A PROCEDURE FOR ESTIMATING REAERATION COEFFICIENTS FOR MASSACHUSETTS STREAMS

By Gene W. Parker and Frederick B. Gay

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FACTORS FOR CONVERTING INCH-POUND UNITS TO INTERNATIONAL SYSTEM

For the convenience of readers who may prefer to use metric (International System) units rather than the inch-pound units used in this report, values may be converted by using the following factors:

Multiply inch-pound units	By	To obtain SI Units
	Length	
foot (ft)	0.3048	meter (m)
mile (mi)	1.609	kilometer (km)
	Flow	
foot per second (ft/s)	0.3048	meter per second (m/s)
square foot per hour (ft^2/h)	0.09294	square meter per hour (m^2/h)
cubic foot per second (ft^3/s)	0.02832	cubic meter per second (m^3/s)
	Acceleration	
foot per second squared (ft/s^2)	0.3048	meter per second squared (m/s^2)

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ABSTRACT

An equation for estimating stream reaeration coefficients from easily measured physical characteristics was developed for moderately sloped streams in Massachusetts. To define the equation, multiple-regression techniques were applied to 30 data sets containing 9 physical, hydrologic, and water-quality characteristics. Data on mean depth, water-surface slope, and mean streamflow velocity ranged from 0.4 to 6.3 feet, from 0.00017 to 0.015 feet per foot, and from 0.13 to 2.15 feet per second, respectively. Reaeration coefficients were measured using a steady-state, propane-gas tracer technique during medium- and low-flow periods in 1983 and 1984. Measured reaeration coefficients ranged from 0.4 to 67.7 base e units per day at 20 degrees Celsius. The regression analysis defines the relation between the reaeration coefficient, K_2, *and the independent variables mean depth, D, water-surface slope, SL, and mean streamflow velocity, V, given by:* $K_2 = 252.2\ D^{-0.176}\ V^{0.355}\ SL^{0.438}$, *in which 252.2 is the regression constant for the equation. The equation was limited to three variables that were the most significant at the 95-percent confidence level because of the small size of the data base. The standard error of estimate for the reaeration equation is 37 percent. An error-analysis technique was used to compare the proposed equation with 19 published reaeration coefficient equations. The error analysis indicates that the proposed equation has the second lowest error (37 percent) for 20 stream reaches with slopes greater than 0.002 feet per foot.*

INTRODUCTION

Stream reaeration coefficients are used in stream-water quality models to forecast the effects of organic loadings on DO (dissolved-oxygen) concentrations in streams. State water-pollution-control agencies, such as the MDWPC (Massachusetts Department of Environmental Quality Engineering, Division of Water Pollution Control) rely on stream DO models for decisions relative to the maintenance of stream water-quality standards set by each state in conjunction with the U.S. Environmental Protection Agency.

The self-cleaning capacity of a river depends on DO concentrations and the capacity to replace oxygen removed by the reduction of organic wastes. When attempting to model the concentration levels of a nonconservative substance such as DO in open-channel flow, allowances must be made for dispersion, decay, and reaeration at the surface, and for deoxygenation resulting from biochemical demands, algal respiration, and interaction with benthic deposits. The use of a steady-state river model is based on the assumption that the dispersion coefficient is constant for a given stream reach over the time period being simulated. All of the other processes can be measured directly, except for reaeration, which can be determined indirectly or estimated from equations.

Measured values for the reaeration coefficient can be determined from the dissolved-oxygen balance, distributed equilibrium, and by tracer methods. The dissolved-oxygen balance method consists of measuring the various sources and sinks of DO and determining the amount of reaeration needed to balance the equation. The disturbed-equilibrium method consists of artificially producing DO deficits by adding sodium sulfite to the stream and subsequently measuring upstream and downstream concentrations of DO at two different concentration levels. The tracer method consists of correlating the rate of desorption of a tracer gas with the rate of absorption of oxygen. However, in addition to being costly and time consuming, the balance and equilibrium methods are indirect determinations of oxygen transfer and are subject to measurement errors. These indirect methods of calculating the DO contribution from reaeration may be no more accurate than reaeration coefficients calculated from theoretical or empirical equations (Bennett and Rathbun, 1972). Recent advances in the tracer method, such as the U.S. Geological Survey's steady-state propane-gas tracer method (Yotsukura and others, 1983; 1984), offer a less costly, more accurate, reliable, and reproducible method of measuring reaeration coefficients in place. The gas-tracer method is useful for determining reaeration coefficients, because it also eliminates interferences of photosynthetic oxygen production and respiration of the suspended and attached aquatic plants (Bennett and Rathbun, 1972; Rathbun and others 1978; Yotsukura and others, 1983; 1984).

The reaeration coefficients used in predictive models generally are estimated from theoretical, semiempirical, or empirical equations. Bennett and Rathbun (1972) state that the theoretical models of the dissolved-oxygen absorption process generally are not suitable to predict the reaeration coefficient in streams because the model parameters have been inadequately related to bulk-flow hydraulic variables. Semiempirical and empirical equations developed from experimental data predict reaeration coefficients adequate for streams of the type on which the equations are based, but large errors may occur when the equations are applied to other types of streams or to conditions outside the range of independent variables considered in the equations derived from empirical data.

Because of stringent water-quality standards imposed on State streams and the high costs of determining stream-reaeration coefficients, there is a need to develop an equation that will estimate reaeration coefficients reliably from easily measured physical, hydraulic, and water-quality characteristics. In response to this need, the U.S. Geological Survey, in cooperation with the MDWPC, has developed regionalized predictive equations using the Survey's steady-state propane-gas tracer method. This gas-tracer method provides the data needed to derive a reaeration-estimating equation.

Purpose and Scope

This report describes the multiple-regression techniques used to derive an equation for estimating reaeration coefficients of Massachusetts streams. The equation is based on easily measured physical and hydraulic characteristics of stream channels. The equation is compared to other commonly used estimating equations by using measured data to determine the accuracy of all the equations to predict reaeration coefficients in Massachusetts streams.

Tracer studies were performed on 16 stream reaches representative of most streams in Massachusetts. The Survey's newly developed steady-state, gas-tracer technique was was used to measure *in-situ* reaeration coefficients. Reaeration coefficients, mean streamflow velocities, and nine easily measured physical, hydraulic, and water-quality characteristics were measured in 30 studies during medium- and low-flow periods between August 1983 and December 1984.

Approach

As a result of searching reaeration literature, nine physical, hydraulic, and water quality characteristics were selected to be correlated with the reaeration coefficient: Water-surface slope, mean velocity, depth, width, roughness coefficient, color, methylene blue active substances concentration, specific conductance, and suspended-solids concentration. Sixteen reaches on 11 rivers (fig. 1) were selected for combined time-of-travel and reaeration-tracer studies on the basis of their regional location, consistency of reach characteristics, and accessibility. Reach and study-site descriptions can be found in Appendix A. Thirty combined tracer studies were conducted on the 16 reaches. When possible, two or more tracer studies were performed on the same reach at different discharge rates. All studies were conducted at a steady discharge with little or no wind. Reach physical, hydraulic, and water-quality characteristics were measured during each tracer study and samples were collected for water-quality analysis. Water-surface slope was determined using differential leveling between reference points established at the ends of each study reach.

All studies were initially evaluated for completeness and accuracy of time-of-travel data. Initial propane-gas desorption coefficients were determined using steady-state, gas-tracer injection techniques outlined in Yotsukura and others (1983). For each tracer study, the transfer of measurement error was estimated.

The reaeration-coefficient estimating equation was developed from reaeration coefficients and the corresponding reach characteristics determined for the 30 combined tracer studies. Step-forward, multiple-regression analyses were conducted to relate reaeration coefficients to channel characteristics. Only those characteristics significant at a 95-percent confidence level were retained.

Reaeration coefficients were estimated from widely used equations using the channel characteristics determined for all 30 tracer studies. All estimating equations, including the equation developed for this report, were ranked according to the accuracy of their predicted reaeration values compared to measured values from the 30 tracer studies.

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The authors wish to acknowledge and thank the many persons and organizations who have kindly given their time, information, and guidance to this study—in particular, personnel from the Massachusetts Division of Water Pollution Control's Westborough, Massachusetts, office who assisted with the

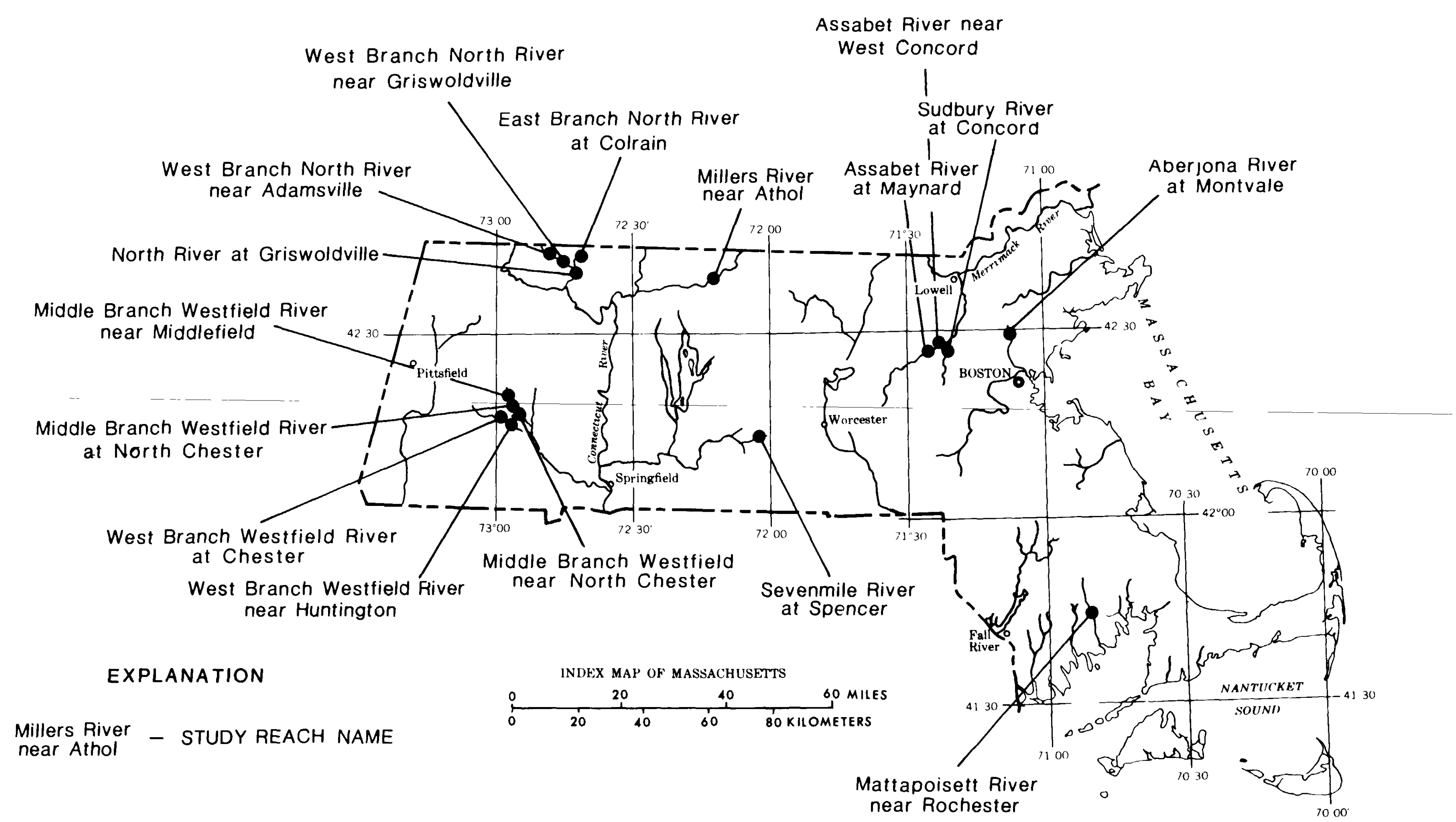

Figure 1.--Stream-reach locations.

selection of study reaches, collection of data, and provided information available from previous dye-tracer studies. In addition, the authors also are indebted to the landowners and municipal officials who allowed access through their property to the various stream reaches. The authors gratefully acknowledge technical assistance and advice received throughout this study from Nobuhiro Yotsukura and David Stedfast of the U.S. Geological Survey, who were responsible for the development and initial evaluation of the steady-state gas-tracer method of determining reaeration coefficients.

THEORY AND METHODOLOGY

Theory

Reaeration is gas transfer that occurs at the stream water surface and air boundary. The rate of absorption of oxygen from air into water is controlled by the thin water film at the stream surface. Based on mixing-tank experiments, the desorption of gases including propane through the air and water interface can be considered a first-order transfer mechanism described by the equation (Rathbun and others, 1978):

$$dC \frac{24}{dT} = -K\ C \qquad (1)$$

where

$dC \frac{24}{dT}$ = the rate of change in gas concentration in time, in micrograms per liter per day;
C = the dissolved gas concentration, in micrograms per liter;
T = time, in hours; and
K = the desorption coefficient, in base e units per day at 20 degrees Celsius.

Equation 1 can be rewritten for the absorption of oxygen or other similar gases by making K positive to indicate the flow of the gases across the air and water interface in the opposite direction. This sorption process is driven by the gas-concentration deficit below saturation for the medium into which the gas is moving.

The absorption and desorption coefficient (K) in equation 1 is defined as:

$$K = \frac{KL}{D} \qquad (2)$$

where

KL = the surface-film absorption or desorption coefficient, in feet per day; and
D = the mean depth, in feet.

Note that the formation of equation 1 treats gas absorption or desorption, which only occurs at the water surface, as if it were equivalent to a first-order decay that occurs throughout the total water column. The surface-film coefficient in equation 2 is a more fundamental term as it represents the transfer rate through the surface film and no the total water column.

The absorption or desorption gas process is maintained by the imbalance of gas concentration across the stream water-surface film (Yotsukura and others, 1983). This process is enhanced by (1) turbulence in the water-column extending from the surface film to the riverbed, and (2) wind shear at the water surface that disperses the gas that has pass through the air and water interface. Turbulence in and at the surface of a river is influenced by changes in physical and hydraulic channel characteristics, such as water-surface slope, mean velocity, depth, width, and roughness coefficient. The gas-transfer process can also be affected by changes in water-quality characteristics such as methylene blue active substances as an indicator of detergent concentrations, color as an indicator of the concentration of organic acids, specific conductance as an indicator of dissolved-solids concentrations, and suspended solids as an indicator of suspended inorganic concentrations in the water column (Bennett and Rathbun, 1972).

Methods

Measurement of Reach Characteristics

For each tracer study, the water-surface slope, stream discharge, mean streamflow velocity, channel width and depth, and Manning's roughness coefficient were determined. Also, water-quality characteristics including methylene blue active substances, color, specific conductance, and suspended solids were determined. The specific details of how each characteristic was measured or determined are described as follows.

1. Water-surface slope (SL) was determined in feet per foot, by the ratio of the change in elevation of the water surface and the study reach length (L) between the sampling sites. The change in water-surface elevation was determined by differential leveling between the sampling sites or from bench marks to the sampling sites. Study-reach lengths between sample sites were measured from topographic maps.

2. Discharge (Q), in cubic feet per second, was determined by averaging measured discharge at each sample site for the same parcel of water as identified by the dye tracer. Discharge measurements were made using the method outlined by Buchanan and Somers (1969) and confirmed using the total dye-recovery method (Kilpatrick and Cobb, 1984). Discharge was not included in the data matrix used for the regression and correlation analyses.

3. Mean streamflow velocity (V) was determined in feet per second, by solving the equation:

$$V = \frac{L}{3{,}600(\overline{T}_d - \overline{T}_u)} \qquad (3)$$

where

L = the length of the reach studied, in feet;

3,600 = a constant to convert hours to seconds; and

$\overline{T}$ = the traveltime in hours of the centroid of a response curve resulting from a slug-injection of dye-tracer (the slug injection method is explained in more detail in the next section); and

u and d = subscripts which designate the upstream and downstream sample sites, respectively.

4. Width (W) was determined in feet, by averaging widths at 10 to 30 locations along a study reach as measured with a cloth tape or Ranging 100 Optical Tape Measure[1] (an optical rangefinder).

5. Depth (D) was determined in feet, by solving the continuity equation:

$$D = \frac{Q}{W\ V} \tag{4}$$

6. Manning's roughness coefficient (N) was determined using the guidelines outlined by Benson and Dalrymple (1966) and Arcement and Schneider (1984), and from notes and photographs taken while examining the individual study reaches.

7. Methylene blue active substances (MBAS), in milligrams per liter, was measured in a water sample collected at the tracer-injection site during each study. The sample was analyzed at the Survey's Central Laboratory in Doraville, Georgia, according to the method outlined by Goerlitz and Brown (1972).

8. Color, in platinum-cobalt units, was measured in a water sample collected at the tracer-injection site during each study.

9. Specific conductance (SC), in microsiemens per centimeter, was measured at the tracer-injection site for each study using a conductance meter.

10. Suspended solids (SS) was estimated by the equation:

$$SS = ROE - (0.54\ SC + 2.7) \tag{5}$$

where

ROE = the total solids concentration, in milligrams per liter, after evaporation of a water sample collected at the tracer-injection site for each study; and

(0.54SC+2.7) = an equation to estimate the dissolved solids concentration, in milligrams per liter, by the method outlined by Delaney and Gay (1980).

Measurement of Time of Travel and Mean Streamflow Velocity

The slug-injection, dye-tracer method (Hubbard and others, 1982) was used to determine the time of travel and mean streamflow velocity of water through a study reach. Steady streamflow conditions were required. A measured volume of rhodamine-WT, 20-percent fluorescent-dye solution was slug injected at a point sufficiently far upstream of the study reach to ensure complete transverse mixing of the dye cloud before it entered the study reach. Water samples were collected at the upstream and downstream ends of the study reach to determine changes in dye concentration with time. Water-sample collection continued at each site until field analysis indicated that the dye concentration had dropped to 2 percent of the maximum concentration observed. All water samples were retained and reanalyzed at a constant temperature in the Survey's Massachusetts Office. Graphs showing

[1]Use of brand names in this report is for identification purposes only and does not constitute endorsement by the U.S. Geological Survey.

changes in dye concentration over time since injection defines the dye-response curve for each sample site. An example of dye-response curves and their characteristics for the May 3, 1984, study conducted on the Sevenmile River at Spencer is illustrated in figure 2.

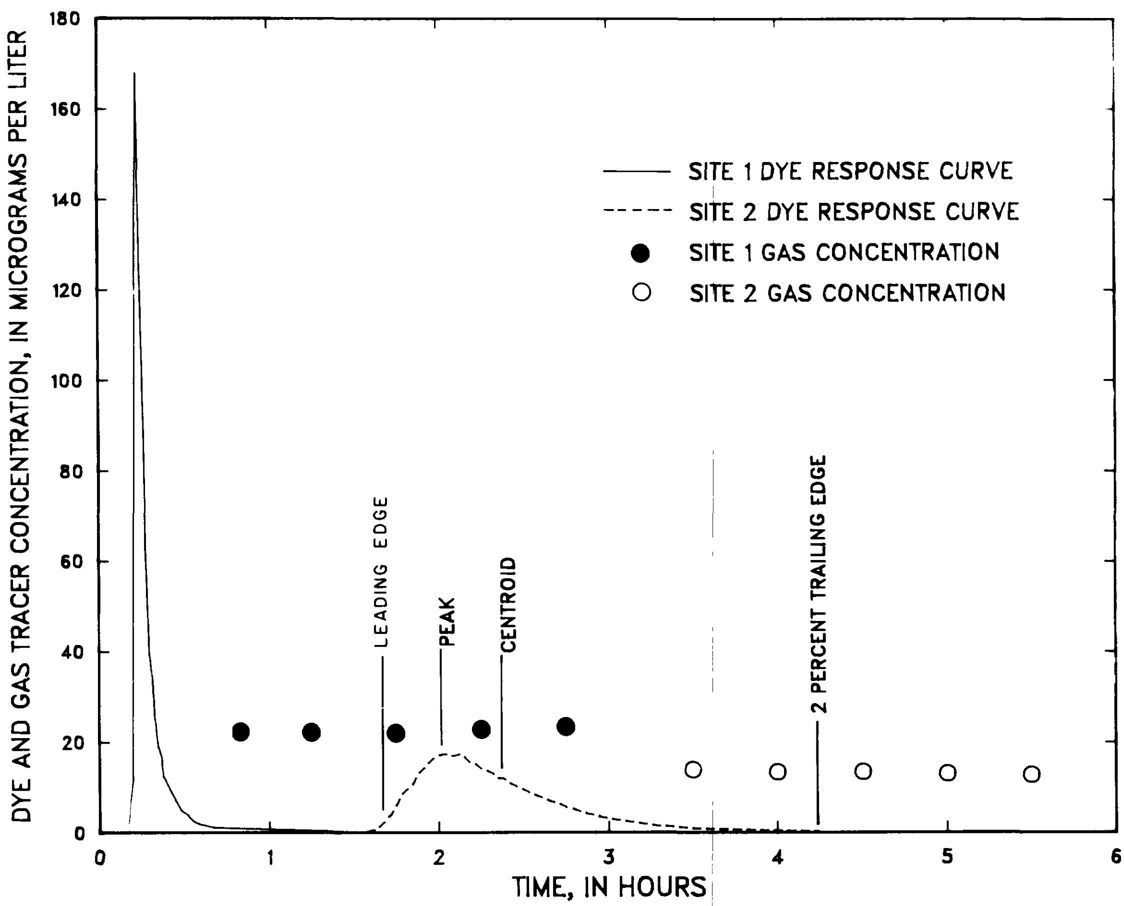

Figure 2.—Dye-response curves for the May 3, 1984, study of Sevenmile River at Spencer, Massachusetts.

At a sample site, the time of travel was determined for four dye-response curve characteristics as outlined by Parker and Hunt (1983). The characteristics are:

Leading edge - The first detectable dye concentration observed at a sample site;

Peak - The maximum dye concentration observed at a sample site;

Centroid - The center of mass of the dye response curve observed at a sample site; and

Trailing edge - The point on the falling limb of the dye response curve that is equal to 2 percent of the peak concentration observed at a sample site.

Using the centroid traveltimes from the dye-response curves, the mean streamflow velocity for a reach is determined using equation 3. The mean streamflow velocity was used to calculate other channel characteristics as previously described.

Measurement of Reaeration Coefficients

The steady-state propane-gas-tracer method described by Yotsukura and others (1983; 1984) measures the desorption coefficient of propane gas through the surface film directly. For uniformity of results, all studies were conducted under little- or no-wind conditions to minimize the effects of wind shear on the water surface and on the desorption of gas through the surface. The steady-state method is a combined procedure involving the concurrent slug injection of a dye tracer and continuous injection of a gas tracer. The dye-tracer study involves slug injection and analysis of water samples for dye concentration. The gas-tracer study involves the long-term, continuous injection of a commercial grade, propane through a flat-plate, porous-tile gas diffuser with a 2-micron-diameter pore size. Gas diffusers were placed on the river bottom within the middle 50-percent of the total streamflow for that cross section. From two to five diffusers were used in a study, depending on cross-section dimensions and flow conditions encountered. The gas- and dye-tracer injections were made at the same upstream location to ensure complete transverse mixing of the tracers before they entered the study reach, so the dye-response curves could be used as a guide to sampling the gas as well as provide time-of-travel data. Commercial-grade propane was injected from a 100-lb tank through a single-stage regulator and a CO_2 (carbon dioxide) rotameter. A CO_2 rotameter was used to allow direct readings of the injection flow rate, inasmuch as the specific gravity of the two gases are nearly identical. The gas injection was maintained at the lowest flow rate that just allowed gas bubbles to break the water surface. In all studies, there were no interruptions in tracer-gas injection, and the injection period ranged from 8 to 48 hours.

The slug-injected dye-tracer was used to identify a volume of water as it passed through the study reach. In most studies, the gas-tracer injection was started 2 to 12 hours prior to the dye-tracer injection to allow the gas-concentration plateau to become fully developed before the passage of the dye cloud at a sampling site. The gas-injection rate was maintained until the water samples for propane analysis had been collected. Using the trailing edge of the dye cloud as an indicator, gas-tracer and water samples were collected at each reach sampling site for 2 hours and late enough to ensure that the measured gas concentrations were from a fully developed concentration plateau. Water samples for gas were collected using a sewage-type sampler. Samples were preserved with 1 mL (milliliter) of a 37-percent solution of formaldehyde. Because dye samples do not require any special preservation, automatic samplers were used for the nighttime collection of water samples for dye concentration during two long-duration studies. This greatly reduced each study's manpower requirements. It should be noted that steady-state gas injection was maintained for the duration of each study. The long-duration gas injection allowed gas samples to be collected at any time after the gas-concentration plateau was fully formed, as indicated by the passage of the dye cloud at a sampling site (Yotsukura and Kilpatrick, 1973; Kilpatrick and Cobb, 1984). With the exception of the two long-duration studies previously mentioned, the gas-tracer samples were collected from the same parcel of water as identified by the passage of the dye-tracer cloud. For the two long-duration studies, the gas samples for the most downstream sample site were collected the morning after the complete passage of the dye-tracer cloud.

The gas desorption coefficient was calculated as outlined by Yotsukura and others (1983) using the gas-plateau concentration and dye response curves determined for each tracer study. The initial approximation of the propane desorption coefficient (Kp) was determined using the following equation:

$$Kp = \left(\frac{24}{\overline{T}_d - \overline{T}_u}\right) \left(\ln \frac{C_u\ Q_u}{C_d\ Q_d}\right) \tag{6}$$

where

C = the propane-gas plateau concentration, in micrograms per liter; and

24 = a constant to convert hours to days.

The initial approximation of Kp does not take into account the effects of longitudinal dispersion (Nobuhiro Yotsukura, U.S. Geological Survey, written commun., 1985). The actual propane-desorption coefficient is determined through trial-and-error balancing of both sides of the equation:

$$\frac{C_u\ Q_u}{C_d\ Q_d} = \frac{\int_0^{\infty} \left[\left(\frac{C_{c,i,u}}{A_u}\right) \exp\left(-Kp\ T_{i,u}\right) dT_{i,u}\right]}{\int_0^{\infty} \left[\left(\frac{C_{c,i,d}}{A_u}\right) \exp\left(-Kp\ T_{i,d}\right) dT_{i,d}\right]} \tag{7}$$

where

$C_{c,i}$ = the dye concentration at T_i, in micrograms per liter;

T_i = the i^{th} hour since injection; and

A = the area under the dye-response curve for the indicated sample site.

The trial-and-error process used the Kp approximated from equation 6 as a starting point for the final determination of the propane tracer-gas desorption coefficient.

The propane desorption coefficient was converted to the reaeration coefficient in two steps. First, the desorption coefficient was standardized by correcting it to a base temperature of 20 °C (Rathbun, 1979) by the equation:

$$Kp_{20} = Kp_t\ 1.024^{(20\ °C\ -t)} \tag{8}$$

where

Kp_{20} = the standard-temperature desorption coefficient; and

t = the field water temperature, in degrees Celsius.

Second, the reaeration coefficient (K_2) is calculated from the standardized desorption coefficient (Rathbun, 1979) by the equation:

$$K_2 = 1.39\ Kp_{20} \tag{9}$$

The surface-film reaeration coefficient (KL) is defined by equation 3 for use in the regression analysis.

The transfer of error from measurement to calculation is controlled by the nondimensional number $Kp(\overline{T}_d-\overline{T}_u)$ (Yotsukura and others, 1983). This measurement error (E) for a reaeration study can be estimated by:

$$E = \frac{Re}{Kp\left(\overline{T}_d-\overline{T}_u\right)} \tag{10}$$

where

Re = the relative combined error of gas-concentration measurement and discharge measurements (estimated to be 10 percent for all studies).

When $Kp(\overline{T}_d-\overline{T}_u)$ is less than 1, the error for the calculated reaeration coefficient is greater than the relative errors in measuring gas concentrations and discharge.

STREAM-REACH DATA USED TO DEVELOP ESTIMATING EQUATIONS

The nine study-reach characteristics, time-of-travel data, and reaeration coefficients determined for the 30 time-of-travel and reaeration studies are summarized in table 1. The maximum and minimum values for each channel characteristic have been underscored to help in comparing studied and unstudied river reaches.

Estimates of water-quality values for MBAS or total solids were made for three tracer studies because of sample loss. The MBAS concentration for the October 6, 1983, study of the Assabet River at Maynard was assumed to be 0.01 μg/L (microgram per liter), according to the MBAS levels determined for a study made 16 days earlier on a downstream reach. Estimates of the total solid concentrations for the April 12, 1984, study of the Aberjona River at Montvale, and the April 27, 1984, study of the Assabet River near West Concord were made using total-solid to specific-conductance ratios from other studies on the same rivers. Suspended-solids concentrations were calculated using these estimates.

The estimated measurement error of the reaeration coefficient for each study was also determined (table 1). Each study's reaeration coefficient should be viewed with it's own relative error in comparison with the error of the other studies conducted.

ESTIMATING EQUATION FOR REAERATION COEFFICIENTS

Development

The relation between reaeration coefficient (a dependent variable) and the channel characteristics (independent variables) was developed by a step-forward, multiple-regression technique. In this technique, a sequence of multiple linear-regression equations are computed by adding one independent variable at each step. Only those independent variables that are statistically significant at greater than the 95-percent confidence level, and that make the greatest reduction in the standard error of estimate, are shown in the final estimating equation. Because of the sample size generated for this study, the regression analysis was limited to a maximum of three independent variables in the final equations.

Table 1.—Reach characteristics and results for 30 tracer studies in Massachusetts, 1983—84

(Maximum and minimum values are underscored)

Study date	Mean width (ft)	Mean depth (ft)	Mean discharge (ft^3/s)	Water surface slope	Mannings' roughness coefficient	Color (Platinum-cobalt units)	Methylene blue active substances (mg/L)	Specific conductance (μS/cm)	Total solids (mg/L)	Suspended solids (mg/L)
Aberjona River at Montvale										
04/12/84	21	1.8	32.	0.00180	0.048	35	0.11	494	186	0
07/17/84	19	1.4	10.	.00180	.081	13	.07	449	169	0
Assabet River near West Concord										
09/20/83	60	1.5	21.	0.00017	0.056	9	0.01	276	153	1
04/27/84	87	3.7	446	.00036	.036	35	.09	145	86	5
Assabet River at Maynard										
09/01/83	58	1.9	89.	0.00407	0.067	12	0.10	302	176	10
10/06/83	32	2.7	29.	.00443	.077	10	.01	320	189	14
07/26/84	85	1.3	50.	.00435	.072	22	.01	203	129	17
Millers River near Athol										
06/27/84	121	1.3	144.	0.00691	0.083	40	0.01	128	104	32
08/29/84	116	1.0	48.	.00694	.086	23	.08	289	161	2
Mattapoisett River near Rochester										
05/11/84	19	3.7	30.	0.00044	0.054	150	0.18	53	64	33
08/22/84	11	2.2	8.2	.00039	.044	48	.01	72	141	99
North River at Griswoldville										
10/18/83	54	1.1	22.	0.00436	0.035	23	0.26	210	159	43
06/19/84	67	1.2	87.	.00435	.036	5	.01	106	73	13
East Branch North River at Colrain										
05/17/84	66	1.8	153.	0.00700	0.070	15	0.11	52	41	10
11/28/84	53	.7	18.	.00822	.072	1	.01	66	40	2
West Branch North River near Griswoldville										
10/20/83	30	0.4	5.9	0.00696	0.062	2	0.25	82	25	0
06/21/84	47	.4	22.	.00638	.062	5	.01	62	91	55
West Branch North River at Adamsville										
06/13/84	33	0.8	25.	0.01500	0.059	5	0.05	45	89	62
10/17/84	28	.6	5.0	.01500	.057	5	.01	64	45	8
Sevenmile River at Spencer										
05/03/84	44	1.7	81.	0.00183	0.031	25	0.10	62	36	0
07/25/84	40	1.1	25.	.00186	.079	27	.02	85	64	15
Sudbury River at Concord										
05/22/84	148	6.3	403.	0.00047	0.036	40	0.18	207	212	98
07/31/84	141	3.5	151.	.00055	.031	17	.03	210	129	13
Middle Branch Westfield River near North Chester										
08/07/84	24	0.9	6.0	0.00877	0.059	3	0.01	52	36	5
Middle Branch Westfield River at North Chester										
11/28/84	41	0.5	8.7	0.00810	0.051	5	0.01	42	38	13
Middle Branch Westfield River near Middlefield										
06/07/84	37	1.3	58.	0.00982	0.063	15	0.01	31	34	15
10/11/84	24	1.1	3.4	.01010	.063	1	.01	52	187	156
West Branch Westfield River near Huntington										
06/05/84	82	1.8	323.	0.00450	0.046	10	0.01	52	46	15
08/10/84	76	1.0	13.	.00450	.046	2	.01	111	74	11
West Branch Westfield River at Chester										
06/07/84	51	1.7	136.	0.00805	0.035	15	0.02	56	64	31

Table 1.—Reach characteristics and results for 30 tracer studies in Massachusetts, 1983—84—Continued

Study date	Time of travel (hours)								Mean streamflow velocity (ft/s)	Reaeration coefficient (t/d)	Estimated error (percent)
	Site 1				Site 2						
	Leading edge	Peak	Centroid	Trailing edge	Leading edge	Peak	Centroid	Trailing edge			
				Aberjona River at Montvale							
04/12/84	0.20	0.30	0.30	0.60	1.6	2.0	2.1	3.2	0.83	3.7	38.5
07/17/84	.40	.50	.60	1.20	3.5	4.5	4.6	7.2	.37	10.1	9.3
				Assabet River near West Concord							
09/20/83	0.70	0.80	1.00	2.00	6.0	8.8	9.7	18.6	0.24	4.2	10.0
04/27/84	.10	.20	.20	.40	1.1	1.3	1.7	3.7	1.37	11.1	16.9
				Assabet River at Maynard							
09/01/83	0.40	0.60	0.60	1.10	1.4	1.8	2.0	3.2	0.81	15.3	17.2
10/06/83	.60	.80	.90	1.80	2.1	3.1	3.6	7.7	.33	14.1	9.7
07/26/84	.70	1.10	1.10	2.00	2.2	3.0	3.0	4.3	.47	14.3	13.9
				Millers River near Athol							
06/27/84	1.20	1.30	1.40	2.20	2.9	3.4	3.7	5.8	0.92	20.7	7.3
08/29/84	3.20	3.90	4.30	6.80	7.2	8.9	9.5	14.0	.40	17.2	4.0
				Mattapoisett River near Rochester							
05/11/84	0.80	1.20	1.40	2.80	3.5	4.9	5.9	13.5	0.43	5.1	13.7
08/22/84	1.50	2.30	2.60	4.60	6.0	7.8	8.4	14.5	.33	4.2	13.5
				North River at Griswoldville							
10/18/83	1.00	1.20	1.40	2.60	3.2	4.2	4.8	8.8	0.38	10.0	8.7
06/19/84	.40	.50	.60	1.20	1.3	1.5	1.8	3.2	1.08	20.4	13.7
				East Branch North River at Colrain							
05/17/84	0.30	0.40	0.40	0.60	0.9	1.2	1.3	2.0	1.65	22.8	11.8
11/28/84	2.40	3.20	3.60	6.50	4.8	6.0	6.7	10.9	.48	15.7	4.6
				West Branch North River near Griswoldville							
10/20/83	0.60	0.80	0.80	1.40	3.8	4.7	4.9	8.2	0.44	25.3	2.7
06/21/84	.30	.40	.40	.80	1.8	2.2	2.2	3.2	1.10	42.9	4.1
				West Branch North River at Adamsville							
06/13/84	1.20	1.50	1.60	2.50	2.0	2.4	2.6	4.2	1.00	67.7	4.8
10/17/84	.70	1.00	1.10	2.20	3.0	4.0	4.5	10.0	.31	32.9	2.7
				Sevenmile River at Spencer							
05/03/84	0.20	0.23	0.30	0.60	1.6	2.0	2.3	4.1	1.06	7.9	17.2
07/25/84	.40	.70	.70	1.50	2.8	4.0	4.4	8.0	.57	12.3	7.8
				Sudbury River at Concord							
05/22/84	2.10	2.80	3.20	6.50	4.9	5.8	7.4	16.3	0.43	1.6	52.6
07/31/84	3.80	4.80	5.30	9.80	7.5	9.2	11.2	21.2	.31	.4	181.8
				Middle Branch Westfield River near North Chester							
08/07/84	0.70	1.00	1.30	3.20	4.5	5.7	6.4	10.8	0.27	36.9	2.1
				Middle Branch Westfield River at North Chester							
11/28/84	1.20	1.60	1.80	3.20	3.1	4.3	4.6	8.4	0.45	17.8	4.7
				Middle Branch Westfield River near Middlefield							
06/07/84	0.08	0.14	0.16	0.38	1.2	1.6	1.7	2.8	1.18	43.6	4.7
10/11/84	.70	2.10	2.30	4.30	10.2	13.8	18.5	50.2	.13	14.5	3.5
				West Branch Westfield River near Huntington							
06/05/84	0.17	0.20	0.20	0.25	0.9	0.9	1.0	1.6	2.15	40.0	8.9
08/10/84	1.80	2.20	2.60	4.80	7.2	10.2	12.6	27.0	.17	19.2	2.0
				West Branch Westfield River at Chester							
06/07/84	0.20	0.30	0.30	0.60	1.2	1.6	1.7	2.5	1.58	33.0	6.2

The general form of the equation involving dependent and independent variables is of the form:

$$\log X = \log a + b \log (A) + c \log (B) + \ldots \qquad (11)$$

or:

$$X = a\, A^{b}\, B^{c} \ldots$$

where

X = the dependent variable;
a = a regression constant;
b and c = regression coefficients; and
A and B = independent variables.

The regression analysis for reaeration coefficients used a weighted correlation matrix (table 2), which included each study's measured reaeration coefficient, mean depth, mean width, discharge, mean velocity, water-surface slope, Manning's roughness coefficient, specific conductance, and concentration of methylene blue active substances, color, and concentration of suspended solids. The weight assigned to each study within the reaeration correlation matrix was determined by:

$$\text{weight} = 30 \left[\frac{\dfrac{1}{(LM + LE)^2}}{\sum \dfrac{1}{(LM + LE)^2}} \right] \qquad (12)$$

where

LM = an estimate model error (determined to be 12 percent through a convergence process); in base 10 log units;
LE = the measurement error for each study, in base to log units; and
30 = the number of tracer studies included in the analysis.

Base 10 log units were used for both model and measurement error due to the base 10 log form of equation 11.

The reaeration coefficient may be estimated by the equation:

$$K_2 = 252.2\ D^{-0.176}\ V^{0.355}\ SL^{0.438} \qquad (13)$$

The estimating relation for reaeration coefficients (eq. 13) has a correlation coefficient of 0.85 and a standard error of estimate of 37.5 percent. None of the remaining independent variables have sufficient degrees of freedom to be included in the equation until a larger data base becomes available for the regression analysis. Regression analysis was attempted using groupings of independent variables (V/D, Q/W, and W/D), and reduced sample size but did not improve the standard error of estimate. The regression analysis was sensitive to sample size in that the order that the independent variables entered the relation differed from that when the full data set was used. The full data set was used to maintain the maximum range in stream-reach characteristic values for the analysis. A plot showing the relation of measured to predicted reaeration coefficients is shown in figure 3. Equation 13 was actually developed using KL as the dependent variable and then converted to K_2 using equation 2 in order to be compatible with other commonly used reaeration equations.

Table 2.—Weighted correlation coefficient matrix used in the reaeration-coefficient regression analysis

Variables	Independent variables									Dependent variable
	Log of mean stream-flow velocity	Log of mean width	Log of mean depth	Log of water-surface slope	Log of Manning's roughness coeffi-cient	Log of color	Log of methylene blue active substances	Log of spe-cific conduct-tance	Log of sus-pended solids	Log of reaera-tion coeffi-cient
Independent										
Log of:										
Mean streamflow velocity	1.000	0.329	0.155	0.094	-0.169	0.405	0.138	-0.264	0.050	0.576
Mean width	.329	1.000	.077	.041	.014	.164	-.020	.273	.212	.149
Mean depth	.155	.077	1.000	-.587	-.183	.619	.105	.328	.146	.340
Water-surface slope	.094	.041	-.587	1.000	.198	-.494	-.156	-.423	.135	.351
Manning's Roughness Coefficient	-.169	.014	-.183	.198	1.000	-.044	-.083	.237	-.077	-.061
Color	.405	.164	.619	-.494	-.044	1.000	.304	.286	.097	.140
Methylene Blue Active Substances	.138	-.020	.105	-.156	-.083	.304	1.000	.282	-.508	-.118
Specific conductance	-.264	.273	.328	-.423	.237	.286	.282	1.000	-.290	-.294
Suspended solids	.050	.212	.146	.135	-.077	.097	-.508	-.290	1.000	.310
Dependent										
Log of: Reaeration coefficient	0.576	0.149	0.340	0.351	-.061	0.140	-.118	-.294	0.310	1.000

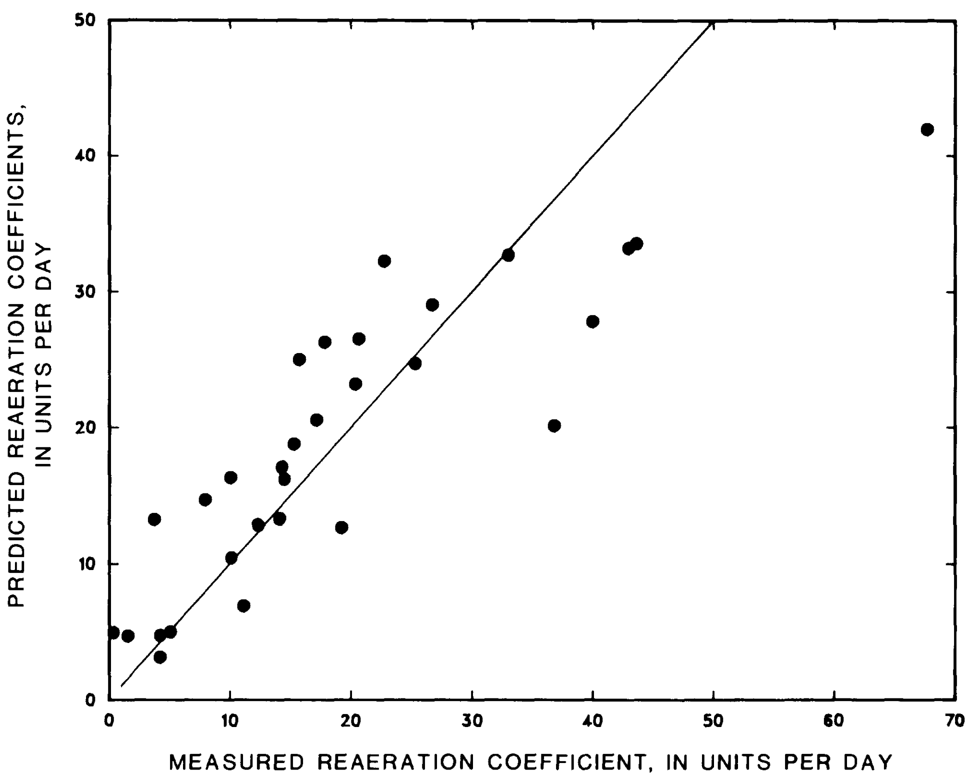

Figure 3.—Comparison of measured to predicted reaeration coefficients for 30 tracer studies in Massachusetts, 1983–84.

Limitations

Equation 13 may be used to estimate reaeration coefficients on streams in Massachusetts whose channel characteristics fall within the range of values given in table 1. Reaeration coefficients estimated by using equation 13 are for a river having steady-flow conditions and under little or no wind conditions.

A review of the residuals from the reaeration-coefficient regression analysis as a function of slope (fig. 4) illustrates the relatively wide range of the standard error of estimate. Of the 30 tracer studies, 13 had residuals within 0.1 log unit of zero and 19 studies were within 0.2 log units. Seven of 10 studies having slopes less than or equal to 0.002 feet per foot have negative residuals up to -1.15 log units from the zero residual. Of the residuals for the 20 studies with slopes greater than 0.002 feet per foot, 11 are negatively distributed from the zero residual; however, all but 4 are within 0.2 log units of zero. The maximum deviation for the 20 highest-sloped studies is -0.26 log units from the zero residual. This difference in the residual distribution about the slope of 0.002 feet per foot suggests that the reaeration regression equation (eq. 13) is more reliable for rivers with slopes greater than 0.002 feet per foot then for those with slopes less than 0.002 feet per foot. This shift also suggests that additional factors other than those identified in this equation are affecting or controlling the reaeration coefficient on stream reaches having water-surface slopes less than 0.002 feet per foot. Residual errors also were plotted as a function of mean depth and mean streamflow velocity showed no significant trend. Residual errors were plotted on a statewide map to examine for possible areal bias for the reaeration equation. This areal plot showed no significant regional trend. Improved regression estimates can be obtained when a larger number of data from rivers with slopes both above and below 0.002 feet per foot are collected from future studies.

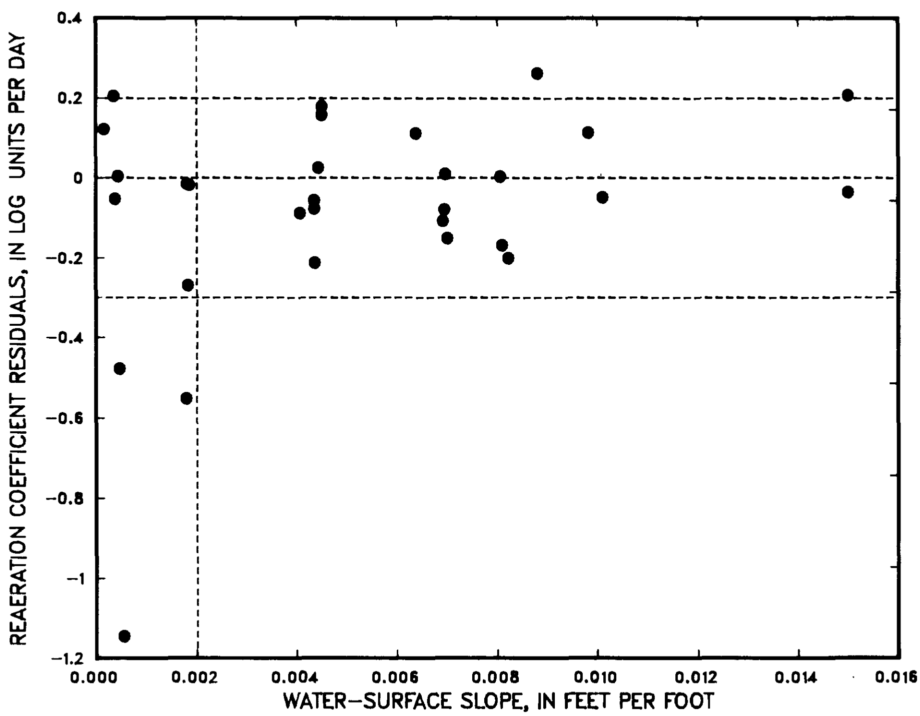

Figure 4.—Comparison of residuals from the reaeration regression analysis and water-surface slope.

COMPARISON OF ESTIMATING EQUATIONS WITH OTHER EQUATIONS

Predictive Error Analysis

Comparison of measured and predicted values indicate the degree of uncertainty that is inherent in individual, predictive equations. The regression equation developed for this report was compared with other widely used estimating equations, and this comparison was used to rank equations by their predictive error for individual or groups of tracer studies.

Reaeration Coefficient

To predict reaeration coefficients, most equations relate the coefficient to the physical characteristics of a stream. All 19 of the widely used predictive equations included in the error analysis require determination of mean streamflow velocity and mean depth, with the exception of the Tsivoglou and Neal (1976) equation. In all cases, the predicted reaeration coefficient is expressed in base e units of per day corrected to 20 °C using equation 8. The 19 reaeration coefficient predictive equations are as follows:

Dobbins (1965):

$$K_2 = 116.6 \frac{1 + F^2}{(0.9 + F)^{0.5}} \frac{(V\ SL)^{0.375}}{D} \coth\left[\frac{4.10\ (V\ SL)^{0.125}}{(0.9 + F)^{0.5}}\right] \quad (14)$$

where

coth = the hyperbolic cotangent angle, in radians, and
F = the Froude number which is defined as the dimensionless ratio:

$$\frac{V}{\sqrt{(g\ D)}} \quad (15)$$

O'Connor and Dobbins (1958):

$$K_2 = 12.81 \frac{V^{0.5}}{D^{1.5}} \quad (16)$$

Krenkel and Orlob (1963):

$$K_2 = 234.5 \frac{(V\ SL)^{0.404}}{D^{0.66}} \quad (17)$$

Cadwallader and McDonnell (1969):

$$K_2 = 336.8 \frac{\sqrt{V\ SL}}{D} \quad (18)$$

Parkhurst and Pomeroy (1972):

$$K_2 = 48.39 \frac{(1 + 0.17\ F^2)\ (V\ SL)^{0.375}}{D} \quad (19)$$

Bennett and Rathbun (1972):

$$K_2 = 106.16 \frac{V^{0.413}\ SL^{0.273}}{D^{1.408}} \quad (20)$$

Churchill and others (1962):

$$K_2 = 0.03453 \frac{V^{2.695}}{D^{3.085} SL^{0.823}} \qquad (21)$$

Lau (1972):

$$K_2 = 2{,}515 \left(\frac{u^*}{V}\right)^{3.0} \frac{V}{D} \qquad (22)$$

where

u^* = the average sheer velocity in feet per second;

$$u^* = \sqrt{(g\ D\ SL)};\ \text{and} \qquad (23)$$

g = the acceleration by gravity, in feet per second squared.

Thackston and Krenkel (1969):

$$K_2 = 24.94 \frac{(1 + \sqrt{F})\ u^*}{D} \qquad (24)$$

Langbein and Durum (1967):

$$K_2 = 7.61 \frac{V}{D^{1.33}} \qquad (25)$$

Owens and others (1964):

$$K_2 = 23.23 \frac{V^{0.73}}{D^{1.75}} \qquad (26)$$

Owens and others (1964):

$$K_2 = 21.74 \frac{V^{0.67}}{D^{1.85}} \qquad (27)$$

Churchill and others (1962):

$$K_2 = 11.57 \frac{V^{0.969}}{D^{1.673}} \qquad (28)$$

Isaac and Gaudy (1968):

$$K_2 = 8.62 \frac{V}{D^{1.5}} \qquad (29)$$

Negulescu and Rojanski (1969):

$$K_2 = 10.92 \left(\frac{V}{D}\right)^{0.85} \qquad (30)$$

Padden and Gloyna (1971):

$$K_2 = 6.87 \frac{V^{0.703}}{D^{1.054}} \qquad (31)$$

Bansal (1973):

$$K_2 = 4.67 \frac{V^{0.6}}{D^{1.4}} \qquad (32)$$

Bennett and Rathbun (1972):

$$K_2 = 20.19 \frac{V^{0.607}}{D^{1.689}} \qquad (33)$$

Tsivoglou and Neal (1976):

$$K_2 = 1.296 \frac{dh}{d\overline{T}} \qquad (34)$$

where

dh = the change in elevation between the start and end of the study reach, in feet; and
$d\overline{T}$ = the change in centroid time of travel from the start to the end of the study reach, in hours.

The unweighted predictive error analysis indicates that the water-surface slope of a river must be considered when choosing the equation that will estimate the most reliable reaeration coefficient. The error analyses are summarized in table 3 for the reaeration estimating equations 13 through 34 (excluding eqs. 15 and 23) by predictive error for the individual studies and average absolute error for three groups of studies. The study groups are: all 30 studies; 20 studies with slopes greater than 0.002 feet per foot; and 10 studies with slopes less than 0.002 feet per foot. The top three ranking equations for all 30 studies together are: equation 34 (Tsivoglou and Neal, 1976) with 49-percent average absolute error; equation 18 (Cadwallader and McDonnell, 1969) with 50-percent average absolute error; and equation 14 (Dobbins, 1965) with 51-percent average absolute error. The average absolute errors improve when considering the group of 20 studies with slopes greater than 0.002 feet per foot. The top three ranking equations for this group are: equation 13 of this study with 27-percent average absolute error; equation 17 (Krenkel and Orlob, 1963) with 36-percent average absolute error; and equation 34 (Tsivoglou and Neal, 1976) with 38-percent average absolute error equation. The top three ranking equations for the 10 studies with slopes less than 0.002 feet per foot are: equation 27 (Owens and others, 1964) with 53-percent average absolute error; equation 33 (Bennett and Rathbun, 1972) with 57-percent average absolute error; and equation 26 (Owens and others, 1964) with 58-percent average absolute error. The error analyses show an improvement in prediction when studies are divided into groups greater than and less than 0.002 feet per foot slope. Best results are obtained when equation 13, 17, or 34 is used for river reaches with slopes greater than 0.002 feet per foot and when equation 27, 33, or 26 is used for reaches with slopes less than 0.002 feet per foot.

Application of Estimating Equations to Stream Reaches in Massachusetts

The following examples describe the step-by-step application of the appropriate reaeration coefficient equations to an unstudied reach in Massachusetts. The reaches chosen for examples are two studies for this project so that predicted reaeration coefficients may be compared with observed values. For the purpose of these examples, both rivers are considered to be unstudied reaches.

Table 3.—Unweighted predictive error for 20 reaeration estimating equations

Study date	Equation 13: Reaeration coefficient (base-e 1/d)	Equation 13: Estimated reaeration coefficient (base-e 1/d)	Equation 13: Error (percent)	Equation 14: Estimated reaeration coefficeint (base-e 1/d)	Equation 14: Predictive error (percent)	Equation 16: Estimated reaeration coefficient (base-e 1/d)	Equation 16: Predictive error (percent)	Euqation 17: Estimated reaeration coefficient (base-e 1/d)	Euqation 17: Predictive error (percent)	Euqation 18: Estimated reaetion coefficient (base-e 1/d)	Euqation 18: Predictive error (percent)	Water slope (ft/ft)
Aberjona River at Montvale												
04/12/84	3.7	13.33	260	5.90	59.5	4.68	26.6	11.04	198.5	7.09	91.6	0.00180
07/17/84	10.1	10.46	4	5.62	-44.3	4.53	-55.2	9.35	-7.4	6.06	-40.0	.00180
Assabet River near West Concord												
09/20/83	4.2	3.17	-25	2.13	-49.3	3.46	-17.7	2.92	-30.4	1.45	-65.6	0.00017
04/27/84	11.1	6.94	-37	1.99	-82.1	2.08	-81.3	4.40	-60.4	2.00	-81.9	.00036
Assabet River at Maynard												
09/01/83	15.3	18.78	23	7.58	-50.4	4.44	-71.0	14.96	-2.2	10.23	-33.1	0.00407
10/06/83	14.1	13.35	-5	3.93	-72.1	1.67	-88.2	8.52	-39.6	4.79	-66.0	.00443
07/26/84	14.3	17.11	20	9.52	-33.4	6.21	-56.6	16.09	12.5	12.09	-15.5	.00435
Millers River near Athol												
06/27/84	20.7	26.56	28	14.35	-30.7	8.50	-58.9	25.33	22.4	21.01	1.5	0.00691
08/29/84	17.1	20.61	20	13.03	-24.2	7.79	-54.7	20.91	21.6	17.31	.7	.00694
Mattapoisett River near Rochester												
05/11/84	5.1	5.03	-1	1.43	-72.0	1.20	-76.5	3.01	-41.0	1.26	-75.2	0.00044
08/22/84	4.0	5.02	26	2.19	-45.3	2.27	-43.2	3.80	-5.1	1.85	-53.7	.00039
North River at Griswoldville												
10/18/83	10.0	16.36	64	10.41	4.1	7.14	-28.6	16.47	64.7	12.83	28.3	0.00436
06/19/84	20.4	23.21	14	13.86	-32.1	10.14	-50.3	23.36	14.5	19.26	-5.6	.00435
East Branch North River at Colrain												
05/17/84	22.8	32.29	42	16.70	-26.3	9.92	-56.5	30.40	33.3	25.82	13.3	0.00700
11/28/84	15.7	25.04	60	20.88	33.0	14.20	-9.6	30.12	91.8	28.92	84.2	.00822
West Branch North River near Griswoldville												
10/20/83	25.3	24.78	-2	35.25	39.31	29.21	15.5	38.10	50.6	42.50	68.0	0.00696
06/21/84	42.9	33.25	-22	47.93	11.7	49.45	15.3	55.04	28.3	67.30	56.9	.00638
West Branch North River at Adamsville												
06/13/84	67.7	42.01	-38	33.49	-50.5	19.31	-71.5	50.60	-25.3	54.19	-19.9	0.01500
10/17/84	32.9	29.06	-12	28.10	-14.6	16.47	-50.0	37.78	14.8	40.03	21.7	.01500
Sevenmile River at Spencer												
05/03/84	7.9	14.73	86	6.85	-13.3	5.68	-28.1	12.66	60.2	8.46	7.0	0.00183
07/25/84	12.3	12.90	5	8.62	-29.9	8.26	-32.9	13.36	8.7	9.86	-19.8	.00186
Sudbury River at Concord												
05/22/84	1.6	4.73	195	0.85	-46.8	0.53	-66.6	2.17	35.8	0.76	-52.3	0.00047
07/31/84	.4	4.97	1,141	1.44	259.9	1.09	171.7	2.97	641.4	1.25	213.0	.00055
Middle Branch Westfield River near North Chester												
08/07/84	36.9	20.13	-45	13.52	-63.4	7.41	-79.9	20.85	-43.5	17.59	-52.3	0.00877
Middle Branch Westfield River at North Chester												
11/28/84	17.8	26.74	50	32.13	80.5	26.67	49.8	39.64	122.7	44.06	147.5	0.00810
Middle Branch Westfield River near Middlefield												
06/07/84	43.6	33.60	-23	17.45	-60.0	9.17	-79.0	31.65	-27.4	27.44	-37.1	0.00982
10/11/84	14.5	16.24	12	9.71	-33.0	4.31	-70.3	15.20	4.8	11.67	-19.5	.01010
West Branch Westfield River near Huntington												
06/05/84	40.0	27.87	-30	12.15	-69.6	7.52	-81.2	23.62	-40.9	18.00	-55.0	0.00450
08/10/84	19.2	12.72	-34	8.43	-56.1	5.28	-72.5	12.62	-34.3	9.36	-51.3	.00450
West Branch Westfield River at Chester												
06/07/84	33.0	32.74	-1	14.34	-56.5	7.39	-77.6	28.04	-15.0	22.59	-31.6	0.00805
Average absolute error for 30 studies			77		51		58		60		50	
Average absolute error for 20 studies with water surface slopes greater than 0.002			27		42		57		36		40	
Average absolute error for 10 studies with water surface slopes less than 0.002			177		70		60		109		70	

Table 3.—Unweighted predictive error for 20 reaeration estimating equations—Continued

Study date	Equation 19: Reaeration coefficient (base-e 1/d)	Equation 19: Estimated reaeration coefficient (base-e 1/d)	Equation 19: Error (percent)	Equation 20: Estimated reaeration coefficeint (base-e 1/d)	Equation 20: Predictive error (percent)	Equation 21: Estimated reaeration coefficient (base-e 1/d)	Equation 21: Predictive error (percent)	Equation 22: Estimated reaeration coefficient (base-e 1/d)	Equation 22: Predictive error (percent)	Equation 24: Estimated reaertion coefficient (base-e 1/d)	Equation 24: Predictive error (percent)	Water slope (ft/ft)
Aberjona River at Montvale												
04/12/84	3.7	2.30	-37.8	7.43	100.8	0.58	-84.2	68.25	1,744.5	5.87	58.7	0.00180
07/17/84	10.1	2.17	-78.5	7.53	-25.4	.14	-98.6	303.01	2,900.1	6.18	-38.8	.00180
Assabet River near West Concord												
09/20/83	4.2	0.74	-82.5	3.15	-25.1	0.28	-93.4	21.19	404.4	1.79	-57.4	0.00017
04/27/84	11.1	.75	-93.3	2.17	-80.4	.95	-91.5	3.24	-70.8	1.88	-83.0	.00036
Assabet River at Maynard												
09/01/83	15.3	3.01	-80.3	8.84	-42.2	0.25	-98.3	250.49	1,537.2	8.69	-43.2	0.00407
10/06/83	14.1	1.56	-89.0	3.79	-73.1	.01	-99.9	2,011.69	41,673.3	6.82	-51.6	.00443
07/26/84	14.3	3.77	-73.6	12.72	-11.0	.19	-98.6	669.45	4,581.5	10.57	-26.1	.00435
Millers River near Athol												
06/27/84	20.7	5.70	-72.5	18.66	-9.9	0.78	-96.2	351.20	1,596.6	14.35	-30.7	0.00691
08/29/84	17.2	5.19	-69.8	18.03	4.8	.16	-99.0	1,654.48	9,519.1	14.70	-14.5	.00694
Mattapoisett River near Rochester												
05/11/84	5.1	0.53	-89.6	1.46	-71.4	0.04	-99.3	44.46	771.8	1.86	-63.5	0.00044
08/22/84	4.0	.80	-79.9	2.70	-32.5	.09	-97.8	57.45	1,337.3	2.40	-40.0	.00039
North River at Griswoldville												
10/18/83	10.0	4.11	-58.9	14.69	46.9	0.18	-98.2	945.80	9,358.0	11.35	13.5	0.00436
06/19/84	20.4	5.44	-73.4	19.23	-5.7	2.14	-89.5	123.15	503.7	12.08	-40.8	.00435
East Branch North River at Colrain												
05/17/84	22.8	6.55	-71.3	20.96	-8.1	2.78	-87.8	117.70	416.2	14.97	-34.2	0.00700
11/28/84	15.7	8.32	-47.0	32.87	109.3	.65	-95.9	1,284.45	8,081.2	19.75	25.8	.00822
West Branch North River near Griswoldville												
10/20/83	25.3	12.88	-49.1	62.08	145.4	2.89	-88.6	28.26	11.7	9.04	-64.3	0.00696
06/21/84	42.9	18.24	-57.5	94.38	120.0	41.93	-2.3	124.85	191.0	27.02	-37.0	.00638
West Branch North River at Adamsville												
06/13/84	67.7	13.25	-80.4	49.58	-26.8	2.53	-96.3	740.89	994.4	28.81	-57.4	0.01500
10/17/84	32.9	11.28	-65.7	45.64	38.7	.25	-99.2	6,818.64	20,625.4	29.11	-11.5	.01500
Sevenmile River at Spencer												
05/03/84	7.9	2.66	-66.3	8.83	11.8	1.27	-83.9	42.71	440.6	6.29	-20.3	0.00183
07/25/84	12.3	3.34	-72.8	13.04	6.0	.96	-92.2	120.61	880.6	7.58	-38.3	.00186
Sudbury River at Concord												
05/22/84	1.6	0.32	-80.1	0.70	-56.5	0.01	-99.6	62.92	3,832.5	1.43	-10.1	0.00047
07/31/84	.4	.53	33.3	1.44	260.8	.01	-96.4	118.81	29,603.5	2.08	420.5	.00055
Middle Branch Westfield River near North Chester												
08/07/84	36.9	5.39	-85.4	18.79	-49.2	0.06	-99.8	5,016.03	13,493.6	16.78	-54.5	0.00877
Middle Branch Westfield River at North Chester												
11/28/84	17.8	12.74	-28.4	59.94	236.8	2.10	-88.2	1,132.97	6,265.0	24.89	39.8	0.00810
Middle Branch Westfield River near Middlefield												
06/07/84	43.6	6.92	-84.1	21.75	-50.1	1.02	-97.7	370.14	749.0	17.39	-60.1	0.00982
10/11/84	14.5	3.84	-73.5	12.21	-15.8	.01	-100.0	27,799.98	191,624.0	15.95	10.0	.01010
West Branch Westfield River near Huntington												
06/05/84	40.0	4.68	-88.3	14.11	-64.7	3.52	-91.2	40.70	1.8	10.69	-73.3	0.00450
08/10/84	19.2	3.29	-82.9	11.66	-39.3	.03	-99.9	4,568.11	23,692.2	11.11	-42.1	.00450
West Branch Westfield River at Chester												
06/07/84	33.0	5.64	-82.9	16.55	-49.9	1.26	-96.2	173.06	424.4	14.33	-56.6	0.00805
Average absolute error for 30 studies												
			71		61		92		11,661		54	
Average absolute error for 20 studies with water surface slopes greater than 0.002												
			71		57		91		15,392		39	
Average absolute error for 10 studies with water surface slopes less than 0.002												
			71		67		94		4,199		83	

Table 3.—Unweighted predictive error for 20 reaeration estimating equations—Continued

Study date	Reaeration coefficient (base-e 1/d)	Equation 25: Estimated reaeration coefficient (base-e 1/d)	Equation 25: Error (percent)	Equation 26: Estimated reaeration coefficeint (base-e 1/d)	Equation 26: Predictive error (percent)	Equation 27: Estimated reaeration coefficient (base-e 1/d)	Equation 27: Predictive error (percent)	Euqation 28: Estimated reaeration coefficient (base-e 1/d)	Euqation 28: Predictive error (percent)	Euqation 29: Estimated reaertion coefficient (base-e 1/d)	Euqation 29: Predictive error (percent)	Water slope (ft/ft)
					Aberjona River at Montvale							
04/12/84	3.7	2.82	-23.9	6.99	89.0	6.23	68.3	3.49	-5.6	2.88	-22.2	0.00180
07/17/84	10.1	1.75	-82.7	5.97	-40.9	5.72	-43.4	2.42	-76.1	1.86	-81.6	.00180
					Assabet River near West Concord							
09/20/83	4.2	1.08	-74.3	4.09	-2.6	4.01	-4.6	1.50	-64.4	1.14	-72.8	0.00017
04/27/84	11.1	1.81	-83.7	2.92	-73.7	2.35	-78.8	1.73	-84.4	1.64	-85.2	.00036
					Assabet River at Maynard							
09/01/83	15.3	2.64	-82.7	6.54	-57.3	5.81	-62.0	3.25	-78.7	2.69	-82.4	0.00407
10/06/83	14.1	.68	-95.2	1.83	-87.0	1.66	-88.2	.76	-94.6	.65	-95.4	.00443
07/26/84	14.3	2.63	-81.6	8.93	-37.5	8.55	-40.2	3.78	-73.6	2.86	-80.0	.00435
					Millers River near Athol							
06/27/84	20.7	5.06	-75.6	14.22	-31.3	13.05	-37.0	7.08	-65.8	5.49	-73.5	0.00691
08/29/84	17.2	2.96	-82.8	11.38	-33.8	11.21	-34.8	4.58	-73.4	3.33	-80.6	.00694
					Mattapoisett River near Rochester							
05/11/84	5.1	0.58	-88.6	1.29	-74.7	1.12	-78.1	0.58	-88.6	0.53	-90.0	0.00044
08/22/84	4.0	.89	-77.9	2.62	-34.4	2.43	-39.4	1.06	-73.4	.88	-78.1	.00039
					North River at Griswoldville							
10/18/83	10.0	2.65	-73.5	10.20	2.0	10.05	0.5	4.06	-59.4	2.97	-70.3	0.00436
06/19/84	20.4	6.47	-68.3	17.90	-12.3	16.37	-19.8	9.21	-54.8	7.10	-65.2	.00435
					East Branch North River at Colrain							
05/17/84	22.8	8.01	-64.9	18.55	-18.6	16.29	-28.6	10.68	-53.2	8.57	-62.4	0.00700
11/28/84	15.7	5.53	-64.8	23.51	49.7	23.73	51.2	9.58	-39.0	6.61	-57.9	.00822
					West Branch North River near Griswoldville							
10/20/83	25.3	10.05	-60.3	53.94	113.2	57.54	127.4	20.76	-17.9	13.08	-48.3	0.00696
06/21/84	42.9	26.63	-37.9	113.95	165.6	115.62	169.5	54.36	26.7	34.96	-18.5	.00638
					West Branch North River at Adamsville							
06/13/84	67.7	10.93	-83.9	37.47	-44.7	36.05	-46.8	18.26	-73.0	12.97	-80.8	0.01500
10/17/84	32.9	4.92	-85.1	26.17	-20.4	27.81	-15.4	9.41	-71.4	6.13	-81.4	.01500
					Sevenmile River at Spencer							
05/03/84	7.9	3.81	-51.7	9.07	14.8	8.00	1.3	4.78	-39.5	3.93	-50.3	0.00183
07/25/84	12.3	3.76	-69.4	12.81	4.1	12.27	-.2	5.62	-54.3	4.19	-66.0	.00186
					Sudbury River at Concord							
05/22/84	1.6	0.29	-82.2	0.50	-68.5	0.41	-74.2	0.24	-85.2	0.24	-85.2	0.00047
07/31/84	.4	.44	10.4	1.10	174.5	.97	143.4	.45	13.4	.40	1.1	.00055
					Middle Branch Westfield River near North Chester							
08/07/84	36.9	2.25	-93.9	10.11	-72.6	10.31	-72.0	3.66	-90.1	2.59	-93.0	0.00877
					Middle Branch Westfield River at North Chester							
11/28/84	17.8	9.35	-47.5	48.61	173.1	51.47	189.1	18.87	6.0	12.04	-32.4	0.00810
					Middle Branch Westfield River near Middlefield							
06/07/84	43.6	6.20	-85.8	16.11	-63.1	14.52	-66.7	8.52	-80.5	6.70	-84.6	0.00982
10/11/84	14.5	.93	-93.6	4.84	-66.6	5.09	-64.9	1.49	-89.7	1.05	-92.8	.01010
					West Branch Westfield River near Huntington							
06/05/84	40.0	7.26	-81.8	13.96	-65.1	11.74	-70.6	8.75	-78.1	7.42	-81.5	0.00450
08/10/84	19.2	1.31	-93.2	6.40	-66.7	6.64	-65.4	2.10	-89.1	1.49	-92.3	.00450
					West Branch Westfield River at Chester							
06/07/84	33.0	6.02	-81.8	13.07	-60.4	11.30	-65.8	7.55	-77.1	6.24	-81.1	0.00805
					Average absolute error for 30 studies							
			73		61		62		63		70	
					Average absolute error for 20 studies with water surface slopes greater than 0.002							
			77		62		66		65		73	
					Average absolute error for 10 studies with water surface slopes less than 0.002							
			64		58		53		58		63	

Table 3.—Unweighted predictive error for 20 reaeration estimating equations—Continued

Study date	Equation 30			Equation 31		Equation 32		Euqation 33		Euqation 34		Water slope (ft/ft)
	Reaer-ation coeffi-cient (base-e 1/d)	Esti-mated reaer-ation coeffi-cient (base-e 1/d)	Error (per-cent)	Esti-mated reaer-ation coeffi-ceint (base-e 1/d)	Pre-dic-tive error (per-cent)	Esti-mated reaer-ation coeffi-cient (base-e 1/d)	Pre-dic-tive error (per-cent)	Esti-mated reaer-ation coeffi-cient (base-e 1/d)	Pre-dic-tive error (per-cent)	Esti-mated reaer-tion coeffi-cient (base-e 1/d)	Pre-dic-tive error (per-cent)	
				Aberjona River at Montvale								
04/12/84	3.7	5.57	50.5	3.18	-14.1	1.78	-51.8	6.45	74.4	7.17	93.9	0.00180
07/17/84	10.1	3.46	-65.7	2.34	-76.8	1.55	-84.6	5.99	-40.7	3.13	-69.0	.00180
				Assabet River near West Concord								
09/20/83	4.2	2.32	-44.7	1.66	-60.5	1.14	-72.9	4.34	3.3	0.19	-95.5	0.00017
04/27/84	11.1	4.66	-58.0	2.14	-80.7	.89	-92.0	2.64	-76.2	2.30	-79.3	.00036
				Assabet River at Maynard								
09/01/83	15.3	5.31	-65.3	3.03	-80.2	1.69	-89.0	6.06	-63.4	15.51	1.4	0.00407
10/06/83	14.1	1.84	-86.9	1.11	-92.1	.60	-95.7	1.94	-86.3	6.89	-51.1	.00443
07/26/84	14.3	4.72	-67.0	3.17	-77.9	2.15	-85.0	8.64	-39.6	9.59	-32.9	.00435
				Millers River near Athol								
06/27/84	20.7	8.27	-60.0	5.01	-75.8	3.15	-84.8	12.67	-38.8	29.76	43.8	0.00691
08/29/84	17.2	4.93	-71.3	3.52	-79.5	2.60	-84.9	11.08	-35.6	13.10	-23.9	.00694
				Mattapoisett River near Rochester								
05/11/84	5.1	1.76	-65.5	0.96	-81.1	0.46	-91.0	1.35	-73.6	0.88	-82.8	0.00044
08/22/84	4.0	2.19	-45.4	1.38	-65.5	.80	-80.0	2.74	-31.5	.61	-84.7	.00039
				North River at Griswoldville								
10/18/83	10.0	4.54	-54.6	3.25	-67.6	2.38	-76.2	10.03	0.3	7.75	-22.5	0.00436
06/19/84	20.4	10.01	-50.9	6.00	-70.6	3.80	-81.4	15.57	-23.7	22.06	8.2	.00435
				East Branch North River at Colrain								
05/17/84	22.8	12.53	-45.0	6.84	-70.0	3.93	-82.8	15.48	-32.1	53.52	134.7	0.00700
11/28/84	15.7	7.62	-51.5	5.70	-63.7	4.66	-70.3	21.95	39.8	18.32	16.7	.00822
				West Branch North River near Griswoldville								
10/20/83	25.3	10.98	-56.6	9.21	-63.6	9.04	-64.3	49.28	94.8	14.39	-43.1	0.00696
06/21/84	42.9	24.83	-42.1	18.38	-57.2	16.69	-61.1	92.80	116.3	32.82	-23.5	.00638
				West Branch North River at Adamsville								
06/13/84	67.7	13.75	-79.7	9.16	-86.5	6.85	-89.9	32.04	-52.7	70.35	3.9	0.01500
10/17/84	32.9	6.44	-80.4	5.40	-83.6	5.04	-84.7	25.43	-22.7	21.37	-35.0	.01500
				Sevenmile River at Spencer								
05/03/84	7.9	7.10	-10.1	3.96	-49.9	2.20	-72.1	8.11	2.6	9.01	14.1	0.00183
07/25/84	12.3	6.18	-49.8	4.14	-66.1	2.87	-76.6	12.01	-2.4	4.91	-60.1	.00186
				Sudbury River at Concord								
05/22/84	1.6	1.12	-29.8	0.55	-65.7	0.22	-86.5	0.54	-66.0	0.95	-40.7	0.00047
07/31/84	.4	1.38	244.8	.80	100.1	.40	-.3	1.19	197.9	.78	94.8	.00055
				Middle Branch Westfield River near North Chester								
08/07/84	36.9	3.80	-89.7	2.95	-92.0	2.35	-93.6	10.29	-72.1	10.97	-70.3	0.00877
				Middle Branch Westfield River at North Chester								
11/28/84	17.8	10.52	-40.9	8.69	-51.2	8.32	-53.2	44.51	150.1	17.68	-0.7	0.00810
				Middle Branch Westfield River near Middlefield								
06/07/84	43.6	9.91	-77.3	5.75	-86.8	3.49	-92.0	13.95	-68.0	53.92	23.7	0.00982
10/11/84	14.5	1.86	-87.2	1.56	-89.2	1.28	-91.1	5.41	-62.7	6.17	-57.4	.01010
				West Branch Westfield River near Huntington								
06/05/84	40.0	12.46	-68.9	6.18	-84.5	3.15	-92.1	11.47	-71.3	44.93	12.3	0.00450
08/10/84	19.2	2.46	-87.2	2.00	-89.6	1.62	-91.6	6.89	-64.1	3.65	-81.0	.00450
				West Branch Westfield River at Chester								
06/07/84	33.0	10.35	-68.6	5.48	-83.4	2.97	-91.0	11.08	-66.4	59.08	79.0	0.00805
				Average absolute error for 30 studies								
			67		74		79		59		49	
				Average absolute error for 20 studies with water surface slopes greater than 0.002								
			67		77		83		60		38	
				Average absolute error for 10 studies with water surface slopes less than 0.002								
			66		66		71		57		71	

Problem 1, Sevenmile River at Spencer, Massachusetts

Estimate the reaeration coefficient for the Sevenmile River at Spencer, Massachusetts (fig. 5).

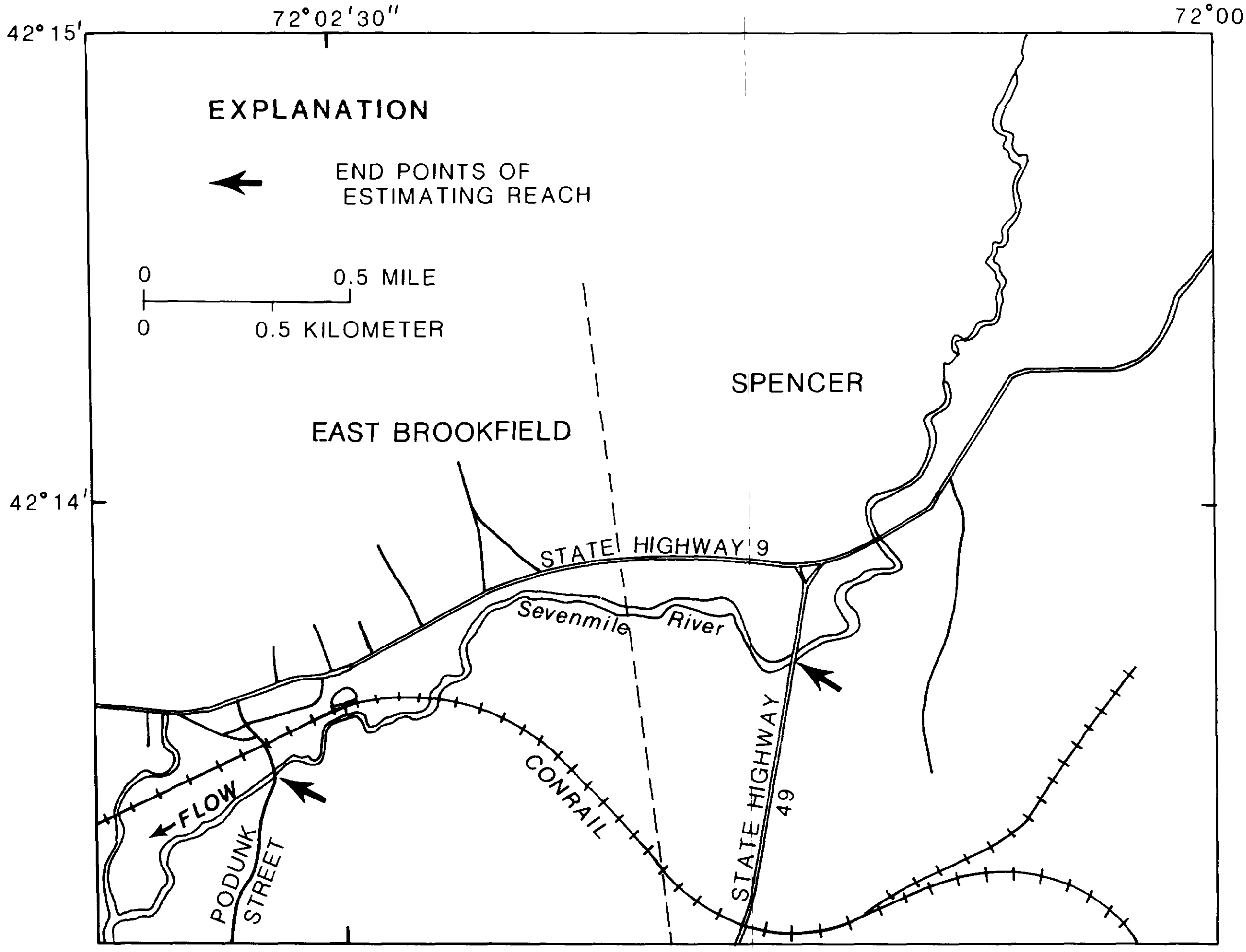

Figure 5.—Problem 1 area: stream reach on Sevenmile River at Spencer, Massachusetts.

1. From the East Brookfield topographic map (U.S. Geological Survey, 1969, photorevised 1979), choose the stream reach for which the estimations will be made. In this case, for convenience of identification in the field, use the reach of river between the bridge on State Highway 49 in Spencer and the bridge on Podunk Street in East Brookfield.

2. From the topographic map, determine the length of the reach. The problem reach length is 1.6 miles or 8,450 feet.

3. From the topographic map, calculate the channel slope by dividing the change in elevation at each end of the reach by it's length. The elevation at the Route 49 bridge is 620 feet above sea level and the elevation at the Podunk Street bridge is 610 feet. The reach slope is 0.0012 feet per foot.

4. Conduct a field reconaissance of the stream reach to measure the channel characteristics needed to solve the estimating equations. Measure the discharge at each end of the reach and measure the width of the channel in at least 10 places along the length of the reach and calculate an average channel width. For the purpose of this problem, assume that the discharge measured at the Route 49 bridge is 80 ft^3/s and at the Podunk Road bridge is 82 ft^3/s. Also assume that the average channel width is 44 feet.

5. Conduct a dye-tracer time-of-travel study to determine the mean streamflow velocity of the study reach. The procedures for conducting time-of-travel studies are explained in detail by Hubbard and others (1982). During the planning stages of a time-of-travel study, an estimation of the mean velocity is needed to determine both the volume of dye tracer to be injected, and the distance upstream of the study reach the dye injection has to be made to ensure complete lateral mixing of the dye before it reaches the study reach. This velocity may be estimated from the equation:

$$V = 3.646\ Q^{0.666}\ SL^{0.272}\ W^{-0.699} \qquad (35)$$

   Equation 35 was developed by a regression analysis of the 30 tracer studies conducted for this project. The equation has a correlation coefficient of 0.88 and a standard error of estimate of 35 percent. Water samples should be collected until the dye concentration recedes to a level less than 10 percent of the peak concentration. Calculate the centroid time of travel for the response curve and determine the mean streamflow velocity using equation 3. This procedure can be simplified by determining the dye peak time of travel and velocity only, but the reader should be aware that for an average of the 30 tracer studies conducted, the peak velocity was 30 percent slower than the mean streamflow velocity. For the purpose of this problem, assume that a time-of-travel study determined that the mean streamflow velocity is 1.1 ft/s.

6. Use equation 4 to calculate a reach mean depth, D:

$$D = \frac{81}{(1.1)(44)} = 1.7 \text{ feet}$$

7. Because the reach slope is under 0.002 feet per foot, use equation 27 (Owens and others, 1964) to estimate the reaeration coefficient K_2.

$$K_2 = 21.74\ (1.1)^{0.67}\ (1.7)^{-1.85}$$

$$= 8.7 \text{ per day}$$

Problem 2, West Branch Westfield River near Huntington, Massachusetts

Estimate the mean streamflow velocity and reaeration coefficient for the West Branch Westfield River below Chester, near Huntington, Massachusetts (fig. 6).

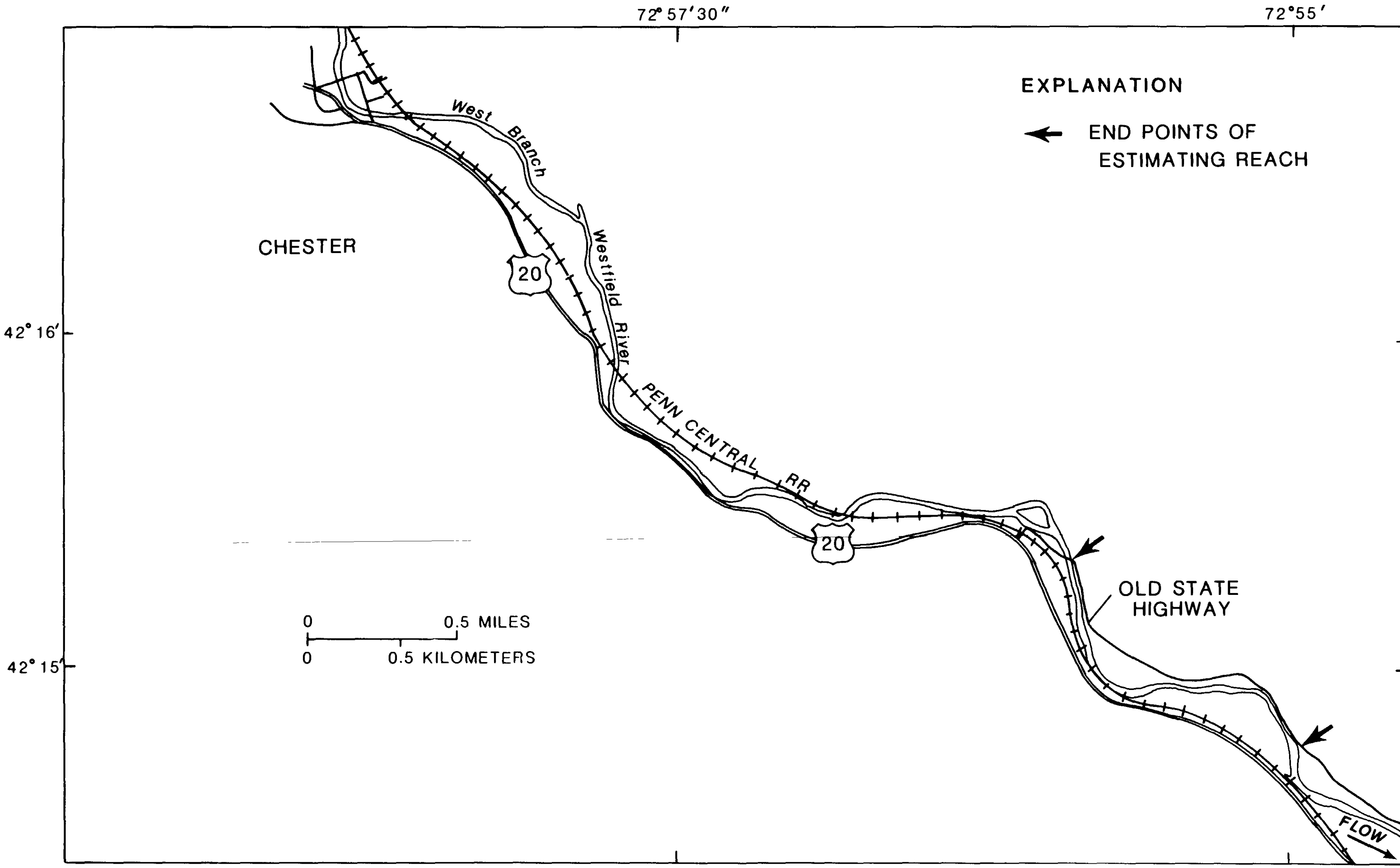

Figure 6.--Problem 2 area: stream reach on West Branch Westfield River below Chester, Massachusetts.

1. From the Blandford topographic map (U.S. Geological Survey, 1972a) and Chester topographic map (U.S. Geological Survey, 1972b), choose the reach for which the estimations will be made. In this problem, for convenience of identification in the field, use the reach of river below the bridge on the Old State Highway and a point downstream where the river runs close to the road but starts to bend southward from the Old State Highway.

2. From the topographic maps estimate the reach length. The problem reach length is 1.4 miles or 7,400 feet.

3. From the maps, calculate the channel slope by dividing the change in elevation at the extremes of the reach by its length. The elevation at the bridge on the Old State Highway is 520 feet. The elevation at the point where the river bends is 485 feet; the reach slope is 0.0047 feet per foot.

4. Conduct a field reconnaissance of the reach to measure the channel characteristics needed to solve the estimating equations. Measure the discharge at each end of the reach and measure channel width at least 10 locations along the length of the reach. For the purpose of this problem, assume the discharge measured at the Old State Highway bridge is 13 cubic feet per second and at the point the river bends South is 13 cubic feet per second. Also assume that the average of 10 measured widths is 75 feet.

5. Conduct a dye-tracer time-of-travel study to determine the mean streamflow velocity. For the purpose of this problem, assume a time-of-travel study determined that the mean streamflow velocity is 0.17 feet per second.

6. Use equation 4 to calculate a reach mean depth:

$$D = \frac{13}{(0.17)(75)} = 1.0 \text{ foot}$$

7. Because the reach slope is over 0.002 feet per foot, use equation 13 to estimate the reaeration coefficient.

$$K_2 = 252.2\ (1.0)^{-0.176}\ (0.17)^{0.355}\ (0.0047)^{0.438}$$

$$= 12.8 \text{ per day}$$

SUMMARY

A weighted multiple-regression technique was applied to 30 data sets collected during medium-flow and low-flow periods during 1983–84 from 16 stream reaches in Massachusetts. The data set of each study includes reaeration coefficient and nine easily measured physical, hydraulic, and water-quality characteristics: streamflow velocity, water-surface slope, mean width, mean depth, Manning's roughness coefficient, water color, concentrations of methylene blue active substances, suspended solids, and specific conductance. The reaeration coefficients were computed using the Survey's steady-state, propane-gas tracer method. The mean streamflow velocity was determined using the Survey's slug-injection dye-tracer time-of-travel technique. The regression analysis yielded an equation that relates reaeration coefficient to the stream's mean depth, water-surface slope, and mean

velocity with a standard error of estimate of 37.5 percent. Only these three variables were significant at the 95-percent confidence level, of the nine easily measured stream characteristics.

The applicability of the reaeration-coefficient estimating equation was graphically determined from plots of residuals against water-surface slope for each study. A shift in the residual scatter is evident for those studies with water-surface slopes less then 0.002 feet per foot. The residuals for 20 studies with slopes greater than 0.002 feet per foot primarily are within 0.2 log units of zero residual. The residuals for the 10 studies with slopes less than 0.002 feet per foot are mostly negative and increase to almost 1.15 log units of zero. This shift suggest that additional factors other than those variables identified in the equation are affecting the reaeration coefficient on stream reaches with the lower water-surface slopes.

From the unweighted error analysis, the regression equation developed in this study (eq. 13), was found to be the best estimator of the reaeration coefficient for the 20 studies having water-surface slopes greater than 0.002 feet per foot with an average absolute error of 27 percent when compared with 19 other commonly used equations available in the literature. The next ranking equation is equation 17 (Krendel and Orlob, 1963), which has an average absolute error of 36 percent. Equation 13 also ranked 17th for the comparisons using the 30 studies and ranked 19th for the comparisons using the 10 studies with water-surface slopes less than 0.002 feet per foot. Equation 34 (Tsivoglou and Neal, 1976) had the lowest average absolute error of 49 percent for the comparison using the 30 studies. Equation 27 (Owens and others, 1964) had the lowest absolute error of 53 percent for the comparison using the 10 studies having water-surface slopes less than 0.002 feet per foot.

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Appendix A: Study-reach locations and descriptions

Study reach	Study site	Location	Study date
Aberjona River near Woburn, Mass.	Injection site near Woburn	Lat. 42°29'06", Long. 071°07'15", Middlesex County, 30 ft downstream from bridge on Washington Street, 0.2 mi upstream from site 1.	04/12/84 07/17/84
	Site 1 at Montvale	Lat. 42°28'56", Long. 071°07'08", Middlesex County, 0.2 mi upstream from bridge on Monvale Avenue	04/12/84 07/17/84
	Site 2 near Winchester	Lat. 42°28'12". Long. 071°07'30", Middlesex County, 300 ft upstream from bridge on Washington Street	04/12/84 07/17/84
Study reach length:	1.0 mile		
Study reach description:	Control - pool and riffle type Bottom description - clay, sand, gravel, and a few cobbles Remarks - debris in channel		
Assabet River near West Concord, Mass.	Injection site near So. Acton	Lat. 42°26'25", Long. 071°25'56", Middlesex County, 50 ft downstream from High Street Dam, 0.4 mi upstream from site 1	09/20/83 04/27/84
	Site 1 near W. Concord	Lat. 42°26'31", Long. 071°25'37", Middlesex County, at bridge on State Highway 62 and Mill Road, 0.1 mi upstream from mouth of Second Division Brook	09/20/83 04/27/84
	Site 2 near W. Concord	Lat. 42°27'12", Long. 071°24'40", Middlesex County, at Damondale Dam, 1.5 mi upstream from Second Division Brook	09/20/83 04/27/84
Study reach length:	1.4 mile		
Study reach description:	Control - channel control type Bottom description - mud, sand, gravel, and cobbles Remarks - aquatic vegetation		
Assabet River at Maynard, Mass.	Injection site at Maynard	Lat. 42°25'40", Long. 071°28'11", Middlesex County, 150 ft upstream from bridge on State Highway 62 and 117, 0.3 mi upstream from site 1	09/01/83 10/06/83 07/26/84
	Site 1 at Maynard	Lat. 42°25'51", Long. 071°28'02", Middlesex County, at bridge on Mill Street, 0.2 mi downstream from bridge on State Highway 62 and 117, 0.4 mi downstream from the mouth of Taylor Brook	09/01/83 10/06/83 07/26/84
	Site 2 at Maynard	Lat. 42°25'51", Long. 071°27'15", Middlesex County, upstream side of bridge on Walnut Street	09/01/83
	Site 2 at Maynard	Lat. 42°25'54", Long. 071°27'20", Middlesex County, at bridge on Main Street and State Highway 62, 0.1 mi downstream from Taylor Brook	10/06/83 07/26/84
Study reach length:	0.7 mile - 9/83 0.6 mile - 10/83 and 7/84		
Study reach description:	Control - pool and riffle type Bottom description - sand, gravel, cobbles, and boulders Remarks - aquatic vegetation		
Mattapoisett River near Rochester, Mass.	Injection site near Rochester	Lat. 41°43'11", Long. 070°51'32", Plymouth County, 100 ft upstream from culvert on Perry Hill Road, 0.4 mi upstream from site 1	05/11/84 08/22/84
	Site 1 near Rochester	Lat. 41°42'54", Long. 070°51'28", Plymouth County, 0.4 mi downstream from culvert on Perry Hill Road	05/11/84 08/22/84
	Site 2 near Rochester	Lat. 41°42'47", Long. 070°50'46", Plymouth County, 1.4 mi downstream from culvert on Perry Hill Road	05/11/84 08/22/84
Study reach length:	1.3 mile		
Study reach description:	Control - channel control type Bottom description - sand and silt Remarks - algae and aquatic vegetation		

Appendix A: Study-reach locations and descriptions--Continued

Study reach	Study site	Location	Study date
Millers River near Athol, Mass.	Injection site near So. Royalston	Lat. 42°37'22", Long. 072°08'20", Worcester County, downstream side of railroad bridge, 1.0 mi downstream from mouth of Beaver Brook, 0.7 mi downstream of bridge in So. Royalston, 0.9 mi upstream from site 1	06/27/84
	Injection site at So. Royalston	Lat. 42°37'47", Long. 072°09'03", Worcester County, 500 ft downstream of bridge in So. Royalston, 0.4 mi downstream from mouth of Beaver Brook, 1.7 mi downstream from Birch Hill Dam, 1.5 mi upstream from site 1	08/29/84
	Site 1 near Athol	Lat. 42°37'22", Long. 072°09'53", Worcester County, at mouth of Rich Brook	06/27/84 08/29/84
	Site 2 near Athol	Lat. 42°37'24", Long. 072°10'51", Worcester County, 1.5 mi downstream from mouth of Rich Brook	06/27/84 08/29/84

Study reach length: 1.5 mile
Study reach description: Control - pool and riffle type
Bottom description - cobbles, large smooth rocks, and boulders
Remarks - benthic invertebrates

Study reach	Study site	Location	Study date
Sevenmile River near Spencer, Mass.	Injection site at Spencer	Lat. 42°13'43", Long. 072°01'09", Worcester County, 300 ft upstream from bridge on State Highway 49, 0.2 mi upstream from site 1	05/03/84 07/25/84
	Site 1 near Spencer	Lat. 42°13'43", Long. 072°01'21", Worcester County, 0.2 mi downstream from bridge on State Highway 49, 2.0 mi upstream from mouth	05/03/84 07/25/84
	Site 2 at E. Brookfield	Lat. 42°13'26", Long. 072°02'42", Worcester County, at E. Brookfield, at bridge on Podunk Street, 0.6 mi upstream from mouth	05/03/84 07/25/84

Study reach length: 1.5 mile
Study reach description: Control - pool and riffle type
Bottom description - sand and gravel
Remarks - algae and aquatic vegetation

Study reach	Study site	Location	Study date
Sudbury River at Concord, Mass.	Injection site at Concord	Lat. 42°26'28", Long. 071°22'06", Middlesex County, downstream of bridge on Sudbury Road, 0.9 mi upstream from site 1	05/22/84 07/31/84
	Site 1 at Concord	Lat. 42°27'06", Long. 071°22'20", Middlesex County, at bridge on State Highway 2, 1.6 mi upstream from mouth	05/22/84 07/31/84
	Site 2 at Concord	Lat. 42°26'28", Long. 071°22'06", Middlesex County, at Nashawtuc Bridge on Nashawtuc Street, 0.5 mi upstream from mouth	05/22/84 07/31/84

Study reach length: 1.3 mile
Study reach description: Control - channel control type
Bottom description - sand and gravel
Remarks - aquatic vegetation

Study reach	Study site	Location	Study date
North River at Griswoldvile, Mass.	Injection site at Griswoldville	Lat. 42°39'13", Long. 072°42'55", Franklin County, upstream side of bridge on State Highway 112 in Griswoldville, 0.5 mi upstream from site 1	10/18/83 06/19/84
	Site 1 at Griswoldville	Lat. 42°38'53", Long. 072°42'50", Franklin County, 20 ft downstream from mouth of McClellan Brook	10/18/83 06/19/84
	Site 2 at Shattuckville	Lat. 42°38'18", Long. 072°43'32", Franklin County, in Shattuckville, 1.3 mi upstream from mouth	10/18/83 06/19/84

Study reach length: 1.5 mile
Study reach description: Control - pool and riffle type
Bottom description - sand, gravel, and cobbles

Appendix A: Study-reach locations and descriptions--Continued

Study reach	Study site	Location	Study date
North River - East Branch - at Colrain, Mass.	Injection site at Colrain	Lat. 42°40'49", Long. 072°41'26", Franklin County, 0.5 mi upstream from site 1 and bridge on State Highway 112 in Colrain	05/17/84
	Injection site near Colrain	Lat. 42°40'20", Long. 072°41'30", Franklin County, 1.3 mi upstream from site 1 and bridge on State Highway 112 in Colrain	11/28/84
	Site 1 at Colrain	Lat. 42°40'29", Long. 072°41'46", Franklin County, at Colrain, at bridge on State Highway 112	05/17/84 11/28/84
	Site 2 at Foundry Village	Lat. 42°40'34", Long. 072°42'48", Franklin County, 0.4 mi upstream from mouth of Foundry Brook at old dam site	05/17/84 11/28/84
	Site 2 at Foundry Village	Lat. 42°40'33", Long. 072°42'48", Franklin County, upstream side of bridge 0.4 mi upstream from mouth of Foundry Brook	11/28/84

Study reach length: 1.0 mile
Study reach description: Control - pool and riffle type
Bottom description - sand, gravel, cobbles, and boulders
Remarks - 11/84: shore ice

Study reach	Study site	Location	Study date
North River - West Branch - near Griswoldvile, Mass.	Injection site near Griswoldville	Lat. 42°40'47", Long. 072°44'20", Franklin County, 50 ft downstream from mouth of Taylor Brook, 0.5 mi upstream from site 1	10/20/83 06/21/84
	Site 1 near Griswoldville	Lat. 42°40'41", Long. 072°44'08", Franklin County, 0.2 mi downstream from mouth of Taylor Brook	10/20/83 06/21/84
	Site 2 near Griswoldville	Lat. 42°40'04", Long. 072°43'34", Franklin County, 1.3 mi upstream from mouth of Cary Brook	10/20/83 06/21/84

Study reach length: 0.9 mile
Study reach description: Control - pool and riffle type
Bottom description - cobbles and boulders
Remarks - algae

Study reach	Study site	Location	Study date
North River - West Branch - near Adamsville, Mass.	Injection site near Adamsville	Lat. 42°41'56", Long. 072°46'26", Franklin County, 200 ft downstream from mouth of Vincent Brook, 1.1 mi upstream from site 1	06/13/84
	Injection site near Adamsville	Lat. 42°41'38", Long. 072°45'46", Franklin County, 0.6 mi upstream from mouth of Tissdell Brook, 0.5 mi upstream from site 1	10/17/84
	Site 1 near Adamsville	Lat. 42°41'27", Long. 072°45'32", Franklin County, 0.1 mi upstream from mouth of Tissdell Brook	06/13/84 10/17/84
	Site 2 near Adamsville	Lat. 42°40'58", Long. 072°45'05", Franklin County, 0.7 mi downstream from mouth of Tissdell Brook at bridge on Archambo Road	06/13/84 10/17/84

Study reach length: 0.9 mile
Study reach description: Control - pool and riffle type
Bottom description - cobbles and boulders
Remarks - gravel bars

Study reach	Study site	Location	Study date
Westfield River - Middle Branch - near North Chester, Mass.	Injection site near N. Chester	Lat. 42°19'05", Long. 072°55'35", Hampden County, 0.8 mi downstream from bridge in North Chester, 0.4 mi upstream from site 1	08/07/84
	Site 1 near N. Chester	Lat. 42°18'51", Long. 072°55'23", Hampden County, 1.3 mi upstream from mouth of Day Brook	08/07/84
	Site 2 near N. Chester	Lat. 42°18'23", Long. 072°54'29", Hampden County, 0.4 mi upstream from mouth of Day Brook	08/07/84

Study reach length: 1.0 mile
Study reach description: Control - pool and riffle type
Bottom description - sand, gravel, cobbles, boulders, and bedrock

Appendix A: Study-reach locations and descriptions--Continued

Study reach	Study site	Location	Study date
Westfield River - Middle Branch - at North Chester, Mass.	Injection site near N. Chester	Lat. 42°20'48", Long. 072°57'21", Hampden County, 0.6 mi downstream from mouth of Glendale Brook, 0.5 mi upstream from site 1	11/28/84
	Site 1 near N. Chester	Lat. 42°20'30", Long. 072°57'05", Hampden County, 1.1 mi downstream from mouth of Glendale Brook on upstream side of bridge	11/28/84
	Site 2 at N. Chester	Lat. 42°20'15", Long. 072°56'12", Hampden County, 2.0 mi downstream from mouth of Glendale Brook on upstream side of bridge	11/28/84
Study reach length:	0.9 mile		
Study reach description:	Control - pool and riffle type Bottom description - sand, cobbles, and boulders Remarks - shore ice		
Westfield River - Middle Branch - near Middlefield, Mass.	Injection site near Middlefield	Lat. 42°22'24", Long. 072°58'11", Hampden County, 1.8 mi upstream from mouth of Glendale Brook, 0.2 mi upstream from site 1	06/07/84
	Site 1 near Middlefield	Lat. 42°20'30", Long. 072°57'05", Hampden County, 1.6 mi upstream from mouth of Glendale Brook	06/07/84
	Site 2 near Middlefield	Lat. 42°20'15", Long. 072°56'12", Hampden County, 0.4 mi upstream from mouth of Glendale Brook	06/07/84
	Injection site near Middlefield	Lat. 42°22'34", Long. 072°58'24", Hampden County, 2.1 mi upstream from mouth of Glendale Brook, 0.3 mi upstream from site 1	10/11/84
	Site 1 near Middlefield	Lat. 42°22'24", Long. 072°58'11", Hampden County, 1.8 mi upstream from mouth of Glendale Brook, also the injection site for the first tracer study on this reach	10/11/84
	Site 2 near Middlefield	Lat. 42°21'26", Long. 072°57'50", Hampden County, 0.3 mi upstream from mouth of Glendale Brook	10/11/84
Study reach length:	1.2 mile - 6/84 1.5 mile - 10/84		
Study reach description:	Control - pool and riffle type Bottom description - cobbles, rocks, boulders, and bedrock, some sand and gravel Remarks - some breached, manmade rock dams		
Westfield River - West Branch - near Huntington, Mass.	Injection site near Chester	Lat. 42°15'21", Long. 072°55'56", Hampden County, 300 ft upstream from bridge on Old State Highway, 0.3 mi upstream from site 1	06/05/84
	Injection site near Chester	Lat. 42°15'37", Long. 072°56'00", Hampden County, 0.2 mi upstream from bridge on Old State Highway, 0.4 mi upstream from site 1	08/10/84
	Site 1 near Chester	Lat. 42°15'09", Long. 072°55'52", Hampden County, 0.3 mi downstream from bridge on Old State Highway, 1.2 mi downstream from the mouth of Sanderson Brook	06/05/84 08/10/84
	Site 2 near Huntington	Lat. 42°21'26", Long. 072°57'50", Hampden County, 0.9 mi upstream from mouth of Roaring Brook	06/05/84 08/10/85
Study reach length:	1.2 mile		
Study reach description:	Control - pool and riffle type Bottom description - sand, gravel, cobbles, rocks, and bedrock Remarks - long deep pools		
Westfield River - West Branch - at Chester, Mass.	Injection site near Chester	Lat. 42°18'13", Long. 072°59'18", Hampden County, 0.3 mi upstream from bridge on Middlefield Road, 0.7 mi upstream from the mouth of Otis Wait Brook, 0.3 mi upstream from site 1	06/07/84
	Site 1 near Chester	Lat. 42°18'02", Long. 072°59'06", Hampden County, at bridge on Middlefield Road,, 0.4 mi upstream from the mouth of Otis Wait Brook	06/07/84
	Site 2 at Chester	Lat. 42°16'09", Long. 072°55'53", Hampden County, at bridge on Main Street	06/07/84
Study reach length:	1.5 mile		
Study reach description:	Control - pool and riffle type Bottom description - cobbles, rocks, and boulders Remarks - some bedrock exposed in channel		