

**DOWNSTREAM EFFECTS OF RESERVOIR RELEASES TO THE  
POTOMAC RIVER FROM LUKE, MARYLAND, TO  
WASHINGTON, D.C.**

By Thomas J. Trombley

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## CONVERSION OF MEASUREMENT UNITS

For those readers who may prefer to use metric units instead of inch-pound units, the conversion factors for units used in this report are listed below.

<u>To convert from</u>	<u>Multiply by</u>	<u>To obtain</u>
foot (ft)	0.3048	meter (m)
square foot per second (ft <sup>2</sup> /s)	0.0929	square meter per second (m <sup>2</sup> /s)
cubic foot per second (ft <sup>3</sup> /s)	0.02832	cubic meter per second (m <sup>3</sup> /s)
mile (mi)	1.609	kilometer (km)
square mile (mi <sup>2</sup> )	2.590	square kilometer (km <sup>2</sup> )

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**ABSTRACT**

A digital computer flow-routing model was used to determine the downstream effects on the Potomac River of flow releases from the Bloomington and Savage River Reservoirs. Both reservoirs are located upstream from Luke, Maryland, approximately 230 miles upstream from Washington, D.C.

The downstream effects of reservoir releases were determined by using the unit-response method of flow routing implemented by a diffusion analogy. Results are in the form of unit-response coefficients which are used to route flows downstream from Luke, Maryland.

A 24-hour sustained reservoir release input at Luke will result in 35 percent of the flow arriving at Washington, D.C., during the fourth day after the beginning of the release, followed by 61 percent and 4 percent arriving on the fifth and sixth days, respectively. For a 7-day sustained reservoir release, 47 percent of the flow will arrive during the first week and 53 percent will arrive during the second week.

## INTRODUCTION

### Background

The Potomac River basin (fig. 1) has a drainage area of 11,560 mi<sup>2</sup> upstream from the gaging station near Washington, D.C. (station 01646500). Mean daily discharge (adjusted for diversions) at that gaging station was 11,490 ft<sup>3</sup>/s for the period March 1930 through September 1980. A mean daily diversion of approximately 500 ft<sup>3</sup>/s provides over 60 percent of the water supply for the Washington metropolitan area. These diversions are less than 5 percent of the mean daily flow.

The lowest observed streamflow at Washington, D.C., (adjusted for diversions) occurred in 1966 with 610 and 601 ft<sup>3</sup>/s observed on September 9 and 10, respectively. Diversions for those 2 days were 489 and 449 ft<sup>3</sup>/s, which is approximately three-fourths of the total flow. Obviously, if water-supply demands should increase and/or more severe droughts should occur, it may be impossible to satisfy the demands with the available streamflow. In addition, the remaining streamflow may not be adequate to prevent water-quality problems from developing downstream from Washington. To augment streamflow at Washington during low flow periods, the Bloomington Reservoir on the Potomac River and the Savage Reservoir on the Savage River are available. Both reservoirs are located upstream from Luke, Md., about 230 mi upstream from Washington (see fig. 1).

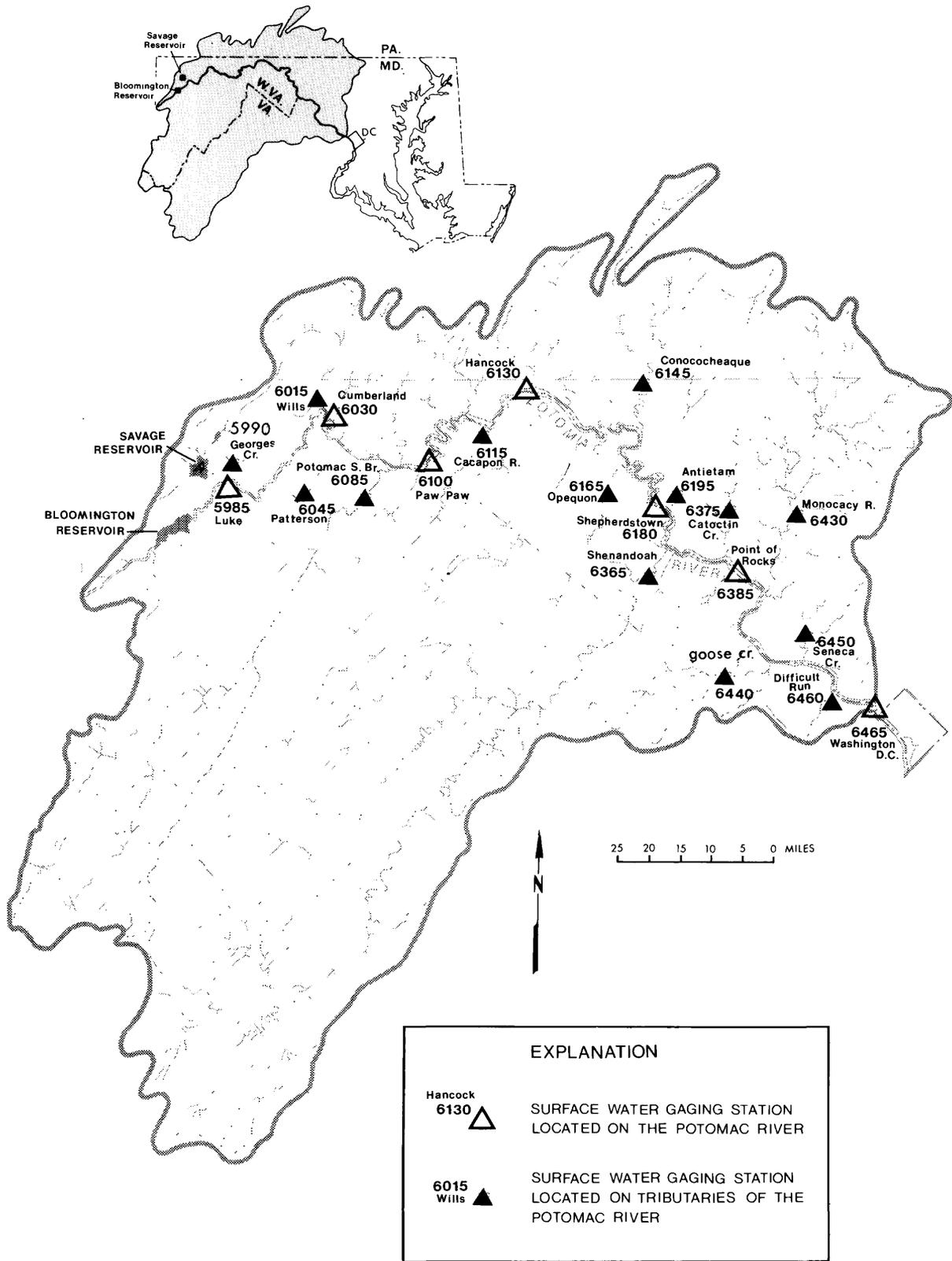


Figure 1.- Location of study area and gaging stations used to model streamflow.

## Purpose and Scope

This report describes a method of estimating downstream responses of reservoir releases from the Bloomington and Savage Reservoirs in the upper Potomac River basin. A flow-routing model is used to route reservoir releases down the river to Washington, D.C. The model yields unit-response coefficients that provide a simple method of estimating the time at which unit releases from the reservoirs will arrive at each of the following downstream stations:

Cumberland, Md.  
Paw Paw, W.Va.  
Hancock, Md.  
Shepherdstown, W.Va.  
Point of Rocks, Md.  
Washington, D.C.

This study was conducted by the U.S. Geological Survey, in cooperation with the U.S. Army Corps of Engineers, Baltimore District.

## MODELING APPROACH

A flow-routing model was applied to six subreaches on the Potomac River using the unit-response method. The subreach models were then calibrated and linked together to produce a final model. The computer program used to model streamflow was developed by J. O. Shearman, Gloria Stiltner, and W. H. Doyle, Jr. (Shearman, 1980, written commun.).

### Unit Response

Streamflow was modeled using the unit-response method of flow routing (Sauer, 1973). Unit response is defined as the downstream response to a unit flow input at the upstream end of the reach (fig. 2). It is analagous to the unit-hydrograph method of surface runoff. In this method, a unit-response function in the form of daily routing coefficients is applied to the input flow at the upstream end of the reach to route that flow to the downstream end of the reach (fig. 3). The discrete equation for the unit-response method of flow routing is:

$$y_t = \sum_{k=0}^{\infty} U_k X_{(t-k)}$$

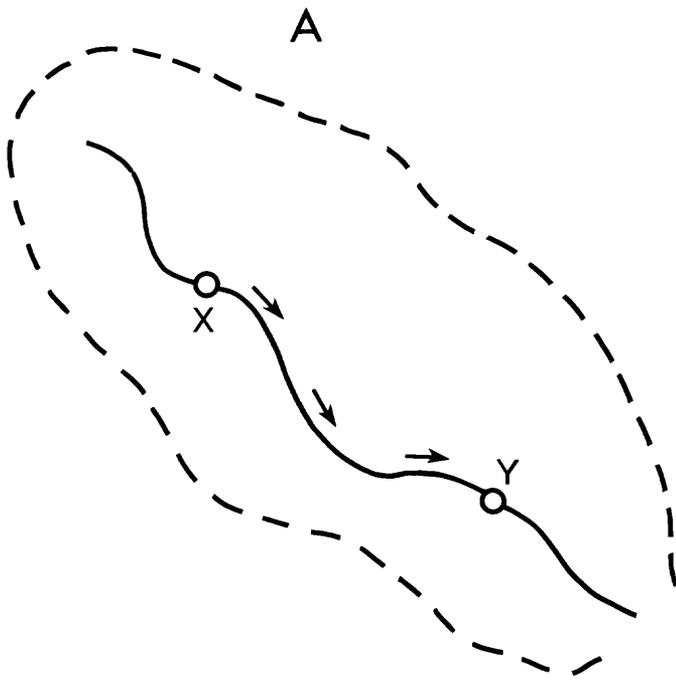
where

$y_t$  = outflow at time (t);

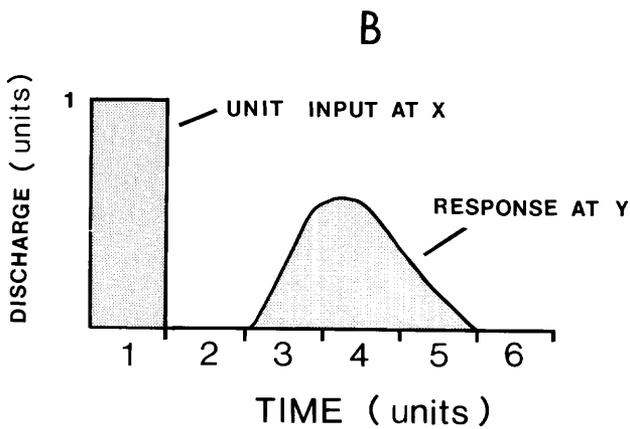
$X_{(t-k)}$  = inflow at time (t-k); and

$U_k$  = unit response coefficient for lag (k).

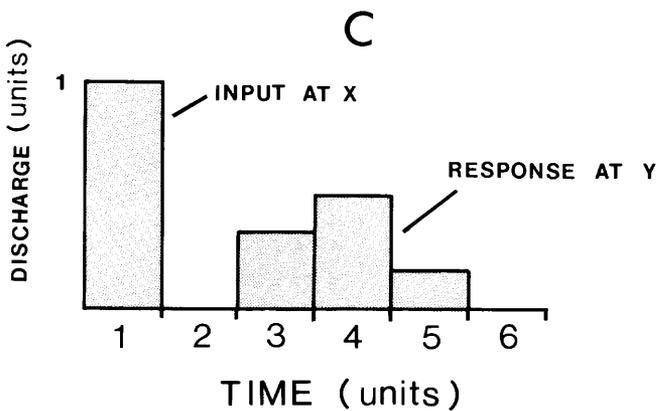
Any ungedged intervening flow or other gains or losses must be explicitly accounted for and added to, or subtracted from the routed flow.



A. Hypothetical drainage basin showing reach X - Y.



B. Unit flow input at x with unit duration, analog unit response function shows the Instantaneous flow at Y.



C. Same unit input at X, but unit response functions in terms of flow/unit time.

Figure 2.-- Development of the unit - response function.

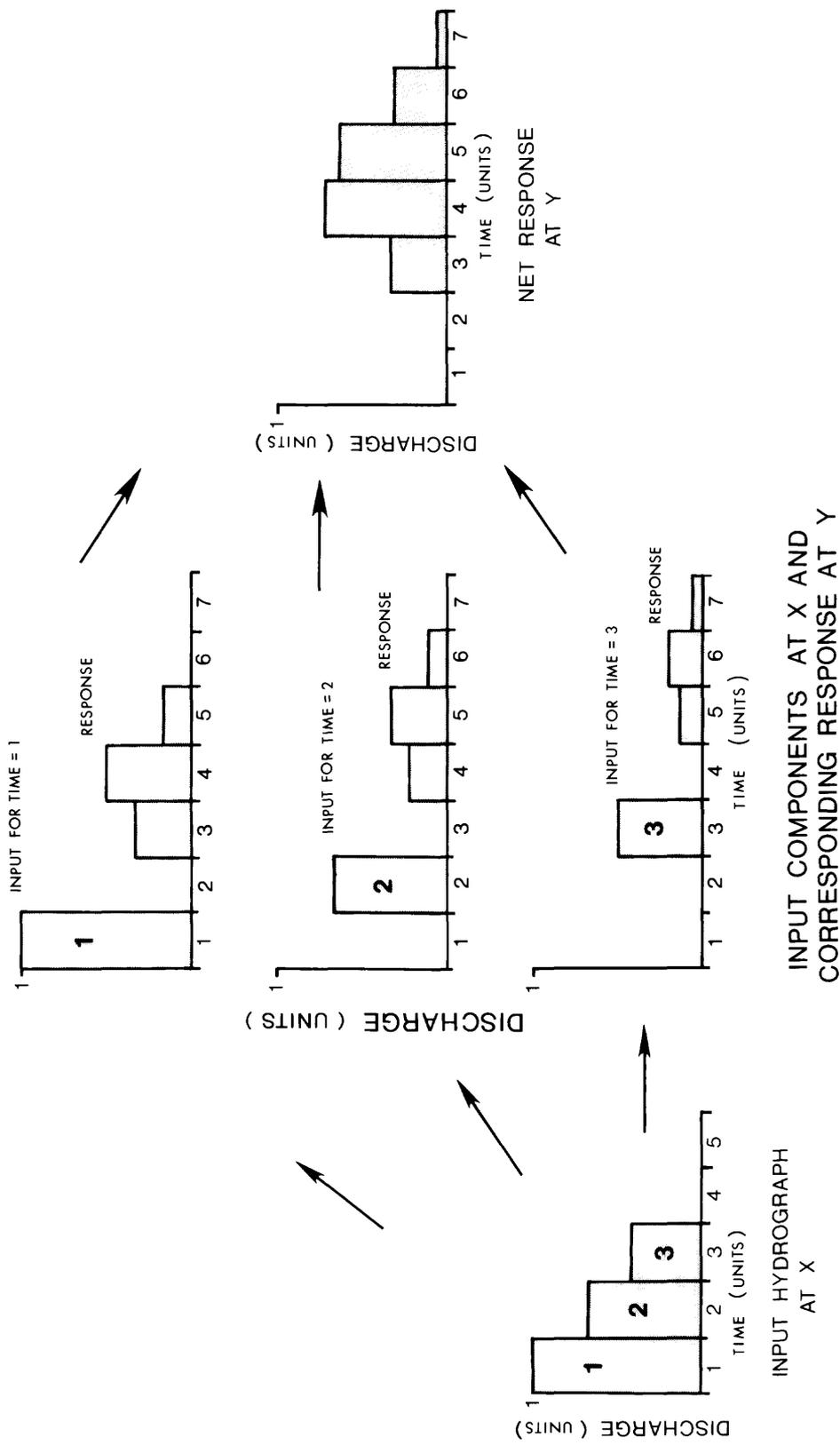


Figure 3 -- Application of the unit-response function to an input hydrograph to simulate the downstream response.

Unit-response functions were calculated using the diffusion approximation to the dynamic equations of open-channel flow (Keefer and McQuivey, 1974). This approximation describes the flow in terms of an input pulse that travels down the channel, spreading out or diffusing as it travels. Three parameters are needed to apply the diffusion analogy:

- 1.) Wave dispersion (K), which defines damping of the wave or flow pulse as it moves downstream,

$$K = \frac{Q}{(2SW)}$$

where

Q = reference discharge in ft<sup>3</sup>/s;  
S = average surface slope at Q; and  
W = average channel width at Q.

- 2.) Wave celerity (C), which is the downstream velocity of the wave,

$$C = \frac{1}{W} \frac{dQ}{dy}$$

where  $\frac{dQ}{dy}$  = slope of discharge/stage at Q.

- 3.) Reach length (X), which is the distance, in miles, that the flow has to travel.

The method used in this study combines system inputs with a unit-response function to produce a system output. In the final linked model, system input is the streamflow at Luke plus gaged tributary inflows, and inflows from ungaged areas between Luke and Washington, D.C. The unit-response function is a series of routing coefficients which convey daily flows through the system from Luke to Washington, D.C, and to intermediate points, with proper accounting for traveltime and dispersion. The system output is the total streamflow at Washington, D.C., and at the intermediate points. This model treats the system as if the unit-response is independent of discharge. That is, the response is the same for all flows.

## Hydrologic Input Data

Streamflow records from 7 mainstem Potomac River gaging stations and 16 tributary gaging stations were used in the modeling process. Table 1 lists the station numbers, names, and drainage areas above the stations as well as the water years for which flow data were used for model calibration. The locations of these gaging stations within the Potomac River basin are shown in figure 1.

Table 1. -- Gaging stations used in modeling process

Station No. <sup>1</sup>	Station name <sup>1</sup>	Drainage area <sup>1</sup> (mi <sup>2</sup> )	Period of record used
01598500	North Branch Potomac River at Luke, Md.	404	1950-78
01599000	Georges Creek at Franklin, Md.	72.4	"
01601500	Wills Creek near Cumberland, Md.	247	"
01603000	North Branch Potomac River near Cumberland, Md.	875	"
01604500	Patterson Creek near Headsville, W. Va.	219	"
01608500	South Branch Potomac River near Springfield, W. Va.	1,471	"
01610000	Potomac River at Paw Paw, W. Va.	3,109	"
01611500	Cacapon River near Great Capon, W. Va.	677	"
01613000	Potomac River at Hancock, Md.	4,073	"
01614500	Conococheague Creek at Fairview, Md.	494	"
01616500	Opequon Creek near Martinsburg, W. Va.	272	"
01618000	Potomac River at Shepherdstown, W. Va.	5,936	1950-53;1965-78
01619500	Antietam Creek near Sharpsburg, Md.	281	1950-78
01636500	Shenandoah River at Millville, W. Va.	3,040	"
01637500	Catoctin Creek near Middletown, Md.	66.9	"
01638500	Potomac River at Point of Rocks, Md.	9,651	"
01643000	Monocacy River at Jug Bridge near Frederick, Md.	817	"
01644000	Goose Creek near Leesburg, Va.	336	"
01645000	Seneca Creek at Dawsonville, Md.	101	"
01646000	Difficult Run near Great Falls, Va.	56	"
01646500	Potomac River near Washington, D.C.	11,500	1950-79

<sup>1</sup> U.S. Geological Survey (1981).

## SUBREACH MODELS

A flow-routing model was applied to six subreaches on the Potomac River. The endpoints of each subreach are at U.S. Geological Survey stream-gaging stations. The six subreaches modeled are:

Luke, Md., to Cumberland, Md.  
Cumberland, Md., to Paw Paw, W. Va.  
Paw Paw, W. Va., to Hancock, Md.  
Hancock, Md., to Shepherdstown, W. Va.  
Shepherdstown, W. Va., to Point of Rocks, Md.  
Point of Rocks, Md. to Washington, D.C.

Luke, Md., was used as the most upstream input station because it is the furthest upstream gaging station below both Bloomington and Savage Reservoirs. The subreach models permitted maximum use of available observed streamflow data, and minimized modeling errors.

The subreach models were calibrated using the following steps:

- 1.) Each subreach model was run using initial values (table 2) for dispersion and celerity that were computed using methods suggested by Keefer (1974).
- 2.) Differences (errors) between simulated and observed flows for the 1950-78 water years were evaluated. Daily volume errors, total volume errors, and root mean square (rms) errors were considered.
- 3.) Adjustments were made to the input parameter values and estimates of the flow from the ungaged area. Additional model runs were made to:
  - (a) reduce the total volume error as much as possible,
  - (b) distribute the daily errors evenly about zero, and to
  - (c) reduce the rms error as much as possible.
- 4.) Finally, a visual comparison was made of the simulated and observed hydrographs for the water years in which the errors were the highest, for 1966, which was a low flow year, and for 1972, which was a high flow year.

Table 2. -- Initial values for modeling parameters

Reach	$X^1$ <sup>2</sup> (mi)	$Q^1$ <sup>3</sup> (ft <sup>3</sup> /s)	$s^1$ <sup>4</sup>	$W^1$ (ft)	K (ft <sup>2</sup> /s)	$\frac{dQ}{dy}$ <sup>5</sup> (ft <sup>2</sup> /s)	C (ft/s)
Luke to Cumberland	33.7	990	0.0020	150	1,600	760	5.1
Cumberland to Paw Paw	27.6	2,300	.00067	220	7,800	1,200	5.5
Paw Paw to Hancock	38.0	3,700	.00052	320	11,000	1,500	4.7
Hancock to Shepherdstown	55.0	5,200	.00035	510	15,000	2,300	4.5
Shepherdstown to Point of Rocks	24.5	7,800	.00062	800	2,000	3,400	3.9
Point of Rocks to Washington	42.1	10,000	.00016	1,300	24,000	7,000	5.4

<sup>1</sup> U.S. Geological Survey (1981).

<sup>2</sup> Difference between downstream river mile and upstream river mile.

<sup>3</sup> Mean of the mean flow for period of record of upstream and downstream stations.

<sup>4</sup> Difference between gage datum for upstream and downstream stations divided by the reach length.

<sup>5</sup> Estimate based on stream widths shown on 1:24000 USGS topographic maps.

<sup>6</sup> Determined from rating tables for upstream and downstream stations.

### Luke to Cumberland Calibration

The segment of the Potomac River between Luke and Cumberland, Md., is the most upstream subreach that was modeled (fig. 4). A detailed description of the calibration of this subreach follows in order to illustrate calibration of all the subreaches.

The drainage area upstream from Luke is 404 mi<sup>2</sup>. At Cumberland the drainage area is 875 mi<sup>2</sup>; therefore, the intervening drainage area of the subreach is 471 mi<sup>2</sup>. There are two gaged tributaries which were used in the model. Georges Creek, which flows into the North Branch Potomac just downstream from Luke, has a gaged area of 72.4 mi<sup>2</sup>. Wills Creek, with a gaged area of 247 mi<sup>2</sup>, flows into the North Branch Potomac just upstream from Cumberland. The ungaged area upstream from Cumberland is 152 mi<sup>2</sup>.

Georges Creek flow was added to the observed flow at Luke and the summed flow was then routed to Cumberland. Flow from Wills Creek and ungaged flow were then added to the routed flow.

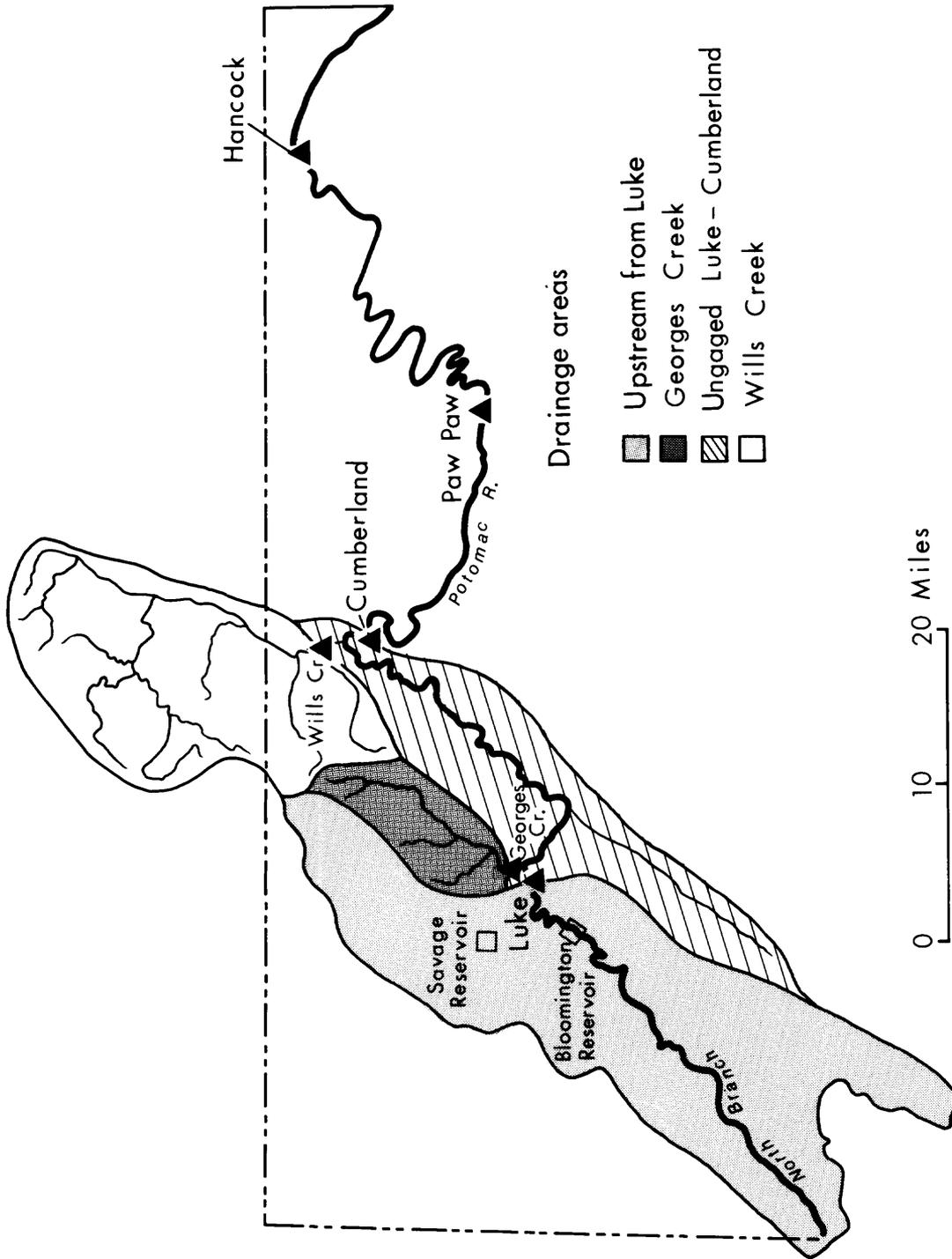


Figure 4.-- Potomac River basin upstream from Cumberland, Md.

The most significant problem with subreach calibration is determining the flow contribution from the ungaged area. Ungaged intervening flow was initially estimated by multiplying the flows from Georges Creek and Wills Creek by index values based on the areal ratio of their drainage basins to the ungaged area. Using the Georges Creek drainage area, a straight drainage-area ratio yields an index value of  $152 \text{ mi}^2 / 72.4 \text{ mi}^2 = 1.10$ . Using the Wills Creek drainage area, the index is  $152 \text{ mi}^2 / 247 \text{ mi}^2 = 0.615$ . Neither of the above index values accurately simulated ungaged flow. By adjusting the index values for which both Wills and Georges Creeks flows were multiplied, volume errors were eliminated, but the distribution of positive and negative errors was still unacceptable.

Because of the above factors, it was necessary to use a more complex method to estimate intervening ungaged flow. The result was an estimation of part of the ungaged flow by multiplying Georges Creek flows by an index of 1.2 before routing. The rest of the ungaged flow was accounted for by applying power curves to Wills Creek flows as follows:

$$\begin{aligned} \text{QINTR1} &= 1.348 \text{ WILLS}^{0.758} && \text{if WILLS} \leq 290 \\ &0.175 \text{ WILLS}^{1.12} && \text{if WILLS} > 290. \end{aligned}$$

where

$$\begin{aligned} \text{QINTR1} &= \text{part of ungaged flow, in ft}^3/\text{s}; \text{ and} \\ \text{WILLS} &= \text{discharge at Wills Creek, in ft}^3/\text{s}. \end{aligned}$$

The calibrated subreach model for Luke to Cumberland is:

$$\text{CUMBERLAND} = (\text{LUKE} + 1.2 \text{ GEORGES})\text{route} + \text{WILLS} + \text{QINTR1}$$

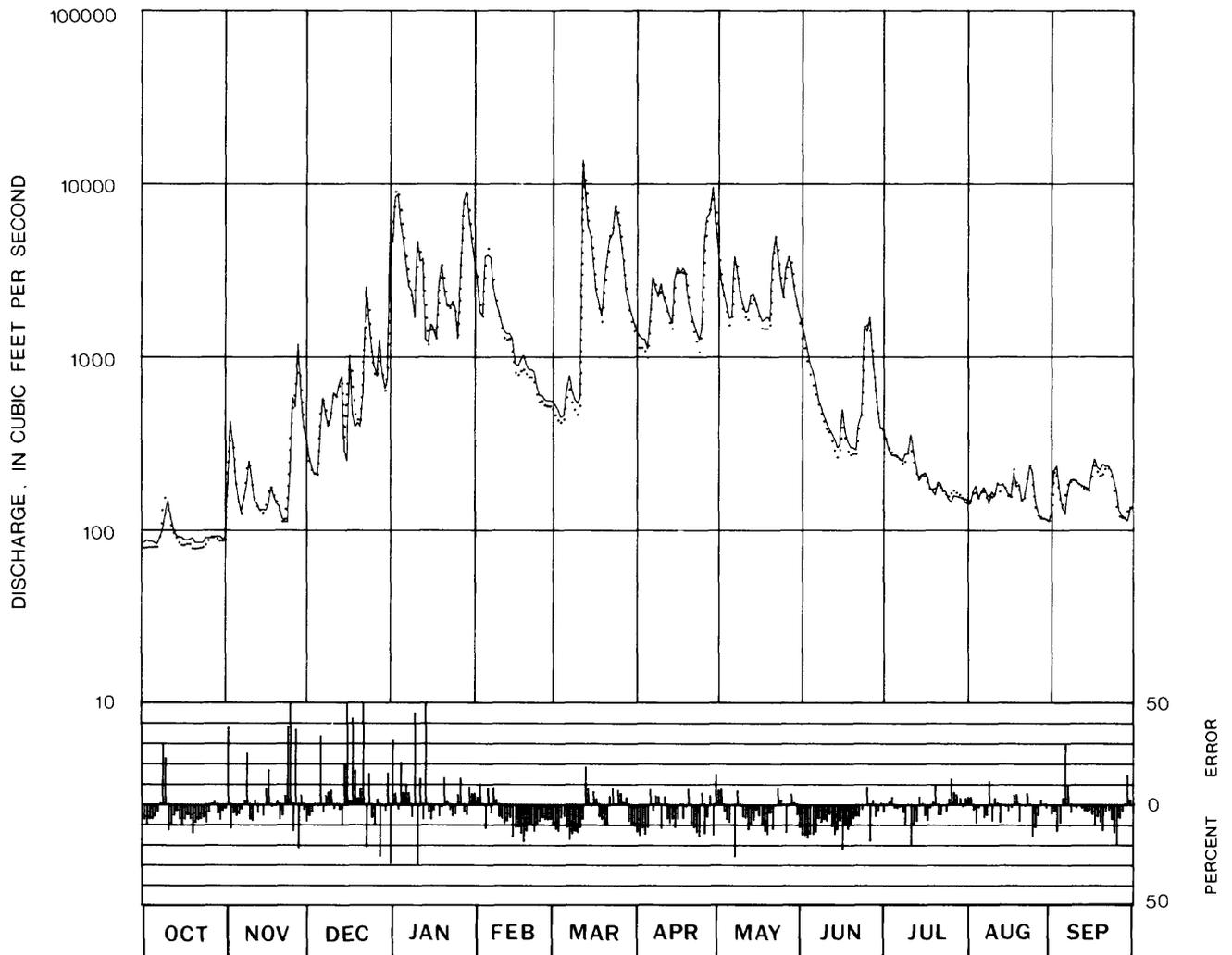
where

$$\begin{aligned} \text{CUMBERLAND} &= \text{discharge at the Cumberland gage;} \\ \text{LUKE} &= \text{discharge at the Luke gage;} \\ \text{GEORGES} &= \text{discharge at the Georges Creek gage;} \text{ and} \\ \text{route} &= \text{routing process demonstrated in} \\ &\text{figure 3.} \end{aligned}$$

The input parameters used for routing streamflow in this model are: Reach length (X) = 33.7 mi, dispersion (K) = 1,500 ft<sup>2</sup>/s, and celerity (C) = 3.60 ft/s. The resulting routing coefficients for the unit-response function are: 0.42 for day 1, and 0.58 for day 2. This means that for any given day, 42 percent of the observed flow at Luke will pass Cumberland on the 1<sup>st</sup> day and 58 percent will pass Cumberland on the 2d day.

Figures 5 through 8 are hydrographs of observed and simulated flows and modeling errors for the Luke to Cumberland subreach for the water years 1952, 1960, 1966, and 1972, respectively. Observed flows are plotted as lines on the hydrographs and simulated flows are plotted as points. Below each hydrograph, daily modeling errors are plotted as percentage deviations of simulated flow from observed flow.

The rms error for 1952 is the highest of all the water years modeled and most of the daily errors (66 percent) were negative. In 1960, most of the daily errors were positive. Flow was abnormally low in 1966 and was abnormally high in 1972. These hydrographs show that for the model of the Luke to Cumberland subreach, simulated flows closely match observed flows.



**EXPLANATION**

- SIMULATED FLOW    .....
- OBSERVED FLOW      —————
- ERROR ( Percent )     $\frac{(\text{Simulated} - \text{Observed}) (100)}{\text{Observed}}$

Figure 5.-- Observed and simulated flow and modeling errors at Cumberland Md., for the 1952 water year.

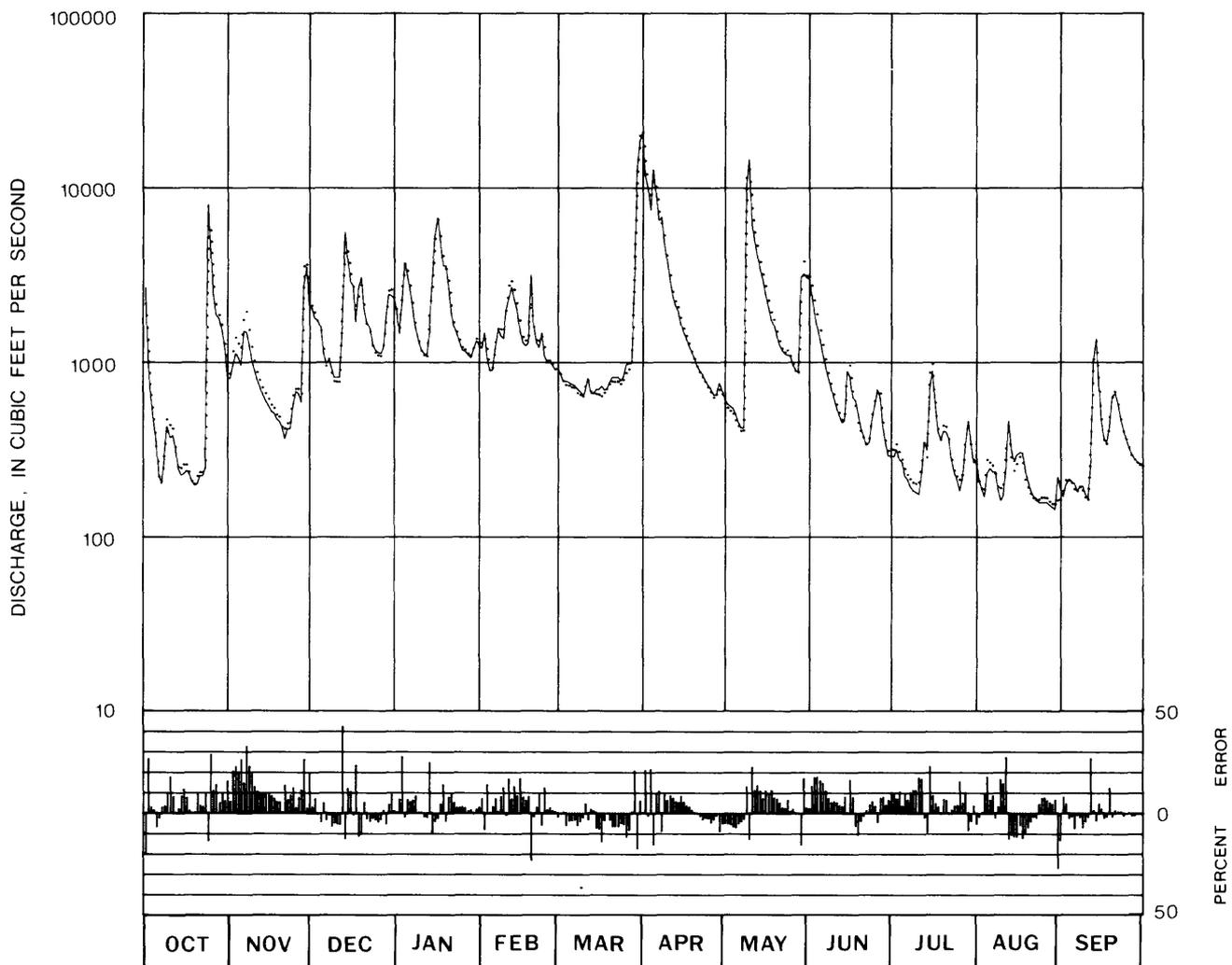
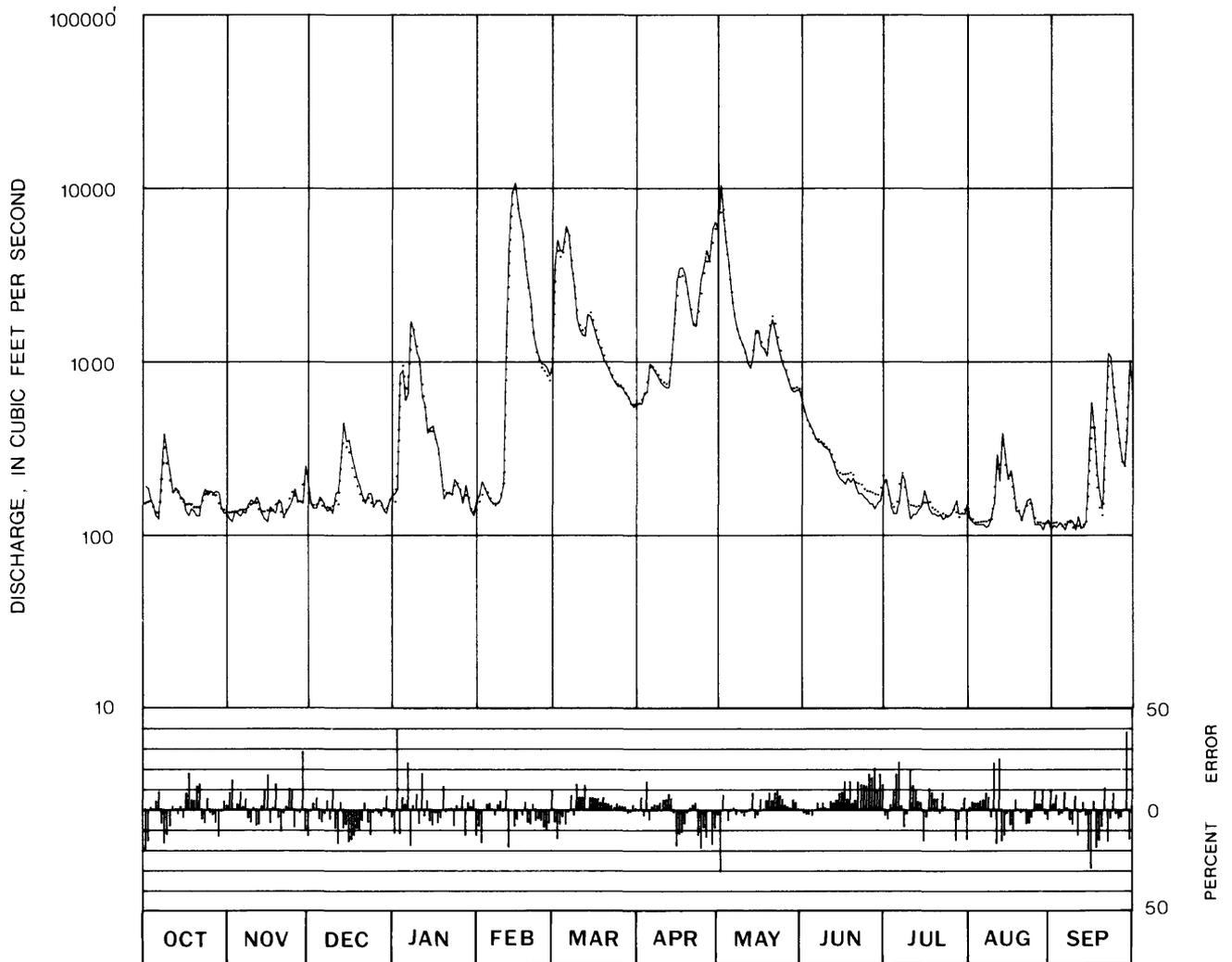


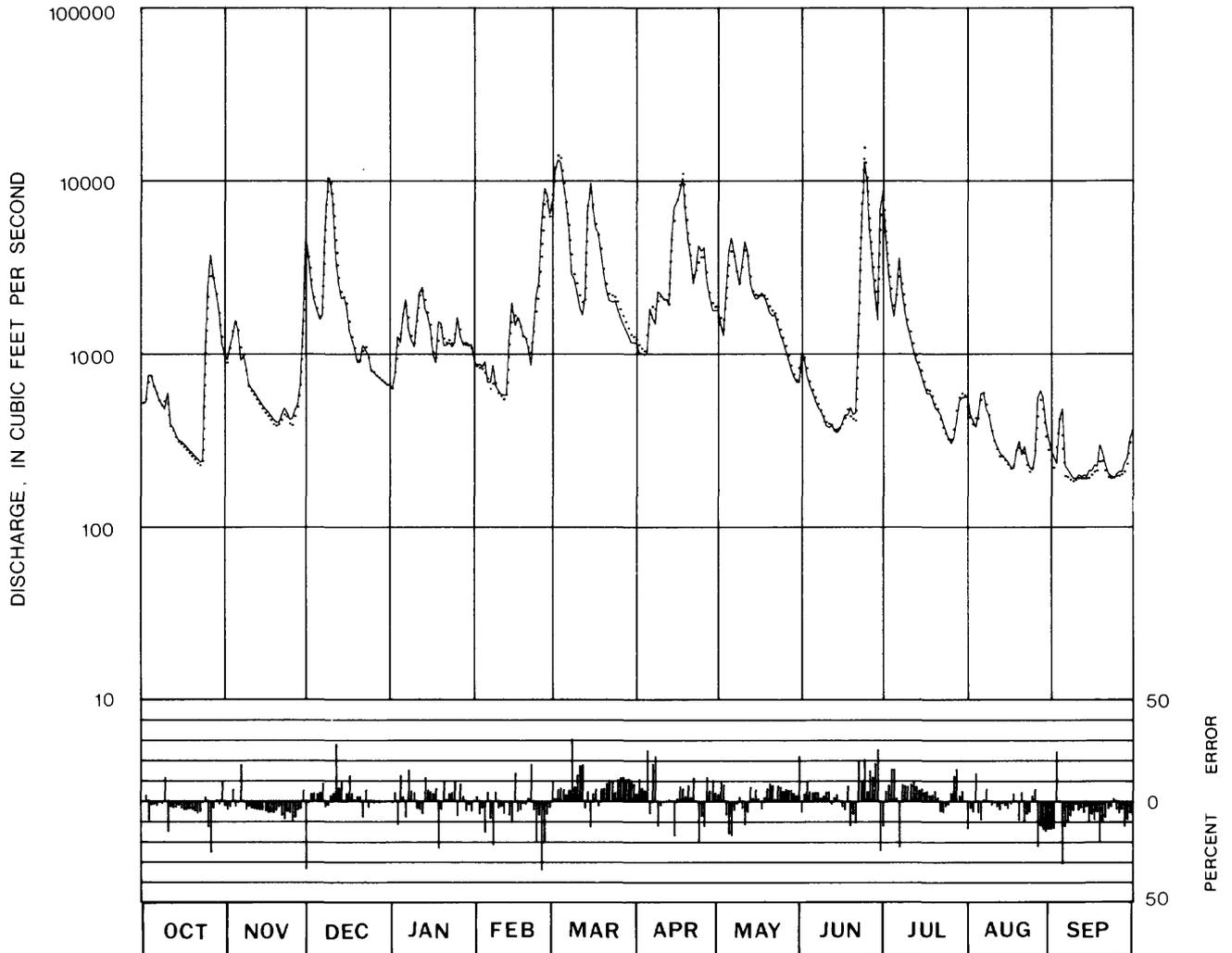
Figure 6 .-- Observed and simulated flow and modeling errors at Cumberland Md., for the 1960 water year.



**EXPLANATION**

SIMULATED FLOW    .....  
 OBSERVED FLOW    \_\_\_\_\_  
 ERROR ( Percent )     $\frac{(\text{Simulated} - \text{Observed}) (100)}{\text{Observed}}$

Figure 7.-- Observed and simulated flow and modeling errors at Cumberland Md. , for the 1966 water year.



**EXPLANATION**

SIMULATED FLOW    .....  
 OBSERVED FLOW    \_\_\_\_\_  
 ERROR ( Percent )     $\frac{(\text{Simulated} - \text{Observed}) (100)}{\text{Observed}}$

Figure 8.-- Observed and simulated flow and modeling errors at Cumberland Md., for the 1972 water year.

Table 3 summarizes modeling errors for each of the years modeled. During the 29 years over which the model was calibrated, approximately half of the daily errors are negative and half are positive; also, the mean absolute error is 7.33 percent, and the net yearly volume error is negative for 15 years and positive for 14 years, with a total volume error of -0.70 percent. The root mean square (rms) error for the final Luke-to-Cumberland subreach model over the 29-year calibration period appears to be minimized and is equal to 10.74 percent.

Table 4 summarizes the daily errors or deviations in terms of number of deviations between the indicated percentages over different discharge ranges. It is important to note that overall, the distribution of positive and negative errors is approximately equal, and that 46 percent of the time, deviations are between plus and minus 5 percent, and that 76 percent of the time, deviations are between plus and minus 10 percent.

The final routing model for the subreach between Luke and Cumberland was accepted because:

- 1.) Hydrographs of simulated and observed flows compare well for selected water years.
- 2.) Total volume error is small and the number of years in which net volume error was negative compares well with the number of years in which the net volume error was positive.
- 3.) Daily volume errors or deviations are evenly distributed with more than three-fourths of them falling between plus and minus 10 percent.
- 4.) The rms errors appear to be minimized.

Table 3. -- Annual summary of modeling errors (percent) for the Luke to Cumberland, Md., subreach.

Year	Mean error	Negative error days	Mean negative error	Positive error days	Mean positive error	Volume error	Rms error
1950	7.99	61	-7.25	39	9.14	-3.07	13.36
1951	6.80	62	-6.02	38	8.07	0.16	9.38
1952	8.81	67	-7.91	33	10.61	-1.50	14.97
1953	5.30	43	-4.80	57	5.68	1.48	7.61
1954	5.74	50	-5.68	50	5.79	-1.59	8.41
1955	8.00	64	-7.71	36	8.50	-1.68	13.84
1956	8.08	51	-7.54	49	8.65	-1.28	15.24
1957	6.99	52	-7.49	48	6.47	1.72	9.09
1958	7.29	42	-5.53	58	8.56	1.51	10.27
1959	7.46	50	-6.78	50	8.14	3.17	11.08
1960	7.42	37	-5.22	63	8.72	3.56	9.84
1961	7.43	53	-6.93	47	7.99	1.63	9.92
1962	8.67	63	-8.82	37	8.41	-4.42	11.84
1963	7.20	42	-7.28	58	7.15	-3.21	9.74
1964	6.38	38	-6.31	62	6.43	1.92	8.77
1965	5.93	52	-6.58	48	5.22	-3.42	9.00
1966	6.67	46	-6.94	54	6.44	-3.39	9.03
1967	6.53	35	-5.79	65	6.94	-0.54	9.09
1968	7.03	44	-5.26	56	8.40	2.11	9.83
1969	8.32	32	-6.99	68	8.94	3.16	10.93
1970	10.28	30	-6.78	70	11.79	4.43	13.83
1971	9.31	23	-7.30	77	9.91	2.44	12.83
1972	6.92	51	-6.68	49	7.19	0.55	9.21
1973	8.62	81	-8.99	19	7.01	-6.28	11.05
1974	7.70	69	-7.37	31	8.44	-4.34	11.12
1975	6.86	69	-7.36	31	5.75	-2.88	9.76
1976	5.83	60	-5.44	40	6.42	-2.41	8.57
1977	6.20	56	-5.56	44	7.01	-2.03	8.38
1978	6.72	50	-6.51	50	6.92	-1.98	9.94
Total	7.33	51	-6.84	49	7.83	-0.70	10.74

Table 4. -- Distribution of daily error with discharge for Luke-to-Cumberland, Md. subreach model.

Discharge (ft <sup>3</sup> /s)	Number of days that daily error was between specified percentages															Total
	<	-30	-25	-20	-15	-10	-5	0	5	10	15	20	25	30	>	
0 -																
83 -	0	0	0	0	3	19	7	3	0	1	1	0	0	0		
98 -	0	0	0	1	2	6	16	59	34	7	3	1	7	1		
120 -	0	0	0	3	7	26	89	125	71	17	7	1	2	1		
140 -	0	0	1	1	20	45	104	142	63	17	17	5	4	4		
160 -	0	0	0	10	47	101	125	151	105	42	23	12	2	11		
190 -	1	2	4	20	53	103	176	167	76	40	15	8	2	7		
230 -	0	2	4	16	41	102	150	106	64	22	15	8	2	8		
270 -	0	0	3	13	45	99	132	114	52	21	10	8	4	7		
320 -	0	1	1	12	29	89	127	116	46	25	11	11	1	8		
380 -	1	0	1	9	25	95	139	116	70	30	12	5	6	6		
450 -	2	2	4	9	27	81	149	125	78	36	14	6	2	8		
540 -	1	2	4	13	23	91	134	126	70	50	13	3	3	9		
640 -	0	0	9	7	26	89	158	118	67	37	16	11	4	1		
750 -	0	0	5	16	49	91	116	114	41	32	15	6	2	4		
890 -	3	1	11	21	60	112	135	117	75	42	16	15	9	6		
1,100 -	3	3	9	15	40	92	110	97	73	31	17	8	4	3		
1,300 -	0	4	5	12	30	52	86	73	61	36	15	5	4	3		
1,500 -	0	2	4	10	33	79	120	105	56	33	17	5	3	10		
1,800 -	2	2	4	8	27	63	21	85	55	25	11	4	4	6		
2,100 -	3	4	7	11	24	57	85	87	42	30	14	8	4	1		
2,500 -	3	1	5	6	21	32	66	56	43	11	9	6	2	3		
2,900 -	3	2	4	12	30	40	57	59	57	25	8	4	0	4		
3,500 -	2	4	8	8	18	25	34	47	47	19	2	2	2	1		
4,100 -	3	3	5	6	9	27	32	45	30	16	5	1	4	3		
4,900 -	3	3	0	6	13	12	24	19	24	13	2	4	1	2		
4,900 -	1	2	4	7	5	21	18	18	13	6	3	3	0	3		
5,800 -	0	3	2	4	10	11	18	26	5	5	1	2	1	2		
6,800 -	3	1	3	2	7	5	12	11	11	4	2	0	0	0		
8,100 -	1	0	0	2	3	3	4	6	4	1	0	1	0	0		
9,600 -	1	0	2	6		5	6	3	3	2	0	2	0	0		
11,000 -	0	2	1	1	3	4	1	3	2	0	1	0	0	0		
13,000 -	1	1	0	0	1	2	0	1	1	0	0	0	0	0		
16,000 -	0	1	1	0	0	0	0	1	0	0	0	0	0	0		
19,000 -	0	1	0	0	1	0	0	0	0	0	0	0	0	0		
22,000 -																
SUM	38	49	106	267	739	1674	2511	2447	1439	676	295	155	74	122	10592	
Percent of total	.4	.5	1.0	2.5	7.0	15.8	23.7	23.1	13.6	6.4	2.8	1.5	.7	1.2	100.2	

### Summary of Subreach Results

The other five subreaches were calibrated in the same manner as the Luke to Cumberland subreach. The results of the subreach calibration are shown in table 5, which lists the calibrated subreach models, input parameters, and the unit-response coefficients used for routing flows in each subreach.

The Hancock to Shepherdstown subreach was divided into two parts. In the first part, a reach length of 55 mi is used to route flows from the upper portion of the subreach to Shepherdstown. In the second part, a reach length of 25 mi is used to route flows from the central and lower portions of the subreach.

On the Shepherdstown to Point of Rocks subreach, flow from Catoctin Creek is lagged 6 hours. This is accomplished by the following:

$$Q_{lag_n} = 0.75 Q_{n-1} + 0.25 Q_n.$$

where

$Q_{lag}$  = lagged flow;

$n$  = day in which flow occurs; and

$Q$  = observed flow.

This lagging feature is a method of accounting for the traveltime of flows from the mouth of Catoctin Creek through the Potomac River to Point of Rocks.

Table 5. -- Subreach model summary

Subreach	Model <sup>1</sup>	Routing parameters			Routing coefficient		
		Dispersion K <sub>2</sub> /s ft <sup>2</sup> /s	Celerity C ft/s	Length X mi	Day 1	Day 2	Day 3
Luke to Cumberland Paw Paw	(LUKE + 1.2 GEORGES)routed <sup>2</sup> + WILLS + QINTR1  QINTR1 = 1.35 WILLS <sup>0.750</sup> if WILLS ≤ 290 ft <sup>3</sup> /s 0.175 WILLS <sup>1.12</sup> if WILLS > 290 ft <sup>3</sup> /s	1,500	3.60	33.7	0.418	0.502	
Cumberland to Paw Paw	(CUMBERLAND + 2.0 PATTERSON + SOUTH BRANCH)routed + QINTR3  QINTR3 = 26.4 PATTERSON if PATTERSON ≤ 40 ft <sup>3</sup> /s 4.60 PATTERSON if PATTERSON > 40 ft <sup>3</sup> /s	6,000	3.50	27.6	.518	.402	
Paw Paw to Hancock	(PAW PAW) routed + 1.3 CACAPON	15,000	3.75	36.0	.364	.636	
Hancock to Shepherdstown	(HANCOCK + CONOCOCHIEAGUE + 1.5 OPEQUON) routed (55 mi) + 0.90 CONOCOCHIEAGUE + OPEQUON)routed (25 mi)	20,000	4.25	55.0 25.0	.216 .640	.770 .360	0.006
Shepherdstown to Point of Rocks	(SHEPHERDSTOWN + 1.5 ANTIETAM + SHENANDOAH)routed + CATOCTIN (lagged 6 hours) + 2.60 CATOCTIN	15,000	4.25	24.5	.647	.353	
Point of Rocks to Washington	(POINT OF ROCKS + MONOCACY + GOOSE)routed + SENECA + DIFFICULT + QINTR7  QINTR7 = 55.7 GOOSE <sup>0.362</sup> if GOOSE ≤ 350 ft <sup>3</sup> /s 0.254 GOOSE <sup>1.32</sup> if GOOSE > 350 ft <sup>3</sup> /s and if GOOSE ≤ 1,600 ft <sup>3</sup> /s 2.15 GOOSE if GOOSE > 1,600 ft <sup>3</sup> /s	30,000	3.00	42.1	.193	.756	.051

<sup>1</sup> Station name used to represent flow at gage.

<sup>2</sup> Routed = routing process demonstrated in figure 3.

Table 6 summarizes the modeling errors over the indicated calibration period for each subreach, and table 7 summarizes the daily errors. Four of the subreaches were calibrated for the 1950-78 water years. The Hancock, Md., to Shepherdstown, W. Va., and Shepherdstown to Point of Rocks, Md., subreaches could not be calibrated for this period because Shepherdstown has an incomplete record. These two subreaches were calibrated for the periods 1950-53 and 1965-78. As table 7 shows, mean error for each of the six subreaches is less than 10 percent. In each subreach, there are approximately the same percentage of negative-error and positive-error days, and the mean positive and mean negative errors have an absolute value of less than 10 percent. The rms errors all appear to be minimized.

Although all of the volume errors in table 6 are relatively low, they are all negative except for the 1950-53 water years on the Shepherdstown to Point of Rocks subreach. Volume errors are negative because the model tends to underestimate peak flows. Hence, there are many days of high flow showing a negative volume error. Because a small negative error for a day with high flow can represent a large quantity of water, a net negative volume error can occur with a few high flow periods.

Table 6. -- Summary of modeling errors (percent) for subreach models

Subreach	Calibration period (water years)	Mean error	Negative error days	Mean negative error	Positive error days	Mean positive error	Volume error	RMS error
Luke to Cumberland	1950 - 78	7.33	51	-6.84	49	7.83	-0.70	10.74
Cumberland to Paw Paw	1950 - 78	8.11	48	-7.46	52	8.71	-1.88	12.14
Paw Paw to Hancock	1950 - 78	6.16	49	-5.82	51	6.48	-1.48	12.38
Hancock to Sheperdstown	1950 - 53	8.55	47	-7.28	53	9.89	-3.40	15.40
	1965 - 78	8.22	47	-7.82	53	8.78	-3.81	13.10
Shepherdstown to Point of Rocks	1950 - 53	4.38	49	-3.52	51	5.17	0.10	6.70
	1965 - 78	6.33	48	-5.02	54	7.47	-0.17	9.42
Point of Rocks to Washington	1950 - 78	6.43	50	-6.20	50	6.66	-0.20	10.00

Table 7. -- Summary of daily errors for subreach models

Subreach	Calibration period	Percentage of days that daily error was between indicated percent error													
		< -30	-25	-20	-15	-10	-5	0	5	10	15	20	25	30	>
Luke to Cumberland	1950 - 78	0.4	0.5	1.0	2.5	7.0	15.8	23.7	23.1	13.6	6.4	2.8	1.5	0.7	1.2
Cumberland to Paw Paw	"	.9	.8	1.2	2.6	5.8	14.2	22.5	22.5	14.2	7.2	3.6	1.8	.9	1.6
Paw Paw to Hancock	"	.7	.3	.8	2.0	4.2	11.5	30.2	29.8	12.3	4.2	1.5	.8	.4	1.1
Hancock to Shepherdstown	1950 - 53	1.0	.9	1.5	3.6	4.7	10.4	25.1	28.0	10.5	5.1	2.6	1.6	1.2	3.6
	1965 - 78	1.7	1.2	1.8	2.9	4.5	11.3	26.1	27.1	11.9	6.8	3.5	1.8	.9	2.8
Shepherdstown to Point of Rocks	1950 - 53	0	0	.1	.5	2.1	7.9	38.3	32.8	11.6	3.4	1.6	.5	.5	.4
	1965 - 78	0	0	.2	.8	3.5	15.8	27.7	25.3	14.8	6.1	2.4	1.1	.9	1.2
Point of Rocks to Washington	1950 - 79	.5	.6	1.0	1.6	5.1	14.4	27.0	27.3	13.0	4.7	2.1	.9	.5	.9

### LINKED MODEL

To implement the flow-routing model for the Potomac River between Luke, Md., and Washington, D.C., it was necessary to link the subreach models together. The linked model uses only the observed flow at Luke, observed tributary flow, and estimated ungaged intervening flow to simulate the flow at the downstream end of each subreach. The results of the model are the routing coefficients used to transport water downstream from Luke. The model was calibrated for the 1950-78 water years.

The standard method used to link subreach models is to use the simulated daily flow from an upper subreach model as input to the next lower subreach model. One of the problems with this method is that it can cause significant timing errors. These errors could be as much as 12 hours per reach.

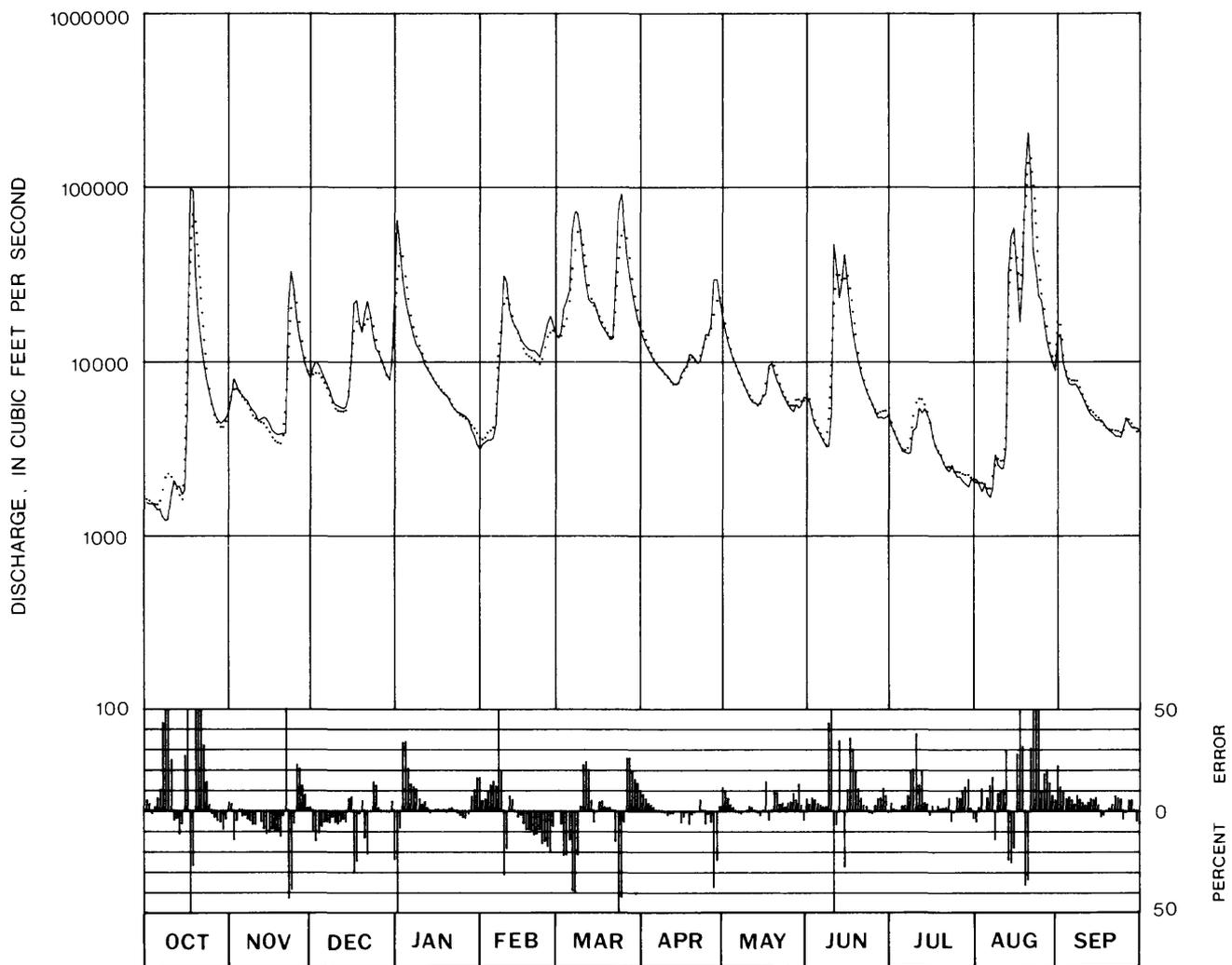
To link the subreaches in this study, hourly routing coefficients were generated by each of the subreach models. These hourly coefficients were then combined and re-expressed as daily routing coefficients for the linked model. Table 9 gives the daily coefficients resulting from the combined hourly coefficients.

Figures 9 through 12 are hydrographs of observed and simulated flows at Washington, D.C., for the 1955, 1966, 1972, and 1978 water years, respectively. For 1955, the rms errors are the highest of the water years modeled and most of the daily errors (60 percent) are positive. The hydrograph for 1966 is shown because that was the year of lowest flow. In 1972, flows were high. For 1978, most of the daily errors (57 percent) were negative. The fit of the simulated flows to the observed flows is generally good. Most of the excessively high errors occurred at or near peaks in the flow and were the result of 1-day timing errors. Because daily flows are used in the model, there is a loss of resolution, which results in these timing errors.

Table 8 summarizes the modeling errors for the linked model from Luke, Md., to Washington, D.C. As would be expected, the errors and deviations are somewhat higher in the linked model than in the subreach model. However, the difference does not appear to be significant.

Table 8. -- Summary of modeling errors  
(percent) for linked model  
(Luke, Md., to Washington, D.C.)

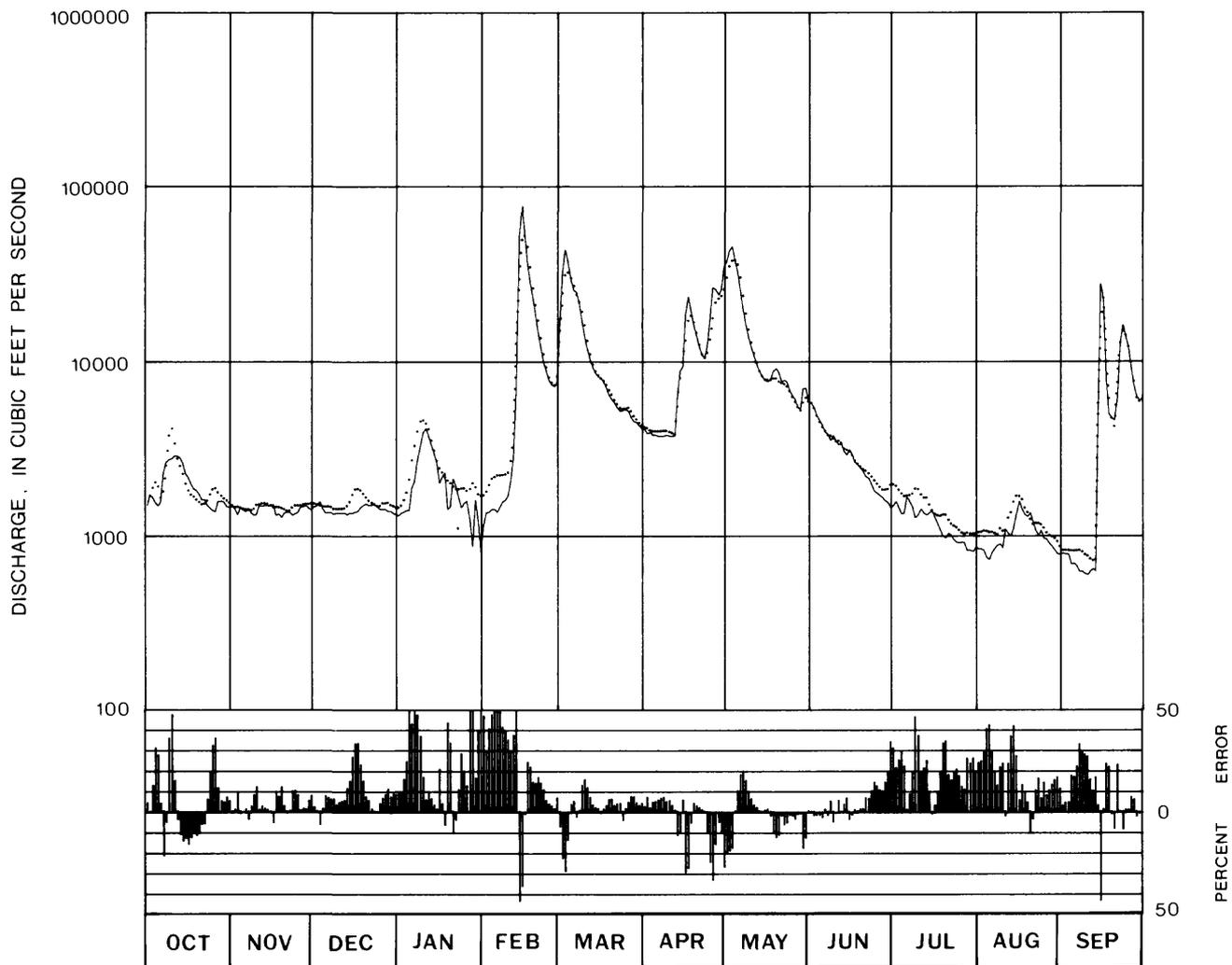
Calibration period	1950 - 78
Mean error	9.57
Negative error days	50
Negative error	-8.77
Positive error days	50
Positive error	10.36
Volume error	-3.66
RMS error	14.69



**EXPLANATION**

- SIMULATED FLOW    .....
- OBSERVED FLOW    \_\_\_\_\_
- ERROR ( Percent )     $\frac{(\text{Simulated} - \text{Observed}) (100)}{\text{Observed}}$

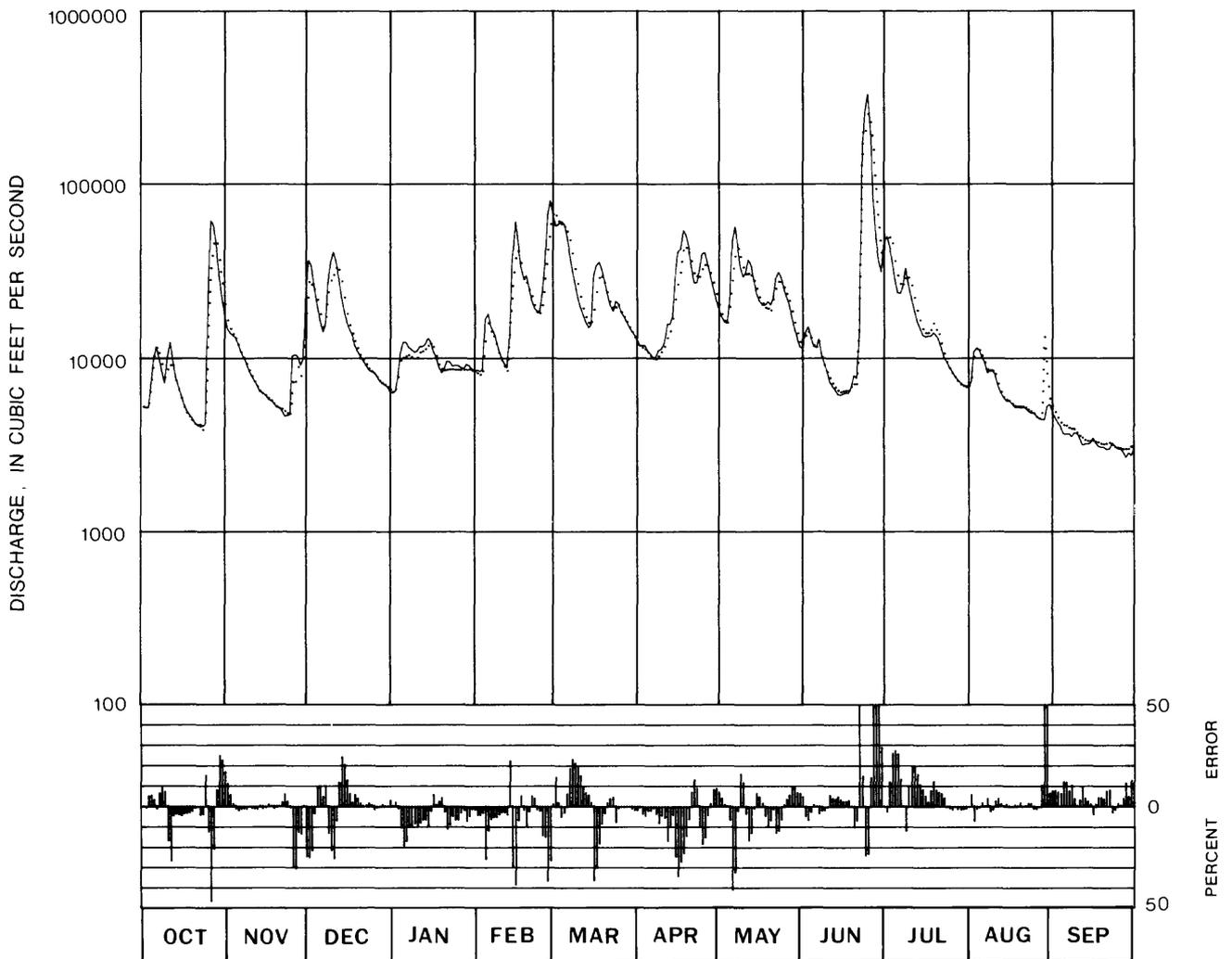
Figure 9.-- Observed and simulated flow and modeling errors at Washington, D.C., using the linked model for the 1955 water year.



**EXPLANATION**

- SIMULATED FLOW    .....
- OBSERVED FLOW    \_\_\_\_\_
- ERROR ( Percent )     $\frac{(\text{Simulated} - \text{Observed}) (100)}{\text{Observed}}$

Figure 10.-- Observed and simulated flow and modeling errors at Washington, D.C., using the linked model for the 1966 water year.



**EXPLANATION**

- SIMULATED FLOW    .....
- OBSERVED FLOW    \_\_\_\_\_
- ERROR ( Percent )     $\frac{(\text{Simulated} - \text{Observed}) (100)}{\text{Observed}}$

Figure 11.-- Observed and simulated flow and modeling errors at Washington, D.C., using the linked model for the 1972 water year.

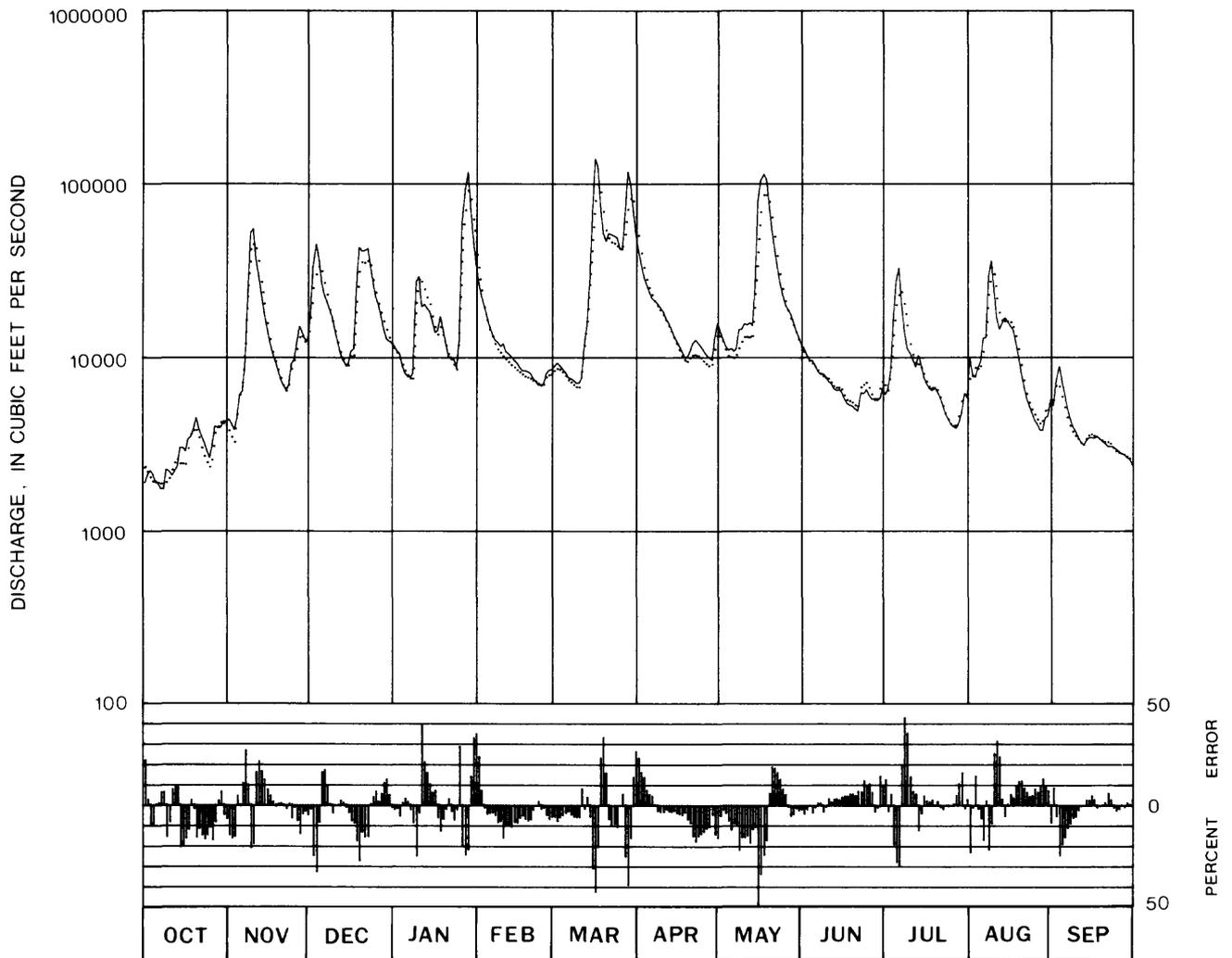


Figure 12.-- Observed and simulated flow and modeling errors at Washington, D.C., using the linked model for the 1978 water year.

## UNIT RESPONSE TO RESERVOIR RELEASES

Unit response to reservoir releases developed by the linked model are expressed in table 9 as daily routing coefficients. The same unit response is expressed in table 10 as 12-hour routing coefficients, and in table 11 as weekly routing coefficients.

Daily coefficients can be used to estimate the response at each of the listed stations for releases made on a daily schedule. For example, if  $100 \text{ ft}^3/\text{s}$  is released for one day,  $35 \text{ ft}^3/\text{s}$  will pass Washington, D.C., on the 4th day,  $61 \text{ ft}^3/\text{s}$  on the 5th day and  $4 \text{ ft}^3/\text{s}$  on the 6th day. The 12-hour and weekly routing coefficients can be used to determine downstream response in the same manner as the daily routing coefficients. The choice of which set of coefficients are used depends on the application and degree of precision required.

Figure 13 illustrates the results of routing a 3-day unit reservoir release from Luke to Washington using a 12-hour unit-response function. Releases for each 12-hour period are shaded to indicate the distribution of flows at Washington. The total 12-hour response is the summation of component responses for that 12-hour period. The daily response, again, is the mean flow for the two 12-hour periods for each day.

Using the model, figure 14 illustrates movement of a 7-day unit input at Luke, spreading out as it flows to Washington. Approximate hourly response is indicated by the curvature at the beginning and end of the response period. It should be noted that essentially all the water has passed Washington by the end of day 11 (4th day of 2d week).

Table 9. -- Daily routing coefficients for linked models

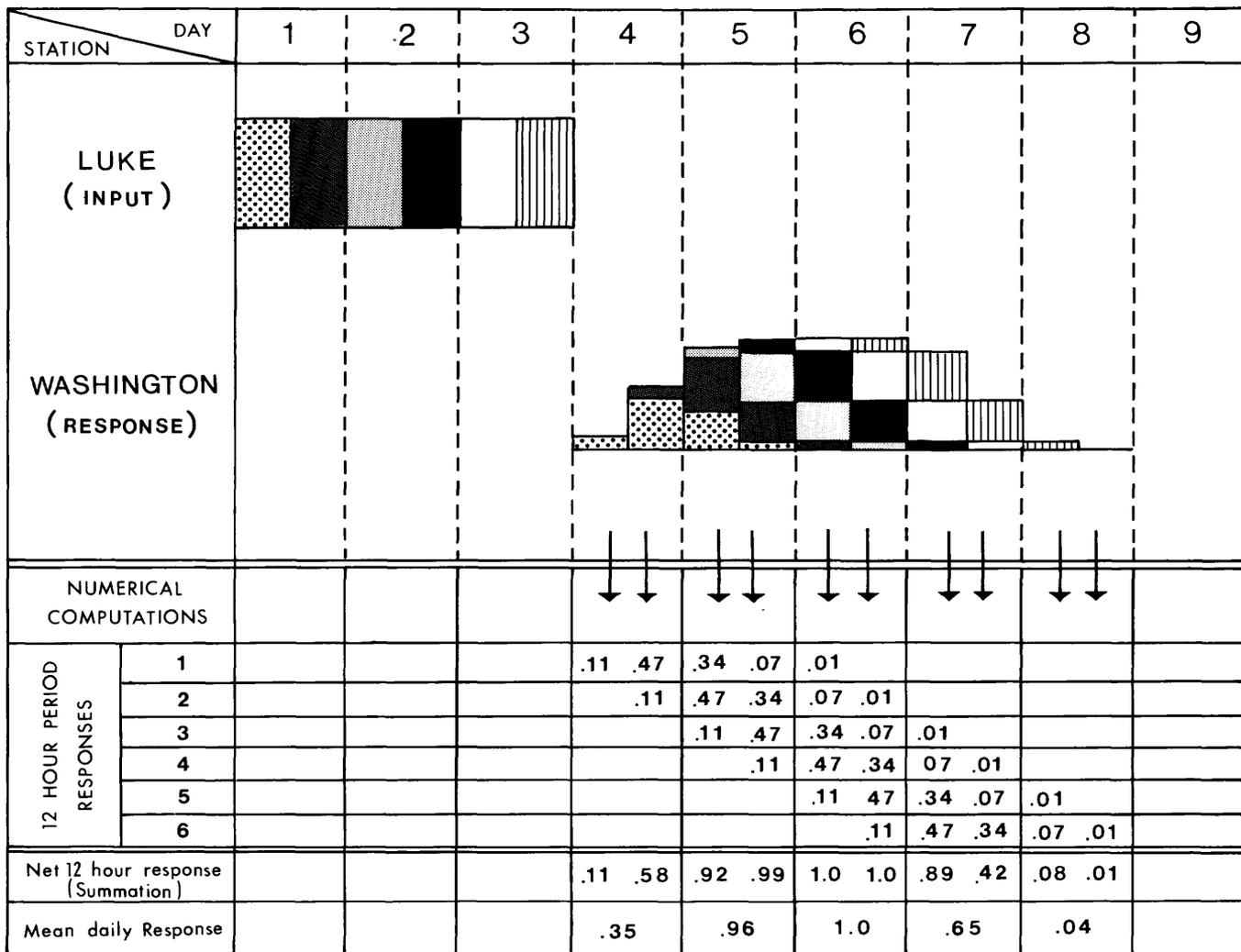
Reach Luke to:	Routing coefficients for day						
	1	2	3	4	5	6	7
Cumberland	0.42	0.58	--	--	--	--	--
Paw Paw	.02	.91	0.07	--	--	--	--
Hancock	--	.31	.69	--	--	--	--
Shepherdstown	--	--	.52	0.48	--	--	--
Points of Rocks	--	--	.20	.77	0.03	--	--
Washington	--	--	--	.35	.61	0.04	--

Table 10. -- Twelve-hour routing coefficients for linked models

Reach Luke to:	Routing coefficients for indicated 12-hour period										
	0-12	12-24	24-36	36-48	48-60	60-72	72-84	84-96	96-108	108-120	120-132
Cumberland	--	0.86	0.14	--	--	--	--	--	--	--	--
Paw Paw	--	.03	.83	0.14	--	--	--	--	--	--	--
Hancock	--	.01	.60	0.38	0.01	--	--	--	--	--	--
Shepherdstown	--	--	--	.19	.66	0.15	--	--	--	--	--
Point of Rocks	--	--	--	--	.01	.38	0.54	0.07	--	--	--
Washington	--	--	--	--	--	--	.11	.47	0.34	0.07	0.01

Table 11. -- Weekly routing coefficients for linked model

Reach Luke to:	Routing coefficients for week	
	1	2
Cumberland	0.92	0.08
Paw Paw	.85	.15
Hancock	.76	.24
Shepherdstown	.65	.35
Point of Rocks	.60	.40
Washington	.47	.53



EXPLANATION

FLOW AT LUKE FOR HOURS

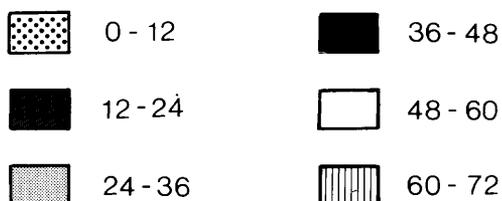


Figure 13.-- 12-Hour response at Washington, D.C. to a 3-day unit input to Luke, Md.



## CONCLUSIONS

A 24-hour sustained reservoir release input at Luke, Md., will result in 35 percent of the flow arriving at Washington, D.C., during the 4th day after the beginning of the release, followed by 61 percent and 4 percent arriving during the 5th and 6th days, respectively. A 7-day sustained reservoir release at Luke will result in 47 percent of the flow arriving at Washington during the 1st week and 53 percent of the flow arriving during the 2d week.

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