HYDROGEOLOGY AND SIMULATION OF WATER FLOW IN STRATA ABOVE THE BEARPAW SHALE AND EQUIVALENTS OF EASTERN MONTANA AND NORTHEASTERN WYOMING

By W. R. Hotchkiss and Julianne F. Levings

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CONVERSION FACTORS

The following factors can be used to convert inch-pound units in this report to the International System of units (SI).

Multiply inch-pound unit	By	To obtain SI unit
acre	4,047	square meter
cubic foot per minute	0.02832	cubic meter per minute
cubic foot per second (ft^3/s)	0.02832	cubic meter per second
foot (ft)	0.3048	meter
foot per day (ft/d)	0.3048	meter per day
foot per second (ft/s)	0.3048	meter per second
foot squared	0.09290	meter squared
foot squared per day (ft^2/d)	0.09290	meter squared per day
gallon per minute (gal/min)	0.06309	liter per second
inch (in.)	25.40	millimeter
mile (mi)	1.609	kilometer
pound per square inch (lb/in^2)	6.895	kilopascal
square mile (mi^2)	2.590	square kilometer

Temperature in degrees Celsius (°C) can be converted to degrees Fahrenheit (°F) by the equation:

$$°F = 9/5\ (°C) + 32$$

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ABSTRACT

The hydrogeology of shallow units in the Powder River, Bull Mountains, and western Williston basins was investigated to improve understanding of the regional ground-water flow system, hydraulic characteristics, and interaction between hydrogeologic units. Five major hydrogeologic units were delineated above the Upper Cretaceous Bearpaw Shale. In ascending order, they include the Fox Hills-lower Hell Creek aquifer, the upper Hell Creek confining layer, the Tullock aquifer, the Lebo confining layer, and the Tongue River aquifer.

Potentiometric-surface maps prepared for the five hydrogeologic units indicate generally increasing potentiometric head with depth. Thus, except near outcrop areas, the study area appears to be a discharge area.

The major sources of ground-water recharge are infiltration of water from precipitation and streamflow on areas of outcrop. Infiltration of water from losing streams is also a factor. Discharge from the area is principally by ground-water outflow, by loss to gaining streams, springs, and seeps, by evapotranspiration, and by pumpage.

Simulation of the regional flow system required estimation of several hydrologic properties that were used as input to the model. Aquifer transmissivities were estimated from the ratio of sandstone and shale in each hydrogeologic unit. A reasonable estimate of vertical hydraulic conductance per unit area between hydrogeologic layers was made by dividing interlayer-thickness values into a single estimated value of vertical hydraulic conductance per unit area of 4.8×10^{-5} foot per day. The initial estimate of recharge to the hydrogeologic units was about 0.5 percent of the precipitation on the outcrop.

The three-dimensional finite-difference model developed by Trescott and Larson was used in the simulation. The model was calibrated by adjusting the initial data and data estimates over the model area and operating the model. A series of simulations was performed in which parameter values were perturbed systematically to determine the magnitude and direction of change necessary for these values to provide improved fit of computed heads relative to the observed values. The improved parameter values were used for the next stage of input to the model for perturbation runs. Five stages of model simulations reduced the standard error of estimate in hydraulic head from 135 to 110 feet for 739 observation nodes located throughout the five layers.

The calibrated mean transmissivity, in feet squared per day, was 443 for the Fox Hills-lower Hell Creek, 191 for the upper Hell Creek, 374 for

the Tullock, 217 for the Lebo, and 721 for the Tongue River hydrogeologic units. Calibrated mean vertical hydraulic conductances per unit area, in feet per day, between the units mentioned above were, respectively, 1.40 x 10^{-4}, 3.18 x 10^{-5}, 2.27 x 10^{-5}, and 1.58 x 10^{-5}. Mean annual recharge across the study area was about 2.45 x 10^{-2} inch or about 0.26 percent of average annual precipitation.

A hydrologic mass balance was calculated at the end of steady-state simulation. The balance contained the computed water additions and subtractions for precipitation, leakage, inflow, and outflow. Recharge from precipitation ranged from 4.19 to 38.16 cubic feet per second. Leakage between aquifers and confining layers ranged from 2.25 to 24.38 cubic feet per second. Inflow was 1.02 cubic feet per second for one confining layer and outflow ranged from 1.91 to 30.87 cubic feet per second for three aquifers and one confining layer. Large rates of intra- and interlayer flow indicate that vertical flow between aquifers, especially along stream valleys, may be significant.

Acquisition of additional data might greatly improve the standard error of estimate of hydraulic head in future model runs. However, the most useful results might be obtained from specific studies targeted at special problem areas.

INTRODUCTION

The development of ground-water resources in eastern Montana and northeastern Wyoming will probably increase with the future development of local energy resources, industry, power generation, irrigation, and domestic and municipal water supplies. Consequently, in 1978 the U.S. Geological Survey began a 4-year study of the northern Great Plains to define the hydrologic system, to determine availability and quality of ground water, and to understand the regional hydrologic system. Specifically, this report presents the results of a study of the flow system in shallow Upper Cretaceous and Tertiary strata in the Powder River, Bull Mountains, and western Williston basins.

Purpose and scope

The purpose of this report is to (1) describe the shallow ground-water flow system and (2) describe the hydraulic characteristics of the hydrogeologic units and the flow system likely to exist between individual units. The result provides a generalized perspective of the regional hydrogeologic system.

Data from many sources were synthesized to form a simulation model. Geophysical logs (Feltis and others, 1981; Lewis and Hotchkiss, 1981) were interpreted to determine the thickness, configuration, and percent sand of the model layers. Historic water-level data supplemented by data from well inventories in 1979 and 1980 (Levings, 1981) were used to construct steady-state potentiometric-surface maps of the model layers. Kriging techniques (Karlinger and Skrivan, 1981) were employed as a means of smoothing the data for interlayer thicknesses and sand-shale ratios prior to including the data in the flow model. Finally, the three-dimensional finite-difference digital flow model by Trescott (1975) and Trescott and Larson (1976) was used in conjunction with parameter-estimation techniques to test the conceptual model and to modify it by calibration until the criteria established for agreement were achieved between the observed steady-state data and simulated results.

Location and general features of the area

The study area is in the unglaciated part of the Missouri Plateau of the northern Great Plains and spans about 370 mi from north to south and 215 mi from east to west (fig. 1). The area contains about 42,000 mi^2.

The topography is characterized by flat or rolling uplands, badlands, and stream valleys. Topographic relief ranges from 300 to 500 ft in the southern part of the area to about 500 to 1,000 ft in the northern part.

Previous investigations

Numerous geologic and hydrologic studies have been completed in various parts of the study area during the past approximately 70 years. Most geologic studies were conducted principally for the purpose of coal-bed definition and correlation of strata above the Bearpaw Shale of Late Cretaceous age. Numerous studies and maps of Tertiary uranium deposits in Wyoming provide excellent stratigraphic or water-quality information (for example, Santos, 1981). Reports by Hodson and others (1973), Lewis and Roberts (1978), Stoner and Lewis (1980), and Lewis and Hotchkiss (1981) were the principal hydrogeologic guides for the present work. Each of the references cited above contains maps or bibliographies describing reports that encompass various parts of the study area. Finally, a selective annotated bibliography is available for the northern Great Plains in Montana (Levings and others, 1981).

Acknowledgments

Thanks are extended to numerous agencies and cooperators for supplying data used in this report. Specifically, preparation of the hydrogeologic information used to derive model input data was aided by use of various types of geophysical logs on file at the Montana Oil and Gas Conservation Commission in Billings, Mont., and the State Engineer's Office in Cheyenne, Wyo. The authors are especially indebted to W. D. Grundy, U.S. Geological Survey, for his invaluable assistance and advice on the application of kriging techniques to hydrologic parameters.

Well-numbering system

In this report, wells are numbered according to geographic position within the rectangular grid system used by the U.S. Bureau of Land Management (fig. 2). The location number consists of 14 characters. The first three characters specify the township and its position north (N) or south (S) of the Base Line. The next three characters specify the range and its position east (E) or west (W) of the Principal Meridian. The next two characters are the section number. The next four characters designate the quarter section (160-acre tract), quarter-quarter section (40-acre tract), quarter-quarter-quarter section (10-acre tract), and quarter-quarter-quarter-quarter section (2.5-acre tract), respectively, in which the well is located. The subdivisions of the section are designated A, B, C, and D in a counterclockwise

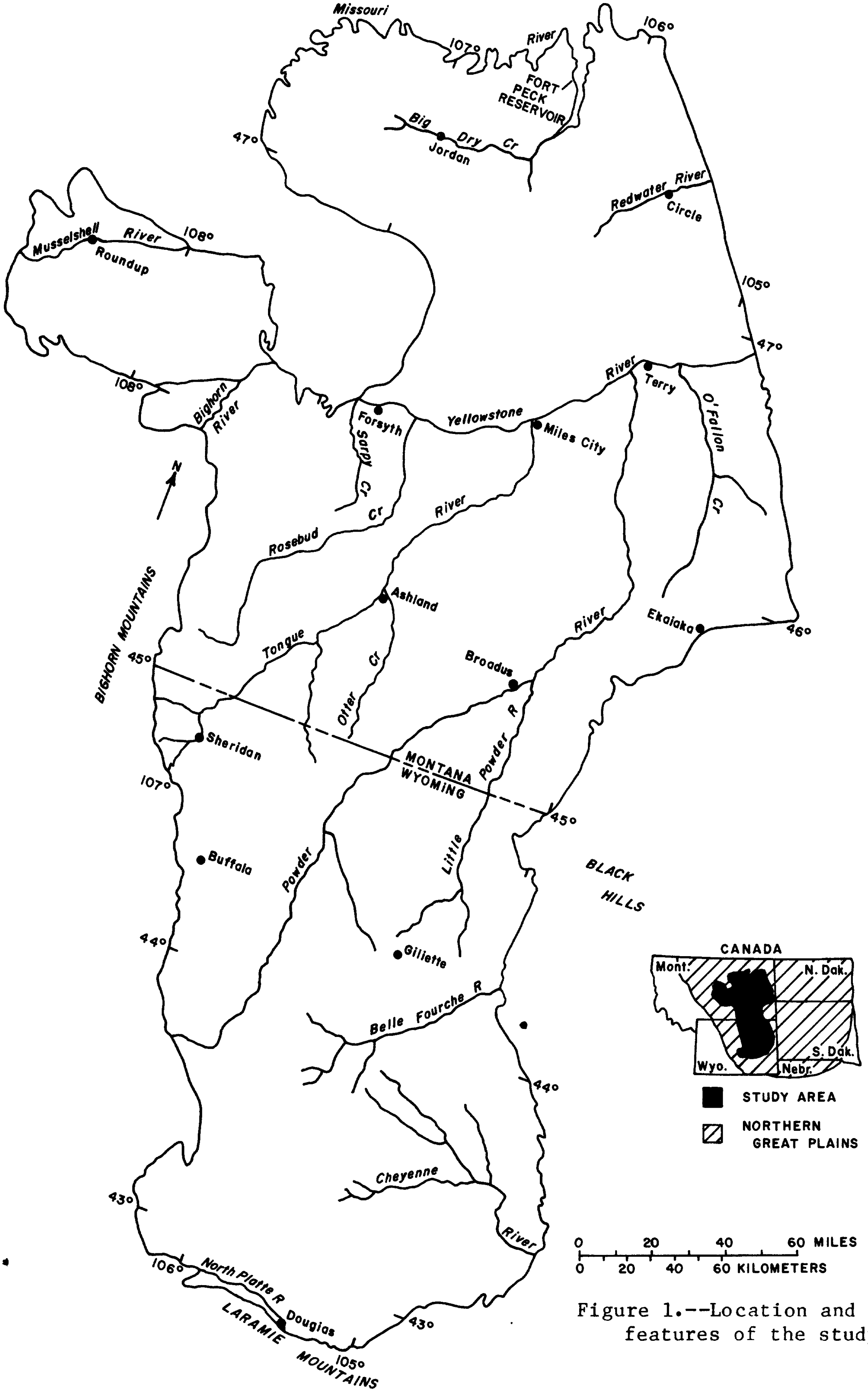


Figure 1.--Location and general features of the study area.

direction, beginning in the northeast quadrant. The last two characters form a sequential number based on order of inventory. Well locations in the Montana part of the study area extend from townships 26 N. to 9 S. of the Montana Base Line and ranges 22 E. to 62 E. of the Montana Principal Meridian; well locations in the Wyoming part of the study area extend from townships 58 N. to 33 N. of the Wyoming Base Line and ranges 87 W. to 62 W. of the Wyoming Principal Meridian. For example, as shown in figure 2, well 06N44E36CACD01 is the first well inventoried in the SE1/4 SW1/4 NE1/4 SW1/4 sec. 36, T. 6 N., R. 44 E., in Montana.

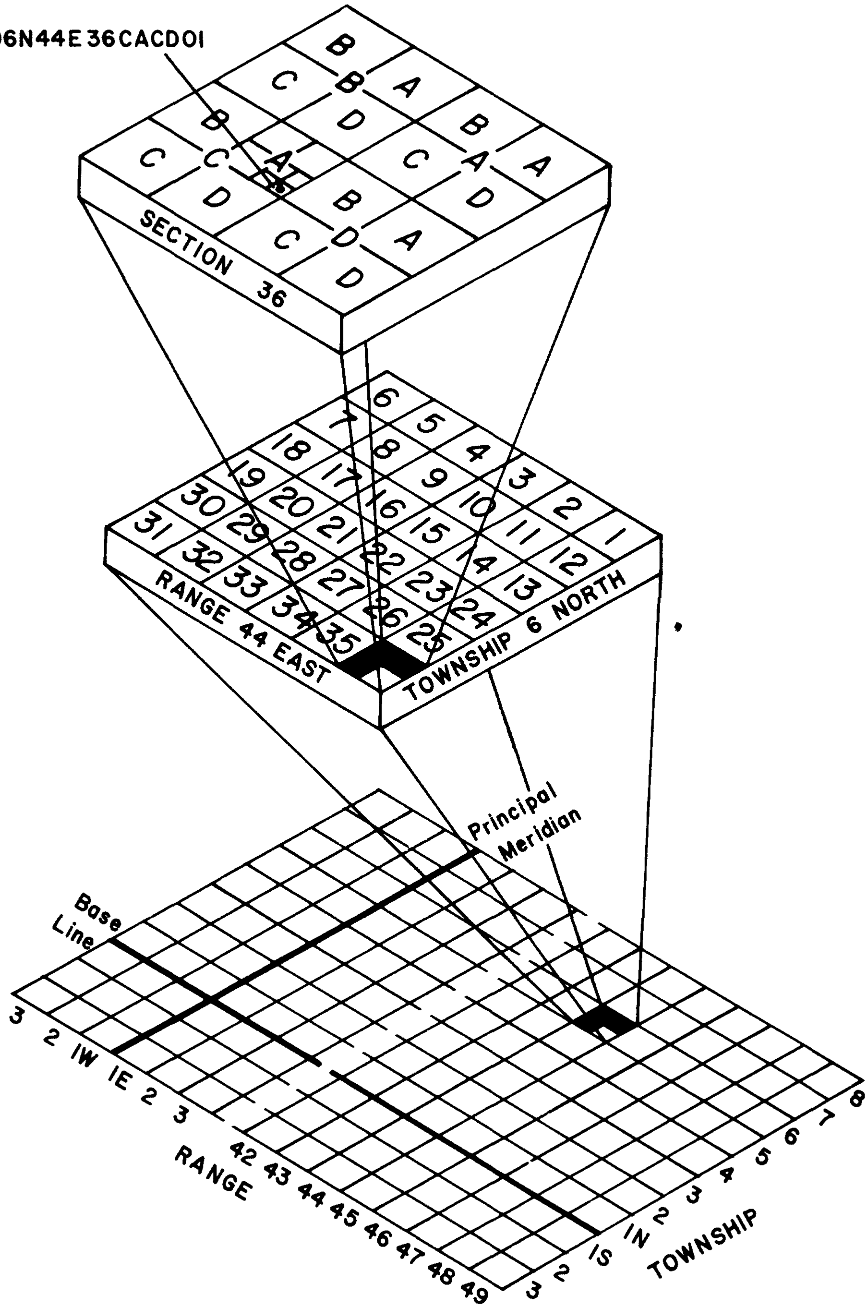


Figure 2.--Well-numbering system.

GEOLOGY

The shallow geologic units of the study area are defined herein as those that are stratigraphically above the regionally widespread and relatively impermeable Bearpaw Shale and its geologic equivalents. The geologic units include the Fox Hills Sandstone and Hell Creek Formation (or Lance Formation in Wyoming) of Late Cretaceous age; the Fort Union, Wasatch, White River, and Arikaree Formations of Tertiary age; and terrace deposits and alluvium of Quaternary age. No attempt has been made to compile a geologic map of these rocks. Instead, the reader is directed to Hodson and others (1973), Lewis and Roberts (1978), Stoner and Lewis (1980), and Lewis and Hotchkiss (1981).

Stratigraphy

The Bearpaw Shale is generally medium-gray to dark-gray marine shaly claystone and shale with thin beds of siltstone, silty sandstone, and bentonite. Concretions of various types occur throughout the formation. Sandy shale and shaly sandstone compose a transitional zone between open-water marine deposits of the Bearpaw and near-shore marine sandstone deposits of the overlying Fox Hills Sandstone (Johnson and Smith, 1964). The Bearpaw is equivalent to the upper part of the Pierre Shale in the Montana Plains (Levings and others, 1981) and eastern part of the Powder River basin in Wyoming (Hodson and others, 1973). Its thickness ranges from 800 to 1,100 ft based on four measurements cited in Schultz and others (1980). The Bearpaw is equivalent to the Lewis and underlying Mesaverde Formations (also the upper part of the Pierre Shale) of the western part of the Powder River basin in Wyoming where the thickness ranges from 900 to 1,300 ft (Hodson and others, 1973).

The Fox Hills Sandstone is composed of fine- to medium-grained marine sandstone containing thin beds of sandy shale. The Fox Hills conformably overlies the Bearpaw Shale and represents shoreline deposits of a regressive sea (Johnson and Smith, 1964). The Fox Hills is equivalent to the Lennep Sandstone of central Montana (Balster, 1971). Locally, it is difficult to differentiate sandstone of the Fox Hills from sandstones in the overlying Hell Creek Formation. The Fox Hills ranges in thickness across the study area from about 200 ft in the northeast to about 700 ft in the southwest.

The Hell Creek Formation is composed of nonmarine sandstone, siltstone, and shale with carbonaceous and bentonitic sandy shale and siltstone. Locally, a fine- to medium-grained fluvial silty sandstone, which may contain thin coal beds, is present in the section. The Hell Creek Formation of Montana overlies the Fox Hills Sandstone and is equivalent to the lower part of the Lance Formation of northeastern Wyoming (Balster, 1971). Thickness of the Hell Creek Formation across the study area ranges from about 400 ft in the northeast to about 2,000 ft in the southwest.

The Fort Union Formation, a diverse sequence of continental sediments of Paleocene age, conformably overlies the Hell Creek Formation. The Fort Union consists of the Tullock, Lebo Shale, and Tongue River Members in ascending order in the Bull Mountains basin, western parts of the Powder River basin, and Williston basin and has been extended across the rest of the study area by Lewis and Hotchkiss (1981). Near the Montana-North Dakota border, the formation consists of the Ludlow and Tongue River Members. The basal part of the Tullock Member is composed of interbedded sandstone, siltstone, shale, and thin coal beds that grade upward into sandstone and silty or sandy shale. The thickness of the Tullock ranges from

180 ft in the northeast to about 1,900 ft in the southwest. The Lebo Shale Member is predominantly massive dark shale with interbedded carbonaceous shale, siltstone, and locally thin coal beds. In places, the Lebo may contain a basal channel sandstone (Lewis and Hotchkiss, 1981). The thickness of the Lebo ranges from about 260 ft in the northeast to about 3,700 ft in the southwest. The uppermost Tongue River Member consists of fine- to medium-grained thick-bedded to massive sandstone and siltstone that are locally lenticular and crossbedded (Balster, 1971). The sandstone is of fluvial origin, composed of crossbedded deltaic channel sands, flood-plain silts, and splay deposits of fine sand intermixed with lacustrine clays and silts. Numerous thick and widespread coal beds are present, some of which have burned and formed conspicuous clinker and baked-shale outcrops. Thickness of the Tongue River Member ranges from about 360 ft in the northeast to about 2,500 ft in the southwest. The Ludlow Member is composed of light-gray to tan claystone, siltstone, silty claystone, and sandstone locally containing bentonite, limonite concretions, and clinker (Balster, 1971). In eastern Montana the Ludlow Member of the Fort Union Formation is equivalent to the Tullock and Lebo Shale Members, undifferentiated. The thickness of the Ludlow ranges from 460 ft in the west to about 550 ft in the east.

The Wasatch Formation of Eocene age comformably overlies the Tongue River Member of the Fort Union Formation and consists of continental fine- to coarse-grained lenticular sandstone interbedded with shale and coal. Coal beds are as thick and laterally persistent as those in the Tongue River Member of the Fort Union Formation and exhibit the same outcrops of clinker where they have burned. Thickness of the Wasatch across the study area ranges from about 400 ft near the Montana-Wyoming border to 2,000 ft in the southwest. The Wasatch Formation is removed by erosion just north of the State line.

The White River Formation of Oligocene age uncomformably overlies the Fort Union or Wasatch Formation and grades upward from lenticular fine-grained sandstone to tuffaceous and bentonitic claystone and siltstone (Balster, 1971). In the study area, this unit is restricted to a few isolated buttes in Wyoming (Hodson and others, 1973).

The Arikaree Formation of Miocene age unconformably overlies the Fort Union or White River Formation and is composed of greenish-gray fine-grained continental sandstone interbedded with light-gray volcanic ash. The Arikaree contains abundant small green clay concretions and fossil plant material. It is capped with green orthoquartzite (Balster, 1971). Small exposures of the unit are found southwest of the Cedar Creek anticline and on isolated buttes in the extreme southeastern part of the study area (Hodson and others, 1973).

The youngest geologic units in the study area are terrace deposits and alluvium of Pleistocene and Holocene age. Terrace deposits are restricted primarily to the valley sides and uplands along the Missouri, Yellowstone, Musselshell, Tongue, and Powder Rivers. Terrace deposits are composed mainly of lenses of gravel, sand, silt, and clay. Alluvium is present mostly beneath the river valleys and principal tributaries. It consists of unconsolidated and interbedded gravel, sand, silt, and clay.

Structure

The study area includes two structural basins and part of a third: the Powder River basin, the Bull Mountains basin, and the westernmost extension of the Williston basin (fig. 3). All three basins are stratigraphically similar. A detailed discussion of the structure of these basins is beyond the scope of this report. However, a brief description of the structural features and their origin is provided as background for understanding the hydrology of the area.

The Powder River basin (fig. 3) is an asymmetric trough, with a generally north-trending fold axis. Beds on the west limb adjacent to the Bighorn uplift have a steep (10-25 degrees or greater) eastward dip, and beds on the east limb contiguous to the Black Hills have a gentle (2-3 degrees) westward dip (Glass, 1976). The Powder River basin axial plane dips steeply to the west (Dahl and Hagmaier, 1976). The Tongue River syncline is the northernmost extension of the basin axis. Structural features enclosing the basin include the Miles City arch to the northeast, the Black Hills uplift to the east, the Hartville uplift to the southeast, the Laramie uplift to the south, the Casper arch to the southwest, and the Bighorn and Pryor uplifts to the west. These anticlinal structures bounding the basin were associated with, or reactivated during, the Laramide orogeny and provided source areas for the sediments that were deposited in the ancestral Powder River basin (Seeland, 1976; Santos, 1981). Outflow from the basin was generally to the north during the Late Cretaceous and Paleocene Epochs. Although many faults of limited lateral extent occur in parts of the basin, large faults are relatively rare. The west flank of the Powder River basin, particularly in northern Wyoming, is an exception (Glass, 1976).

The Bull Mountains basin is bounded by the Pryor uplift to the south, the Big Coulee-Hailstone dome to the west, and various structures of the Big Snowy anticlinorium to the north. The Porcupine dome partly separates the Bull Mountains basin from the Williston basin. The Bull Mountains basin (fig. 3) is connected to the northwestern edge of the Powder River basin by the Ashland syncline. The northwest-trending synclinal axis of the Ashland syncline (Dobbin and Erdmann, 1955) splits as it extends westward into the Bull Mountains basin. The southernmost syncline west of the split forms a broad downwarp in the central part of the basin, whereas the northernmost syncline extends toward the Willow Creek syncline.

The study area north of the Yellowstone River and the Porcupine dome are the westernmost extensions of the Williston basin of North Dakota and eastern Montana (fig. 3). The Williston basin is separated from the Powder River basin by the Miles City arch and the Black Hills uplift. The north-trending fold axis of the Cedar Creek anticline and the associated Redwater anticline form part of a natural northeast boundary to the study area. Significant structural features within the Williston basin include the Blood Creek syncline, the Cedar Creek anticline, and the Weldon fault (or Weldon fault zone). The Blood Creek syncline, north of Porcupine dome, has a northwest- to west-trending fold axis that gently plunges to the east into the Williston basin (Dobbin and Erdmann, 1955). The steep westward dip of the western limb of the Cedar Creek anticline also has resulted in associated vertical faulting (Osterwald and Dean, 1958). The northeast-striking Weldon fault (or Weldon fault zone) has an associated northeast-trending monocline ([illegible] and [illegible], 1980).

HYDROGEOLOGIC UNITS

Definition of hydrogeologic units

In this report, aquifers are considered to be rocks or unconsolidated sediments that contain 50 percent or greater composite sandstone content and yield signifi-

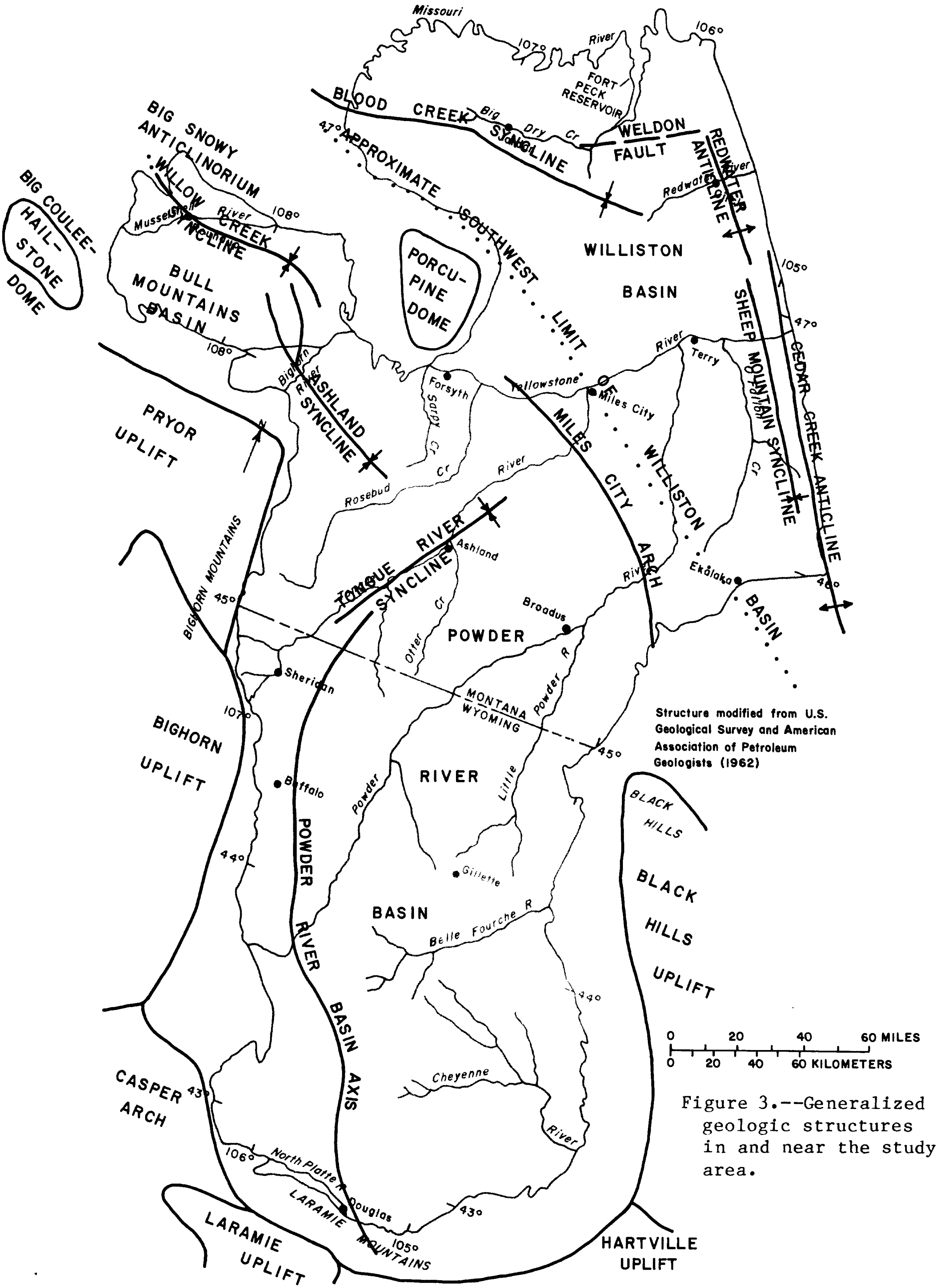


Figure 3.--Generalized geologic structures in and near the study area.

cant quantities of water to wells and springs. This definition follows that of Stoner and Lewis (1980) and Lewis and Hotchkiss (1981), who identified hydrogeologic units as aquifers or confining layers in the Powder River basin on the basis of sandstone content (percentage of the unit composed of sandstone beds 5 ft or greater in thickness) as determined from geophysical logs. Confining layers are beds of predominantly fine-grained rocks or sediments that have less than 50 percent sandstone content, are relatively impermeable to water movement, and yield little or no water to wells or springs.

On the basis of sandstone content and permeability, the authors differentiated five major hydrogeologic units overlying the relatively impermeable Bearpaw Shale at any given location (fig. 4). In most of the area three aquifers are separated by two confining layers. However, in easternmost Montana the middle (Tullock) aquifer cannot be differentiated on geophysical logs from the upper (Lebo) confining layer; this combined unit (lower Fort Union aquifer) was mapped separately by Stoner and Lewis (1980).

In ascending order, the shallow hydrogeologic units identified in the study area are:

Fox Hills-lower Hell Creek aquifer--Composed of the Fox Hills Sandstone and lower part of the Hell Creek or Lance Formation. The fine- to medium-grained Fox Hills Sandstone, which contains thin beds of sandy shale, together with the lower Hell Creek or Lance, which contains interbedded fluvial sandstone, siltstone, and shale, form the basal aquifer.

Upper Hell Creek confining layer--Composed of the silty and shaly upper part of the Hell Creek Formation in southeastern Montana or the Lance Formation in northeastern Wyoming. The massive shale in Montana together with the interbedded shale and fine- to medium-grained sandstone in Wyoming form the lower confining layer.

Tullock aquifer--Composed of the basal channel sandstone of the Lebo Shale Member, where present, and the Tullock Member of the Fort Union Formation. The interbedded sandstone, siltstone, and shale grade upward to sandstone and sandy to silty shale and form the middle aquifer.

Lebo confining layer--Composed of the Lebo Shale Member of the Fort Union Formation. The generally massive shale of the Lebo forms the upper confining layer.

Lower Fort Union aquifer--Composed of the Lebo confining layer and the Tullock aquifer where they cannot be differentiated in the northeastern part of the study area. This unit is equivalent to the Ludlow Member of the Fort Union Formation in easternmost Montana. The Ludlow is geologically more like the Tullock than the Lebo and, therefore, has been classified as an aquifer.

The Tongue River aquifer--Composed primarily of the Tongue River Member of the Fort Union Formation and the Wasatch Formation. The unit includes the overlying White River and Arikaree Formations, and alluvium and terrace deposits where they are present. It also includes channel sandstone and siltstone of the upper part of the Lebo Shale Member of the Fort Union Formation. The thick-bedded fine- to medium-grained massive sandstones

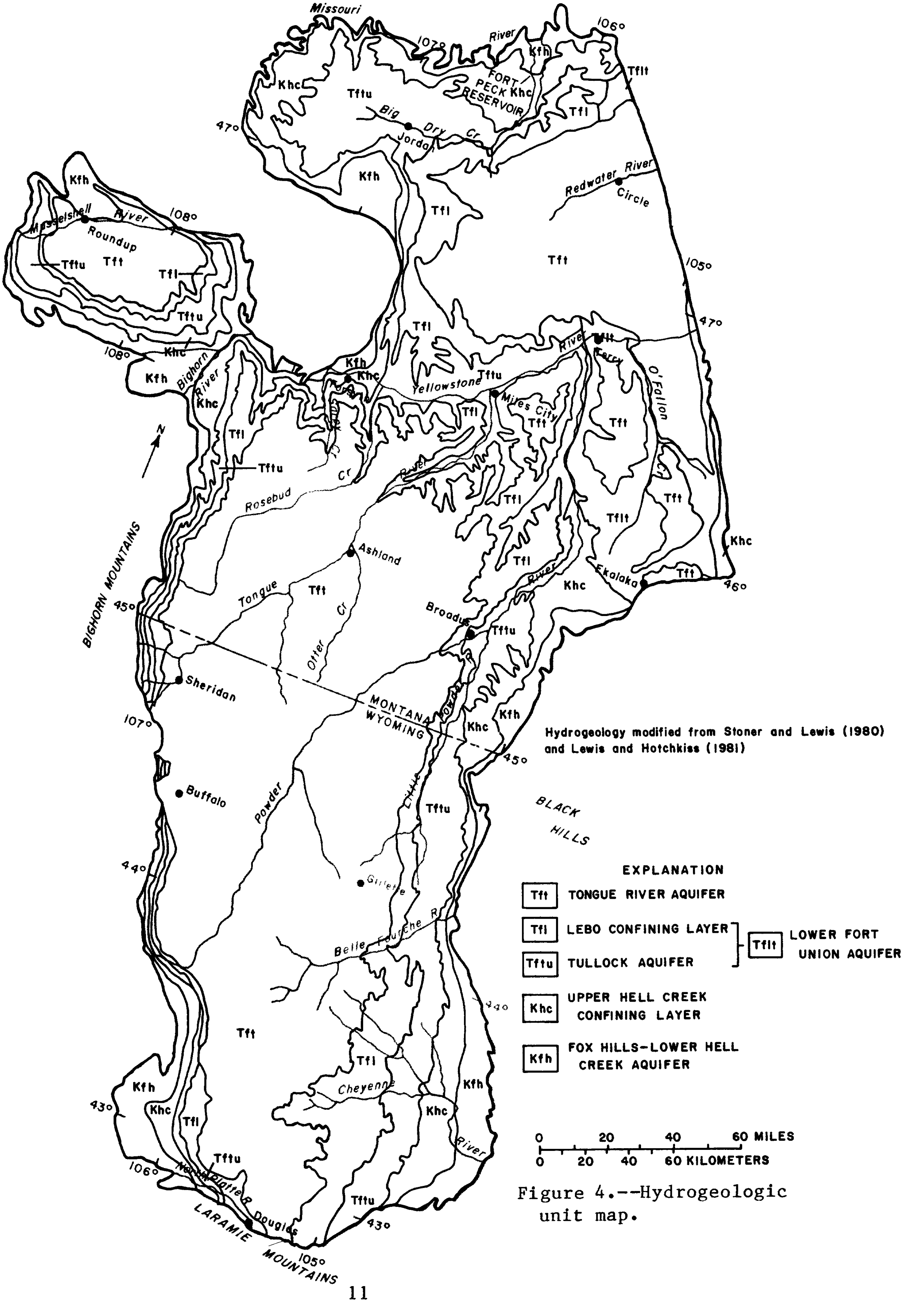


Figure 4.--Hydrogeologic unit map.

and siltstones are similar in the Tongue River Member and Wasatch Formation. The Tongue River and Wasatch together with the isolated White River and Arikaree Formations form the uppermost part of the aquifer.

Anomalies occur at various locations where confining layers may contain extensive sandstone lenses and function as aquifers, or aquifers may locally contain thick shale beds and function as confining layers. In addition, the lenticularity of sandstone units within all aquifers limits the confidence with which the hydrologic properties can be extrapolated.

The tops of hydrogeologic units were identified from geophysical logs of oil test wells. The most commonly used geophysical logs were standard electric logs. Where available, other logs such as natural gamma were used for correlation. Lithologic logs provided by water-well drillers also were used.

Representative electric logs from the northwestern and northeastern (fig. 5) and southern (fig. 6) parts of the study area illustrate the subsurface relationships of hydrogeologic to geologic units. The Bearpaw Shale, the base of the shallow aquifer system, exhibits relatively large or positive spontaneous potential and relatively small resistivity values, causing the curves (as normally presented) to converge. Curves representing the upper Hell Creek and Lebo confining layers have a similar appearance but do not converge as much, owing to a greater silt or fine-sand content. The Fox Hills-lower Hell Creek, Tullock, lower Fort Union, and Tongue River aquifers exhibit relatively small or negative spontaneous potential and large resistivity deflections, causing the curves to diverge in response to lithologies of sandstone, siltstone, sandy or silty shale, and sometimes coal. The varying degrees of response of these aquifers to the logging techniques indicate that these deposits interfinger and locally restrict the vertical flow of water between the aquifers. Similarly, the lack of regionally persistent sandstone units indicates that the horizontal interconnection and flow of water within the aquifers also may be locally restricted.

Physical characteristics of hydrogeologic units

Electric resistivity and self-potential geophysical logs generally describe hydrologic units better than geologic formation boundaries because variations in grain size and sorting--physical characteristics which greatly affect the water-yielding property of a unit--can be inferred from changes in porosity and clay content seen on logs. Thus, more than one hydrologic unit may be distinguished within a geologic formation on this basis. Alternately, a hydrologic unit may overlap geologic-formation boundaries. For example, the Fox Hills Sandstone and the lower part of the Hell Creek Formation (or the lower part of the Lance Formation) compose the Fox Hills-lower Hell Creek aquifer (figs. 5 and 6).

Fox Hills-lower Hell Creek aquifer

The Fox Hills-lower Hell Creek aquifer ranges in thickness from 0 ft at the base of its outcrop to 2,550 ft in the southern part of the Powder River basin in Wyoming. The mean thickness of the aquifer in the study area is 666 ft. The sandstone content in this aquifer ranges from 10 to 94 percent and has a mean value of 50 percent.

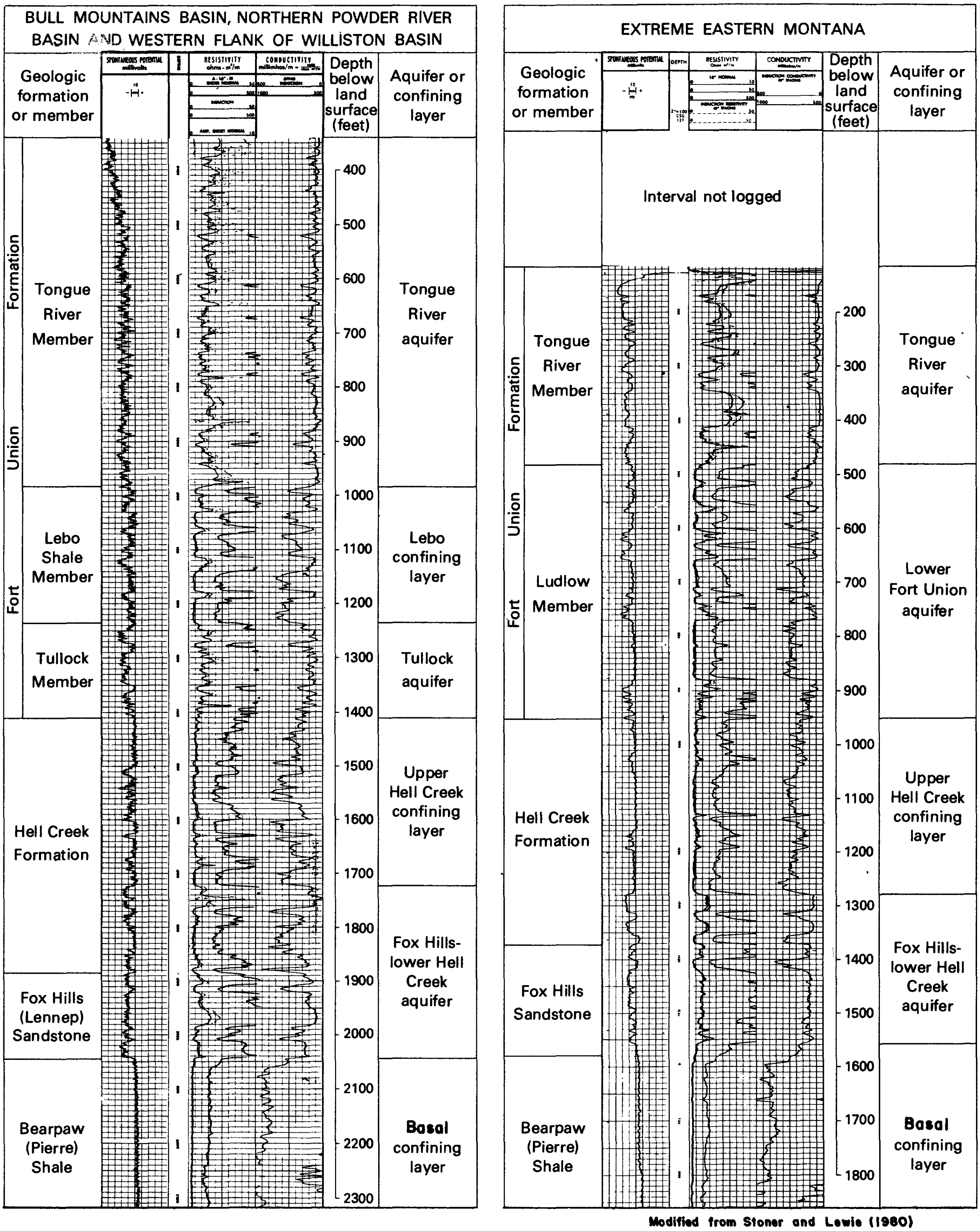


Figure 5.--Representative electric logs showing correlation between selected geologic and hydrogeologic units, Montana.

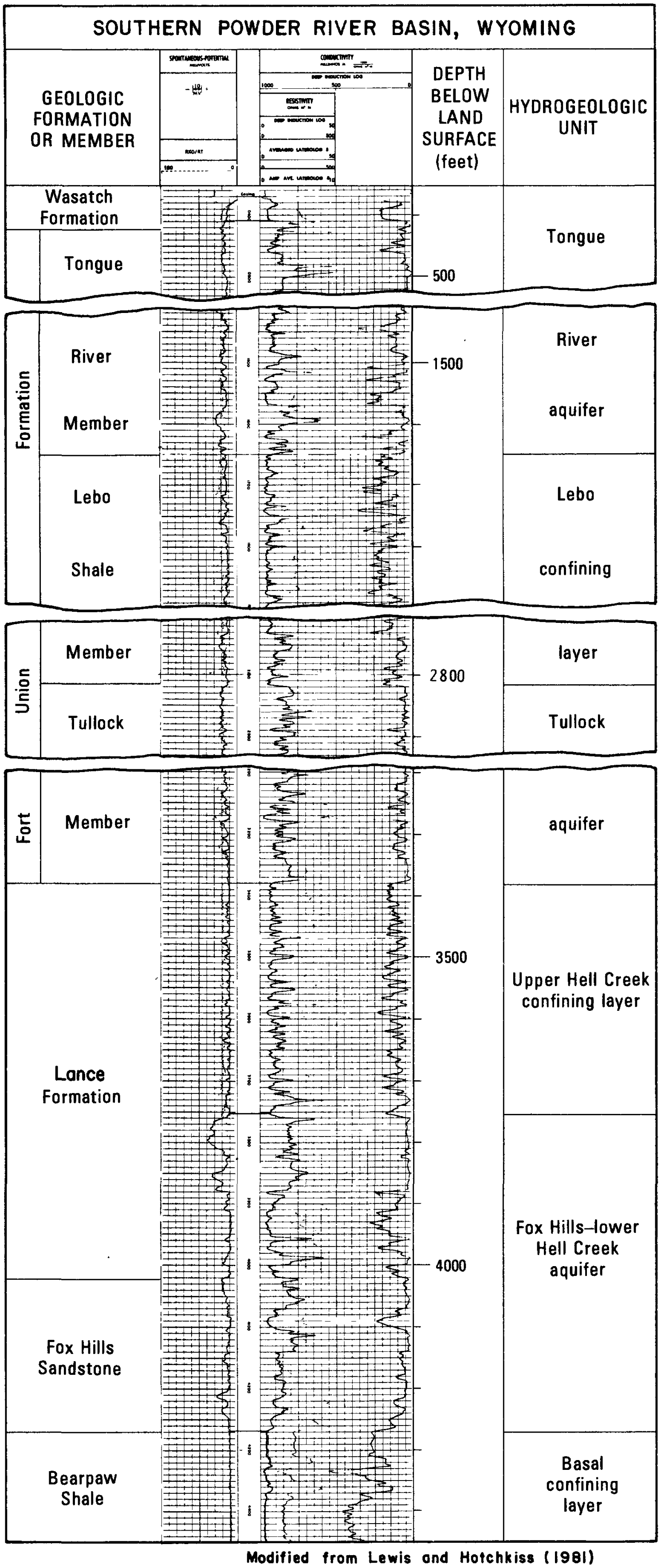


Figure 6.--Representative electric log showing correlation between selected geologic and hydrogeologic units, Wyoming.

The Fox Hills-lower Hell Creek aquifer is a significant source of water throughout the study area. The mean sandstone content of 50 percent indicates that the unit will yield water to wells in most areas. Yields are generally less than 100 gal/min to wells (Lewis and Hotchkiss, 1981), but yields greater than 200 gal/min are locally available for municipal and oil-field waterflooding use (Hodson and others, 1973). Yields of 20 gal/min have been measured in flowing wells along major stream valleys in the central part of the Powder River basin.

Upper Hell Creek confining layer

The upper Hell Creek confining layer ranges in thickness from 0 ft at the base of its outcrop to 2,000 ft in the extreme southern part of the study area. The mean thickness of the layer is about 514 ft. Lewis and Hotchkiss (1981) postulate that the anomalously large thickness of the upper Hell Creek confining layer near the western, southwestern, and southern edges of the Powder River basin was caused by rates of basin subsidence during deposition that were greater than in adjacent areas.

Sandstone content in this confining layer ranges from 9 percent near outcrop areas to 88 percent where channel deposits occur in the section; the mean is 35 percent. The relatively small mean value indicates that the unit will function regionally as a confining layer and retard water movement toward wells. Yields to the few wells completed in this unit may be as much as 4 gal/min (Lewis and Hotchkiss, 1981). Some wells flow at land surface in the major stream valleys. The large number of shale stringers and beds, as illustrated on geophysical logs (figs. 5 and 6), restricts vertical flow between beds and flow to adjacent aquifers.

Tullock aquifer

Because the Tullock aquifer lies stratigraphically between the underlying upper Hell Creek and overlying Lebo confining layers, it is hydrologically confined, except near outcrop areas. The thickness of the unit ranges from 0 ft at the base of its outcrop to 1,960 ft in the deepest parts of the area. The mean thickness is 633 ft.

Sandstone content in the Tullock aquifer ranges from 21 to 88 percent. The mean sandstone content is 53 percent, which indicates that regionally the unit functions as an aquifer. In fact, wells completed in fine-grained sandstone and coal beds may yield as much as 40 gal/min, but yields of 15 gal/min are more common (Lewis and Hotchkiss, 1981). Locally, the aquifer is most productive where thick stream-channel deposits are present.

Lebo confining layer

The Lebo confining layer ranges in thickness from 0 to 3,780 ft and has a mean thickness of 630 ft. The wide variability in thickness may be due to prolonged and continued deposition of volcanic material at lacustrine sites, and the formation of channel-deposit sandstones near the upper surface and base of the unit (Lewis and Hotchkiss, 1981).

The sandstone content of the Lebo confining layer is extremely variable, ranging from 6 to 93 percent. The mean sandstone content is 31 percent, less than the underlying Tullock aquifer. The small mean value indicates that regionally the Lebo confining layer will generally retard water movement. Yields to wells are small; however, where the sandstone content is locally large, this confining layer may yield as much as 10 gal/min to wells. The sandstone content increases in the northeastern part of the area near the Miles City arch, where channel sandstones in the unit are more abundant (Lewis and Hotchkiss, 1981).

Lower Fort Union aquifer

The lower Fort Union aquifer ranges in thickness from 0 ft at the base of its outcrop to 550 ft in the area between the Yellowstone and Missouri Rivers. The mean and range in sandstone content are believed to be similar to that of the Tullock aquifer. Shaly sandstones, siltstones, and coal beds are the water-yielding horizons. Typical wells may yield as much as 12 gal/min (Stoner and Lewis, 1980).

Tongue River aquifer

The Tongue River aquifer, the uppermost (surficial) unit of the shallow aquifer system, is considerably thicker than the underlying units. It is hydrologically confined except near land surface. The aquifer is as much as 3,910 ft thick in the southern part of the Powder River basin and has a mean thickness of 1,240 ft. The wide variation in thickness over short distances has resulted from erosion by streams and rivers of the present drainage system.

The sandstone content of the Tongue River aquifer ranges from 21 to 91 percent and has a mean value of 54 percent. The variation may be due in part to erosion but is mostly due to differences in thickness at the time the unit was deposited. The mean sandstone content indicates that regionally the unit is an aquifer and will yield water to wells. Yields to wells completed in thick saturated sequences of sandstone may be as large as 160 gal/min in the Montana part of the study area. In Wyoming the additional thickness of the included Wasatch Formation and other units locally increases the maximum yields to about 500 gal/min (Lewis and Hotchkiss, 1981).

GROUND WATER

Aquifer properties

Horizontal flow

Transmissivity is a property of an aquifer and the water it contains that describes the ability of a given saturated thickness of rock or unconsolidated material to transmit water. Hydraulic conductivity is related to transmissivity in the following way:

$$T = Kb \qquad (1)$$

where

T is transmissivity (L^2T^{-1}, where L is length and T is time),
K is hydraulic conductivity (LT^{-1}), and
b is aquifer thickness (L).

Specifically, hydraulic conductivity is defined as the volume of water at the existing kinematic viscosity that will move in unit time under a unit hydraulic gradient through a unit area measured at right angles to the direction of flow. In a heterogeneous aquifer, the hydraulic conductivity (K) in equation 1 becomes a cumulative hydraulic conductivity over the thickness. Thus, the transmissivity can be expressed by:

$$T = \sum_{i=1}^{n} (K_i \; b_i) \tag{2}$$

where i increments of Kb are summed over n intervals within the aquifer.

Hydraulic conductivity is also related to kinematic viscosity and intrinsic permeability in the following way:

$$K = \frac{kg}{\nu} \tag{3}$$

where

K is hydraulic conductivity (LT^{-1}),
k is intrinsic permeability (L^2),
g is the acceleration due to gravity (LT^{-2}), and
ν is the kinematic viscosity (L^2T^{-1}).

Intrinsic permeability is a function of the size, shape, and interconnection of pore space of the aquifer. Kinematic viscosity is a property of the fluid, and is temperature dependent.

Overburden pressure or effective stress (Poland and others, 1972) causes compaction of the pore volume in rocks at depth, thereby decreasing intrinsic permeability and related values of hydraulic conductivity and transmissivity. The relationship of the ratio of permeability with various overburden pressures to permeability with zero overburden pressure has been measured by Fatt and Davis (1952). The permeability ratio becomes a compaction correction factor for adjusting transmissivity for effective stress. Specifically,

$$T_C = C_{MP} \; T \tag{4}$$

where

T_C is depth-corrected transmissivity (L^2T^{-1})
C_{MP} is compaction correction factor at the midpoint of the hydrogeologic unit (dimensionless), and
T is original transmissivity (L^2T^{-1}).

The actual equations used to determine the compaction correction factor are presented in a subsequent section of this report.

An additional property important to the study of aquifers and aquifer systems is the capability of the porous medium to store water. Three terms related to storage need to be understood even though the present study utilizes a steady-state model with no storage input. The terms are storage coefficient, specific storage, and hydraulic diffusivity:

Storage coefficient, S (dimensionless), is the volume of water a confined aquifer releases from or takes into storage per unit surface area of the aquifer per unit change in head (Lohman, 1972).

Specific storage, S_s (L^{-1}), is the volume of water released from a unit volume of the saturated medium as the result of a unit decline in head (Poland and others, 1972).

Hydraulic diffusivity, α (L^2 T^{-1}), is the ratio of the hydraulic conductivity of the porous medium to the specific storage or K/S_s (Poland and others, 1972).

These concepts are important in subsequent parts of this report.

The most reliable technique for determining the transmissivity or hydraulic conductivity of saturated media is analysis of data from aquifer tests in which a known thickness of aquifer is stressed (usually by pumping), and the response of the aquifer to that stress is carefully observed and recorded. Even when such tests are carefully conducted, numerous complexities both within the aquifer and within the analysis technique may lead to inaccurate values. Where aquifer lithology is extremely variable, as in the study area, few aquifer tests over a 42,000-mi^2 area can give little more than estimates of minimum aquifer transmissivities.

A statistical summary of the known aquifer transmissivity listed in U.S. Geological Survey files from the study area is given in table 1. The small number of tests, together with the fact that the standard deviations of the tests in any given aquifer are similar to or greater than the mean, indicate large variability of transmissivity within each aquifer. This result may be due in part to the fact that most wells only partly penetrate an aquifer. In most instances, these tests were performed for special purposes and were not designed to give a transmissivity representative of the full thickness of the aquifer.

A summary of aquifer tests at wells drilled, developed, and tested by the Montana Bureau of Mines and Geology is given in table 2. The tests were designed to target horizontal hydraulic conductivity and vertical hydraulic conductance per unit area in the Fox Hills-lower Hell Creek aquifer, upper Hell Creek confining layer, and Tullock aquifer in areas generally near the Yellowstone River.

Vertical hydraulic conductance per unit area

One purpose of the aquifer testing conducted by the Montana Bureau of Mines and Geology was to determine vertical hydraulic conductance per unit area by measuring response in observation wells in aquifer units above and below the unit that was stressed, but no response was observed during the tests. However, the drawdown-ratio method (Neuman and Witherspoon, 1972) was used on four of these tests to identify the approximate maximum limit for vertical hydraulic conductance per unit area. This method can be applied to the early drawdown part of an aquifer test in

Table 1.--*Summary of transmissivity values for the shallow hydrogeologic units from existing U.S. Geological Survey data files*

Hydrogeologic unit [1]	State	Transmissivity, in feet squared per day: Minimum	Maximum	Mean	Standard deviation	Number of aquifer tests
FHCC	Montana	3.6	388	133	118	26
	Wyoming	23	1,470	319	441	10
HLCK	Montana	.15	630	112	191	11
	Wyoming	9.3	281	77.4	101	7
TLCK [2]	Montana	1.0	130	67.0	64.6	3
	Wyoming	70.4	70.4	70.4	--	1
LEBO [2,3]	Montana	--	--	--	--	--
	Wyoming	.04	.61	.32	.40	2
TGRV	Montana	.11	400	115	184	14
	Wyoming	5.1	869	209	274	9

[1]Unit codes--FHHC is Fox Hills-lower Hell Creek aquifer, HLCK is upper Hell Creek confining layer, TLCK is Tullock aquifer, LEBO is Lebo confining layer, and TGRV is Tongue River aquifer.

[2]Small number of aquifer tests make statistics suspect.

[3]Aquifer tests in the Lebo confining layer and, to a lesser extent, the upper Hell Creek confining layer were only two targeted on intervals of little transmissivity.

Table 2.--*Summary of transmissivity values from wells drilled by the Montana Bureau of Mines and Geology*

Well location	Hydro-geologic unit [1]	Tested interval (feet below land surface)	Pumping duration (minutes)	Pumping rate (cubic feet per minute)	Transmissivity (feet squared per day)
16N44E25BBAB01	FHHC	1,135-1,460	480	2.8	83.6
13N51E31BDCB01	FHHC	778-973	200	4.8	74.0
13N51E31BCDD01	HLCK	440-565	160	.12	.15
13N51E31BDCB02	TLCK	243-330	600	1.1	70.4
10N45E28BBBA01	FHHC	832-1,012	1,920	2.1	56.3
10N45E28BBAA01	HLCK	678-762	480	1.4	30.0
08N31E36DDDD01	FHHC	904-1,175	500	.30	8.5
08N31E36DDDD02	HLCK	803-850	240	.11	2.1
08N31E36DDDD03	HLCK	431-486	36	.27	.6
06N44E36CACD01	FHHC	760-902	517	2.7	45.1
06N44E36CACD02	HLCK	508-609	300	2.1	19.4
06N44E36CACD03	HLCK	290-314	60	.17	2.0
05N25E16CCCC01	FHHC	1,118-1,350	451	.14	3.6
05N25E16CCCC02	TLCK	362-427	118	.08	1.0

[1]Unit codes--FHHC is Fox Hills-lower Hell Creek aquifer, HLCK is upper Hell Creek confining layer, and TLCK is Tullock aquifer.

any aquifer and its adjacent confining beds. In this application, a maximum value for vertical hydraulic conductance per unit area (K') within each of the tested shallow hydrogeologic units was estimated assuming a minimum measurable drawdown of 0.01 ft in the confining layer, although no clearly defined drawdown could be discerned. The calculated estimate of K', therefore, represents a possible maximum value if such a drawdown had actually been measured. Because no drawdown was measured by the end of the pumping period, the early time requirement of the method was fulfilled. Storage coefficients for calculation of hydraulic diffusivity of the aquifer were estimated to be 5.0×10^{-4} in each instance and specific storage of the confining layers was assumed to be 5.0×10^{-7} per foot (F. S. Riley, U.S. Geological Survey, oral commun., 1981). Data and the results of the drawdown-ratio method calculations are presented in table 3. A generalized diagram of one of the field tests is shown in figure 7. Although the drawdown-ratio method assumes strictly vertical flow in all but the pumped stratum, horizontal flow in the confining bed may provide flow that decreases observed drawdown in the confining bed. Horizontal flow would result in an unrealistically small calculated maximum vertical hydraulic conductance per unit area, because in this instance no horizontal flow was assumed. Nevertheless, the smallest value obtained from the tests, 4.8×10^{-5} ft/d, was used as a guide to the initial estimate of vertical hydraulic conductance per unit area.

Table 3.--*Estimates of maximum vertical hydraulic conductance per unit area for selected test sites*

[<, less than]

Well location	Site use	Hydrogeologic unit[1]	Drawdown[2] (feet)	Confining bed thickness (feet)	Horizontal hydraulic conductivity[3] (feet per day)	Vertical hydraulic conductance[4] per unit area (feet per day)
10N45E28BBBA01	Pumped	FHHC	52.4	--	0.31	--
10N45E28BBAA01	Observation	HLCK	5.4	70	--	<0.000048
10N45E28BBAA01	Pumped	HLCK	63.5	--	.36	--
10N45E28BBBA01	Observation	FHHC	6.7	70	--	<.00019
10N45E28BBBA02	Observation	TLCK	1.7	318	--	<.0067
06N44E36CACD01	Pumped	FHHC	70.5	--	.32	--
06N44E36CACD02	Observation	HLCK	3.6	151	--	<.0010
06N44E36CACD02	Pumped	HLCK	81.6	--	.19	--
06N44E36CACD01	Observation	FHHC	2.5	151	--	<.0018
06N44E36CACD03	Observation	HLCK	3.8	194	--	<.00043

[1]Unit codes--FHHC is Fox Hills-lower Hell Creek aquifer, HLCK is upper Hell Creek confining layer, and TLCK is Tullock aquifer.

[2]Drawdown in the pumped well was measured by Montana Bureau of Mines and Geology; drawdown in the confining bed was assumed to be 0.01 foot.

[3]Calculated by Montana Bureau of Mines and Geology.

[4]Calculated by the drawdown-ratio method of Neuman and Witherspoon (1972).

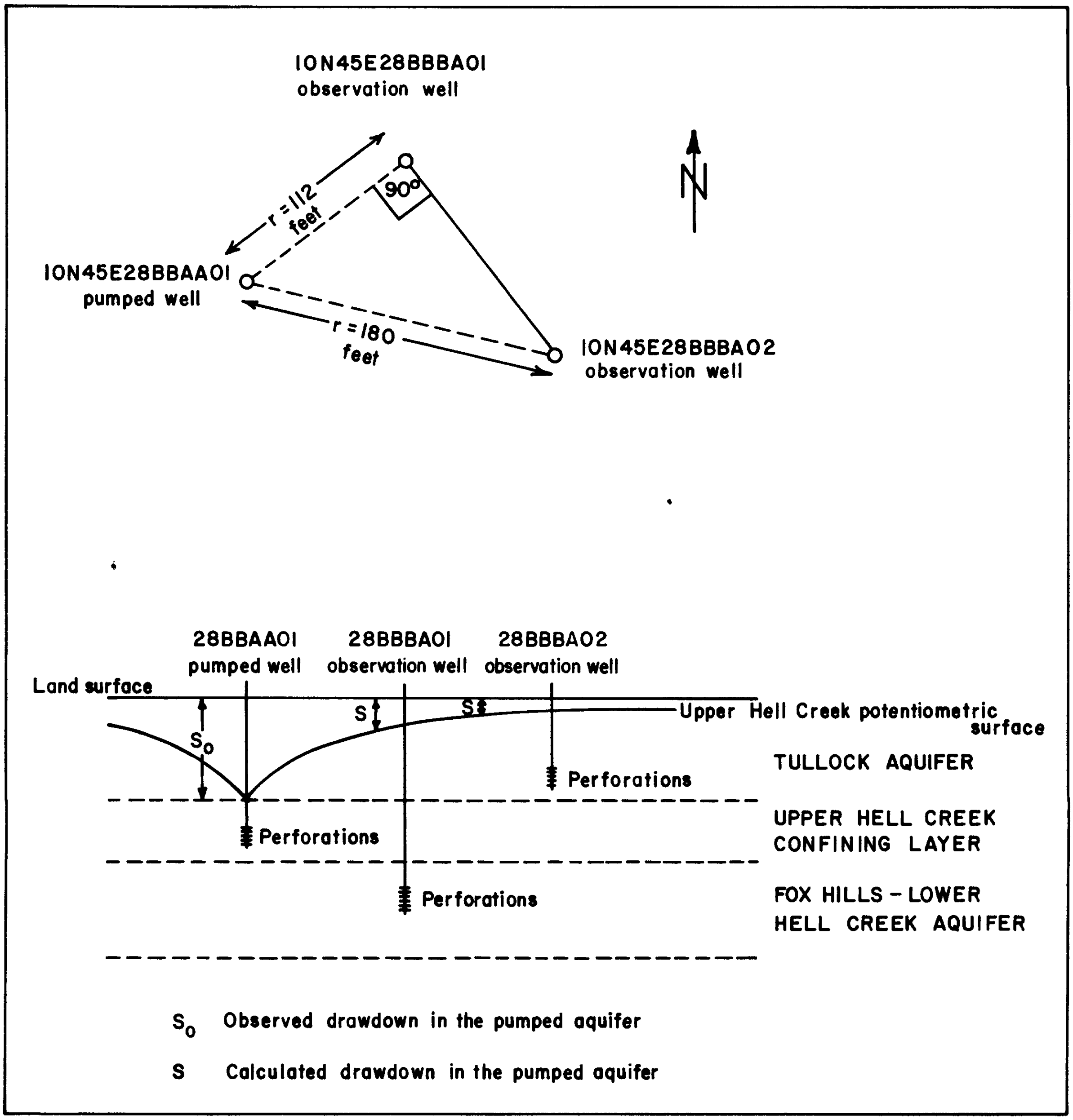


Figure 7.--Generalized diagram of well test used in estimation of vertical hydraulic conductance per unit area.

Potentiometric surfaces

The Fox Hills-lower Hell Creek aquifer potentiometric-surface map (fig. 8) was generated from data whose distribution varied widely across the study area. Data for about 50 observation sites, mostly from the area of outcrop on the east side of the Powder River basin, were available for Wyoming (M. G. Croft, U.S. Geological Survey, written commun., 1978), as compared to about 850 sites in the Montana part of the study area (Levings, 1982). Because of the paucity of data in Wyoming, potentiometric contours in the south-central part of the Powder River basin are speculative.

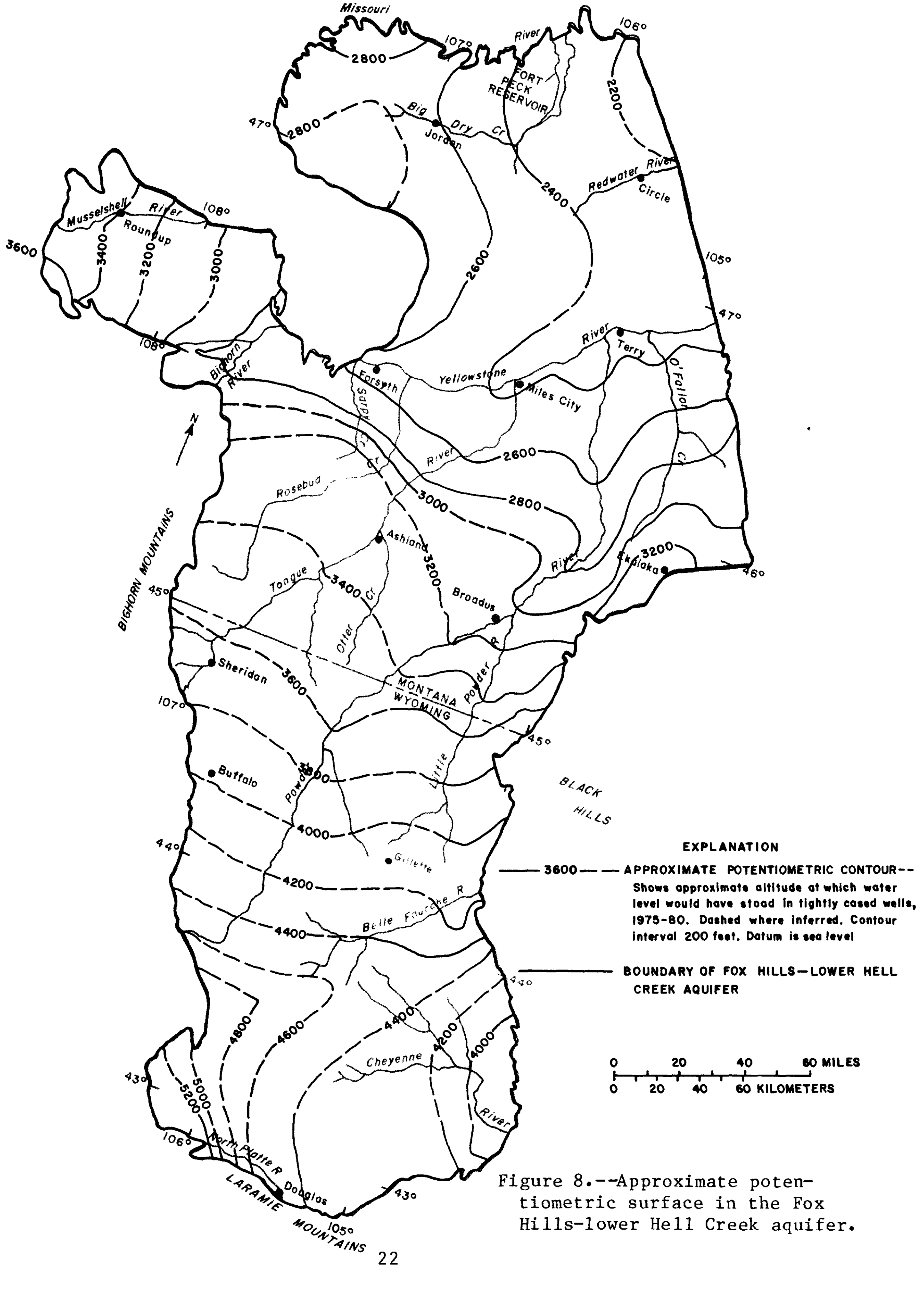


Figure 8.--Approximate potentiometric surface in the Fox Hills-lower Hell Creek aquifer.

The potentiometric-surface maps of the upper Hell Creek confining layer (fig. 9), the Tullock aquifer (fig. 10), and the Lebo confining layer (fig. 11) have similar data-distribution problems, with even fewer total data points than the underlying Fox Hills-lower Hell Creek aquifer. The upper Hell Creek confining layer has about 350 points; the Tullock aquifer, about 400 points; and the Lebo confining layer, about 270 points. In each instance, the potentiometric contours are not very reliable in the central and western parts of the Powder River basin in Wyoming.

The Tongue River aquifer potentiometric surface was mapped from 1,900 observation sites, 177 of which were located in the Powder River basin of Wyoming (fig. 12). The potentiometric data for the Tongue River aquifer are more evenly distributed across the study area than for the other hydrogeologic units. Even so, the thickness of the unit (1,240 ft) together with the possibility of perched aquifers overlying the fine-grained sedimentary deposits, especially those associated with coal beds, may lead to inaccuracies in depicting the Tongue River potentiometric surface.

Potentiometric-surface maps indicate hydraulic gradients, which can be affected by water use, geometry of the areas of recharge and discharge, and variations in hydraulic conductivity and leakage. The Fox Hills-lower Hell Creek and Tongue River aquifers are the most productive and reliable sources of ground water in the study area. The expense of drilling to the Fox Hills-lower Hell Creek aquifer prevents its wide use, except along major stream valleys and in the outcrop areas where the depth to the aquifer is least. The Tullock aquifer has little more than incidental use as a water source except in its area of outcrop and along the stream valleys. Use of the upper Hell Creek and Lebo confining layers has been almost completely limited to domestic and stock wells in the areas of outcrop. Throughout the study area, potentiometric surfaces of the deeper aquifers seem to be affected by the location of the river, showing a discharge toward the topmost aquifer; however, the effects of the larger streams and rivers on deeper aquifers are subdued. Such effects may be caused by deep, abandoned flowing wells drilled to the Fox Hills-lower Hell Creek aquifer along the major drainages, such as in the central part of the Powder River area of Montana (Miller, 1979).

Potentiometric-surface maps are useful for determining areas of ground-water recharge and discharge. The approximate potentiometric surface in figure 8 implies discharge to the Powder and Little Powder Rivers, and inflow to the aquifer along the extreme southwest and southeast edges. The potentiometric surfaces in figures 9-12 show increasingly subdued versions of the same general pattern observed in figure 8. The effects may be caused by flowing wells or by leakage upward from aquifers deeper than the Tongue River aquifer in the vicinity of major streams where the uppermost aquifer locally has depressed hydraulic heads and has been thinned by erosion. These factors tend to increase the upward gradient in discharge areas and promote leakage upward.

The limited data generally indicate that the deeper aquifers have higher potentiometric heads, in relation to shallow aquifers, in the topographically higher southwestern part of the study area. The same is true for the northeastern part of the area, except that the hydraulic head in the upper Hell Creek confining layer appears to be locally higher than the hydraulic head in either the aquifer above or below. Thus, except near outcrops, the entire area appears to be a discharge area. The regional pattern of ground-water flow is undoubtedly complicated by lenticular beds and local differences in hydraulic conductivity. Consequently, localized hydraulic properties of the study area are probably important and need to be included in models of the area where such detailed information is available.

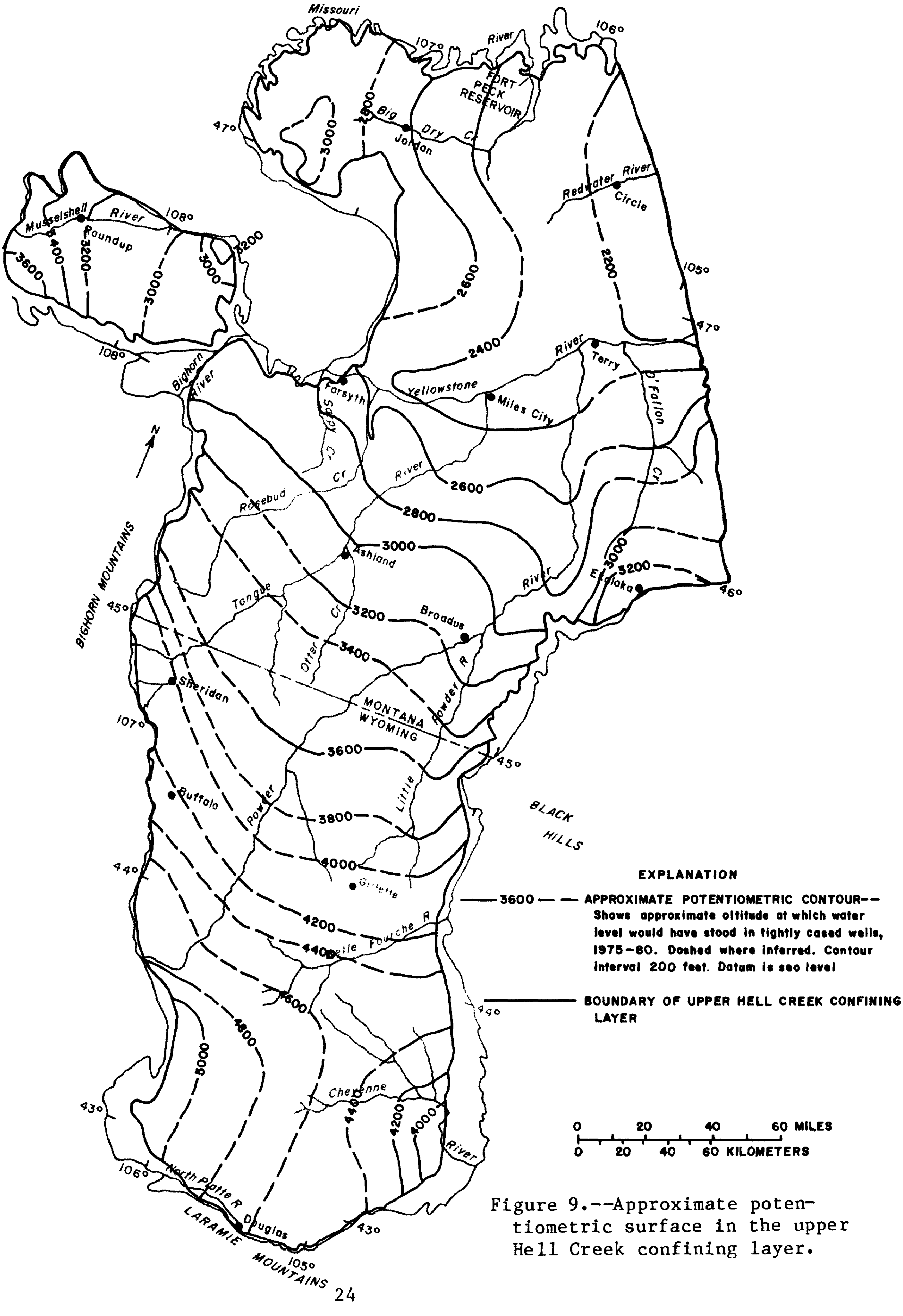


Figure 9.--Approximate potentiometric surface in the upper Hell Creek confining layer.

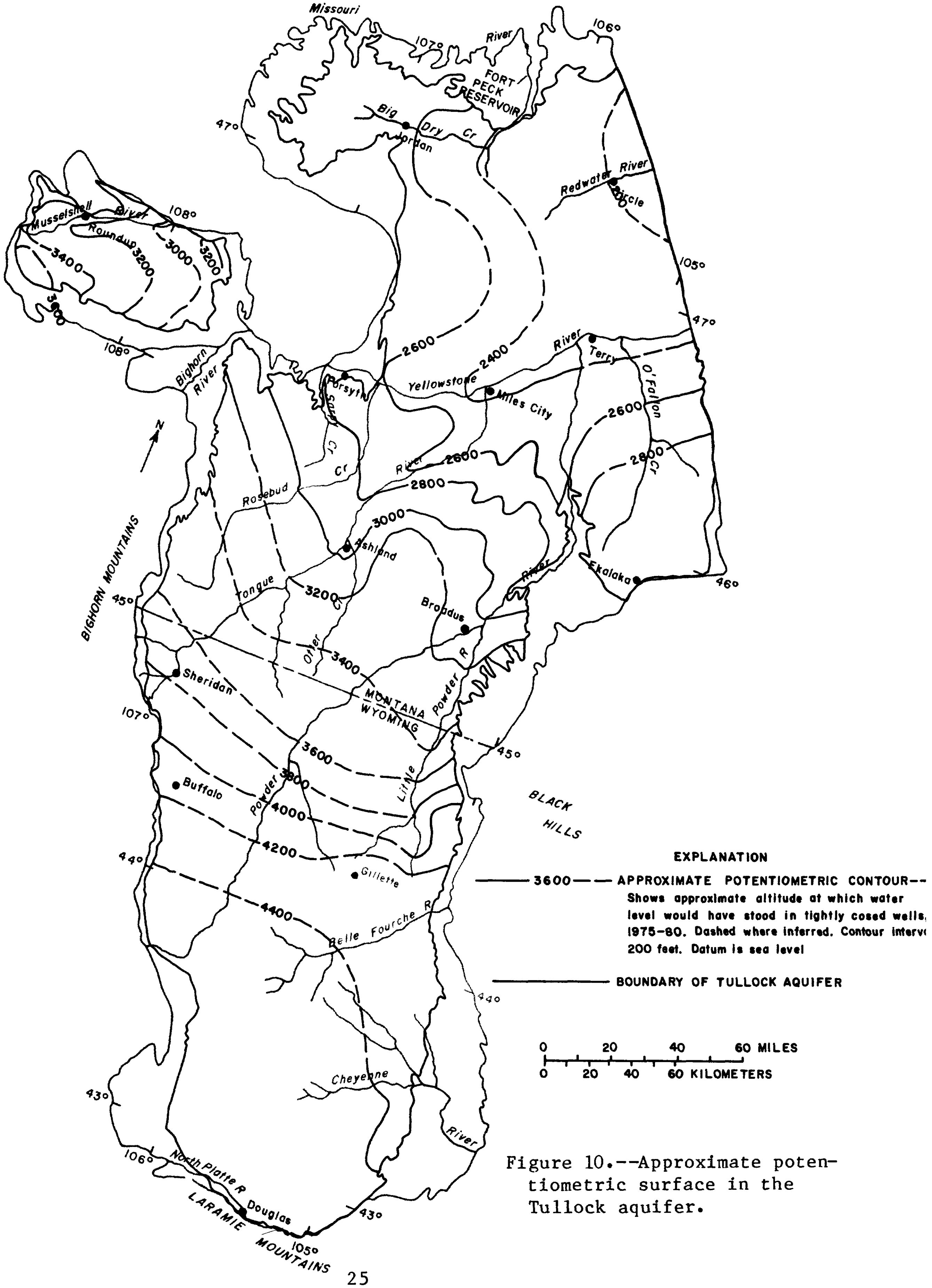


Figure 10.--Approximate potentiometric surface in the Tullock aquifer.

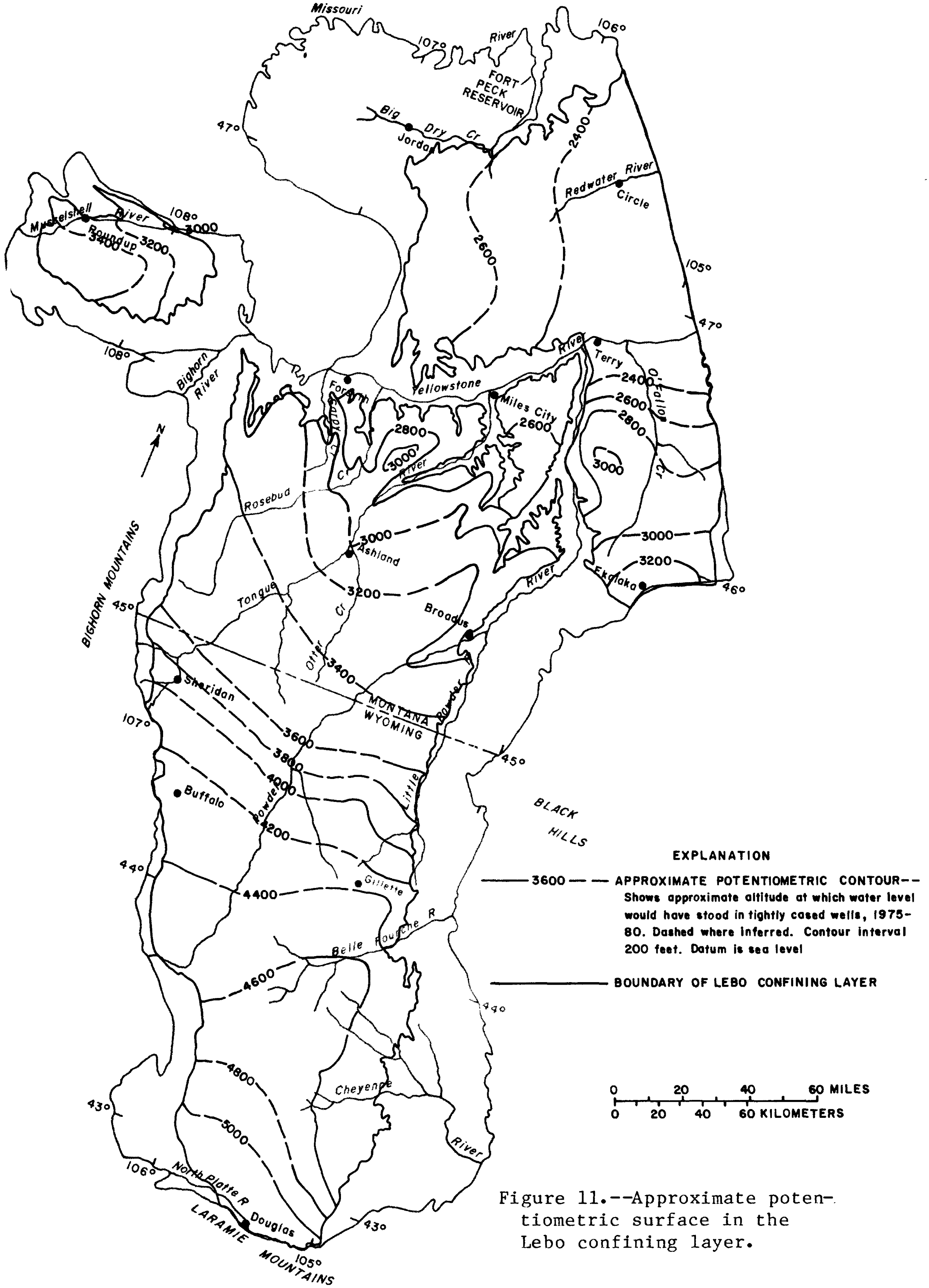


Figure 11.--Approximate potentiometric surface in the Lebo confining layer.

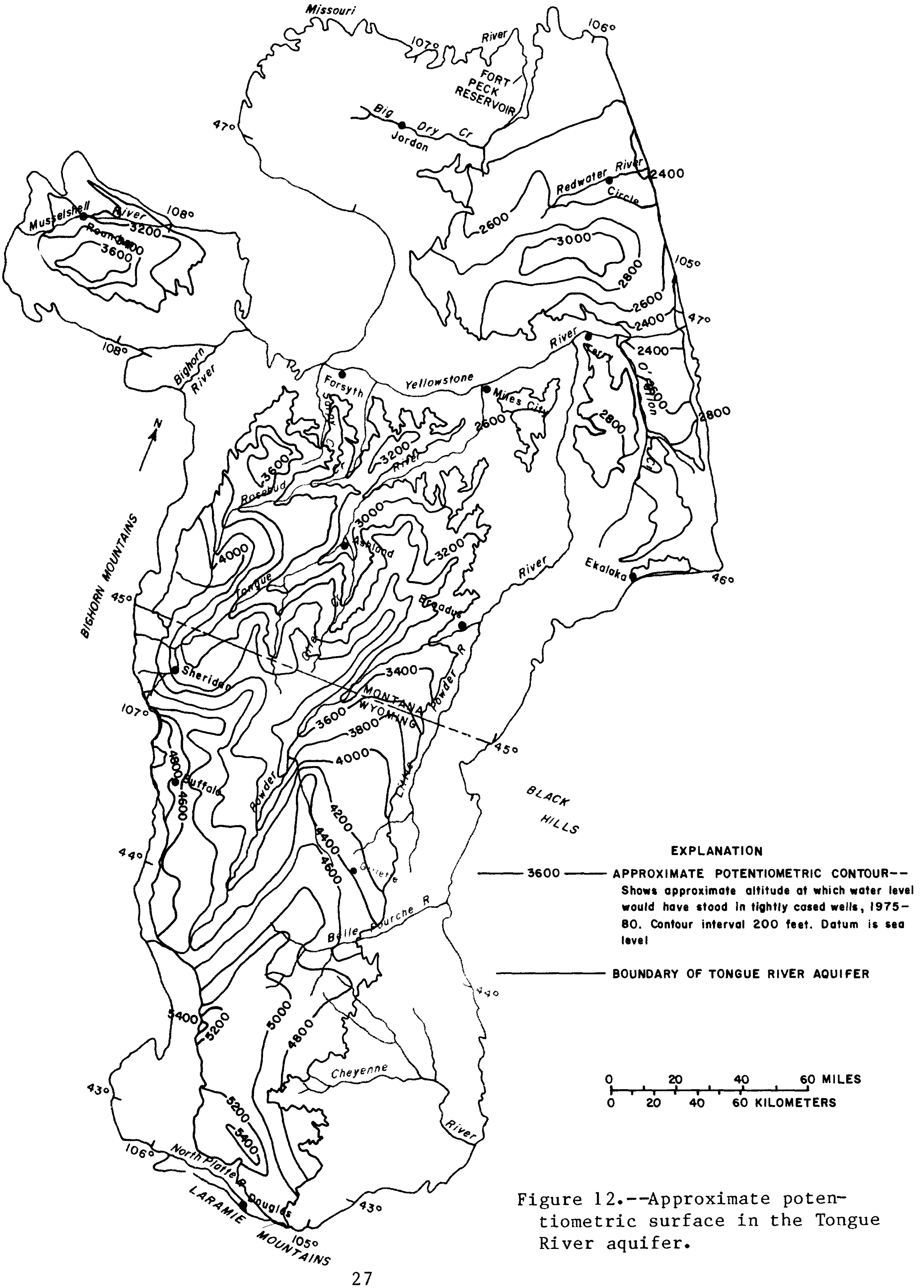


Figure 12.--Approximate potentiometric surface in the Tongue River aquifer.

CONCEPTUAL FLOW MODEL

The major source of recharge to the shallow hydrogeologic units is infiltration of water from precipitation and streamflow on areas of outcrop. Infiltration of water from losing streams within the area boundaries (Druse and others, 1981; Dodge and Levings, 1980) and especially along the mountain front perimeter also accounts for some recharge. No data are available to support suggestions that water leaks upward from or through the basal confining layer into the shallow hydrogeologic system in the Powder River basin (D. T. Hoxie, U.S. Geological Survey, oral commun., 1981). Therefore, although recharge from below the shallow hydrogeologic system is possible, it is not considered in this study.

Distribution of mean annual precipitation in the Montana part of the study area was shown on a map by Johnson and Omang (1976). A similar map including the Powder River basin in Montana and Wyoming was prepared by Toy and Munson (1978). These maps were adjusted to fit at their common boundary and combined to provide a map of mean annual precipitation encompassing the present study area (fig. 13).

Estimates of the amount of precipitation that infiltrates and recharges aquifers on areas of outcrops are dependent upon such factors as temporal distribution of precipitation, presence and type of vegetation, soil type, soil depth, and soil moisture deficit at the outcrop. In preliminary modeling of the southern High Plains of Texas, where rainfall and geology are comparable to those in the study area, J. B. Weeks (U.S. Geological Survey, written commun., 1982) reported that infiltration was 0.086 in. or 0.5 percent of annual precipitation. Rahn and Gries (1973) reported that about 3.6 percent of annual precipitation infiltrated into a carbonate aquifer in a 332-mi^2 drainage area in the Black Hills. Annual recharge in the study area was expected to be in the range of 0.5 to 3.6 percent of annual precipitation on the outcrop area.

Discharge from the area takes the form of underground outflow along the northeastern boundary; loss of ground water to gaining streams, springs, and seeps; evapotranspiration; and pumpage of ground water. Annual potential evapotranspiration from the central Powder River area of southeastern Montana was 51 in. during 1950-66 (Miller, 1979). This value is more than 4 times the precipitation in some parts of the study area. Little data are available regarding these losses except in localized areas (Miller, 1979). Well discharge from the shallow hydrogeologic units is small and is mostly limited to domestic use, stock use, and widely scattered irrigation or abandoned flowing wells. This limited discharge of ground water is too small to be of regional importance. As a generalization, discharge in the area is assumed to about equal recharge.

DIGITAL SIMULATION MODEL

Description of the model

Trescott (1975) and Trescott and Larson (1976) wrote and documented a computer program to numerically compute the hydraulic head at any location in an aquifer using a specified set of hydraulic properties, boundaries, and stresses. This digital simulation model, with several time-saving minor modifications, was used to simulate the shallow flow system in the study area.

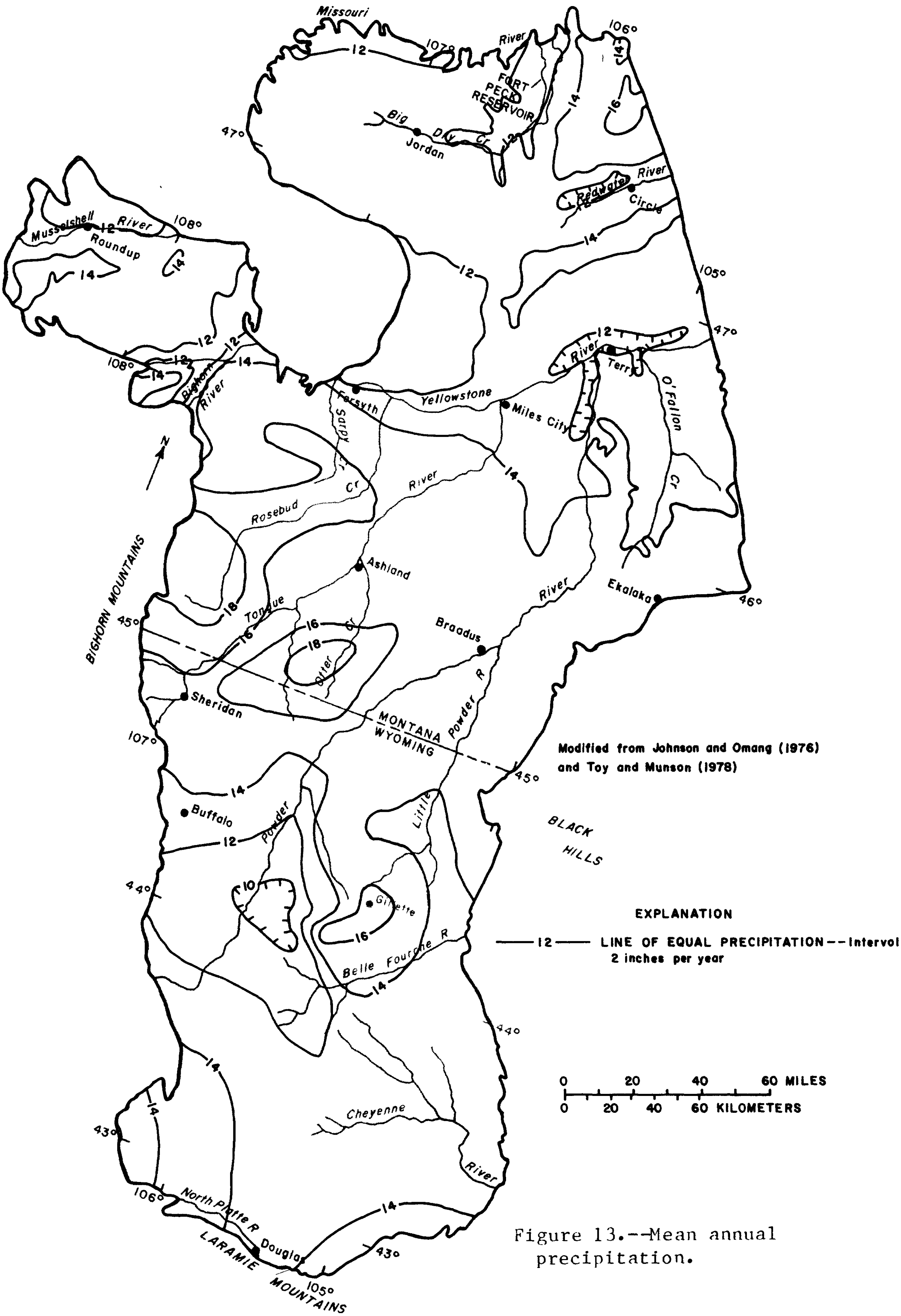


Figure 13.--Mean annual precipitation.

Model modifications

To aid in the interpretation of the model results, the format for the arrays of data has been standardized for ease of comparison of data values at a particular node. An acceleration parameter similar to that in the two-dimensional model of ground-water flow (Trescott and others, 1976, p. 51) was included to improve the efficiency of the iteration procedure. Positive and negative signs were added to the printout of maximum simulated potentiometric hydraulic-head change at each iteration so that the direction and magnitude of the change could be evaluated. When the vertical leakage option is specified in the program, an internal search is activated that checks potentiometric heads above and below all non-zero values of vertical leakage coefficient (TK) to ensure that they do not contain values of zero. The constant recharge from precipitation option (RECH) has been modified to allow recharge to all layers of the model so that recharge may be applied to the outcrop area of all aquifers. Finally, a statistical subroutine was added to aid in calibration. The mean and standard deviation of the difference between initial and calculated potentiometric head were calculated by model layer for all active nodes in the model.

Assumptions

Simplifying assumptions are necessary to construct a ground-water model. Assumptions either are inherent in the equations of flow on which the model is based or are required to simplify the hydrologic complexities of the prototype system. The assumptions inherent in the equations of flow include:

(1) Ground-water flow is completely described by Darcy's law.
(2) Within any active cell, the unit represented is homogeneous and horizontally isotropic.
(3) Recharge to the various units is instantaneous and constant with time.
(4) Discharge at a node occurs at a constant rate over the entire cell.

The simplifying assumptions relating to the prototype system include:

(1) The potentiometric surfaces of water (starting potentiometric heads) in the five modeled units are currently in a state of equilibrium.
(2) Each of the units in the system is considered to be confined everywhere.

In addition to the foregoing assumptions, the following conventions were adopted for this regional simulation:

(1) Recharge from precipitation has been distributed to the five units on the basis of the areal percentage of each node that is covered by outcrop.
(2) In easternmost Montana where the lower Fort Union aquifer represents the undifferentiated Lebo confining layer and the Tullock aquifer, transmissivity values were estimated for the Lebo and Tullock parts of the unit, but recharge was added only to the Lebo part.

Finite-difference grid

The finite-difference grid for the model consists of five layers, each with a maximum of 62 rows and 37 columns of nodes. The nodes were uniformly spaced 6 mi apart to approximate the geophysical data density and because regional rather than local hydrologic trends were to be modeled. Thus, the area of each cell is 36 mi^2 and the active grid represents an area of about 42,000 mi^2. Each layer (fig. 14) represents the mean thickness of a hydrogeologic unit over the area of occurrence. In ascending order, layer 1 is equivalent to the Fox Hills-lower Hell Creek aquifer; layer 2 is equivalent to the upper Hell Creek confining layer; layer 3 is equivalent to the Tullock aquifer (or the lower part of the lower Fort Union aquifer in easternmost Montana); layer 4 is equivalent to the Lebo confining layer (or the upper part of the lower Fort Union aquifer in easternmost Montana); and layer 5 is equivalent to the Tongue River aquifer.

Boundary conditions

The boundaries of the modeled layers within the study area have been selected to coincide with outcrops of the hydrogeologic units, structural limits of the basins, or line segments perpendicular to the flow direction. Boundary nodes are input either as constant heads or as constant flow; the flow may be zero (no flow).

Most of the boundary nodes around the Powder River and Bull Mountains basins occur where the shallow hydrogeologic units cease to exist and have been treated as no-flow boundaries. The boundaries of the study area where major rivers cross the outcrop of the hydrogeologic unit are represented by constant-head nodes (figs. 15-19). Also included as constant-head nodes are selected locations along the southwestern and western edges of the Powder River basin where streamflow from the west in addition to recharge from precipitation recharges aquifers at their outcrop.

Boundary nodes surrounding most of the Williston basin are constant-head nodes (figs. 15-19). The northeast boundary is modeled as constant head because it generally is perpendicular to the major outflow from the area. The north and northwestern boundaries are modeled as constant heads because the hydrogeologic units are bounded by the Musselshell and Missouri Rivers, or because the units crop out adjacent to the rivers and have potentiometric surfaces nearly identical to stream altitude. Along the western boundary of the Williston basin, constant heads were used to simulate recharge from the topographically higher strata of the Porcupine dome. Constant heads were used at the southern boundary of the Williston basin to simulate discharge toward the Yellowstone River in all layers that terminate north of the Yellowstone River. Finally, the complexity of aquifer outcrops required subdivision of recharge to the isolated groups of nodes in the area north and east of Ekalaka, Mont., in the southeastern corner of the Williston basin. Constant-head nodes were utilized to stabilize this area.

Hydrologic stresses

The system appears to be in a steady-state condition, stable for the period of record, and affected primarily by natural conditions. The river systems provide most of the control on the aquifer system for steady-state simulation and these systems have been simulated by selected constant-head nodes (figs. 15-19). Recharge is primarily from precipitation and streamflow on areas of outcrop. Stress or pumpage resulting in water-level decline from municipal pumpage at Gillette, Wyo., has been recognized and documented in at least one well in the Fox Hills-lower Hell Creek aquifer through 1979 (Jeff Smith, Assistant City Engineer, Gillette, Wyo., written commun., 1979). However, this stress is not known to have a broad, regional

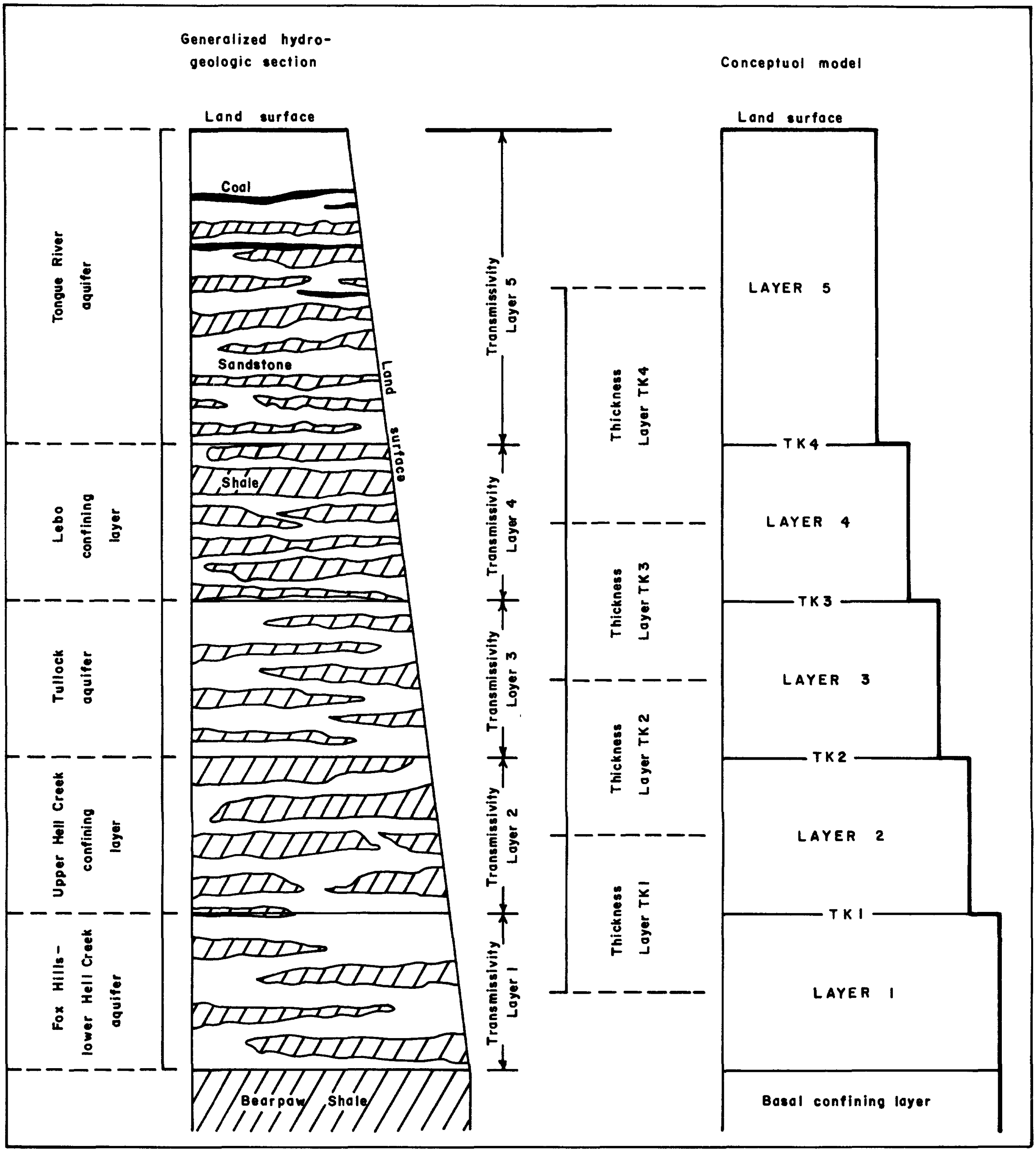


Figure 14.--Correlation between the hydrogeologic units and the conceptual model.

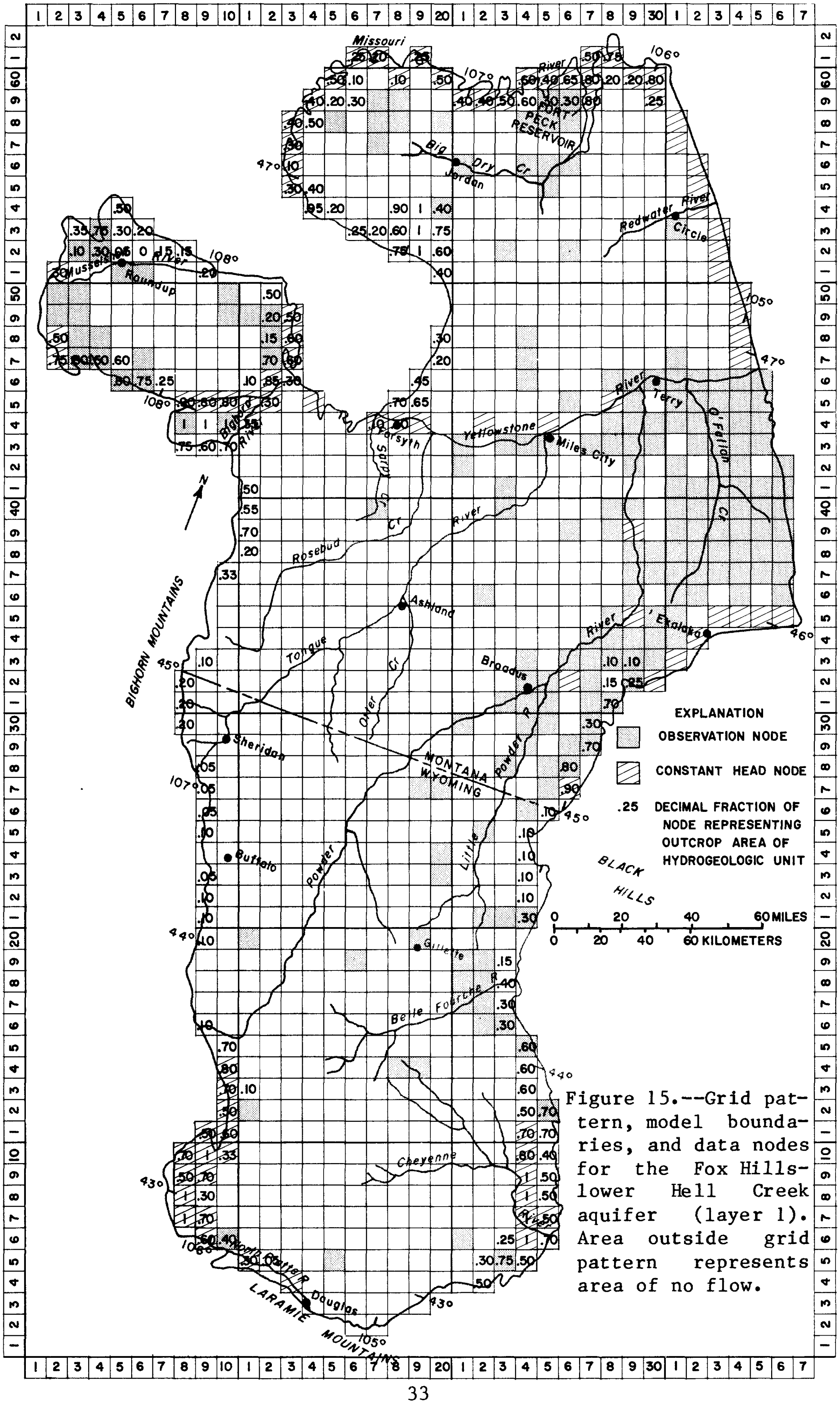


Figure 15.--Grid pattern, model boundaries, and data nodes for the Fox Hills-lower Hell Creek aquifer (layer 1). Area outside grid pattern represents area of no flow.

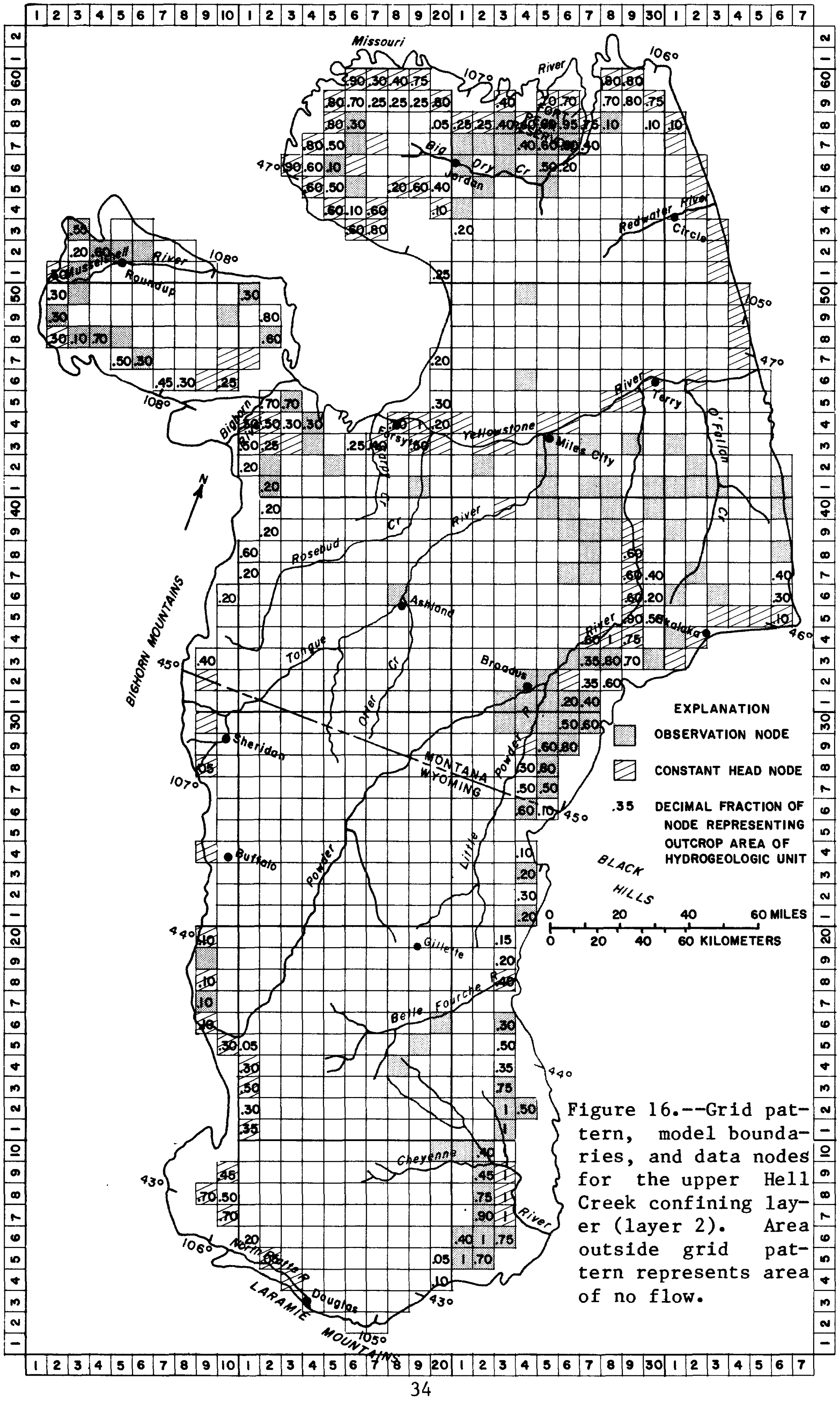


Figure 16.--Grid pattern, model boundaries, and data nodes for the upper Hell Creek confining layer (layer 2). Area outside grid pattern represents area of no flow.

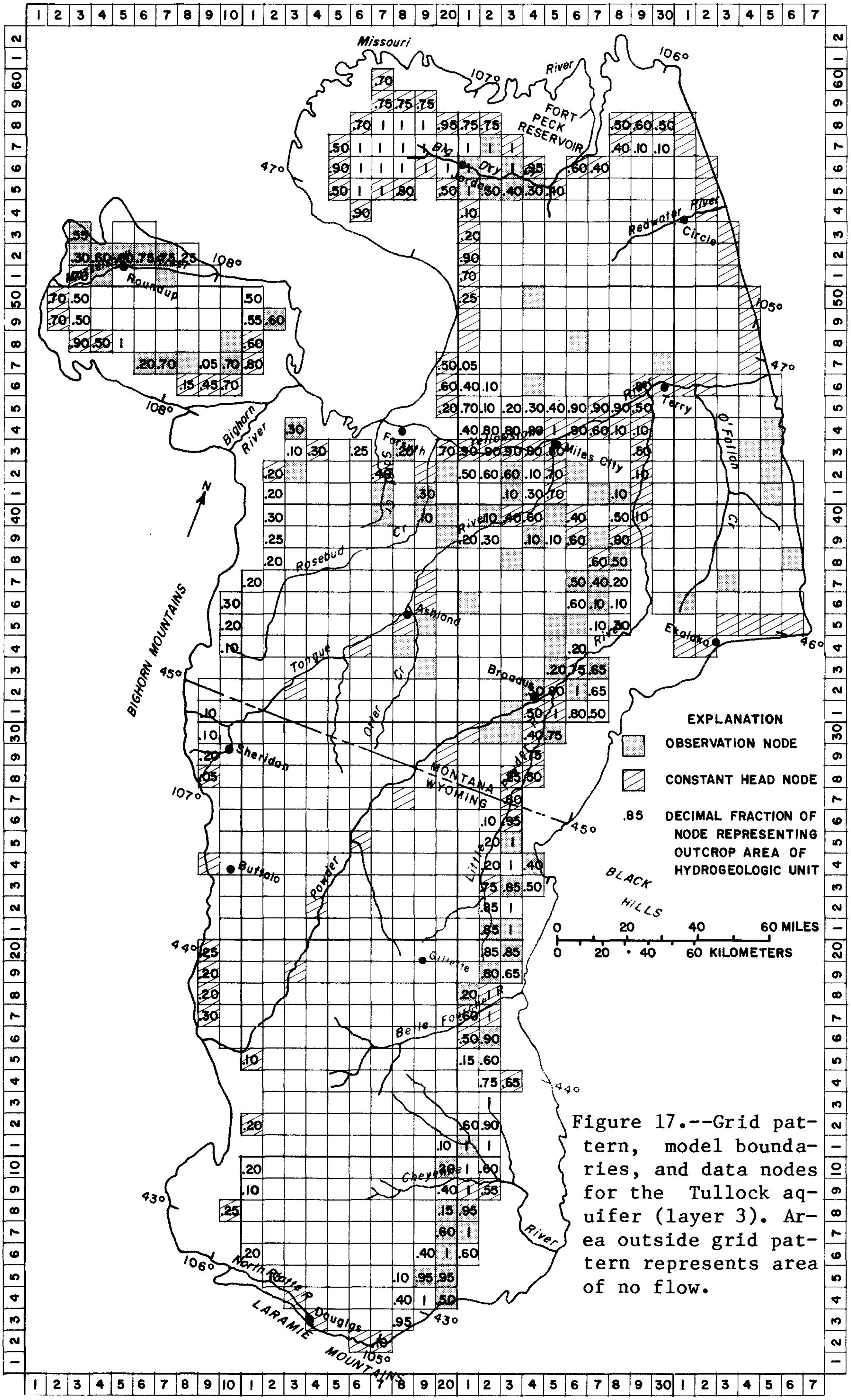


Figure 17.--Grid pattern, model boundaries, and data nodes for the Tullock aquifer (layer 3). Area outside grid pattern represents area of no flow.

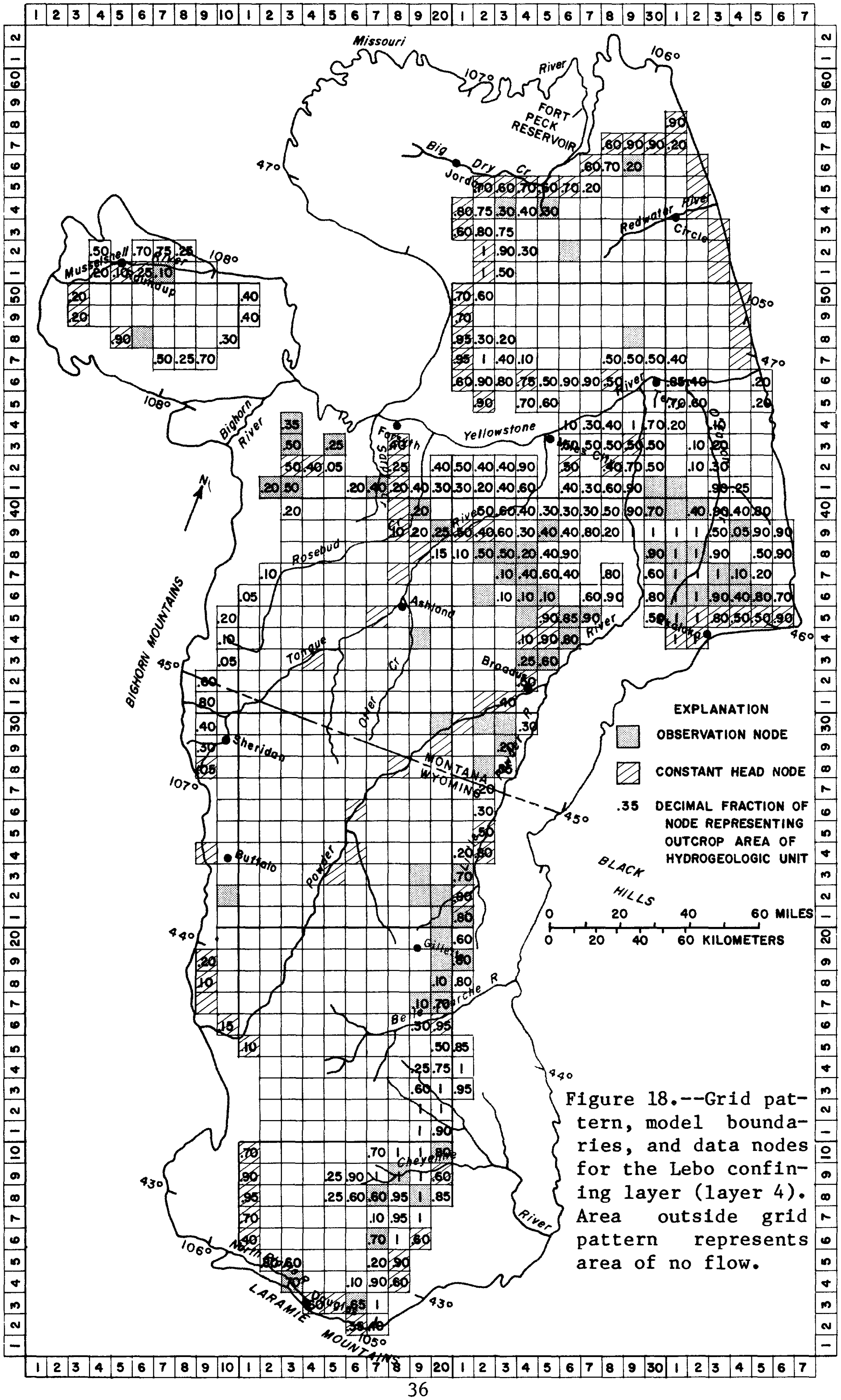


Figure 18.--Grid pattern, model boundaries, and data nodes for the Lebo confining layer (layer 4). Area outside grid pattern represents area of no flow.

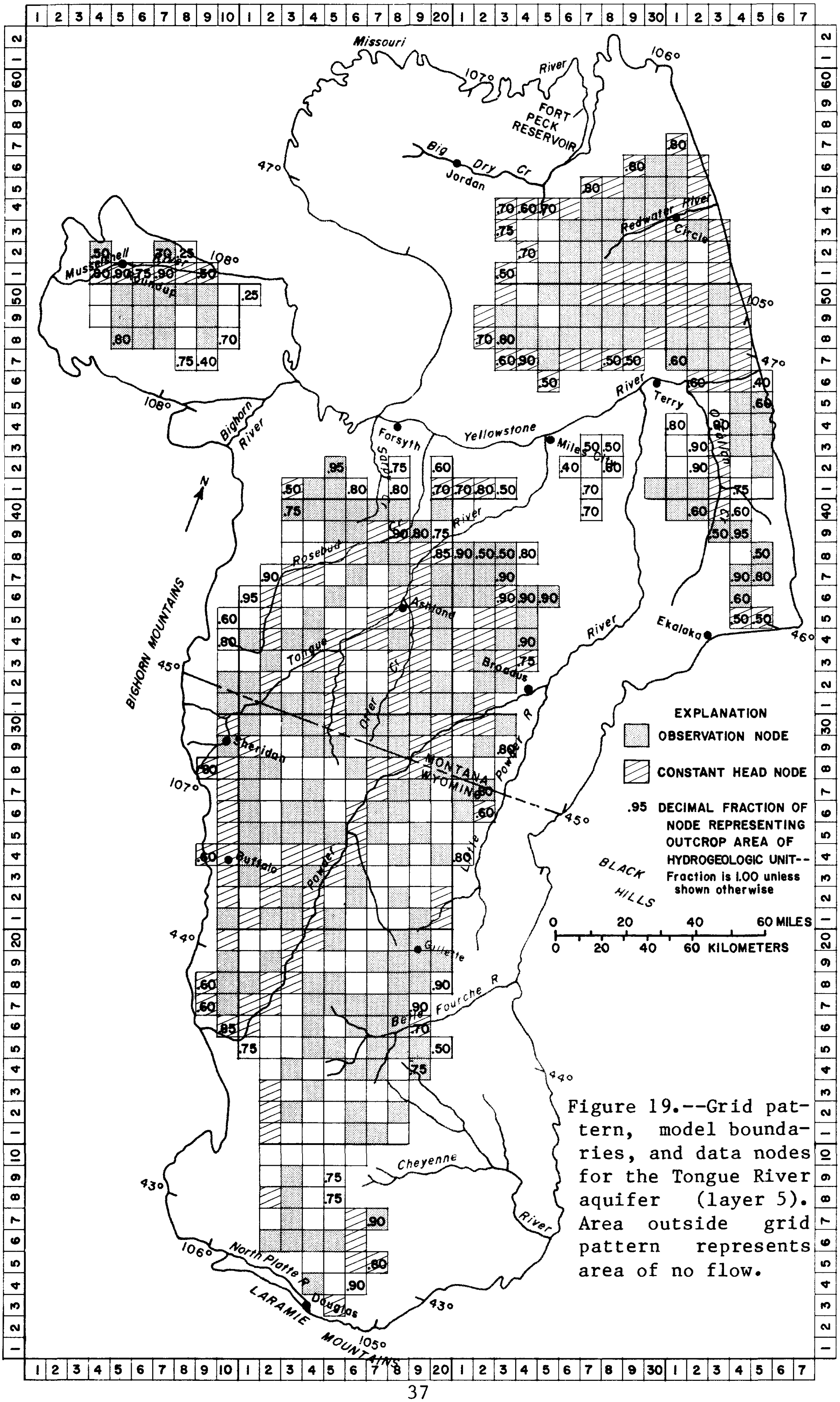


Figure 19.--Grid pattern, model boundaries, and data nodes for the Tongue River aquifer (layer 5). Area outside grid pattern represents area of no flow.

effect at this time (Jeff Smith, oral commun., 1985). Since 1980 about 95 percent of Gillette's water has been pumped from a well completed in the Madison aquifer (Ron Doba, Superintendent of Water and Waste Water, Gillette, Wyo., oral commun., 1982). Stresses caused by domestic and stock uses are evenly distributed over the area, and generally are inconsequential on a regional scale. Stresses caused by domestic pumping from the upper part of the Fort Union Formation in the vicinity of Gillette have resulted in about 150 feet of head decline during the past 20 years (Marvin A. Crist, U.S. Geological Survey, oral commun., 1985). Uncontrolled flowing wells in the river valleys probably have had regional effects on the system, but these effects cannot be evaluated analytically from the available data.

Constant heads in the buried aquifers were utilized to help calibrate the model along stream courses where upstream displacement of potentiometric-surface contours in deeper aquifers could have been caused by flowing wells or by greater-than-normal upward leakage from the deeper aquifers. The streams represented in part by constant-head nodes in the various aquifers include the North Platte, Cheyenne, Belle Fourche, Musselshell, and Yellowstone Rivers and their major tributaries located within the study area.

Aquifer-characteristics estimation

The paucity and variability of values determined from aquifer tests necessitated estimating aquifer transmissivity by utilizing interpretations of geophysical logs by Stoner and Lewis (1980), Lewis and Hotchkiss (1981), and Feltis and others (1981). A description of the estimating procedure follows.

Transmissivities of the five hydrogeologic units were estimated at 597 wells by considering intervals of both sandstone and shale. Composite sandstone thicknesses at each well were multiplied by a representative sandstone hydraulic conductivity to arrive at a sandstone transmissivity. To this value was added the shale transmissivity, determined as the remaining thickness multiplied by a representative shale hydraulic conductivity. It was assumed that sandstone units picked from the various aquifers could be assigned the same hydraulic conductivity and that everything not sandstone was shale. The sandstone hydraulic conductivity for the study area was estimated to be about 1.55×10^{-5} ft/s and the shale hydraulic conductivity about 1.55×10^{-9} ft/s (R. D. Feltis, U.S. Geological Survey, oral commun., 1981). These values occur within or near the ranges of hydraulic conductivities of consolidated materials determined by the U.S. Bureau of Reclamation (1977). The equation to derive the transmissivity estimates from sandstone and shale thicknesses at a given location in a given aquifer is as follows:

$$T = K_s\, b_s + K_{sh}\, b_{sh} \qquad (5)$$

where

T is estimated transmissivity of the hydrogeologic unit, in feet squared per second;
K_s is hydraulic conductivity of sandstone, in feet per second;
K_{sh} is hydraulic conductivity of shale, in feet per second;
b_s is composite thickness of sandstone, in feet; and
b_{sh} is composite thickness of shale, in feet.

Since

$$b_{sh} = b - b_s \qquad (6)$$

where

b is total thickness of the aquifer unit, then

$$T = K_s\, b_s + K_{sh}\,(b - b_s).$$

Rearranging, $$T = (K_s - K_{sh})b_s + K_{sh}\, b$$

$$\begin{aligned} \text{or } T &= (1.55 \times 10^{-5} - 1.55 \times 10^{-9})\, b_s + 1.55 \times 10^{-9}\, b \\ &= 1.55 \times 10^{-5}\, b_s + 1.55 \times 10^{-9}\, b \end{aligned} \qquad (7)$$

The aquifers occur at a great range of depths throughout the study area; therefore, it seemed appropriate to correct the resulting transmissivity estimates for large variations in kinematic viscosity and compaction due to overburden pressures. To determine kinematic viscosity, bottom-hole temperature from each well and an assumed top-hole temperature (mean annual air temperature of 8°C) were used to interpolate the temperature at the midpoint of each hydrogeologic unit. Temperature at the aquifer midpoint was used to determine kinematic viscosity of pure water using the relationship given for variation of properties of pure water with temperature in Lohman and others (1972, p. 20-21). As kinematic viscosity varies inversely with hydraulic conductivity (equation 3), its reciprocal was factored into the transmissivity estimates. The changes in mean transmissivity due to kinematic viscosity calculations ranged from about a 26-percent increase in the Fox Hills-lower Hell Creek aquifer to about a 5-percent decrease in the Tongue River aquifer. The greatest increases were found in the deep, thick units of the Powder River basin. The greatest decreases were found in the thin, shallow part of the study area in eastern Montana.

To determine overburden pressure and the associated reduction in transmissivity, the thickness of strata overlying the midpoint of each layer was calculated at each well site. Overburden thickness was used to determine overburden pressure O_T (in pounds per square inch) for calculating the compaction correction factor C_{MP} (equations 8-11) according to the method devised by Weiss (1982) for general use on thick rock sequences.

For overburden thickness less than 1,500 ft: $$C_{MP} = 1.0 - 0.2\,\frac{O_T}{1{,}500 \text{ ft}} \qquad (8)$$

For overburden thickness 1,501 - 3,000 ft: $$C_{MP} = 0.8 - 0.1\,\frac{(O_T - 1{,}500 \text{ ft})}{1{,}500 \text{ ft}} \qquad (9)$$

For overburden thickness 3,001 - 12,000 ft: $$C_{MP} = 0.7 - 0.2\,\frac{(O_T - 3{,}000 \text{ ft})}{9{,}000 \text{ ft}} \qquad (10)$$

For overburden thickness greater than 12,000 ft: $$C_{MP} = 0.45 \qquad (11)$$

where

C_{MP} is ratio of intrinsic permeability at the midpoint of a unit at depth to intrinsic permeability at zero depth (dimensionless), and

O_T is pressure (lb/in^2) caused by the overburden thickness at the midpoint of a unit computed by multiplying overburden thickness by 0.5.

This compaction correction factor, C_{MP}, was factored into the transmissivity estimates, already corrected for kinematic viscosity, according to equation 4 to obtain the depth-corrected transmissivity, T_C.

The heterogeneity of the five hydrogeologic units causes variability in the way water is transmitted both horizontally and vertically. To simulate the vertical flow of water effectively, a vertical-leakage coefficient, TK (vertical hydraulic conductance per unit area K' divided by thickness b'), was computed between each of the layers a total of four times. The vertical hydraulic conductance per unit area used in each of these computations was the 4.8×10^{-5} ft/d value estimated from the drawdown-ratio method. The thickness, to be explained in detail in a subsequent section of this report, was calculated as the distance between the midpoints of two successive units (fig. 14). For example, interlayer-array TK1 is equivalent to vertical hydraulic conductance per unit area divided by the sum of the thicknesses from the upper one-half of layer 1 plus the lower one-half of layer 2. The grid pattern of array TK1 is the same as that for layer 2. Likewise, arrays TK2, TK3, and TK4 have grid patterns identical to layers 3, 4, and 5 respectively.

Regional distribution of parameters

Determination of vertical-leakage coefficient, TK, requires knowledge about vertical hydraulic conductance per unit area and thickness for two units per layer. The initial upper limits of vertical hydraulic conductance per unit area for each hydrogeologic unit were approximated from the drawdown-ratio method of Neuman and Witherspoon (1972). Interaquifer thickness was interpolated from the oil-well test holes at the nodes of the model. Together they provided the vertical-leakage coefficients for the model. These coefficients were distributed to the nodes of the model by kriging.

Kriging is a linear interpolating and extrapolating technique based on the premise that the relationship between point values of data has continuity within a region of study. Specifically, the corrected point values of interpretations from geophysical logs a short distance apart tend to be more similar than point values taken at greater distances; thus, the point values are autocorrelated.

The mean transmissivities resulting from the interpolation and extrapolation by kriging were 441 ft^2/d for layer 1, 190 ft^2/d for layer 2, 372 ft^2/d for layer 3, 216 ft^2/d for layer 4, and 717 ft^2/d for layer 5. Transmissivity shown in figures 20-24 is largest where the unit thicknesses are greatest, as expected. The mean interlayer thicknesses interpolated by kriging were 601 ft for TK layer 1, 564 ft for layer 2, 635 ft for layer 3, and 750 ft for layer 4. Thickening from north to south through the study area is generally indicated in figures 25-28.

The transmissivity values were entered as arrays in the model of the study area. For the TK arrays, estimates of interlayer thickness were divided into the initial estimates of vertical hydraulic conductance per unit area to provide values of the vertical-leakage coefficient TK for the model.

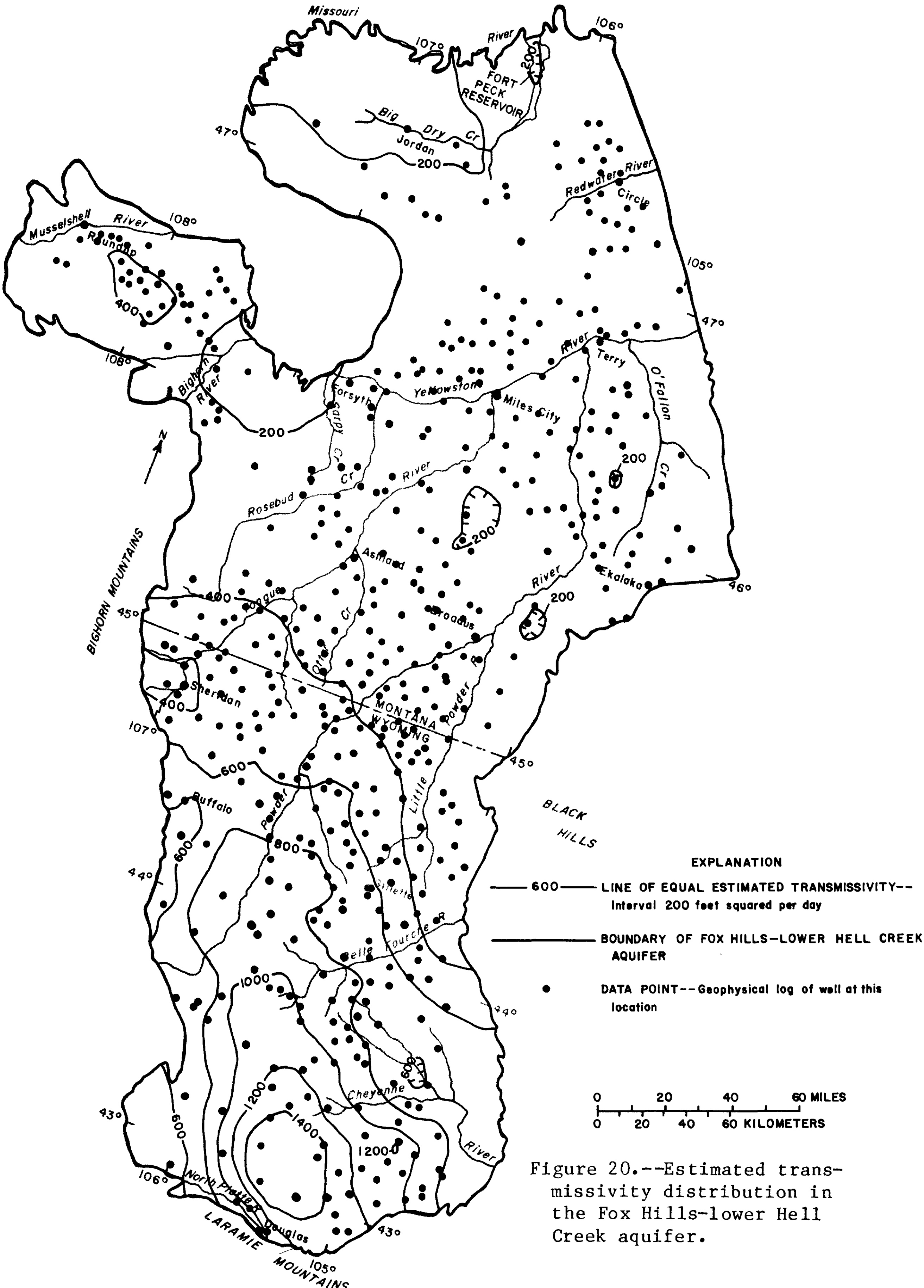


Figure 20.--Estimated transmissivity distribution in the Fox Hills-lower Hell Creek aquifer.

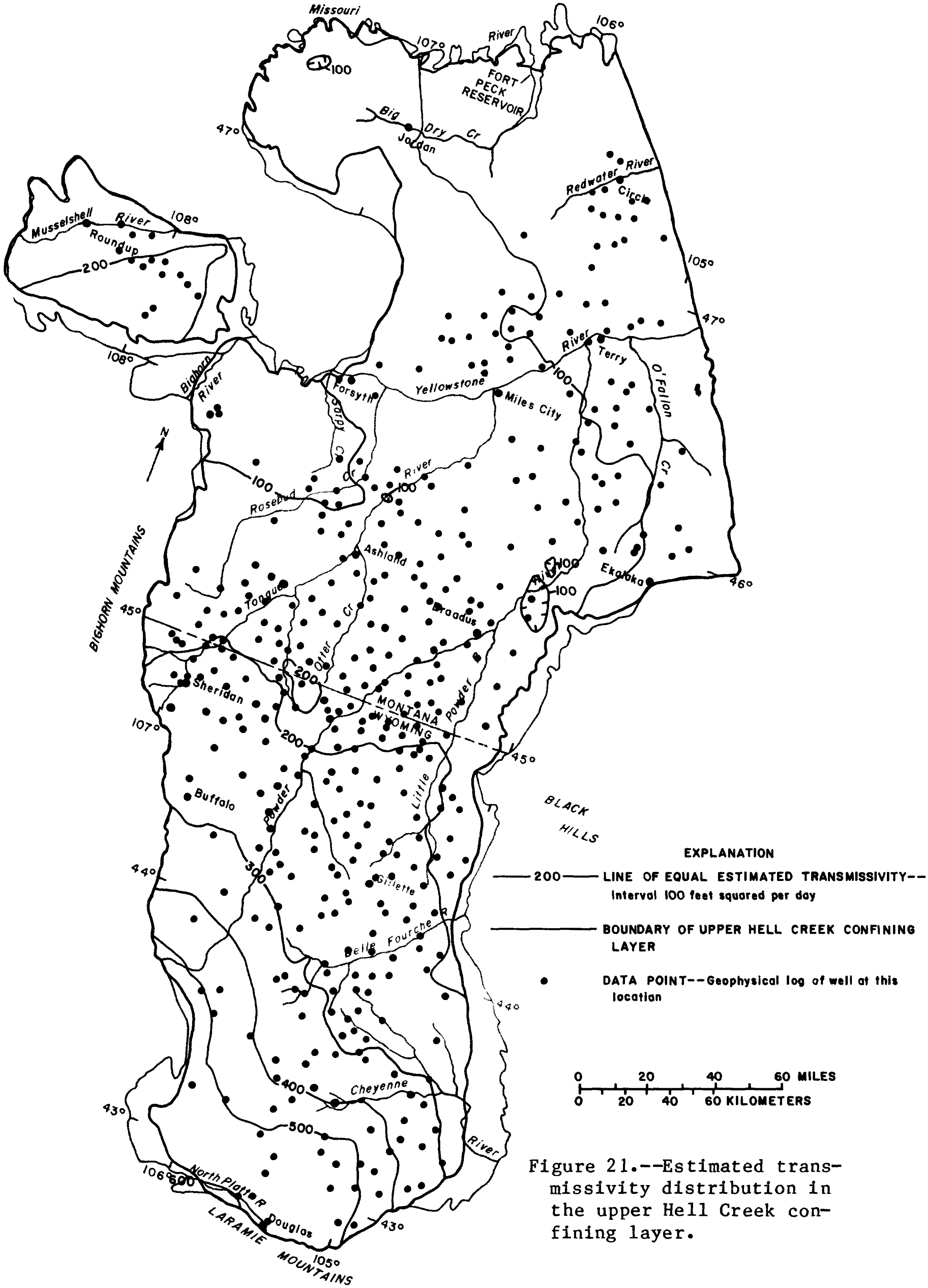


Figure 21.--Estimated transmissivity distribution in the upper Hell Creek confining layer.

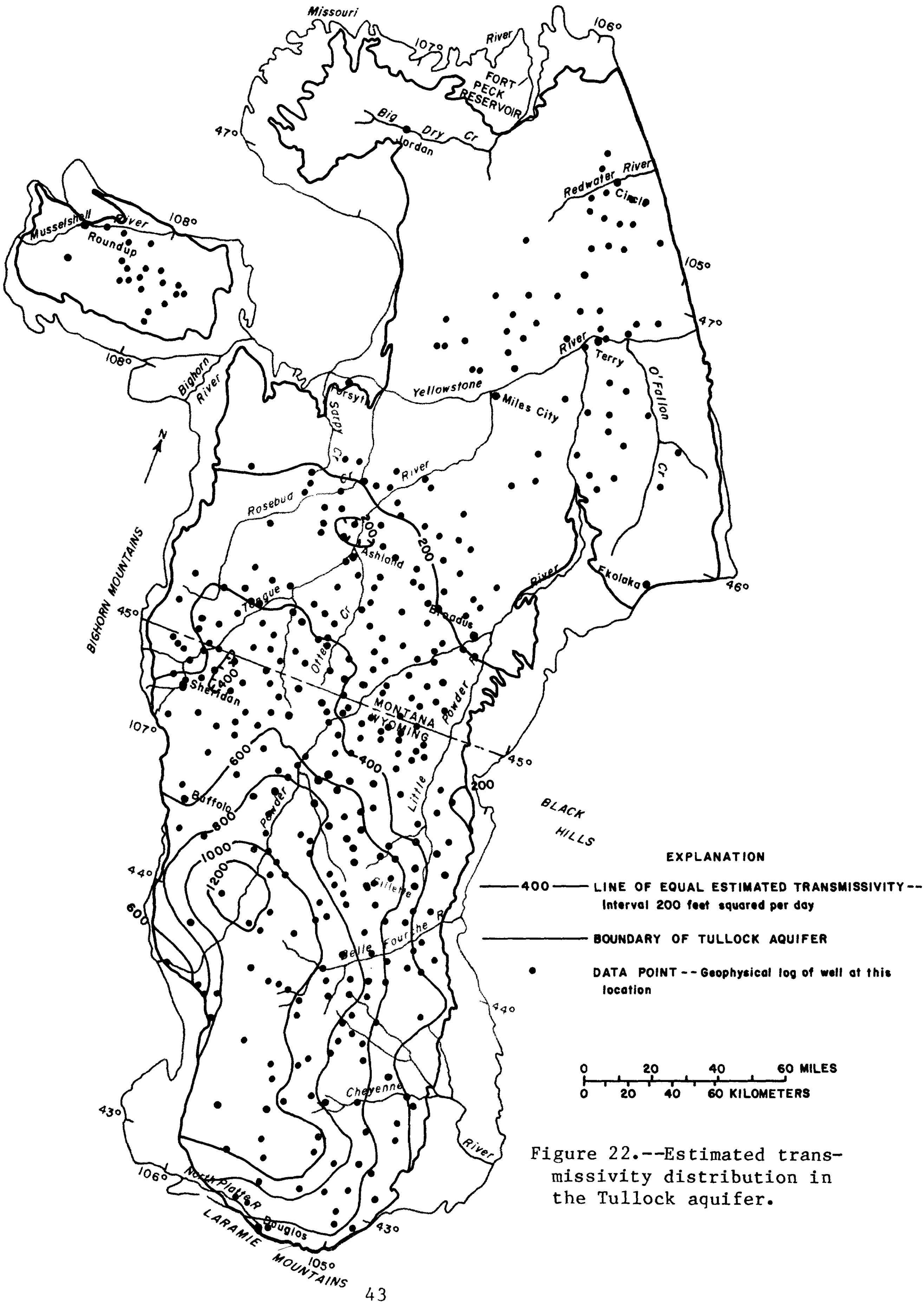


Figure 22.--Estimated transmissivity distribution in the Tullock aquifer.

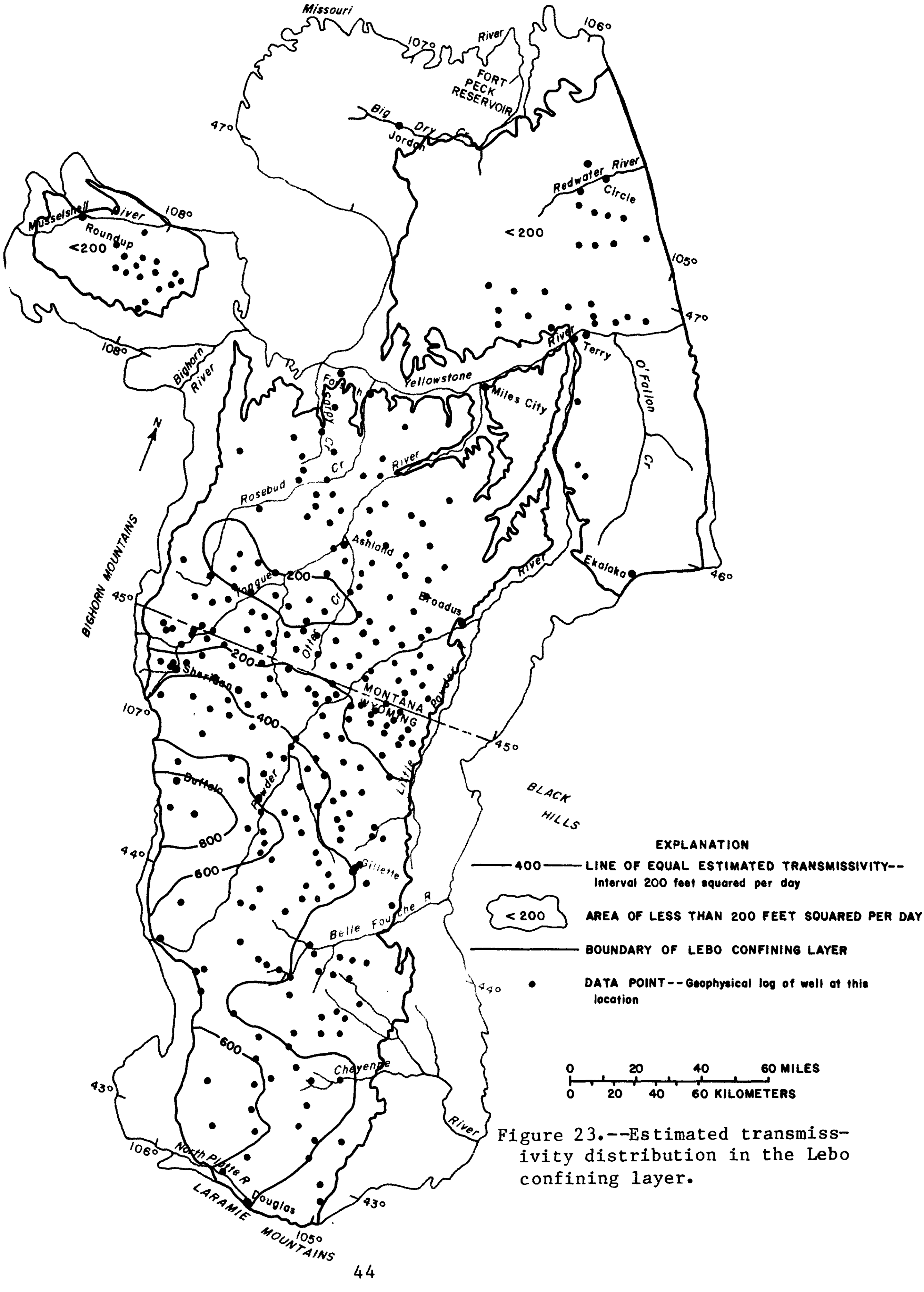


Figure 23.--Estimated transmissivity distribution in the Lebo confining layer.

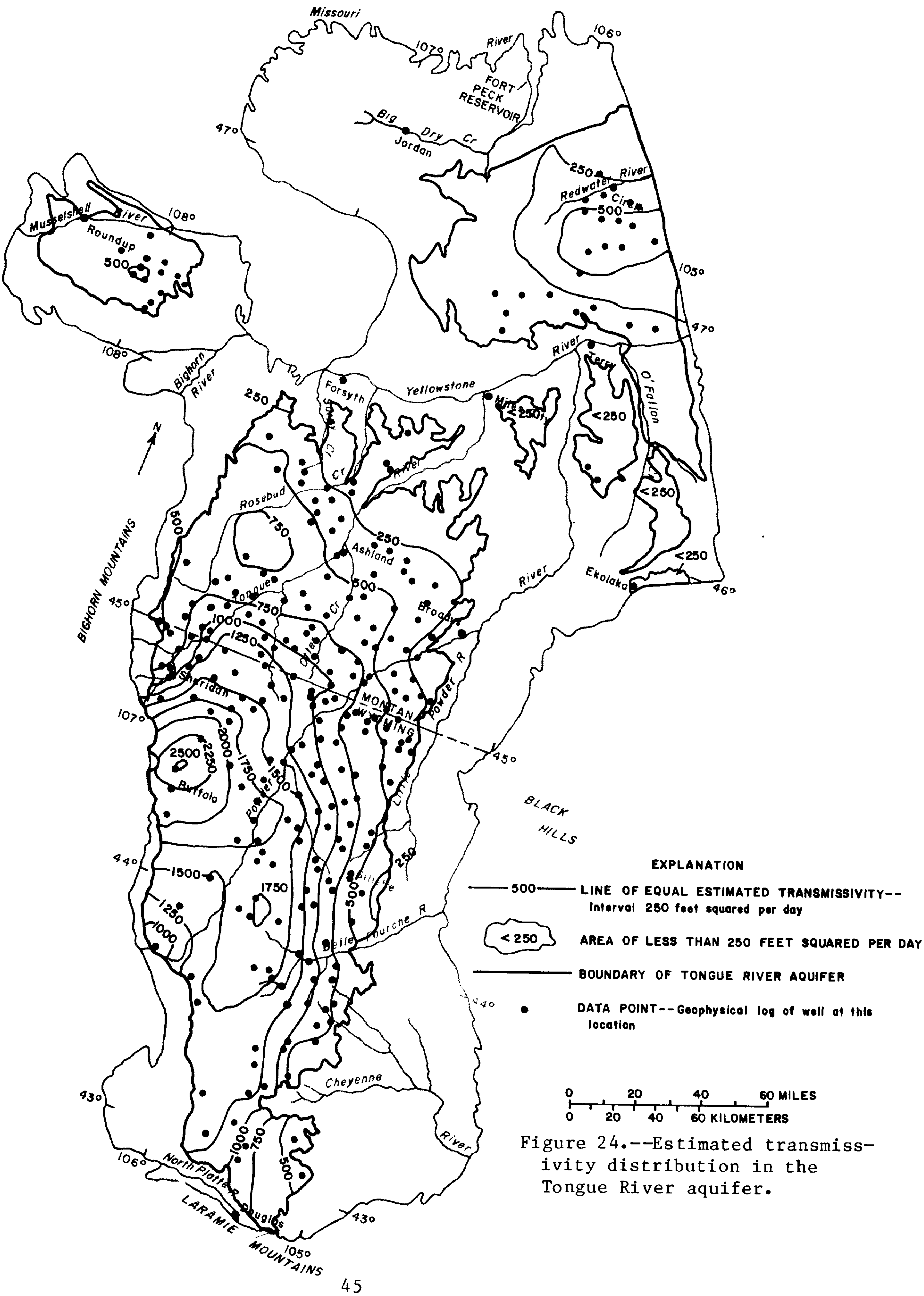


Figure 24.--Estimated transmissivity distribution in the Tongue River aquifer.

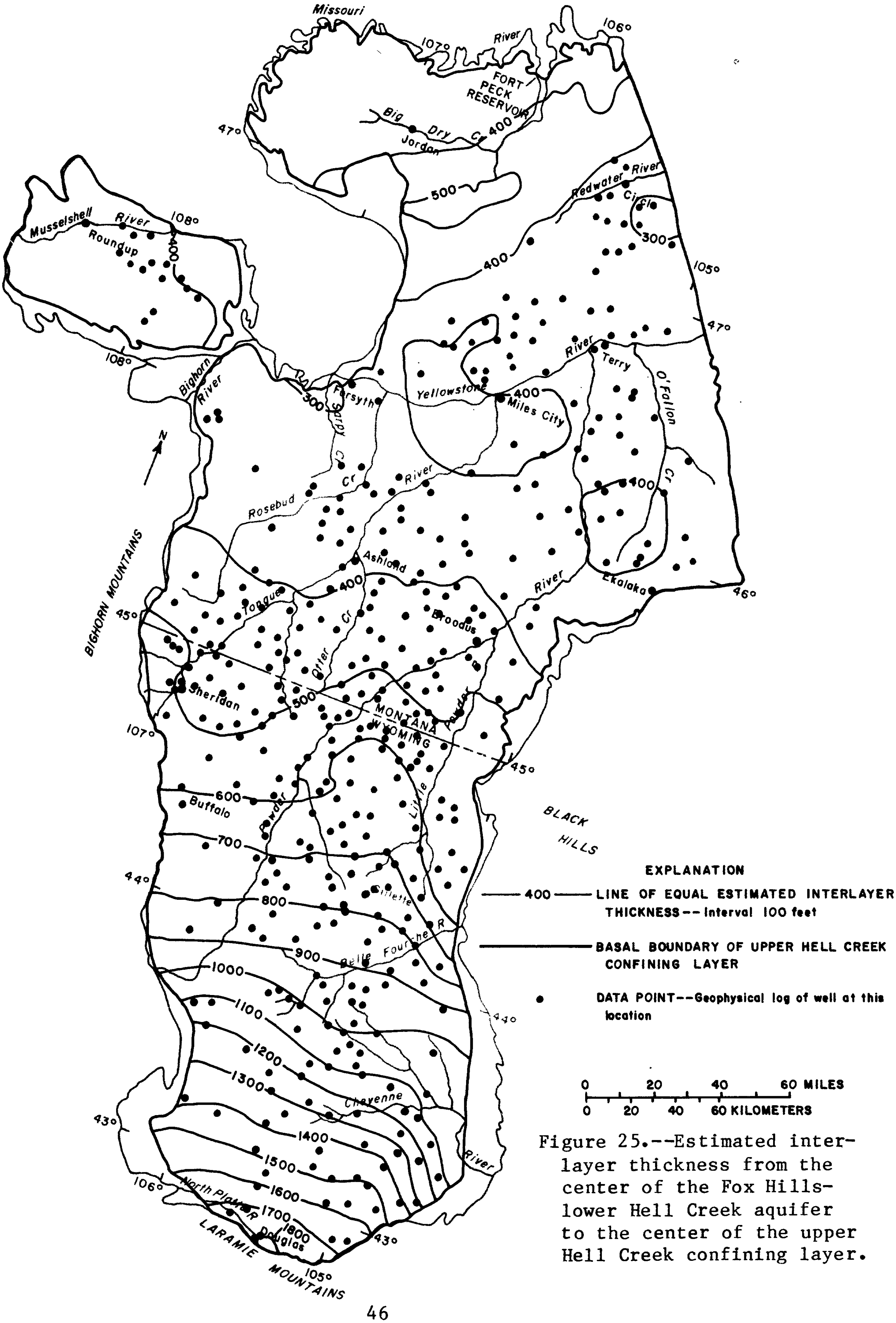


Figure 25.--Estimated interlayer thickness from the center of the Fox Hills-lower Hell Creek aquifer to the center of the upper Hell Creek confining layer.

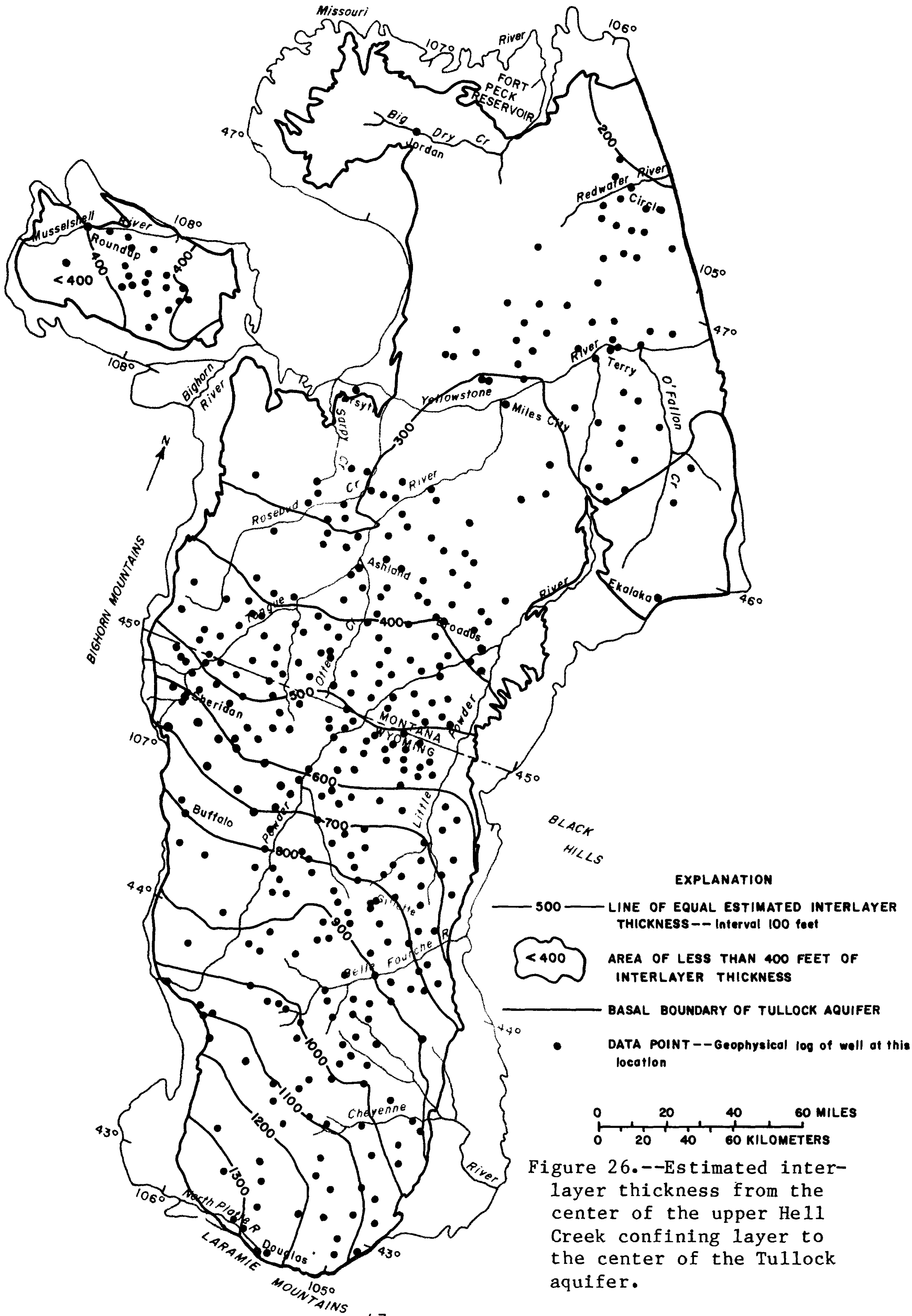


Figure 26.--Estimated interlayer thickness from the center of the upper Hell Creek confining layer to the center of the Tullock aquifer.

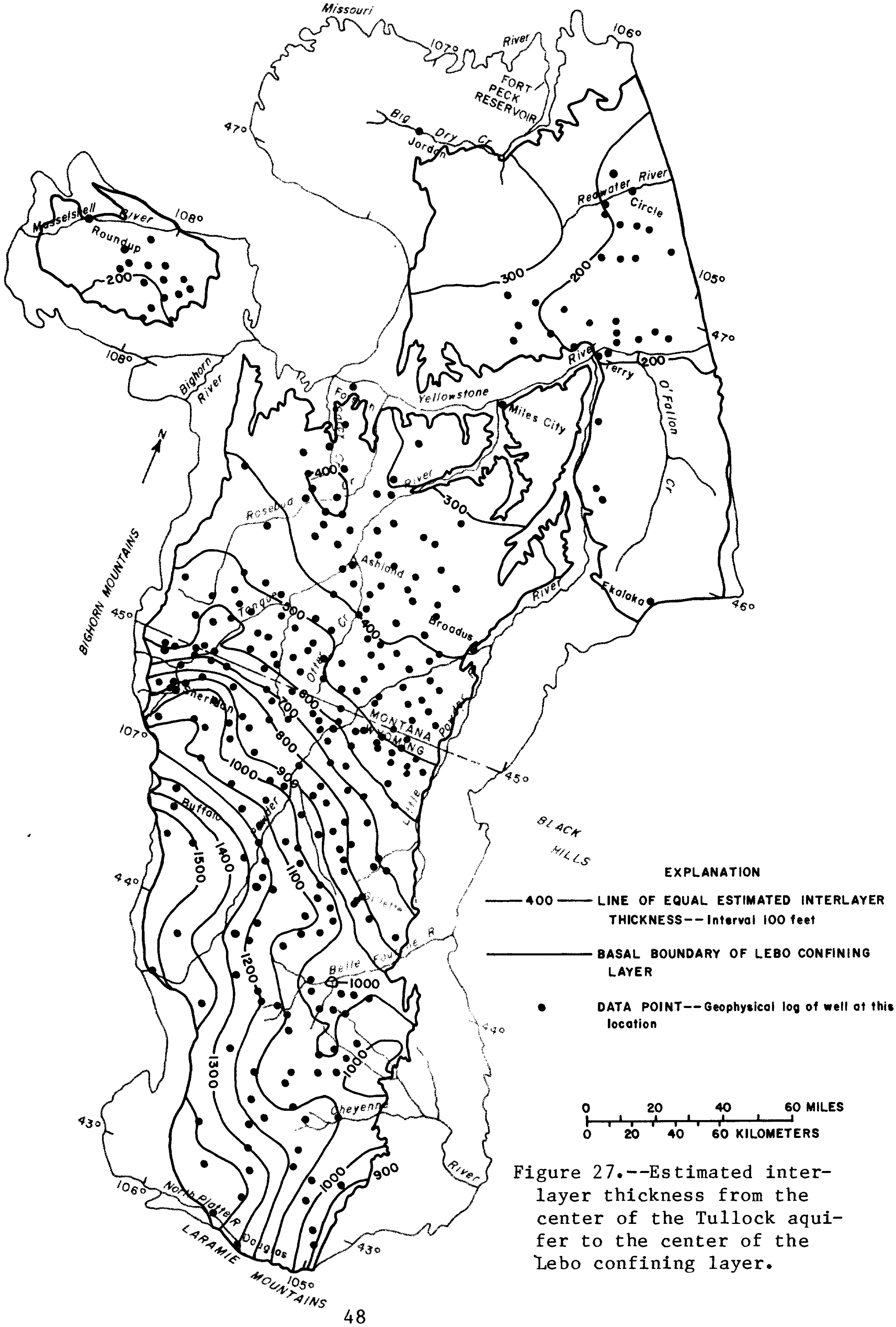


Figure 27.--Estimated interlayer thickness from the center of the Tullock aquifer to the center of the Lebo confining layer.

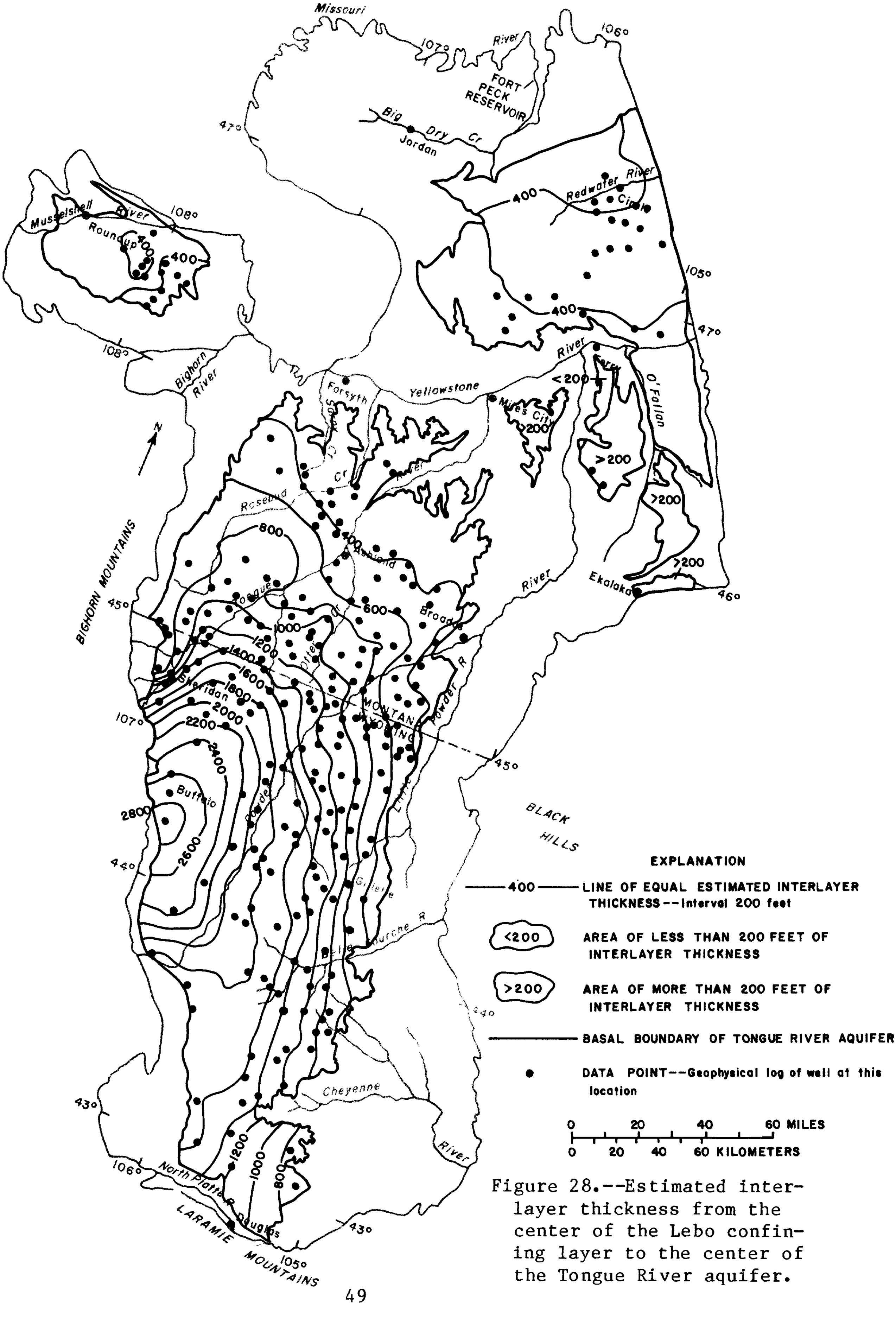


Figure 28.--Estimated interlayer thickness from the center of the Lebo confining layer to the center of the Tongue River aquifer.

Steady-state calibration

Calibration is the process by which a digital model tests the validity of certain hydrologic parameters to affect the flow of ground water in a conceptual model of the flow system. Parameters such as hydrologic boundaries, stresses, and aquifer properties are systematically adjusted within hydrologically reasonable limits to minimize the difference between computed and observed potentiometric heads and flows (where known). The goodness of fit expected from calibration depends upon both the accuracy of the observed or estimated data and the hydrologic credibility of the conceptual flow model. The calibration of a steady-state model is independent of storage properties and the starting heads supplied to the model. However, the input of initial head conditions as starting heads in this model allows model drawdowns to be used as a measure of divergence between actual steady-state conditions as perceived in the prototype and steady-state conditions calculated by the model.

Achieving an optimum steady-state solution to the digital model included two steps. The first step was a preliminary, trial and error, general-best-fit solution of the finite-difference equations based on adjustments of input data and utilization of the statistical subroutine included within the model. The final step was a sensitivity-calibration analysis using a least-squares algorithm.

Trial and error approach

Heads were supplied to the preliminary steady-state model by discretizing potentiometric-surface maps (figs. 8-12). These maps were derived from measured water levels. Other input data include arrays of transmissivity estimates for each of the five layers. Three percent of precipitation on the outcrop was applied as recharge. Vertical hydraulic conductances per unit area within each active layer were internally calculated by multiplying the horizontal hydraulic conductivity of each layer by a factor of 0.001. All boundaries in the preliminary models were input as no-flow boundaries. Calculated potentiometric heads across the study area compared poorly with the observed data, from too low at all but the northeastern and no-flow boundaries to too high in the central parts of the basins. The mean and standard deviation of calculated potentiometric head showed a large disparity between observed and model-calculated values.

To improve the fit, the conceptual model was changed to include transmissivity estimated from geophysical logs and corrected for kinematic viscosity and compaction due to overburden pressure. These data were then distributed to the model nodes by kriging. Again, the model was allowed to calculate its own vertical hydraulic conductance per unit area as 0.001 of the horizontal hydraulic conductivity. All other data remained the same. This simulation showed some improvement of fit, but basin centers still tended to show extreme increases in potentiometric heads.

In an attempt to make continued improvement of fit, the percentage of precipitation as recharge to the model area was decreased to about 0.5 percent. This revision decreased the overall mean difference between observed and calculated hydraulic heads; however, the disparity remained between computed and observed heads in the basin centers and along the basin edges. The problem appeared to be related, in part, to the internal calculation of vertical-leakage coefficients by the model.

Input for internally calculated vertical-leakage coefficients allows only one value per model layer for average thickness. Therefore, the approximation of vertical-leakage coefficient was good where the thickness was about average, but the vertical-leakage coefficient was overestimated in thicker-than-average sections and underestimated in thinner-than-average sections. For this reason and on the recommendation of L. J. Torak (U.S. Geological Survey, oral commun., 1981), the available geophysical data were utilized to derive interaquifer thicknesses. These data then were distributed to all active nodes by kriging in the same way as the transmissivity. Four interlayer (TK) arrays of vertical-leakage coefficient were entered into the model to replace the internally calculated vertical-leakage coefficient as described earlier. Replacement of internally calculated vertical-leakage coefficient by the TK arrays decreased the great disparity between calculated and observed potentiometric head in the basin centers and edges. This improvement in fit also resulted in smaller standard deviations in calculated potentiometric head in each layer than was previously obtained.

At this point it seemed appropriate to switch from the trial-and-error approach of adding variables like the TK arrays to a more objective statistical approach. Thus, the next step in simulating the study area included sensitivity-calibration analyses.

Sensitivity-calibration analyses

Values for transmissivity and recharge for the five modeled layers together with values for vertical-leakage coefficient between the five layers were systematically adjusted within hydrologically reasonable limits in the sensitivity-calibration analyses. The analyses were performed in five stages by adjusting parameter values to obtain a best fit (least-squares algorithm) to observed water levels at 739 nodes in the model layers.

Interpolated transmissivity values were equally well applied among the five layers, because the same estimation and interpolation techniques (kriging) were used for each. Precipitation distribution was available for the study area. Although little is known about how much of the precipitation reaches the saturated zones of the five aquifers as recharge, about 0.5 percent of precipitation is recharged in a geologically similar area and seemed hydrologically reasonable in earlier trial-and-error simulations. Finally, vertical hydraulic conductances per unit area are relatively unknown in the study area and only a maximum value could be estimated. Therefore, vertical-leakage coefficient also is a relatively unknown parameter in the model. The five transmissivity parameters, five recharge parameters, and four vertical-leakage coefficient parameters were adjusted to calibrate the ground-water model (table 4). Because the same methods of estimating recharge and transmissivity were applied uniformly to all layers, values for these parameters were assumed to be known with the same degree of uncertainty. Hence, the five transmissivity and the five recharge parameters were each perturbed (adjusted) as one unit (table 4), except for the final sensitivity analysis.

Each of the flow parameters is represented for input to the digital model by an array of data and an array factor--an operator--that multiplies each value within its respective array prior to using that parameter in the model. The data in the arrays are fixed; that is, one value within an array will not change relative to another. However, the whole array may be altered by changing the array factor. Uniform array-factor changes were used in the sensitivity-calibration analyses.

Table 4.--*Model parameters used in sensitivity-calibration analyses*

Number of model input array factor	Sensitivity parameter	Number of analysis unit	Hydro-geologic unit[1]	Model input variable
1	Transmissivity	1	FHHC	T1
2	Transmissivity	1	HLCK	T2
3	Transmissivity	1	TLCK	T3
4	Transmissivity	1	LEBO	T4
5	Transmissivity	1	TGRV	T5
6	Vertical-leakage coefficient	2	FHHC-HLCK	TK1
7	Vertical-leakage coefficient	3	HLCK-TLCK	TK2
8	Vertical-leakage coefficient	4	TLCK-LEBO	TK3
9	Vertical-leakage coefficient	5	LEBO-TGRV	TK4
10	Recharge	6	FHHC	QRE1
11	Recharge	6	HLCK	QRE2
12	Recharge	6	TLCK	QRE3
13	Recharge	6	LEBO	QRE4
14	Recharge	6	TGRV	QRE5

[1]Unit codes--FHHC is Fox Hills-lower Hell Creek aquifer, HLCK is upper Hell Creek confining layer, TLCK is Tullock aquifer, LEBO is Lebo confining layer, and TGRV is Tongue River aquifer; hyphenated codes indicate interlayer units.

The sensitivity-calibration analyses involved systematically adjusting the 14 array factors for the 6 analysis units (table 4) to obtain the best match (least-squares error) between computed and observed potentiometric heads at the 739 observation nodes. These 739 calibration points (figs. 15-19) were selected on the basis of water-level information at the well sites.

A modified non-linear regression analysis was used to systematically arrive at the best estimate of the array factors. This approach involved using the best trials at several stages of systematic trial-and-error model runs. Values of the array factor were perturbed by a small amount and the system's response was observed. The application of these techniques to the analysis of ground-water systems has been discussed by Draper and Smith (1966), Cooley (1977), and Durbin (1978) and has been documented by Garabedian (1984). Garabedian applied a finite-difference based parameter-estimation model to the Snake River plain aquifer. These analyses were conducted by increments until optimal estimates of the 14 array factors were found and the sensitivity calibration was terminated.

New array factors were computed numerically from the output of the regression program. Each stage of the regression analysis required from four to seven model simulations, involving one simulation using the current-best estimate of array factor (base run), and one simulation for each of the array-factor combinations to be perturbed (perturbation runs). The regression analysis yielded coefficients showing the optimal change for each array factor as a percentage of the current-best estimates used in the base run. Normalized equations were formed from the coefficients and solved simultaneously to determine the necessary adjustments to the array factors that would reduce the sum of squares of the difference between calculated and observed heads, a procedure used by Gardner (1981). Array factors were adjusted according to the solution obtained from the normalized equations, thus concluding one stage of the regression analysis. A simulation involving the new parameter values served as a new base run for another regression analysis, if the analysis had not been terminated by meeting the selected criteria. The selected criteria for terminating this analysis included improvement of the standard error of estimate of less than 2 feet and smaller improvements on two consecutive runs.

To ensure that the array factors remained physically realistic and that the regression procedure was not diverging, new estimates of array factors at each stage of the analysis were examined subjectively for hydrologic implication. If the estimates resulting from considering all array factors were not reasonable, alternative estimates involving fewer array factors were used. Generally, seven combinations of parameters were considered at each stage of the analysis. In the last stage, 26 or more combinations of parameters were evaluated to determine their potential for reducing the sum of the squared differences between observed and computed heads. New array-factor estimates that were reasonable and tended to reduce the potentiometric head differences were obtained from these array-factor combinations during the final stage of the analysis.

Results

The standard error of estimate of starting (observed) head minus calculated head was reduced from 135 to 110 ft over five stages of model simulations. In each of the stages, the perturbations were accomplished by increasing the base run estimated array factor by 25 percent. The sum of squares obtained from stage 5 was the result of applying the array-factor adjustments as indicated from stage 4 to parameters 1-14. The statistical results of the five stages of the sensitivity-calibration analyses are given in table 5 by layer and for all 739 calibration points.

The standard errors of estimate for the base and perturbation runs for stage 5 of the sensitivity-calibration analyses are compared in table 6. Stage 5 was the last sensitivity-calibration analysis because the criteria for terminating the analyses were achieved.

The model was very sensitive to changes in combinations of similar parameters (composite factors). Data from stage 5 (table 6) indicate that recharge to all layers (QRE1-5) was the most sensitive composite factor, followed by transmissivity (T1-5) and vertical-leakage coefficients (TK1-4). An increase of 25 percent in array factors for QRE1-5, T1-5, and TK1-4 produced changes in the standard error of estimate of 44.7 percent for QRE1-5, 29.5 percent for T1-5, and 18.9 percent for TK1-4.

Individual factors were generally less sensitive, although 25-percent increases in recharge applied to the Tongue River aquifer (QRE5) and Lebo confining layer (QRE4) both produced a change in the standard error of estimate greater than 20 per-

Table 5.--*Summary of statistics from the sensitivity-calibration analyses*

Stage	Starting head minus calculated head, in feet, for active nodes, by layer: Layer 1 Mean	Layer 1 Standard deviation	Layer 2 Mean	Layer 2 Standard deviation	Layer 3 Mean	Layer 3 Standard deviation	Layer 4 Mean	Layer 4 Standard deviation	Layer 5 Mean	Layer 5 Standard deviation	Observation nodes, all layers: Sum of squares (feet squared)	Observation nodes, all layers: Standard error of estimate (feet)
1	-74	93	-66	98	-49	73	-66	137	-23	175	13,500,000	135
2	-56	88	-45	102	-36	70	-19	110	-77	146	9,480,000	113
3	-54	87	-43	103	-35	70	-15	109	-84	144	9,440,000	113
4	-51	87	-34	103	-27	69	-2	107	-78	147	9,150,000	111
5	-55	90	-35	103	-31	69	-11	106	-70	150	9,020,000	110

Table 6.--*Summary of stage 5 sensitivity analysis*

Model input variable	Parameter[1]	Change in sum of squares, in feet squared[2]	Change in standard error of estimate, in feet[3]	Percent change in standard error of estimate
T1	Transmissivity factor for FHHC aquifer	119,000	12.7	11.5
T2	Transmissivity factor for HLCK confining layer	48,000	8.06	7.29
T3	Transmissivity factor for TLCK aquifer	161,000	14.8	13.4
T4	Transmissivity factor for LEBO confining layer	27,900	6.14	5.56
T5	Transmissivity factor for TGRV aquifer	226,000	17.5	15.8
T1-5	Composite transmissivity factor	784,000	32.6	29.5
TK1	Vertical-leakage coefficient for FHHC-HLCK interlayer	20,600	5.28	4.78
TK2	Vertical-leakage coefficient for HLCK-TLCK interlayer	71,500	9.84	8.90
TK3	Vertical-leakage coefficient for TLCK-LEBO interlayer	84,100	10.7	9.66
TK4	Vertical-leakage coefficient for LEBO-TGRV interlayer	108,000	12.1	11.0
TK1-4	Composite vertical leakage coefficient	324,000	20.9	18.9
QRE1	Recharge from precipitation to FHHC aquifer	224,000	17.4	15.8
QRE2	Recharge from precipitation to HLCK confining layer	38,300	7.12	6.44
QRE3	Recharge from precipitation to TLCK confining layer	43,200	7.65	6.92
QRE4	Recharge from precipitation to LEBO confining layer	379,000	22.6	20.5
QRE5	Recharge from precipitation to TGRV aquifer	800,000	32.9	29.8
QRE1-5	Composite recharge from precipitation	1,800,000	49.4	44.7

[1]All array factors for parameters were increased by 25 percent for each perturbation run. Unit codes--FHHC is Fox Hills-lower Hell Creek aquifer, HLCK is upper Hell Creek confining layer, TLCK is Tullock aquifer, LEBO is Lebo confining layer, and TGRV is Tongue River aquifer.

[2]Base (unperturbed) run sum of squares equals 9.02×10^6 feet squared.

[3]Base (unperturbed) run standard error of estimate equals 110 feet.

cent. A 25-percent increase in the array factor for the Fox Hills-lower Hell Creek aquifer (QRE1) produced an increase of 15.8 percent in the standard error of estimate.

Among the parameters of transmissivity and vertical-leakage coefficient, the most sensitive were transmissivities of the Tongue River aquifer (T5), Tullock aquifer (T3), and Fox Hills-lower Hell Creek aquifer (T1), and the vertical-leakage coefficient between the Lebo confining layer and the Tongue River aquifer (TK4). An increase of 25 percent in the array factors for these parameters produced changes in the standard error of estimate of 15.8 percent for T5, 13.4 percent for T-3, 11.5 percent for T1, and 11.0 percent for TK4. Increases of 25 percent to the remaining array factors produced changes in the standard error of estimate of less than 10 percent. The least sensitive parameter was the vertical-leakage coefficient between the Fox Hills-lower Hell Creek aquifer and the upper Hell Creek confining layer (TK1). An increase of 25 percent to the array factor for TK1 produced a change of 4.78 percent in the standard error of estimate.

Mean, minimum, and maximum values of transmissivity (T), vertical leakage coefficient (TK), and recharge (QRE), determined from the sensitivity-calibration analyses are given in table 7. Array factors resulting from those analyses are also given.

The mean transmissivities were 443 ft^2/d for layer 1, 191 ft^2/d for layer 2, 374 ft^2/d for layer 3, 217 ft^2/d for layer 4, and 721 ft^2/d for layer 5. Mean interlayer values of vertical-leakage coefficients were 2.33×10^{-7}, 5.64×10^{-8}, 3.58×10^{-8}, and $2.11 \times 10^{-8} day^{-1}$ for interlayers TK1 to TK4, respectively. The mean vertical-leakage coefficient for TK1 corresponds to a mean thickness between the Fox Hills-lower Hell Creek aquifer and the upper Hell Creek confining layer of 601 ft and a vertical hydraulic conductance per unit area of 1.40×10^{-4} ft/d. Other mean interlayer thicknesses and vertical hydraulic conductances per unit area are 564 ft and 3.18×10^{-5} ft/d for array TK2, 634 ft and 2.27×10^{-5} ft/d for array TK3, and 749 ft and 1.58×10^{-5} ft/d for array TK4. Mean annual recharge from precipitation was 2.45×10^{-2} in., corresponding to about 0.26 percent of average annual precipitation across the study area.

The values of vertical hydraulic conductance per unit area and recharge obtained from the sensitivity calibration were significantly different from the initial values, whereas the values of transmissivity increased less than 0.5 percent of the initial values. The vertical hydraulic conductance per unit area for interlayer 1 increased by about 192 percent. Vertical hydraulic conductance per unit area decreased by 33.8 percent for interlayer 2, 52.7 percent for interlayer 3, and 67.1 percent for interlayer 4. These changes were acceptable because of the lack of knowledge of the vertical flow regime; one estimated value was used to generate the initial model input. The recharge values obtained from calibration showed a decrease of about 48 percent. This decrease was accepted because very little was known about the quantity of precipitation that actually percolates to the ground-water body and the extreme sensitivity of this parameter to change.

Hydraulic heads derived from the final (stage 5) sensitivity-calibration model run, together with the original measured or estimated heads, are mapped in figures 29-33. As would be expected, a model with 6-mi by 6-mi node spacing cannot reproduce the detail of the Tongue River aquifer (layer 5) where the bulk of the potentiometric data was available. In addition, the deeper layers (layers 1-4) in the western part of the Powder River basin in Wyoming show divergence between calcu-

Table 7.--*Summary of transmissivity, vertical-leakage coefficient, and recharge data from stage 5 of the sensitivity calibration*

Geohydrologic unit[1]	Model input variable	Transmissivity, Feet squared per day: Mean	Minimum	Maximum	Array factor[2]
FHHC aquifer (layer 1)	T1	443	105	1,810	1.01×10^{-5}
HLCK confining layer (layer 2)	T2	191	33.6	641	1.01×10^{-5}
TLCK aquifer (layer 3)	T3	374	71.3	1,390	1.01×10^{-5}
LEBO confining layer (layer 4)	T4	217	.348	1,035	1.01×10^{-5}
TGRV aquifer (layer 5)	T5	721	47.3	2,530	1.01×10^{-5}

Geohydrologic unit[1]	Model input variable	Vertical-leakage coefficient, Units of day^{-1}: Mean	Minimum	Maximum	Mean inter-aquifer thickness, in feet	Mean vertical hydraulic conductance per unit area, in feet per day	Array factor[2]
FHHC-HLCK (inter-layer 1)	TK1	2.33×10^{-7}	7.63×10^{-8}	5.08×10^{-7}	601	1.40×10^{-4}	1.62×10^{-15}
HLCK-TLCK (inter-layer 2)	TK2	5.64×10^{-8}	2.29×10^{-8}	1.55×10^{-7}	564	3.18×10^{-5}	3.68×10^{-16}
TLCK-LEBO (inter-layer 3)	TK3	3.58×10^{-8}	1.40×10^{-8}	1.65×10^{-7}	634	2.27×10^{-5}	2.63×10^{-16}
LEBO-TGRV (inter-layer 4)	TK4	2.11×10^{-8}	5.47×10^{-9}	1.15×10^{-7}	749	1.58×10^{-5}	1.83×10^{-16}

Geohydrologic unit[1]	Model input variable	Recharge from precipitation, Inches per year: Mean	Minimum	Maximum	Array factor[2]
Exposed aquifers and confining layers	QRE1-5	2.45×10^{-2}	1.55×10^{-2}	5.10×10^{-2}	6.88×10^{-14}

[1]Unit codes -- FHHC is Fox Hills-lower Hell Creek, HLCK is upper Hell Creek, TLCK is Tullock, LEBO is Lebo, and TGRV is Tongue River.

[2]From sensitivity calibration. Includes conversion to model units of feet and seconds.

lated and measured head in several areas. Very little potentiometric data were available for this area. However, in general, the potentiometric surface calculated by the model approximates the measured regional potentiometric surface. The regional pattern of northward flow from the southern part of the Powder River Basin, joining with the eastward flow from the Bull Mountains basin, then northeastward flow out of the area parallel to the Yellowstone River is accurately described by the digital model. This general trend is reproduced correctly in each layer with greater detail in successively shallower layers. The model also reproduces a small eastward trend in flow in the Fox Hills-lower Hell Creek and the upper Hell Creek layers along the Cheyenne River in the southeastern part of the study area.

A mass balance or water budget of the hydrologic model was calculated at the end of the steady-state simulation. The balance contains the computed flow, recharge, and discharge from each hydrologic component, which permits comparison of the relative importance of the components. The generalized hydrologic mass balance from the layers of the calibrated steady-state model is depicted in figure 34.

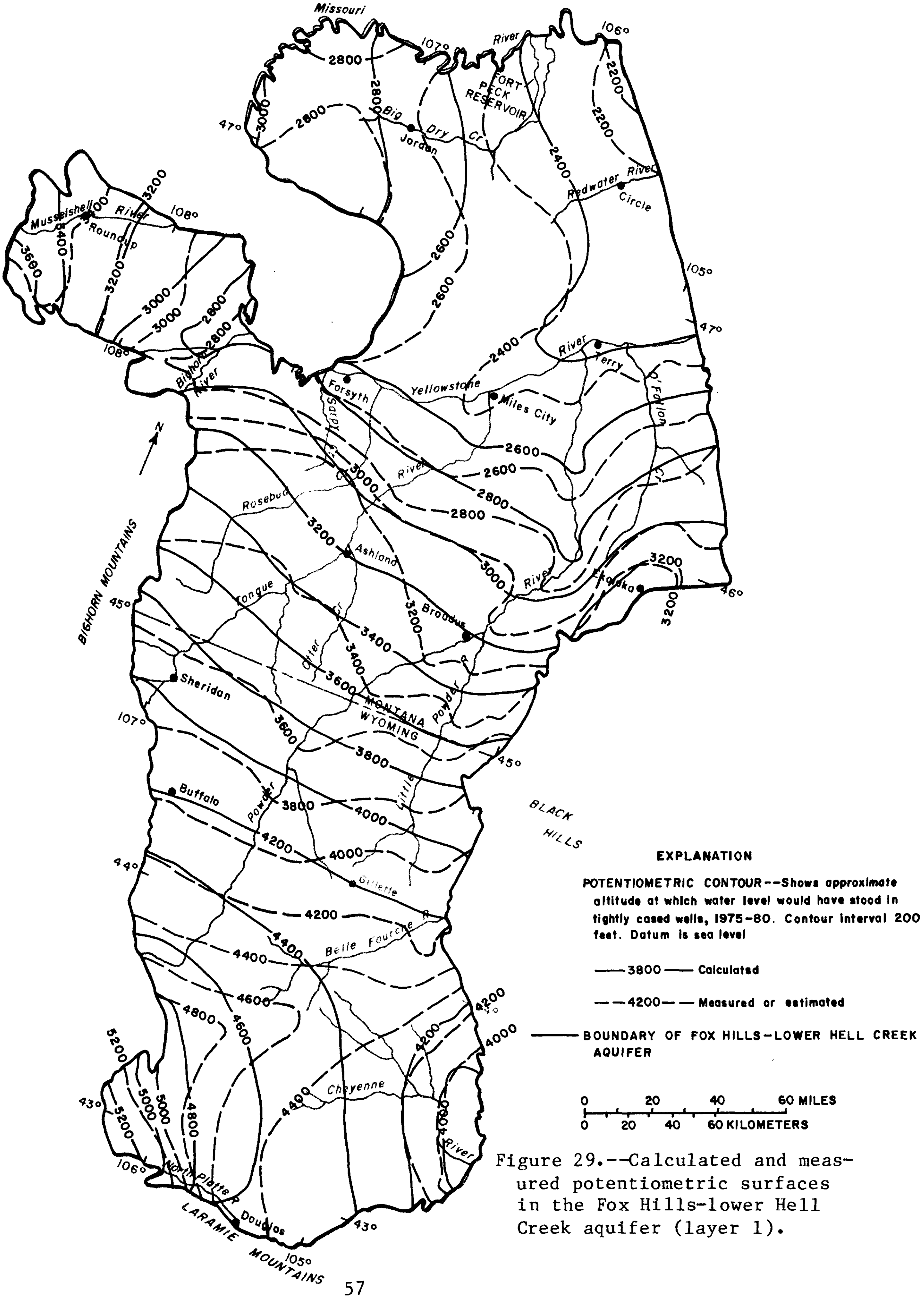


Figure 29.--Calculated and measured potentiometric surfaces in the Fox Hills-lower Hell Creek aquifer (layer 1).

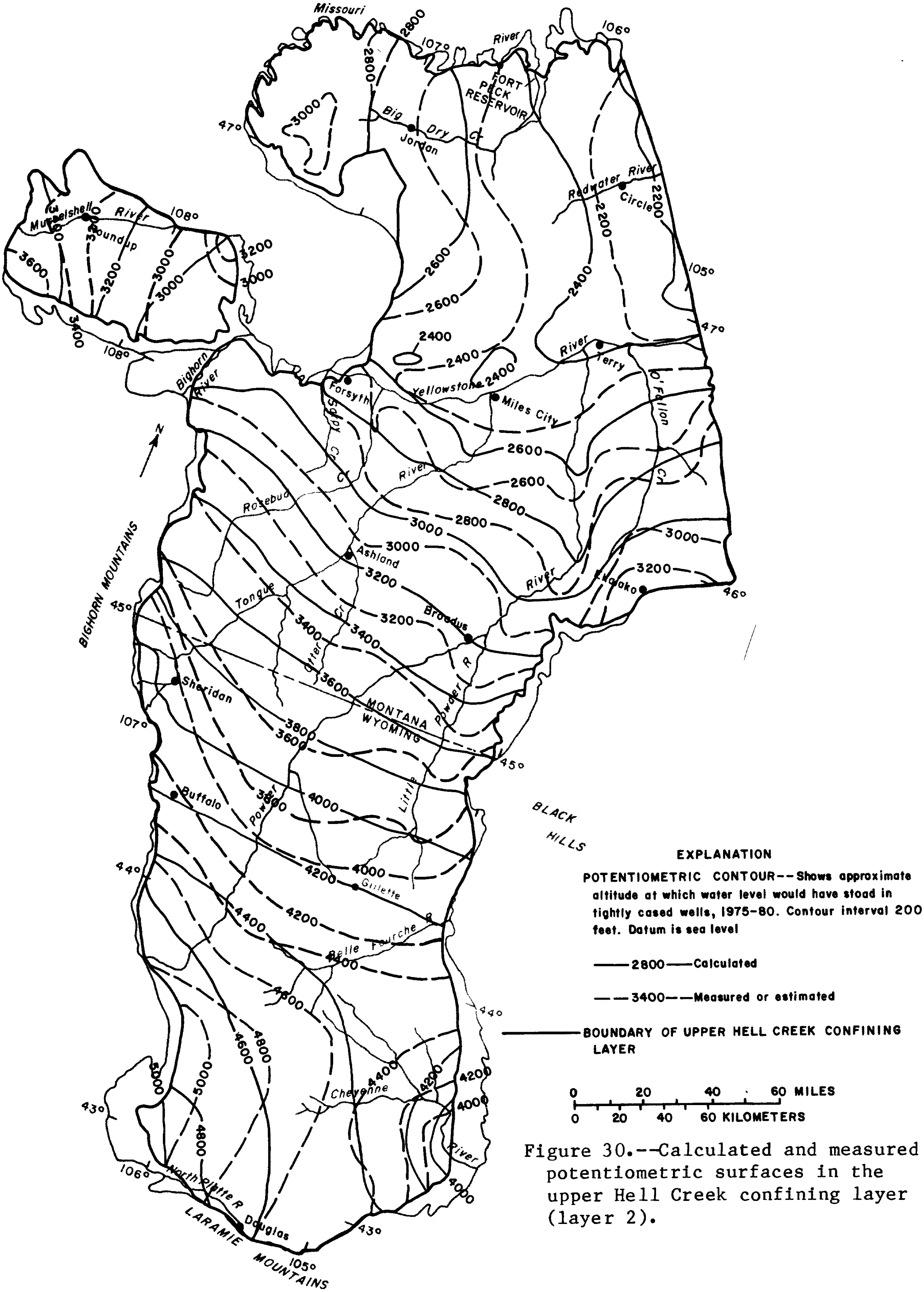


Figure 30.--Calculated and measured potentiometric surfaces in the upper Hell Creek confining layer (layer 2).

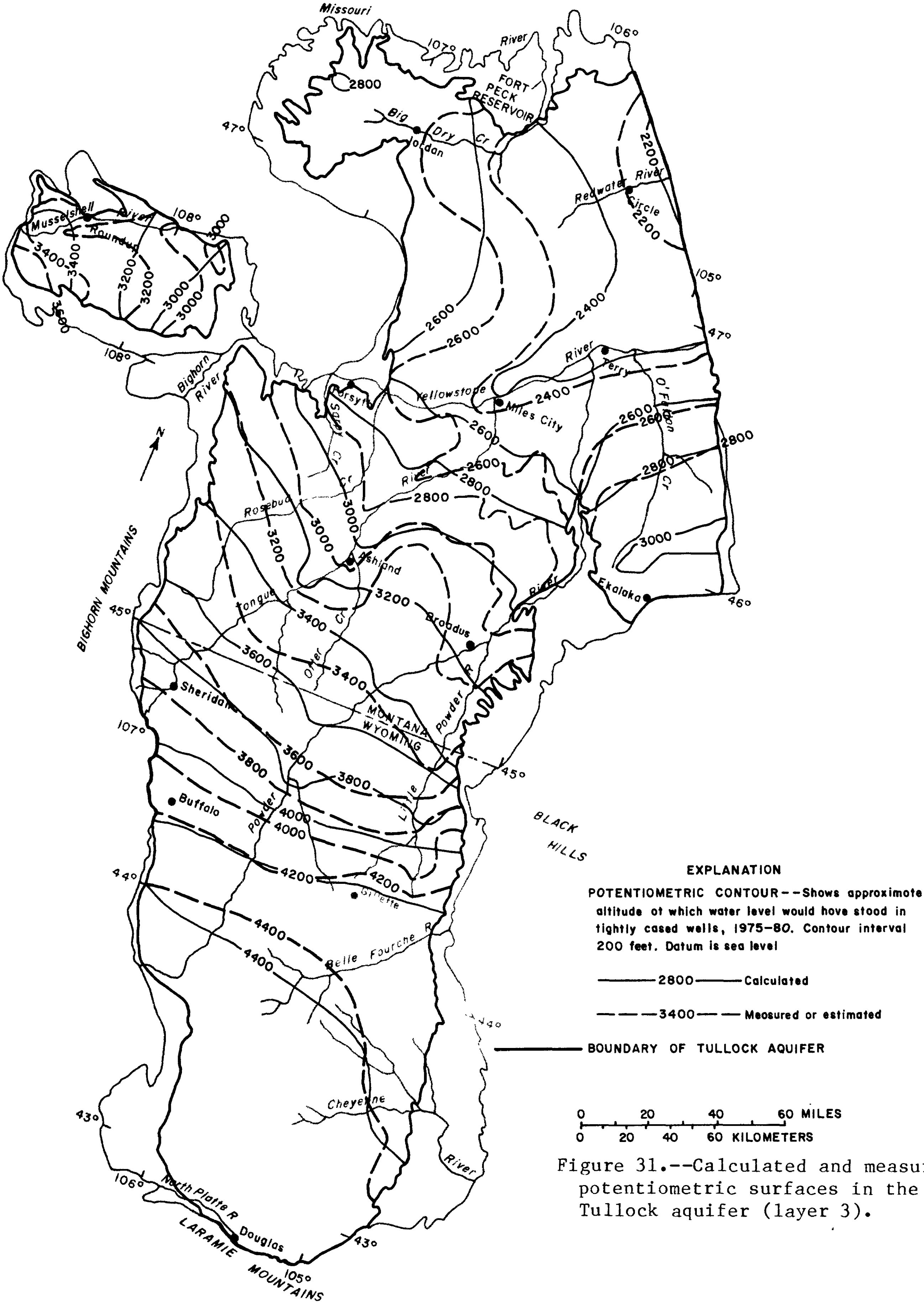


Figure 31.--Calculated and measured potentiometric surfaces in the Tullock aquifer (layer 3).

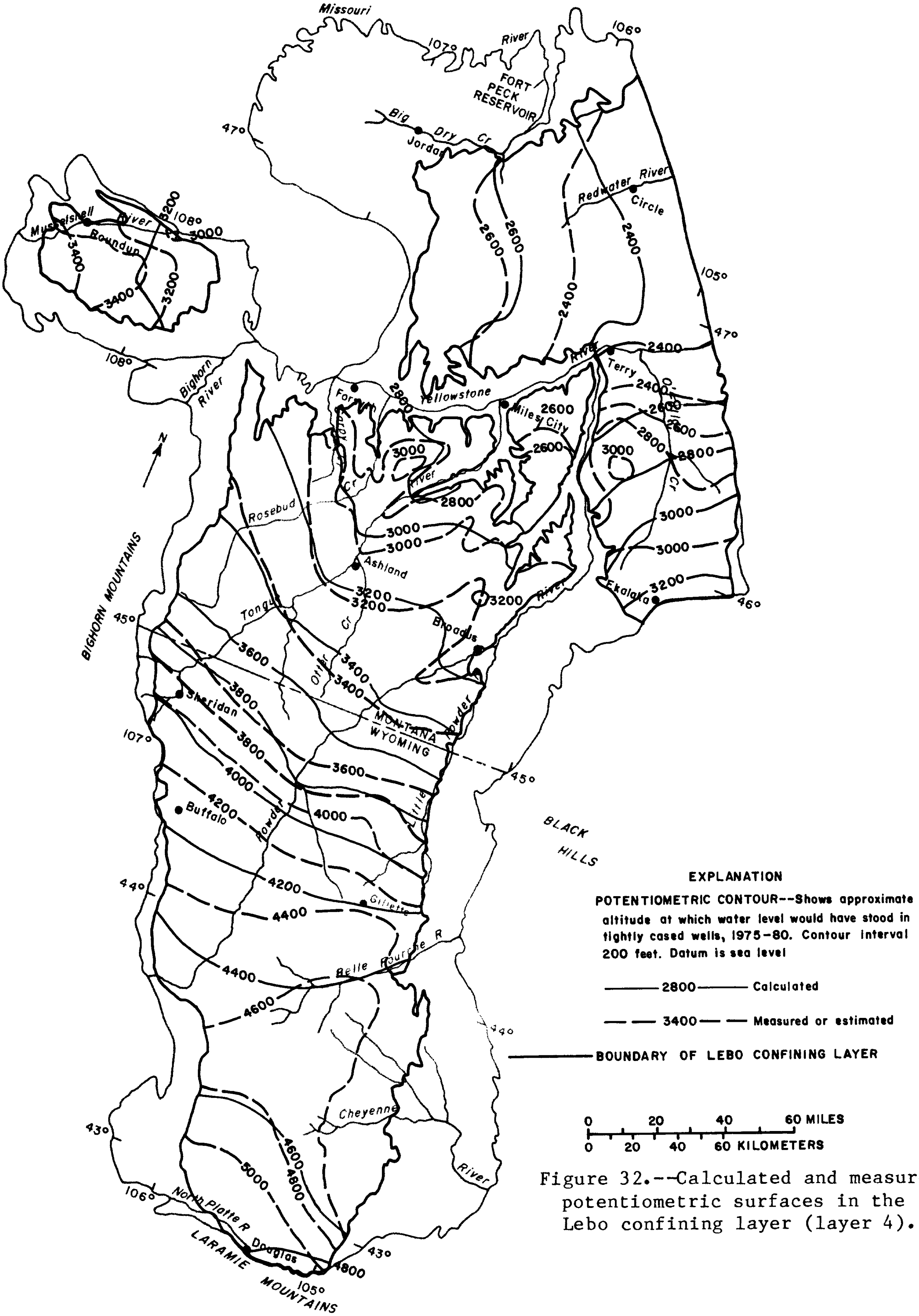


Figure 32.--Calculated and measured potentiometric surfaces in the Lebo confining layer (layer 4).

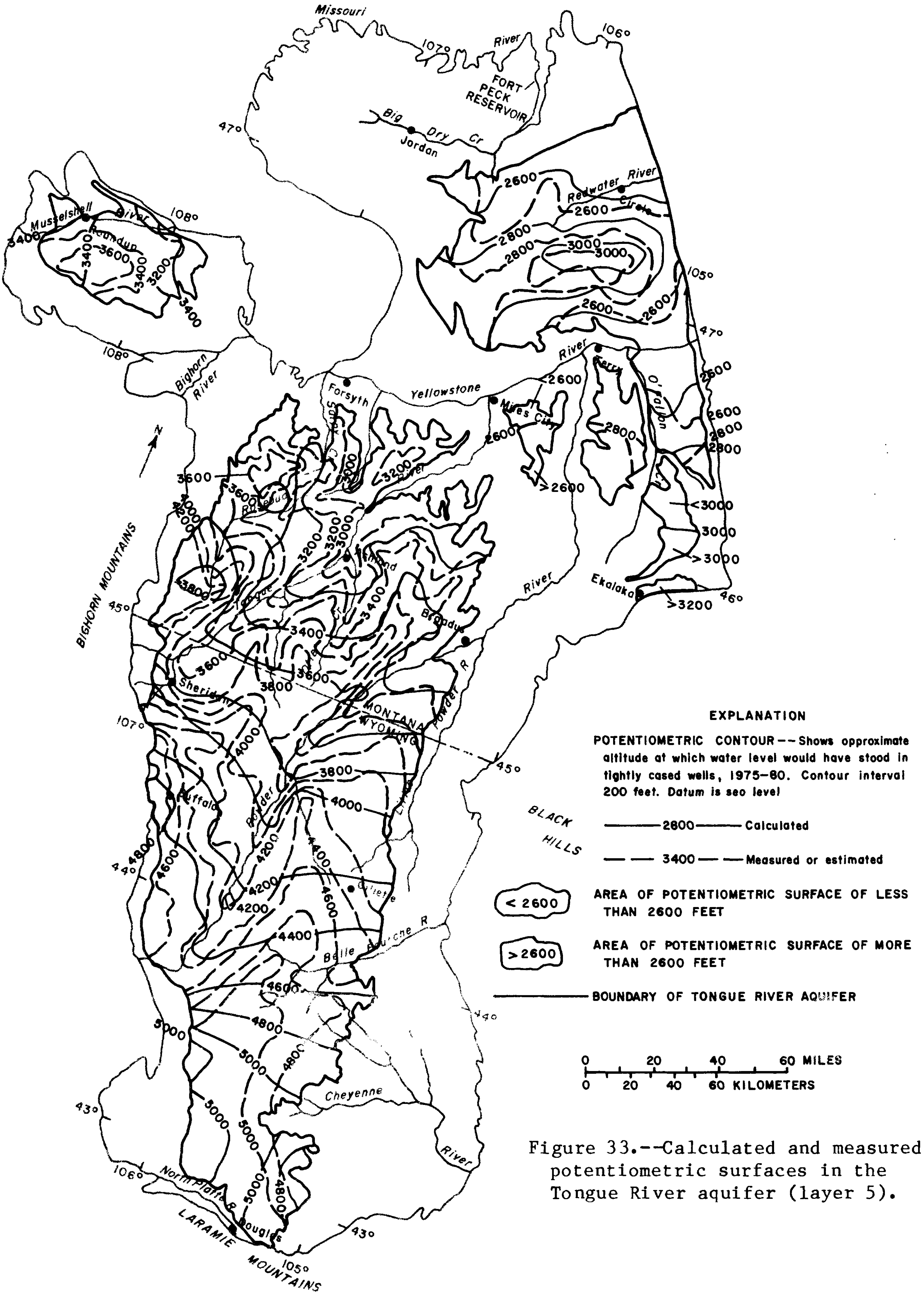
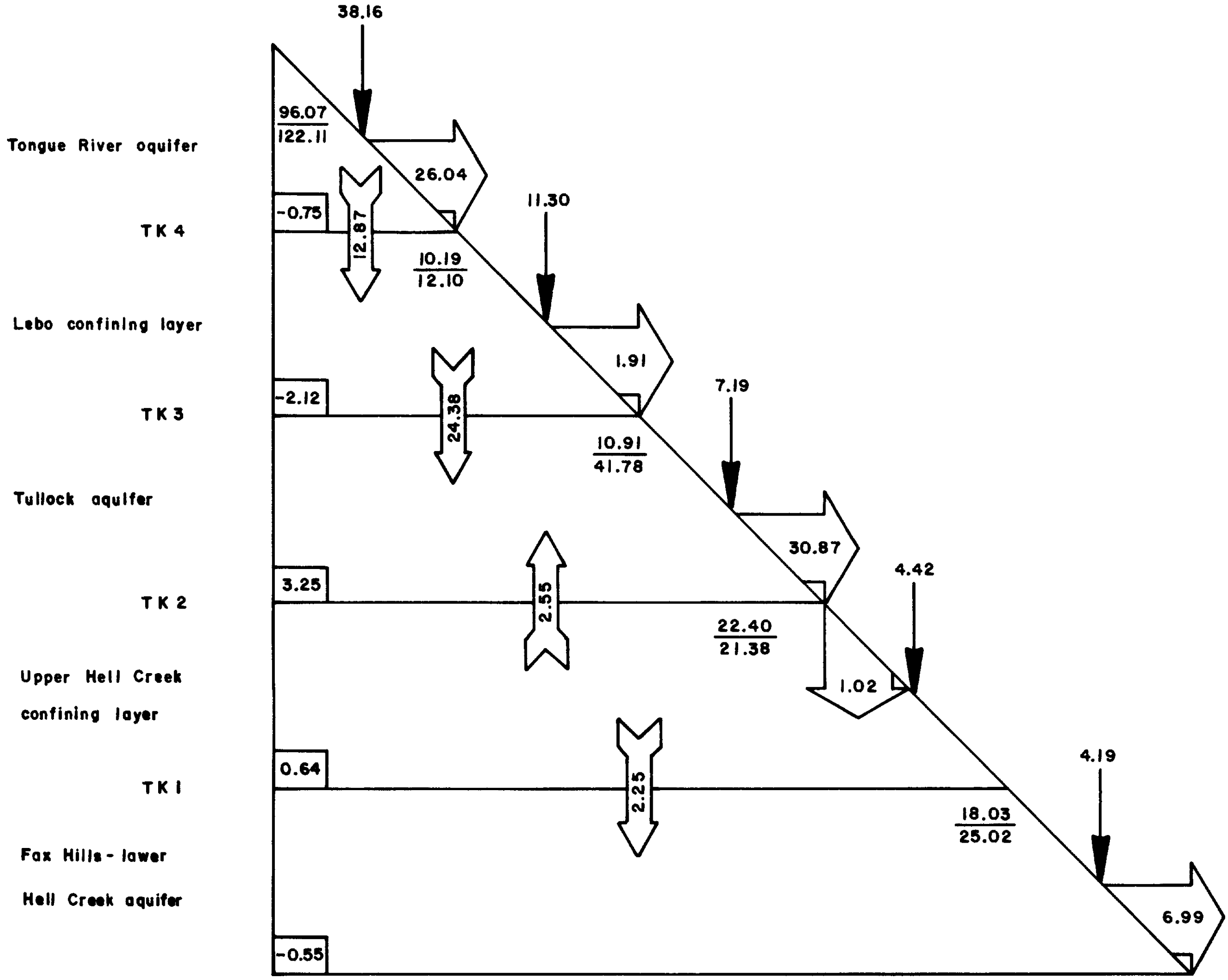


Figure 33.--Calculated and measured potentiometric surfaces in the Tongue River aquifer (layer 5).



EXPLANATION

38.16 PRECIPITATION RECHARGE TO LAYER, IN CUBIC FEET PER SECOND

24.38 LEAKAGE BETWEEN LAYERS, IN CUBIC FEET PER SECOND

1.91 NET CONSTANT HEAD FLOW OUT OF LAYER, IN CUBIC FEET PER SECOND

1.02 NET CONSTANT HEAD FLOW INTO LAYER, IN CUBIC FEET PER SECOND

$\frac{22.40}{21.38}$ GROSS CONSTANT HEAD INPUT OVER OUTPUT FROM LAYER, IN CUBIC FEET PER SECOND

-0.55 NET INFLOWS MINUS OUTFLOWS (BUDGET) FOR LAYER, IN CUBIC FEET PER SECOND

Figure 34.--Generalized diagram of model mass balance by layer.

The total recharge to the modeled system includes 65.26 ft^3/s (29.3 percent) from precipitation and 157.60 ft^3/s (70.7 percent) from inflow at constant head nodes, for a total inflow of 222.86 ft^3/s. The actual percentage of recharge as precipitation on the study area is somewhat larger because precipitation on constant-head nodes is not considered by the model. Constant-head recharge to the model generally occurs at the model boundaries, which represent topographically high areas at the edge of the study area, such as the areas of losing streams along the front of the Bighorn Mountains.

The total discharge from the modeled system is 222.39 ft^3/s, all from constant-head nodes. Because a steady-state condition necessitates that discharge equals recharge, resulting in no change in storage, the net constant-head discharge should equal the recharge to the system. The sum of the net constant-head flow from each of the layers is 65.81 ft^3/s (fig. 34). Constant-head discharge from the model generally occurs at the model boundaries, which represent topographic low areas at the edge of the study area and along the nodes that are in the vicinity of major streams.

The difference between total recharge and discharge amounts to 0.75 percent of the total recharge. The source of the error is related in part to the error criterion of 1 ft used in the model. Additional error may have come from vertical discretization of the aquifer system into model layers, where thickness of adjacent layers is greater than a factor of 1.5. The difference between model layers 4 and 5 is about 1.91.

Interlayer flow, which is simulated using the TK layers, is depicted by the vertical interlayer arrows shown in figure 34. Constant-head nodes also were used to route water in and out of layers at selected locations, generally boundaries and streams, where water levels could be inferred easily from the data. Constant-head flow in these areas represents departure from the TK-layer flow at preferred locations. Such locations include routing of water to areas of aquifers where the scale of the model precluded recharge (boundaries) and localized thinning of layers at streams occurred but could not be generalized across a 6-mi-square grid block. The selected constant heads also could indicate water movement along vertical zones in proximity to streams.

The water budget within the layers is generally balanced. The largest departure from a balance occurs in layers 3 and 4, which are combined in the prototype in eastern Montana. Combining the water budget for the two layers greatly improves the balance of the budget for the two layers from +3.25 ft^3/s and -2.12 ft^3/s, respectively, to +1.13 ft^3/s.

The large number of constant-head nodes required together with the large amount of intralayer and interlayer flow, especially along stream courses, imply that local effects are important in regional modeling efforts. Specifically, local variations in confining-unit thicknesses and the possible presence of preferred flow paths along fractures, such as those postulated for rocks below or within the Pierre Shale by Downey (1984) and Leonard and others (1983), may be important in modeling head distributions in the shallow aquifers and confining layers of the northern Great Plains.

ADDITIONAL STUDY

Additional study would be needed before specific or detailed questions related to water availability or movement within the study area could be answered. The standard error of estimate of the potentiometric surface of 110 ft makes the existing model insensitive to localized anomalies of about 100 ft. Even though the model reproduces broad regional trends, it does not provide a means for assessing or managing problems related to dewatering in mining operations or declining water levels in individual wells.

Additional study of the area could include improving the present regional model of the entire area or selecting specific subareas to be modeled with more detail. Improvement of the 110-ft standard error in the present model might be possible through acquisition of additional data such as basinwide discharge and recharge, refinement of the capabilities of the present digital model, or improvements in the conceptual model. Additional data would include transmissivity data especially for the Tongue River, Tullock, and Fox Hills-lower Hell Creek aquifers away from aquifer outcrops. Data on vertical flow and fracture flow also could be sought. If a smaller grid spacing were used, more of the variability in thickness of the Lebo-Tongue River interlayer could be incorporated to improve the model fit on the upper layer. In addition, a larger number of layers would allow water levels to be assigned to nodes with greater accuracy.

Insights into the regional flow system gained during the present study would be helpful in modeling these smaller areas. Specifically, the hydraulic heads calculated by this model could be used along the boundaries of smaller scale model studies where existing data are scarce.

SUMMARY

The hydrogeology of shallow units in the Powder River, Bull Mountains, and western Williston basins was investigated to improve understanding of the regional ground-water flow system. As part of this study, a three-dimensional digital model was constructed from existing geologic and hydrologic data.

Five major hydrogeologic units overlying the relatively impermeable Bearpaw Shale were differentiated on the basis of sandstone content, permeability, and previous studies. The Fox Hills Sandstone and hydrologically similar overlying lower part of the Hell Creek Formation or their geologic equivalents together form the Fox Hills-lower Hell Creek aquifer (layer 1), the lowermost layer. The generally less permeable upper part of the Hell Creek Formation or its geologic equivalents forms the upper Hell Creek confining layer (layer 2). The Tullock Member of the Fort Union Formation or its geologic equivalents forms the Tullock aquifer (layer 3). The Lebo Shale Member of the Fort Union Formation or its geologic equivalents forms the Lebo confining layer (layer 4). Where the Lebo confining layer and Tullock aquifer cannot be differentiated in the northeastern part of the study area, they are combined into the lower Fort Union aquifer. Finally, the Tongue River Member of the Fort Union Formation, its geologic equivalents, and all younger formations taken together form the Tongue River aquifer (layer 5), the uppermost layer. These five layers formed the basis for modeling.

All five hydrogeologic units crop out in the study area; therefore, the minimum thickness of each unit is 0 ft at the base of outcrops where the units pinch out. The Fox Hills-lower Hell Creek aquifer is confined, has a maximum thickness of 2,550 ft and a mean thickness of 666 ft, and yields generally less than 100 gal/min to wells. The upper Hell Creek confining layer has a maximum thickness of 2,000 ft and a mean thickness of 514 ft, and can yield as much as 4 gal/min from channel-sandstone deposits. The Tullock aquifer is confined, has a maximum thickness of 1,960 ft and a mean thickness of 633 ft, and generally yields about 15 gal/min to wells. The Lebo confining layer has a maximum thickness of 3,780 ft and a mean thickness of 630 ft, and can yield as much as 10 gal/min to wells locally where the sandstone content is large. The Tongue River aquifer generally is confined, has a maximum thickness of 3,910 ft and a mean thickness of 1,240 ft, and provides maximum well yields of about 500 gal/min.

Potentiometric-surface maps prepared for the five hydrogeologic units indicate that, in general, the deeper aquifers have higher potentiometric heads than the shallow aquifers. Thus, the entire study area appears to be a discharge area, except near outcrop areas.

The major sources of recharge to the shallow hydrogeologic units are infiltration of water from precipitation and streamflow on areas of outcrop. Infiltration of water from losing streams is also a component of recharge. Discharge from the area is principally outflow along the northeastern boundary; loss of ground water to gaining streams, springs, and seeps; evapotranspiration; and pumpage of ground water.

Measured transmissivity values for the hydrogeologic units were sparse and in only a few instances represented the complete thickness of the aquifer. Therefore, the aquifer properties for modeled units, specifically transmissivity, were estimated from the ratio of sandstone and shale in each hydrogeologic unit as interpreted from geophysical logs of oil-well test holes. The estimates of transmissivity at each oil-well test site were distributed to model nodes by the regionalization technique of kriging. Original mean transmissivity values were estimated to be 441 ft^2/d for layer 1, 190 ft^2/d for layer 2, 372 ft^2/d for layer 3, 216 ft^2/d for layer 4, and 717 ft^2/d for layer 5. These estimates were adjusted for kinematic viscosity and for compaction at the center of each unit. In a like manner, the thicknesses from the center of each layer to the center of the layer above were distributed to each model node for determining the four interlayer vertical-leakage coefficients.

The three-dimensional finite-difference model developed by Trescott (1975) and Trescott and Larson (1976) was used in the simulation. The model was calibrated by adjusting the available data and data estimates across the model area and operating the model. After a series of trial-and-error simulations, array factors which multiply input parameters to the model were perturbed systematically. The purpose of the perturbation was to determine the magnitude and direction of change necessary for the parameters to improve the fit of predicted head values relative to 739 observed head values. The improved parameters then were used for the next stage of input to the model. In this manner, five stages of model simulations were attempted to determine new factors that would reduce the standard error of estimate in computed heads. The standard error of estimate was reduced from 135 to 110 feet, the amount of improvement becoming very small at stages 2 through 5.

The calibrated mean transmissivity for the hydrogeologic units was 443 ft^2/d for the Fox Hills-lower Hell Creek, 191 ft^2/d for the upper Hell Creek, 374 ft^2/d for the Tullock, 217 ft^2/d for the Lebo, and 721 ft^2/d for the Tongue River hydrogeologic units. Calibrated mean vertical hydraulic conductance per unit area between the units listed above was, respectively, 1.40×10^{-4}, 3.18×10^{-5}, 2.27×10^{-5}, and 1.58×10^{-5} ft/d. Mean annual recharge across the study area was about 2.45×10^{-2} in. or about 0.26 percent of average-annual precipitation.

A hydrologic mass balance (water budget) was calculated at the end of the steady-state simulation. The balance contained the computed water additions and subtractions for each hydrologic component. Water in the Tongue River aquifer is gained from precipitation (38.16 ft^3/s) and is lost by outflow (26.04 ft^3/s) and leakage (12.87 ft^3/s) to the underlying confining layer. Water in the Lebo confining layer is gained from precipitation (11.30 ft^3/s) and leakage from the overlying aquifer and is lost by outflow (1.91 ft^3/s) and leakage (24.38 ft^3/s) to the underlying aquifer. Water in the Tullock aquifer is gained from precipitation (7.19 ft^3/s) and leakage from the overlying and underlying (2.55 ft^3/s) confining layers and is lost by outflow (30.87 ft^3/s). Water in the upper Hell Creek confining layer is gained from precipitation (4.42 ft^3/s) and inflow (1.02 ft^3/s) and is lost by leakage to the overlying and underlying (2.25 ft^3/s) aquifers. Water in the Fox Hills-lower Hell Creek aquifer is gained from precipitation (4.19 ft^3/s) and leakage from the overlying confining layer and is lost by outflow (6.99 ft^3/s).

The mass balance of the steady-state simulation indicated that recharge caused by precipitation accounted for about 65.26 ft^3/s, or 29.3 percent of total recharge. The remaining 70.7 percent (157.60 ft^3/s) of the total is recharge to constant heads such as would occur in areas of losing streams along the front of the Bighorn Mountains. The total discharge from the model of 222.39 ft^3/s is virtually equal to total recharge (222.86 ft^3/s) and is all from constant-head nodes. Net constant-head discharge from the model generally occurs at the model boundaries representing topographically low areas at the edge of the study area and at nodes in the vicinity of major streams.

Additional study would be needed to address detailed and localized mining or declining water-level problems. The large standard error of estimate precludes this model from use as a means for assessing or managing local water problems. Future studies could concentrate on understanding the flow system of specific subareas. Insights into the regional flow system gained during the present modeling effort might be helpful in modeling these smaller areas.

SELECTED REFERENCES

Armstrong, Margaret, and Jabin, Romain, 1981, Variogram models must be positive-definite: Mathematical Geology, v. 13, no. 5, p. 455-459.

Balster, C. A., ed., 1971, Catalog of stratigraphic names for Montana: Montana Bureau of Mines and Geology Special Publication 54, 448 p.

______1980, Stratigraphic nomenclature chart for Montana and adjacent areas: Montana Bureau of Mines and Geology Geologic Map 8.

Bredehoeft, J. D., and Pinder, G. F., 1970, Digital analysis of areal flow in multi-aquifer groundwater systems; a quasi three-dimensional model: Water Resources Research, v. 6, no. 3, p. 883-888.

Coffin, D. L., Reed, T. E., and Ayres, S. D., 1977, Water-level changes in wells along the west side of the Cedar Creek anticline, southeastern Montana: U.S. Geological Survey Water-Resources Investigations 77-93, 11 p.

Cooley, R. L., 1977, A method of estimating parameters and assessing reliability for models of steady state groundwater flow--Theory and numerical properties: Water Resources Research, v. 13, no. 2, p. 318-324.

Crist, M. A., 1977, Hydrologic evaluation of the Arikaree Formation near Lusk, Niobrara and Goshen Counties, Wyoming: U.S. Geological Survey Water-Resources Investigations 77-111, 23 p.

Curry, W. H., 1971, Laramide structural history of the Powder River Basin, Wyoming: Wyoming Geological Association Guidebook 13th Annual Field Conference, p. 49-60.

Dahl, A. R., and Hagmaier, J. L., 1976, Genesis and characteristics of the southern Powder River Basin uranium deposits, Wyoming: Wyoming Geological Association Guidebook 28th Annual Field Conference, p. 243-252.

David, Michael, 1977, Developments in geomathematics 2, geostatistical ore reserve estimation: Amsterdam, Elsevier Scientific Publishing Co., 364 p.

Davis, J. C., 1973, Statistics and data analysis in geology: New York, John Wiley and Sons, 54 p.

Delfiner, Pierre, 1976, Linear estimation of nonstationary spatial phenomena, *in* NATO Advanced Study Institute on advanced geostatistics in the mining industry, Rome, Italy, 1975: Boston, D. Reidel Publication Co., p. 49-68.

Delfiner, Pierre, and Delhomme, J. P., 1975, Optimum interpolation by kriging, *in* NATO Advanced Study Institute on display and analysis of spatial data, Nottingham, England, 1973: New York, John Wiley and Sons, p. 96-114.

Delhomme, J. P., 1978, Kriging in the hydrosciences: Advances in Water Resources, v. 1, no. 5, p. 251-266.

Denson, N. M., and Horn, G. H., 1975, Geologic and structure map of the southern part of the Powder River Basin, Converse, Niobrara, and Natrona Counties, Wyoming: U.S. Geological Survey Miscellaneous Investigations Map I-877.

Dinwiddie, G. A., and others, 1979, Plan of study for the Northern Great Plains regional aquifer-system analysis in parts of Montana, North Dakota, South Dakota, and Wyoming: U.S. Geological Survey Water-Resources Investigations 79-34, 20 p.

Dobbin, C. E., and Erdmann, C. E., 1955, Structure contour map of the Montana plains: U.S. Geological Survey Oil and Gas Investigations Map OM-178B.

Dodge, K. A., and Levings, G. W., 1980, Measurements of discharge, gain or loss in flow, and chemical quality of the Poplar and Redwater Rivers, northeastern Montana, October 24-25, 1979: U.S. Geological Survey Open-File Report 80-1210, 16 p.

Downey, J. S., 1984, Geohydrology of the Madison and associated aquifers in parts of Montana, North Dakota, South Dakota, and Wyoming: U.S. Geological Survey Professional Paper 1273-G, 47 p.

Draper, N. R., and Smith, Harry, 1966, Applied regression analysis: New York, John Wiley and Sons, 407 p.

Druse, S. A., Dodge, K. A., and Hotchkiss, W. R., 1981, Base flow and chemical quality of streams in the Northern Great Plains area, Montana and Wyoming, 1977-78: U.S. Geological Survey Water-Resources Investigations Open-File Report 81-692, 60 p.

Durbin, T. J., 1978, Calibration of a mathematical model of the Antelope Valley ground-water basin, California: U.S. Geological Survey Water-Supply Paper 2046, 51 p.

Eardley, A. J., 1951, Structural geology of North America: New York, Harper and Brothers, 624 p.

Ellis, A. J., and Meinzer, D. E., 1924, Ground water in Musselshell and Golden Valley Counties, Montana: U.S. Geological Survey Water-Supply Paper 518, 92 p.

Fatt, I., and Davis, D. H., 1952, Reduction in permeability with overburden pressure: American Institute of Mining and Metallurgical Engineers Petroleum Transactions, v. 195, p. 329.

Feltis, R. D., Lewis, B. D., Frasure, R. L., Rioux, R. P., Jauhola, C. A., and Hotchkiss, W. R., 1981, Selected geologic data from the northern Great Plains area of Montana: U.S. Geological Survey Water-Resources Investigations Open-File Report 81-415, 63 p.

Freeze, R. A., and Cherry, J. A., 1979, Groundwater: Englewood Cliffs, N. J., Prentice-Hall, Inc., 604 p.

Gambolati, Guiseppe, and Volpi, Giampiero, 1979, Groundwater contour mapping in Venice by stochastic interpolators 1 theory: Water Resources Research, v. 15, no. 2, p. 281-290.

Garabedian, S. P., 1984, Application of a parameter-estimation technique to modeling the regional aquifer underlying the eastern Snake River plain, Idaho: U.S. Geological Survey Open-File Report 84-461, 119 p.

Gardner, R. A., 1981, Model of the ground-water flow system of the Gordo and Eutaw aquifers in west-central Alabama: Geological Survey of Alabama Bulletin 118, 30 p.

Glass, G. B., 1976, Update on the Powder River coal basin: Wyoming Geological Association Guidebook 28th Annual Field Conference, p. 209-220.

Grose, L. T., 1972, Tectonics, *in* Geologic atlas of the Rocky Mountain Region: Denver, Colo., Rocky Mountain Association of Geologists, p. 35-47.

Hall, G. M., and Howard, C. S., 1929, Ground water in Yellowstone and Treasure Counties, Montana: U.S. Geological Survey Water-Supply Paper 599, 118 p.

Hancock, E. T., 1918, Geology and oil and gas prospects of Lake Basin field, Montana: U.S. Geological Survey Bulletin 691-D, 147 p.

Hardy, R. C., and Cottingham, R. L., 1949, Viscosity of deuterium oxide and water in the range 5° to 125° centigrade: U.S. National Bureau of Standards, Journal of Research, Research Paper 1994, v. 42, p. 573-578.

Heald, K. C., 1926, The geology of the Ingomar anticline, Treasure and Rosebud Counties, Montana: U.S. Geological Survey Bulletin 786-A, p. 1-37.

Hittman Associates, Inc., 1981, Monitoring and modeling of shallow groundwater systems in the Powder River basin: Englewood, Colo., Fourth Annual Technical Report, 269 p.

Hodson, W. G., Pearl, R. H., and Druse, S. A., 1973, Water resources of the Powder River basin and adjacent areas, northeastern Wyoming: U.S. Geological Survey Hydrologic Investigations Atlas-465.

Hotchkiss, W. R., 1978, Developing a hydrologic model of the shallow aquifers of the Powder River basin, Montana and Wyoming: Proceedings of the Fifth National Conference, Energy and the Environment, p. 212-217.

Johnson, M. V., and Omang, R. J., 1976, A method for estimating magnitude and frequency of floods in Montana: U.S. Geological Survey Open-File Report 75-650, 46 p.

Johnson, W. D., Jr., and Smith, H. R., 1964, Geology of the Winnett-Mosby area, Petroleum, Garfield, Rosebud, and Fergus Counties, Montana: U.S. Geological Survey Bulletin 1149, 91 p.

Journel, A. G., and Huijbregts, C. J., 1978, Mining geostatistics: London, Academic Press, 600 p.

Kanizay, S. P., Obernyer, S. L., and Cattermole, J. M., 1976, Preliminary geologic map of the Buffalo area, northwest Powder River basin, Wyoming: U.S. Geological Survey Miscellaneous Field Studies Map MF-806.

Karlinger, M. R., and Skrivan, J. A., 1981, Kriging analysis of mean annual precipitation, Powder River Basin, Montana and Wyoming: U.S. Geological Survey Water-Resources Investigations 80-50, 25 p.

Kim, Y. C., Martino, Francis, and Chopra, I. K., 1981, Application of geostatistics in a coal deposit: Mining Engineering, v. 33, no. 10, p. 1476-1481.

Konikow, L. F., 1976, Preliminary digital model of ground-water flow in the Madison Group, Powder River basin and adjacent areas, Wyoming, Montana, South Dakota, North Dakota, and Nebraska: U.S. Geological Survey Water-Resources Investigations 63-75, 44 p.

Lee, R. W., 1979, Ground-water-quality data from the Northern Powder River basin, southeastern Montana: U.S. Geological Survey Water-Resources Investigations Open-File Report 79-1331, 55 p.

Lenfest, L. W., Jr., 1981, Evapotranspiration rates at selected sites in the Powder River Basin, Wyoming and Montana: U.S. Geological Survey Water-Resources Investigations 82-4105, 35 p.

Leonard, R. B., Signor, D. L., Jorgensen, D. G., and Helgesen, J. D., 1983, Geohydrology and hydrochemistry of the Dakota aquifer, central United States: American Water Resources Association, Water Resources Bulletin, v. 19, no. 6, p. 903-911.

Levings, G. W., 1981, Selected hydrogeologic data from the Northern Great Plains area of Montana: U.S. Geological Survey Open-File Report 81-534, 241 p.

______1982, Potentiometric-surface map of water in the Fox Hills-lower Hell Creek aquifer in the Northern Great Plains area of Montana: U.S. Geological Survey Open-File Report 82-564, scale 1:1,000,000.

Levings, J. F., 1983, Hydrogeology and simulation of water flow in the Kootenai aquifer of the Judith basin, central Montana: U.S. Geological Survey Water-Resources Investigations Report 83-4146, 39 p.

Levings, J. F., Levings, G. W., Feltis, R. D., Hotchkiss, W. R., and Lee, R. W., 1981, Selective annotated bibliography of geology and ground-water resources for the Montana part of the Northern Great Plains regional aquifer-system analysis: U.S. Geological Survey Water-Resources Investigations Open-File Report 81-401, 91 p.

Lewis, B. D., and Hotchkiss, W. R., 1981, Thickness, percent sand, and configuration of shallow hydrogeologic units in the Powder River Basin, Montana and Wyoming: U.S. Geological Survey Miscellaneous Investigations Map I-1317.

Lewis, B. D., and Roberts, R. S., 1978, Geology and water-yielding characteristics of rocks of the Northern Powder River Basin, southeastern Montana: U.S. Geological Survey Miscellaneous Investigations Map I-847-D.

Lohman, S. W., 1972, Ground-water hydraulics: U.S. Geological Survey Professional Paper 708, 70 p.

Lohman, S. W. and others, 1972, Definitions of selected ground-water terms--Revisions and conceptual refinements: U.S. Geological Survey Water-Supply Paper 1988, 21 p.

Matheron, Georges, 1971, The theory of regionalized variables and its application: Fontainebleau, France, Les Cahiers du Centre de Imorphologie Mathematics No. 5, 211 p.

Meyer, W. R., and Carr, J. E., 1979, A digital model for simulation of ground-water hydrology in the Houston area, Texas: Texas Department of Water Resources LP-103, 27 p.

Miller, W. R., 1979, Water resources of the central Powder River area of southeastern Montana: Montana Bureau of Mines and Geology Bulletin 108, 65 p.

______1981, Water resources of the southern Powder River area of southeastern Montana: Montana Bureau of Mines and Geology Memoir 47, 53 p.

Morrissey, D. J., Lines, G. C., and Bartholoma, S. D., 1980, Three-dimensional digital computer model of the Ferron Sandstone aquifer near Emry, Utah: U.S. Geological Survey Water-Resources Investigations 80-62, 15 p.

Neuman, S. P., and Witherspoon, P. A., 1968, Theory of flow in aquicludes adjacent to slightly leaky aquifers: Water Resources Research, v. 4, no. 1, p. 103-112.

______1972, Field determination of the hydraulic properties of leaky multiple aquifer systems: Water Resources Research, v. 8, no. 5, p. 1284-1298.

Olea, R. A., 1975, Optimum mapping techniques using regionalized variable theory: Kansas Geological Survey Series of Spatial Analysis No. 2, 137 p.

Osterwald, F. W., and Dean, B. A., 1958, Preliminary tectonic maps showing distribution of uranium deposits, eastern Montana: U.S. Geological Survey Miscellaneous Field Studies Map MF-126.

Poland, J. F., Lofgren, B. E., and Riley, F. S., 1972, Glossary of selected terms useful in studies of aquifer systems and land subsidence due to fluid withdrawal: U.S. Geological Survey Water-Supply Paper 2025, 9 p.

Rahn, P. H., and Gries, J. P., 1973, Large springs in the Black Hills, South Dakota and Wyoming: South Dakota Geological Survey Report of Investigations No. 107, 46 p.

Rendu, J. M., 1979, Normal and lognormal estimation: Journal of the International Association for the Mathematical Geology, October, p. 407-422.

Robinson, C. S., Mapel, W. J., and Bergendahl, M. H., 1964, Stratigraphy and structure of the northern and western flanks of the Black Hills uplift, Wyoming, Montana, and South Dakota: U.S. Geological Survey Professional Paper 404, 134 p.

Santos, E. S., 1981, Facies distribution in uranium host rocks of the southern Powder River Basin, Wyoming: U.S. Geological Survey Open-File Report 81-741, 15 p.

Schultz, L. G., Tourtelot, H. A., Gill, J. R., and Boerngen, J. G., 1980, Composition and properties of the Pierre Shale and equivalent rocks, Northern Great Plains region: U.S. Geological Survey Professional Paper 1064-B, 114 p.

Seeland, D. A., 1976, Relationships between early Tertiary sedimentation patterns and uranium mineralization in the Powder River basin, Wyoming: Wyoming Geological Association Guidebook 28th Annual Field Conference, p. 53-64.

Skrivan, J. A., and Karlinger, M. R., 1980, Semi-variogram estimation and universal kriging program: Tacoma, Wash., U.S. Geological Survey Computer Contribution, 98 p.

Stone, H. L., 1968, Iterative solution of implicit approximations of multidimensional partial differential equations: Society for Industrial and Applied Mathematics, Journal of Numerical Analysis, v. 5, no. 3, p. 530-558.

Stoner, J. D., and Lewis, B. D., 1980, Hydrogeology of the Fort Union coal region, eastern Montana: U.S. Geolgical Survey Miscellaneous Investigations Map I-1236.

Strickland, J. W., 1958, Habitat of oil in the Powder River Basin: Wyoming Geological Association Guidebook 13th Annual Field Conference, p. 132-147.

Taylor, A. E., 1955, Advanced calculus: Boston, Ginn and Co., 786 p.

Taylor, O. J., 1978, Summary appraisals of the Nation's ground-water resources--Missouri basin region: U.S. Geological Survey Professional Paper 813-Q, 41 p.

Thom, W. T., Jr., Hall, G. M., Wegemann, C. H., and Moulton, G. F., 1975, Geology of Big Horn County and the Crow Indian Reservation, Montana, with special reference to the water, coal, oil, and gas resources: U.S. Geological Survey Bulletin 856, 200 p.

Toy, T. J., and Munson, B. E., 1978, Climate appraisal maps of the rehabilitation potential of strippable coal lands in the Powder River basin, Wyoming and Montana: U.S. Geological Survey Miscellaneous Field Studies Map MF-932.

Trescott, P. C., 1975, Documentation of finite-difference model for simulation of three-dimensional ground-water flow: U.S. Geological Survey Open-File Report 75-438, 32 p.

Trescott, P. C., and Larson, S. P., 1976, Supplement to Open-File Report 75-438--Documentation of finite-difference model for simulation of three-dimensional ground-water flow: U.S. Geological Survey Open-File Report 76-591, 21 p.

Trescott, P. C., Pinder, G. F., and Larson, S. P., 1976, Finite-difference model for aquifer simulation in two dimensions with results of numerical experiments: U.S. Geological Survey Techniques of Water-Resources Investigations, Book 7, Chapter C1, 116 p.

U.S. Bureau of Reclamation, 1977, Ground water manual--A guide for the investigation, development, and management of ground-water resources: Washington, D.C., U.S. Government Printing Office, 024-003-00106-6, 480 p.

U.S. Geological Survey and American Association of Petroleum Geologists, 1962, Tectonic map of the United States, exclusive of Alaska and Hawaii: scale 1:2,500,000.

Volpi, Giampiero, and Gambolati, Giuseppe, 1978, On the use of a main trend for the kriging technique in hydrology: Advances in Water Resources, v. 1, no. 6, p. 345-349.

Weast, R. C., ed., 1968, CRC handbook of chemistry and physics: Cleveland, Ohio, Cleveland Rubber Publishing Co., 2092 p.

Weiss, Emanuel, 1982, A computer program for calculating relative-transmissivity inputs to aid model calibration: U.S. Geological Survey Open-File Report 82-447, 18 p.

Wienstein, H. C., Stone, H. L., and Kwan, T. V., 1969, Iterative procedure for solution of systems of parabolic and elliptic equations in three dimensions: Industrial Engineering Chemistry Fundamentals, v. 8, no. 2, p. 281-287.

Woessner, W. W., Osborne, T. J., Heffern, E. L., Andrews, Charles, Whiteman, Jason, Spotted Elk, Wesley, and Morales-Brink, Daniel, 1981, Hydrologic impacts from potential coal strip mining, Northern Cheyenne Reservation: U.S. Environmental Protection Agency Northern Cheyenne Research Project, 303 p.

Woolsey, L. H., Richards, R. W., and Lupton, C. T., 1917, The Bull Mountains coal field, Musselshell and Yellowstone Counties, Montana: U.S. Geological Survey Bulletin 647, 218 p.