

FLOW ROUTING IN THE SUSQUEHANNA RIVER BASIN:

PART V - FLOW-ROUTING MODELS FOR THE WEST

BRANCH SUSQUEHANNA RIVER BASIN, PENNSYLVANIA

By Stan A. Brua

U.S. GEOLOGICAL SURVEY

Water-Resources Investigations Report 82-4049

Prepared in cooperation with the
SUSQUEHANNA RIVER BASIN COMMISSION



Harrisburg, Pennsylvania
1984

UNITED STATES DEPARTMENT OF THE INTERIOR

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FACTORS FOR CONVERTING INCH-POUND UNITS TO INTERNATIONAL
SYSTEM OF UNITS (SI)

The following factors may be used to convert the inch-pound units published herein to the International System of units (SI).

<u>Multiply inch-pound units</u>	<u>By</u>	<u>To obtain SI units</u>
	<u>LENGTH</u>	
foot (ft)	0.3048	meter (m)
mile (mi)	1.609	kilometer (km)
	<u>AREA</u>	
square foot (ft ²)	0.09290	square meter (m ²)
square mile (mi ²)	2.590	square kilometer (km ²)
acre	4047 0.4047	square meter (m ²) square hectometer (hm ²)
	<u>VOLUME</u>	
cubic foot (ft ³)	28.32 0.02832	cubic decimeter (dm ³) cubic meter (m ³)
cubic foot per second-day (ft ³ /s-day)	2447	cubic meter per second-day (m ³ /s-day)
acre-foot (acre-ft)	1233	cubic meter (m ³)
	<u>FLOW</u>	
cubic foot per second (ft ³ /s)	28.32 0.02832	liter per second (L/s) cubic meter per second (m ³ /s)
	<u>VELOCITY</u>	
foot per second (ft/s)	0.3048	meter per second (m/s)

FLOW ROUTING IN THE SUSQUEHANNA RIVER BASIN:
PART V - FLOW-ROUTING MODELS FOR THE WEST BRANCH
SUSQUEHANNA RIVER BASIN, PENNSYLVANIA

By Stan A. Brua

ABSTRACT

Digital-computer, daily-flow routing models were developed for four consecutive reaches of the West Branch Susquehanna River between Curwensville and Lewisburg, Pennsylvania. These models will enable water-resources managers to evaluate efficiently the effect of present and future water-resources developments on streamflows at six locations along the West Branch Susquehanna River.

The models utilize a unit-response, convolution technique of flow-routing based on diffusion analogy and multilinearization. Estimates of streamflow from the intervening areas of each reach, wave celerity, and wave-dispersion coefficients are the model parameters. These were adjusted until simulated outflow hydrographs and flow statistics adequately approximated those for observed flow data.

The overall accuracy of the models is considered good. Average, absolute, daily flow errors between observed and simulated flows ranged from 8.82 to 13.70 percent for the periods that were evaluated. Volume errors for the same periods were between -0.01 and 1.61 percent. Estimates of the 7-day, 10-year low flows for simulated conditions were within 5 percent of those computed for observed flows.

INTRODUCTION

Agricultural, domestic, and industrial demands for water have greatly increased in recent years, placing a strain on the available water resources of many areas of the Nation. Since current trends indicate that demands for water will increase even more rapidly in the future, severe water shortages, which already exist in some areas, will become more commonplace if water resources are not further developed or better utilized. To assess the impact of future demands, an area's present water resources must be evaluated, thoroughly. An efficient and practical means of doing so is through the use of computer models.

In 1975, the U.S. Geological Survey and the Susquehanna River Basin Commission (SRBC) began a series of studies to develop computerized, flow-routing models for all major streams in the Susquehanna River basin. This study is the fifth of the series and covers most of the West Branch Susquehanna River, hereinafter referred to as the West Branch. Studies by Armbruster (1977) and (1979), Bingham (1979), and Zembrzuski (1980) provide models for the Chenango, Tioughioga, Tioga, Chemung, and Juniata Rivers and

for the main stem Susquehanna River from Unadilla, N.Y., to Conowingo, Md. Bingham (1979) also provides coverage for the lower 8 miles of the West Branch Susquehanna River downstream from Lewisburg, Pa. A study by Karplus and Dickey (1980) resulted in modified models for the Juniata and lower Susquehanna Rivers.

The purpose of this study is to develop flow-routing models for four consecutive reaches of the West Branch between Curwensville and Lewisburg, Pa. These models can be used to determine the effects of present and proposed water-resources development at various locations on the West Branch. Furthermore, the results can be applied to previously developed models by Armbruster (1977) as modified by Karplus and Dickey (1980) to extend the evaluation to locations on the Susquehanna River downstream from the West Branch.

Each model and summaries of their calibration and verification are described in this report. Model adequacy is assessed by comparing volume errors and average errors of daily flows of observed and simulated flow sequences and by comparing flow-frequency and flow-duration characteristics computed from simulated and observed flow data.

DESCRIPTION OF STUDY REACHES

The West Branch flows generally eastward across north-central Pennsylvania. At its confluence with the Susquehanna River at Sunbury, the West Branch is 237 mi long and has a drainage area of 6,981 mi² (fig. 1).

Just northeast of Lock Haven, the West Branch crosses the Allegheny Front, which forms the boundary between two of the State's five physiographic provinces. North and west of the Front is the Appalachian Plateau Province. Here, the West Branch has eroded a deep and sinuous path through the plateau, forming a relatively narrow and steep-sided valley. Downstream from the Front to a point about 10 mi downstream from Williamsport, the West Branch meanders through a wide, flat valley in the northernmost part of the Valley and Ridge Province, the ridges forming part of the Appalachian Mountains. It then turns southward across the province to its confluence with the Susquehanna at Sunbury, meandering slightly as it flows around several ridges. Flat flood plains have formed inside the meanders along this part of the river.

This study is concerned with that part of the West Branch between Curwensville and Lewisburg, a reach of 174 mi. Continuous-record gaging stations at five sites on the West Branch serve as end points for the four reaches for which flow-routing models were developed. These gaging stations are at Curwensville (01541200), Karthaus (01542500), Renovo (01545500), Williamsport (01551500), and Lewisburg (01553500) and provide the data needed to calibrate and evaluate the routing models.

The second largest of five reservoirs being studied, completed in 1965, is on the West Branch just upstream from the gaging station at Curwensville. The other four reservoirs are on tributary streams in the central part of the

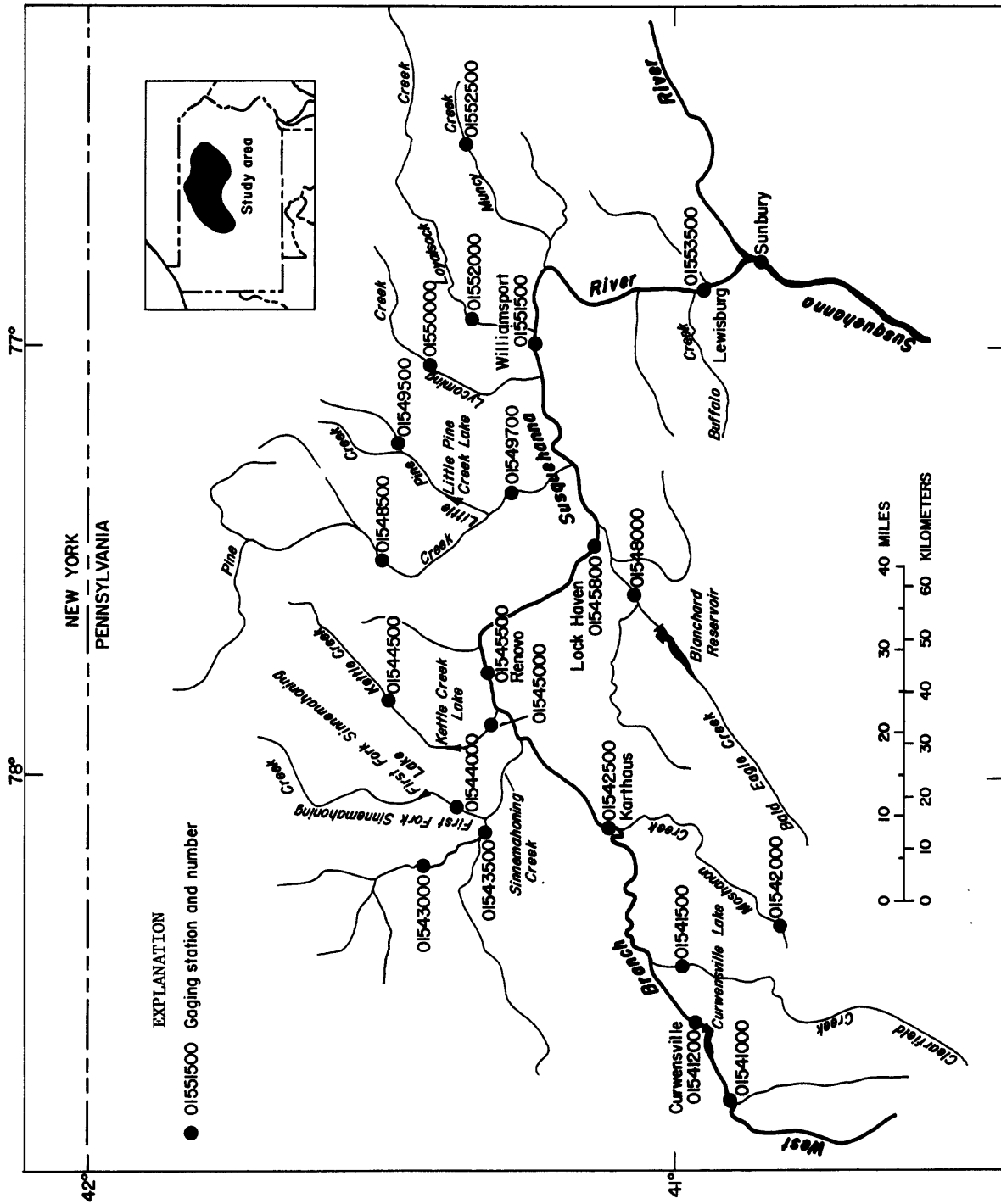


Figure 1.--Study area.

basin between Karthaus and Williamsport. These reservoirs are described in the next section.

The following is a description of each study reach.

Reach 1.--Curwensville (01541200) to Karthaus (01542500), length 50.8 mi. The reach begins 0.85 mi downstream from Curwensville Lake or 1.1 mi upstream from Curwensville and terminates at Karthaus. Drainage areas at Curwensville and Karthaus are 367 mi² and 1,462 mi², respectively. Clearfield Creek, the largest tributary to the reach, draining 393 mi², enters the West Branch 11 mi downstream from Curwensville. It is estimated from recorded stream flow data that Clearfield Creek accounts for one-half of the total flow at the confluence. Another major tributary, Moshannon Creek, drains 274 mi² and enters the reach 3 mi upstream from Karthaus.

Reach 2.--Karthaus (01542500) to Renovo (01545500), length 33.7 mi. Drainage area at Renovo is 2,975 mi². Major tributaries are Sinnemahoning Creek, draining 1,035 mi² and Kettle Creek, draining 246 mi². Sinnemahoning Creek, the largest West Branch tributary, enters near the middle of the reach and has been partly regulated by a small reservoir on First Fork Sinnemahoning Creek since 1956. Kettle Creek flows into the West Branch 6 mi upstream from Renovo and has also been regulated by a small reservoir 8 mi upstream from its mouth since 1962.

Reach 3.--Renovo (01545500) to Williamsport (01551500), length 57.6 mi. Because of the large flow contribution of tributaries entering near the middle of the reach and the abrupt change in valley geometry downstream from the Allegheny Front, Reach 3 was subdivided. A crest-stage, partial-record gaging station on the West Branch at Lock Haven (01545800) was selected as the breakpoint.

Reach 3a.-- refers to the 27.2-mi reach from Renovo to Lock Haven. The drainage area at Lock Haven is 3,345 mi². There are no major tributaries to the West Branch along this reach.

Reach 3b.-- refers to the remaining 30.4-mi reach from Lock Haven to Williamsport. Drainage area at Williamsport is 5,682 mi². Two large tributaries, Bald Eagle Creek, draining 770 mi², and Pine Creek, draining 986 mi², enter along the upper half of the subreach, 2 and 11 miles, respectively, downstream from Lock Haven. Flow from Bald Eagle Creek has been regulated since 1971 by Blanchard Reservoir, the largest reservoir in the study area. Bald Eagle Creek is, to a large extent, spring fed and, compared with other streams in the basin, contributes a disproportionately large flow during low-flow periods. Since 1950, flows from Pine Creek have been partly regulated by a small reservoir on Little Pine Creek. A third significant tributary, Lycoming Creek, has a drainage area of 272 mi² and flows into the West Branch just upstream from Williamsport.

Reach 4.--Williamsport (01551500) to Lewisburg (01553500), length 31.6 mi. Drainage area at Lewisburg is 6,847 mi². Loyalsock Creek, the largest tributary draining 494 mi², and Muncy Creek, draining 194 mi², enter the West Branch 3 and 11 mi, respectively, downstream from Williamsport. Another

tributary, Buffalo Creek, drains 134 mi² and enters just upstream from Lewisburg.

RESERVOIRS STUDIED

As part of its responsibility for managing the water resources of the Susquehanna River basin, the SRBC requires new consumptive water users in the basin to provide additional water in an amount equal to their consumptive use whenever the streamflow from which water is drawn falls below the 7-day, 10-year low-flow and the consumptive use. The five reservoirs listed in table 1 are considered to have sufficient storage to provide additional water during low-flow periods, including makeup for consumptive uses. The routing models developed for this study will, thus, enable the SRBC to evaluate the effectiveness of these reservoirs to meet water supply needs. Streamflow data for the first gaging stations downstream from the reservoirs were used as input for the models.

The five reservoirs being studied are authorized to be operated only for flood control, recreation, and water quality. Legislation may be required to reallocate the storage in the reservoirs to provide water supply.

DATA USED IN MODELING

Daily streamflow data for 20 continuous-record gaging stations, shown in figure 1, were used in the modeling. The stations, their period of record, and their drainage areas are listed in table 2.

Instantaneous streamflow data for the West Branch at Lock Haven (01545800) for the 1958-77 water years (unpublished data, NOAA River Forecast Center, Harrisburg, Pa.) also were used. These data, collected irregularly during the period, generally consisted of once- or twice-daily streamflow values. More frequent data were available for many high-flow periods. Daily streamflow hydrographs, constructed from some of these data, were used as a guide for developing the relation to route streamflow through reach 3a.

Additional streamflow data for five gaging stations used directly in the routing models were simulated by the relations given in table 3. These equations relate an appropriate period of streamflow at the stations requiring simulation to streamflow for one or more index stations.

The relationships shown in table 3 were developed by the following procedure:

1. Approximately 10 ranges of flows were selected for both index stations and stations to be simulated. The ranges were selected arbitrarily with more increments selected for low flows.
2. A linear equation was fit to each flow range, using least squares regression with the fitted line forced to pass through zero.
3. These regressions were used to select the best index station(s), based on significance of the regression.

Table 1.--Major reservoirs in the West Branch Susquehanna River basin

Reservoir	Date storage began	Drainage area (mi ²)	Normal pool elevation (above NGVD) (ft)	Surface area at normal pool (acres)	Volume at normal pool (acre ft)	Use
Foster Joseph Sayers	March 1971	339	630	1,730	28,800	Flood control, water quality, recreation.
Curwensville	November 1965	365	1,162	790	9,000	Flood control, recreation.
First Fork Sinnemahoning Creek	January 1956	243	920	142	2,000	do
Kettle Creek	February 1962	226	840	160	1,590	do
Little Pine Creek	October 1950	165	710	93.7	1,100	do

Table 2.--Data available for use in routing study

Station number	Station name	Water years of record ^{1/}	Drainage area (mi ²)
01541000	West Branch Susquehanna River at Bower	1914-77	315
01541200	West Branch Susquehanna River at Curwensville	1956-77	367
01541500	Clearfield Creek at Dimeling	1914-77	371
01542000	Moshannon Creek at Osceola Mills	1941-77	68.8
01542500	West Branch Susquehanna River at Karthaus	1941-77	1,462
01543000	Driftwood Branch Sinnemahoning Creek at Sterling Run	1914-77	272
01543500	Sinnemahoning Creek at Sinnemahoning	1939-77	685
01544000	First Fork Sinnemahoning Creek at Sinnemahoning	1954-77	245
01544500	Kettle Creek at Cross Fork	1941-77	136
01545000	Kettle Creek near Westport	1955-77	233
01545500	West Branch Susquehanna River at Renovo	1908-77	2,975
01548000	Bald Eagle Creek at Beech Creek Station	1911-77	559
01548500	Pine Creek at Cedar Run	1919-77	604
01549500	Blockhouse Creek near English Center	1941-77	37.7
01549700	Pine Creek below Little Pine Creek near Waterville	1958-77	944
01550000	Lycoming Creek near Trout Run	1915-77	173
01551500	West Branch Susquehanna River at Williamsport	1896-77	5,682
01552000	Loyalsock Creek at Loyalsockville	1926-74 1976-77	443
01552500	Muncy Creek near Sonestown	1941-77	23.8
01553500	West Branch Susquehanna River at Lewisburg	1940-77	6,847

^{1/} October 1 - September 30.

Table 3.--Relations used to simulate missing streamflow data

Station number	Period	Calibration errors, in percent		Simulation period	Relation
		Daily flows	Flow volume		
01541200	1956-64	8.94	-0.25	1942-55	$Q5412 = 0.8 \times Q5410 + 0.3 \times Q5410(\text{lag } 1)$ ^{2/}
01544000	1954-55	20.93	-2.32	1942-53	$Q5440 = \text{Fctn}(Q5430) + \text{Fctn}(Q5445)$ where, $\text{Fctn}(Q5430) = 0.9640 \times Q5430^{.970}$, for $Q5430 < 100 \text{ ft}^3/\text{s}$ $= 2.1200 \times Q5430^{.799}$, for $Q5430 > 100 \text{ ft}^3/\text{s}$ and $\text{Fctn}(Q5445) = 0.0300 \times Q5445^{1.397}$, for $Q5445 < 20 \text{ ft}^3/\text{s}$ $= 0.0055 \times Q5445^{1.970}$, for $20 > Q5445 > 150 \text{ ft}^3/\text{s}$ $= 0.2470 \times Q5445^{1.208}$, for $Q5445 > 150 \text{ ft}^3/\text{s}$
01545000	1955-61	12.24	-1.53	1942-54	$Q5450 = 1.35 \times Q5445^{1.054}$, for $Q5445 < 37 \text{ ft}^3/\text{s}$ $= 1.67 \times Q5445^{0.996}$, for $Q5445 > 37 \text{ ft}^3/\text{s}$
01549700	1958-77 ^{3/}	15.42	1.45	1942-57 ^{4/}	$Q5497 = 1.295 \times Q5485^{1.036}$, for $Q5485 < 2900 \text{ ft}^3/\text{s}$ $= 1.726 \times Q5485$, for $Q5485 > 2900 \text{ ft}^3/\text{s}$
01552000	1942-74	22.04	.02	1975	$Q5520 = 1.78 \times Q5500 + 5.30 \times Q5525$

1/ The terms used in the relations are daily flows for the applicable gaging stations. The four digits following the "0"'s are the middle four numbers of the station numbers for these gaging stations.
 For example: Q5412 is the daily flow for station 01541200.

2/ Daily flows for station 01541000 are lagged 1 day.

3/ Regulated flows during these water years.

4/ Natural and regulated flows during these water years.

NOTE.--Four significant figures in some relations were used in order to minimize the difference between computed daily flows at the break points.

4. The average coefficient for each index station was then adjusted by trial and error in order to balance the number of positive and negative errors between observed and simulated flows.

This procedure yielded suitable linear relations for West Branch at Curwensville (01541200) and Loyalsock Creek at Loyalsockville (01552000). Similar relations could not be developed for the other three stations and efforts were redirected to developing exponential relations for First Fork Sinnema-honing Creek at Sinnemahoning (01544000), Kettle Creek near Westport (01545000), and Pine Creek near Waterville (01549700). The index station(s) that produced the most significant linear regression were used to develop the exponential equation(s) by the following procedure:

1. The coefficients from the regressions for each index station were plotted against the discharge corresponding to the midpoint of each range of the index-station flow.
2. A least squares curve-fitting program was used to fit an exponential function through these points.
3. These exponential equations were then substituted for the simple coefficient in the linear relation between the flows at the index station(s) and the flows at the station to be simulated.
4. The resulting equation(s) were used to compute simulated flows which then were compared to observed flows. Coefficients were adjusted by trial and error to balance the number of positive and negative flow errors.

The procedure described above was used instead of a straight forward regression because regression analyses of daily streamflow data provide mathematically sound relationships which frequently are hydrologically inadequate. A hydrologically acceptable relationship may sacrifice some reduction in flow volume error or average daily flow errors to achieve a satisfactory balance between the number and distribution of positive and negative daily flow errors. It is often impossible to obtain a hydrologically acceptable relationship when giving equal weight to each and every daily flow value as is the case in linear regression analyses. Therefore, the regression analyses were used only to determine the most significant index stations and provide the first approximation of the coefficients to be used in the final relations.

The final daily flow and flow volume errors are included in table 3. The equations used to compute these errors are given in the section, "Model Calibration."

Four of the five stations requiring simulation of streamflow data are stations used to represent reservoir outflows. For 01541200, 01544000, and 01545000, streamflow data were missing only for pre-reservoir or natural conditions. Natural conditions also existed during the 2-year period of missing record for 01552000. However, at least 2 years of concurrent

observed natural flow data were available for developing relations for these stations. The above techniques could not be used to estimate the missing natural flow data for the 1942-50 water years for Pine Creek near Waterville because there are no observed unregulated flow data. Since 1950, streamflows at this site have been regulated by a reservoir on Little Pine Creek, a tributary which joins Pine Creek 4 miles upstream from the gaging station. A reservoir-regulation model for Little Pine Creek Reservoir is needed before valid estimates of the missing regulated flow data for the 1950-57 water years at 01549700 can be made. Such a model probably could also be used indirectly to develop an adequate relation for simulating the missing unregulated flow data. The relation given in table 3 for simulating streamflow at 01549700 is based on a comparison of observed, regulated flows at 01549700 to unregulated flows at two other gaging stations in the Pine Creek basin for the 1958-77 water years. Flow data generated by this relation were used only as input to the verified model for reach 3 in an effort to provide simulated streamflow data on the West Branch at Williamsport and Lewisburg for the 1942-57 water years. The consequences of using these simulated Pine Creek data in the routing models is discussed in the section, "Application of Models."

Except for station 01549700, streamflows generated by the relations in table 3 will hereafter be considered as observed flow data. Simulated data, as subsequently discussed, will refer to data generated by the flow-routing models or that produced by the relation for 01549700.

DESCRIPTION OF FLOW-ROUTING MODELS

Daily flow-routing models were developed for each of the four study reaches by the U.S. Geological Survey computer program J351 (Shearman and others, 1979). The program applies unit-response, convolution techniques to route flows through each reach. The unit-response functions determined by the program, are computed by the diffusion-analogy method discussed by Keefer and McQuivey (1974). The two parameters used to determine the response functions are wave celerity and wave dispersion. Wave celerity accounts for the travel time of streamflow through the reach. Wave dispersion accounts for the attenuation or damping of the wave by the effects of channel storage. The multiple linearization option offered by the program was used to account for the variation of celerity and dispersion with discharge.

The routing process is illustrated in figure 2. First, inflow to the reach is separated into several flow segments. A single linear response function is then applied to each segment to route it through the reach. The routed segments are then summed to obtain the outflow from the reach.

Estimates of the intervening streamflow to each reach were added before, and (or) after routing, depending on the configuration of tributary streams to the reach. Streamflow at gaging stations downstream from the reservoirs were included as a separate part of the intervening flow.

Losses and gains in streamflow to and from bank storage were not treated as a separate modeling component in any of the study reaches. However, they are implicitly accounted for since total streamflow is accounted for in the

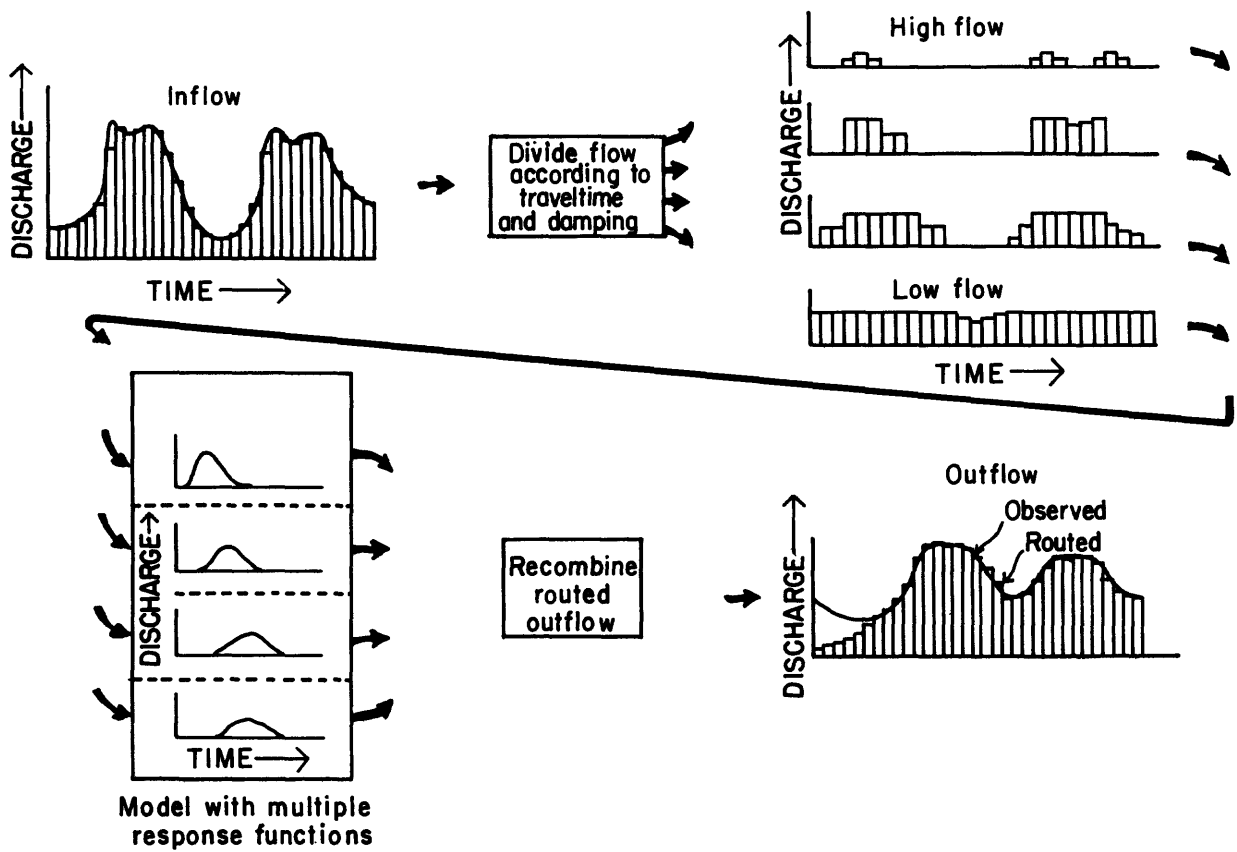


Figure 2.--Schematic diagram of the multiple linearization flow-routing model. (From Keefer and McQuivey, 1974.)

modeling process. The modeling results do not reveal trends at any flow range that would indicate that bank storage is a significant factor in this part of the basin. Specifically, there is no pattern of less runoff and more baseflow occurring between reaches.

MODEL CALIBRATION

Each of the four flow-routing models were calibrated by the following trial and error procedure.

1. Preliminary estimates of the celerity and dispersion coefficients for each reach were determined by the methods suggested by Keefer and McQuivey (1974).
2. Initial estimates of the flow from ungaged streams in the intervening area of each reach were made by multiplying the flow for a suitable gaged tributary stream (index station) by the ratio of total intervening area to gaged intervening area. Where two or more index stations were available the ungaged area was distributed arbitrarily to those stations for the purpose of the initial estimates.
3. The gaged flow from tributary streams and the estimates of flow from the ungaged areas were distributed on the basis of their contribution to the upper or lower ends of the reach.
4. Outflow hydrographs were generated using the following as input to the routing models: streamflows for the gaging station at the upstream end of the reach, estimates of total intervening flow, and the estimated celerity and dispersion coefficients.
5. Observed and simulated hydrographs were plotted for several periods of at least one year's duration. Periods were selected so as to have sustained low flows and several significant rises.
6. The adequacy of the calibration was then evaluated by:
 - a. Visual examination of observed and simulated outflow hydrographs.
 - b. Magnitude of daily flow errors.
 - c. Volume errors.
 - d. Distribution of daily flow errors.
7. The celerity and dispersion coefficients were adjusted and steps 4, 5, and 6 repeated until the timing of peaks and troughs of the simulated hydrographs matched those of the observed hydrographs.
8. The differences between observed and simulated daily flows were computed. Logarithms of the differences were then regressed against logarithms of observed daily flows at one or more index stations to refine the coefficients for the index stations. The index stations

with highest predictive capability were selected for this purpose.

9. The routing coefficients and the coefficients applied to index stations were further adjusted and steps 4 through 6 repeated until the best possible equation was developed.

The criteria for evaluating the adequacy of calibration were as follows:

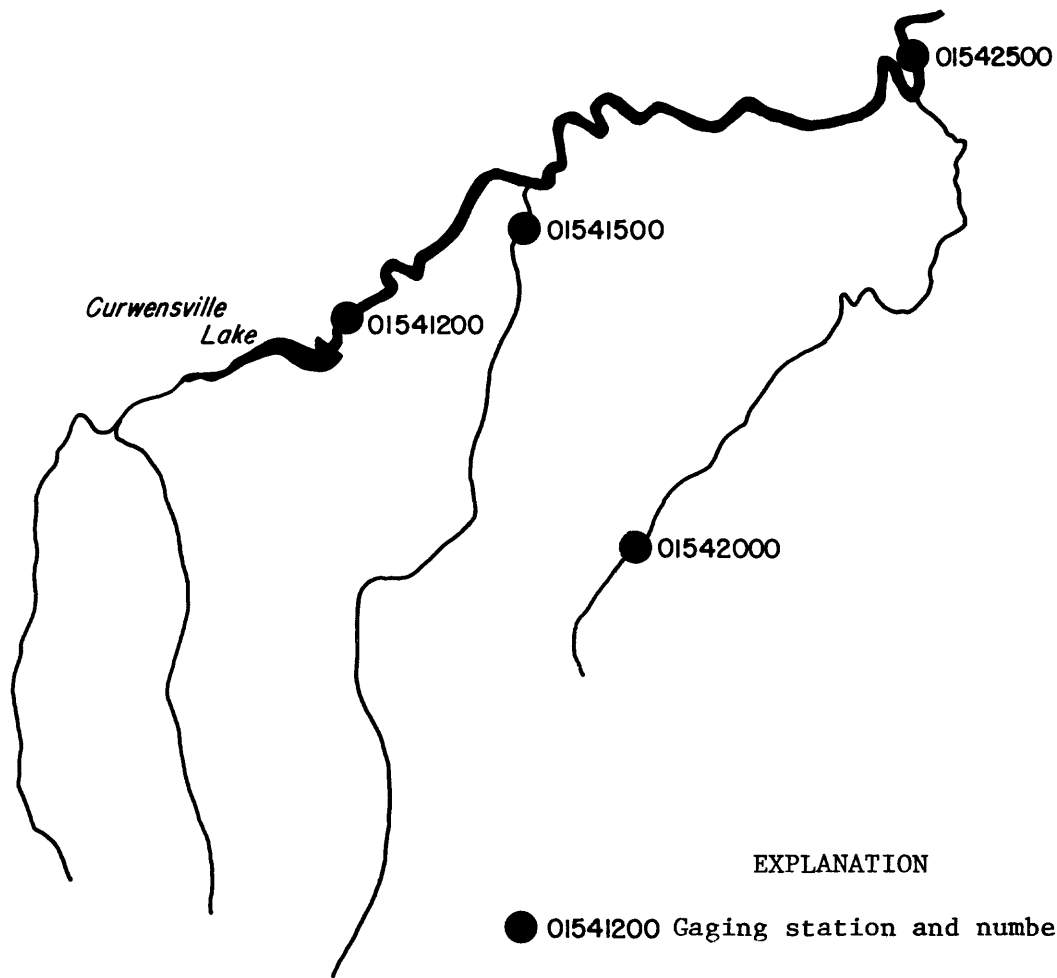
1. Total volume error less than 2 percent;
2. Daily flow error less than 15 percent;
3. Distribution of daily flow errors approximately normal both within and among ranges of flows;
4. Very few individual daily flow errors in excess of 30 percent;
5. Timing of peaks and troughs approximately correct;
6. Balance of positive and negative daily flow errors.

For reach 3, the plot of differences in daily flow errors versus flow at several index stations showed a straight line, so that a single equation could be used to represent the flow from the intervening area. For reaches 1, 2, and 4 those plots and the distribution of flow errors showed that different relations were needed for different flow ranges. Divisions of flow were established and separate equations developed for each division. Exponential relations were used in some cases to represent the flow ranges, resulting in a substantial reduction of modeling errors.

Relations describing the final calibrated flow-routing models and schematic diagrams of each reach are shown in figures 3-6. The final celerity and dispersion coefficients are listed in table 4. Multiple linearization was not used in the model developed for reach 3a. The best method developed for simulating streamflow for the West Branch at Lock Haven (01545800) was a simple addition of an unlagged and lagged component of the streamflow at Renovo (01545500). The final coefficients applied to these components, determined by a trial and error procedure, are given in the relation presented in figure 5. In effect, this is analogous to a single unit-response function determined from a single celerity and dispersion coefficient.

Figures 7 and 8 show observed and simulated outflow hydrographs for the West Branch at Karthaus (01542500) and Renovo (01545500) for parts of a calibration period. Both show relatively good agreement between observed and simulated flows indicating good calibration of the models for reaches 1 and 2. Comparisons of observed and simulated hydrographs for the West Branch at Williamsport (01551500) and Lewisburg (01553500) were generally more favorable.

Additional evaluation of the adequacy of the calibrated models was accomplished by simulating outflow for each reach for the longest time period for which both inflow and outflow had been observed. Using a combination of



EXPLANATION

● 01541200 Gaging station and number

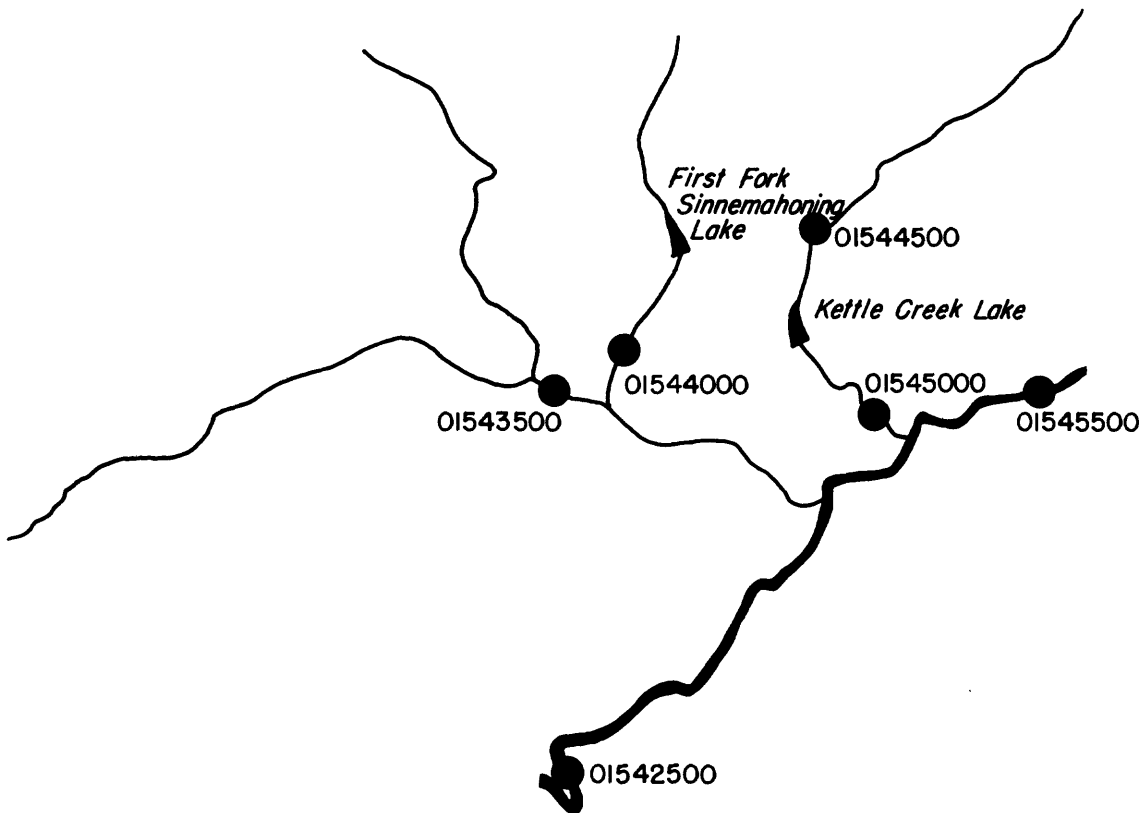
Q₅₄₂₅ Daily flow at station 01542500

$$Q_{5425} = \underbrace{Q_{5412} + 1.89 \times Q_{5415}}_{\text{routed}} + \text{fctn}(Q_{5420})$$

$$\text{where, } \text{fctn}(Q_{5420}) = 0.235(Q_{5420})^{1.991}, \text{ for } Q_{5420} \leq 30 \text{ ft}^3/\text{s}$$

$$= 6.820(Q_{5420}), \text{ for } Q_{5420} > 30 \text{ ft}^3/\text{s}$$

Figure 3.--Schematic diagram of reach 1 showing the relations used in model calibration.



EXPLANATION

● 01545500 Gaging station and number

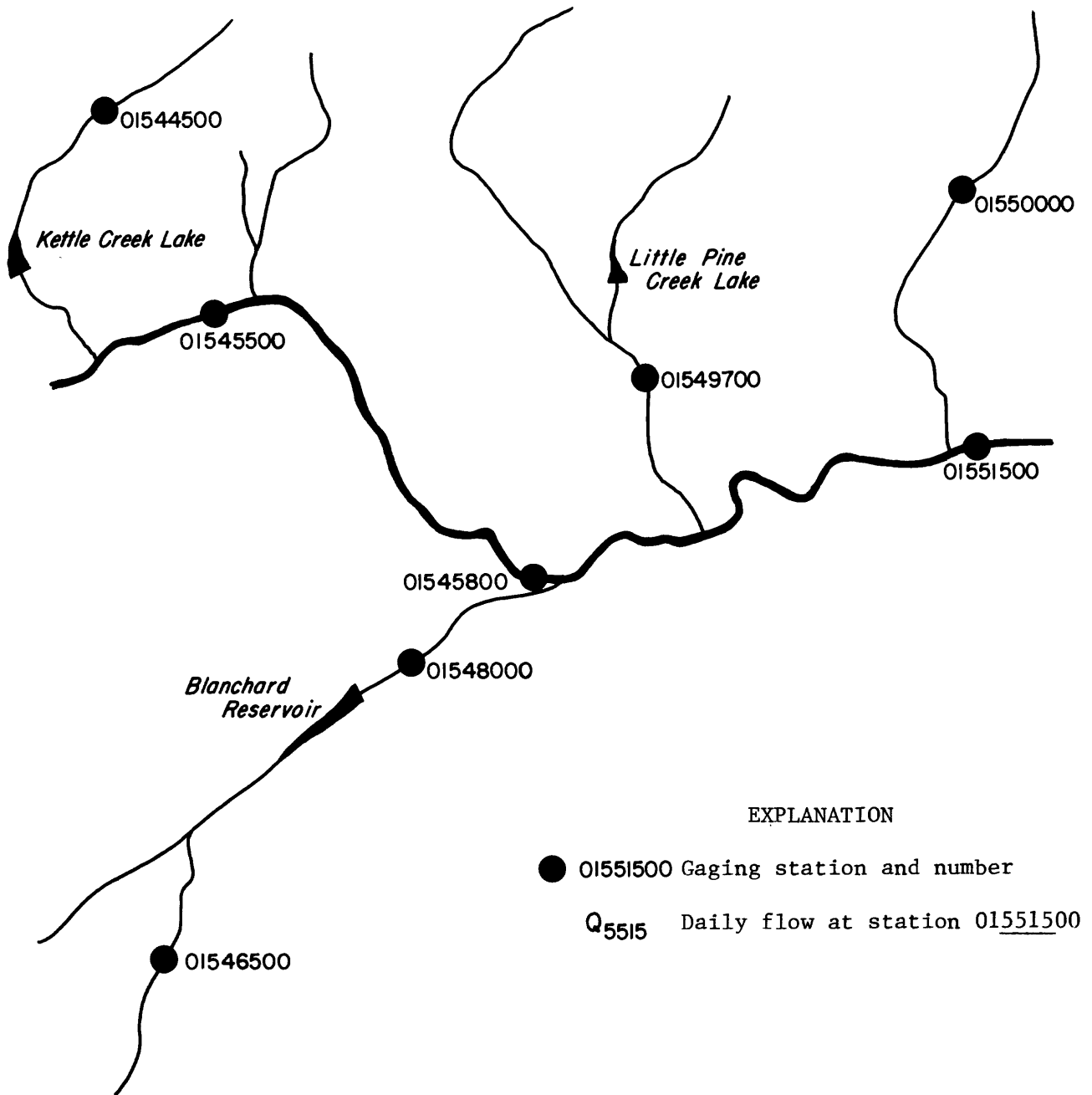
Q_{5455} Daily flow at station 01545500

$$Q_{5455} = \underbrace{Q_{5425} + \text{fcn}(Q_{5435}) + Q_{5440}}_{\text{routed}} + Q_{5450} + Q_{5445}$$

where, $\text{fcn}(Q_{5435}) = 0.358(Q_{5435})^{1.218}$, for $Q_{5435} \leq 500 \text{ ft}^3/\text{s}$

$= 1.176(Q_{5435})^{1.026}$, for $Q_{5435} > 500 \text{ ft}^3/\text{s}$

Figure 4.--Schematic diagram of reach 2 showing the relations used in model calibration.



EXPLANATION

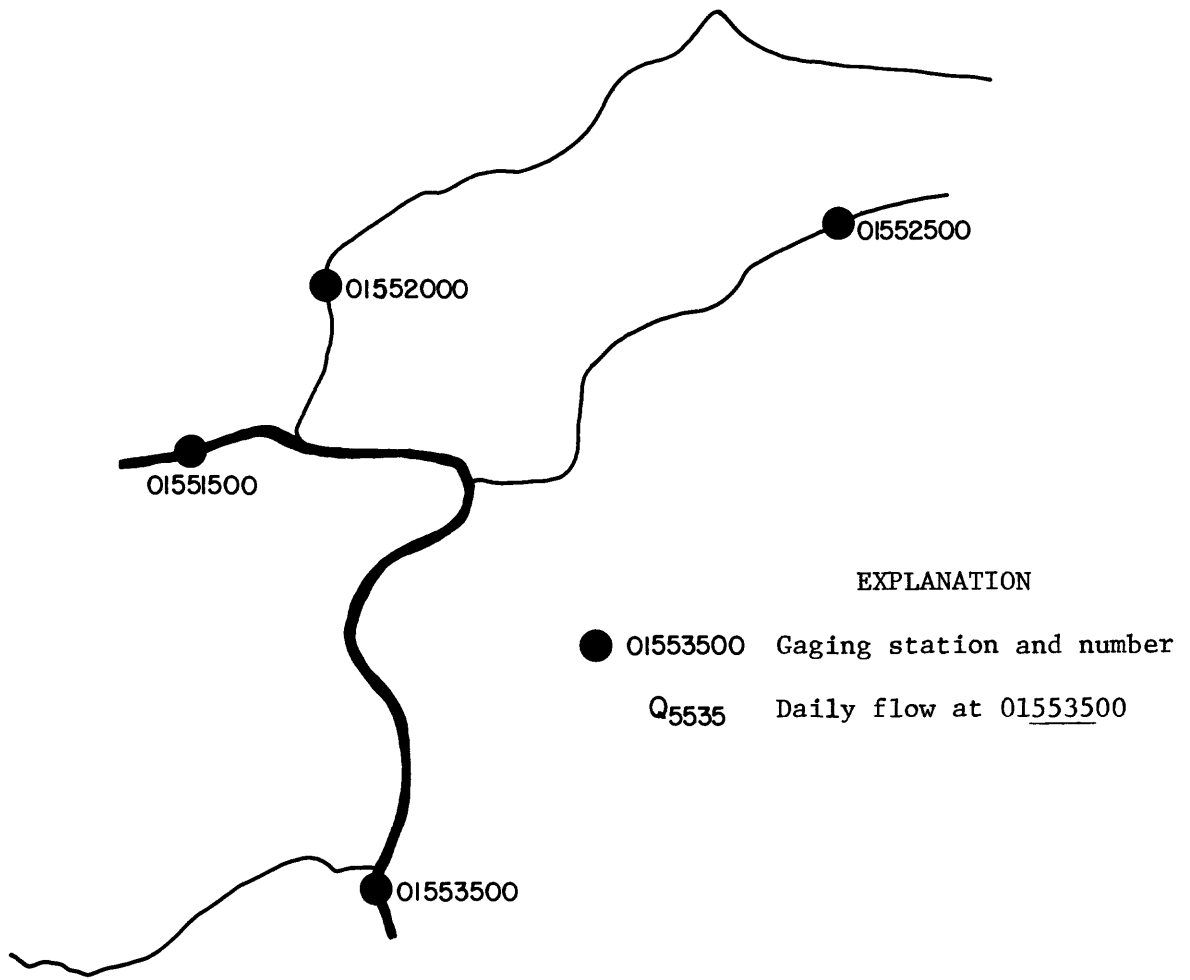
- 01551500 Gaging station and number
- Q_{5515} Daily flow at station 01551500

$$Q_{5458} = 0.8 \times (Q_{5455} + 2.7 \times Q_{5445}) + 0.2 \times (Q_{5455} + 2.7 \times Q_{5445}) \text{ LAG } 1$$

where, LAG 1 indicates that the quantity is lagged 1 day

$$Q_{5515} = \underbrace{Q_{5458} + 4.30 \times Q_{5465} + Q_{5480} + Q_{5497}}_{\text{routed}} + 2.70 \times Q_{5500}$$

Figure 5.--Schematic diagram of reach 3 showing the relations used in model calibration.



$$Q_{5535} = \underbrace{Q_{5515} + \text{fctn}(Q_{5520})}_{\text{routed}} + 2.00(Q_{5525})$$

where, $\text{fctn}(Q_{5520}) = 10.455 (Q_{5520})^{0.680}$, for $Q_{5520} \leq 150 \text{ ft}^3/\text{s}$
 $= 2.10 (Q_{5520})$, for $Q_{5520} > 150 \text{ ft}^3/\text{s}$

Figure 6.--Schematic diagram of reach 4 showing the relations used in model calibration.

Table 4.--Model parameters used in final flow-routing models

[Q, flow in cubic feet per second; C, celerity in feet per second; and K, dispersion coefficient in square feet per second]

Q	Reach 1		Reach 2		Reach 3B		Reach 4				
	C	K	Q	C	K	Q	C	K			
100	2.00	1,000	100	1.50	200	400	1.30	2,000	400	2.70	2,000
400	2.20	1,100	400	1.60	500	1,000	1.50	2,400	1,000	2.80	2,700
1,000	2.50	1,300	1,000	2.50	1,200	2,000	1.90	4,600	2,000	2.90	4,100
2,500	4.00	3,000	2,000	3.40	2,300	3,000	2.30	6,700	3,000	3.10	5,400
5,000	5.40	5,700	3,000	4.10	3,200	5,000	2.90	10,800	5,000	3.30	8,000
10,000	6.80	10,100	5,000	5.20	5,300	10,000	4.10	21,000	10,000	3.80	14,000
20,000	8.70	18,200	10,000	6.70	9,700	20,000	5.70	40,400	20,000	4.70	26,000
30,000	9.60	24,800	20,000	8.70	18,200	50,000	9.00	98,000	50,000	6.40	63,000
50,000	11.40	37,900	50,000	12.00	40,000	250,000	11.50	155,000	310,000	9.00	130,000

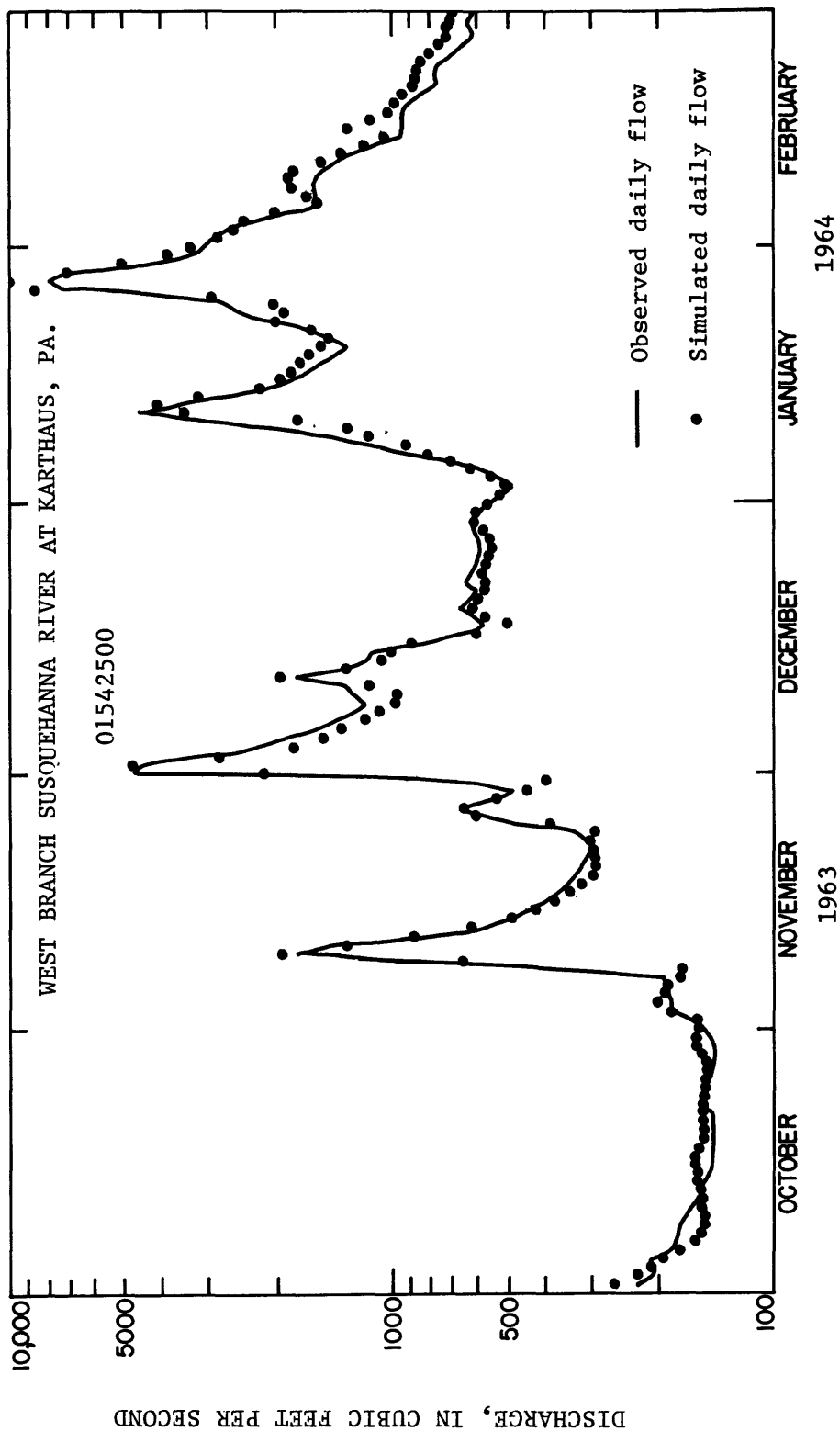


Figure 7.--Hydrographs of observed and simulated flows at station 01542500, October 1, 1963 to February 28, 1964.

observed data and estimated data, streamflows were simulated for the 1942-77 water years for reaches 1 and 2. Since neither observed nor estimated flow data were available for Pine Creek near Waterville (01549700) prior to the 1958 water year, simulations for reaches 3 and 4 were limited to the 1958-77 water years. Daily flow and flow volume errors for these data (table 5) were computed using the following equations:

$$\text{Daily flow error (in percent)} = \left(\frac{\sum_{i=1}^n \left| \frac{Q_o - Q_s}{Q_o} \right|}{n} \right) \times 100$$

where Q_o and Q_s are observed and simulated flows in ft^3/s , respectively, for the i^{th} day, and n is the number of days in the calibration period; and

$$\text{Volume error (in percent)} = \left(\frac{V_o - V_s}{V_o} \right) \times 100$$

where V_o and V_s are observed and simulated flow volumes in $\text{ft}^3/\text{s} - \text{days}$, respectively, for a calibration period.

As shown in figure 9, the accuracy of the flow-routing models seem to be largely dependent upon the proportion of ungaged tributary flow in a reach to total drainage area. Data compiled by Zembrzuski (1980) in Part IV of this series of flow-routing studies are also plotted and shows a similar trend.

MODEL EVALUATION

To evaluate the models, presented in the previous section, each was rerun using the outflows generated by the upstream model as the inflow to the next reach downstream. Daily flow and volume errors between the outflows simulated in this manner and observed flows are less than 14 and 2 percent, respectively, and are not highly cumulative (table 6).

A primary objective of the study was to develop reliable models for modeling streamflow during low-flow periods. Favorable comparisons of flow-duration and low-flow frequency curves developed from the observed and simulated data were, therefore, part of the final criteria for evaluating the accuracy of the calibrated models.

Flow-duration curves, which show the percentage of time that specified discharges are equalled or exceeded, are shown in figures 10-13 for the station at the downstream end of each reach. These curves show excellent agreement throughout the range of flows encountered.

The 7-day, low-flow frequency curves were developed from series of annual minimum, 7-day average flows for both simulated and observed conditions. The annual low-flows were determined on the basis of climatic years, which run from April 1 to March 31, the year being designated by the calendar year in which the climatic year ends. Water years are not

Table 5.--Model calibration errors

Reach	Period	Errors, in percent	
		Daily flows	Flow volume
1	October 1, 1941 to September 30, 1977	13.70	-0.01
2	October 1, 1941 to September 30, 1977	7.35	1.62
3	October 1, 1957 to September 30, 1977	7.45	- .06
4	October 1, 1957 to September 30, 1977	6.66	.57

Table 6.--Model evaluation errors

Reach	Period	Errors, in percent	
		Daily flows	Flow volume
1	October 1, 1941 to September 30, 1977	13.70	-0.01
2	October 1, 1941 to September 30, 1977	10.13	1.61
3	October 1, 1957 to September 30, 1977	8.82	.09
	October 1, 1941 to September 30, 1977	10.39	1.69
4	October 1, 1957 to September 30, 1977	9.50	.65
	October 1, 1941 to September 30, 1977	9.80	1.25

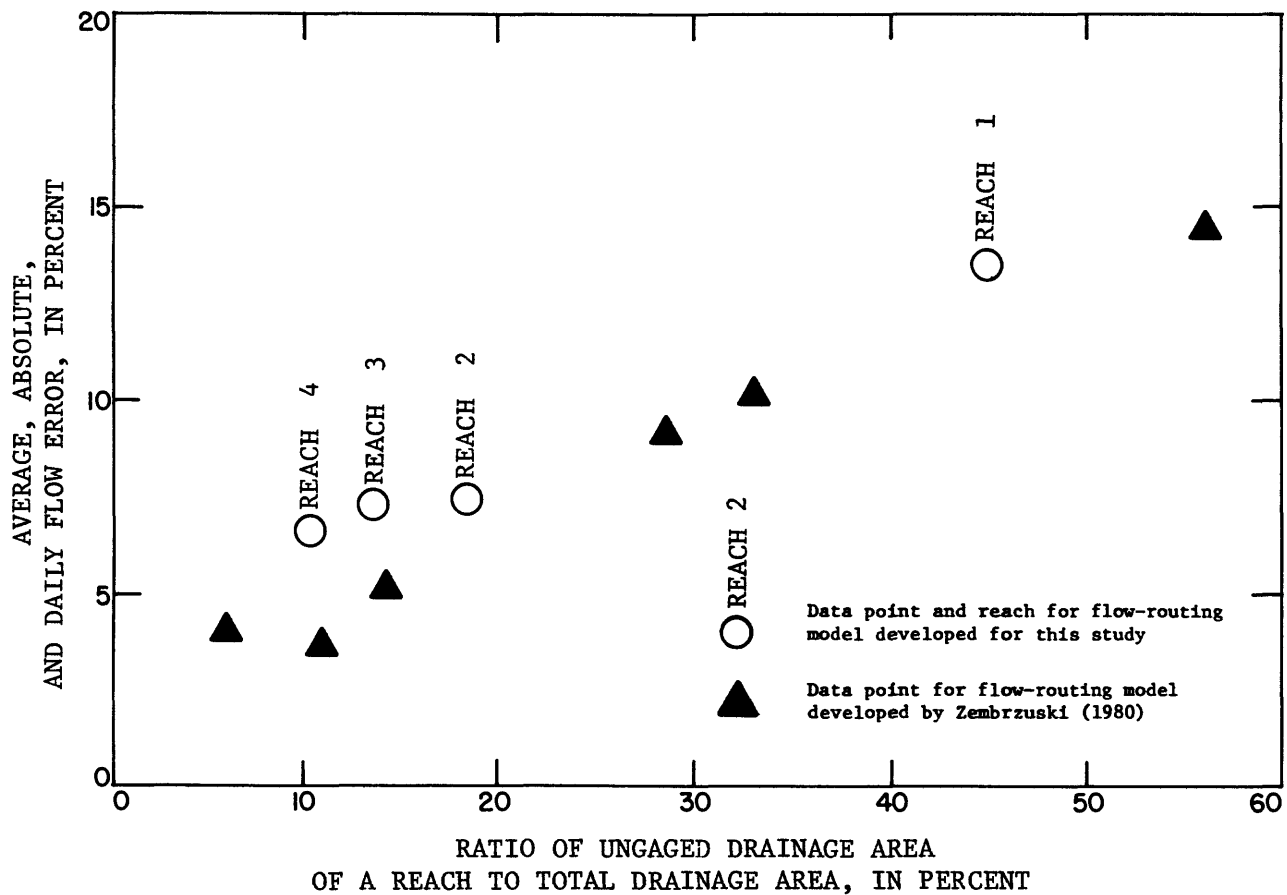


Figure 9.--Relation between daily flow errors and the ungaged drainage area of each reach.

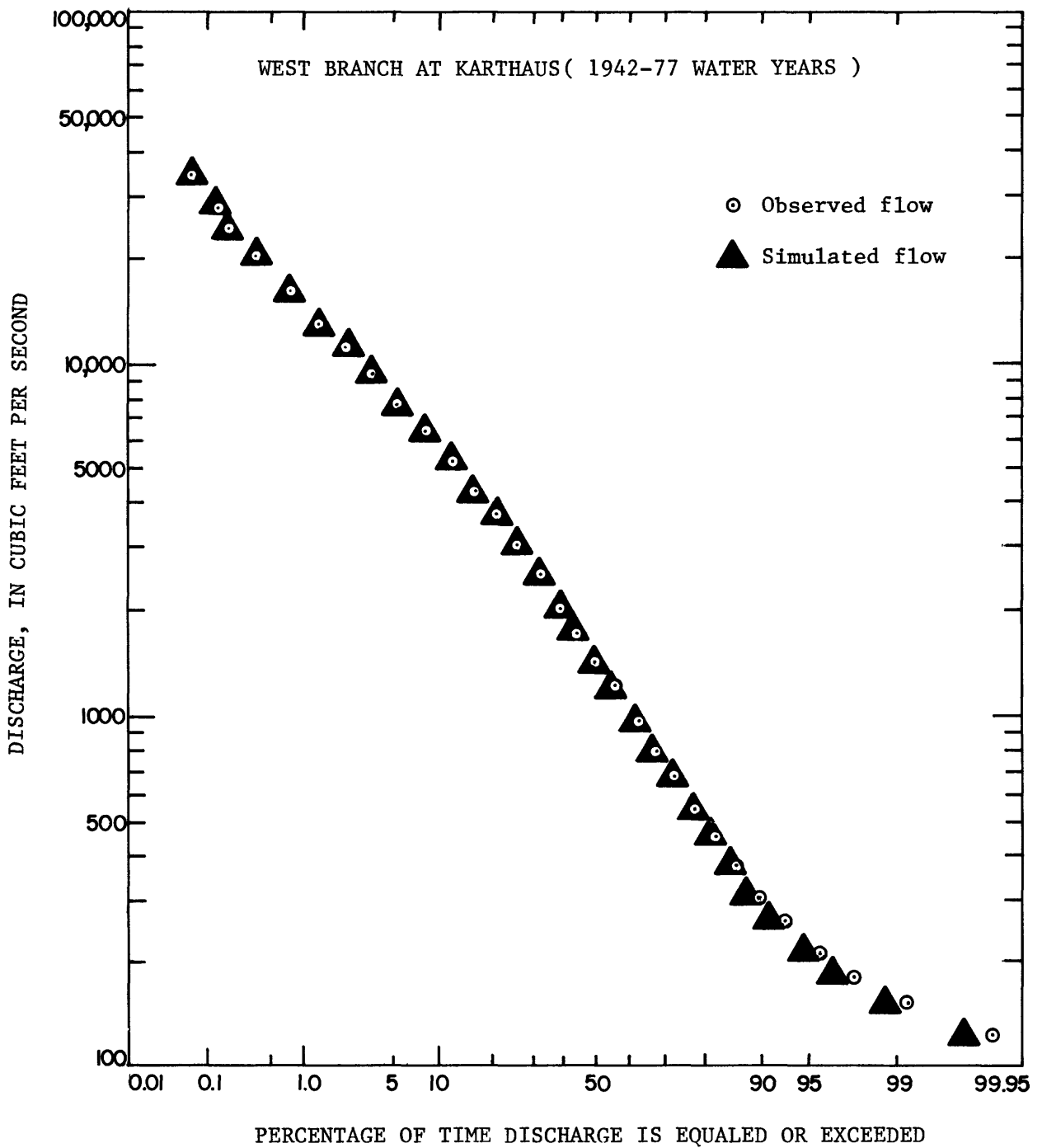


Figure 10.--Flow-duration curves for station 01542500 for observed and simulated conditions, 1942-77 water years.

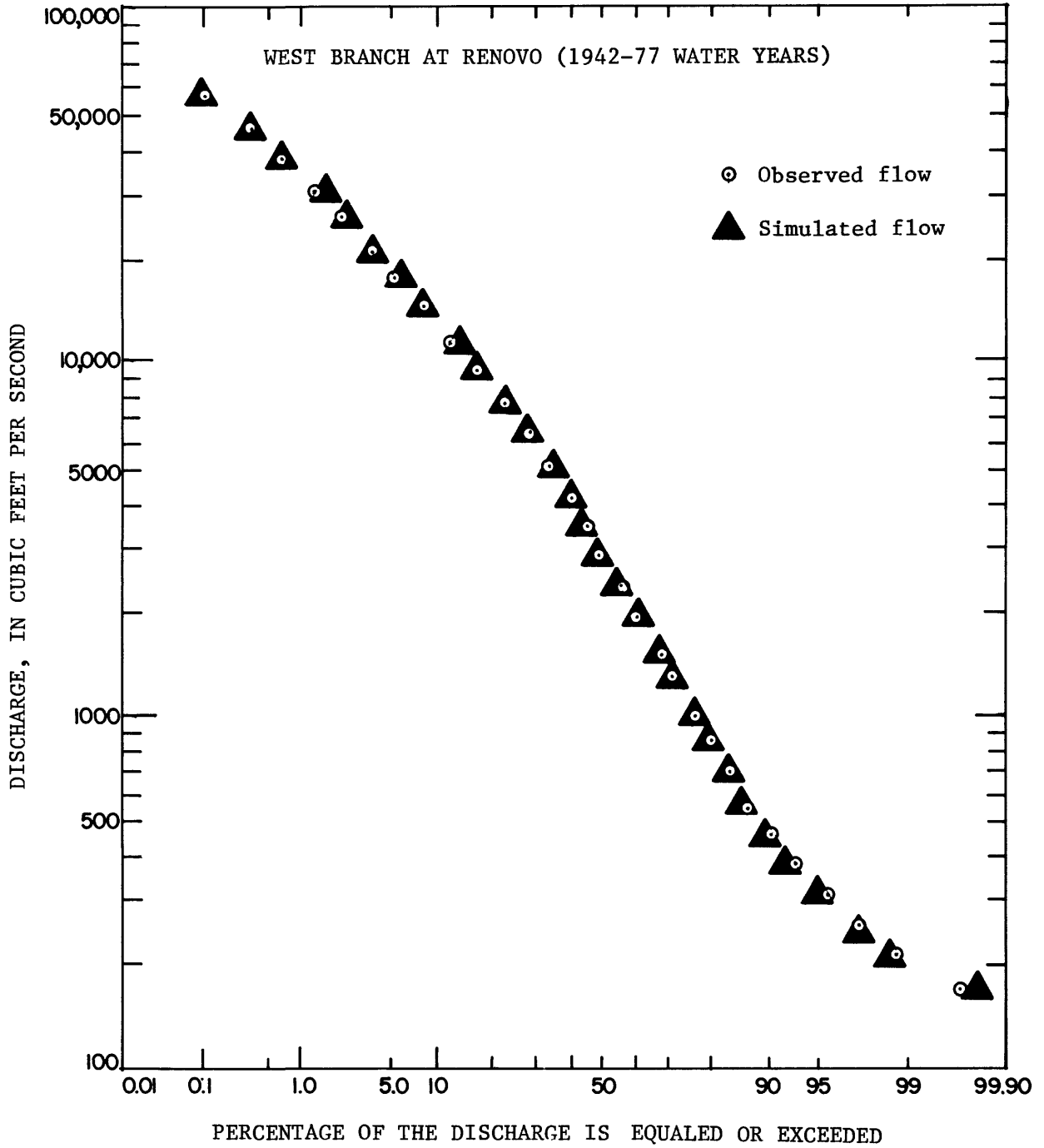


Figure 11.--Flow-duration curves for station 01545500 for observed and simulated conditions, 1942-77 water years.

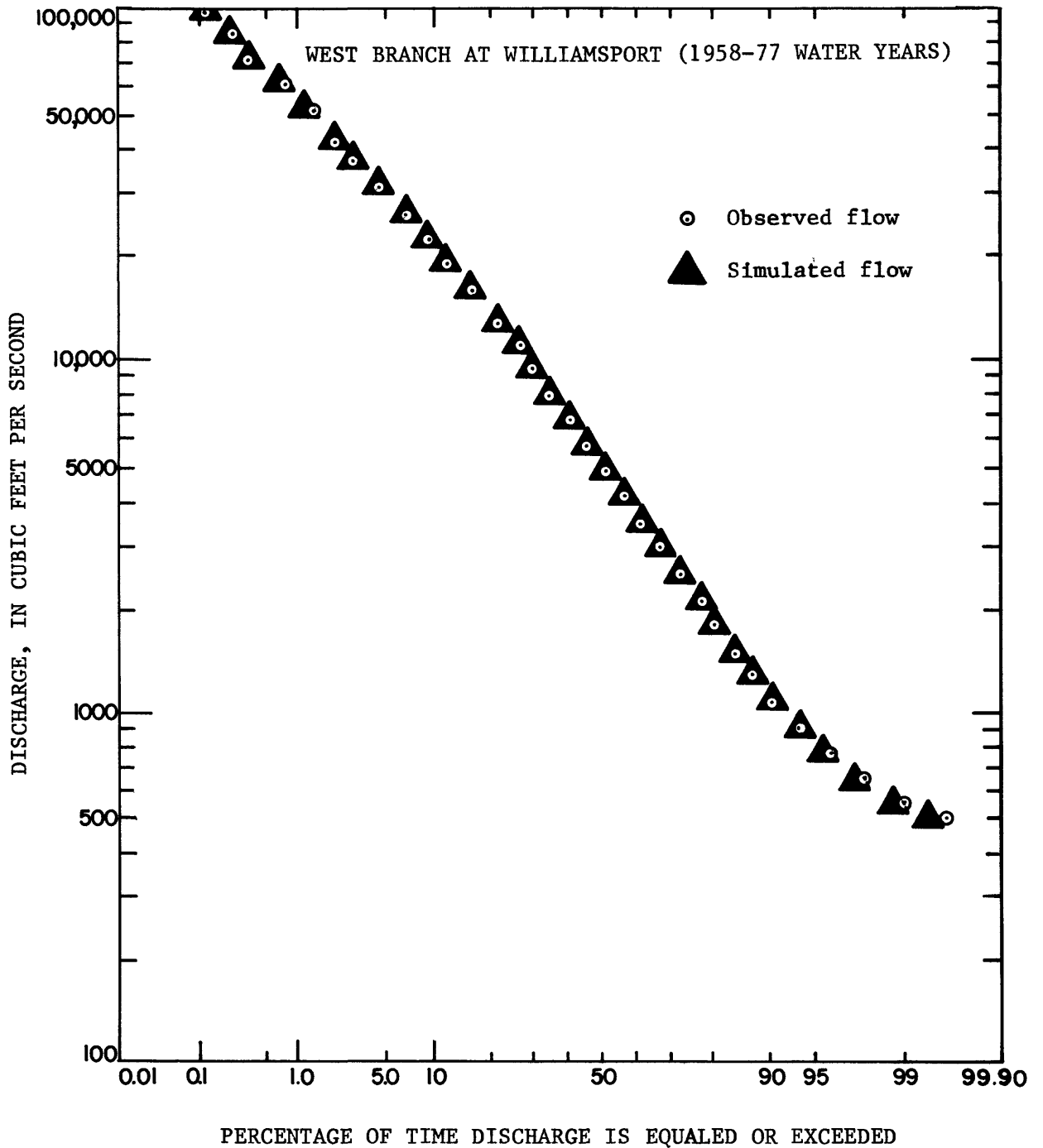


Figure 12.--Flow-duration curves for station 01515500 for observed and simulated conditions, 1958-77 water years.

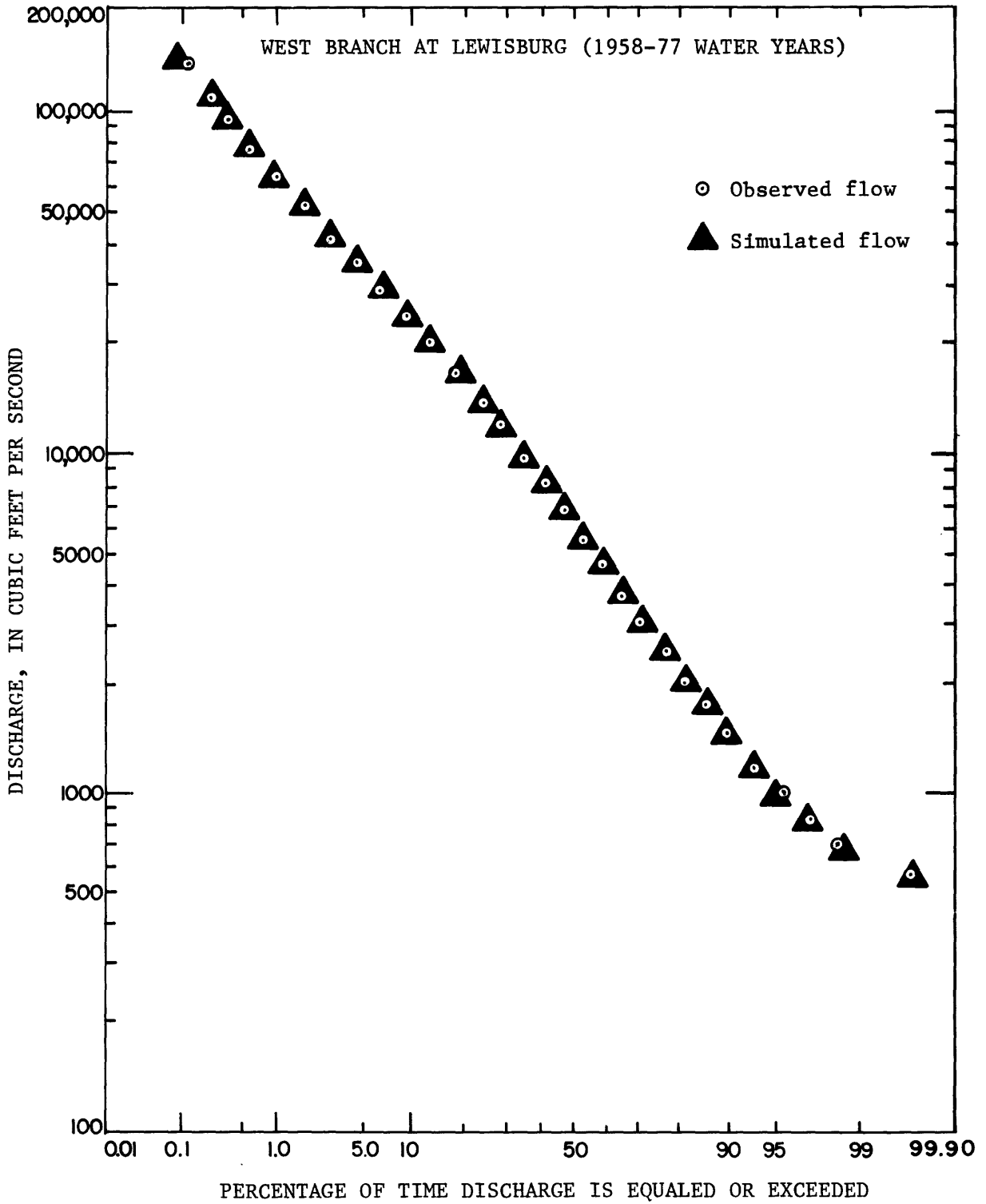


Figure 13.--Flow-duration curves for station 01553500 for observed and simulated conditions, 1958-77 water years.

acceptable for use in these analyses, as the possibility is good that a significant low flow may span from the end of one water year into the beginning of the next water year. The analyses, therefore, use climatic years, which number one less than the number of water years of data available.

Visual comparisons of the 7-day, low-flow frequency curves presented in figures 14-17, also show excellent agreement. The recurrence interval, given on the abscissa of each figure, is the average interval of years during which flows equal to or less than a given value would be expected only once.

Values of the 7-day low flow having a recurrence interval of 10 years ($Q_{7,10}$), commonly used in establishing water resources policies for low-flow periods, are given in table 7. Each of the $Q_{7,10}$ values computed from simulated data are within 5 percent of those computed from observed data.

The streamflow data used in the aforementioned analyses are nonhomogeneous, reflecting a progressive change from unregulated to highly regulated streamflow. As these analyses should generally be performed only for homogeneous conditions, use of these analyses for purposes other than this study may not be valid.

APPLICATION OF MODELS

The model was used for several applications. In evaluating the flow-routing model for reach 3, a record of streamflow for the 1942-77 water years was generated for the West Branch at Lock Haven (01545800), a key location for which continuous, daily flow data were not previously available. Flow-duration and 7-day, low-flow frequency curves, similar to those presented in the previous section, were developed from the simulated data and are shown in figures 18 and 19.

As part of this study, the SRBC required that continuous flow simulations for the 1942-77 water years be made for the station at the downstream end of each of the four routing reaches. This has already been accomplished for Karthaus (01542500) and Renovo (01545500) as part of evaluating the models for reaches 1 and 2. As another application of the routing models, flow data for Pine Creek near Waterville (01549700) for the 1942-57 water years were determined by the relation given in table 3. These were merged with the observed data for this station and used as input to the model for reach 3 to produce a continuous record of simulated daily flows at Williamsport (01551500) for the period required. Simulated flows at Williamsport were then used as input to the reach 4 model to simulate flows at Lewisburg (01553500) for the same period.

A comparison of the daily flow and volume errors for reaches 3 and 4 for 1957-77 and 1942-77 is shown in table 6. The addition of the simulated Pine Creek data, which are in effect "regulated" flows being used to represent unregulated conditions during 1942-50, resulted in greater errors at both Williamsport and Lewisburg. The effect of using the simulated data is illustrated further by the low-flow frequency curves for the two stations (figs. 20 and 21). The $Q_{7,10}$ values for these simulated conditions at Williamsport and Lewisburg for the 1942-77 climatic years are 11.8 percent

and 4.2 percent higher, respectively, than those computed from observed data. For comparison, $Q_{7,10}$ values determined from simulated flows for the 1958-77 water years, using only observed data for Pine Creek as input to the routing model for reach 3, are 0.6 percent lower and 1.3 percent higher than those determined from the observed data at Williamsport and Lewisburg, respectively, during this period. Differences between flow-duration curves for simulated and observed conditions at Williamsport and Lewisburg, presented in figures 22 and 23, for the 1942-77 water years are not as discernible.

Plans for future use of the flow-routing models are also being considered. To effectively evaluate the operation of reservoirs over a given period, concurrent streamflow data for both unregulated and regulated conditions must be simulated for that period. The relations given in table 3 for West Branch Susquehanna River at Curwensville (01541200), First Fork Sinnemahoning Creek at Sinnemahoning (01544000), and Kettle Creek near Westport (01545000) were used for simulating unregulated flows downstream from Curwensville, First Fork Sinnemahoning, and Kettle Creek Reservoirs, respectively. These simulated flows are available on computer files of the SRBC. In addition, the SRBC plans to develop similar relations for Bald Eagle Creek at Beech Creek Station (01548000) and Pine Creek near Waterville (01549700) to simulate unregulated conditions downstream from Blanchard and Little Pine Creek Reservoirs. The SRBC also plans to develop reservoir-regulation models for simulating regulated conditions at each of these sites. Alternative reservoir-operating schemes may also be applied and their effects easily evaluated by the flow-routing models developed by this study.

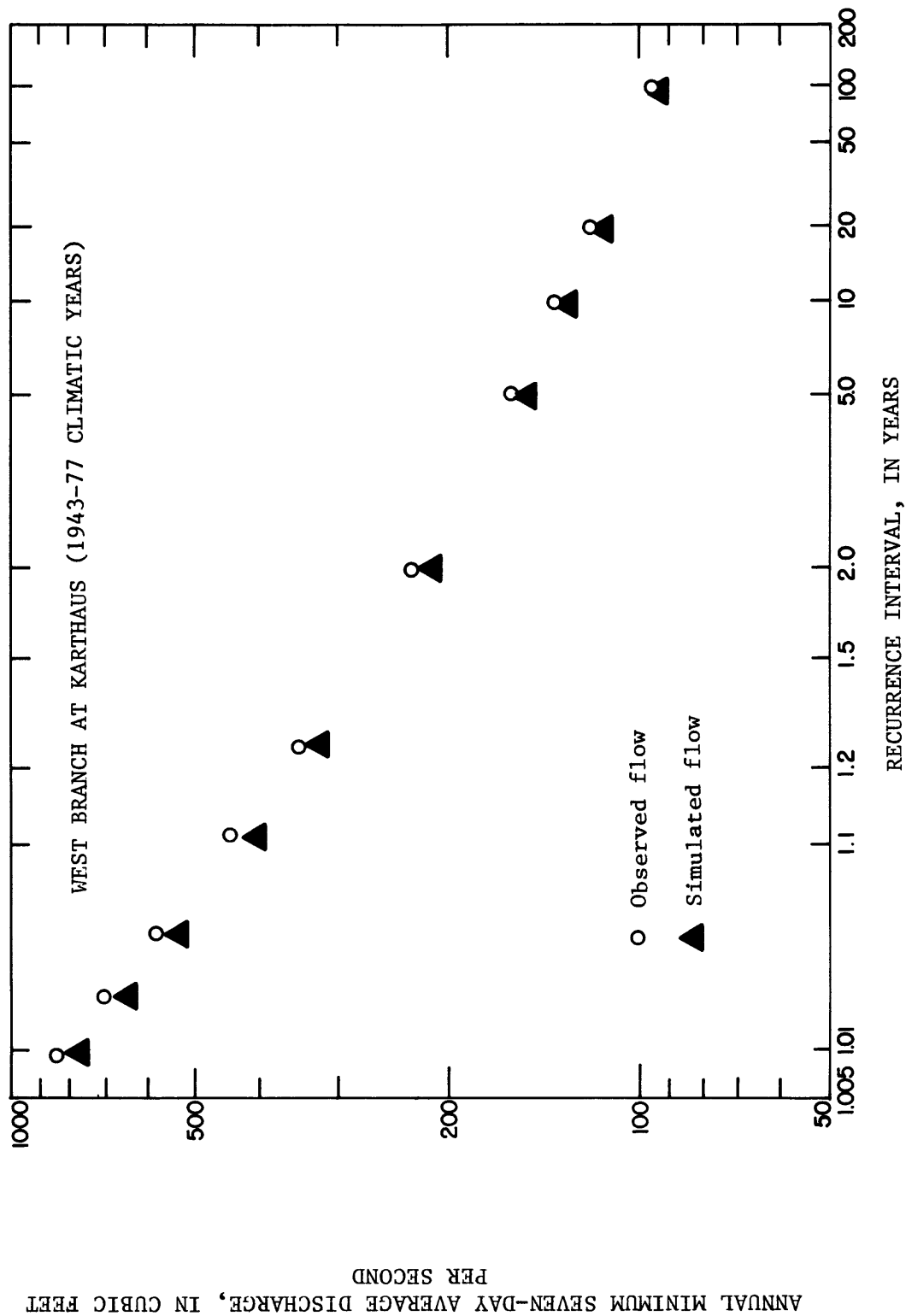


Figure 14.--Seven-day, low-flow frequency curves for station 01542500 for observed and simulated conditions, 1943-77 climatic years.

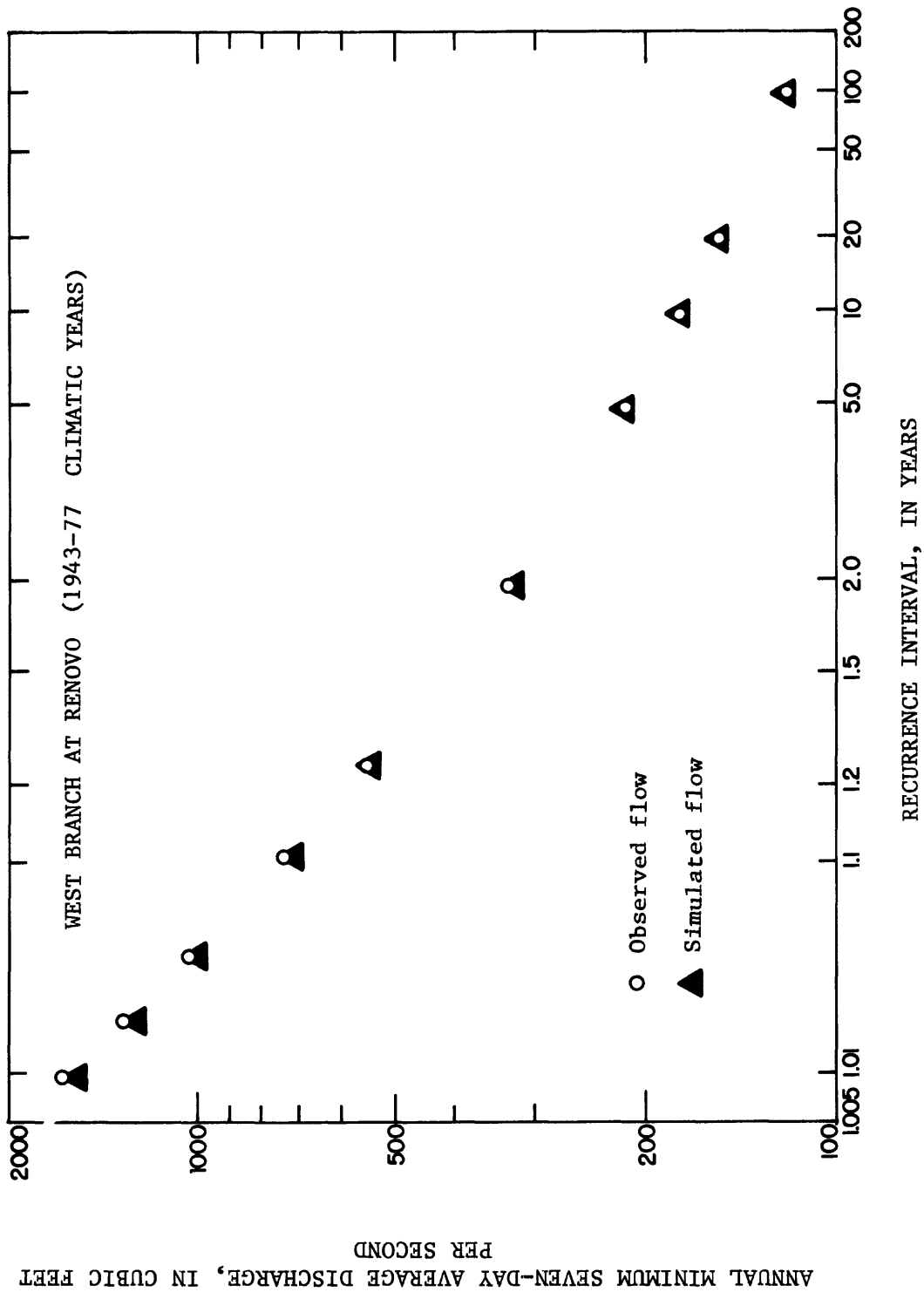


Figure 15.---Seven-day, low-flow frequency curves for station 01545500 for observed and simulated conditions, 1943-77 climatic years.

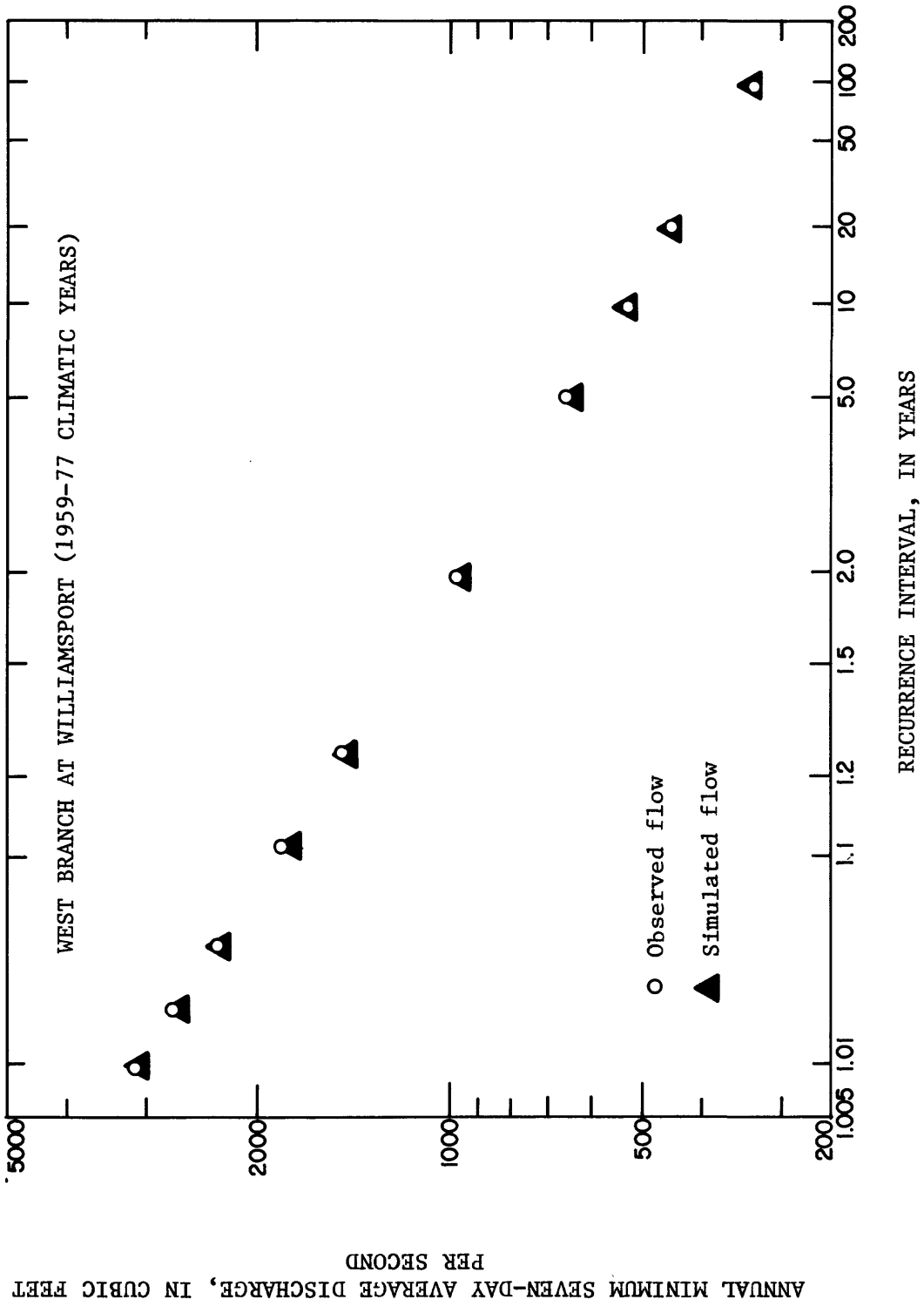


Figure 16.--Seven-day, low-flow frequency curves for station 01551500 for observed and simulated conditions, 1959-77 climatic years.

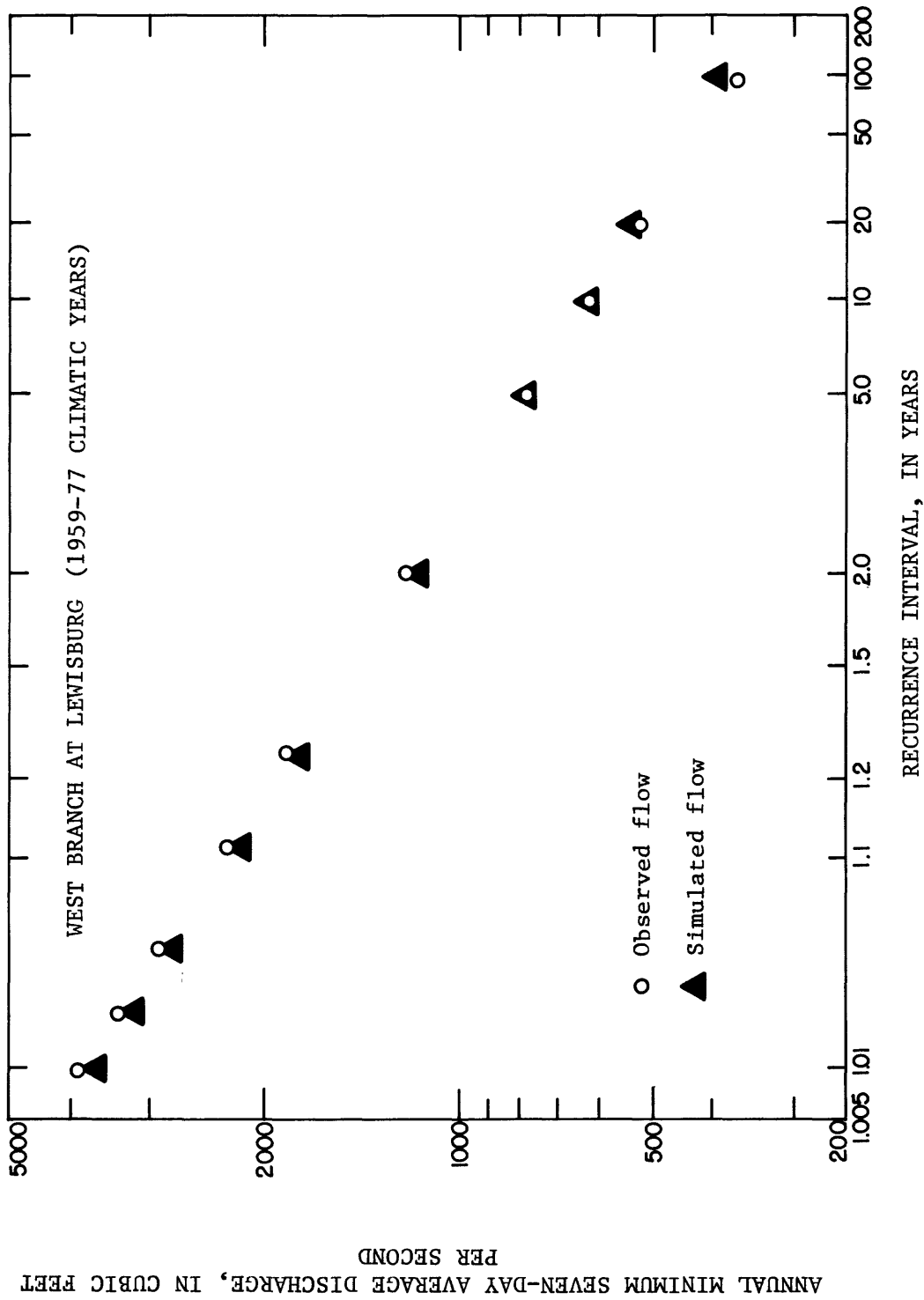


Figure 17. ---Seven-day, low-flow frequency curves for station 01553500 for observed and simulated conditions, 1959-77 climatic years.

Table 7.--Seven-day, 10-year, low flows for gaging stations
at the downstream end of each reach

Station	Period (climatic years)	Q _{7,10} (ft ³ /s)		Error (percent)
		Observed	Simulated	
01542500	1943-77	135	129	-4.4
01545500	1943-77	175	176	.6
01551500	1959-77	525	522	-.6
01553500	1959-77	626	634	1.3

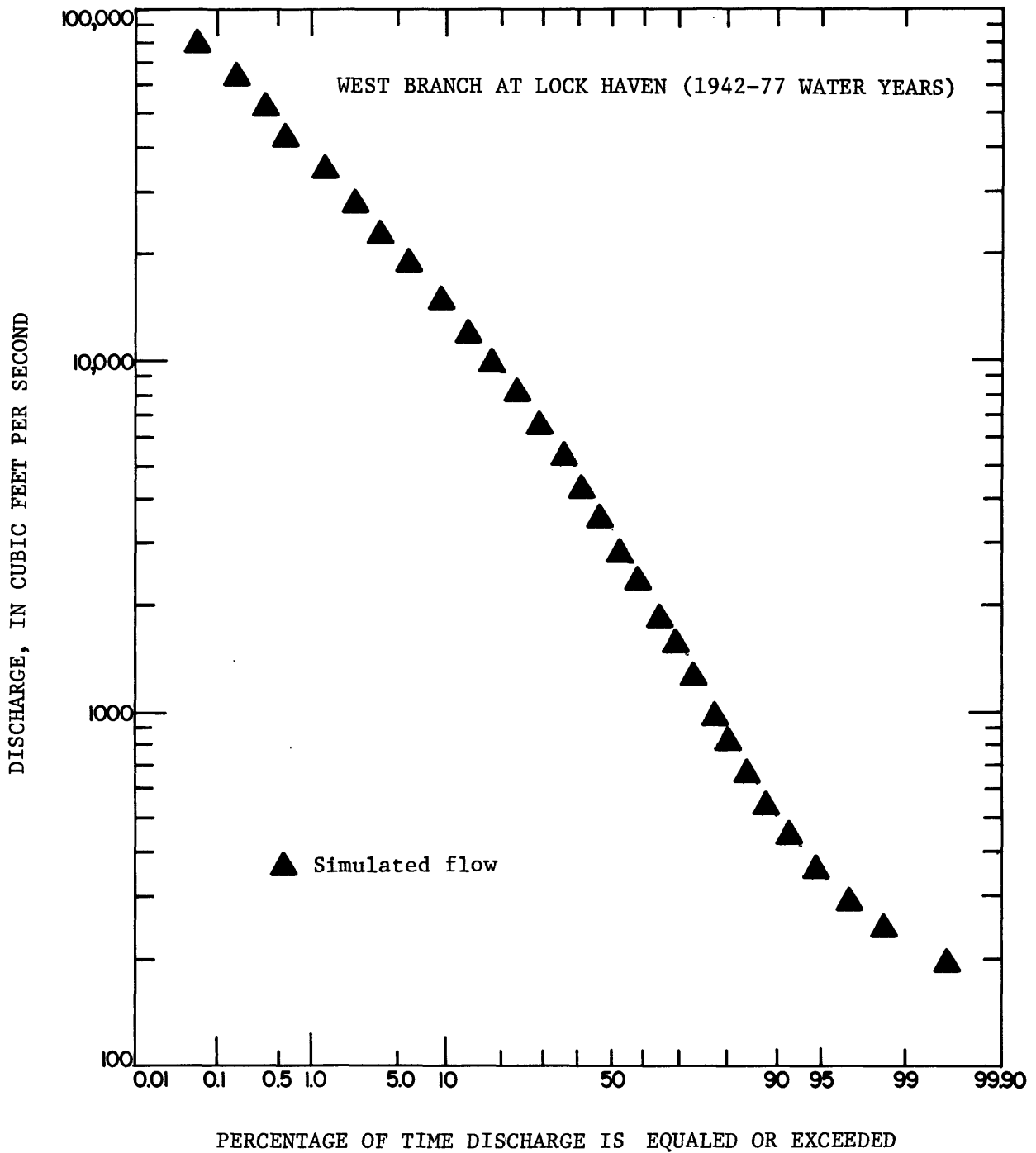


Figure 18.--Flow-duration curve for station 01545800 for simulated conditions, 1942-77 water years.

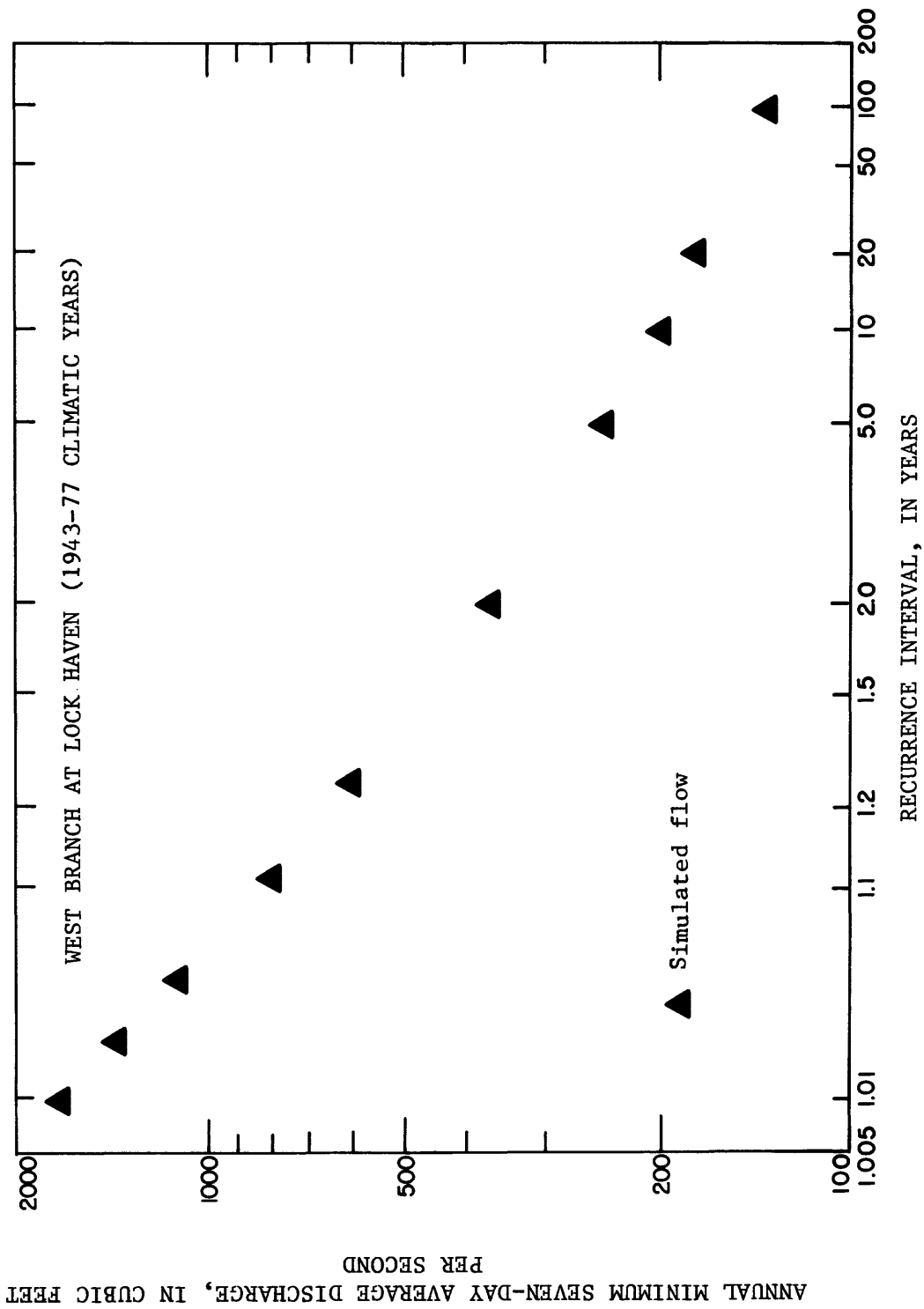


Figure 19.--Seven-day, low-flow frequency curve for station 01545800 for simulated conditions, 1943-77 climatic years.

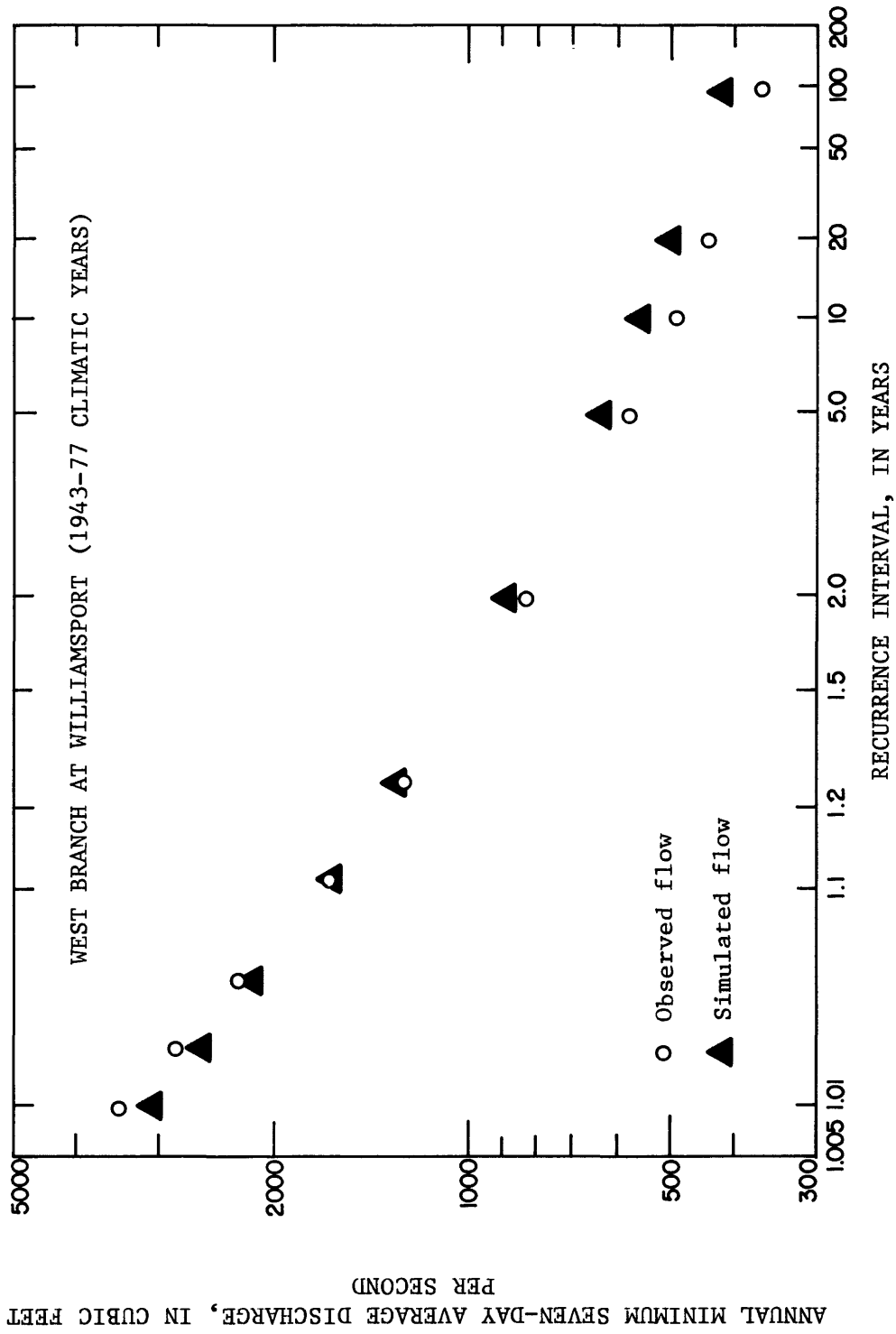


Figure 20.--Seven-day low-flow frequency curves for station 01551500 for observed and simulated conditions, 1943-77 climatic years.

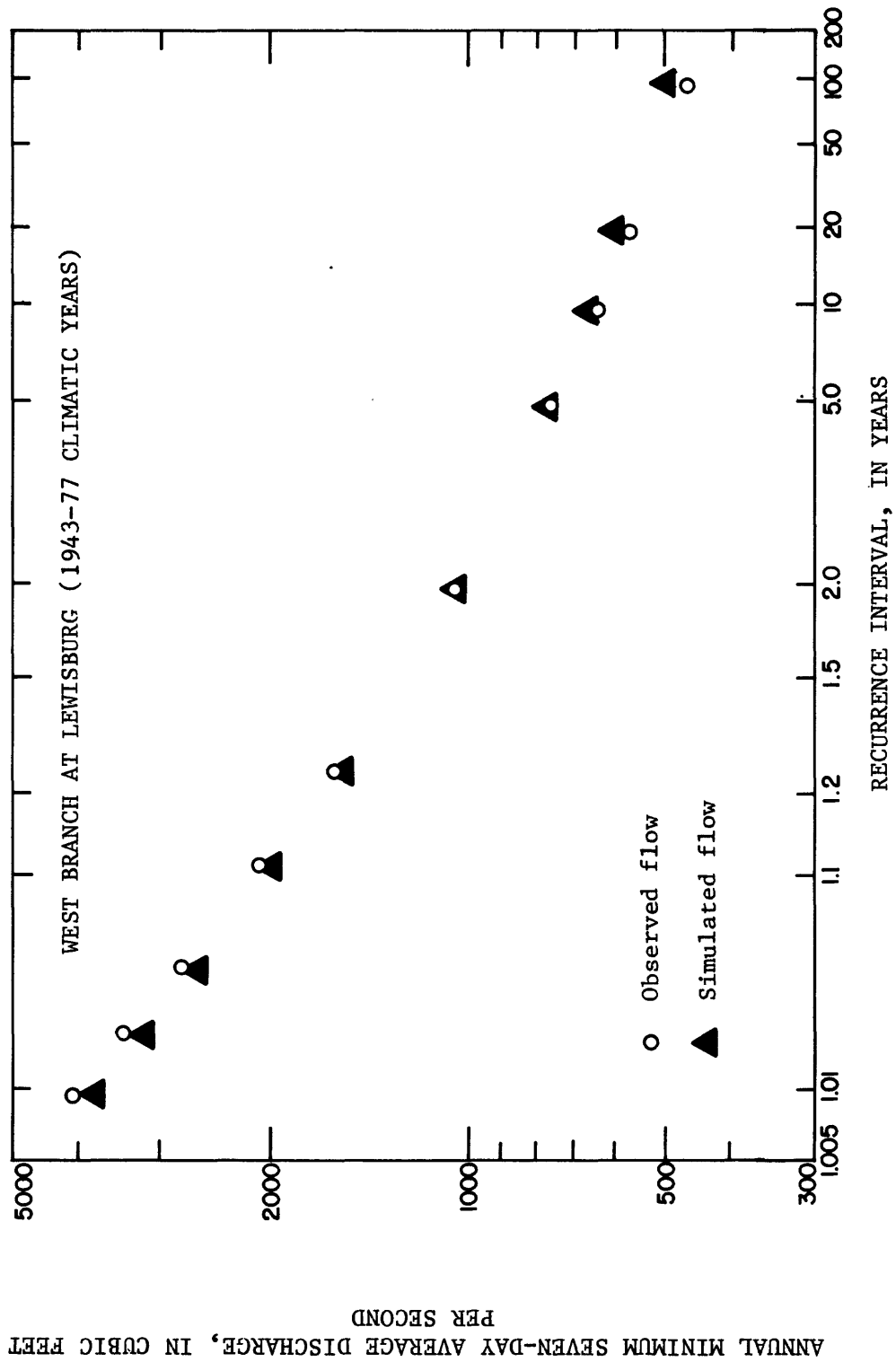


Figure 21.--Seven-day, low-flow frequency curves for station 01553500 for observed and simulated conditions, 1943-77 climatic years.

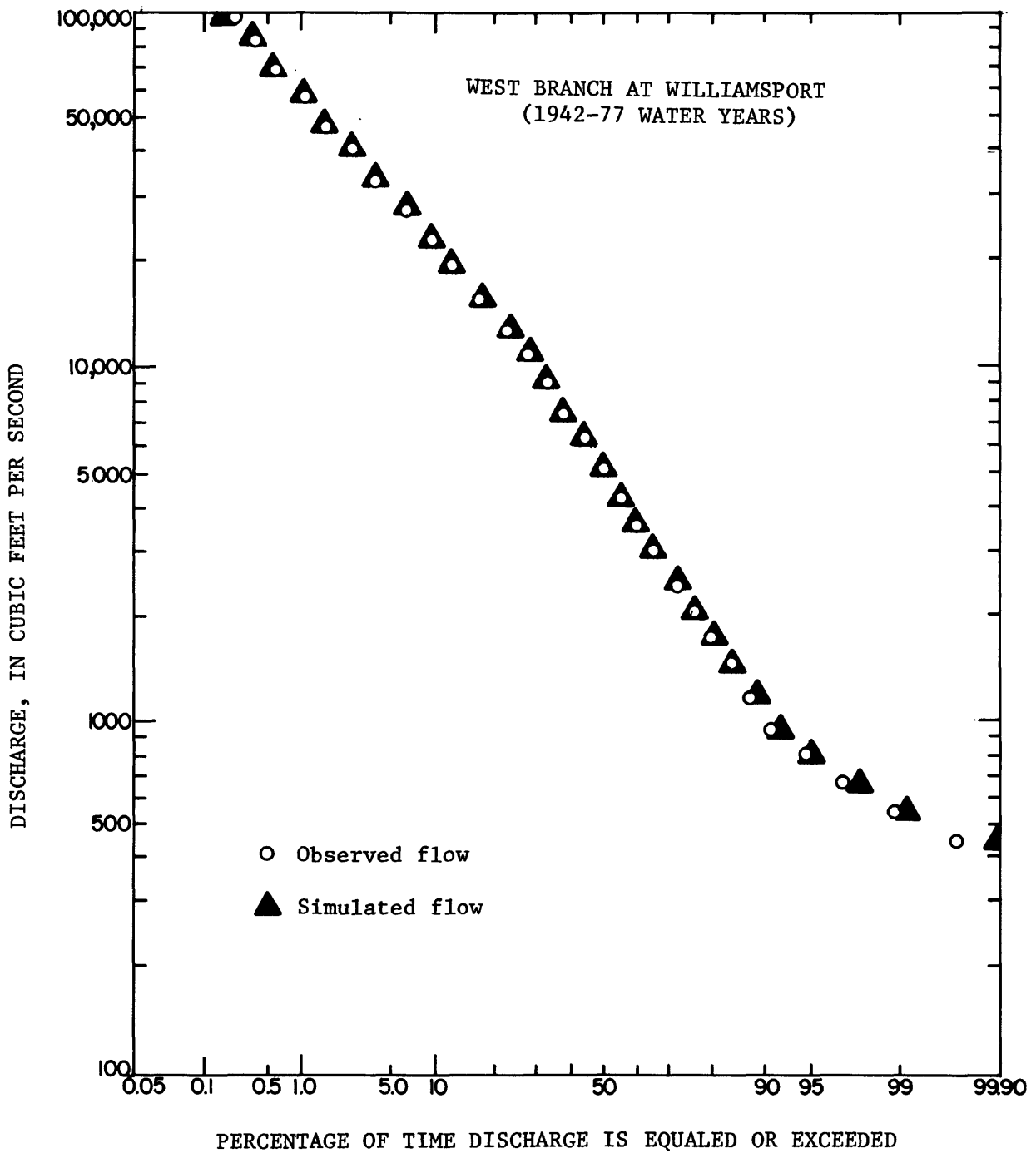


Figure 22.--Flow-duration curves for station 01551500 for observed and simulated conditions, 1942-77 water years.

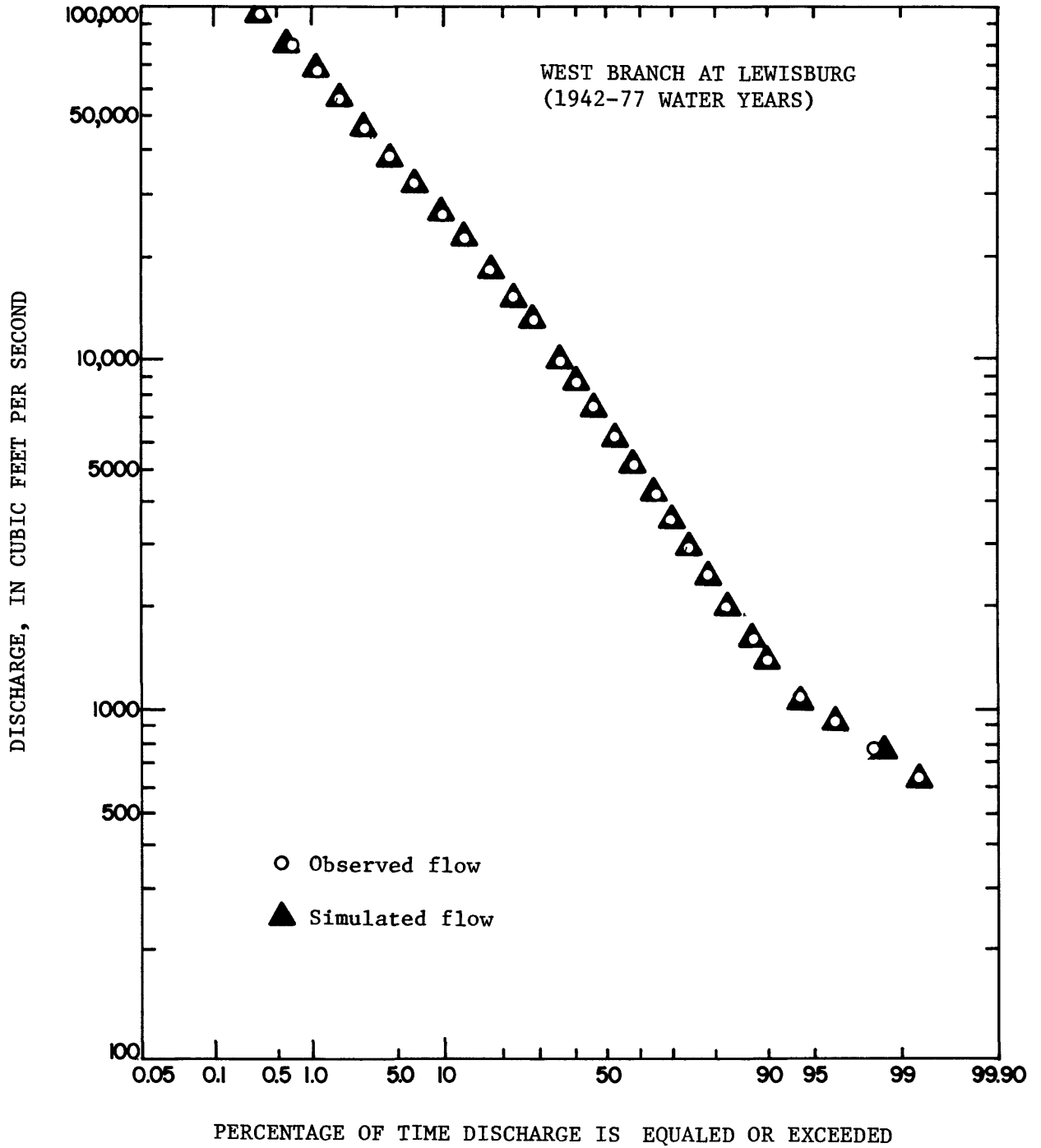


Figure 23.--Flow-duration curves for station 01553500 for observed and simulated conditions, 1942-77 water years.

SUMMARY

Digital computer, daily flow-routing models have been developed, calibrated, and evaluated for four reaches of the West Branch Susquehanna River between Curwensville and Lewisburg, Pennsylvania. The models will enable water-resources managers to evaluate the operating schemes of five reservoirs in the study area, particularly as they relate to low flows. They have already been used to simulate 36 years of daily flow data for conditions at Lock Haven, a site where continuous flow data were not previously available.

Although modeling errors are inherent in all the simulated data, accuracy of the routing models is considered to be adequate. Average absolute daily flow errors between observed and simulated flows ranged from 8.82 to 13.70 per cent for the periods evaluated. Volume errors for the same periods, were between -0.01 and 1.6 percent. Estimates of the 7-day, 10-year, low flows for simulated conditions were within 5 percent of those computed for observed conditions.

REFERENCES

- Armbruster, J. T., 1977, Flow routing in the Susquehanna River basin: Part I-Effects of Raystown Lake on low-flow frequency characteristics of the Juniata and lower Susquehanna Rivers, Pennsylvania: U.S. Geological Survey Water-Resources Investigations 77-12, 35 p.
- _____, 1979, Flow routing in the Susquehanna River basin: Part III-Routing reservoir releases in the Tioga and Chemung Rivers System, Pennsylvania and New York: U.S. Geological Survey Water-Resources Investigations 79-85, 34 p.
- Barker, J. L., 1978, Characteristics of Pennsylvania Recreational Lakes: Pennsylvania Dept. of Environmental Resources, Water Resources Bulletin No. 14, 226 p.
- Bingham, D. L., 1979, Flow routing in the Susquehanna River basin: Part II-Low-flow frequency characteristics of the Susquehanna River between Waverly, New York and Sunbury, Pennsylvania: U.S. Geological Survey Water-Resources Investigations 79-52, 31 p.
- Karplus, A. K., and Dickey, D. R., 1980, Flow routing in the Susquehanna River Basin: Modified flow-routing models for Juniata and lower Susquehanna Rivers, Pennsylvania, Susquehanna River Basin Commission Technical Report No. 3., 33 p.
- Keefer, T. N., 1974, Desktop computer flow routing: American Society of Civil Engineers Proceedings, Journal of the Hydraulics Division, v. 100, no. HY7, p. 1047-1058.
- Keefer, T. N., and McQuivey, R. S., 1974, Multiple linearization flow routing model: American Society of Civil Engineers Proceedings, Journal of the Hydraulics Division, v. 100, no. HY7, p. 1031-1046.
- Shearman, J. O., Stiltner, G. J., and Doyle, W. H. Jr., 1979, A digital model for streamflow routing by convolution methods: Unpublished data on file in office of Gulf Coast Hydrosience Center, NSTL Station, Mississippi, 118 p.
- Zembrzuski, T. J., Jr., 1980, Flow routing in the Susquehanna River Basin: Part IV- Routing reservoir releases in the eastern part of the Susquehanna River basin in New York State, U.S. Geological Survey Water-Resources Investigations 80-117, 38 p.