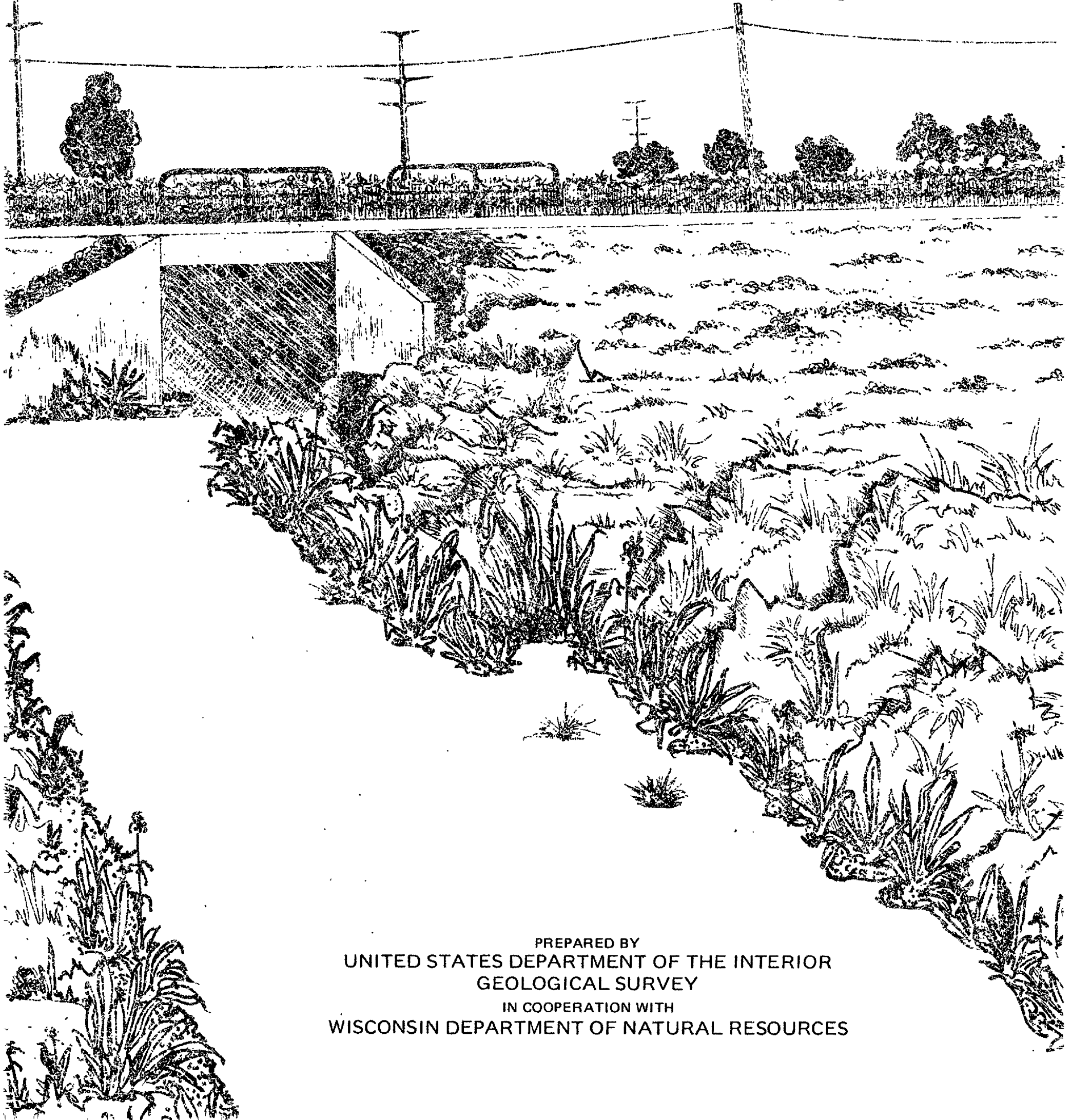


Comparison of the Propane-Area Tracer Method and Predictive Equations for Determination of Stream-Reaeration Coefficients on Two Small Streams in Wisconsin



PREPARED BY
UNITED STATES DEPARTMENT OF THE INTERIOR
GEOLOGICAL SURVEY
IN COOPERATION WITH
WISCONSIN DEPARTMENT OF NATURAL RESOURCES

REPORT DOCUMENTATION PAGE		1. REPORT NO.	2.	3. Recipient's Accession No.
4. Title and Subtitle COMPARISON OF THE PROPANE-AREA TRACER METHOD AND PREDICTIVE EQUATIONS FOR DETERMINATION OF STREAM-REAERATION COEFFICIENTS ON TWO SMALL STREAMS IN WISCONSIN			5. Report Date January 1981	
7. Author(s) Leo B. House and Steven Skavroneck			6.	
9. Performing Organization Name and Address U.S. Geological Survey, Water Resources Division 1815 University Avenue Madison, Wisconsin 53706			8. Performing Organization Rept. No. USGS/WRI-80-105	
12. Sponsoring Organization Name and Address U.S. Geological Survey, Water Resources Division 1815 University Avenue Madison, Wisconsin 53706			10. Project/Task/Work Unit No.	
			11. Contract(C) or Grant(G) No. (C) (G)	
15. Supplementary Notes Prepared in cooperation with the Wisconsin Department of Natural Resources			13. Type of Report & Period Covered Final	
			14.	
16. Abstract (Limit: 200 words) This study was made to identify the best predictive equations for a stream's reaeration-rate coefficient. Reaeration-rate information is needed in dissolved-oxygen modeling work, but an actual tracer measurement is not always possible. The propane-area gas-tracer method and predictive equations were compared for determination of stream-reaeration coefficients (K_2) for reaches of two small streams in Wisconsin. The study was made by the U.S. Geological Survey in cooperation with the Wisconsin Department of Natural Resources. The reaeration-rate coefficients actually measured by the propane-area tracer technique were 14.0 per day and 10.5 per day for two reaches of Honey Creek near Monroe, Wisconsin, with 6.98 per day and 0.98 per day measured at separate reaches on Mill Creek near Marshfield, Wisconsin. Of 20 predictive equations evaluated, the top five ranking equations were as follows: Tsivoglou-Neal with 34 percent mean error, Foree with 34.8 percent, Cadwallader with 45.5 percent, Isaacs-Gaudy with 45.8 percent, and Langbein-Durum with 49 percent.				
17. Document Analysis a. Descriptors Reaeration, Tracers, Measurement, Streamflow, Oxygenation, Dissolved oxygen, Water pollution b. Identifiers/Open-Ended Terms Reaeration coefficient, Dye tracing, Hydrocarbon gas tracers, Predictive equations, Wisconsin c. COSATI Field/Group				
18. Availability Statement No restriction on distribution		19. Security Class (This Report) UNCLASSIFIED		21. No. of Pages 23
		20. Security Class (This Page) UNCLASSIFIED		22. Price

Comparison of the Propane-Area Tracer Method and Predictive Equations for Determination of Stream-Reaeration Coefficients on Two Small Streams in Wisconsin

Leo B. House, *U.S. Geological Survey*

Steven Skavroneck, *Wisconsin Department of Natural Resources*

U.S. GEOLOGICAL SURVEY
Water-Resources Investigations 80-105

Prepared in cooperation with the
Wisconsin Department of Natural Resources



January 1981

UNITED STATES DEPARTMENT OF THE INTERIOR

CECIL D. ANDRUS, *SECRETARY*

GEOLOGICAL SURVEY

H. William Menard, *Director*

For additional information write to:

**U. S. Geological Survey
1815 University Avenue
Madison, Wisconsin 53706**

CONTENTS

	<u>Page</u>
Abstract-----	1
Introduction-----	1
Field-data collection-----	3
Description of streams studied-----	3
Honey Creek at Monroe, Wis.-----	3
Mill Creek at Marshfield, Wis.-----	5
Channel geometry surveys and predictive equation input-----	9
Tracer methods and predictive equations-----	10
Propane-area modified-tracer method-----	11
Predictive equations-----	11
Comparison of propane-tracer method and predictive equation results-----	14
Conclusions-----	17
Cited references-----	17

ILLUSTRATIONS

	<u>Page</u>
Figure 1. Map showing location of study sites in Wisconsin-----	2
2. Map showing location of sampling stations on Honey Creek near Monroe-----	4
3. Map showing location of sampling stations on Mill Creek near Marshfield-----	6

TABLES

	<u>Page</u>
Table 1. Hydraulic data for Honey Creek study reaches, Monroe, Wis., November 8, 1978-----	7
2. Hydraulic data for Mill Creek study reaches, Marshfield, Wis., November 15, 1978-----	8
3. Hydraulic geometry data for Honey Creek subreaches-----	9
4. Hydraulic geometry data for Mill Creek subreaches-----	10
5. Percentage error of predictive equations versus propane- tracer method, corrected to 25° Celsius-----	15
6. Absolute average percentage errors and predictive equation rank-----	16

CONVERSION FACTORS

<u>Multiply</u>	<u>By</u>	<u>To obtain</u>
inch (in.)	2.540	centimeter (cm)
foot (ft)	30.48	centimeter (cm)
foot (ft)	0.3048	meter (m)
foot per second (ft/s)	30.48	centimeter per second (cm/s)
mile (mi)	1.609	kilometer (km)
mile per hour (mi/h)	1.609	kilometer per hour (km/h)
cubic foot per second (ft ³ /s)	0.02832	cubic meter per second (m ³ /s)

Comparison of the Propane-Area Tracer Method and Predictive Equations for Determination of Stream-Reaeration Coefficients on Two Small Streams in Wisconsin

Leo B. House, *U.S. Geological Survey*

Steven Skavroneck, *Wisconsin Department of Natural Resources*

ABSTRACT

This study was made to identify the best predictive equations for a stream's reaeration-rate coefficient. Reaeration-rate information is needed in dissolved-oxygen modeling work, but an actual tracer measurement is not always possible.

The propane-area gas-tracer method and predictive equations were compared for determination of stream-reaeration coefficients (K_2) for reaches of two small streams in Wisconsin. The study was made by the U.S. Geological Survey in cooperation with the Wisconsin Department of Natural Resources.

The reaeration-rate coefficients actually measured by the propane-area tracer technique were 14.0 per day and 10.5 per day for two reaches of Honey Creek near Monroe, Wisconsin, with 6.98 per day and 0.98 per day measured at separate reaches on Mill Creek near Marshfield, Wisconsin.

Of 20 predictive equations evaluated, the top five ranking equations were as follows: Tsivoglou-Neal with 34 percent mean error, Foree with 34.8 percent, Cadwallader with 45.5 percent, Isaacs-Gaudy with 45.8 percent, and Langbein-Durum with 49 percent.

INTRODUCTION

Evaluation of stream-reaeration capacity is used to determine the self-purification capacity of a stream receiving oxygen-depleting wastes. Stream-reaeration coefficients can be measured by tracer techniques, such as radioactive tracer and modified tracer, or calculated by predictive equations. This study was made to aid in selecting an appropriate predictive equation for use in waste-load-allocation studies and for general dissolved-oxygen modeling of small streams in Wisconsin. The State has more than 100 streams that require such an allocation study periodically. This report is the third in a series intended to aid in predictive-equation selection. A summary report comparing all the data and results collected will be published later.

During the past several years use of mathematical models in water-quality planning has increased. In these models, a stream's dissolved oxygen is typically depleted by its biochemical oxygen demand (BOD) loads and by benthic or sediment oxygen demand of the channel substrate. The stream's dissolved oxygen is primarily replenished by atmospheric reaeration. The formula for atmospheric reaeration is usually expressed as:

$$\frac{dC}{dt} = K_2 (C_S - C)$$

where: K_2 = the reaeration-rate coefficient;

C = the dissolved-oxygen concentration at time t ; and

C_S = the temperature dependent dissolved-oxygen saturation concentration.

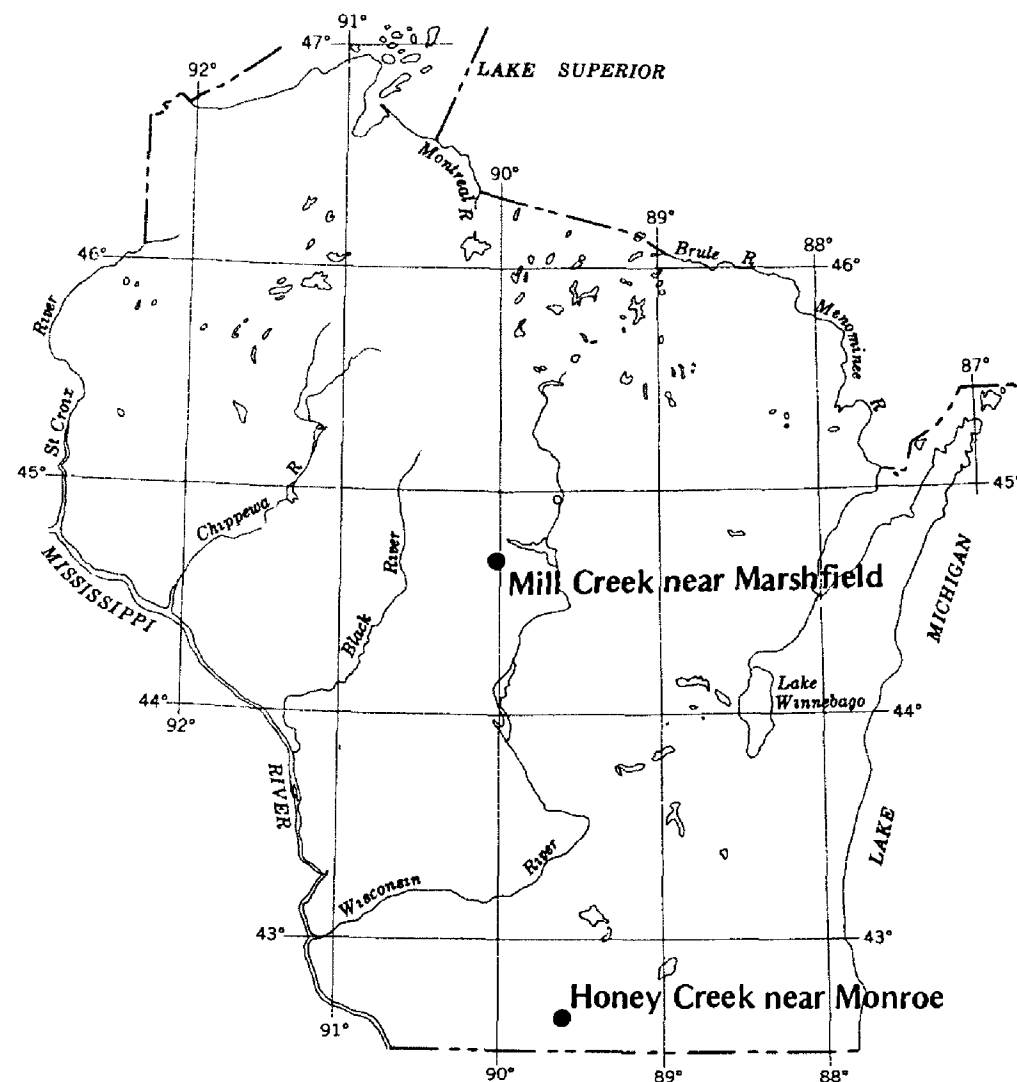


Figure 1. Location of study sites in Wisconsin.

This report specifically evaluates the accuracy of equations used to determine reaeration coefficients (K_2) of small streams at Monroe and Marshfield, Wis., with reaches just downstream from sewage-treatment-plant outfalls (fig. 1). This study, made by the U.S. Geological Survey in cooperation with the Wisconsin Department of Natural Resources (DNR), will be useful in establishing effluent limits that will maintain water-quality standards in receiving streams.

The propane-area tracer method and predictive equations were applied to identical stream reaches, and the predicted reaeration coefficients were compared with the measured values. It was assumed that the propane-tracer method provides a stream's true reaeration coefficient (Grant and Skavronek, 1980), and all predictive equations were compared with it.

FIELD-DATA COLLECTION

Two small streams at Monroe and Marshfield, Wis., were studied in November 1978. Tracer injections were made by the Survey by the propane-area modified-tracer technique to determine reaeration rates. For a thorough discussion of this technique, the reader is referred to the report by Rathbun and Grant (1978). Channel geometry was surveyed concurrently with the tracer studies by DNR to provide input data for the 20 predictive equations.

The slope of the energy gradient for each reach was calculated by dividing the difference in stream-surface elevation between the start and end of the reach (measured in the field) by the length of the reach (map distance). Contributing drainage areas were measured from 7.5-minute topographic maps.

DESCRIPTION OF STREAMS STUDIED

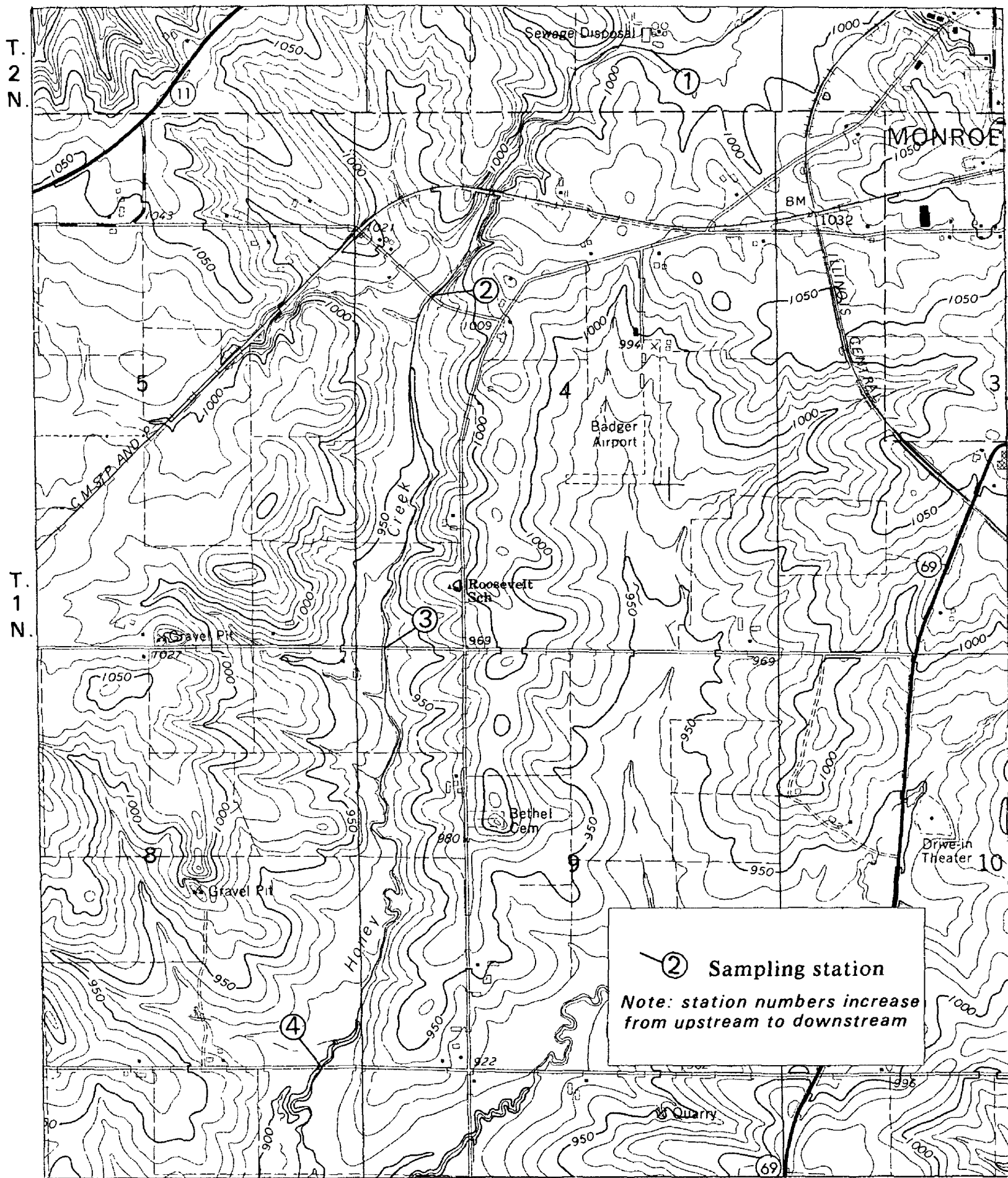
Honey Creek at Monroe, Wis.

Honey Creek flows generally southward through the west side of Monroe toward the Illinois border (fig. 2). The propane gas and dye tracer was injected at Site No. 1, about 150 ft downstream from the sewage-treatment-plant outfall.



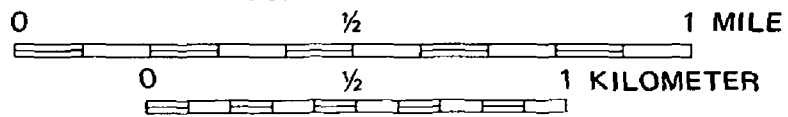
Honey Creek at Monroe, Wis.

R. 7 E.



Base from U S Geological Survey
Monroe, 1962

SCALE 1:24000



CONTOUR INTERVAL 10 FEET (3 METERS)
NATIONAL GEODETIC VERTICAL DATUM OF 1929

Figure 2. Location of sampling stations on Honey Creek near Monroe.

The channel is pool and riffle and is composed of soft to firm mud, gravel, and cobbles. The stream meanders through pastureland and the water was turbid. The stream was divided into two reaches for study purposes. The two consecutive reaches studied have a combined length of 3.15 mi. Hydraulic data for Honey Creek are given in table 1.

Mill Creek at Marshfield, Wis.

Mill Creek flows eastward through the south side of Marshfield to the Wisconsin River (fig. 3). The propane gas and dye tracer was injected at Site No. 1, about 20 ft downstream from the Marshfield sewage-treatment-plant outfall. The stream flows primarily through pastureland. The channel is fairly straight because it is an excavated ditch. However, actual traveltime at low flows is lengthy because of low velocities. The stream discharge was almost all sewage-



Mill Creek at Marshfield, Wis.

treatment-plant effluent, although the water was fairly clear. The stream was divided into two reaches for study purposes. The two consecutive reaches studied have a combined length of 3.4 mi. Hydraulic data for Mill Creek are given in table 2.

Table 1.--Hydraulic data for Honey Creek study reaches,
Monroe, Wis., November 8, 1978¹

Station	Distance downstream from outfall (mi)	Time of peak dye concentration	Water temperature (°C)	Stream discharge (ft ³ /s)	Average depth (ft)	Top width (ft)
1	0.03	----	12.0	4.40	0.25	17.2
2	.89	1235	12.0	5.44	.40	13.5
3	1.83	1600	11.0	6.36	1.25	5.1
4	3.18	1952	9.0	4.54	.38	12.0

Reach	Slope (ft/mi)	Traveltime (hours)	Velocity (ft/s)	Reaeration coefficient (K ₂)	
				Observed	² @25°C
2-3	30	3.42	0.40	14.0	19.0
3-4	20	3.87	.51	10.5	14.8
2-4	24	7.39	.45	12.0	16.5

	Reach 2-3	Reach 3-4
Length (ft)	4,670	6,551
Average velocity (ft/s)	.35	.52
Average depth (ft)	.93	.62
Slope (ft/ft)	.00568	.00379
Froude number	.0645	.1167
Shear velocity (ft/s)	.413	.274
Elevation change (ft)	26.53	24.83
Specific discharge {(ft ³ /s)/mi ² }	.98	.71

¹Slope and distance data taken from R. S. Grant (1976).

²Observed K₂ values were adjusted to 25°C for comparative purposes using equation 3 of this report.

Table 2.--Hydraulic data for Mill Creek study reaches,
Marshfield, Wis., November 15, 1978¹

Station	Distance downstream from outfall (mi)	Time of peak dye concentration	Water temperature (°C)	Stream discharge (ft ³ /s)	Average depth (ft)	Top width (ft)
1	0.01	----	16.0	5.46	0.65	8.4
2	1.30	1250	13.5	6.25	.46	13.7
3	2.34	1715	10.0	5.45	.60	9.0
4	3.41	2345	7.0	8.62	1.01	8.5

Reach	Slope (ft/mi)	Traveltime (hours)	Velocity (ft/s)	Reaeration coefficient (K ₂)	
				Observed	² @25°C
2-3	4.6	4.42	0.34	6.98	9.61
3-4	3.3	6.50	.24	.98	1.45
2-4	3.9	10.92	.28	3.41	4.75

	Reach 2-3	Reach 3-4
Length (ft)	5,595	5,866
Average velocity (ft/s)	.37	.19
Average depth (ft)	1.91	2.19
Slope (ft/ft)	.00087	.00063
Froude number	.0466	.0222
Shear velocity (ft/s)	.231	.211
Elevation change (ft)	4.87	3.70
Specific discharge {(ft ³ /s)/mi ² }	.73	.70

¹Slope and distance data taken from R. S. Grant (1976).

²Observed K₂ values were adjusted to 25°C for comparative purposes using equation 3 of this report.

CHANNEL GEOMETRY SURVEYS AND PREDICTIVE EQUATION RESULTS

Velocity and depth were determined by dividing each study reach into a series of subreaches, each with fairly consistent hydraulic characteristics. Average subreach velocities and depths were calculated from the channel geometry measured by DNR and discharge measurements made by the Survey. Actual reach traveltimes were measured by the Survey using dye-tracer techniques.

For each subreach, a characteristic stream-channel cross section was measured. From these measurements, average velocity and depth in a subreach can be calculated as follows:

$$H = A/W \text{ and}$$

$$V = Q/A$$

where: H is mean depth;
 A is mean cross-sectional area;
 W is mean width of water surface;
 V is mean velocity; and
 Q is mean reach discharge.

Subreach traveltimes were calculated by dividing the subreach length by the average subreach velocity. The traveltime of each reach was calculated by adding the individual subreach traveltimes together.

Tables 3 and 4 contain the hydraulic geometry data collected for the two streams studied.

Table 3.--Hydraulic geometry data for Honey Creek subreaches

Subreach	Length (ft)	Cross-section area (ft ²)	Top width (ft)	Measured flow (ft ³ /s)	Cumulative dye traveltime (hours)
1	923	13.34	14	5.44	----
2	775	16.54	25	----	----
3	920	10.51	14.5	----	----
4	778	30.22	23	----	----
5	467	11.06	13.5	----	----
6	807	10.36	18	6.36	3.42
7	3,438	10.36	18	----	----
8	3,113	7.55	11.3	4.54	7.39

Table 4.--Hydraulic geometry data for Mill Creek subreaches

Subreach	Length (ft)	Cross-section area (ft ²)	Top width (ft)	Measured flow (ft ³ /s)	Cumulative dye traveltime (hours)
1	161	40.00	20	6.25	----
2	2,287	16.00	8	----	----
3	736	34.00	17	----	----
4	439	18.00	9	----	----
5	1,972	9.08	5.8	----	----
6	1,047	11.06	13.5	5.45	4.42
7	509	10.61	7.5	----	----
8	1,627	23.79	20	----	----
9	1,429	60.00	20	----	----
10	1,254	16.09	7.5	8.62	10.92

TRACER METHODS AND PREDICTIVE EQUATIONS

The methods used in this study to determine or predict a stream's reaeration coefficient fall into two basic categories--tracer methods and mathematical equations. The tracer methods employ a gas that is injected into the stream and then monitored with a dye tracer as it moves downstream. Water samples are taken at specified sites to determine the change in the dissolved-gas concentration. This difference in concentration is then related to the reaeration coefficient. See Rathbun and Grant (1978) for further details.

The predictive equations generally relate the reaeration coefficient to the stream's depth and velocity or other physical factors.

Reaeration coefficients (K_2) for each subreach were calculated by the various predictive equations. The total K_2 for the entire reach then was estimated by averaging these subreach K_2 values over the entire reach. A weighted average based on calculated traveltime within each subreach was used:

$$K_2 \text{ reach} = \frac{\sum\{(K_2 \text{ subreach})(TT \text{ subreach})\}}{TT \text{ reach}}$$

where: TT = traveltime.

Some predictive equations did not require subdivision of the reach parameters and used one or more of the following input parameters: average reach depth, average reach velocity, slope, Froude number, shear velocity, change in elevation, and specific discharge.

PROPANE-AREA MODIFIED-TRACER METHOD

The modified-tracer technique used for this study utilizes propane as the tracer for oxygen. The gas is injected into the stream by bubbling it through porous stone diffusers. Fluorescent dye is ordinarily injected simultaneously at a constant rate. In the propane-area method it is not necessary to use the dye tracer for dispersion-dilution correction calculations. The dye serves only as a field-sampling indicator. However, when using this method, the stream is sampled to an endpoint of 10 percent of the peak dye concentration to assure a well-defined gas-concentration curve.

The computation of the desorption coefficient, K_T , is made by:

$$K_T = \frac{1}{t} \log_e \left(\frac{A_u Q_u}{A_d Q_d} \right) \quad (1)$$

where: K_T is the base e desorption coefficient for the tracer gas, in units per day,

t is the traveltime of the peak concentration between sampling sites, in days,

A_u and A_d are areas under the gas concentration versus time curves at the upstream and downstream sampling sites, respectively, determined by a planimeter, and

Q_u and Q_d is stream discharge at each end of the reach, in cubic feet per second.

The reaeration coefficient, K_2 (in units per day), is computed as:

$$K_2 = K_T / 0.72 \text{ for propane.} \quad (2)$$

The reaeration coefficient is adjusted to 25°C by the following equation:

$$K_{2_{25^{\circ}\text{C}}} = K_{2_T} (1.024)^{(25^{\circ}-T)} \quad (3)$$

where 'T' is the observed temperature, in degrees Celsius.

PREDICTIVE EQUATIONS

The predictive equations evaluated typically relate a stream's physical characteristics to the reaeration coefficient. With only two exceptions (Tsivoglou-Neal and Foree), all the 20 predictive equations to be tested require determinations of mean stream velocity and depth. Other variables which appear in these models (and the number of models in which they appear) are: the slope of the energy gradient (8), Froude number (3), shear velocity (2), traveltime (1), and mean specific discharge (1). The Froude number and the shear velocity are both functions of the other parameters. The specific discharge is a function of both streamflow and drainage area. Thus, there were eight independent parameters to be determined.

The following symbols are used in the equations listed:

$$F = \text{Froude number} = V/\sqrt{gH}$$

$$g = \text{acceleration of gravity (ft/s}^2\text{)}$$

$$H = \text{average hydraulic depth (ft)}$$

$$\Delta h = \text{change in elevation between the start and end of the study reach (ft)}$$

$$Q = \text{average streamflow (ft}^3\text{/s)}$$

$$q = \text{specific discharge \{ (ft}^3\text{/s)/mi}^2\text{ \} = streamflow divided by the total drainage area}$$

$$R = \text{hydraulic radius (ft)}$$

$$s = \text{slope of the energy gradient (ft/ft)}$$

$$t = \text{traveltime in the study reach (hours)}$$

$$u^* = \text{average shear velocity (ft/s)} = \sqrt{gRs}$$

$$v = \text{average stream velocity (ft/s)}$$

coth = hyperbolic cotangent angle, in radians

A list of the various predictive equations, both empirical and semiempirical, which were considered in this study appears below. In all cases the reaeration-rate coefficient is expressed in base e units of days⁻¹. For comparative purposes, all are corrected to 25°C using the temperature correction equation number 3.

1. Dobbins (1965)

$$K_2 = 131.28 \frac{1 + F^2}{(0.9 + F)^{1.5}} \frac{(VS)^{0.375}}{H} \coth \left[\frac{4.10 (VS)^{0.125}}{(0.9 + F)^{0.5}} \right]$$

2. O'Connor-Dobbins (1958)

$$K_2 = 14.42 v^{0.5} H^{-1.5}$$

3. Krenkel-Orlob (1963)

$$K_2 = 264. (VS)^{0.408} H^{-0.66}$$

4. Cadwallader-McDonnell (1969)

$$K_2 = 379.2 (VS)^{0.5} H^{-1}$$

5. Parkhurst-Pomeroy (1972)

$$K_2 = 54.48 (1 + 0.17 F^2) (VS)^{0.375} H^{-1}$$

6. Bennett-Rathbun I (1972)

$$K_2 = 119.52 V^{0.413} S^{0.273} H^{-1.408}$$

7. Churchill and others I (1962)

$$K_2 = 0.03888 V^{2.695} H^{-3.085} S^{-0.823}$$

8. Lau (1972)

$$K_2 = 2832. \left(\frac{u^*}{v} \right)^{3.0} VH^{-1}$$

9. Thackston-Krenkel (1969)

$$K_2 = 28.08 (1 + F^{0.5}) u^* H^{-1}$$

10. Langbein-Durum (1967)

$$K_2 = 8.57 VH^{-1.33}$$

11. Owens and others I (1964)

$$K_2 = 26.16 V^{0.73} H^{-1.75}$$

12. Owens and others II (1962)

$$K_2 = 24.48 V^{0.67} H^{-1.85}$$

13. Churchill and others II (1962)

$$K_2 = 13.03 V^{0.969} H^{-1.673}$$

14. Isaacs-Gaudy (1968)

$$K_2 = 9.70 VH^{-1.5}$$

15. Negulescu-Rojanski (1969)

$$K_2 = 12.29 (V/H)^{0.85}$$

16. Padden-Gloyna (1971)

$$K_2 = 7.73 v^{0.703} H^{-1.054}$$

17. Bansal (1973)

$$K_2 = 5.26 v^{0.6} H^{-1.40}$$

18. Bennett-Rathbun II (1972)

$$K_2 = 22.73 v^{0.607} H^{-1.689}$$

19. Tsivoglou-Neal (1976)

$$K_2 = 0.124 \frac{(\Delta h)}{t} \text{ for } 1 \leq Q \leq 10 \text{ ft}^3/\text{s}$$

20. Foree (written commun., 1977)

$$K_2 = (0.63 + 0.4s^{1.15}) q^{0.25}$$

if $q > 1.0$, use $q = 1.0$

if $q < 0.05$, use $q = 0.05$

COMPARISON OF PROPANE-TRACER METHOD AND PREDICTIVE EQUATION RESULTS

The results of an error analysis are shown in table 5 for the two streams using the equations listed previously in this report. These tables indicate a wide range of values for all streams.

The percentage error for each predicted K_2 was calculated using:

$$\text{Percent error} = \frac{(K_2 \text{ eq} - K_2 \text{ tracer})}{K_2 \text{ tracer}} \times 100 \quad (4)$$

where: $K_2 \text{ eq}$ = equation K_2 value at 25°C and

$K_2 \text{ tracer}$ = measured K_2 value using propane-tracer method corrected to 25°C.

Each of the predictive equations has been rated based upon the absolute value of the percentage errors averaged for each stream and for both streams together. These data are presented in table 6. Using this rating scheme, the five best predictive equations (in order of increasing average absolute value of percent error) are: Tsivoglou-Neal (34 percent), Foree (35 percent), Cadwallader-McDonnell (46 percent), Isaacs-Gaudy (46 percent), and Langbein-Durum (49 percent). For Honey Creek the equation of Foree (13 percent) provided the best estimates of the measured K_2 values, and for Mill Creek the equation of Tsivoglou-Neal (39 percent) provided the best estimates.

Table 5.--Percentage error of predictive equations versus propane-tracer method, corrected to 25° Celsius

Equation ¹	Honey Creek at Monroe		Mill Creek at Marshfield	
	Reach 2-3	Reach 3-4	Reach 2-3	Reach 3-4
1. Dobbins (1965)	-20	44	-58	81
2. O'Connor-Dobbins (1958)	-35	58	-66	168
3. Krenkel-Orlob (1963)	15	93	-33	171
4. Cadwallader-McDonnell (1969)	- 4	85	-63	30
5. Parkhurst-Pomeroy (1972)	-70	-42	-86	-42
6. Bennett-Rathbun I (1972)	- 3	152	-55	266
7. Churchill and others I (1962)	-58	22	-82	46
8. Lau (1972)	8,914	2,269	1,329	23,953
9. Thackston-Krenkel (1969)	-18	13	-57	114
10. Langbein-Durum (1967)	-76	-33	-86	- 1
11. Owens and others I (1964)	4	188	-57	265
12. Owens and others II (1964)	6	191	-60	257
13. Churchill and others II (1962)	-99	-80	-99	-99
14. Isaacs-Gaudy (1968)	-71	-18	-86	8
15. Negulescu-Rojanski (1969)	-66	-18	-69	88
16. Padden-Gloyna (1971)	-74	-39	-80	30
17. Bansal (1973)	-79	-48	-88	-10
18. Bennett-Rathbun II (1972)	10	166	-52	83
19. Tsivoglou-Neal (1976)	13	43	-65	-13
20. Foree (1977)	8	-18	-72	41

¹Note: A negative error indicates the equation underpredicted the measured K_2 value.

Table 6.--Absolute average percentage errors and predictive equation rank

Equation	Honey Creek at Monroe		Mill Creek at Marshfield		Both streams	
	$ \overline{E} ^1$	Rank	$ \overline{E} ^1$	Rank	$ \overline{E} ^1$	Rank
1. Dobbins (1965)	32	4	69.5	11	50.8	7
2. O'Connor-Dobbins (1958)	46.5	9	117	16	81.8	15
3. Krenkel-Orlob (1963)	54	10	102	15	78	14
4. Cadwallader-McDonnell (1969)	44.5	7/8	46.5	3	45.5	3
5. Parkhurst-Pomeroy (1972)	56	12	64	8/9	60	11
6. Bennett-Rathbun I (1972)	77.5	15	160.5	18	119	17
7. Churchill and others I (1962)	40	5	64	8/9	52	8
8. Lau (1972)	5,591	20	12,641	20	9,116	20
9. Thackston-Krenkel (1969)	15.5	2	85.5	13	50.5	6
10. Langbein-Durum (1967)	54.5	11	43.5	2	49	5
11. Owens and others I (1964)	96	18	161	19	128.5	18/19
12. Owens and others II (1964)	98.5	19	158.5	17	128.5	18/19
13. Churchill and others II (1962)	89.5	17	99	14	94.3	16
14. Isaacs-Gaudy (1968)	44.5	7/8	47	4	45.8	4
15. Negulescu-Rojanski (1969)	42	6	78.5	12	60.3	12
16. Padden-Gloyna (1971)	56.5	13	55	6	55.8	9
17. Bansal (1973)	63.5	14	49	5	56.3	10
18. Bennett-Rathbun II (1972)	88	16	67.5	10	77.8	13
19. Tsivoglou-Neal (1976)	28	3	39	1	33.5	1
20. Foree (1977)	13	1	56.5	7	34.8	2

¹ $|\overline{E}|$ symbolizes the absolute average percentage error.

The Foree equation relies on a specific discharge term that would not appear to have much meaning when most of the flow is treatment-plant effluent. However, this study was done at low-flow conditions when the natural flow was negligible at the study sites. It may be that the effluent flow is close to the natural average flow that shapes the channel.

CONCLUSIONS

The data collected in this and a previous study (Grant and Skavroneck, 1980) indicate that the Tsivoglou-Neal (1976) predictive equation for stream-re-aeration coefficients is the most accurate and most consistent of the 20 predictive equations evaluated so far. It produced the lowest mean absolute error in the study. However, the Foree (1977, written commun.) equation had less than half the mean absolute error (13 percent versus 28 percent) on Honey Creek.

CITED REFERENCES

- Bansal, M. K., 1973, Atmospheric reaeration in natural streams: Water Research, Pergamon Press, Oxford, England, v. 7, no. 5, p. 769-782.
- Bennett, J. P., and Rathbun, R. E., 1972, Reaeration in open-channel flow: U.S. Geological Survey Professional Paper 737, 75 p.
- Cadwallader, T. E., and McDonnell, A. J., 1969, A multivariate analysis of reaeration data: Water Research, Pergamon Press, Oxford, England, v. 3, p. 731-742.
- Churchill, M. A., Elmore, H. L., and Buckingham, R. A., 1962, The prediction of stream reaeration rates: Journal of the Sanitary Engineering Division, Proceedings of the American Society of Civil Engineers, v. 88, no. SA4, Proceedings Paper 3199.
- Dobbins, W. E., 1965, BOD and oxygen relationships in streams: Journal of the Sanitary Engineering Division, Proceedings of the American Society of Civil Engineers, v. 91, no. SA5, Proceedings Paper 4442, p. 49-55.
- Grant, R. S., 1976, Reaeration-coefficient measurements of 10 small streams in Wisconsin using radioactive tracers...with a section on the energy-dissipation model: U.S. Geological Survey Water-Resources Investigations 76-96, 50 p.
- Grant, R. S., and Skavroneck, Steven, 1980, Comparison of tracer methods and predictive equations for determination of stream-re-aeration coefficients on three small streams in Wisconsin: U.S. Geological Survey Water-Resources Investigations 80-19, 36 p.
- Krenkel, P. A., and Orlob, G. T., 1963, Turbulent diffusion and the reaeration coefficient: Transactions of the American Society of Civil Engineers, v. 128, part III, Paper No. 3491, p. 293-334.

- Isaacs, W. P., and Gaudy, A. F., 1968, Atmospheric oxygenation in a simulated stream: Journal of the Sanitary Engineering Division, Proceedings of the American Society of Civil Engineers, v. 94, no. SA2, Proceedings Paper 5905, p. 319-344.
- Langbein, W. B., and Durum, W. H., 1967, The aeration capacity of streams: U.S. Geological Survey Circular 542, 6 p.
- Lau, Y. L., 1972, Prediction equations for reaeration in open-channel flow: Journal of the Sanitary Engineering Division, Proceedings of the American Society of Civil Engineers, v. 98, no. SA6, Proceedings Paper 9398, p. 1,063-1,068.
- Negulescu, M., and Rojanski, V., 1969, Recent research to determine reaeration coefficient: Water Research, Pergamon Press, Oxford, England, v. 3, no. 3, p. 189-202.
- O'Conner, D. J., and Dobbins, W. E., 1958, Mechanism of reaeration in natural streams: Transactions of the American Society of Civil Engineers, v. 123, p. 641-684.
- Owens, M., Edwards, R. W., and Gibbs, J. W., 1964, Some reaeration studies in streams: International Journal of Air and Water Pollution, Oxford, England, v. 8, no. 819, p. 469-486.
- Padden, T. J., and Gloyna, E. F., 1971, Simulation of stream processes in a model river: University of Texas, Austin, Report No. EHE-70-23, CRWR-72, 130 p.
- Parkhurst, J. D., and Pomeroy, R. D., 1972, Oxygen absorption in streams: Journal of the Sanitary Engineering Division, Proceedings of the American Society of Civil Engineers, v. 98, no. SA1, Proceedings Paper 8701, p. 101-124.
- Rathbun, R. E., and Grant, R. S., 1978, Comparison of the radioactive and modified techniques for measurement of stream reaeration coefficients: U.S. Geological Survey Water-Resources Investigations 78-68, 57 p.
- Thackston, E. L., and Krenkel, P. A., 1969, Reaeration prediction in natural streams: Journal of the Sanitary Engineering Division, Proceedings of the American Society of Civil Engineers, v. 95, no. SA1, Proceedings Paper 6407, p. 65-94.
- Tsivoglou, E. C., and Neal, L. A., 1976, Tracer measurement of reaeration: III. Predicting the reaeration capacity of inland streams: Water Pollution Control Federation Journal, v. 48, no. 12, p. 2,669-2,688.

