A Comparison of Analog and Digital Modeling Techniques for Simulating Three-Dimensional Ground-Water Flow on Long Island, New York

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A COMPARISON OF ANALOG AND DIGITAL MODELING TECHNIQUES FOR SIMULATING THREE-DIMENSIONAL GROUND-WATER FLOW ON LONG ISLAND, NEW YORK

By Thomas E. Reilly and Arlen W. Harbaugh

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1980

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CONVERSION FACTORS AND ABBREVIATIONS

Multiply Metric (SI)[1] units	By	To obtain inch-pound units
cubic meters per second (m^3/s)	35.31	cubic feet per second (ft^3/s)
cubic meters per second (m^3/s)	22.82	million gallons per day (Mgal/d)
kilometers (km)	.6214	miles (mi)
meters (m)	3.281	feet (ft)

Multiply inch-pound units	By	To obtain Metric (SI)[1] units
cubic feet per second (ft^3/s)	.0283	cubic meters per second (m^3/s)
million gallons per day (Mgal/d)	.0438	cubic meters per second (m^3/s)
miles (mi)	1.609	kilometers (km)
feet (ft)	.3048	meters (m)

[1] International System of units

A COMPARISON OF ANALOG AND DIGITAL MODELING TECHNIQUES FOR SIMULATING THREE-DIMENSIONAL GROUND-WATER FLOW ON LONG ISLAND, NEW YORK

By

T. E. Reilly and A. W. Harbaugh

ABSTRACT

A three-dimensional electric-analog model of the Long Island ground-water system constructed by the U.S. Geological Survey in the early 1970's was used as the basis for developing a digital, three-dimensional finite-difference model. The digital model was needed to provide faster modifications and more rapid solutions to water-management questions. Results generated by the two models are depicted as potentiometric-surface maps of the upper glacial and Magothy aquifers. Results compare favorably for all parts of Long Island except the northwestern part, where hydrologic discontinuities are most prevalent and which the two models represent somewhat differently. The mathematical and hydrologic principles used in development of ground-water models, and the procedures for calibration and acceptance, are presented in nontechnical terms.

INTRODUCTION

Long Island, N.Y., a part of the New York metropolitan area, has grown rapidly since World War II. Although the western part of the island obtains water from elsewhere, Nassau, Suffolk, and parts of Queens Counties (fig. 2) are totally dependent on local ground water for their supply. Because acreage and natural resources needed for growth and development on Long Island are restricted, as on any island, land managers and planners are faced with many types of decisions as to the best use of these resources. At present, much work is being done to develop accurate methods and equipment to provide information and to evaluate future alternatives in environmental questions.

Since 1932, the U.S. Geological Survey, in cooperation with local and State agencies, has been studying and monitoring the water resources of Long Island. In the mid-1960's, the Geological Survey began to develop an electric analog model of the Long Island aquifer system to predict how the ground-water system would respond to alternative water-management proposals. The resulting information has been important in answering questions concerning local ground-water resources.

Before 1965, the most reliable method of ground-water modeling was the electric-analog model (a physical-mathematical model), which is based on the analogy between the flow of electricity and the flow of water. With sufficient data and proper calibration, such a model can indicate ground-water flow at any locality within the modeled area. Subsequently, the Geological Survey constructed and tested a large, complex electric-analog model of the Long Island ground-water system (Getzen, 1977). This model was used successfully for several years to predict the response of the ground-water system to a variety of management alternatives (Kimmel and Harbaugh, 1976; Harbaugh and Reilly, 1976). Although the model proved useful and yielded satisfactory results, it has inherent disadvantages. It was large and cumbersome, the measurement of electrical values on the model was time consuming, and modifying the model electrically for alternative types of tests was even more time consuming.

Since the late 1960's, the speed and versatility of digital computers in solving complex mathematical problems has increased, and the cost of such computations has decreased accordingly. At the same time, the quantity of data that can be stored and manipulated has increased. Thus, it soon became evident that the electric-analog model's capability to predict the response of the ground-water system to stresses could be enhanced by use of a digital model, which, in addition, would provide greater versatility and wider applications.

The hydraulic values used in the electric-analog model were used as a basis for developing the digital model. The digital model, which solves hydrologic equations to indicate water levels at selected localities, provides more rapid solutions to hydrologic questions than the analog model and is more easily modified as new information becomes available. In addition, the digital model can simulate certain aspects of the ground-water system more accurately than the analog model.

Purpose and Scope

The purpose of this report is to (1) explain the concepts and procedures used in developing a ground-water model, and (2) compare the operation and results of the analog and digital models of the Long Island ground-water system.

The first sections of this report describe the principles of modeling and explain the similarities and differences between the electric analog-model representation of a real hydrologic system and the numerical (digital) model representation. The steps in designing a model are introduced to explain the development of a model from problem definition to solution; a short discussion of ground-water hydrology is included also. Those interested in a more comprehensive discussion of the principles of ground-water flow may refer to Bennett (1976).

The latter sections of the report trace the transition of ground-water modeling on Long Island from the analog to the digital technique. Although the analog representation was able to answer many questions about the Long Island ground-water system, it gave rise to certain questions as to (a) reliability of the data that had been used in constructing the model, and (b) the lack of data from some localities. Getzen (1977, p. 19-31) gives a detailed discussion of the Long Island analog-model design principles and the simulation of Long Island ground-water hydrology. This report includes a comparison of results from the digital and the analog models and a discussion of the acceptability of the numerical model as evidence that the transition to the numerical simulation technique was done accurately and that reliable results can be obtained. The result of this conversion is an acceptable digital model that can yield results faster and allow for more flexibility in stressing and updating the model.

Acknowledgments

The authors are deeply indebted to O. L. Franke, whose enthusiasm and technical knowledge were invaluable in quantitatively describing the Long Island flow system and in preparing this general report, and to R. T. Getzen, whose work (Getzen, 1977) forms the foundation of this report.

GROUND-WATER MODELING ON LONG ISLAND

Developing a Ground-Water Model

A model is an approximate representation of some aspect of a real system. A ground-water model of the type discussed in this report, is designed to indicate the patterns of flow within the ground-water system by solving equations that describe the system. Thus, a ground-water flow model incorporates only those aspects of the natural system that directly affect the movement of ground water.

Hydrologic systems are modeled for two main reasons--to provide information on how the system functions and to predict how the system will respond to given stresses. Thus, a model constructed properly can be useful in planning for the management of water resources. However, because a model is based on approximate, rather than exact, values, its predictions are only as accurate as the data and assumptions used in its construction and operation.

A general procedure for developing a ground-water flow model from concept to final calibrated model is shown as a flow chart in figure 1. In step 1, the problem to be modeled is defined. The problem must be formulated in adequate detail to allow systematic development of the model. Because the problem cannot be solved through direct measurements, the solution must rely on mathematical equations which will describe the ground-water flow.

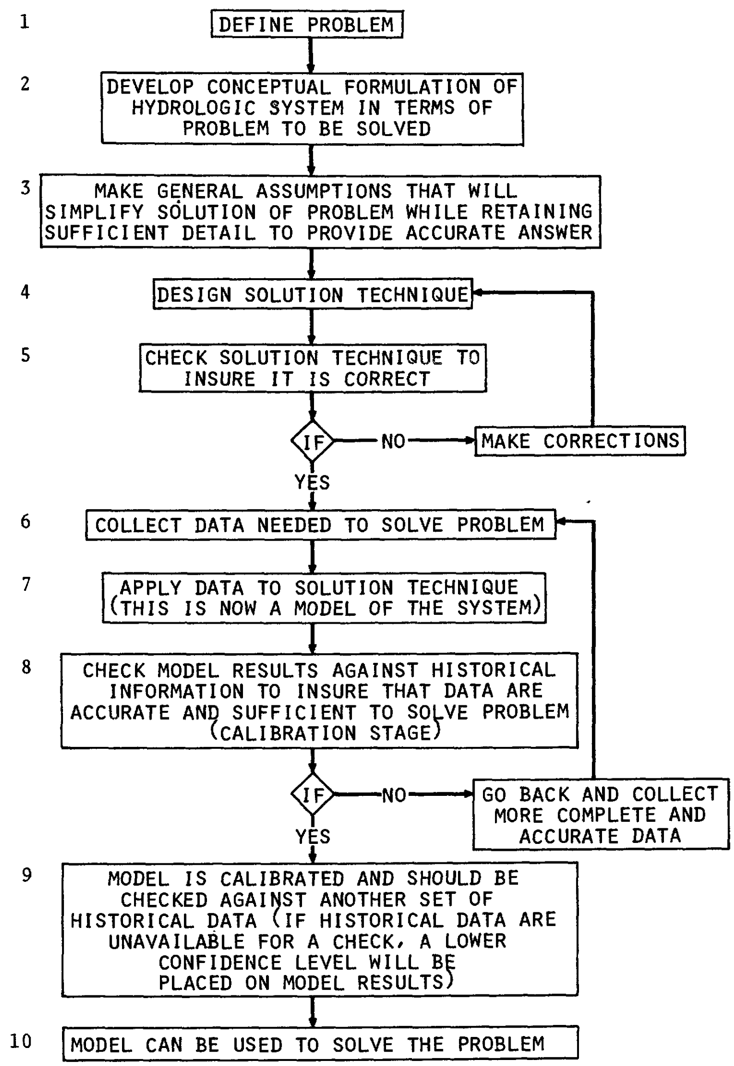


Figure 1.--Flow chart showing steps in designing a ground-water model.

The second step, the conceptual formulation of the hydrologic system and its components, is the most important one in the modeling process because this step determines which characteristics of the flow system are to be incorporated. A correct concept allows for a true, although approximate, mathematical representation of the flow system; otherwise the model will produce an incorrect solution to the problem.

Once a concept of the hydrologic system has been developed, the mathematical formulation and solution techniques can be derived (steps 2-5) and values representing the hydraulic characteristics of the system (step 6) can be input into the model. The combination of the solution technique and the hydraulic data (step 7) constitute an approximate representation (or model) of the natural system. Model results are then checked against actual historical measurements to see if the model's solutions are accurate; if the model reproduces the historical information incorrectly, more complete or more accurate data are needed. This phase of the model development, in which the model response is compared with historical data to check the applicability of the selected hydrologic coefficients, is called the calibration stage (step 8). This phase is repeated and the data adjusted until a good match between values measured in the real system and those produced by the model is attained. However, if possible, the model should be checked against another set of historical data (other than the calibration data) to assure that the calibration match was not coincidental.

The difference between a calibrated model and an accepted model is subtle but important. The calibration stage involves a "tuning" process in which repeated runs are made to match historical information. The acceptance of a model requires a prediction of historical data (independent of the calibration data) within acceptable limits of accuracy. Konikow (1978) discusses the calibration and acceptance of models in more detail.

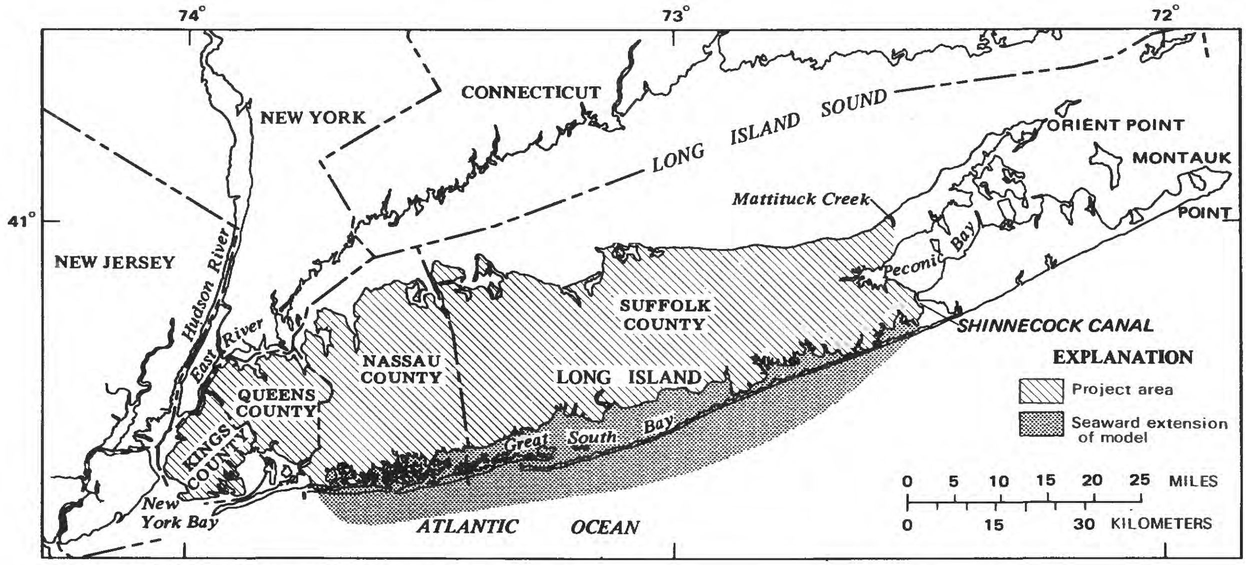
In simulating the ground-water system of Long Island on the electric-analog model, Getzen (1977) used the approach outlined in figure 1. The digital model described in this report uses the same hydrologic data as described in Getzen (1977) but uses a different solution technique (fig. 1, step 4).

Hydrology of Long Island

Long Island is bounded by Long Island Sound on the north, the Atlantic Ocean on the east and south, and the East River and New York Bay on the west. The area represented on the electric-analog and digital models inludes all of Long Island except the eastern peninsulas, or "forks." The major geographic features of Long Island and the modeled area are shown in figure 2. The model's seaward extension, as shown on figure 2, is necessary to represent the confined Magothy aquifer, which extends under the ocean. The specific hydrogeologic information that was used in the Long Island analog model was documented by Getzen (1977); general descriptions of the hydrologic system are given in Cohen and others (1968), Jensen and Soren (1974), and McClymonds and Franke (1972). A listing and characterization of the hydrogeologic units are given in table 1; a generalized geologic section showing the relative positions of most of the units listed in table 1 is shown in figure 3.

In general, Long Island's ground-water system is made up of three major aquifers--the upper glacial, the Magothy, and the Lloyd--separated by two confining units--the Gardiners Clay and the Raritan clay. (See table 1 and fig. 3.) The saturated upper glacial aquifer has a high hydraulic conductivity and is generally somewhat less than 30 meters thick; the underlying Magothy aquifer has a moderate to high hydraulic conductivity and ranges in thickness from zero at some north-shore locations to 300 meters along the south shore. The Lloyd aquifer and Raritan clay are the deepest units in the hydrologic system on Long Island and are underlain by bedrock. (See fig. 3.) Neither the Lloyd nor the Raritan is simulated in the analog model because the Raritan clay, of low hydraulic conductivity, effectively isolates the Lloyd aquifer from the rest of the ground-water reservoir.

At several places, especially near the south shore, the Gardiners Clay and other associated clay beds lie between the upper glacial and Magothy aquifers. (See fig. 3.) These clay beds confine water in the underlying Magothy aquifer and restrict flow between the upper glacial and Magothy aquifers. However, these clay beds are not continuous; therefore, water may flow from one aquifer to the other where clay beds are absent. This vertical flow between aquifers is significant in many localities and must, therefore, be included along with horizontal flows in any model.



Base from U.S. Geological Survey, 1:250,000 series: Scranton, Hartford, 1962; New York, 1957; Newark, Providence, 1947

Figure 2.--Location and extent of modeled area, Long Island, N.Y.

Table 1.--Summary of the rock units and their water-bearing properties, Long Island

[From McClymonds and Franke, 1972, p. E5-E6]

System	Series	Geologic unit [1]	Hydro-geologic unit	Approx-imate maximum thickness (feet)	Depth from land surface to top (feet)	Character of deposits	Water-bearing properties
Quaternary	Holocene	Recent deposits: Artificial fill, salt marsh deposits, stream alluvium, and shoreline deposits.	Recent deposits	50	0	Sand, gravel, clay, silt, organic mud, peat, loam, and shells. Colors are gray, brown, green, black, and yellow. Recent artificial-fill deposits of gravel, sand, clay, and rubbish.	Permeable sandy beds beneath barrier beaches yield fresh water at shallow depths, brackish to salty water at greater depth. Clay and silt beneath bays retard salt-water encroachment and confine underlying aquifers. Stream floodplain and marsh deposits may yield small quantities of water but are generally clayey or silty and much less permeable than the underlying upper glacial aquifer.
	Pleistocene	Upper Pleistocene deposits	Upper glacial aquifer	600	0-50	Till (mostly along north shore and in moraines) composed of clay, sand, gravel, and boulders forms Harbor Hill and Ronkonkoma terminal moraines. Outwash deposits (mostly between and south of terminal moraines, but also interlayered with till) consist of quartzose sand, fine to very coarse, and gravel, pebble to boulder sized. Glaciolacustrine deposits (mostly in central and eastern Long Island) and marine clay (locally along south shore) consist of silt, clay, and some sand and gravel layers; includes the "20-foot clay" in southern Nassau and Queens Counties. Colors are mainly gray, brown, and yellow; silt and clay locally are grayish green. Contains shells and plant remains, generally in finer grained beds; also contains Foraminifera. Contains chlorite, biotite, muscovite, hornblende, olivine, and feldspar as accessory minerals; "20-foot clay" commonly contains glauconite.	Till is poorly permeable; commonly causes perched-water bodies and impedes downward percolation of water to underlying beds. Outwash deposits are moderately to highly permeable; specific capacities of wells tapping them range from about 10 to more than 200 gpm per ft (gallons per minute per foot) of drawdown. Good to excellent infiltration characteristics. Glaciolacustrine and marine clay deposits are mostly poorly permeable but locally have thin, moderately permeable layers of sand and gravel; generally retard downward percolation of ground water. Contains fresh water except near the shore lines. Till and marine deposits locally retard salt-water encroachment.
		—— Unconformity? ——					
		Gardiners Clay	Gardiners Clay	300	50-400	Clay, silt, and few layers of sand and gravel. Colors are grayish green and brown. Contains marine shells, Foraminifera, and lignite; also locally contains glauconite. Altitude of top generally is 50-80 feet below mean sea level. Occurs in Kings, Queens, and southern Nassau and Suffolk Counties; similar clay occurs in buried valleys near north shore.	Poorly permeable; constitutes confining layer for underlying Jameco aquifer. Locally, sand layers yield small quantities of water.
		—— Unconformity? ——					
		Jameco Gravel	Jameco aquifer	300	50-550	Sand, fine to very coarse, and gravel to large-pebble size; few layers of clay and silt. Gravel is composed of crystalline and sedimentary rocks. Color is mostly dark brown. Contains chlorite, biotite, muscovite, hornblende, and feldspar as accessory minerals. Occurs in Kings, Queens, and southern Nassau Counties; similar deposits occur in buried valleys near north shore.	Moderately to highly permeable; contains mostly fresh water, but brackish water and water with high iron content occurs locally in southeastern Nassau and southern Queens Counties. Specific capacities of wells in the Jameco range from about 20 to 150 gpm per ft of drawdown.

System	Series	Geologic unit [1]		Hydrogeologic unit	Approximate maximum thickness (feet)	Depth from land surface to top (feet)	Character of deposits	Water-bearing properties
		Unconformity						
Tertiary (?)	Pliocene(?)	Mannetto Gravel		(Commonly included with upper glacial aquifer.)	300	0–120	Gravel, fine to coarse, and lenses of sand; scattered clay lenses. Colors are white, yellow, and brown. Occurs only near Nassau-Suffolk County border near center of island.	Highly permeable, but occurs mostly above water table. Excellent infiltration characteristics.
		Unconformity						
Cretaceous	Upper Cretaceous	Magothy(?) Formation [2]		Magothy aquifer	1,100	0–600	Sand, fine to medium, clayey in part; interbedded with lenses and layers of coarse sand and sandy and solid clay. Gravel is common in basal 50–200 feet. Sand and gravel are quartzose. Lignite, pyrite, and iron oxide concretions are common; contains muscovite, magnetite, rutile, and garnet as accessory minerals. Colors are gray, white, red, brown, and yellow.	Most layers are poorly to moderately permeable; some are highly permeable locally. Specific capacities of wells in the Magothy generally range from 1 to about 30 gpm per ft of drawdown, rarely are as much as 80 gpm per ft of drawdown. Water is unconfined in uppermost parts, elsewhere is confined. Water is generally of excellent quality but has high iron content locally along north and south shores. Constitutes principal aquifer for public-supply wells in western Long Island except Kings County, where it is mostly absent. Has been invaded by salty ground water locally in southwestern Nassau and southern Queens Counties and in small areas along north shore.
		Unconformity						
		Raritan Formation	Clay member	Raritan clay	300	70–1,500	Clay, solid and silty; few lenses and layers of sand; little gravel. Lignite and pyrite are common. Colors are gray, red, and white, commonly variegated.	Poorly to very poorly permeable; constitutes confining layer for underlying Lloyd aquifer. Very few wells produce appreciable water from these deposits.
			Lloyd Sand Member	Lloyd aquifer	500	200–1,800	Sand, fine to coarse, and gravel, commonly with clayey matrix; some lenses and layers of solid and silty clay; locally contains thin lignite layers and iron concretions. Locally has gradational contact with overlying Raritan clay. Sand and most of gravel are quartzose. Colors are yellow, gray, and white; clay is red locally.	Poorly to moderately permeable. Specific capacities of wells in the Lloyd generally range from 1 to about 25 gpm per ft of drawdown, rarely are as much as 50 gpm per ft of drawdown. Water is confined under artesian pressure by overlying Raritan clay; generally of excellent quality but locally has high iron content. Has been invaded by salty ground water locally in necks near north shore, where aquifer is mostly shallow and overlying clay is discontinuous. Called "deep confined aquifer" in some earlier reports.
		Unconformity						
Precambrian		Bedrock		Bedrock		0–2,700	Crystalline metamorphic and igneous rocks; muscovite-biotite schist, gneiss, and granite. A soft, clayey zone of weathered bedrock locally is more than 100 feet thick.	Poorly permeable to virtually impermeable; constitutes virtually the lower boundary of ground-water reservoir. Some hard, fresh water is contained in joints and fractures but is impractical to develop at most places; however, a few wells near the western edges of Queens and Kings Counties obtain water from the bedrock.

[1] Names are those used in reports by the Geological Survey.
[2] The use of the term "Magothy(?) Formation" has been abandoned. The post-Raritan Cretaceous deposits are divided into the Magothy Formation and Matawan Group undifferentiated and the Monmouth Group undifferentiated.

Other significant hydrologic features that affect the movement of ground water on Long Island are streamflow and subsea outflow. The base flow, or ground-water contribution to Long Island streams, constitutes approximately 95 percent of the total streamflow (Harbaugh and Getzen, 1977, p. 3), and it is estimated that about 50 percent of the water that leaves the Long Island ground-water system naturally does so by way of streams (Cohen and others, 1968, p. 58). The local ground-water flow systems that discharge to the streams (shallow flow system) occur mostly in the upper part of the upper glacial aquifer, but a deeper regional system (deep flow system) discharges from the lower part of the upper glacial aquifer and the underlying Magothy aquifer directly to the ocean or bays.

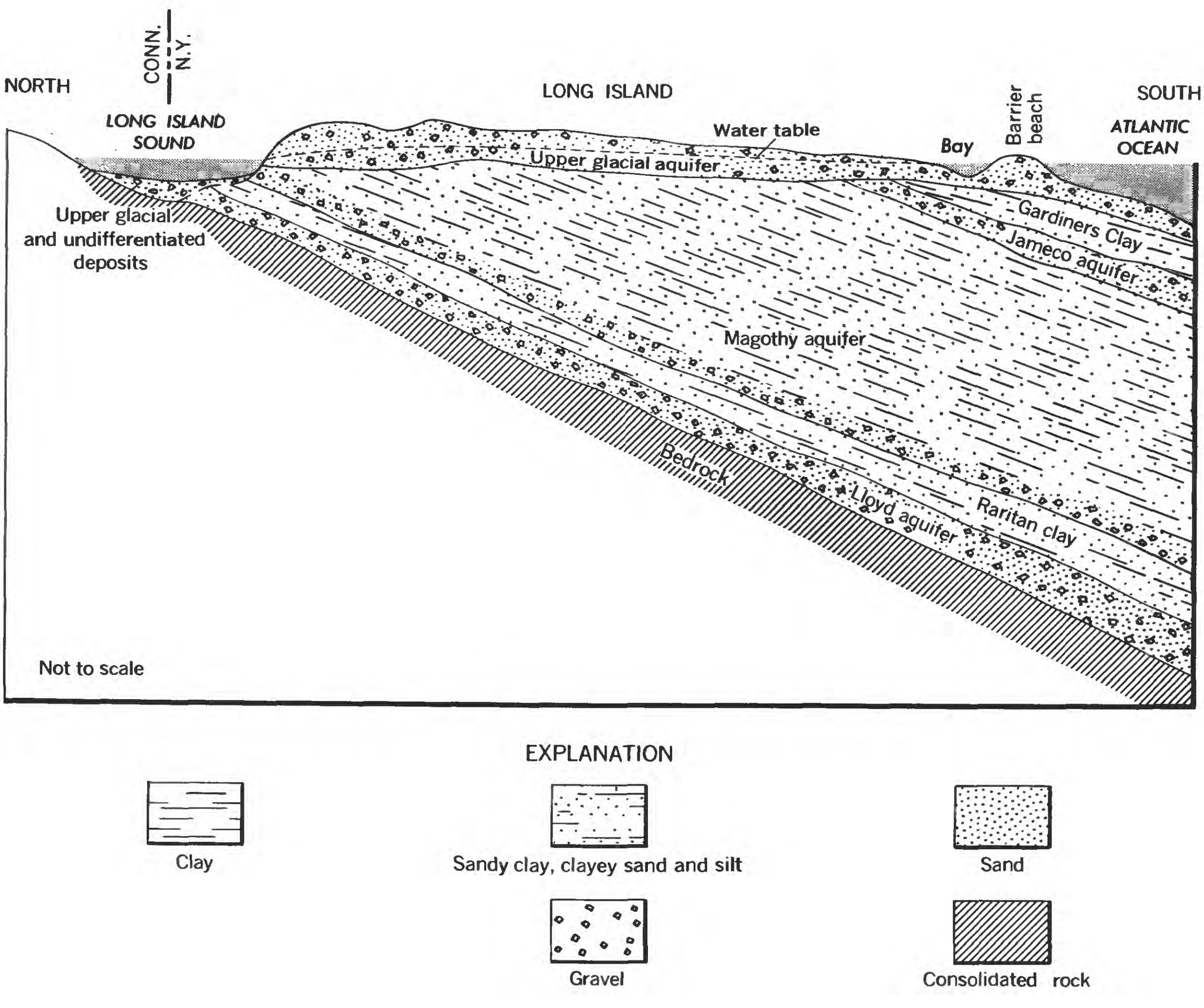


Figure 3.--Generalized geologic section of Long Island. (From McClymonds and Franke, 1972, p. E4.)

PRINCIPLES OF GROUND-WATER FLOW IN MODELING

Movement and Modeling

The most important concept that governs ground-water movement is the principle of conservation, which states that matter cannot be created or destroyed. In terms of water movement, this concept can be expressed by the following formula:

$$\text{Inflow} = \text{Outflow} \pm \text{Change in storage} \tag{1}$$

where each term is expressed in units of volume per time, for example, cubic meters per second. This concept is often used in water-resources studies to make water budgets and is also the basis for defining rates of ground-water movement within a given area. Thus, ground-water flow models use this basic equation to answer the question of how much water moves in the aquifer.

Both analog and digital models represent ground water through a method known as "finite differences." A model is made by arbitrarily dividing a volume representing the natural ground-water system into a system of separate but interconnected geometric shapes, referred to as blocks. The quantity of water flowing through each segment of aquifer represented by a model block is calculated by a mathematical formula in the digital model or is represented by the flow of electricity in the analog model. Calculating results from a digital model is similar to bookkeeping in that the quantity of water represented as moving in and out of each block must be accounted for simultaneously and balanced so that no water is gained or lost from the system through computational errors.

Because most models consist of many interconnected blocks, a detailed examination of one block representing a volume of aquifer material can reveal how one of the basic equations of ground-water flow is derived. The following development (not a rigorous mathematical proof) is a simplified explanation of the basic equation of ground-water flow.

Let us first examine the flow into and out of a typical block of aquifer material (fig. 4). Q represents the quantity of water moving into or out of the block at any face, and ΔQ represents the total gain or loss of water along any axis. Neglecting changes in storage for the moment, the block must keep the same volume of water at all times. Thus, the hydrologic formula becomes simply

$$\text{Inflow} = \text{Outflow}, \tag{2}$$

which is referred to as the steady-state continuity equation (that is, the relationship is not altered with time). In algebraic terms, this continuity equation can be expressed as:

$$\text{quantity in} = \text{quantity out, or}$$

$$\text{quantity in} - \text{quantity out} = 0$$

From figure 4, where Q may be positive at some faces and negative at others, depending on flow patterns:

$$Q_{x_{left}} + Q_{y_{front}} + Q_{z_{top}} + Q_{x_{right}} + Q_{y_{back}} + Q_{z_{bottom}} = 0 \qquad (3)$$

and $\Delta Q_x = Q_{x_{left}} + Q_{x_{right}}$ = Quantity of flow gained or lost in X direction

$\Delta Q_y = Q_{y_{front}} + Q_{y_{back}}$ = Quantity of flow gained or lost in Y direction

$\Delta Q_z = Q_{z_{top}} + Q_{z_{bottom}}$ = Quantity of flow gained or lost in Z direction

then:

$$\Delta Q_x + \Delta Q_y + \Delta Q_z = 0 \qquad (4)$$

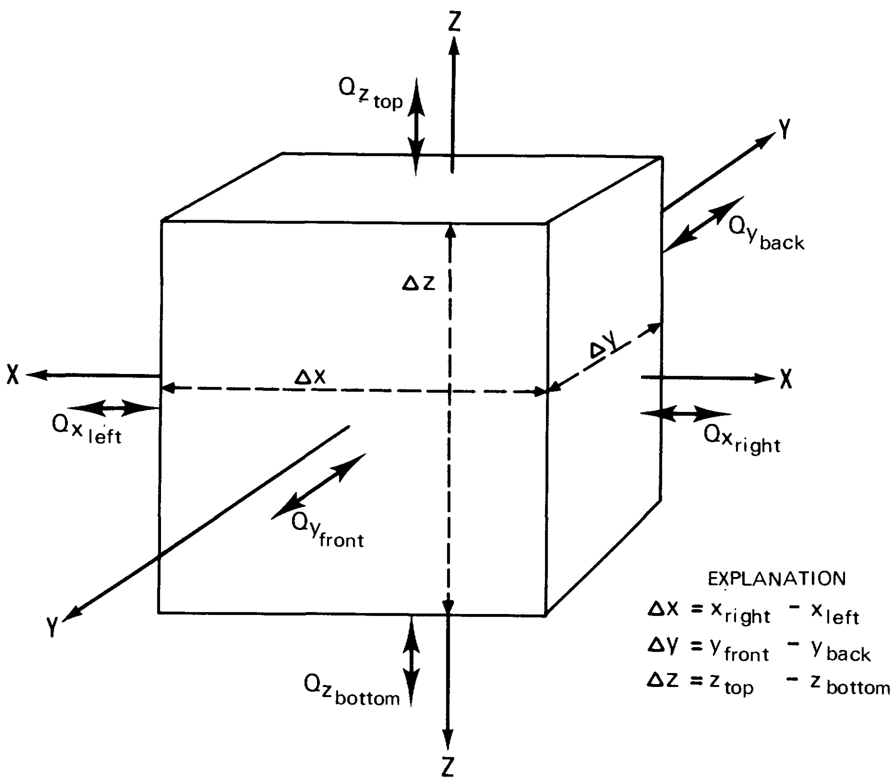


Figure 4.--Paths of ground-water flow through block of aquifer material under steady-state conditions, where quantity in equals quantity out (conservation of mass). X, y, and z axes are paths of flow; each axis involves two faces of the block (x = left and right; y = front and back; z = top and bottom). Flow (Q) may occur at all six faces but will be positive (inward) at some, negative (outward) at others.

In digital models, solution of the continuity equation for the entire model is obtained by simultaneously solving the continuity equation for each block (eq. 4). Often this is difficult because of the large number of algebraic equations (one for each block). The mathematical solution of all simultaneous equations makes the use of a computer necessary.

In analog models, the continuity equation is solved continuously because electrical current (I) is proportional to quantity of water (Q). In an analog model, each intersection of wires, termed a model node, represents a block of aquifer material, and electrical resistors form the paths that the electrical current (quantity of water) follows. Figure 5 compares a node in an analog model with a block in a mathematical model.

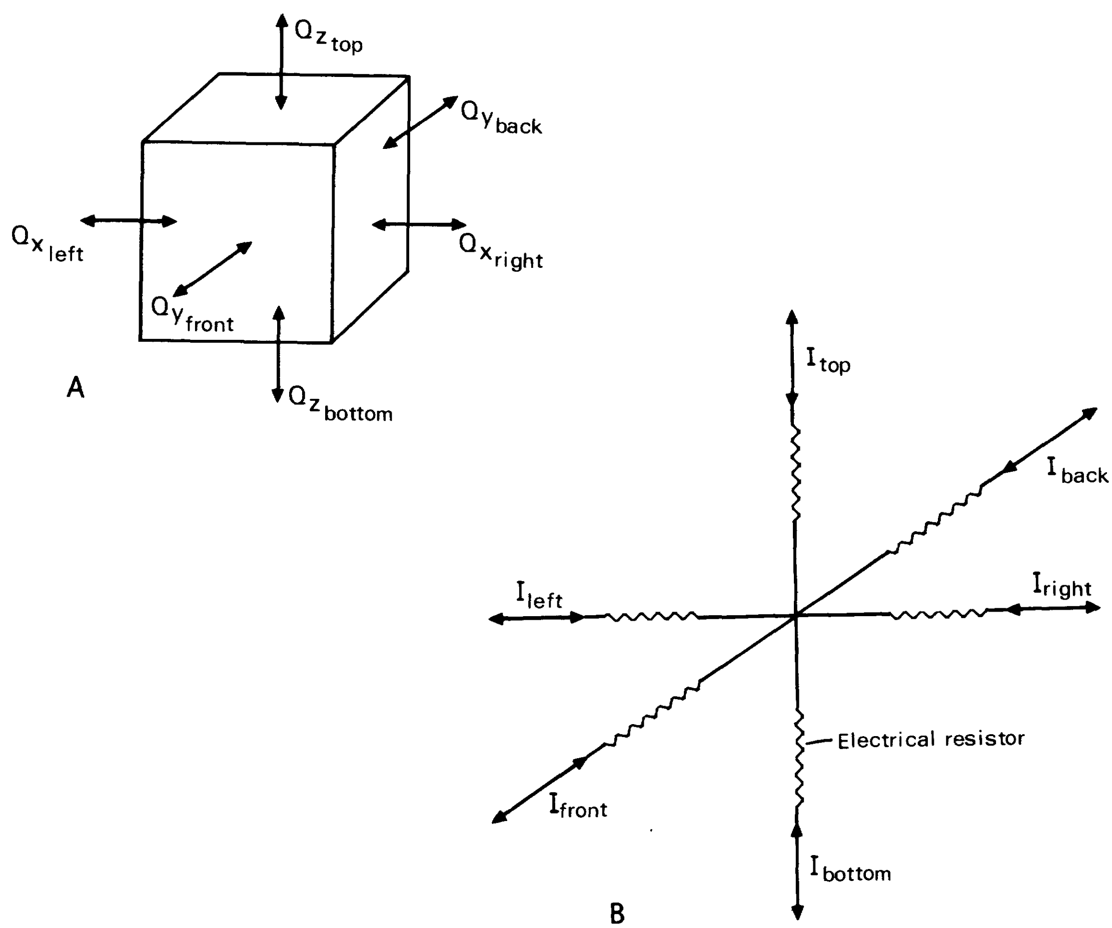


Figure 5.--Comparison of flows to model elements: A, in a block of a mathematical model; and B, in a node of an electric-analog model.

A basic principle of electricity is Kirchhoff's law, which states that the sum of electrical currents at a given point must equal zero, or that inflow must equal outflow. This is equivalent to the continuity equation (eq. 2), in which flow or current in must equal flow or current out. Thus, both models are designed to solve the continuity equation--the digital model by equations, the analog model by a system of electrical components.

Although the continuity equation (eq. 4) for a single block of aquifer material requires that total inflow equal total outflow, it does not indicate how to determine the volume of water that should enter or leave the block of aquifer material (node). The means of determining these quantities was first developed experimentally in 1856 by Henry Darcy, who discovered that the quantity of flow of water through a porous medium is directly proportional to the hydraulic gradient. Darcy's experimental law was theoretically derived by M. King Hubbert (1940); Darcy's statement of the law is:

$$Q = KA \frac{h_1 - h_o}{L} \tag{5}$$

where:

Q is the quantity of flow through the material, in units of volume per time (L^3/T);

K is the hydraulic conductivity of the material, in units of length per time (L/T);

A is the cross-sectional area through which the flow is moving, in units of length squared (L^2);

h is hydraulic head at a given point in units of length (L); and

L is the distance between the points where h_o and h_1 were measured, in units of length (L).

The hydraulic conductivity (K) is a measureable property of the fluid and the porous medium; and it represents the capability of the material to allow water to pass through it. To apply Darcy's law in modeling a ground-water system, at least an approximate value of the hydraulic conductivity of each hydrogeologic unit must be known.

As indicated by Darcy's law, the hydraulic head, or potential (h), is the force causing ground-water movement. The head represents the energy of water at a specific location and equals the sum of (1) the potential energy corresponding to the water's vertical location above an arbitrary plane of reference (usually mean sea level), and (2) the energy due to the water pressure. Specifically, the components of the hydraulic head (Domenico, 1972) are:

$$h = Z + \frac{P}{\rho_w g} \tag{6}$$

where:

h = head (L)

Z = elevation relative to arbitrary datum (L)

P = pressure ($M/T^2\ L$)

g = acceleration of gravity (L/T^2)

ρ_w = density of water (M/L^3)

At the water table, where water pressure (P) equals zero, the head (h) is equal to the altitude (Z) of the water surface above datum. Thus, according to Darcy's law, ground water at zero pressure (or near the water table) moves from areas of higher elevation to areas of lower elevation--in other words, downgradient. In addition, Darcy's law provides a rule for calculating the volume and rate of flow from an area of higher hydraulic potential to an area of lower hydraulic potential.

On Long Island under natural conditions, the lowest natural hydraulic potentials within the fresh ground-water system are at sea level. Thus, the fresh ground water moves from land, the area of higher potential, to the sea.

As stated earlier, the continuity equation (eq. 4) for both water and electricity is based on the principle of the conservation of mass and energy. Darcy's law for ground-water flow, on the other hand, is a derived equation for a specific application and is analogous to the electrical equation known as Ohm's law:

$$I = \frac{1}{R}(V_1 - V_o) \tag{7}$$

where:

I = electrical current, in amperes;

R = electrical resistance, in ohms; and

$V_1 - V_o$ = change in voltage across electrical resistance.

Comparing equation 5 with equation 7, the analogy between Darcy's law and Ohm's law is:

I (current) $\backsim$ Q (flow)

$\frac{1}{R}$ (1 ÷ resistance) $\backsim$ $\frac{KA}{L}$ (hydraulic conductivity x cross-sectional area ÷ distance)

$V_1 - V_o$ (change in voltage) $\backsim$ $h_1 - h_o$ (change in head)

This analogy between the variables in the two laws allows an electric-analog model to represent ground-water flow. Because the development of the Long Island mathematical model is analogous to that of the analog model, the same equations that describe the ground water flow are solved. Thus, if the same hydraulic parameters and block dimensions are used in both models, the results should be the same.

In summary, the continuity equation (eq. 4) in conjunction with Darcy's law defines the movement of ground water. Therefore, the solution of a ground-water flow model is the head distribution that satisfies the continuity equation.

In the theory described above and in the following section, two simplications are made. The first is that the volume of water entering the block is the same as the volume leaving the block, so that, in accordance with the conservation of mass, we assume that balancing the volume of water is equivalent to balancing the mass of water. To satisfy this assumption it is also necessary to assume the density of ground water is constant. Because the compressibility of water is small, this assumption is usually valid. A more rigorous development of the flow equation using mass balance instead of volume balance is given by C. E. Jacob (1950).

The second simplification, which is used primarily in the following section, involves the manner in which Darcy's Law is used to calculate the flow through each side of a block of aquifer material. The hydraulic conductivity of an aquifer can vary with direction--for example, the horizonal hydraulic conductivity may be considerably greater than the vertical hydraulic conductivity. In the development of the flow equation, we assume that the orthogonal principal axes of the block lie in the directions of the maximum and minimum principal components of hydraulic conductivities. This assumption is necessary to ensure that the equations used for each face of the block accurately describe the flow through it. In mathematical terms, the hydraulic conductivity is a second-order tensor that becomes diagonal when the three axes (x, y, and z) are orthogonal and lie in the direction of the principal components of hydraulic conductivities. This paper is not intended to explain this complex principle, but the reader should be aware that the set of equations developed are based on this assumption. The physical significance of this hydraulic-conductivity tensor is briefly discussed by Lohman and others (1972, p. 5).

Ground-Water Storage

An aquifer, or the saturated porous medium, can be thought of as a huge underground reservoir in which ground water is stored in the spaces between the grains of the aquifer material. As more water flows into the aquifer than out of it, (or vice versa) since the time of the previous measurement, water levels in the aquifer change and the volume of water in storage changes proportionally. Thus, any change in ground-water storage is a time-dependent phenomenon.

Thus far we have discussed only the simplified steady-state form of the continuity equation (neglecting storage)--that is:

$$\text{Inflow} = \text{Outflow, or} \tag{2}$$

$$\Delta Q_x + \Delta Q_y + \Delta Q_z = 0 \tag{4}$$

Because changes in storage can affect both the rate and direction of ground-water flow, the effect of storage must be accounted for in equations that describe flow. As stated at the beginning of the section "Movement and Modeling," the continuity equation that includes the effect of storage is:

$$\text{Inflow} = \text{Outflow} \pm \text{Change in storage} \tag{1}$$

In this equation, flow is no longer steady because a time-dependent, or transient, factor has been introduced. Modeling tests that use this transient factor are known as "transient state" as opposed to "steady state."

Ground-water storage occurs in either of two forms--as water-table storage or elastic (artesian or confined) storage. In both cases, the quantity of water gained or released from storage is dependent upon the change in hydraulic head as measured by a change in ground-water level.

Changes in water-table storage occur primarily at the upper boundary of the aquifer (the water table), whereas changes in elastic (artesian) storage occur throughout the aquifer. In artesian storage, as the hydraulic head changes, the water pressure that helps support the weight of the unconsolidated medium also changes, and the particles forming this medium change position in response to the pressure change. The resulting shift in the physical framework of the unconsolidated medium allows a change in the volume of fluid that can be stored in the pore space. Ferris and others (1962) summarized elastic (artesian) storage in stating that "the water released or taken into storage, in response to a change in head, is attributed solely to compressibility of the aquifer material and of the water."

To include the effect of storage in the continuity equation, we will reexamine a block of saturated aquifer material. We already know that for a saturated block of aquifer in equilibrium

$$Q_{in} = Q_{out}, \text{ or}$$

$$\Delta Q = Q_{in} - Q_{out} = 0$$

Let us assume that during some time interval, the hydraulic head in the block drops in response to a stress on the system, for example, a pumping well. As the aquifer material adjusts geometrically in response to the lowered hydraulic head, the quantity of water stored in the interstitial space decreases. In examining the components of flow during this process, we see that

$\Delta Q = Q_{in} - Q_{out}$ (where $Q_{out} > Q_{in}$) = quantity of water that is released from storage due to head decrease.

In this example, more water is leaving the block than is flowing into it. Because this displaced water represents outflow not accounted for by the flow into the block, another term must be included in the continuity equation (eq. 4) to account for this water. This additional term represents the volume of water that is gained or lost from storage, divided by the time period in which the head change took place. The difference between the inflows and outflows must equal the change in storage in the block of aquifer. This storage term must be added to the steady-state continuity equation (eq. 4), which then becomes:

$$\Delta Q_z + \Delta Q_y + \Delta Q_x = \frac{V_2 - V_1}{t_2 - t_1} \tag{8}$$

where:

V_1 = the initial volume of water in the block of aquifer material;

t_1 = the initial time, the time at which volume V_1 existed;

V_2 = the final volume of water in the block; and

t_2 = the final time, the time at which volume V_2 existed.

Therefore, the term $(V_2 - V_1)$ equals the change in volume (ΔV), and the term $(t_2 - t_1)$ equals the elapsed time (Δt) between measurement of V. The continuity equation can now be written:

$$\Delta Q_z + \Delta Q_y + \Delta Q_x = \frac{\Delta V}{\Delta t} \tag{9}$$

which states simply that

Inflow - Outflow = Change in storage (rate of storage accumulation) (1)

Because changes in elastic (artesian) storage are related to hydraulic head, as are inflow and outflow, the only dependent variable in the final continuity equation (eq. 9) is hydraulic head. This can be shown by first substituting Darcy's law for Q_z, Q_y, Q_x:

$$Q_x = K_x \, \Delta y \, \Delta z \, \frac{\Delta h}{\Delta x}$$

$$Q_y = K_y \, \Delta x \, \Delta z \, \frac{\Delta h}{\Delta y}$$

$$Q_z = K_z \, \Delta x \, \Delta y \, \frac{\Delta h}{\Delta z}$$

where:

$K_{x, y, z}$ = hydraulic conductivity along x, y, z axes

Δx = length of the block in x direction (change in length along x axis from one side of block to the other)

Δy = length of the block in y direction (change in length along y axis from one side of block to the other)

Δz = length of the block in z direction (change in length along z axis from one side of block to the other)

Δh = change in hydraulic head

Then by substituting for ΔV using the following formula which relates ΔV (where ΔV is the volume of water per unit area of aquifer that is drained from or gained by a block of aquifer) to the hydraulic head (Bennett, 1976, p. 53-68):

$$\Delta V = S \, \Delta x \Delta y \Delta h \tag{10}$$

where:

ΔV = change in volume of water in storage

S = storage coefficient (dimensionless) for block of aquifer material defined by Δx, Δy, Δz

Substituting these equations into the transient continuity equation and dividing by $\Delta x \Delta y \Delta z$ leaves the equation:

$$\frac{\Delta}{\Delta x}\left(K_x \frac{\Delta h}{\Delta x}\right) + \frac{\Delta}{\Delta y}\left(K_y \frac{\Delta h}{\Delta y}\right) + \frac{\Delta}{\Delta z}\left(K_z \frac{\Delta h}{\Delta z}\right) = \frac{S}{\Delta z}\left(\frac{\Delta h}{\Delta t}\right) \tag{11}$$

If we take the limit of this equation as the block of aquifer material and the time interval get smaller and smaller (Δx, Δy, Δz, and Δt approach zero), the equation may be written in terms of partial derivatives as,

$$\frac{\partial}{\partial x}\left(K_x \frac{\partial h}{\partial x}\right) + \frac{\partial}{\partial y}\left(K_y \frac{\partial h}{\partial y}\right) + \frac{\partial}{\partial z}\left(K_z \frac{\partial h}{\partial z}\right) = S_s \frac{\partial h}{\partial t} \tag{12}$$

where S_s is termed the specific storage with dimensions (L^{-1}).

The continuity equation in partial derivative form is presented here to complete the generalized theoretical development of the ground-water flow equations and to show that the movement of water is dependent on the hydraulic head.

The storage coefficients mentioned briefly in the development of equations 10, 11, and 12 are of particular significance in ground-water modeling. The storage coefficient (S) for elastic (artesian) conditions represents the percentage of water that the block of aquifer material releases in response to a unit head drop per unit area. The specific storage (S_s), which is used in the three-dimensional differential equation (eq. 12), is defined as the volume of water released from or taken into storage per unit volume (instead of planar area, as in the other storage coefficient) of aquifer material per unit change in head (Lohman and others, 1972).

The storage term in equation 12 applies to any small volume in any aquifer. In aquifers characterized by elastic (artesian) storage, changes in the volume of water in storage occur throughout the thickness of aquifer in response to changes in head. In contrast, however, a change in storage in an unconfined (water-table) aquifer is reflected as a change in the elevation of the top boundary of the aquifer (the water table or free surface). Boundary conditions will be discussed in the following section. However, the water-table boundary, because it is specifically related to unconfined storage, is discussed in this section.

The response of a water-table aquifer near a pumping well provides an example of the manner in which changes in storage in a water-table aquifer take place. A series of cross sections of the well, aquifer, and position of the water table, both before and during pumping, are shown in figure 6. The directions of local water movement within the aquifer as water is pumped are shown by idealized flow lines in figures 6B, 6C, and 6D.

Before the well begins pumping (fig. 6A), the aquifer is in its natural state with the water table sloping gently. When the well is not pumping, the water level in the well is the same as that surrounding the well. When the pump is turned on, however, water is drawn out of the well casing so that the hydraulic head (water level) in the well is lower than that in the aquifer. Because water flows from the areas of higher head to the areas of lower head, it moves from the aquifer into the well.

As the water moves into the well, the hydraulic head (water level) in the aquifer near the well is lowered. This drop in head is felt throughout the depth of the aquifer but is manifested at the top boundary (water table) as a lowering of the water table, as depicted in figure 6B. As a result of this lowering, part of the saturated earth materials near the well become unsaturated, so that water that was originally held in storage in the interstitial spaces of the saturated materials drains out and becomes a component of flow to the well. The idealized flow lines in figure 6B illustrate the path of water as it drains from the top boundary and then flows toward the well. This draining continues until equilibrium is reached (fig. 6D).

As the aquifer system begins to stabilize in response to the continued pumping of the well at a constant rate, the decreases in hydraulic head near the well become progressively smaller, although small declines in hydraulic head will take place at ever greater distances from the well, and water will continue to be released from storage as long as the water table moves downward. The flow lines in figure 6C indicate how flow patterns near the well are being affected by the release of water from storage as the water table continues downward.

Eventually, the well-aquifer system may reach equilibrium (fig. 6D), where inflow equals outflow at each block of aquifer material. At this stage, as the idealized flow lines indicate, all drainage from the dewatered top of the aquifer has ceased. Because the system is now in steady state (water levels are no longer declining), the upper boundary (water table) is not changing position, and no water is being drained from or put into storage. The four stages of aquifer response in figure 6 illustrate the changes in water-table storage at the top boundary (free surface) of the unsaturated flow system during pumping.

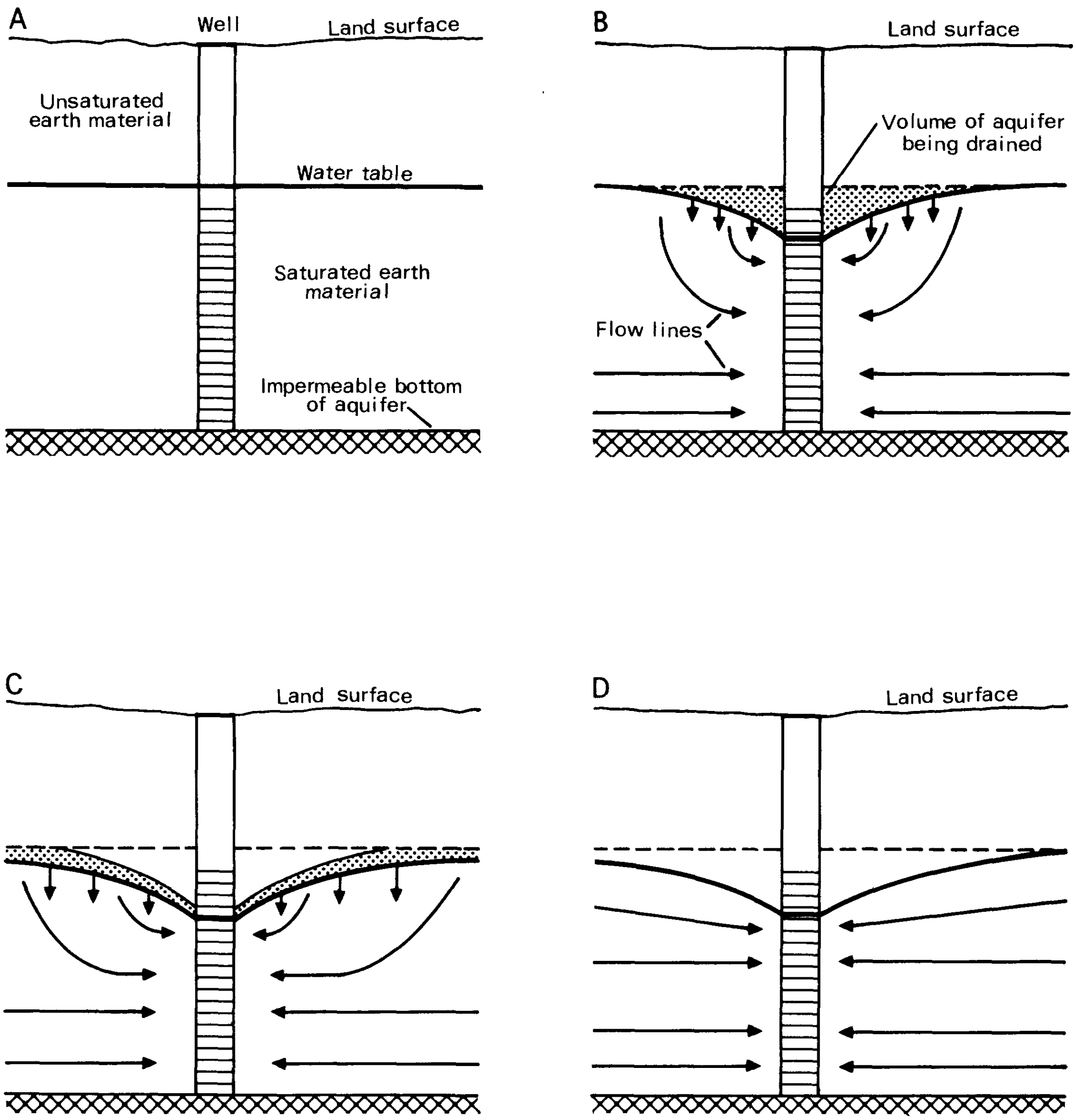


Figure 6.--Configuration of water table in an unconfined aquifer near a pumping well: A, before pumping; B, after short period of pumping; C, after medium-length period of pumping; D, after long period of pumping; well-aquifer system has achieved equilibrium (inflow = outflow). Shaded area represents volume drained since previous investigation.

Because the exact mathematical description of this drainage at the water table is highly complex, most numerical models that simulate water-table conditions must be simplified. To do this, most models, including those discussed in this report, average the amount of water gained or lost from water-table storage over the entire model block containing the section of water-table aquifer. Thus, storage obtained at the free surface is represented by a single storage coefficient for the entire model block of aquifer in the same manner as explained previously for elastic (artesian) storage.

To examine the general effect of water-table storage in an aquifer, we will refer to a block of saturated aquifer material that is at steady-state equilibrium, that is, with inflow of water equal to outflow (fig. 7A). If the hydraulic head in the block drops in response to some stress on the system such as pumping, the elevation of the water table will also drop. As the

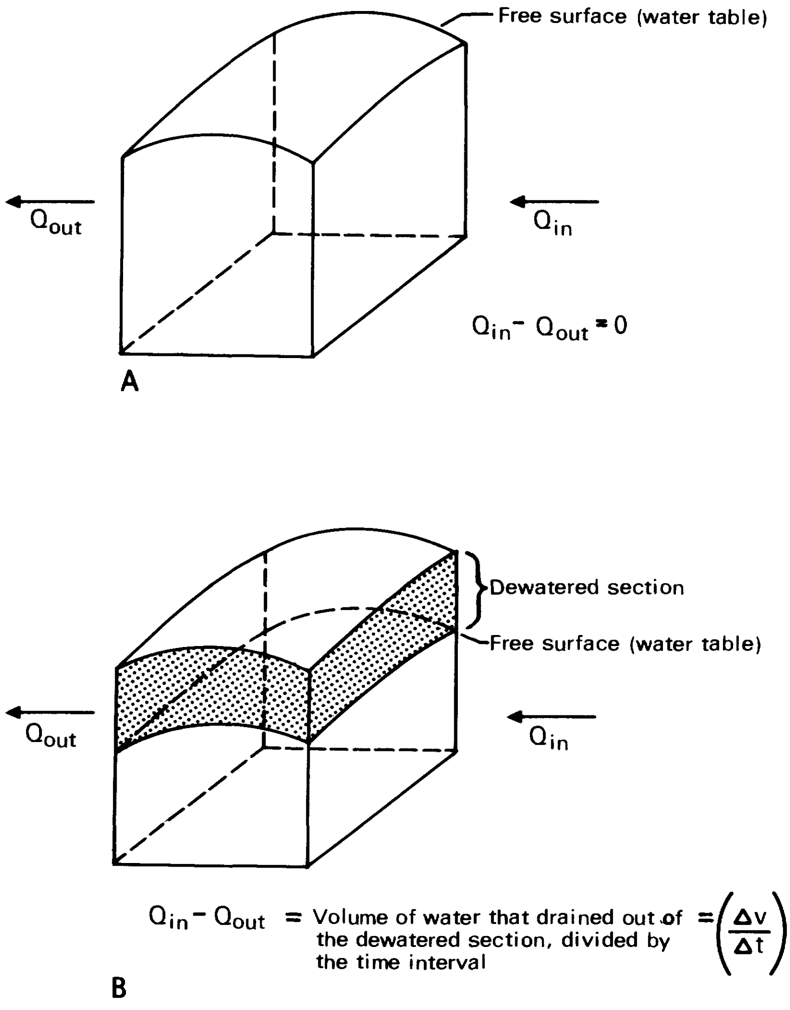


Figure 7.--Storage in a block of water-table aquifer material: A, under unconfined, steady-state flow conditions (inflow equals outflow); and B, under partially dewatered conditions.

water level in the block declines (fig. 7B), water drains out of the interstitial space at or immediately above the new top boundary (new free surface) and leaves the block through one or more of the faces as part of the discharge (Q_{out}).

In the case of an unconfined (water-table) aquifer, the storage coefficient (S) represents the amount of water that can be stored or released in a block of aquifer material. Consider, for example, a block of earth made up of 70 percent solid material and 30 percent water. If all saturated pore spaces were interconnected and the block was then drained, the maximum amount of water that could be released from storage would be 30 percent of the volume of the block of aquifer, and the storage coefficient (also called specific yield) would be 0.3. However, some of the water is held in small pores after the block has been drained so that the actual storage coefficient is less than the bulk porosity (the proportion of pore space to the total volume of aquifer material). Water-table storage coefficients (specific yields) range from about 0.05 to 0.30, whereas artesian (confined) storage coefficients generally range from about 0.0001 to 0.001 (Ferris and others, 1962). This large difference in magnitude is attributed solely to the two different mechanisms of water storage. Water-table storage involves either a draining or filling of pore spaces, whereas artesian (confined) storage involves a slight change in the size, shape, and configuration of the porous medium and water.

In the electric-analog model, storage is accounted for by capacitors, which store electrical charge. The electrical expression for a change in storage analogous to $\Delta V/\Delta t$ is $\Delta \varepsilon/\Delta t$, where ε equals electrical charge. Each node, which represents a specific block of aquifer material, is connected to a capacitor. The electric-analog circuit representing storage (fig. 8) is equivalent to the discrete transient-flow equation (eq.11), except that the change of electrical charge with time in the capacitor is continuous.

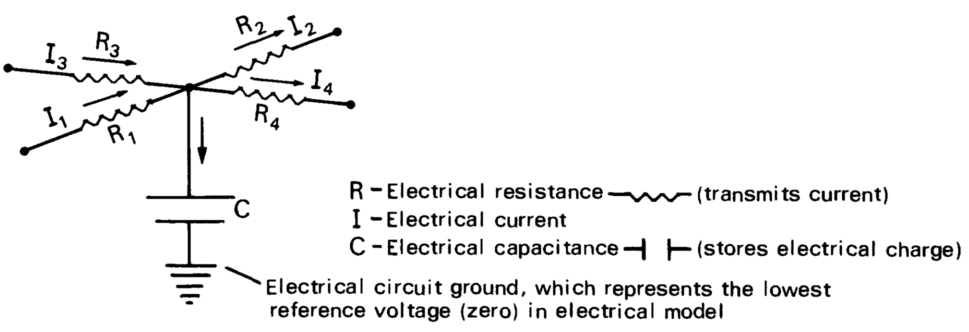


Figure 8.--Electrical elements connected to representative node of two-dimensional, transient-state electric-analog model, including storage element (capacitor).

Boundary Conditions

All ground-water systems are of finite areal and vertical extent--that is, they possess natural boundaries. The mathematical models described in previous sections of this report solve a series of differential equations that describe ground-water flow in space and time. However, without definition of boundary conditions, only general solutions can be obtained for these equations, and these solutions are not applicable to physical systems. The conditions of flow at natural boundaries of an aquifer system are controlled by other physical systems that affect the movement of ground water, for example, oceans, lakes, impermeable rock, or precipitation. To obtain accurate model solutions for a ground-water system, the boundary conditions must be represented correctly.

Models of ground-water flow systems use a set of interconnected blocks, representing sections of the aquifer, to simulate the aquifer system. The set of blocks ends at the edge of the model aquifer system. Natural boundaries are ideal for representation in a model; however, when model size and other constraints frequently make it impossible to apply the natural boundary conditions, appropriate fictitious boundary conditions must be applied.

Two common conditions at an aquifer boundary are constant flow and constant head. Constant-head boundaries are used to represent areas at which the hydraulic head is assumed not to change with time, and constant-flow boundaries represent areas at which the inflow or outflow of water is held constant. Constant-flow boundaries can be further classified into three categories:

(1) impermeable boundary (zero flow).--No flow may cross this boundary; this boundary also defines a flow path for the ground water.

(2) recharge boundary.--A constant inflow enters the boundary.

(3) discharge boundary.--A constant outflow leaves the boundary.

Flow and head boundaries in a model can also be functions of position, time, or aquifer conditions. A hypothetical ground-water system exhibiting some typical boundaries is shown in figure 9.

As indicated by the arrows in figure 9, boundary conditions determine both the direction and the amount of flow through the system; thus, a model of the ground-water system requires that these boundary conditions be represented accurately. Once the boundary conditions are established and the hydraulic parameters determined, the model solves the flow equations for the interior blocks accordingly.

The boundary conditions used in the Long Island electric analog model are described in detail by Getzen (1977, p. 27-31) and are summarized as follows:

I. Constant-head boundary.--Mean sea level is represented as a constant hydraulic head of zero. This boundary is the area of discharge for flow from the shoreline and from the Magothy aquifer across the Gardiners Clay (subsea outflow to surrounding bays and ocean).

II. Constant-flow boundaries

1. Impermeable boundary (zero flow):

   a. The top of the Raritan clay, beneath the Magothy aquifer, is represented as a no-flow boundary on the assumption that the vertical hydraulic conductivity of the Raritan clay is extremely low and that only negligible quantities of ground water move through the clay.

   b. The saltwater interface at the north and south shores is represented as a no-flow boundary because the difference in density between fresh and salty water tends to keep the two separated. However, this boundary condition is only an approximation and must be adjusted to reflect movement of the interface.

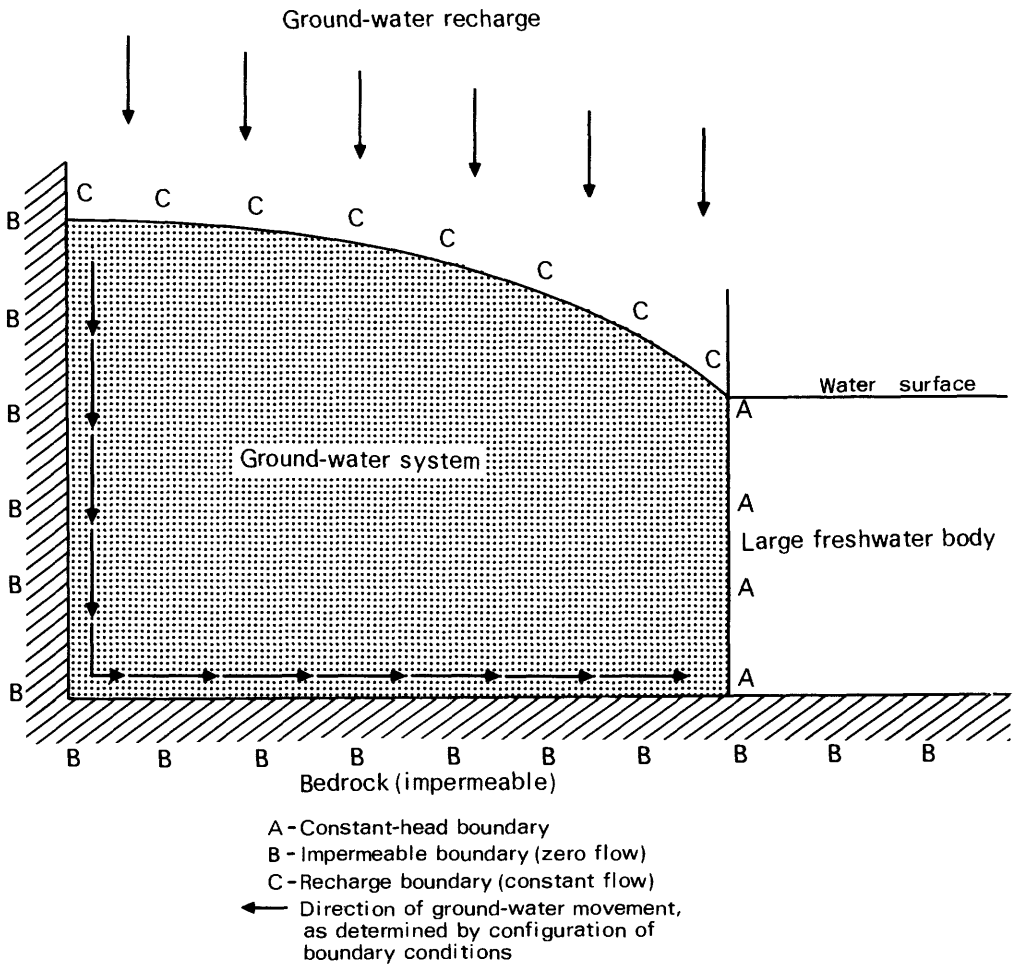


Figure 9.--Representative ground-water boundaries within a natural hydrologic system.

2. Recharge boundary (constant inflow).--Recharge from precipitation is assumed to occur at the water table everywhere on Long Island and is represented at a uniform rate with varying areal distribution across the model.

III. Flow boundary dependent upon aquifer conditions.--The streams on Long Island, which are fed mostly by ground water, are represented as a discharge boundary at the model water table. This boundary affects the movement of ground water only when the streams are gaining seepage. When the streams dry up (when the water table drops below stream-bed elevation), the boundary is cut off and no longer affects the movement of ground water. Both models are able to simulate this effect.

Because the water movement in an aquifer system is determined by the hydrologic boundaries, the boundaries must be represented in the model as accurately as possible.

OPERATION OF A GROUND-WATER MODEL

Both an analog and a numerical model use Darcy's law and the continuity equation to compute hydraulic heads (water levels) that would occur at different locations and times in response to stresses imposed. Erroneous model results generally occur either from incorrect representation of the boundary conditions, as discussed in the preceding section, or from use of inaccurate hydrologic coefficients. The degree of accuracy and detail of the data needed for specific modeling situations may vary according to need or to complexity of the hydrologic system; therefore, the data to be used must be studied at the onset of the modeling effort.

Accuracy of Model Data

The variables, or hydrologic coefficients, needed for a ground-water flow model are (1) areal extent and thickness of the aquifers and confining units to be modeled, (2) vertical and horizontal hydraulic conductivity of the aquifers and confining units, and (3) storage coefficients of all units. The values assigned to these terms define the geometric boundaries of the system to be modeled and also the hydrologic coefficients that will affect the flow paths predicted by the model. Because the hydrologic coefficients of a natural system can never be known exactly, a main problem in modeling is to decide what degree of accuracy is necessary to produce acceptable answers. Since a finite-difference model uses average values to represent each block, the size and number of the blocks must be considered in determining the degree of accuracy required. If the model is to be of regional scope, with relatively large blocks, the hydrologic data can be regional averages. However, if the model pertains to a specific site, the data must be accurate to the scale required.

Method of Calibration

Because most aquifers and confining beds are heterogeneous in composition, their hydraulic conductivity and storage coefficients may range considerably within a given area, which makes it difficult to establish widely applicable average values. For this reason, the calibration stage (fig. 1, step 8) is extremely important. At first, initial estimates of the hydrologic coefficients to be used are obtained through aquifer-test analyses, laboratory analyses of the aquifer material, and other techniques. The initial estimates are then used to calculate head values, and the model values are compared with measured water-level data. If the model does not reproduce the measured water level adequately, the initial data must be reevaluated and corrections made in the hydrologic coefficients. If possible, additional data should be collected. Model calculations obtained from the revised data set are then compared with measured water levels, and the calibration procedure is continued until a satisfactory match with the historical record is achieved.

In the development of the Long Island electric-analog regional model, initial estimates of the hydrologic coefficients were available from McClymonds and Franke (1972), Jensen and Soren (1974), and others. Getzen (1977, p. 31-45) calibrated the model by comparing model results with the historical Long Island hydraulic-head record. In a steady-state calibration, model results were compared with water levels on (1) a map of the 1903 water table on western Long Island (Veatch and others, 1906), (2) the 1970 water table on eastern Long Island (Kimmel, 1972), and (3) a 1971 potentiometric map of the Magothy aquifer (Jensen and Soren, 1974). A transient-state model run was then checked against a map of hydraulic head declines observed during the 1962-66 drought on Long Island (Cohen and others, 1969). The final data set of hydrologic coefficients, which achieved the closest correspondence between model results and historical water levels, is documented in Getzen (1977); the hydrologic coefficients used in the final model calibration are acceptable on a regional basis. In a model for a smaller area of Long Island, however, these coefficients would need further refinement to give reliable results.

Application of Stress

In a finite-difference ground-water model of either the electric-analog or the numerical type, stresses are applied to the interconnected system of blocks at the appropriate location. (A stress is any natural or man-induced change in inflow or outflow of the ground-water system). Typical stresses applied to the Long Island ground-water systems are pumping from wells, recharge through wells and basins, and drought. Any of these stresses may induce a change in the inflow-outflow relations of the ground-water system and therefore change the hydraulic heads in the system. The prediction of these changes in hydraulic heads is the result of a model run.

DEVELOPMENT OF DIGITAL MODEL FROM ANALOG MODEL

The Long Island electric-analog regional model (Getzen, 1977) is a five-layer, three-dimensional model in which the top two layers represent the upper glacial aquifer, the bottom three layers the Magothy aquifer. The horizontal spacing between nodes on the top layer represents 914 meters; spacing on the other four layers is 1,829 meters. The vertical distance represented between nodes varies according to hydrogeologic conditions.

Figure 10 depicts (a) the horizontal (areal) grid with the 1,829-meter node spacing, (b) a representative geologic section of Long Island; and (c) the vertical (cross-section) grid used by Getzen (1977).

The Long Island electric-analog regional model was used as the basis for development of a finite-difference numerical model. A digital computer program designed specifically for modeling ground-water problems (Trescott, 1975) was used for the digital model. Trescott's finite-difference program allowed the hydraulic conductivities that were used in the analog model to be transferred directly into the digital model because the same horizontal spacing between nodes (1,829 meters) was used in both models. The hydrologic data used to simulate the Long Island flow system are examined in detail in the analog-model report (Getzen, 1977). The only differences in the way the two models were discretized (ground-water system broken into blocks) were in the uppermost layer. The upper layer of the analog model had a smaller spacing between nodes (914 meters), whereas the digital technique requires that all layers have the same spacing. Therefore some minor adjustments were made in the data matrix for the uppermost layer of the digital model. Despite these changes, the models are based on virtually the same data, and the only major differences between the models is in the method of solution.

Both the digital and analog models are three dimensional and can simulate recharge, discharge, and flow to streams. The streams are simulated so that the flow to the model stream is proportional to the difference between the head at the node nearest the stream and the head in the stream (which in Long Island streams is nearly the same as the elevation of the stream bottom). The simulation of the streams allows them to dry up (decrease in length) when ground-water levels drop sufficiently. The concept of how the streams were simulated in both models is documented by Harbaugh and Getzen (1977).

The digital model contains 11,020 nodes (76 x 29 x 5). The main problem in developing the digital model was in transferring the large number of hydrologic coefficients. In large models, the manipulation of the extensive data sets requires considerable time and effort to insure that all information is compiled correctly. However, the processing of data for transfer to the digital model was done quickly and accurately by a minicomputer. (The model runs were made on a larger, faster digital computer).

Method of Testing the Acceptability of the Model

The set of hydrologic coefficients used in the Long Island analog model was assumed to be accurate because that model had been checked (Getzen, 1977) by comparison with historical field data. Because the digital model uses the same data set as the analog model, results from the two models should be the same. From this it follows that if the digital-model results match the analog-model results for the same series of selected problems, the digital model can be assumed to be accepted. This allows future digital model results to be compared with past predictive work done with the electric analog model. Thus, the purpose of the model comparisons is not to recalibrate a new model but simply to prove that the digital model will give the same results as the electric-analog model.

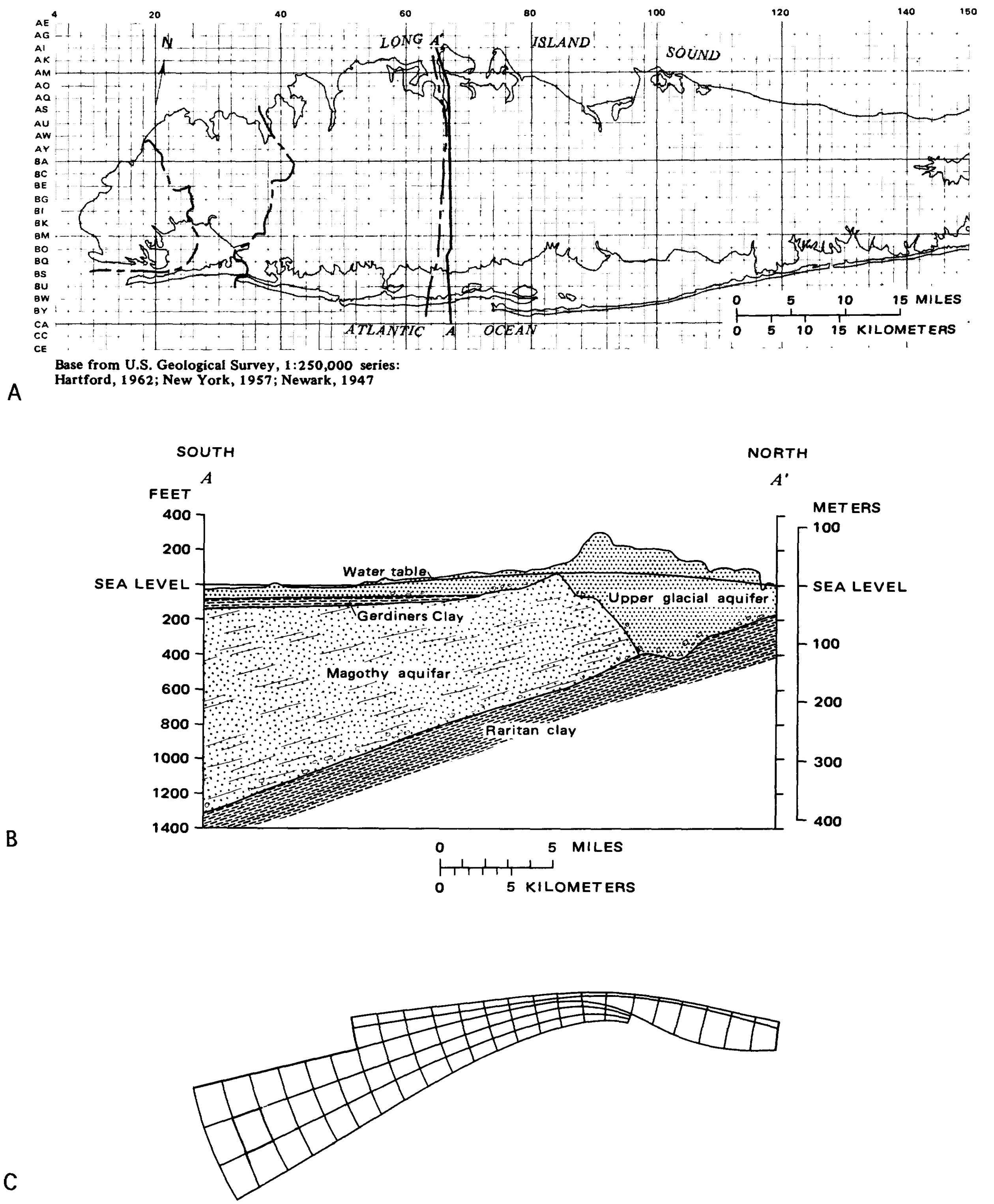


Figure 10.--Grid (block setup) on electric-analog model of Long Island (from Getzen, 1977). A, Grid in the x-y plane; B, Generalized cross section of Long Island; C, Typical grid for above cross section in the y-z plane.

To assess the acceptability of the digital model, three test problems were run on both models, and the results were compared. These tests were designed to calculate (1) steady-state heads at the water table and in the Magothy aquifer under natural conditions (before urbanization); (2) heads near four large hypothetical pumping centers after 20 years of pumping, and (3) heads and streamflow near a large hypothetical pumping center in the Magothy aquifer on eastern Long Island after 5 years of pumping. These three problems were designed to test all aspects of both models rigorously. Thus, comparison of the digital-and analog-model results for these problems allows us to evaluate the validity of the digital model.

Results of the first tests, comparison of heads under steady-state conditions before the influence of man (a) at the water table, and (b) in the Magothy aquifer, are shown on potentiometric-surface maps in figures 11 and 12, respectively. In the water-table map (fig. 11), results from the two models are similar with few small discrepancies. In the Magothy aquifer map (fig. 12), results from the two model runs are also similar.

Results of the second tests, comparison of predicted changes in water levels near four hypothetical pumping centers after 20 years of pumping at 1.25 m^3/s, are given in figure 13 and 14 for the water-table and the Magothy aquifers, respectively. Except in the far western part of the island, the match between the analog and digital predictions for both aquifers was excellent.

The streams on Long Island, as explained in the section "Boundary Conditions" are a significant factor in Long Island's ground-water system. Results of the third test, comparison of losses in streamflow after 5 years of ground-water withdrawal from a pumping center in the Magothy aquifer on eastern Long Island, are given in table 2; locations of stream areas listed and pumping center are shown in figure 15. Drawdowns in the Magothy aquifer associated with the above stress, as predicted by both models, are compared in figure 16.

Table 2.--Losses in Long Island streamflow resulting from 5-year pumping stress, as predicted by analog and digital models

[All values are in percent]

Stream group (see fig. 15)	Digital model (loss of flow/ average flow)	Analog model (loss of flow/ average flow)
A	7	6
B	10	10
C	2	2
D	2	2
E	6	6

As can be seen from the results of the three test problems (figs. 11-16), the two solution techniques produced highly similar results, which indicates that because the analog model was checked (Getzen, 1977), the digital model can also be considered acceptable. The small discrepancies in the map contours are attributed to (1) the difference in block spacing in the upper layer of the two models; (2) inaccuracies inherent in some of the electrical components of the analog model, and (3) errors in electrical readings from the analog model. These factors are described in detail in the following section.

Reasons for Discrepancies

The main cause of differences in results from the two models is probably the difference in node spacings in the upper (water-table) layer. In parts of western Long Island, the upper glacial aquifer is discontinuous. As discussed earlier, the analog-model nodes represent a 1,829-meter spacing in all but the top layer, which represents a 914-meter spacing. In the digital model, however, the node spacing is a constant 1,829 meters in all five layers. Thus, the boundary conditions and discontinuities of the upper glacial aquifer on the two models could not be matched exactly. This results in model discrepancies that are most noticeable in western Long Island because there the island is relatively narrow and the aquifer discontinuities most prevalent.

Getzen (1977, p. 44) points out that hydrologic boundaries in northeastern Queens County have fine-scale variations that the analog model cannot match and concludes the analog-model results for this area are less reliable than for the rest of the island. Because the digital model requires even greater generalizations for these fine-scale variations than the analog model, comparison of digital and analog models results may reveal pronounced differences in this area. Thus, any predictive modeling in the Brooklyn-Queens area should entail new model development to incorporate the local variations in hydrogeology.

A second reason for the differences between results from the two models is that the resistors used in the analog model to represent the hydraulic conductances give values that may be in error by as much as 10 percent. This is well within the tolerance limits of the input data, but variations due to this feature could cause discrepancies in comparative tests.

The third reason for differences in model results is the precision of the electronic equipment used to operate the analog model. Stresses applied to the analog model are generated and measured by electronic instruments, and the resulting heads (voltages) are read by instruments. This allows the possibility of (1) errors in measurement over the entire model as a result of the precision limits of the instruments, and (2) random errors due to poor electrical contacts between the measurement probes and the model. These errors should not diminish the accuracy of the entire solution and are well within the accuracy of the hydrologic coefficients but may show up as small discrepancies at a few locations when the two model solutions are compared.

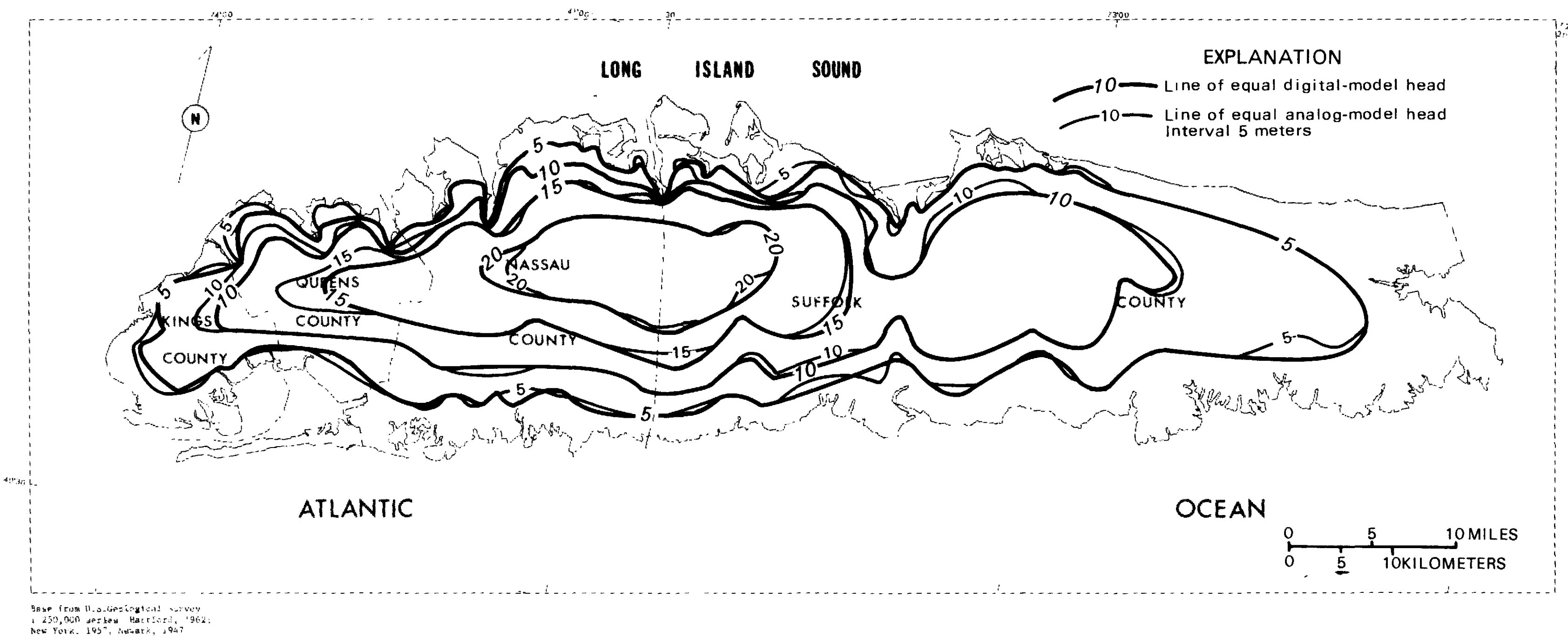


Figure 11.--Comparison of digital- and analog-model results at water table under steady-state conditions before urbanization.

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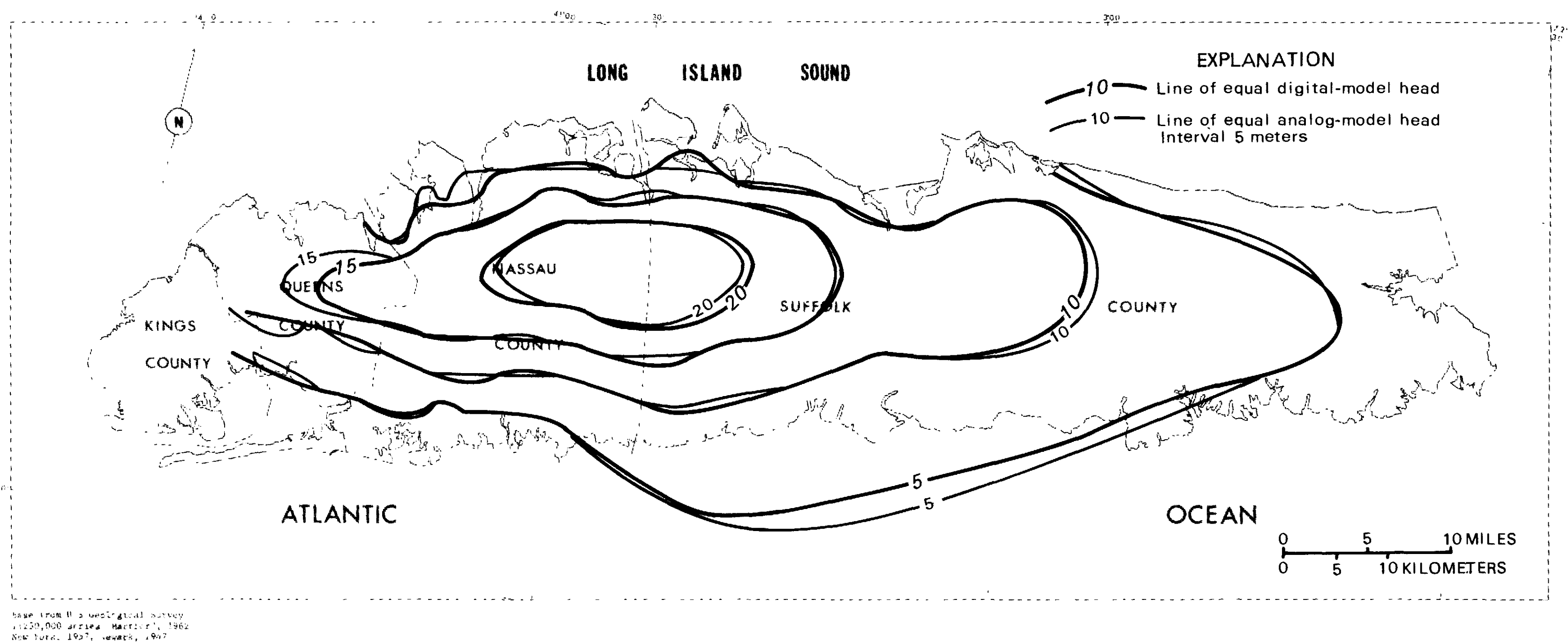
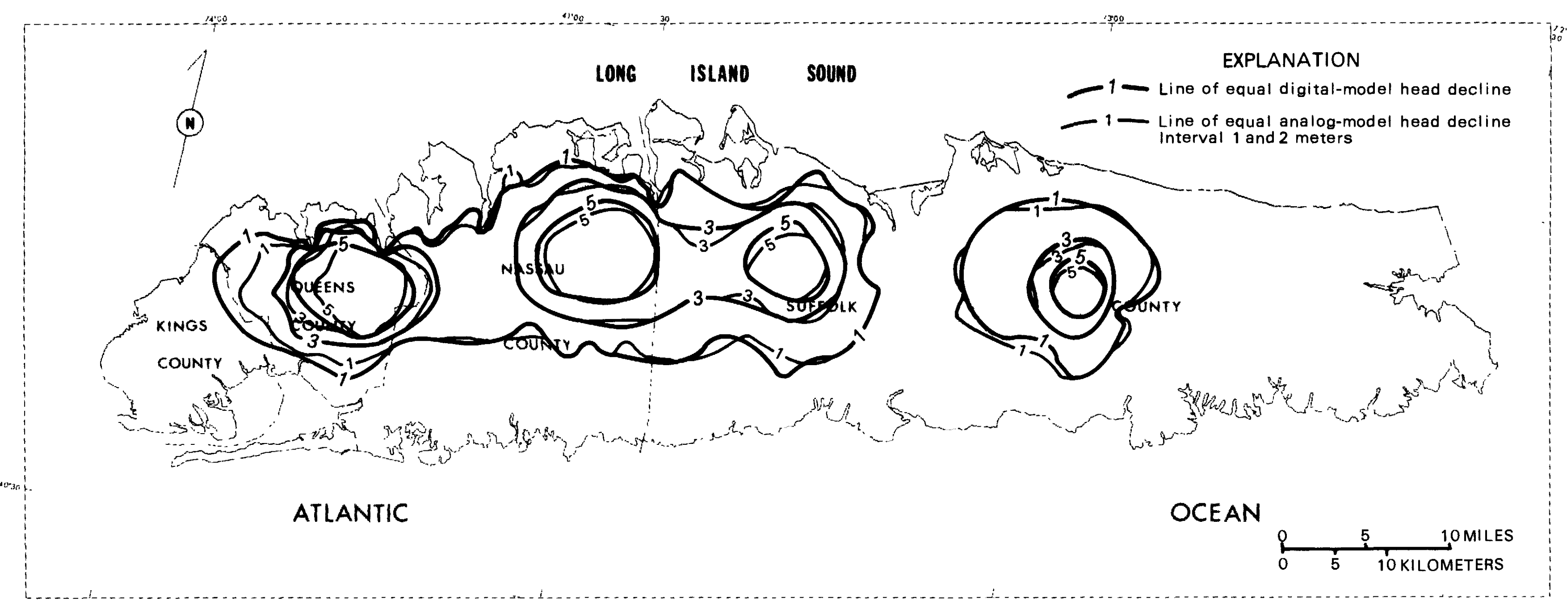


Figure 12.--Comparison of digital- and analog-model results for Magothy aquifer under steady-state conditions before urbanization.



Base from U.S.Geological Survey 1:250,000 series. Hartford, 1962, New York, 1957, Newark, 1947

Figure 13.--Comparison of digital- and analog-model results at water table in response to 20 years of pumping at 1.25 m^3/s at four hypothetical pumping centers.

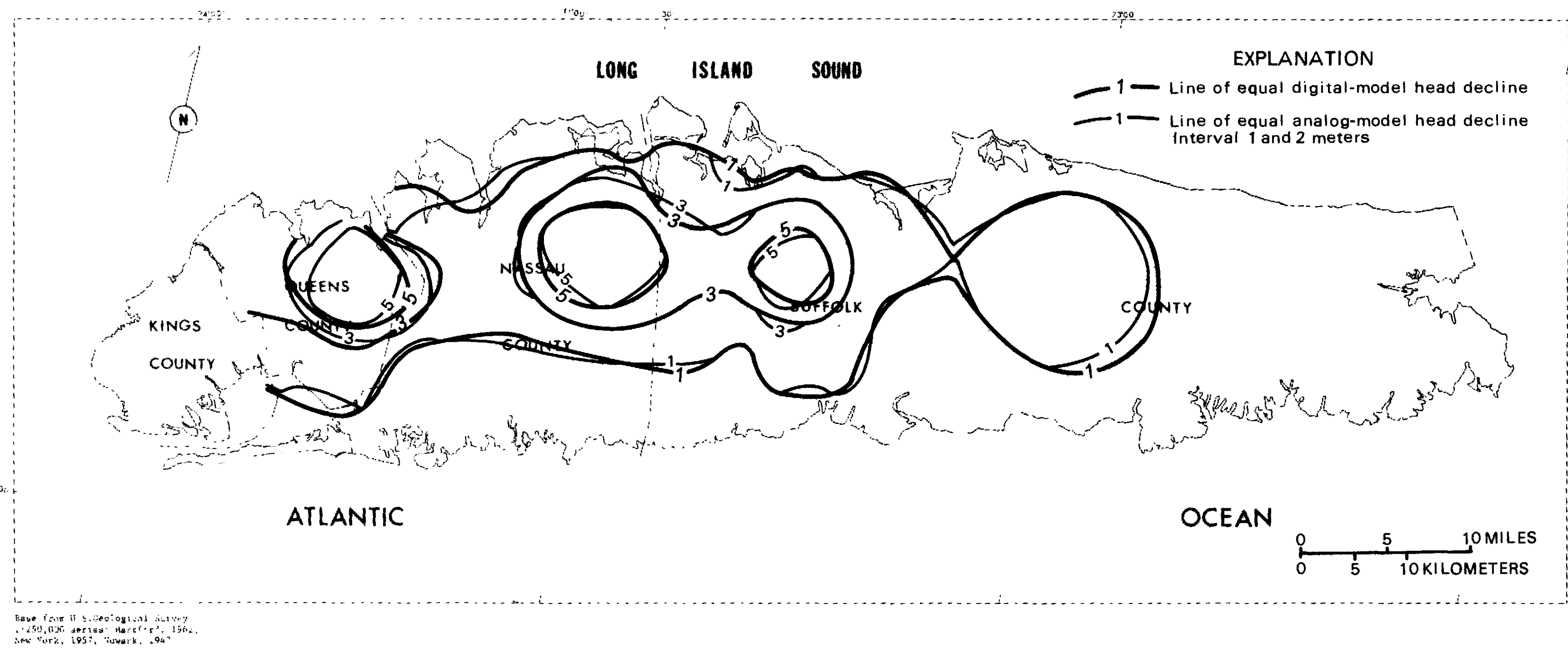


Figure 14.--Comparison of digital- and analog-model results in Magothy aquifer in response to 20 years of pumping at 1.25 m^3/s at four hypothetical pumping centers.

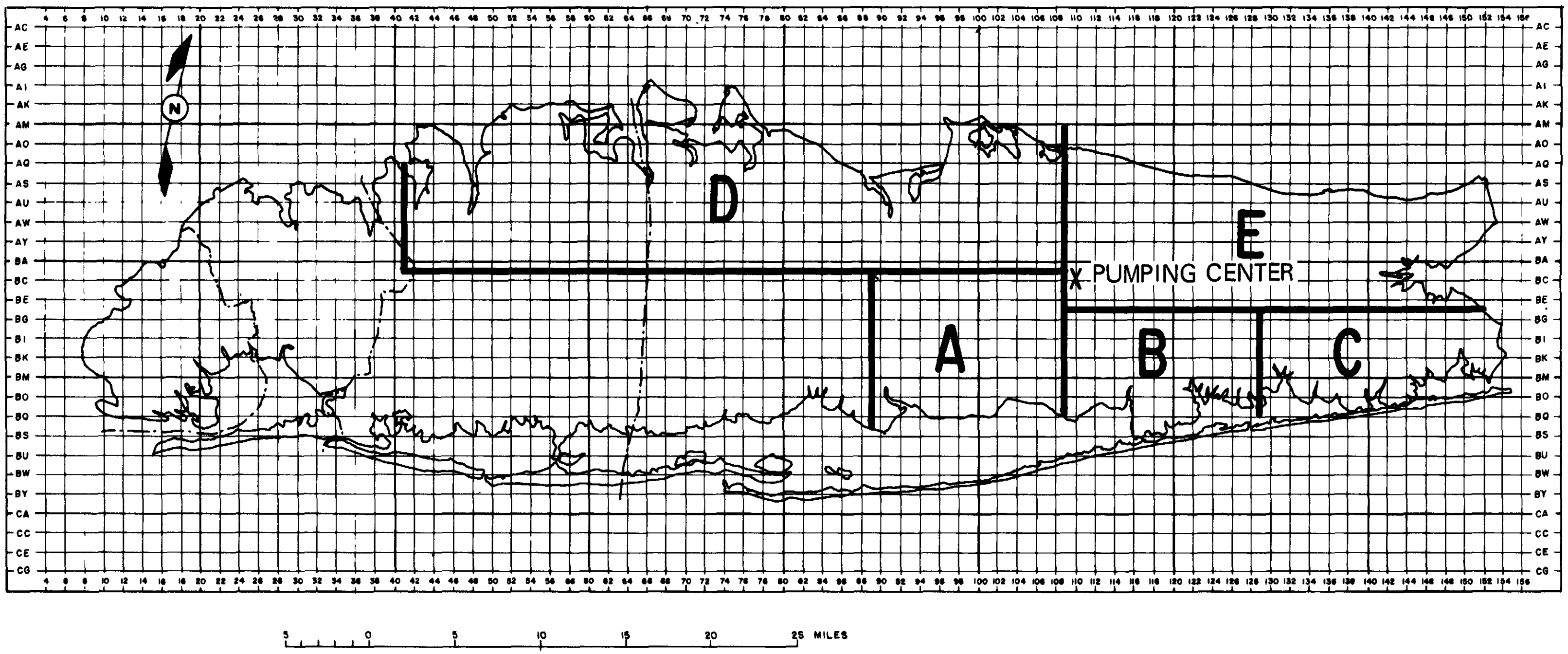


Figure 15.--Location of pumping center and stream areas mentioned in table 2.

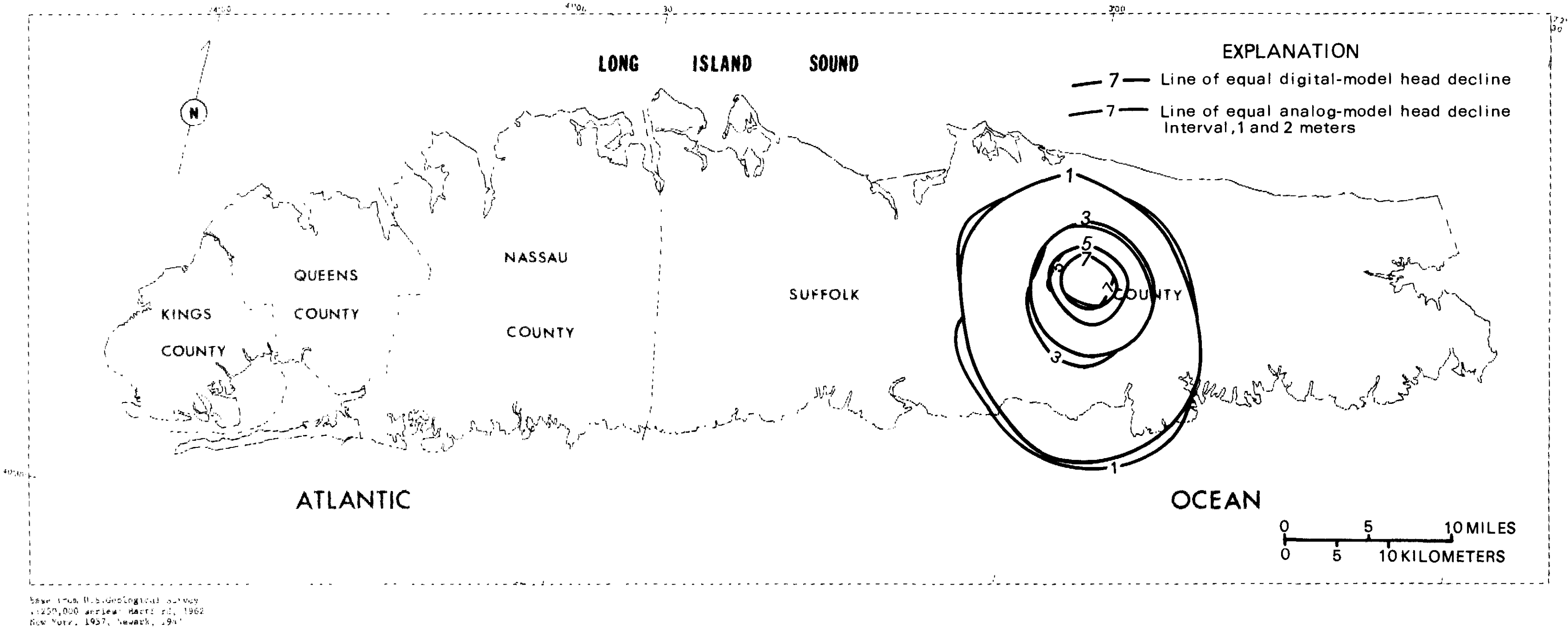


Figure 16.--Comparison of digital- and analog-model results in the Magothy aquifer after 5 years of pumping at hypothetical pumping center in Suffolk County.

SUMMARY AND CONCLUSIONS

Ground-water flow in an aquifer system may be represented by mathematical models of either the numerical (digital) type or the physical mathematical (electric-analog) type. The accuracy of the model depends principally on how the boundary conditions are represented and on the accuracy of the hydrologic coefficients (such as hydraulic conductivity) selected for the area.

The Long Island regional analog model developed in the early 1970's was used as a basis for developing a digital model. The digital model has several advantages over the analog model:

(1) The digital model may be more easily modified and recalibrated than the analog model, which will allow increasingly accurate simulations as more data become available.

(2) The digital model should provide faster responses to complex water-management problems.

(3) The digital model program has the capability to simulate changes in the saturated aquifer thickness resulting from water-level fluctuations. This option will be included in future development, which will enable more detailed simulation.

A series of tests were run on both the analog and the digital models to compare solutions and to verify that the digital version was accurate and could produce the same results as the electric-analog model.

Acceptability of the digital model was based on the assumption that the hydrologic coefficients used in the analog model were representative of the system. The analog model, in turn, had been calibrated against actual historical performance of the aquifer system. The solution techniques of the two models produced almost identical results.

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