Application of Transient-Flow Model to the Sacramento River at Sacramento, California

Prepared in cooperation with the California Department of Water Resources
**Title and Subtitle**
APPLICATION OF TRANSIENT-FLOW MODEL TO THE SACRAMENTO RIVER AT SACRAMENTO, CALIFORNIA

**Author(s)**
Richard N. Oltmann

**Performing Organization Name and Address**
U.S. Geological Survey, Water Resources Division
345 Middlefield Road
Menlo Park, California 94025

**Sponsoring Organization Name and Address**
U.S. Geological Survey, Water Resources Division
345 Middlefield Road
Menlo Park, California 94025

**Abstract**
The transient-flow simulation model was applied to the Sacramento River at Sacramento, Calif. An alternative is needed to the empirical stage-fall-discharge computational method, which provides only daily mean discharges and requires frequent measurement over a 25- to 26-hour period of the tide-affected flow. Measurement of tide-affected flow involves use of a cable suspended across the river; this presents a hazardous navigational condition. Use of the model does not eliminate the hazardous navigational problem, but it significantly reduces the time that this condition exists.

The model has demonstrated that it can provide reliable daily mean as well as instantaneous discharge data. The model also has the capability of providing discharge data for locations farther downstream in the tidal zone from the Sacramento stream gage; this is not possible with the stage-fall-discharge computational method.

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- Stream gages, Computer models, Open channel flow, Non-uniform flow, Flow rates

- Sacramento River, Sacramento-Freeport tidal reach

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By Richard N. Oltmann

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CONVERSION FACTORS

For readers who may prefer to use metric units (International System of Units) rather than inch-pound units, the conversion factors for the terms used in this report are listed below.

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<th>Inch-pound unit</th>
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<tr>
<td>mi² (square mile)</td>
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APPLICATION OF TRANSIENT-FLOW MODEL TO
THE SACRAMENTO RIVER AT SACRAMENTO, CALIFORNIA

By Richard N. Oltmann

ABSTRACT

The transient-flow simulation model was applied to the Sacramento River at Sacramento, Calif. An alternative is needed to the empirical stage-fall-discharge computational method, which provides only daily mean discharges and requires frequent measurement over a 25- to 26-hour period of the tide-affected flow. Measurement of tide-affected flow involves use of a cable suspended across the river; this presents a hazardous navigational condition. Use of the model does not eliminate the hazardous navigational problem, but it significantly reduces the time that this condition exists.

The model has demonstrated that it can provide reliable daily mean as well as instantaneous discharge data. The model also has the capability of providing discharge data for locations farther downstream in the tidal zone from the Sacramento stream gage; this is not possible with the stage-fall-discharge computational method.
INTRODUCTION

Location and General Information

Hydrologic data for the Sacramento River have been collected at or in the vicinity of the stream-gaging station at Sacramento, Calif., since 1879. The stream gage is approximately 70 mi northeast of San Francisco, and the drainage area at the gage is 23,508 mi². It is the last streamflow gage on the Sacramento River before it discharges into the San Francisco Bay system. For the period of continuous streamflow record (1949-76) the daily mean discharge at the gage has ranged from 5,590 ft³/s to 99,400 ft³/s and has averaged 24,330 ft³/s. Stage-discharge relations for streamflows less than about 40,000 ft³/s are affected by the tide.

The gage is a Federal-State cooperative station, operated by the U.S. Geological Survey for the purposes of flood forecasting and water inventory. Data collected at the site are used by Federal agencies, the State of California, and the Sacramento community.

Description of Problem

Because the flow of the Sacramento River at Sacramento is tide affected for flows of less than 40,000 ft³/s, a conventional stage-discharge relation cannot be used to compute the discharge for these unsteady-flow conditions that exist about 80 percent of the time. The daily mean discharge for unsteady-flow periods has been computed by using the empirical stage-fall-discharge method (R. W. Carter and Jacob Davidian, U.S. Geological Survey, written commun., 1965), where the fall or slope of the water surface is determined by using a 10.8-mi reach of the river. A conventional stage-discharge relation has been used for the periods not affected by the tide.

The stage-fall-discharge computational method provides a reliable daily mean discharge, but it is not reliable with respect to instantaneous discharge, especially for flows less than 14,000 ft³/s, which prevail about 40 percent of the time. Also, the method requires frequent measurement of the discharge. Because the flow is tide affected, tidal-cycle discharge measurements are required to define the stage-fall-discharge relation. A tidal-cycle discharge measurement consists of about 20 conventional discharge measurements made over a 25- to 26-hour period from which a daily mean discharge is computed. The 20 discharge measurements are made from a boat attached to a cable that moves the boat from section to section across the 650-foot-wide channel. The cable spans the river about 20 ft above the water surface, thereby allowing small boats to travel the river without obstruction. In order for larger vessels to pass the measuring site, however, the cable must be dropped to the river bottom. This suspended cable constitutes a hazardous navigational situation, especially after sundown.
Purpose and Scope

The U. S. Geological Survey, in cooperation with the California Department of Water Resources, undertook this study to evaluate the feasibility of computing the discharge of the Sacramento River at the Sacramento stream gage by using a digital-computer transient-flow simulation model.

The scope of the study included (1) collecting and processing input data and (2) using the data in conjunction with discharge-measurement data to calibrate a flow-simulation model for a reach of the Sacramento River.

DESCRIPTION OF RIVER REACH

The reach of the Sacramento River to which the flow-simulation model was applied extends 10.8 mi downstream from the stream gage at Sacramento (station 11447500) to the auxiliary stage gage near Freeport, Calif. (fig. 1). The river channel in this reach has a mild, uniform slope and is not very sinuous. Also, it is channelized throughout the reach with large areas of the levied river banks riprapped. The nonriprapped areas are fairly free of flow-resisting vegetation. The river bed is composed predominately of sand-size material and has 3- to 5-ft dunes. At low flows (10,000 to 16,000 ft³/s) the water depth above the crests of the dunes is about 15 ft.

The fall in the water surface through the reach ranges from about 4.9 ft at 90,000 ft³/s to about a negative 0.2 ft at 4,000 ft³/s. No reversal of streamflow because of tidal inflow was observed at the Sacramento gage during the period of gage operation, but the flow at the downstream end of the reach approached reversal when the flow at the upstream end of the reach is about 4,000 ft³/s. Because of tidal effects, the maximum difference between high and low water stage at the upstream end of the reach, during a day, is 2.5 ft. This 2.5-ft difference in stage results in a difference between maximum and minimum discharge of 6,000 ft³/s for a day which has a daily mean discharge of 7,000 ft³/s.

Throughout the reach, numerous small irrigation pumps extract water from the river and some small drains discharge water into the river, but the net effect upon stream discharge is believed to be insignificant. A set of navigational locks connecting the Sacramento River and the Port of Sacramento deepwater ship channel, is located 2.1 mi downstream from the Sacramento stream gage. The operation of the locks periodically affects the Sacramento River at the Sacramento gage, but not significantly.
FIGURE 1.--Location of Sacramento River study area.
DESCRIPTION OF MODEL

The flow-simulation model that was applied to the Sacramento River was the multiple-reach method-of-characteristics digital-computer transient-flow simulation model developed by Lai (1967). This one-dimensional flow-simulation model is based on the equations of continuity (conservation of mass) and motion (conservation of momentum). These two partial-differential equations are transformed into characteristic equations and are solved by using finite-difference approximations with a specified time interval (Lai, 1967). The assumptions used in the derivation of this numerical model are: (1) The velocity is uniform in any given channel cross section; (2) the water in the channel is substantially of homogeneous density; (3) the channel slope is mild and uniform over the reach; (4) the reach does not have a high degree of sinuosity; (5) the reach geometry is relatively simple; and (6) the flow-resistance coefficient, eta, used with unsteady flow is the same as that for steady flow.

As the model name implies, the model can be applied to a river reach which is subdivided because of significant variation throughout the reach with respect to channel geometry or the flow-resistance coefficient. Because it was not necessary to subdivide the study reach of the Sacramento River, the model was applied to a single reach and not used as a multiple-reach model.

A further explanation of the unsteady-flow equations and the method-of-characteristics numerical method has been abstracted from Lai and Onions (1976) and can be found in the section "Mathematical Equations and Computation Procedures," at the end of this report. The primary difference between the model used on the Sacramento River (version 27) and that described by Lai and Onions (1976) (version 13) is in the treatment of the flow-resistance coefficient, eta. Version 13 treats eta as a constant, but version 27 has the capability of varying eta either as a linear or quadratic function of discharge.

A flow-simulation model based on a power-series solution of the unsteady-flow equations was also investigated. The power-series model had previously been used on this reach of the Sacramento River in a limited test (9,000 to 19,000 ft³/s) by Baltzer and Shen (1961, 1964). Preliminary computations using both the multiple-reach method-of-characteristics solution and the power-series solution indicated that the flow-resistance coefficient varied throughout the range of discharge and could not be treated as a constant. Only the method-of-characteristics model was modified to accommodate a variable eta; the power-series model was not modified and was therefore not tested beyond the preliminary computations.

The model has eight output options. The options include stage and discharge hydrographs, computed versus observed stage and discharge hydrographs, 15-minute discharge or daily mean discharge tables, and a summation of positive and negative discharge for each day. The output options can be computed for either end of the reach or for the junction of two segment lengths as stated on page 18 and illustrated in figure 5. This allows the user to obtain flow-characteristic data at or very near any location within the reach.
DESCRIPTION OF INPUT DATA

Input data consist of synchronous-stage data (boundary-condition data) collected at each end of the reach and a physical description of the channel configuration. The channel-description data include the average channel bottom elevation at each end of the reach, length of the reach, and the relation between eta and discharge. The channel cross-section properties for the reach are simulated in the form of a table of depth versus area and depth versus top width, which were developed from cross-section data collected within the reach.

Processing System for Input Boundary-Condition Data

The input boundary-condition data for the Sacramento River reach are 15-minute stage data punched on 16-channel paper tape. Precision, temperature-compensated, crystal-oscillator timers with a ±5 seconds per month accuracy are used to actuate the digital recorders. After the punched paper tape is removed from the field, the stage readings are transmitted by a tape reader (Mitron) and stored on magnetic tape at the Geological Survey's computer center at Reston, Va. The data set is then processed through a system of two to four computer programs (R. A. Baltzer and R. W. Schaffranek, written commun., 1976).

TAPTRN, the first program used in the processing system, translates data sets from the magnetic tape to an on-line magnetic disk pack and flags data sets containing apparent inconsistencies. The data sets are then processed by the ZEDIT program which checks the stage readings against user specified tolerances. Three stage-reading tolerance checks are used: (1) Upper and lower stage reading boundaries, (2) number of consecutive unchanging stage readings, and (3) rate of change between two consecutive stage readings. Stage readings which exceed the specified tolerances are flagged and plotted for inspection. The plot details the part of the stage hydrograph that contains the suspect data. Each plot consists of 51 stage readings and is constructed by the line printer. If the user's specified number of allowable tolerance check errors is not exceeded, the data set is transferred to a disk-pack output file from which it can be retrieved and processed through the flow-simulation model; otherwise, the data set is placed in an error file on an on-line magnetic disk pack. Data sets that pass to the output file and contain flagged stage readings can be updated by the program DAFIX. Data sets that are placed in the error file often can be retrieved and rectified by using the program ZERREDIT.

The use of brand names in this report is for identification purposes only and does not imply endorsement by the Geological Survey.
Collection and Processing of Channel-Property Data

When the investigation began, channel cross-section data collected in 1957 and 1966 were available for use as channel-simulation input data for preliminary model computations. After the preliminary computations revealed that the use of a flow-simulation model was feasible, current channel property data were collected. Level lines were run between the stage gages at the ends of the reach to insure that the two gages were referenced to the same datum. Channel cross-sectional data were collected at 13 locations along the reach for use in simulating the channel configuration. The number of cross sections and their locations were selected so as to define the actual channel configuration as accurately as feasible. The cross sections were surveyed from top of levee to top of levee, using the same datum as the two stage gages. The submerged channel configuration was determined by using a fathometer.

Cross-sectional data were processed by using PARS (a program for analyzing channel geometry data, developed by R. W. Schaffranek of the Geological Survey). This program calculates the top width and cross-sectional area for selected water-surface elevations for each of the 13 cross sections. These data were then combined to construct a table for top width versus depth and cross-sectional area versus depth that is used in the model to simulate the actual channel configuration for the reach.

The comparison of the channel cross-sectional data collected in 1957 and 1966 showed that the channel had been scoured 2 to 3 ft. This scour probably was brought about by construction of numerous storage reservoirs on the upstream tributaries of the Sacramento River during that period. The comparison of the 1966 data to those collected as part of this study showed that the channel has stabilized. Another indication that the channel has stabilized is that the model was calibrated with stage and measured discharge data collected over a 4-year period.
Calibration was achieved by comparing discharge data generated by the flow-simulation model with measured discharge data. Because the stage-fall-discharge method also required simultaneously collected stage data at the ends of the reach, there was a good supply of current (within 4 years) synchronized-stage data periods that also included discharge-measurement data. The stage data for these periods were processed as previously described. Each period of record was then processed by the model, varying eta until good agreement was obtained between the computed discharge and the measured discharge for that particular period. Each of these eta values was then plotted against the corresponding discharge to determine the eta-versus-discharge relation. For the Sacramento River reach it was found that a quadratic relation would best represent the eta-versus-discharge relation (fig. 2). The form of the equation is

\[ \text{eta} = C_1 + C_2Q + C_3Q^2 \]

where \(C_1\), \(C_2\), and \(C_3\) are coefficients determined by curve fitting.

For transient (unsteady) flow conditions, figures 3 and 4 show the comparison between measured discharge and model-computed discharge, using the eta and discharge relation shown in figure 2. Table 1 shows the comparison between measured and computed discharge for the nontransient flow condition. The comparison between the model-computed daily mean discharge and the summation of the corresponding daily mean discharges for the upstream stations is shown in table 2. The summation consists of the daily mean discharges for the Sacramento River at Verona, Calif. (station 11425500), 19.6 mi upstream of the Sacramento stage gage, and for the American River at Fair Oaks, Calif. (station 11446500), 22.8 mi upstream of the Sacramento stage gage (fig. 1). The gage at Fair Oaks is on the only tributary between the Sacramento and Verona gages. The data in figures 3 and 4 and tables 1 and 2 show that the flow-simulation model provides accurate instantaneous and daily mean discharge data throughout the entire discharge range for the Sacramento River at Sacramento.
CALIBRATION OF MODEL

RELATION EQUATION:
\[ \eta = 2.620 \times 10^{-2} + 1.283 \times 10^{-7} Q - 4.167 \times 10^{-13} Q^2 \]

FIGURE 2.--\( \eta \) and discharge relation.

Table 1.--Measured versus model-computed discharge, nontransient flow condition

<table>
<thead>
<tr>
<th>Date of discharge measurement</th>
<th>Measured discharge ((\text{ft}^3/\text{s}))</th>
<th>Model-computed discharge ((\text{ft}^3/\text{s}))</th>
<th>Percentage difference</th>
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<td>Jan. 15, 1973</td>
<td>89,800</td>
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<td>Jan. 20, 1973</td>
<td>88,800</td>
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<td>59,100</td>
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### Table 2. -- Summation of upstream daily mean discharges versus Sacramento River at Sacramento model-computed daily mean discharge

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FIGURE 4.—Measured versus model-computed discharge, transient flow condition, May-December 1976.
The driving function of the flow-simulation model is synchronized-stage data. These data define the fall or slope of the water surface within the reach, and errors in the collection of these stage data can significantly affect model results. Errors in stage data can be caused by intake lag or partially plugged intakes, nonsynchronized collection of the stage readings, and incorrect recordings of the water stage by the digital recorder.

Intake lag or partially plugged intake errors of a few hundredths of a foot can cause a 10 to 20 percent error in the computation of instantaneous discharge during highly transient flow conditions when the fall in the reach approaches zero. The effect on the daily mean discharge for this flow condition is minimized because of a balancing of the instantaneous-discharge errors over a 24-hour period, which is also nearly equal to the period of time for a full tidal cycle (24.8 hours). To summarize: The error in discharge on the two rising-stage segments of the tidal cycle is balanced by the error on the two falling-stage segments of the tidal cycle; the actual fall in the reach is increased for two periods of the day and decreased for two periods of the day.

The rate of change in stage at the gages has been observed to be as high as 0.75 ft in an hour, or a little more than 0.01 ft per minute. Therefore, if one digital recorder is not synchronized with the other stage recorder, the fall between the gages could be in error and the model results would be affected in the same manner as described for intake lag.

If the digital recorder is not recording the water-stage data correctly at one gage, for example 0.02 ft low at the downstream gage, an error will result that is nonbalancing. In the example above, the fall between the two gages would be increased by 0.02 ft for all the conditions and, therefore, the error in the computed results would not balance over a tidal cycle.

The effects of the stage-data errors mentioned above are decreased as the discharge increases. When the discharge increases, the fall between the two stage gages increases; therefore, a 0.02-foot error in recording the river stage is not as critical if there is 4.5 ft of fall between the gages compared to 0 ft or negative fall within the reach.

If model results begin to deviate from the measured data, and the stage data used are valid, the channel may be scouring or filling. This can be verified by resurveying some of the channel cross sections that were originally surveyed and used for simulation of the channel configuration. Recalibration of the model would be necessary if the cross-section data revealed a significant change in channel configuration.
Calibration of the model involved varying the roughness coefficient, \( \eta \); an increase in \( \eta \) resulted in a decrease in discharge, and a decrease in \( \eta \) resulted in an increase in discharge. A percentage change in \( \eta \), however, did not result in the same percentage change in discharge. For a specified percentage change in \( \eta \), the resultant percentage change in discharge differed with the magnitude of the discharge being computed.

**SUMMARY**

The purpose of this study was to evaluate the potential of computing the discharge for the Sacramento River at the Sacramento stream gage, using a flow-simulation model rather than the empirical stage-fall-discharge method. Two flow-simulation models were studied, but only the multiple-reach method-of-characteristics digital-computer transient-flow simulation model was modified, calibrated, and applied to the Sacramento River reach. The evaluation has shown that the flow-simulation model can provide reliable discharge data on a daily mean as well as an instantaneous basis.

The use of the flow-simulation model has not eliminated the hazardous discharge-measuring conditions that exist when using the suspended cable, but the time during which the cable is in use has been significantly reduced. The 25- to 26-hour tidal cycle measurements that were necessary for the stage-fall-discharge method to determine the daily mean discharge have been reduced to 8 to 9 hours and can be made in daylight hours. Discharge measurements are currently being made bimonthly, but the number of measurements required in the future will depend upon flow conditions and on the stability of the calibration.

Additional benefits gained by using the flow-simulation model rather than the stage-fall-discharge method are that the model has the capability of providing instantaneous-stage, velocity, and (or) discharge data, as well as daily mean discharge data at or near any location within the reach. The model can also be used over a wider range of flow conditions (the stage-fall-discharge method cannot be used if there is zero or negative fall in the reach). The model could also be incorporated into a real-time data system using telemetering equipment in the two stream-gaging stations and a small programmable office computer.

At the present time there are plans for the extension of the model reach 10.5 mi downstream from Freeport to Hood (fig. 1). The extension of the model would provide flow-characteristics data at Hood, which is the location of the proposed State of California peripheral canal intake structure.
REFERENCES CITED


The following description of the mathematics and the computational procedures, together with illustrations, is taken from Lai and Onions (1976, p. 3-11).

**Review of Mathematical Equations**

**Basic partial differential equations.**—The following set of partial differential equations that represent one-dimensional transient open-channel flows of homogeneous density is used as the basic equations for the numerical simulation.

**Equation of continuity**

\[ \bar{H} \frac{\partial u}{\partial x} + u \frac{\partial Z}{\partial x} + \frac{\partial Z}{\partial t} + uS_o - \frac{q}{B} = 0 \]  

\[ (1) \]

**Equation of motion**

\[ \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + g \frac{\partial Z}{\partial x} + g \frac{k}{R^{4/3}} u \left| u \right| + \frac{qu}{A} = 0 \]  

\[ (2) \]

in which

- \( x \) = distance along the longitudinal axis (x-axis) of channel
- \( t \) = time
- \( u \) = flow velocity (averaged over the cross section)
- \( Z \) = elevation of water surface
- \( A \) = cross-sectional area
- \( B \) = top width of cross section
- \( \bar{H} \) = average depth of flow (=A/B)
- \( R \) = hydraulic radius of cross section
- \( S_o \) = slope of the channel bottom
- \( g \) = acceleration of gravity
- \( k \) = parameter depending on flow-resistance coefficient, \( \eta \) (eta).

\[ k = \eta^2 \text{ for metric system} \]
\[ = (\frac{\eta}{1.486})^2 \text{ for inch-pound system} \]

\( q \) = lateral inflow per unit length of channel
The above equations are a set of hyperbolic, first-order, quasi-linear partial differential equations of two dependent variables \((u,Z)\) and two independent variables \((x,t)\).

**Characteristic equations.**—The type of partial differential equations shown above can be transformed to the following simple characteristic equations each of which contains only total derivatives.

\[
\text{d}t - \frac{\text{d}x}{u \pm c} = 0 \quad \text{along } C_\pm \text{ curve},
\]

\[
dZ + \frac{c}{g} \text{d}u + F_\pm \text{d}t = 0
\]

in which

\[
F_\pm = u_0 \pm \frac{ck}{R^{1/3}} u \left| u \right| - \frac{g}{B} \left( 1 \mp \frac{u}{c} \right)
\]

\[
c = \sqrt{gH} = \text{the celerity of gravity wave}
\]

\(C_+, C_- = (+) \text{ and } (-) \text{ characteristic curves, also called forward and backward characteristics}

**Finite-difference approximations.**—For the numerical solution by computer, equations 3 and 4 can be expressed in their corresponding finite difference forms:

\[
(t_p - t_L) - \left( \frac{1}{u+c} \right)_L (x_p - x_L) = 0
\]

\[
(Z_p - Z_L) + \frac{c_L}{g} (u_p - u_L) + (F_+)_L (t_p - t_L) = 0
\]

\[
(t_p - t_R) - \left( \frac{1}{u-c} \right)_R (x_p - x_R) = 0
\]

\[
(Z_p - Z_R) - \frac{c_R}{g} (u_p - u_R) + (F_-)_R (t_p - t_R) = 0
\]

in which the subscript indicates the point in figure 5 to which each quantity is referred. These equations will be solved by the specified-time-interval scheme using \(\Delta t\) as the time interval and \(\Delta x\) as the segment length on a rectangular grid system.
By linear interpolation, the values of \( u_L, u_R, Z_L \) and \( Z_R \) may be expressed in terms of the corresponding values at points A, B, and C, as follows:

\[
\begin{align*}
u_L &= u_B[1-\theta(u+c)_B] + u_A \theta(u+c)_B \\
u_R &= u_B[1+\theta(u-c)_B] - u_C \theta(u-c)_B \\
Z_L &= Z_B[1-\theta(u+c)_B] + Z_A \theta(u+c)_B \\
Z_R &= Z_B[1+\theta(u-c)_B] - Z_C \theta(u-c)_B
\end{align*}
\]

in which \( \theta = \frac{\Delta t}{\Delta x} \) and the assumption that \((u+c)_L \approx (u+c)_B\), \((u-c)_R \approx (u-c)_B\), is made. From these values (equations 10-13) and further assuming that \( c_L \approx c_B \), \( c_R \approx c_B \), \((F_+)_L \approx (F_+)_B\), \((F_-)_R \approx (F_-)_B\),

\[
Z_P = \frac{c_B}{2g} (u_L - u_R) + \frac{1}{2}(Z_L + Z_R) - (uS_o - \frac{g}{B}B\Delta t)
\]

\[
u_P = \frac{1}{2}(u_L + u_R) + \frac{g}{2c_B}(Z_L - Z_R) - \left[\frac{gk}{4/3u} \right] \frac{u|u|}{A} + \frac{ug}{B}\Delta t
\]

In the multiple-reach scheme, the above equations have to be expressed in two-dimensional form, that is, two subscripts should be added to variables and parameters to indicate what nodal point or segment, in what subreach, they belong to. For example, if point B is on the \( i \)th node of the \( j \)th subreach, then \( u_B \) may be expressed as \( u_{i,j} \), \( Z_A \) as \( Z_{i-1,j} \), \( Z_P \) as \( Z_{i,j}^{k+1} \), \( u_P \) as \( u_{i,j}^{k+1} \) or \( u_{i,j}^{k+1} \), and so forth. The length of the \( j \)th subreach is expressed by \( \Delta L_j \). (See figs. 5 and 6; Point \( x_{i,j} \) is expressed as \( (x_{i,j})_i \) in fig. 5.)
FIGURE 5.—A rectangular net and characteristic curves for numerical solution by the multiple-reach method of characteristics.
FIGURE 6.—Schematization of a long nonuniform channel.

From Lai and Onions (1976, fig. 2)
Boundary conditions.—Four boundary equations, derived from equations 7 and 9, are: at the left boundary \((x = 0)\),

\[
    u_P = u_R + \frac{g}{c_B}(Z_P - Z_R) + \frac{g}{c_B}(F^-)_B \Delta t
\]  

or

\[
    Z_P = Z_R + \frac{c_B}{g}(u_P - u_R) - (F^-)_B \Delta t
\]  

at the right boundary \((x = L)\), \((L = \text{total reach length})\)

\[
    u_P = u_L - \frac{g}{c_B}(Z_P - Z_L) - \frac{g}{c_B}(F^+)_B \Delta t
\]  

or

\[
    Z_P = Z_L - \frac{c_B}{g}(u_P - u_L) - (F^+)_B \Delta t
\]  

One equation from each end of the reach \((x = 0 \text{ and } x = L)\) should be chosen for boundary value computation (for subcritical flow, of course). The choice depends on which type of boundary-value data (stage or velocity) is obtainable at each end.

For any junction between two subreaches, say the \((j + 1)\)th junction, the following four boundary equations may be used.

\[
    (Z_p)_{n+1,j} = (Z_p)_{1,j+1}
\]  

\[
    (u_P)_{n+1,j} = (u_P)_{1,j+1} \quad \text{i.e.} \quad Q_{n+1,j} = Q_{1,j+1}
\]  

\[
    (u_P)_{n+1,j} = (u_L)_{n+1,j} - \frac{g}{c_{n+1,j}}[(Z_p)_{n+1,j} - (Z_L)_{n+1,j}]
\]

\[
    - \frac{g}{c_{n+1,j}} (F^+)_{n+1,j} \Delta t
\]  

\[
    (u_P)_{1,j+1} = (u_R)_{1,j+1} + \frac{g}{c_{1,j+1}}[(Z_p)_{1,j+1} - (Z_R)_{1,j+1}]
\]

\[
    + \frac{g}{c_{1,j+1}} (F^-)_{1,j+1} \Delta t
\]  

in which \(Q\) is the discharge.
To solve these equations, a value $Z_p$ at the junction is estimated by quadratic extrapolation from the previous elevations. Substitute the assumed $Z_p$ value into both equations 22 and 23 and compute $(u_p)_{n+1,j}$ and $(u_p)_{1,j+1}$. Using the same $Z_p$ value, $(A_P)_{n+1,j}$ and $(A_P)_{1,j+1}$ can be evaluated. Then $(u_P A_p)_{n+1,j}$ and $(u_P A_p)_{1,j+1}$ are computed and the difference of the two is tested. The computation is iterated until the difference becomes less than a maximum tolerable error, $\varepsilon$.

**Computation Procedure**

The machine computation is carried out according to the following steps:

(a) Read reach geometry data.

(b) Organize or compile depth versus cross-sectional area, depth versus top width relationships.

(c) Read data describing the initial values and other parameters.

(d) Divide each subreach into $n$ segments (or elements) of length $\Delta x$ (that is, $\Delta x_j = \Delta L_j/n_j$), and call pivotal points between segments, including end points, as nodal points. Assign the initial values for $u$ and $Z$ at each nodal point throughout the entire reach.

(e) Prepare the boundary values for each end of the entire reach for designed time intervals.

(f) Estimate $(Z_p)_j$-values at each junction between subreaches.

(g) From upstream (left) to downstream (right) calculate $u_p$ and $Z_p$ for a time interval $2\Delta t$ at each nodal point for each subreach, using equations 10 through 19. At the upstream end (the left end of the 1st subreach) use equation 16 or 17 and at the downstream end (the right end of the last subreach) use 18 or 19 to calculate $u_p$ or $Z_p$, and at each junction use equations 22 and 23 to calculate $u_p$.

(h) Use equations 20 through 23 repeatedly to find satisfactory $u_p$ and $Z_p$ values at each junction. Call steps f through h as "m = 1 cycle".

(i) Calculate $u$ and $Z$ for a time interval $\Delta t$ using the same equations and following the same steps described in f through h. Call this step "m = 2 cycle".

(j) Using the results of step i, calculate $u$ and $Z$ for $\Delta t$ again. Call this step "m = 3 cycle".

(k) Repeat steps f through j as far as desired.

(l) Whenever appropriate, combine the results of g, h, and j, through the use of equation 24, and obtained extrapolated values of $u$ and $Z$.

When only the first-order approximation is required, steps i, j, and l are bypassed and only f through h are repeated; that is, only m = 1 cycle is needed.