Application of Transient-Flow Model to the Sacramento River at Sacramento, California

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UNITED STATES DEPARTMENT OF THE INTERIOR

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CONVERSION FACTORS

For readers who may prefer to use metric units (International System of Units) rather than inch-pound units, the conversion factors for the terms used in this report are listed below.

Inch-pound unit	Multiply by	Metric (SI) unit
ft (foot)	0.3048	m (meter)
ft^3/s (cubic foot per second)	0.02832	m^3/s (cubic meter per second)
mi (mile)	1.609	km (kilometer)
mi^2 (square mile)	2.590	km^2 (square kilometer)

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ABSTRACT

The transient-flow simulation model was applied to the Sacramento River at Sacramento, Calif. An alternative is needed to the empirical stage-fall-discharge computational method, which provides only daily mean discharges and requires frequent measurement over a 25- to 26-hour period of the tide-affected flow. Measurement of tide-affected flow involves use of a cable suspended across the river; this presents a hazardous navigational condition. Use of the model does not eliminate the hazardous navigational problem, but it significantly reduces the time that this condition exists.

The model has demonstrated that it can provide reliable daily mean as well as instantaneous discharge data. The model also has the capability of providing discharge data for locations farther downstream in the tidal zone from the Sacramento stream gage; this is not possible with the stage-fall-discharge computational method.

INTRODUCTION

Location and General Information

Hydrologic data for the Sacramento River have been collected at or in the vicinity of the stream-gaging station at Sacramento, Calif., since 1879. The stream gage is approximately 70 mi northeast of San Francisco, and the drainage area at the gage is 23,508 mi^2. It is the last streamflow gage on the Sacramento River before it discharges into the San Francisco Bay system. For the period of continuous streamflow record (1949-76) the daily mean discharge at the gage has ranged from 5,590 ft^3/s to 99,400 ft^3/s and has averaged 24,330 ft^3/s. Stage-discharge relations for streamflows less than about 40,000 ft^3/s are affected by the tide.

The gage is a Federal-State cooperative station, operated by the U.S. Geological Survey for the purposes of flood forecasting and water inventory. Data collected at the site are used by Federal agencies, the State of California, and the Sacramento community.

Description of Problem

Because the flow of the Sacramento River at Sacramento is tide affected for flows of less than 40,000 ft^3/s, a conventional stage-discharge relation cannot be used to compute the discharge for these unsteady-flow conditions that exist about 80 percent of the time. The daily mean discharge for unsteady-flow periods has been computed by using the empirical stage-fall-discharge method (R. W. Carter and Jacob Davidian, U.S. Geological Survey, written commun., 1965), where the fall or slope of the water surface is determined by using a 10.8-mi reach of the river. A conventional stage-discharge relation has been used for the periods not affected by the tide.

The stage-fall-discharge computational method provides a reliable daily mean discharge, but it is not reliable with respect to instantaneous discharge, especially for flows less than 14,000 ft^3/s, which prevail about 40 percent of the time. Also, the method requires frequent measurement of the discharge. Because the flow is tide affected, tidal-cycle discharge measurements are required to define the stage-fall-discharge relation. A tidal-cycle discharge measurement consists of about 20 conventional discharge measurements made over a 25- to 26-hour period from which a daily mean discharge is computed. The 20 discharge measurements are made from a boat attached to a cable that moves the boat from section to section across the 650-foot-wide channel. The cable spans the river about 20 ft above the water surface, thereby allowing small boats to travel the river without obstruction. In order for larger vessels to pass the measuring site, however, the cable must be dropped to the river bottom. This suspended cable constitutes a hazardous navigational situation, especially after sundown.

Purpose and Scope

The U. S. Geological Survey, in cooperation with the California Department of Water Resources, undertook this study to evaluate the feasibility of computing the discharge of the Sacramento River at the Sacramento stream gage by using a digital-computer transient-flow simulation model.

The scope of the study included (1) collecting and processing input data and (2) using the data in conjunction with discharge-measurement data to calibrate a flow-simulation model for a reach of the Sacramento River.

DESCRIPTION OF RIVER REACH

The reach of the Sacramento River to which the flow-simulation model was applied extends 10.8 mi downstream from the stream gage at Sacramento (station 11447500) to the auxiliary stage gage near Freeport, Calif. (fig. 1). The river channel in this reach has a mild, uniform slope and is not very sinuous. Also, it is channelized throughout the reach with large areas of the levied river banks riprapped. The nonriprapped areas are fairly free of flow-resisting vegetation. The river bed is composed predominately of sand-size material and has 3- to 5-ft dunes. At low flows (10,000 to 16,000 ft^3/s) the water depth above the crests of the dunes is about 15 ft.

The fall in the water surface through the reach ranges from about 4.9 ft at 90,000 ft^3/s to about a negative 0.2 ft at 4,000 ft^3/s. No reversal of streamflow because of tidal inflow was observed at the Sacramento gage during the period of gage operation, but the flow at the downstream end of the reach approached reversal when the flow at the upstream end of the reach is about 4,000 ft^3/s. Because of tidal effects, the maximum difference between high and low water stage at the upstream end of the reach, during a day, is 2.5 ft. This 2.5-ft difference in stage results in a difference between maximum and minimum discharge of 6,000 ft^3/s for a day which has a daily mean discharge of 7,000 ft^3/s.

Throughout the reach, numerous small irrigation pumps extract water from the river and some small drains discharge water into the river, but the net effect upon stream discharge is believed to be insignificant. A set of navigational locks connecting the Sacramento River and the Port of Sacramento deepwater ship channel, is located 2.1 mi downstream from the Sacramento stream gage. The operation of the locks periodically affects the Sacramento River at the Sacramento gage, but not significantly.

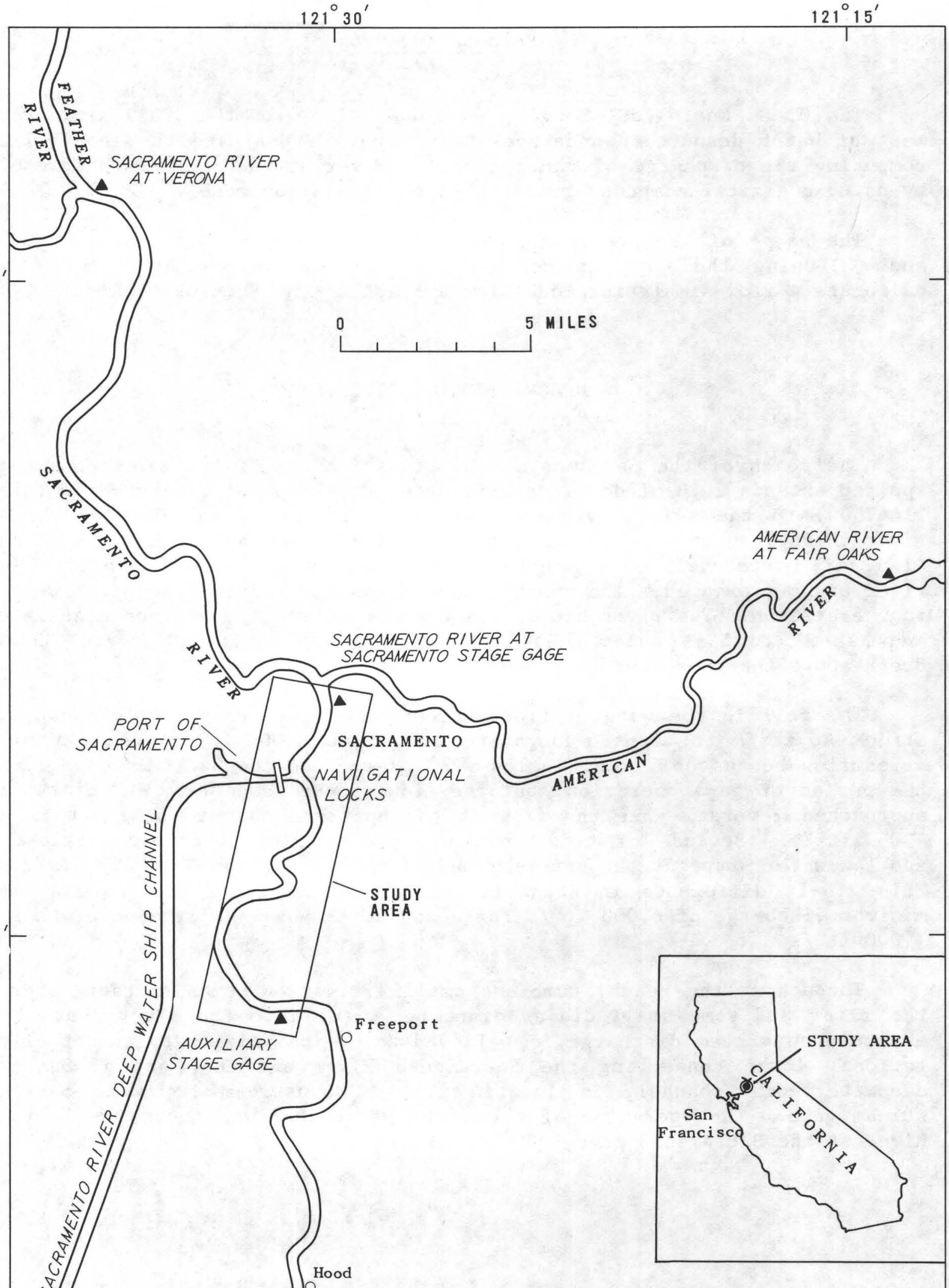

FIGURE 1.--Location of Sacramento River study area.

DESCRIPTION OF MODEL

The flow-simulation model that was applied to the Sacramento River was the multiple-reach method-of-characteristics digital-computer transient-flow simulation model developed by Lai (1967). This one-dimensional flow-simulation model is based on the equations of continuity (conservation of mass) and motion (conservation of momentum). These two partial-differential equations are transformed into characteristic equations and are solved by using finite-difference approximations with a specified time interval (Lai, 1967). The assumptions used in the derivation of this numerical model are: (1) The velocity is uniform in any given channel cross section; (2) the water in the channel is substantially of homogeneous density; (3) the channel slope is mild and uniform over the reach; (4) the reach does not have a high degree of sinuosity; (5) the reach geometry is relatively simple; and (6) the flow-resistance coefficient, eta, used with unsteady flow is the same as that for steady flow.

As the model name implies, the model can be applied to a river reach which is subdivided because of significant variation throughout the reach with respect to channel geometry or the flow-resistance coefficient. Because it was not necessary to subdivide the study reach of the Sacramento River, the model was applied to a single reach and not used as a multiple-reach model.

A further explanation of the unsteady-flow equations and the method-of-characteristics numerical method has been abstracted from Lai and Onions (1976) and can be found in the section "Mathematical Equations and Computation Procedures," at the end of this report. The primary difference between the model used on the Sacramento River (version 27) and that described by Lai and Onions (1976) (version 13) is in the treatment of the flow-resistance coefficient, eta. Version 13 treats eta as a constant, but version 27 has the capability of varying eta either as a linear or quadratic function of discharge.

A flow-simulation model based on a power-series solution of the unsteady-flow equations was also investigated. The power-series model had previously been used on this reach of the Sacramento River in a limited test (9,000 to 19,000 ft^3/s) by Baltzer and Shen (1961, 1964). Preliminary computations using both the multiple-reach method-of-characteristics solution and the power-series solution indicated that the flow-resistance coefficient varied throughout the range of discharge and could not be treated as a constant. Only the method-of-characteristics model was modified to accommodate a variable eta; the power-series model was not modified and was therefore not tested beyond the preliminary computations.

The model has eight output options. The options include stage and discharge hydrographs, computed versus observed stage and discharge hydrographs, 15-minute discharge or daily mean discharge tables, and a summation of positive and negative discharge for each day. The output options can be computed for either end of the reach or for the junction of two segment lengths as stated on page 18 and illustrated in figure 5. This allows the user to obtain flow-characteristic data at or very near any location within the reach.

DESCRIPTION OF INPUT DATA

Input data consist of synchronous-stage data (boundary-condition data) collected at each end of the reach and a physical description of the channel configuration. The channel-description data include the average channel bottom elevation at each end of the reach, length of the reach, and the relation between eta and discharge. The channel cross-section properties for the reach are simulated in the form of a table of depth versus area and depth versus top width, which were developed from cross-section data collected within the reach.

Processing System for Input Boundary-Condition Data

The input boundary-condition data for the Sacramento River reach are 15-minute stage data punched on 16-channel paper tape. Precision, temperature-compensated, crystal-oscillator timers with a ±5 seconds per month accuracy are used to actuate the digital recorders. After the punched paper tape is removed from the field, the stage readings are transmitted by a tape reader (Mitron[1]) and stored on magnetic tape at the Geological Survey's computer center at Reston, Va. The data set is then processed through a system of two to four computer programs (R. A. Baltzer and R. W. Schaffranek, written commun., 1976).

TAPTRN, the first program used in the processing system, translates data sets from the magnetic tape to an on-line magnetic disk pack and flags data sets containing apparent inconsistencies. The data sets are then processed by the ZEDIT program which checks the stage readings against user specified tolerances. Three stage-reading tolerance checks are used: (1) Upper and lower stage reading boundaries, (2) number of consecutive unchanging stage readings, and (3) rate of change between two consecutive stage readings. Stage readings which exceed the specified tolerances are flagged and plotted for inspection. The plot details the part of the stage hydrograph that contains the suspect data. Each plot consists of 51 stage readings and is constructed by the line printer. If the user's specified number of allowable tolerance check errors is not exceeded, the data set is transferred to a disk-pack output file from which it can be retrieved and processed through the flow-simulation model; otherwise, the data set is placed in an error file on an on-line magnetic disk pack. Data sets that pass to the output file and contain flagged stage readings can be updated by the program DAFIX. Data sets that are placed in the error file often can be retrieved and rectified by using the program ZERREDIT.

[1]The use of brand names in this report is for identification purposes only and does not imply endorsement by the Geological Survey.

Collection and Processing of Channel-Property Data

When the investigation began, channel cross-section data collected in 1957 and 1966 were available for use as channel-simulation input data for preliminary model computations. After the preliminary computations revealed that the use of a flow-simulation model was feasible, current channel property data were collected. Level lines were run between the stage gages at the ends of the reach to insure that the two gages were referenced to the same datum. Channel cross-sectional data were collected at 13 locations along the reach for use in simulating the channel configuration. The number of cross sections and their locations were selected so as to define the actual channel configuration as accurately as feasible. The cross sections were surveyed from top of levee to top of levee, using the same datum as the two stage gages. The submerged channel configuration was determined by using a fathometer.

Cross-sectional data were processed by using PARS (a program for analyzing channel geometry data, developed by R. W. Schaffranek of the Geological Survey). This program calculates the top width and cross-sectional area for selected water-surface elevations for each of the 13 cross sections. These data were then combined to construct a table for top width versus depth and cross-sectional area versus depth that is used in the model to simulate the actual channel configuration for the reach.

The comparison of the channel cross-sectional data collected in 1957 and 1966 showed that the channel had been scoured 2 to 3 ft. This scour probably was brought about by construction of numerous storage reservoirs on the upstream tributaries of the Sacramento River during that period. The comparison of the 1966 data to those collected as part of this study showed that the channel has stabilized. Another indication that the channel has stabilized is that the model was calibrated with stage and measured discharge data collected over a 4-year period.

CALIBRATION OF MODEL

Calibration was achieved by comparing discharge data generated by the flow-simulation model with measured discharge data. Because the stage-fall-discharge method also required simultaneously collected stage data at the ends of the reach, there was a good supply of current (within 4 years) synchronized-stage data periods that also included discharge-measurement data. The stage data for these periods were processed as previously described. Each period of record was then processed by the model, varying eta until good agreement was obtained between the computed discharge and the measured discharge for that particular period. Each of these eta values was then plotted against the corresponding discharge to determine the eta-versus-discharge relation. For the Sacramento River reach it was found that a quadratic relation would best represent the eta-versus-discharge relation (fig. 2). The form of the equation is

$$\text{eta} = C_1 + C_2Q + C_3Q^2$$

where C_1, C_2, and C_3 are coefficients determined by curve fitting.

For transient (unsteady) flow conditions, figures 3 and 4 show the comparison between measured discharge and model-computed discharge, using the eta and discharge relation shown in figure 2. Table 1 shows the comparison between measured and computed discharge for the nontransient flow condition. The comparison between the model-computed daily mean discharge and the summation of the corresponding daily mean discharges for the upstream stations is shown in table 2. The summation consists of the daily mean discharges for the Sacramento River at Verona, Calif. (station 11425500), 19.6 mi upstream of the Sacramento stage gage, and for the American River at Fair Oaks, Calif. (station 11446500), 22.8 mi upstream of the Sacramento stage gage (fig. 1). The gage at Fair Oaks is on the only tributary between the Sacramento and Verona gages. The data in figures 3 and 4 and tables 1 and 2 show that the flow-simulation model provides accurate instantaneous and daily mean discharge data throughout the entire discharge range for the Sacramento River at Sacramento.

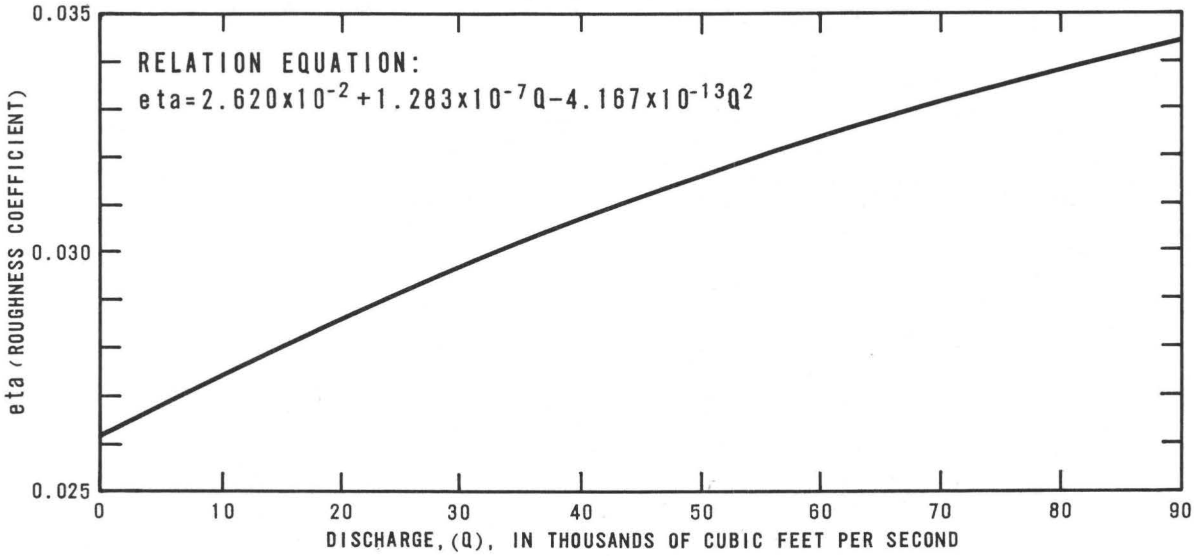

FIGURE 2.--eta and discharge relation.

Table 1.--Measured versus model-computed discharge, nontransient flow condition

Date of discharge measurement	Measured discharge (ft^3/s)	Model-computed discharge (ft^3/s)	Percentage difference
Dec. 21, 1972	42,700	42,400	-0.7
Jan. 15, 1973	89,800	91,000	+1.3
Jan. 20, 1973	88,800	90,700	+2.1
Feb. 5, 1973	59,100	58,200	-1.5
Feb. 27, 1973	53,600	51,800	-3.4
Mar. 22, 1974	65,800	67,700	+2.9
Apr. 25, 1974	42,400	41,400	-2.4
Feb. 18, 1975	67,000	65,500	-2.2
Mar. 14, 1975	59,200	58,600	-1.0

Table 2.--Summation of upstream daily mean discharges versus Sacramento River at Sacramento model-computed daily mean discharge

	Daily mean discharge, in cubic feet per second			
	March 1973		March 1976	
Day	Summation	Model	Summation	Model
1	71,000	73,100	20,200	19,700
2	71,600	73,700	25,200	25,200
3	70,900	73,200	26,600	27,500
4	71,100	73,300	23,800	24,800
5	70,400	72,500	21,400	22,200
6	69,400	71,400	19,000	19,800
7	69,400	71,200	16,900	17,700
8	69,400	71,800	15,100	15,800
9	69,000	71,000	13,900	14,400
10	67,800	69,600	13,100	13,700
11	66,400	68,100	12,900	13,200
12	64,600	66,200	12,800	13,000
13	61,900	63,300	12,800	13,000
14	57,900	59,200	13,200	13,200
15	52,600	54,000	13,000	13,100
16	44,300	45,900	12,600	12,700
17	40,700	40,600	12,400	12,400
18	37,600	37,200	12,300	12,400
19	35,000	34,600	12,200	12,300
20	33,400	33,100	12,000	12,100
21	34,300	33,500	11,900	12,000
22	40,000	38,500	11,700	11,900
23	44,800	43,800	11,500	11,700
24	44,900	44,800	11,500	11,700
25	40,900	41,000	11,100	11,200
26	37,100	36,800	10,800	10,900
27	34,100	33,700	10,500	10,600
28	30,500	30,400	10,400	10,500
29	27,700	27,700	10,800	10,900
30	26,800	26,500	10,900	10,900
31	26,700	26,300	11,200	11,300

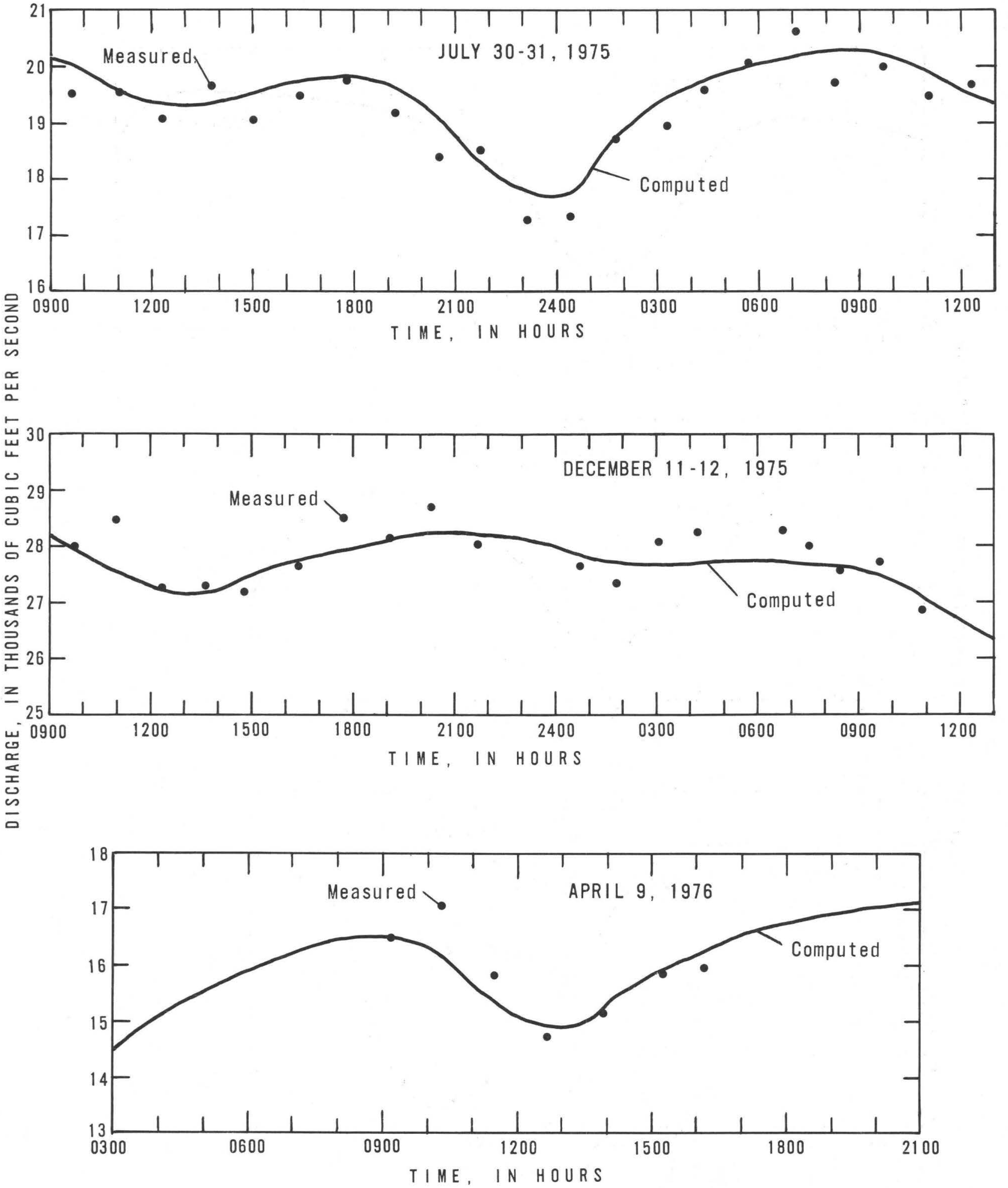

FIGURE 3.--Measured versus model-computed discharge, transient flow condition, July 1975-April 1976.

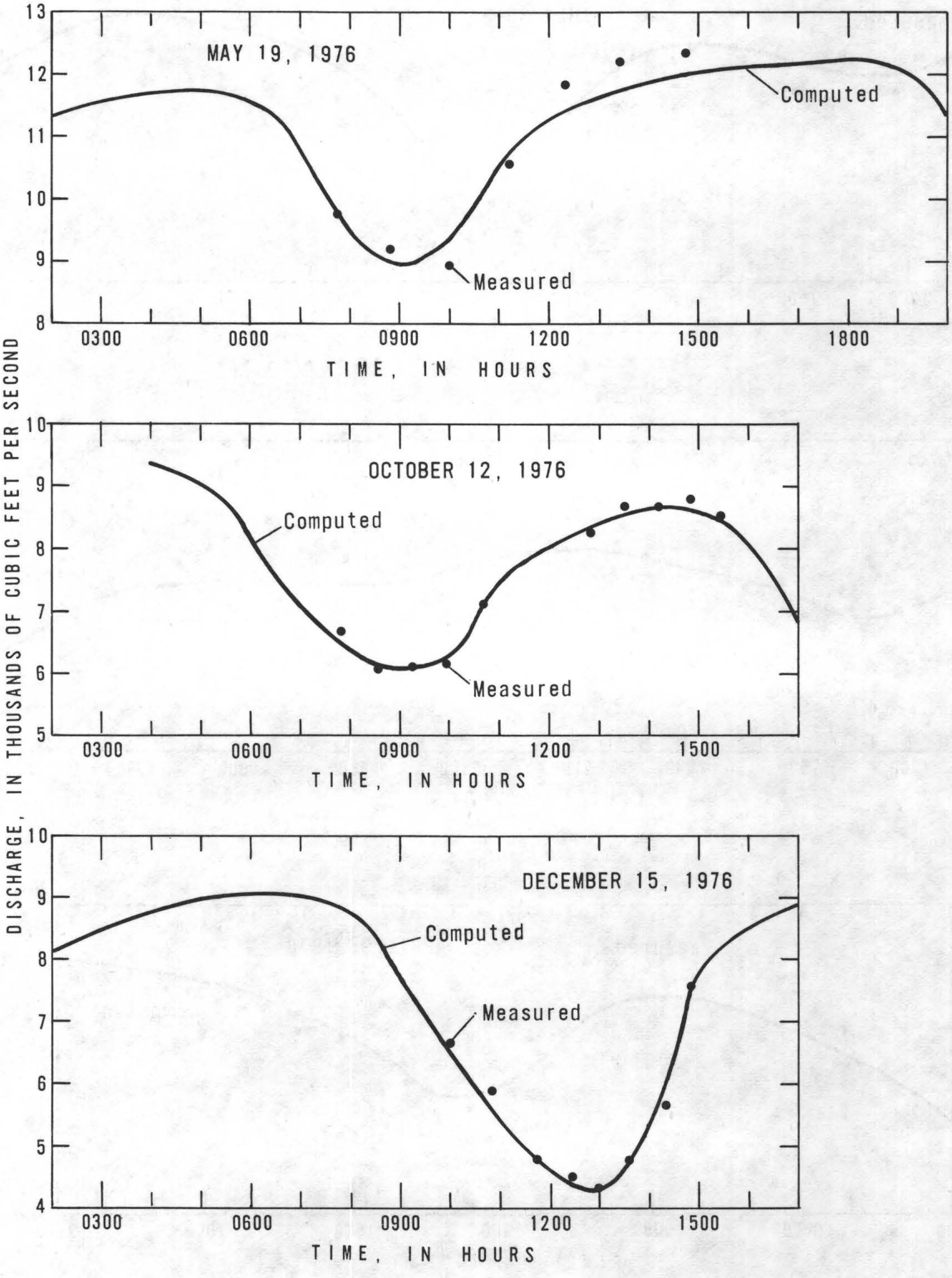

FIGURE 4.--Measured versus model-computed discharge, transient flow condition, May-December 1976.

SOURCES OF ERROR

The driving function of the flow-simulation model is synchronized-stage data. These data define the fall or slope of the water surface within the reach, and errors in the collection of these stage data can significantly affect model results. Errors in stage data can be caused by intake lag or partially plugged intakes, nonsynchronized collection of the stage readings, and incorrect recordings of the water stage by the digital recorder.

Intake lag or partially plugged intake errors of a few hundredths of a foot can cause a 10 to 20 percent error in the computation of instantaneous discharge during highly transient flow conditions when the fall in the reach approaches zero. The effect on the daily mean discharge for this flow condition is minimized because of a balancing of the instantaneous-discharge errors over a 24-hour period, which is also nearly equal to the period of time for a full tidal cycle (24.8 hours). To summarize: The error in discharge on the two rising-stage segments of the tidal cycle is balanced by the error on the two falling-stage segments of the tidal cycle; the actual fall in the reach is increased for two periods of the day and decreased for two periods of the day.

The rate of change in stage at the gages has been observed to be as high as 0.75 ft in an hour, or a little more than 0.01 ft per minute. Therefore, if one digital recorder is not synchronized with the other stage recorder, the fall between the gages could be in error and the model results would be affected in the same manner as described for intake lag.

If the digital recorder is not recording the water-stage data correctly at one gage, for example 0.02 ft low at the downstream gage, an error will result that is nonbalancing. In the example above, the fall between the two gages would be increased by 0.02 ft for all the conditions and, therefore, the error in the computed results would not balance over a tidal cycle.

The effects of the stage-data errors mentioned above are decreased as the discharge increases. When the discharge increases, the fall between the two stage gages increases; therefore, a 0.02-foot error in recording the river stage is not as critical if there is 4.5 ft of fall between the gages compared to 0 ft or negative fall within the reach.

If model results begin to deviate from the measured data, and the stage data used are valid, the channel may be scouring or filling. This can be verified by resurveying some of the channel cross sections that were originally surveyed and used for simulation of the channel configuration. Recalibration of the model would be necessary if the cross-section data revealed a significant change in channel configuration.

Calibration of the model involved varying the roughness coefficient, eta; an increase in eta resulted in a decrease in discharge, and a decrease in eta resulted in an increase in discharge. A percentage change in eta, however, did not result in the same percentage change in discharge. For a specified percentage change in eta, the resultant percentage change in discharge differed with the magnitude of the discharge being computed.

SUMMARY

The purpose of this study was to evaluate the potential of computing the discharge for the Sacramento River at the Sacramento stream gage, using a flow-simulation model rather than the empirical stage-fall-discharge method. Two flow-simulation models were studied, but only the multiple-reach method-of-characteristics digital-computer transient-flow simulation model was modified, calibrated, and applied to the Sacramento River reach. The evaluation has shown that the flow-simulation model can provide reliable discharge data on a daily mean as well as an instantaneous basis.

The use of the flow-simulation model has not eliminated the hazardous discharge-measuring conditions that exist when using the suspended cable, but the time during which the cable is in use has been significantly reduced. The 25- to 26-hour tidal cycle measurements that were necessary for the stage-fall-discharge method to determine the daily mean discharge have been reduced to 8 to 9 hours and can be made in daylight hours. Discharge measurements are currently being made bimonthly, but the number of measurements required in the future will depend upon flow conditions and on the stability of the calibration.

Additional benefits gained by using the flow-simulation model rather than the stage-fall-discharge method are that the model has the capability of providing instantaneous-stage, velocity, and (or) discharge data, as well as daily mean discharge data at or near any location within the reach. The model can also be used over a wider range of flow conditions (the stage-fall-discharge method cannot be used if there is zero or negative fall in the reach). The model could also be incorporated into a real-time data system using telemetering equipment in the two stream-gaging stations and a small programmable office computer.

At the present time there are plans for the extension of the model reach 10.5 mi downstream from Freeport to Hood (fig. 1). The extension of the model would provide flow-characteristics data at Hood, which is the location of the proposed State of California peripheral canal intake structure.

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MATHEMATICAL EQUATIONS AND COMPUTATION PROCEDURES

The following description of the mathematics and the computational procedures, together with illustrations, is taken from Lai and Onions (1976, p. 3-11).

Review of Mathematical Equations

Basic partial differential equations.--The following set of partial differential equations that represent one-dimensional transient open-channel flows of homogeneous density is used as the basic equations for the numerical simulation.

Equation of continuity

$$\bar{H}\frac{\partial u}{\partial x} + u\frac{\partial Z}{\partial x} + \frac{\partial Z}{\partial t} + uS_o - \frac{q}{B} = 0 \qquad (1)$$

Equation of motion

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + g\frac{\partial Z}{\partial x} + g\frac{k}{R^{4/3}}u|u| + \frac{qu}{A} = 0 \qquad (2)$$

in which

x = distance along the longitudinal axis (x-axis) of channel
t = time
u = flow velocity (averaged over the cross section)
Z = elevation of water surface
A = cross-sectional area
B = top width of cross section
$\bar{H}$ = average depth of flow (=A/B)
R = hydraulic radius of cross section
S_o = slope of the channel bottom
g = acceleration of gravity
k = parameter depending on flow-resistance coefficient, η(eta).

$k = \eta^2$ for metric system

$= (\frac{\eta}{1.486})^2$ for inch-pound system

q = lateral inflow per unit length of channel

The above equations are a set of hyperbolic, first-order, quasi-linear partial differential equations of two dependent variables (u,Z) and two independent variables (x,t).

Characteristic equations.--The type of partial differential equations shown above can be transformed to the following simple characteristic equations each of which contains only total derivatives.

$$dt - \frac{dx}{u \pm c} = 0 \qquad (3)$$

along $C_{\pm}$ curve,

$$dZ \pm \frac{c}{g}\, du + F_{\pm} dt = 0 \qquad (4)$$

in which

$$F_{\pm} = uS_o \pm \frac{ck}{R^{4/3}}\, u|u| - \frac{q}{B}\,(1 \mp \frac{u}{c}) \qquad (5)$$

$c = \sqrt{gH}$ = the celerity of gravity wave

C_+, C_- = (+) and (-) characteristic curves, also called forward and backward characteristics

Finite-difference approximations.--For the numerical solution by computer, equations 3 and 4 can be expressed in their corresponding finite difference forms:

$$(t_P - t_L) - (\frac{1}{u+c})_L \, (x_P - x_L) = 0 \qquad (6)$$

$$(Z_P - Z_L) + \frac{c_L}{g}(u_P - u_L) + (F_+)_L(t_P - t_L) = 0 \qquad (7)$$

$$(t_P - t_R) - (\frac{1}{u-c})_R(x_P - x_R) = 0 \qquad (8)$$

$$(Z_P - Z_R) - \frac{c_R}{g}(u_P - u_R) + (F_-)_R(t_P - t_R) = 0 \qquad (9)$$

in which the subscript indicates the point in figure 5 to which each quantity is referred. These equations will be solved by the specified-time-interval scheme using Δt as the time interval and Δx as the segment length on a rectangular grid system.

By linear interpolation, the values of u_L, u_R, Z_L and Z_R may be expressed in terms of the corresponding values at points A, B, and C, as follows:

$$u_L = u_B[1-\theta(u+c)_B] + u_A\,\theta(u+c)_B \tag{10}$$

$$u_R = u_B[1+\theta(u-c)_B] - u_C\theta(u-c)_B \tag{11}$$

$$Z_L = Z_B[1-\theta(u+c)_B] + Z_A\theta(u+c)_B \tag{12}$$

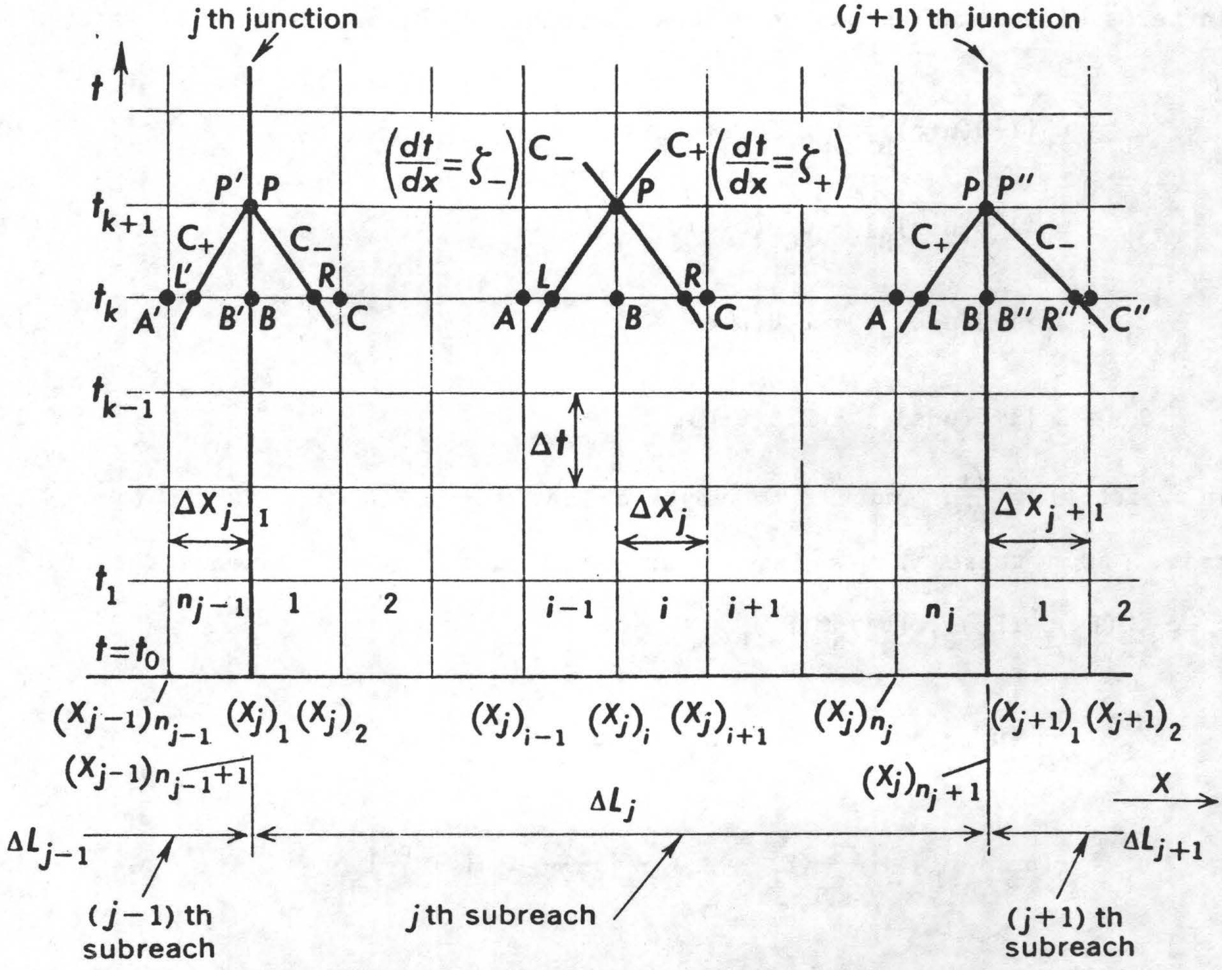
$$Z_R = Z_B[1+\theta(u-c)_B] - Z_C\theta(u-c)_B \tag{13}$$

in which $\theta = \frac{\Delta t}{\Delta x}$, and the assumption that $(u+c)_L \cong (u+c)_B$, $(u-c)_R \cong (u-c)_B$, is made. From these values (equations 10-13) and further assuming that $c_L \cong c_B$, $c_R \cong c_B$, $(F_+)_L \cong (F_+)_B$, $(F_-)_R \cong (F_-)_B$,

$$Z_P = \frac{c_B}{2g}(u_L - u_R) + \frac{1}{2}(Z_L + Z_R) - (uS_o - \frac{q}{B})_B\Delta t \tag{14}$$

$$u_P = \frac{1}{2}(u_L + u_R) + \frac{g}{2c_B}(Z_L - Z_R) - [\frac{gk}{R^{4/3}}\,u|u| + \frac{uq}{A}]_B\Delta t \tag{15}$$

In the multiple-reach scheme, the above equations have to be expressed in two-dimensional form, that is, two subscripts should be added to variables and parameters to indicate what nodal point or segment, in what subreach, they belong to. For example, if point B is on the ith node of the jth subreach, then u_B may be expressed as $u_{i,j}$, Z_A as $Z_{i-1,j}$, Z_P as $(Z_P)_{i,j}$ or $Z^{k+1}_{i,j}$, u_P as $(u_P)_{i,j}$ or $u^{k+1}_{i,j}$ and so forth. The length of the jth subreach is expressed by ΔL_j. (See figs. 5 and 6; Point $x_{i,j}$ is expressed as $(x_j)_i$ in fig. 5.)



From Lai and Onions (1976, fig.1)

FIGURE 5.--A rectangular net and characteristic curves for numerical solution by the multiple-reach method of characteristics.

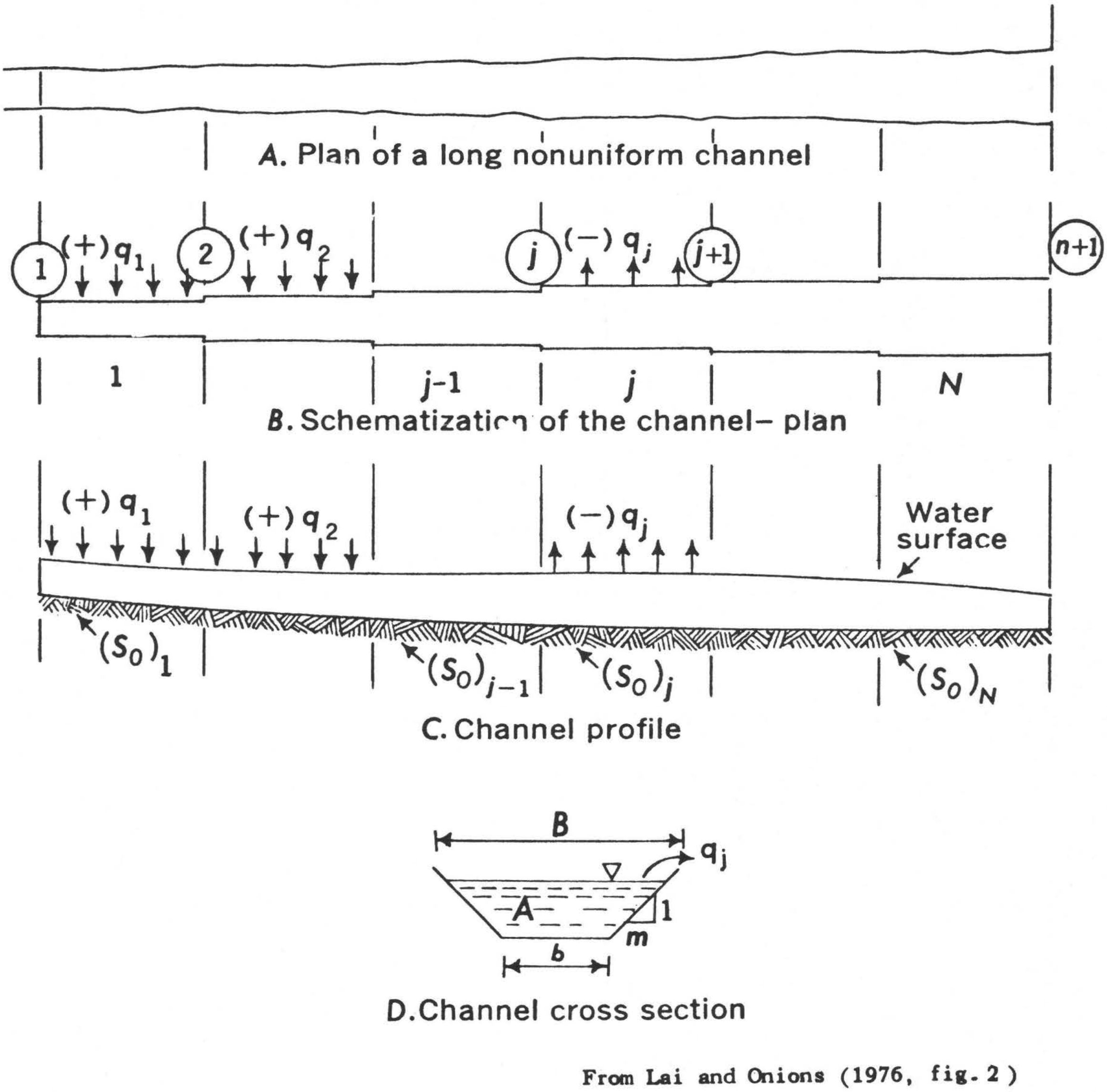


FIGURE 6.--Schematization of a long nonuniform channel.

Boundary conditions.--Four boundary equations, derived from equations 7 and 9, are: at the left boundary (x = 0),

$$u_P = u_R + \frac{g}{c_B}(Z_P - Z_R) + \frac{g}{c_B}(F_-)_B \Delta t \tag{16}$$

or

$$Z_P = Z_R + \frac{c_B}{g}(u_P - u_R) - (F_-)_B \Delta t \tag{17}$$

at the right boundary (x = L), (L = total reach length)

$$u_P = u_L - \frac{g}{c_B}(Z_P - Z_L) - \frac{g}{c_B}(F_+)_B \Delta t \tag{18}$$

or

$$Z_P = Z_L - \frac{c_B}{g}(u_P - u_L) - (F_+)_B \Delta t \tag{19}$$

One equation from each end of the reach (x = 0 and x = L) should be chosen for boundary value computation (for subcritical flow, of course). The choice depends on which type of boundary-value data (stage or velocity) is obtainable at each end.

For any junction between two subreaches, say the (j + 1)th junction, the following four boundary equations may be used.

$$(Z_P)_{n_j+1,j} = (Z_P)_{1,j+1} \tag{20}$$

$$(u_P A_P)_{n_j+1,j} = (u_P A_P)_{1,j+1} \quad \text{i.e.} \quad Q_{n_j+1,\ j} = Q_{1,\ j+1} \tag{21}$$

$$(u_P)_{n_j+1,j} = (u_L)_{n_j+1,j} - \frac{g}{c_{n_j+1,j}}[\ (Z_P)_{n_j+1,j} - (Z_L)_{n_j+1,j}]$$

$$- \frac{g}{c_{n_j+1,j}} (F_+)_{n_j+1,j} \Delta t \tag{22}$$

$$(u_P)_{1,j+1} = (u_R)_{1,j+1} + \frac{g}{c_{1,j+1}}[\ (Z_P)_{1,j+1} - (Z_R)_{1,j+1}]$$

$$+ \frac{g}{c_{1,j+1}}(F_-)_{1,j+1} \Delta t \tag{23}$$

in which Q is the discharge.

To solve these equations, a value Z_P at the junction is estimated by quadratic extrapolation from the previous elevations. Substitute the assumed Z_P value into both equations 22 and 23 and compute $(u_P)_{n_j+1,j}$ and $(u_P)_{1,j+1}$. Using the same Z_P value, $(A_P)_{n_j+1,j}$ and $(A_P)_{1,j+1}$ can be evaluated. Then $(u_PA_P)_{n_j+1,j}$ and $(u_PA_P)_{1,j+1}$ are computed and the difference of the two is tested. The computation is iterated until the difference becomes less than a maximum tolerable error, ε.

Computation Procedure

The machine computation is carried out according to the following steps:

(a) Read reach geometry data.

(b) Organize or compile depth versus cross-sectional area, depth versus top width relationships.

(c) Read data describing the initial values and other parameters.

(d) Divide each subreach into n segments (or elements) of length Δx (that is, $\Delta x_j = \Delta L_j/n_j$), and call pivotal points between segments, including end points, as nodal points. Assign the initial values for u and Z at each nodal point throughout the entire reach.

(e) Prepare the boundary values for each end of the entire reach for designed time intervals.

(f) Estimate (Z_P)-values at each junction between subreaches.

(g) From upstream (left) to downstream (right) calculate u_P and Z_P for a time interval $2\Delta t$ at each nodal point for each subreach, using equations 10 through 19. At the upstream end (the left end of the 1st subreach) use equation 16 or 17 and at the downstream end (the right end of the last subreach) use 18 or 19 to calculate u_P or Z_P, and at each junction use equations 22 and 23 to calculate u_P.

(h) Use equations 20 through 23 repeatedly to find satisfactory u_P and Z_P values at each junction. Call steps f through h as "m = 1 cycle".

(i) Calculate u and Z for a time interval Δt using the same equations and following the same steps described in f through h. Call this step "m = 2 cycle".

(j) Using the results of step i, calculate u and Z for Δt again. Call this step "m = 3 cycle".

(k) Repeat steps f through j as far as desired.

(l) Whenever appropriate, combine the results of g, h, and j, through the use of equation 24, and obtained extrapolated values of u and Z.

When only the first-order approximation is required, steps i, j, and l are bypassed and only f through h are repeated; that is, only m = 1 cycle is needed.