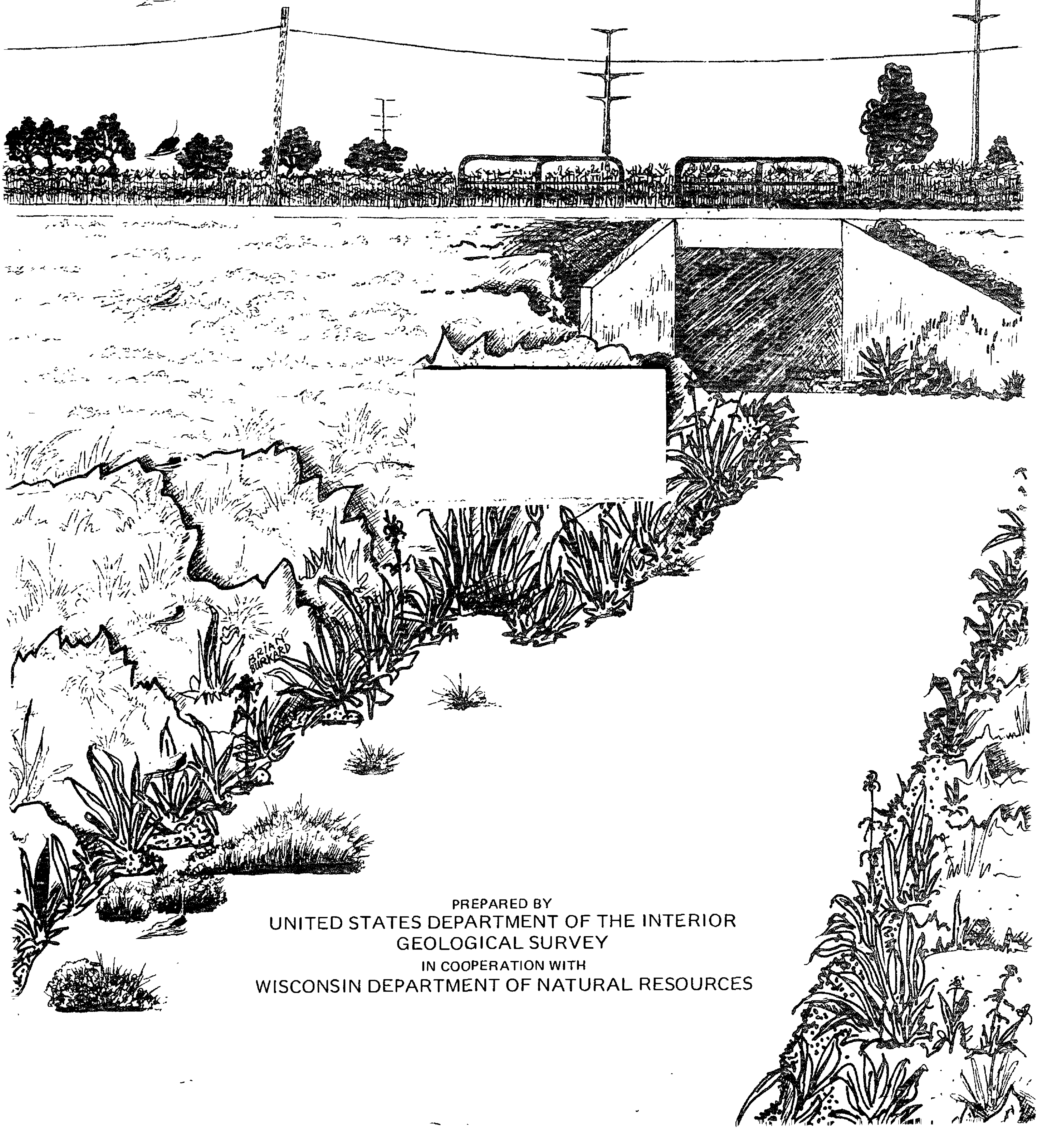
WATER-RESOURCES INVESTIGATIONS
80-19

Comparison of Tracer Methods and Predictive Equations for Determination of Stream-Reaeration Coefficients on Three Small Streams in Wisconsin



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Of the four modified-tracer techniques, the propane-area technique, which measures the total weight of propane gas passing stream-sampling stations, yielded the least mean absolute difference of 11.0 percent compared with the radioactive-tracer method. The propane peak concentration, ethylene peak concentration, and ethylene total weight methods gave mean absolute differences of 18, 21, and 26 percent, respectively.

The top five ranking predictive equations were as follows: Tsivoglou-Neal with 18 percent mean error, Negulescu-Rojanski with 21 percent, Padden-Gloyna with 23 percent, Thackston-Krenkel with 29 percent, and Bansal with 32 percent.

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by

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CONTENTS

ILLUSTRATIONS

TABLES

CONVERSION FACTORS

Multiply	By	To obtain
inch (in.)	2.540	centimeter (cm)
foot (ft)	30.48	centimeter (cm)
foot (ft)	0.3048	meter (m)
foot per second (ft/s)	30.48	centimeter per second (cm/s)
mile (mi)	1.609	kilometer (km)
mile per hour (mi/h)	1.609	kilometer per hour (km/h)
cubic foot per second (ft^3/s)	0.02832	cubic meter per second (m^3/s)

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ABSTRACT

Four modified nonradioactive-tracer methods and 20 predictive equations for determination of stream-reaeration coefficients (K_2) in three small Wisconsin streams were compared with the radioactive-tracer method developed by Tsivoglou.

Of the four modified-tracer techniques, the propane-area technique, which measures the total weight of propane gas passing stream-sampling stations, yielded the least mean absolute difference of 11.0 percent compared with the radioactive-tracer method. The propane peak concentration, ethylene peak concentration, and ethylene total weight methods gave mean absolute differences of 18, 21, and 26 percent, respectively.

The top five ranking predictive equations were as follows: Tsivoglou-Neal with 18 percent mean error, Negulescu-Rojanski with 21 percent, Padden-Gloyna with 23 percent, Thackston-Krenkel with 29 percent, and Bansal with 32 percent.

INTRODUCTION

Evaluation of stream-reaeration capacity is used to determine the self-purification capacity of a stream receiving oxygen-depleting wastes. Stream-reaeration coefficients can be measured using tracer techniques such as radioactive tracer and modified-tracer methods or calculated using predictive equations. All these methods have limitations. Predictive equations can be very inaccurate on one stream and more accurate on another. Tracer methods are accurate and reliable, but the difficulties of implementation and expense can be much greater than those for predictive equations. The method selected for use in a waste-load-allocation study, therefore, depends on the accuracy required.

The purpose of this report is to evaluate the relative accuracy of methods used to determine reaeration coefficients (K_2) when applied to reaches of three small streams just downstream from sewage-treatment-plant outfalls at Belmont, Cross Plains, and Holmen, Wis. (figs. 1-4). The study, by the U.S. Geological Survey in cooperation with the Wisconsin Department of Natural Resources (DNR), will be useful in establishing effluent standards. The various tracer methods and predictive equations were applied to identical stream reaches in this study, and the predicted reaeration coefficients were compared with the measured values. For this study, the radioactive-tracer method was used as a reference base from which to compare other methods.

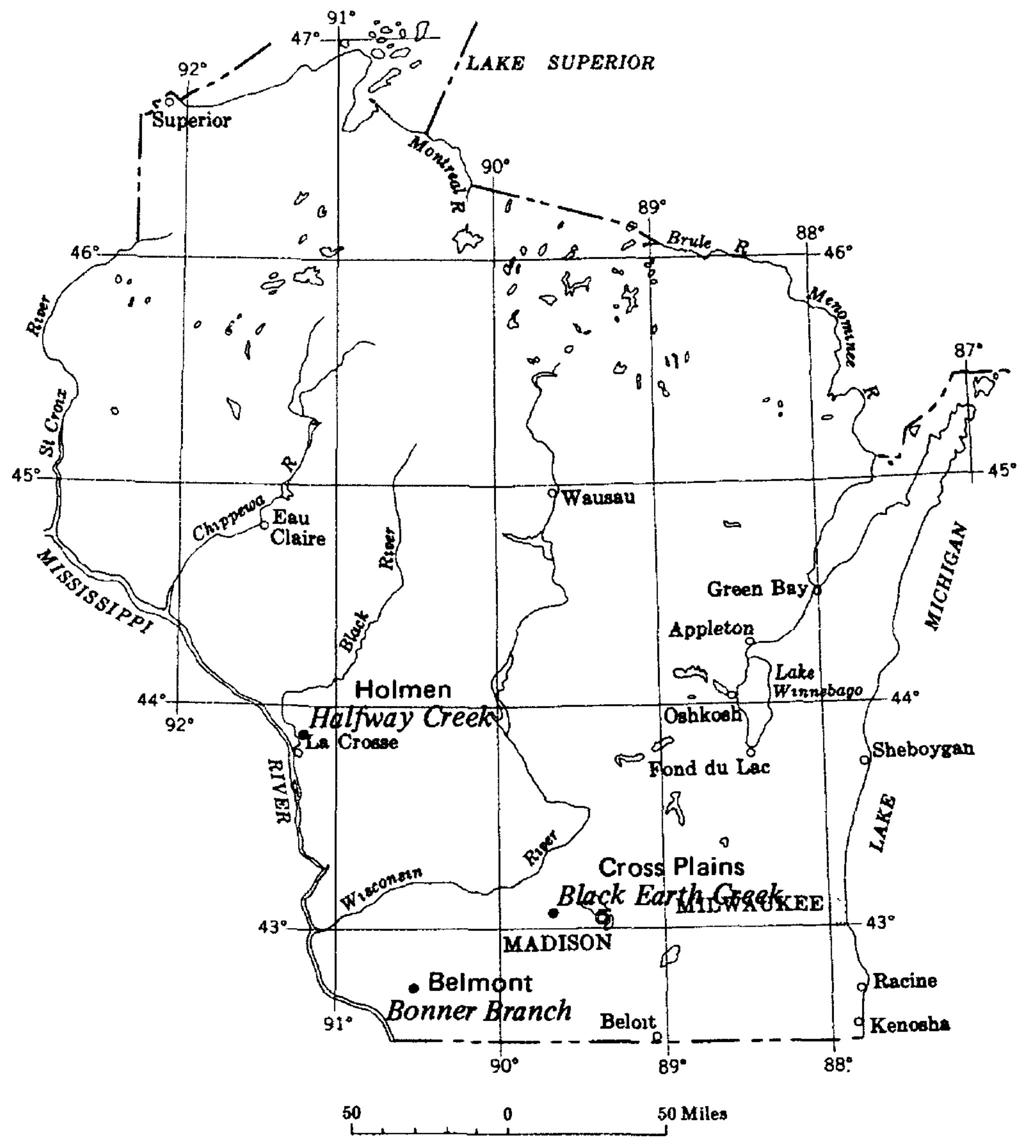

Figure 1. Location of study sites in Wisconsin.

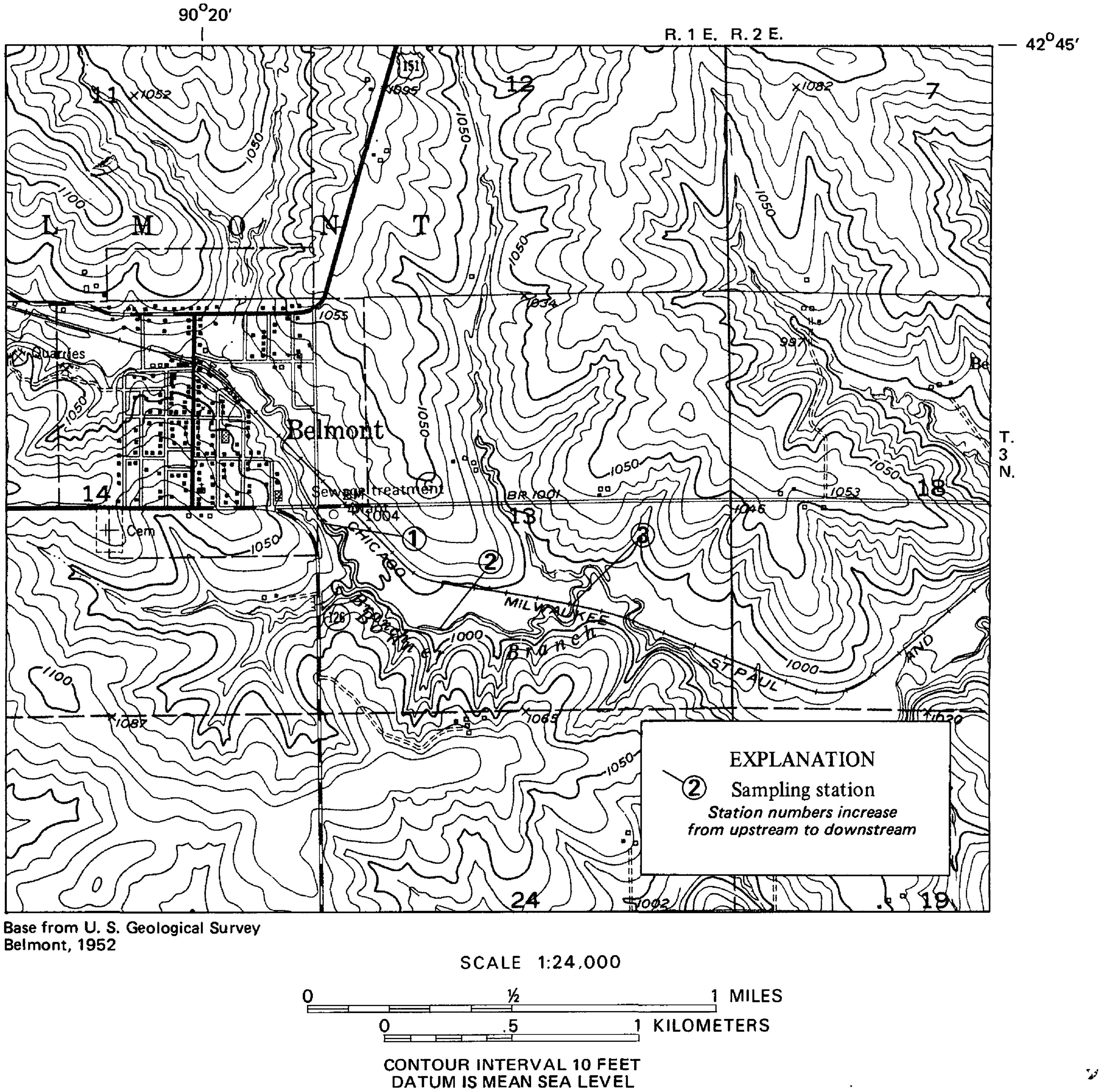

Figure 2. Location of tracer-sampling stations on Bonner Branch.

FIELD-DATA COLLECTION

Fieldwork on Bonner Branch at Belmont, Black Earth Creek at Cross Plains, and Halfway Creek at Holmen, Wis., was conducted in mid-October 1977. Tracer injections were made for both the radioactive and modified techniques. Channel geometry was surveyed concurrently with the tracer studies.

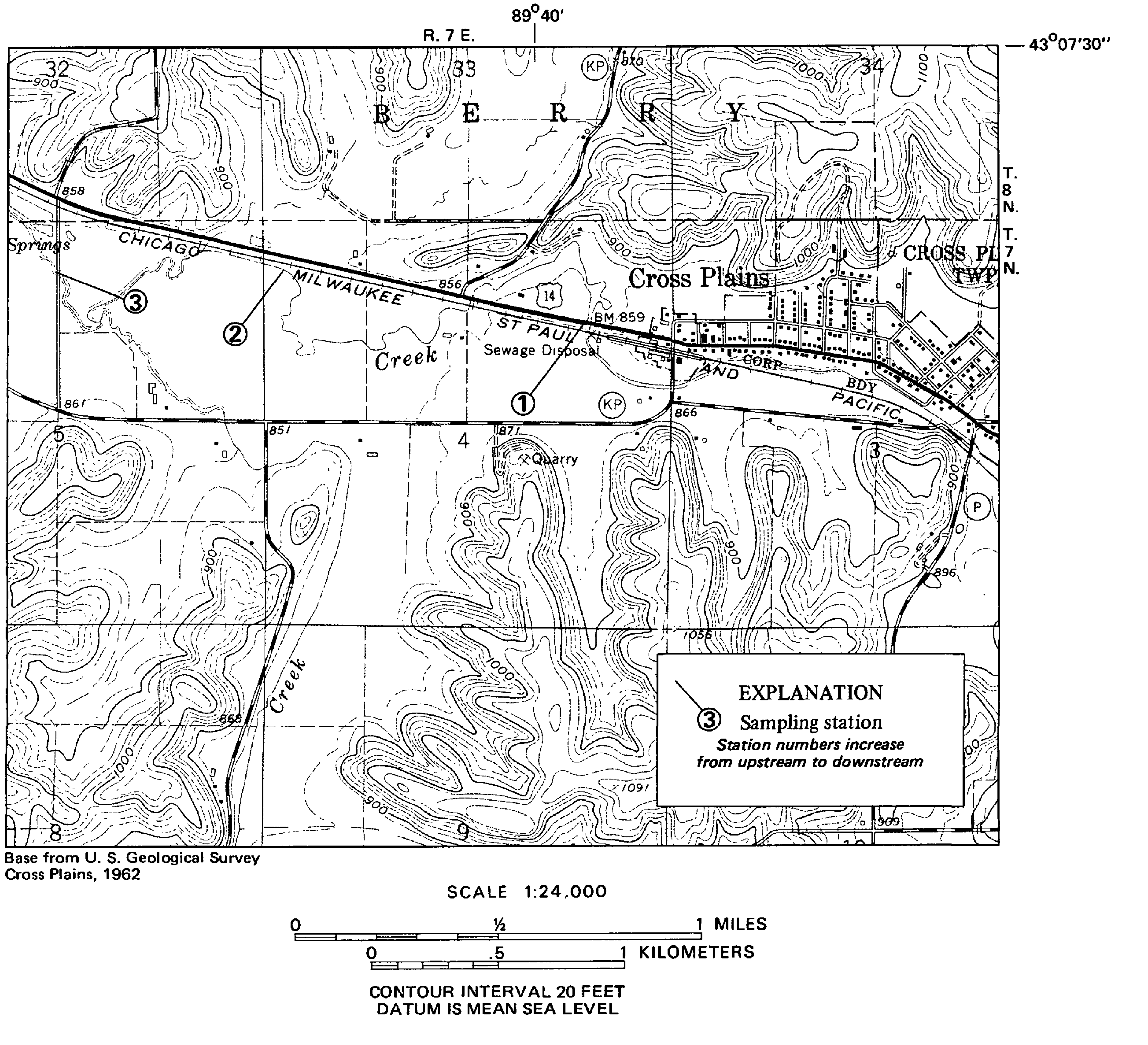

Figure 3. Location of tracer-sampling stations on Black Earth Creek.

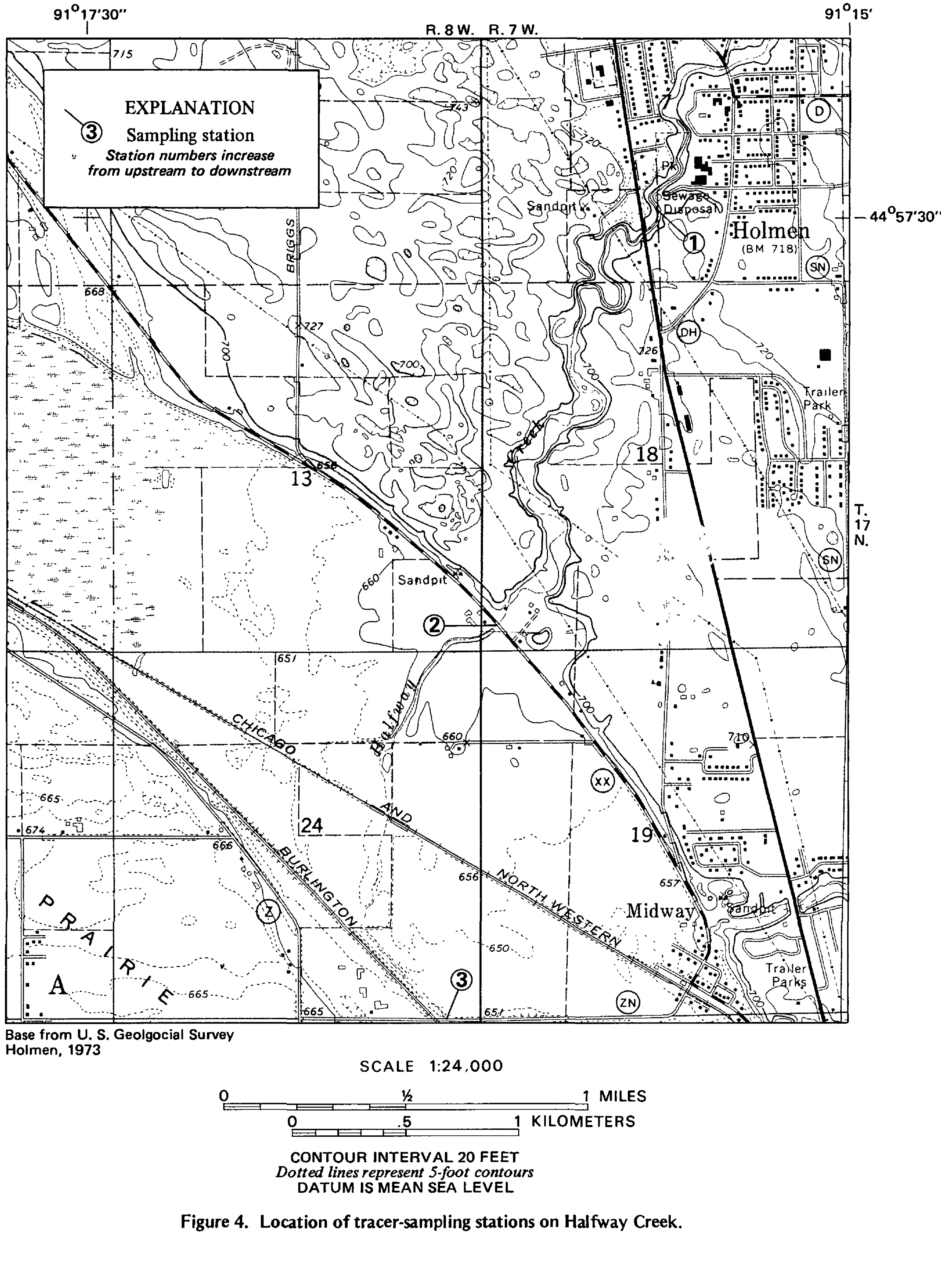

Figure 4. Location of tracer-sampling stations on Halfway Creek.

TRACER INJECTION

The radioactive and modified techniques cannot be conducted simultaneously in each stream reach because of differences in the tracer-injection procedures. The radioactive method uses an instantaneous slug injection whereas the modified method uses a constant rate injection.

In this study, the radioactive tracers were injected first and the ethylene and propane gas were injected 2 to 3 hours later. The ethylene and propane gas were injected into the stream using three porous stone diffusers. Propane is less soluble in water than ethylene, therefore two diffusers were used to inject the propane gas, and one diffuser was used to inject the ethylene gas. The diffusers used were of finer porosity than normally used to aerate waste water in sewage-treatment plants because the standard treatment plant type diffusers did not result in measurable concentrations of gas being dissolved in the water.

The finer porosity diffusers actually used for this study produced smaller gas bubbles that were more conducive to transferring the gas into solution. However, the rate of gas injection is much slower using the fine porosity stone diffusers. In order to inject the same rate of gas as a standard type diffuser, multiple fine-porosity diffusers may have to be used.

DESCRIPTION OF STREAMS STUDIED

Bonner Branch at Belmont, Wis., drains about 5.6 mi^2 of upland in the southwest part of the State. The study reach starts at the municipal sewage-treatment plant and ends about 1 mi downstream (fig. 2). See table 1 for additional information.

The stream is pool and riffle at low flows, having numerous long deep pools that discharge over fairly short gravel bars into the next pool. The study reach meanders through almost exclusively wide open pastureland. Stream depth averaged from 0.7 to 0.9 ft and water-surface width about 7 ft. Stream slope was about 14 ft/mi and stream velocity averaged about 0.2 ft/s. Stream discharge ranged from 1.7 to 1.9 ft^3/s. The $Q_{7,10}$ (annual minimum 7-day mean flow below which the flow will fall on the average of once in 10 years) is 0.74 ft^3/s just upstream from the sewage-treatment plant. The study reach was not obstructed by debris or affected by backwater from any artificial structures. Macrophytes were observed growing mostly along the streambanks with very little growth elsewhere in the channel. Periphytic algae were not apparent in the reach.

Black Earth Creek at Cross Plains, Wis., drains about 27 mi^2 of gently rolling to hilly terrain in the south-central part of the State. The study reach starts at the municipal sewage-treatment plant and extends about 2.5 mi downstream (fig. 3). Descriptive data for Black Earth Creek are in table 2.

Table 1. Descriptive data for Bonner Branch, October 18, 1977.

Reach[1]	Mean water temperature (°C)	Elevation change (ft)	Travel-time (h)	Slope (ft/mi)	Velocity (ft/s)
1-2	9.5	6.77	4.15	15.0	0.16
2-3	11.0	6.01	3.08	12.3	.23
1-3	10.0	12.78	7.23	13.6	.19

Station	Mile[1]	Time of peak dye concentration	Water temperature (°C)	Stream discharge (ft^3/s)	Time discharge measured
1	0.00	1002	8.0	1.66	0957
2	.45	1411	11.0	1.70	1402
3	.94	1716	11.0	1.89	1725

[1]A reach is the section of the stream between the two sampling stations. Mileage length is measured on U.S. Geological Survey 7.5-minute topographic map.

The stream is pool and riffle at low flows. The bed is composed primarily of sand, gravel, and cobbles. The channel meanders through agricultural land and is relatively free of debris. Stream depths during the field studies averaged from 0.9 to 1.1 ft, and water-surface width about 18 ft. Stream slope is about 7 ft/mi and stream velocity averaged 0.5 ft/s. Discharge ranged from about 9 to 18 ft^3/s. The $Q_{7,10}$ discharge is 4.3 ft^3/s just upstream from the sewage-treatment plant.

Macrophytes grow mostly along the banks and some growth was observed at isolated locations in the channel. There is no significant flow-retarding effect by vegetation. A small amount of periphytic algae was observed.

Halfway Creek at Holmen, Wis., drains about 31 mi^2 of hilly terrain in the southwest part of the State. The study reach starts at the municipal sewage-treatment plant and extends about 3.5 mi downstream (fig. 4). Descriptive data for Halfway Creek are in table 3.

Table 2. Descriptive data for Black Earth Creek, October 17, 1977.

Reach[1]	Mean water temperature (°C)	Elevation change (ft)	Travel-time (h)	Slope (ft/mi)	Velocity (ft/s)
1-2	9.5	12.67	4.68	7.7	0.52
2-3	11.0	5.26	2.48	5.8	.53
1-3	10.0	17.93	7.16	7.0	.52

Station	Mile[1]	Time of peak dye concentration	Water temperature (°C)	Stream discharge (ft^3/s)	Time discharge measured	Drainage area (mi^2)
1	0.05	0924	8.0	9.14	0929	26.6
2	1.70	1401	11.0	15.9	1510	----
3	2.60	1634	11.0	18.2	1646	39.1

[1]A reach is the section of stream between the two sampling stations. Mileage length is measured on U.S. Geological Survey 7.5-minute topographic map.

The stream is pool and riffle at low flows and the channel is composed of gravel and cobbles in the bars and sand and finer materials in the pools. The stream meanders through a wooded area between sampling stations 1 and 2. Reach 2-3 is partly channelized through open farmland with wetland near the downstream end. There was some debris in the channel in reach 1-2. During the field studies, stream depth averaged from 0.3 to 0.6 ft and width about 11 ft. Stream slope was about 8 ft/mi and velocity averaged 1.0 ft/s. Stream discharge decreased from 8.5 to 6.9 ft^3/s between stations 1 and 2, probably due to ground-water outflow. The $Q_{7,10}$ discharge is 3.1 ft^3/s just upstream from the sewage-treatment plant. There was very little macrophyte growth observed in the channel and no apparent periphytic algae.

CHANNEL GEOMETRY SURVEYS

Channel geometry data were collected to compare 20 reaeration-coefficient (K_2) models with measured K_2's determined using the radioactive-tracer method. With only 2 exceptions all of the 20 models to be tested

Table 3. Descriptive data for Halfway Creek, October 20, 1977.

Reach[1]	Mean water temperature (°C)	Elevation change (ft)	Travel-time (h)	Slope (ft/mi)	Velocity (ft/s)
1-2	8.0	17.80	3.18	8.4	0.98
2-3	11.0	10.87	1.97	7.9	1.02
1-3	9.0	28.67	5.15	8.2	1.00

Station	Mile[1]	Time of peak dye concentration	Water temperature (°C)	Stream discharge (ft^3/s)	Time discharge measured
1	0.00	0846	7.5	8.53	0850
2	2.13	1157	9.0	6.87	1201
3	3.50	1355	13.0	6.83	1405

[1]A reach is the section of stream between the two sampling stations. Mileage length is measured on U.S. Geological Survey 7.5-minute topographic map.

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require determinations of mean stream velocity and depth. Other parameters which appear in these models (and the number of models in which they appear) are: the slope of the energy gradient (8), Froude number (3), shear velocity (2), traveltime (1), and mean specific discharge (1). The Froude number and the shear velocity are both functions of the other parameters. The specific discharge is a function of both streamflow and drainage area. Thus, there were eight independent parameters to be determined.

The determination of velocity and depth was accomplished by dividing each study reach into a series of subreaches, each with fairly consistent hydraulic parameters. The average number of subreaches in each study reach was 10 with a maximum of 20 and a minimum of 3. Subreach lengths were determined by pacing. The accuracy of this method was checked for straight segments using the topographic maps. The agreement was very good. For those subreaches with many bends, pacing probably was more accurate than map measurements.

For each subreach, a characteristic stream-channel cross section was measured. From these measurements average velocity and depth in a subreach can be calculated as follows:

$$H = A/W \text{ and}$$

$$V = Q/A$$

where: H is mean depth;

A is mean cross-sectional area;

W is mean width of water surface;

V is mean velocity; and

Q is mean subreach discharge.

The slope of the energy gradient for each reach was calculated by dividing the difference in stream-surface elevation between the start and end of the reach (measured in the field) by the length of the reach (map distance). The traveltime for each reach was determined using rhodamine-WT dye, which was injected into the stream at the same time as the other tracers. Contributing drainage areas were measured from 7.5-minute topographic maps.

Tables 4-6 contain the hydraulic data collected for the three streams studied.

Table 4. Hydraulic data for Bonner Branch.

Subreach	Length (ft)	Area (ft^2)	Top width (ft)	Measured flow (ft^3/s)	Cumulative traveltime (h)
1	50	2.78	5.8	1.66	----
2	250	20.19	16	----	----
3	90	3.95	7	----	----
4	75	6.37	10.5	----	----
5	450	17.65	20	----	----
6	165	19.65	21	----	----
7	300	5.63	10.2	----	----
8	120	15.96	16	----	----
9	210	2.64	10.8	----	----
10	375	13.98	14	1.70	4.68
[1] 11	1,470	3.77	10.5	----	----
12	210	5.99	11.5	----	----
13	450	2.38	9	----	----
14	820	7.15	8.2	----	----
15	245	14.22	16	1.89	7.22

[1]For computational purposes subreach 11 was divided into two segments (300 and 1,170 ft long) because the tracer-sampling station was not located at the start of a segment.

Table 5. Hydraulic data for Black Earth Creek.

Subreach	Length (ft)	Area (ft^2)	Top width (ft)	Measured flow (ft^3/s)	Cumulative traveltime (h)
1	290	19.11	26.5	9.14	----
2	290	19.49	21	-----	----
3	330	25.17	20.3	-----	----
4	225	14.90	16	-----	----
5	105	21.39	17.2	-----	----
6	620	13.58	17.5	-----	----
7	150	11.05	15	-----	----
8	500	24.01	22.5	-----	----
9	195	28.5	29	-----	----
10	210	24.0	20	-----	----
11	480	16.65	15.5	-----	----
12	440	23.66	22.3	-----	----
13	1,350	22.02	28.2	-----	----
14	400	29.27	27.5	-----	----
15	635	12.45	22	-----	----
16	400	20.99	25.5	-----	----
17	100	9.39	15.5	[2]14.6	4.68
18	1,800	22.70	25	-----	----
19	275	12.55	24	17.9	----
[1]20	1,300	32.49	30.3	-----	----
21	1,140	20.14	24	[2]16.9	----
22	210	39.50	24	-----	----
23	1,305	29.80	31	-----	----
24	480	22.25	22	-----	----
25	270	28.85	17	-----	----
26	750	38.48	30	18.2	7.16

[1]For computational purposes subreach 20 was divided into two segments (100 and 1,200 ft long) because the tracer-sampling station was not located at the start of a segment.

[2]All flow measurements were made using a Gurley flow meter except for these measurements which were made using a Model 201 Marsh McBirney portable current meter.

PREDICTIVE EQUATION INPUT DATA

Average subreach velocities and depths were calculated from the channel geometry and discharge measurements made by the Wisconsin Department of Natural Resources.

The traveltime of each reach was calculated by adding the individual subreach traveltimes together. Subreach traveltimes were calculated by

Table 6. Hydraulic data for Halfway Creek.

Subreach	Length (ft)	Area (ft^2)	Top width (ft)	Measured flow (ft^3/s)	Cumulative traveltime (h)
1	540	15.95	21	8.53	----
2	700	8.35	13	----	----
3	630	8.10	18	----	----
4	660	6.55	13	----	----
5	600	7.25	17	----	----
6	1,620	6.62	13.9	----	----
7	960	7.20	12	----	----
8	855	7.32	14.5	----	----
9	210	8.25	20	----	----
10	630	7.95	17	----	----
11	1,650	8.50	16	----	----
12	2,130	7.45	10	----	----
13	690	6.50	17	6.87	3.18
14	5,010	6.91	23.5	----	----
15	1,440	6.27	15	----	----
16	600	6.08	13	6.83	5.15

dividing the subreach length by a cross section average subreach velocity. Table 7 compares these calculated traveltimes used in the predictive models with the traveltimes observed in the dye-tracer methods.

Reaeration coefficients (K_2) for each subreach were calculated using the various predictive equations. The total K_2 for the entire reach then was estimated by averaging these subreach K_2 values over the entire reach. A weighted average based on traveltime within each subreach was used:

$$K_2 \text{ reach} = \frac{\Sigma(K_2 \text{ subreach})(\text{TT subreach})}{\text{TT reach}}$$

where: TT = traveltime.

The predictive equations that did not require subdivision of the reach parameters by subreach used the following input parameters: average reach depth, average reach velocity, slope, Froude number, shear velocity, change in elevation, and specific discharge. Summaries of these model parameters are in tables 8-10.

Table 7. Comparison of calculated and observed traveltime data.

	Calculated traveltime (h)	Observed traveltime (h)
Bonner Branch		
Reach 1-2	4.55	4.15
Reach 2-3	2.58	3.08
Black Earth Creek		
Reach 1-2	4.40	4.68
Reach 2-3	2.58	2.48
Halfway Creek		
Reach 1-2	3.36	3.18
Reach 2-3	1.92	1.97

Table 8. Reach data for Bonner Branch.

	Reach 1-2	Reach 2-3
Length (ft)	2,385	2,895
Average velocity (ft/s)	.16	.27
Average depth (ft)	.92	.66
Slope (ft/ft)	.00285	.00232
Froude number	.029	.057
Shear velocity (ft/s)	.29	.22
Elevation change (ft)	6.80	6.72
Specific discharge $\{(ft^3/s)/mi^2\}$	.30	.31

Table 9. Reach data for Black Earth Creek.

	Reach 1-2	Reach 2-3
Length (ft)	8,895	5,355
Average velocity (ft/s)	.53	.60
Average depth (ft)	.92	1.10
Slope (ft/ft)	.00145	.00111
Froude number	.097	.101
Shear velocity (ft/s)	.208	.198
Elevation change (ft)	12.90	5.94
Specific discharge $\{(ft^3/s)/mi^2\}$	.44	.43

Table 10. Reach data for Halfway Creek.

	Reach 1-2	Reach 2-3
Length (ft)	11,875	7,050
Average velocity (ft/s)	1.04	.99
Average depth (ft)	.57	.33
Slope (ft/ft)	.00159	.00159
Froude number	.242	.304
Shear velocity (ft/s)	.17	.13
Elevation change (ft)	18.88	10.57
Specific discharge {$(ft^3/s)/mi^2$}	.25	.22

TRACER METHODS AND PREDICTIVE EQUATIONS

The methods used to determine or predict a stream's reaeration coefficient fall into two basic categories--tracer methods and mathematical equations. The tracer methods employ a gas that is injected into the stream and then monitored with a dye tracer as it moves downstream. Water samples are taken at specified sites to determine the change in the dissolved-gas concentration. This difference in concentration is then related to the reaeration coefficient. In some applications, the dye tracer is used as a dispersion indicator.

The mathematical equations generally relate the reaeration coefficient to the stream's depth and velocity or other physical factors. All equations presented have some theory behind their development.

RADIOACTIVE-TRACER METHOD

The radioactive-tracer technique used in this study to measure reaeration coefficients was developed by Tsivoglou (1967). The method employs two radioisotopes, krypton-85 (^{85}Kr), a tracer for oxygen, and tritium (^{3}H), the dispersion-dilution tracer. A third tracer, rhodamine-WT fluorescent dye, is used with the ^{85}Kr and ^{3}H for time-of-travel determination and also as a radioactive-tracer sampling indicator. The tritium is assumed to be conservative and the krypton is assumed to be lost from the stream only by desorption through the surface of the stream. The tracers are assumed to be fully mixed with the stream water when they arrive at the first sampling station. This sample established the baseline concentration for further samples.

The tracers are released by breaking a glass container under water to simulate an instantaneous slug injection. The tracer concentrations decrease downstream because of dispersion and dilution. Some ^{85}Kr gas escapes from the water surface into the atmosphere. Increased turbulence increases the rate at which ^{85}Kr escapes and the rate at which atmospheric oxygen can enter the water.

The ^{85}Kr concentration decreases faster than the ^{3}H concentration and the difference is the rate of desorption of the ^{85}Kr. Laboratory studies by Tsivoglou (1967) showed that the rate coefficient for ^{85}Kr escaping from water into the atmosphere is 83 percent of that for oxygen entering the water. Determination of the desorption coefficient of ^{85}Kr in a reach of stream is readily converted into the reaeration coefficient (K_2).

At sites downstream from the tracer-injection point, the stream water is sampled near the center of the flow. Presence of the dye tracers is detected by a fluorometer. Water samples taken during the dye peaks are used in laboratory determinations of ^{85}Kr and ^{3}H concentrations using a liquid scintillation counter.

The desorption coefficient for ^{85}Kr is computed using the following equation.

$$K_{kr} = \frac{1}{t_d - t_u} \log_e \frac{(C_{kr}/C_H)_u}{(C_{kr}/C_H)_d}$$

where: K_{kr} is the base e desorption coefficient of ^{85}Kr;

t is the time flow of the peak concentrations;

(C_{kr}/C_H) is the ratio of the concentration of ^{85}Kr and ^{3}H; and the

u and d subscripts indicate the upstream and downstream ends of the reach, respectively.

The reaeration coefficient, K_2 to the base e, is computed from

$$K_2 = \frac{K_{kr}}{0.83}$$

A more complete discussion of the theory, field, laboratory, and computation procedures has been published by Tsivoglou and others (1965); Tsivoglou and others (1968); Tsivoglou and Wallace (1972); and Tsivoglou (1967, 1974).

MODIFIED-TRACER METHODS

The modified techniques use a hydrocarbon gas tracer and rhodamine-WT dye as the dispersion-dilution tracer (Rathbun and Grant, 1978). In some studies, including this study, two tracer gases, ethylene and propane, are used simultaneously. Use of the two gases, which have different desorption characteristics, permits two independent determinations of the reaeration coefficient in one experiment with little additional work.

The measurement of a reaeration coefficient by the modified technique requires injecting the tracers into the stream by bubbling the gases through porous tube diffusers. Rhodamine-WT dye is injected simultaneously at a constant rate. Samples are obtained at various points downstream so that concentrations of the gas and dispersion-dilution tracers can be determined. Gas concentrations are determined by use of gas chromatography analysis. Details of the analytical method have been presented by Shultz and others (1976).

In this study K_2 was computed two ways for each tracer gas. The first was based on peak concentrations of tracers at sampling stations and the second on total weight of tracers passing each station. Thus, K_2 can be computed twice for ethylene and twice for propane.

The peak concentration method is as follows:

$$K_T = \frac{1}{t_d - t_u} \log_e \frac{\left(\frac{C_T}{C_D}\right)_u}{\left(\frac{C_T}{C_D}\right)_d}$$

where: K_T is the base e desorption coefficient for the tracer gas;

t is the time of travel of the peak concentrations;

(C_T and C_D) are the peak concentrations of the gas and rhodamine-WT dye respectively, and the

d and u subscripts indicate the downstream and upstream ends of the reach, respectively.

Because the rhodamine-WT dye is not a conservative tracer, it is necessary to measure the total quantity of dye passing each sampling station. The dye loss is then calculated so that the observed dye concentrations can be corrected for use in the above equation.

In the total-weight method it is not necessary to use the dye tracer data in the calculations. The dye serves only as a field-sampling indicator. The computation of K_T made using the total-weight method is as follows:

$$K_T = \frac{1}{t_d - t_u} \log_e \frac{A_u Q_u}{A_d Q_d}$$

where: A_u and A_d are areas under the gas concentration-versus-time curves at the upstream and downstream ends of the reach, respectively, and

Q_u and Q_d is stream discharge at each end of the reach.

The reaeration coefficient K_2 is computed

$$K_2 = K_T/R$$

where R is 0.87 for ethylene and 0.72 for propane.

PREDICTIVE EQUATIONS

During the past several years there has been a trend toward the increased use of mathematical models in water-quality planning. Basic to the application of these water-quality models is the assumption that dissolved oxygen is a good indicator of the pollution status of a stream. To model the dissolved oxygen of a stream, one must attempt to identify the various chemical, physical, and biological processes that affect dissolved-oxygen levels and to express these in terms of mathematical equations. The formula for atmospheric reaeration is usually expressed as:

$$\frac{dC}{dt} = K_2 \left(C_S - C\right)$$

where: K_2 = the reaeration-rate coefficient;

C = the dissolved-oxygen concentration at time t; and

C_S = the temperature dependent dissolved-oxygen saturation concentration.

The choice of an equation to predict K_2 is extremely important in the overall water-quality analysis and usually there is little basis for making such a choice. Most often K_2 values are computed for the low-flow condition because this is the most critical time in terms of dissolved-oxygen levels in the stream. The problem arises because of the large discrepancies among the various predictive equations at low flow. The major reason for this is each of the predictive equations was developed using a specific range of values for the hydraulic parameters. Any application of a predictive equation outside of the range of values for which it was developed can produce large errors in predicted K_2 values. With few exceptions predictive equations have been developed using data from streams much larger than the streams in this study.

The DNR is typically faced with the problem of applying water-quality models for very small streams receiving municipal waste. Thus, one of the goals of this study was to determine which, if any, of the equations can adequately predict K_2 values similar to those measured by the radioactive-tracer method for these small streams.

A list of the various predictive equations, both empirical and semi-empirical, which were considered in this study appears below. In all cases the reaeration-rate coefficient is expressed in base e units of days^{-1}. All are corrected to 25°C using the temperature correction equation:

$$K_{2,25} = K_{2,T}\ \theta^{(25 - T)}$$

where: $\theta = 1.024$, and

T = stream temperature, in degrees Celsius.

The following symbols are used in the equations listed:

F = Froude number = $V/\sqrt{gH}$

g = acceleration of gravity (ft/s^2)

H = average hydraulic depth (ft)

Δh = change in elevation between the start and end of the study reach (ft)

Q = average streamflow (ft^3/s)

q = specific discharge $\{(ft^3/s)/mi^2\}$ = streamflow divided by the total drainage area

R = hydraulic radius (ft)

s = slope of the energy gradient (ft/ft)

t = traveltime in the study reach (hours)

$u*$ = average shear velocity (ft/s) = $\sqrt{gRs}$

v = average stream velocity (ft/s)

coth = hyperbolic cotangent angle, in radians

1. Dobbins (1965)

$$K_2 = 131.28\ \frac{1 + F^2}{(0.9 + F)^{1.5}}\ \frac{(VS)^{0.375}}{H}\ \coth\left[\frac{4.10\ (VS)^{0.125}}{(0.9 + F)^{0.5}}\right]$$

2. O'Connor-Dobbins (1958)

$$K_2 = 14.42\ V^{0.5}\ H^{-1.5}$$

3. Krenkel-Orlob (1963)

$$K_2 = 264.\ (VS)^{0.408}\ H^{-0.66}$$

4. Cadwallader-McDonnell (1969)

$$K_2 = 379.2\ (VS)^{0.5}\ H^{-1}$$

5. Parkhurst-Pomeroy (1972)

$$K_2 = 54.48\ (1 + 0.17\ F^2)\ (VS)^{0.375}\ H^{-1}$$

6. Bennett-Rathbun I (1972)

$$K_2 = 119.52\ V^{0.413}\ S^{0.273}\ H^{-1.408}$$

7. Churchill and others I (1962)

$$K_2 = 0.03888\ V^{2.695}\ H^{-3.085}\ S^{-0.823}$$

8. Lau (1972)

$$K_2 = 2832.\ \left(\frac{u^*}{v}\right)^{3.0}\ VH^{-1}$$

9. Thackston-Krenkel (1969)

$$K_2 = 28.08\ (1 + F^{0.5})\ u^*\ H^{-1}$$

10. Langbein-Durum (1967)

$$K_2 = 8.57\ VH^{-1.33}$$

11. Owens and others I (1964)

$$K_2 = 26.16\ V^{0.73}\ H^{-1.75}$$

12. Owens and others II (1964)

$$K_2 = 24.48\ V^{0.67}\ H^{-1.85}$$

13. Churchill and others II (1962)

$$K_2 = 13.03\ V^{0.969}\ H^{-1.673}$$

14. Isaacs-Gaudy (1968)

$$K_2 = 9.70\ VH^{-1.5}$$

15. Negulescu-Rojanski (1969)

$$K_2 = 12.29\ (V/H)^{0.85}$$

16. Padden-Gloyna (1971)

$$K_2 = 7.73\ V^{0.703}\ H^{-1.054}$$

17. Bansal (1973)

$$K_2 = 5.26\ V^{0.6}\ H^{-1.40}$$

18. Bennett-Rathbun II (1972)

$$K_2 = 22.73\ V^{0.607}\ H^{-1.689}$$

19. Tsivoglou-Neal (1976)

$$K_2 = 0.124\ \frac{(\Delta h)}{t} \text{ for } 1 \leq Q \leq 10 \text{ ft}^3/\text{s}$$

20. Foree (written commun., 1977)

$$K_2 = (0.63 + 0.4S^{1.15})\ q^{0.25}$$

if $q > 1.0$, use $q = 1.0$

if $q < 0.05$, use $q = 0.05$

COMPARISON OF RESULTS

COMPARISON OF RADIOACTIVE-TRACER AND MODIFIED-TRACER METHODS

Table 11 presents the reaeration coefficients determined by the two tracer methods and a comparison of the percent difference in the modified methods. The modified method K_2 is presented for the peak-concentration technique and also for the total-weight technique, which will be referred to as the area technique in this report because total weight was determined measuring the areas under the time-concentration curves for each tracer. Figures 5-8 compare the four modified-tracer methods with the radioactive-tracer method.

Table 11. Reaeration coefficients determined using the radioactive-tracer and modified-tracer methods.

Stream	Reach	Reaeration coefficient, K_2 (base e), at 25°C, in $days^{-1}$				
			Propane		Ethylene	
		Radioactive	Area method	Peak method	Area method	Peak method
Bonner Branch	1-2	5.72	7.39	8.14	6.39	5.88
	2-3	9.20	9.09	8.98	10.8	9.45
Black Earth Creek	1-2	9.03	8.52	7.25	10.2	8.37
	2-3	6.49	7.49	7.86	12.7	11.8
Halfway Creek	1-2	16.2	15.3	13.1	14.3	12.2
	2-3	24.4	26.6	22.8	26.7	23.0

Stream	Reach	Percent difference versus radioactive method			
		Propane		Ethylene	
		Area technique	Peak technique	Area technique	Peak technique
Bonner Branch	1-2	29.2	42.3	11.7	2.80
	2-3	-1.20	-2.39	17.4	2.72
Black Earth Creek	1-2	-5.65	-19.7	13.0	-7.31
	2-3	15.4	21.1	95.7	81.8
Halfway Creek	1-2	-5.56	-19.1	-11.7	-24.7
	2-3	9.02	-6.56	9.43	-5.74
Algebraic mean		6.87	2.61	22.6	8.26
Absolute value mean		11.0	18.5	26.5	20.8
Standard deviation		10.1	14.0	34.0	31.0

COMPARISON OF RADIOACTIVE-TRACER METHOD AND PREDICTIVE EQUATIONS

Predicted reaeration-rate coefficients are presented in tables 12-14 for the three streams using the equations listed previously in this report. These tables indicate a wide range of values for all streams. For Bonner Branch predicted K_2 values ranged between 2,970 and 0.04 days^{-1}, for Black Earth Creek values ranged between 98.45 and 1.93 days^{-1}, and for Halfway

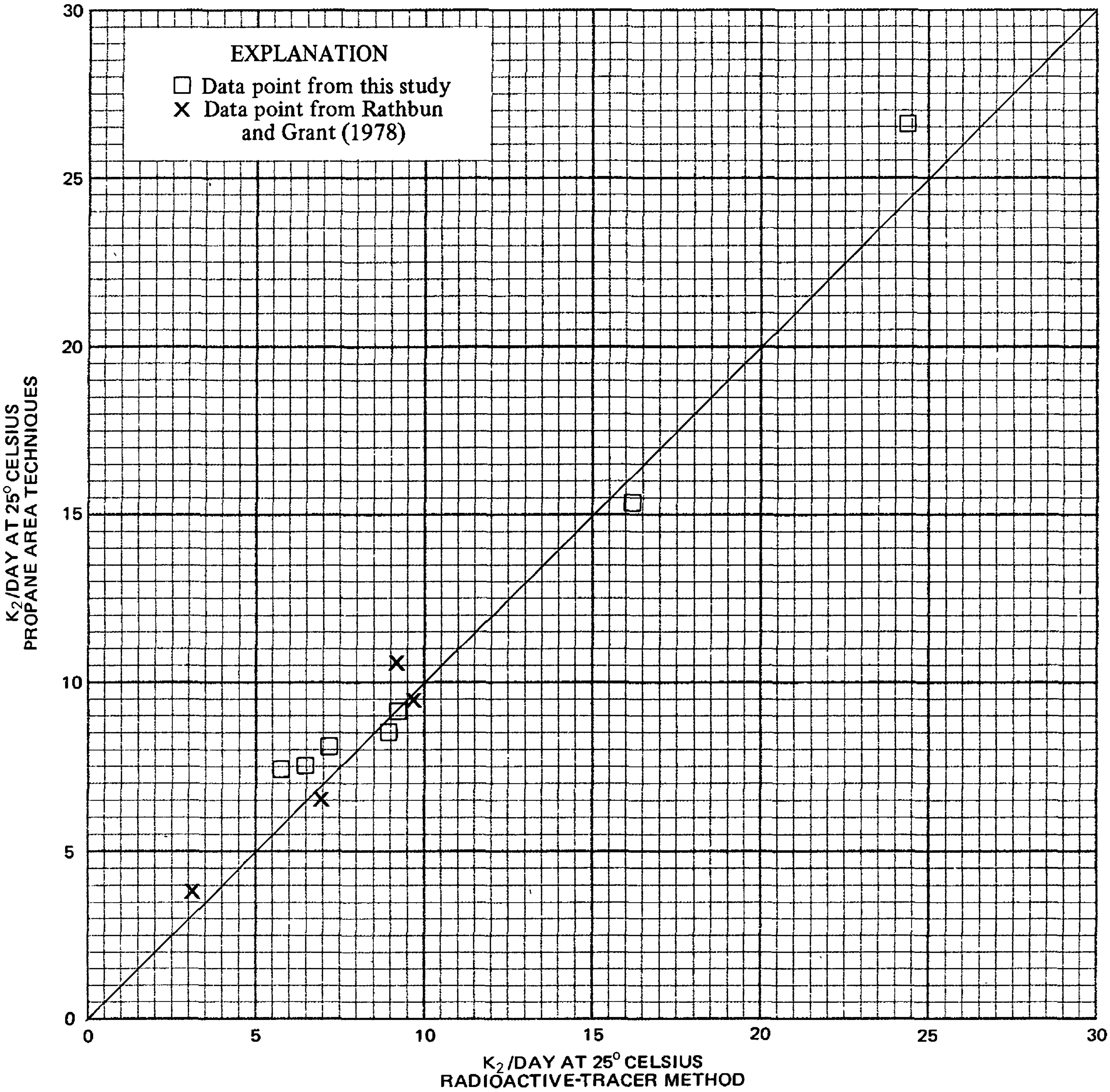

Figure 5. Comparison of radioactive-tracer and propane-area methods.

Creek values ranged between 244.22 and 3.39 $days^{-1}$. Eight equations overpredicted all six K_2 values but only one equation underpredicted all six K_2 values.

The percentage error for each predicted K_2 was calculated using:

$$\text{Percent error} = \frac{(K_2\ eq - K_2\ meas)}{K_2\ meas} \times 100$$

where: K_2 eq = equation K_2 value and

K_2 meas = measured K_2 value using radioactive-tracer method.

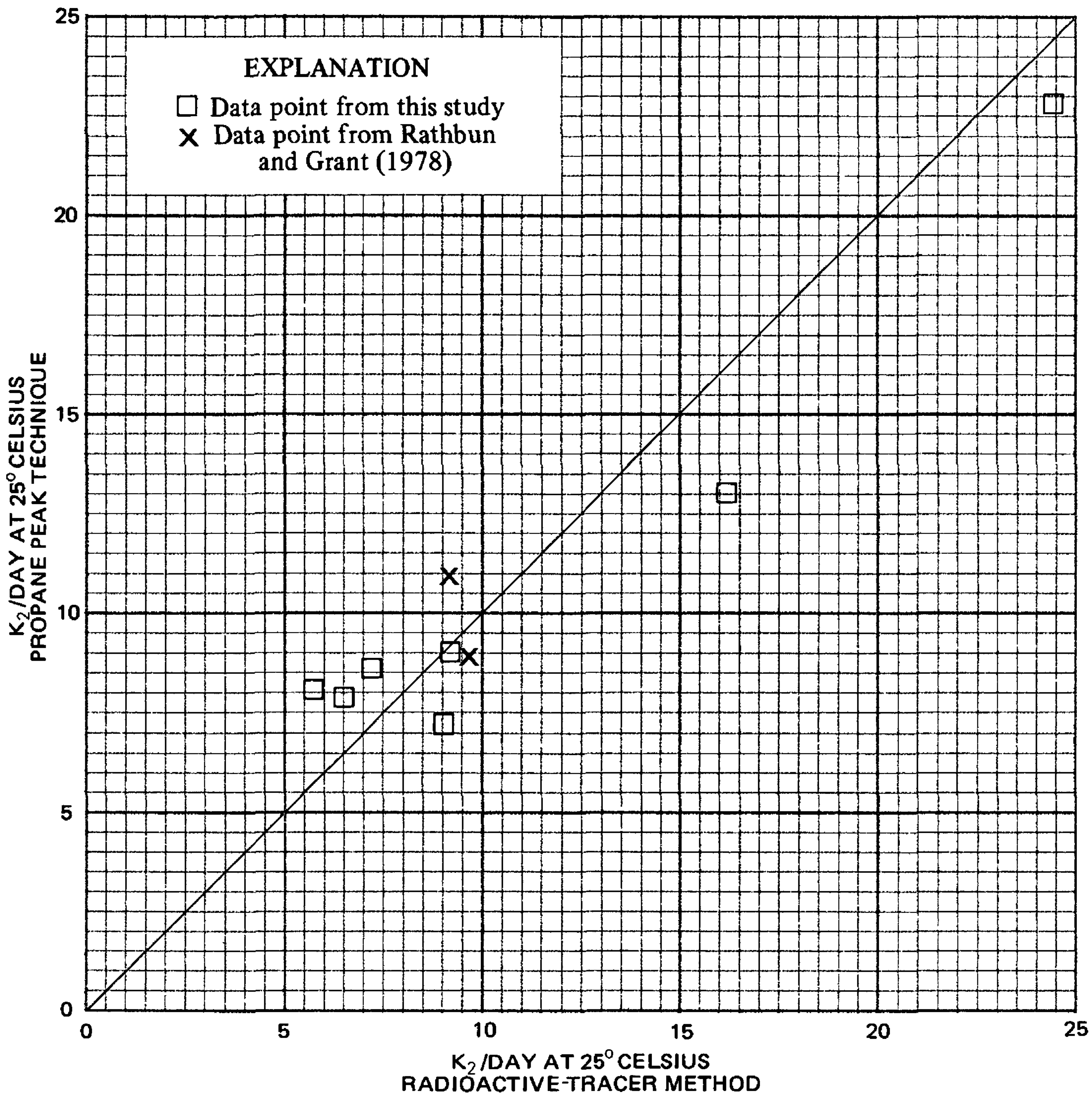

Figure 6. Comparison of radioactive-tracer and propane-peak methods.

Each of the predictive equations was then rated based upon the absolute value of the percentage errors averaged over the six measurements. Using this rating scheme, the five best predictive equations (in order of increasing average absolute value of percent error) are: Tsivoglou-Neal (17 percent), Negulescu-Rojanski (21 percent), Padden-Gloyna (23 percent), Thackston-Krenkel (29 percent), and Bansal (32 percent). For Bonner Branch the equation of Tsivoglou (20 percent) provided the best estimates of the

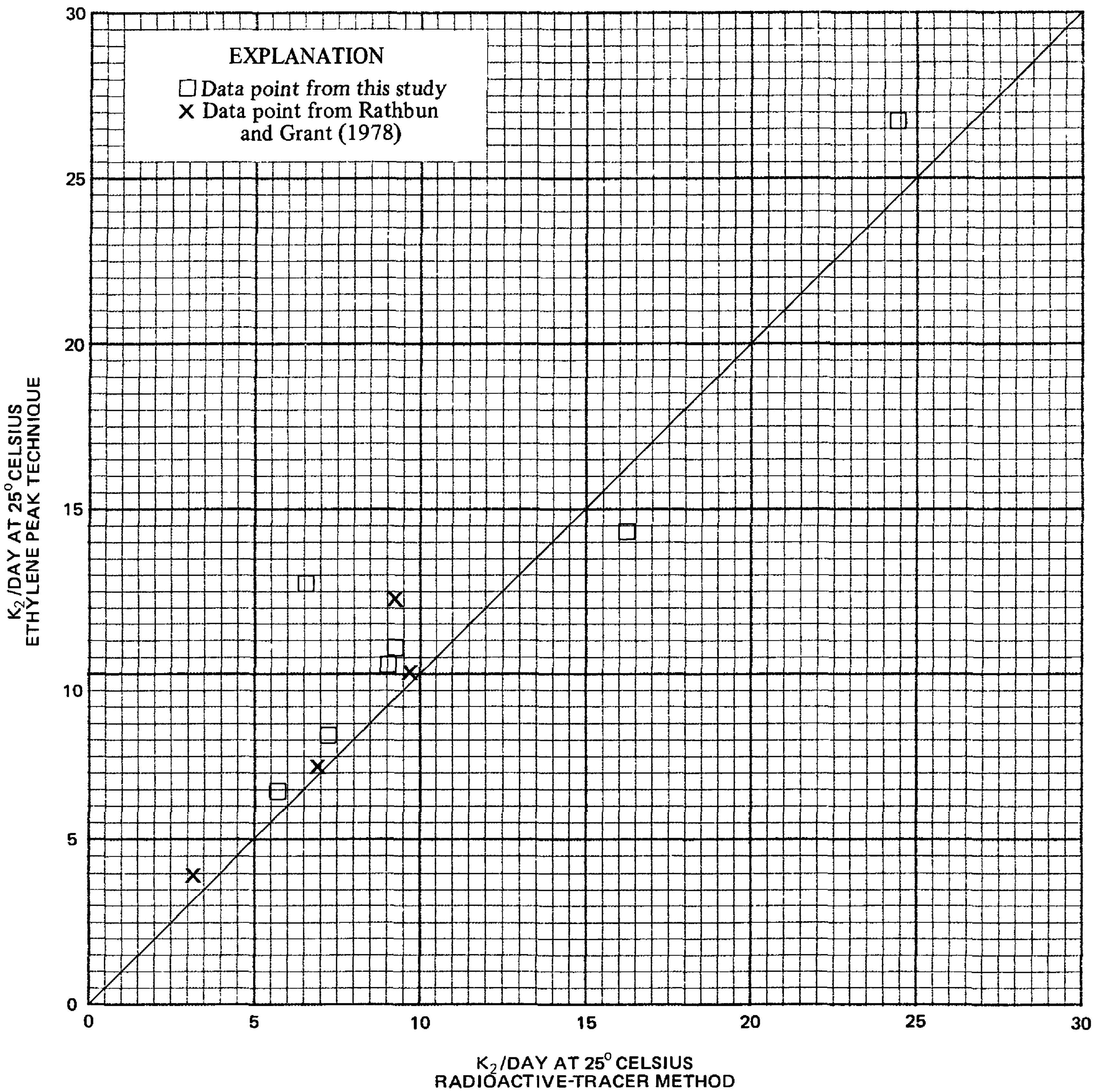

Figure 7. Comparison of radioactive-tracer and ethylene-area methods.

measured K_2 values, for Black Earth Creek the equation of Thackston-Krenkel (5 percent) provided the best estimates, and for Halfway Creek the equation of Padden-Gloyna (8 percent) provided the best estimates. The large variability associated with the Lau (1972) equation clearly indicates the dangers of using equations developed for larger streams on small Wisconsin streams. The average absolute value of the percentage error and the ranking for each equation are listed in table 15. Figures 9-12 show a graphic comparison of these predictive equations and the radioactive-tracer method.

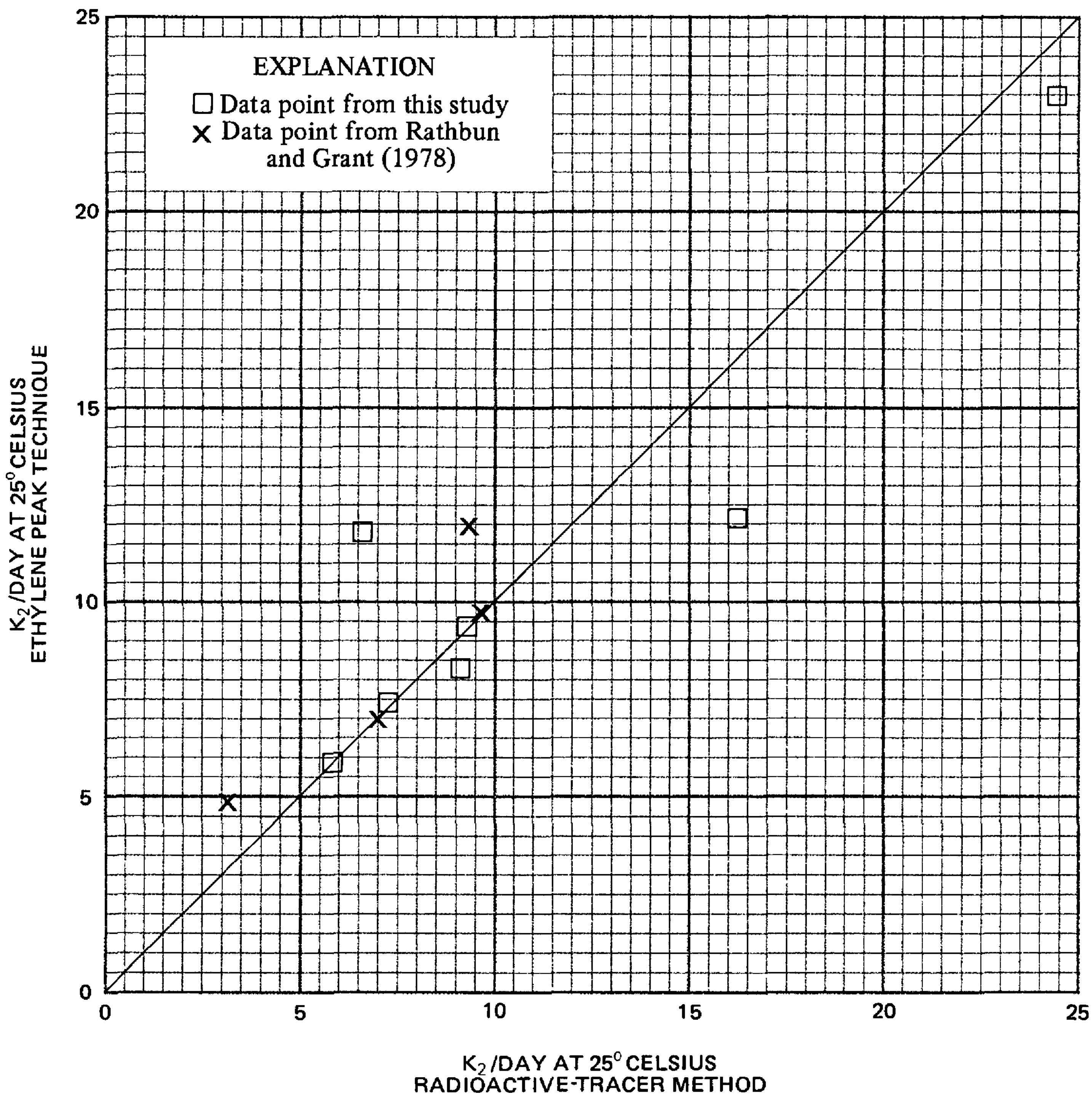

Figure 8. Comparison of radioactive-tracer and ethylene-peak methods.

Table 12. **Reaeration coefficients and percentage error for Bonner Branch** ***using predictive equations.***

Equation name	Reach 1		Reach 2	
	K_2 (25°C)	Percentage error	K_2 (25°C)	Percentage error
1. Dobbins (1965)	9.66	71	14.2	57
2. O'Connor-Dobbins (1958)	9.51	68	24.6	170
3. Krenkel-Orlob (1963)	12.0	113	17.0	86
4. Cadwallader-McDonnell (1969)	8.75	55	14.2	56
5. Parkhurst-Pomeroy (1972)	3.29	-42	5.13	-44
6. Bennett-Rathbun I (1972)	12.6	123	23.5	158
7. Churchill and others I (1962)	.04	-99	.57	-94
8. Lau (1972)	2,970	52,300	665	7,200
9. Thackston-Krenkel (1969)	10.4	83	11.7	28
10. Langbein-Durum (1967)	2.65	-53	8.30	-9
11. Owens and others I (1964)	15.3	170	47.0	416
12. Owens and others II (1964)	16.6	193	50.9	459
13. Churchill and others II (1962)	5.50	-3	18.1	99
14. Isaacs-Gaudy (1968)	3.43	-39	11.1	22
15. Negulescu-Rojanski (1969)	3.37	-40	8.69	-5
16. Padden-Gloyna (1971)	2.94	-48	7.43	-18
17. Bansal (1973)	2.84	-50	7.52	-17
18. Bennett-Rathbun II (1972)	14.2	151	40.4	344
19. Tsivoglou-Neal (1976)	4.87	-14	6.62	-27
20. Foree (1977)	7.18	27	5.79	-36

Table 13. Reaeration coefficients and percentage error for Black Earth Creek using predictive equations.

Equation name	Reach 1		Reach 2	
	K_2 (25°C)	Percentage error	K_2 (25°C)	Percentage error
1. Dobbins (1965)	10.5	18	8.38	31
2. O'Connor-Dobbins (1958)	13.4	50	10.3	60
3. Krenkel-Orlob (1963)	14.9	67	12.6	96
4. Cadwallader-McDonnell (1969)	11.4	27	8.92	39
5. Parkhurst-Pomeroy (1972)	4.01	-55	3.20	-50
6. Bennett-Rathbun I (1972)	17.3	93	13.3	107
7. Churchill and others I (1962)	1.93	-78	2.00	-69
8. Lau (1972)	98.4	1,000	55.7	767
9. Thackston-Krenkel (1969)	8.28	-7	6.68	4
10. Langbein-Durum (1967)	6.06	-32	4.75	-26
11. Owens and others I (1964)	22.9	156	16.6	158
12. Owens and others II (1964)	22.5	152	16.0	149
13. Churchill and others (1962)	10.1	13	7.34	14
14. Isaacs-Gaudy (1968)	7.11	-20	5.38	-16
15. Negulescu-Rojanski (1969)	8.53	-5	7.41	15
16. Padden-Gloyna (1971)	5.99	-33	5.00	-22
17. Bansal (1973)	4.57	-49	3.56	-45
18. Bennett-Rathbun II (1972)	20.6	131	15.3	139
19. Tsivoglou-Neal (1976)	8.20	-8	7.13	11
20. Foree (1977)	3.89	-56	2.99	-53

Table 14. Reaeration coefficients and percentage error for Halfway Creek using predictive equations.

	Equation name	Reach 1		Reach 2	
		K_2 (25°C)	Percentage error	K_2 (25°C)	Percentage error
1.	Dobbins (1965)	19.0	19	30.1	24
2.	O'Connor-Dobbins (1958)	36.5	128	80.3	232
3.	Krenkel-Orlob (1963)	28.0	75	38.5	59
4.	Cadwallader-McDonnell (1969)	27.0	69	44.2	83
5.	Parkhurst-Pomeroy (1972)	8.74	-45	14.6	-40
6.	Bennett-Rathbun I (1972)	46.1	188	95.8	296
7.	Churchill and others I (1962)	48.9	206	244	910
8.	Lau (1972)	23.0	44	17.5	-28
9.	Thackston-Krenkel (1969)	12.6	-22	16.6	-31
10.	Langbein-Durum (1967)	19.5	22	39.4	63
11.	Owens and others I (1964)	78.4	390	194	706
12.	Owens and others II (1964)	78.4	390	205	747
13.	Churchill and others (1962)	37.3	133	89	268
14.	Isaacs-Gaudy (1968)	24.7	54	54.2	124
15.	Negulescu-Rojanski (1969)	20.4	28	32.6	35
16.	Padden-Gloyna (1971)	14.6	-9	25.8	7
17.	Bansal (1973)	12.5	-22	26.1	8
18.	Bennett-Rathbun II (1972)	63.2	294	148	515
19.	Tsivoglou-Neal (1976)	17.7	10	16.0	-34
20.	Foree (1977)	3.70	-77	3.39	-86

CONCLUSIONS

The data collected in this and previous studies indicate that the propane-area method for measurement of stream-reaeration coefficients is more accurate and more consistent than any of the predictive equations evaluated in this investigation. The mean absolute difference, 11.0 percent, suggests that it may be as accurate as the radioactive-tracer method because experimental errors inherent in both methods may prevent determination of K_2 with zero error.

This study found that the maximum measured differences in K_2 were for ethylene determinations. The larger differences were 58, 82, and 96 percent. The largest difference in a propane measurement was 42 percent. There are insufficient data to compare the peak and area methods.

Of 20 predictive equations evaluated, the Tsivoglou-Neal equation produced the lowest mean absolute error (17.5 percent) in the 3-stream study. The Padden-Gloyna equation ranked third, but appears to be more consistent over the entire range of K_2's than the other equations. Data from this study and a previous one (Grant, 1976) for 11 small streams in Wisconsin show that the number 1 ranking Tsivoglou-Neal equation had a mean absolute error of 37 percent, much higher than that for the 3-stream study. Additional research is necessary to reduce equation errors to an acceptable level for use on small streams.

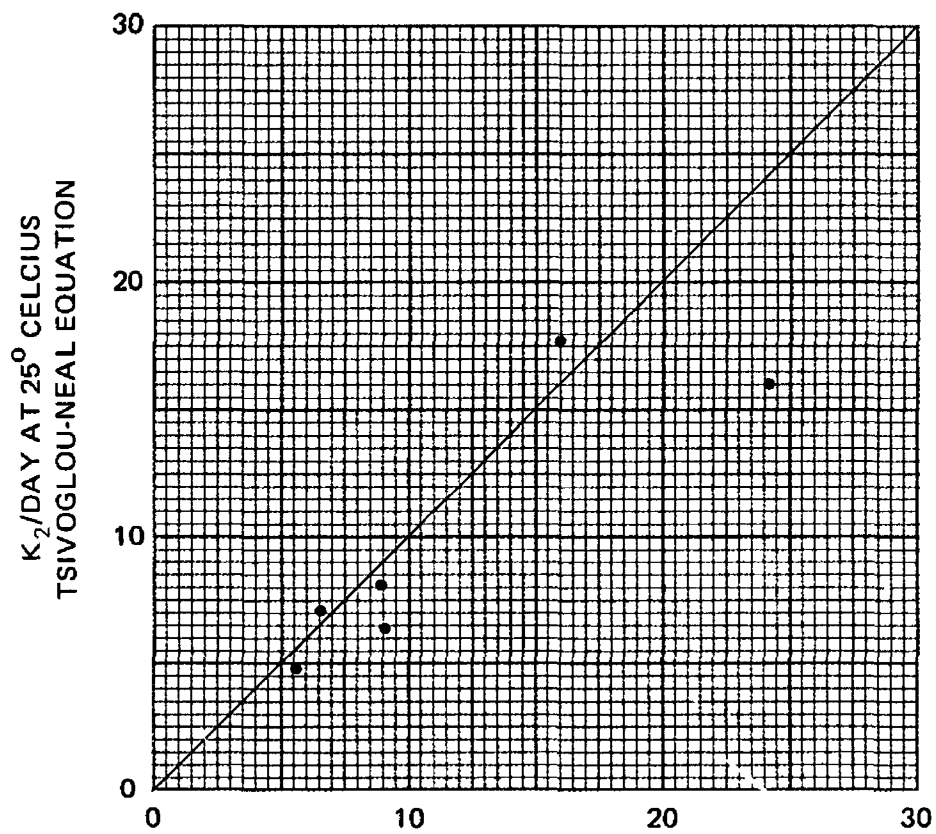

Figure 9. Comparison of Tsivoglou-Neal equation and radioactive-tracer method.

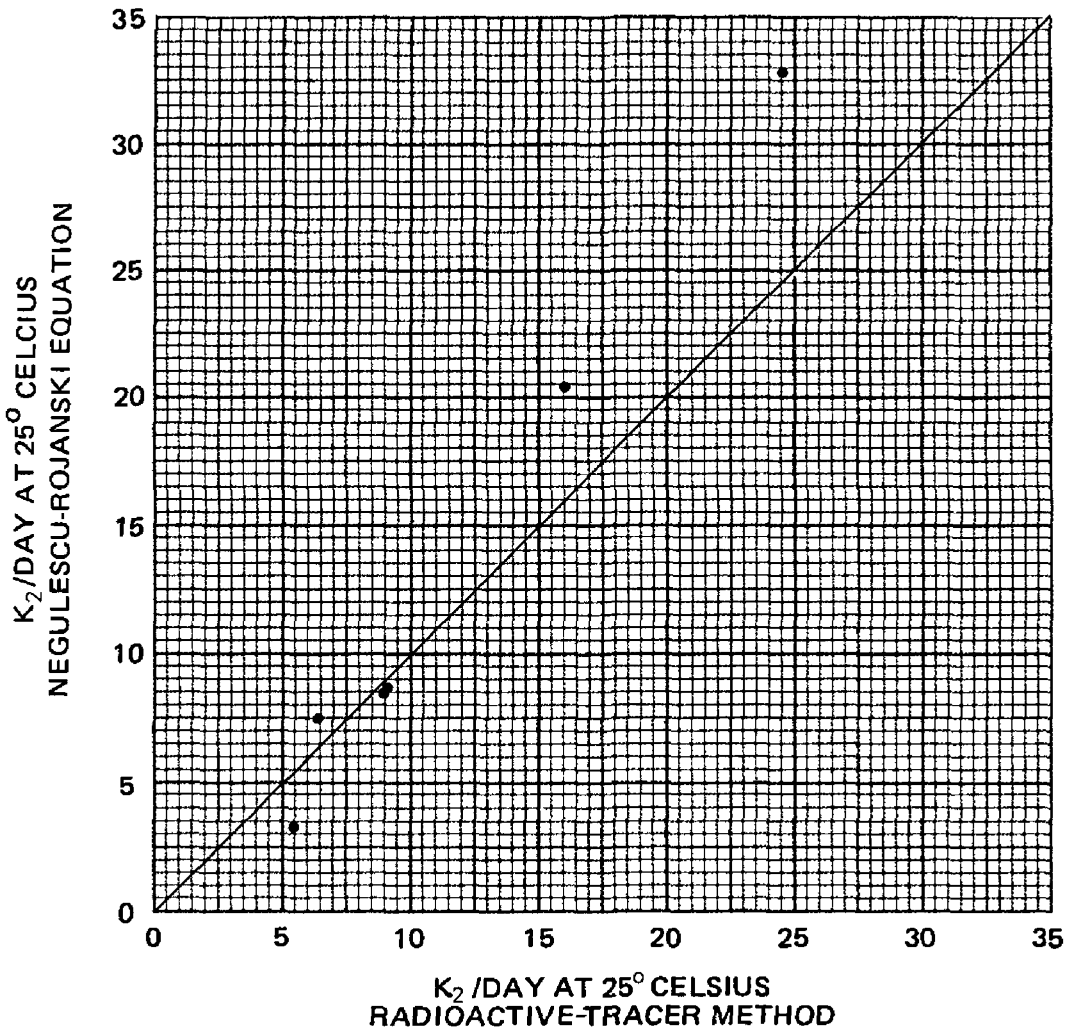

Figure 10. Comparison of Negulescu-Rojanski equation and radioactive-tracer method.

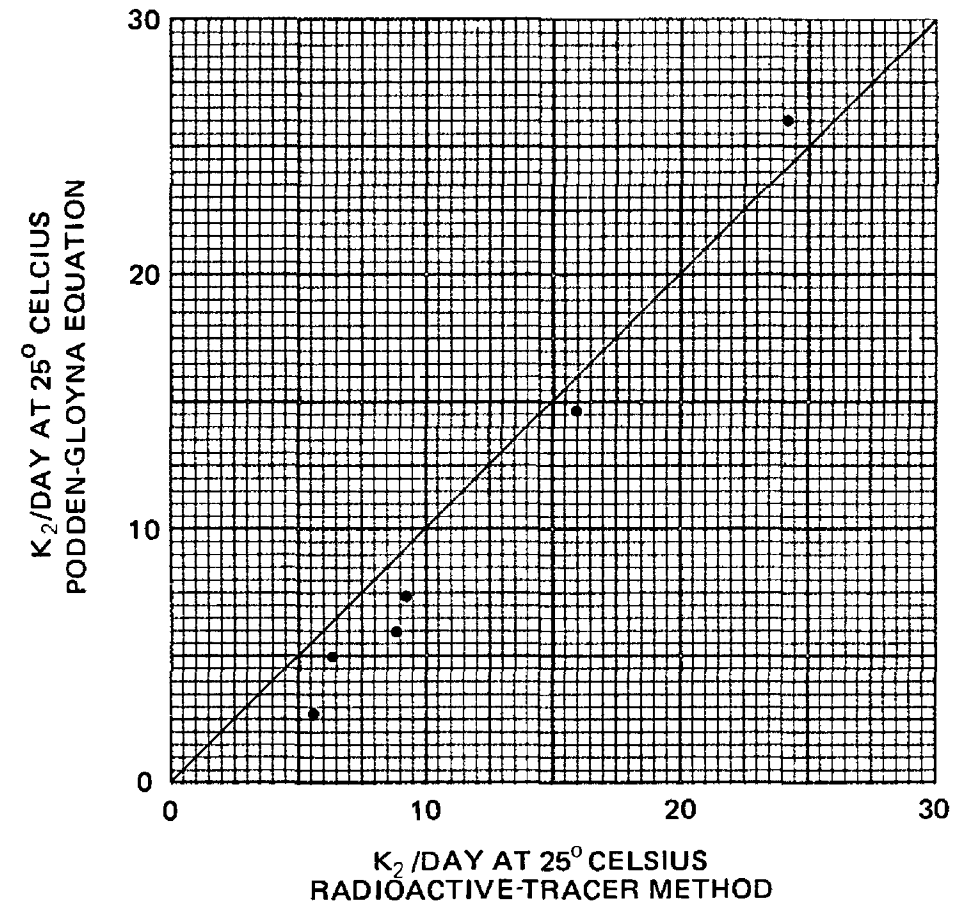

Figure 11. Comparison of Padden-Gloyna equation and radioactive-tracer method.

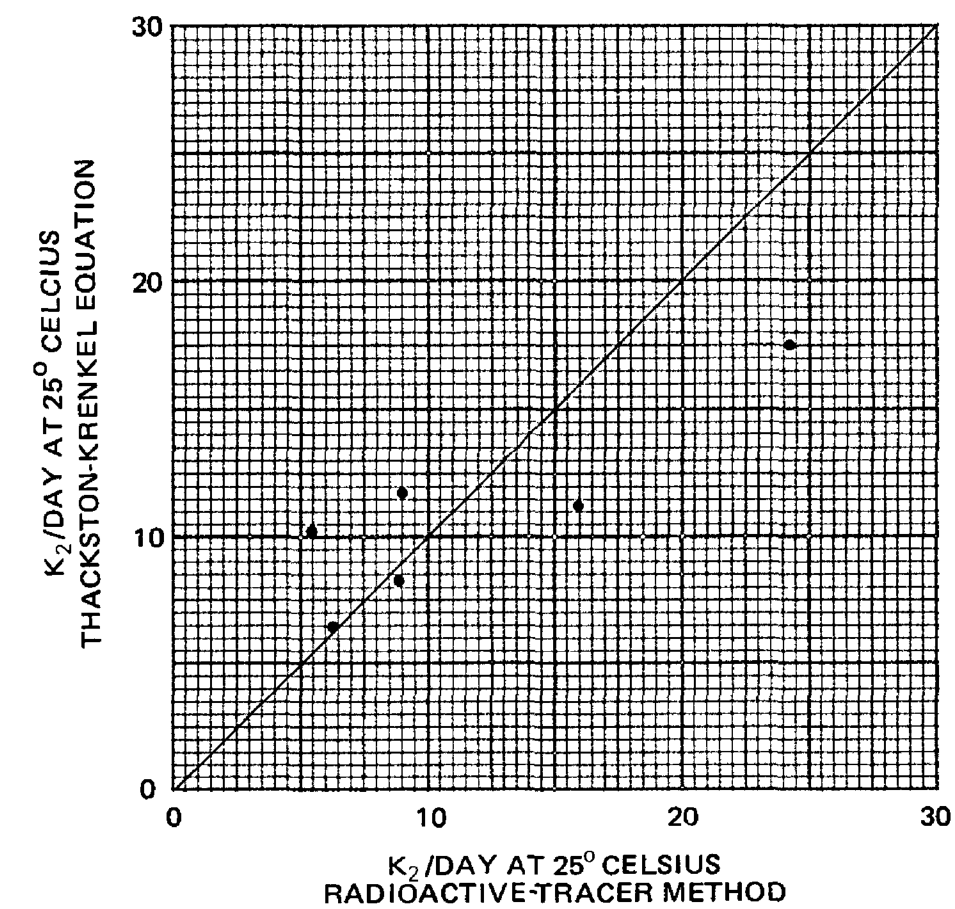

Figure 12. Comparison of Thackston-Krenkel equation and radioactive-tracer method.

Table 15. Average error in reaeration coefficients by predictive equations.

Equation name	Average absolute value of the percent error	Rank
Tsivoglou-Neal	17	1
Negulescu-Rojanski	21	2
Padden-Gloyna	23	3
Thackston-Krenkel	29	4
Bansal	32	5
Langbein-Durum	34	6
Dobbins	37	7
Isaacs-Gaudy	46	8
Parkhurst-Pomeroy	46	9
Cadwallader-McDonnell	55	10
Foree	56	11
Krenkel-Orlob	83	12
Churchill and others II	88	13
O'Connor-Dobbins	118	14
Bennett-Rathbun I	161	15
Churchill and others I	243	16
Bennett-Rathbun II	262	17
Owens and others I	333	18
Owens and others II	348	19
Lau	10,233	20

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STREAM-REAERATION COEFFICIENTS ON THREE SMALL STREAMS IN WISCONSIN