Englishtown. The volume of water that could be obtained from steady leakage from the Mount Laurel can be demonstrated by considering those parameters that control steady leakage across confining beds, in particular, the head difference between the Englishtown aquifer and the overlying Mount Laurel aquifer.

The head in the Mount Laurel aquifer ranged from about 10 ft to about 40 ft higher than the head in the Englishtown aquifer in 1959. The ratio of head difference to confining bed thickness, $\Delta h/\ell$, ranged from about 0.3 to 1.5, and locally was as high as 2.5. The approximate average ratio was about 0.9. A restricted amount of comparative head data collected in November 1970 suggests that the same general differences and ratios existed in 1970 as in 1959. It is, therefore, concluded that the head difference across the confining layer remained fairly constant throughout the 12-year period of the analysis and that the head in the Mount Laurel aquifer has been declining at approximately the same rate as the head in the Englishtown since 1959.

The "constant" head gradient across the confining bed greatly simplified the calculation of steady leakage into the Englishtown aquifer. Because the gradient remains constant, the rate of flow has remained constant during the entire time. The rate of flow is then given by Darcy's equation

$$q' = -K' \frac{\partial h}{\partial \ell}$$

which, when solved for the following values

$$K' = 1.7 \times 10^{-10} \text{ ft/s}$$

$$\frac{\partial h}{\partial \ell} = 0.9$$

leads to

$$q'_s = (1.7 \times 10^{-10})(9 \times 10^{-1}) = 1.53 \times 10^{-10} \text{ ft/s}$$

where q'_s is the rate of steady leakage per square foot of confining layer-aquifer interface. The volume per unit area of steady leakage is given by

$$Q'_s = q'_s t$$

where t is the total elapsed time and is equal to 3.78×10^8 s. The volume of steady leakage per square foot is

$$Q'_s = (1.53 \times 10^{-10})(3.78 \times 10^8)$$

$$= 5.8 \times 10^{-2} \text{ ft}$$

Finally, the total volume of steady leakage through the overlying confining layer over the 12-year period is

$$Q'_{ST} = (5.8 \times 10^{-2} \text{ ft})(2.79 \times 10^7 \text{ ft}^2/\text{mi}^2)(8.75 \times 10^2 \text{ mi}^2)$$

$$= 1.4 \times 10^9 \text{ ft}^3$$

$$= 10.5 \times 10^9 \text{ gallons}$$

(10.6 billion gallons)

where Q'_{ST} is the total volume of steady leakage.

This is perhaps a surprisingly large quantity of water to be obtained through steady leakage from the Mount Laurel into the Englishtown. It represents an average rate of flow from the Mount Laurel aquifer of about 2.4 Mgal/d for the 12 years of the analysis. The possible sources of such a large quantity of water should, therefore, be identified before concluding the analysis. Because the head decline in the Mount Laurel aquifer from 1959 through 1970 was about the same as in the Englishtown aquifer for the same period of time and because the reported storage coefficient of the Mount Laurel (Rush, 1968) is similar to that of the Englishtown, it is reasonable to assume that no more than about 2 billion gallons of water were removed from storage in the Mount Laurel aquifer. This leaves about 8 billion gallons unaccounted for; to this quantity must be added approximately 3.5 billion gallons of water pumped from the Mount Laurel from 1959 through 1970. The total volume of water to be accounted for then is estimated to be about 11.5 billion gallons.

A number of possibilities should be considered in any attempt to account for the leakage from the Mount Laurel into the Englishtown. All the possibilities must involve sources from which the Mount Laurel might in turn obtain recharge. These sources could and may include the captured water discharged to sources other than the Englishtown, some increase in recharge from the outcrop area between 1959 and 1970 (B. Nemickas, written commun., 1975), leakage from overlying aquifers, and stored water released from confining beds overlying the Mount Laurel.

The outcrop area of the Mount Laurel Formation in Monmouth County was both a discharge and recharge area for the aquifer in 1960 (B. Nemickas, written commun.). The same conditions existed in 1970 with the possibility of somewhat less discharge from the confined part of the aquifer to the outcrop area and a small increase in the recharge to the aquifer from the outcrop area. The discharge from the Mount Laurel to the outcrop is supported by vertical leakage from or through the overlying confining layer. Any decrease in this discharge reflects either a decrease in the rate of leakage or an increase in the amount of leakage captured by the aquifer. The latter case is the most likely. The total volume of direct recharge to the confined part of the Mount Laurel aquifer from the outcrop area is estimated to have been about 1.0 Mgal/d in 1959. This may have increased to about 1.5 Mgal/d by 1970, leaving an average total of about 6 billion gallons in recharge to the Mount Laurel that would appear to have come from sources other than the outcrop area.

B. Nemickas (oral commun., 1975) has developed some data on the Mount Laurel aquifer system for a simulation model. At the present stage of development, the confining layer above the Mount Laurel ranges from about 50 ft to about 1,200 ft over the area modeled, which covers much of the Coastal Plain and exceeds the area covered by the Englishtown model. The average thickness in the area coincident with the Englishtown model is about 350 ft. Much of this sequence consists of fine-grained sediments, but there are several sand units of locally significant thickness.

Nevertheless, the head declines in the Mount Laurel aquifer in Monmouth and part of northern Ocean County can be expected to cause release of stored water from the silty and clayey sediments in the overlying sequence with minor contributions of water from storage in the intercalated coarse-grained sediments.

The plausibility of obtaining all or part of the required 6 billion gallons of water from storage in fine-grained sediments over a 12-year period can be shown using the following approximate average values of thickness and other pertinent parameters for the confining layer overlying the Mount Laurel aquifer:

$$K''' = 2.0 \times 10^{-10} \text{ ft/s}$$

$$S_s''' = 3.5 \times 10^{-5} \text{ ft}^{-1}$$

$$\ell''' = 350 \text{ ft}$$

$$t = 1.89 \times 10^{8} \text{ s}$$

$$H_o = 40 \text{ ft}$$

The value of dimensionless time is

$$T_D = 8.8 \times 10^{-3}$$

From figure 23, the total flow per square foot is

$$Q''' = 5.14 \times 10^{-2} \text{ ft}$$

and the total flow into the Mount Laurel aquifer is then

$$Q'''_T = 1.25 \times 10^{9} \text{ ft}^3$$

$$= 9.4 \times 10^{9} \text{ gal}$$

(9.4 billion gallons)

This volume of water combined with recharge from the outcrop is more than the volume needed to support or replace the water discharged from the Mount Laurel aquifer into the underlying Englishtown aquifer. This evaluation is not intended to be a rigorous analysis of all recharge to and discharge from the Mount Laurel aquifer from 1959 through 1970. Rather, it is intended to demonstrate that water stored in the confining bed overlying the Mount Laurel could be a significant indirect source of water that discharges from the Mount Laurel through the Wenonah-Marshalltown confining layer into the Englishtown aquifer.

The analysis of leakage, both transient and steady, into the Englishtown aquifer has demonstrated several points mentioned at the beginning of this discussion. First, and most obvious, is the significance of low permeability sediments as a source of stored water. About 60 percent of the water withdrawn from the Englishtown from 1959 through 1970 was water released from storage in the adjacent confining layers (table 5). An additional 34 percent of the withdrawals, which contributed directly as discharge from the Mount Laurel through the confining layer into the Englishtown, may have been derived or supported significantly from storage in the confining layer above the Mount Laurel aquifer.

Table 5.--Source of water removed from storage to sustain withdrawals from the Englishtown aquifer, 1959-70, and percentage of total withdrawal from each source.

Source	Volume (billion gallons)	Percent of total withdrawn
Aquifer; storage	1.8	6
Lower confining layer; storage	8.4	27
Upper confining layer; storage	10.4	33
Upper confining layer; steady leakage*	10.6	34
Total	31.2	100

*Discharge from the Mount Laurel aquifer across the confining bed into the Englishtown aquifer.

The second point demonstrated by the analysis is the effect that pumpage from the Englishtown aquifer has had on the total ground-water flow system. Withdrawals from the Englishtown have caused the movement and removal of water through or from as much as 300 ft of the Coastal Plain sedimentary sequence. This thickness includes 1) about 60 ft of the Merchantville-Woodbury Formations, 2) an average of about 100 ft (30 m) of Englishtown Formation, 3) an average of 40 ft of Marshalltown and the finer-grained part of the Wenonah Formation, 4) an average of 50 ft of coarser-grained Wenonah and Mount Laurel Formations, and 5) possibly as much as 50 ft of fine-grained sediments overlying the Mount Laurel Sand. Furthermore, it is obvious that water levels in the overlying Mount Laurel aquifer would have declined significantly because of leakage into the Englishtown even if there had been no withdrawals from the Mount Laurel during this time. Increased gradients in the Mount Laurel downdip from the outcrop area caused increased recharge from and decreased discharge to the outcrop. Clearly, a meaningful analysis of ground-water flow in the Englishtown aquifer cannot be made by considering only the response of Englishtown water levels to pumpage from the aquifer and treating the aquifer as an isolated source of water.

Analytical techniques can be used to evaluate the effects of withdrawals from one or more aquifers in a multiaquifer system if the problem can be sufficiently simplified and idealized. The solutions assist in providing better understanding of the mechanics of the total ground-water flow system. The analytical approach is severely limited, however, when system geometry and boundary conditions are complex, as in the case of the confining layer overlying the Englishtown aquifer. The same equations and techniques used to evaluate the effects of past development of the Englishtown aquifer also could be used to estimate the effects of future development. Such predictive evaluations would require the use of assumed head declines in the aquifer, and the results would be largely speculative. More reliable predictions of aquifer response can be obtained by analog or digital computer methods of aquifer simulation. Either of these simulation techniques can be used to solve equations of flow for reasonably complex problems, and digital computer simulation models can include the effects of transient as well as steady leakage. Aquifer simulation model results can be used to evaluate the impact of pumping stress on the total system, if further analysis is required.

SIMULATION OF A MULTILAYERED GROUND-WATER SYSTEM

Introduction

A digital computer simulation model of a ground-water system is essentially a mathematical model based on the equations of transient flow of a homogeneous fluid in a nonhomogeneous porous medium. The computer is nothing more than a tool used to solve these equations for a given set of aquifer characteristics and boundary conditions. In single layered aquifer systems the problems involved in solving the three-dimensional flow equation are simplified by assuming that vertical components of flow in the aquifer are negligible and that only the components of flow in the plane of the aquifer need be considered. The problem is then one of two-dimensional flow. In multilayered aquifer systems, where the aquifer is bounded by leaky confining layers and flow is not restricted to the aquifer, the problem is again one of three-dimensional flow, if vertical transient flow in the adjacent confining beds is to be adequately represented. The problem can be simplified by considering only the vertical component of flow in the confining layer and assuming that the horizontal components of flow in the plane of the confining bed are negligible. This assumption is considered valid because of the usually large contrast in permeability between the aquifer and confining beds (Hantush, 1960, p. 3713). The problem is then reduced to evaluating two-dimensional flow in the aquifer, which is modified by one-dimensional steady and transient leakage.

Differential and Finite-difference Equations of Flow

Aquifer

The general equation of two-dimensional ground-water flow in an elastic confined anisotropic aquifer is given by

$$\frac{\partial}{\partial x}\left(T_{xx}\frac{\partial h}{\partial x}\right)+\frac{\partial}{\partial y}\left(T_{yy}\frac{\partial h}{\partial y}\right)= S\frac{\partial h}{\partial t}+W(x,y,t) \qquad (19)$$

where T_{xx} and T_{yy} are the principle components of the transmissivity tensor in an anisotropic aquifer (Papadopulos, 1965), h is the head in the confined aquifer, S is the storage coefficient, and W(x,y,t) is the flux into or out of the aquifer through a source or sink. It is assumed that the coordinate axes of the finite difference grid of the model are colinear with the principle axes, or components, of the transmissivity tensor.

Analytical solutions of the differential form of the flow equation are usually either very cumbersome or impossible to obtain unless the geometry and boundary conditions of the aquifer are very simple. Approximate solutions of the differential equation may be obtained by numerical methods. The most popular approach currently in use in ground-water studies is the method of finite differences, which involves the replacement of the continuous derivatives by values of the difference quotients of the function and solving the resulting difference equations (Volynskii and Bukhman, 1965; Richtmyer and Morton, 1967). The resulting algebraic equations are solved for the unknown function at specified points in time and space (Volynskii and Bukhman, 1965; Richtmyer and Morton, 1967). Pinder and Bredehoeft (1968) have given a detailed discussion of the derivation of the finite-difference approximation of the ground-water flow equation (equation 19).

The finite-difference equations are commonly developed for a set of points or nodes, which form a rectangular grid in the x - y plane (fig. 24). For the template indicated in figure 24 the finite-difference approximation of equation 1 may be given by

$$
\begin{aligned}
&T_{xx(i-1/2,j)}\left[\frac{h_{i-1,j,k}-h_{i,j,k}}{\Delta x_i}\right]+T_{xx(i+1/2,j)}\left[\frac{h_{i+1,j,k}-h_{i,j,k}}{\Delta x_i}\right]\\
&+T_{yy(i,j-1/2)}\left[\frac{h_{i,j-1,k}-h_{i,j,k}}{\Delta y_j}\right]\\
&+T_{yy(i,j+1/2)}\left[\frac{h_{i,j+1,k}-h_{i,j,k}}{\Delta y_i}\right]\\
&=S_{i,j}\left[\frac{h_{i,j,k}-h_{i,j,k-1}}{\Delta t}\right]+W_{i,j,k}
\end{aligned}
\tag{20}
$$

where $T_{xx(i-1/2,j)} = \dfrac{2T_{xx(i,j)}\,T_{xx(i-1,j)}}{(T_{xx(i,j)}\,\Delta x_{i-1})+(T_{xx(i-1,j)}\,\Delta x_i)}$

and is the weighted harmonic mean of

$$\frac{T_{xx(i-1,j)}}{\Delta x_{i-1}} \text{ and } \frac{T_{xx(i,j)}}{\Delta x_i}.$$

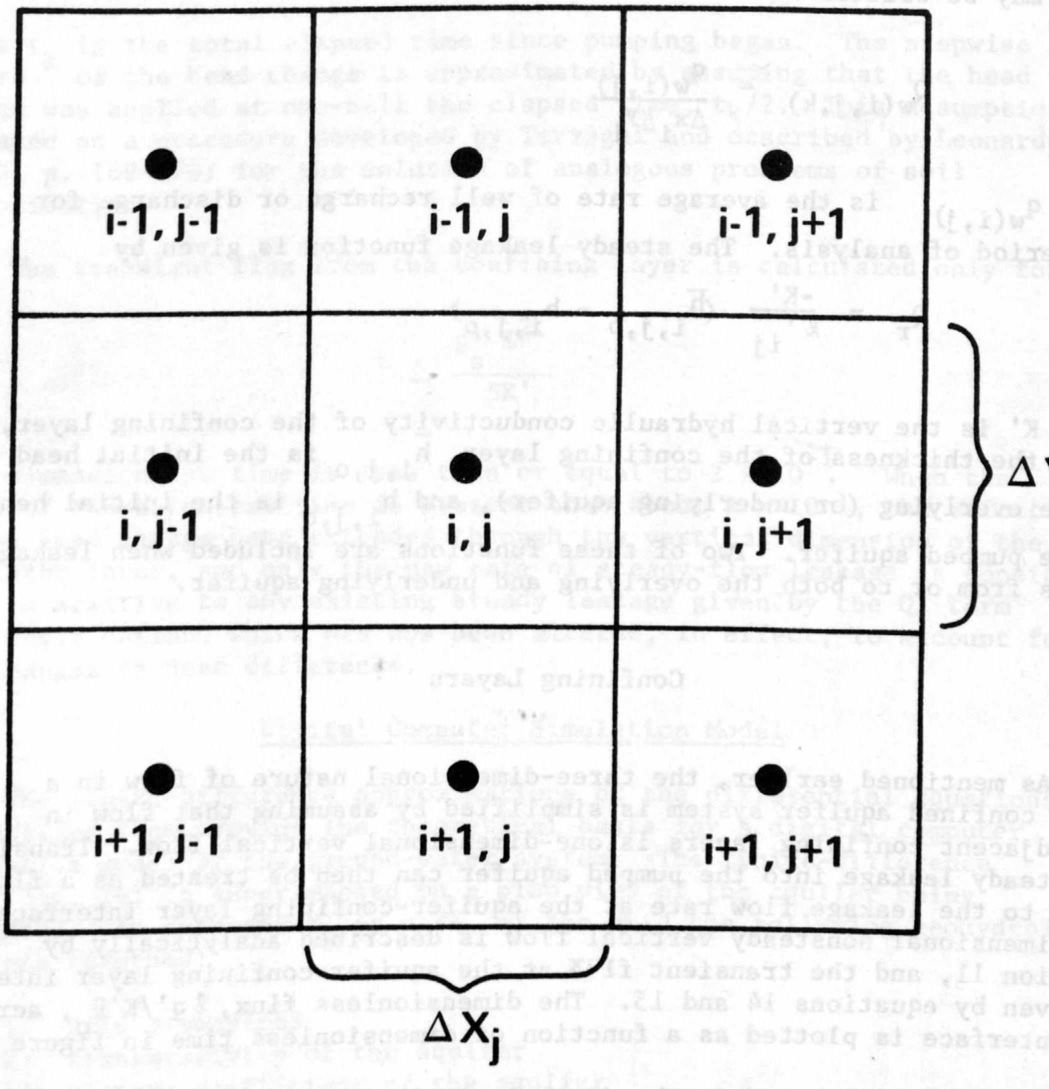


FIGURE 24.—FINITE-DIFFERENCE GRID AND NODE ARRAY FOR DIGITAL MODEL.

i,j	are the indexes in the y and x direction respectively
k	is the time index
Δt	is the time increment
$\Delta x, \Delta y$	are the distance increments in the x and y directions respectively.

The term, $W_{(i,j,k)}$ may include steady leakage and withdrawals and recharge through wells. The function for discharge or recharge through wells may be written

$$Q_{w(i,j,k)} = \frac{q_{w(i,j)}}{\Delta x_i \Delta y_i}$$

where $q_{w(i,j)}$ is the average rate of well recharge or discharge for the period of analysis. The steady leakage function is given by

$$Q_r = \frac{-K'}{\ell'_{ij}} (\bar{h}_{i,j,o} - h_{i,j,o})$$

where K' is the vertical hydraulic conductivity of the confining layer, ℓ' is the thickness of the confining layer, $\bar{h}_{i,j,o}$ is the initial head in the overlying (or underlying aquifer), and $h_{i,j,o}$ is the initial head in the pumped aquifer. Two of these functions are included when leakage occurs from or to both the overlying and underlying aquifer.

Confining Layers

As mentioned earlier, the three-dimensional nature of flow in a leaky confined aquifer system is simplified by assuming that flow in the adjacent confining layers is one-dimensional vertical flow. Transient and steady leakage into the pumped aquifer can then be treated as a flux equal to the leakage flow rate at the aquifer-confining layer interface. One-dimensional nonsteady vertical flow is described analytically by equation 11, and the transient flux at the aquifer-confining layer interface is given by equations 14 and 15. The dimensionless flux, $\ell q'/K'H_o$, across the interface is plotted as a function of dimensionless time in figure 22.

Equation 14 is used in the digital simulation model to compute an approximation of the flux into the aquifer at any time. The stepwise head change is approximated by solving the transient flux equation for each time step, Δt, of the finite-difference approximation equation of flow in the aquifer (equation 20).

The transient flow from the confining layer, using equation 14, is then approximated by (after Bredehoeft and Pinder, 1970, p. 887)

$$q_{i,j,k} \approx (h_{ijo} - h^n_{ijo}) \left[\frac{K'}{\ell_{ij} \sqrt{\dfrac{\pi K'(t_k/2)}{S_s' \ell^2_{ij}}}} \right] \left\{ 1 + 2 \sum_{n=1}^{\infty} \exp \left[\frac{-m^2}{K'(t_k/2)/S_s \ell^2_{ij}} \right] \right\} \quad (21)$$

where t_k is the total elapsed time since pumping began. The stepwise nature of the head change is approximated by assuming that the head change was applied at one-half the elapsed time, $t_k/2$. This assumption is based on a procedure developed by Terzaghi and described by Leonards (1962, p. 169-170) for the solution of analogous problems of soil consolidation.

The transient flux from the confining layer is calculated only for

$$t \leq \frac{S_s' \ell^2}{5K'}$$

when dimensionless time is less than or equal to 2×10^{-1}. When the value of dimensionless time is greater than about 2×10^{-1}, the effects of the head change have extended through the vertical dimension of the confining layer, and only the new rate of steady-flow leakage is considered. This is additive to any existing steady leakage given by the Q_r term previously defined which has now been altered, in effect, to account for new changes in head difference.

Digital Computer Simulation Model

The finite-difference approximations of the differential equations of confined flow provide the theoretical basis for a digital computer simulation model of the ground-water system. The finite-difference grid (fig. 24) is superimposed on a plan view of the aquifer being simulated. At each point, or node, of the grid the following geohydrologic data are recorded:

1) grid dimensions
2) transmissivity of the aquifer
3) storage coefficient of the aquifer
4) pumping rate (as required)
5) initial hydraulic head
6) thickness of overlying and underlying confining layers
7) hydraulic head in the overlying and underlying aquifers.

The physics of the flow system is then approximately described by using these data to write the finite-difference equations for each node of the grid. This leads to N equations in N unknowns, where N is the number of points in the grid.

The approximation equations are solved simultaneously using an iterative alternating-direction procedure. The alternating-direction technique was developed by Peaceman and Rachford (1955), and Douglas and Peaceman (1955). The technique was adopted by Pinder (1970; also Pinder and Bredehoeft, 1968) for his digital model for aquifer evaluation. An improved technique, using an iterative scheme for the alternating-direction solution was subsequently applied to aquifer models by Bredehoeft and Pinder (1970). The computer program developed for the present study makes extensive use of mathematical and programming techniques and procedures developed by Pinder and Bredehoeft.

Solution of Finite-difference Approximation Equations

The alternating-direction techniques of Douglas and Peaceman used to solve the approximation equations of ground-water flow have been discussed in detail by Pinder and Bredehoeft (1968). Briefly, the procedure requires a separate solution of the set of finite-difference equations for each of the coordinate directions that corresponds to the row and column directions of the finite difference grid (fig. 24). When the iterative scheme is used, the row and column calculations are repeated at each time step until convergence is achieved. The finite-difference equation for the column computations is

$$T_{xx(i-1/2,j)}\left[\frac{h^{n}_{i-1,j,k}-h^{n}_{i,j,k}}{\Delta x_i}\right]+T_{xx(i+1/2,j)}\left[\frac{h^{n}_{i+1,j,k}-h^{n}_{i,j,k}}{\Delta x_i}\right]$$

$$+T_{yy(i,j-1/2)}\left[\frac{h^{n+1/2}_{i,j-1,k}-h^{n+1/2}_{i,j,k}}{\Delta y_j}\right]+T_{yy(i,j+1/2)}\left[\frac{h^{n+1/2}_{i,j+1,k}-h^{n+1/2}_{i,j,k}}{\Delta y_j}\right]$$

$$=S_{ij}\left[\frac{h^{n+1/2}_{i,j,k}-h_{i,j,k-1}}{\Delta t}\right]+Q_{w(i,j,k)}+\frac{K'_{ij}}{\ell'_{ij}}\left[h_{i,j,o}-h_{i,j,o}\right]$$

$$+\frac{K''_{ij}}{\ell''_{ij}}\left[\overline{\overline{h}}_{i,j,o}-h_{i,j,o}\right]+Q'_{i,j,k}+Q''_{i,j,k}+I\left[h^{n+1/2}_{i,j,k}-h^{n}_{i,j,k}\right] \quad (22)$$

where $\bar{h}$ is the head in the overlying aquifer, $\bar{\bar{h}}$ is the head in the underlying aquifer, n is the index indicating the cycle of iteration, W_{ijk} is well discharge or recharge, Q'_{ijk} and Q''_{ijk} is the transient leakage from the overlying and underlying confining beds given by equation 21, and I is a normalized iteration parameter, for which the maximum value is

$$I_{max} = 1$$

and the minimum value is given by

$$I_{min} = \mathrm{Min}\left\{\frac{\pi^2}{2N_x^2}\left[\frac{1}{1 + \frac{C_{i,j+1/2}}{C_{i+1/2,j}}}\right] ; \frac{\pi^2}{2N_y^2}\left[\frac{1}{1 + \frac{C_{i+1/2,j}}{C_{i,j+1/2}}}\right]\right\}$$

where

$$C_{i+1/2,j} = \frac{2\ T_{ij}\ T_{i+1,j}}{(T_{ij}\ \Delta x_{i+1}) + (T_{i+1,j}\ \Delta x_i)}$$

$$C_{ij+1/2} = \frac{2\ T_{ij}\ T_{ij+1}}{(T_{ij}\ \Delta y_{j+1}) + (T_{ij+1}\ \Delta y_j)}$$

N_x = number of nodes in x direction

N_y = number of nodes in y direction

Intermediate values of the iteration parameter are spaced as a geometric series between the minimum and maximum values.

Pinder and Bredehoeft (1968) compared the numerical results obtained by the alternating-direction procedure with analytical solutions for problems in homogeneous media and with electric-analog results for problems that were hydrologically more complex. Their analysis indicates the numerical technique gives good results. Bredehoeft and Pinder (1970) have compared the numerical solution for aquifer flow problems involving transient leakage from a confining layer with Hantush's (1960) modified leaky aquifer theory. Their results are shown in figure 25. Similar results were obtained during this study for the case involving two confining beds with constant head boundaries above and below (Hantush, 1960, case 1). The results are shown in figure 26.

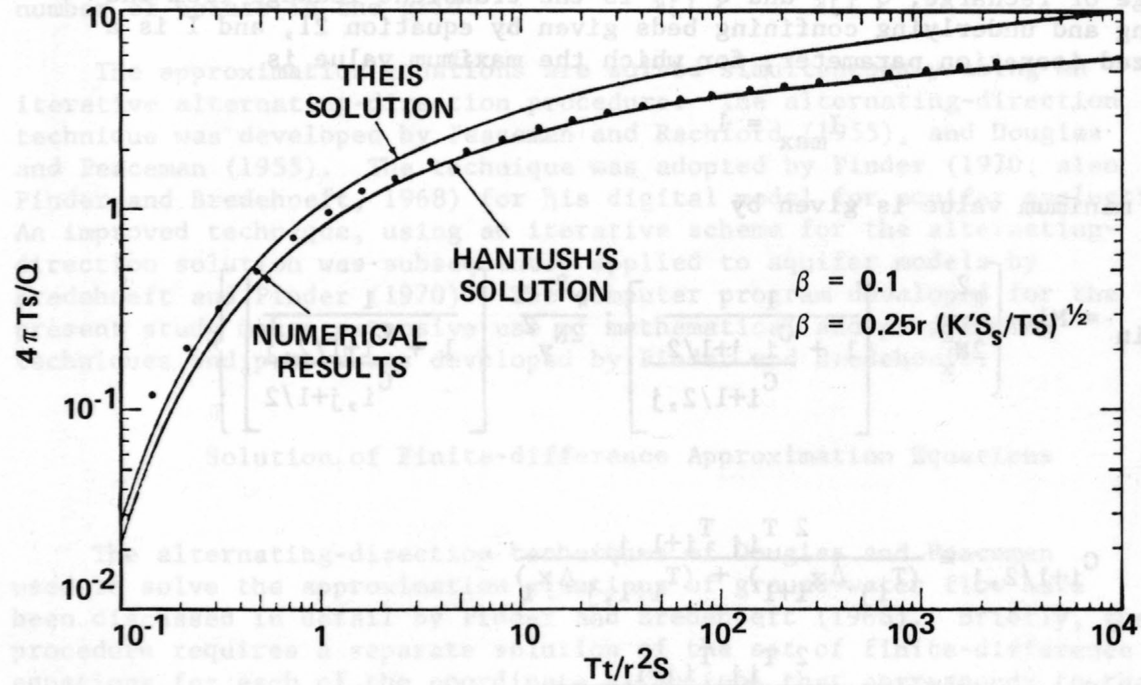


FIGURE 25. —COMPARISON OF NUMERICAL VERSUS HANTUSH'S THEORETICAL RESULTS FOR THE DRAWDOWN IN A PUMPED INFINITE AQUIFER OVERLAIN (AND/OR UNDERLAIN) BY A LEAKY CONFINING LAYER WITH STORAGE. (AFTER BREDEHOEFT AND PINDER, 1970, P. 887, FIG. 5)

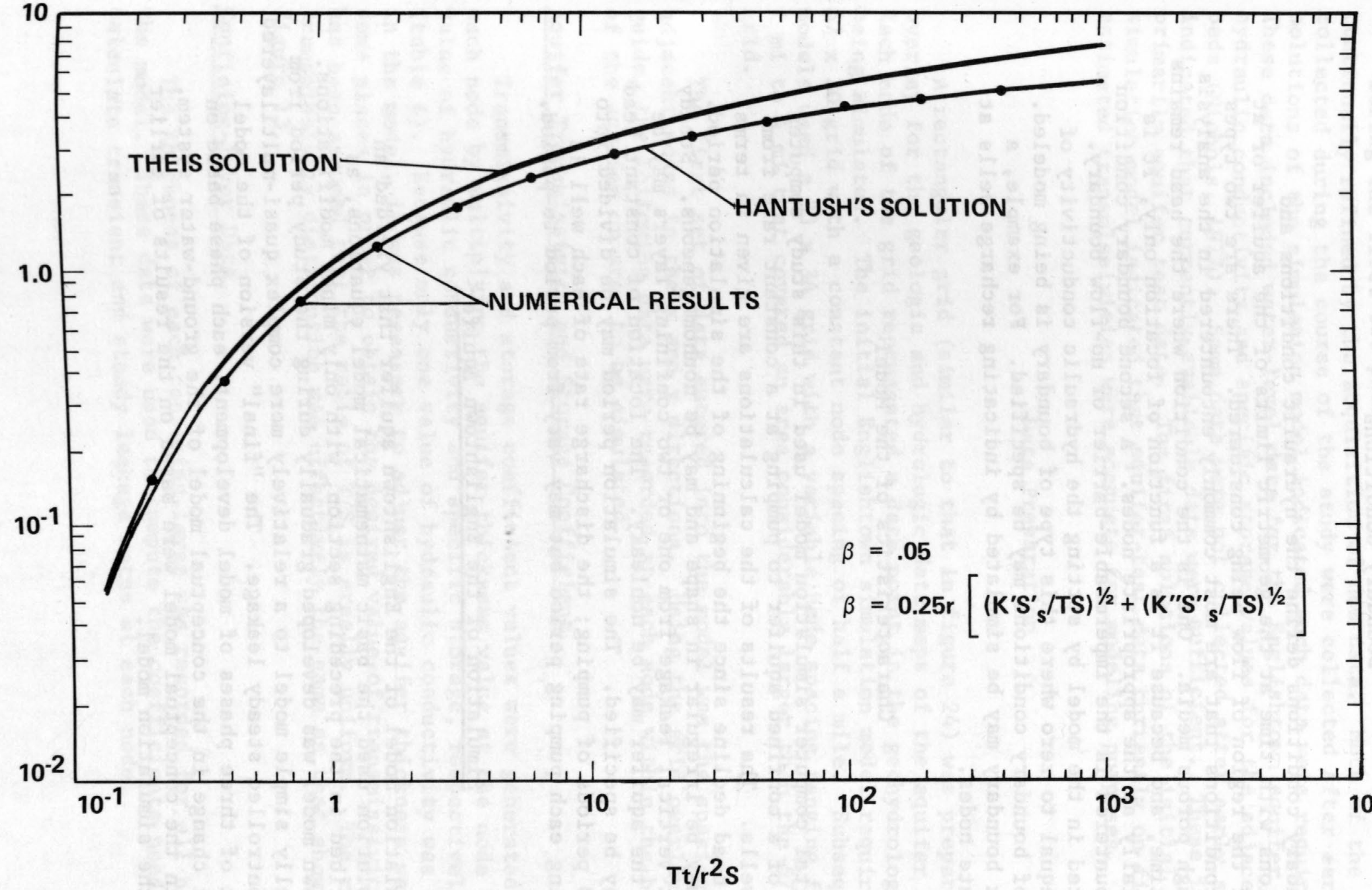


FIGURE 26. ––COMPARISON OF NUMERICAL RESULTS OBTAINED WITH MODEL USED IN THIS STUDY VERSUS HANTUSH'S THEORETICAL RESULTS FOR THE DRAWDOWN IN A PUMPED INFINITE AQUIFER OVERLAIN AND UNDERLAIN BY A LEAKY CONFINING LAYER WITH STORAGE.

Boundary Conditions

The boundary conditions define the hydraulic conditions and their variations with time at the geometric limits of the aquifer or at the limits of the region of flow being considered. There are two types of boundary conditions that are most commonly encountered in the analysis of flow through porous media. One is the condition where the head remains constant in time, and because it is a function of location only, it is defined initially at the appropriate nodes. A second boundary condition commonly encountered is the impermeable-barrier or no-flow boundary. This is treated in the model by setting the hydraulic conductivity of the aquifer equal to zero where this type of boundary is being modeled. Other types of boundary conditions may be specified. For example, a constant flux boundary may be simulated by indicating recharge wells at the appropriate nodes.

Characteristics of the Model

The digital computer simulation model used in this study computes the response of a confined aquifer to pumping at a constant rate from one or more wells. The results of the calculations are given in terms of hydraulic head decline since the beginning of the simulation period. The aquifer may be irregular in shape and may be nonhomogeneous. Steady and transient vertical leakage from one or two confining layers may be included, or the aquifer may be nonleaky. The location of constant head boundaries may be specified. The simulation period may be divided into as many as 10 periods of pumping; the discharge rate of each well is constant during each pumping period but may vary from period to period.

Simulation of the Englishtown Aquifer

The simulation model of the Englishtown aquifer that yielded an acceptable solution used the basic mathematical model equations, as briefly described in the preceding section, with only minor modifications. The Englishtown model was developed gradually during the study period from a hydrologically simple model to a relatively more complex quasi-multilayered model with controlled steady leakage. The "final" version of the model is the result of three phases of model development, each phase based on a significant change in the conceptual model of the ground-water system. The changes in the conceptual model were based on the results of earlier versions of the simulation model.

During the first two phases of model development, the model was continually refined by the acquisition of new data. Much of the data collected during the course of the study were collected after earlier solutions of the simulation model indicated such data were required. These data included water-level data for the Englishtown aquifer and hydraulic conductivity and specific storage values for the adjacent confining beds (Nichols, 1977). Some data were not collected after early solutions indicated that currently available data were sufficient. These included primarily aquifer test data. The first solutions of the initial Englishtown simulation model indicated that aquifer-test data currently on file and estimated transmissivities were adequate for modeling purposes.

Data Preparation

A rectangular grid (similar to that in figure 24) was prepared as an overlay for the geologic and hydrologic data maps of the aquifer system. Each node of the grid represented a data point in the geohydrologic system being simulated. The initial Englishtown simulation model required a 57 x 70 grid with a constant node spacing of half a mile. Subsequent models used a 33 x 34 grid with a variable node spacing ranging from 1 mi to 4 mi that covered an area considerably larger than the 57 x 70 grid.

The first set of data entered into the model was the spacing between adjacent nodes in the x and y directions. Using the grid overlay as a guide, the following data were recorded at each node within the boundaries of the aquifer or area to be simulated.

Aquifer Transmissivity and Storage Coefficient

Transmissivity and storage coefficient values were generated at each node by multiplying the aquifer thickness value at the node by a value of hydraulic conductivity and specific storage, respectively (table 6). Because only one value of hydraulic conductivity was used in the model, it was necessary to adjust the aquifer thickness value in some places in order to obtain the desired value of transmissivity. It has been assumed that the ratio of T/S is areally uniform. The resulting transmissivity distribution was effectively the same as that shown in figure 13.

Confining-layer Data

The thickness of each confining layer was recorded for all nodes in the model. These data were used to compute the coefficients needed to calculate transient and steady leakage rates at each node.

Table 6.--Hydraulic coefficients used in the Englishtown aquifer simulation models, and extreme and average values of coefficients obtained from field tests and laboratory tests of core samples.

	Field data			Prototype model	Phase II model	Phase III model
	Maximum	Minimum	Average			
Aquifer:[1/]						
K (ft/s)	2.3×10^{-4}	1.3×10^{-4}	1.6×10^{-4}	2.32×10^{-4}	1.6×10^{-4}	1.2×10^{-4}
S_s (1/ft)	3.0×10^{-6}	8.5×10^{-7}	--[2/]	4.0×10^{-5}	3.2×10^{-6}	3.0×10^{-6}
Upper confining layer:						
K' (ft/s)	5.7×10^{-9}	6.5×10^{-11}	1.7×10^{-10}	3.5×10^{-10}	1.7×10^{-10}	1.7×10^{-10}
S_s' (1/ft)	9.2×10^{-5}	5.1×10^{-6}	7.1×10^{-5}	--[3/]	6.0×10^{-5}	7.1×10^{-5}
Lower confining layer:						
K'' (ft/s)	6.9×10^{-10}	4.2×10^{-11}	7.6×10^{-11}	--[3/]	5.0×10^{-11}	5.0×10^{-11}
S_s'' (1/ft)	4.6×10^{-4}	6.5×10^{-5}	8.8×10^{-5}	--[3/]	2.2×10^{-5}	9.0×10^{-5}

1/ Calculated from aquifer test data.

2/ Insufficient field data.

3/ Not included in model.

Hydraulic Head Data

Actual field values of the altitude of hydraulic head in the Englishtown aquifer were not used in the model. Instead, the head decline from about 1900 to 1959 (fig. 27) was used as the initial head condition in the aquifer for the period of simulation. Initial head conditions in the overlying and underlying aquifers were entered as a function of the initial head in the Englishtown aquifer. The rationale for using these data as the initial condition is discussed in the following section of this report dealing with the assumptions made in the model.

Pumpage Data

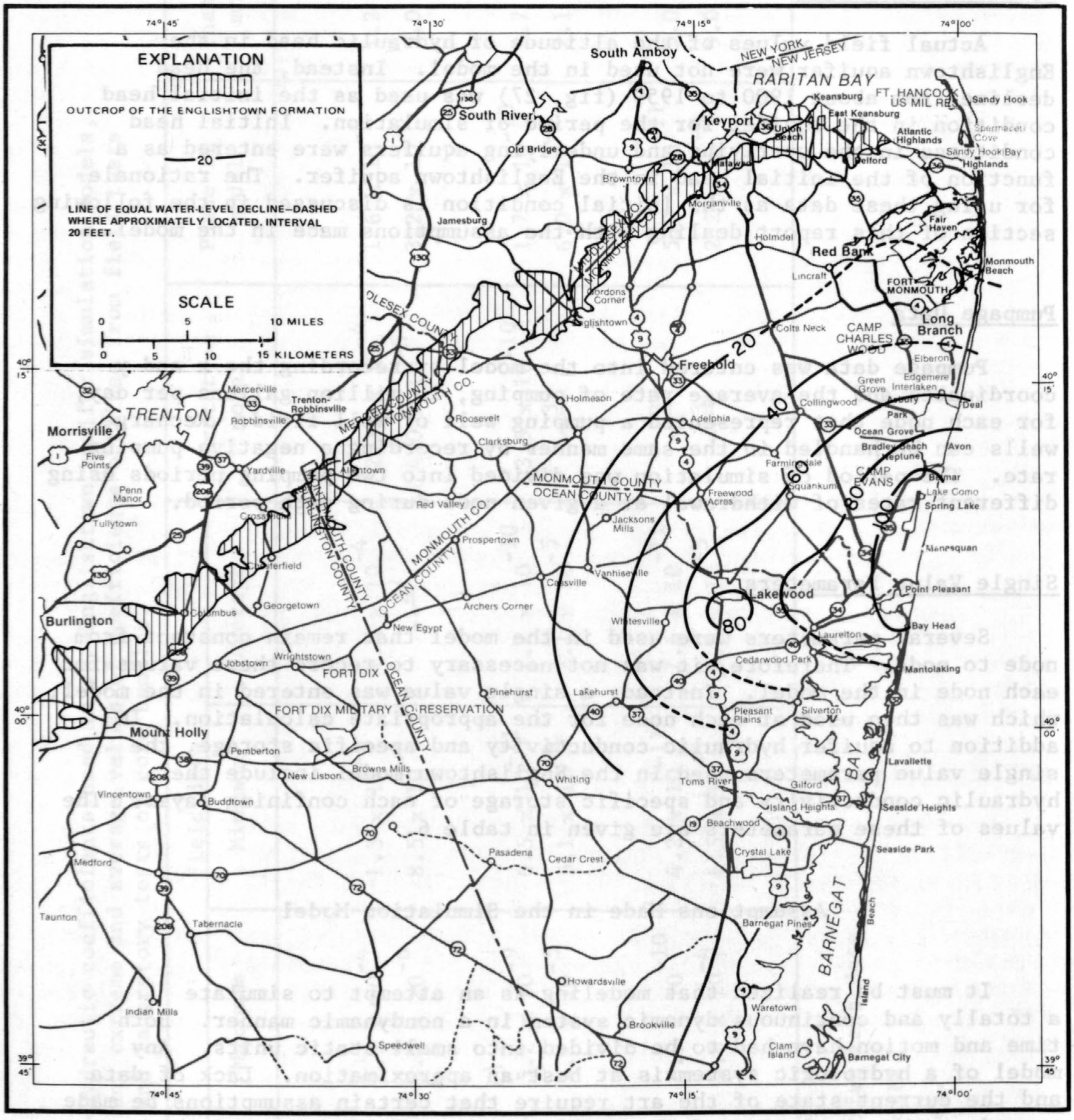
Pumpage data was entered into the model by recording the x and y coordinates and the average rate of pumping, in million gallons per day, for each node that represented a pumping well or well field. Recharge wells can be handled in the same manner by recording a negative pumping rate. The period of simulation was divided into two pumping periods using different rates of withdrawal at a given node during each period.

Single Value Parameters

Several parameters were used in the model that remain constant from node to node. Therefore, it was not necessary to record these values for each node in the model. Instead, a single value was entered in the model, which was then used at each node for the appropriate calculation. In addition to aquifer hydraulic conductivity and specific storage, the single value parameters used in the Englishtown model include the hydraulic conductivity and specific storage of each confining layer. The values of these parameters are given in table 6.

Assumptions Made in the Simulation Model

It must be realized that modeling is an attempt to simulate a totally and continuous dynamic system in a nondynamic manner. Both time and motion have had to be divided into small static units. Any model of a hydrologic system is at best an approximation. Lack of data and the current state of the art require that certain assumptions be made to reduce the complexity of the problem to be solved. Some of the complexities of the hydrologic system, such as three-dimensional flow, aquifer anisotropism, and time-dependent changes in hydraulic coefficients, can be included in a digital simulation model. As the problem becomes more complex, however, more data are needed, and the size of the model soon becomes unmanageable and may exceed computer storage capabilities. In addition, the inclusion of some complex characteristics in a simulation model may not increase the accuracy of the solution significantly with respect to the particular purpose for which the model was developed.



Base from Army Map Services, Newark, N.J., 1964 and Wilmington, Del., NJ-PA-MD 1966, 1:250000.
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FIGURE 27. —WATER-LEVEL DECLINE BETWEEN ABOUT 1900 AND JANUARY 1959 IN THE ENGLISHTOWN AQUIFER IN MONMOUTH AND NORTHERN OCEAN COUNTIES, N.J.

The present analysis employs several simplifying assumptions. Some of these have been made to reduce the amount of data, computer programming, and computer storage required. Other assumptions were made because of lack of data, in which case the simpler conditions have been assumed to exist.

Boundary Conditions

Little information is available to determine precisely the boundaries of the modeled area. The northern border of the model is located in Raritan and Sandy Hook Bays. Interpretation of available geologic data precludes locating the northern border of the model any farther north than the position chosen, and, as such, is an artificial boundary. Several small well fields are located near this boundary, and based on several trial boundary conditions, the northern boundary of the model is specified as a constant head boundary; an impermeable barrier boundary condition produced excessive declines around the nearby pumping wells.

The eastern border of the model has been located considerably east of the New Jersey Coast. Numerous well fields are situated along the coast; and consequently, the shoreline nearly bisects one of the major cones of depression developed in the Englishtown aquifer. Aquifer conditions and continuity offshore are not known. The eastern boundary of the model was, therefore, extended from 18 to 20 mi east of the shoreline, sufficiently far from the pumping centers that any influence of this boundary on the areas of primary dynamic response was insignificant, and specified as an impermeable barrier boundary. It is assumed the aquifer extends this far offshore. This configuration produces a reasonably symmetric cone, with minimum decline at the boundary.

The southern boundary and the north-south trending part of the western boundary of the model extends across as area with scant geologic and hydrologic data. Available data suggest the aquifer is thinner and finer-grained throughout much of this region. Consequently, the southern boundary was located at about latitude 39°45' or about 22 mi south of the southernmost well fields and designated as a no-flow boundary. In addition, the aquifer transmissivity has been gradually decreased across this distance. Again, results are good in the vicinity of the southern well fields, and declines along the model border are small.

The diagonal northwestern border of the model coincides as nearly as possible with the upper contact of the Englishtown Formation along its outcrop area. The outcrop area is known to be a discharge area, and the configuration of the potentiometric surface (fig. 15) suggests that the upper contact of the formation is a discharge boundary for the confined part of the aquifer. Discharge rates along the boundary are not known; these rates could be computed and handled by the simulation model in several ways. However, the streams and water table at the upper boundary of the aquifer function as a constant-head boundary, and this is the most reasonable way to model it. This representation of the boundary was used and does not seem to have resulted in any significant error in the solution.

Initial Conditions

The head decline in the Englishtown aquifer from about 1900 to 1959 shown in figure 27 is used as the initial head condition in the aquifer. That is, the initial heads were taken as equal to the drawdowns generated by pumpage during the period 1900-59. This approach served to incorporate the effects of the earlier pumping (pre-1959) into the analysis without the necessity of going through a simulation of this earlier pumpage, the data for which were not available. The final head values computed by the model for 1970 actually represent drawdowns due to the entire history of pumping from 1900 to 1970. The various flow terms obtained in the model analysis accordingly represent changes from pre-pumping flow rates.

Initial conditions in the adjacent aquifers also are induced so that the leaky nature of the Englishtown aquifer system can be more fully simulated. The initial heads in the underlying and overlying aquifers are treated, in the model, as a function of the initial head in the Englishtown. This requires that heads in these aquifers be expressed in terms of decline since predevelopment, with respect to decline in the Englishtown, so that changes in leakage since predevelopment are more correctly expressed.

Initial head conditions in the Mount Laurel aquifer are approximations of the drawdown in the aquifer between about 1900 and 1959 as they relate to similar decline in the Englishtown over the same period. Actual head changes in the Mount Laurel from 1900 to 1959 are not well known, but neither they nor the corresponding actual difference between Mount Laurel and Englishtown heads in 1959 can be used directly. Rather, the drawdown differences relative to predevelopment head differences are the appropriate values to be used in calculating steady leakage across the confining layer. Identification of appropriate initial drawdown difference values could not be made from existing data but could be determined by trial and error using 1959 drawdown differences as a guide. The calculated gradient $\Delta h/\ell'$, where Δh is the drawdown difference between the Mount Laurel and Englishtown aquifers, across the upper confining layer varies from 0.17 to 1.7 at the beginning of the simulation period.

Appropriate initial head differences between the Englishtown aquifer and the underlying Magothy-Raritan aquifer in 1959 are more difficult to determine. Sufficient data were not available to evaluate the history of head decline in the Magothy-Raritan and its relationship to the history of head changes in the Englishtown. It is intuitively obvious from the time lag values given in table 4 that the head differences and, therefore, any drawdown differences between the two aquifers in 1959 do not bear a direct relationship to the rate of steady leakage flow across the

Merchantville-Woodbury confining layer in 1959. The long time needed for head changes to be felt across this confining layer (table 4) makes it virtually impossible to define the appropriate values of Magothy-Raritan drawdowns that could be related to the 1959 drawdowns used as the initial condition in the Englishtown aquifer. The problem is circumvented for purposes of the computer analysis by assuming that there was no drawdown difference for the 1900-59 period between the Englishtown and Magothy-Raritan aquifers, implying there has been no change in predevelopment flux between these aquifers. (The implications of this assumption with respect to steady and transient leakage are considered in more detail in the following discussion of the leakage regimen.) The initial drawdown values in the Magothy-Raritan aquifer are, therefore, set in the model equal to those in the Englishtown so that there is no difference between initial drawdown values in the two aquifers.

Aquifer Homogeneity

On a regional scale, the model represents a nonhomogeneous aquifer, as the transmissivity and storage coefficient can vary from node to node. The hydraulic coefficients of the aquifer vary linearly between nodes in the directions of the coordinate axes of the model grid.

Well Discharge Rate

The pumping rate used at each node representing a discharging well or wells is an average rate for the entire period of simulation or pumping. If the actual discharge rate of a pumping well does not change greatly over the simulation period, then averaging the pumping rate over the period will not lead to significant error. In the case of Englishtown, however, the discharge of several wells increased more over the period of analysis than could be averaged over the simulation period for a satisfactory solution. Consequently, the period of analysis was divided into two pumping periods, and the average rate of withdrawal for each period was used.

Confining-layer Coefficients

The hydraulic conductivity and specific storage of each of the confining layers are assumed to remain constant with time. It has been shown (Riley, 1970) that the specific storage of a compressible clayey sediment is approximately given by the coefficient of volume compressibility, as determined from clay consolidation tests in the laboratory. The hydraulic conductivity also can be calculated with data obtained from these tests. Both parameters, specific storage (coefficient of volume compressibility) and hydraulic conductivity, decrease rapidly with decreasing void ratio during drainage and consolidation, but the ratio of K/S_s is fairly constant (Terzaghi and Peck, 1967, p. 86). As a result, the equations that use the hydraulic diffusivity will give solutions that are correct, at least theoretically. Those equations that use only a value of hydraulic conductivity, specifically the equations used to calculate steady and transient leakage rates, will give solutions that are somewhat in error during the later part of the simulation period. However, the error is probably small, as the values of the confining-layer parameters used in the simulation model are nearly the same as the average values obtained from laboratory consolidation tests.

Leakage Regimen

The regional analysis of steady and transient leakage in the Englishtown aquifer system previously presented provides a conceptual model of the leakage regimen that can be incorporated in the simulation model. Two components of leakage must be considered, steady flow through the confining layer and transient flow from the confining layer. These elements have been considered separately in the regional analysis and can, with some simplifying assumptions, be included in a simulation of the aquifer system.

Consider first steady leakage through the adjacent confining layers. It has been assumed that there is an initial difference between the adjusted drawdowns in the Mount Laurel and Englishtown aquifers. The steady flux from the Mount Laurel at the beginning of the simulation period approximates the difference between actual 1900 fluxes and actual 1959 fluxes. The initial drawdown difference, which has a gradient $\Delta h/\ell'$ ranging from 0.17 to 1.7 across the upper confining bed, is maintained from the beginning of the simulation period until new drawdown changes in the Englishtown can be transmitted through the confining bed. This is an attempt to simulate the lag time during which transient leakage is occurring (table 4). The time during which the constant gradient is maintained is determined by the confining layer parameters or by drawdown in the Englishtown, Δh_E, and is equal to $K't/S'_s\ell'^2 = 2 \times 10^{-1}$ or

$$\Delta h_E = .75\ \ell' \text{ for } \ell' \leq 30 \text{ ft or}$$

$$\Delta h_E = 10 \text{ ft for } \ell' > 30 \text{ ft},$$

whichever comes first. At the new time, t, or when the drawdown in the Englishtown satisfies the above criteria, a new gradient is established across the upper confining layer that ranges from 0.27 to 2.45 and averages 0.83. This gradient, and the flux from the Mount Laurel, remains constant for the rest of the simulation period. The change in steady flux is assumed to approximate the changes in the difference between actual steady flux components during the 1959-70 period. These changes occur constantly over the period of analysis as drawdown increases in the Englishtown aquifer and new head changes are transmitted slowly across the overlying confining layer (table 4). The above described method of computing steady flux from the Mount Laurel aquifer is an attempt to simulate the complex changes in drawdown differences and interactions that affect steady leakage between the Mount Laurel and Englishtown aquifers. At the same time, as will be described, provision is also made to simulate transient leakage from the overlying confining layer.

The assumption that there is no drawdown difference, for the 1900-59 period, between the Englishtown and Magothy-Raritan aquifers implies that the rate of steady flux into and out of each of the aquifers in 1959 is everywhere equal to the predevelopment flux between the two aquifers. This condition is maintained during the entire period of simulation analysis. The assumption and its underlying implication, while not consistent with the assumptions made in the analytical analysis and perhaps not entirely valid, is based on the long lag time required to transmit head changes across the thick lower confining layer (table 4). It is believed to be a not unreasonable approximation of changes in the steady flux component of leakage across the lower confining bed in 1959 relative to predevelopment. The total leakage flux into the Englishtown has changed since predevelopment, but it is believed to be largely the result of changes in the transient component of leakage as will be described.

The second element of leakage that must be considered is the transient component; the flux from the confining layer in response to head changes in the adjacent aquifer. Drawdown change response time across the upper bed is small enough (table 4) that the magnitude of transient fluxes in 1959 are assumed to be insignificant compared to transient fluxes caused by drawdown in the Englishtown during the early part of the simulation period. Consequently, only transient leakage in response to drawdown increases in the Englishtown during the 12-year period of analysis need be considered. This simplification still leaves computational problems involved in the computer simulation techniques. The generally short response time across the upper confining bed and the nearly simultaneous continuous decline changes in the Englishtown and Mount Laurel aquifers are complications that require additional simplification. Transient flux from the overlying confining layer is, therefore, computed only during the time when simulated drawdown differences between the two aquifers are not changing. This time interval extends from time $t = 0$ until $K't/S'_s \ell'^2 = 2 \times 10^{-1}$,

or until drawdown in the Englishtown, Δh_E, is such that

$$\Delta h_E = .75\ \ell' \text{ for } \ell' \leq 30 \text{ feet or}$$

$$\Delta h_E = 10 \text{ feet for } \ell' > 30 \text{ feet}$$

whichever occurs first. The drawdown in the Englishtown Δh_E together with the initial drawdown differences establishes a new gradient across the upper confining layer that ranges from .27 to 2.45 and is given by

$$\Delta h = 27 \text{ feet for } \ell' > 30 \text{ feet or}$$

$$\Delta h = 17 \text{ feet} + .75\ \ell' \text{ for } \ell' \leq 30 \text{ feet}$$

where Δh is the total drawdown difference between the Mount Laurel and Englishtown aquifers. After the time defined by these limitations, drawdowns in the Englishtown and Mount Laurel are assumed to occur concurrently with equal magnitude and only the steady flux is computed; transient fluxes from the confining layer are no longer calculated. This is an oversimplification involving some inconsistency with existing theory, and it is believed to result in a lower estimate of the volume of transient leakage released from the overlying confining layer than is probably the case. The result is a more conservative estimate of leakage from and through the overlying confining bed because the steady flux component represents the minimum rate of leakage.

The simulation of transient flux from the underlying Merchantville-Woodbury confining layer presents fewer problems, but some simplification is still required. This component of leakage into the Englishtown has been assumed to be the only change in leakage conditions from below with respect to predevelopment. The rate of transient flow into the Englishtown at the beginning of and immediately before 1959 is not known. However, the significant factor when considering transient discharge from a confining layer is the magnitude of head change at the aquifer-confining layer interface and the time elapsed since the head change occurred, because the highest flow rates occur during the early time of transient conditions.

The history of head changes in the Englishtown aquifer before 1959 is poorly known. However, it is known that the declines from 1959 to 1970 are equal to, and in some cases greater than, the declines from about 1900 to 1959. It is possible that the greatest percentage of transient discharge into the Englishtown caused by 1900-59 head declines had already occurred by 1959. The actual transient flux in 1959 caused by pre-1959 drawdowns is not known, but could be as much as 100 times smaller than that caused by drawdown changes in the early part of the 1959-70 period (fig. 22). Consequently, it is assumed that initial transient flux from the lower confining layer is negligible and it is not included in the analysis. Only transient flux caused by declines from 1959 to 1970 are considered.

Calibration of the Englishtown Simulation Model

The simulation model of the Englishtown aquifer developed gradually over a 2-year period from a simple model with uncontrolled steady leakage to a more complex, quasi-multilayered aquifer model with controlled steady leakage. There were three main phases of model development; each phase produced a calibrated model that more closely simulated the aquifer system. Each model is discussed briefly, but the results of only the third model are discussed in detail.

Prototype Model

The initial phase of model construction started shortly after the study began. In this way, it was possible to evaluate the adequacy and reliability of available data early in the investigation and to begin testing alternate concepts of the hydrologic regimen. About four months were spent in assembling data then on hand. Two months of this time were required to assemble and interpret geophysical log data and to define aquifer geometry. The period 1959-64 was selected as the base period of historical record for calibration. Several basic-data reports were available that identified all significant pumping wells for the period, and a well inventory was not required. The years 1959 and 1964 were the only ones for which enough water-level data were available to base even a tentative calibration. Within 7 months, a tentatively calibrated prototype model of the Englishtown aquifer had been developed.

The prototype model was a simplistic representation of the hydrologic system. The significance of leakage was not known, and the first version of the prototype did not include recharge. The nonleaky model computed decline at pumping nodes that significantly exceeded the field decline after 6 years of actual pumping and strongly suggested that leakage was significant. Subsequent versions of the prototype model included steady leakage through an overlying confining bed. Various uniform average values of confining bed thickness were used, but a satisfactory solution was not obtained until areally variable thickness values, corresponding to actual confining layer thickness, were used in the model. The confining layer hydraulic conductivity used in the model was 3.024×10^{-5} ft/d; this value was obtained by trial and error.

The initial estimated transmissivity distribution used in the prototype model produced a regional pattern of water-level decline caused by pumping that closely approximated field observations. The distribution was based on a constant average hydraulic conductivity of 20 ft/d, somewhat larger than supported by field data. Changes in this distribution lead to solutions significantly different from the observed regional pattern of water-level decline. These results strongly suggested that the initial estimates of transmissivity were satisfactory for the purpose of developing a regional simulation model of the Englishtown aquifer.

The only other hydraulic parameter used in the prototype model was the aquifer storage coefficient. The only available field data suggested an average storage coefficient of about 3×10^{-4}. This value was used in the initial version of the prototype and was maintained as an areally constant parameter, even though the storage coefficient probably varied as much as half an order of magnitude over the study area. However, a satisfactory solution was not obtained until the storage coefficient was increased to 3×10^{-3}, or one order of magnitude greater than suggested by field data. The solution using this value was then accepted as a tentatively calibrated simulation model of the Englishtown aquifer until the results could be more completely evaluated and additional data could be collected.

The results of the prototype model provided considerable insight into the mechanics of the aquifer system and specifically indicated the additional data needed. First, the results strongly suggested that recharge from vertical leakage is a significant factor in the response of the aquifer to pumping. More particularly, the solution was especially sensitive to the rate and volume of leakage through the overlying confining bed, and satisfactory results could not be achieved by using areally constant average thickness values for this confining layer, nor could they be achieved by using hydraulic conductivity values as little as half an order of magnitude larger or smaller than the value of 3.024×10^{-5} ft/d that was used.

The need to increase the aquifer storage coefficient from 3×10^{-4} to 3×10^{-3} is an additional consequence of the need to include a more accurate simulation of leakage. The larger value of the storage coefficient was, in effect, simulating the transient component of leakage, particularly from the lower confining bed, which was not explicitly included in the model. Furthermore, the relative importance of the aquifer as a source of stored water is suggested by the magnitude of the increase in the storage coefficient, which represents the gross average storage coefficient of the aquifer and both confining layers. In the prototype model, the component of the storage coefficient that represents storage in the confining beds is roughly nine times larger than the component representing storage in the aquifer.

The data requirements for further development and refinement of the simulation model were fairly well defined by the evaluation of the prototype model results, and several changes were made in the data-collection program developed during project planning. The most obvious deficiency in the prototype was the manner in which transient leakage was simulated. A more accurate method of including this quantity in the model would be to solve equation 14 at each node of the grid. This approach, however, requires the availability of hydraulic conductivity and specific storage values for each confining layer. As these parameters can be calculated easily from soil consolidation test data, undisturbed core samples were collected at three localities. These samples, the collection of which had not been included in the earlier data-collection program, provided sufficient data for continued development of the model. The results achieved with the initial estimate of transmissivity distribution strongly suggested that the investigation would not have benefited materially by the collection of detailed aquifer test data at selected localities. Planned aquifer tests were postponed and eventually abandoned.

Phase 2 Model

The second phase of model development was concerned mainly with making major modifications of the computer program so that transient leakage could be more adequately simulated. The base period for calibration was extended to cover 1959 through 1970. Values of confining layer hydraulic conductivity and specific storage were incorporated as they became available. The accuracy of the model, which now included transient and steady leakage from two confining beds, was tested by comparing the model results for a hypothetical aquifer with analytical solutions obtained with Hantush's modified leaky aquifer equations (Hantush, 1960, Case II). The results of this analysis are shown in figure 26.

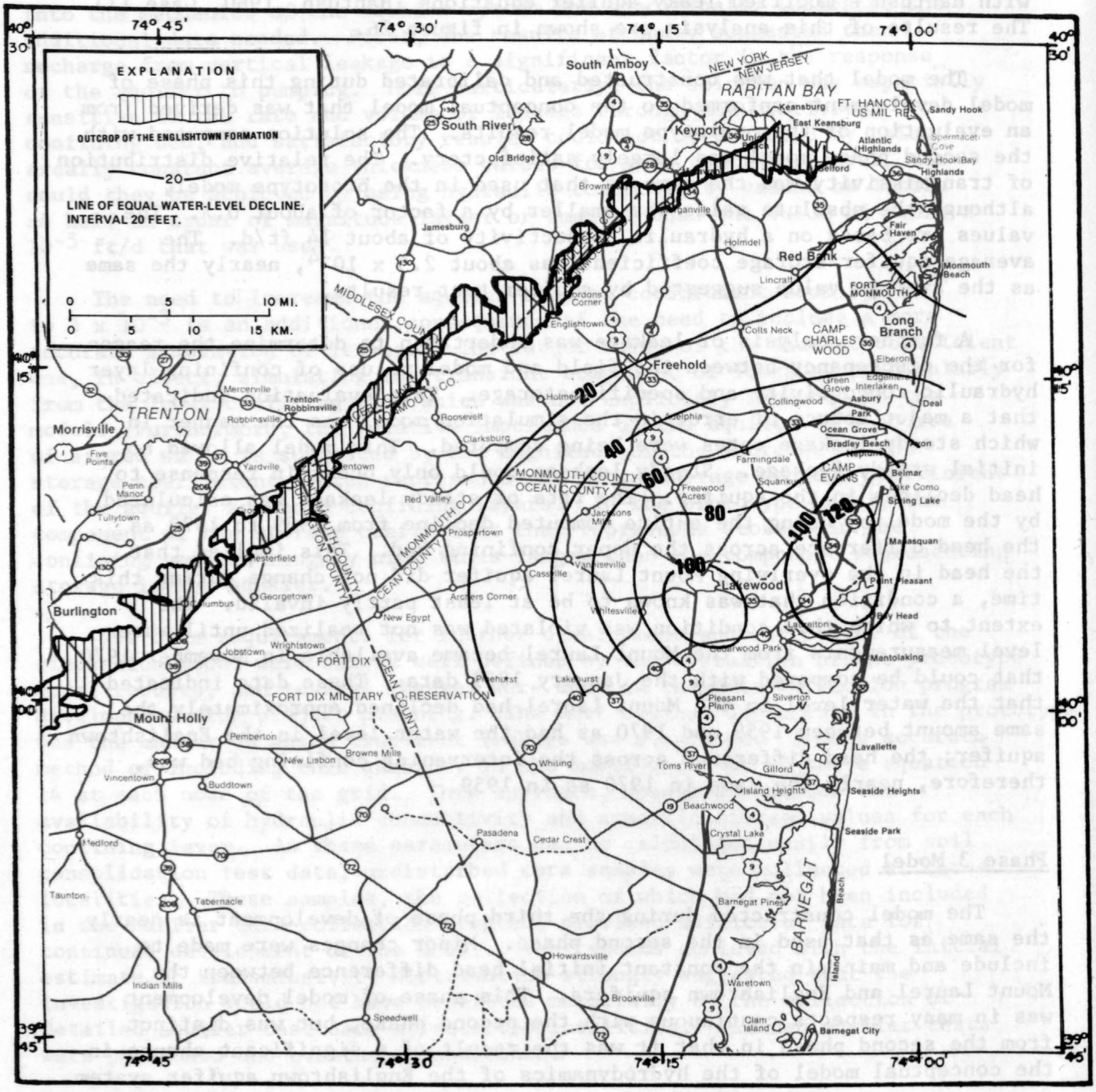
The model that was constructed and calibrated during this phase of model development conformed to the conceptual model that was derived from an evaluation of the prototype model results. The solution computed with the second phase model was largely satisfactory. The relative distribution of transmissivity was the same as that used in the prototype model, although the absolute value was smaller by a factor of about 0.7. The values are based on a hydraulic conductivity of about 14 ft/d. The average aquifer storage coefficient was about 2.5×10^{-4}, nearly the same as the average value suggested by aquifer test results.

A further analysis of leakage was undertaken to determine the reason for the discrepancy between the field and model values of confining layer hydraulic conductivity and specific storage. This evaluation indicated that a major source of error in the simulation model was the manner in which steady leakage rates were being computed. This model allowed no initial steady leakage. Steady leakage could only occur in response to head declines in the aquifer. The rate of steady leakage was calculated by the model by using the entire computed decline from 1959 to 1970 as the head difference across the upper confining bed. This implied that the head in the overlying Mount Laurel aquifer did not change during this time, a condition that was known to be at least partly invalid. The extent to which this condition was violated was not realized until water-level measurements from the Mount Laurel became available in November 1970 that could be compared with the January 1959 data. These data indicated that the water level in the Mount Laurel had declined approximately the same amount between 1959 and 1970 as had the water level in the Englishtown aquifer; the head difference across the intervening confining bed was, therefore, nearly the same in 1970 as in 1959.

Phase 3 Model

The model constructed during the third phase of development is nearly the same as that used in the second phase. Minor changes were made to include and maintain the constant initial head difference between the Mount Laurel and Englishtown aquifers. This phase of model development was in many respects continuous with the second phase, but was distinct from the second phase in that it was the result of a significant change in the conceptual model of the hydrodynamics of the Englishtown aquifer system.

The water-level decline computed by this version of the simulation model for the period 1959-70 is shown in figure 28. A similar solution was obtained with the model configuration used in the second phase of development. The significant differences between the two models are the values of confining layer specific storage used in each model and the simulated hydrodynamics of the system. The third version of the model uses



Base from Army Map Services, Newark, N.J., 1964 and Wilmington, Del., NJ-PA-MD 1966, 1:250000.
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FIGURE 28. —WATER-LEVEL DECLINE IN THE ENGLISHTOWN AQUIFER FOR THE PERIOD 1959-70 COMPUTED BY THE AQUIFER SIMULATION MODEL.

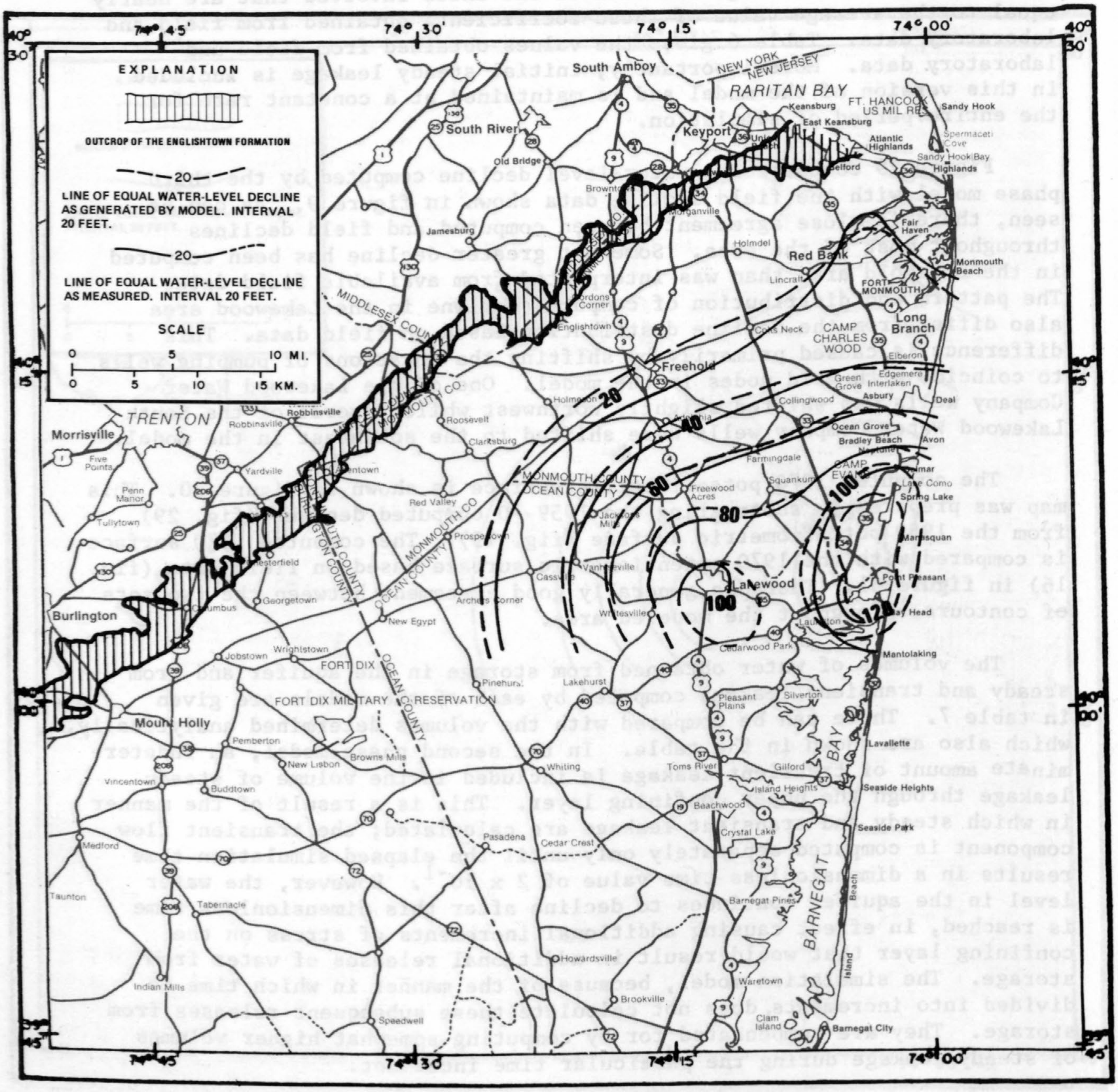
values of specific storage for each of the units involved that are nearly equal to the average value of these coefficients obtained from field and laboratory data. Table 6 gives the values obtained from field and laboratory data. More importantly, initial steady leakage is included in this version of the model and is maintained at a constant rate for the entire period of simulation.

Figure 29 compares the water-level decline computed by the third phase model with the field decline data shown in figure 9. As can be seen, there is close agreement between computed and field declines throughout most of the area. Somewhat greater decline has been computed in the Freehold area than was interpreted from available field data. The pattern and distribution of computed decline in the Lakewood area also differ from the decline distribution based on field data. This difference is caused primarily by shifting the locations of pumping wells to coincide with grid nodes in the model. One of the Lakewood Water Company wells was shifted slightly northwest while several of the South Lakewood Water Company wells were shifted to the southeast in the model.

The computed 1970 potentiometric surface is shown in figure 30. This map was prepared by subtracting the 1959-70 computed decline (fig. 29) from the 1959 potentiometric surface (fig. 15). The computed 1970 surface is compared with the 1970 potentiometric surface based on field data (fig. 16) in figure 31. There is generally good agreement between the two sets of contours throughout the modeled area.

The volumes of water obtained from storage in the aquifer and from steady and transient leakage computed by each of the models are given in table 7. These can be compared with the volumes determined analytically, which also are shown in the table. In the second phase model, an indeterminate amount of transient leakage is included in the volume of steady leakage through the upper confining layer. This is a result of the manner in which steady and transient leakage are calculated; the transient flow component is computed separately only until the elapsed simulation time results in a dimensionless time value of 2×10^{-1}. However, the water level in the aquifer continues to decline after this dimensionless time is reached, in effect causing additional increments of stress on the confining layer that would result in additional releases of water from storage. The simulation model, because of the manner in which time is divided into increments, does not calculate these subsequent releases from storage. They are compensated for by computing somewhat higher volumes of steady leakage during the particular time increment.

The third phase model handles leakage in a different manner. Transient leakage, as before, is computed until the elapsed simulation time reaches dimensionless time 2×10^{-1}. In this model, however, the rate of steady leakage is held constant in the manner described in the discussion of the leakage regimen used in the model, and additional releases from storage are not included. A better comparison of the analytically and numerically computed flux from and through the upper confining layer can be obtained by combining the two components given



Base from Army Map Services, Newark, N.J., 1964 and Wilmington, Del., NJ-PA-MD 1966, 1:250000.
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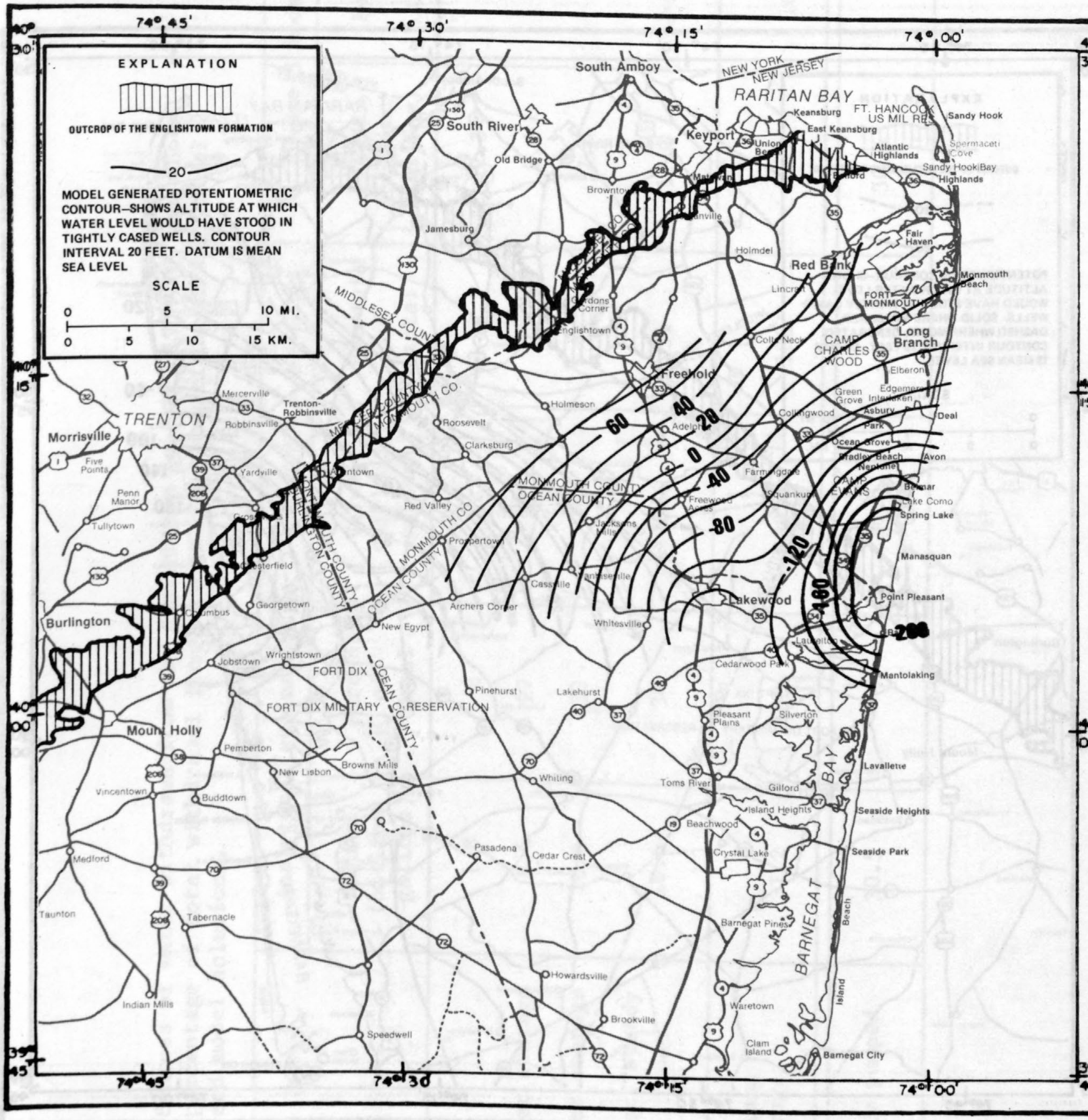
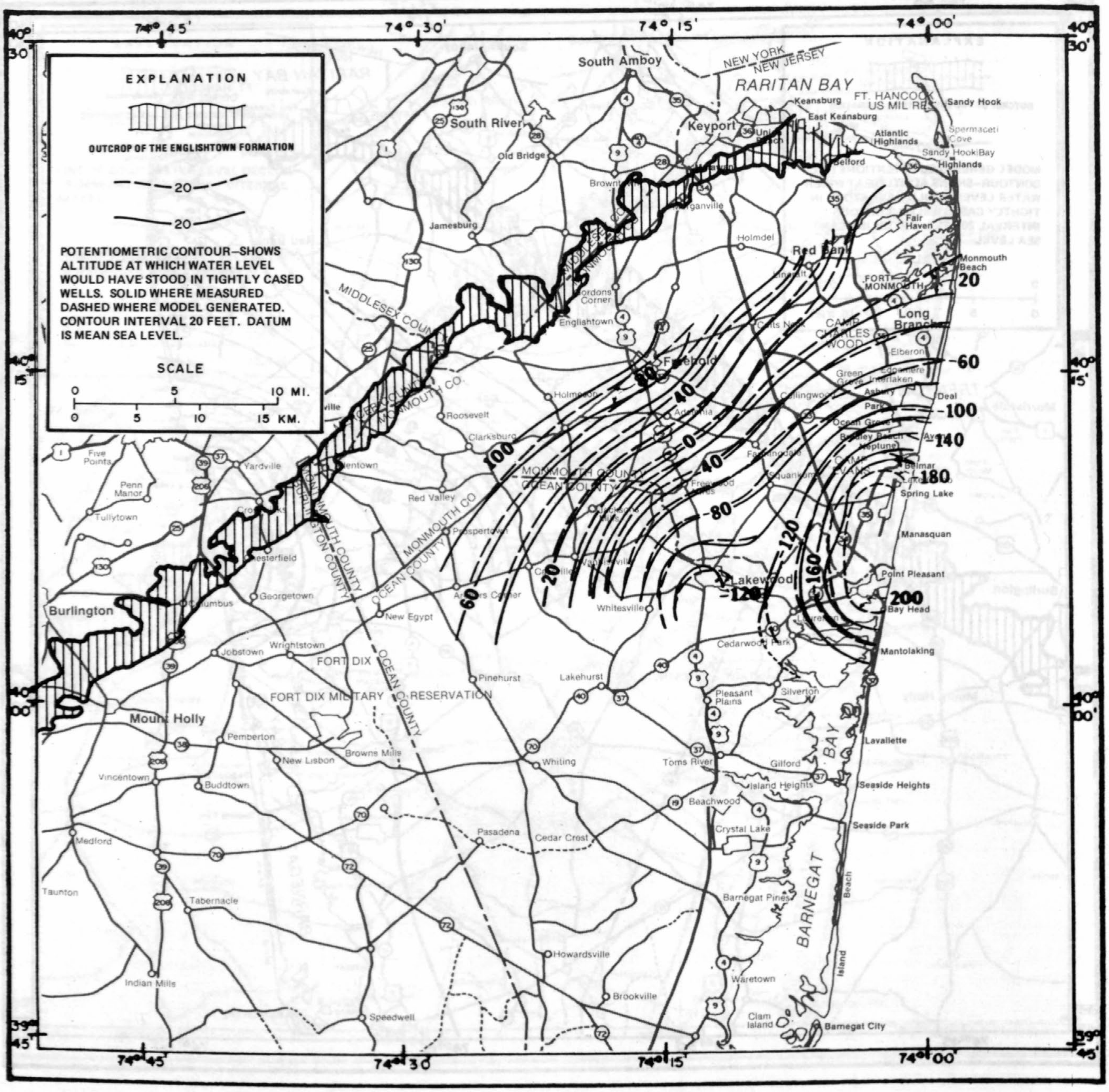
FIGURE 29. —COMPUTED AND MEASURED WATER-LEVEL DECLINES IN THE ENGLISHTOWN AQUIFER FOR THE PERIOD 1959-70.


EXPLANATION
OUTCROP OF THE ENGLISHTOWN FORMATION
20
MODEL GENERATED POTENTIOMETRIC CONTOUR–SHOWS ALTITUDE AT WHICH WATER LEVEL WOULD HAVE STOOD IN TIGHTLY CASED WELLS. CONTOUR INTERVAL 20 FEET. DATUM IS MEAN SEA LEVEL
SCALE
0 5 10 MI.
0 5 10 15 KM.
74° 45'
74° 30'
74° 15'
74° 00'
40° 30'
40° 15'
40° 00'
39° 45'
RARITAN BAY
NEW YORK
NEW JERSEY
South Amboy
Keyport
Red Bank
Freehold
Lakewood
Long Branch
TRENTON
Burlington
Mount Holly
FORT DIX MILITARY RESERVATION
MIDDLESEX COUNTY
MONMOUTH COUNTY
OCEAN COUNTY
BARNEGAT BAY
60
40
20
0
-40
-80
-120
-160
-200



Base from Army Map Services, Newark, N.J., 1964 and Wilmington, Del., NJ-PA-MD 1966, 1:250000.
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FIGURE 31. —ALTITUDE OF THE COMPUTED AND MEASURED POTENTIOMETRIC SURFACE FOR 1970 IN THE ENGLISHTOWN AQUIFER.

Table 7.--Sources of water withdrawn from the Englishtown aquifer, 1959-70, and the percentage of total withdrawal from each source--comparison of analytical and model solutions.

Source	Analytical evaluation		Phase 2 model		Phase 3 model	
	Volume (billion gallons)	Percent of total	Volume (billion gallons)	Percent of total	Volume (billion gallons)	Percent of total
Aquifer storage	1.8	6	2.5	8	2.3	7
Lower confining layer storage	8.4	27	5	17	8.6	28
Upper confining layer storage	10.4	33	7.4	24	2.0	7
Upper confining layer, steady leakage	10.6	34	15.6	51	17.6	58
Total calculated volume released from storage	31.2	100	30.5	100	30.5	100
Total volume actually pumped	30.5		30.5		30.5	

in table 7. The analytical solutions show a combined total of 21.0 billion gallons of steady and transient leakage from the overlying confining layer; this amounts to 67 percent of the total calculated pumpage. The second phase model computed a total of 23 billion gallons of leakage from above--75 percent of the total withdrawals. The third phase model computed 19.6 billion gallons of steady and transient leakage from the upper confining bed, which represents 65 percent of the total withdrawal for the period of the analysis. The third phase model also computes a closer approximation of the analytically estimated volume of transient leakage from the lower confining layer than does the second phase model (table 7). This general correspondence strongly suggests that reasonable estimates of the regional volume of interaquifer leakage can be determined analytically using regional average values of limiting parameters as opposed to constructing a simulation model with numerous point and time segments.

The second- and third-phase models demonstrate the care that must be taken in simulating the hydrodynamic configuration of an aquifer system. The 12-year declines computed by each model agreed closely with the declines observed in the field, but the main purpose of the simulation model is not simply to reproduce some period of historical record; rather its principal use is in making reasonably reliable predictions of future declines. The second phase model, although "calibrated" for the 1959-70 test period, is not a particularly good simulation of the hydrodynamics of the Englishtown aquifer system. Once equilibrium conditions are established through the upper confining layer, this model calculates steady leakage rates by using the total decline computed for each time increment of the simulation period as the head difference across the upper confining bed. The rate and volume of steady leakage, therefore, increases from one time increment to the next as the head declines, and, as a result, smaller contributions of water from storage in the lower confining bed are required to assist in maintaining the water level in the aquifer. Consequently, the specific storage of the lower confining layer used in the second phase model is lower than suggested by field data.

The third-phase model sets an upper limit on the rate of steady leakage from the overlying Mount Laurel aquifer. The actual head gradient across the upper confining bed varies throughout the model but maintains an average value of $\Delta h/\ell = 0.83$. The rate of steady leakage in this model does not increase as the head continues to decline. The limited volume of steady leakage requires a greater volume of water to be released from storage in the lower confining layer to support the withdrawals from the aquifer. As a result of the differences in the simulated hydrodynamics of the two models, the third phase model predicts smaller declines for periods of less than about 12 years and greater declines for periods of more than 12 years than the second phase model. For example, the 1959-70 pumpage rates were used in both models to predict the decline caused by this pumping regimen by 1980, a period of 22 years. The declines computed by the third phase model were from 7 to 10 percent greater than those computed by the second phase model.

CONCLUSIONS

Digital simulation modeling of aquifers is both a means and an end for the analysis and evaluation of a complex ground-water system. The technique provides an excellent vehicle for the investigator to determine the adequacy of existing data, to identify data deficiencies, and to define areas where new or additional data are required. For this purpose, construction and development of a simulation model based on the currently held conceptual model of the system's hydrodynamic characteristics should take place early in the study. Every attempt should be made, at all stages of model development, to use system coefficients and parameters that are substantiated by field data; nevertheless, an approximate solution using hypothetical data that may be erroneous can give the investigator valuable insight into the hydrodynamics of the aquifer. In the case of the Englishtown aquifer study, a working simulation model had been completed within seven months of the start of the investigation. The prototype Englishtown model was developed with only immediately available data. The model at this stage was a fairly crude simulation of the aquifer. The parameters used in some parts of the aquifer were chosen primarily to achieve the best possible match to observed aquifer response and were, in many cases, unsubstantiated by measurement in the field. Evaluation of the results of the prototype model and of the data used to obtain these results led to a modification of the conceptual model of the mechanics of the aquifer system. This, in turn, largely determined the data requirements for further model development.

Refinement of the initial model can proceed as additional or new data become available or as new concepts evolve. A digital computer model is extremely flexible and well adapted to this kind of refinement. Aquifer system mechanics are modified easily by the change of perhaps only a few computer program statements. Aquifer system data can be quickly changed or modified. Frequently, an idea can be tested by changing only a single input parameter that is then used by the computer to change an entire set of data.

As a model becomes more refined and complex, greater care should be taken to use system coefficient and parameter values that are supported by field and laboratory values, and mathematical equations and techniques that are theoretically sound. It may not always be possible to satisfy these requirements, as a model, after all, can only approximate a geohydrologic system. Nevertheless, the investigator will generally have more confidence in model predictions when the model uses data values that closely approximate field values. When it becomes necessary to depart from field or laboratory data values, the investigator must be certain of the reasons for the departure and must be fully aware of the significance of the change and the effect on the results. Again, the development of the Englishtown simulation model provides an example that demonstrates the care that must be taken in selecting system coefficients. Confining layer coefficients used in the second phase model were approximations of the values obtained from laboratory test data, yet the model results after 12 years of simulated pumping were satisfactory. A more detailed analysis of the leakage regimen of the aquifer system caused significant changes in the concept of leakage in the system and resulted in the development of the third phase model, which yielded substantially the same solution. This of course, does not prove that predictions made with the third phase model are more accurate and reliable than predictions made with earlier versions. It does suggest, however, that more confidence may be placed in predictions made by the third phase model, as this version more closely approximates the hydrodynamic characteristics of the aquifer system than any of the other models. It also demonstrates the viability of analytic analysis of interaquifer leakage using regional average values of limiting parameters.

The analytical estimate of transient and steady leakage is one of the most significant elements of the investigation. It demonstrates the dependence of withdrawals from the Englishtown aquifer on leakage from the overlying Mount Laurel aquifer, even though the leakage is ultimately derived from the confining beds above the Mount Laurel and recharge from the Mount Laurel Formation outcrop area. Increased withdrawal from the Englishtown will cause increased declines of water level in the aquifer and induce higher rates and volumes of vertical leakage, particularly across the confining layer separating the Englishtown from the Mount Laurel aquifers. The increased leakage from the Mount Laurel will, in turn, cause increased declines of the water level in this aquifer, which then will induce additional releases of stored water from the confining beds above, and more recharge from the outcrop area of the Mount Laurel. In the same manner, increased withdrawal from the Mount Laurel aquifer will affect the rate and magnitude of water-level decline in the Englishtown aquifer. Any decrease in head in the Mount Laurel in response to pumping from that aquifer will cause a decrease or even reversal of the head difference between the two aquifers. The decrease in head difference will, in turn, cause a reduction in the rate and volume of leakage into the Englishtown; leakage that is now supporting a large part of the withdrawal from the aquifer. As a result, there will be an increase in the rate of water-level decline in the Englishtown aquifer, even with no increase in direct withdrawal.

The Englishtown aquifer and adjacent confining layers are an integral part of the ground-water system of the New Jersey Coastal Plain. Experience has shown that the response of the aquifer to pumping stresses must be viewed not as an isolated entity but one that extends beyond the vertical limits of the aquifer being evaluated. Thus, the response of the Englishtown must be evaluated in the context of the hydrodynamics of the large coastal plain aquifer system. It follows, therefore, that future modeling efforts should be directed toward the development of a multilayered model of the coastal plain that can simulate the varied response of each of the aquifers simultaneously. With such a calibrated model, the yield potential of the total aquifer system, and the consequence of alternative schemes of development can best be predicted.

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