ROCK RIPRAP DESIGN FOR PROTECTION OF STREAM CHANNELS NEAR HIGHWAY STRUCTURES

VOLUME 2 -- EVALUATION OF RIPRAP DESIGN PROCEDURES

By J.C. Blodgett and C.E. McConaughy

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LIST OF SYMBOLS

Symbol	Term
a	Constant
A	Area
B	Channel bottom width
C	Coefficient; Chezy resistance coefficient
d	Depth
d_a	Mean depth
d_m	Maximum depth
D	Diameter of rock particles
D_s	Spherical diameter of stone
F	Froude number
g	Acceleration of gravity
G_s	Specific gravity of stone
k	Stone diameter; equivalent roughness; constant

LIST OF SYMBOLS (continued)

Symbol	Term
K	Shape factor of stone
k_w	Stone size for stone of w pounds per cubic foot
ℓ	Long axis of stone
L	Length
n	Manning's roughness coefficient
P	Wetted perimeter
Q	Total discharge
Q_D	Design discharge
r	Centerline radius of channel bend
R	Hydraulic radius
R_d	Mean radius of outside bank of bend
R_o	Mean radius of channel centerline
S_e	Energy slope
S_f	Friction slope
SF	Safety factor for riprap on side slope with horizontal flow
SF_m	Safety factor for riprap on side slope with no flow
S_o	Channel bed slope
S_w	Water-surface slope
t	Thickness of riprap normal to face slope
T	Top width of channel; water-surface width of channel
v	Point velocity; kinematic viscosity
V	Velocity
V_a	Average (mean) velocity in cross section
V_m	Maximum point velocity in cross section
V_s	Velocity against stone
w	Unit weight of stone; water-surface width at upstream end of bend
W	Unit weight of stone
W_c	Class weight of stone
W_s	Weight of stone
y	Depth above boundary corresponding to v
z	Side slope, ratio of horizontal to vertical
γ	Unit weight of water
γ_s	Specific weight of stone

LIST OF SYMBOLS (continued)

Symbol	Term
Δ	Internal angle of channel bend, in degrees
Δc	Internal angle which differentiates between a long and short bend
Δy	Magnitude of superelevation
η	Stability factor for particle on plane horizontal bed
θ	Angle of repose of riprap; angle of bed slope; side slope angle
λ	Angle between horizontal and velocity vector at a point
ξ	Converts safety factor from nonhorizontal to horizontal side-slope flow
ρ	70° constant for broken rock
τ	Actual or design shear on channel bed
τ_b	Shear at bend
τ_c	Critical shear stress
τ_o	Mean boundary shear acting over wetted perimeter; critical shear stress
τ_s	Average tractive force on side slope in vicinity of particle
Φ	Angle of side slope to horizontal; angle of repose of riprap
ψ	Angle of side slope

CONVERSION FACTORS

For readers who prefer metric units rather than inch-pound units, the conversion factors for the terms used in this report are listed below:

Multiply	By	To obtain
ft (feet)	0.3048	m (meters)
ft (feet)	304.8	mm (millimeters)
ft/s (feet per second)	0.3048	m/s (meters per second)
ft/s^2 (feet per square second)	0.3048	m/s^2 (meters per square second)
ft^2 (square feet)	0.0929	m^2 (square meters)
ft^3/s (cubic feet per second)	0.0283	m^3/s (cubic meters per second)
inches	25.4	mm (millimeters)
lb (pound)	0.454	kg (kilogram)
lb/ft^2 (pounds per square foot)	4.882	kg/m^2 (kilograms per square meter)
lb/ft^3 (pounds per cubic foot)	16.02	kg/m^3 (kilograms per cubic meter)
mi (miles)	1.609	km (kilometers)
ton	0.907	megagram

ROCK RIPRAP DESIGN FOR PROTECTION OF

STREAM CHANNELS NEAR HIGHWAY STRUCTURES

VOLUME 2--EVALUATION OF RIPRAP DESIGN PROCEDURES

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ABSTRACT

Volume 1, "Hydraulic Characteristics of Open Channels," discusses the hydraulic and channel properties of streams, based on data from several hundred sites. Streamflow and geomorphic data were collected and developed to indicate the range in hydraulic factors typical of open channels, to assist design, maintenance, and construction engineers in preparing rock riprap bank protection. Typical channels were found to have a maximum-to-mean depth ratio of 1.55 and a ratio of hydraulic radius to mean depth of 0.98, which is independent of width. Most stable channel characteristics for a given discharge are the slope, maximum depth, and hydraulic radius.

In volume 2, seven procedures now being used for design of rock riprap installations were evaluated using data from 26 field sites. Four basic types of riprap failures were identified: Particle erosion, translational slide, modified slump, and slump. Factors associated with riprap failure include stone size, bank side slope, size gradation, thickness, insufficient toe or endwall, failure of the bank material, overtopping during floods, and geomorphic changes in the channel. A review of field data and the design procedures suggests that estimates of hydraulic forces acting on the boundary based on flow velocity rather than shear stress are more reliable. Several adjustments for local conditions, such as channel curvature, superelevation, or boundary roughness, may be unwarranted in view of the difficulty in estimating critical hydraulic forces for which the riprap is to be designed. Success of riprap is related not only to the appropriate procedure for selecting stone size, but also to reliability of estimated hydraulic and channel factors applicable to the site.

Further identification of channel properties and the development of a new procedure for estimating stone size are presented in volume 3, "Assessment of Hydraulic Characteristics of Streams at Bank Protection Sites."

INTRODUCTION

The need to evaluate the various procedures being used to design rock riprap has been indicated by the diverse results that may be obtained depending on the procedure used and assumptions concerning hydraulic and geomorphic conditions at a site. Failure at a site is usually attributed to excessive hydraulic forces acting on the bank and causing displacement of the stones that comprise the riprap (fig. 1). However, other factors, such as improper gradation or placement of stone, inadequate assessment of probable morphologic changes, or failure of the original bank material may contribute to the riprap failure.

Methods available to protect highway structures from streamflow hazards include armoring, retards, and spurs. Armoring is the surfacing of a channel bed or banks, or an embankment slope; retards are a permeable or impermeable structure in a channel to deflect flow; and spurs are linear structures projecting into a channel to induce deposition along the bank. A description of these (and other) protective measures (countermeasures) and an evaluation of their performance at various field installations is given in a report by Brice and Blodgett (1978).

FIGURE 1. Erosion of rock riprap on left bank of Pinole Creek at Pinole, California, following flood of January 4, 1982. Note deposition of displaced riprap in channel bed (photographed March 1982).

This study presents an analysis of various procedures commonly used for armoring a bank by placement of flexible rock revetment, also known as riprap or riprap lining, to protect highway structures from damage caused by channel erosion. The study was funded by the Federal Highway Administration (FHWA) as part of their effort to develop streambank stabilization measures in the federally coordinated program of Highway Research, Development, and Technology. Dr. Roy E. Trent is the FHWA's technical representative for the study.

Procedures for design of riprap have been prepared by a number of agencies, such as FHWA, U.S. Army Corps of Engineers (USCE), U.S. Bureau of Reclamation (USBR), and California Department of Transportation (CALTRANS, formerly California Department of Public Works, Division of Highways). The various design procedures have been identified in table 1, using an abbreviation for brevity.

The purpose of this research effort is to evaluate inconsistencies or possible deficiencies in the procedures described in HEC-11 and HEC-15 for design of rock riprap. Other procedures for riprap design, such as "Hydraulic Design of Flood Control Channels," (EM-1601) and the "Bank and Shore Protection in California Highway Practice" manual (Cal-B&SP) were also evaluated. All of the design procedures included in this study were reviewed with the intent of providing information needed for interim and long-term approaches for the practical and functional design of riprap.

Table 1. Agency, publication title, and abbreviated title of various rock riprap design procedures.

Agency	Title and date of riprap design procedure	Procedure abbreviation
Federal Highway Administration (FHWA)	Hydraulic Engineering Circulars:	
	Use of riprap for bank protection (Searcy, 1967),	HEC-11
	Design of stable channels with flexible linings (Normann, 1975)	HEC-15
California Department of Transportation (CALTRANS)	Bank and shore protection in California highway practice (1970)	Cal-B&SP
U.S. Army Corps of Engineers (USCE)	Hydraulic design of flood control channels, EM 1110-2-1601 (1970)	EM-1601
American Society of Civil Engineers (ASCE)	Sedimentation Engineering, Manual No. 54 (Vanoni, 1975)	Man-54
Simons and Senturk	Sediment transport technology (1977)	Simons-STT
U.S. Bureau of Reclamation (USBR)	Hydraulic design of stilling basins and energy dissipators (Peterka, 1958, Engineering monograph No. 25)	USBR-EM-25
Oregon Department of Transportation	Keyed riprap (no date given)	ODOT

Procedures for riprap and channel design have been developed using theoretical concepts to define the magnitude of hydraulic stress (force) in the boundary zone at which movement of individual stones becomes imminent. These procedures were then empirically confirmed or extended to prototype conditions on the basis of laboratory flume study data. HEC-15 summarizes the status of some of these procedures and indicates that very few actual data points are available for the study of flow at bends. Although the lack of actual data was noted in reference to estimation of stresses at channel bends, it applies to many of the other procedures for designing bank protection. A similar observation was also made in a report, "Practical Riprap Design," by Maynord (1978).

It was determined that the best approach for evaluating the various riprap design procedures would be to utilize prototype data. Three sources of data were used: (1) Field surveys made specifically for this project, (2) the ongoing U.S. Geological Survey stream-gaging program, and (3) reports that include detailed tabulations of hydraulic and channel data. Field surveys for this project were made at 26 sites in Washington, Arizona, Oregon, California, and Nevada. Many of the sites, referred to as pilot study sites, were selected because rock riprap had been installed. Data obtained as part of the stream-gaging program generally were from sites without riprap but were selected to provide flow and channel data.

REVIEW OF RIPRAP DESIGN TECHNOLOGY

A number of approaches have been developed to relate the magnitude and direction of forces acting on the boundary of a channel with the passive forces that tend to prevent erosion of the boundary material. These approaches can be categorized as follows:

- o Relationship of permissible velocity to particle size for cohesive and noncohesive soils lining the channel (HEC-11, Cal-B&SP, EM-1601, Man-54, USBR-EM-25).

- o Relationship of permissible velocity to grasses or other channel linings (HEC-15).

- o Relationship of boundary shear to the size of particles that comprise the channel boundary (HEC-15, EM-1601, Simons-STT).

All of these approaches assume uniform and subcritical flow conditions in the reach, although the procedures in EM-1601 consider supercritical flow also. The channel shape is usually assumed to be trapezoidal with a constant cross section and bed slope.

In the hydraulic analysis of a site for design of riprap to prevent scour, the evaluation generally assumes uniform or gradually varied flow conditions. Uniform flow has a constant depth for all cross sections in a reach. Varied flow conditions occur if the depth of flow changes along the length of channel. The evaluation of a site is generally based on a design discharge and uniform flow conditions. A large magnitude change in channel size or gradient may cause the state of flow to change. The depth is less and the velocity greater in supercritical flow than in subcritical flow. Abrupt changes may cause hydraulic drops or jumps. A hydraulic drop will occur where flow changes abruptly from subcritical to supercritical, and a hydraulic jump will occur where flow changes from supercritical to subcritical. Severe turbulence accompanies a hydraulic jump. If supercritical flow occurs in a reach between two reaches of subcritical flow, a hydraulic drop may occur at the upstream end of the critical reach and a hydraulic jump may occur at the downstream end.

In several design procedures, the shear stress (also referred to as the tractive force, Chow, 1959) is used as a quantitative indicator of the forces acting on the channel bed and banks. The magnitude of shear stress is dependent on the depth of flow and channel gradient; therefore, values of shear stress in a reach with supercritical conditions may be less than those for subcritical flow.

Shear Stress Related to Permissible Flow Velocity

The maximum permissible velocity is the highest mean velocity that will not cause erosion of the boundary. Procedures for design of channels based on permissible velocity are described in EM-1601, HEC-11, and USBR-EM-25. Chow (1959) presents a summary of several design procedures that are based on maximum permissible (mean) velocity for channels with vegetative linings. Figure 2 (adapted from EM-1601) shows a comparison of design curves used to relate permissible velocity to stone size. The Isbash (USBR) procedure gives the largest stone size for a given velocity. The curve for an isolated cube, which is not generally used for riprap design, is given for comparison.

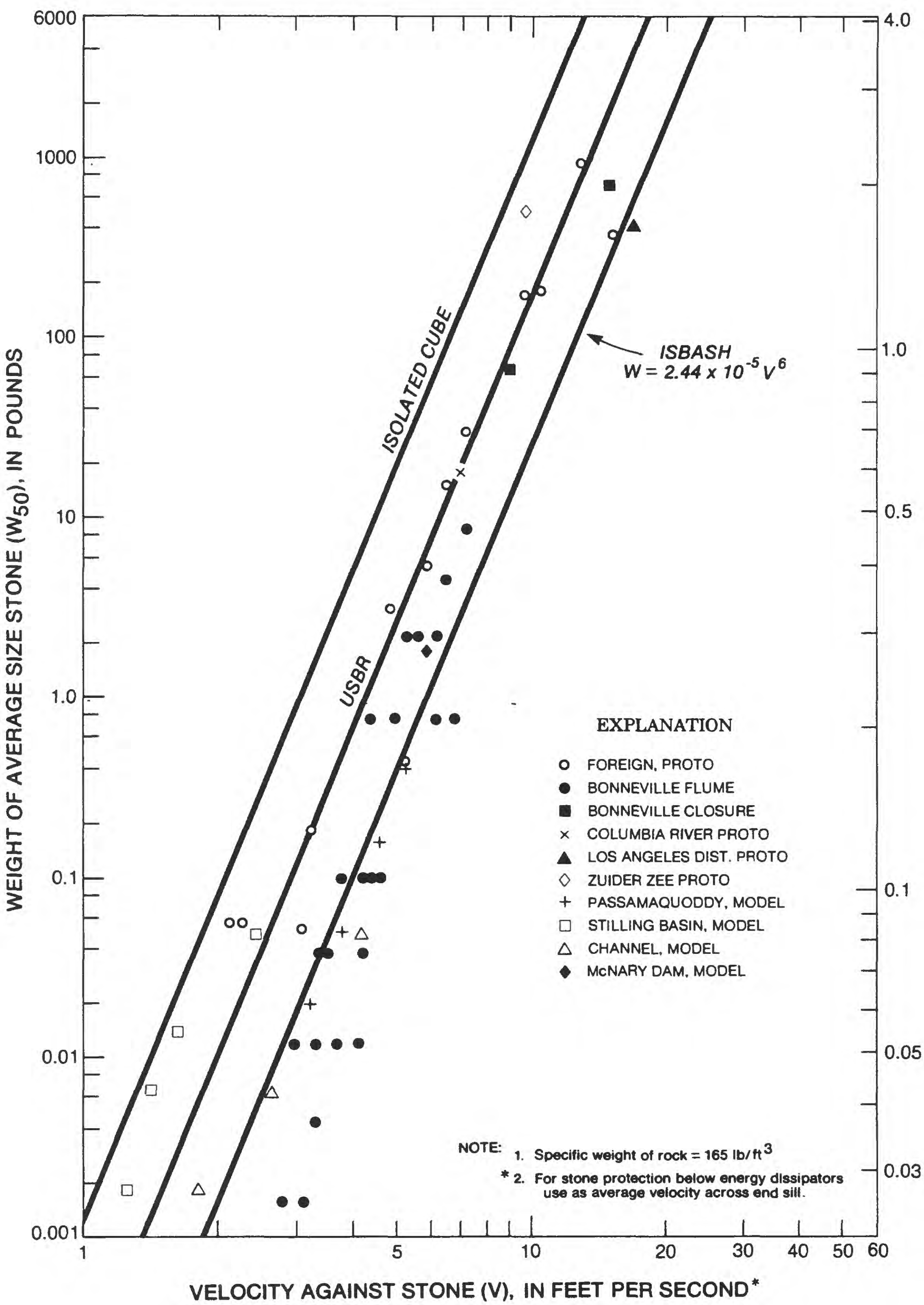


FIGURE 2. Comparison of procedures relating velocity to stone weight (adapted from EM-1601).

Shear Stress Related to Hydraulic Radius and Gradient

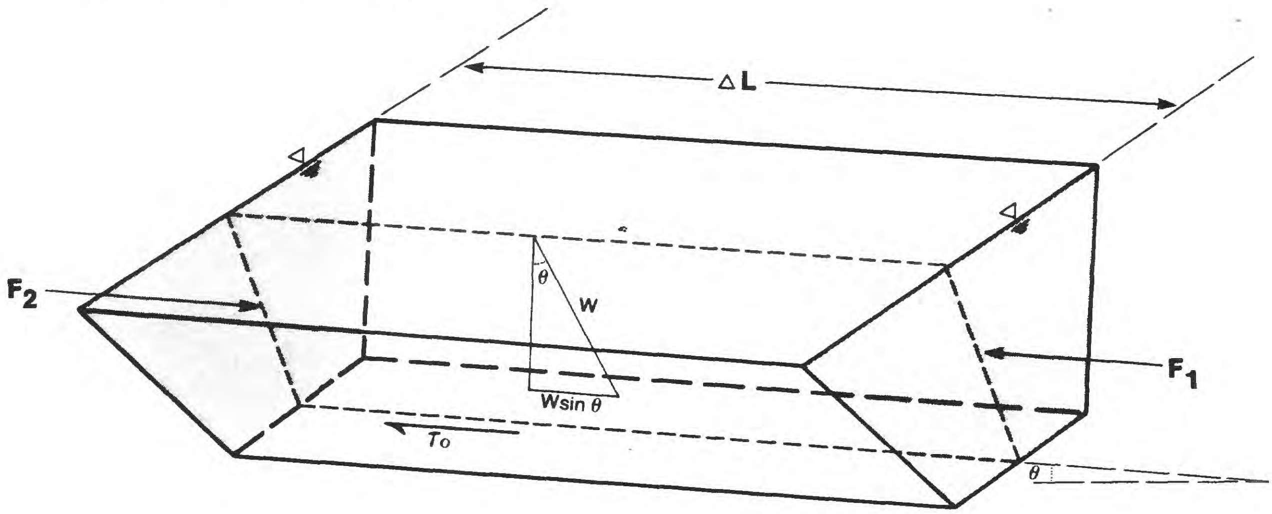
Water flows in a channel as a function of gravity and develops forces that act in many directions depending on the amount of turbulence, but with the primary vector in the direction of flow. Friction forces act in the opposite direction to the flow and, when considered over an area, are referred to as boundary shear. The forces acting on a body of water are shown in figure 3. For uniform flow, the pressure forces are equal and act in opposite directions, and all accelerations are zero. Analyzing the flow condition shown in figure 3 and converting forces to shear stresses gives the equation:

$$\tau_0 P \Delta L = \gamma A \Delta L \sin\theta \qquad (1)$$

For small angles, $\sin\theta = \tan\theta = S_0$. By dividing by $P\Delta L$ and substituting S_0, the average shear stress on the boundary is given by:

$$\tau_0 = \gamma R S_0 \qquad (2)$$

where τ_0 = mean boundary shear acting over the wetted perimeter
P = wetted perimeter
L = length
γ = unit weight of water
A = cross-sectional area
θ = angle of bed slope
R = hydraulic radius
S_0 = channel bed slope



Note: d_m is measured in the vertical and is a close approximation (0.5 percent) as $d_m = d_m \cos\theta$ for channel slopes less than 0.1.

F_1, F_2 = Forces of static pressure

W = Weight of water

FIGURE 3. Three-dimensional free body diagram of forces acting on a water mass.

Other analytical procedures (Chow, 1959) derive the shear stress (tractive force per unit of wetted area) for gradually varied flow conditions in which S_0 is replaced by the energy slope S_e, giving the equation:

$$\tau_0 = \gamma R S_e \tag{3}$$

In uniform flow, the energy slope (S_e) and water-surface or bed slope (S_0) are equal; in gradually varied flow, the difference between the energy and water-surface slopes is generally small and either value can be used in estimating shear stress. The energy slope is less than the water-surface slope when flow is contracting. This suggests that use of the water-surface slope will give larger than actual values of energy slope when estimating the shear stress unless the reach is expanding or is at a bend. In an expanding reach, energy losses are usually assumed to be 50 percent of the change in velocity head.

For wide channels, where the mean depth (d_a) is approximately equal to the hydraulic radius, the average shear stress may be determined by the equation from Chow (1959):

$$\tau_0 = \gamma d_a S_0 \tag{4}$$

To determine the maximum stress at a cross section, the maximum depth (d_m) is substituted for mean depth. Equation 4 may then be modified to estimate critical shear stress needed for riprap design procedures by HEC-15.

$$\tau_0 = \gamma d_m S_0 \tag{5}$$

Boundary shear is also a function of channel velocity, and equation 4 may be expressed in terms of mean velocity (V_a) and Manning's roughness coefficient n. Using the following procedure, the Chezy equation $V_a = C\sqrt{RS_e}$ may be rearranged and modified for bed slope so that $RS_e = RS_0 = (V_a/C)^2$ and substituting in equation 3 gives:

$$\tau_0 = \gamma (V_a/C)^2 \tag{6}$$

The relationship between Manning's n and the Chezy C can be expressed by the equation:

$$C = \frac{1.486\ R^{1/6}}{n} \tag{7}$$

where C is a coefficient that varies with the hydraulic radius (R) and roughness (n) of the channel. The boundary shear on the wetted perimeter is given by the equation:

$$\tau_0 = \frac{\gamma V_a^{\ 2} n^2}{2.21 R^{0.333}} \tag{8}$$

For wide channels, the hydraulic radius and mean depth are assumed to be approximately equal (table 2). Grouping constants and simplifying yields:

$$\tau_0 = \frac{28.2\ V_a^{\ 2} n^2}{d_a^{\ 0.333}} \tag{9}$$

Table 2. Hydraulic properties and channel geometry of streams as a function of channel slope (adapted from table 1 in volume 1 of this report).

Average value of variable for sample (N varies from 44 to 763)	Water-surface slope (ft/ft)			
	<0.001	>0.001-<0.005	>0.005	All slopes
Maximum point velocity, V_m (ft/s)	[1]9.77	[1]14.8	[1]16.7	16.7
Average velocity, V_a (ft/s)	3.97	4.94	7.4	4.4
Maximum depth, d_m (ft)	[1]50.4	[1]23.4	[1]19.2	10.3
Average depth, d_a (ft)	14.8	4.1	4.1	6.9
Froude number, F	0.22	0.45	0.68	0.36
V_m/V_a	1.56	1.62	1.71	1.61
d_m/d_a	1.61	1.68	1.73	1.55
T/d_m (T=top width)	19.4	27.8	19.5	19.8
R/d_a (R=hydraulic radius)	0.979	1.03	0.965	0.975

[1]Maximum value for sample.

Application of the hydraulic factors in equations 8 and 9 indicates that accurate estimates of Manning's n and velocity are needed. The hydraulic radius or mean depth may be defined by measuring the cross section.

There are difficulties in applying the concepts of permissible velocity or shear stress to determine the riprap material required to resist erosion. Permissible velocities (such as given in figure 2) and shear stresses are usually expressed as mean values for the cross section. Estimates of shear stress based on gradient are not considered reliable because in localized areas of turbulence, the gradient may be negative, and at channel banks, the gradient along each bank may be dissimilar. The problems in analyzing boundary stresses based on shear stress are discussed in detail in later sections of the report. The actual point values that effectively contribute to erosion of the bank material are difficult to determine and are estimated from relationships established using laboratory data. These data are then extended to accommodate the magnitude of hydraulic conditions that occur in the field.

CHARACTERISTICS OF RIPRAP FAILURE

Inadequate recognition of the type of erosion process that is occurring or improper riprap design may lead to failure of the riprap, as shown in figure 1. Types of erosion that can be successfully controlled by riprap include channel degradation, bank erosion, scour, and changes in alinement associated with meandering, branching, and braiding of streams. The rate of channel erosion varies with time, but is primarily a function of the magnitude of streamflow. Other factors that affect channel erosion are stream control works, sand and gravel pit operations, and land-use developments.

A discussion of (1) geomorphic factors and the classification of stream properties, (2) assessment of stability as related to sediment transport and hydraulics, and (3) effectiveness of countermeasures for hydraulic problems at bridges is given by Brice and Blodgett (1978). An overview of streambank stabilization measures is given by Brown (1985).

Classification of Failures

During this study, the authors identified four basic types of riprap failure along streambanks: Particle erosion, translational slide, modified slump, and slump. The cause of each type of failure is different and certain riprap design procedures will be needed that consider each type of failure. A sketch of each type of failure is shown in figure 4.

Particle Erosion

Particle erosion is the transport of riprap stones to the channel bed near the installation or to a point downstream. Particle erosion is considered the most common type of failure, and the mechanics of impending movement are documented in the literature. A mathematical analysis of particle erosion is presented by Simons and Senturk (1977). The sketch in figure 4 shows an advanced stage of failure caused by particle erosion. Displaced riprap usually comes to rest on the bed near the eroded areas and for some distance downstream. A mound of displaced riprap on the channel bed indicates that the transport capability of the stream is insufficient to move all of the eroded riprap from the site. This situation occurred on Pinole Creek at Pinole, California (see fig. 1). A detrimental effect of the mound is the tendency to confine flows of high velocity between the mound and the toe of the embankment, causing additional bank and bed erosion.

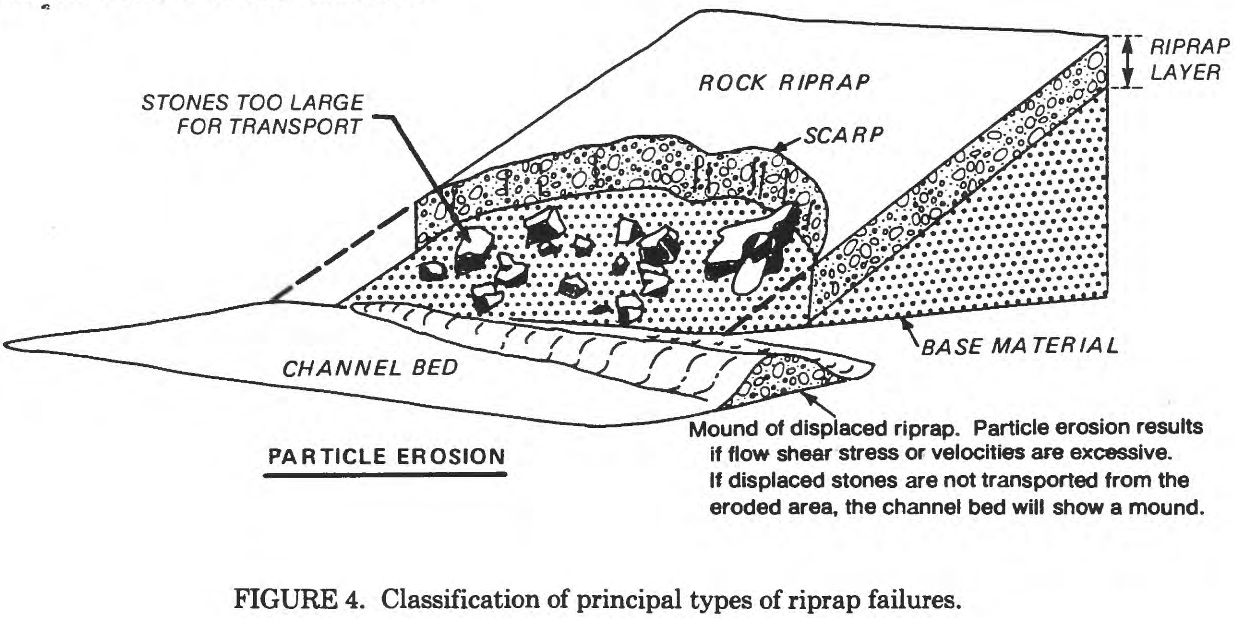


FIGURE 4. Classification of principal types of riprap failures.

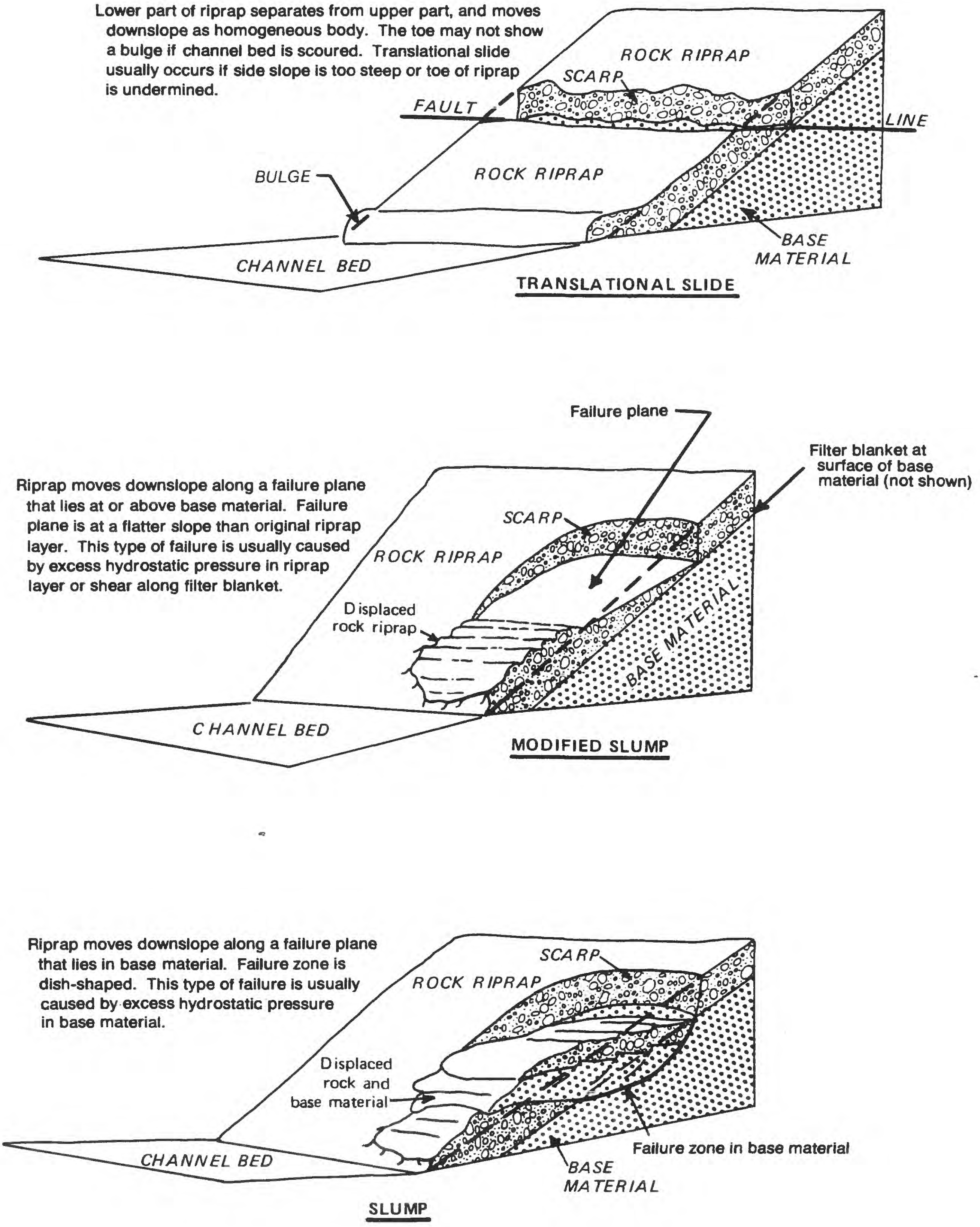


FIGURE 4. Classification of principal types of riprap failures (Continued).

A method to study the stability and movement of individual particles (stones) comprising the riprap layer was developed for the Sacramento River at site E-10 near Chico, California, by painting a red stripe along the top face of the riprap layer during the low flow season (fig. 5). Any movement of individual stones was noted following flood events. The initial failure by particle erosion in February 1983 is illustrated in figure 6. At the time of the photograph, the red stripe had been destroyed in the area shown but was still visible near the upper end of the rod. Following a number of flood events between February and May 1983 during which the entire bank and riprap was subject to inundation, particle erosion progressed to the condition shown in figure 7. All but the largest stones were subsequently transported from the eroded area.

The scarp at the upslope end of the failure (fig. 6) is not related to a slump failure as described by Schuster and Krizek (1978). A slump is a mass movement of material along a slip surface with the terminus of the upslope face of the failure designated a scarp. The scarp observed with failures caused by particle erosion is related to angular impingement of flow. Progressive scour during eddy action of the streamflow at flow expansions also tends to erode the exposed bank after the protective layer of riprap has been damaged. During the initial stages of failure related to particle erosion, the height of the scarp face is small.

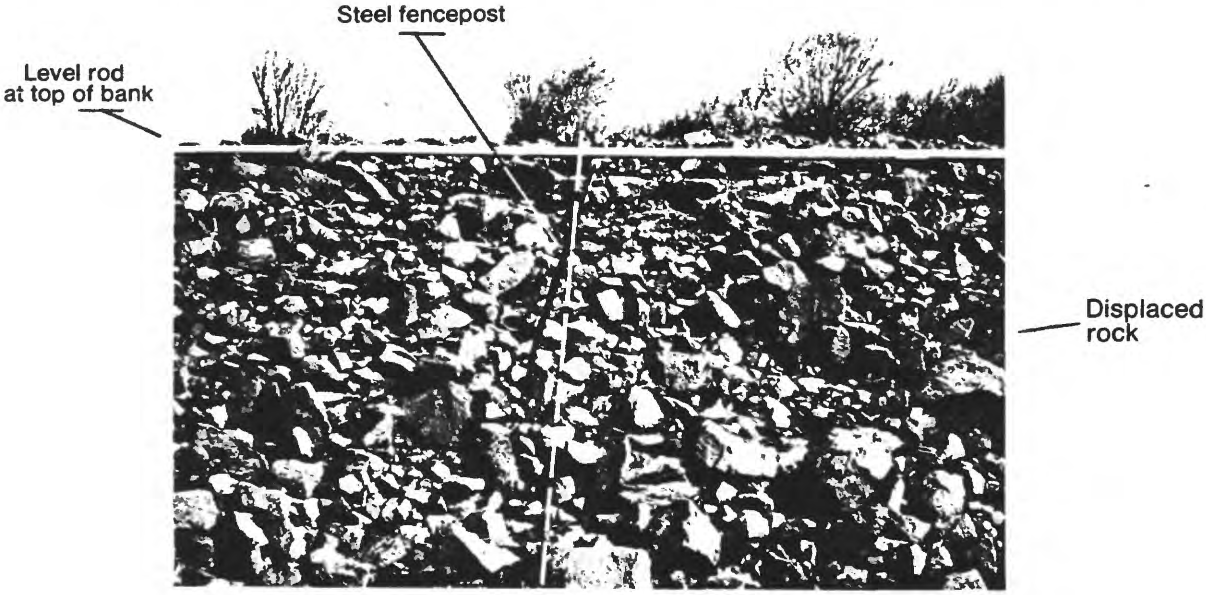


FIGURE 5. Riprap on left bank of Sacramento River at E-10 near Chico, California. A reference line shows location of stones in December 1981. Flow is from left to right. Note displaced stone near steel fencepost (photographed March 4, 1982).

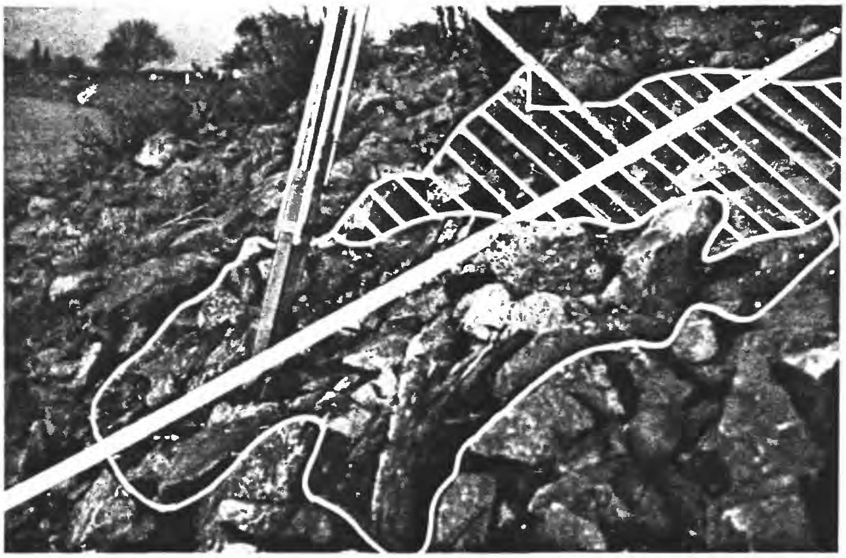


FIGURE 6. Riprap on left bank of Sacramento River at E-10 near Chico, California, showing initial effect of particle erosion. Survey rod at approximate location of original bank line. Scarp face (cross-hatched area, between 15-foot mark on rod and tripod leg) is approximately 1 foot high (photographed February 1, 1983).

FIGURE 7. Riprap on left bank of Sacramento River at E-10 near Chico, California, showing advanced stage of particle erosion. Survey rods held at approximate location of original bank line. Note only the largest stones, as compared with the well-graded distribution in figure 5, have not been displaced (photographed May 2, 1983).

An example of progressive riprap failure and increased height of the scarp caused by the particle erosion is given in table 3. Data for failures at three sites on the Sacramento River at E-10 near Chico were collected in May 1983. The larger failure was first observed in January 1983, after the initial damage occurred during a flood in December 1982. This failure subsequently increased in size, and the other two failures occurred during floods that overtopped the bank in January and March 1983. All of these failures exhibit the geometric shape of a horseshoe, shown in figure 8, during the beginning phase of riprap failure. The riprap failure on Pinole Creek at Pinole represents the advanced stage of particle erosion, in which the individual areas of failure are combined. A significant characteristic of the Pinole Creek failure is that the new side slope is flatter than the original side slope of 2.4:1, thus leaving an exposed face (scarp) that is easily eroded.

The probable causes of particle erosion are:

o Median size (D_{50}) stone not large enough to resist the shear stress of the stream.

o Abrasion or removal by impact of individual stones. For this and the previous situation, individual stones are removed, and in time, the cumulative effect results in failure of the riprap.

o Side slope of the bank so steep that the angle of repose of the riprap is easily exceeded, causing instability of the individual stones.

o Gradation of riprap may be too uniform (all stones near the median size). Without enough smaller diameter stones that tend to fill the voids and provide lateral support for larger material, failure may occur even though the median size is adequate and the bank side slope is not too steep.

Table 3. Geometry of progressive riprap failures related to particle erosion on the Sacramento River at Site E-10 near Chico, California (surveyed May 2, 1983).

Failure site	New side slope in area of erosion (feet/feet)	Maximum width[1] (feet)	Length (feet) at stated intervals (in percent) of width[2]: 0	25	50	75	90	Height of scarp[3] (feet)
1	0.54	5.5	12	12	11.5	9	7	1.6
2	.62	10.5	25	24	22	19.5	16	1.9
3	.59	11.0	44	43	40	30	14	2.4

[1]Width is maximum slope distance, measured perpendicular to shoreline.

[2]Length is the distance across failure, measured parallel to shoreline. The zero percent length is at the downslope end of the failure. The 100 percent length is at the face of the scarp, as shown in figure 8.

[3]Maximum height of scarp. The height decreases to zero at the 0 percent width interval.

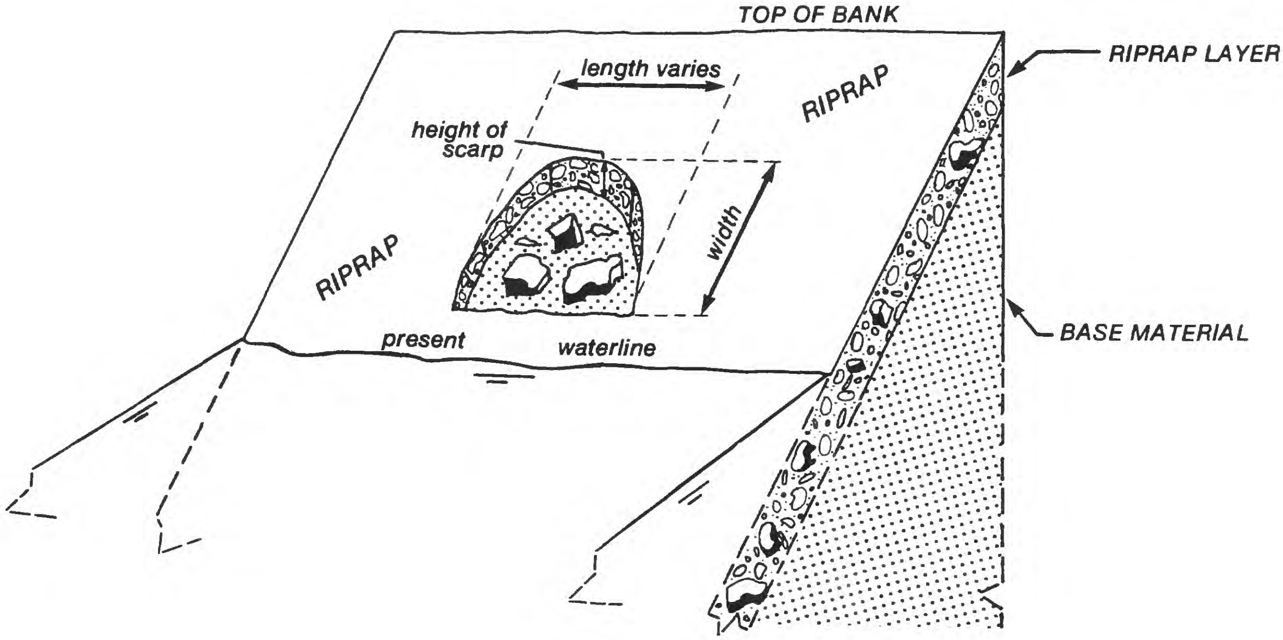


FIGURE 8. Typical riprap failure area in the shape of a horseshoe, caused by particle erosion.

Translational Slide

A translational slide is a riprap failure caused by the downslope movement of a mass of stones that consists of a single or closely related units with the fault line usually on the horizontal plane (Schuster and Krizek, 1978), as shown in figure 4. The riprap is undisturbed except at the fault line and a bulge at the toe. If the moving mass is not greatly deformed, it may be called a block slide. The initial phases of a translational slide are indicated by cracks in the upper part of the riprap bank that extend parallel to the channel. The movement of translational slides is controlled by (1) variations in shear strength along the interface between the riprap and the base material, and (2) stability of the riprap at the junction point with the channel bed. A translational slide is initiated if the channel bed scours and undermines the toe of the riprap layer, or particle erosion of the toe material occurs, reducing the support of the upslope material. In either case, the shear resistance of the interface between the bed material and riprap may be insufficient to resist translational movement. The translational slide may progress downslope indefinitely if erosion of the riprap material at the toe (which constitutes the bulge shown in fig. 4) continues. Continued downslope creep of the riprap may also occur if the base material underlying the riprap is sufficiently saturated with water and the shear resistance along the interface is less than the gravitational force.

A translational slide with the fault line located high on the embankment suggests that extensive channel bed scour or particle erosion undermined the toe of the embankment material. In this situation, the slide would occur only when the mass of riprap was sufficiently large for downslope forces to exceed the shear strength at the interface. The occurrence of translational slides is also related to the presence of excess hydrostatic (pore) pressure in the base material that causes reduced frictional resistance of the riprap at the interface. Excess pore pressure may develop during periods of high precipitation, flooding, or rapid fluctuation of water levels in the stream. The presence of a filter blanket placed on the base material probably would not prevent this type of failure and may actually provide a potential failure plane. Figure 9 shows an example of translational slide failure during the winter of 1982-83 on the Cosumnes River at site 2 near Sloughhouse, California.

The probable causes of translational slide failure are:

- Bank side slope too steep.
- Loss of foundation support at the toe of the riprap caused by scour or degradation of the channel bed, or by particle erosion of the lower part of the riprap.
- Presence of excess hydrostatic (pore) pressure that reduces the frictional resistance along the interface between the riprap and base material.

Modified Slump

The riprap failure referred to as a modified slump is a mass movement along an internal slip surface. Slumps are described by Schuster and Krizek (1978) as rotational slides along a concave surface of rupture. The modified slump is different, however, from the various types of slumps discussed by Schuster and Krizek because the failure plane is located in the riprap, and the underlying material supporting the riprap does not fail. As a result, the surface of the rupture is not concave, but is a relatively flat plane. This type of failure is similar in many respects to the translational slide, but the geometry of the damaged riprap (fig. 4) is similar in shape to initial stages of failure caused by particle erosion. The new side slope within the modified slump area is flatter than the slope of the interface between the base material and the riprap. Material that is dislodged from the failure area usually comes to rest on the bank just downslope from the failure, as shown in figure 10, similar to what occurs in a typical slump failure on hilly terrain. The displaced stones may cause increased turbulence of flow and eddy action along the bank in the area of the slump. The secondary currents may then cause additional riprap failure by particle erosion of smaller materials, especially those exposed at the scarp. An interesting factor concerning modified slump failures is that the median stone size (D_{50}) may be adequate for the site, but movement of certain (key) stones (possibly due to poor gradation) leads to a localized failure of the riprap.

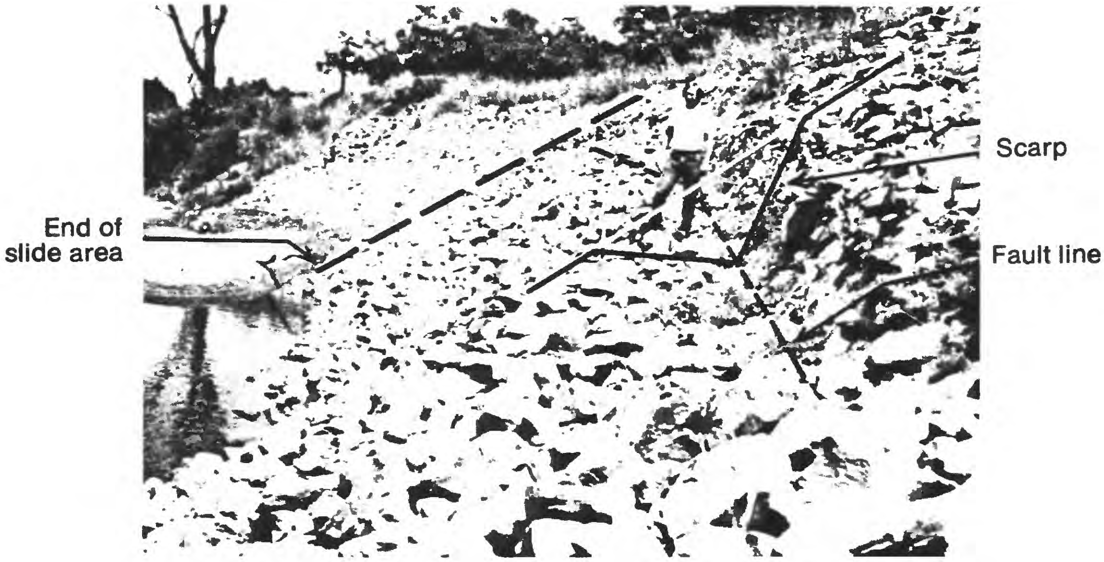


FIGURE 9. Riprap on Cosumnes River at site 2 near Dillard Road Bridge near Sloughhouse, California, showing translational slide failure (photographed May 31, 1983).

FIGURE 10. Riprap on Cosumnes River at site 3 near Dillard Road Bridge near Sloughhouse, California, looking downstream, showing modified slump failure (photographed May 31, 1983).

The probable causes of modified slump failures are:

- o Bank side slope is so steep that the riprap is resting very near the angle of repose. Any imbalance or movement of individual stones creates a situation of instability for other stones in the riprap.

- o Certain stones, critical in supporting upslope riprap, are dislodged by settlement of the submerged riprap, impact, abrasion, or particle erosion. The loss of support provided by the key stones results in the downslope movement within a local area near the point of the dislodged stones. This cause of failure may be reduced in frequency if the riprap material is of proper size gradation.

Slump

A slump is a rotational-gravitational movement of material along a concave surface of rupture. This type of failure is unlike a modified slump in that the failure zone is dish-shaped rather than a relatively flat plane (fig. 4). The cause of the slump failure is related to shear failure of the underlying base material that supports the riprap. As discussed by Schuster and Krizek (1978), the rupture may not occur simultaneously over the failure area, but propagates from a local point. The displaced mass, including the riprap, moves downslope beyond the original failure area onto the surface of the riprap (fig. 11). The primary feature of a slump failure is the localized displacement of base material along a slip surface, which is usually caused by excess pore pressure that reduces friction along a fault line in the base material. The scarp at the head of the slump is located in both the base and riprap material and may be almost vertical. With progressive slump failures along the face of the riprap, the areas of instability may enlarge until the entire bank has failed and a new lower gradient bank slope is present. As with a modified slump, once a failure has occurred, displaced rock in an area of slump tends to create turbulence that accelerates the action of particle erosion.

The probable causes of slump failures are:

- o Nonhomogeneous base material with layers of impermeable material that act as fault planes when subject to excess pore pressure.

- o Side slope too steep, and gravitational forces exceed the inertia forces of the riprap and base material along a friction plane.

- o Too much overburden at the top of the slope; may be caused in part by the riprap.

Hydraulics Associated with Riprap Failures of Selected Streams

The hydraulics of streams associated with the four types of riprap failure (discussed in the preceding section) were documented for this study at five sites. Table 4 lists the hydraulic properties and riprap material for three of the four types of failure. Shear stress was determined by applying equation 5. The depths used are from surveyed peak water-surface elevations for one specific flood, even though several periods of high flow may have contributed to the observed riprap failure at some sites. All of the failures studied occurred in reaches where the channel was contracting. A ratio termed "flow contraction" was computed for each site in order to have a common basis for comparison among the various sites. The flow contraction is the ratio of flow area at the most contracted section in the study reach to the area at the largest upstream section. The contraction of flow may be caused by a lateral constriction, as on Pinole Creek, or a vertical constriction, as on the Sacramento River at E-10.

The ratio of particle sizes D_{85} to D_{50} (sizes at which 85 and 50 percent of the particles, by number, are finer than the indicated size) is used to indicate the size gradation of the riprap material. The recommended ratio, from data given in HEC-11, is about 1.4. If the ratio is too large, passage of flow through the voids in the riprap is relatively easy and may undermine the riprap or base material, ultimately causing a modified slump failure. The design D_{50} that would be obtained from procedures outlined in HEC-11, HEC-15, Cal-B&SP, and EM-1601, is presented for comparison with the median (D_{50}) stone size used in each failed installation. The Cal-B&SP procedure gave the largest size D_{50} for all but one site. For the Truckee River at Sparks, Nevada, the D_{50} from the EM-1601 procedures was larger than that from the Cal-B&SP methods because of the combined effect of depth and channel slope.

FIGURE 11. Riprap on left bank of Cosumnes River at site 1 near Dillard Road Bridge near Sloughhouse, California, showing slump failure (photographed May 31, 1983).

Table 4. Hydraulic, riprap, and damage characteristics of selected sites with riprap failure.

Parameter	Pinole Creek at Pinole, CA (cross-section 3)	Sacramento River at E-10 near Chico, CA	Hoh River at site 1, cross section 3, near Forks, WA	Cosumnes River at site 3 near Dillard Road Bridge near Sloughhouse, CA	Truckee River at Sparks, NV
Hydraulic:					
Date	1-04-82	1-27-83	10-22-82	3-13-83	3-13-83
Discharge (ft^3/s)	2,250	[1]98,000	22,000	26,100	7,340
Water-surface slope	0.0054	0.000556	0.0014	0.00070	0.0030
Manning's n	0.030	0.033	0.035	0.030	0.035
Mean velocity (ft/s)	7.7	6.7	7.94	4.08	5.19
Maximum depth (ft)	7.7	34.5	19.1	31.0	17.5
Depth of flow above toe of damaged riprap (ft)	7.7	13.0	19.1	10.2	17.5
Mean depth (ft)	4.9	20.2	3.52	18.6	10.5
Curvature angle (°)	66	11	80.5	99	18
Curvature radius (ft)	150	4,280	991	458	646
Flow contraction[2]	0.87	0.63	0.76	1.25	0.88
Shear stress[3] (lb/ft^2)	2.59	0.90	1.67	0.812	3.27
Froude number, F	0.61	0.26	0.76	0.17	0.28
Revetment material:					
D_{85} (ft)	0.84	0.66	2.5	1.00	1.14
D_{50} (ft)	0.60	0.51	1.3	0.78	0.71
D_{15} (ft)	0.42	0.31	0.58	0.50	0.46
Ratio D_{85}/D_{50}	1.40	1.29	1.92	1.28	1.61
Specific gravity, G_s	2.85	2.60	2.59	2.92	2.68
Design side slope, z	2:1	2:1	1.2:1	1.8:1	1.8:1
D_{50} (ft) computed from following procedures					
HEC-11	0.43	0.30	0.40	0.20	0.20
HEC-15	[4]0.98/[5]0.5	([6])	([6])	([6])	([6])
Cal-B&SP	0.85	0.70	1.4	0.3	0.4
EM-1601	0.60	0.23	0.40	0.195	0.82
Cause of failure:	Particle erosion	Particle erosion	Translational slide	Modified slump	Particle erosion

[1]Main channel discharge.
[2]Ratio of approach to contracted section, as described in text.
[3]Maximum shear stress in cross section, $\tau_0=\gamma d_m S_0$.
[4]D_{50} from chart 27 and appropriate adjustments.
[5]D_{50} from chart C-1.
[6]Chart 27 of HEC-15 is not applicable to depths above 16 ft.

The largest computed shear stress at any of the sites listed in table 4 is 3.3 lb/ft^2, and requires a stone size that is in the range of sizes normally available from quarries.

Procedures in HEC-15 are limited to depths of less than 16 ft and are not applicable to four of the five sites in table 4 because of greater depths. The procedures in HEC-11 and EM-1601 give a D_{50} size similar to the existing material for the sites in table 4 with particle erosion problems, except the Truckee River near Sparks, Nevada. These data suggest that shear stresses derived for the channels in table 4 are not a good indicator of the actual forces acting on the boundary, and procedures for riprap design given in HEC-11 and EM-1601 may yield riprap sizes that are too small.

Pinole Creek at Pinole, California

Damage to the riprap at Pinole Creek (table 4), which was designed by the U.S. Army Corps of Engineers (construction plans dated April 1965) using procedures given in EM-1601, resulted from particle erosion of the riprap from the lower part of the channel banks (fig. 12). A small zone of riprap near the top of the bank remained intact, indicating shear stresses were insufficient to remove the upper material. The fact that the upper zone material remained in place even though vertical support had been removed indicates the side slope of the banks for this riprap was less than the angle of repose. Much of the eroded riprap was found on the channel bed and acted as a flow diverter that directed some of the flow towards the newly unprotected bank. Failure of the riprap at this site is attributed to a particle size D_{50} that is too small for the hydraulic stresses created by this size of flood.

Sacramento River at Site E-10 near Chico, California

The flood plain at this site (figs. 5, 6, and 7) is low and subject to frequent and prolonged inundation. As a result, the entire riprap layer is subject to shear stress. Displacement of individual stones at the site has been documented, and the submerged weight of the largest rock moved was 14.6 pounds (6.63 kg); the intermediate axis was 0.60 ft (0.18 m).

Three localized areas of riprap failure caused by particle erosion were surveyed during the 1983 water year. A unique hydraulic condition at this site is the contraction of flow caused by a vertical rise in the channel bed rather than a reduction in channel width. The vertical constriction is caused by a delta built up in the riverbed by a tributary entering the Sacramento River downstream from the site. The channel bed slope at the site is -0.004815, in comparison with the water-surface slope of 0.000556. This site illustrates the problems in estimating the effective shear stress when slopes are estimated from topographic maps or from the water surface.

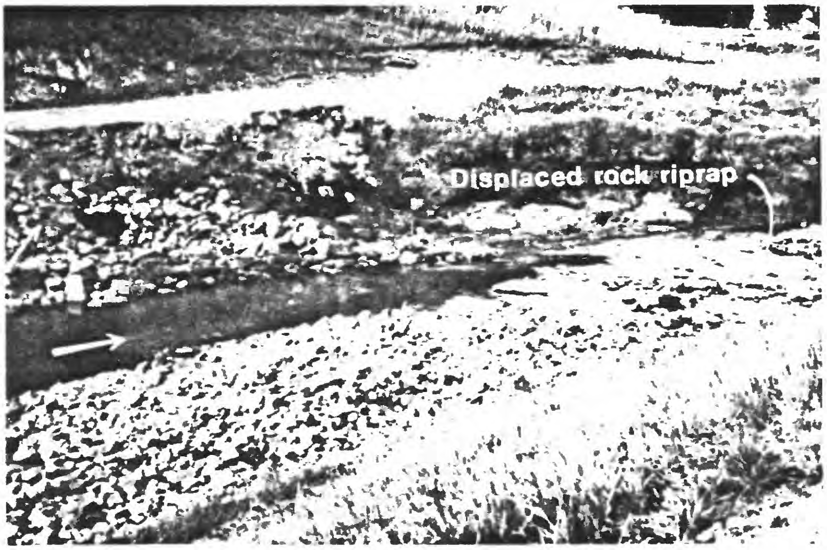


FIGURE 12. Damaged riprap on left bank of Pinole Creek at Pinole, California, following flood of January 4, 1982. Note deposition of displaced riprap from upstream locations in channel bed (photographed March 1982).

Failure of the riprap at this site was initiated by displacement of individual stones (particle erosion). After repeated periods of high water, the riprap lining was eroded to the original base material; however, there was no evidence of base material failure at the site. The gradation of the riprap (ratio of D_{85}/D_{50} = 1.29) is close to the recommended ratio of 1.4 given in HEC-11 and HEC-15 and is within the range specified in EM-1601. Failure of the riprap is attributed to the rock size being too small, and side slope of the bank being too steep.

Hoh River at Site 1 near Forks, Washington

The procedure used for riprap design at this site (figs. 13 and 14) is not known. Particle erosion occurred at two locations during the first several floods after the riprap was installed during summer 1982.

FIGURE 13. New riprap placed on left bank (upstream view) of Hoh River at site 1 near Forks, Washington. Riprap was damaged by modified slump at a location near truck tires during flooding in autumn of 1982 (photographed August 1982).

FIGURE 14. Damaged riprap on left bank (downstream view) of Hoh River at site 1 near Forks, Washington. Site is near bulldozer (top of fig. 13) and about 400 feet upstream from the foreground of site shown in figure 13. Damage is attributed to particle erosion by impinging flows that overtopped bank during flood of December 3, 1982, and extends about 4 feet below top of bank (photographed December 1982).

Riprap damage occurred at cross section 3, located near the trailer shown in figure 13, during October 1982. The damage is attributed to (1) channel bed scour that undermined the toe of the riprap and caused modified slump; (2) poor size gradation of the riprap that allowed erosion of the supporting smaller material in the riprap; and (3) a steep side slope that reduced the amount of force required to displace individual stones. The ratio of D_{85}/D_{50} was 1.92 (table 4), and exceeds the recommended ratio given in all design procedures. Most of the larger stones still in position at the site were at a precarious state of balance. Cracks along the top of the embankment parallel to the channel were observed in November 1982 after the first flood of the winter season 1982-83. These cracks indicated the mass of riprap was unstable and near the angle of repose.

Riprap damage on the left bank at an upstream location (cross section 2, near the bulldozer shown in figure 13) during the flood of December 3, 1982, is attributed to particle erosion. The damaged riprap shown in figure 14 was overtopped about 3 ft (0.9 m) during the flood. Most of the damage occurred near the top of the bank next to the low elevation access road. Riprap erosion may have been caused by irregular patterns of overbank flow in the vicinity of the low bank access road.

Cosumnes River at Site 3 near Dillard Road Bridge near Sloughhouse, California

The riprap at this site (fig. 10) was constructed to prevent lateral migration of the channel. The design procedure is not known. A modified slump failure about 15 ft (4.6 m) wide was noted about 1 month after flooding and 6 months after construction of the riprap. The riprap is subject to impinging flows. Individual pieces of riprap in the slump area were displaced downslope, with the toe of the slump ending up 13 ft (4.0 m) below the top of the bank. The failure is attributed to failure of the interface between the base material and riprap and possible excess hydrostatic pressure in the base material. The location of the riprap failure, which is about 21 ft (6.4 m) above the channel bed, indicates that stresses near the top of the bank may be more critical than stresses defined for the channel bed.

Truckee River at Sparks, Nevada

The riprap at this site was placed to prevent lateral migration of the channel toward the right bank. The age of trees and brush growing along the channel indicates that the riprap was installed more than 10 years prior to the site survey. The channel is curved 18° at the site, and flows are impinging. The riprap shows evidence of overall failure by particle erosion at the outside of the bend (fig. 15).

Site surveys in June 1983 indicated some rocks had recently been displaced (fig. 15). This damage is attributed to the flood of March 13, 1983 (table 4). Field surveys in May 1982 showed extensive riprap damage had occurred during an earlier flood. Streamflow records show a large flood (discharge 8,690 ft^3/s or 245.9 m^3/s) on December 20, 1981. This flood caused peak stages at the site to be about 6 ft (1.8 m) higher than the stage at the time of the survey in May 1982 (measured discharge 3,880 ft^3/s or 109.8 m^3/s).

A right-bank highwater profile of the December 1981 flood was surveyed in May 1982. However, no marks could be found on the left bank. The right-bank profile, shown in figure 16, demonstrates the need for careful surveys to determine the hydraulic properties of a channel, and the difficulty in applying riprap design procedures based on shear stress to sites with bends. For example, in the vicinity of the channel bend, the right-bank profile of the December 20, 1981, peak had a negative gradient. The water-surface elevation increased from 97.8 to 98.3 ft (29.8 to 30.0 m) in a reach 90 ft (27.4 m) long, which included the area where riprap was damaged.

FIGURE 15. Damaged riprap on right bank of Truckee River at Sparks, Nevada. Damage is attributed to particle erosion by impinging flow at channel bend. Area of damage is limited to upper 6 feet of bank (photographed June 15, 1983).

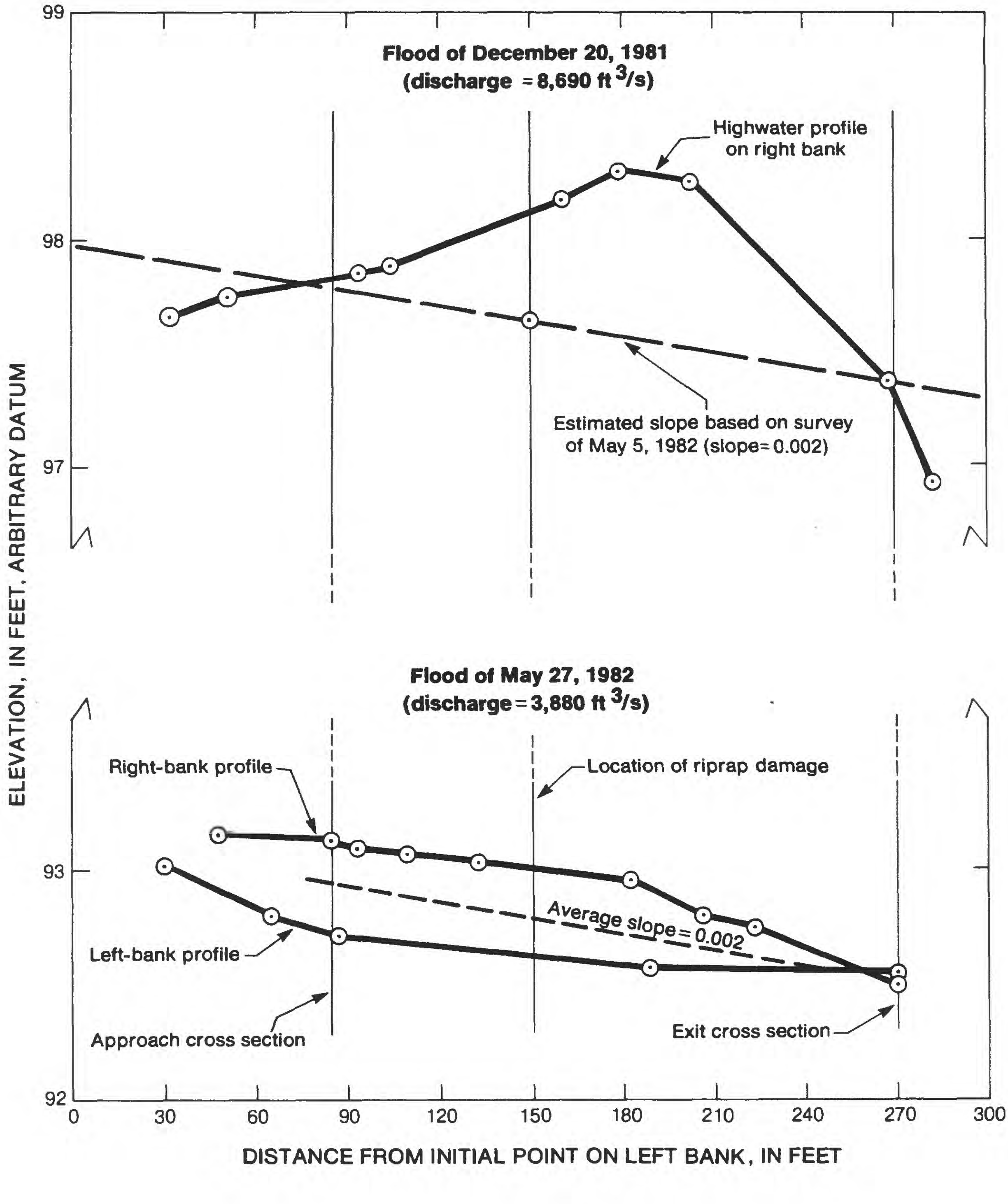


FIGURE 16. Water-surface profiles of Truckee River at Sparks, Nevada, for floods of December 20, 1981, and May 27, 1982.

As an alternative, the slope of the water surface surveyed in May 1982 was applied at the stage of the December 1981 flood. This approach yielded a shear stress of about 2.2 lb/ft^2 (10.7 kg/m^2), which is lower than the stress listed in table 4 for the March 13, 1983, flood. For the March 13, 1983, flood, procedures in EM-1601 indicate a D_{50} of 0.8 ft (0.2 m), which is 0.1 ft (0.03 m) larger than the D_{50} for the material installed. Even if the riprap had met the design requirements of EM-1601, it probably would not have provided adequate protection because (1) the actual water-surface slope is probably steeper than estimated, and (2) the velocity may have been greater than estimated.

There was also localized displacement of individual stones on the shoreward side of a 1-ft-diameter tree that is 7 ft (2.1 m) from the top of the bank, as shown in figure 17. The displacement of riprap near the tree is attributed to localized shear stresses that exceed the critical shear stress of the stones. The failure area was oblong in shape, about 5 ft (1.5 m) long, 1 ft (0.3 m) wide, and 1 ft (0.3 m) deep. Similar localized scour of riprap on channel banks in the vicinity of bridge piers has been noted at several sites.

The study of riprap failure on the Truckee River illustrates that care must be exercised in selecting the slope to be used for design of riprap. Bed slope, average water-surface slope, local water-surface slopes in areas of turbulence, and energy slope differ considerably. The study also illustrates that additional protection may be needed in the vicinity of piers and vegetation which cause local stresses greater than those estimated from design procedures.

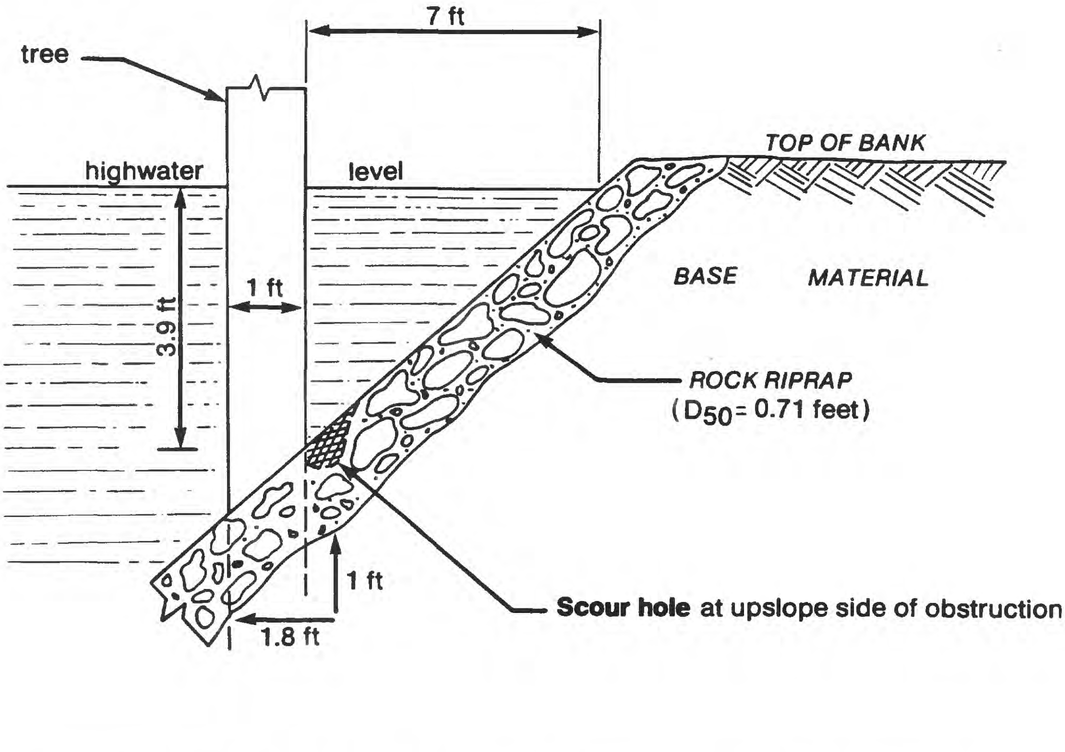


FIGURE 17. Sketch of scour hole in riprap adjacent to obstruction on streambank.

Summary of Factors Contributing to Riprap Failures

Certain hydraulic factors are associated with each of the four types of riprap failure (particle erosion, translational slide, modified slump, and true slump), as indicated by the field data collected at five sites (table 4). The specific mechanism causing failure of the riprap is difficult to determine, and a number of factors, acting either individually or combined, may be involved. Several reasons for riprap failures are identified and grouped below:

- Particle size was too small because:
  - a. Shear stress was underestimated.
  - b. Velocity was underestimated.
  - c. Inadequate allowance was made for channel curvature.
  - d. Design channel capacity was too low.
  - e. Design discharge was too low.
  - f. Inadequate assessment was made of abrasive forces.
  - g. Inadequate allowance was made for effect of obstructions.

- Riprap material had improper gradation.

- Material was placed improperly.

- Side slopes were too steep.

- No filter blanket was installed or blanket was inadequate or damaged.

- Excess hydrostatic pressure caused failure of base material.

- Channel changes caused:
  - a. Impinging flow.
  - b. Flow to be directed at ends of protected reach.
  - c. Decreased channel capacity or increased depth.
  - d. Scour of toe of riprap.

- Differential settlement occurred during submergence or periods of excessive precipitation.

Estimates of particle stability serve as the basis for most riprap design procedures, such as HEC-11, HEC-15, and EM-1601. This approach seems sound because particle erosion is involved in most of the causes of failure described above.

EVALUATION OF RIPRAP DESIGN PROCEDURES

Most publications on riprap design compare results obtained from different methods. For example, results from several methods relating stone size to allowable velocity are compared in appendix A of HEC-11. The methods are those recommended by the California Division of Highways, Bureau of Public Roads, HEC-11, U.S. Bureau of Reclamation, and U.S. Army Corps of Engineers. That comparison is reproduced here as figure 18. Anderson and others, in National Cooperative Highway Research Program (NCHRP) report 108 (1970), compare results from methods that relate stone size to shear stress, as used by Lane and Carlson (1953), and Shields (1936). That comparison is reproduced here as figure 19. In his figure C-1, Normann (HEC-15) compares the results from the methods that relate stone size to critical shear stress. His comparison is a simplified version of the one by Anderson and others. The U.S. Army Corps of Engineers compares design procedures in EM-1601. In their report, stone size is related to velocity, and the relationships suggested by Isbash and the USBR are shown. The Corps expands on the concept of relating stone size to velocity by use of additional hydraulic factors, such as depth, channel curvature, and equivalent roughness. Various relationships of stone size to critical tractive force were compared by Simons and Senturk (1977). The relationships given in their figure 7.8 represent the results of laboratory flume tests and are limited to sizes of stone riprap (D_{50}) less than 0.13 ft (0.040 m) diameter.

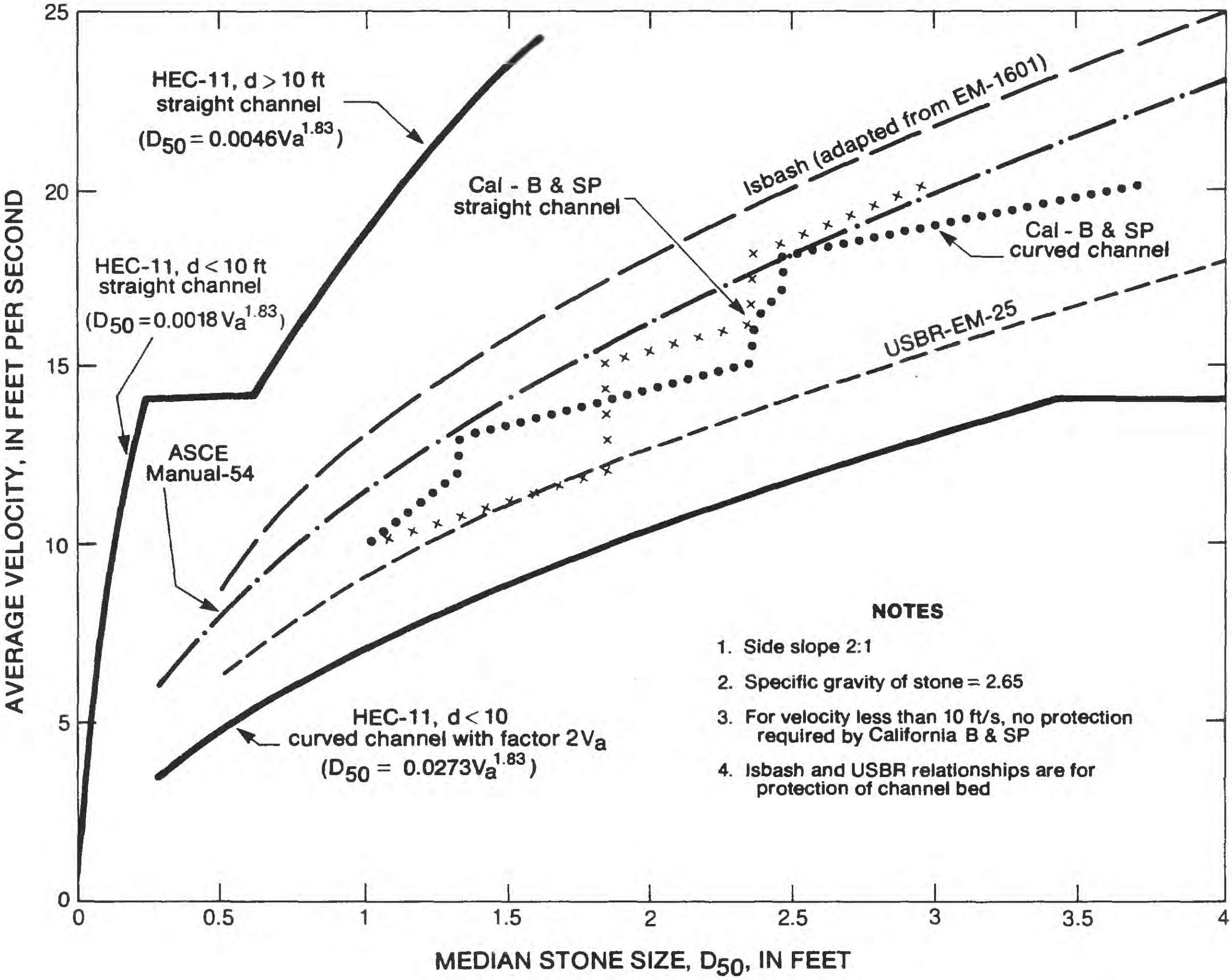


FIGURE 18. Comparison of procedures for estimating stone size on channel bank based on permissible velocities. (Adapted from appendix A of HEC-11, Searcy, 1967).

For this study, the Federal Highway Administration requested that procedures in their Hydraulic Engineering Circulars 11 and 15 be evaluated in regard to (1) technical accuracy, (2) adequacy of design specifications obtained by using the method, (3) ease of application, and (4) method of presentation. Methods recommended in HEC-11 and HEC-15 were to be compared to those recommended by other government agencies. A large part of this evaluation involved a study of field sites where riprap had failed. The failures, such as on Pinole Creek at Pinole, California, are described in previous sections of this report. The size and gradation of material at those sites are compared to those that would be obtained by applying the various methods. The results of this evaluation are to be the basis for new design guidelines.

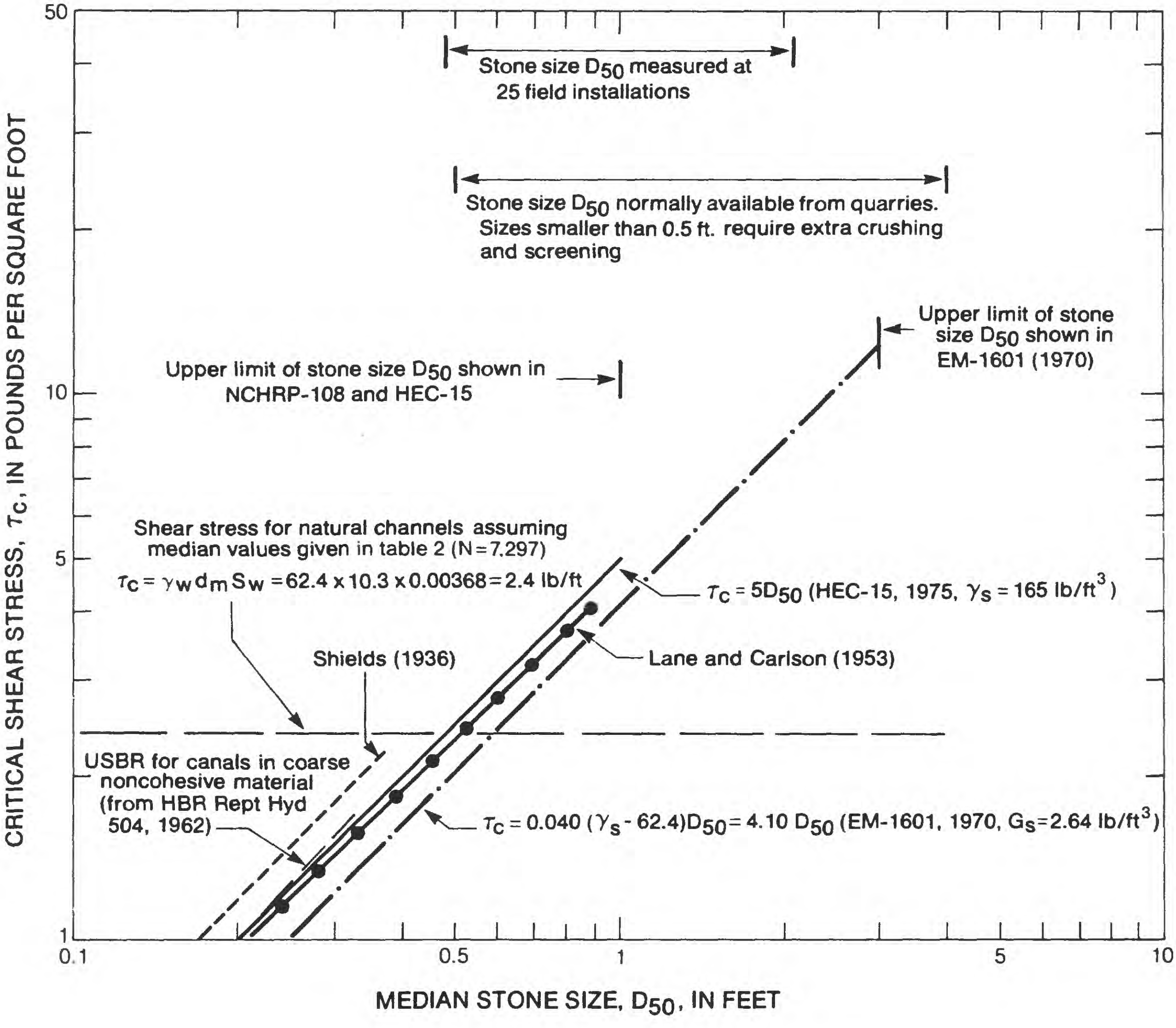


FIGURE 19. Comparison of procedures for estimating stone size on channel bed using critical shear stress (adapted from report by Anderson and others, 1970).

Procedures recommended by Man-54, Cal-B&SP, USBR-EM-25, and HEC-11 for curved channels give stone sizes that may have been adequate to prevent the riprap failures on Pinole Creek at Pinole, California, and the Sacramento River at E-10 near Chico, California (table 4). In terms of applicability and ability to handle situations such as variable side slopes and channel curves, the methods by USBR-EM-25 and HEC-11 for curved channels appear to be the most appropriate.

Three of the five methods compared in figure 19 do not consider median stone size (D_{50}) larger than 1.0 ft (0.30 m), but the median size available from rock quarries ranges from about 0.5 to 4 ft (0.15 to 1.2 m). To obtain a D_{50} smaller than 0.5 ft (0.15 m) requires extra crushing and screening. Materials of this size are usually obtained from road paving stockpiles.

The average shear stress for open channels, based on the median slope (0.00368) for 297 sites in table 1 of volume 1, is 2.4 lb/ft^2 (11.7 kg/m^2). This value of shear would indicate a D_{50} of 0.5 to 0.6 ft (0.15 to 0.18 m), which is near the lower size limit of commercially available rock riprap.

The channel slope for about 25 percent of the sites listed in table 2 is steeper than 0.010. The shear stress for a slope of 0.010 and a flow depth of 10.3 ft (3.14 m) is 6.43 lb/ft^2 (31.4 kg/m^2). Only EM-1601 provides a stone size for a shear stress this large.

Data for Comparison of Design Procedures

For this report, the various methods for designing riprap were compared using a set of hydraulic data for Pinole Creek at Pinole, California. Data for the Pinole Creek site (fig. 20 and table 5) were used because the cause of riprap failure was known and an extensive set of hydraulic data was available. Pinole Creek is located approximately 17 mi (27.4 km) northeast of San Francisco on Interstate 80. The reach of channel studied was protected with riprap by the Corps of Engineers in cooperation with Contra Costa County in 1966. Several failures occurred over the surveyed reach as a result of high flows from the flood of January 4, 1982. Two indirect measurements of the peak flow, high-water profile data, and 17 cross sections were surveyed in March 1982 to determine the peak discharge and channel geometry. Original design data were obtained to supplement the field data.

The channel of Pinole Creek was designed for a flood discharge of about 2,500 ft^3/s (70.75 m^3/s). The channel had a bottom width of 20 ft (6.1 m) for most of the reach that is over 1,400 ft (426.7 m) long. Channel banks were designed with a 2:1 side slope. Rock riprap placed throughout the reach in 1966 was severely damaged during the January 4, 1982, flood (discharge 2,250 ft^3/s) by particle erosion. The Manning's roughness coefficient n ranges from 0.027 to 0.048 based on verification studies made after the January 1982 flood. A chute structure, built to reduce the channel gradient in the vicinity of cross section 0.4 (D_{50} = 2.3 ft or 0.70 m), caused supercritical flow. Cross section 0.2 in table 5 is in a straight reach of channel, and cross section 3 is just downstream from the apex of a 66° curve, as shown on the aerial photograph of the site in figure 20. The data in table 5 summarize the hydraulic and riprap characteristics at cross sections 0.2 and 3 and were used as a common base for evaluating the various riprap design procedures in subsequent sections of this report.

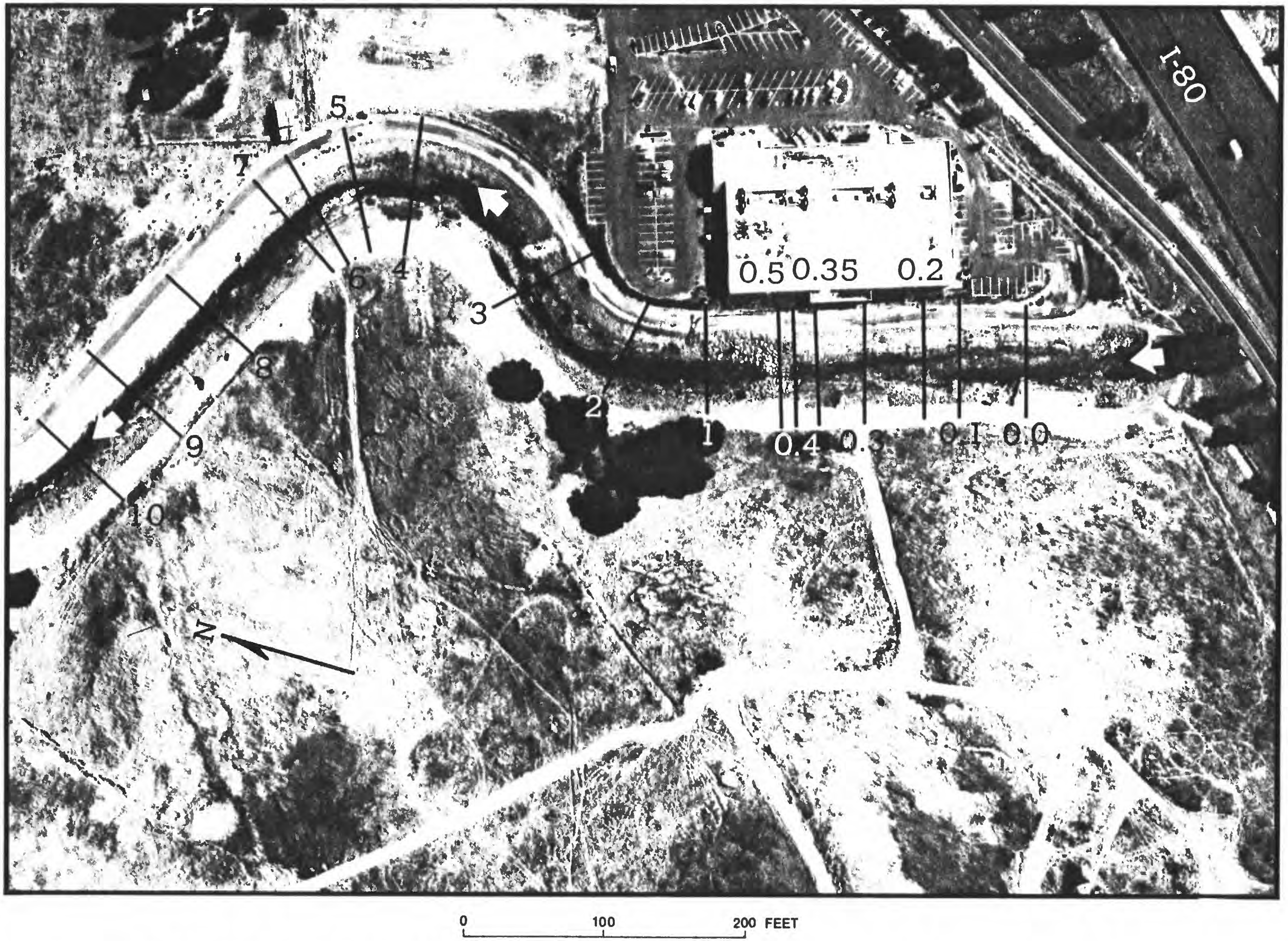


FIGURE 20. Aerial photograph of Pinole Creek at Pinole, California, showing study reach (October 1, 1982).

Table 5. Hydraulic properties of the January 4, 1982, flood at cross sections 0.2 and 3 on Pinole Creek at Pinole, California.

[Locations of cross sections are shown in figure 20. Values are based on cross-section geometry surveyed March 2, 1982]

	Cross-section 0.2 (straight reach)	Cross-section 3 (curved reach)
Discharge (ft^3/s)	2,250	2,250
Water-surface elevation (ft)	92.59	88.29
Water-surface slope (S_w)(ft/ft)	0.0036	0.0054
Area (ft^2)	275	293
Width of water surface (ft)	54	61
Mean velocity (ft/s)	8.2	7.7
Maximum depth (ft)	8.3	7.7
Average depth (ft)	5.1	4.8
Hydraulic radius (ft)	4.74	4.62
D_{50} (ft)	0.47	0.60
Manning's n	0.035	0.030
Ratio D_{85}/D_{50}	1.74	1.40
Side slope, z	1.6:1	2:1
Angle of curvature (°)	0	66
Radius of curvature (ft)	0	150
Specific weight of rock (γ_s) (lb/ft^3)	172	178

The failure of the riprap at section 0.2 is related to (1) the steep bank slope, and (2) the expansion of flow, and associated turbulence that resulted when flow velocities decreased downstream from the culvert under Interstate 80. Hydraulic data for cross section 0.2 were not used in the comparison of design methods, but are included for comparison with hydraulic data at cross section 3. The riprap failure at cross section 3 is attributed to excessive shear stress and inadequate size of rock in the vicinity of a channel bend.

The actual size riprap (in terms of D_{50}) that would be required to adequately protect the bank near cross section 3 for a discharge of 2,250 ft^3/s (63.67 m^3/s) is not precisely known. It may be safely assumed, however, that those methods that result in a median stone size smaller than the size that was installed (D_{50} =0.47 ft or 0.14 m at section 0.2 and 0.60 ft or 0.18 m at section 3) (table 5) would not have provided adequate protection of the banks.

Evaluation of Hydraulic Engineering Circular No. 11 (HEC-11)

The procedures in HEC-11 and HEC-15 for designing rock riprap each require different hydraulic data and provide different results for the same design situation, as indicated in table 4. The procedure in HEC-11 was developed on the basis of slope protection methods in use prior to 1948 and on research on the protection of upstream slopes of earth dams, which was current in 1967. The principal sources of material used in preparation of the circular include the subcommittee report "Review of Slope Protection Methods" by the American Society of Civil Engineers (1948), and procedures developed by the Corps of Engineers for the hydraulic design of rock riprap (Campbell, 1966). The circular also discusses the types of riprap in common use and compares methods for determining stone size and design of filter blankets. Although not stated in the circular, the procedures are applicable for subcritical flow conditions only (Froude number less than 1.0).

Description

The basic design procedure outlined in HEC-11 is illustrated by the flow chart in figure 21. The size of rock (stone) riprap needed to protect the streambank and bed is determined on a trial-and-error basis using an estimated stone size, design flow velocity, and depth.

Two graphs in HEC-11 are used to relate the velocity of flow against the channel bed to the size of stone needed to resist displacement from the banks for various side slopes. Depths of flow that exceed 10 ft (3.05 m) are multiplied by 0.4. For impinging flow, the velocity is multiplied by a factor ranging from 1 to 2 to obtain an estimate of the flow velocity against the rock.

The rock diameter, k, is equivalent to the median diameter (D_{50}) of the riprap. Procedures outlined in HEC-11 for determining rock size are based on the unit weight of stone equal to 165 lb/ft^3 (2,640 kg/m^3). A procedure is presented to adjust the rock size required for other specific weights of stone. This procedure, described on page 11-4 in HEC-11, should be shown as:

$$k_w = \frac{102.5k}{w-62.5} \tag{10}$$

where k = stone size from figure 2 of HEC-11

k_w = stone size for stone of w pounds per cubic foot

w = unit weight of proposed stone riprap, in pounds per cubic foot

The basic requirement in riprap design is to relate the forces of streamflow to the resisting forces of inertia and friction of the riprap. HEC-11 uses stream velocity as a measure of streamflow forces, which are then related to the required rock size that will resist displacement.

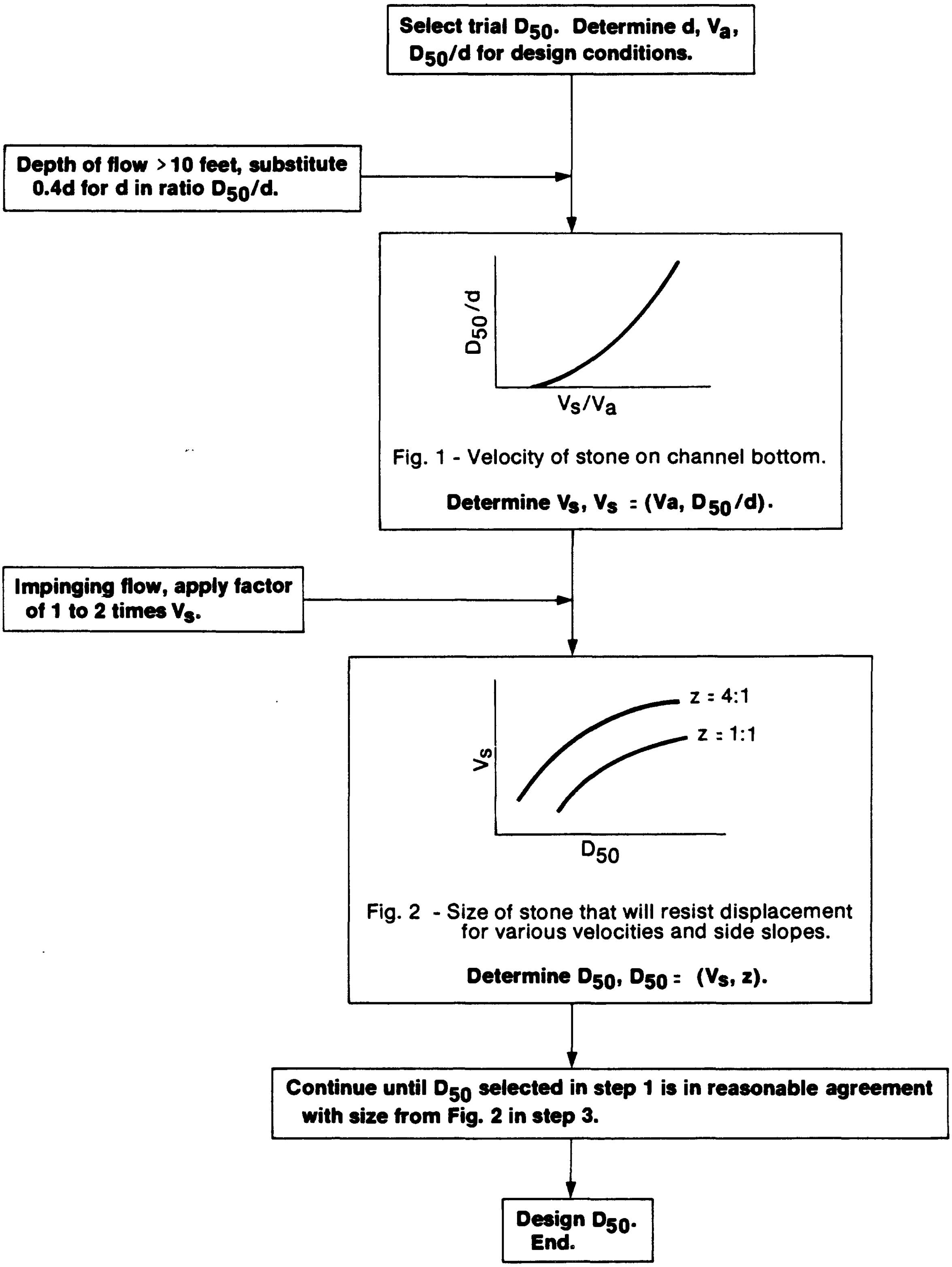


FIGURE 21. Flow chart of riprap design procedure for HEC-11. (The factor k in figure 1 of HEC-11 equals the median particle size, D_{50}).

Hydraulic Factors

Channels designed and built as trapezoids develop a primary flow path within the trapezoid that generally results in local velocities that are higher than anticipated in the design. Estimates of average velocity for main channel flow of streams with overbank flow may be low if large areas of overbank flow are included in the calculations. In addition, the accuracy of the estimated average velocity is dependent on the reliability of the computed water-surface elevation and flow area. For example, in a trapezoidal channel of widths normally encountered in the field, with side slopes of 3:1 and depths of 5 to 10 ft (1.5 to 3.05 m), a 20 percent error in water-surface elevation can result in a cross-sectional area that is as much as 40 percent in error.

Curves of velocity against stone, V_s, given in figure 2 of HEC-11 are extended to velocities greater than those generally encountered. In a straight reach of subcritical flow, maximum point velocities (at any location in the cross section) exceeding 16 ft/s (4.9 m/s) and mean velocities exceeding about 8 ft/s (2.4 m/s) are unusual (table 2) for a typical gravel and cobble bed channel. Higher velocities may occur in sand bed streams.

A relationship of velocity versus stone weight, similar to figure 2 of HEC-11, is presented in EM-1601. The relationship labeled Isbash in EM-1601, plate 29, is similar to the curve in figure 2 of HEC-11 for a channel with 2:1 side slope. The data used to develop the relationships shown in plate 29 of EM-1601 and figure 2 of HEC-11 are from hydraulic model experiments and observations of special events such as dam closures or stilling basin analyses. These relationships, therefore, may not be realistic indicators of the interaction between velocity and stone weight (size) for typical streamflow conditions.

In HEC-11, figure 1 relates D_{50}/d_m to V_s/V_a and figure 2 relates V_s to stone size. Figure 1 is reproduced here in modified form as figure 22. The curve for a 2:1 side slope from figure 2 is reproduced here as figure 23. For straight channels, no hydraulically reasonable values of d and V_s can be selected that result in a D_{50} larger than about 0.5 ft (0.15 m). This size is too small to resist displacement during most floodflows.

The factor of 1 to 2 for adjusting velocity against stone for impinging flow at bends is arbitrary and no guidelines for applying this factor are given. The only justification for extending the ordinate in figure 2 of HEC-11 above 10 ft/s (3.05 m/s) is to accommodate velocities that may occur near energy dissipaters or velocities that have been derived to account for channel curvature.

Application of the riprap design procedure requires an initial estimate of D_{50}. The trial-and-error procedure continues until the estimated and design D_{50} are in reasonable agreement. However, no guidelines are given to indicate the level of reasonable agreement.

Procedures for designing riprap in HEC-11 are generally straightforward and easy to use but some of the details for applying the procedure are presented in the appendix of HEC-11. This inhibits the logical sequence in the design procedure. Not all graphs are presented in consistent units of measure; some use units of feet for stone size, and others use units of inches.

Riprap Stability Factors

The effects of channel side slope and stone size on riprap stability are considered in HEC-11. Using the concept that the maximum permissible velocity (and shear) for a stable channel should be less than critical values, relationships have been prepared for various side slopes and presented in figure 2 of HEC-11. The curve for a side slope of 12:1 or bottom is the same relationship derived by Isbash and shown in EM-1601, plate 29. The other relationships shown in figure 2 of HEC-11 are for steeper side slopes. As expected, for a given stone size (D_{50}), material on flatter side slopes will resist shear stresses better than material on steep slopes, due to gravity helping to dislodge the material.

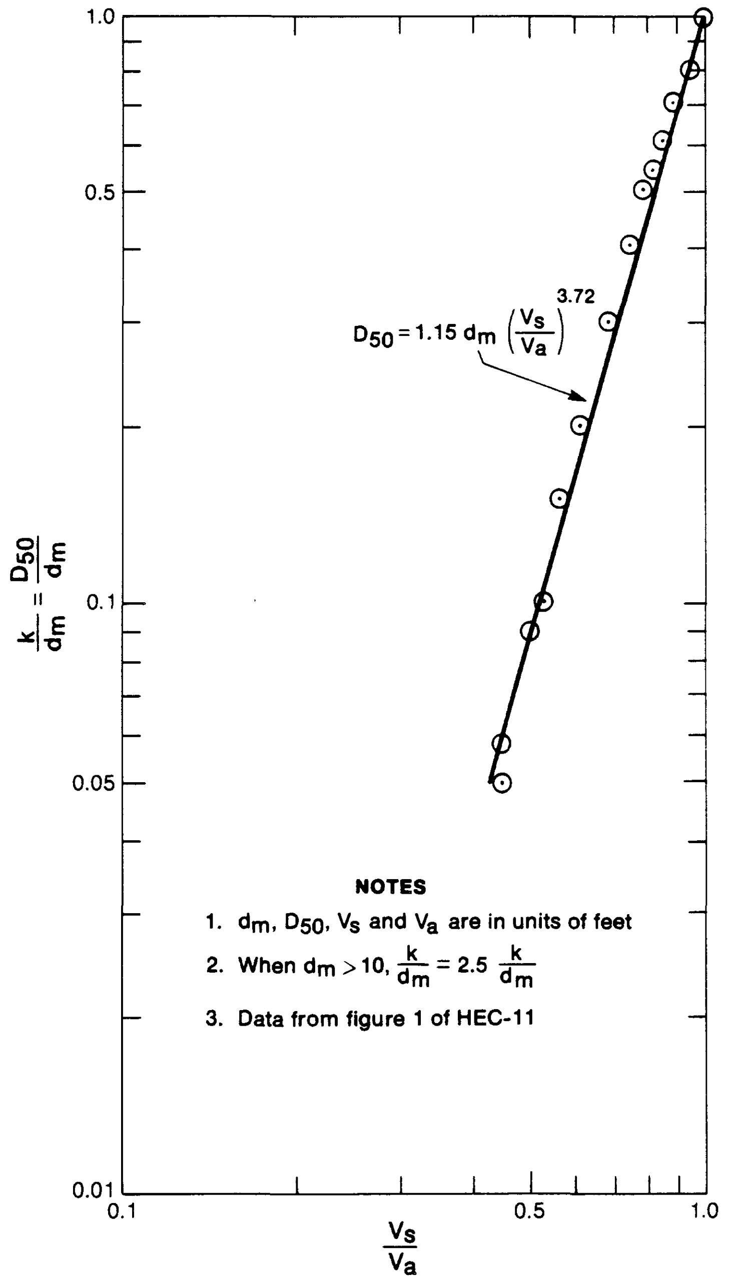


FIGURE 22. Relationship of D_{50} stone size on channel bottom to velocity against stone.

Other procedures discussed in HEC-11 are for:

- o Thickness of stone layer
- o Design of filter blanket
- o Gradation requirements of riprap.

HEC-11 procedures apparently assume the stone shape of the riprap material is angular. There is some confusion in interpreting figure 8 of HEC-11 to determine which cases failed and which gradations were satisfactory in laboratory tests. It is suggested that only the curves that represent satisfactory gradations (that is, A_1 and B) be shown in a revised manual. A family of curves that represent different classes of riprap designated by the 50 percentile size (D_{50}) could be shown. These curves could then be related to the discussion of riprap specifications and class sizes on pages 11-37, -38, and -39. The mean (median) diameter of stone (D_{50}) on figure 8 of HEC-11 should be in units of feet instead of inches.

Application

A detailed analysis of the design procedures given in HEC-11 required the conversion of the various graphical relationships to regression equations. Because the relationships in figures 1 and 2 of HEC-11 are curvilinear, regression equations were developed using the data transformed on the basis of logarithms, as shown in figures 22 and 23. To simplify notation of the variables in this analysis, k in figure 1 of HEC-11, which is considered equivalent to median stone size, is denoted as D_{50}. For streams with maximum depths greater than 10 ft (3.05 m), the depth used in computing the ratio in figure 1 of HEC-11 is adjusted by multiplying depth (d) by a factor of 0.4. This adjustment is to increase the stone size near the water surface. To further simplify the analysis, the regression equation was developed for a channel with an assumed side slope of 2:1 because most design criteria recommend side slopes between 1.5:1 and 3:1. Because the stone size, D_{50}, is the unknown variable, the regression equations derived from figures 22 and 23 (figs. 1 and 2 of HEC-11) were combined, and include the hydraulic factors of mean velocity and maximum

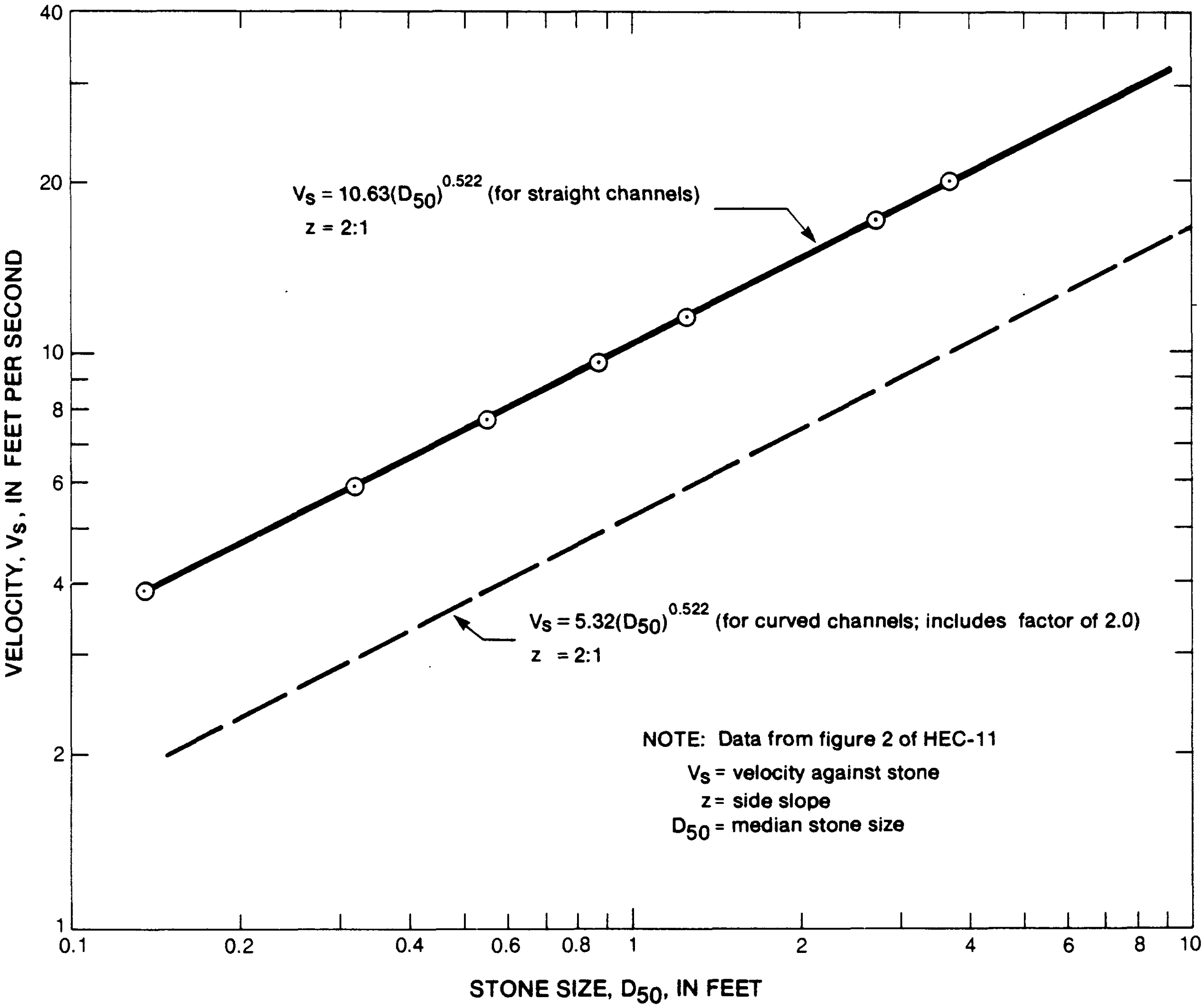


FIGURE 23. Relationship of stone size to velocity.

depth. For channels with impinging flow, the California Division of Highways (in HEC-11) recommends doubling the velocity against stone (V_s), and Lane (in HEC-11) recommends increasing the velocity against stone by 22 percent. In summary, HEC-11 recommends that a factor varying from 1 to 2, depending upon the severity of attack by the current, be applied to determine the velocity against stone (V_s). The coefficient, C, for determining D_{50} is based on an adjustment factor of 2 for curved channels.

The regression equation is:

$$D_{50} = \frac{CV_a^{3.95}}{d_m^{1.06}} \tag{11}$$

where C = 0.000076 for z = 2:1, d_m<10 ft, and straight channel

C = 0.000202 for z = 2:1, d_m>10 ft, and straight channel

C = 0.00117 for z = 2:1, d_m<10 ft, and curved channel

C = 0.00310 for z = 2:1, d_m>10 ft, and curved channel

D_{50} = stone diameter at 50 percentile point on a size distribution curve

V_a = mean velocity (ft/s)

d_m = total depth (ft)

z = side slope

The regression equation (eq. 11) for determining D_{50} using HEC-11 procedures includes the hydraulic factors of velocity and depth of flow. To combine these factors, and assuming subcritical flow conditions, the velocity and depth components of this equation can be related to the Froude number. The Froude number is related to the forces of gravity, and is represented by the equation:

$$F = \frac{V_a}{\sqrt{gd_a}} \tag{12}$$

where F = Froude number

V_a = mean velocity in cross section

g = gravity

d_a = mean depth of flow in cross section

Hydraulic data presented in table 4 indicate flow conditions were subcritical at all sites. In general, when the Froude number is less than 0.95, flows are subcritical. The Corps of Engineers, in EM-1601, indicates supercritical flow may occur when the Froude number exceeds 0.86. For purposes of this analysis, flows are considered subcritical up to a Froude number of 0.95. When the Froude number exceeds this value, flow conditions are borderline or supercritical. With supercritical flow, the depth is less than when flows are subcritical, the applicability of Manning's n becomes questionable, and a flow expansion represented by a hydraulic jump and turbulent flow may occur at some location in the reach. For borderline subcritical flow conditions ($F \lesssim 0.95$), which are considered to be the worst case for riprap design, the mean depth is related to mean velocity by the equation:

$$d_a = 0.0344\, V_a^2 \tag{13}$$

The data in table 2 also indicate the maximum depth in a channel is about 50 percent greater than the mean depth. Therefore, the maximum depth when supercritical flow is about to occur may be represented by the equation:

$$d_m = 0.0516\ V_a^{2} \tag{14}$$

Combining equation 14 with equation 11 results in the following expressions that were used to determine the median stone size, D_{50}, directly for worst case design conditions of borderline subcritical (in other words, almost critical) flow conditions and a bank slope (z) of 2:1.

$$D_{50} = CV_a^{1.83} \tag{15}$$

where C = 0.0018 for z = 2:1, d_m<10 ft, $F\lesssim 0.95$, and straight channel

C = 0.0046 for z = 2:1, d_m>10 ft, $F\lesssim 0.95$, and straight channel

C = 0.0273 for z = 2:1, d_m<10 ft, $F\lesssim 0.95$, and curved channel

C = 0.0718 for z = 2:1, d_m>10 ft, $F\lesssim 0.95$, and curved channel

The mean velocity in the cross section rather than an adjusted velocity against the stone was used in the development of equation 15 and used in the comparisons shown in figure 18. The break points of the HEC-11 curves in the figure at a velocity of 14.0 ft/s (4.27 m/s) represent the transition in the use of figure 2 of HEC-11 when depths exceed 10 ft (3.05 m).

The relationships presented in figures 1 and 2 of HEC-11 (figs. 22 and 23 of this report) require an estimate of the velocity against stone (V_s). No data are presented in HEC-11, however, to estimate these velocities, and because abscissa values of the ratio V_s/V_a of figure 1 of HEC-11 are less than 1, velocities against the stone are always shown to be less than average velocities in the channel. However, the streamflow acting against the streambank or bed is in a state of turbulence, with the severity of turbulence depending on the size of boundary material, shape of channel, bank irregularities, and velocity of flow. As a result, effective velocities (in causing scour) near the stream boundary may be greater than average for the cross section. This suggests use of the relationship shown in figure 1 of HEC-11 may give unsatisfactory results.

The analysis of the design procedures recommended in HEC-11 indicates that rock sizes obtained using this circular are too small for straight reaches of channels (no velocity adjustment factor) and may be larger than needed for curved channels (velocity adjustment factor of 2.0). Curve number 2 in figure 9 of HEC-11 represents an adjustment factor to increase the velocity against the stone by about 50 percent over that of other design methods.

The results of procedures given in HEC-11 for design of rock riprap for channel conditions measured at Pinole Creek at Pinole, California, are given in table 4. The D_{50} (0.43 ft or 0.13 m) computed with HEC-11 procedures (eq. 11) was smaller than the D_{50} in place (0.60 ft or 0.18 m), which failed; therefore, the computed design size is considered too small to protect the streambank.

Summary Discussion of Circular HEC-11

Page 11-4: Adjustment of stone size on basis of specific gravity. The specific gravity of the riprap at 23 sites ranged from 2.36 to 2.95. These values are within 12 percent of the assumed condition given in HEC-11 where weight of stone, W = 165 lb/ft^3 (2,640 kg/m^3). Because errors in our ability to assess the hydraulic forces and the resisting forces to erosion are of the same magnitude, this adjustment procedure may be unwarranted unless the specific gravity differs from 2.65 by more than 10 percent.

Page 11-4 and 11-5: The design procedure described on these pages for both straight and sinuous channels is not comparable, as shown in figure 18. The use of the adjustment factor of 2 for sinuous channels results in a permissible velocity relationship that is closer to the field data obtained during this study, but still provides stone sizes that are too small for bank protection.

Page 11-6: The relationships in figure 2 for side slopes between 1½:1 and 4:1 are considered reasonable; it is recommended that the figure be redrafted to expand the detail for velocities between 3 to 10 ft/s (0.9 to 3.0 m/s).

Page 11-7, second paragraph: The statement is made that bank protection should extend a minimum of 5 ft (1.5 m) below the thalweg elevation of the channel. The designer should assume the location of the thalweg may occur at any point in the active waterway. The source of the 5-ft (1.5-m) depth is unknown, but data in table 15 of volume 1 address the magnitude of scour depths and indicate that a constant value of 5 ft will not necessarily be adequate.

Page 11-8, figure 3A: The toe trench detail should be modified to show riprap extending below the streambed, as indicated in figure 24.

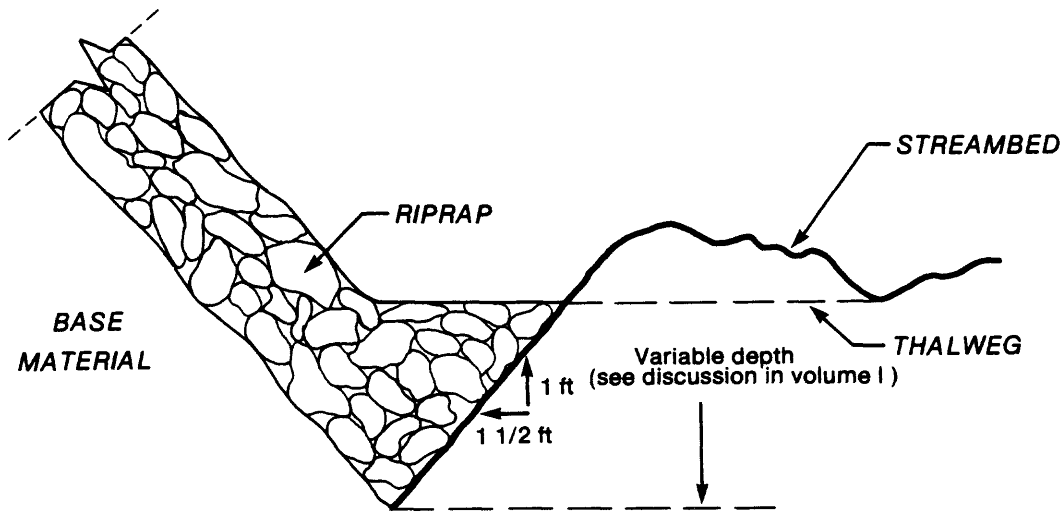


FIGURE 24. Toe trench detail for riprap protection.

Page 11-16, design of filter blanket: The suggested criteria for the particle size requirement is:

$$\frac{D_{15} \text{ (of riprap)}}{D_{85} \text{ (of bank)}} < 5 < \frac{D_{15} \text{ (of riprap)}}{D_{15} \text{ (of bank)}} < 40 \tag{16}$$

The discussion indicates that a filter blanket is usually needed to prevent water from removing base material through voids in the riprap. It appears that the requirement for a filter blanket should apply when the base material is non-cohesive material, such as gravel or sand, or when the filter ratio is outside the limits given in equation 16. A filter blanket may not be necessary because:

- A filter blanket will not reduce the effect of excessive shear stresses that act to remove individual riprap stones.

- A fiberglass or plastic filter blanket may actually be detrimental to the stability of the riprap layer. This type of filter blanket will tend to reduce the amount of friction that prevents slippage of the riprap stones along the face of the bank material and may increase the potential for failure by a translational slide.

- The pervious filter blanket will not prevent excessive pore pressure that is conducive to the planar or rotational slippage that accompanies a slump failure.

One benefit of a filter blanket is that a relatively smooth base material surface should be graded prior to placement of the filter blanket and riprap. If the material is compacted and bank perturbations minimized, there is less possibility of stone erosion caused by local areas of turbulence.

Evaluation of Hydraulic Engineering Circular No. 15 (HEC-15)

The procedure for designing riprap presented in HEC-15 by Normann (1975), is based on the concept of maximum permissible depth of flow and hydraulic resistance of the lining material. The basic concept for evaluating bed and bank stresses, which are then related to permissible depth of flow, was developed by Anderson at the University of Minnesota. Results of his investigation are presented in the National Cooperative Highway Research Program (NCHRP) report number 108 (Anderson and others, 1970). Stated in general terms, HEC-15 presents procedures to determine the hydraulic factors of the proposed channel and procedures to prevent bed and bank erosion. These procedures were derived from the report by Anderson and others (1970) and surveys of other literature, such as a study on flood protection at bridge crossings by Simons and Lewis (1971), which were then incorporated into a new design manual for triangular and trapezoidal channels. The methods presented in HEC-15 were verified by laboratory tests of hydraulic models and field observations at four sites. The field evaluation of the tentative design procedure for riprap linings is discussed in project report number 146 by Anderson (1973).

Presented in HEC-15 are procedures for the design of granular filter blankets, use of plastic filter cloth, design of rock riprap, and a new procedure for design of channel protection at bends. Because the hydraulic factors of a proposed channel are needed in the design procedure, methods are provided to determine hydraulic factors such as channel geometry and roughness coefficient.

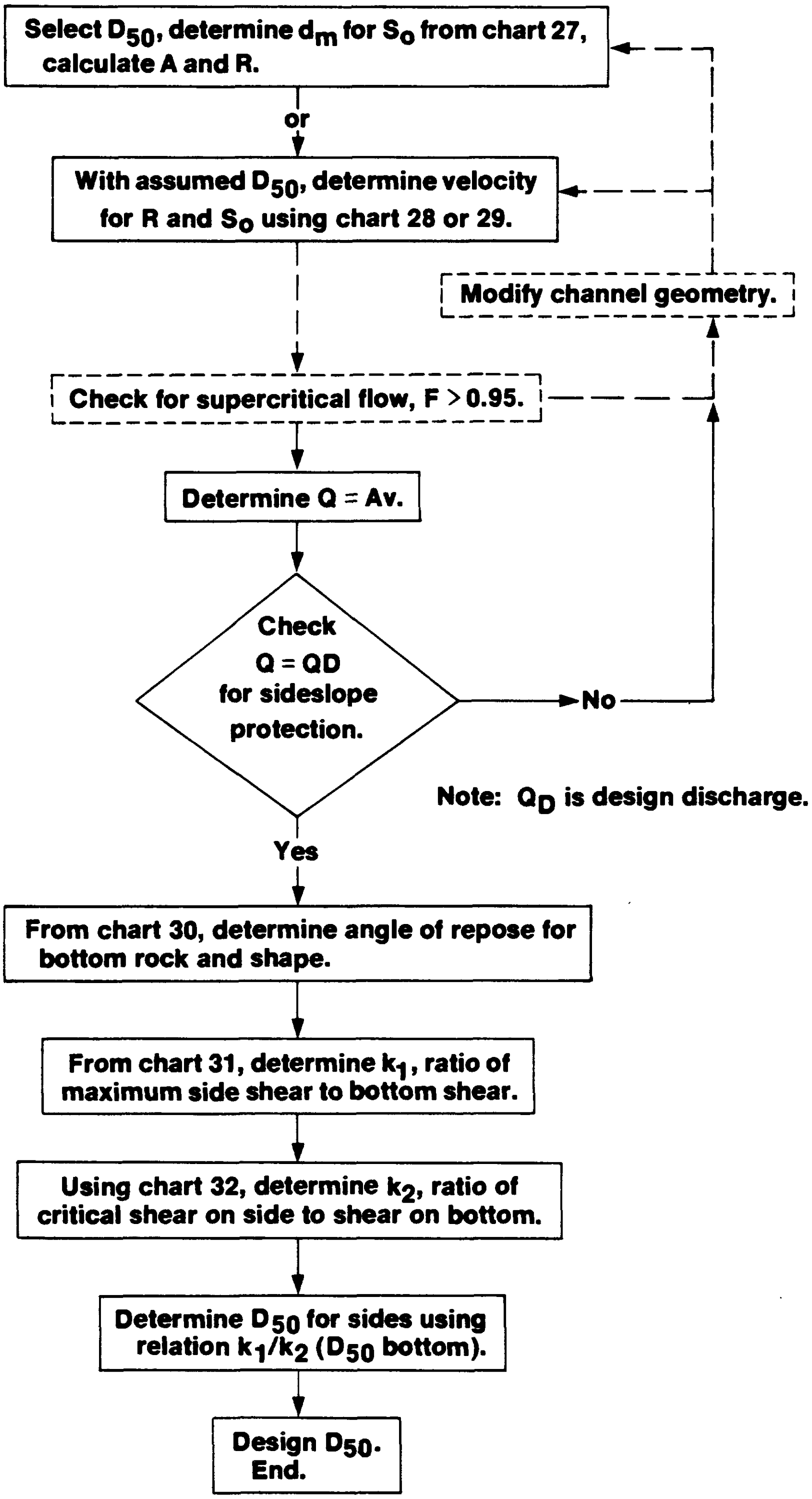
Protective lining procedures presented in HEC-15 do not apply to rigid linings, such as concrete and grouted riprap. Hand-placed riprap such as concrete-filled sandbags is also considered to be a rigid lining because it cannot accommodate minor movement without causing discontinuities in the protective layer. The design procedures also do not apply to cases where the flow is rapidly varied, such as at bridge abutments, weirs, or near culvert entrances or exits. Although stated only in the foreword, HEC-15 is applicable only to fully lined channels which carry discharges up to 1,000 ft^3/s (28.3 m^3/s), triangular channels with maximum discharge of 100 ft^3/s (2.83 m^3/s), and channels with maximum slopes of 0.10 ft/ft. These are the limits of the research data used in developing the design procedures.

Description

The size of riprap needed to protect the channel bed and banks is determined using a trial-and-error procedure. The basic flow chart and design procedures outlined in HEC-15 for straight and curved channels are illustrated in figures 25 and 26. The general method for design of riprap starts with a selection of the design discharge, proposed channel geometry, amd median rock size (D_{50}). If the rock size is not adequate for the design discharge, the channel geometry is adjusted or a new D_{50} is selected. Then, adjustments for steep side slopes and channel curvature are made. The check for supercritical flow conditions shown in figures 25 and 26 has been added to the flow charts because the procedures outlined in HEC-15 are not applicable for supercritical flow conditions.

Channels with side slopes steeper than 3:1 require the use of a supplemental procedure to determine the size of rock needed. Factors affecting the stability of riprap on side slopes are weight (size) of rock, angularity of shape, and slope of the channel bank (z). The minimum and maximum adjustment (ratio of k_1/k_2, figs. 25 and 26) to the riprap size chosen for the channel bed varies between 1.2 and 2. This range in adjustment is based on crushed rock or gravel riprap linings and an assumed minimum D_{50} of 0.1 ft (0.0305 m) with side slopes between 1.5:1 and 4.0:1.

Channels that are curved may require riprap protection on both the bed and banks. If the stream is wide (width to depth ratio is greater than 10), only the outside bank needs to be protected. The procedures outlined in HEC-15 (p. 16 and 17) make a distinction between short and long bends, based on the internal angle of the bend. The adjustment procedure for long bends provides a larger D_{50} than required on the channel bed. For short bends, the adjustment using chart 34 will result in a smaller D_{50} than required for long bends but not smaller than the D_{50} estimated for the channel bed.



(Note: The check for supercritical flow conditions is not included in HEC-15 instructions.)

FIGURE 25. Flow chart for application of HEC-15 procedures to straight channel with side slopes flatter than 3:1.

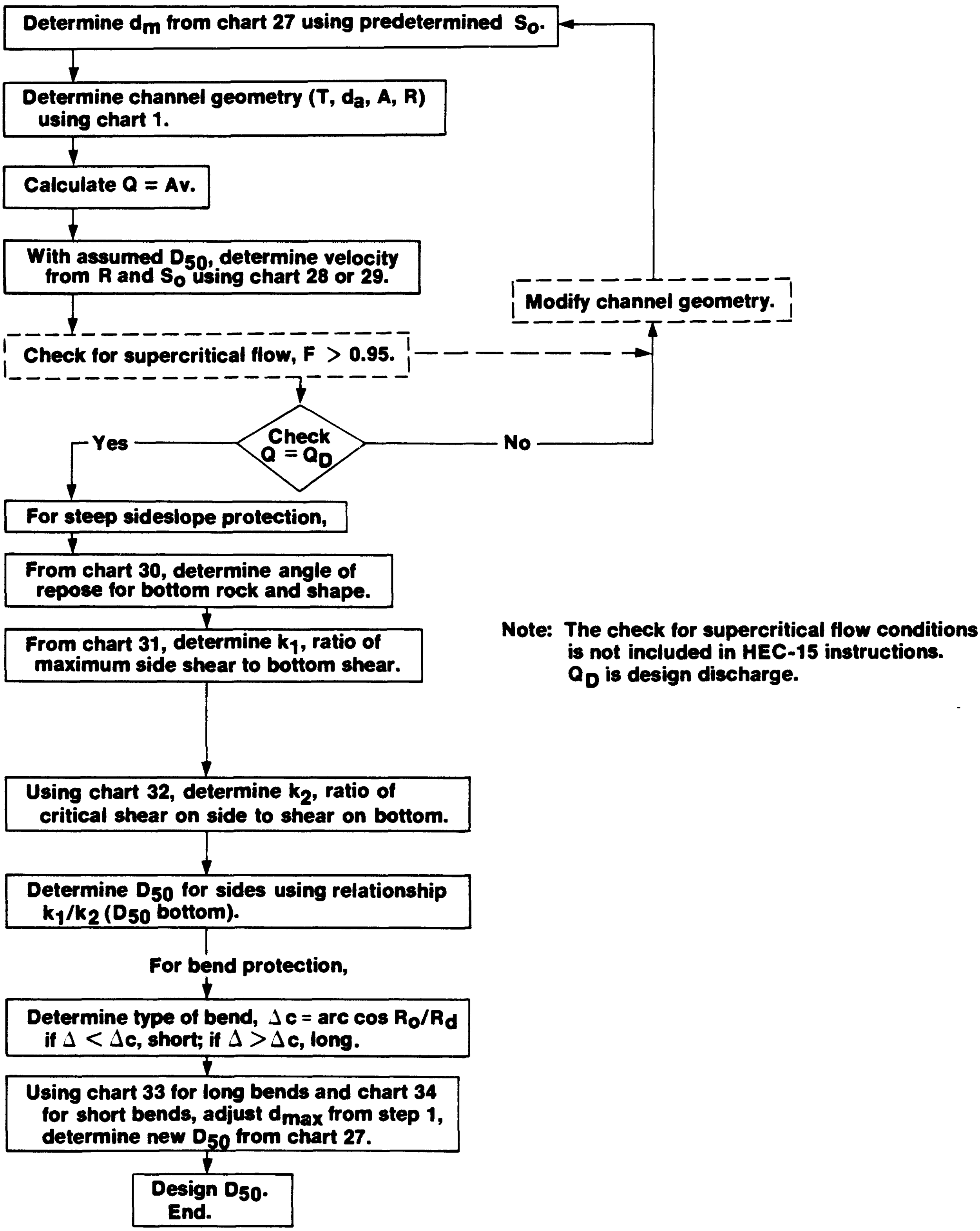


FIGURE 26. Flow chart for application of HEC-15 procedures to curved channel with side slopes steeper than 3:1.

Hydraulic Factors

To define the hydraulic resistance of the channel lining, the following relationship given by Anderson and others in NCHRP report 108 (1970) is presented:

$$n = 0.0395\ D_{50}^{1/6} \tag{17}$$

This relationship is the same as the equation given in EM-1601 for determining the boundary roughness. Based on hydraulic data in figure 7 of volume 1 of this report, equation 17 gives low estimates of n and will indicate a more efficient channel than actually occurs. For a given discharge and channel slope, the water-surface elevation will be lower; the cross-sectional area will be smaller; velocities will be higher; and the depth of flow, an essential hydraulic factor in the use of chart 27 in HEC-15, will be less than actually occurs, indicating a size D_{50} that is smaller than needed by boundary stability.

There are two considerations in the use of equation 17 for estimating Manning's roughness coefficient:

1. The riprap material may not cover the entire channel perimeter. Depending on the size of riprap and bed material, estimates of roughness for the entire channel that are larger than actual may result, which gives greater depths than required for the design discharge. For design procedures based on shear stress, this condition would result in a D_{50} size that may be larger than needed. Design procedures based on flow velocity, however, will result in an undersize D_{50}.

2. Other hydraulic variables affect the value of Manning's roughness coefficient besides the size of bed material. These variables include the hydraulic radius, channel curvature and slope, size and frequency of bank protuberances, water temperature (viscosity or Reynolds number), and size and density of submerged vegetal growth on the channel banks and amount of suspended material in transport. The effect of these variables should be added to any estimates of Manning's roughness coefficient n determined by use of equation 17 or by other methods, such as those described by Arcement and Schneider (1984).

Another procedure for estimating the hydraulic resistance of channels is discussed in volume 1 of this report. The new procedure suggests values of Manning's roughness coefficient n that are larger than those derived using equation 17 (Anderson and others, 1970, fig. C-4 of NCHRP 108).

The V-R (velocity-hydraulic radius) relationships shown in charts 28 and 29 of HEC-15 are used to estimate the velocity of flow that may be expected for various combinations of channel geometry. Factors such as hydraulic radius, channel slope, and size of rock riprap are expressed for various values of D_{50}. Charts 28 and 29 were developed from laboratory data, using the Manning equation and roughness relationships described by Anderson and others in NCHRP report 108 (1970). The relationships shown on these charts are considered suitable for use in fully lined channels with uniform flows less than 1,000 ft^3/s (28.3 m^3/s), but are not generally applicable for natural channels because the relationships are based on inadequate estimates of Manning's n.

Channel Geometry

As a part of the design procedure, the area and hydraulic radius of the study channel are related to the maximum permissible depth of flow using the graphical relationships shown in chart 1 of HEC-15. The plots in chart 1 of HEC-15 were developed for trapezoidal channels, which are the most common geometry used in channel design.

The relationships in chart 1 include a range in channel geometry that is much greater than needed. For example, most channels have a d/B ratio (B, bottom width) between 0.02 and 0.4, thus making the upper part of the curve of limited use for natural channels. Also, the use of the relationships in chart 1 is highly dependent on the channel bottom width B, as used in the ratios d_a/B and A/Bd_a. Because natural channels are not generally trapezoidal, it is difficult to estimate B. For wide channels or those with point bars, the value of B will be difficult to determine or will have no relation to the stresses acting on the revetted bank.

Although most channels are designed as trapezoids, the action of bed scour and bank erosion eventually creates a channel shaped as a parabola or trapezoid with rounded corners, as shown in figure 4 in volume 1. The design of a channel should consider the possibility of such changes in channel geometry as well as changes in alinement that will affect the capacity of the channel or the performance of the riprap.

Estimation of Stone Size Based on Depth of Flow (Chart 27)

When designing rock riprap, the general procedure is to determine the size of the channel, usually shaped as a trapezoid, for a given channel slope. With an estimated channel geometry, the riprap stone size (D_{50}) is selected from chart 27 of HEC-15 (fig. 27). If the channel is curved or side slopes are greater than 3:1, adjustments to the assumed depth of flow (d_m) are made and a revised stone size (D_{50}) selected from chart 27. Final estimates of channel size and corresponding stone size are made by trial-and-error procedure.

While procedures for design of flexible linings given in HEC-15 are primarily for use in roadside (highway drainage) channels, they are also applied to stream channels. The method of estimating suitable channel geometry for a given site is a trial-and-error procedure, resulting in excess time required to design a channel. Another problem is the limitation of chart 27 (fig. 27) when data for subcritical flow conditions are to be analyzed. In general, the range in factors given in chart 27 appears to be appropriate for design of highway drainage channels since depths of flow are frequently less than 1 ft (0.305 m) and slopes are similar to the roadway slope, which ranges from 0.02 to 0.06 ft/ft.

For example, with a roughness coefficient of 0.040 or less, only the shaded part of figure 27 (near the left margin) is applicable for design of open channels, assuming subcritical flow conditions. In addition, only that part of the figure in which the maximum depth of flow is greater than about 1 ft (0.305 m) is likely to be used in design of rock riprap. The relationships in figure 27 need to be extended to handle channel slopes less than 0.01, as indicated by various channel slopes given in table 2.

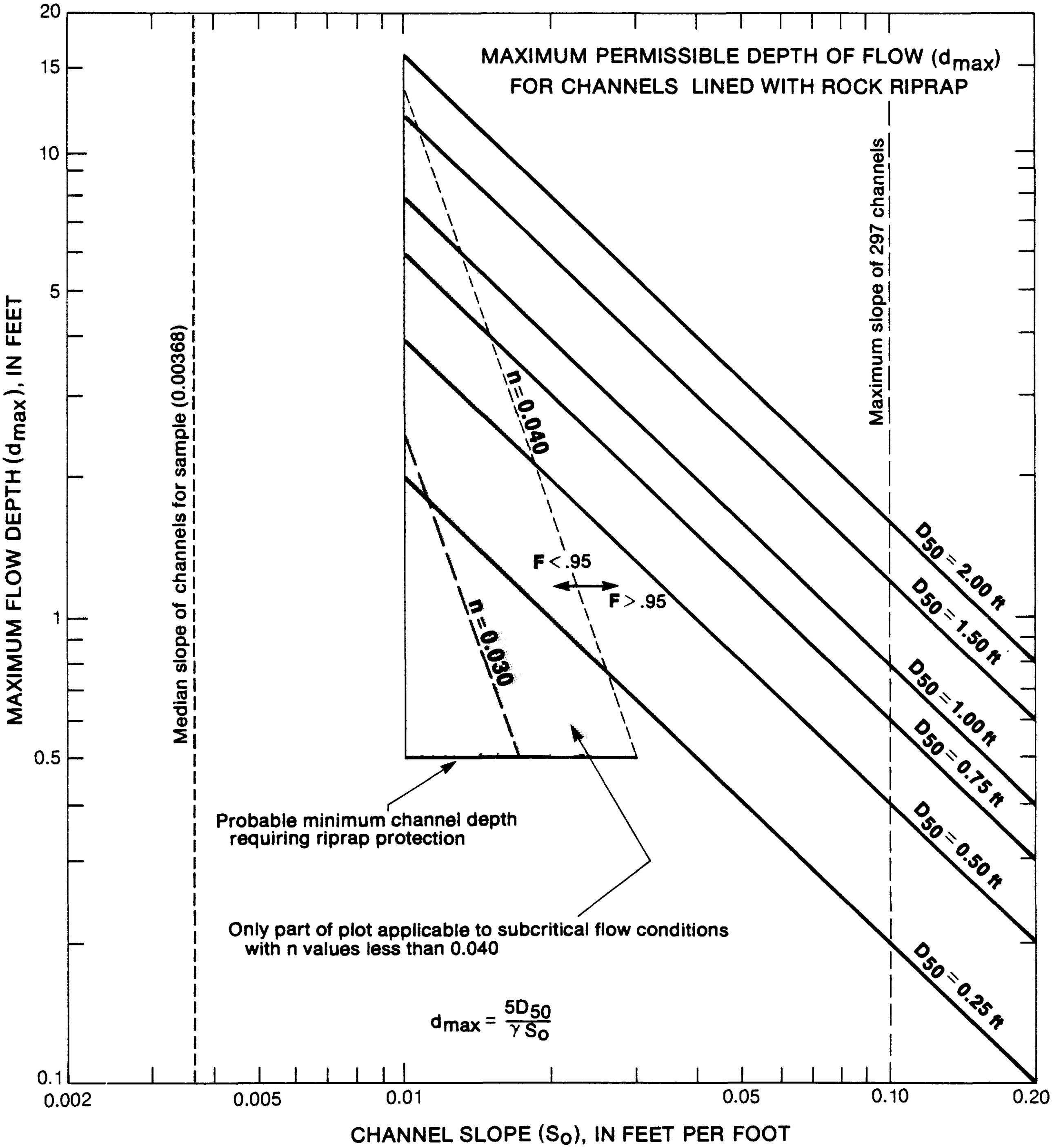


FIGURE 27. Applicability of chart 27 in HEC-15 to open channels (adapted from Normann, 1975).

Channel Bends

Flow around bends creates secondary currents which impose higher stresses on the channel sides and bottom than in straight reaches. Procedures are presented in HEC-15, based on findings by Anderson and others (1970) and an analysis by Watts (written commun., Federal Highway Admin., 1975), to determine the necessary modifications to riprap size at bends.

The procedure to modify the size of riprap at bends is to first estimate a correction factor (k_3) that varies linearly from 1 to 4 as a function of the ratio V_a^2/R_d, where V_a is the mean velocity of flow and R_d is the average radius of the outside bank of the bend. The correction factor is applied to the design depth of flow for a straight reach to obtain an adjusted (greater) depth of flow that is then used to select the size of riprap from chart 27. The relationships in HEC-15 that describe the ratio of boundary shear at bends to straight reaches are based on these assumptions:

- The main hydraulic factor causing rock riprap movement is direct impingement of the flow.
- The greater the degree of deflection or angle of curvature, the greater the amount of superelevation of the water surface. Therefore, the boundary shear correction factor is inversely related to the radius of channel curvature.

To derive the shear adjustment factor, a basic assumption concerning the difference in flow velocity in straight reaches and at bends was adapted from a procedure described on page 110 of Cal-B&SP. Cal-B&SP states that impinging flow on the outside bank (in line with the central thread) has velocities that are 4/3 the average stream velocity, and the tangent velocity (velocity at the bank of a straight channel) is 2/3 the average stream velocity. This suggests velocity of flow impinging on the bank of a curved channel is two times as great as the velocity of flow along the bank of a straight channel.

There is some question whether the velocity of impinging flow (assumed to be directed normal to the channel bank) is a critical factor in estimating the erosion potential. Flow velocities acting tangentially with the bank but affected by the rough boundary material are also effective in moving stone particles.

Procedures in HEC-15 assume the median rock size, D_{50}, is proportional to the mean velocity squared (V_a^2), because the superelevation (Δy) and drag force exerted on the bank surface is proportional to velocity squared:

$$\Delta y = \frac{TV_a^2}{gR_o} \tag{18}$$

where T = water-surface width
V_a = mean velocity
g = gravitational acceleration (32.2 ft/s^2 or 9.81 m/s^2)
R_o = mean radius of bend

On the basis of data presented in Cal-B&SP and the assumed effect of velocity on bank stress, a concept was developed that indicates the shear in a bend should vary from 1 to 4 times that in a straight channel. Chart 33 of HEC-15 was prepared on the basis that the adjustment factor for bends should vary linearly between 1 and 4, depending on magnitude of velocity and channel curvature.

Velocity data assembled from field surveys of sites with bends and straight reaches (table 2 in volume 1) indicate that the ratio of maximum point velocities to average velocities in a cross section is not much different at channel bends than in straight reaches. The distribution of the velocity across the section, however, is different at bends than at straight reaches. This suggests that the adjustment factors for impinging flow at banks need to be based on the channel shape as well as degree of curvature.

Chart 34 of HEC-15 is a procedure to further adjust the depth of streamflow as a function of D_{50} in chart 27 for short versus long bends. Short and long bends are defined on the basis of criteria presented in figure 2 of HEC-15 where

$$\Delta_c = \text{arc cos } (R_0/R_d) \tag{19}$$

where Δ_c = internal angle which differentiates between a long and short bend
R_0 = mean radius of bend
R_d = radius of outside bank of bend

Because the adjustment for shear at bends presented in chart 33 is of questionable validity, further refinements in estimating shear stresses, as given in chart 34, are considered unnecessary.

Application

Application of procedures given in HEC-15 for riprap design is described in a section on design procedures, and further explained by examples. As the manual was not developed for design of riprap protection of natural open channels, the biggest shortcoming of HEC-15 is the lack of design guidelines for many prototype stream conditions, as indicated by the comparison of the various design procedures in table 4. Also, limitations in application of the graphical relationships, notably chart 27, and the detailed adjustment of the D_{50} rock size give misleading concepts concerning the accuracy of the selected riprap.

The results of applying procedures in HEC-15 to Pinole Creek are given in table 4. The rock size, D_{50}, is $0.98 \cong 1.0$ ft (0.30 m), derived using chart 27 with appropriate adjustments; the in-place material that failed had a D_{50} of 0.60 ft (0.18 m).

Summary Discussion of Circular HEC-15

Most of the discussion and design procedures given in this circular were developed assuming trapezoidal shaped (man-made) channels.

Page 1: The introductory paragraph indicates this report is intended for the design of linings for drainage channels and prevention of erosion on the right-of-way. There is no indication that the procedures given in HEC-15 should be limited to small drainage channels and not applied to open channels located across or along the right-of-way. Several examples in HEC-15, discussions about the role of HEC-11, and concepts outlined in NCHRP Report 108 (Anderson and others, 1970) that were adapted in HEC-15 suggest rock riprap design procedures given in HEC-15 are applicable to a wide variety of stream and drainage channels. The limitations of this report need to be redefined.

The statement is made that effective flexible linings are limited by the depth of flow. Data collected for this study do not substantiate this statement.

Page 2: The last paragraph indicates that HEC-11 is obsolete for dumped rock riprap channel linings. This paragraph needs revision because the statement has been taken, understandably, to mean that HEC-11 is obsolete.

Page 5: This circular presents two procedures for design of riprap: (1) Maximum permissible depth of flow, presented in chart 27, and (2) critical boundary shear, presented in figure C-1. Results obtained from the two methods are not consistent. Both methods are related because the maximum permissible depth is based on shear stress (tractive force) theory. The report should be clarified to indicate the procedure most applicable and delete alternative method(s).

Page 6, paragraph 3: The location of first scour is not necessarily at the bottom of the channel. Also, it is immaterial whether scour occurs on the channel bed first or last. Field surveys indicate bank scour may be of greatest concern in protecting highway embankments, as the channel bed is continually undergoing fill or scour.

Data obtained during field surveys suggest strongly that many riprap failures are not related to particle erosion. Instead, many failures occur by translational slides or slumps. These problems are largely related to soil mechanics, not size of riprap or gradation.

Page 6, last paragraph: The concept of maximum depth (d_m) as the critical factor in limiting scour for all wide channels does not consider the influence of angular flow near the boundary. Eddies, reverse flow, and perturbations along the bank may cause flow discontinuity and more potential for scour than the depth of flow.

Page 8: The base material under a riprap layer can be protected with a filter blanket or cloth to prevent leaching of the underlying soil. Another bank stability problem may be related to saturation with water and negative pore pressure that contributes to translational slides or slumps.

Page 9: The relationship for computing Manning's n gives a much lower roughness coefficient than actually occurs for a given D_{50}. Use of this relationship indicates a channel more efficient than it actually is, and results in a channel size smaller than needed for the design discharge. See figure 7 of volume 1.

Page 10: Although channels may be designed as a trapezoid or other shape, they usually transform to natural channel shape if scour and fill occur. The most common channel geometry has a maximum depth 1.5 times greater than mean depth. See table 2.

Page 11, channel bends: The adjustment in the maximum depth (d_m) factor for bends assumes the depth is a function of channel curvature, that is, greatest depths occur at channel curves; however, this assumption may not always be true. See data for maximum depth in table 5.

In addition, the elaborate procedure provided for riprap design at channel bends could be simplified by use of the depth ratio (d_m/d_a) such as 1.7 (see table 2 of vol. 1). For many streams, the amount of channel curvature is in a continual state of flux, and varies with time and discharge. Detailed procedures, as given in HEC-15, to adjust riprap size for channel bends probably provide a false sense of security to the designer.

Page 13: Field data (table 13 of vol. 1) suggest that the relationship given in HEC-15 for estimating superelevation of the water surface at bends provides results that are usually less than normally occur. Superelevation Δy may be estimated using the following equation (see eq. 18 and discussion in vol. 1):

$$\Delta y = C \frac{TV_a^2}{gR_0} \tag{20}$$

where C = coefficient, with an average value of 1.5 for 36 sites
T = width of channel
V_a = mean velocity
g = gravitational acceleration
R_0 = mean radius of channel centerline

Page 15, item 8: For natural streams, the slope, S_0, is usually fixed for long reaches. Only for drainage ditches, median ditches, meandering channels with cutoffs, or channels that are to be constructed would S_0 be a variable (see table 17 of vol. 1).

Page 15, last paragraph: Most natural and design channels will have side slopes that vary between 1.5:1 and 3:1 (table 6); thus, the majority of channels will have steep side slopes.

Page 17: The definition of a wide river is one that has a width-depth ratio greater than 10 (Chow, 1959). For the sample of data in table 2, 80 percent of all width-depth ratios were greater than 10, indicating most channels would be classed as wide.

Page 20: This relationship is cumbersome to use. The various factors could be set up for computer processing. For a simplified approach, the relationship for estimating channel geometry as a function of discharge may be used (see table 2, and eq. 4 of vol. 1). This assumes the channel shape will eventually approximate a natural channel with maximum depth equal to 1.5 mean depth.

Page 56, chart 27: The lower limit of channel slope presented in this chart is 0.01. As shown in table 2, the relationships given in this table need to be extended to include slopes less than 0.001. Also, only the relationships on the left margin of this figure apply to uniform flow. Generally, slopes steeper than 0.02 and depths greater than about 2 ft (0.6 m) represent supercritical flow conditions. Some of the limitations of channel slope as an

Table 6. Steepest suggested side slopes by design procedure and placement method.

[ND = not determined]

Procedure	Placement method: Dumped	Machine-placed	Hand-placed	Remarks
FHWA (HEC-11)	2:1	2:1	2:1	No distinction made between placement types. 2:1 applies to sand slopes; no further criteria given.
FHWA (HEC-15)	3:1	3:1		Procedure given for sizing riprap on banks steeper than 3:1. Hand-placed revetment considered "rigid" and not discussed.
U.S. Army Corps of Engineers (EM-1601)	2:1	2:1	1.5:1	
California Bank and Shore Protection (Cal-B&SP)	1.5:1	1.5:1	ND	No specific discussion given on hand placing as a placement method. Assume some hand placing may occur in machine placing (method A, p. 116) procedure.
Sediment Transport Technology (Simons-SST)				Side slope variable with changes in particle size and water-surface or bed slope. No distinction is made for different placement method.
Oregon Dept. of Transportation (ODOT)				No discussion given.

indicator of hydraulic stresses are given in the discussion of HEC-11 (see fig. 27) and in volume 1. Although crushed rock of smaller size (D_{50}) than 0.4 ft (0.1 m) is usually available as part of the highway construction, the effective use of this material to prevent erosion is limited to lining small drainage channels. A lower limit of effective riprap size D_{50} for shallow depths (about 0.5 ft or 0.15 m) needs to be defined by modification of existing relationships, such as chart 27 in HEC-15.

Page 57, chart 28: These relationships are based on the equation $n = 0.0395\ D_{50}$. This equation is not appropriate for open channels (see fig. 7 of vol. 1); thus, the curves on this chart are of limited applicability.

Page 58, chart 29: Same comments as above.

Page 59, chart 30: This chart is not practical. The smallest D_{50} that can normally be obtained from quarries is about 0.4 ft (0.1 m) unless road surfacing material is used. For D_{50} larger than about 0.4 ft, the difference in angle of repose of crushed rock and very angular rock is insignificant.

Page 61, chart 32: Curves for angles of repose of 20° and 25° should not be shown. Chart 30 indicates angles from 31° to 43° only.

Pages 62 and 63, charts 33 and 34: These relationships provide adjustment of maximum depths on the basis of channel curvature. A more simplified approach to consider the effects of channel curvature (the degree and radius of curvature is difficult to measure for open channels) would be to use the relationship, maximum depth = 1.7 mean depth (table 2 of vol. 1). See previous discussion of page 11 of HEC-15.

Page 103, figure C-1: This comparison is shown in figure 19 of this report. A new comparison of critical shear stress and median stone size, D_{50}, based on field data obtained during this study (table 7), is given in figure 28. Because of the scatter of data points in the figure, a revised relationship has not been prepared. The field data in figure 28 indicate, however, that the shear equation, $\tau = 4D_{50}$, recommended by Anderson and others (1970), gives a larger value of tau than the relationship adopted for use in HEC-15.

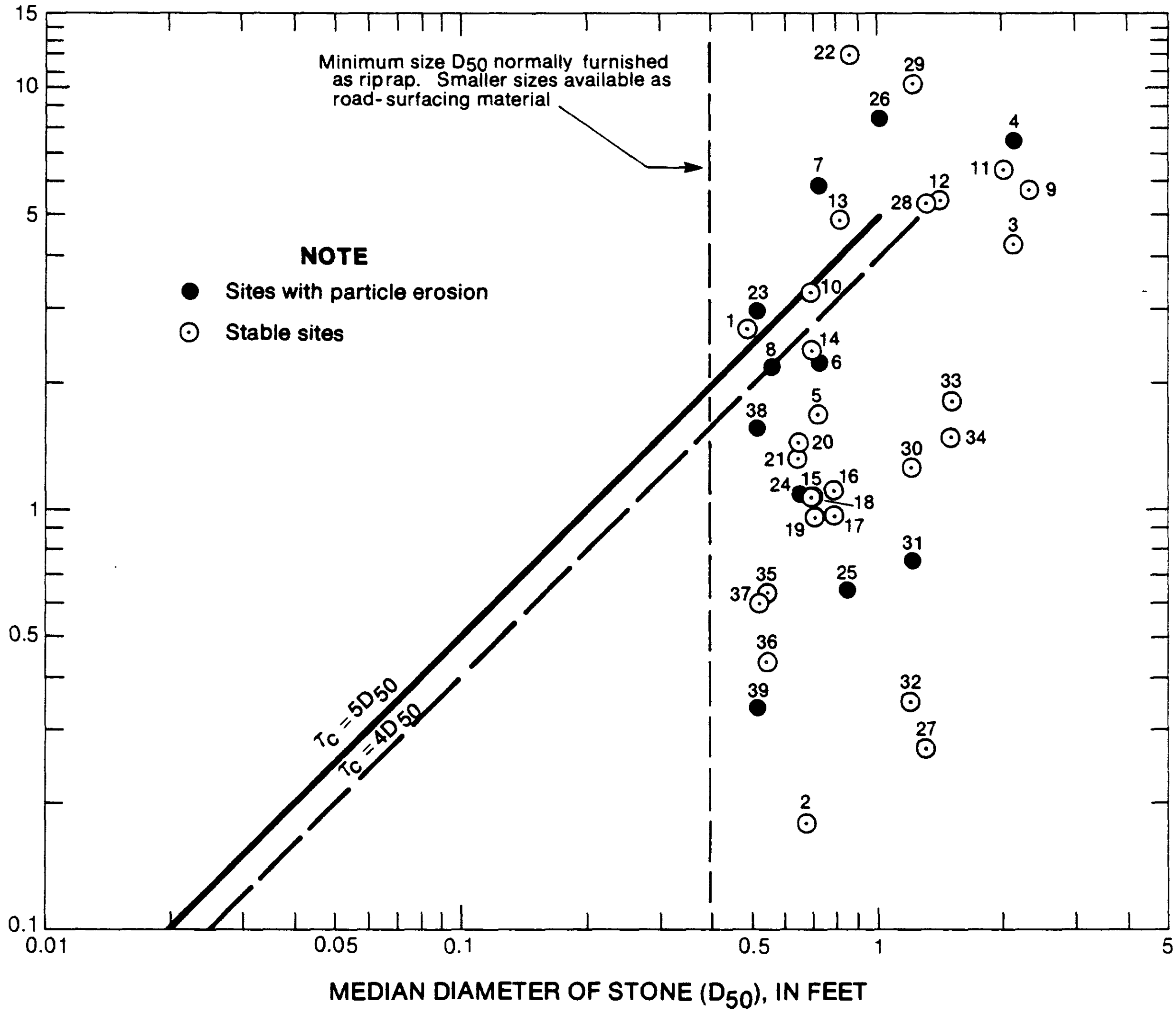


FIGURE 28. Comparison of median stone size (D_{50}) estimated on basis of shear stress (HEC-15 procedures) and performance at field sites.

Table 7. Estimates of flow velocity and shear stress related to riprap performance.

[Prt erosn, particle erosion; trnsl slide, translational slide]

Site	Measurement number	D_{50} (ft)	Specific gravity, G_s	Date of flood or survey	Corresponding discharge, Q (ft^3/s)	Water surface, slope, S_w	Depth, max, d_m (ft)	Side slope, z	Area, A (ft^2)	Velocity Mean, V_a (ft/s)	Velocity Max, V_m (ft/s)	Shear, τ (lb/ft^2)	Performance
Sacramento River:													
at Princeton, CA	1	0.48	2.95	2/17/83	[1]71,200	0.00091	48.7	1.3:1	12,700	5.61	7.97	2.77	No damage
at Colusa, CA	2	.67	2.77	12/22/81	[1]40,600	.00006	47.0	1.9:1	10,300	3.93	6.17	.176	No damage
Truckee River at	3	2.1	--	6/14/83	[1]6,550	.00625	11.0	1.5:1	780	8.40	12.95	4.29	No damage
Reno, NV				3/13/83	[2]7,230								
	4	--	--	12/20/81	[2]8,690	.0089	13.8	--	946	9.19	--	7.66	Prt erosn
Truckee River at	5	.71	2.68	5/27/82	[1]3,880	.0022	12.1	1.8:1	744	5.22	8.17	1.66	No damage
Sparks, NV	6	--	--	6/15/83	[1]5,850	.00219	16.5	1.8:1	994	5.89	7.97	2.25	Prt erosn
	7	--	--	12/20/81	[2]8,670	.0055	17.2	1.8:1	1,420	6.10	--	5.90	Prt erosn
Pinole Creek at Pinole, CA:													
(cross sec. 3)	8	.55	2.85	1/03/82	[2]2,250	.0049	7.3	2:1	293	7.68	--	2.23	Prt erosn
(cross sec. 0.4)	9	2.3	2.80	1/03/82	2,250	.0172	6.4	2.1:1	212	10.6	--	6.87	No damage
Donner Creek at Truckee, CA	10	.68	--	6/13/83	[1]519	.00720	5.6	2.1:1	112	4.63	7.43	2.52	No damage
E.F. Carson River near Markleeville, CA	11	2.0	2.36	6/16/83	[1]2,150	.01224	8.5	1.5:1	260	8.27	13.59	6.49	No damage
W. Walker River near Coleville,NV:													
at #2	12	1.4	--	6/11/82	[1]1,450	.0193	6.4	1.3:1	146	9.93	15.91	7.71	No damage
at #4	13	.8	2.61	6/10/82	[1]1,280	.01636	4.8	1.5:1	140	9.14	16.69	4.90	Prt erosn
Russian River near	14	.69	2.78	1/13/80	[2]28,800	.00312	12.3	2.1:1	6,833	[3]4.22	--	2.39	No damage
Cloverdale, CA	15	--	--	12/23/81	[1]6,970	.0018	9.7	--	1,140	6.11	9.46	1.09	No damage
Cosumnes River at Dillard Rd near Sloughhouse, CA:													
at #1	16	.78	2.79	3/13/83	[2]26,100	[5].00073	[4]24.6	2.3:1	9,975	2.62	--	1.12	No damage
	17	--	--	12/22/82	[2]18,800	[5].00076	[4]20.4	--	7,301	2.52	--	.97	No damage
at #2	18	[5].7	--	3/13/83	[2]26,100	[5].00073	[4]24.5	1.6:1	6,750	3.87	--	1.09	Trnsl slide
	19	[5].7	--	12/22/82	[2]18,800	[5].00076	20.5	--	5,360	3.51	11	.97	Trnsl slide
at #3	20	.64	--	3/13/83	[2]26,100	[5].00073	[4]31.8	1.8:1	6,400	4.08	--	1.45	Slump
	21	--	--	12/22/82	[2]18,800	[5].00076	[4]27.8	--	5,070	3.71	--	1.32	Slump

Table 7. Estimates of flow velocity and shear stress related to riprap performance (Continued).

Site	Measurement number	D_{50} (ft)	Specific gravity, G_s	Date of flood or survey	Corresponding discharge, Q (ft^3/s)	Water surface, slope, S_w	Depth, max, d_m (ft)	Side slope, z	Area, A (ft^2)	Velocity Mean, V_a (ft/s)	Velocity Max, V_m (ft/s)	Shear, τ (lb/ft^2)	Performance
Rillito Cr at I-10, AZ													
(cross sec. 1)	22	0 .84	2.74	10/03/83	[2] [6]29,000	0.0162	12.1	2:1	2,790	10.4	--	12.2	No damage
above SPRR (cross sec. 3)	23	.51	2.69	10/03/83	[2] [6]29,000	.0038	12.5	2.5:1	3,870	7.49	--	2.96	Prt erosn
Santa Cruz R. at I-19, AZ													
(cross sec. 3)	24	.65	2.71	10/03/83	[2] [7]50,000	.0019	[4]9.1	2.3:1	3,430	14.6	--	1.09	Prt erosn
(cross sec. 1)	25	.84	--	10/03/83	[2] [7]50,000	.0011	[4]9.5	1.9:1	2,780	18.0	--	.65	Prt erosn
Esperanza Creek	26	1.0	2.63	10/03/83	[2]12,700	.0171	8.1	1.5:1	743	17.1	--	8.64	Trnsl slide & prt erosn
Santiam River	27	1.3	2.66	10/27/82	[1]7,440	.0002	21.5	2:1	2,180	3.41	5.20	.27	No damage
near Albany, OR	28	--	--	12/26/80	[2]61,000	.0030	28.9	2:1	4,190	14.6	--	5.41	No damage
Hoh River near Forks, WA													
site #1 (old)	29	1.2	2.69	10/22/82	[2]22,000	.00867	19.10	1.2:1	2,772	7.94	--	10.3	Prt erosn
	30	--	--	11/04/82	[1]5,060	.0014	14.6	1.2:1	814	6.22	9.26	1.28	No damage
site #1 (new)		1.3	2.59	Data not available									Prt erosn
site #2	31	1.2	2.48	1/12/79	[2]51,600	.00058	20.9	1.6:1	4,943	10.44	--	.76	Prt erosn
	32	--	--	11/03/82	[1]2,140	.0006	9.4	1.6:1	608	3.52	5.57	.35	No damage
Yakima R. at	33	1.5	2.82	2/22/82	[2]22,200	.0024	12.2	2:1	1,783	12.45	--	1.83	No damage
Cle Elum, WA	34	--	--	11/05/82	[1]2,660	.0037	6.5	2:1	266	10.0	--	1.50	No damage
Sacramento River	35	.54	2.72	12/23/82	[2]77,000	.000364	28.10	3:1	12,651	6.10	--	.638	No damage
at Peterson Ranch near Chico, CA	36	--	--	4/14/82	[2]56,000	.00030	23.5	3:1	9,610	5.83	7.92	.44	No damage
Sacramento River	37	.51	2.60	12/15/81	[1]27,700	.00076	20.3	1.8:1	4,320	6.41	8.54	.963	No damage
at E-10 near	38	.51	2.60	12/23/83	[2]78,000	.00081	31.3	1.8:1	10,700	7.3	--	1.58	Prt erosn
Chico, CA	39	.51	2.60	1/27/83	[2]98,000	.00042	[4]13.0	1.8:1	14,600	6.7	--	.341	Prt erosn

[1]Discharge at time of survey.
[2]Discharge at time of flood.
[3]Mean velocity decreased because of wide overflow area.
[4]Depth of flow above toe of damaged riprap area.
[5]Estimated values.
[6]Discharge at site 4 miles upstream equals 29,700 ft^3/s.
[7]Estimated from flood routing between gages at Continental and Tucson, AZ.

Evaluation of California Department of Transportation "Bank and Shore Protection" Manual

Description

According to the California Department of Transportation (Cal-B&SP, 1970), wave action generated by wind or vessels is the dominant process in bank erosion. The statement is made that "a 2-ft (0.6 m) wave is more damaging than direct impingement of a cross current flowing at 10 ft/s (3.05 m/s)." Their approach to developing a formula for protection of highway embankments from streams thus began with an analysis of wave theory and the force-energy relationships necessary to dislodge a riprap particle. From these relationships, an equation for minimum rock weight was derived for both shallow- and deep-water waves approaching embankments. Consequently, a maximum value of velocity head was substituted for wave height in the shallow-water wave equation, and the equation for stone weight for streambank protection was thus derived. After the minimum stone weight is calculated, specifications for both stone (shape, durability, and specific gravity) and riprap layer characteristics (filter blanket, thickness, gradation, and placement) are given. A flow diagram for the procedure is shown in figure 29.

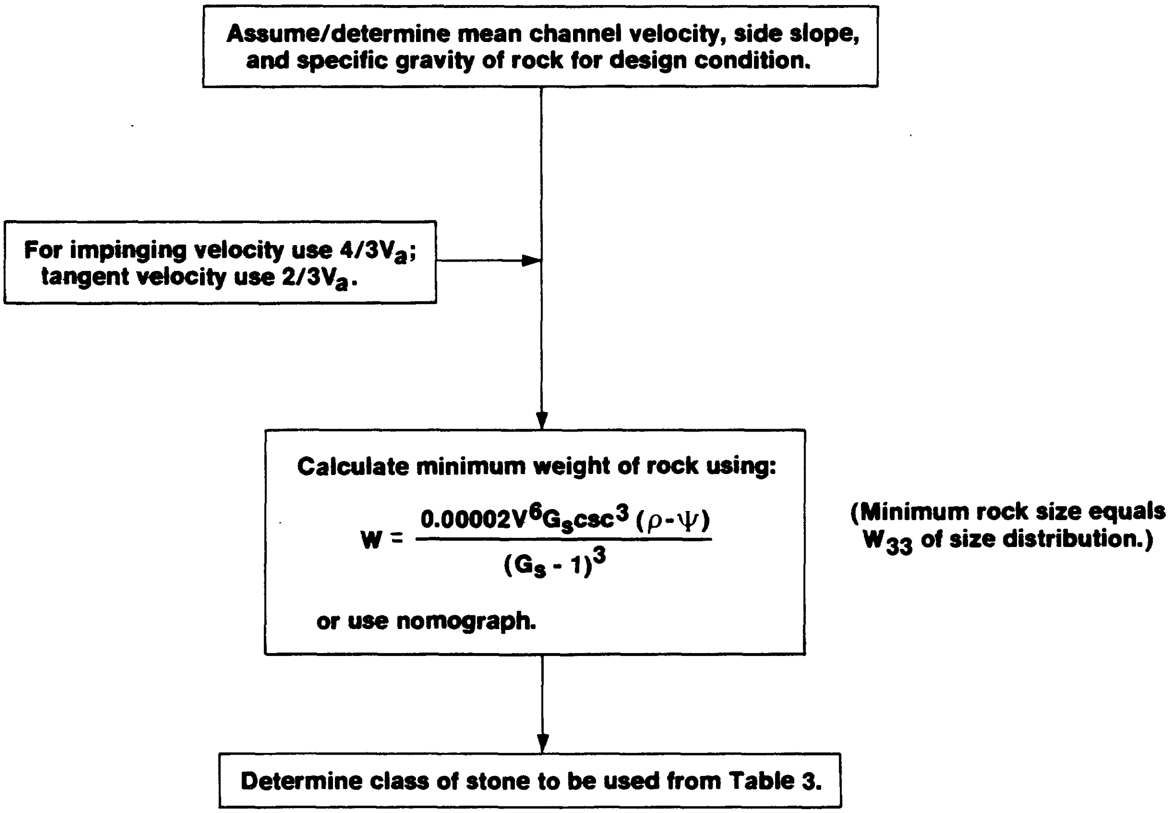


FIGURE 29. Flow chart for bank protection procedure in 'Bank and Shore Protection in California Highway Practice' (California Department of Public Works, Division of Highways, 1970).

In our assessment, supplementary notes on the development of the Cal-B&SP equation, assembled by R. M. Carmany (1963) of CALTRANS, were used. The following discussion applies to the supplementary notes:

- The University of California conducted laboratory experiments to determine the minimum force necessary to dislodge a stone from the bank. They constructed a model streambank on which small stones were arranged as riprap, and underlying stones were cemented in a plaster of paris base. The side slope was increased until the first outer stone was displaced. It was determined that 65° to 70° was the maximum angle attained before a stone fell out. The concept seems valid; however, no mention was made as to the actual size or shape of stones used. Both size and shape have an effect on natural angle of repose. Also, it may not be valid to apply the laboratory condition of a cemented base rock to the field application in which underlying base rocks may sit on a smooth filter blanket or less rigid bank. Data showing the actual angle of repose for given rock sizes are available and would better fit field conditions. Riprap is considered "flexible" and the underlying rocks may in fact move.

- A constant of 0.003 is incorporated in the wave equation. This value is based on only one observed failure, and if it is not accurate, this error is transmitted into the estimated weight of stone based on this equation.

- It is assumed, but nowhere shown, that velocity head, $V_a^2/2g$, is equal to the height of a breaking wave.

Cal-B&SP is specific as to the procedure for placement of riprap. Two methods (designated A and B) are employed: "A" is a machine-placement method and "B" is a dumping method. Method A is for the larger stone classes but there is some overlap between A and B in the 1/2-ton (454-kg) (2.3-ft or 0.70-m) and 1-ton (908-kg) (2.9-ft or 0.88-m) classes. Method A requires that stones be placed with their longitudinal axis normal to the alinement of the embankment face, either in horizontal position or dipping slightly inward. Outer rocks must be placed so as to have a three-point bearing on underlying rocks. No dumping is permitted in method A. Precise placement should contribute to greater particle stability but is costly. In view of the Cal-B&SP premise that wave action is the dominant erosive process, this method may be more appropriate for shore than for streambank protection.

In method B (placement by dumping), segregation of stones is advocated in the following quote: "Larger rocks shall be placed in the foundation course and on the outside surface of the protection. The rock may be placed by dumping and may be spread in layers by bulldozers or other similar equipment." Whether this segregation and layering is ultimately detrimental to riprap performance is not presently known, but other procedures do not recommend it and some discourage it. Spreading by bulldozers may cause substantial breakage, thus reducing the size of rocks on the outside layer. Possible detrimental effects of this type of placement may be offset, however, by the compaction and relatively smooth surface caused by bulldozer work and the requirement of an additional 25 percent thickness (for method B).

Both methods require that the surface irregularities of slope protection not extend from the surface of the designed slope by more than 1 ft (0.305 m), as measured at right angles to the slope. This is an important criterion because large irregularities in the slope may set up undue turbulence and, hence, boundary stresses. Guidelines and principles for designing the vertical and lateral extent of the revetment protection are well presented. A typical construction example is presented showing side slope angle, riprap thickness, filter blanket guidelines, and toe construction. Specifications are thorough for specific gravity and rock quality and include specific American Association of State Highway Transportation Officials (AASHTO) and California Department of Transportation tests.

The Cal-B&SP requirements were analyzed for the size gradation of stone by developing characteristic limiting size gradations and showing them as a percent of the D_{50} size. Table 8 shows a relatively wide range for the ratios of D_{15}, D_{85}, and D_{100} to D_{50} ratios. The low size of the D_{15} ratio requires the largest particle sizes of all procedures. This relatively wide range of sizes allows for numerous possibilities in terms of acceptable distributions. This may aid in utilizing a source of acceptable stone but may not be suitable for providing a distributed size range that will contribute to an interlocking arrangement of stone with few or no pockets.

Riprap thickness criteria for the Cal-B&SP procedure are based on (1) the orientation of the stone on the bank, (2) the side slope, (3) a stone shape factor, and (4) the class weight of the stone. The equation for selecting riprap layer thickness for method A is:

$$t = 1.5K \sin \psi \sqrt[3]{W_c} \qquad (21)$$

where t = thickness normal to the face slope

K = shape factor of stone (for common shapes use 0.40)

ψ = angle of side slope (in the Cal-B&SP report, α is used to define angle of side slope)

W_c = class weight of stone

The 1.5 coefficient is from the Cal-B&SP recommendation that, as a minimum for machine-placed riprap (method A), thickness (t) should be 1.5 times the long axis (ℓ) of the critical stones.

Thickness for method B is determined by the same equation except that the 1.5 coefficient is increased 25 percent to 1.875. An example thickness calculation is presented in Cal-B&SP for extremes of size distributions and for ℓ-axis dimensions. The presentation is somewhat confusing and, owing to assumptions for values of ℓ and the amount of overlap between stones, the result is likely to be no better than a more practical estimate for thickness as a function of the median particle size.

Table 8. Particle-size gradation and riprap data.

Procedure/site	Ratios of sizes D_{15}/D_{50}	D_{85}/D_{50}	D_{100}/D_{50}	Actual D_{50} (ft)	Probable designer	Specific gravity
	Ratios recommended by various methods					
FHWA (HEC-11 & -15)	0.50	1.38	1.5			
CALTRANS Bank and Shore (Cal-B&SP)	.88-1.3	1.0-1.4	1.2-1.8			
Simons & Senturk	.42	1.67	2.0			
Corps of Eng. (EM-1601)	.62-.97	1.1-1.38	1.18-1.6			
Oregon Dept. of Transportation (keyed riprap)	.48-.83	1.1-1.4	1.26-1.59			
	Actual ratios measured at selected field sites					
Clackamus River near Estacada, OR:						
Site 1	.50	2.1	3.8	[1]1.7	FHWA	2.72
Site 2 (New)	.46	2.1	3.3	[1]1.8	FHWA	2.50
Site 2 (Old)	.56	1.9	4.0	[1]1.2	FHWA	2.76
Site 4	.47	2.3	5.2	[1].90	FHWA	2.73
Santiam River at I-5, OR	.77	1.2	1.8	[1]1.3	FHWA	2.66
Hoh River near Forks, WA:						
Site 1 (Old)	.42	2.1	3.1	[1]1.2	USCE	2.69
Site 1 (New)	.45	1.9	4.2	[1]1.3	USCE	2.59
Site 2	.53	2.4	5.2	[1]1.2	USCE	2.48
Yakima River at Cle Elum, WA	.53	1.5	2.1	[1]1.5	USCE	2.82
Sacramento River, CA:						
Near Nord	.61	1.2	1.5	.54	USCE	([2])
At E-10, Chico Landing	.61	1.3	1.6	.51	USCE	2.60
At Princeton	.71	1.4	1.6	.48	USCE	2.95
At Colusa	.69	1.4	1.6	.67	USCE	2.77
Truckee River at Reno, NV	.62	1.3	2.1	2.1	([2])	([2])
Truckee River at Sparks, NV	.65	1.6	2.5	.71	U. Nev. at Reno	2.68
Pinole Creek at Pinole (cross section 3), CA	.62	1.7	2.5	[1].60	USCE	2.85
Donner Creek nr Truckee, CA	.43	1.3	3.9	[1]1.3	Shell Oil	([2])
East Fork Carson River near Markleeville, CA	.50	1.6	2.6	[1]2.0	Cal. DOT	2.36
West Walker River near Coleville, CA:						
Site 2	.72	2.2	6.5	[1]1.4	Cal. DOT	2.54
Site 4	.63	2.1	5.9	[1].80	Cal. DOT	2.61
					Average	2.67

[1]Particle count by field grid.
[2]Not determined.

Application

Application of the Cal-B&SP procedure for determining riprap size is straightforward. As shown on the flow diagram (fig. 29), the following equation is used to calculate the stone size, W (weight), which is based on stream bank velocity, side slope angle, and specific gravity of the stones. No background is given for its derivation.

$$W = \frac{0.00002V^6 \; G_s \; \csc^3 \; (\rho-\psi)}{(G_s-1)^3} \tag{22}$$

where W = minimum weight of outside stone for no damage, equivalent to W_{33}
V = stream velocity to which bank is exposed
G_s = specific gravity of stones
ρ = 70° for randomly placed rubble
ψ = side slope (degrees)

An assumption is made that velocity of flow near the bank is 2/3 the mean velocity in tangential reaches and 4/3 the mean velocity for flows impinging on the bank. For rough calculations, Cal-B&SP advises the use of estimated values of 2.65 for specific gravity and 1.5:1 for side slope. The side slope and specific gravity values are reasonable but no data or references are given to substantiate the use of the 2/3 and 4/3 mean velocity adjustment factors. The weight of stone computed from the equation is the minimum weight (pounds) of outside stone for no damage. Cal-B&SP terms this the critical stone weight, W, and states that 2/3 of the stone should be heavier. On a size distribution basis, the critical stone weight would be equal to W_{33}, the weight for which 33 percent of the total sample would be smaller.

Some vagueness arises in determining the proper class to use after calculating W. It is difficult to estimate from table 3 (p. 115) in Cal-B&SP the size class for which 67 percent are larger than the calculated W. Also, if one wishes to know the W_{50} associated specifically with the calculated W, it must be interpolated from the given classes.

The following is a sample calculation for stone weight, W, using Cal-B&SP streambank equation (22) and necessary input data (table 5) from cross section 3 at the Pinole Creek study site:

$$W = \frac{0.00002V^6 \; G_s \; \csc^3 \; (\rho-\psi)}{(G_s-1)^3}$$

Input data are:

V_a = 7.7 ft/s (2.3 m/s); use 4/3 adjustment factor for impinging flow, so V_m = 10.3 ft/s (3.14 m/s)

G_s = 2.65 (actual field determination)

ρ = 70° for randomly placed rubble

ψ = 26.6°

$$W_{33} = \frac{0.00002(10.3)^6(2.65)\;\csc^3\;(70-26.6)}{(2.65-1)^3} = 43.3 \cong 43 \text{ lbs } (19.5 \text{ kg})$$

According to Cal-B&SP, 2/3 of the stone should be heavier than the value for W, so this weight would be equal to the W_{33} value in a size distribution. By plotting particle size distribution data from table 3 in Cal-B&SP, a W_{50} of 54 lbs (24.5 kg) can be determined for the calculated W_{33} of 43 lbs (19.5 kg) by interpolating between size distributions for the standard classes (table 3 in Cal-B&SP).

The equivalent spherical diameter of the stone in feet, D_s, may be related to the stone weight in pounds, W_s, by the following equation:

$$D_s = \left(\frac{6W_s}{\pi \gamma_s}\right)^{1/3} \tag{23}$$

where W_s = weight of stone, in pounds

γ_s = specific weight of stone = 165 lb/ft^3 (2,640 kg/m^3)

The equivalent spherical diameter of W_{50} (54 lb or 24.5 kg) is 0.85 ft (0.26 m). Actual field determination shows the D_{50} for Pinole Creek to be 0.6 ft (0.18 m).

The corresponding thickness of protection for the class stone size of 75 lbs (34.0 kg) and a method B placement in table 3 of Cal-B&SP is calculated using the following equation:

$t = 1.875\ K \sin \psi \sqrt[3]{W_c}$ (see discussion of thickness on p. 59)

$t = 1.875\ (.40)(.448)\ \sqrt[3]{60} = 1.3$ ft (0.40 m) (normal to embankment face).

Evaluation of U.S. Army Corps of Engineers, Bulletin EM-1601

Description

The Corps of Engineers "Hydraulic Design of Flood Control Channels Manual" (EM-1601, 1970) presents an organized procedure for the design of rock riprap installations. Procedures outlined in the manual have been updated with Engineering Technical Letters (ETL) dated May 14, 1971, and July 14, 1975. The ETL's provide clarification of procedures in EM-1601 with a flow chart (fig. 30) and an example of riprap design. Procedures and illustrations are oriented to aid the designer, and details of hydraulic theory are well referenced. The procedures outlined in EM-1601 (see flow chart, fig. 30), however, require a considerable amount of channel and bank data that is analyzed in some detail. Procedures are given for estimating flow stability for near-supercritical flows, channel geometry with supercritical flow, flow at bends, and at channel transitions such as bridge piers and abutments. The arrangement of the manual is summarized as follows:

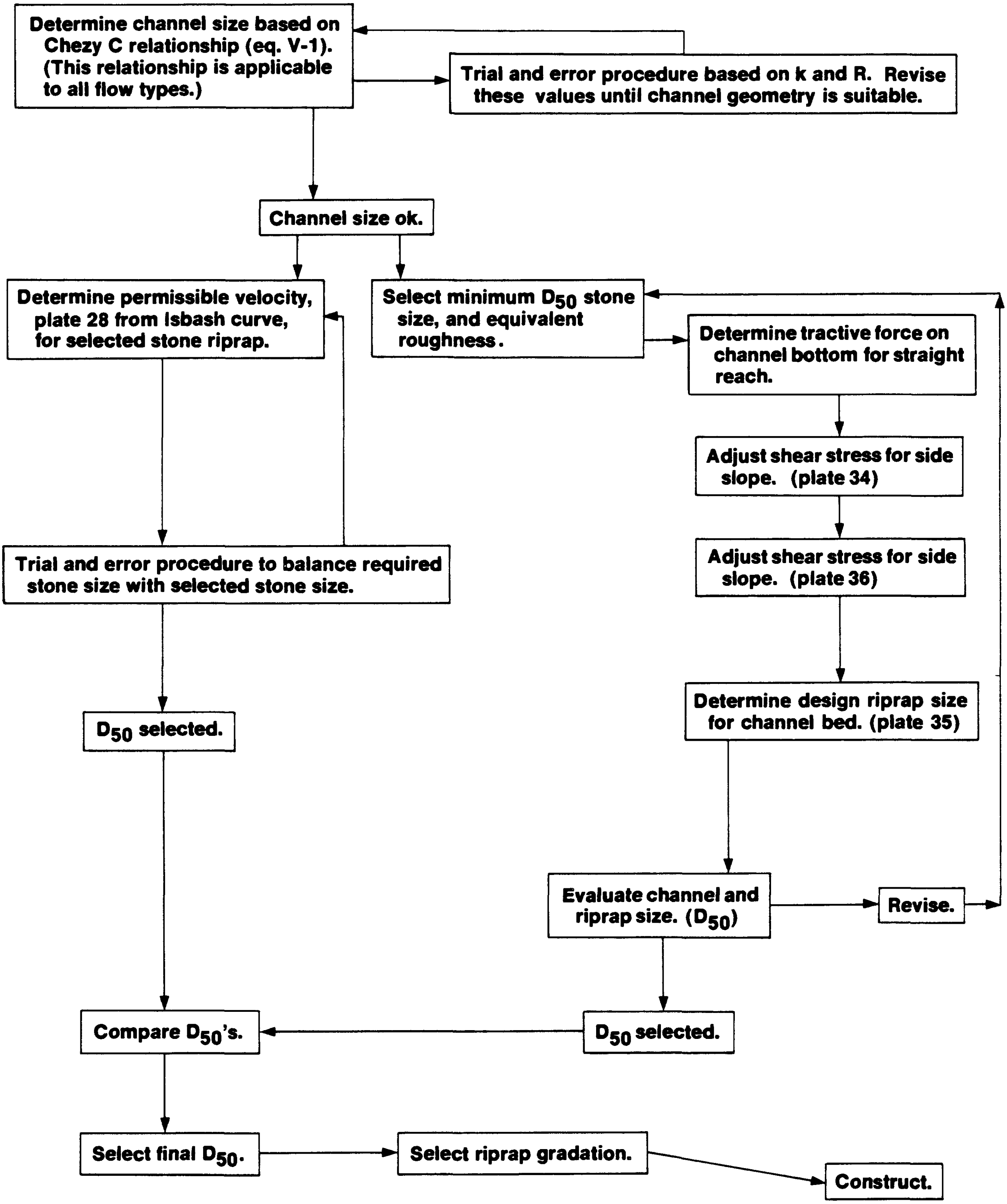


FIGURE 30. Flow chart for application of U.S. Army Corps of Engineers Manual EM-1601 in design of rock riprap.

1. Channel classification -- Flow is classified as subcritical, which is equivalent to tranquil flow, with a Froude number less than 1 ($F<1$). Supercritical or rapid flow has a Froude number greater than 1 ($F>1$). Referring to the specific-energy relationship presented by Chow (1959), it is apparent that a large change in depth may occur with a small variation in the specific energy at a cross section. Thus, flows that are near the transition are unstable and wave action, turbulence, and a hydraulic jump may occur. Studies by the Corps (EM-1601) indicate that the region of unstable flows may occur in the range of Froude numbers between 0.86 and 1.13.

2. Physical features -- This part of the study is site specific and includes evaluation of geomorphic, cultural, historical, and economic factors that may affect the existing or proposed channel.

3. Physical hydraulic elements -- This part of the procedure addresses the channel cross sectional size, configuration, length, roughness, and hydraulic efficiency. Design charts and equations are provided to estimate the required channel geometry and details of the riprap protection.

4. The riprap design procedures include two concepts in the estimation of a stable channel. The two concepts are estimation of permissible velocity, and estimation of permissible shear.

The Corps channel and riprap design procedure is keyed to a determination of the actual or proposed channel roughness. From this, boundary shear is determined and then compared with the design shear that the riprap is able to resist. Friction losses, which dictate the size of channel required to convey the design discharge, are based on estimates of friction slope (S_f), which may be determined by use of one of three equations--Chezy's, Manning's, or Darcy's. Procedures in EM-1601 are based largely on Chezy C resistance coefficients. Friction coefficients represent energy losses, related friction loss, turbulence, and eddy losses. Energy losses caused by turbulence and eddies are difficult to estimate and require special procedures.

EM-1601 then presents the concept of equivalent roughness, k, which is a measure of effective surface roughness within the boundary layer. Chow (1959) defines the roughness height, k, as the effective height of irregularities forming the boundary. The roughness height is a measure of the linear dimension of the roughness factor, but is not necessarily equal to the actual or average height of the boundary material. The ratio R/k or the hydraulic radius to roughness height is known as relative roughness. The equivalent roughness of a channel boundary is a function of the kinematic viscosity, v, and mean velocity, V_a, of flow. EM-1601 shows that for hydraulically rough channels, the Chezy friction coefficient C may be estimated by the equation:

$$C = 32.6 \log_{10} (12.2R/k) \tag{24}$$

For river channels, Chow (1959, table 8-1) indicates the value of k usually falls between 0.1 and 3.0. In the design of riprap, EM-1601 indicates equivalent roughness k may be taken as the theoretical spherical diameter of the median size stone. For the design of riprap, this diameter is assumed to be equivalent to D_{50}. The discussion in EM-1601 (p. 7) also indicates that the use of k in computational procedures is emphasized because results are relatively insensitive to errors in assigned k. The difficulty with the use of equivalent roughness k in riprap design is the problem of identifying realistic values of k, which is an index of boundary roughness that is similar to Manning's n. The tie between Manning's n and equivalent roughness k is given by the following equation from EM-1601:

$$n = \frac{R^{1/6}}{23.85+21.95 \log_{10} R/k} \tag{25}$$

The local boundary shear, τ_0, at any point in the cross section is estimated by the following equation:

$$\tau_0 = \frac{\gamma V_a^{\,2}}{\left(32.6 \log_{10} \dfrac{12.2d}{D_{50}}\right)^2} \tag{26}$$

For this equation,

τ_0 = local boundary shear (lb/ft^2)
γ = unit weight of water (lb/ft^3)
V_a = average velocity in vertical
d = depth of vertical
D_{50} = median stone diameter

If velocities near the riprap boundary are available from measurements or model tests, another equation from Chow (1959) is given in the ETL dated May 27, 1971, for estimating the local boundary shear stress:

$$\tau_0 = \gamma\left(\frac{v}{32.6 \log_{10} 30y/k}\right)^2 \tag{27}$$

where τ_0 = local boundary shear
γ = unit weight of water
v = local velocity near riprap (from model studies, prototype measurements)
y = depth above boundary corresponding to v
k = equivalent roughness

Equation 27 is the same as equation 26 except the function that governs the local velocity distribution, $30y/k$, is used instead of $12.2d/D_{50}$. Use of equation 27 will result in a larger boundary shear stress because equation 26 represents an integration of velocities over the vertical rather than the local velocity near the boundary.

Values of Manning's n and equivalent values of k based on equation 25 have been determined for 31 current-meter measurements of streams with banks protected with rock riprap. Estimates of Manning's n for bank roughness are based on that part of the channel that is covered by riprap, and the hydraulic radius is equivalent to the area divided by the wetted perimeter of the rock riprap, as shown in figure 31. Only the measured discharge in the shaded area of the figure was used in estimating Manning's n. The water-surface profile was obtained by field surveys at the time of the discharge measurement. None of the sites selected had riprap installations on both banks or on the channel bed.

In the estimation of equivalent roughness k by application of equation 25, all variables were measured except k. The resulting relationship between the roughness factors, n and k, for the 31 measurements is shown in figure 32. A comparison of Manning's n and corresponding values of k in the figure indicates the values of k computed using equation 25 must be in the order of 16 ft (4.9 m) for a corresponding n value of 0.08. These values of k are far greater than the maximum value of 3 suggested by Chow (1959, table 8-1). A relationship between n and k was determined by least squares regression for flow in the wedge to be

$$n = 0.0355k^{0.293} \tag{28}$$

Both Manning's n and the equivalent roughness k are quantitative assessments of boundary roughness. Plates 4 and 5 in EM-1601 provide a four-way relationship between Chezy C, Manning's n, R, and k for estimating two of the four factors. On the basis of the relationship shown in figure 32, estimates of equivalent roughness k obtained by application of plates 4 and 5 in EM-1601 may be questionable.

The relationship shown in figure 32 and equation 28 provides a means to estimate the local boundary roughness of a riprap installation in terms of Manning's roughness coefficient n on equivalent roughness k. These data can then be used to compare the estimated capacity of the channel with the proposed design discharge and to estimate the water-surface profile. These values of Manning's n represent the effect of local bank roughness and may be used to determine local shear stresses. Average shear stresses based on average depths or the hydraulic radius for the cross section may be estimated using equation 9. In estimating Manning's n, it should be recognized that median values of Manning's n in the flow wedge next to the riprap for 28 measurements were a ratio of 1.4 greater than values of n determined for the overall channel.

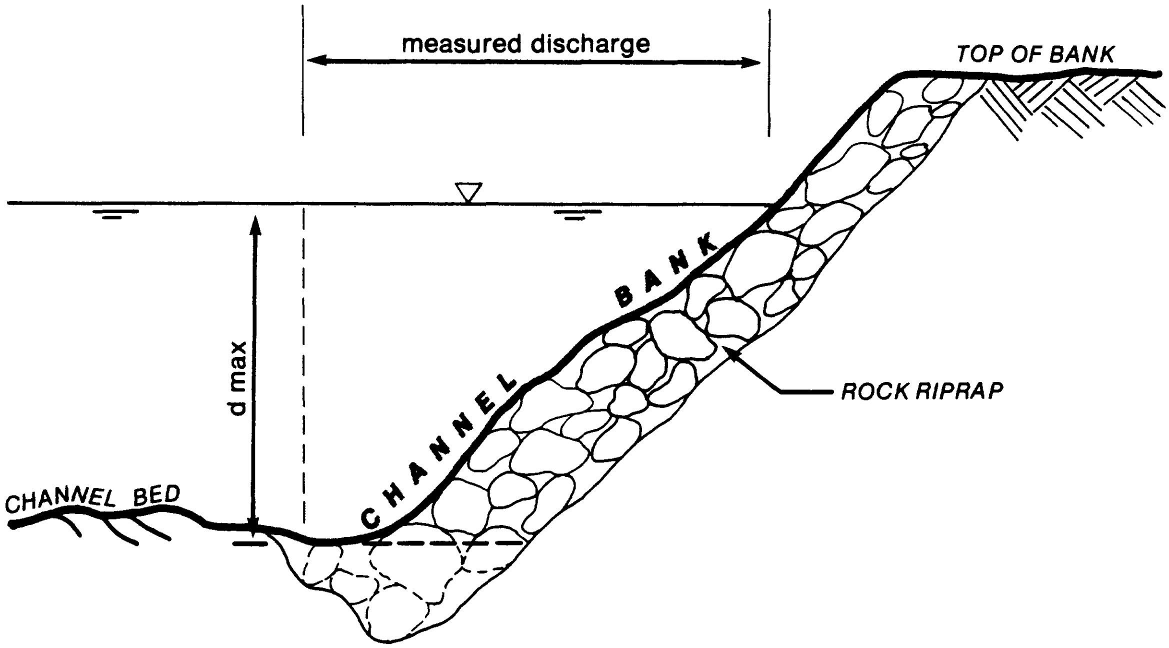


FIGURE 31. Sketch of channel bank used to measure boundary resistance in the form of Manning's n and equivalent roughness k.

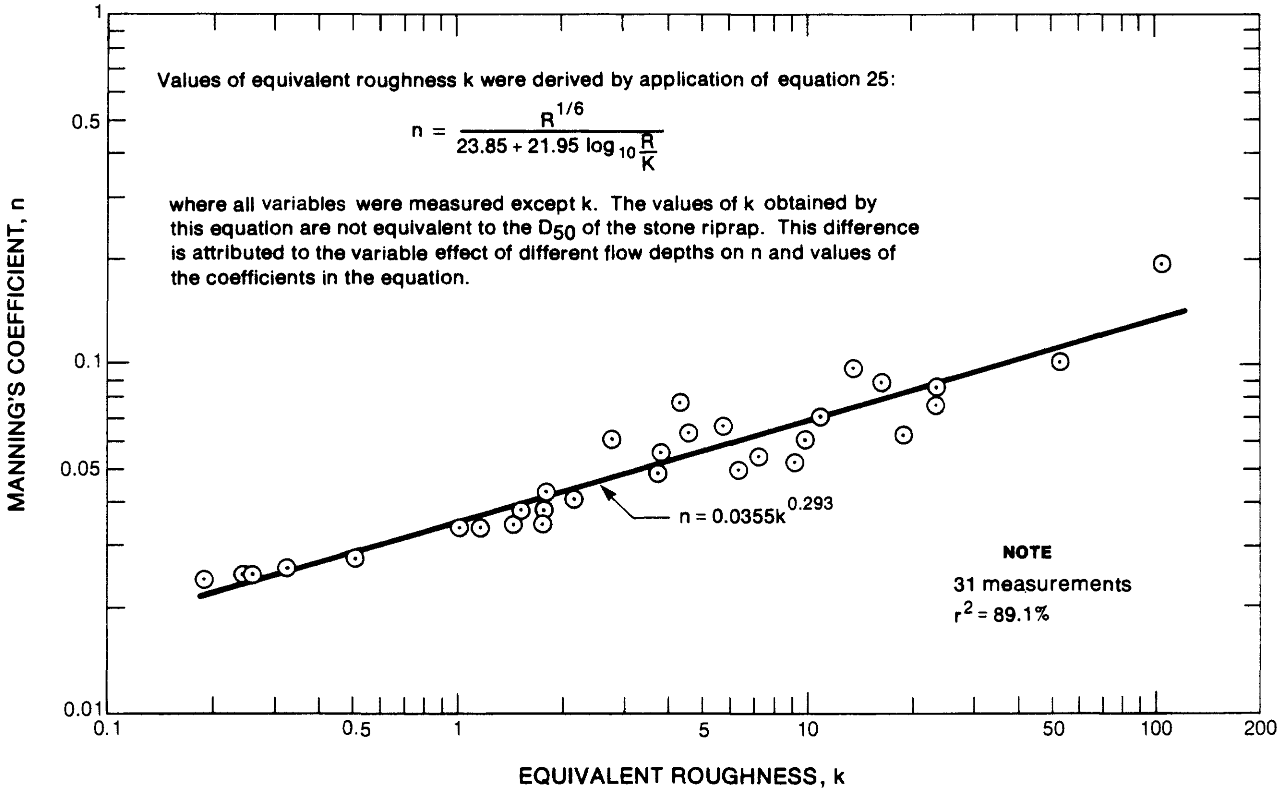


FIGURE 32. Relationship of Manning's n and equivalent roughness k for channel banks.

In the design of riprap, the local boundary shear on the bed is estimated using equation 26, which is then adjusted by a factor of 1.05 to determine local shear at the toe of the side slope. In the ETL dated May 14, 1971, it is recommended that stresses computed by equation 26 be adjusted by a factor of 1.5 if the channel shape is not uniform, and localized, higher boundary shear conditions are expected. Other adjustments for the effect of banks and bends are made as described in EM-1601.

Adjustments of the boundary shear stress at banks are applied to determine the design shear on the channel side slope (τ'), based on the angle of side slope (ϕ) and angle of repose (θ) of the riprap, as given by the equation:

$$\tau' = \tau\left(1-\frac{\sin^2\phi}{\sin^2\theta}\right)^{0.5} \tag{29}$$

The adjusted local shear stress is then compared with the allowable design shear. The design shear is determined using the minimum D_{50} stone size (EM-1601 allows a range in stone size depending on riprap thickness). The D_{50} stone size for the channel bed is estimated using the relationship (EM-1601, plate 35):

$$\tau = a(\gamma_s-\gamma)\ D_{50} \tag{30}$$

where τ = design shear on channel bed

γ_s = specific weight of stone

γ = unit weight of water

D_{50} = theoretical spherical diameter of average size stone (actually the median size)

a = coefficient equal to 0.040, which indicates the ability of the riprap to resist boundary shear. No data are presented indicating the source of the coefficient or range in values of design shear that may be appropriate.

Application

Application of the riprap design procedures in EM-1601 involves these considerations:

- o The design of the channel size for a specified discharge is based on estimates of boundary roughness in the form of Chezy C.
- o The roughness of riprap is expressed in terms of equivalent roughness k, which is considered equivalent to the stone size D_{50} (as discussed previously, values of k and stone size (D_{50}) are not equivalent).
- o Boundary shear is determined for the channel bed on the basis of local velocity, but to account for variations in velocity, an arbitrary factor of 1.05 is used to increase the shear stress at the toe of the side slope.
- o Boundary shear at bends is adjusted from bed shear by a factor greater than one. Shear on the side slopes is adjusted from bed shear by a factor less than one.
- o The angle of repose of riprap is given as about 40°. This angle is about half the angle of 70° recommended by Cal-B&SP.
- o The ability of riprap to resist shear is a function of stone weight, size, and the coefficient a. The value of a is arbitrarily set at 0.040.
- o The design of a riprap installation is a trial-and-error procedure with different stone size, in terms of D_{50}, and riprap thickness being adjusted so that the estimated boundary shear is less than the riprap design shear.
- o Estimates of local boundary shear based on a subdivided channel are difficult to make because the local velocities are not known or are difficult to determine.
- o New procedures, given in 1971, adjust bed shear by a factor of 1.5 for irregular shaped channels subject to flow turbulence, localized velocities, and pressure pulsations.

Design procedures given in EM-1601 and the ETL of May 14, 1971, were applied to the flood of January 4, 1982, on Pinole Creek at Pinole, California, at cross section 3 (table 5). Results are described on the basis of three procedures. Number 1 is the use of EM-1601 for average flow conditions; number 2 assumes an irregular shaped channel as described in the 1971 ETL; and number 3 uses procedures for estimating local shear as described in the 1971 ETL.

Number 1: Using equation 32 from EM-1601 (equation 26 of this report; d in eq. 32 is equivalent to d_a used here), the estimated average shear stress is

$$\bar{\tau}_{o\ bed} = \frac{62.4x(7.7)^2}{\left(32.6\ \log_{10}\ \frac{12.2x4.9}{0.6}\right)^2} = 0.87\ \mathrm{lb/ft^2}\ (4.3\ \mathrm{kg/m^2})$$

Adjusting shear to toe of the side slope, $\tau_{o\ toe} = 1.05\ \tau_{o\ bed}$ (plate 31 in EM-1601) = 0.87x1.05 = 0.91. From Plate 34 in EM-1601 for channel bends:

$$r/w = 188/54 = 3.48;\ \frac{\bar{\tau}_{0\ bed}}{\tau_b} = 1.60;\ \tau_0 = 1.6\ x\ 0.87 = 1.39\ \mathrm{lb/ft^2}\ (6.8\ \mathrm{kg/m^2})$$

where r = centerline radius of channel bend
w = water-surface width at upstream end of bend
τ_b = shear at bend

Number 2: Adjusting the bed shear stress for an irregular shaped channel by a factor of 1.5, as recommended in the 1971 ETL, gives:

$$\tau_{0\ bed} = 0.87x1.5 = 1.31\ \mathrm{lb/ft^2}\ (6.37\ \mathrm{kg/m^2})$$

Then, adjusting for bends, but not at the toe, gives:

$$\tau_0 = 1.6x1.31 = 2.10\ \mathrm{lb/ft^2}\ (10.2\ \mathrm{kg/m^2})$$

Number 3: Riprap design procedures given in EM-1601 were modified in the ETL dated May 27, 1971, so that local boundary shear could be used in the design process. The local shear stress is estimated using equation 27, in which v is local velocity, obtained by adjusting mean velocity in the cross section by a factor of 1.6 (see table 2). Applying the equation:

$$\tau_0 = 62.4\left(\frac{7.7x1.6}{32.6\ \log_{10}\ 30x4.9/0.60}\right)^2 = 62.4(0.158)^2 = 1.56\ \mathrm{lb/ft^2}\ (7.62\ \mathrm{kg/m^2})$$

$$\tau_0\ \text{adj. bed} = 1.56x1.5 = 2.34\ \mathrm{lb/ft^2}\ (11.4\ \mathrm{kg/m^2})$$

$$\tau_0\text{adj. bend and side slope} = 2.34x1.60x0.72 = 2.70\ \mathrm{lb/ft^2}\ (13.2\ \mathrm{kg/m^2})$$

In the EM-1601 procedure, flows in each subarea and associated velocities are estimated on the basis of estimated equivalent roughness k for each subarea. If the bed or bank is submerged, determination of k for underwater parts of the cross section is difficult. Estimation of local velocities is a laborious task unless the design is set up for computer processing. The development of considerable detail, in which each cross section is subdivided up to six times, seems to be unnecessary when the channel geometry is subject to change during future flooding. Further, the procedure for subdividing a channel assumes the energy gradient has the same slope throughout the entire cross section.

The design or allowable shear stress on the channel bed based on equation 30 and a stone size D_{50} of 0.60 ft (equation 33 of EM-1601) is:

$$\tau_{allowable} = 0.040(165-62.4)0.60 = 2.46 \text{ lb/ft}^2 \text{ } (12.0 \text{ kg/m}^2)$$

Application of riprap design procedures based on equation 32 in EM-1601 (procedure number 1, which uses average velocities) suggests that the estimated shear stress on the channel bed of 0.87 lb/ft^2 (4.3 kg/m^2) or the adjusted stress for bends, τ_0 = 1.39 lb/ft^2 (6.8 kg/m^2), is less than the allowable design stress (2.46 lb/ft^2 or 12.0 kg/m^2). Procedure number 3, which is the application of equation 8-13 from Chow (1959) and recommended in the 1971 ETL, to estimate local shear stresses and then adjust for the effect of channel side slope and bends, gives an estimated shear stress of 1.56 lb/ft^2 (7.62 kg/m^2), which is less than the allowable stress of 2.46 lb/ft^2 (12.0 kg/m^2). In the use of equation 32 from EM-1601 and equation 8-13 from Chow (1959), the velocity component is not the mean for the cross section but rather is an estimate of the average local velocity that may occur in the vertical, which is assumed to be the maximum velocity in the cross section. The maximum velocity (V_m) used in this comparison was estimated using the ratio V_m/V_a = 1.6, based on data from table 2.

On the basis of the estimates of shear stress computed by the various methods and compared with the allowable stress, the use of a riprap size D_{50} of 0.60 ft (0.18 m) would be considered adequate for this site on the basis of the three procedures, as indicated by the comparison given in table 4.

Evaluation of "Sedimentation Engineering," American Society of Civil Engineers (Manual No. 54)

Description

Manual Number 54, "Sedimentation Engineering" (Man-54, Vanoni, 1975), presents a section on channel control structures designed to prevent erosion. A layer of rock riprap is one form of protection from erosion.

The discussion on riprap refers to positive experiences in the use of graded material to protect banks on the Missouri River. The median size rock (D_{50}) is determined on the basis of flow velocity, and the gradation of the material is specified so that the maximum size stone is limited to about 1.5 times the median size. The thickness of the riprap layer should be 1.5 times the median size (D_{50}) rock. The toe of the riprap is protected by placement of a large quantity of stone with the bottom of the trench 7 ft (2.1 m) below normal low water. A sketch of toe riprap placement in the manual does not specifically indicate the channel bed, but it appears the bottom of the trench is below the channel bed. It is indicated that the use of rock riprap is restricted to flow depths of not more than 40 ft (12.2 m) and preferably less than 30 ft (9.1 m), and the angle of attack should not exceed 30°. If depths of flow exceed 30 to 40 ft (9.1 to 12.2 m), a mattress is needed to assist in placement of the riprap and to reduce separation of the material.

The size of rock required to protect the bank is a function of flow velocity at a distance 10 ft (3.05 m) from the bank. No information is provided on the procedure for estimating the velocity at this location. The median size stone, D_{50}, is determined in units of weight, using the following equation, which was proposed by Isbash (plate 29, EM-1601) and modified for sloping banks in Man-54:

$$W = \frac{0.000041\ G_s V^6}{(G_s-1)^3\ \cos^3\phi} \tag{31}$$

where W = weight of stone in pounds

G_s = specific gravity of the stone

V = velocity; it was assumed V is equivalent to V_m

ϕ = angle of side slope to horizontal

In order to compare this equation with other methods, G_s is assumed to be 2.65, which gives a rock density of 165 lb/ft^3 (2,640 kg/m^3), and the side slope is 2:1 or ϕ = 26.6°. The equation for computing stone size on the bank then takes the form:

$$W_{50} = \frac{0.000041\text{x}2.65\ V^6}{(2.65-1)^3\ \cos^3 26.6^\circ} = 3.43\text{x}10^{-5}V^6$$

The equation uses the velocity as an indicator of hydraulic stresses on the bed and bank rather than depth or hydraulic radius. This seems to be a preferable approach because estimates of slope that are applicable to the site and used to determine boundary shear may be difficult to determine.

Application

Using Pinole Creek at Pinole, California, for comparison, the mean velocity was 7.7 ft/s (2.4 m/s) during the flood of January 4, 1982, and discharge was 2,250 ft^3/s (63.67 m^3/s). The mean velocity during this flood was equivalent to a maximum velocity of about 12.2 ft/s (3.72 m/s), based on the relationship V_m/V_a = 1.6 from table 2. Applying equation 31 and using the assumed velocity of 12.2 ft/s (3.72 m/s) at a location 10 ft (3.05 m) from the bank gives a W_{50} of 113 lb (51.3 kg). The following relationship given on plate 30 in EM-1601 was used to determine the equivalent stone diameter for a given stone weight (see eq. 23, p. 62).

$$D_{50} = (6W_s/\pi\gamma_s)^{1/3}$$

where W_s = weight of rock

γ_s = specific weight of stone

In this case, $D_{50} = \left(\frac{6\text{x}113}{\pi\text{x}165}\right)^{1/3}$ = 1.1 ft (0.34 m).

Riprap with a D_{50} of 0.6 ft (0.2 m) was installed at this site and failed during the 1982 flood. Application of equation 31 with the mean velocity increased by a factor of 1.6 results in a stone with D_{50} of 1.1 ft (0.34 m), which may be a more appropriate D_{50} size for this channel, as indicated by the data in table 4. A plot of this equation relating stone size to average velocity is shown in figure 18. This equation is easy to apply, but guidelines for estimating the velocity are needed. There are no procedures that consider the effects of channel curvature. The differences in stone size required to protect the channel bed or bank are considered in the factor $\cos^3 \phi$, which becomes 1 for flat slopes or the channel bed. The illustration (fig. 5.13 in Man-54) describing toe trench revetment should be modified to show the trench depth below the channel bed for a proper installation.

Evaluation of "Sediment Transport Technology" (Simons and Senturk)

Description

The procedure in "Sediment Transport Technology" (Simons-STT) by Simons and Senturk (1977) involves solving a set of four equations that describe the stability of a riprap particle for a given set of channel and hydraulic conditions. The equations are based on an analysis of (1) the passive forces affecting particle stability on a side slope, and (2) active fluid forces tending to rotate the particle out of position. The analysis uses the beginning-of-motion concept in a tractive force approach for determining particle stability. The forces affecting the stability of the particle are expressed as a ratio (termed the factor of safety, SF). If this ratio is greater than 1, the riprap supposedly will be safe from failure; if equal to 1, incipient motion conditions exist; and if less than 1, the riprap will fail. The Pinole Creek failure occurred with a safety factor of 1.06. Equations for determining the safety factor are given for horizontal and nonhorizontal flow along side slopes and for plane bed conditions. A complete discussion of the forces affecting the riprap and the derivation of the equations is given. Several design options are possible by preselecting either rock size, side (or bed) slope, or the safety factor.

Several other channel design and riprap procedures are presented in Simons-STT for informative and comparative purposes, as well as mathematical comparisons of the Simons-STT approach with approaches presented in EM-1601, Cal-B&SP, Man-54, USBR-EM-25, HEC-11, and Campbell (1966).

Riprap stone characteristics (shape, durability, and specific gravity) are discussed in Simons-STT, as well as guidelines for the gradation, placement, and thickness of riprap.

The Simons-STT riprap design procedure is presented through a rigorous mathematical analysis of forces affecting riprap particles. Some assumptions are inevitable in a theoretical treatment of a practical problem. The validity of these assumptions is critical to the applicability of the equations derived for riprap design, but two of several assumptions seem somewhat unfounded. The first concerns the ratio of two moment arms for a given riprap particle on a side slope. The text indicates the ratio to be equal to 2 and cites the definition sketch as a reference. It is not apparent from the sketch (their fig. 7.9, p. 420) that the value assumed is reasonable. Because the magnitude of moment arms for a given particle depends on the magnitude and direction of flow in the vicinity of the particle and on the point (on the particle) through which the force is acting, it seems that the value of the ratio must vary. In the second assumption, the drag force is said to be twice the lift force (for the same particle) and the product of the two assumed ratios is approximately equal to 1, but no basis for these values is given.

Additional observations concerning development and use of the Simons-STT procedure are:

- Downstream from drop structures, or for hydraulic conditions where the flow is decelerating, use of the average shear stress in calculating the stability factor for side slope or bed particles may yield values of the safety factor much too small.

- Some adjustment is needed for water-surface profile conditions with high turbulence (and shear stress) and small slope.

- No apparent corrections for determining riprap size (or safety factor) are made for channels with bends or where flows impinge directly upon the bank.

- No criteria are given for determining possible combinations of hydraulic conditions where riprap lining may be necessary.

- Specific guidelines for vertical or lateral extent of the riprap lining or for toe construction are not given.

- Equations are presented for determining the safety factor either for flow along a channel side slope or for flow on a plane sloping bed. However, for use as a practical design procedure, the presentation is confusing because of frequent interjections of theory and supplemental equations.

- A recommended or suitable safety factor is not directly given in the discussion, although the value of 1.5 appears in an example problem.

An example problem is given in the riprap design section of Simons-STT (chap. 7) in which $\lambda = 20^\circ$ (where λ = angle between horizontal and velocity vector at a point; see their fig. 7.9). Velocities with angles to the horizontal this large are usually localized, and an illustration to show the assumed hydraulic condition would aid in explanation. A better explanation and figure is given for an identical example by Richardson and others (1975, section 6.4.0). A suitable illustration and example for the more common case of small λ values (commonly 2°) and nearly horizontal flows (τ=0) on side slopes would also be helpful. Solved examples at the end of the riprap design section provide a minimum of relevance to the practical use of the equations they present for determining safety factors for rock riprap.

Application

The determination of a safety factor for rock riprap on a channel side slope subject to horizontal flow involves solving the following equations (see fig. 33 for definition sketch and fig. 34 for flow diagram of the procedure):

$$SF_m = \frac{\tan \phi}{\tan \theta} \tag{32}$$

$$\eta = \frac{21\ \tau_s}{(G_s - 1)\gamma D} \cong \left[\frac{SF_m{}^2 - SF^2}{SF x SF_m{}^2}\right] \cos \theta \tag{33}$$

$$\xi = SF_m\ \eta \sec \theta$$

$$SF = \frac{SF_m}{2} \left[(\xi^2 + 4)^{1/2} - \xi\right]$$

where SF = safety factor for riprap on a side slope with horizontal flow

SF_m = safety factor for riprap on a side slope with no flow

ϕ = natural angle of repose of riprap particles (degrees)

θ = side slope angle (degrees) (note that the angles of θ and ϕ refer to opposite parameters when compared with procedures discussed in EM-1601; see equation 29)

η = stability factor for a particle on a plane horizontal bed

τ_s (=γRS) = average tractive force (shear stress) on side slope in vicinity of particle (1.56 lb/ft^2 or 7.62 kg/m^2)

G_s = specific gravity of riprap material

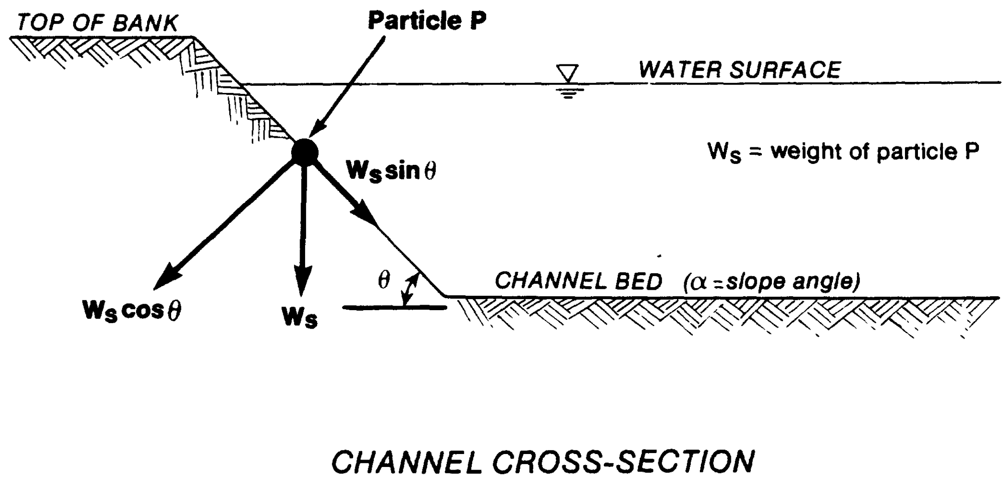
γ = unit weight of water (lb/ft^3)

D = assumed median particle size or D_{50} of riprap

ξ = converts the safety factor from nonhorizontal to horizontal side-slope flow

Similar equations are also provided for hydraulic conditions with nonhorizontal flow along side slopes.

In the procedure, several design options are possible by preselection of values for rock size, side slope, or safety factor. The simplest solution involves solving the equations for given values of rock size, and side or bed slope, and accepting or rejecting the assumed design on the basis of the calculated value of the safety factor.



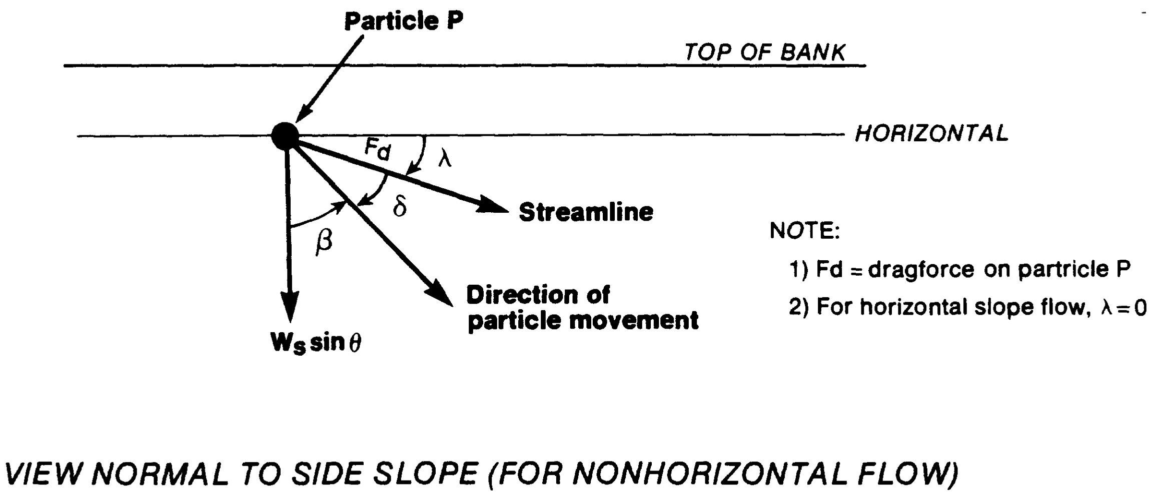


FIGURE 33. Definition sketch of variables used by Simons and Senturk for design of bank protection.

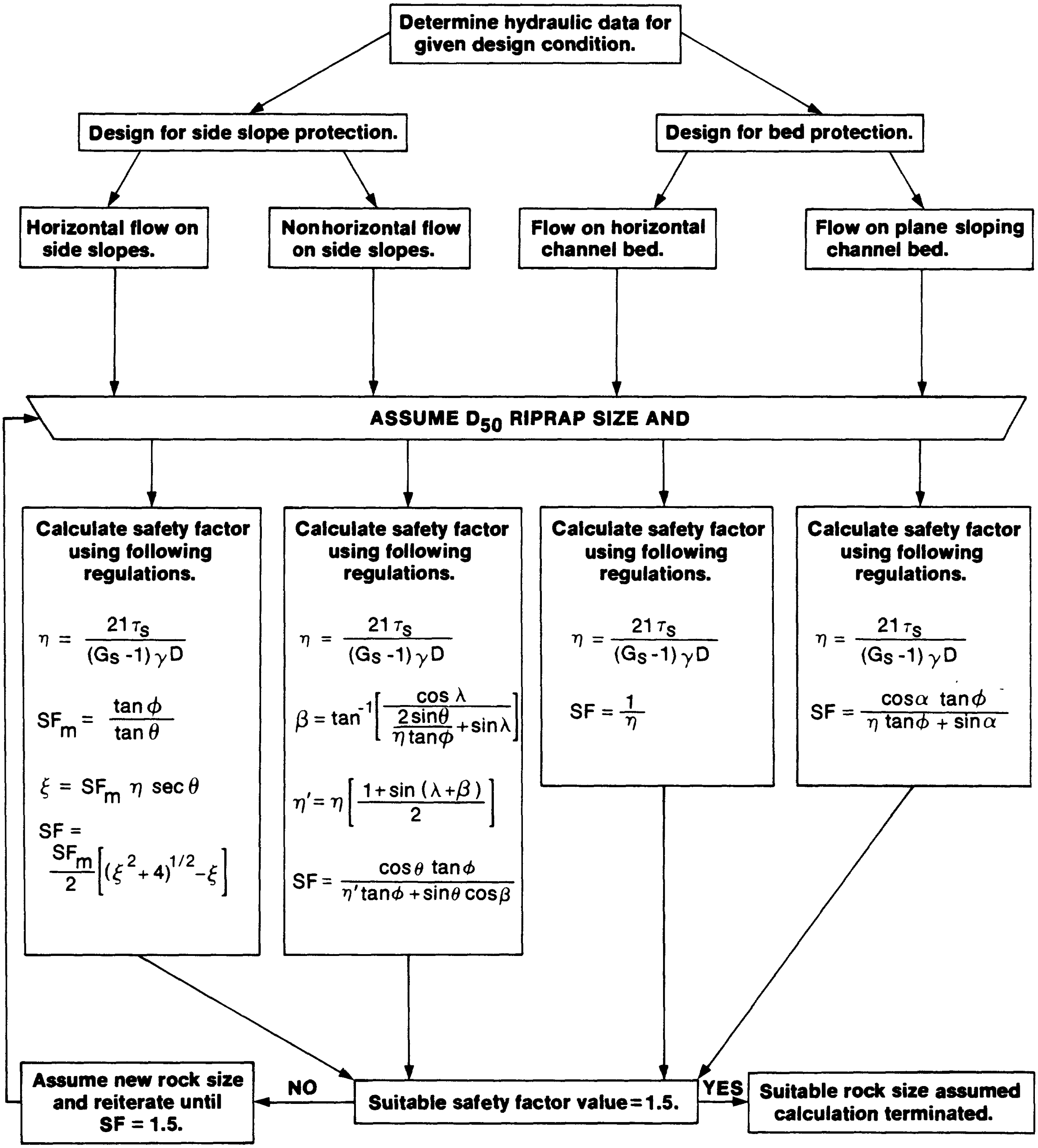


FIGURE 34. Flow chart for Simons and Senturk bank protection procedure.

Applying the horizontal flow, side-slope procedure to the Pinole Creek site, cross section 3 (table 5), a safety factor can be computed from:

Q = 2,250 ft^3/s (63.67 m^3/s) (discharge)
R = 4.62 ft (1.41 m) (hydraulic radius)
S_w = 0.0054 (slope of water surface)
G_s = 2.65
θ = 26.6° (2:1 side slope)
D = 0.6 ft (0.2 m) (assume $D = D_{50}$)
ϕ = 41° natural angle of repose of riprap
γ = 62.4 lb/ft^3 (1,000 kg/m^3)

$$SF_m = \frac{\tan 41}{\tan 26.6} = 1.74$$

$$\eta = \frac{21\,\tau_s}{(G_s-1)\gamma D} = \frac{21(2.59)}{(1.65)(62.4)(0.6)} = 0.53$$

$$\xi = SF_m \eta \sec\theta = 1.74(0.53)(1.12) = 1.03$$

$$SF = \frac{SF_m}{2}\left[(\xi^2+4)^{\frac{1}{2}}-\xi\right] = \frac{1.74}{2}\left[(1.03^2+4)^{\frac{1}{2}}-1.03\right] = 1.06$$

Because 1.06 is only slightly greater than unity, a design particle size of 0.6 ft (0.2 m) would be close to the condition of incipient motion (table 4). Several trial-and-error calculations were made by changing the particle size, D, while holding the other variables constant. As indicated in table 9, the minimum safe size is a D_{50} of about 0.9 ft (0.27 m), which has a safety factor of 1.24.

Table 9. Safety factors for various sizes of riprap, Pinole Creek at Pinole, California (cross-section 3).

D_{50} (ft)	ϕ (deg)	tan ϕ	SF_m	η	ξ	SF	Remarks
0.3	40	0.84	1.68	1.06	1.99	0.70	D_{50} too small
.6	41	.87	1.74	.53	1.03	1.06	Incipient motion condition
.9	41	.87	1.74	.35	.68	1.24	Minimum safe size
1.2	42	.90	1.80	.27	.54	1.39	
1.5	42	.90	1.80	.21	.42	1.46	
1.8	42	.90	1.80	.18	.36	1.50	

D_{50} = median particle size of riprap.
ϕ = natural angle of repose of riprap particles (Note that the angles of θ and ϕ refer to opposite parameters when used in EM-1601).
SF_m = safety factor for riprap on a side slope with no flow.
η = stability factor for a particle on a plane horizontal bed.
$\xi = SF_m \eta \sec\theta$. θ = constant side slope 2:1 or 26.6 degrees.
SF = safety factor for riprap on a side slope with horizontal flow.

Evaluation of U.S. Bureau of Reclamation, Engineering Monograph No. 25

Description

The procedure in Engineering Monograph No. 25 (USBR-EM-25, Peterka, 1958, p. 207) for estimating the size of riprap to be used downstream from stilling basins, involves relating stone size to flow velocity on the channel bottom. The relationship of stone size to velocity, shown in figure 35, is based on laboratory flume test data reported in the literature between 1786 and 1948, and additional laboratory tests by the Bureau of Reclamation prior to 1974. An attempt was made to verify the validity of the relationship shown in figure 35 by investigation and analysis of several prototype stilling basins subsequently constructed by the Bureau on the basis of the procedures given in USBR-EM-25.

The riprap design procedure described in USBR-EM-25 was evaluated using data from 11 prototype installations. The scatter of prototype data in figure 35 is about ±5 ft/s (1.5 m/s) throughout the range of measured velocities. The velocity data indicated to be bottom velocities in figure 35 are actually average velocities determined by dividing discharge by flow area at the end sill. The end sill is the riprapped part of the channel, shaped as a trapezoid and located just downstream from an irrigation outlet structure. The flow area at the end sill was estimated using geometry of the outlet structure and from surveyed water-surface elevations. Flow velocities at the outlet structure are very high (as releases from an upstream impoundment) and are nearly supercritical.

On the basis of the field tests of the constructed stilling basins, the Bureau concluded that a well-graded riprap layer containing 40 percent of the rock pieces smaller than the required size was more stable than a blanket of stones consisting entirely of the required size. This was attributed to interlocking of the various size stones and to turbulence of flow near the rough boundary surface.

Unit weight of the stone (γ_s) was assumed to be 165 lb/ft^3 (2,640 kg/m^3) (G_s = 2.64), which is similar to the unit weight of riprap stones measured at five sites in the western United States (table 4).

In the vicinity of the end sill, flows are decelerating and are highly turbulent. As such, the erosive potential of flow, which is nearly free of suspended sediment, is considered high compared to the erosive potential of streamflow in open channels. Therefore, the riprap performance data determined from prototype installations at stilling basins should provide a good indication of the stone size required to resist displacement by particle erosion.

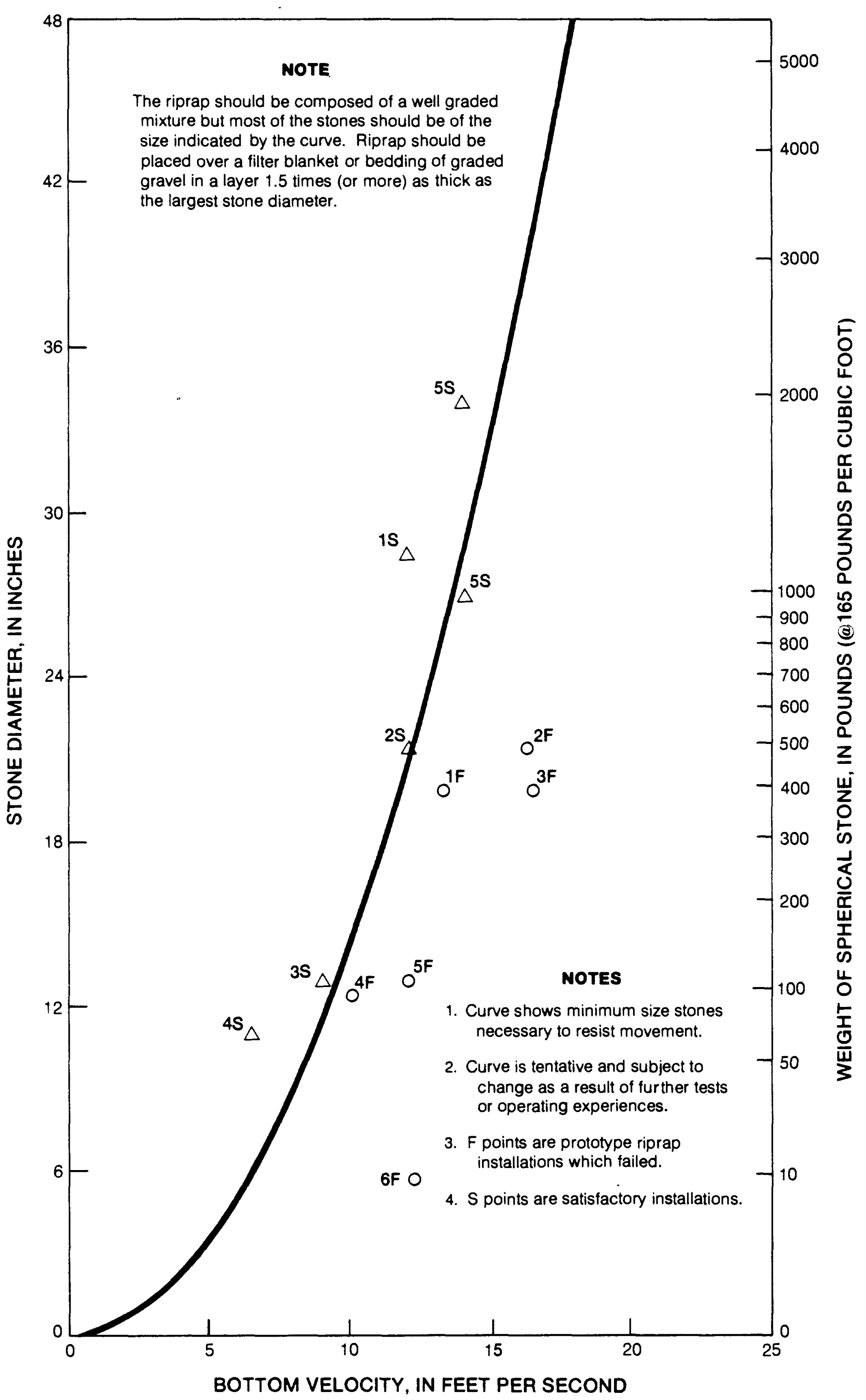


FIGURE 35. Curve to determine maximum stone size in riprap mixture. (Adapted from figure 165 in USBR Engineering Monograph No. 25, 1974).

Application

Adapting the relationship between velocity and stone size given in figure 35 by logarithmic transformation gives the relationship shown as curve A in figure 36. Because the size of stone derived using curve A represents material containing 40 percent of the stones smaller than the required size, then 60 percent of the stones should be larger. Because the comparison of the different design procedures in this study is based on a D_{50}, it was necessary to adjust the relationship from D_{40} to D_{50}, as shown by curve B in figure 36. This adjustment was made using the size gradation for riprap given in HEC-11 and HEC-15 in which $D_{50} = 1.16\ D_{40}$.

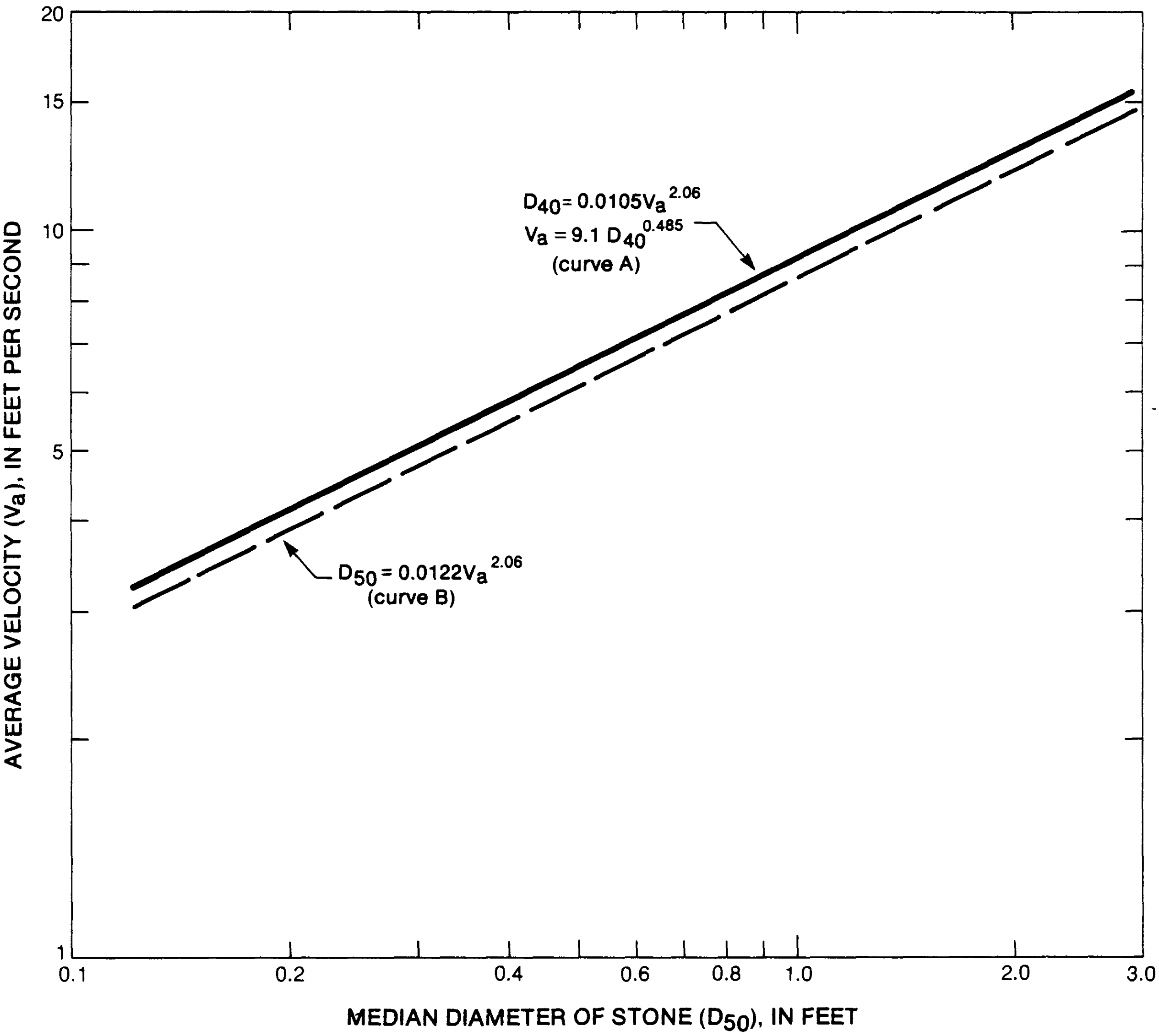


FIGURE 36. Relationship to determine median stone size based on average velocity (adapted from USBR-EM-25).

Application of the procedure as given in USBR-EM-25 for the design of rock riprap involves these steps:

1. Estimate average velocity at the study site. The selected cross section for determining the velocity should be at a constriction so the velocity factor is maximized.
2. Determine the median stone size (D_{50}), using curve B in figure 36. This curve is expressed as

$$D_{40} = 0.0105\ V_a^{2.06} \tag{34}$$

Since $D_{50} = 1.16\ D_{40}$, (35)

then $D_{50} = 0.0122\ V_a^{2.06}$ (36)

where D_{40} = diameter of rock size at which 60 percent of stones are larger
D_{50} = median diameter of stones
V_a = average velocity

The median stone size necessary to resist displacement for a discharge of 2,250 ft^3/s (63.67 m^3/s) on Pinole Creek at cross section 3 (table 5) is computed using equation 36:

$$D_{50} = 0.0122\ V_a^{2.06} = 0.0122\ (2250/298)^{2.06} = 0.79 \text{ ft } (0.24 \text{ m})$$

The D_{50} of 0.79 ft (0.24 m) is larger than the median size riprap installed at the site (0.60 ft or 0.18 m), which failed during the January 1982 flood. This analysis suggests the method presented by USBR-EM-25 would result in a slightly larger rock size than the material that failed during the January 1982 flood.

RIPRAP SPECIFICATIONS

After the selection of a median diameter rock size suitable to protect the channel, specifications for both the physical characteristics of the rock and the riprap layer (or blanket) must be determined. A general comparison of the physical characteristics of the rock and the riprap layer is presented here to illustrate the similarities and differences in specifications for the following design procedures: FHWA HEC-11 and HEC-15; EM-1601; Cal-B&SP; and Simons-STT. These procedures are primarily "loose" riprap designs. A fifth procedure, the Oregon Department of Transportation (ODOT) method for "keyed riprap," is also examined to include a different construction technique.

Rock Specifications

Rock specifications, as discussed here, refer to the physical characteristics of the rock particles that make up the bank protection. The fundamental shape, durability, and specific gravity criteria are summarized for each procedure. All but the Simons-STT procedure include detailed rock specifications and most state that specifications may be altered at the discretion of the engineer. Sample rock specifications presented in HEC-11 are the same as those given in HEC-15 (p. 113), and are referred to as FHWA procedures.

Shape

Simons-STT offers the least detail on shape criteria and gives the single criterion that "riprap consisting of angular stones is more suitable than rounded stones." FHWA and ODOT procedures give two basic shape criteria: (1) Stones should be angular (rounded stones are not acceptable), and (2) neither breadth nor thickness of a single stone should be less than one-third its length. Cal-B&SP and EM-1601 offer the most detailed shape criteria. In addition to the two criteria given by FHWA, EM-1601 adds, "not more than 25 percent of the stones shall have a length more than 2.5 times the breadth or thickness." The Cal-B&SP procedure also gives the basic criterion that length should not exceed 3 times either width or breadth. In addition, it presents a dimensional analysis of the effect of extreme range of shape on stone size in order to determine the maximum allowable long-axis dimension for a given stone and density. Results of the analysis are applied to the determination of thickness.

Durability

Where riprap must withstand abrasive action or is subject to freezing, all procedures recommend tests for durability. Typically, the procedures recommend qualitatively a stone that is hard, dense, durable, and resistant to weathering and water action. In addition, FHWA, Cal-B&SP, and ODOT recommend records of previous use and laboratory tests to determine the acceptability of a selected stone. Simons--STT indicates that "visual inspection is most often adequate to judge quality, but laboratory tests may be made to aid the judgment of the field inspector." Durability specifications are not mentioned in EM-1601.

Specific Gravity

The design stone size for a given channel depends partly on specific gravity. Cal-B&SP and ODOT recommend a minimum apparent specific gravity value of 2.5. The FHWA circulars recommend a minimum specific stone weight of 165 lb/ft^3 (2,640 kg/m^3), which converts to a specific gravity of 2.64. A small discrepancy occurs between Cal-B&SP and FHWA specifications in the basis for specific gravity determination: Cal-B&SP uses apparent specific gravity while FHWA uses bulk saturated surface-dry specific gravity. EM-1601 also uses saturated surface-dry specific gravity, but gives no minimum value of specific gravity. The degree to which the discrepancy in determination of specific gravity of stone may affect required stone sizes is not known. No criteria for specific gravity are presented in Simons-STT.

Riprap-Layer Specifications

The major characteristics of the riprap layer include: (1) thickness, (2) method of placement, (3) toe construction, (4) gradation of stone, and (5) filter blankets. In table 10, a summary of these and additional stone characteristics is given as they are recommended in each riprap design procedure. Following is a discussion of the key points presented in each procedure for the major revetment characteristics.

Table 10. Summary of riprap criteria used in design procedures.

[AASHTO, American Association of State Highway Transportation Officials; ND, not determined; max, maximum; min, minimum]

Procedure	Rock characteristics			Riprap layer characteristics					
	Shape	Durability	Specific gravity	Stone gradation	Placement	Thickness	Toe construction	Extent	Filter blanket
FHWA (HEC-11)	3:1, max ratio for stone dimensions	Specific AASHTO tests cited	2.5 min (AASHTO T-85)	Specific criteria	Guidelines given for dumped & hand placed	Equal to max stone size as min	yes	yes	yes
FHWA (HEC-15)	do.	do.	do.	do.	Dumped	ND	ND	ND	yes
Corps of Engineers (EM-1601)	3:1	ND	No min given	do.	Dumped & hand placed	Equivalent spherical diameter of W_{100} stone	yes	yes	yes
California Bank and Shore Protection (Cal-B&SP)	3:1	Visual inspection by experienced personnel and CA test 229E	2.5 min (AASHTO T-85)	do.	Machine placed & dumped	Min of 1.5ℓ where ℓ=long axis of calculated stone size	yes	yes	yes
Sediment Transport Technology (Simons-STT)	3:1	Visual inspection by experienced personnel, with lab tests if needed	No min given	do.	Dumped and hand placed	$2D_{50}$	ND	ND	yes
Oregon Dept. of Transportation (ODOT)	3:1	Visual inspection	2.5 min (AASHTO T-85)	do.	yes	no	yes	ND	yes

Thickness

No definite specification is given in HEC-15 as to thickness of riprap. Under "Construction Requirements" (section 612.04, p. 119), the statement is made that "the entire mass of stone shall be placed so as to be in conformance with the lines, grades, and thicknesses shown on the plans." The same is said for riprap (and filter blanket) that is dumped underwater. This statement also appears in HEC-11 (section 2.1.3, p. 11-7), along with the specification that thickness should be at least equal to the maximum stone size (D_{100}).

In EM-1601, a minimum practical thickness of 12 inches (0.305 m) is specified for placement. Their basic criterion is that the riprap layer thickness "should not be less than the spherical diameter of the upper limit W_{100} stone or less than 1.5 times the spherical diameter of the upper limit of the W_{50} stone, whichever is greater." Adjustments are given for additional thickness when riprap is placed underwater or where the bank is subject to impact by heavy drift or waves.

The Cal-B&SP procedure uses a more complex calculation for thickness which is based on an assumed orientation of stone, the angle of the side slope, assumed stone shape factor, and the class weight of stone. The general thickness equation given in the manual is for 1.5:1 side slopes, and it assumes a stone shape factor of 0.40. If actual conditions are something different, the equation must be manipulated accordingly. Development of the thickness equation involves a discussion on dimensional ratios of revetment stone. This, in combination with the thickness section, is a little confusing, and, because of assumptions concerning estimated hydraulic conditions at the site, is not likely to provide a greater degree of safety to the revetment protection. For machine placement, the general recommendation for minimum thickness (t) is 1.5 times the long-axis (ℓ) dimension of the stone, or $t=1.5\ell$. For dumped stone, an additional 25 percent thickness is recommended, or $t=1.875\ell$.

In the Simons-STT procedure, the following thickness guideline is given: "Thickness should be sufficient to accommodate the largest stones in the riprap. With a well-graded riprap with no voids, this thickness should be adequate. If strong wave action is of concern, the thickness should be increased 50 percent." We assume that the referenced thickness is approximately equal to the equivalent spherical diameter of the upper limit stone size (D_{100}).

The ODOT suggests that in the keyed riprap procedure, thickness requirements can be significantly reduced from those applying to loose riprap. Specifications for keyed riprap have been determined through experience and are given for each class of rock. Reasons cited for the decreased thickness are greater stability through reduced drag on individual stones, and an increase in the angle of repose produced by the compact mass of rock.

Method of Placement

HEC-11 and HEC-15 discuss placement of riprap, but HEC-15 considers hand-placed riprap to be rigid lining and does not address that method. In EM-1601, three common forms of riprap placement are described: (1) Machine placement, usually from a skip, bucket, or dragline; (2) dumping from trucks and spreading by bulldozer; and (3) hand placement. Of these, hand placement is said to produce the best revetment, but it is usually the most expensive. Dumping and spreading is considered least desirable in EM-1601, because of possible segregation and breakage. Cal-B&SP presents two methods: Machine-placement, and dumping and spreading. According to ODOT, rock placement is critical to the keyed riprap method because segregation and breakage prior to the tamping process could result in additional construction time or in a less effective bank protection. In all procedures other than Cal-B&SP, any dumping or spreading that may cause segregation or breakage of particles is disapproved or prohibited.

Toe Construction

No discussion is given in HEC-15 or Simons-STT for design of toe in riprap protected banks. HEC-11 gives specifications for toe design where: (1) The channel bed is movable, (2) scour may occur due to bends, and (3) a toe trench cannot be dug. A vertical depth of 5 ft (1.5 m) of toe below streambed is recommended.

EM-1601 acknowledges that the toe of the riprap is subject to greater erosive forces than other areas of the bank, thus additional thickness or extension into the streambed is required. Five different hydraulic and channel conditions are presented, and methods for their required protection are discussed and illustrated.

Cal-B&SP (fig. 153, p. 103) gives a general guideline for toe design in the form of a typical riprap section, but states that the figure is not a standard design and should be modified as required. Toe depth is shown in the figure as "below scour depth," but no guideline is given to determine the depth of scour.

ODOT also indicates the need for additional protection at the toe. Example specifications in the keyed riprap manual indicate that toe trenches should be dug and backfilled with riprap. Typical construction details and sections are illustrated in the manual.

Gradation of Stone

In general, it may be said that all procedures recommend a dense, uniform mass of durable angular stone with no apparent voids or pockets as a final product for the riprap layer. In HEC-11 and HEC-15, three general classes of stone are given (light, medium, and heavy). It is noted that these classes may not suffice for all cases, so a single gradation referenced to the D_{50} size is also given. The recommended gradations were based on performance of completed installations and tests conducted by the Corps of Engineers on the Arkansas River (Murphy and Grace, 1963, p. 47-55). Figure 37 shows how suggested gradations for each procedure vary with respect to the single gradation given in HEC-11 and HEC-15. In addition, table 8 shows the range of ratios of D_{15}, D_{85}, and D_{100} to D_{50} for each procedure that was determined for the various study sites. ODOT, Cal-B&SP, and EM-1601 present their gradation specifications by weight, and for comparison were converted to equivalent-volume spherical diameter using equation 23.

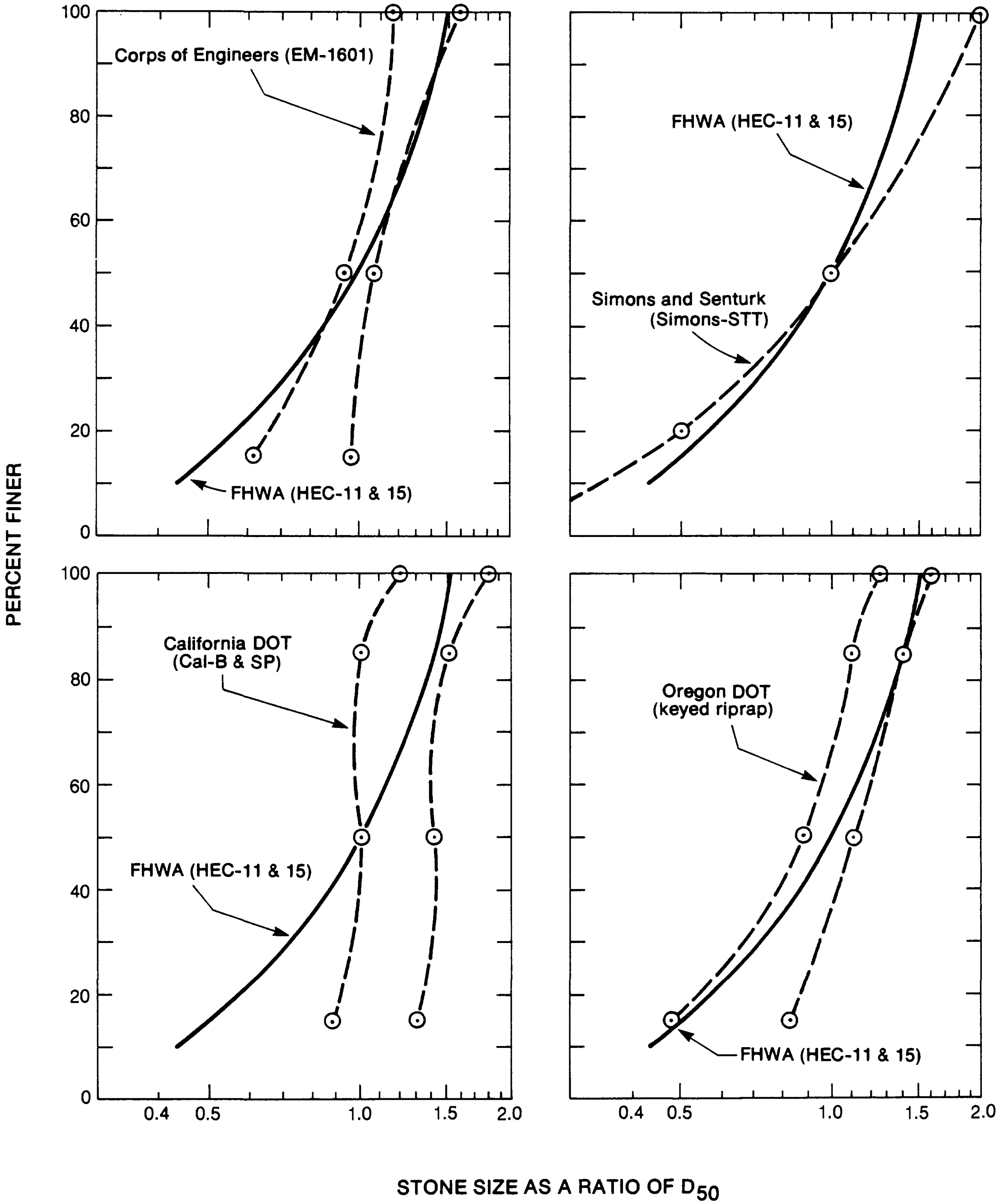


FIGURE 37. Comparison of stone gradations specified in different design procedures.

The EM-1601 specifications are quite detailed but do provide flexibility for limitations in stone availability. Basically, upper and lower limits are provided for W_{15}, W_{50}, and W_{100} stone sizes. Figure 37 shows that above the 50 percent size, the upper limiting curve agrees almost identically with the HEC-11 and HEC-15 curve. Below the 40 percent size, the EM-1601 gradation indicates a need for larger particles than does HEC-11 and HEC-15.

Simons-STT recommends the largest range in particle sizes for a given D_{50} (fig. 37). Appropriate size gradations are easily determined by the use of a frequency distribution curve which presents particle sizes as a ratio of the D_{50} size. Recommended maximum stone size is $2D_{50}$; median stone size is $2D_{20}$; and the minimum stone size is $0.2D_{50}$. The concept behind the wide range in sizes is to "key" the protection together by filling voids between the larger particles, in an interlocking fashion, to form a well-distributed protection. Given sufficient thickness of protection, subsequent erosion or selective removal of particles should leave a more resistant layer much the same as the channel-bed process of armoring.

Cal-B&SP presents eight classes of riprap with corresponding distributions. Their size distribution criterion is quite flexible. Theoretical maximum and minimum size distributions for a given class were determined from criteria given in table 3 of Cal-B&SP and plotted in figure 37. Good agreement between Cal-B&SP and FHWA distributions appear above the median size. Below the median size, Cal-B&SP indicates a need for significantly coarser particles. Cal-B&SP shows the largest D_{15} to D_{50} ratio (see table 8) of those examined.

For loose riprap designs, ODOT uses stone gradations in accordance with FHWA specifications. Five classes of riprap are presented in the manual. As a result of breakage during the tamping process, ODOT recommends using a larger percentage of heavier rock sizes in their keyed riprap procedure. This is not evident from figure 37 because the FHWA curves for the larger sizes (greater than D_{50}) closely parallel the upper limit ODOT gradation.

Filter Blankets

Granular filter blankets underlying riprap serve two basic purposes. First, they protect the natural base (riverbank) material from washing out through the riprap, and secondly, they provide a "bedding" on which the riprap will rest.

Filter blanket criteria for the FHWA circulars and the Simons-STT procedure are identical. The need for a filter blanket is a function of particle-size ratios between the riprap and riverbank material. If the particle size ratios meet the two requirements, <5,<40, then no filter blanket is required. If there are very large differences, multiple filter blanket layers may be required. The particle-size ratios or "filter ratios" are as follows:

$$\frac{D_{15}\ \text{(riprap)}}{D_{85}\ \text{(bank)}} < 5 < \frac{D_{15}\ \text{(riprap)}}{D_{15}\ \text{(bank)}} < 40$$

Example gradation requirements for filter material are given in both FHWA circulars; however, only in HEC-11 are minimum thickness criteria (for filter) given. Simons-STT suggests filter thicknesses should vary depending on riprap size, but indicates that "filters one-half the thickness of the riprap are quite satisfactory." Simons-STT also suggests limits of particle sizes for filter material.

Cal-B&SP considers the filter blanket (termed "backing" in the manual) an integral part of the bank protection structure. A general discussion is given concerning function and design principles. Specific sizes or gradings of filter material are not given, but it is suggested that "it should be uniformly graded to a size that will not work through the voids of the rock, or placed in two or more layers of progressively coarser sizes." In the case of end-dumped protection where particles are sufficiently nonuniform in size, Cal-B&SP suggests use of a filter may be avoided due to differential settling of particle sizes.

The ODOT keyed riprap procedure indicates that a filter blanket 1 ft (0.3 m) thick of native river gravels appeared to be adequate, based on experience of the Corps of Engineers. No filter blanket is required where the embankment is made of gravel, but guides are not given for suitable size ranges of native gravel. For installation where the embankment is not native gravel, ODOT suggests a particle-size distribution for the filter material as established by the Soils Laboratory of the Portland District, Army Corps of Engineers.

The use of plastic filter cloths as a substitute for granular filter blankets is discussed in HEC-15 (pp. 53 and 54). These recommendations are based on studies of plastic filter cloths by the Corps of Engineers and Colorado State University and are considered reasonable. In general, plastic filter cloths should be used on streambanks with flat side slopes (such as 3:1) in order to prevent lateral movement such as translational slides. Use of plastic filter blankets should be discouraged if the streamflow is highly regulated, causing rapid fluctuations in hydrostatic pressure in the streambank or levee. This condition could cause slump or modified slump failures of the riprap.

DEVELOPMENT OF A NEW PROCEDURE FOR ESTIMATING MEDIAN STONE SIZE

The various design procedures provide considerably different estimates of median stone size (D_{50}), as shown in figure 18 and discussed previously. To relate D_{50} to velocity and to determine the shear stress typical of open channels, field data on the performance of rock riprap installations at 26 sites in Washington, Oregon, California, Nevada, and Arizona were investigated. Inspection of these sites indicated that, in some cases, the riprap performed as intended, but in other cases, it failed. Surveys were made to relate the hydraulic conditions at the site to the performance of the riprap. In some cases, more than one flow event was surveyed at a site; thus, there were 39 events available for evaluation. Results of these surveys are given in table 7. Of the 39 flow events, 22 resulted in no apparent damage to the riprap. In the other 17 cases, the probable cause of riprap damage was identified, with particle erosion involved in the majority (14) of the cases.

Figure 38, which is a re-plot of the velocity/D_{50} relationship from HEC-11, also includes data from field sites surveyed during this study (table 7) and from USBR-EM-25. Using these data, a new velocity/D_{50} relationship is tentatively defined, based on a recognition of those sites with riprap failure caused by particle erosion. This relationship is:

$$D_{50} = 0.01\, V_a^{2.44} \tag{37}$$

where D_{50} = median stone size
V_a = average velocity in cross section

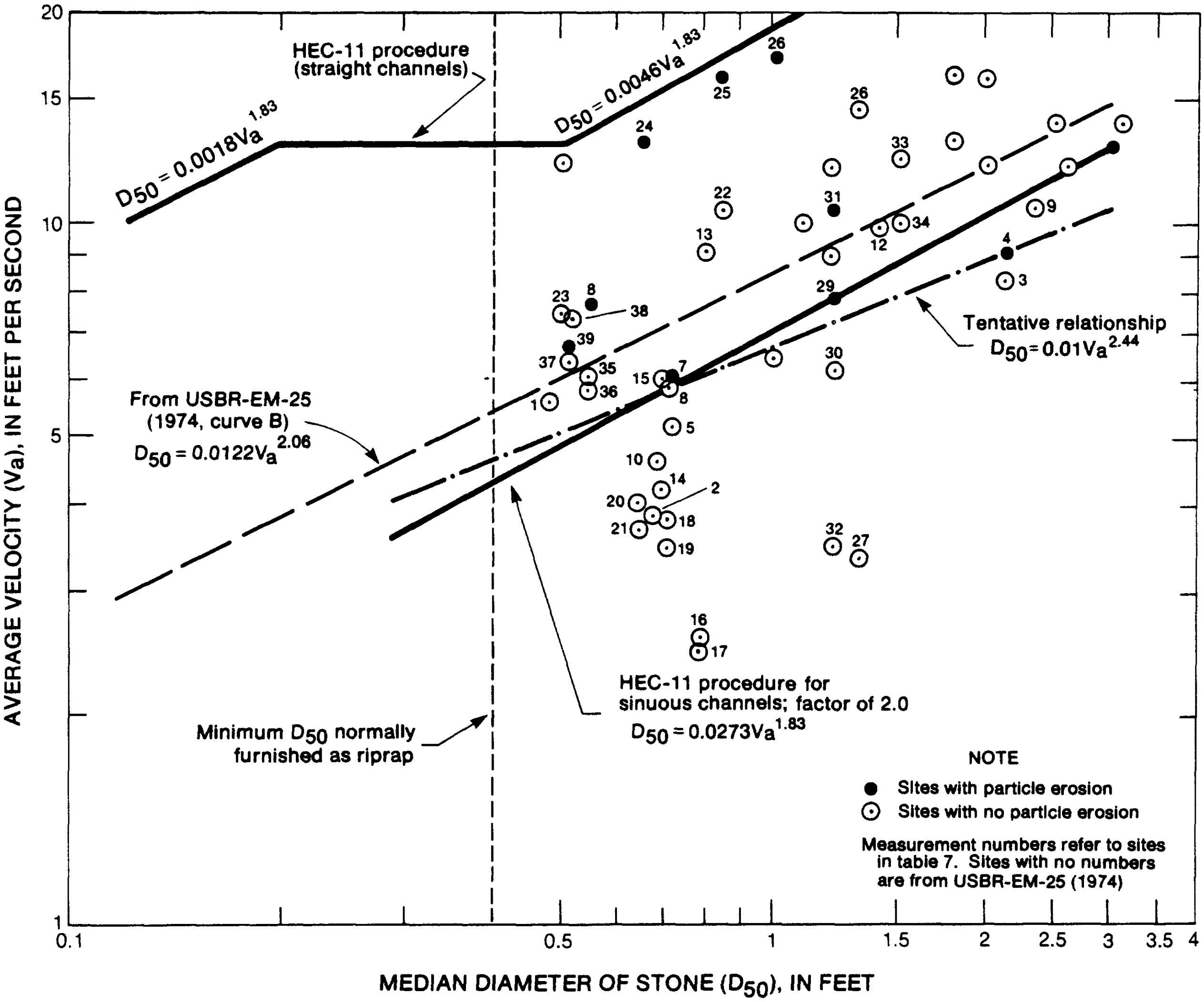


FIGURE 38. Comparison of median stone size (D_{50}) estimated on basis of mean velocity (HEC-11 procedures) and performance at field sites.

For all cases described in table 7, the damaged riprap was placed on the bank, and is generally on a channel bend. The slope of the relationship expressed by equation 37 is slightly steeper than that derived by the USBR in USBR-EM-25. The relationship expressed by equation 37 has been extended down to average velocities of 3.5 ft/s (1.1 m/s) on the basis of data presented in USBR-EM-25. Sufficient data have not yet been obtained to define this relationship with accuracy. Note that flow depth apparently is not a factor in defining the relationship. The relationship evidently applies for all channels, whether curved or straight, with side slopes equal to or flatter than 1.5:1.

Estimates of boundary shear based on flow velocity rather than depth or gradient, are considered preferable because of three factors:

- o Unless the water-surface or energy slope is accurately defined for the locality of interest, the associated shear stress derived by the equation $\tau_0 = \gamma d_m S_0$ may be greatly in error. Although the channel gradient may be estimated from maps or surveys of water surface at low flow, these data usually provide average slopes for long reaches of channel and do not identify localized slopes for short reaches that may be critical in estimating the appropriate shear stress.

- o It is easier to estimate the velocity of flow needed for analysis because design discharge and channel cross-sectional data are usually developed or obtained as part of the site survey for highway design purposes. Water-surface or energy slope data are more difficult to obtain.

- o Values of flow velocity that are hydraulically reasonable are easier for the engineer to relate to a site than different values of channel gradient.

SUMMARY COMPARISON OF VARIOUS DESIGN PROCEDURES

A comparison of the different procedures for estimating median riprap stone size (D_{50}) was made by estimating stresses on the bed or bank that would be imposed by average flow velocities ranging from 2 to 10 ft/s (0.6 to 3.0 m/s). Because the EM-1601 and Simons-STT procedures are based on stresses in the form of shear, adjustments were made so all hydraulic factors expressed in terms of velocity or shear are comparable. Table 11 compares results using the different design procedures, and table 12 summarizes the equations used to prepare table 11. In general, data in table 11 were developed assuming worst case conditions--that is, curved channels, depths of 9 ft (2.7 m), the need for bank protection, and a channel slope only slightly less than that needed for supercritical flow.

In preparing the data and subsequent comparisons in tables 11 and 12, the following problems or conditions in application of the various riprap designs were considered:

- o Estimates of stone size D_{50} by methods in EM-1601, Man-54, and proposed in this report (eq. 37) provide similar results.

- o Application of procedures by EM-1601 and Simons-STT are the most complex to follow and time consuming.

Table 11. Comparison of D_{50} stone size determined by various riprap design procedures.

Average veloc-ity, V_a (ft/s)	Maximum point velocity, V_m (ft/s)	Maximum depth, d_m (ft)	D_{50} stone size to protect bank with impinging flow						
			HEC-11[1]	HEC-15[2]	Cal-B&SP	USCE EM-1601	ASCE Man-54	Simons and Senturk[3]	USBR EM-25
2	3.2	9	0.002	1.3	0.06	0.07	0.08	0.038	0.05
4	6.4	9	0.03	1.3	0.22	0.38	0.30	0.21	0.21
6	9.6	9	0.14	1.3	0.50	0.92	0.68	0.50	0.49
8	12.8	9	0.42	1.3	0.88	1.70	1.20	0.92	0.88
10	16.0	9	1.02	1.3	1.4	2.78	1.88	1.49	1.40

[1]Curved channel, equation 11 in this report.
[2]Chart 27 in HEC-15 report.
[3]Assumes factor of safety (SF) = 1.00.

- Use of HEC-11 and HEC-15 resulted in riprap that is undersize, or the procedures were not applicable to the hydraulic conditions evaluated.

- For an average velocity of 6 ft/s (1.8 m/s), there is a small range in estimated values of D_{50} obtained by most procedures; five of eight procedures gave a D_{50} of 0.68 ft ±0.29 ft (0.21 m ±0.088 m) (table 11). Although the range in D_{50} size obtained by most procedures is small, other factors contribute to failures of riprap. These include:

  (a) Bank side slope too steep
  (b) Poor gradation of material
  (c) Insufficient toe or endwall depth or length
  (d) Failure of base material by slump
  (e) Incorrect assessment of design flood
  (f) Inadequate assessment of future channel alinement or other geomorphic changes.

- Many of the riprap design procedures require channel and hydraulic factors that are difficult to determine. Inaccuracies in estimating these factors usually have an impact on the computation of the desired size of riprap. For example, in HEC-11, the velocity against stone must be estimated. In Cal-B&SP, the stream velocity to which the bank is exposed must be determined; this factor is then expanded to the 6th power so any error in velocity is greatly magnified. Further, estimates of rock size are expressed by weight, with 67 percent of the stones a larger size. To use ASCE Man-54, the flow velocity at a distance 10 ft (3.0 m) from the bank is needed. In EM-1601, estimates of maximum shear at channel bends are divided into rough versus smooth channels but a quantitative definition of rough versus smooth is lacking. In Simons-STT, the relationship between reference velocity and shear is applicable only for uniform flow in wide prismatic channels in which the flow is fully turbulent; no data are provided to estimate shear for flow or channel conditions not meeting this requirement.

- o Some of the adjustment procedures that account for such factors as channel curvature and superelevation may be unwarranted in view of the difficulty involved in estimating the critical hydraulic forces for which the riprap material is designed.

There is, therefore, a considerable amount of evidence that the success or failure of a rock riprap installation is related not only to proper selection of a flow stress/stone size (D_{50}) in the design procedure, but also to the evaluation of the hydraulic and channel factors that are typical or potentially applicable to the site.

Table 12. Riprap D_{50} stone size design equations.

Procedure	Equation[1]	Remarks
Vol. 2	$D_{50} = 0.01\ V_a^{2.44}$	Based on field studies.
FHWA, HEC-11	$D_{50} = \dfrac{0.00117\ V_a^{3.95}}{d_m^{1.06}}$	Assumed z=2:1, d_m<10 ft, curved channel
FHWA, HEC-15	Chart 27	Assumed S_0=0.012 ft/ft, n=0.040, d_m=9.0 ft
Cal-B&SP	$W_{33} = 0.0000364\ V_m^6$	V_m=1.6 V_a, G_s=2.65
USCE, EM-1601	$\tau_{o(V_a)} = \gamma\left(\dfrac{V_a}{32.6\ \log_{10} 30y/k}\right)^2$	$\sqrt{a}=V_m=1.6V_a$, k=D_{50}, R=d_a, y=d_m=1.6d_a, n=0.040, S_o=0.012
ASCE, Man-54	$W = \dfrac{0.000041\ G_s V^6}{(G_s-1)^3\ \cos^3\phi} = 0.0000343V_m^6$	V_m=1.6V_a, G_s=2.65 0=26.6 degrees or z=2.1
Simons & Senturk	$D_{50} = \dfrac{0.204\tau_s}{\eta}$	τ_0 side =0.72 τ_0 bed, ϕ=26.6, angle of repose θ=41 degrees, SF=1.00
USBR-EM-25	$D_{40} = 0.0105\ V_a^{2.06}$	Adapted from fig. 165 in USBR-EM-25

[1]Where applicable, a channel bed slope of 0.012 ft/ft is used. Adjustments were made where needed to convert W_{33} or D_{40} to D_{50} stone size.

LITERATURE SURVEY AND REFERENCES

A search of the literature on streambank protection methods was conducted by Keown and others (1977) of the Corps of Engineers and published in Technical Report H-77-9. This literature search was very extensive and included an assessment of stone riprap, vegetation, gabions, and other methods of streambank protection. Since completion of this literature survey, additional studies have been published. In 1981, the Corps published a final report to Congress evaluating streambank control, which consisted of a main report and appendices A through H (U.S. Army Corps of Engineers, 1981). Appendix A is a literature survey and, for single-component revetments, which includes stone (rock) revetment, repeats a list of references identical to the list in the literature survey by Keown and others (1977). The following list of references was assembled as part of the technical analysis for this study and includes studies of rock riprap and gabion streambank protection published since 1977.

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