UNITED STATES
DEPARTMENT OF THE INTERIOR
GEOLOGICAL SURVEY

TRAVELTIME, UNIT-CONCENTRATION, LONGITUDINAL-DISPERSION, AND REAERATION CHARACTERISTICS OF UPSTREAM REACHES OF THE YAMPA AND LITTLE SNAKE RIVERS, COLORADO AND WYOMING

WATER RESOURCES INVESTIGATIONS 78-122

50272-101

REPORT DOCUMENTATION PAGE

1. REPORT NO.	2.	3. Recipient's Accession No

4. Title and Subtitle
TRAVELTIME, UNIT-CONCENTRATION, LONGITUDINAL-DISPERSION, AND REAERATION CHARACTERISTICS OF UPSTREAM REACHES OF THE YAMPA AND LITTLE SNAKE RIVERS, COLORADO AND WYOMING

5. Report Date November 1979

6.

7. Author(s)
Daniel P. Bauer, Ronald E. Rathbun, and Hugh W. Lowham

8. Performing Organization Rept. No. USGS/WRI 78-122

9. Performing Organization Name and Address
U.S. Geological Survey, Water Resources Division
Denver Federal Center, Box 25046, Mail Stop 415
Lakewood, Colorado 80225

10. Project/Task/Work Unit No.

11. Contract(C) or Grant(G) No.
(C)
(G)

12. Sponsoring Organization Name and Address
U.S. Geological Survey, Water Resources Division
Denver Federal Center, Box 25046, Mail Stop 415
Lakewood, Colorado 80225

13. Type of Report & Period Covered Final

14.

15. Supplementary Notes

16. Abstract (Limit: 200 words)
Two traveltime, unit-concentration, and dispersion analyses were made along a 58-mile reach of the Yampa River in Colorado when the average streamflow was approximately 100 and 3,400 cubic feet per second; three traveltime, unit-concentration, and dispersion analyses were made along a 72-mile reach of the Little Snake River in Wyoming and Colorado when the average streamflow was approximately 200, 600, and 1,600 cubic feet per second. Reaeration coefficients also were determined for the Yampa River reach when the average streamflow was approximately 100 cubic feet per second.

Traveltime and unit-concentration simulations were made using a mathematical model. Data collected for the Little Snake River when the average streamflow was approximately 600 cubic feet per second were used as a check of model-simulation accuracy. Traveltime simulations compared to within 5 percent, and unit-concentration simulations were within 30 to 40 percent of the measured flow data. Longitudinal-dispersion coefficients ranged from 400 to 6,050 square feet per second for the two streams. Reaeration coefficients for the Yampa River, adjusted to 20 degrees Celsius, ranged from 6.04 to 33.4 day^{-1}.

17. Document Analysis a. Descriptors

Colorado, Wyoming, Velocity, Streamflow, Simulation analysis, Dispersion, Aeration, Computer model, Regression analysis, Shear drag, Tracers, Dye releases

b. Identifiers/Open-Ended Terms

Traveltime, Longitudinal-dispersion coefficient, Reaeration coefficient, Stream-reaeration capacity, Unit concentration, Yampa River basin (Colo.-Wyo.)

c. COSATI Field/Group

18. Availability Statement	19. Security Class (This Report)	21. No. of Pages
No restriction on distribution		73
	20. Security Class (This Page)	22. Price

(See ANSI-Z39.18) See *Instructions on Reverse* OPTIONAL FORM 272 (4-77) (Formerly NTIS-35) Department of Commerce

TRAVELTIME, UNIT-CONCENTRATION, LONGITUDINAL-DISPERSION, AND REAERATION CHARACTERISTICS OF UPSTREAM REACHES OF THE YAMPA AND LITTLE SNAKE RIVERS, COLORADO AND WYOMING

By Daniel P. Bauer, Ronald E. Rathbun, and Hugh W. Lowham

U.S. GEOLOGICAL SURVEY

Water-Resources Investigations 78-122

November 1979

UNITED STATES DEPARTMENT OF THE INTERIOR

CECIL D. ANDRUS, Secretary

GEOLOGICAL SURVEY

H. William Menard, Director

For additional information write to:

District Chief
U.S. Geological Survey
Mail Stop 415
Box 25046, Denver Federal Center
Lakewood, CO 80225

CONTENTS

	Page
Metric conversion table	VI
Abstract	1
Introduction	2
Purpose and scope	2
Acknowledgments	3
Location and extent of study reaches	3
Determination of traveltime, unit concentration, and longitudinal dispersion	5
Dye-tracer technique	5
Methodology for simulating traveltime and unit concentration	15
Traveltime results	17
River conditions during traveltime measurements	18
Traveltime simulations using index-discharge stations and computer-model techniques	18
Traveltime simulations using linear-regression relationships	27
Traveltime simulations using graphical relationships	29
Unit-concentration simulation results	29
Longitudinal-dispersion coefficients	35
Determination of reaeration coefficients using a modified tracer technique	36
Peak method	39
Area method	40
Experimental procedure	41
Injection of the tracers	41
Sampling the tracers	42
Sample analysis	42
Reaeration-coefficient results	42
Dye loss, flow accrual, and dye-correction factors	46
Computation of reaeration coefficients	48
Comparisons of results to empirical formulas	49
Semiempirical (energy-dissipation) equations	51
Empirical (velocity-depth) equations	54
Result comparisons of measured reaeration coefficients versus predicted values using empirical and semiempirical equations	55
Comparison of traveltime and reaeration coefficients with results of analysis of waste-load assimilative capacity	55
Applications	60
Summary	62
Selected references	64

ILLUSTRATIONS

		Page
Figures 1-3.	Maps showing:	
	1. Location of study reaches in the Yampa River basin.	4
	2. Location of injection and sampling sites along the study reach of the Yampa River.	7
	3. Location of injection and sampling sites along the study reach of the Little Snake River	9
4.	Graph and sketch showing downstream movement and dispersion of dye cloud, Little Snake River, May 1977 .	11
5-8.	Graphs showing:	
	5. Mean velocity, traveltime of dye clouds, and river profile for the Yampa River, June and September 1976.	19
	6. Mean velocity, traveltime of dye clouds, and river profile for the Little Snake River, June and September 1976, and May 1977.	20
	7. Mean velocity and shear velocity versus discharge, Yampa River at Steamboat Springs, Colo., station 09239500, and below diversion, near Hayden, Colo., station 09244410	21
	8. Mean velocity and shear velocity versus discharge, Little Snake River near Slater, Colo., station 09253000, and near Dixon, Wyo., station 09257000.	22
9-12.	Simulated cumulative traveltime curves:	
	9. For the Yampa River, using index station 09239500, Yampa River at Steamboat Springs, Colo.	23
	10. For the Yampa River, using index station 09244410, Yampa River below diversion, near Hayden, Colo. .	24
	11. For the Little Snake River, using index station 09253000, Little Snake River near Slater, Colo. .	25
	12. For the Little Snake River, using index station 09257000, Little Snake River near Dixon, Wyo. . .	26
13.	Cumulative traveltime curves for the Yampa River, based on linear-regression relationships	28
14.	Cumulative traveltime curves for the Little Snake River, based on mean velocity versus discharge relationships	30
15-18.	Simulated unit-concentration curves:	
	15. For the Yampa River, using index station 09239500, Yampa River at Steamboat Springs, Colo.	31
	16. For the Yampa River, using index station 09244410, Yampa River below diversion, near Hayden, Colo. .	32
	17. For the upper Little Snake River, 16 miles east of Slater, Colo., to Dixon, Wyo., using index station 09253000, Little Snake River near Slater, Colo.	33

CONTENTS

Page

Figures 15-18. Simulated unit-concentration curves--Continued
18. For the lower Little Snake River, Dixon, Wyo., to 33 miles southwest of Baggs, Wyo., using index station 09257000, Little Snake River near Dixon, Wyo. 34
19-22. Graphs showing:
19. Sample computation of the longitudinal-dispersion coefficient for the Yampa River, 5 miles southeast of Steamboat Springs to Milner, Colo 37
20. Concentrations of ethylene and propane tracer gases, and rhodamine-WT dye, Yampa River, river-mile 191.0 to 185.2, September 1976. . . 43
21. Concentrations of ethylene and propane tracer gases, and rhodamine-WT dye, Yampa River, river-mile 183.4 to 177.3, September 1976. . . 44
22. Concentrations of ethylene and propane tracer gases, and rhodamine-WT dye, Yampa River, river-mile 175.5 to 171.0, September 1976. . . 45

TABLES

Table 1. Injection and sampling sites, Yampa River, June and September 1976 6
2. Injection and sampling sites, Little Snake River, June and September 1976, and May 1977 8
3. Data collected during traveltime measurements. 12
4. Longitudinal-dispersion coefficients for varying streamflow conditions, Yampa and Little Snake Rivers 38
5. Data determined from curves of concentrations versus time for ethylene and propane tracer gases, and rhodamine-WT dye, Yampa River, September 1976 47
6. Peak concentrations and traveltime of peak concentrations for ethylene and propane tracer gases, and rhodamine-WT dye, Yampa River, September 1976 49
7. Reaeration coefficients for ethylene and propane determined using area and peak computation methods, selected subreaches of Yampa River, September 1976 50
8. Geometry of selected subreaches of the Yampa River, and traveltime and velocity data collected in the subreaches during September 1976. 51
9. Comparison between reaeration coefficients determined using measured data (September 1976) and those determined using energy-dissipation equations, selected subreaches of the Yampa River. 56

CONTENTS

Page

Table 10. Comparison between reaeration coefficients determined using measured data (September 1976) and those determined using velocity-depth equations, selected subreaches of the Yampa River. 57

11. Error analysis of reaeration coefficients. 58

12. Comparison between traveltime simulations determined by Bauer, Steele, and Anderson (1978) and those determined during this study using rhodamine-WT dye, Yampa River, Steamboat Springs to Hayden, Colo. 59

METRIC CONVERSION TABLE

Inch-pound units used in this report may be converted to metric units by the following conversion factors:

Multiply inch-pound unit	*By*	*To obtain metric unit*
foot (ft)	0.3048	meter (m)
mile (mi)	1.609	kilometer (km)
square foot per second (ft^2/s)	0.0929	square meter per second (m^2/s)
cubic foot per second (ft^3/s)	0.02832	cubic meter per second (m^3/s)
mile per hour (mi/h)	1.609	kilometer per hour (km/h)
foot per second (ft/s)	0.3048	meter per second (m/s)
fluid ounce	29.57	milliliter (mL)
foot per mile (ft/mi)	0.1894	meter per kilometer (m/km)
pound (lb)	0.4536	kilogram (kg)

TRAVELTIME, UNIT-CONCENTRATION, LONGITUDINAL-DISPERSION, AND REAERATION CHARACTERISTICS OF UPSTREAM REACHES OF THE YAMPA AND LITTLE SNAKE RIVERS, COLORADO AND WYOMING

By Daniel P. Bauer, Ronald E. Rathbun, and Hugh W. Lowham

ABSTRACT

Measurements in the Yampa and the Little Snake Rivers were made to determine traveltime, unit-concentration, and longitudinal-dispersion characteristics of upstream reaches during high- and low-streamflow conditions. Reaeration coefficients were measured for the Yampa River, using a modified tracer technique to quantify the process of reaeration by which the stream replaces the dissolved oxygen consumed in the oxidation of organic wastes. Stream reaches studied included a 58-mile (93-kilometer) reach of the Yampa River and a 77-mile (124-kilometer) reach of the Little Snake River. Field data were collected during June and September 1976, and May 1977.

Two traveltime measurements using a fluorescent dye were made on the Yampa River when discharges were approximately 100 and 3,400 cubic feet per second (2.8 and 5.2 cubic meters per second), and three traveltime measurements were made on the Little Snake River when discharges were approximately 20, 600, and 1,600 cubic feet per second (0.56, 16.8, and 44.8 cubic meters per second). Measured stream velocities varied as follows: 0.26 to 3.6 miles per hour (0.42 to 5.8 kilometers per hour) for the Yampa River and 0.04 to 3.5 miles per hour (0.06 to 5.6 kilometers per hour) for the Little Snake River.

Simulations of traveltime and unit concentrations for the Little Snake River were made using a mathematical model. Medium-flow data (600 cubic feet per second or 16.8 cubic meters per second) were used as a check of the model accuracy. Traveltime simulations compared within 5 percent, and unit-concentration simulations were within 30 to 40 percent of the measured flow data. An analysis of the waste-load assimilative capacity of the Yampa River, completed earlier, used traveltime estimates based on cross-sectional properties of streams and velocity and linear-regression relationships. A comparison of the traveltime simulations from this study agreed within 6 percent of the estimates from the waste-load analysis.

Longitudinal-dispersion coefficients were computed for low- and high-flow conditions in the Yampa River and for medium- and high-flow conditions in the Little Snake River. Longitudinal-dispersion coefficients ranged from 400 to 6,050 square feet per second (37.2 to 560 square meters per second) for the two rivers.

Reaeration coefficients measured for low-flow conditions on the Yampa River, and adjusted to 20 degrees Celsius, ranged from 6.04 to 33.4 day^{-1}. Two semiempirical (energy-dissipation) equations gave coefficients in best agreement with the measured coefficients. Absolute errors of estimate for these equations were 11.8 and 17.3 percent.

Management activities that could use results of this study include predicting arrival time and concentration of soluble contaminants accidentally spilled in streams, length of stream affected by a municipal wastewater discharge, and traveltime required for reservoir water released for given downstream needs.

INTRODUCTION

The Yampa River basin is undergoing substantial economic development, spurred predominantly by mining, transport, and conversion of coal resources in the basin. The impacts of this development pose existing and potential stresses on the basin's limited water resources. This report is part of a series of multidisciplinary studies conducted by the U.S. Geological Survey in the Yampa River Basin Assessment project (Steele and others, 1976a, 1976b). The multidisciplinary studies include a wide range of existing conditions and anticipated changes in the availability and uses of the basin's water resources (Steele and others, 1979).

Impending population growth resulting from energy development and greater recreational use in areas along principal streams of the Yampa River basin may increase the discharge of wastes to these streams. A knowledge of existing streamflow conditions will aid State and local officials in determining how fast wastes move downstream, how they are dispersed vertically and horizontally in streams, and how rapidly streams can assimilate certain forms of treated wastes (Bauer and others, 1978).

Purpose and Scope

The first purpose of the study was to determine traveltime, unit-concentration, and longitudinal-dispersion characteristics for specified reaches of the Yampa and the Little Snake Rivers for a range of stream-discharge conditions. A second purpose was to measure reaeration coefficients (K_2) for three reaches of the Yampa River and then to compare measured K_2 values with those computed using various empirical equations.

Studies were conducted on the following stream reaches: (1) The Yampa River from about 5 mi (8 km) southeast of Steamboat Springs, Colo., downstream to Craig, Colo., and (2) the Little Snake River from 16 mi (26 km) east of Slater, Colo., downstream to 33 mi (53 km) southwest of Baggs, Wyo. (fig. 1). Studies of the Yampa River included determinations of traveltime, unit-concentration, longitudinal-dispersion, and reaeration coefficients. The study of the Little Snake River included only determinations of traveltime, unit concentration, and longitudinal dispersion.

This report describes results of an analysis of two sets of traveltime, unit-concentration, and longitudinal-dispersion data for the Yampa River and three sets for the Little Snake River. Determinations of traveltime were made for high-flow (June) conditions to low-flow (September) conditions. These data were used to develop approximate relationships of traveltime and unit concentration versus discharge for each of the stream reaches. Longitudinal-dispersion coefficients were also determined for each of the stream reaches.

Reaeration coefficients were measured for low-flow (September) conditions in the Yampa River study reach, using a modified tracer technique. The measured reaeration coefficients were compared with various empirically determined coefficients and the equation providing the best comparison was determined.

Acknowledgments

The authors thank the many residents along the Yampa and the Little Snake Rivers for permitting access to stream sites during periods of field-data collection for these studies. The authors also are grateful to Philip E. Stark, Routt County Department of Environmental Health, and David J. Shultz, U.S. Geological Survey, for assistance in the planning and collection of the data. The contributions of James F. Kircher, U.S. Geological Survey, Cheyenne, Wyo., in the preparation of this report are gratefully acknowledged. Assistance in collecting the data provided by personnel of the Colorado and Wyoming offices of the U.S. Geological Survey is appreciated. Laboratory assistance of Doreen Y. Tai of the U.S. Geological Survey in the determination of the tracer-gas concentrations for the reaeration study is also gratefully acknowledged.

LOCATION AND EXTENT OF STUDY REACHES

The general location and extent of the study reaches of the Yampa and the Little Snake Rivers are shown on figure 1. The length of the study reach along the Yampa River is 58 mi (93 km), and the study reach along the Little Snake River is 77 mi (124 km). Data used for the studies reported here were collected during June and September 1976, and May 1977. During September, flow conditions for the two rivers generally can be characterized as low with approximately 90-percent duration (flow duration is defined as the percentage of time the flow was equaled or exceeded), and as high during June with approximately 10-percent duration. During September, only 20 to 30 percent of each stream reach was sampled, due to the low streamflow velocities. A third

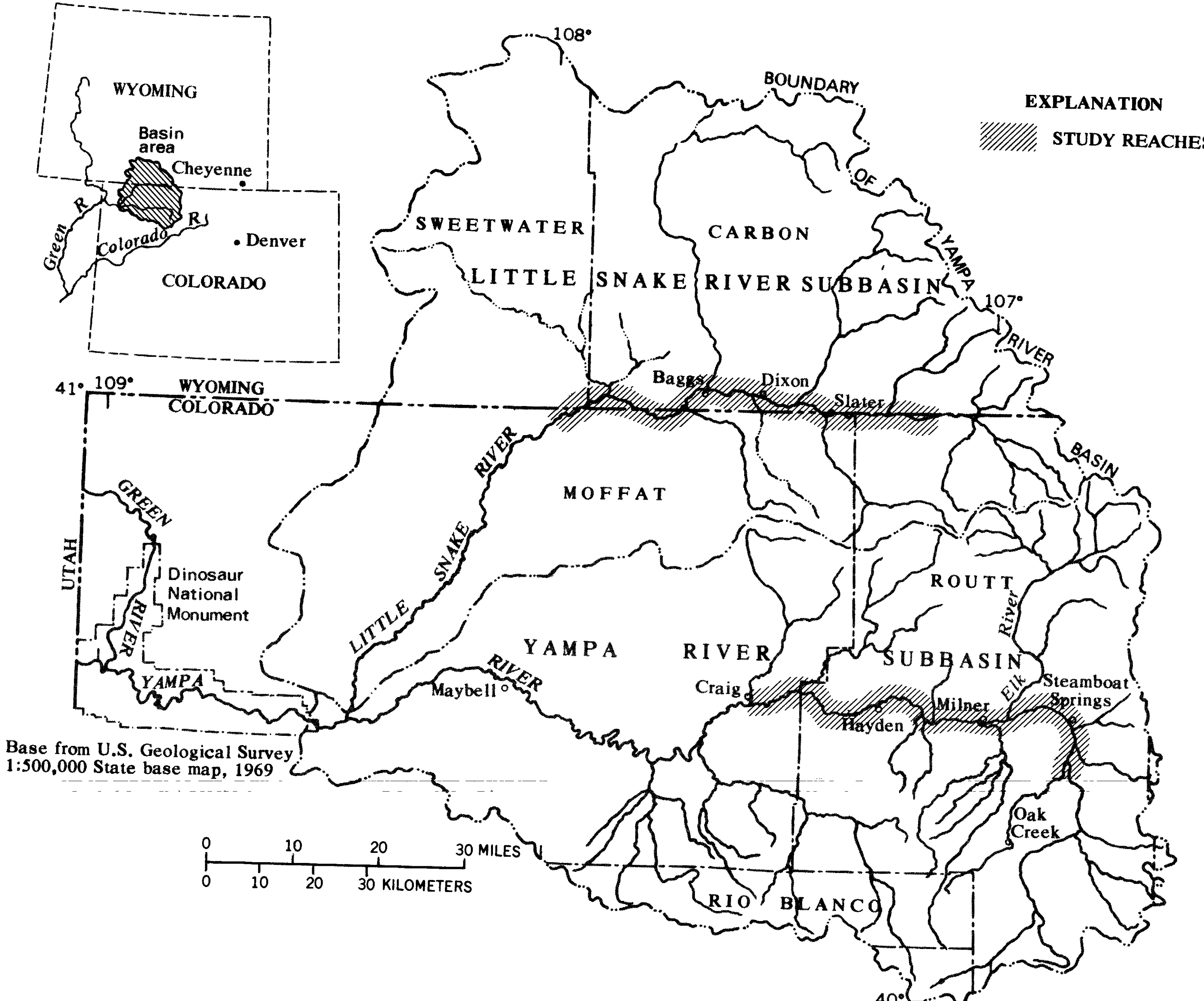

Figure 1.--Location of study reaches in the Yampa River basin.

field measurement was made on the Little Snake River during May 1977 for a medium-flow condition with approximately 25-percent duration. Because of resource constraints, a similar May 1977 measurement was not done for the Yampa River.

A general summary of the injection and sampling sites for the various measurements are contained in table 1 and on figure 2 for the Yampa River and in table 2 and on figure 3 for the Little Snake River. Many of the injection- and sampling-site locations described in tables 1 and 2 also were used in other studies of the Yampa River Basin Assessment (Steele and others, 1976a). The "Ym" reference code used in table 1 refers to a sampling site used in an analysis of the waste-load assimilative capacity of the Yampa River (Bauer and others, 1978) and the "Y" reference code used in tables 1 and 2 refers to a sampling site used in a reconnaissance study to determine the quality of surface water in the Yampa River basin (Wentz and Steele, 1978).

DETERMINATION OF TRAVELTIME, UNIT CONCENTRATION, AND LONGITUDINAL DISPERSION

Traveltime, unit-concentration, and longitudinal-dispersion characteristics of a stream vary with the magnitude of its flow. Measurements of the rate of movement and dispersion of a substance injected into a stream are necessary to define these characteristics throughout a range of flows of interest. Two series of measurements on the Yampa River and three series of measurements on the Little Snake River were made at different flow conditions. A mathematical model then was used to estimate the characteristics for magnitudes of flow other than those measured.

Dye-Tracer Technique

The measurements of traveltime, unit concentration, and longitudinal dispersion were made by injecting a fluorescent dye, rhodamine-WT, into the river and tracing the shape and speed of the resultant dye cloud as it moved downstream. Dye is a solute when injected into the water; that is, it mixes completely with the water and it moves in the same manner as the water molecules. Measurement of the movement, concentration, and dispersion of the dye cloud depicts the characteristics of other soluble contaminants that might be introduced into the stream. An extensive description of the methods, procedures, dyes, and equipment used in making measurements of traveltime and dispersion has been completed (E. F. Hubbard, F. A. Kilpatrick, L. A. Martens, and J. F. Wilson, written commun., 1978).

The dye was injected at several locations along both the Yampa and the Little Snake Rivers. The movements of the resultant dye clouds were monitored at 14 sampling sites along the Yampa River (table 1 and fig. 2) and at 10 sites along the Little Snake River (table 2 and fig. 3). The samples

Table 1.--*Injection and sampling sites, Yampa River, June and September 1976*

Site number	Reference code[1]	Flow conditions[2]	Site type[3]	Distance from mouth (river miles)	Name
1	Ym-0, Y-65A	H	I	195.5	Yampa River below Oak Creek, near Steamboat Springs, Colo.
2	-----------	L	I	191.0	Yampa River at railroad crossing, above Steamboat Springs, Colo.
3	Ym-1	H,L	S	190.5	Yampa River at Steamboat Springs, Colo.
4	Ym-2	L	S	189.3	Yampa River above wastewater-treatment plant, below Steamboat Springs, Colo.
5	Ym-4	L	S	186.7	Yampa River below KOA campground, near Steamboat Springs, Colo.
6	Ym-5	L	S	185.2	Yampa River below Steamboat II development, near Steamboat Springs, Colo.
7	Ym-6, Y-63	H,L	I,S	183.4	Yampa River above Elk River, near Milner, Colo.
8	-----------	L	S	182.0	Yampa River 0.4 mile above Elk River, near Milner, Colo.
9	-----------	L	S	180.1	Yampa River 1.6 miles below Elk River, near Milner, Colo.
10	Ym-8	H	I,S	179.0	Yampa River at Milner, Colo.
11	Ym-9, Y-50	L	S	177.3	Yampa River below Trout Creek, at Milner, Colo.
12	-----------	L	I	175.5	Yampa River 3.5 miles below Milner, Colo.
13	Ym-10	L	S	174.6	Yampa River above Tow Creek oilfield, near Milner, Colo.
14	-----------	L	S	171.0	Yampa River above diversion, near Hayden, Colo.
15	Ym-11, Y-47	H	S	169.2	Yampa River below diversion, near Hayden, Colo.
16	Ym-13,Y-45	H	S	159.6	Yampa River at Hayden, Colo.
17	Y-39	H	S	137.3	Yampa River at Craig, Colo.

[1]Ym, sampling site used in a study to determine the capacity of the Yampa River to assimilate wastes (Bauer and others, 1978); Y, sampling site used in a reconnaissance study to determine the quality of surface water in the Yampa River basin (Wentz and Steele, 1978).

[2]H, high flow, June 1976; L, low flow, September 1976.

[3]I, injection site; S, sampling site.

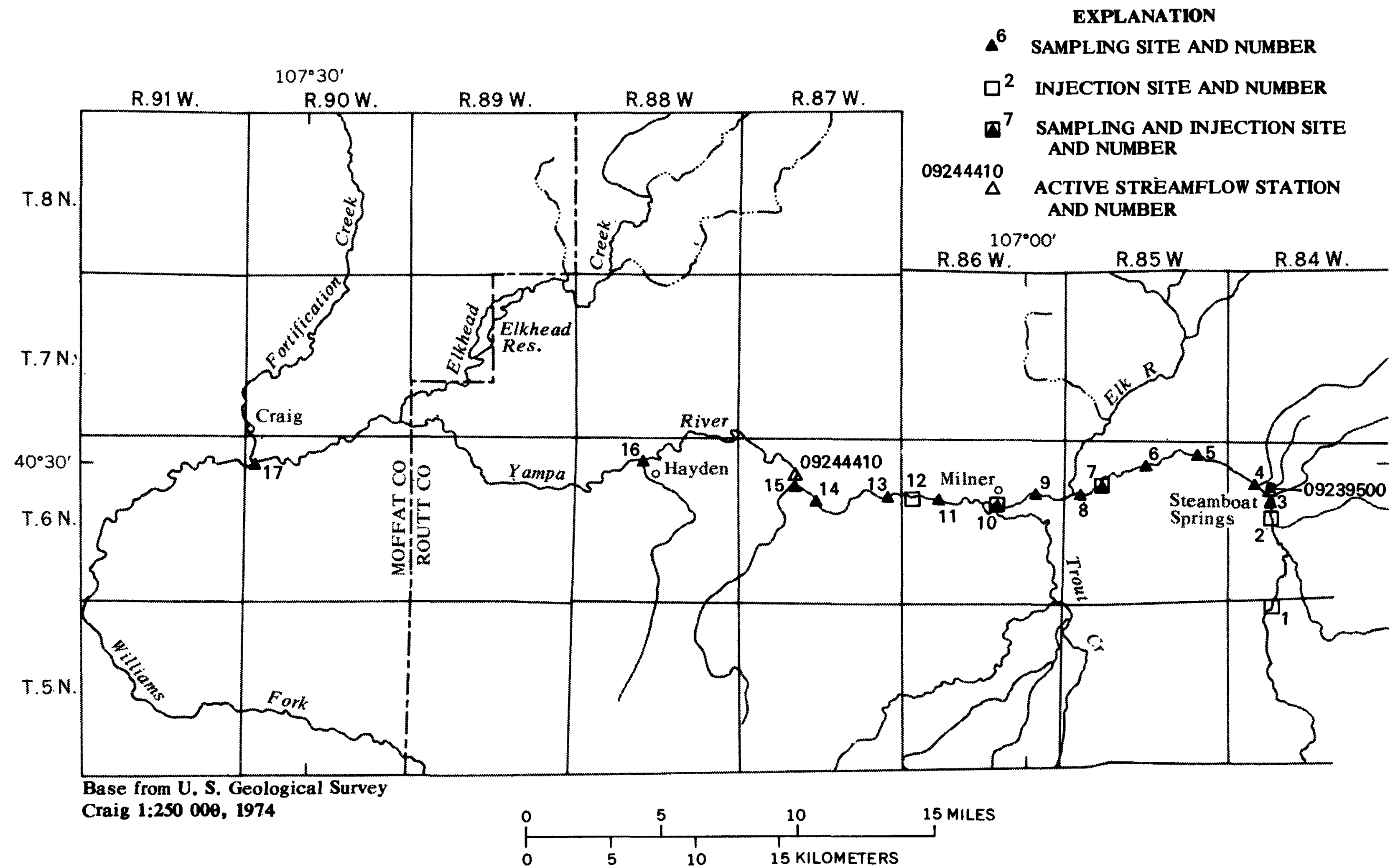

Figure 2.--Location of injection and sampling sites along the study reach of the Yampa River.

Table 2.--*Injection and sampling sites, Little Snake River, June and September 1976, and May 1977*

Site number	Reference code[1]	Flow conditions[2]	Site type[3]	Distance from mouth (river miles)	Name
1	Y-12	H,M,L	I	154.7	Little Snake River 5.4 miles below North Fork Little Snake River, near Slater, Colo.
2	----	M,L	S	151.3	Little Snake River 8.8 miles below North Fork Little Snake River, near Slater, Colo.
3	----	H,M	S	148.7	Little Snake River 10.2 miles above Slater, Colo.
4	----	M,L	I,S	143.4	Little Snake River 4.9 miles above Slater, Colo.
5	----	M,L	S	138.5	Little Snake River at Slater, Colo.
6	----	H,M	S	137.0	Little Snake River 1.5 miles below Slater, Colo.
7	----	L	I	121.2	Little Snake River at Dixon, Wyo.
8	Y-8	H,M,L	I,S	120.3	Little Snake River near Dixon, Wyo.
9	----	H,M,L	I,S	110.7	Little Snake River at Baggs, Wyo.
10	----	L	S	107.2	Little Snake River 3.5 miles below Baggs, Wyo.
11	----	L	I	102.8	Little Snake River 7.9 miles below Baggs, Wyo.
12	----	L	S	99.8	Little Snake River 10.9 miles below Baggs, Wyo.
13	Y-2	H,M	S	77.4	Little Snake River near Baggs, Wyo.

[1]Y, sampling site used in a reconnaissance study to determine the quality of surface water in the Yampa River basin (Wentz and Steele, 1978).

[2]H, high flow, June 1976; M, medium flow, May 1977; L, low flow, September 1976.

[3]I, injection site; S, sampling site.

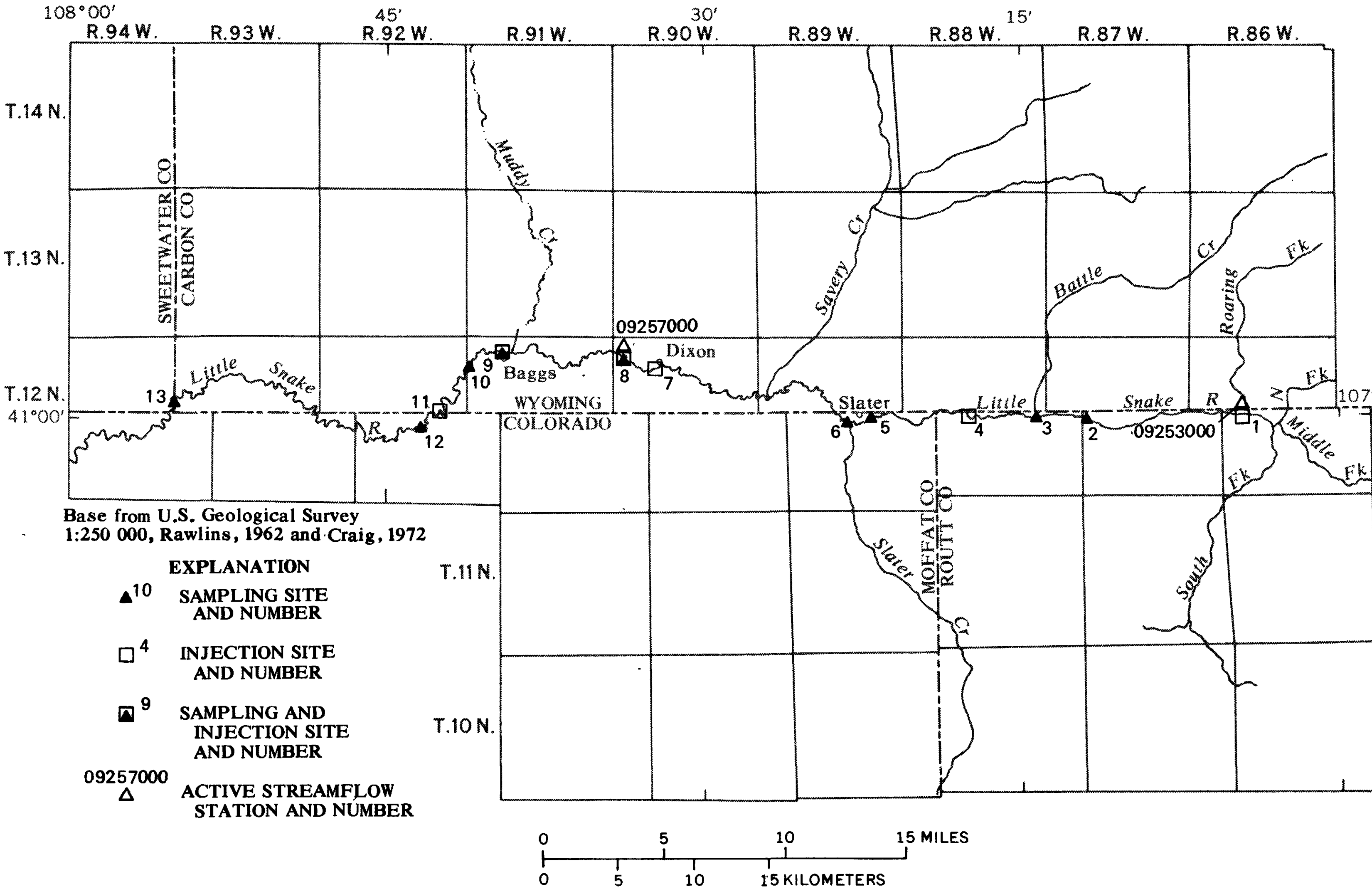

Figure 3.--Location of injection and sampling sites along the study reach of the Little Snake River.

collected at each site were analyzed using a fluorometer (Wilson, 1968), which is an instrument that measures fluorescence. The amount of fluorescence measured is directly proportional to the concentration of dye contained in each sample.

As the dye clouds traveled downstream, they dispersed, taking longer to pass each successive site, while the peak concentrations gradually decreased. As an example, a graph and sketch depicting the downstream movement and dispersion of the dye cloud for the May 1977 measurement on the Little Snake River are shown in figure 4. During the May 1977 measurement, the dye cloud took 1.75 hours to pass sample site 3 (table 2) with a peak concentration of 4.00 μg/L (micrograms per liter). When the dye cloud reached site 6, it took 3.67 hours to pass that sampling site, and the peak concentration had decreased to 1.10 μg/L (table 3). The times required for the dye clouds to pass each site, from the arrival of the leading edge to the approximate trailing edge of detectable concentrations of dye, are listed in table 3. Different combinations of injection and sampling sites were used for each of the measurements. In general, shorter subreaches were used during the low-flow measurements. The lower limit of detectability of the dye is about 0.05 μg/L.

The traveltime of the leading edge of the dye cloud can be estimated by using the approximate relation (E. F. Hubbard, F. A. Kilpatrick, L. A. Martens, and J. F. Wilson, written commun., 1978):

$$T_1 = \frac{T_p}{1.25}, \qquad (1)$$

where T_1=traveltime of leading edge, and
T_p=traveltime of peak concentration.

The sketch in figure 4 indicates the lateral and longitudinal mixing patterns of the dye cloud as it moves downstream from the injection site. As noted by Hubbard, Kilpatrick, Martens, and Wilson (written commun., 1978), "The mixing action, or dispersion, of the tracer in the receiving stream takes place in all three dimensions of the channel. Complete mixing normally occurs first in the vertical direction. Lateral mixing is completed later depending upon the width of the stream and velocity variations. Longitudinal dispersion, having no boundaries, continues indefinitely and is the dispersion component of primary interest." As noted in the sketch, dye-tracer particles at the center of the stream travel faster than those near the edges. The actual distributions of dye concentrations versus time for sites 2, 3, 4, and 5 on the Little Snake River, shown in figure 4, are based on data collected at approximately the center of the stream.

Traveltime-data presentations are enhanced by reporting in terms of unit concentrations. Unit concentration is defined by Hubbard, Kilpatrick, Martens, and Wilson (written commun., 1978) as the concentration, in micrograms per second of flow, produced in 1 cubic foot per second of flow due to the injection of 1 pound of a conservative solute.

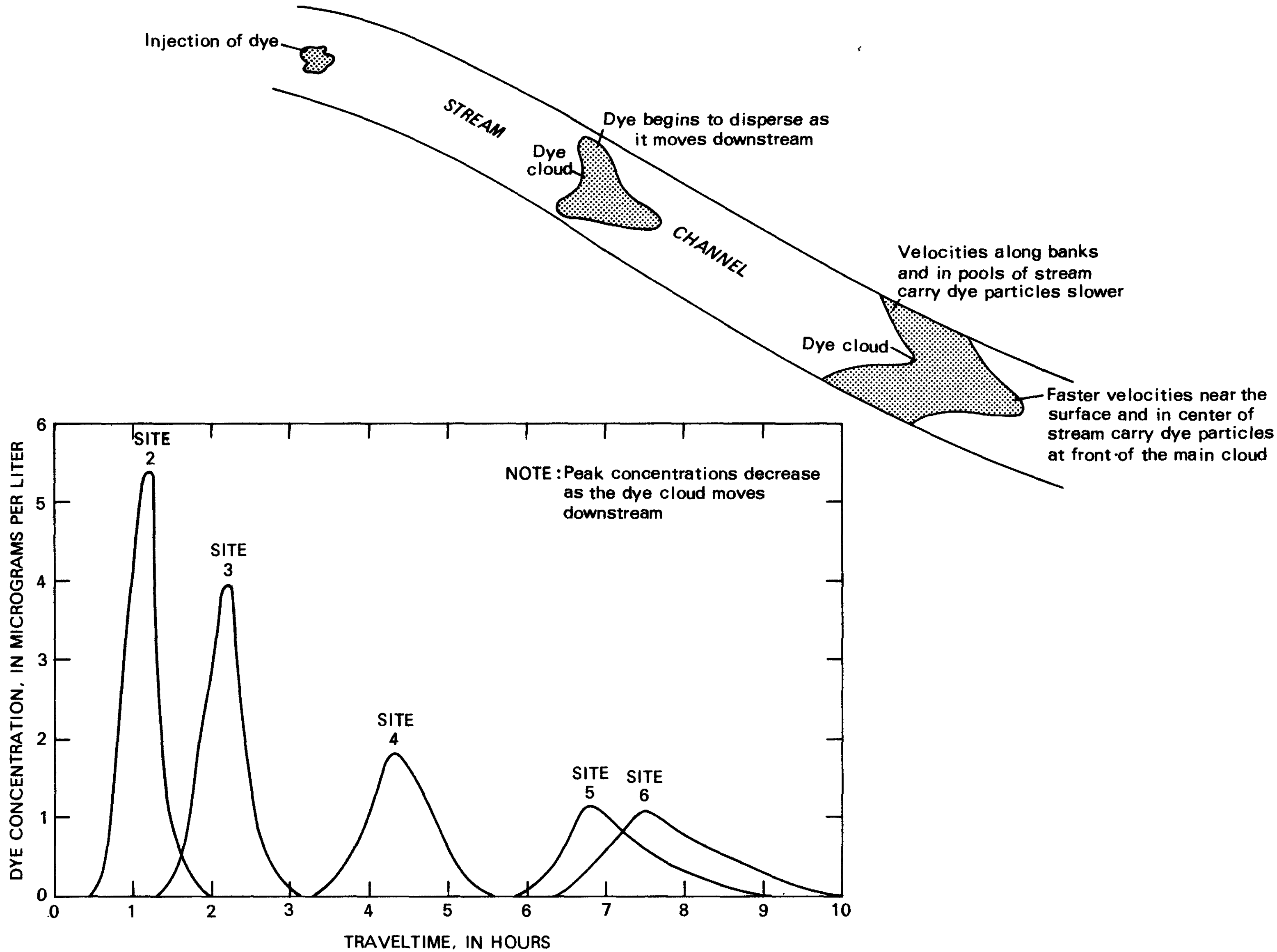

Figure 4.--Downstream movement and dispersion of dye cloud, Little Snake River, May 1977.

Table 3.--*Data collected during traveltime measurements*

Site number	Distance downstream from point of injection (miles)	Elevation of streambed (feet)	Stream discharge (cubic feet per second)	Cumulative traveltime of dye clouds: Leading edge (hours)	Cumulative traveltime of dye clouds: Peak (hours)	Mean velocity of dye cloud (miles per hour)	Time for dye cloud to pass site (hours)	Peak dye concentration (micrograms per liter)
Yampa River, traveltime measurement 1, slug injection of 1,000 milliliters of 20-percent dye solution at 2100 hours, June 3, 1976, at site 1								
1	0.0	6,780	2,300	-----	-----	----	-----	------
3	5.0	6,698	2,300	1.70	2.00	2.24	2.25	1.68
7	12.1	6,545	2,380	3.70	4.10	3.60	2.25	.93
10	16.5	6,485	4,560	4.80	5.50	3.03	2.50	.34
Yampa River, traveltime measurement 1, slug injection of 2,910 milliliters of 20-percent dye solution at 2230 hours, June 2, 1976, at site 10								
10	0.0	6,485	4,560	-----	-----	----	-----	------
15	9.8	6,395	4,470	2.50	2.95	3.13	2.35	2.20
16	19.4	6,315	4,470	5.40	6.00	2.97	3.40	1.10
17	41.7	6,165	4,610	12.4	13.6	2.86	5.00	.44
Yampa River, traveltime measurement 2, continuous injection of 4,200 milliliters of 2.73-percent dye solution for 90 minutes beginning at 0915 hours, September 21, 1976, at site 2								
2	0.0	6,715	78.5	-----	-----	----	-----	------
3	.50	6,698	78.5	0.50	1.44	0.36	2.40	10.7
4	1.70	6,670	76.3	1.50	2.60	.93	3.17	9.56
5	4.30	6,620	83.1	4.00	5.55	----	3.67	6.74
6	5.80	6,574	72.2	7.50	10.0	.46	8.33	4.01
Yampa River, traveltime measurement 2, continuous injection of 4,040 milliliters of 4.0-percent dye solution for 59 minutes beginning at 0931 hours, September 22, 1976, at site 7								
7	0.0	6,540	73.0	-----	-----	----	-----	------
8	1.4	6,525	73.0	1.67	2.48	0.55	3.00	14.7
9	3.2	6,501	152	3.83	4.83	.68	5.33	4.51
11	6.1	6,470	171	7.17	8.73	.68	7.83	2.53
Yampa River, traveltime measurement 2, continuous injection of 4,040 milliliters of 3.6-percent dye solution for 85 minutes beginning at 0943 hours, September 23, 1976, at site 12								
12	0.0	6,450	204	-----	-----	----	-----	------
13	.9	6,443	204	0.67	1.70	0.58	2.50	5.01
14	5.1	6,410	210	5.00	6.80	.78	6.00	3.21

Table 3.--*Data collected during traveltime measurements*--Continued

Site number	Distance downstream from point of injection (miles)	Elevation of streambed (feet)	Stream discharge (cubic feet per second)	Cumulative traveltime of dye clouds		Mean velocity of dye cloud (miles per hour)	Time for dye cloud to pass site (hours)	Peak dye concentration (micrograms per liter)
				Leading edge (hours)	Peak (hours)			
Little Snake River, traveltime measurement 1, slug injection of 1,500 milliliters of 20-percent dye solution at 2300 hours, June 1, 1976, at site 1								
1	0.0	6,760	1,400	-----	-----	----	-----	------
3	6.8	6,660	1,400	1.33	1.63	4.00	1.00	7.20
6	17.7	6,510	1,620	3.75	5.10	3.07	2.80	2.00
8	32.4	6,340	2,070	8.00	10.2	2.71	6.50	.78
Little Snake River, traveltime measurement 1, slug injection of 1,500 milliliters of 20-percent dye solution at 0000 hours, June 1, 1976, at site 8								
8	0.0	6,340	1,690	-----	-----	----	-----	------
9	9.6	6,220	1,690	2.7	3.12	3.01	2.50	2.80
13	42.9	6,050	1,690	14.6	16.3	2.48	4.00	.78
Little Snake River, traveltime measurement 2, slug injection of 500 milliliters of 20-percent dye solution at 0920 hours, September 30, 1976, at site 1								
1	0.0	6,760	20.0	-----	-----	----	-----	------
2	3.4	6,740	20.0	6.33	7.83	0.43	5.00	45.0
Little Snake River, traveltime measurement 2, slug injection of 500 milliliters of 20-percent dye solution at 2210 hours, September 29, 1976, at site 4								
4	0.0	6,580	30.0	-----	-----	----	-----	------
5	4.8	6,540	30.0	13.8	18.1	0.25	15.0	52.0
Little Snake River, traveltime measurement 2, slug injection of 500 milliliters of 20-percent dye solution at 2200 hours, September 28, 1976, at site 7								
7	0.0	6,350	30.0	-----	-----	----	-----	------
8	.9	6,340	30.0	11.8	19.3	0.04	35.0	195
Little Snake River, traveltime measurement 2, slug injection of 500 milliliters of 20-percent dye solution at 2200 hours, September 27, 1976, at site 9								
9	0.0	6,260	32.4	-----	-----	----	-----	------
10	3.5	6,220	32.4	9.50	13.0	0.29	12.5	92.0

Table 3.--*Data collected during traveltime measurements*--Continued

Site number	Distance downstream from point of injection (miles)	Elevation of streambed (feet)	Stream discharge (cubic feet per second)	Cumulative traveltime of dye clouds: Leading edge (hours)	Cumulative traveltime of dye clouds: Peak (hours)	Mean velocity of dye cloud (miles per hour)	Time for dye cloud to pass site (hours)	Peak dye concentration (micrograms per liter)
Little Snake River, traveltime measurement 2, slug injection of 500 milliliters of 20-percent dye solution at 0000 hours, September 27, 1976, at site 11								
11	0.0	6,200	25.0	-----	-----	----	-----	------
12	3.0	6,180	25.0	13.0	23.8	0.14	19.5	64.0
Little Snake River, traveltime measurement 3, slug injection of 500 milliliters of 20-percent dye solution at 0552 hours, May 18, 1977, at site 1								
1	0.0	6,760	596	-----	-----	----	-----	------
2	3.4	6,740	596	0.47	1.22	2.66	1.58	5.45
3	6.0	6,670	596	1.30	2.20	2.52	1.75	4.00
4	11.3	6,580	711	3.33	4.30	2.48	2.17	1.85
5	16.2	6,540	711	5.80	6.80	1.78	3.33	1.20
6	17.7	6,495	711	6.33	7.47	2.24	3.67	1.10
Little Snake River, traveltime measurement 3, slug injection of 500 milliliters of 20-percent dye solution at 0540 hours, May 18, 1977, at site 4								
4	0.0	6,580	711	-----	-----	----	-----	------
8	23.1	6,340	666	9.00	10.4	2.19	4.00	0.65
Little Snake River, traveltime measurement 3, slug injection of 995 milliliters of 20-percent dye solution at 0519 hours, May 13, 1977, at site 9								
9	0.0	6,260	668	-----	-----	----	-----	------
13	33.3	6,050	668	17.2	20.3	1.59	8.50	0.84

Unit concentrations (*UC*) were computed at each high-flow sampling site for the Yampa and the Little Snake Rivers, using the following equation:

$$UC=\frac{C_{cpk} \cdot Q}{W_d}, \tag{2}$$

where Q=mean discharge, in cubic feet per second;

w_d=dry weight of injected solute, rhodamine-WT dye, in pounds; and

C_{cpk}=peak concentration in the stream adjusted for measured dye losses, in micrograms per liter, and defined by the following equation:

$$C_{cpk}=\frac{100 \cdot C_m}{PR}, \tag{3}$$

where C_m=measured peak concentration of rhodamine-WT dye at sampling site, in micrograms per liter; and

PR=percentage recovery of the rhodamine-WT dye injected upstream.

The use of unit-concentration predictions provides a convenient means of predicting the peak concentrations of contaminants at various points downstream. The percentage recovery (*PR*) of the rhodamine-WT dye usually decreases with distance downstream from an upstream injection site. The decrease in dye mass can be a result of several factors: Dye loss as a result of absorption on bottom and suspended sediments, adsorption on vegetation and debris, and photochemical decay; dye lag, in which the sampling period is not long enough to obtain the entire dye-concentration versus relative time-curve; flow accrual, which is defined as an increase in streamflow rate with distance downstream; and chemical reaction, in which the dye reacts chemically with some substance in the water (for example, chlorine).

The mean velocities listed in table 3 were computed using the traveltime of the center of mass of the dye clouds. A similar computation could be made using the traveltime of the peak concentrations. The mean velocities were computed using the distance and traveltime between each pair of adjacent sampling sites. The general trend on the Yampa and the Little Snake Rivers is a decrease in velocity in the downstream direction.

Methodology for Simulating Traveltime and Unit Concentration

A computer model developed by McQuivey and Keefer (1976) was used to simulate traveltime and unit concentration. Their technique was devised to model longitudinal dispersion in streams as a convective process. For this report, the technique was modified (T. N. Keefer, written commun., 1976) to simulate traveltime and unit concentrations in streams, in addition to longitudinal dispersion. This model contains two major parameters, defined as follows:

damping coefficient=$D_*=U_*/k$, in meters per second; and (4)

mean stream velocity=$\bar{U}$, in meters per second;

where U_*=shear velocity, in meters per second, and

k=von Karman's constant.

The stream in which traveltime or unit concentration is to be modeled is broken into a given number of subreaches. The number of subreaches is manually determined on the basis of the stream-reach hydraulic properties.

Predictions of relative dye concentration [$C_{(x,t)}$] at the end of each subreach are given by the following formula:

$$C_{(x,t)}=\int C_{(n-1)}(Z_n) \cdot h(x_n,t_n-Z_n)dZ_n, \tag{5}$$

where n=1, 2, 3, 4, ...,

$C_{(n-1)}(Z_n)$=concentration at the downstream end of the (n-1) subreach,

t_n=coordinate in time direction,

x_n=coordinate in downstream direction,

z_n=arbitrary coordinate system, and

$h(x_n,t_n-Z_n)$=model unit-response function.

For a logarithmic-velocity profile, the integral of equation 5, as derived by Sayre (1977), is as follows:

$$C(x,t)=\left(\frac{W}{BY\bar{U}}\right)\cdot\left(\frac{\bar{U}}{D_*t}\right)\exp\left[\frac{x}{D_*}\left(\frac{1}{t}-\frac{U_{max}}{X}\right)\right], \tag{6}$$

where W=amount, in milligrams, of dye tracer injected;
B=channel width, in meters;
Y=depth of stream, in meters; and
U_{max}=stream velocity at water surface, in meters per second.

The unit-response function portion of equation 5 is as follows:

$$h(x,t)=\left(\frac{\bar{U}}{D_*t}\right)\exp\left[\frac{X}{D_*}\left(\frac{1}{t}-\frac{U_{max}}{X}\right)\right]. \tag{7}$$

Equation 7 indicates that when dye is introduced as an instantaneous pulse into a stream reach, the maximum dye concentration occurs immediately and then decreases exponentially as a function of time. The actual solution of equation 5 with equation 7 as the response function over a time period of interest is accomplished in the computer model by replacing the integral in equation 5 by a sum of finite delta time increments as follows:

$$C(x,t)= \frac{W}{BY\bar{U}} \sum_{i=0}^{t} h(x,t). \tag{8}$$

Traveltime predictions of the dye-cloud peak concentration, leading edge, or trailing edge, can then be determined using the desired time of occurrence from the relative dye-concentration curve.

The use of a computer model to simulate traveltime and unit concentration for hypothetical discharges of varying magnitude involves calibrating the model. Calibration is achieved by varying U and D_* in the model until they match the measured dye-concentration curves. Once the model has been calibrated, simulations of traveltime and unit concentration for hypothetical discharges may be made by the procedures described below:

1. Develop a relationship of shear velocity ($\bar{U}_*$ gage) and mean velocity ($\bar{U}$ gage) versus discharge from discharge measurements at index discharge stations located in the study reaches.
2. Determine the ratio of $\bar{U}_*$ gage and $\bar{U}$ gage versus the damping coefficient (D_*) determined by the model mean velocity ($\bar{U}$ comp) computed for the reach, as $\bar{U}_*$ gage/D_* and $\bar{U}$ gage/$\bar{U}$ comp.
3. Choose different index-discharge values and determine corresponding $\bar{U}$ gage and $\bar{U}_*$ gage values (figs. 7 and 8, p. 21 and 22).
4. Compute respective $\bar{U}$ comp and D_* values for the reach from the ratios in procedure 2. This assumes the ratios are constant for different flow conditions.
5. Use these new parameter values in the model to obtain simulated traveltimes or unit concentrations.

The index discharges, $\bar{U}_*$ gage, and $\bar{U}$ gage, were obtained from discharge measurements made during the last 10-year period at each index site.

TRAVELTIME RESULTS

Traveltime results, using three different methods of traveltime estimation, are presented. A greater emphasis has been placed on estimating traveltime using the computer model and index-discharge stations because they provide a more practical method for application. The other two methods, linear-regression and graphical relationships, are presented primarily as cross checks for the computer-model technique and also to provide general information on the different traveltime-estimation methods.

River Conditions during Traveltime Measurements

The river profiles, mean velocities, and dye-cloud traveltimes for the Yampa River are shown in figure 5, and for the Little Snake River in figure 6. The regimes of flow for both rivers were channel control during June 1976 and May 1977 and pool-and-riffle control during September 1976. Both streams have fairly steep gradients as shown by the stream profiles in figures 5 and 6. Average gradients for the Yampa and the Little Snake Rivers are 10 ft/mi (1.9 m/km) for the study reaches. Upstream from the study reaches, the average gradient is 73 ft/mi (14 m/km) for the Yampa River and 184 ft/mi (35 m/km) for the Little Snake River.

The dye-cloud traveltimes shown in figures 5 and 6 are for only the high-flow (June) conditions. The traveltimes are shown for the leading edge, peak, and trailing edge of the dye cloud. In some instances, the leading- and trailing-edge traveltimes had to be estimated because of insufficient data.

Mean-velocity data for high-, medium-, and low-flow conditions are shown in figures 5 and 6 and also in table 3. During both high- and low-flow conditions, the discharges in the study reach of the Yampa River were greater than those for the study reach of the Little Snake River. A general summary of the measured discharges is given in table 3. Velocities during the high-flow period generally decreased in a downstream direction except for one 5-mi (8-km) reach of the Yampa River upstream from Steamboat Springs (fig. 5). A minimum velocity of 0.26 mi/h (0.42 km/h) was measured during the low-flow period in the Yampa River near a sand-and-gravel business 5 mi (8 km) downstream from Steamboat Springs. A minimum low-flow velocity of 0.04 mi/h (0.06 km/h) occurred in the Little Snake River in a reach downstream from a large irrigation ditch, which diverted 85 percent of the streamflow near sites 7 and 8 [river-mile 121 (river-km 195)] (table 2 and fig. 3).

Traveltime Simulations Using Index-Discharge Stations and Computer-Model Techniques

To simulate traveltime for other flow conditions in the study reaches using the computer-model techniques (see p. 17), four index-discharge stations were designated. Two index stations were located within the study reach of each stream. The index stations were as follows: Yampa River at Steamboat Springs, Colo., station 09239500; Yampa River below diversion, near Hayden, Colo., station 09244410; Little Snake River near Slater, Colo., station 09253000; and Little Snake River near Dixon, Wyo., station 09257000. Relationships of mean velocity and shear velocity versus stream discharge are shown for these four sites in figures 7 and 8. The index stations are located at sites 3 and 15 on the Yampa River (fig. 2 and table 1) and sites 1 and 8 on the Little Snake River (fig. 3 and table 2). Bauer (1968) made a similar use of index-discharge stations.

The results of the traveltime simulation for the four index-discharge stations are shown in figures 9 and 10 for the Yampa River and in figures 11 and 12 for the Little Snake River. Site numbers given on the figures refer

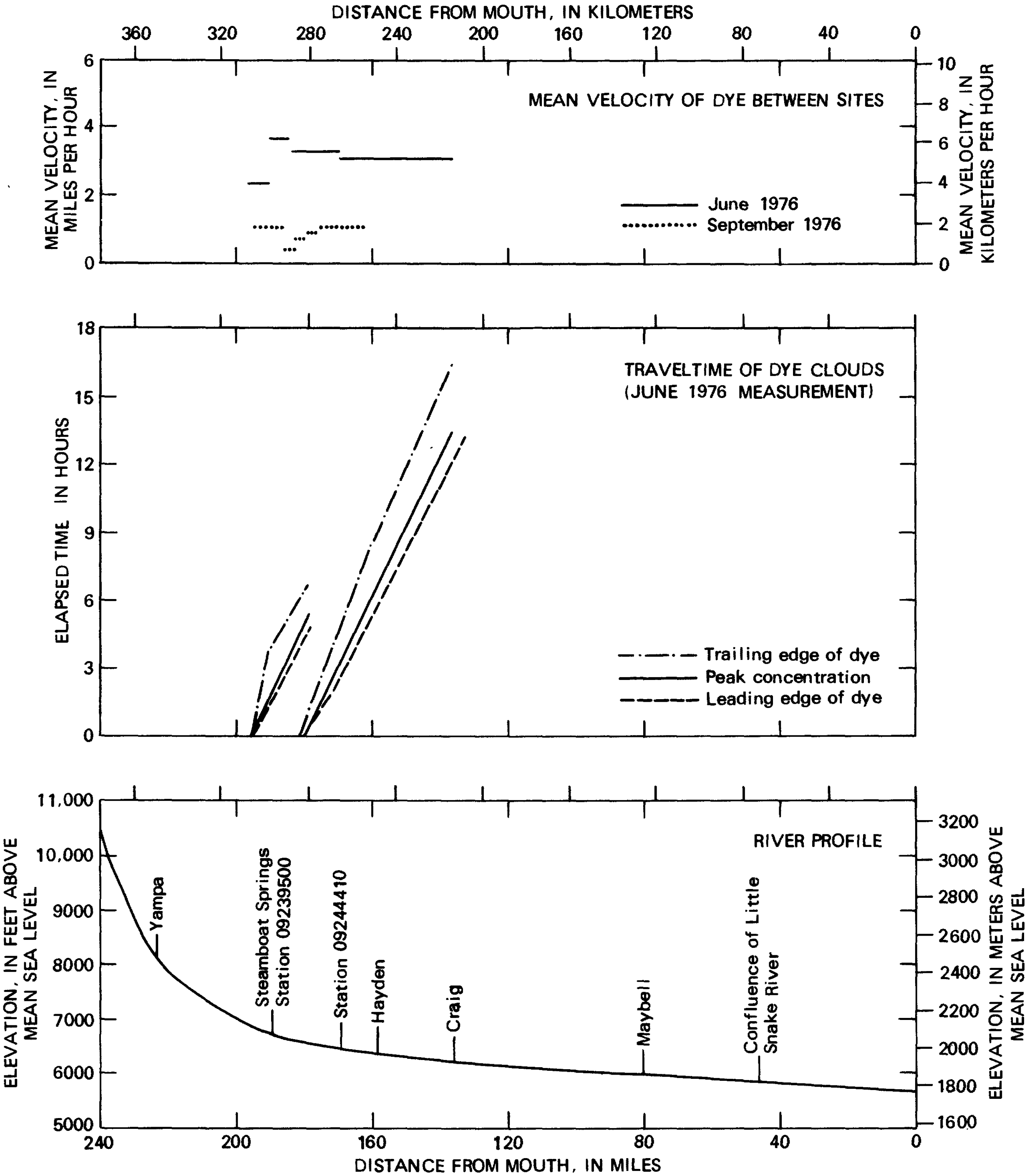

Figure 5.--Mean velocity, traveltime of dye clouds, and river profile for the Yampa River, June and September 1976.

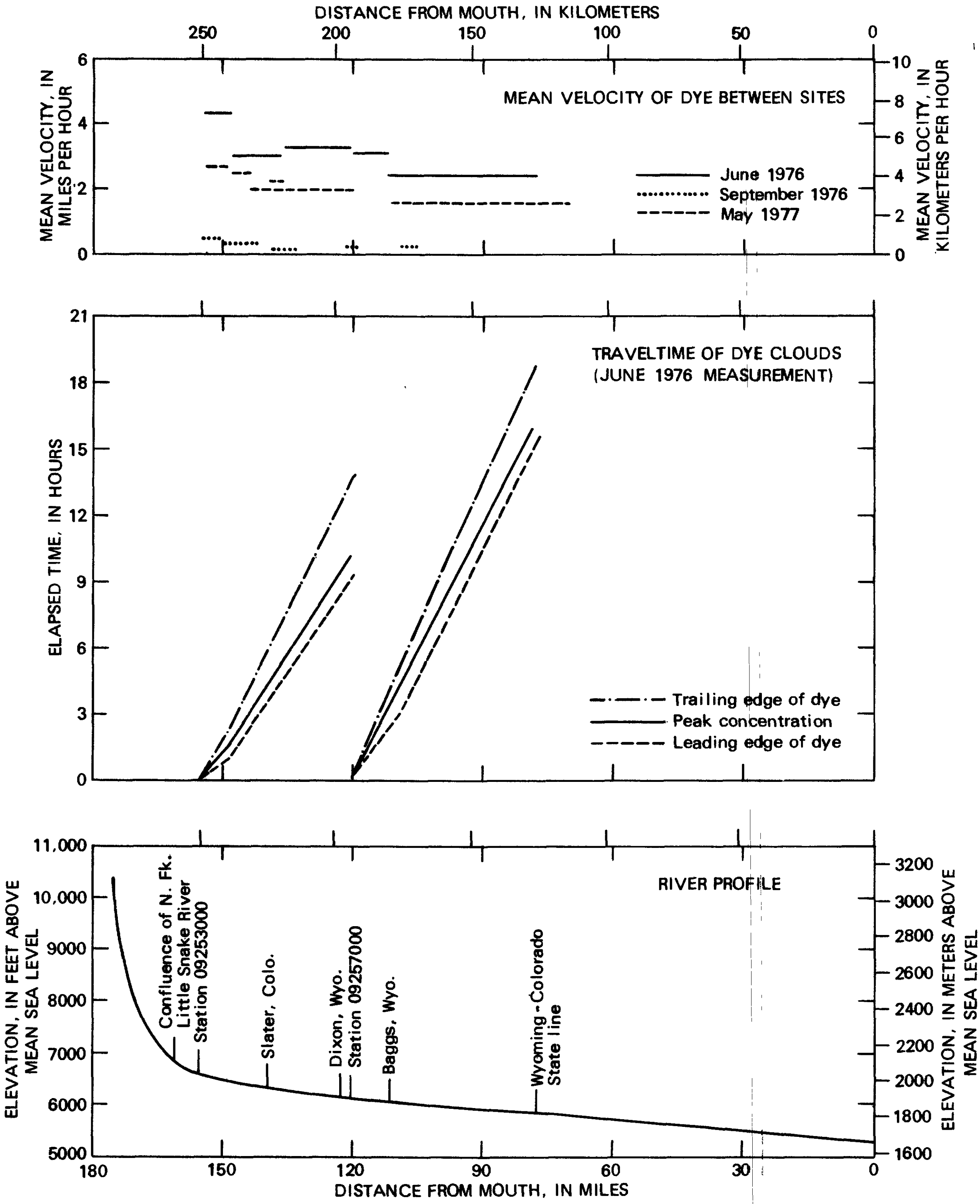

Figure 6.--Mean velocity, traveltime of dye clouds, and river profile for the Little Snake River, June and September 1976, and May 1977.

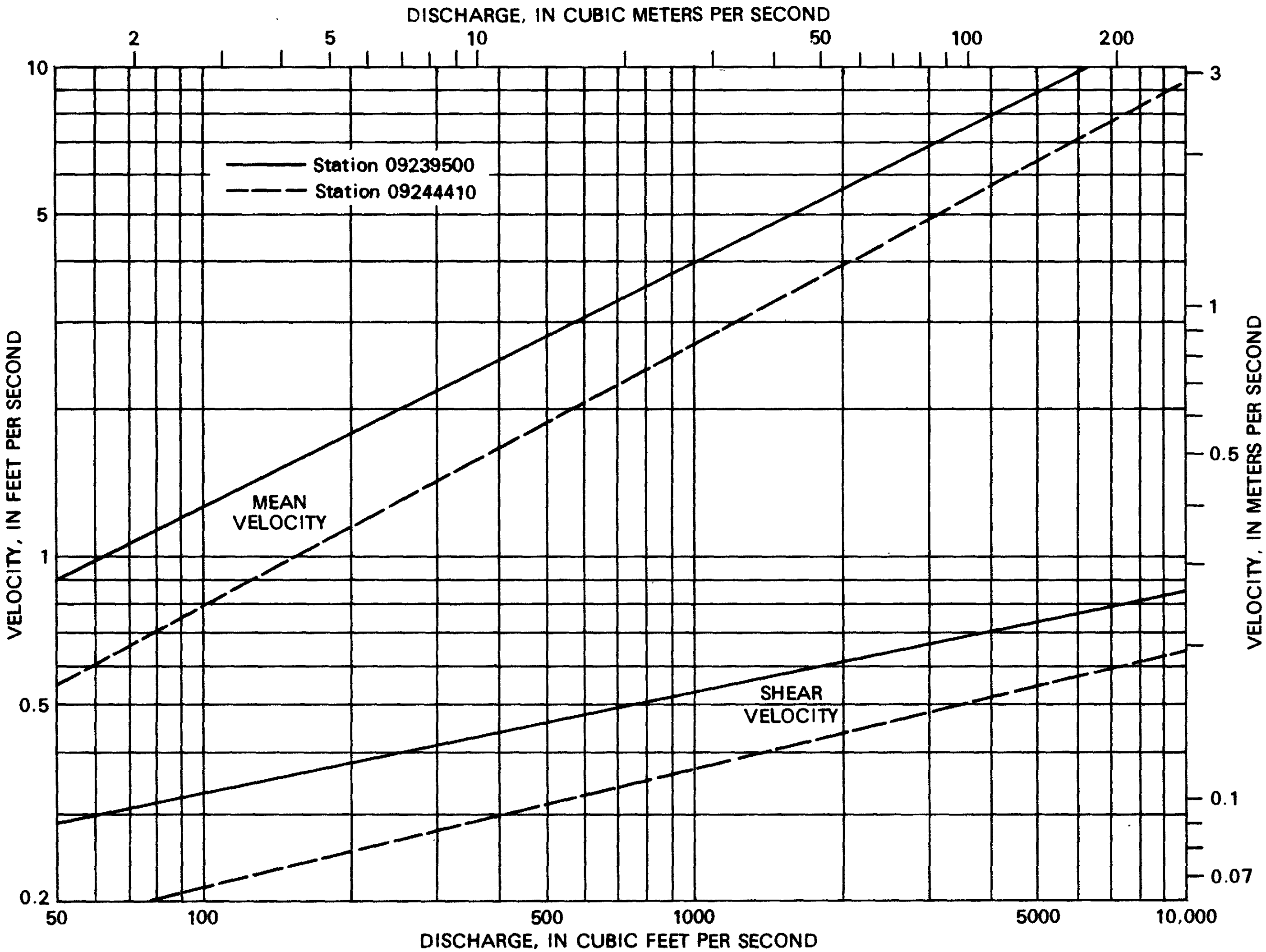

Figure 7.--Mean velocity and shear velocity versus discharge, Yampa River at Steamboat Springs, Colo., station 09239500, and below diversion, near Hayden, Colo., station 09244410.

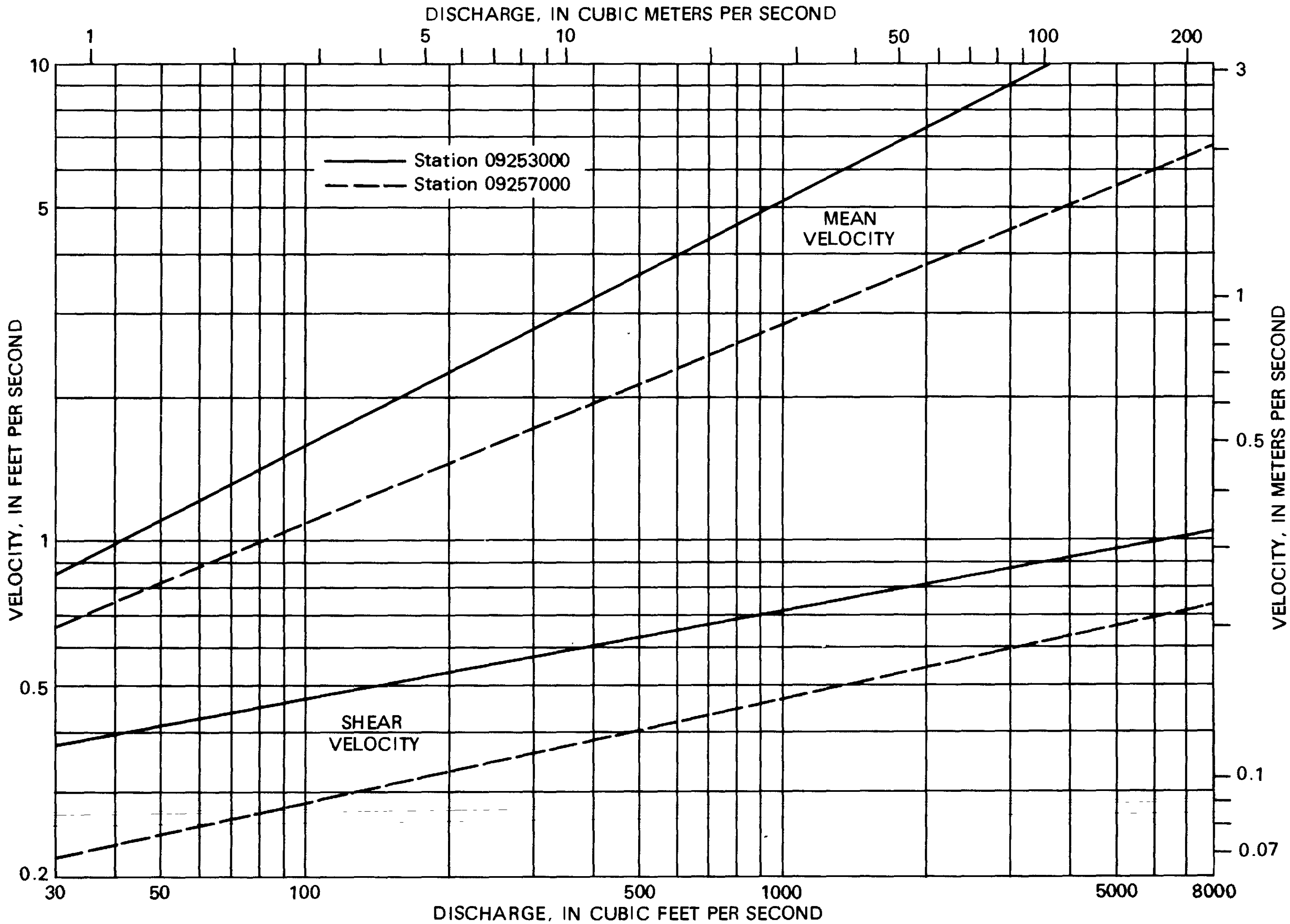

Figure 8.--Mean velocity and shear velocity versus discharge, Little Snake River near Slater, Colo., station 09253000, and near Dixon, Wyo., station 09257000.

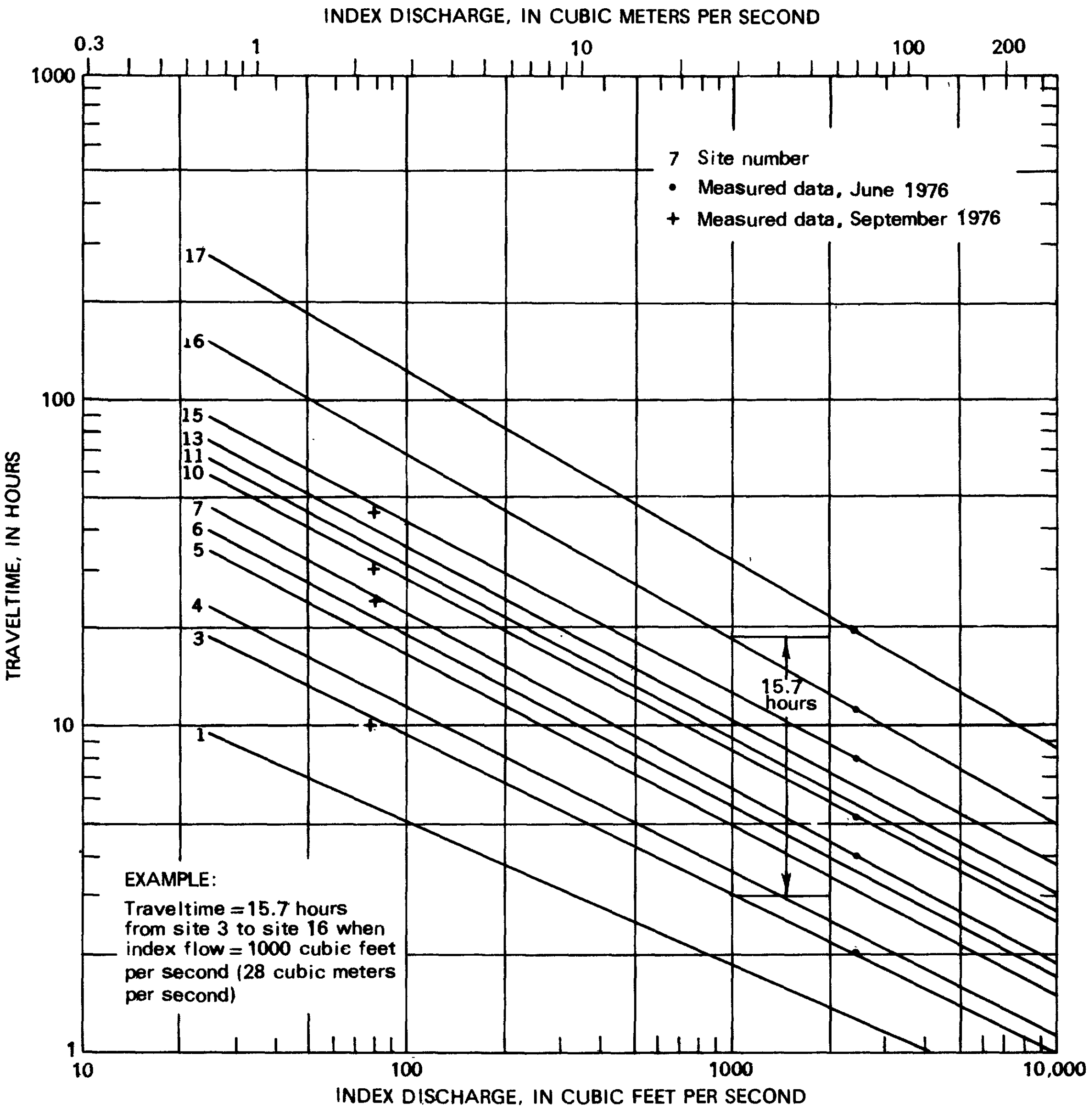

Figure 9.--Simulated cumulative traveltime curves for the Yampa River, using index station 09239500,Yampa River at Steamboat Springs,Colo.

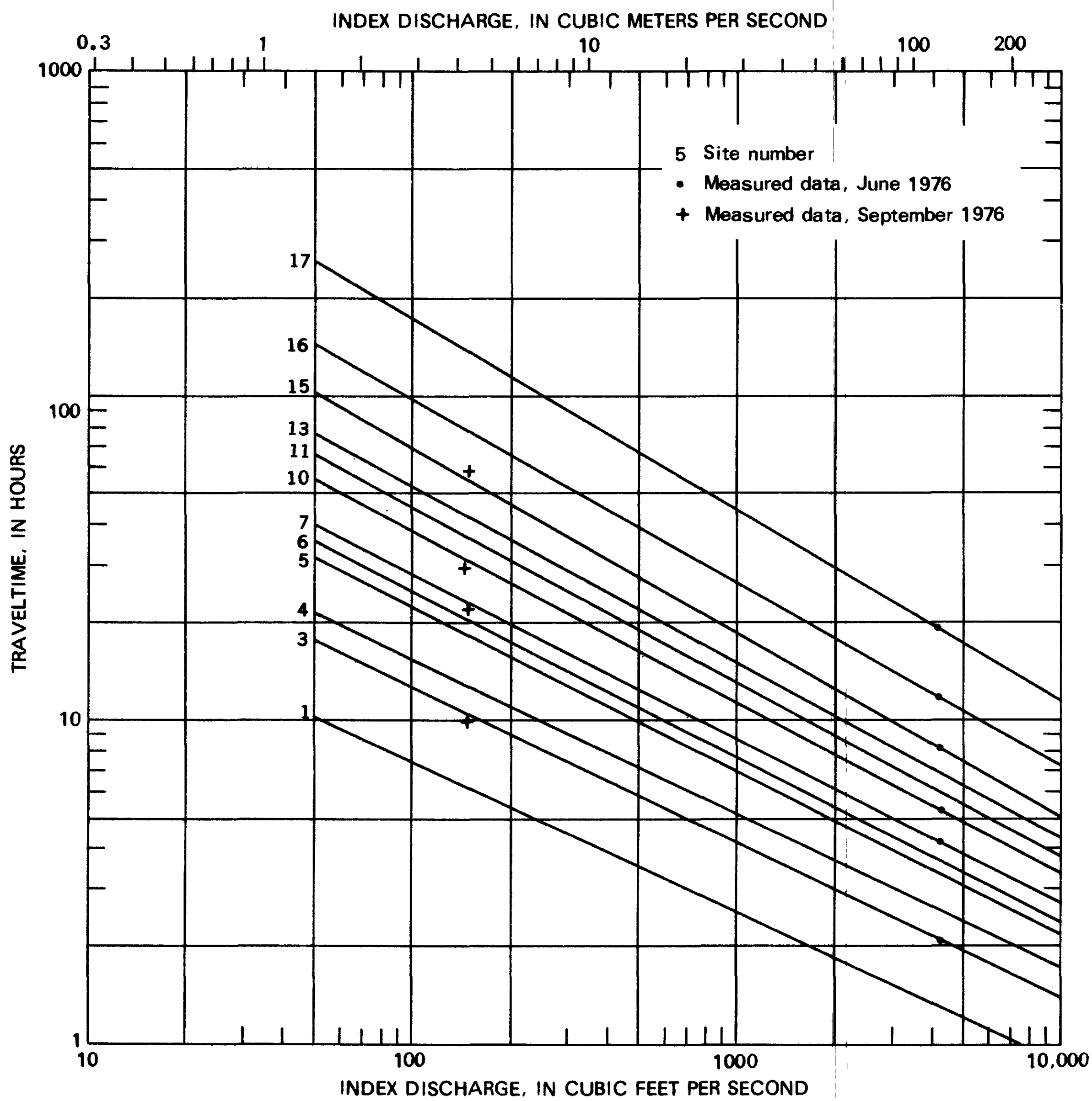

Figure 10.--Simulated cumulative traveltime curves for the Yampa River, using index station 09244410, Yampa River below diversion, near Hayden, Colo.

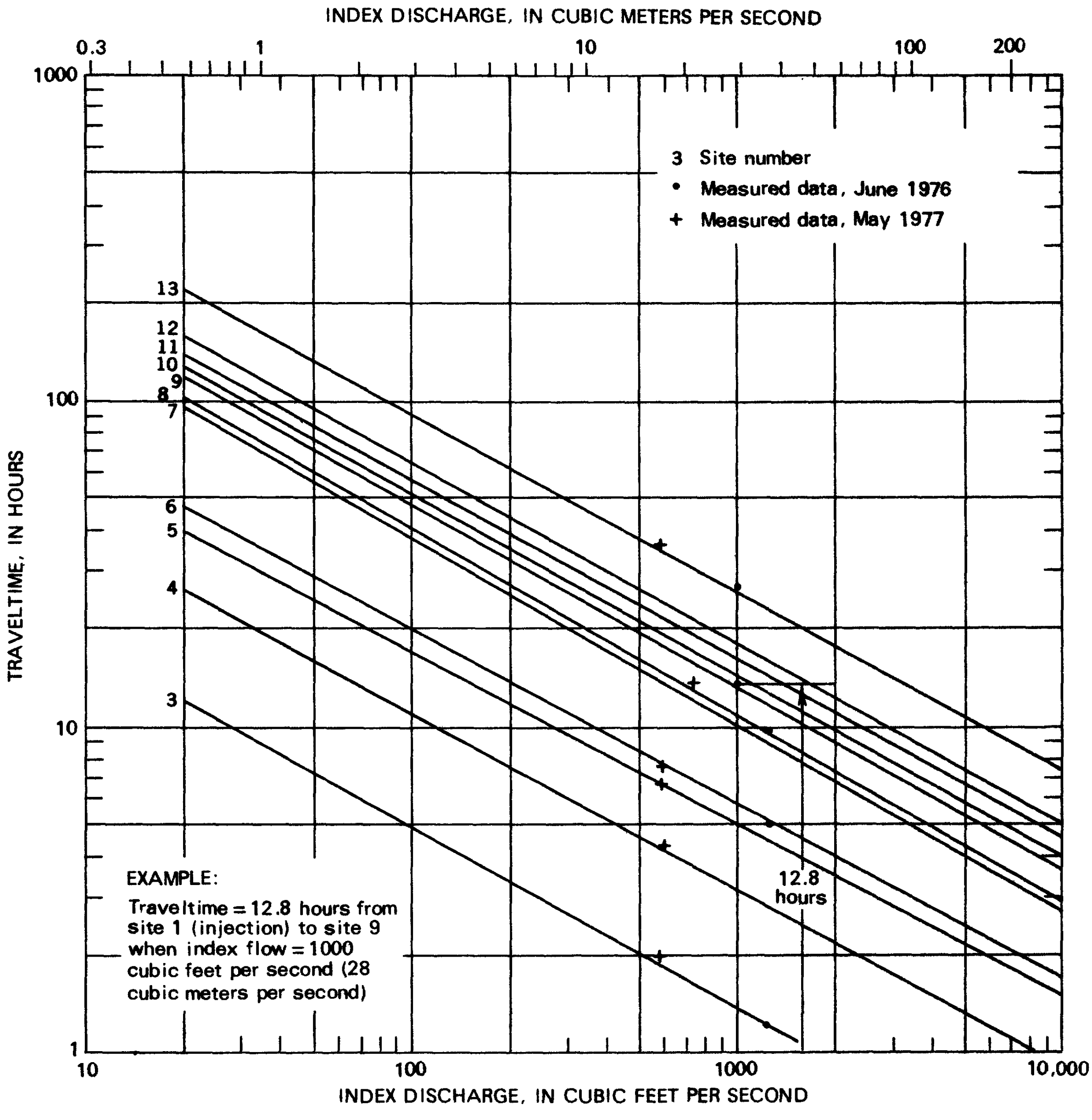

Figure 11.--Simulated cumulative traveltime curves for the Little Snake River, using index station 09253000, Little Snake River near Slater, Colo.

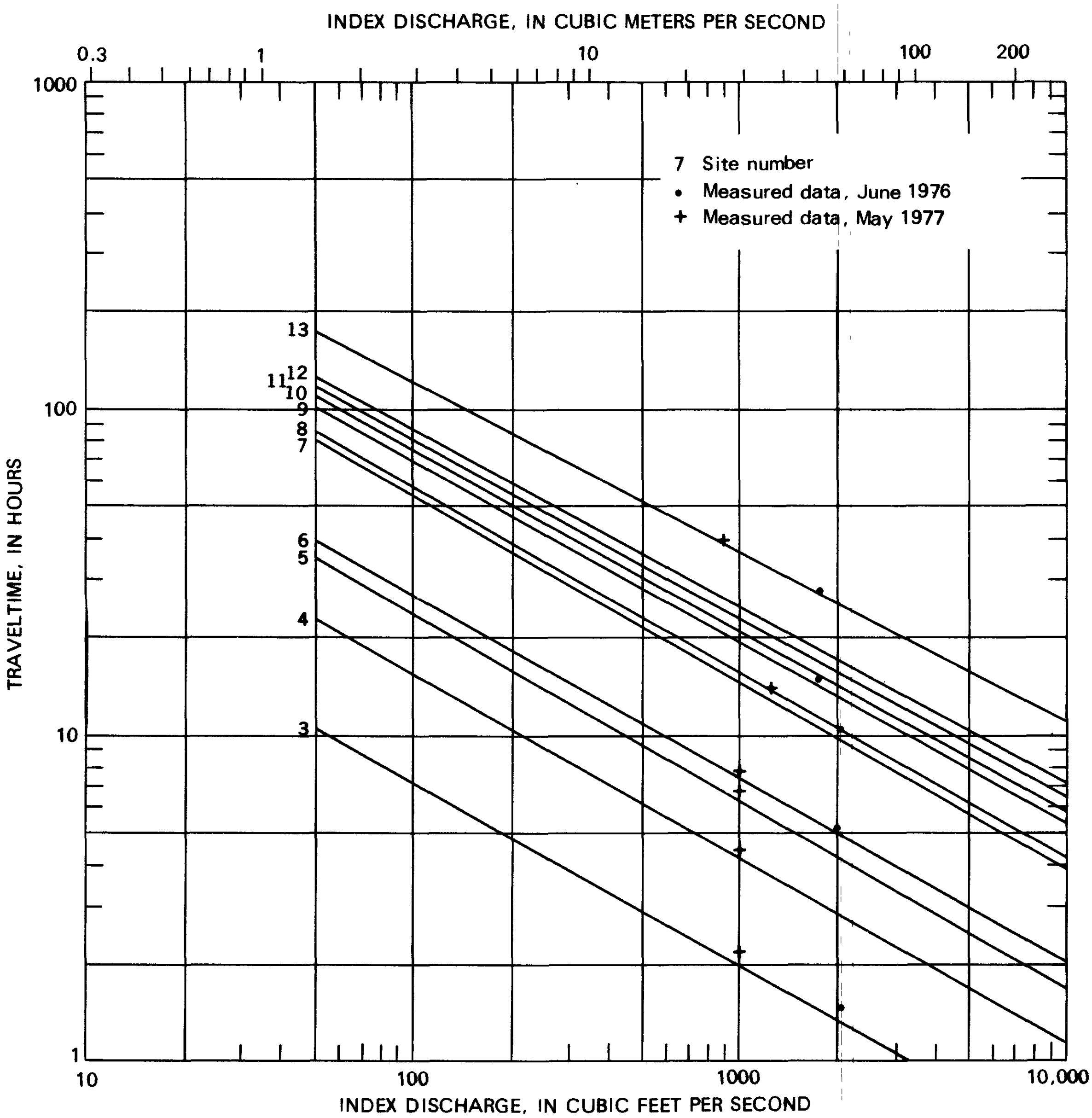

Figure 12.--Simulated cumulative traveltime curves for the Little Snake River, using index station 09257000, Little Snake River near Dixon, Wyo.

to the sites described in table 1 and located on figure 2 (Yampa River) and described in table 2 and located on figure 3 (Little Snake River). The simulations for the Yampa River were developed using measurements for June and September 1976, and the simulations for the Little Snake River were developed using measurements for June 1976 only. Measurements for the Little Snake River made during September 1976 were affected by diversion of approximately 85 percent of the flow; hence, these were not used to make estimates. The measurements for May 1977 on the Little Snake River (figs. 11 and 12) were used to check the simulation developed from the high-flow measurements made in June 1976. The measured and simulated traveltimes agree within 5 percent for the various sampling sites.

The simulation of traveltime using index-discharge stations assumes the existence of a relationship of index discharge versus a given reach discharge. Diversion of water for irrigation, which commonly occurs during summer months, can significantly affect this assumed relationship in medium- and low-flow conditions.

The traveltime simulations using the index-discharge stations for the Yampa and the Little Snake Rivers (figs. 9 to 12) were made on the basis of the dye-peak traveltime. As noted previously, the contaminant leading edge will travel at a faster rate than the peak (figs. 5 and 6). The traveltime of the leading edge of a contaminant can be estimated by using equation 1.

The traveltime simulations were developed using two index-discharge stations for each river (figs. 9 to 12). To simulate traveltime between two sites along a reach, the index discharge must first be known. The current index discharges may be obtained by contacting personnel in the U.S. Geological Survey's offices, in Meeker, Colo., for stations 09239500, 09244410, and 09253000; and in Cheyenne, Wyo., for station 09257000. The most accurate traveltime simulations for medium- and low-flow periods can be obtained by using the index-discharge site nearest the reach of interest (figs. 5 and 6). Either index station can, however, be used. If the index station farthest from the reach of interest is used, the amount of effect on the traveltime estimated cannot be stated.

Traveltime Simulations Using Linear-Regression Relationships

Dye-peak traveltime simulations for the Yampa River also were developed on the basis of a linear-regression equation. Traveltime simulations made using this method are shown in figure 13. The traveltime in this method is related to the mean discharge of the stream reach and distance downstream from the dye injection site. The resulting equation has the following form:

$$T=aQ^{b}M^{c}, \tag{9}$$

where T=traveltime, in hours;
a,b,c=coefficients determined by the linear-regression program, and had values of: a=16.38, b=-0.565, and c=1.237;
Q=mean discharge, in cubic feet per second; and
M=distance downstream from injection site, in miles.

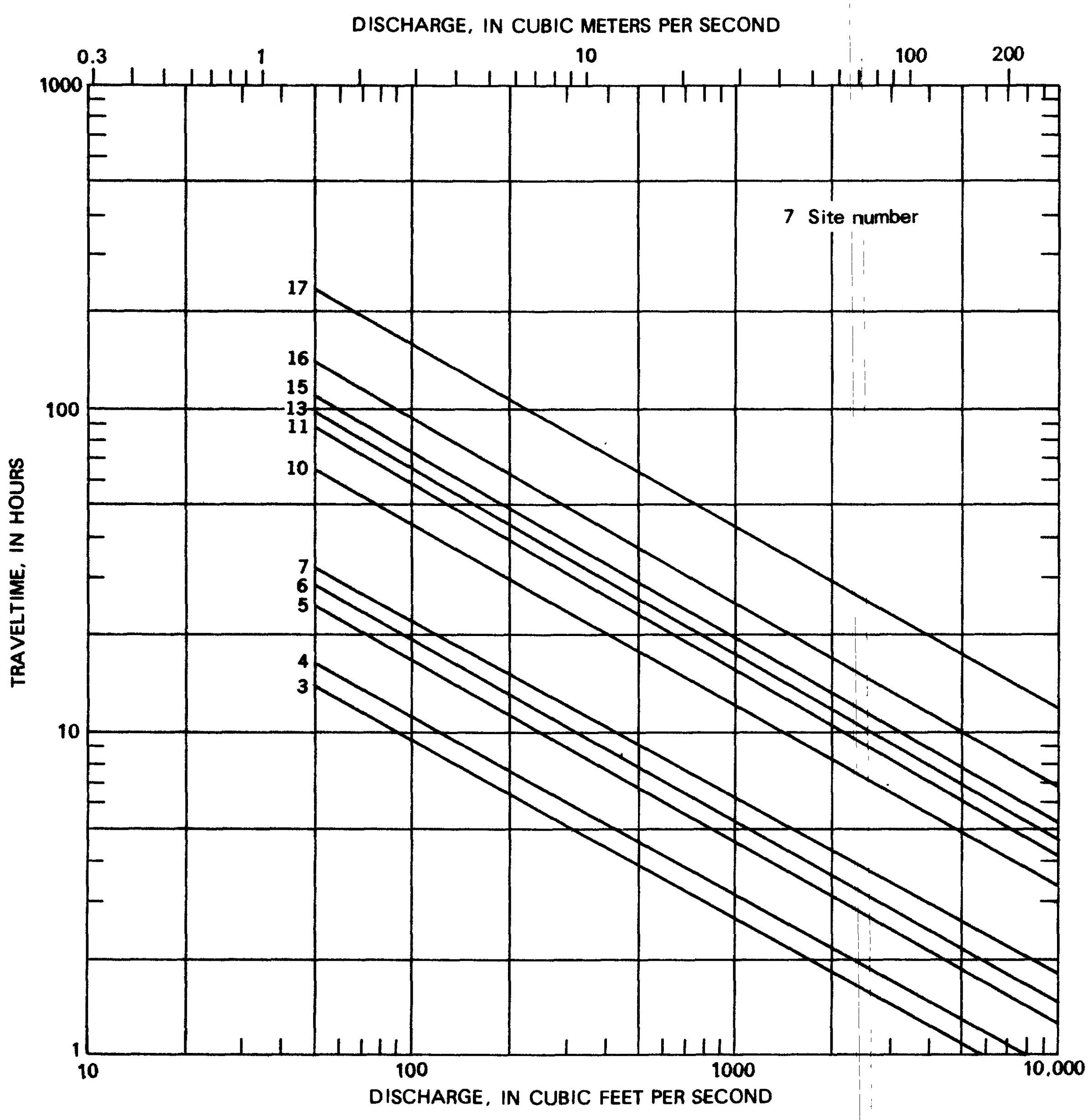

Figure 13.--Cumulative traveltime curves for the Yampa River, based on linear-regression relationships.

To use this method for traveltime simulations in reaches with large variations in discharge, the reach needs to be subdivided into smaller subreaches, each having nearly constant discharges. A traveltime simulation then can be made for each subreach.

Traveltime Simulations Using Graphical Relationships

Dye-peak traveltime simulations for the Little Snake River also were developed on the basis of a two-step graphical approach. The first step involved plotting mean velocities versus the corresponding mean discharges in the reaches using the three measurements from the Little Snake River and visually fitting velocity curves through the measurement points. The second step involved determining the traveltime versus mean discharge in the reaches, shown in figure 14, using the velocity curves. It is noted that the traveltime versus mean-discharge relationship is curvilinear on log-log paper (fig. 14). This resulted because of the small velocity measurements (fig. 6) obtained during the low-flow (September) period. The same technique of dividing the reach into subreaches, as described for the linear-regression approach, is applicable to this method of traveltime simulation.

UNIT-CONCENTRATION SIMULATION RESULTS

Unit concentrations (*UC*) were computed from the measured high-flow data (June 1976) and then simulated, in the same manner as the traveltime data, using the computer model and index-discharge stations for other index-flow conditions (figs. 15 to 18).

The mathematical model described earlier by McQuivey and Keefer (1976) was used to determine *UC* values for other traveltime and index-discharge values. The *UC* simulations were developed using data from the following four index-discharge stations: Stations 09239500 and 09244410 for the Yampa River, and 09253000 and 09257000 for the Little Snake River. These are the same stations used for the traveltime simulations described earlier. During medium- and low-flow periods, the index-discharge station located nearest the reach of interest should be used, as described for the traveltime projections.

For use in the mathematical model, values of peak *UC* (equation 3) were determined at each of the sampling sites using the model-calibration results from the reaches modeled on the Yampa and the Little Snake Rivers. A ratio of the *UC* determined from the measured data and the peak relative dye concentration (equation 8) computed by the model was determined for each sampling site. The ratios computed were assumed to be constant and were used to simulate *UC* for other index discharges. The simulation procedure used the peak relative dye concentration determined for the traveltime simulation. Graphical representations of simulated *UC* versus index discharges and traveltimes are shown in figures 15 and 16 for the Yampa River and in figures 17 and 18 for the Little Snake River. The relationships based on measured data of *UC* versus traveltime and discharge are indicated by dashed curves in figures 15 to 18. The dashed curves are based on data from four or five sampling sites for each subreach.

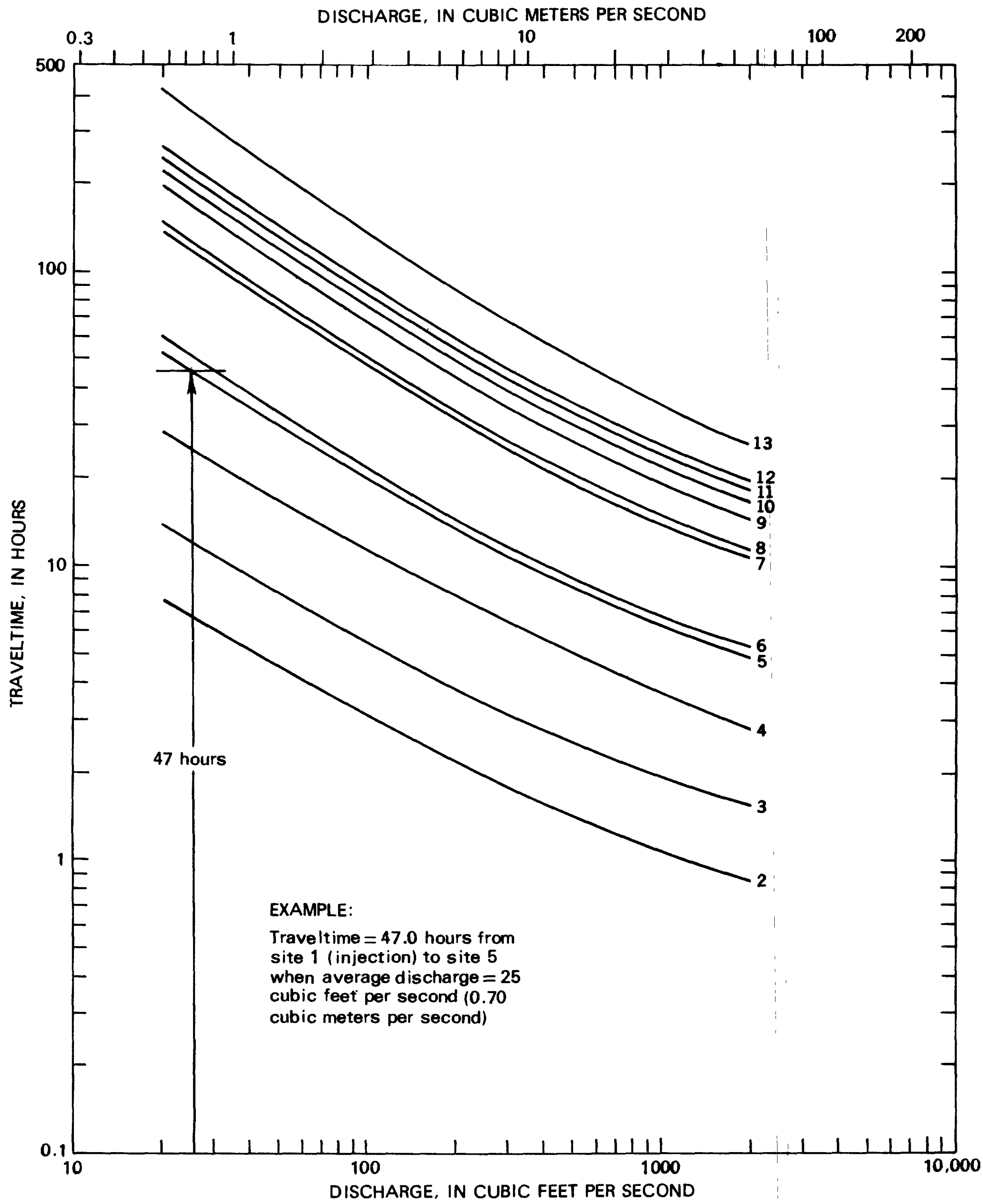

Figure 14.--Cumulative traveltime curves for the Little Snake River, based on mean velocity versus discharge relationships.

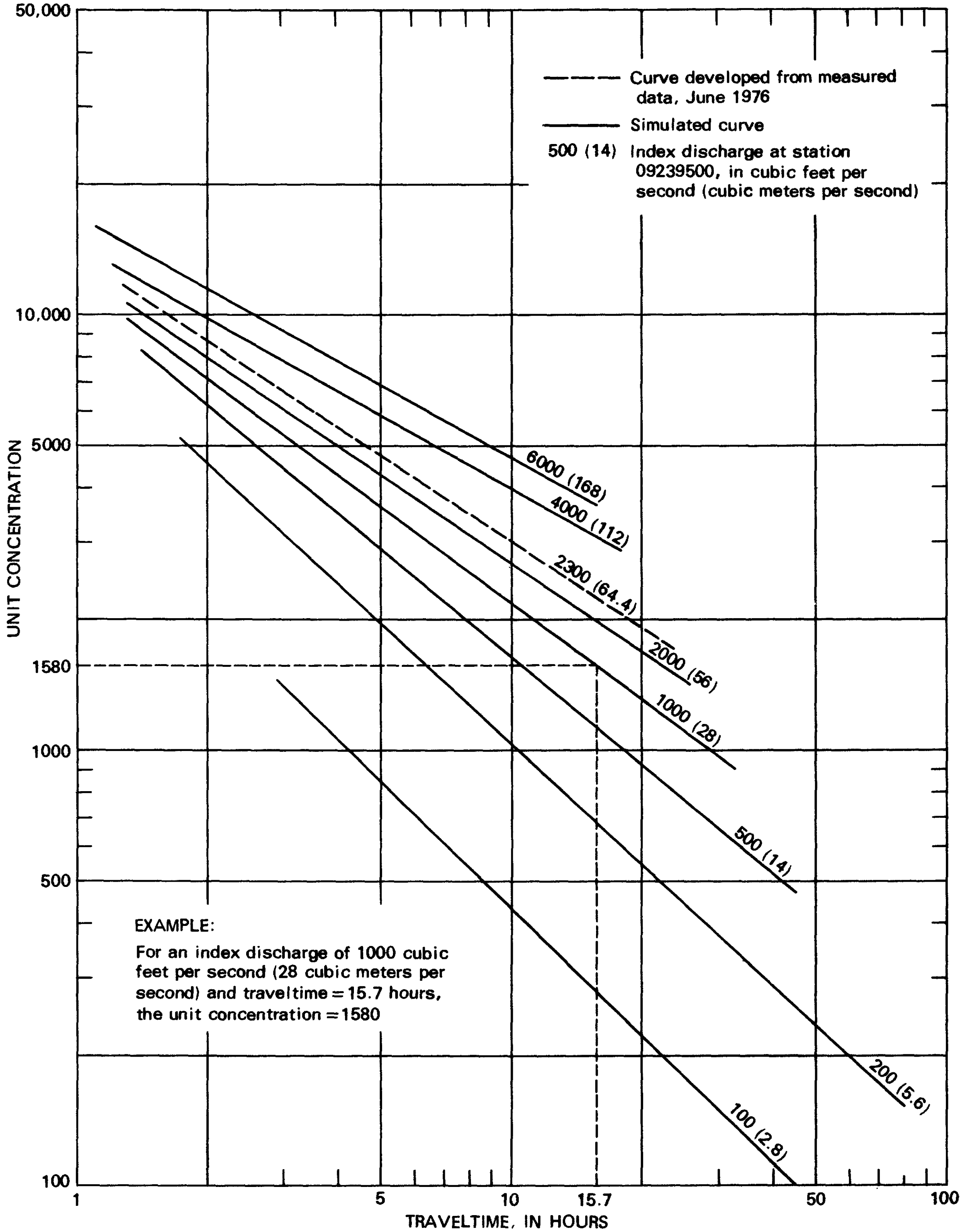

Figure 15.--Simulated unit-concentration curves for the Yampa River, using index station 09239500, Yampa River at Steamboat Springs, Colo.

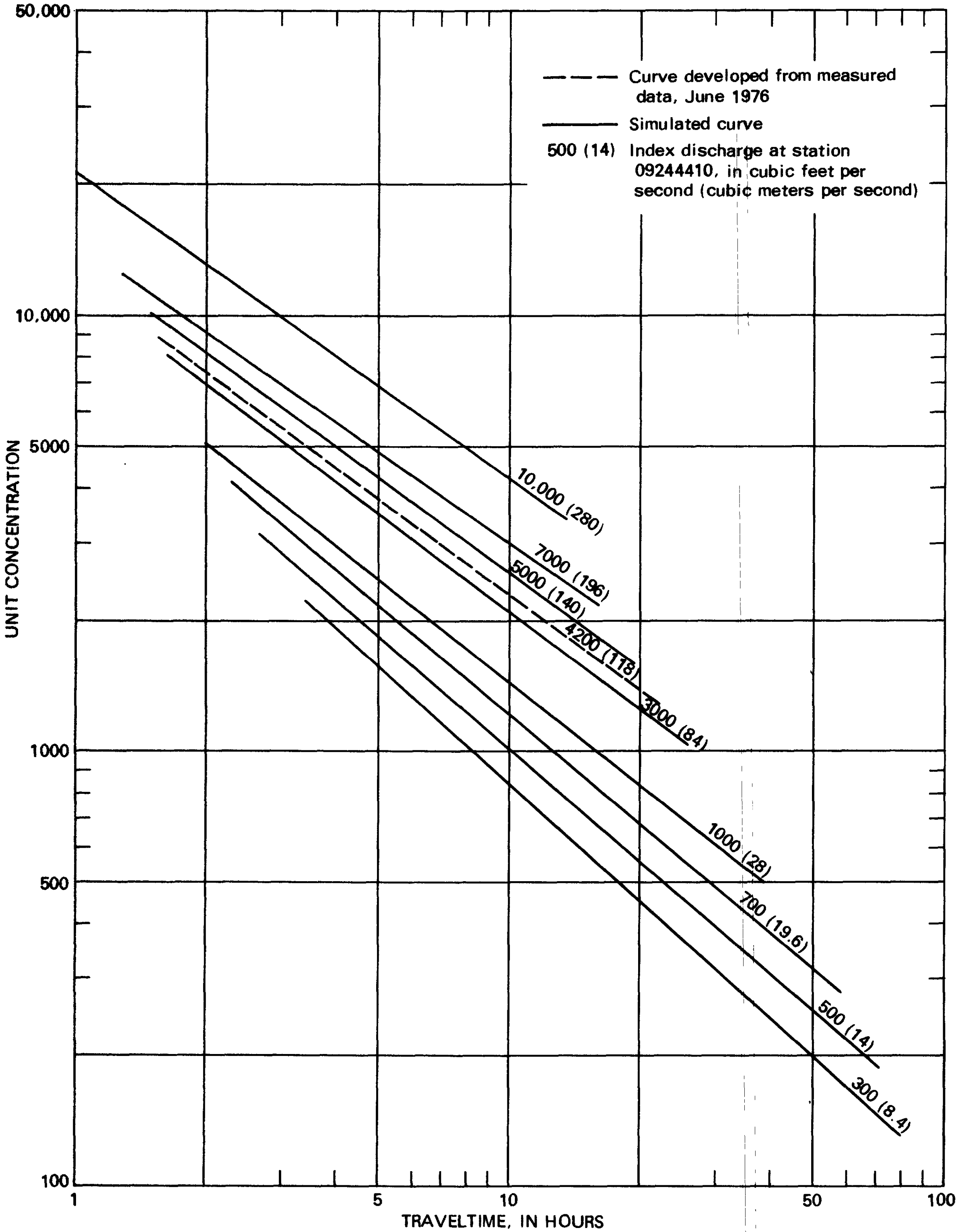

Figure 16.--Simulated unit-concentration curves for the Yampa River, using index station 09244410, Yampa River below diversion, near Hayden, Colo.

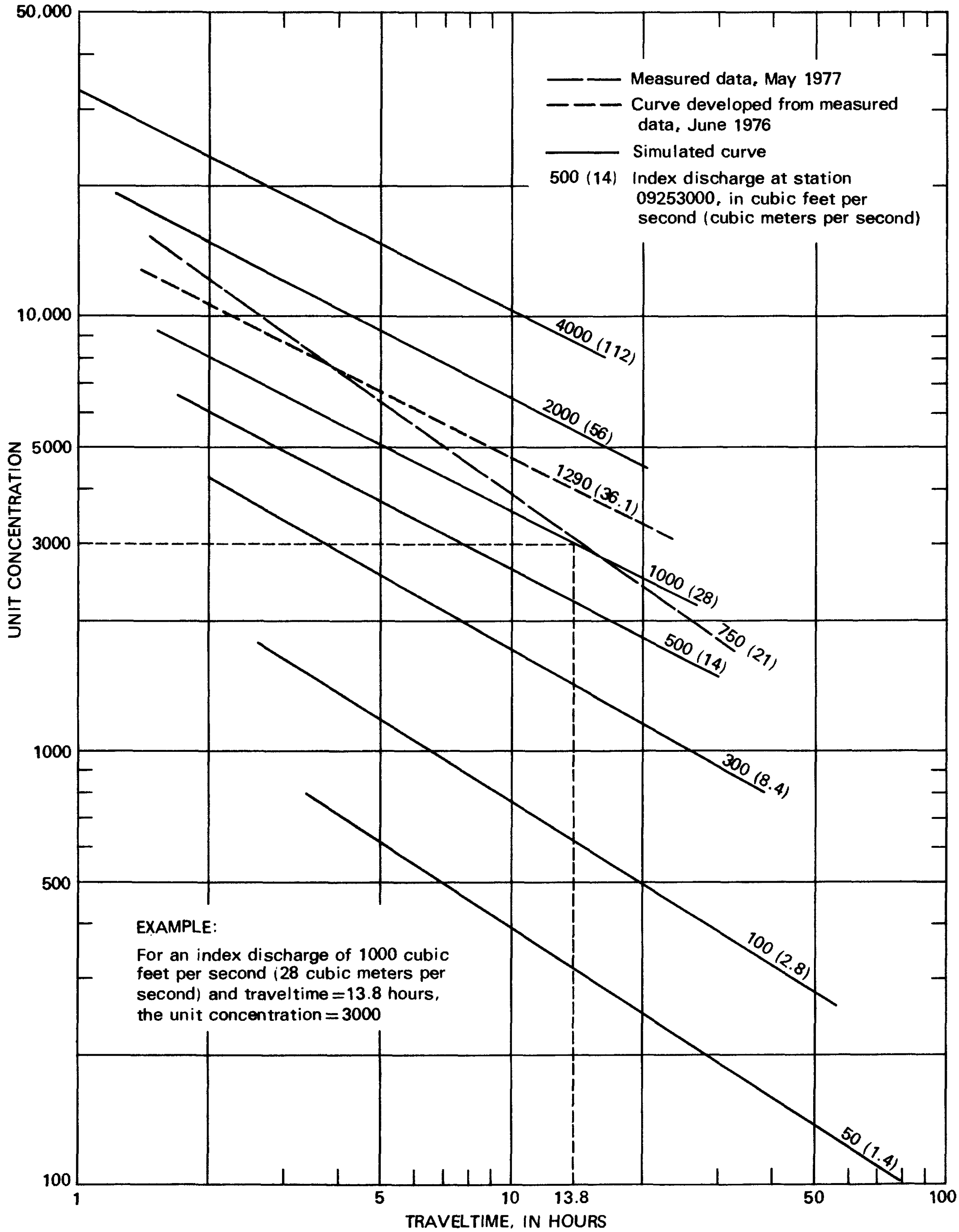

Figure 17.--Simulated unit-concentration curves for the upper Little Snake River, 16 miles east of Slater, Colo., to Dixon, Wyo., using index station 09253000, Little Snake River near Slater, Colo.

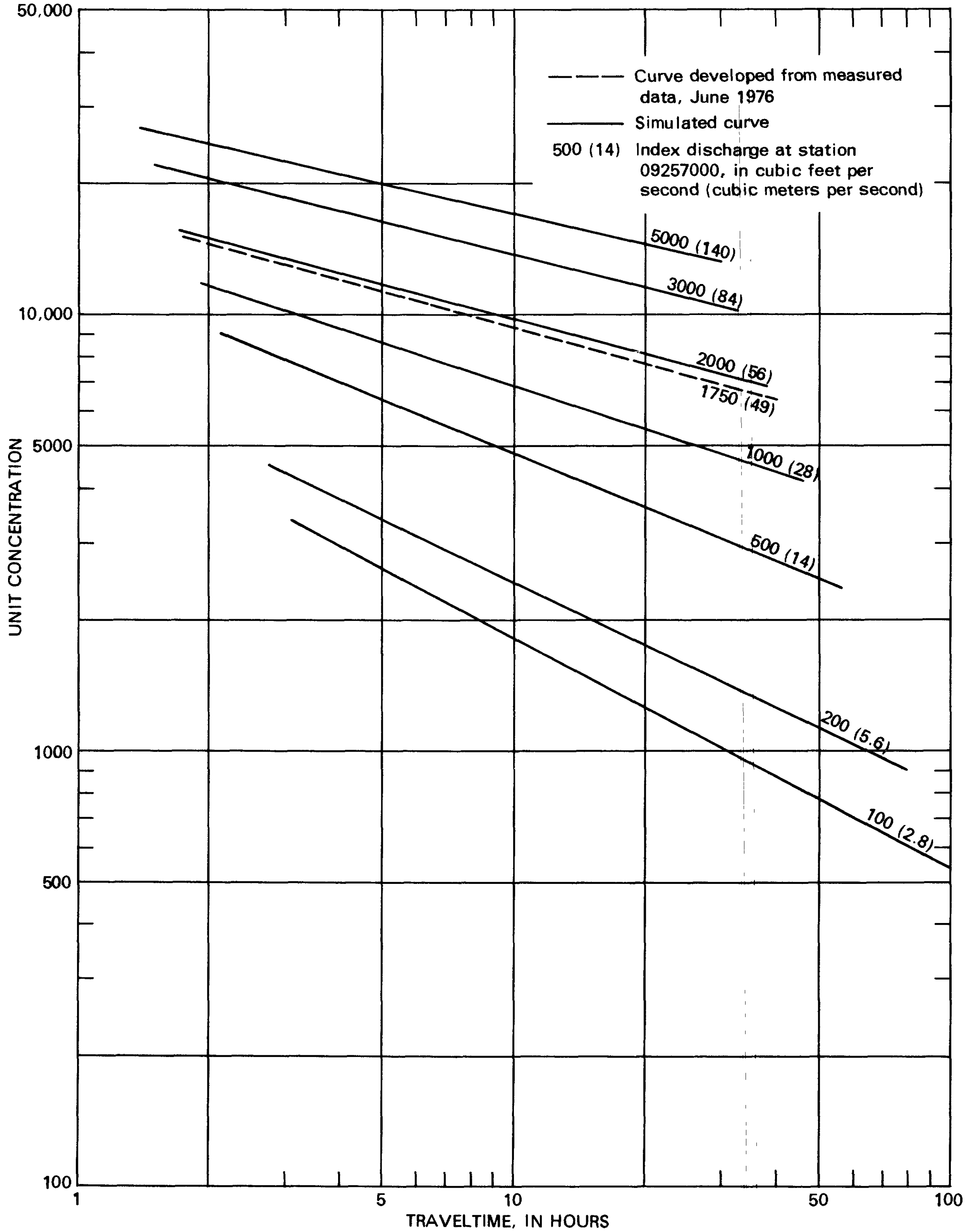

Figure 18.--Simulated unit-concentration curves for the lower Little Snake River, Dixon, Wyo., to 33 miles southwest of Baggs, Wyo., using index station 09257000, Little Snake River near Dixon, Wyo.

The data collected during September 1976 were not used for the *UC* simulations for either the Yampa or the Little Snake Rivers. The low-flow study on the Yampa River was conducted using a continuous dye-injection procedure; whereas, the *UC* computations assume an instantaneous contaminant injection. The low-flow study on the Little Snake River was affected by irrigation diversions and had only one sampling site for each dye-injection site.

The *UC* data for the Little Snake River, collected during medium-flow conditions in May 1977, are shown in figure 17. These data were not used in the model calibration but were used to check the accuracy of the *UC* simulations. Simulated values of *UC* for the Little Snake River agree within 30 to 40 percent of the measured data collected in May 1977 (fig. 17). The *UC* curve for May 1977 has a steeper slope than the simulated curves (fig. 17). The curve for May 1977 has a steeper slope because it is based on data measured only in the upstream one-half of the subreach used in developing the *UC* relationship. Because of the decrease in stream slope (fig. 5) and channel aggregate roughness in the downstream direction of the Little Snake River, a larger change of unit concentration versus index discharge would be expected in the upstream part of the reach.

LONGITUDINAL-DISPERSION COEFFICIENTS

Longitudinal-dispersion coefficients were computed for the Yampa River using low-flow data, and for the Little Snake River using high- and medium-flow data. The longitudinal-dispersion coefficient (K_x) is calculated based on a procedure described by Nordin and Sabol (1974). The basic equation used is as follows:

$$K_x = \frac{\bar{U}^2}{2} \cdot \frac{d\sigma_t^2}{dt}, \qquad (10)$$

where $\bar{U}$=mean velocity, in feet per second; and
σ_t^2=variance of concentration with respect to time, in hours^2.

Fischer (1973) determined that the use of equation 10 resulted in a close approximation of the longitudinal-dispersion coefficient if

$$t > \frac{1.8 \cdot L^2}{rU_*}, \qquad (11)$$

where t=mixing time, in hours;
L=distance from the point of maximum surface velocity to the farthest bank, approximately one-half the stream width, in feet;
r=hydraulic radius, in feet; and
U_*=shear velocity, in feet per second.

A sample computation of a longitudinal-dispersion coefficient is shown in figure 19. This example uses high-flow data collected during June 1976 from the subreach of the Yampa River extending from 5 mi (8.1 km) southeast of Steamboat Springs to Milner, Colo. The required mixing time determined for this subreach using equation 11 was 1.2 hours. An approximate slope of 0.1 hour, defined by the upper part of the curve (variance greater than 0.6 hours^2), was used as an estimate of $d\sigma_t{}^2/dt$ for the reach. The computation of K_x for the subreach then was made directly by using mean velocity, $\bar{U}$, value of 4.33 ft/s (1.32 m/s) and $d\sigma_t{}^2/dt$ into equation 10. A value for K_x of 3,400 ft^2/s (315 m^2/s) was determined.

The computed K_x coefficients for the seven subreaches of the Yampa and the Little Snake Rivers are presented in table 4. The corresponding geometry and flow characteristics for the subreaches also are given. Fischer (1973) presented similar results from experimental measurements of longitudinal dispersion in open channels. His study presented K_x values for a range of shear velocities from 0.46 to 0.06 ft/s (0.14 to 0.02 m/s), stream depths from 26.5 to 0.07 ft (8.07 to 0.02 m), and stream widths from 656 to 0.43 ft (200 to 0.13 m). The corresponding range of longitudinal-dispersion coefficients determined by Fischer (1973) ranged from 16,100 to 1.3 ft^2/s (1,500 to 0.123 m^2/s).

DETERMINATION OF REAERATION COEFFICIENTS USING A MODIFIED TRACER TECHNIQUE

A modification of the tracer technique developed by Tsivoglou (1967) was used to measure the reaeration coefficients of a part of the study reach of the Yampa River. Ethylene and propane were used as tracer gases and rhodamine-WT dye was used as the dispersion and dilution tracer. Only a brief synopsis of the modified tracer technique is included here. Details of the technique have been given by Rathbun, Shultz, and Stephens (1975) and Rathbun and Grant (1978).

The basic procedure consists of injecting a quantity of the tracer gas into the stream and determining a desorption coefficient for the gas from measurements of the gas concentration at various points downstream. The desorption coefficient for the tracer gas is then converted to a reaeration coefficient for oxygen, using a constant determined in the laboratory. An advantage of the modified technique is that two tracer gases can be used simultaneously, thus permitting two measurements of the reaeration coefficient in a single experiment.

The three assumptions inherent in the tracer technique are presented by Tsivoglou (1967). These are as follows: It is assumed, first, that the ratio of the desorption coefficient for the tracer gas to the absorption coefficient for oxygen is independent of mixing conditions, temperature, and the presence of pollutants for the range of ambient conditions in streams; second, that the dispersion and dilution tracer is conservative; and third, that the tracer gas undergoes the same dispersion and dilution as the conservative tracer and is lost from the stream only by desorption through the water surface to the atmosphere.

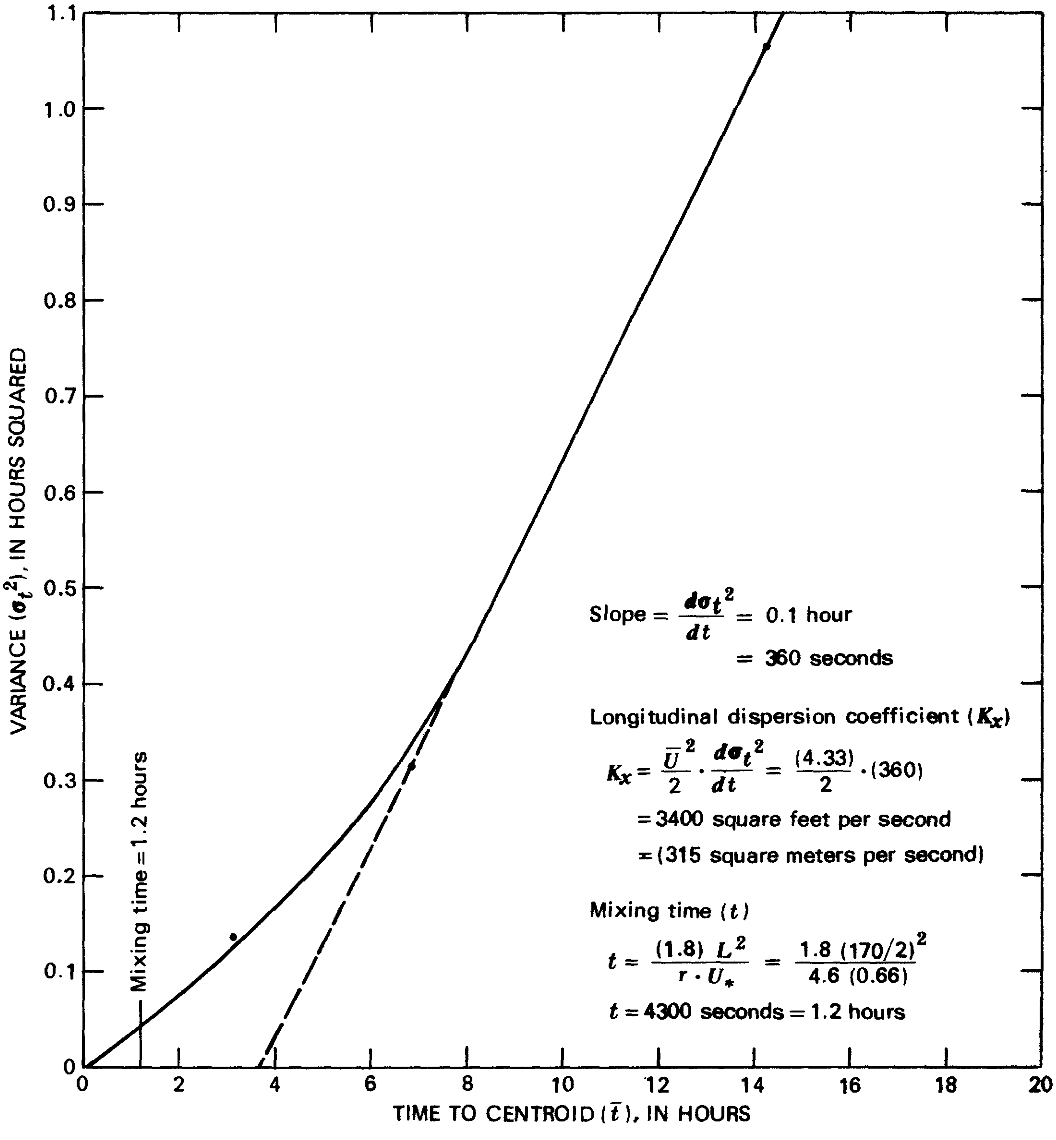

Figure 19.--Sample computation of the longitudinal-dispersion coefficient for the Yampa River, 5 miles southeast of Steamboat Springs to Milner, Colo.

Table 4.--*Longitudinal-dispersion coefficients for varying streamflow conditions, Yampa and Little Snake Rivers*

Stream	Subreach	River-mile (start)	River-mile (end)	Discharge (ft^3/s)	Depth (ft)	Width (ft)	Shear velocity (ft/s)	Mean velocity (ft/s)	Longitudinal-dispersion coefficient (ft^2/s)
Yampa----	Steamboat Springs to Steamboat II development.	191.0	185.2	80	1.3	80	0.30	0.98	540
Yampa----	1.6 miles east from Elk River confluence to Milner, Colo.	183.4	177.3	180	2.0	150	.18	.79	400
Yampa----	5.0 miles southeast of Steamboat Springs to Milner, Colo.	195.5	179.0	2,300	3.6	130	.59	4.33	3,400
Yampa----	Milner to Craig, Colo.	179.0	137.3	4,500	4.6	170	.66	5.12	2,400
Little Snake.	16 miles east of Slater, Colo. to Dixon, Wyo.	154.7	120.3	1,400	3.3	120	.56	4.49	6,050
Little Snake.	Dixon to 33 miles southwest of Baggs, Wyo.	120.3	77.4	1,700	4.6	180	.30	2.99	700
Little Snake.	16 miles east of Slater, Colo. to 1.5 miles west of Slater.	154.7	137.0	590	1.7	100	.48	3.50	2,700

Reaeration coefficients are usually computed from the peak concentration of the tracer gases and rhodamine-WT dye, although the coefficients also can be computed from the areas under the curves of gas-tracer concentration versus time for those cross sections where sufficient samples are obtained to define the complete curve.

Peak Method

The basic equation for the tracer-gas desorption coefficient (K_G) using the peak method is as follows:

$$K_G = \frac{1}{t_d - t_u} \, ln\left(\frac{C_{G_U} / C_{D_U}}{C_{G_D} / C_{D_D}}\right), \tag{12}$$

where C_{G_U}, C_{G_D} =peak concentration of the tracer gas at the upstream and downstream ends of the reach, in micrograms per liter;

C_{D_U}, C_{D_D} =peak concentration of dye at the upstream and downstream ends of the reach, in micrograms per liter; and

t_d, t_u =traveltime of the peak concentrations of dye at the downstream and upstream ends of the reach, in hours; and

ln=natural logarithm, base e.

Because rhodamine-WT dye is not completely conservative in streams, the rhodamine-WT dye curves must be corrected for dye loss before the reaeration coefficients are computed. Also, the curves must be corrected for any flow accrual that occurred.

It can be shown from the conservation of mass that

$$Q_1A_1=Q_2A_2=Q_3A_3=. \; . \; .Q_nA_n, \tag{13}$$

where Q=discharge at each cross section where samples are collected, and

A=corresponding area under the curves of the dye concentration versus time for each sample cross section where samples are collected.

If there is dye loss, then Q_2A_2 will be less than Q_1A_1 and Q_nA_n will be less than $Q_{n-1}A_{n-1}$. The correction procedure is to multiply each point on the curve of dye concentration versus time curve by a correction factor, J. Hence, equation 13 becomes:

$$Q_1A_1=Q_2A_2J_2=. \; . \; .Q_nA_nJ_n. \tag{14}$$

Therefore,

$$J_2 = \frac{Q_1 A_1}{Q_2 A_2},$$

and

$$J_n = \frac{Q_1 A_1}{Q_n A_n}.$$

Equation 12, for the subreach between cross sections 1 and 2, then becomes:

$$K_{G_{1-2}} = \frac{1}{t_2 - t_1} ln\left(\frac{C_{G_1} / C_{D_1}}{C_{G_2} / C_{D_2} \cdot J_2}\right), \quad (15)$$

with the same form for the remaining downstream cross sections.

Area Method

For those reaches where sufficient samples are collected to define the complete curves of tracer-gas concentration versus time, the reaeration coefficient can be computed from the areas under the curves. The basic equation is:

$$K_G = \frac{1}{t_d - t_u} ln\left(\frac{A_u}{A_d}\right), \quad (16)$$

where A_u, A_d=area under the curve of tracer-gas concentration versus time at the upstream and downstream ends of the reach; and

t_d, t_u=traveltime of the centroids of the tracer-gas mass at the downstream and upstream ends of the reach.

If there is flow accrual, the areas must be corrected and equation 16 becomes:

$$K_G = \frac{1}{t_d - t_u} ln\left(\frac{A_u Q_u}{A_d Q_d}\right), \quad (17)$$

where Q_u, Q_d=discharge at the upstream and downstream ends of the reach.

The area method has the advantage of being independent of the measurements of the dye and, therefore, the nonconservative nature of the dye is not critical. The disadvantage is that complete curves of tracer-gas concentration versus time must be determined and discharge measurements must be made. The peak method, in comparison, only requires complete curves of dye concentration versus time and discharge measurements to permit correction for dye loss.

The desorption coefficient computed by the peak or area method is converted to a reaeration coefficient (K_2-base e logarithmic units) with the relation:

$$K_2=R \cdot K_G, \tag{18}$$

where R=the ratio of the absorption coefficient for oxygen to the desorption coefficient for the tracer gas (determined in the laboratory).

From laboratory studies by Rathbun, Stephens, Shultz, and Tai (1978), the following relationships have been determined:

$$\text{Ethylene, } K_2=1.15K_G, \tag{19}$$

and

$$\text{Propane, } K_2=1.39K_G. \tag{20}$$

Experimental Procedure

The experimental procedure consisted of three steps: Injecting the tracers into the stream; sampling the tracers at downstream points; and analyzing the samples for concentrations of the tracers. Each of these steps is briefly described in the following sections. For more details, refer to Rathbun, Shultz, and Stephens (1975) and Rathbun and Grant (1978).

Injection of the Tracers

Ethylene and propane were injected into the stream by bubbling the gases through a porous tube diffuser like those used for aeration in wastewater-treatment plants. The diffusers were mounted in frames and placed on the bottom of the stream, generally at the deepest point or in the area of greatest flow. Ethylene and propane were released directly from high-pressure cylinders through two-stage regulating valves and rotameters for monitoring the flows to the diffusers.

A solution of rhodamine-WT dye and water was injected at the same injection point and for the same injection period as that used for the ethylene and propane gases. A direct-displacement pump was used for continuous injection. Dye-injection rates and concentrations appropriate for the stream discharges were estimated using equations presented by E. D. Cobb and J. F. Bailey (written commun., 1965).

Sampling the Tracers

A dye-detection system consisting of a fluorometer and portable generator was used. The dye concentration was monitored, using the fluorometer, as samples were collected as a function of time at approximately the center of flow. The dye samples were collected in 1.1-fluid ounce (32-mL) bottles with polyseal caps for subsequent analysis in the laboratory.

Samples for gas analysis were collected from the center of flow using a direct-displacement sampler. The sample bottles were placed in 2.0-fluid ounce (about 60-mL) glass bottles with ground-glass stoppers. Samples were preserved for later laboratory analysis by adding 1 mL of 37-percent formalin stock solution to each sample.

Sample Analysis

Ethylene and propane concentrations in the water samples were determined using a modification of the gas-chromatographic technique of Swinnerton and Linnenbom (1967). Basically, the technique consists of: Introducing a known aliquot of the water sample into a stripping column; stripping the ethylene and propane from the water with helium gas; trapping the tracer gases in a cold trap; warming the trap once the stripping process is completed; and flushing the ethylene and propane from the cold trap into a gas chromatograph equipped with a flame-ionization detector. The modified procedure, together with techniques developed for sample storage and preservation, are described in a report by Shultz, Pankow, Tai, Stephens, and Rathbun (1976). Dye samples were analyzed using a fluorometer and standard techniques described by E. D. Cobb (written commun., 1965); Wilson (1968); and E. F. Hubbard, F. A. Kilpatrick, L. A. Martens, and J. F. Wilson (written commun., 1978).

REAERATION-COEFFICIENT RESULTS

Reaeration coefficients were determined for three subreaches of the Yampa River during the low-flow period in September 1976. Because of limited time and funding, a similar analysis was not done for the Little Snake River. The subreaches, as described earlier, were parts of the study reaches from river-mile 190.5 to 171.0 (river-km 306.7 to 275.3) and included three injection sites and nine sampling sites (table 1 and fig. 2). Samples were collected once during the low-flow period from each of the three subreaches. The concentrations of ethylene and propane tracer gases, and rhodamine-WT dye measured in the three subreaches are shown in figures 20 to 22. Dye and gas tracers were injected continuously for 90 minutes at site 2, 59 minutes at site 7, and 85 minutes at site 12 (fig. 2). Approximate constant concentrations of dye and gas were measured at sampling sites 3 and 13 (figs. 20 and 22). Approximate constant concentrations can be obtained using the continuous-injection procedures when the sampling site is located near the injection site. Sampling site 3 is located 0.5 mi (0.8 km) downstream from the injection site and sampling site 13 is located 0.9 mi (1.4 km) downstream from the injection site. Data collected at sampling site 8, located

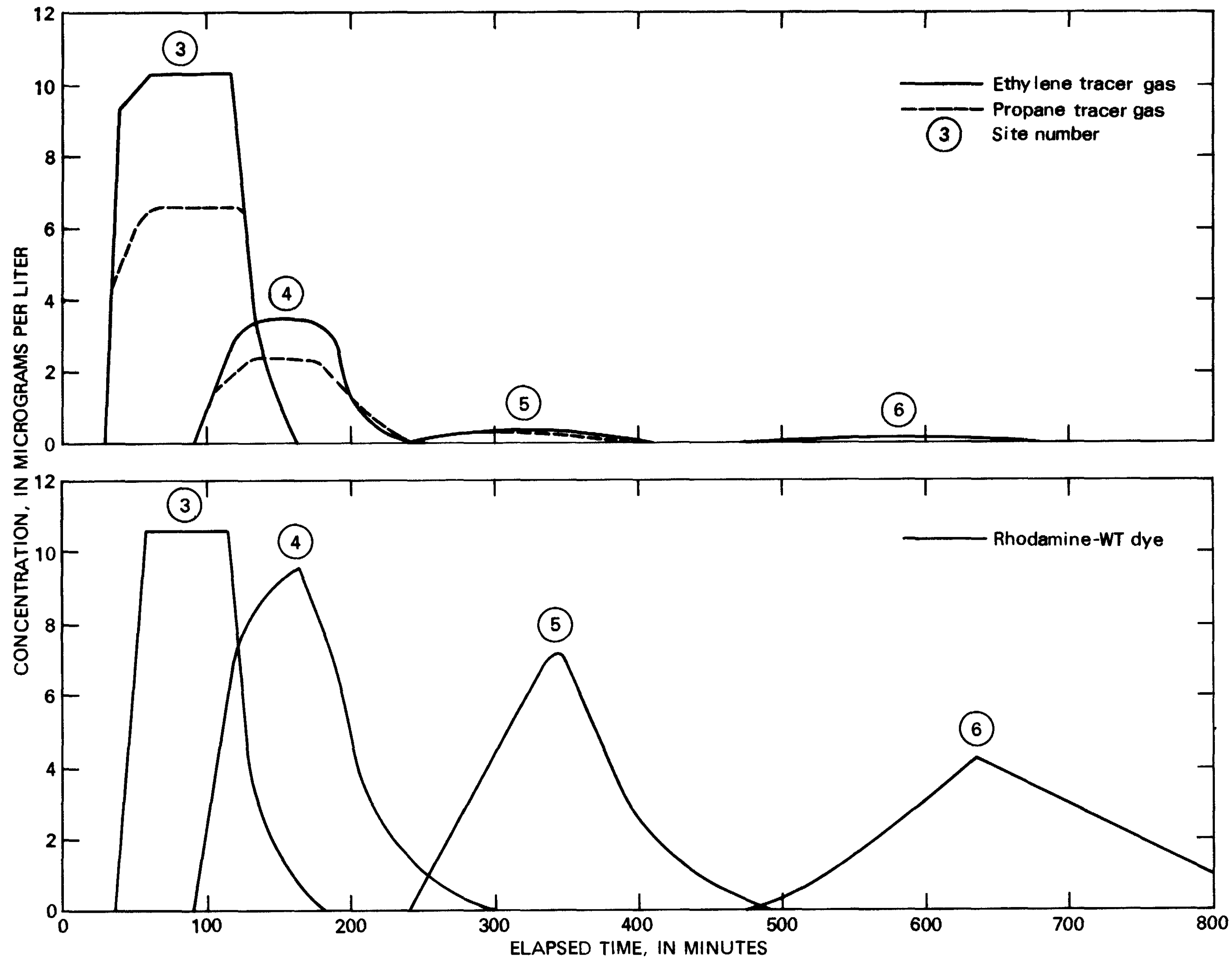

Figure 20.--Concentrations of ethylene and propane tracer gases, and rhodamine-WT dye, Yampa River, river-mile 191.0 to 185.2, September 1976.

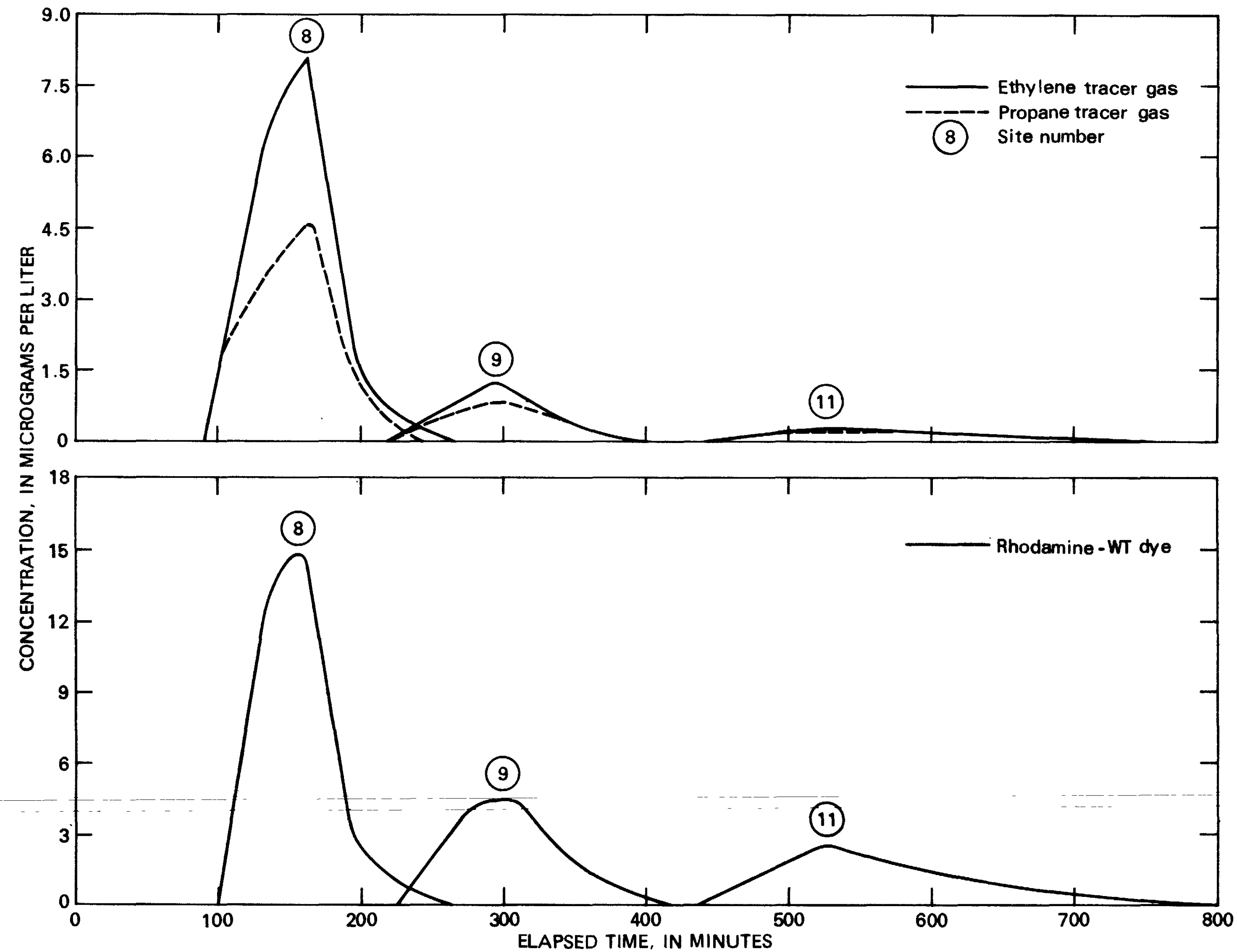

Figure 21.--Concentrations of ethylene and propane tracer gases, and rhodamine-WT dye, Yampa River, river-mile 183.4 to 177.3, September 1976.

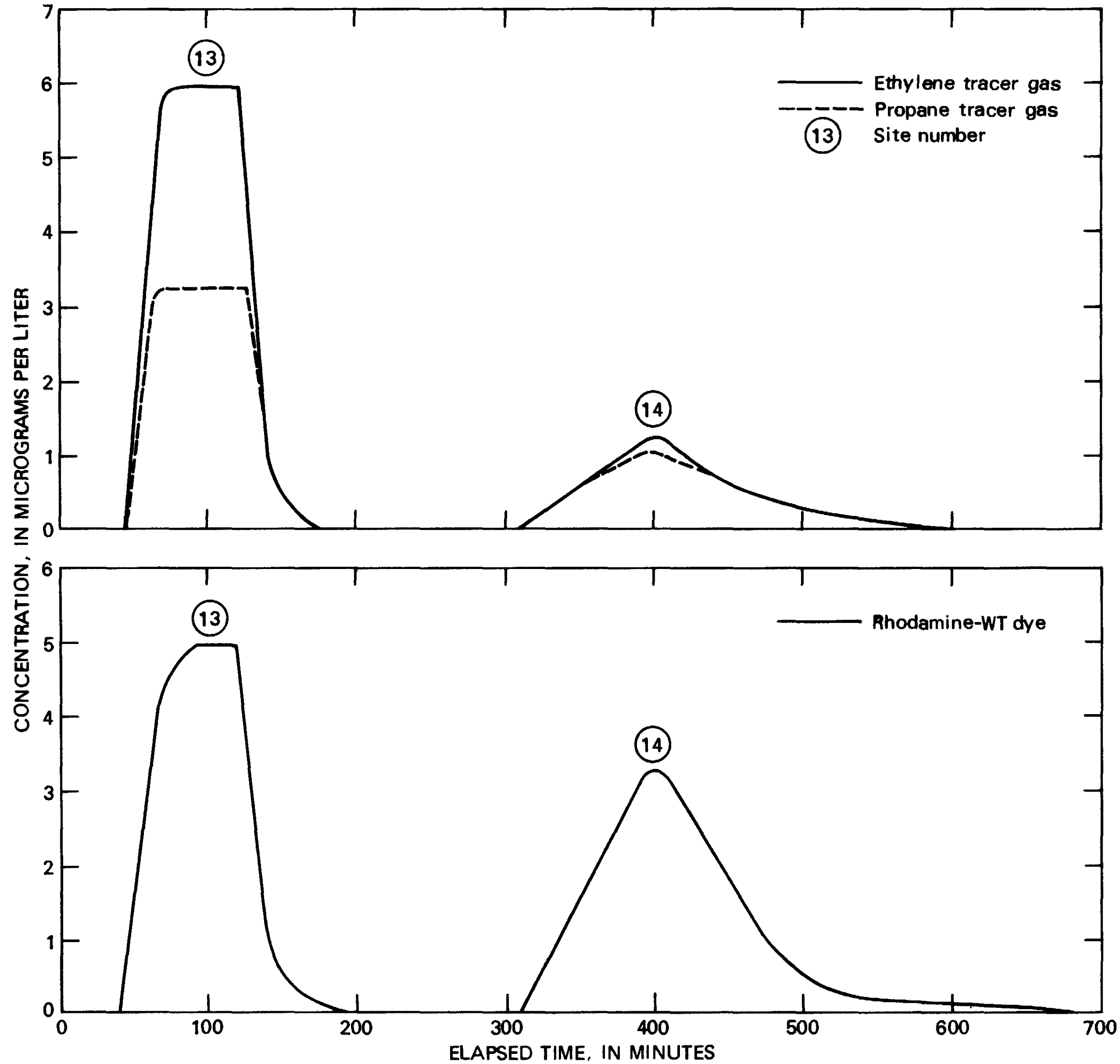

Figure 22.--Concentrations of ethylene and propane tracer gases, and rhodamine-WT dye, Yampa River, river-mile 175.5 to 171.0, September 1976.

2.2 mi (3.5 km) downstream from injection site 7, exhibited no constant concentrations for the tracer gases or dye (fig. 21).

Dye Loss, Flow Accrual, and Dye-Correction Factors

To compute the reaeration coefficients by the peak method, described on page 48, requires adjustment of the curves showing concentrations of rhodamine-WT dye for dye loss and flow accrual. The correcting of the dye concentrations was done using equation 14. A tabulation of the areas under the curves of dye concentration versus time, traveltime of the centroids of the dye clouds, and discharges are given in table 5 for the three subreaches studied. The three subreaches included the following sampling sites: Subreach 1--sample sites 3 to 6; subreach 2--sample sites 8, 9, and 11; subreach 3--sampling sites 13 and 14 (fig. 2). Also given in table 5 are the areas under the curves and the traveltimes of the centroids for the ethylene and propane gas tracers.

The streamflow discharges for subreach 1 (sampling sites 3 to 6) indicate that there was probably no significant flow accrual in the subreach. There may have been a flow loss between sites 5 and 6; however, it was assumed that the gas tracers and dye are lost in proportionate amounts to the flow loss, so that a correction is not necessary. Hence, it was assumed for computing dye-correction factors that there was no increase in discharge, so that equation 14 reduces to:

$$A_3=A_4J_4=A_5J_5=A_6J_6, \qquad (21)$$

where the subscripts denote the sampling sites. The areas under the dye curves (table 5) for sites 3, 4, and 6 were computed using a computer program. The measured area for site 5, however, was found to be less than the measured area for site 6 and was not used. This was the result of insufficient samples at site 5 to correctly define the tail of the dye cloud. Dye-correction factors for the dye loss at sites 4 and 6 were computed using equation 21 and data from table 5:

$$J_4=\frac{A_3}{A_4}=\frac{974}{894}=1.09,$$

and

$$J_6=\frac{A_3}{A_6}=\frac{974}{791}=1.23.$$

To estimate J_5, it was assumed that the dye loss was directly proportional to the traveltime, or:

$$J=a\Delta t^b \qquad (22)$$

where Δt=elapsed traveltime between sampling sites, in minutes; and
a,b=unknown equation coefficients.

Table 5.--*Data determined from curves of concentrations versus time for ethylene and propane tracer gases, and rhodamine-WT dye, Yampa River, September 1976*

Site number	Area under curves (microgram-minutes per liter)			Traveltime of centroid (minutes)			Discharge (cubic feet per second)
	Ethylene	Propane	Dye	Ethylene	Propane	Dye	
3	952	607	974	82.2	82.4	82.7	78.5
4	318	217	894	153	155	160	76.3
5	-----	-----	---	-----	-----	-----	83.1
6	-----	-----	791	-----	-----	692	72.2
8	510	304	911	150	150	153	73.0
9	101	69.1	436	301	305	312	152
11	29.1	27.2	384	540	552	569	171
13	497	266	407	93.1	92.9	93.4	204
14	113	102	363	407	412	418	210

Using known data from sampling sites 3, 4, and 6, a and b were estimated using least squares and a J_5 of 1.17 was computed from the resulting equation.

The flows in subreach 2 (sampling sites 8, 9, and 11) increased significantly between sites 8 and 9 (table 5). The increases in flow resulted largely from the additional flow of the Elk River and also from some local runoff of rainfall that occurred during the day. Dye-correction factors for this subreach, computed by equation 14, were as follows:

$$J_9=\frac{(Q_8)(A_8)}{(Q_9)(A_9)}=\frac{(73)(911)}{(152)(436)}=1.00,$$

and

$$J_{11}=\frac{(Q_8)(A_8)}{(Q_{11})(A_{11})}=\frac{(73)(911)}{(171)(384)}=1.01.$$

These results indicate only a very small dye loss for this subreach.

For subreach 3 (sampling sites 13 and 14), both flow accrual and dye loss were considered (table 5). A dye-correction factor was computed as follows:

$$J_{14}=\frac{(Q_{13})}{(Q_{14})}\cdot\frac{(A_{13})}{(A_{14})}=\frac{(204)}{(210)}\cdot\frac{(407)}{(363)}=1.089.$$

Computation of Reaeration Coefficients

Reaeration coefficients were computed for the Yampa River study reach using both the peak and area methods (see p. 39 and 40). The peak concentrations and times for the ethylene and propane gas tracers and rhodamine-WT dye are given in table 6. These concentrations and times were used in the peak method of the reaeration-coefficient computation. Example reaeration-coefficient computations for the ethylene gas tracer covering a reach between sample sites 3 and 4 are as follows:

Area method:

$$K_{e(3-4)}=\frac{1}{t_d-t_u}ln\left(\frac{A_u}{A_d}\right)=\frac{1}{153-82.2}ln\left(\frac{952}{318}\right)=0.0154\ \text{min}^{-1},$$

$$K_{2(3-4)}=R_e\cdot K_{e(3-4)}=1.15(0.0154)(1{,}440\ \text{min/d})=25.6\ \text{d}^{-1}.$$

Note: min^{-1}=per minute; min/d=minutes per day; and d^{-1}= per day.

Peak method:

$$K_{e(3-4)}=\frac{1}{t_d-t_u}ln\left(\frac{C_{G_U}/C_{D_U}}{C_{C_D}/C_{D_D}/J_D}\right)=\frac{1}{160-86.5}ln\ \frac{10.3/10.7}{3.62/9.56(1.09}=0.0139\ \text{min}^{-1}.$$

$$K_{2(3-4)}=R_e\cdot K_{e(3-4)}=1.15(0.0139)(1{,}440\ \text{min/d})=23.1\ \text{d}^{-1}.$$

The sample computation for the area method used equations 16 and 19 and the peak methods used equations 12 and 19. The remaining reaeration coefficients computed using these equations and the observed field condition are given in table 7. Reaeration coefficients were computed using the area method when sufficient samples were taken to estimate the entire concentration-time curves. The observed field reaeration coefficients were adjusted to a common temperature base of 20°C (Celsius) by the following formula (Elmore and West, 1961):

$$K_{2(20°C)}=K_{2(t)}(1.0241)^{20-t}, \tag{23}$$

where t=mean-reach water temperature, in degrees Celsius.

The mean-subreach water temperatures and adjusted reaeration coefficients are given in table 7. The reaeration coefficients, adjusted to a common 20°C base, ranged from 33.4 to 6.62 d^{-1} using the area method, and from 31.4 to 6.04 d^{-1} using the peak method.

Table 6.--*Peak concentrations and traveltime of peak concentrations for ethylene and propane tracer gases, and rhodamine-WT dye, Yampa River, September 1976*

Site number	Peak concentration (micrograms per liter)			Traveltime of peak concentrations (minutes)		
	Ethylene	Propane	Dye	Ethylene	Propane	Dye
3	10.3	6.64	10.7	85.5	85.5	86.5
4	3.62	2.42	9.56	156	156	160
5	.283	.285	6.74	325	333	340
6	------	.035	4.01	-----	602	642
8	7.96	4.70	14.7	159	159	159
9	1.21	.815	4.51	292	295	290
11	.235	.203	2.53	529	529	524
13	6.06	3.23	5.01	100	100	102
14	1.16	.996	3.21	402	402	408

Comparisons of Results to Empirical Formulas

Comparisons of the experimental reaeration coefficients with coefficients from predictive equations are given in this section. These comparisons give some measure of the degree of uncertainty inherent in the predictive equations. Two types of equations were used in the comparisons: Semi-empirical equations, which include those based on the rate-of-energy dissipation and those in which the reaeration coefficient (K_2) is correlated with the longitudinal-dispersion coefficient; and empirical equations of the form:

$$K_2 = a\bar{U}^b / H^c \qquad (24)$$

where a, b, c=coefficients of a given equation,
$\bar{U}$=mean velocity of stream, and
H=mean depth of stream.

A thorough discussion of the various semiempirical and empirical predictive equations is given by Rathbun (1977).

Table 7.--*Reaeration coefficients for ethylene and propane determined using area and peak computation methods, selected subreaches of Yampa River, September 1976*

Subreach as defined by site numbers indicated	Water temperature degrees Celsius	Reaeration coefficient based on measured water temperatures (day^{-1})				Reaeration coefficient adjusted to 20 degrees Celsius (day^{-1})			
		Peak method		Area method		Peak method		Area method	
		Ethylene	Propane	Ethylene	Propane	Ethylene	Propane	Ethylene	Propane
3-4	13.3	23.1	26.8	25.6	28.5	27.1	31.4	30.0	33.4
4-5	16.2	20.8	20.6	-----	-----	22.8	22.6	-----	-----
5-6	18.0	-----	10.9	-----	-----	-----	11.4	-----	-----
3-6	15.1	-----	16.1	-----	-----	-----	18.1	-----	-----
8-9	13.8	8.93	8.78	9.71	9.69	10.4	10.2	11.2	11.2
9-11	14.4	7.56	7.01	7.82	6.60	8.64	8.01	8.94	7.54
8-11	14.0	8.05	7.65	8.56	7.79	9.29	8.83	9.87	8.99
13-14	14.8	7.00	5.34	7.67	5.85	7.92	6.04	8.68	6.62

To compare the various equation results, it was necessary first to calculate various hydraulic and energy-dissipation properties for the stream subreaches studied (table 8). These hydraulic and energy-dissipation properties then were used in the equation computations. Following is a short summary of the selected equations used.

Semiempirical (Energy-Dissipation) Equations

Lau (1972):

$$k_2=0.0126\left(\frac{U_*}{\bar{U}}\right)^3 \frac{\bar{U}}{H}, \quad (25)$$

where k_2=reaeration coefficient, base 10 units, 20°C, in seconds^{-1};

U_*=mean-reach shear velocity, in feet per second;

$\bar{U}$=mean-reach velocity, in feet per second; and

H=mean-reach stream depth, in feet.

Krenkel and Orlob (1963):

$$k_2=1.141\times10^{-4}(\bar{U}_{fm}Sg_{fm})^{0.408}/H^{0.66}, \quad (26)$$

where k_2=reaeration coefficient, base 10 units, 20°C, in minutes^{-1};

S=slope of energy gradient, in feet per foot;

$\bar{U}_{fm}$=mean stream velocity, in feet per minute; and

g_{fm}=acceleration of gravity, in feet per minute^2.

Parkhurst and Pomeroy (1972):

$$K_2=0.96(1+0.17F^2)T_{co}{}^2(\bar{U}_{ms}S/Hm)^{0.375}, \quad (27)$$

where K_2=reaeration coefficient, base e units, 20°C, in hours^{-1};

F=Froude number, defined as $F=\bar{U}_{ms}/(gH_m)^{0.5}$;

T_{co}=water-temperature correction factor;

$\bar{U}_{ms}$=mean-reach velocity, in meters per second; and

H_m=mean-reach stream depth, in meters.

Table 8.--*Geometry of selected subreaches of the Yampa River, and traveltime and velocity data collected in the subreaches during September 1976*

Subreach as defined by site numbers indicated	Geometry				Traveltime data		Velocity data			
	Decrease in elevation of water surface through subreach (feet)	Length of subreach (feet)	Slope of subreach (feet per feet)	Mean depth of water in subreach (feet)	Traveltime of peak concentration of dye through subreach (minutes)	Traveltime of centroid of dye cloud through subreach (minutes)	Mean velocity of peak concentration of dye (feet per second)	Mean velocity of centroid of dye cloud (feet per second)	Shear velocity (feet per second)	Mean velocity of peak times slope of subreach (feet per second)
3-4	28	6,340	0.00442	0.95	73.5	77.3	1.44	1.36	0.367	0.00636
4-5	50	13,730	.00364	.87	180	-----	1.27	-----	.320	.00462
5-6	46	7,920	.00581	1.23	302	-----	.437	-----	.479	.00254
3-6	124	27,990	.00443	1.09	555	609	.840	.766	.394	.00372
8-9	24	10,030	.00239	.99	131	159	1.28	1.05	.276	.00306
9-11	31	14,780	.00210	1.23	234	257	1.05	.959	.288	.00220
8-11	55	24,810	.00222	1.06	365	416	1.13	.993	.275	.00250
13-14	33	19,010	.00174	1.27	306	325	1.04	.976	.266	.00180

Tsivoglou and Neal (1976):

$$K_{2(20)}=0.054H_{ch}/t_t, \tag{28}$$

where $K_{2(20)}$=reaeration coefficient, base e units, 20°C, in hours^{-1};

H_{ch}=reach elevation change, in feet; and

t_t=reach traveltime, in hours.

Cadwallader and McDonnell (1969):

$$k_2=25.7E^{0.5}/H, \tag{29}$$

where k_2=reaeration coefficient, base 10 units, 20°C, in days^{-1};

$E=\bar{U}Sg$, in feet^2 per second^3; and

g=acceleration of gravity, in feet per second^2.

Bennett and Rathbun (1972):

$$k_2=46.05\bar{U}^{0.413} \cdot S^{0.273}/H^{1.408}. \tag{30}$$

Thackston and Krenkel (1969):

$$k_2=10.8(1+F^{0.5})U_*/H. \tag{31}$$

Churchill, Elmore, and Buckingham (1962):

$$k_2=1.447\bar{U}^{1.049} \cdot H^{-2.262} \cdot f^{-0.823}, \tag{32}$$

where f=flow-resistance factor, defined as $f=8gHS/\bar{U}^2$.

Dobbins (1965):

$$k_2=\frac{0.12C_a(30.0S_1\bar{U})^{0.375}A\ Coth(BE^{0.126}/C_4)}{C_4^{1.5}\ H}, \tag{33}$$

where $C_a=1.0+F^2$;

$C_4=0.9+F$

$A=9.68+0.054(t-20)$; where t=water temperature; in degrees Celsius;

$B=0.976+0.0137(30-t)^{1.5}$;

$E=30.0S_1\bar{U}$;

S_1=slope; in feet per 1,000 feet; and

$Coth$=hyperbolic cotangent angle, in radians.

Empirical (Velocity-Depth) Equations

Churchill, Elmore, and Buckingham (1962):

$$k_2=5.026\bar{U}^{0.969}/H^{1.673}. \tag{34}$$

Langbein and Durum (1967):

$$k_2=3.3\bar{U}/H^{1.33}. \tag{35}$$

Owens, Edwards, and Gibbs (1964):

(equation 1) $$k_2=10.09\bar{U}^{0.73}/H^{1.75}. \tag{36}$$

(equation 2) $$k_2=9.41\bar{U}^{0.67}/H^{1.85}. \tag{37}$$

Isaacs and Gaudy (1968):

$$k_2=3.739\bar{U}/H^{1.5}. \tag{38}$$

Negulescu and Rojanski (1969):

$$k_2=4.74(\bar{U}/H)^{0.85}. \tag{39}$$

Bennett and Rathbun (1972):

$$k_2=8.76\bar{U}^{0.607}/H^{1.689}. \tag{40}$$

O'Connor and Dobbins (1958):

$$k_2=127.6(D_L\bar{U})^{0.5}/H^{1.5}, \tag{41}$$

where D_L=molecular-diffusion coefficient of oxygen in water, in feet^2 per day.

Padden and Gloyna (1971):

$$k_2=2.98(\bar{U}/H^{1.5})^{0.703}. \tag{42}$$

Bansal (1973):

$$K_{2(25)}=0.219\bar{U}^{0.6}/H^{1.4}, \tag{43}$$

where $K_{2(25)}$=reaeration coefficient, base e units, 25°C, in hours^{-1}.

Result Comparisons of Measured Reaeration Coefficients Versus Predicted Values using Empirical and Semiempirical Equations

A summary of the comparisons of the averages of the measured reaeration coefficients, and the reaeration coefficients predicted with the semiempirical and empirical equations, are given in tables 9 and 10. The computed values listed in the tables are in the same order, by column, as the predictive equations listed above. A tabular listing of the individual errors of estimate is given in table 11. The error of estimate (*PE*) is defined as follows:

$$PE = (K_2\text{pred} - K_2\text{exp}) / K_2\text{exp}, \qquad (44)$$

where K_2pred=reaeration coefficient by equation, and
K_2exp=reaeration coefficient measured experimentally.

The average absolute errors of estimate were used in the computation because individual error values can either be negative or positive (table 11). Results of the error analysis indicate that the Tsivoglou and Neal (1976) equation average error of estimate of 0.1181 and the Thackston and Krenkel (1969) equation average error of estimate of 0.1727 gave the lower error results. Both of these equations are semiempirical energy-dissipation equations. The one extreme outlier was the prediction equation by Lau (1972) with an average absolute error of estimate of 15.98.

Comparison of Traveltime and Reaeration Coefficients with Results of Analysis of Waste-Load Assimilative Capacity

An analysis of the waste-load assimilative capacity of the Yampa River from Steamboat Springs to Hayden, Colo., a distance of 38 mi (61 km), was documented by Bauer, Steele, and Anderson (1978). That study was carried out to assess the impacts on the stream reach from existing and projected waste loadings, and involved the application of a steady-state water-quality model (Bauer and Jennings, 1975). Two basic study-reach conditions were considered: Model calibration for measured water quality and flow data, and model simulation based on 7-day low flows with a 10-year recurrence interval. Reach traveltime for the calibration phase was estimated using cross-sectional properties and mean velocities from discharge measurements. The simulation-phase traveltime was related to mean discharges and streambed slope by a regression-equation procedure described by Boning (1974).

A comparison of the simulated traveltime for the Yampa River using results from the rhodamine-WT dye (figs. 9 and 10) and the methods used for the waste-load assimilative-capacity analysis is shown in table 12. The comparisons were made on the same reach boundaries as the Yampa River traveltime study except for the lower Yampa River reach, river-mile 179.0 to 137.3 (river-km 288.2 to 221.1), which included only river-mile 179.0 to 159.6 (river-km 288.2 to 257.0). The index-discharge stations, station numbers 09239500 and 09244410, were also the same stations as described for

Table 9.--*Comparison between reaeration coefficients determined using measured data (September 1976) and those determined using energy-dissipation equations, selected subreaches of Yampa River*

Subreach as defined by site numbers indicated	Reaeration coefficients (day^{-1}), base *e* units									
	Determined using measured data	Determined using energy-dissipation equations								
		Lau (1972)	Krenkel and Orlob (1963)	Parkhurst and Pomeroy (1972)	Tsivoglou and Neal (1976)	Cadwallader and McDonnell (1969)	Bennett and Rathbun (1972)	Thackston and Krenkel (1969)	Churchill, Elmore, and Buckingham (1962)	Dobbins (1965)
3-4	30.5	63.1	30.8	7.76	29.7	28.3	30.3	14.6	9.48	17.5
4-5	22.7	58.3	28.5	7.45	21.6	26.1	30.6	13.6	10.1	17.2
5-6	11.4	1,170	17.8	4.19	11.8	13.8	13.8	12.2	.14	11.8
3-6	18.1	199	22.6	5.47	17.4	18.8	19.9	12.4	1.43	14.0
8-9	10.8	32.5	22.2	5.63	14.3	18.8	23.0	10.2	10.0	13.2
9-11	8.28	44.2	16.8	3.99	10.3	12.8	15.0	8.20	3.33	9.96
8-11	9.25	38.5	19.6	4.86	11.7	15.8	19.4	9.29	6.14	11.8
13-14	7.32	34.4	15.2	3.58	8.42	11.2	13.6	7.31	3.44	9.02

Table 10.--*Comparison between reaeration coefficients determined using measured data (September 1976) and those determined using velocity-depth equations, selected subreaches of Yampa River*

Subreach as defined by site numbers indicated	Reaeration coefficients ($days^{-1}$), base *e* units										
	Determined using measured data	Determined using velocity-depth equations									
		Churchill, Elmore, and Buckingham (1962)	Langbein and Durum (1967)	Owens, Edwards, and Gibbs[1] (1964)	Owens, Edwards, and Gibbs[2] (1964)	Isaacs and Gaudy (1968)	Negulescu and Rojanski (1969)	Bennett and Rathbun (1972)	O'Connor and Dobbins (1958)	Padden and Gloyna (1971)	Bansal (1973)
3-4	30.5	18.1	11.8	33.4	30.6	13.5	15.6	27.6	16.9	9.39	6.27
4-5	22.7	18.3	11.5	35.0	32.6	13.4	15.0	29.3	17.9	9.36	6.50
5-6	11.4	3.67	2.52	8.84	8.48	2.76	4.53	8.67	6.28	3.08	2.12
3-6	18.1	8.47	5.69	17.6	16.4	6.36	8.75	15.7	10.4	5.54	3.73
8-9	10.8	15.0	9.87	28.4	26.1	11.2	13.6	23.9	14.9	8.26	5.50
9-11	8.28	8.59	6.06	16.8	15.3	6.63	9.54	14.6	9.74	5.71	3.60
8-11	9.25	11.8	7.95	22.9	21.1	8.92	11.5	19.7	12.6	7.03	4.63
13-14	7.32	8.06	5.75	15.7	14.3	6.26	9.21	8.69	9.23	5.48	3.42

[1]Equation 1.
[2]Equation 2.

Table 11.--*Error analysis of reaeration coefficients*

[Based on comparisons presented in tables 9 and 10 between reaeration coefficients determined using measured data (September 1976) and those determined using energy-depth and velocity-depth equations for selected subreaches of the Yampa River]

Energy dissipation			Velocity and depth		
Equation used	Range of error of estimate	Average error of estimate (absolute value)	Equation used	Range of error of estimate	Average error of estimate (absolute value)
Lau (1972)----------	101.90 → 1.070	15.98	Churchill, Elmore, and Buckingham (1962).	-0.6779 → 0.3874	0.3271
Krenkel and Orlob (1963).	1.116 → .0108	.6704			
Parkhurst and Pomeroy (1972).	-.7455 → -.4741	.5911	Langbein and Durum (1967).	-.7788 → -.0858	.4098
Tsivoglou and Neal (1976).	.1862 → -.1526	.1181	Owens, Edwards, and Gibbs[1] (1964).	1.627 → -.2246	.7715
Cadwallader and McDonnell (1969).	.7399 → -.0729	.3756	Owens, Edwards, and Gibbs[2] (1964).	1.416 → -.2557	.6605
Bennett and Rathbun (1972).	1.127 → -.0064	.5696	Isaacs and Gaudy (1968).	-.7579 → .0380	.3492
Thackston and Krenkel (1969).	-.5226 → .0735	.1727	Negulescu and Rojanski (1969).	-.6025 → .2590	.3580
Churchill, Elmore, and Buckingham (1962).	-.9881 → -.0744	.5861	Bennett and Rathbun (1972).	1.212 → .2450	.5949
			O'Connor and Dobbins (1958).	-.4491 → .3804	.3393
Dobbins (1965)------	-.4260 → .2754	.2329	Padden and Gloyna (1971).	-.7295 → .2351	.4673
			Bansal (1973)-------	-.8134 → .4907	.6504

[1]Equation 1.
[2]Equation 2.

Table 12.--*Comparison between traveltime simulations determined by Bauer, Steele, and Anderson (1978) and those determined during this study using rhodamine-WT dye, Yampa River, Steamboat Springs to Hayden, Colo.*

Model phase	Index flow station	Index flow (cubic feet per second)	Site numbers in reach between Steamboat Springs and Hayden, Colo.	Reach extent (river miles)	Traveltime simulation (hours)	
					Bauer, Steele, and Anderson (1978)	This study
Calibration---	09239500	100	Ym-0 to Ym-8	195.5 to 179.0	26.0	27.5
Calibration---	09244410	130	Ym-8 to Ym-13	179.0 to 159.6	36.0	38.2
Simulation----	09239500	28	Ym-0 to Ym-8	195.5 to 179.0	51.0	51.0
Simulation----	09244410	50	Ym-8 to Ym-13	179.0 to 159.6	61.0	63.0

time study. Estimates of the traveltime for the waste-load assimilative-capacity analysis agree within 6 percent of the model simulation results (table 12).

The reaeration coefficient for the waste-load assimilative-capacity analysis was computed using a velocity-depth equation described by Bennett and Rathbun (1972). The comparison of the reaeration coefficient computed by this predictive equation with the experimentally determined reaeration coefficient indicated an average error of estimate of 0.5995 (table 11). The results of this analysis (table 11) indicated that the semiempirical energy-dissipation equations by Tsivoglou and Neal (1976) and Thackston and Krenkel (1969) gave the lowest average errors of estimate for the studied subreaches of the Yampa River. The amount of error induced by the use of the Bennett and Rathbun (1972) reaeration equation for the earlier waste-load assimilative-capacity analysis was not calculated.

APPLICATIONS

The traveltime and unit-concentration versus index-discharge simulations (figs. 9 to 12 and 15 to 18) provide a convenient means of predicting the arrival time and concentration of soluble contaminants accidentally spilled in a stream.

As an example, assume that a train accident occurred in Steamboat Springs in which a tanker ruptured and spilled its contents of 1,000 pounds (453 kg) of a soluble contaminant into the Yampa River near site 3 (fig. 2). Assume the index discharge at gaging station 09239500 is 1,000 ft^3/s (28 m^3/s). One immediate concern would be to determine the arrival time and peak concentration of the contaminant at the Hayden water plant, river-mile 159.6 (river-km 257) (site 16). For the index discharge of 1,000 ft^3/s (28 m^3/s), the traveltime of the peak concentration from sites 3 to 16 [river-mile 190.5 to 159.6 (river-km 307 to 257)] would be 15.7 hours (fig. 9). The leading edge of the solute could be expected to arrive at site 16 in 15.7/1.25=12.5 hours (equation 1). The curve of figure 15 for an index discharge of 1,000 ft^3/s (28 m^3/s) at a traveltime of 15.7 hours indicates a unit concentration for the peak of 1,580. The estimated peak concentration (equation 2, p. 15) for a conservative contaminant would be:

$$C_{cpk}1{,}000\ ft^3/s = \frac{1{,}580\ \mu g/L\ ft^3/s}{lb} \cdot \frac{1{,}000\ lb}{1{,}000\ ft^3/s} = 1{,}580\ \mu g/L \text{ at Hayden, Colo.,}$$

at 15.7 hours after the spill. Health standards for a particular contaminant would indicate the potential toxic effects of such a concentration, and dictate whether water use would need to be curtailed.

As noted earlier, the dye-cloud leading edge travels faster than the dye-cloud peak (equation 1) and, as a result, some concentration of the contaminant would arrive sooner than the 15.7-hour peak arrival. Due to the dispersive characteristics of the contaminant, it would also persist for some time after the peak-concentration occurrence.

As a second example for the Little Snake River, assume that an accident occurs near site 1 and 1,000 pounds (453 kg) of soluble contaminant has spilled into the river with an index flow of 1,000 ft^3/s (28 m^3/s) at gaging station 09253000. It is desired to predict the traveltime and concentration of the contaminant at Baggs, Wyo. (site 10), 44.0 mi (70.8 km) downstream. The predicted traveltime (fig. 11) is 13.8 hours and the peak concentration (fig. 17) is 3,000 μg/L. The approximate leading-edge traveltime is 13.8/1.25=11.0 hours (equation 1).

On the main-stem Yampa and the Little Snake Rivers, several reservoir projects have been proposed (Steele and others, 1979; Steele, 1978). The proposed reservoirs have several uses, including irrigation, hydropower, municipal water supply, and industry. Another application of the study results would be to predict the average traveltimes for water released from the reservoirs to given locations downstream. It should be noted that this type of application is restricted to situations where the flow change from reservoir releases is small compared to the present downstream-flow conditions. The relationships developed earlier were developed for mean water velocity and assume steady flow-rate conditions. A large abrupt increase in flow causes a flood wave in the stream channel. Flood waves normally travel approximately 1.5 to 2.0 times faster than the mean water velocity.

There are several wastewater-treatment plants along the Yampa and the Little Snake Rivers. An earlier waste-load assimilative-capacity analysis of the Yampa River from Steamboat Springs to Hayden, Colo. (Bauer and others, 1978) indicated possible nonionized ammonia-nitrogen pollution problems. The length of stream for which a pollution problem of this type could persist is of interest to water managers. Another application of this study would then be to predict the length of stream and waste concentration, as a function of waste-decay rate, with downstream traveltime from the wastewater-treatment plant. For example, the length of stream required to reduce a waste with an initial in-stream concentration of 20 mg/L (milligrams per liter) to a concentration of 2.0 mg/L at some point downstream for the Yampa River is desired. For this computation, assume an index flow of 100 ft^3/s (2.8 m^3/s) at the Steamboat Springs gage, and a waste-decay rate of 1.0 per day. By using an approximate first-order exponential formula of the form:

$$conc_t = conc_{int}\ e^{-kt}, \qquad (45)$$

where t=traveltime downstream of waste discharge, in days;
e=natural logarithm base e;
$conc_t$=concentration remaining after time (t), in milligrams per liter;
$conc_{int}$=initial concentration of waste, after being mixed with the stream, in milligrams per liter; and
k=waste decay rate constant, base e, $days^{-1}$

An approximate traveltime of 2.3 days, by a trial-and-error procedure using equation 45, was determined to reduce the waste concentration from 20 to 2 mg/L. The length of stream required, from figure 9, for an index flow of 100 ft^3/s (2.8 m^3/s), would include a reach approximately from site 3 to site 16, for a traveltime of 55 hours or 2.3 days. This example computation did, however, assume no additional waste or tributary inflow into the stream reach.

SUMMARY

Traveltime, unit-concentration, and longitudinal-dispersion characteristics were determined for stream reaches on the Yampa and the Little Snake Rivers. The Yampa River study reach extended for 58 mi (93 km), from approximately 5 mi (8 km) southeast of Steamboat Springs to Craig, Colo.; whereas, the Little Snake River study reach extended 77 mi (124 km), from approximately 16 mi (26 km) east of Slater, Colo., and ending 33 mi (53 km) southwest of Baggs, Wyo. Two data runs were made on the Yampa River with approximate average stream discharges of 3,400 and 100 ft^3/s (95.2 and 2.80 m^3/s), and three data runs were made on the Little Snake River with approximate average stream discharges of 1,600, 600, and 20 ft^3/s (44.8, 16.8, and 0.56 m^3/s). Measured stream velocities varied as follows: 0.26 to 3.6 mi/h (0.42 to 5.8 km/h) for the Yampa River and 0.04 to 3.5 mi/h (0.06 to 5.64 km/h) for the Little Snake River.

Sand and gravel operations along the Yampa River and large irrigation diversions along the Little Snake River significantly affected stream velocity patterns during low-flow measurements.

Simulations of traveltime and unit concentrations for other stream-discharge conditions were done using a mathematical model described by McQuivey and Keefer (1976). The model includes two parameters--damping coefficient and mean stream velocity. Four index-discharge stations were used in the traveltime and unit-concentration simulations. The index stations were as follows: Yampa River at Steamboat Springs, Colo.; Yampa River below diversion, near Hayden, Colo.; Little Snake River near Slater, Colo.; and Little Snake River near Dixon, Wyo. The simulation of traveltime and unit-concentration data involves calibrating the model. Calibration is achieved by varying mean velocity and damping coefficient until the model-computed traveltime and unit-concentration results match the measured data. Once the model has been calibrated, simulations of traveltime for hypothetical discharges may be made by the procedures described below:

1. Develop a relationship for shear velocity (U_* gage) and mean velocity ($\bar{U}$ gage) versus discharge for each of the index discharge stations.
2. Determine the ratio for U_* gage and $\bar{U}$ gage versus the damping coefficient (D_*) determined by the model and reach-computed mean velocity ($\bar{U}$ comp).
3. Choose different index discharges and determine corresponding U_* gage and $\bar{U}$ gage values.
4. Compute respective D_* and $\bar{U}$ comp values for the reach, assuming ratios in procedure 2 are constant.

5. Use these new parameter values in the model to obtain simulated traveltime or unit concentration.

Traveltime simulations were based on all field-observation runs except the Little Snake River low- and medium-flow runs. Unit-concentration estimates are based only on the high-flow runs. Traveltime simulations for the Little Snake River using the model-analysis scheme were compared to the measured medium-flow Little Snake River data for May 1977. The simulation results agreed within 5 percent of the measured field data. A similar comparison also was made for the unit-concentration data, with agreement within 30 to 40 percent. Traveltime simulations based on linear-regression relationships of mean stream velocity or discharge also are given for both streams.

Traveltime-simulation results from the study were compared to traveltime predictions used for a waste-load assimilative-capacity analysis for the Yampa River from Steamboat Springs to Hayden, Colo. The waste-load assimilative-capacity analysis used two traveltime-estimation approaches: Cross-sectional reach properties and mean stream velocity from discharge measurements, and a regression equation using streambed slope and stream discharge as variables. The two traveltime-estimation techniques used for waste-load assimilative capacity agreed within 6 percent of the model simulation results.

The traveltime and unit-concentration simulations provide a convenient means of predicting the arrival time and concentration of soluble contaminants accidentally spilled in a stream. The length of stream affected by municipal wastewater-treatment discharges can also be estimated by the study-simulation results. A third application is the estimation of traveltime for reservoir water released for downstream irrigation, hydropower, municipal water supply, and industrial uses.

Longitudinal-dispersion coefficients were computed for the Yampa River high- and low-flow runs and the Little Snake River high- and medium-flow runs. Coefficients ranged from 6,050 to 400 ft^2/s (560 to 37 m^2/s) for the two streams.

Stream reaeration coefficients were measured, using a modified tracer technique, for a reach of the Yampa River below Steamboat Springs from river-mile 190.5 to 171.0 (river-km 306.7 to 275.3). Ethylene and propane were used as tracer gases and rhodamine-WT dye was used as a dispersion tracer. The basic premise of the reaeration-coefficient computation is that the ratio of the rate coefficient for a tracer gas desorbing from water to the rate coefficient for oxygen being absorbed by the same water is a constant, and is independent of mixing conditions and water temperature. Measured reaeration coefficients adjusted to 20°C ranged from 33.4 to 6.04 d^{-1} for the stream reaches. A comparison of the measured reaeration coefficients to those computed from semiempirical and empirical equations also was completed for the study reaches. Semiempirical predictive equations by Tsivoglou and Neal (1976) and Thackston and Krenkel (1969) gave the best comparisons with absolute errors of estimate of 0.1181 and 0.1727.

SELECTED REFERENCES

Bansal, M. K., 1973, Atmospheric reaeration in natural streams: Water Research, v. 7, no. 5, p. 769-782.

Bauer, D. P., 1968, Time of travel of water in the Great Miami River, Dayton to Cleves, Ohio: U.S. Geological Survey Circular 546, 15 p.

Bauer, D. P., and Jennings, M. E., 1975, Steady-state segmented dissolved-oxygen model: Bay St. Louis, Miss., U.S. Geological Survey Computer Contribution; available only from U.S. Department of Commerce, National Technical Information Service, Springfield, Va., as Report PB-241 779, 104 p.

Bauer, D. P., Steele, T. D., and Anderson, R. D., 1978, Analysis of waste-load assimilative capacity of the Yampa River, Steamboat Springs to Hayden, Routt County, Colorado: U.S. Geological Survey Water-Resources Investigations 77-119; Available only from Department of Commerce, National Technical Information Service, Springfield, Va. 22151, as Report PB-278 051, 69 p.

Bennett, J. P., and Rathbun, R. E., 1972, Reaeration in open-channel flow: U.S. Geological Survey Professional Paper 737, 75 p.

Boning, C. W., 1974, Generalization of stream travel rates and dispersion characteristics from time-of-travel measurements: U.S. Geological Survey Journal of Research, v. 2, no. 4, p. 495-499.

Cadwallader, T. E., and McDonnell, A. J., 1969, A multivariate analysis of reaeration data: Water Research, v. 3, p. 731-742.

Churchill, M. A., Elmore, H. L., and Buckingham, R. A., 1962, The prediction of stream reaeration rates: American Society of Civil Engineers, Journal of the Sanitary Engineering Division, v. 88, no. SA-4, p. 1-46.

Dobbins, W. E., 1965, Closure to "BOD and oxygen relationship in streams": American Society of Civil Engineers, Journal of the Sanitary Engineering Division, v. 91, no. SA-5, p. 49-55.

Elmore, H. L., and West, W. F., 1961, Effect of water temperature on stream reaeration: American Society of Civil Engineers, Journal of the Sanitary Engineering Division, v. 87, no. SA-6, p. 59-71.

Fischer, H. B., 1973, Longitudinal dispersion and turbulent mixing in open-channel flow, *in* Annual review of fluid mechanics: Palo Alto, Calif., George Banta Co., v. 5, p. 59-78.

Grant, R. S., 1976, Reaeration coefficient measurements of ten small streams in Wisconsin using radioactive tracers, *with a section on* the energy-dissipation model: U.S. Geological Survey Water-Resources Investigations 76-96, 50 p.

Isaacs, W. P., and Gaudy, A. F., 1968, Atmospheric oxygenation in a simulated stream: American Society of Civil Engineers, Journal of the Sanitary Engineering Division, v. 94, no. SA-2, p. 319-344.

Krenkel, P. A., and Orlob, G. T., 1963, Turbulent diffusion and the reaeration coefficient: American Society of Civil Engineers Transactions, v. 128, Paper 3491, p. 293-334.

Langbein, W. B., and Durum, W. H., 1967, The aeration capacity of streams: U.S. Geological Survey Circular 542, 6 p.

Lau, Y. L., 1972, Prediction equations of reaeration in open-channel flow: American Society of Civil Engineers, Journal of the Sanitary Engineering Division, v. 98, no. SA-6, p. 1063-1068.

McQuivey, R. S., and Keefer, T. N., 1976, Convective model of longitudinal dispersion: American Society of Civil Engineers, Journal of the Hydraulics Division, v. 102, no. HY-10, p. 1409-1424.

Negulescu, M., and Rojanski, V., 1969, Recent research to determine reaeration coefficient: Water Research, v. 3, no. 3, p. 189-202.

Nordin, C. F., Jr., and Sabol, G. V., 1974, Empirical data on longitudinal dispersion in rivers: U.S. Geological Survey Water-Resources Investigations 20-74, 334 p.

O'Connor, D. J., and Dobbins, W. E., 1958, Mechanisms of reaeration in natural streams: American Society of Civil Engineers Transactions, v. 123, p. 641-684

Owens, M., Edwards, R. W., and Gibbs, J. W., 1964, Some reaeration studies in streams: Oxford, England, International Journal of Air and Water Pollution, v. 8, no. 8/9, p. 469-486.

Padden, T. J., and Gloyna, E. F., 1971, Simulation of stream processes in a model river: Austin, University of Texas, Report EHE-70-23, CRWR-72, 130 p.

Parkhurst, J. D., and Pomeroy, R. D., 1972, Oxygen absorption in streams: American Society of Civil Engineers, Journal of the Sanitary Engineering Division, v. 98, no. SA-1, p. 101-124.

Rathbun, R. E., 1977, Reaeration coefficients of streams--State-of-the-art: American Society of Civil Engineers, Journal of the Hydraulics Division, v. 103, no. HY-4, p. 409-424.

Rathbun, R. E., and Grant, R. S., 1978, Comparison of the radioactive and modified techniques for measurement of stream reaeration coefficients: U.S. Geological Survey Water-Resources Investigations 78-68, 57 p.

Rathbun, R. E., Shultz, D. J., and Stephens, D. W., 1975, Preliminary experiments with a modified tracer technique for measuring stream reaeration coefficients: U.S. Geological Survey Open-File Report 75-256, 36 p.

Rathbun, R. E., Stephens, D. W., Shultz, D. J., and Tai, D. Y., 1978, Laboratory studies of gas tracers for reaeration: American Society of Civil Engineers, Journal of the Environmental Engineering Division, v. 104,, no. EE-2, p. 215-229.

Sayre, W. W., 1977, Discussion of convective model of longitudinal dispersion, by McQuivey, R. S., and Keefer, T. N., 1976, *in* American Society of Civil Engineers, Journal of the Hydraulics Division, v. 102, no. HY-10, p. 1409-1424: American Society of Civil Engineers, Journal of the Hydraulics Division, v. 103, no. HY-7, p. 820-823.

Shultz, D. J., Pankow, J. F., Tai, D. Y., Stephens, D. W., and Rathbun, R. D., 1976, Determination, storage, and preservation of low-molecular weight hydrocarbon gases in aqueous solution: U.S. Geological Survey Journal of Research, v. 4, no. 2, p. 247-251.

Steele, T. D., 1978, Assessment techniques for modelling water quality in a river basin impacted by coal resource development, *in* Modelling the Water Quality of the Hydrological Cycle Symposium, Baden, Austria, September 11-15, 1978, Proceedings: IAHS-AISH Publication No. 125, p. 322-332.

Steele, T. D., Bauer, D. P., Wentz, D. A., and Warner, J. W., 1976a, An environmental assessment of impacts of coal development on the water resources of the Yampa River basin, Colorado and Wyoming--Phase-I work plan: U.S. Geological Survey Open-File Report 76-367, 17 p.

Steele, T. D., Bauer, D. P., Wentz, D. A., and Warner, J. W., 1979, The Yampa River basin, Colorado and Wyoming--A preview to expanded coal-resource development and its impacts on regional water resources: U.S. Geological Survey Water-Resources Investigations 78-126, 133 p.

Steele, T. D., James, I. C., II, Bauer, D. P., and others, 1976b, An environmental assessment of impacts of coal development on the water resources of the Yampa River basin, Colorado and Wyoming--Phase-II work plan: U.S. Geological Survey Open-File Report 76-368, 33 p.

Swinnerton, J. W., and Linnenbom, V. J., 1967, Determination of the C_1 to C_4 hydrocarbons in sea water by gas chromatography: Journal of Gas Chromatography, v. 5, p. 570-573.

Thackston, E. L., and Krenkel, P. A., 1969, Reaeration prediction in natural streams: American Society of Civil Engineers, Journal of the Sanitary Engineering Division, v. 95, no. SA-1, p. 65-94.

Tsivoglou, E. C., 1967, Tracer measurements of stream reaeration: Federal Water Pollution Control Administrative Report, 86 p.

Tsivoglou, E. C., and Neal, L. A., 1976, Tracer measurement of reaeration, Part III. Predicting the reaeration capacity of inland streams: Journal of the Water Pollution Control Federation, v. 48, no. 12, p. 2669-2689.

Wentz, D. A., and Steele, T. D., 1978, Surface-water quality in the Yampa River basin, Colorado and Wyoming--An area of accelerated coal development, *in* Karadi, G. M., and Krizek, R. J., eds., Water for energy development: Engineering Foundation, Asilomar Conference Grounds, Pacific Grove, Calif., December 5-10, 1976, p. 56-74.

Wilson, J. F., Jr., 1968, Fluorometric procedures for dye tracing: U.S. Geological Survey Techniques of Water-Resources Investigations, book 3, chap. A12, 31 p.

☆ U. S. GOVERNMENT PRINTING OFFICE 1979 - 681-847/131 Reg. 8