

DETERMINATION OF REAERATION  
COEFFICIENTS FOR OHIO STREAMS

By Janet Hren

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## CONVERSION FACTORS

For the benefit of readers who prefer to use the International System of units (SI), conversion factors for terms used in this report are listed below:

<u>Multiply inch-pound units</u>	<u>By</u>	<u>To obtain SI units</u>
foot (ft)	0.3048	meter (m)
foot per second (ft/s)	0.3048	meter per second (m/s)
foot per mile (ft/mi)	0.1864	meter per kilometer (m/km)
mile per hour (mi/h)	1.609	kilometer per hour (km/h)
cubic foot per second (ft <sup>3</sup> /s)	0.02832	cubic meter per second (m <sup>3</sup> /s)

# DETERMINATION OF REAERATION COEFFICIENTS FOR OHIO STREAMS

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## ABSTRACT

The hydrocarbon-gas tracer technique was used to determine reaeration coefficients on 30 reaches of Ohio streams. The studies were done from September 1979 through August 1982 to determine the reaeration coefficients for the individual reaches and to develop general equation that could be used to estimate the coefficients.

Multiple linear regression was used to determine relationships among the reaeration coefficients and physical stream characteristics. Four special equations based on the general equation were developed from various combinations of discharge, slope, width, depth, measured velocity, and estimated velocity. The standard errors of estimate for these equations ranged from 37 to 47 percent. The variables that resulted in the lowest standard error of estimate were discharge, slope, width, depth, and measured velocity. The most significant variables were depth and velocity.

## INTRODUCTION

Reaeration is the physical absorption of oxygen from the atmosphere into a body of water. In streams, reaeration is mainly a function of turbulence due to flow and channel morphology. The rate of dissolved oxygen replenishment is defined as:

$$dc/dt = K_2(Cs-c) \quad (1)$$

where

- c = concentration of dissolved oxygen in a stream;
- Cs = saturation value of dissolved oxygen;
- t = time;
- K<sub>2</sub> = reaeration coefficient.

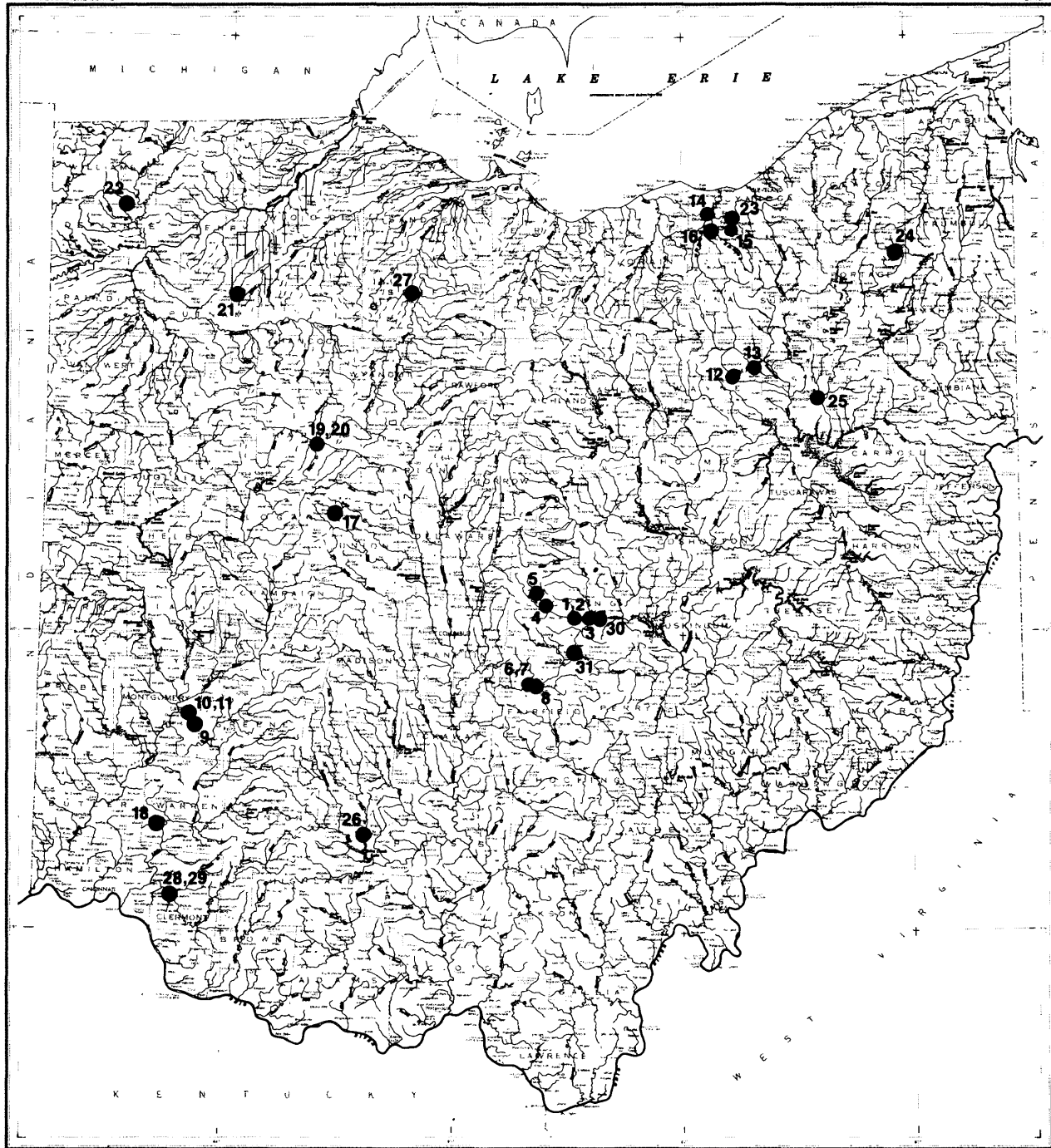
The reaeration coefficient quantifies the process of dissolved-oxygen replenishment, and hence is important in determining the waste-assimilation capacity of streams.

Reaeration coefficients may be estimated from a number of equations or determined in the field. These equations and techniques have been reviewed by Rathbun (1977). Two techniques used for measuring reaeration coefficients are the radioactive and hydrocarbon-gas tracer methods. Comparison tests have indicated that coefficients determined by either of these methods are far more accurate than those estimated by predictive equations, and that the hydrocarbon-gas method shows promise as an alternative to the use of radioactive tracers (Rathbun and Grant, 1978).

This report presents the results of a study to determine reaeration coefficients, by means of the hydrocarbon method, for selected reaches of Ohio streams (fig. 1, table 1), and to use these data to develop a general regression equation to estimate reaeration coefficients.

## HYDROCARBON-GAS TRACER METHOD

The basis for this method is the observation that the hydrocarbon-gas desorption coefficient is related to the oxygen absorption coefficient by a constant (Rathbun and others, 1978). Ethylene and (or) propane gas are used as the tracers for oxygen; rhodamine-WT dye is the dilution/dispersion tracer as well as the indicator for when to sample for the gases. The use of ethylene and propane together provides two independent estimates of the reaeration coefficient. Because the gases are only slightly soluble in water, they are injected for a short period of time through porous diffusers placed on the streambed. The dye also is continuously injected for the same period. The procedure is described in detail by Rathbun (1979).



Base from U.S. Geological Survey  
State Base Map 1975

Scale 1:500,000  
1:500,000  
Scale of map is approximate and does not represent true scale.

OHIO  
1975

**EXPLANATION**

●<sup>24</sup> Location and number  
of study site

**Figure 1. --Location of study sites**

Table 1.--Reach numbers, dates studied, and description of reaches

Reach	Date studied (mo./day/yr)	Description
1	09/26/79	Raccoon Creek at Granville (subreach 1-2)
2	09/26/79	Raccoon Creek at Granville (subreach 2-3)
3	11/05/79	Raccoon Creek at Newark
4	11/06/79	Raccoon Creek near Johnstown
5	11/07/79	Raccoon Creek at Johnstown
6	08/25/80	Paw Paw Creek at Baltimore
7	08/29/80	Paw Paw Creek at Baltimore
8	10/07/80	Walnut Creek at Baltimore
9	10/15/80	Little Miami River near Bellbrook
10	10/16/80	Little Beaver Creek near Dayton (subreach 1-2)
11	10/16/80	Little Beaver Creek near Dayton (subreach 2-3)
12	08/11/81	Little Chippewa Creek at Orrville
13	08/12/81	Silver Creek at Doylestown
14	08/24/81	Rocky River at North Olmsted
15	08/25/81	East Branch Rocky River near Strongsville
16	08/27/81	Unnamed trib to W. B. Rocky River near Berea
17	09/17/81	Unnamed trib to Bokes Creek at West Mansfield
18	09/30/81	Muddy Creek at Mason
19	10/08/81	Scioto River at Kenton (subreach 1-2)
20	10/08/81	Scioto River at Kenton (subreach 2-3)
21	10/14/81	Little Yellow Creek at Leipsic
22	10/15/81	Prairie Creek at Bryan
23	10/22/81	Baldwin Creek near Strongsville
24	11/04/81	Eagle Creek at Garrettsville
25	11/18/81	Nimishillen Creek at North Industry
26	07/21/82	Paint Creek at Greenfield
27	08/04/82	Sandusky River at Tiffin
28	08/12/82	E. F. Little Miami River at Batavia (subreach 1-2)
29	08/12/82	E. F. Little Miami River at Batavia (subreach 2-3)
30	08/18/82	Licking River at Newark
31	08/19/82	Waste Weir Run at Buckeye Lake

The desorption coefficient for each tracer gas ( $K_g$ ) can be calculated by either the peak or the area method. In the peak method, the peak concentration of the tracer is a term in the following equation:

$$K_g = \frac{1}{\bar{t}_d - \bar{t}_u} \log_e \frac{(C_g/C_D)_u}{(C_g/C_D)_d J} \quad (2)$$

where  $t$  is the time of travel of the dye peak,  $C_g$  is the peak concentration of the tracer gas, and  $C_D$  is the peak concentration of the dye. The subscripts  $u$  and  $d$  indicate the upstream and downstream sampling points, respectively. Because the rhodamine-WT is not totally conservative, dye loss must be accounted for. The dye-loss correction factor  $J$  is calculated by:

$$J = \frac{Q_u A_u}{Q_d A_d} \quad (3)$$

where  $Q$  is the discharge and  $A$  is the area under the dye concentration-versus-time curve.

In the area method, the following equation is used:

$$K_g = \frac{1}{\bar{t}_d - \bar{t}_u} \log_e \frac{A_u Q_u}{A_d Q_d} \quad (4)$$

where  $\bar{t}$  is the time of travel of the centroid of the gas tracer mass,  $A$  is the area under the gas concentration versus time curve, and  $Q$  is the discharge.

The desorption coefficients for ethylene and propane are converted to the reaeration coefficient ( $K_2$ ) by the following:

$$K_2 = 1.15K_g \quad (\text{Ethylene}) \quad (5)$$

$$K_2 = 1.39K_g \quad (\text{Propane}) \quad (6)$$

These equations were developed by Rathbun and others (1978).

Reaeration coefficients are adjusted to a common temperature of 20°C by the equation:

$$K_{2,20} = K_{2,T} (1.0241)^{(20-T)} \quad (7)$$

where  $T$  is the water temperature in degrees Celsius. The factor of 1.0241 was determined by Elmore and West (1961). In this study only the peak method was used to calculate reaeration coefficients.



## EXPERIMENTAL PROCEDURE

The U.S. Geological Survey determined reaeration coefficients on 30 separate reaches of 23 streams. One reach, Paw Paw Creek at Baltimore, was studied twice. The reaches and dates studied are listed in table 1. All studies were made between September 1979 and August 1982 at low flow. The reaches ranged from 5 to 191 feet in average water-surface width, 0.2 to 2.2 feet in average depth, 0.05 to 12.59 feet per second in average velocity, and 0.1 to 126 cubic feet per second in average discharge. The data collected at each site are listed in table 2.

Discharge measurements were made, as described by Buchanan and Somers (1969), at each site where complete dye curves were collected. This was usually at the first and second sampling sites. The average of these two measurements was used in the regression equations.

Wind speed was measured by a totalizing cup anemometer placed within 1 to 2 feet of the water surface. The anemometer was placed at a site where bank vegetation was representative of the entire reach.

Slope of the reach was determined from U.S. Geological Survey 7.5-minute topographic maps.

The water temperature used to calculate  $K_2$  was the average of those recorded at each site when the dye concentration peaked. Average reach velocity was determined from the time-of-travel from dye peak to dye peak and the reach length. Average channel width was determined by averaging the stream widths at approximately 20 evenly spaced points along the length of the reach (width measurements were made during low-flow periods in 1983). Average reach depth was calculated by dividing the average discharge by the product of average velocity and width.

Chemically pure-grade ethylene and propane were injected into the stream for 10 to 60 minutes through ceramic porous-plate diffusers. The diffusers were placed on the streambed in the center of the flow. A solution of rhodamine-WT dye and water was injected from a constant-rate-metering pump for the same period of time and at the same point in the stream. Gas and dye were injected at rates determined from equations by Rathbun (1979).

Table 2.--Summary of data collected for each reach

[ft, feet; ft/mi, feet per mile; ft/s, feet per second; ft<sup>3</sup>/s, cubic feet per second; °C, degrees Celsius; mi/h, miles per hour]

Reach number	Reaeration coefficient (days <sup>-1</sup> ) at 20°C		Average discharge (ft <sup>3</sup> /s)	Slope (ft/mi)	Reach's average velocity (ft/s)	Reach's average depth (ft)	Reach's average width (ft)	Water temp. (°C)	Average wind speed (mi/h)
	Ethylene	Propane							
1	3.49	2.27	47.4	7.09	0.53	1.7	52.5	22.0	--
2	3.46	2.84	55.7	7.46	.57	2.23	43.5	22.0	--
3	11.61	15.05	55.4	11.24	1.59	0.75	46.7	10.0	--
4	--	7.09	7.9	17.24	.24	1.29	25.6	7.0	--
5	2.94	3.14	4.4	7.75	.21	1.4	14.9	7.0	--
6	4.42	5.46	7.13	16.67	.97	.37	20	20.5	--
7	3.02	4.43	3.52	16.67	.53	.33	20	21.5	--
8	2.81	1.94	4.92	4.92	.09	1.28	42.8	11.5	0.88
9	5.37	5.70	126	4.28	.66	2.02	93.8	12.5	1.77
10	12.2	9.32	21.9	9.17	.81	1.02	26.6	18.5	1.18
11	10.69	8.46	23.9	14.81	.64	1.28	29.2	19.0	1.18
12	4.96	4.05	2.76	6.08	.35	.49	15.8	24.0	1.52
13	12.36	11.08	2.88	24.51	.3	.79	12.1	19.5	1.0
14	8.64	11.65	32.8	7.07	.71	.49	94.2	22.0	1.9
15	2.44	3.74	9.87	6.24	.34	.84	34.5	19.0	.61
16	6.97	8.32	4.4	29.0	.26	1.16	14.7	19.5	.13
17	7.44	5.0	.10	5.17	.061	.32	5.1	15.0	1.10
18	12.96	10.63	1.30	39.0	.128	.51	19.3	20.4	.72
19	2.97	5.48	19.8	1.25	.286	1.09	63.4	11.3	1.15
20	5.86	4.32	20.6	1.25	.301	1.31	52.4	11.4	1.15
21	12.48	9.58	.27	4.28	.240	.20	5.5	13.4	1.94
22	4.99	4.76	2.92	4.88	.39	.48	15.8	13.9	3.22
23	4.72	2.69	3.15	9.37	.125	1.57	16.1	12.6	.21
24	1.17	2.87	11.7	9.19	.266	1.22	36	8.7	1.02
25	6.22	5.28	87.3	6.71	1.07	1.39	58.7	12.1	1.90
26	5.99	5.36	15.2	8.33	.29	.78	68.2	25.5	--
27	7.72	7.70	58.4	15.0	.704	.43	191	27.5	1.84
28	7.32	7.13	31.2	4.3	.36	.95	89.9	21.0	2.07
29	2.98	2.87	31.0	4.3	.18	2.13	81.7	22.5	2.07
30	2.85	2.07	72.5	3.7	.47	1.13	119	22.0	1.52
31	4.29	2.06	.52	5.0	.048	.49	21.9	22	.56

Samples for dye and the hydrocarbon tracers were collected at two or three points downstream of the injection site. At each site the samples were collected at the center of flow. Samples for hydrocarbon gas analyses were collected with displacement type samplers and 40-milliliter (ml) borosilicate glass vials with Teflon<sup>1</sup> septa screw caps. Samples for ethylene and propane analyses were preserved with 1 ml of formalin. Dye samples were collected in 30-ml vials and analyzed in the field with a Turner model 10 fluorometer by standard techniques described by Wilson (1968).

Sampling was continued at two sites until the dye concentration decreased to 10 percent or less of the peak concentration. Dye curves were then extrapolated to 1 percent of the peak concentration to determine the area under each curve. The curves were extrapolated by plotting the natural logarithm of the dye concentrations in the tail region of the curve versus time. A best-fit straight line was then drawn through the points and extended to 1 percent of the peak dye concentration. If a third sampling site was used, only the peak dye concentration curve was defined, and the dye loss was assumed to be linear with time of travel.

Samples for ethylene and propane concentrations were analyzed by the U.S. Geological Survey laboratory at Doraville, Ga., or by Brehm Laboratory, Wright State University, Dayton, Ohio. Both labs used the gas chromatography procedure described by Shultz and others (1976). Reaeration coefficients were calculated by the peak method and equations described previously.

### REGRESSION ANALYSIS

Regression measures relationships between a response variable and one or more predictor variables. The relationships can then be used to predict values of the response variable.

Multiple regression analysis was used to relate the reaeration coefficient ( $K_2$ ) to physical stream characteristics and wind velocity.  $K_2$  was plotted against each of the independent variables to determine if the relationships were linear. None were, so a  $\log_{10}$  transformation of all the variables was used and a more linear relationship was obtained. A constant of 1 was added to all variables to prevent negative logarithms. This resulted in an equation of the general form:

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<sup>1</sup>The use of brand names in this report is for identification purposes only and does not constitute endorsement by the U.S. Geological Survey.

$$\log(Y+1) = a + b_1 \log(X_1+1) + b_2 \log(X_2+1) + \dots + b_n \log(X_n+1)$$

where Y = dependent variable ( $K_2$ )  
 $X_1, \dots, X_n$  = independent variables  
 $b_1, \dots, b_n$  = regression coefficients  
a = regression constant  
n = the number of independent variables

Table 3 lists the logarithmic regression equations developed from the equation for ethylene and propane, the  $R^2$  (percent variation explained for the dependent variable, based on 31 sets of data), and the standard error of estimate (SEE). The SEE is the square root of the residual mean square and is calculated as a percentage of the mean value of the dependent variable. The SEE was converted from log units by the equations described by Riggs (1968).

Equations 8a and 8b in table 3 are the combinations of independent variables that explain the highest percent variation of the  $K_2$  and have the lowest SEEs (37 and 45 percent for propane and ethylene, respectively). These variables are discharge, slope, average depth, average width, and average velocity. The most significant variables were depth and velocity. The negative coefficient on average depth indicates that as stream depth increases, the  $K_2$  decreases. Wind velocity was not a significant variable for the range of values measured. The effect of wind on the reaeration coefficient was not detected, possibly because of the sheltering effects of the streambanks and vegetation and the small range of wind speeds observed during the studies.

Equations 9a and 9b, which incorporate slope, average velocity, and average depth, are second best in terms of fitting the data and have SEEs of 38 and 45 percent, respectively.

Equations 10a and 10b, which incorporate average discharge, slope, and average velocity, result in SEEs of 41 and 46 percent, respectively. In these equations, average discharge has a negative coefficient, which indicates that discharge may be acting as a substitute for average depth.

In equations 11a and 11b, average discharge, slope, and estimated velocity are used. Average discharge is the only field-measured variable. The estimated velocity was determined by the following equation (Boning, 1974):

$$VP = 0.38Q^{0.40}S^{0.20}$$

where VP = dye peak velocity, in feet per second  
Q = discharge, in cubic feet per second  
S = channel slope, in feet per foot.

Equation 11 has SEEs of 45 and 47 percent, respectively.

Table 3.--Results of regression analysis

Regression equation (log is base 10) at 20°C*		R <sup>2</sup>	Standard error of estimate (in percent)**
(8a)	$K_{2P} = 1.224Q^{.573} S^{.282} V^{-.789} D^{-1.54} W^{-.498}$	0.50	37
(8b)	$K_{2E} = 1.322Q^{.481} S^{.216} V^{-.719} D^{-1.26} W^{-.50}$	.34	45
(9a)	$K_{2P} = .615S^{.224} V^{.853} D^{-.566}$	.43	38
(9b)	$K_{2E} = .708S^{.190} V^{.522} D^{-.522}$	.27	45
(10a)	$K_{2P} = .515Q^{-.103} S^{.208} V^{1.286}$	.37	41
(10b)	$K_{2E} = .643Q^{-.133} S^{.157} V^{1.08}$	.24	46
(11a)	$K_{2P} = .523Q^{-.171} S^{.22} V_e^{2.17}$	.22	45
(11b)	$K_{2E} = .778Q^{-.510} S^{.014} V_e^{5.079}$	.21	47

\*Q = average reach discharge, in cubic feet per second + 1

S = channel slope, in feet per mile + 1

V = average reach velocity, in feet per second + 1

D = average reach depth, in feet + 1

W = average channel width, in feet + 1

V<sub>e</sub> = estimated reach velocity, in feet per second + 1

K<sub>2E</sub> = reaeration coefficient in days<sup>-1</sup> + 1 (at 20°C), using ethylene

K<sub>2P</sub> = reaeration coefficient in days<sup>-1</sup> + 1 (at 20°C), using propane

\*\* Calculated as an average of the positive and negative departures as described by Riggs (1968)

In each set of regression equations the reaeration coefficients obtained using ethylene resulted in lower  $R^2$  values and higher standard errors than those coefficients obtained using propane. Comparison of the coefficients (table 2) show that the ethylene values are higher than the propane values in 20 of 30 reaches where both gases were used. However, there is no statistical difference between the ethylene and propane  $K_2$  values (based on a paired t test) at the 0.01 level of significance.

#### SUMMARY

The hydrocarbon tracer technique was used to determine reaeration coefficients on 30 reaches of Ohio streams. All studies were conducted at approximately low flow. The reaches studied ranged from 0.10 to 126 cubic feet per second in discharge, 1.25 to 39 feet per mile in slope, 5.1 to 191 feet in average width, 0.20 to 2.23 feet in average depth, and 0.048 to 1.59 feet per second in average velocity.

Multiple regression analysis was used to develop a general equation of the relationships among the reaeration coefficient and physical stream characteristics. This equation was used to develop special equations that relate stream characteristics to reaeration coefficient. Using discharge, slope, average depth, average width, and average velocity as the independent variables resulted in equations that fit the data best and have SEEs of 37 and 45 percent for propane and ethylene, respectively. Slope, average velocity, and average depth are used in equations that provide the second best fit and have SEEs of 38 and 45 percent. Using only discharge, slope, and average velocity (determined from the time of travel of dye) produced equations having SEEs of 41 and 46 percent. An estimated velocity (calculated from Boning, 1974) used in combination with discharge and slope, yielded equations having SEEs of 45 and 47 percent; the only field determination required is for discharge. A log base 10 transformation of the dependent and independent variables was used in all equations.

These equations represent an estimate of the average reaeration coefficient expected on the basis of determinations for a large number of streams having similar characteristics. The equations are applicable only to streams whose characteristics are within the range of those presented in this report.

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