DESCRIPTION AND COMPARISON OF SELECTED MODELS FOR HYDROLOGIC ANALYSIS OF GROUND-WATER FLOW, ST. JOSEPH RIVER BASIN, INDIANA

By James G. Peters

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FACTORS FOR CONVERTING INCH-POUND UNITS TO METRIC (INTERNATIONAL SYSTEM) UNITS

Multiply inch-pound unit	by	To obtain Metric units
inch (in)	25.4	millimeter (mm)
foot (ft)	0.3048	meter (m)
square foot (ft^2)	0.09294	square meter (m^2)
foot per day (ft/d)	0.3048	meter per day (m/d)
foot squared per day (ft^2/d)	0.9294	meter squared per day (m^2/d)
mile (mi)	1.609	kilometer (km)
square mile (mi^2)	2.590	square kilometer (km^2)
acre	0.4047	hectare
inch per year (in/yr)	2.54	centimeter per year (cm/a)
cubic foot per second (ft^3/s)	0.02832	cubic meter per second (m^3/s)
cubic foot per day (ft^3/d)	0.02832	cubic meter per day (m^3/d)
million gallons per day (Mgal/d)	0.04381	cubic meter per second (m^3/s)

To convert degree Fahrenheit (°F) to degree Celsius (°C)

$$(0.556)\ (°F - 32°) = °C$$

[Note: The letter s is used in this report as an abbreviation for second only in the term, ft^3/s. All other uses of s are as a symbol for drawdown.]

FACTORS FOR CONVERTING INCH-POUND UNITS TO OTHER INCH-POUND UNITS

Multiply	by	To obtain
foot per day (ft/d)	7.481	gallon per day per foot squared [(gal/d)/ft^2]
foot squared per day (ft^2/d)	7.481	gallon per day per foot [(gal/d)/ft]
foot squared per day (ft^2/d)	0.005194	gallon per minute per foot [(gal/min)/ft]
cubic foot per second (ft^3/s)	448.8	gallon per minute (gal/min)
cubic foot per second (ft^3/s)	0.6463	million gallons per day (Mgal/d)
cubic foot per day (ft^3/d)	7.481×10^{-6}	million gallons per day (Mgal/d)
gallon per day per foot squared [(gal/d)/ft^2]	0.1337	foot per day (ft/d)
gallon per day per foot [(gal/d)/ft]	0.1337	foot squared per day (ft^2/d)
gallon per minute per foot [(gal/min)/ft]	192.5	foot squared per day (ft^2/d)
gallon per minute (gal/min)	0.002228	cubic foot per second (ft^3/s)
million gallons per day (Mgal/d)	1.547	cubic foot per second (ft^3/s)
million gallons per day (Mgal/d)	1.337×10^5	cubic foot per day (ft^3/d)

[Note: The letter s is used in this report as an abbreviation for second only in the term, ft^3/s. All other uses of *s* are as a symbol for drawdown.]

SYMBOLS

[Dimension: L, length; T, time]

Symbol	Explanation	Dimension
B	$(Tb'/K')^{\frac{1}{2}}$	L
C	generalized constant	dimensionless
D	well-loss constant	T/L^5
K_r	radial hydraulic conductivity	L/T
K_z	vertical hydraulic conductivity	L/T
K'	vertical hydraulic conductivity of confining bed	L/T
M	term used in calulating transmissivity from specific-capacity data	dimensionless
Q	rate of discharge from well or stream	L^3/T
Q/s	specific capacity	L^2/T
S	storage coefficient	dimensionless
S_s	specific storage of an aquifer	1/L
S_y	specific yield	dimensionless
S'_s	specific storage of a confining bed	1/L
T	transmissivity	L^2/T
T'	uncorrected transmissivity	L^2/T
W(u)	well function for a well fully penetrating a nonleaky, confined aquifer	dimensionless
W(u, r/B)	well function for a well fully penetrating a leaky, confined aquifer	dimensionless
$W(u_A, u_B, \beta, \ell/b, \ell'/b)$	well function for a well that partially penetrates an unconfined aquifer	dimensionless
$W(\rho)$	well function for a well partially penetrating a leaky, confined aquifer	dimensionless
Y	$r/4b\ (K'S'_s/K_r\ S_s)^{\frac{1}{2}}$	dimensionless
a	distance between pumping well and stream	L

SYMBOLS--Continued

Symbol	Explanation	Dimension
b	aquifer thickness	L
b'	thickness of confining bed	L
c	distance from the top of the aquifer to the top of the well screen of the pumping well	L
e	base of Napierian logarithm (2.7183)	dimensionless
erf	error function	dimensionless
erfc	complimentary error function = 1 - erf	dimensionless
f( )	generalized function	dimensionless
$f(u, hr/b, r/b, d/b, \ell/b, a/b)$	function describing the effects of partial penetration	dimensionless
g	distance from the top of the aquifer to the bottom of the piezometer	L
h	$(K_z/K_r)^{\frac{1}{2}}$	dimensionless
ℓ	distance from the top of the aquifer or static water level to the bottom of the screen of the pumping well	L
ℓ'	distance from the static water level to the bottom of the screen of the pumping well	L
ln	natural logarithm (base = 2.7183)	dimensionless
q	rate of streamflow depletion	L^3/T
q_{max}	maximum rate of streamflow reduction	L^3/T
r	distance from pumping well to point of observation	L
r_w	well radius	L
s	drawdown	L
s_a	drawdown adjusted for partial penetration and (or) dewatering	L
s_1	well loss	L

Symbol	Explanation	Dimension
s_o	observed drawdown	L
s_p	drawdown adjusted for partial penetration	L
s_t	total drawdown	L
s'	drawdown corrected for dewatering	L
srf	streamflow reduction factor, a^2T/S	T
t	time	T
t_i	time since pumping stopped	T
t_p	total time of pumping	T
u	$r^2S/4Tt$	dimensionless
v	volume of streamflow reduction	L^3
y	generalized variable of integration	dimensionless
α	screened interval divided by b	dimensionless
π	pi, approximately 3.1416	dimensionless
ω,τ	generalized aquifer properties	dimensionless

[Note: The letter s is used in this report as an abbreviation for second only in the term, ft^3/s. All other uses of s are as a symbol for drawdown.]

DESCRIPTION AND COMPARISON OF SELECTED MODELS FOR HYDROLOGIC ANALYSIS OF GROUND-WATER FLOW, ST. JOSEPH RIVER BASIN, INDIANA

By James G. Peters

ABSTRACT

Rapid growth of agricultural irrigation in the St. Joseph River basin, northeastern Indiana, in the past decade is likely to continue through the year 2000. The Indiana Department of Natural Resources (IDNR) is developing water-management policies designed to assess the effects of irrigation and other water uses on water supply in the basin. In support of this effort, the U.S. Geological Survey, in cooperation with IDNR, began a study to evaluate appropriate methods for analyzing the effects of pumping on ground-water levels and streamflow in the basin's glacial aquifer systems.

Two types of models were used for hydrologic analysis: (1) analytical models based on analytical flow equations and solved by graphical methods, and (2) numerical models based on numerical flow equations and solved by digital computer.

Four analytical models were used to estimate hydraulic properties of the glacial aquifers. These models describe drawdown for a nonleaky, confined aquifer and fully penetrating well; a leaky, confined aquifer and fully penetrating well; a leaky, confined aquifer and partially penetrating well; and an unconfined aquifer and partially penetrating well. Analytical equations, simplifying assumptions, and methods of application are described for each model. In addition to these four models, several other analytical models were used to predict the effects of ground-water pumping on water levels in the aquifer and on streamflow in local areas with up to two pumping wells. Analytical models for a variety of other hydrogeologic conditions are cited.

A digital ground-water flow model was used to describe how a numerical model can be applied to a glacial aquifer system. The numerical model was used to predict the effects of six pumping plans in a 46.5 square-mile area with as many as 150 wells. Water budgets for the six pumping plans were used to estimate the effect of pumping on streamflow reduction.

Results of the analytical and numerical models indicate that, in general, the glacial aquifers in the basin are highly permeable. Radial hydraulic conductivity calculated by the analytical models ranged from 280 to 600 feet per day, compared to 210 and 360 feet per day used in the numerical model. Maximum seasonal pumping for irrigation produced maximum calculated drawdown of only one-fourth of available drawdown and reduced streamflow by as much as 21 percent.

Analytical models are useful in estimating aquifer properties and predicting local effects of pumping in areas with simple lithology and boundary conditions and with few pumping wells. Numerical models are useful in regional areas with complex hydrogeology with many pumping wells and provide detailed water budgets useful for estimating the sources of water in pumping simulations. Numerical models are useful in constructing flow nets. The choice of which type of model to use is also based on the nature and scope of questions to be answered and on the degree of accuracy required.

INTRODUCTION

BACKGROUND

The water-management responsibilities of the Indiana Department of Natural Resources (IDNR) have increased in the last few years. In preparation, IDNR recently developed an assessment of the State's water-resource needs. This work was done as a contribution to the Governor's Water Resource Study Commission (GWRSC) which delineated several areas of the State where conflicts in water use may occur.

Of the various areas of potential conflicts, IDNR selected the 1,700 mi^2 part of the St. Joseph River basin in Indiana (fig. 1) as a top priority for study. (Throughout the remainder of this report the terms "St. Joseph River basin," or "basin" refer to the part of the St. Joseph River basin in Indiana only.) Results of studies by Purdue University for the GWRSC indicate that agricultural irrigation is extensive in this part of the State and might double by the year 2000 (Governor's Water Resource Study Commission, 1980, p. 179). Many natural lakes, streams, and marshes are used for recreation and wildlife habitat. Summer homes are built in areas adjacent to lakes and marshes, which are sensitive to changes in streamflow and ground-water levels. The State is concerned about possible effects of water withdrawals on surface- and ground-water resources in the basin.

As a first step in preparing for increased responsibilities in water-resource management, the GWRSC and IDNR compiled much of the water-resource information available for the basin, including ground-water availability, irrigation potential of soils, and ground- and surface-water withdrawals. In addition, IDNR updated water-use information, identified and mapped natural lakes and wetlands, and provided estimates of future irrigation. Beyond this preliminary work, the IDNR was interested in developing management tools that effectively use this information to evaluate the effect of ground- and surface-water withdrawals on water supply.

The St. Joseph River basin project began in 1980 at the request of IDNR to provide information on methods for evaluating and managing the basin's water resource and to review the present hydrologic data-collection network in the basin. Two types of simulation models were used in the project--numerical models and analytical models. The numerical models used for the Howe and

Milford study areas are described in two project reports (Bailey and others, 1985; Lindgren and others, 1985). The use of analytical models, as well as a comparison between analytical and numerical models is presented in this report.

PURPOSE AND SCOPE

The purposes of this report are (1) to describe selected analytical and numerical models useful in studying the ground-water flow and hydrology of the St. Joseph River basin, and (2) to compare the characteristics and usefulness of analytical and numerical models.

Four analytical models were used to estimate properties of aquifers in the Howe and Milford areas (fig. 1). These models described drawdown in nonleaky, confined aquifers; leaky, confined aquifers; and unconfined aquifers, as well as the effects of fully penetrating and partially penetrating wells on drawdown. Estimates of aquifer properties obtained from these models were then used to predict aquifer responses in a variety of hypothetical situations that commonly require water-management decisions. The information on analytical models is presented in a detailed step-by-step manner so that it can be easily used by readers who have a working knowledge of analytical models.

A three-dimensional, numerical flow model previously constructed for the Howe area (Bailey and others, 1985) is presented as an example of an application of a numerical model. The analytical and numerical models are compared in terms of (1) methods of simulation, (2) data requirements, (3) type of output (results), (4) applications, and (5) accuracy.

STUDY AREAS

The two areas selected for study are heavily irrigated, have potential for increased irrigation, and have hydrogeologic features common to other irrigated areas in the basin. Both areas contain substantial surface-water and ground-water resources.

The study area at Howe is 46.5 mi^2 in size and is in the northern part of Lagrange County along the Indiana-Michigan border (fig. 2). Fawn River and Pigeon River are the northern and the southern boundaries. A complex of wetlands around Cedar Lake is near the center of the area. The area is underlain by 200 to 350 ft of unconsolidated glacial deposits--mostly sand and gravel but including numerous clay lenses (fig. 3). These deposits are covered with medium-to-coarse-textured soils that respond favorably to irrigation (Chelf, 1983). Muck soils associated with the wetlands are poorly drained and not cultivated. The glacial deposits are underlain by shale of Mississippian age (Johnson and Keller, 1972). In 1982, an estimated 80 percent of the land was used for agriculture, of which 38 percent was irrigated. Irrigation represented 64 percent of the water used throughout 1982 and 88 percent of the water used during June, July, and August, 1982. Additional information about the Howe area is presented by Bailey and others (1985).

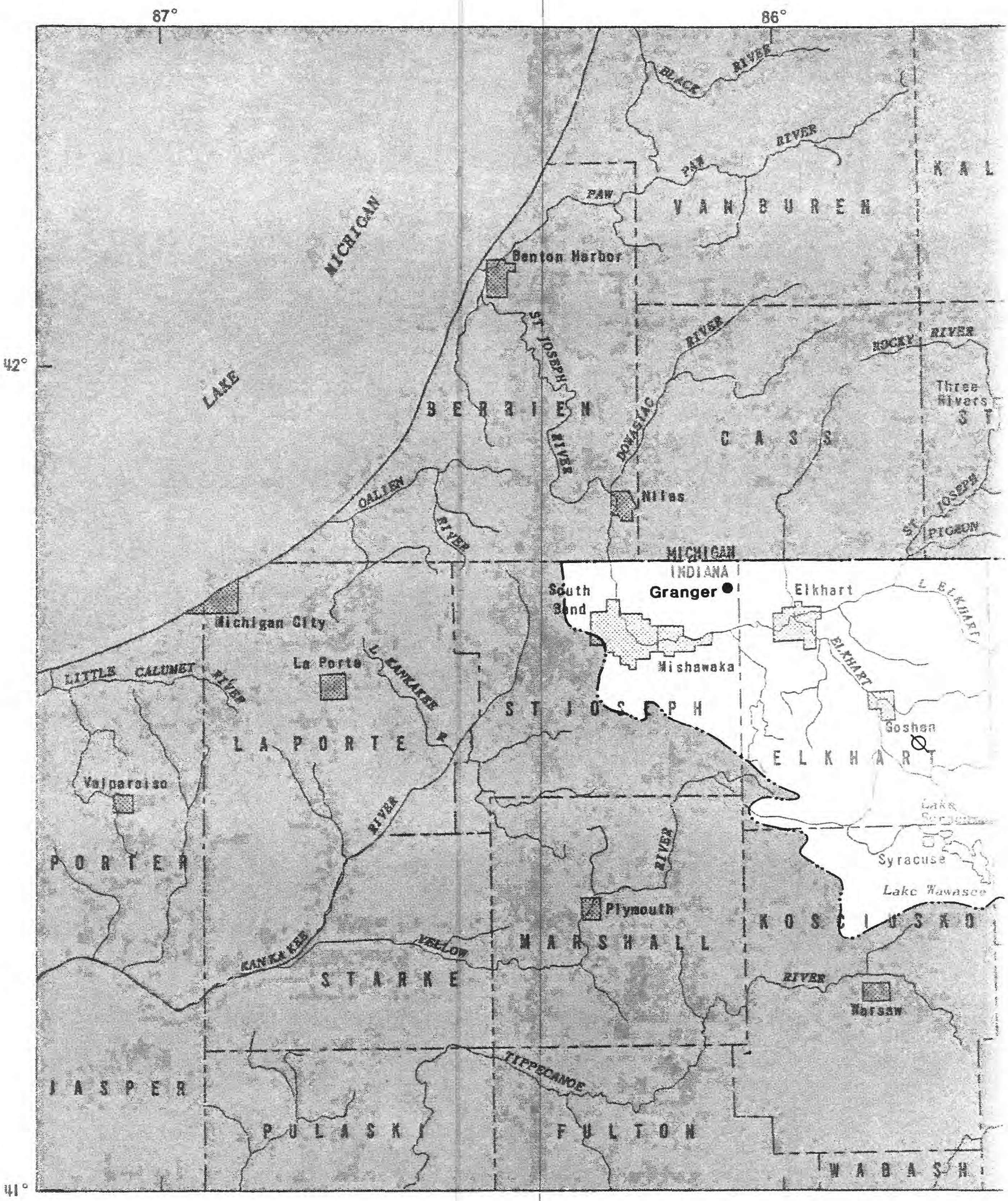
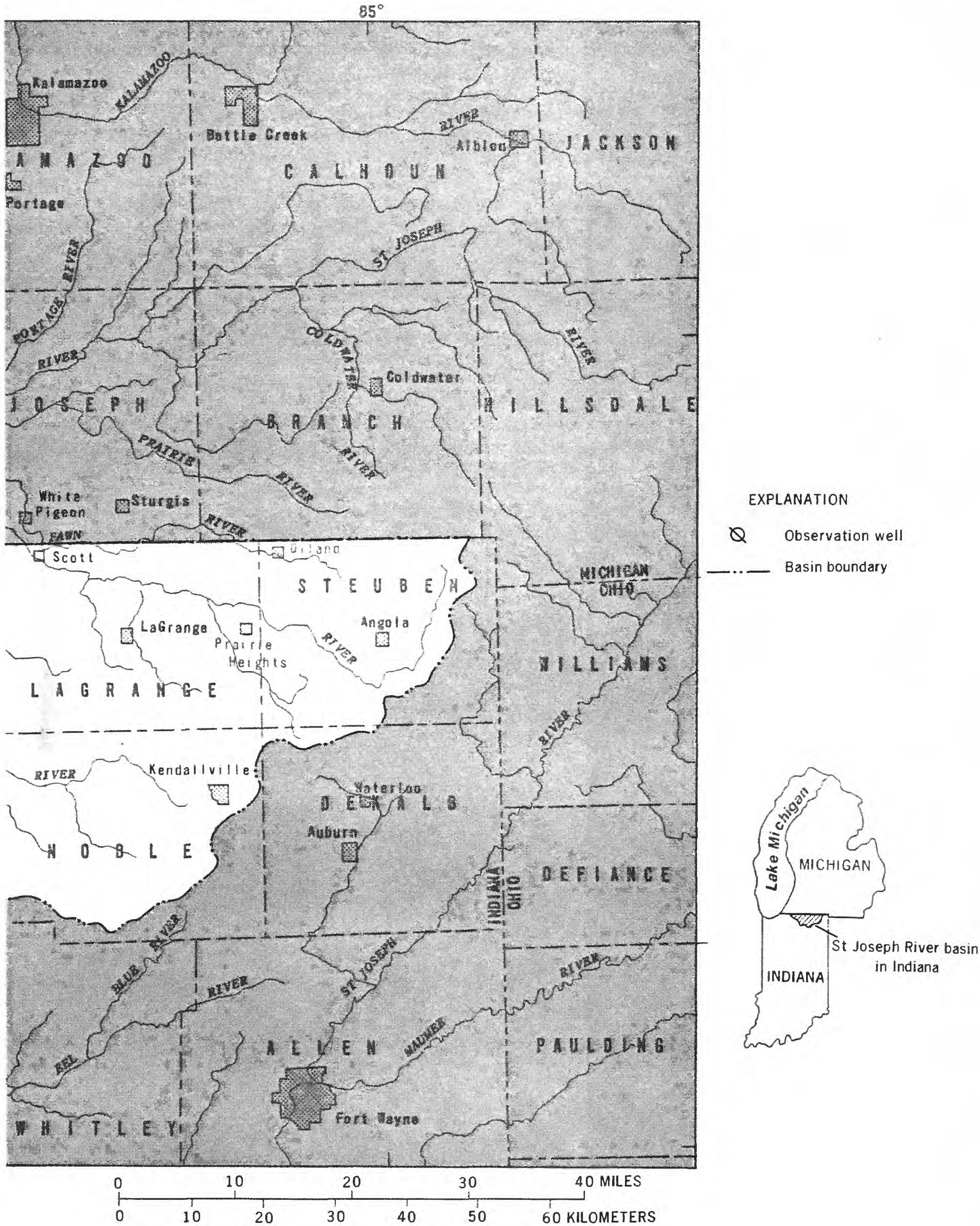


Figure 1.-- The St. Joseph River basin in Indiana.


85°
Kalamazoo
KALAMAZOO
Battle Creek
RIVER
Albion
JACKSON
CALHOUN
Portage
PORTAGE RIVER
ST JOSEPH
COLDWATER
Coldwater
RIVER
HILLSDALE
BRANCH
PRAIRIE
RIVER
White Pigeon
Sturgis
FAWN
RIVER
Scott
Orland
STEUBEN
MICHIGAN
OHIO
LaGrange
Prairie Heights
Angola
RIVER
LAGRANGE
WILLIAMS
RIVER
RIVER
Kendallville
Waterloo
DEKALB
Auburn
NOBLE
INDIANA
OHIO
DEFIANCE
BLUE RIVER
RIVER
ST JOSEPH
RIVER
EEL
ALLEN
MAUMEE
PAULDING
WHITLEY
Fort Wayne
0 10 20 30 40 MILES
0 10 20 30 40 50 60 KILOMETERS
EXPLANATION
Observation well
Basin boundary
Lake Michigan
MICHIGAN
INDIANA
St Joseph River basin in Indiana

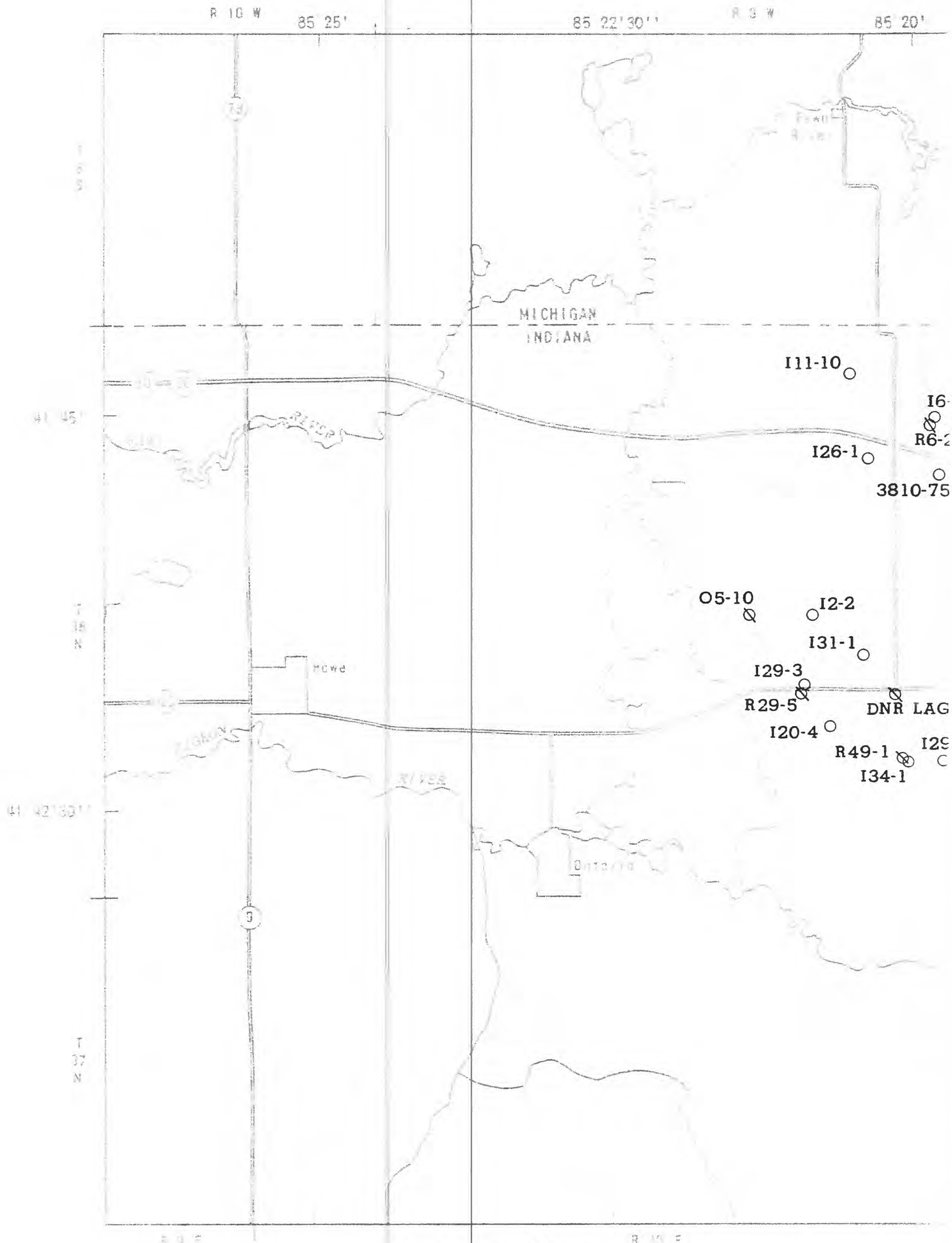
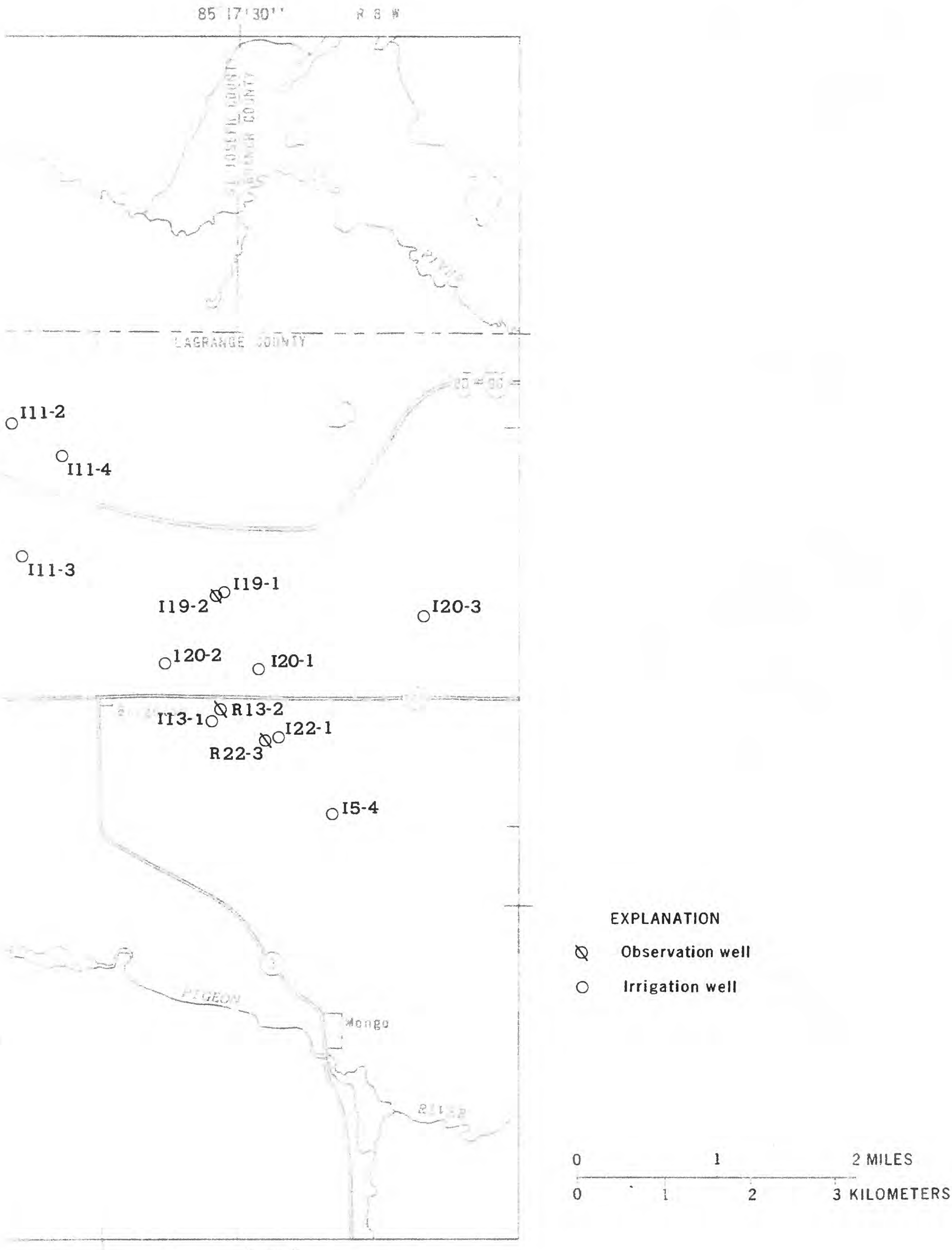


Figure 2.-- Howe area.



Modified from Bailey, Greeman, and Crompton, 1985

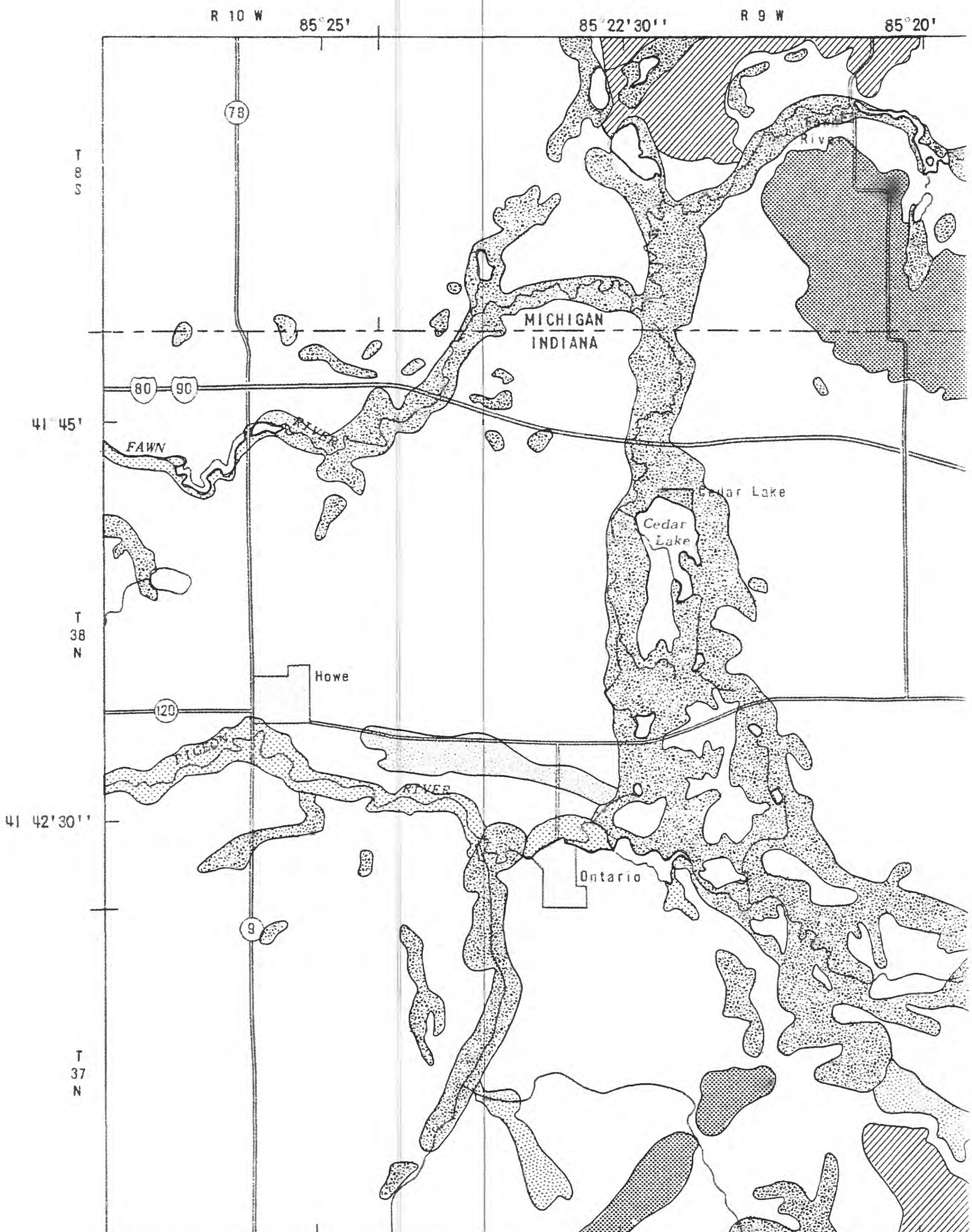
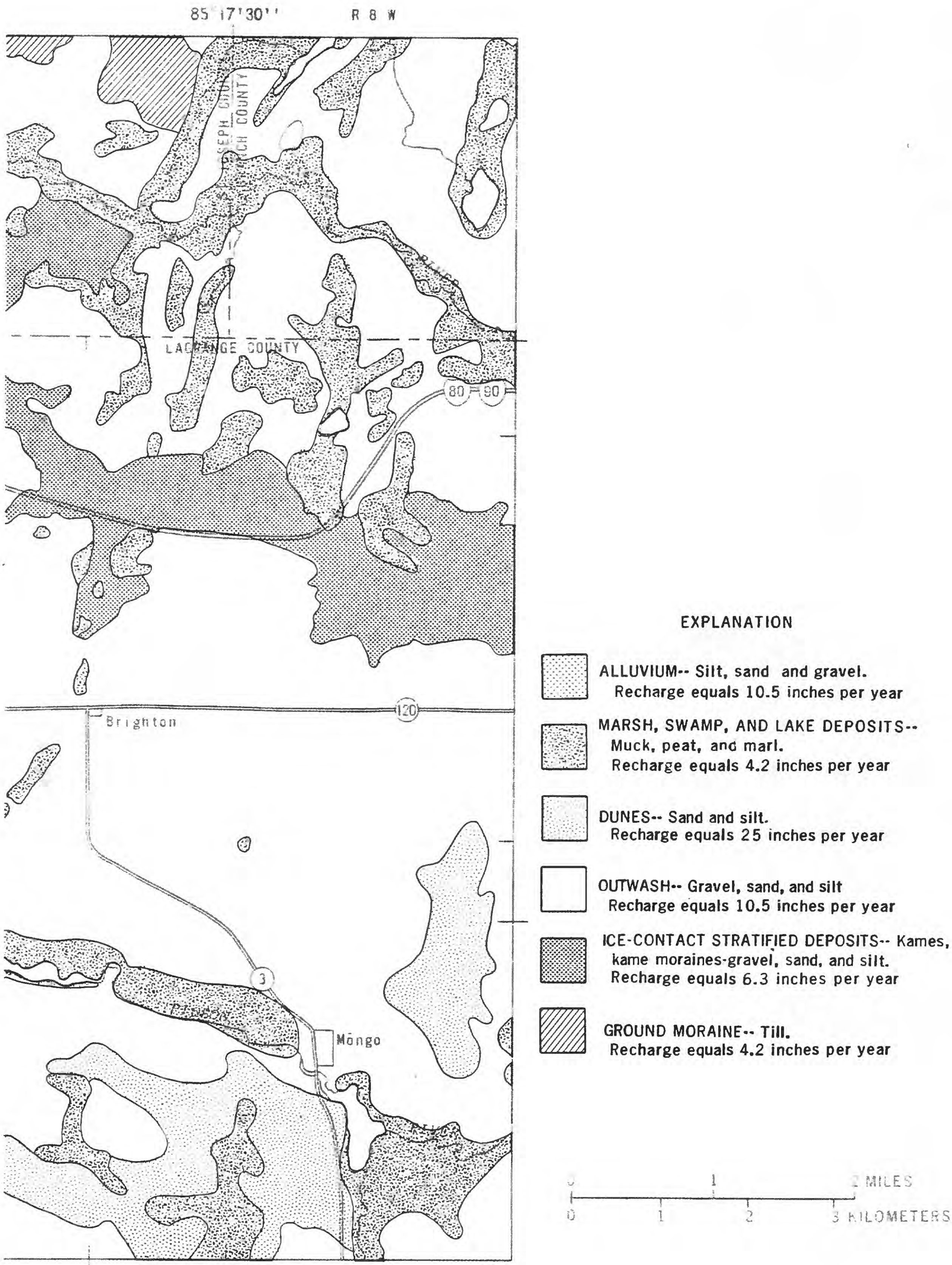


Figure 3.-- Surficial geology of the Howe area.



Geology modified from Johnson and Keller (1972)
Modified from Bailey, Greeman, and Crompton, 1985

The study area near Milford is 16.5 mi^2 in size (fig. 4). Turkey Creek forms the southern and western edges of the area. Glacial deposits in the center of the area are mostly outwash-plain and valley-train deposits near the surface (fig. 5). The thickness of all unconsolidated deposits is as much as 400 ft. These deposits contain several clay lenses and a layer of clay and silt (flowtill) that underlies most of the area (Ned Bleuer, Indiana Geological Survey, oral commun., 1983). The flowtill ranges from 3 to 45 ft in thickness and lies about 40 ft below the surface. Other deposits of till, mostly clay, border the outwash. Soils derived from the outwash deposits are coarse-textured and respond favorably to irrigation (Chelf, 1983). The bedrock underlying the glacial deposits is shale of Devonian and Mississippian age (Johnson and Keller, 1972). In 1982, an estimated 90 percent of the land was used for agriculture, 10 percent of which was irrigated. Irrigation represented 56 percent of the water used throughout 1982 and 73 percent of the water used in June, July, and August. Additional information about the Milford study area is presented by Lindgren and others (1985).

ACKNOWLEDGMENTS

Funds for this project were provided jointly by U.S. Geological Survey and the IDNR. The IDNR, Division of Water, provided much of the information needed for the study by updating an inventory of irrigation wells, supplying driller's logs of high production wells, compiling land-use information, and delineating areas with irrigable soils in the two study areas. The Division of Water also provided valuable suggestions and advice to the project staff at each stage of the project. The IDNR Division of Fish and Wildlife aerially photographed the Howe study area and mapped the wetlands in the Howe study area. Dr. Daniel Wiersma, retired chairman of the Indiana Water Resources Institute at Purdue University, and Dr. Rolland Wheaton, agricultural engineer at Purdue University, provided technical guidance in various aspects of agricultural irrigation. Dr. Wheaton wrote the calculator program for estimating the amount of water needed for irrigation. Dale Redding, Mike Jewett, and Vic Virgil, the agricultural extension agents for Lagrange, Elkhart, and Kosciusko Counties, respectively, coordinated activities and meetings between the project staff and the irrigators. Charles Phillips of Phillips and Sons Irrigation Company[1], Bristol, Ind., and Jim Fizel of J. and J. Irrigation Company, Lagrange, Ind., provided technical information about the irrigation systems used in the two study areas. Driller's logs and pumping-test data were also provided by Peerless Midwest Drilling Company, Granger, Ind. and by George Reid and Sons Drilling Company, Howe, Ind. Thirty-four irrigators in Lagrange, Elkhart, and Kosciusko Counties permitted the project staff to monitor their irrigation systems and to drill observation wells on their properties. Several irrigators collected field data during the 1982 irrigation season. Without this support, much of these data would not have been collected.

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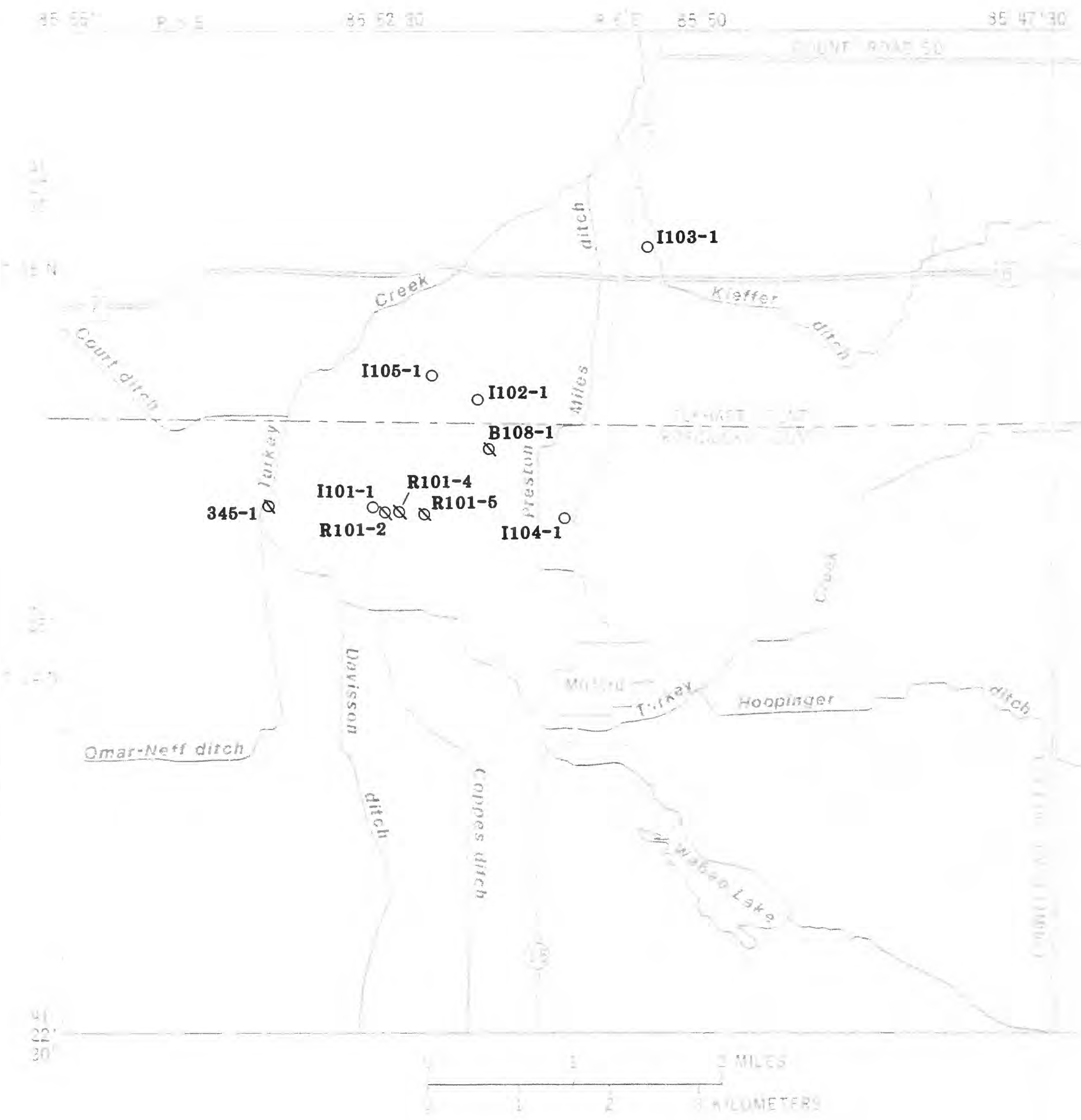
[1]Use of brand and firm trade names in this report is for identification purposes only and does not constitute endorsement by the U.S. Geological Survey.



EXPLANATION

Observation well

Irrigation well

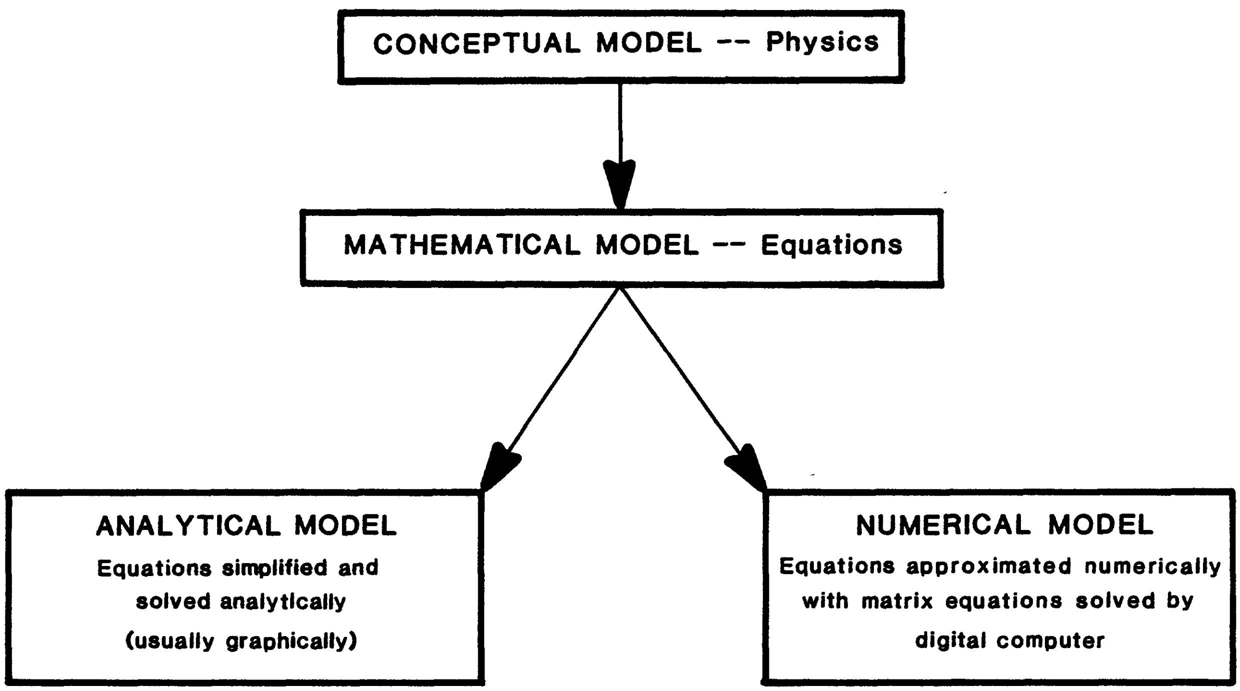
R101-2 Well designation

Figure 4.-- Milford area.

Figure 5.-- Surficial geology of the Milford area.

DESCRIPTION OF HYDROLOGIC MODELS

The first step in modeling is to develop a conceptual model of the system by identifying the physical processes involved (fig. 6). An example for a ground-water system would be the relation of the hydraulic gradient to the rate of flow (Darcy's Law). The laws applicable to these processes are then translated into flow equations, which, together with certain assumptions and boundary conditions, form a mathematical model. The equations in the mathematical model are usually solved by analytical or numerical methods (Mercer and Faust, 1980).



(Modified from Mercer and Faust, 1980)

Figure 6.-- Development of analytical and numerical models.

Analytical methods involve simplification of equations by assuming, for example, that confining beds are nonleaky or that aquifers are infinite in areal extent. The variables in the equations are continuous--that is, the equations have solutions for any value of the variables. The equations and the solution, which is usually done by curve-matching technique, are called an analytical model. A familiar example is the Theis equation and type-curve analysis described in the section "Model of Nonleaky, Confined Aquifer and Fully Penetrating Well."

In many instances, the simplifying assumptions required for analytical models are not valid for the system under consideration. In these cases, the flow equations can be approximated numerically by replacing the continuous variables with discrete variables identified in grid blocks at nodes. A separate algebraic equation is needed for each node. The equations are generally solved by using matrix algebra and a digital computer. Historically, these numerical models have come into use much more recently than analytical models principally because the matrix equations used in numerical models are tedious to solve without digital computers.

The following section presents sources of information about a wide variety of analytical models that are currently available. The calibration of three analytical models using data from the Howe study area and one analytical model using data from the Milford study area is discussed in detail. Use of analytical models for predicting effects of ground-water withdrawal on water levels and streamflow is also described.

ANALYTICAL MODELS

Analytical models were developed as a means of estimating the hydraulic properties of aquifers by use of drawdown data near pumping wells. The first analytical procedure for evaluating aquifer-test data was developed by Gunther Thiem (1906). The Thiem equation can be used to calculate transmissivity from two measurements of drawdown in the vicinity of a pumping well. Application of the Thiem equation was limited because of the many restrictive assumptions that it required:

1. Discharge of the well is constant and has continued for sufficient time so that the rate of drawdown is zero (steady-state).
2. Diameter of the well is infinitesimal and storage in the well casing is negligible.
3. Well is open to the full thickness of the aquifer (fully penetrating).
4. Aquifer is bounded above and below by nonleaky confining beds and remains saturated.
5. Aquifer is infinite in areal extent and uniform in thickness.
6. Aquifer is homogeneous and isotropic.

The many analytical models that were developed after the Thiem equation represented attempts to overcome one or more of these limitations. Information about many of these models was compiled in a table by Stallman (1971) and was updated by Weeks (1977). A modification of Weeks' table is reproduced in table 1.

The models mentioned in table 1 are perhaps most often used for aquifer test analysis--that is, for estimating the hydraulic properties of aquifers. This procedure is called <u>model calibration</u> in this report.

Calibration

The calibration process for any model involves a comparison between two sets of values of some dependent variable (for example, drawdown). One set is calculated by the model, and the other set is measured in the field. If the two sets of values are acceptably close, the model is said to be calibrated, and the corresponding values of model parameters (for example, transmissivity) are accepted as those that adequately describe the system being modeled. If the two sets of values are not acceptably close, the model does not adequately describe the physical system and will provide poor predictions of response to simulated stresses. In this case, the model must be modified or exchanged for another model.

Although it is not commonly described as such in the literature, an aquifer test is actually a calibration process. Measured values of drawdown are compared with calculated values in a procedure called curve matching which is described in detail in the section, "Model of Nonleaky, Confined Aquifer and Fully Penetrating Well." One characteristic of analytical models is that most parameters of the hydrologic system are "fixed" by the model assumptions, especially those relating to aquifer geometry. Therefore, adequate knowledge of the hydrologic system and the selection of the proper analytical model are important before any analysis is attempted.

In designing an aquifer test, the following conditions should be met:

- Enough information about aquifer geometry, probable variations in hydraulic properties, expected changes in water levels, etc. is available so that boundary conditions are known explicitly.
- Response curves that adequately accommodate the boundary conditions are available or can be developed at reasonable cost.
- Equipment to be used can make measurements of water levels with sufficient accuracy to produce definite estimates of aquifer properties.

Stallman (1971) provides a complete description of how to plan an aquifer test.

Of the many analytical models described in the literature cited in table 1, four were selected for use with data collected from the Howe and Milford areas. All four models, beginning with the Theis equation, describe drawdown near wells pumping from unconsolidated aquifers. There is no significant pumping from bedrock in the St. Joseph River basin. The four models simulate confined and unconfined aquifers, leaky and nonleaky confining beds, and fully penetrating and partially penetrating wells. For each of the four models, a detailed description of the application procedure and an example are provided.

Table 1.--Selected references for analyti

[Source: Stallman (1971) as modified by Weel
Q, rate of pump

Category	Confined aquifer																															
	Nonleaky								Leaky			Multiple				Horizontal plane anisotropy				Partial penetration					Well characteristic							
	Thiem (1906)	Theis (1935)	Jacob and Lohman (1952)	Stallman (1963)	Hantush (1964a)	Hantush (1965)	Hantush (1962b)	Streltsova (1976)	Hantush and Jacob (1955)	Hantush (1959)	Hantush (1960)	Papadopulos (1966)	Hantush (1967a)	Neuman and Witherspoon (1969a)	Witherspoon and others (1971)	Hantush (1966a)	Hantush (1966b)	Hantush and Thomas (1966)	Papadopulos (1967a)	Hantush (1961a)	Hantush (1961b)	Hantush (1964a)	Mansur and Dietrich (1965)	Weeks (1969)	Jacob (1947)	Rorabaugh (1953)	Lennox (1966)	Cooper and others (1965)	Bredehoeft and others (1966)	Cooper and others (1967)	Papadopulos and Cooper (1967)	Papadopulos (1967b)
A. Type of control imposed:																																
Step change Q	X	X	-	X	-	X	X	X	X	-	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	-	-	-	X	X
Step change s	-	-	X	-	X	-	X	-	-	X	-	X	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	X	-	-	-
Pulsed Q	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	X	-	-
Pulsed s	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	X	-	-
Variable Q	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	X	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
Variable s	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	X	-	-	-	-
B. Control-well characteristics:																																
Full penetration	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	-	-	-	-	-	X	X	X	X	X	X	X	X
Partial penetration	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	X	-	-	X	X	X	X	X	-	-	-	-	-	-	-	-
Diameter infinitesimal	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	-	-	-	-	-
Diameter finite	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	X	X	X	X	X
Seepage face	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
Well loss	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	X	X	X	-	-	-	-	-
Radial screens	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
C. Conductivity and flow conditions:																																
Homogeneous, isotropic	X	X	X	X	X	X	X	-	X	X	X	X	X	X	X	-	-	-	-	X	X	-	-	-	X	X	X	X	X	X	X	X
Homogeneous, anisotropic	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	X	X	X	-	-	-	-	-	-	-	-
Heterogeneous, isotropic	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
Fracture permeability	-	-	-	-	-	-	-	X	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
Impermeable confining beds	X	X	X	X	X	X	X	-	-	-	-	X	-	-	-	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X
Permeable confining beds (steady)	-	-	-	-	-	-	-	-	X	X	-	-	X	-	-	X	X	-	-	-	-	X	-	-	-	-	-	-	-	-	-	-
Permeable confining beds (nonsteady)	-	-	-	-	-	-	-	-	-	-	X	-	-	X	X	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
Sloping beds	-	-	-	-	-	-	X	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
Areally infinite	X	X	X	-	X	-	X	-	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X
Areally semi-infinite	-	-	-	X	-	X	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
Areally discontinuous	-	-	-	-	-	X	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
Dewatering negligible	X	X	X	X	X	X	-	-	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X
Dewatering significant	-	-	-	-	-	X	X	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
Flow radial	X	X	X	X	X	X	-	-	X	X	X	X	-	-	-	X	X	X	X	-	-	-	-	-	X	X	X	X	X	X	X	X
Flow radial and vertical	-	-	-	-	-	-	X	-	-	-	-	-	X	X	X	-	X	-	-	X	X	X	X	X	-	-	-	-	-	-	-	-
Nonsteady flow	-	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	-	X	X	X	X	X	X	X	X	X
Steady flow	X	-	-	-	-	X	-	-	X	-	X	-	-	-	-	-	-	-	-	-	-	-	X	-	-	-	-	-	-	-	-	-
D. Storage relation:																																
Linear to head	-	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X
Head and time	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
Artesian	-	X	X	X	X	X	-	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X
Unconfined	-	-	-	-	-	-	X	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
E. Emphasis on paper:																																
Q versus time	-	-	X	-	-	-	X	-	-	X	-	X	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
s versus time and space	-	X	-	X	X	X	X	X	X	-	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X
Analytical equation	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	-	X	-	-	X	X	X	X	X	X	X	X
Graphical type curve	-	-	X	X	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
Tables, type-curve	-	-	-	-	X	X	-	-	X	X	X	-	-	-	-	-	-	-	-	-	-	-	-	X	-	-	-	X	X	X	X	X
Numerical or analog techniques	-	-	-	-	-	-	-	-	-	-	-	-	-	X	X	-	-	-	-	-	-	-	X	-	-	-	-	-	X	-	X	-
Theory development	X	X	X	-	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	-	X	-	-	X	X	-	X	X	X	X	X
Application of data	-	-	-	-	-	-	-	-	-	-	-	-	-	-	X	X	-	X	X	-	X	-	X	X	X	X	X	-	-	X	-	-

models and emphasis of each reference

977); X, condition treated in reference;
s, drawdown]

Confined aquifer														Unconfined aquifer																
Variable discharge										Miscelleanous				Water table in aquifer														Water table in overlying aquitard		
Werner (1946)	Stallman (1962)	Hantush (1964b)	Abu-Zied and Scott (1963)	Abu-Zied and others (1964)	Aron and Scott (1965)	Sternberg (1968)	Moench (1971)	Lai and others (1973)	Lai and Su (1974)	Hantush (1962a)	Hantush and Papadopulos (1962)	Bixel and others (1963)	Moench and Prickett (1972)	Boulton (1954a)	Boulton (1954b)	Boulton (1963)	Boulton (1964)	Boulton (1965)	Prickett (1965)	Stallman (1965)	Norris and Fidler (1966)	Dagan (1967)	Neuman (1972)	Streltsova (1972)	Neuman (1974)	Boulton and Streltsova (1975)	Neuman (1975)	Cooley (1972)	Cooley and Case (1973)	Category
																														A. Type of control imposed:
-	-	-	-	-	-	-	-	-	X	X	X	X	X	X	X	X	X	-	X	X	X	X	X	X	X	X	X	X	X	Step change Q
-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	Step change s
X	X	X	X	X	X	X	X	X	X	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	Pulsed Q
-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	Pulsed s
X	X	X	X	X	X	X	X	X	X	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	Variable Q
-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	Variable s
																														B. Control-well characteristics:
X	X	X	X	X	X	X	X	X	X	X	-	X	X	X	X	X	X	X	X	-	-	-	X	X	-	X	X	X	X	Full penetration
-	-	-	-	-	-	-	-	-	-	-	X	-	-	-	-	-	-	-	-	X	X	X	-	-	X	-	-	-	X	Partial penetration
X	X	X	X	X	X	X	X	-	-	X	-	X	X	X	X	X	X	-	X	X	X	X	X	X	X	X	X	X	X	Diameter infinitesimal
-	-	-	-	-	-	-	-	X	X	X	-	-	-	-	-	-	-	X	-	-	-	-	-	-	-	-	-	-	-	Diameter finite
-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	X	-	-	-	-	-	-	-	-	-	-	-	Seepage face
-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	Well loss
-	-	-	-	-	-	-	-	-	-	-	X	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	Radial screens
																														C. Conductivity and flow conditions:
X	X	X	X	X	X	X	X	X	X	-	X	X	X	X	X	X	X	X	X	-	-	-	X	X	-	X	-	-	-	Homogeneous, isotropic
-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	X	X	X	-	-	X	-	X	-	-	Homogeneous, anisotropic
-	-	X	-	-	-	-	-	-	-	X	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	X	X	Heterogeneous, isotropic
-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	Fracture permeability
X	X	X	X	X	X	X	X	X	-	X	X	X	X	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	Impermeable confining beds
-	-	X	-	-	-	-	X	-	X	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	Permeable confining beds (steady)
-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	Permeable confining beds (unsteady)
-	-	X	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	Sloping beds
X	X	X	X	X	X	X	X	X	X	X	X	-	X	X	X	X	X	X	X	X	-	X	X	X	X	X	X	X	X	Areally infinite
-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	X	-	-	-	-	-	-	-	-	Areally semi-infinite
-	-	-	-	-	-	-	-	-	-	-	-	X	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	Areally discontinuous
X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	-	X	X	X	X	X	X	X	X	X	X	X	Dewatering negligible
-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	X	-	-	-	-	-	-	-	-	-	-	-	Dewatering significant
X	X	X	X	X	X	X	X	X	X	X	-	X	X	-	X	X	X	-	X	-	-	-	-	-	-	-	-	X	X	Flow radial
-	-	-	-	-	-	-	-	-	-	-	-	-	-	X	-	-	-	X	-	X	X	X	X	X	X	X	X	-	-	Flow radial and vertical
X	X	X	X	X	X	X	X	X	X	-	X	-	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	Nonsteady flow
-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	Steady flow
																														D. Storage relation:
X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	-	-	-	-	-	X	X	X	X	X	X	X	X	X	X	Linear to head
-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	X	X	X	-	X	-	-	-	-	-	-	-	-	-	X	Head and time
X	X	X	X	X	X	X	X	X	X	X	X	X	X	-	X	X	X	-	X	-	-	-	X	X	X	X	X	-	-	Artesian
-	-	-	-	-	-	-	-	-	-	-	-	-	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	Unconfined
																														E. Emphasis on paper:
X	X	X	X	X	X	X	X	X	X	-	-	-	-	-	-	-	-	X	-	-	-	-	-	-	-	-	-	-	-	Q versus time
X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	-	X	X	X	X	X	X	X	X	X	X	X	s versus time and space
X	-	X	X	X	X	-	X	X	X	X	X	X	X	X	X	X	X	X	-	-	-	X	X	X	X	X	X	X	X	Analytical equation
-	-	-	-	-	-	-	-	-	-	-	-	X	-	X	-	X	-	X	X	X	-	X	X	X	X	-	X	-	-	Graphical type curve
-	-	X	X	X	-	-	-	-	-	-	-	-	X	X	-	X	X	X	-	-	-	X	X	X	X	-	X	-	-	Tables, type-curve
-	-	-	-	-	-	-	X	-	-	-	-	-	X	-	-	-	-	-	-	X	-	-	X	-	X	-	-	X	X	Numerical or analog techniques
X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	-	-	X	-	X	X	X	X	-	-	X	X	Theory development
-	-	X	-	X	-	-	X	-	-	-	-	X	-	-	X	X	X	X	-	-	X	X	-	-	-	X	X	-	-	Application of data

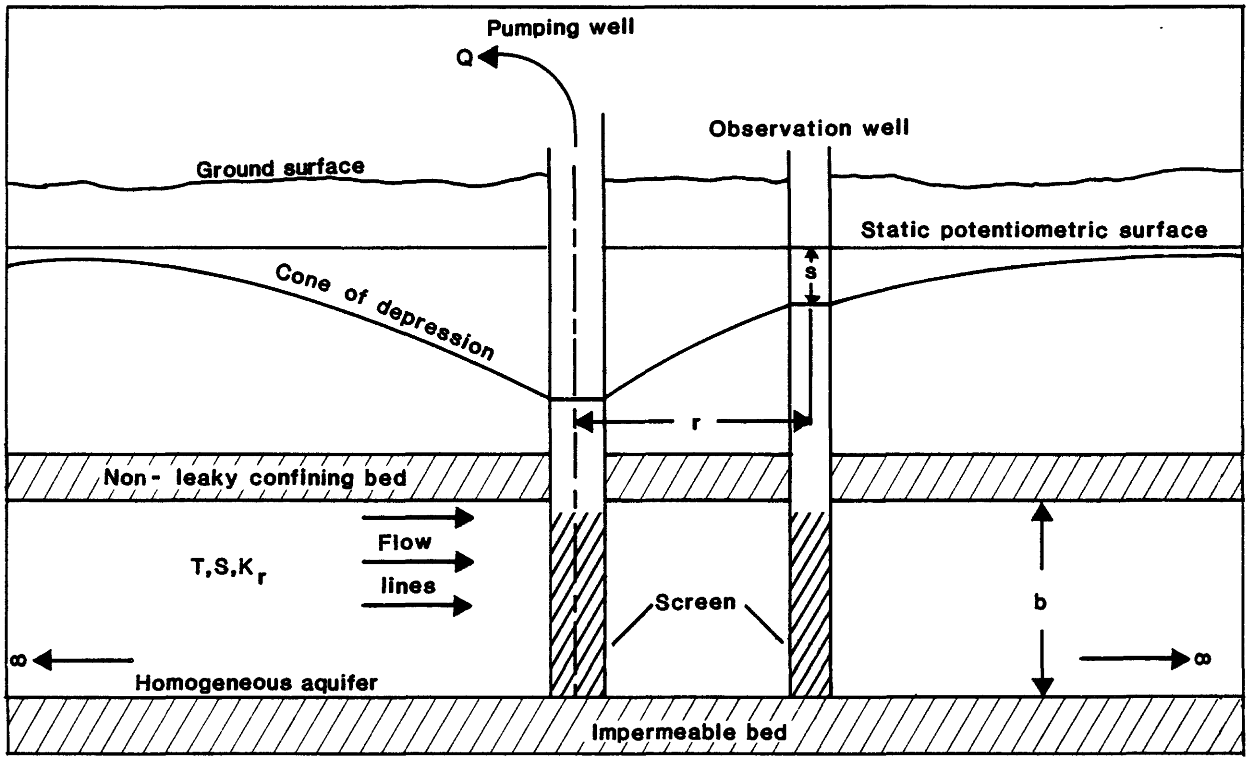
Drawdown data for the examples were collected from observation wells near irrigation wells in the Howe and Milford areas. It was generally not feasible to have an observer at the observation wells to measure drawdown when the irrigation systems were turned on initially. Therefore, water levels in observation wells used for aquifer tests were collected using automatic recorders that stored water-level values every 5 minutes. These data are less than ideal because water levels during the initial moments of the pumping period were usually not recorded, and many of the data collected during the 1982 irrigation season could not be used for aquifer-test analysis. Only drawdown data that provided sufficient curvature to the field-data plots were used for curve matching.

Another problem with using data from recorders was that the time of the clocks in the recorders often differed by several minutes from that of the clocks used by the irrigators, who recorded the times of the day when their pumps were turned on and off. This difference in time resulted in potential errors in the field-data plots, especially for the data plotted early in the pumping period, when small changes in time caused large changes in curvature of the plotted data. Data for which times could not be determined accurately were not used for analysis. Thus some drawdowns recorded early in the pumping period were omitted from the field-data plots.

Model of Nonleaky, Confined Aquifer and Fully Penetrating Well

Prior to 1935, analysis of aquifer-test data was restricted by the requirement that the hydrologic system be in steady state (Thiem, 1906). The development of a nonsteady-state equation (Theis, 1935) greatly improved aquifer-test analysis because: (1) an estimate of aquifer storage was possible, (2) only one observation well was needed, although several improve interpretation, and (3) the required pumping period was usually much shorter than for a steady-state system (Todd, 1980, p. 124).

The Theis equation describes drawdown near a pumping well that fully penetrates a confined aquifer of infinite areal extent bounded on top and bottom by impermeable confining beds (fig. 7). Despite its idealized conditions, the Theis equation has been widely applied to ground-water flow problems (Ferris and others, 1962, p. 93) and is the best known of all analytical models.



(Modified from Reed, 1980, p. 6)

Figure 7.-- Section through a pumping well that fully penetrates a nonleaky, confined aquifer.

Conceptualization

The Theis equation can be written in terms of drawdown as:

$$s = \frac{Q}{4\pi T} \int_u^\infty \frac{e^{-u}}{u} du \qquad (1)$$

where $u = r^2S/4Tt$; (2)

s is drawdown at any point of observation near a discharging well, in feet;
Q is well discharge, in cubic feet per day;
T is transmissivity, in square feet per day;
e is base of Napierian logarithms (2.7183), dimensionless;
r is distance from point of observation to pumping well, in feet;
S is storage coefficient, dimensionless; and
t is time since pumping started, in days.

The integral in equation 1 is called the well function of u and is often represented as W(u).

[Note: Throughout this report, equations are presented so that any consistent units can be used. For example, in equation 1, s is in feet, Q is in cubic feet per day, and T is in square feet per day. For converting to other units, the reader is directed to the two conversion tables at the front of this report. Also, the units used for each variable are given the first time the variable appears in an equation. The units remain the same throughout the rest of the report unless otherwise specified.]

The Theis equation is based on the following assumptions:

1. The well discharge rate is constant.
2. The diameter of the well is infinitesimal, and storage in the well is negligible.
3. The well is open to the full thickness of the aquifer (fully penetrating well).
4. The aquifer is bounded above and below by nonleaky confining beds and remains saturated.
5. The aquifer is infinite in areal extent and is uniform in thickness.
6. The aquifer is homogeneous and isotropic.
7. All water pumped by the well is from aquifer storage and is released instantaneously with head decline.

Using equations 1 and 2, one can estimate T and S if Q is known and if s is measured at one value of r for several values of t or at one value of t for several values of r. That is, drawdown data measured at different times from one observation well or drawdown data from at least two observation wells measured at the same time can be used to estimate T and S. As discussed in the section "Prediction," equations 1 and 2 can also be used to predict the drawdown at any distance from the pumping well, for any pumping rate, and at any time as long as the assumptions remain valid.

Assumption 2 indicates that the Theis solution does not consider water stored in the well casing. During the initial part of aquifer tests, well storage tends to minimize drawdown. Papadopulos and Cooper (1967) state that well storage can be neglected when:

$$t > 250\ r_w^2/T \qquad (3)$$

where r_w is the radius of the pumping well in that part of the casing where drawdown occurs, in feet.

The effect of storage in the pumping well on drawdown in an observation well can be neglected if

$$r/r_w > 300$$

where r is distance between pumping and observation wells (Walton, 1984, p. 181).

The effect of partial penetration on drawdown diminishes with distance from the pumping well. A partially penetrating pumping well can be assumed to be fully penetrating (assumption 3) if the point of observation is at:

$$r > 1.5b(K_r/K_z)^{\frac{1}{2}} \tag{4}$$

where

K_z is the vertical hydraulic conductivity, in feet per day,
K_r is the radial hydraulic conductivity, in feet per day, and
b is the aquifer thickness, in feet (Hantush, 1964a, p. 355).

Though designed for confined aquifers (assumption 4), the Theis solution can also be applied to unconfined aquifers very early or very late in the pumping period when release of water from storage in the aquifer can be assumed to be instantaneous (assumption 5) and if drawdown is slight compared to total saturated thickness of the aquifer. This situation is discussed in the section "Model of Unconfined Aquifer and Partially Penetrating Well."

Two other methods of estimating aquifer properties are based on modifications of the Theis equation. The first takes advantage of the fact that for large values of t and small values of r, u becomes very small and equation 1 can be simplified to a form that does not require curve fitting for solution. This modified method can be used for values of u less than 0.01 (see Ferris and others, 1962, p. 99). In the second method, recovery data rather than drawdown data, are used for estimating T. This method is described by Ferris and others (1962, p. 100-102).

Application

Because T occurs both inside and outside the integral in equation 1, solving the equation for T cannot be done algebraically. However, the equation can be solved graphically using a curve-matching procedure.

Curve matching involves plotting u (or 1/u) versus W (u) on log-log paper and matching the resulting curve to field data also plotted on log-log paper. A concise description of the procedure quoted from Ferris and others (1962, p. 94-98) follows:

Rearranging equations 1 and 2 there follows:

$$s = \left(\frac{Q}{4\pi T}\right) W(u) \tag{5}$$

or

$$\log s = \left(\log \frac{Q}{4\pi T}\right) + \log W(u) \tag{6}$$

and

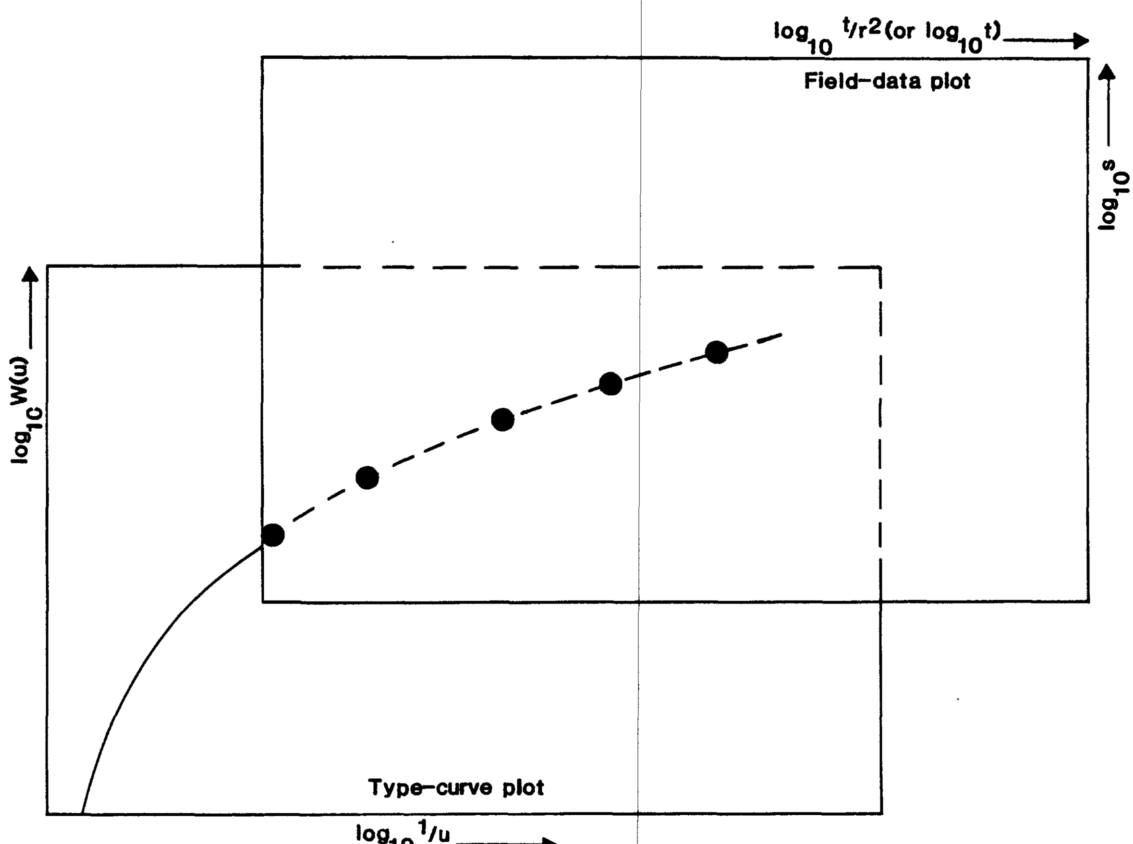
$$\frac{r^2}{t} = \left(\frac{4T}{S}\right)u \tag{7}$$

or

$$\log \frac{r^2}{t} = \left(\log \frac{4T}{S}\right) + \log u \tag{8}$$

If the discharge, Q is held constant, the bracketed parts of equations 6 and 8 are constant for a given pumping test, and W (u) is related to u in the manner that s is related to r^2/t. This is shown graphically in figure 8. Therefore, if values of the drawdown, s, are plotted against r^2/t (or $1/t$ if only one observation well is used), on logarithmic tracing paper to the same scale as the type curve, the curve of observed data will be similar to the type curve. The data curve may then be superposed on the type curve, the coordinate axes of the two curves being held parallel, and translated to a position which represents the best fit of the field data to the type curve. An arbitrary point is selected anywhere on the overlapping portion of the sheets and the coordinates of this common point on both sheets are recorded. It is often convenient to select a point whose type-curve coordinates are both 1. These data are then used with equations 5 and 7 to solve for T and S.

A type curve on logarithmic coordinate paper of W(u) versus 1/u, the reciprocal of the argument, could have been plotted (see fig. 8). Values of the drawdown (or recovery), s, would then have been plotted against t or t/r^2, and superposed on the type curve in the manner outlined above. This method eliminates the necessity for computing 1/t values for the values of s.



(Modified from Ferris and others, 1962)

Figure 8.-- Relations of 1/u to W(u) type-curve plot and t/r^2(or t) to s.

In the preceding quotation, equation and figure numbers and constants were changed to be consistent with this text. Values of $W(u)$ for values of u between 10^{-5} and 9.9 are tabulated in Ferris and others (1962, p. 96-97, table 2).

The curve-matching procedure can be used to solve many analytical equations. Stallman (1971, p. 4) points out that equation 1 can be generalized to

$$s = \frac{Q}{CT} f\left(u,\omega,\tau,\ldots\right) \tag{9}$$

where C, a constant, and f, a well function, depend on aquifer boundaries and hydraulic properties. ω and τ represent dimensionless aquifer properties such as anisotropy or confining-bed leakage. Regardless of the form that equation 9 takes, the curve-matching procedure is the same in that s against t/r^2 (or t) is used for the field-data plot and $1/u$ against $f(u, \omega, \tau,..)$ is used for the type-curve plot.

Example

Well I6-1 in the Howe area (fig. 2) was used to irrigate about 140 acres in 1982. The well is 105 ft in depth, is 1 ft in diameter, and has a 20-ft-long screen. The rate of pumping during irrigation was measured to be 108,760 ft^3/d or 0.81 Mgal/d. Lithologic information from eight wells within a 1-mi radius of well I6-1 indicates that well I6-1 produces from a layer of sand and gravel 21 ft in thickness between two layers of clay (fig. 9). The average thickness of the upper clay layer is about 20 ft and the layer is probably continuous near well I6-1. The lower clay layer is probably continuous east of the well. Lithologic information immediately west of the well is insufficient for estimating the extent of the lower clay in that direction. On the basis of this lithologic information, the author assumed that well I6-1 completely penetrates a nonleaky, confined aquifer.

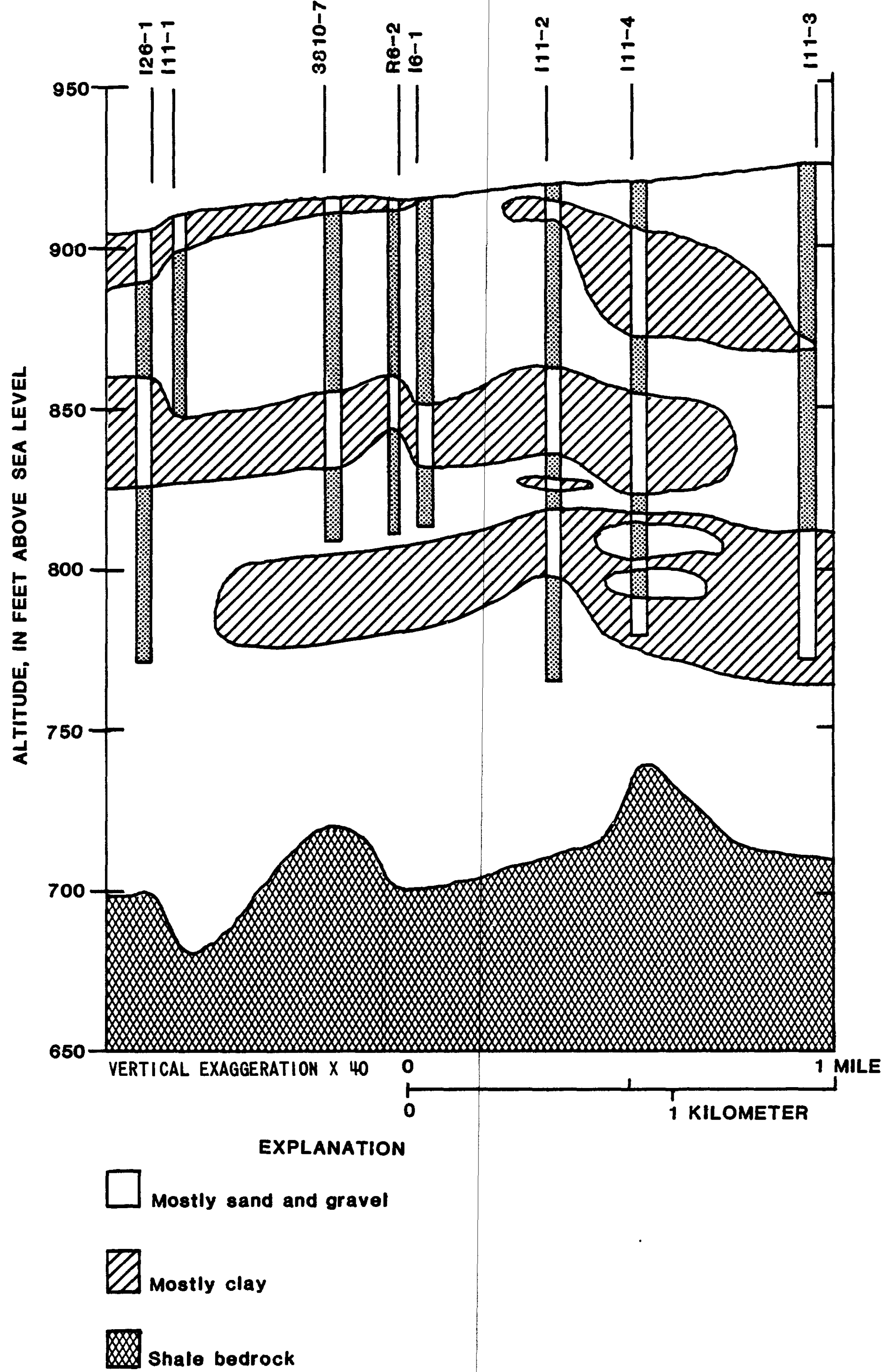


Figure 9.-- Geologic section near well I6-1, Howe area. (See fig. 2.)

Well R6-2 is a 2-in-diameter observation well 154 ft south of well I6-1 (fig. 2). The two wells are cased to the same depth. Well I6-1 was equipped with a 3-ft-long screen and an automatic water-level recorder.

Water levels in well R6-2 were recorded every 5 minutes on June 26, 1982 during the time that I6-1 was being pumped. These water-level data were used to calculate drawdown (table 2), which is plotted versus time (field-data plot) in figure 10. Figure 10 also contains the type curve for a nonleaky, confined aquifer and fully penetrating well. The type curve is plotted in the position that best "fits" the field data. The coordinates of the match point at W(u), 1/u from the type curve plot (axes not shown in fig. 10), were 1.0, 10.0. The corresponding coordinates of s, t on the field-data plot are 0.68 ft and 0.29 minutes.

Table 2.--Drawdown of water level in observation well R6-2, Howe area, June 26, 1982

Time of record[1]	Time since pumping began (minutes)	Drawdown (feet)
1040	5	3.41
1045	10	3.58
1050	15	3.97
1055	20	4.20
1105	30	4.54
1120	45	4.88
1135	60	5.13
1150	75	5.35
1205	90	5.55
1220	105	5.63
1435	240	6.21
1635	360	6.36
1835	480	6.44
2035	600	6.47

[1]Military time

T is calculated by rearranging equation 5 to yield

$$T = \frac{Q}{4\pi s} W(u).$$

By using the match-point values for W(u) and s and noting that Q = 108,760 ft^3/d,

$$T = \frac{(108,760\ ft^3/d)(1)}{4\pi\ (0.68\ ft)} = 12,728\ ft^2/d$$

S is calculated by rearranging equation 7 as follows:

$$S = 4Ttu/r^2 \qquad (9.5)$$

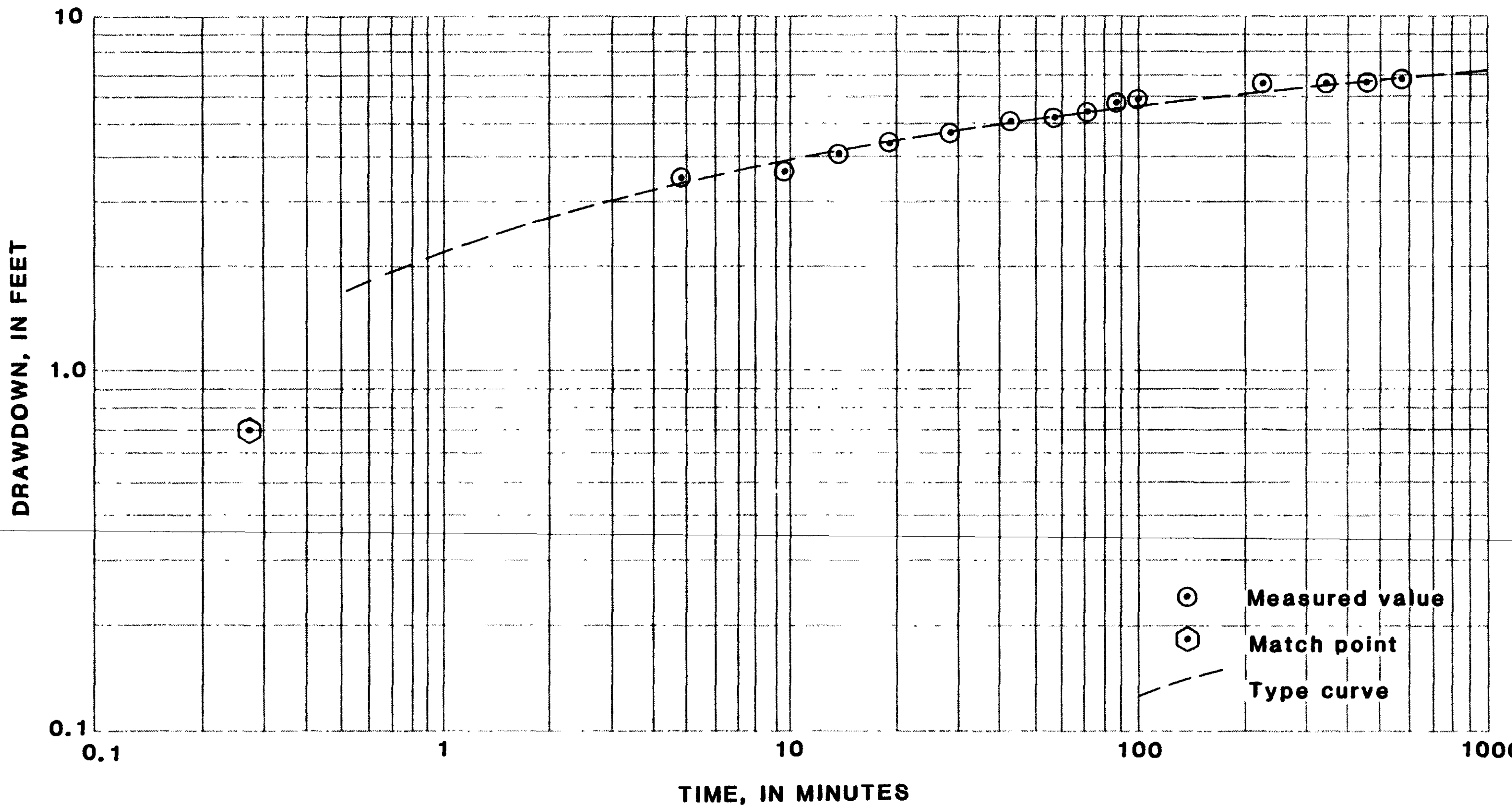


Figure 10.-- Relation of drawdown to time for observation well R6-2, Howe area, June 26, 1982, and type curve for a non-leaky, confined aquifer and fully penetrating well.

Match-point values for 1/u and t are 10 and 0.29 minutes. Thus,

$$u = 0.1,$$
$$t = 2.0 \times 10^{-4} d, \text{ and}$$

$$S = \frac{4(12,728\ ft^2/d)(2.0 \times 10^{-4} d)(0.1)}{(154\ ft)^2} = 4.3 \times 10^{-5}.$$

The radial hydraulic conductivity, K_r, can be found from:

$$K_r = T/b. \tag{10}$$

Thus, if b = 21 ft,

$$K_r = \frac{12,728\ ft^2/d}{21\ ft} = 606\ ft/d$$

Discussion

The hydraulic conductivity of the aquifer material is very high, but the productivity of the aquifer might be limited by its thickness. The hydraulic conductivity is greater than values reported in literature for similar material (table 3); but because the aquifer near well I6-1 is only about 21 ft thick, the value of transmissivity is less than the value of 13,400 ft^2/d suggested by Freeze and Cherry (1979, p. 60) as the threshold value for an aquifer capable of producing significant quantities of water.

Table 3.--Values of hydraulic conductivity of unconsolidated material

Material	Hydraulic conductivity (feet per day)	Source
Clay, montmorillonite	1.3×10^{-5}	Davis and DeWiest, 1966, p. 164
Clay, kaolinite	1.3×10^{-3}	Davis and DeWiest, 1966, p. 164
Clay, silt	$10^{-6} - 10^{-1}$	Walton, 1970, p. 36
Clay	1.0	Lohman, 1972, p. 53
Till, mostly clay	9×10^{-5}	Larson and others, 1975, p. 31
Till	$7 \times 10^{-5} - 1.0$	Freeze and Cherry, 1979, p. 29
Sand, fine-coarse	8.2 - 150	Todd, 1980, p. 71
Sand, fine-coarse	15 - 700	Lohman, 1972, p. 53
Sand	10 - 400	Walton, 1970, p. 36
Sand, clean, fine-coarse	1 - 5,000	Freeze and Cherry, 1979, p. 29
Sand, well sorted	24 - 7,800	Davis and DeWiest, 1966, p. 164
Gravel, fine-coarse	900 - 1,000	Lohman, 1972, p. 53
Gravel	$9 \times 10^{3} - 9 \times 10^{5}$	Freeze and Cherry, 1979, p. 29
Gravel, fine-coarse	500 - 1,500	Todd, 1980, p. 71
Sand and gravel	25 - 650	Walton, 1970, p. 36
Outwash	230	Larson and others, 1975, p. 31
Outwash	323 - 630	Helgesen, 1971, p. 13

The storage coefficient is near the range of 1.5×10^{-5} to 3.1×10^{-5} for confined sand and gravel aquifers suggested by Jumikis (1962) as reported by Walton (1970, p. 627).

Equation 3 can be used to test whether the effect of water stored in the pumping well can be neglected (assumption 2); that is, storage in the pumping well can be neglected if t is greater than $250\ r_w^2/T$.

For the above example:

$$t = 250\ (0.5\ \text{ft})^2/12{,}728\text{ft}^2/\text{d} = 0.0049\ \text{d (about 7 minutes)}.$$

On the basis of equation 4, the effects of partial penetration on drawdown can be neglected (assumption 3) if

$$r > 1.5b\ \left(K_z/K_r\right)^{\frac{1}{2}}.$$

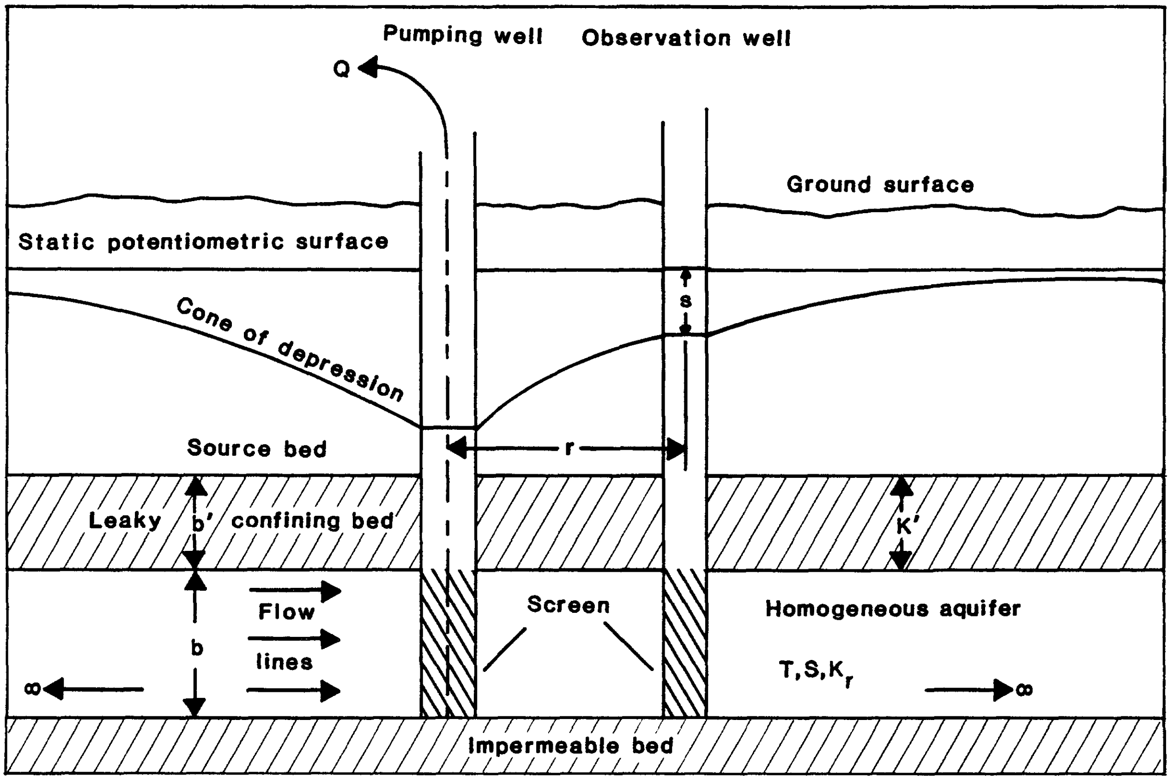
For outwash deposits, K_z/K_r ranges from about 0.1 to 0.5 (Edwin Weeks, U.S. Geological Survey, oral commun., 1982). Analysis of data from another well in the Howe area indicates K_z/K_r equals 0.1 (see p. 57.) If this value is used for the aquifer tapped by I6-1, then

$$1.5(21\ \text{ft})\ \left(K_z/K_r\right)^{\frac{1}{2}} = 10\ \text{ft}.$$

Thus, beyond 10 ft from the pumping well, the effects of partial penetration on drawdown can be neglected.

Model of Leaky, Confined Aquifer and Fully Penetrating Well

A leaky, confined aquifer is bounded above or below by a confining bed that is permeable to water (fig. 11). All confining beds are permeable and provide recharge to the aquifer; however, where the amount of recharge through the confining bed is very small compared to the amount of water released from storage, the confining bed is assumed to be nonleaky. Thus, the Theis equation represents a special case of the more general equation for leaky aquifers discussed here. If the rate of pumping from the leaky aquifer is high enough, water is induced through the confining bed from a source bed which is usually an unconfined, surficial aquifer above the confining bed (fig. 11). The rate of flow through the confining bed is proportional to the difference in head between the aquifer and the source bed. The constant of proportionality is the vertical hydraulic conductivity of the confining bed, K'.



(Modified from Reed, 1980, p. 21)

Figure 11.-- Section through a pumping well that fully penetrates a leaky, confined aquifer.

Conceptualization

The first nonsteady-state equation to describe drawdown near a fully penetrating well that taps a leaky, confined aquifer was developed by Hantush and Jacob (1955). The equation can be abbreviated as

$$s = \frac{Q}{4\pi T} W(u, r/B) \tag{11}$$

where W(u, r/B) is the well function and is equal to

$$\int_u^\infty (1/y) \exp(-y - r^2/4B^2y)dy \text{ (Hantush and Jacob, 1955, p. 98)}$$

where

$B = (\frac{Tb'}{K'})^{1/2}$, in feet; (12)

b' is the thickness of the confining bed, in feet;

K' is the vertical hydraulic conductivity of the confining bed, in feet per day; and

y is the variable of integration.

In addition to assumptions 1, 2, 3, 5, and 6 on page 20, the Hantush-Jacob equation is based on the following assumptions:

1. The aquifer is overlain by a continuous confining bed of uniform vertical hydraulic conductivity and thickness.
2. The confining bed is overlain by an infinite, constant-head source bed.
3. The confining bed is incompressible, and release of water from storage in the confining bed is negligible.
4. The aquifer is underlain by a nonleaky confining bed.
5. The flow toward the pumping well is radial in the aquifer and is vertical in the confining bed.

As the cone of depression around the pumping well expands, water from the source bed is induced to flow through the confining bed. If the pumping rate is high enough and the pumping period is long enough, drawdown in the source bed will reduce the gradient across the confining bed and, in turn, will reduce recharge to the confined aquifer. This reduction becomes significant and violates assumption 2 if

$$\frac{Tt}{r^2 S} < 1.6 \frac{Y^2}{(r/B)^4} \qquad (13)$$

where $Y = (r/4b) \left(\frac{K'S'_s}{K \ S_s}\right)^{\frac{1}{2}}$, dimensionless; (14)

S'_s is specific storage of the leaky confining bed, dimensionless; and

S_s is specific storage of the confined aquifer, dimensionless (Neuman and Witherspoon, 1969a, p. 810).

If assumption 2 is violated, drawdown near the pumping well would be greater than that predicted by the Hantush-Jacob equation.

Storage in the confining bed can be neglected (assumption 3) if the change in hydraulic gradient across the confining bed instantaneously accompanies the decline in head in the aquifer. Factors contributing to a rapid change in head across the confining bed are

- A thin confining bed,
- A slow decline in head in the confined aquifer, and
- A large hydraulic diffusivity, K'/S'_s, in the confining bed (Cooper, 1963, p. C48).

According to Neuman and Witherspoon (1969b, p. 821), water stored in the confining bed is negligible if $\beta < 0.1$, where $\beta = (r/b)\ (K_z/K_r)^{\frac{1}{2}}$. Drawdown in the confined aquifer is less than that predicted by the Hantush-Jacob equation when assumption 3 is violated.

Flow is three-dimensional in the aquifer and in the confining bed. However, assumption 5 can be considered valid when $b/B < 0.1$ (Hantush, 1967b, p. 587). When assumption 5 is violated, drawdown in the aquifer is greater than that predicted by the Hantush-Jacob equation.

Application

The well function for a leaky, confined aquifer, as represented in equation 11, has two parameters--u and r/B. Type curves generated from equation 11 represent three terms--1/u (or u), r/B, and W(u,r/B). W(u,r/B) is plotted on the vertical axis and 1/u on the horizontal axis. Each type curve represents one value of r/B (fig. 12).

The field data for one or more observation wells are plotted on log-log paper exactly as described for nonleaky aquifer analysis--that is, values of s are plotted on the vertical axis, and values of t (or t/r^2 if more than one observation well is used) are plotted on the horizontal axis.

The curve-matching procedure produces coordinate values at [1/u, W(u, r/BO] and at [t (or t/r^2), s] as well as an interpolated value for r/B. Values for hydraulic properties are then calculated by using the following equations:

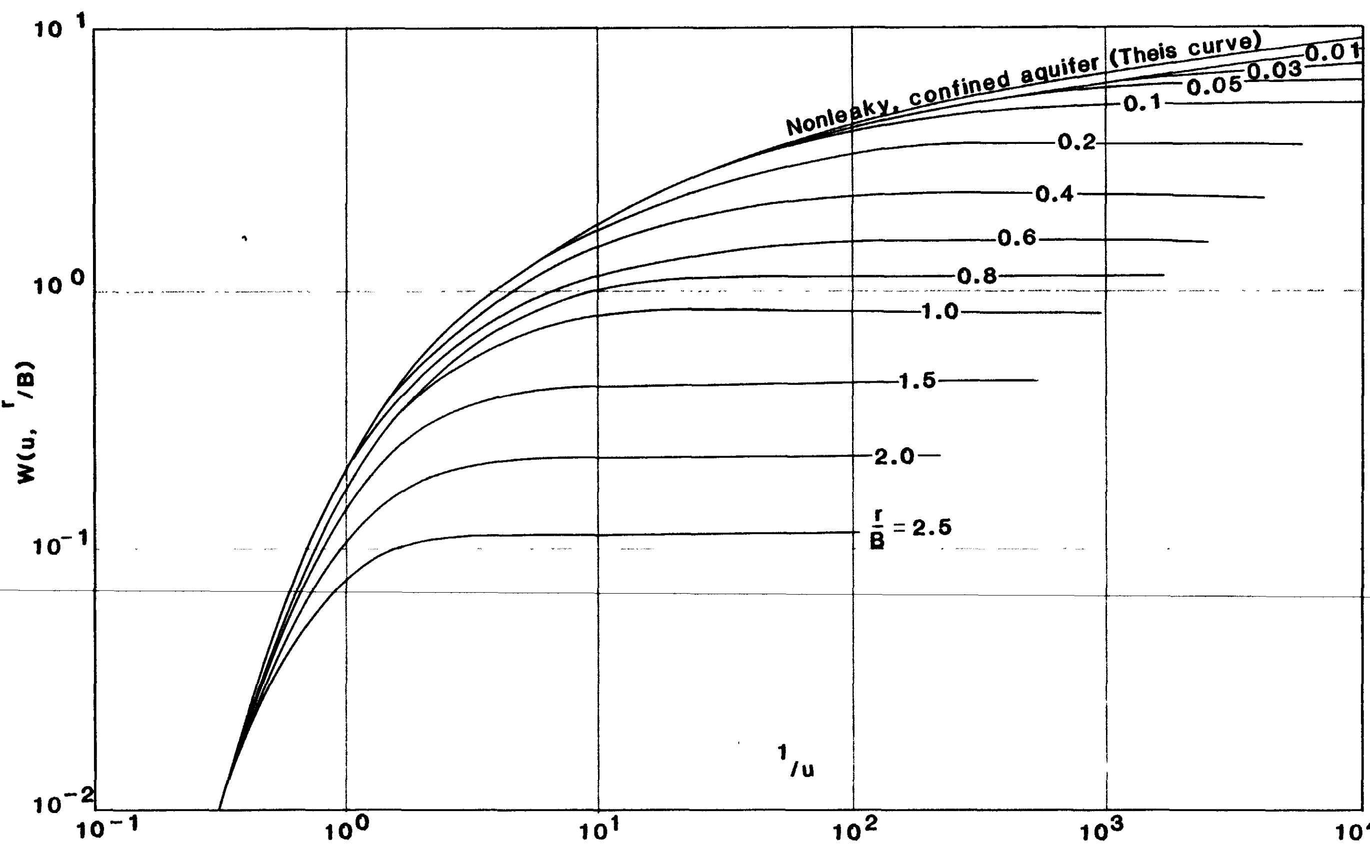
$$T = \frac{Q}{4\pi s} W(u,\ r/B), \text{ and} \tag{15}$$

$$K' = \frac{T\ b'}{B^2}, \tag{16}$$

which follow from equations 11 and 12, respectively. The storage coefficient, S, is calculated from equation 9.5.

Example

Well I101-1 in the Milford area (fig. 4) was used to irrigate about 225 acres of corn and soybeans in 1982. The well is 85 ft deep, is 1 ft in diameter, and has a 20-ft-long screen. The rate of pumping during irrigation was 119,350 ft^3/d (0.89 Mgal/d). Lithologic information from five wells within a 1-mi radius of I101-1 indicates that well I101-1 produces from a layer of sand and gravel 31 ft thick that lies beneath a layer of clay about 8 ft thick near the well (fig. 13). Beneath the sand and gravel is a layer of clay underlain by sand with gravel and clay and bedrock (Lindgren and others, 1985). Three 2-in-diameter observation wells (R101-2, R101-4, and R101-5) were installed at distances of 36, 182, and 845 ft from I101-1. Each observation well was screened in the same aquifer as well I101-1.



(Modified from Walton, 1962, plate 1)

Figure 12.-- Type curves for a leaky, confined aquifer and fully penetrating well.

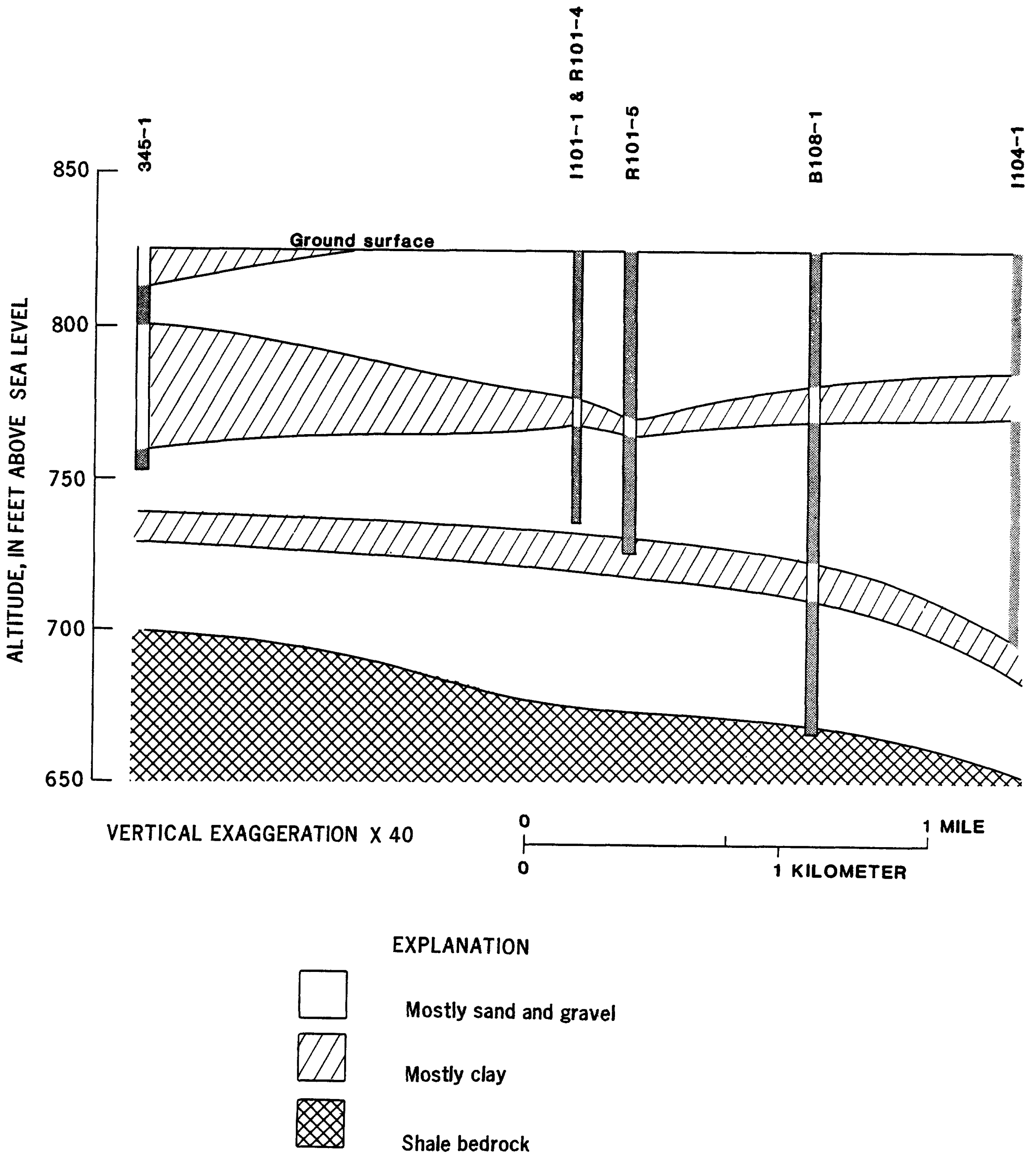


Figure 13.-- Geologic section near well I101-1, Milford area. (See fig. 4).

Drawdown data from the three observation wells were recorded during the time well I101-1 was operating on July 9, 1982 (table 4). To construct the field-data curves (fig. 14), the author used values of t/r^2, rather than t, because data from all three observations wells were used for the analysis. Each field-data curve in figure 14 was individually matched to one of the type curves. Match points and coordinate values for each of the three curve matches are as follows:

Observation well	s	t/r^2	W(u, r/B)	1/u	r/B
R101-2	0.70	1.0×10^{-5}	1.0	1.0	0.05
R101-4	.89	5.6×10^{-6}	1.0	1.0	.20
R101-5	.76	3.8×10^{-6}	1.0	1.0	.80

Table 4.--Drawdown of water level in observation wells R101-2, R101-4, and R101-5, Milford area, July 9, 1982

[t, time since pumping began, in minutes; s, drawdown, in feet; r, distance between pumping and observation wells, in feet]

Time of record[a]	t	R101-2 (r=36)		R101-4 (r=182)		R101-5 (r=845)	
		s	t/r^2	s	t/r^2	s	t/r^2
1010	10	3.92	7.7×10^{-3}	2.64	3.0×10^{-4}	0.55	1.4×10^{-5}
1015	15	3.94	1.2×10^{-2}	2.71	4.5×10^{-4}	.67	2.1×10^{-5}
1020	20	4.05	1.5×10^{-2}	2.85	6.0×10^{-4}	.71	2.8×10^{-5}
1025	25	4.13	1.9×10^{-2}	2.95	7.5×10^{-4}	.78	3.5×10^{-5}
1030	30	4.16	2.3×10^{-2}	2.98	9.1×10^{-4}	.79	4.2×10^{-5}
1040	40	---[b]	3.1×10^{-2}	3.03	1.2×10^{-3}	.83	5.6×10^{-5}
1050	50	---[b]	3.9×10^{-2}	3.07	1.5×10^{-3}	.85	7.0×10^{-5}
1100	60	4.24	4.6×10^{-2}	3.09	1.8×10^{-3}	.86	8.4×10^{-5}
1115	75	4.24	5.8×10^{-2}	3.09	2.3×10^{-3}	.86	1.1×10^{-4}
1130	90	4.25	6.9×10^{-2}	3.14	2.7×10^{-3}	.87	1.3×10^{-4}
1145	105	4.25	8.1×10^{-2}	3.15	3.2×10^{-3}	.87	1.5×10^{-4}
1200	120	4.27	9.3×10^{-2}	3.17	3.6×10^{-3}	.88	1.7×10^{-4}
1230	150	4.28	1.2×10^{-1}	3.19	4.5×10^{-3}	.89	2.1×10^{-4}
1300	180	4.30	1.4×10^{-1}	3.19	5.4×10^{-3}	.90	2.5×10^{-4}
1330	210	4.31	1.6×10^{-1}	3.20	6.3×10^{-3}	.90	2.9×10^{-4}
1400	240	4.32	1.9×10^{-1}	3.21	7.2×10^{-3}	.91	3.3×10^{-4}
1500	300	4.32	2.3×10^{-1}	3.21	9.1×10^{-3}	.91	4.2×10^{-4}
1600	360	4.33	2.8×10^{-1}	3.21	1.1×10^{-2}	.91	5.0×10^{-4}
1700	420	4.33	3.2×10^{-1}	3.21	1.3×10^{-2}	.91	5.8×10^{-4}
1800	480	4.33	3.7×10^{-1}	3.21	1.5×10^{-2}	.91	6.7×10^{-4}

[a]Military time
[b]Missing data

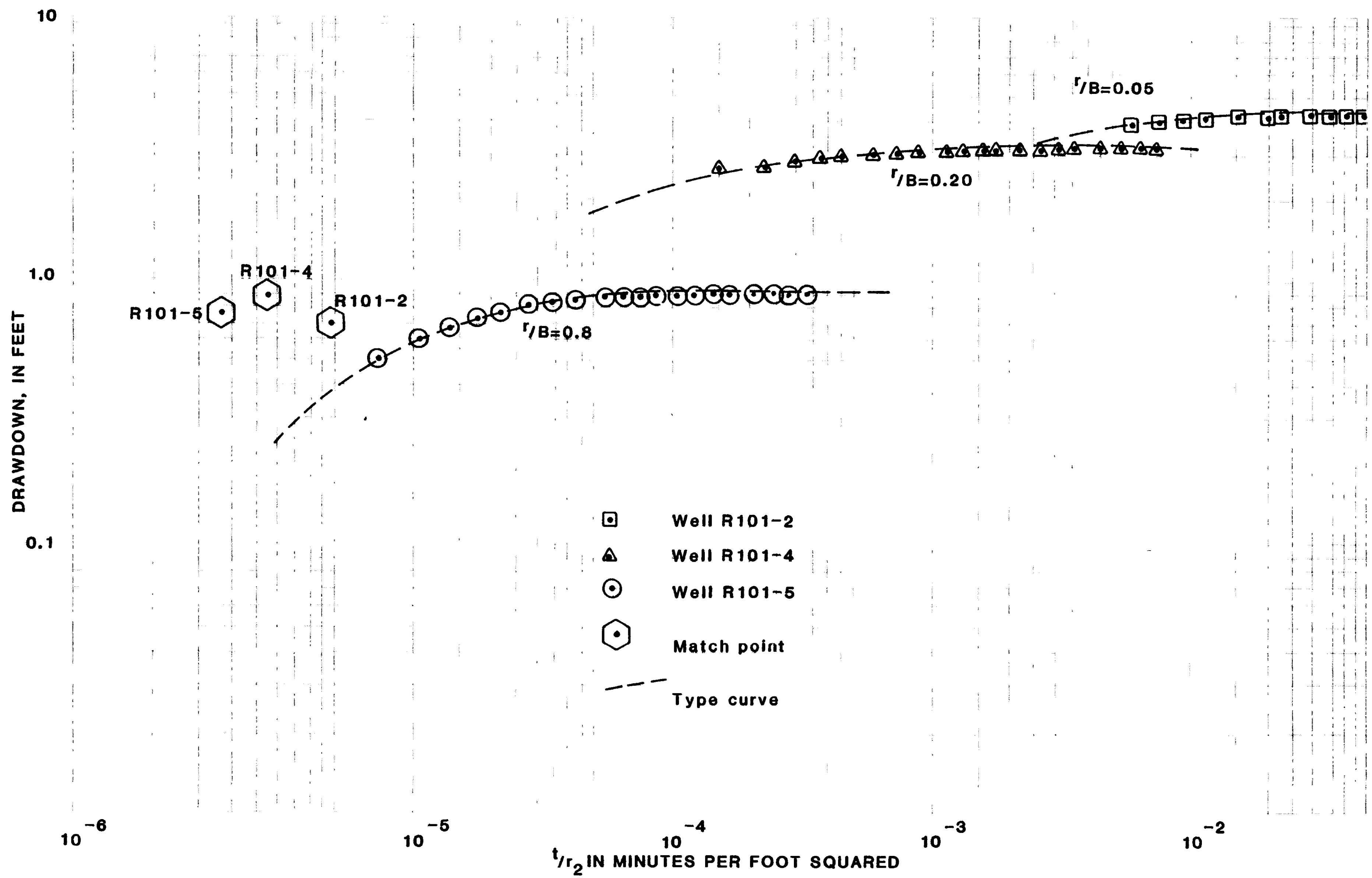


Figure 14.– Relation of drawdown to time for observation wells R101-2, R101-4, and R101-5, Milford area, July 9, 1982, and type curves for a leaky, confined aquifer and fully penetrating well.

Other information needed to calculate hydraulic properties includes

$$Q = 119{,}350 \text{ ft}^3/\text{d} \qquad r_{R101-5} = 845 \text{ ft}$$
$$r_{R101-2} = 36 \text{ ft} \qquad b = 31 \text{ ft}$$
$$r_{R101-4} = 182 \text{ ft} \qquad b' = 8 \text{ ft}$$

Hydraulic properties of the aquifer and confining bed were calculated by using equations 15, 10, 9.5, and 16. Data for well R101-2 illustrate how the calculations were made.

$$T = \frac{(119{,}350 \text{ ft}^3/\text{d})\ (1.0)}{(4)(3.142)(0.7 \text{ ft})} = 13{,}564 \text{ ft}^2/\text{d}$$

$$K_r = (13{,}564 \text{ ft}^2/\text{d})/31 \text{ ft} = 438 \text{ ft/d}$$

$$S = 4(13{,}564 \text{ ft}^2/\text{d})(1.0 \times 10^{-5} \text{ min/ft}^2)\ \left(\frac{1}{1{,}440} \text{ d/min}\right)(1.0) = 3.8 \times 10^{-4}$$

$$K' = \frac{(13{,}564 \text{ ft}^2/\text{d})(8 \text{ ft})}{(720 \text{ ft})^2} = 0.21 \text{ ft/d}$$

Results of calculations for all three sets of data are presented in table 5.

Table 5.--Hydraulic properties of the leaky, confined aquifer near pumping well I101-1 calculated by using drawdown data from observation wells R101-2, R101-4, and R101-5, Milford area

[r, distance between pumping well and observation well, in feet; T, transmissivity, in feet squared per day; K_r, radial hydraulic conductivity of the aquifer, in feet per day; S, storage coefficient, unitless; K', vertical hydraulic conductivity of the confining bed, in feet per day; values for hydraulic properties rounded to two significant digits]

Observation well	r	T	K_r	S	K'
R101-2	36	14,000	440	3.8×10^{-4}	0.21
R101-4	182	11,000	340	1.7×10^{-4}	.10
R101-5	845	12,000	400	1.3×10^{-4}	.090

Discussion

The three field-data plots (fig. 14) would theoretically fall on the same curve if leakage did not occur. The displacement of the plots from one another results because the leakage does not affect equally drawdown from the three wells. The area of the cone of depression, which receives leakage through the confining layer, increases with the square of the distance from the pumping well, r. Thus, as r increases, the volume of leakage, which reduces drawdown, increases. Conversely, the smaller the value of r and, therefore, of r/B, the smaller the effect of leakage on drawdown. As r/B approaches zero, the effect of leakage becomes negligible and the type curves in figure 12 approach the Theis curve.

The three values of T in table 5 are within 27 percent of each other. The values of S differ by more than 100 percent. The greater precision in the estimates of transmissivity can be explained, in part, by the curvature of the field-data plots and the curve-matching process. During curve matching, the field-data plot is moved vertically (up and down) on the type curve to determine values of W(u, r/B) and s used to calculate T (see equation 15). The field-data plot is moved horizontally (back and forth) on the type curve to determine values of 1/u and t/r^2 used to calculate S (see equation 9.5). When early drawdown data are missing from the field-data plots, the curvature is nearly flat, and the matching process for calculation of S is imprecise. In figure 14, the curvature of the field data plot for well R101-5 is greater than that of the other two plots, and 1.3×10^{-4} (table 5) is probably the most accurate value of S. This value is near the middle of the range of 0.00005 to 0.005 reported by Todd (1980, p. 45-46) for all aquifer materials.

The storage coefficient can be calculated as follows:

$$S = S_s \times b \tag{17}$$

where S_s is specific storage, in feet^{-1}. Specific storage is the volume of water removed from (or added to) a unit volume of aquifer per unit decline (or rise) in head. It follows from equation 17 that if

$$S = 1.3 \times 10^{-4},$$

then

$$S_s = 1.3 \times 10^{-4}/31 \text{ ft} = 4.2 \times 10^{-6} \text{ ft}^{-1}.$$

This value falls between an average value of 10^{-6} ft^{-1} suggested by Lohman (1972, p. 53) and a range of 3.1×10^{-5} to 1.5×10^{-5} for sandy gravel determined by Jumikis (1962) as reported in Walton (1970, p. 627).

Calculated values of transmissivity range from 11,000 to 14,000 ft^2/d. These values indicate that the aquifer is capable of producing significant quantities of water (Freeze and Cherry, 1979, p. 60).

Hydraulic conductivity of the aquifer ranges from 340 to 440 ft/d, which is within the range of values reported by Walton (1970, p. 36) for sand and gravel (table 3). Vertical hydraulic conductivity of the confining bed ranges from 0.09 to 0.21 ft/d. These values are higher than most values for clay or till reported in literature (table 3). The relatively high conductivity and low thickness (8 ft) of the confining layer permit a quick response to changes in head between the confined aquifer and the source bed. This quick response might explain why water levels in the three observation wells stabilized after only 6 hours of pumping (table 4). Presumably the system had reached steady state--that is, all the water to the pumping well is induced recharge through the confining bed (Davis and DeWiest, 1966, p. 229). The cone of depression would remain stable until the head in the source bed is lowered sufficiently to cause a decrease in recharge to the confined aquifer and the removal from storage.

The validity of assumption 2 can be tested by using equation 13 for well R101-5. The value for S'_s--the specific storage of the confining bed--was assumed to be 5×10^{-4} d^{-1}, an average value for clay (Jumikis, 1962, as reported in Walton, 1970, p. 627). Values for all other terms in equation 13 have been given previously. This test indicates that reduced head in the source aquifer would not significantly affect drawdown in the confined aquifer for $t < 8.2$ hours. Models that can accommodate reduced head in the source aquifer are described in Hantush (1967a) and in Neuman and Witherspoon (1969b).

Assumption 3 is probably valid because of the thin confining bed and high value of K'. A model that can accommodate storage in confining beds was developed by Hantush (1964a, p. 334-337).

In conformity with assumption 4, the preceding discussion assumes that all leakage to the pumped aquifer was through the upper clay layer. Some leakage probably also moved through the lower clay layer that underlies the aquifer (fig. 13). However, limited geologic data collected near well I101-1 indicate that the material underlying the lower clay layer has much lower hydraulic conductivity than either the pumped aquifer or the upper unconfined aquifer (Lindgren and others, 1985). Thus, the leakage of water through the lower clay layer was probably much less than the leakage through the upper clay layer.

Assumption 5 can be assumed to be valid if b/B is less than 0.1 (Hantush, 1967b, p. 587). Values of b/B were calculated for wells R101-2, R101-4, and R101-5 by using data in table 5 and equation 12. These values are 0.04, 0.03, and 0.03 and indicate that flow is horizontal in the aquifer and vertical in the confining bed.

Model of Leaky, Confined Aquifer and Partially Penetrating Well

Pumping wells are not usually screened throughout the entire thickness of the aquifer. A well is considered to be partially penetrating if (1) it does not extend to the bottom of the aquifer or (2) it is not screened through the entire length of penetration.

The pumping of a partially penetrating well results in vertical flow components in the aquifer (fig. 15). Vertical flow affects the drawdown distribution near the pumping well. Walton (1962, p. 7) described the effects of partial penetration on drawdown as follows:

> The cone of depression is distorted and observed drawdowns in observation wells differ from theoretical drawdowns for a fully penetrating well according to the vertical position of the observation well. If the pumped and observation wells are both open in either the top or the bottom portion of the aquifer, the observed drawdown in the observation well is greater than for fully penetrating conditions. If the pumped well is open to the top of the aquifer and the observation well is open to the bottom of the aquifer, or vice versa, the observed drawdown in the observation well is smaller than for fully penetrating conditions.

The magnitude of the effect on drawdown in observation wells is determined by the degree of penetration, by the proximity to the pumping well, and by the anisotropy of the aquifer.

Conceptualization

The equation describing drawdown near a discharging well that partially penetrates a leaky, confined aquifer was developed by Hantush (1964a, p. 350). The Hantush equation can be written as

$$s = \frac{Q}{4\pi T}\left[W(u,r/B) + f(u,\ hr/b,\ r/B,\ c/b,\ \ell/b,\ g/b)\right] \qquad (18)$$

where

$W(u,r/B)$ is the well function for a fully penetrating well (equation 11), dimensionless;

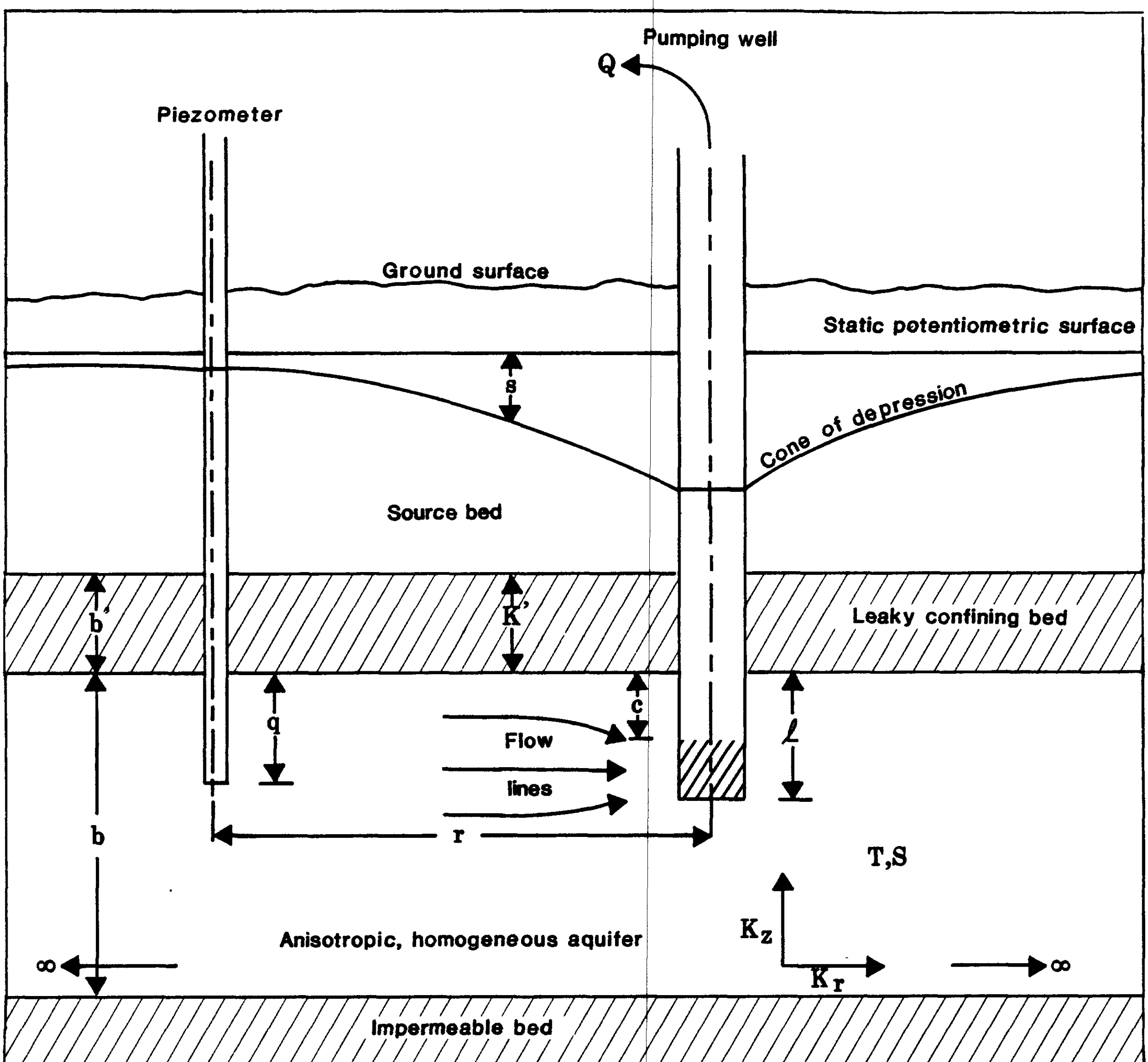
$f(u,\ hr/b,\ r/B,\ d/b,\ \ell/b,\ g/b)$ is an integral function that describes the effect of partial penetration (see Hantush, 1964a, p. 350 for integral form), dimensionless;

$h = (K_z/K_r)^{\frac{1}{2}}$, dimensionless;

c is the distance from the top of the aquifer to the top of the pumping well screen, in feet;

ℓ is the distance from the top of the aquifer to the bottom of the pumping well screen, in feet, and;

g is the distance from the top of the aquifer to the bottom of the piezometer, in feet.



(Modified from Reed, 1980, p. 32)

Figure 15.– Section through a pumping well that partially penetrates a leaky, confined aquifer

The two well functions in brackets in equation 18 are henceforth represented as $W(\rho)$ in this report. That is,

$$W(\rho) = W(u, r/B) + f(u, hr/b, r/B, c/b, \ell/b, g/b).$$

Assumptions for the Hantush model include those listed as 1, 2, and 5 on page 20, those listed as 1, 2, 3, and 4 on page 30, plus the following two assumptions:

1. Flow is three-dimensional in the aquifer and vertical in the confining bed. Vertical flow components in the aquifer result from partial penetration and not leakage.

2. The aquifer is homogeneous but anisotropic.

As might be inferred from the apparent complexity of equation 18, analytical solutions involving partial penetration are difficult to use and should be avoided if possible. Many partially penetrating wells can be treated as fully penetrating wells. Todd (1980, p. 152) provides three cases where the effects of partial penetration can be considered negligible.

- When the well is open to at least 85 percent of the saturated thickness of the aquifer.
- For homogeneous and isotropic aquifers, when the observation well is at least 1.5 to 2 times the saturated thickness away from the pumping well.
- For highly anisotropic aquifers, when the length of the screened interval is taken as the total saturated thickness and the aquifer is assumed to be leaky and confined.

The second case is similar to equation 4, which also includes a term for anisotropy.

The solution to equation 18 requires that the vertical flow components in the confined aquifer result only from partial penetration and not leakage (see assumption 1). Hantush (1967b, p. 587) determined that this requirement is met if

$$b(K'/Tb')^{\frac{1}{2}} < 0.1. \tag{19}$$

Equation 18 applies strictly to drawdown in piezometers. However, the equation can be used for observation wells if one of two criteria is met.

- The observation well is screened above or below the screened interval of the pumping well and the screened interval of the pumping well is not more than 20 percent of the saturated thickness.

- The screened intervals of the pumping and the observation wells overlap and the screened interval of the observation well is not more than 5 percent of the saturated thickness (Weeks, 1969, p. 198).

Hantush (1964a, p. 350) provided another model for use with observation wells that meet neither of the two criteria.

Application

Equation 18 is used to estimate aquifer properties near the pumping well if time-drawdown data are available from at least one nearby piezometer (or observation well).

The solution to equation 18 is simplified by recognizing that drawdown can be partitioned into two components. Expanding the right side of equation 18 yields

$$s = \frac{Q}{4\pi T} W (u, r/B) \pm \frac{Q}{4\pi T} f (u, hr/b, r/B, c/b, \ell/b, g/b) \tag{20}$$

The first term on the right side of the equality sign represents drawdown for full penetration as described by equation 11. The second term represents drawdown resulting from the vertical flow components of partial penetration. The second term is added or subtracted from the first term depending on the relation of the screened intervals of the pumping well and observation well as described on p. 41.

Because of the large number of variables in equations 18 and 20, it would be impractical to maintain type curves to accommodate all situations. Weeks (1969) provided tables that can be used to calculate values of the second term in equation 20. These values can be used to "correct" the observed values of drawdown for the effects of partial penetration. The corrected drawdown values can then be used to plot field-data curves that can be matched to the type curves in figure 12.

Alternatively, Reed (1980, p. 75-83) provided a Fortran program that calculates values of W(u, r/B) and incorporates the effects of partial penetration (the second term in equation 20). The variables needed for the program are g, b, c, ℓ, r, and K_z/K_r. Output from the program can be used to develop type curves for specific well configurations and aquifer characteristics. The match point determines values of s and t (or t/r^2) from the field-data plot and values of W(u, r/B) and 1/u from the type-curve plot. An interpolated value of r/B is obtained from the selection of the appropriate type curve. T, K_r, S, and K' are calculated from equations 11, 10, 9.5, and 16.

Example

In the two previous examples, drawdown of water levels was used in constructing field-data plots. However, the recovery of water levels in an observation well after pumping ceases in a nearby pumping well can also be used for field-data plots. A hydrograph of water levels during the recovery is theoretically an exact inverse to the drawdown hydrograph (fig. 16). In the graphing of recovery data, s becomes s_r, the difference between the extrapolated drawdown curve and the measured water level, and t becomes t_r, the time since pumping ceased (fig. 16). Q (recovery) is the same as Q (drawdown). However, unlike Q (drawdown), Q (recovery) is not subject to fluctuations, and the recovery curve is usually smoother than the drawdown curve. Other variables remain the same. By collecting both drawdown and recovery data, two independent estimates of aquifer properties can be obtained from one aquifer test.

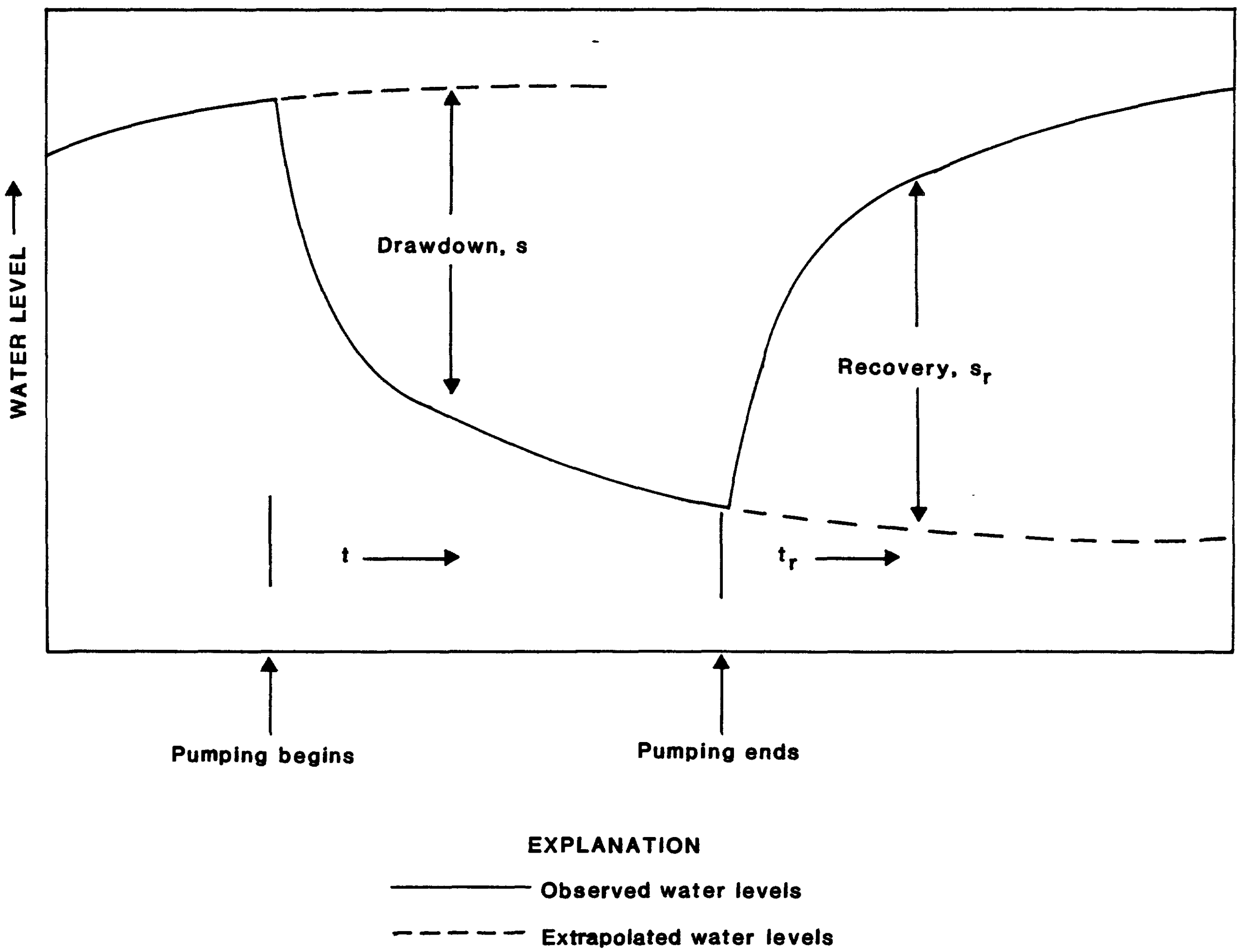


Figure 16.-- Drawdown after pumping begins and recovery after pumping ends in an observation well near a pumping well.

Well I29-3 in the Howe area (fig. 2) was used to irrigate about 160 acres in 1982. The well is 104 ft in depth, is 1 ft in diameter, and has a 20-ft-long screen. The rate of pumping during irrigation was 115,500 ft^3/d (0.86 Mgal/d). Lithologic information from 10 wells within a 1.2-mi radius of well I29-3 indicates that well I29-3 penetrates 31 ft of a sand and gravel aquifer averaging about 100 ft thick near the well (fig. 17). The aquifer is overlain by a clay layer ranging in thickness from 4 to 8 ft. Above the clay are two layers of sand and gravel separated by a layer of clay. The two clay layers are probably continuous within 0.25 mi of well I29-3. Based on this lithologic information, the author assumed that well I29-3 partially penetrates a leaky, confined aquifer.

Observation well R29-5 is 297.5 ft west of and at the same depth as well I29-3. The 2-in-diameter observation well was equipped with an automatic water-level recorder. Recovery data from observation well R29-5 were collected for 5 hours on June 27 and 28, 1982 (table 6), after 6 hours of pumping from well I29-3.

Table 6.--Recovery of water level in observation well R29-5, Howe area June 27 and 28, 1982

[t_r, time since recovery began, in minutes; s_r, recovery, in feet]

Time of record[1]	t_r	s_r
2336	0	0.00
2340	4	.59
2345	9	.72
2350	14	.74
2355	19	.74
0000	24	.74
0005	29	.75
0020	44	.77
0035	59	.78
0050	74	.79
0105	89	.80
0120	104	.81
0205	149	.83
0305	209	.85
0335	239	.87
0435	299	.89

[1]Military time

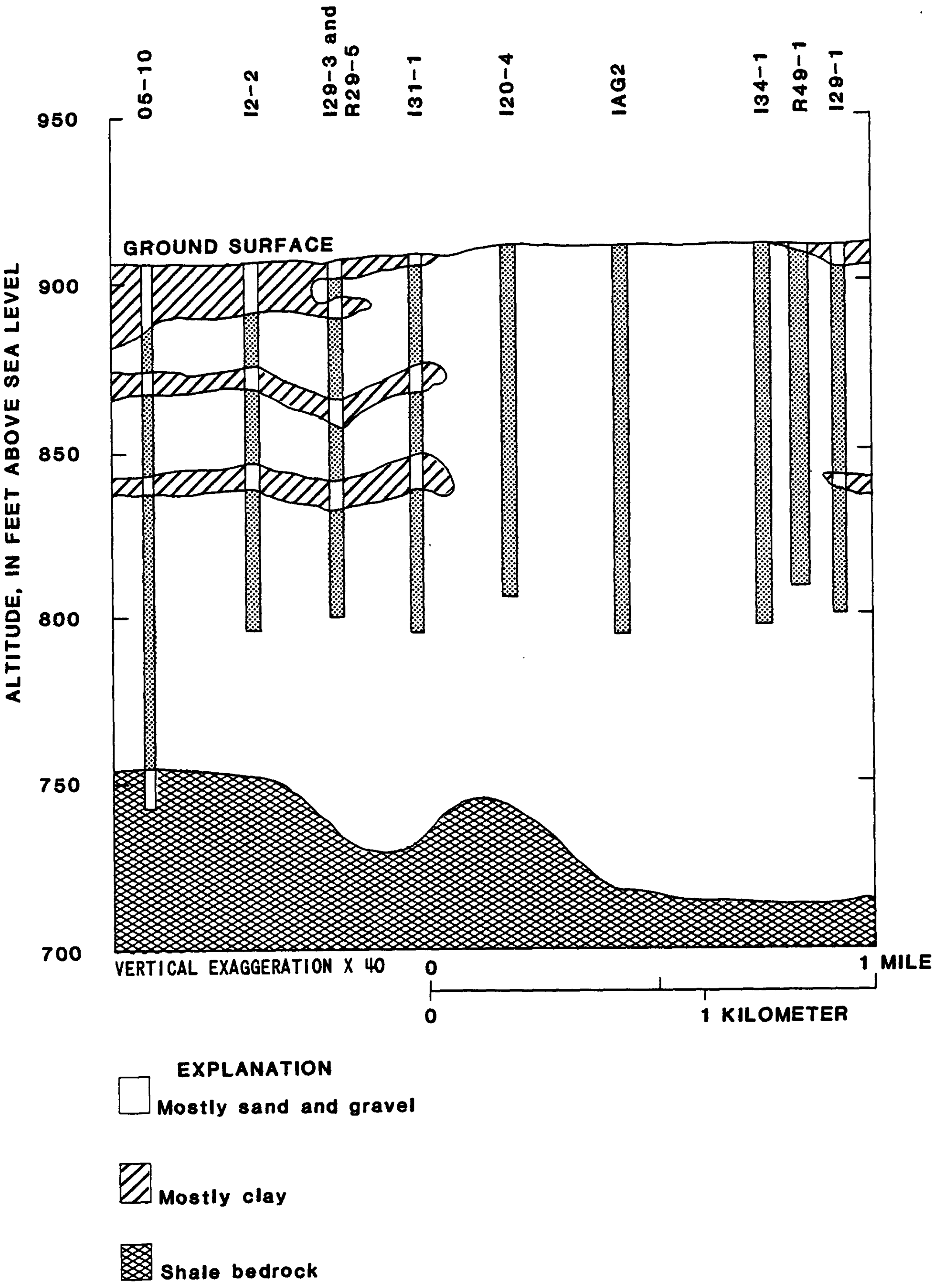


Figure 17.-- Geologic section near well I29-3, Howe area. (See fig. 2)

A family of type curves was constructed by using output from the Fortran program developed by Reed (1980, p. 75-83). Values of the variables used for the program are as follows:

$$b = 100 \text{ ft} \qquad r = 297.5 \text{ ft}$$
$$c = 15 \text{ ft} \qquad q = 32 \text{ ft}$$
$$\ell = 35 \text{ ft} \qquad K_z/K_r = 0.1$$

The value for K_z/K_r was calculated from another aquifer test in the Howe area. (See section "Model of Unconfined Aquifer and Partially Penetrating Well.") Type curves for values of r/B ranging from 0.01 to 2.5 were drawn (fig. 18).

Recovery data from table 6 were used for the field-data plot (fig. 19). The type curve corresponding to r/B = 0.1 was chosen as the one which best fit the field-data plot. The match-point values are as follows: W(u,r/B) = 1.0, 1/u = 10, s_r = 0.16 ft, and t_r = 0.56 min = 3.9×10^{-4} d. Other information needed for calculation is listed for convenience.

$$Q = 115{,}500 \text{ ft}^3/\text{d}$$
$$r = 297.5 \text{ ft}$$
$$b' = 100 \text{ ft}$$
$$b' = 4 \text{ ft}$$
$$B = r/0.1 = 2{,}975 \text{ ft}$$

By rearranging equations 11 and 7, there follows

$$T = \frac{(115{,}500 \text{ ft}^3/\text{d})\ (1.0)}{4\pi\ (0.16 \text{ ft})} = 57{,}445 \text{ ft}^2/\text{d, and}$$

$$S = \frac{4(57{,}445 \text{ ft}^2/\text{d})\ (3.9 \times 10^{-4}\text{d})\ (0.1)}{(297.5)^2} = 1.0 \times 10^{-4}.$$

And from equations 10 and 16,

$$K_r = T/b = \frac{57{,}445 \text{ ft}^2/\text{d}}{100 \text{ ft}} = 574 \text{ ft/d, and}$$

$$K' = \frac{(57{,}445 \text{ ft}^2/\text{d})\ (8 \text{ ft})}{(2{,}975 \text{ ft})^2} = 0.052 \text{ ft/d.}$$

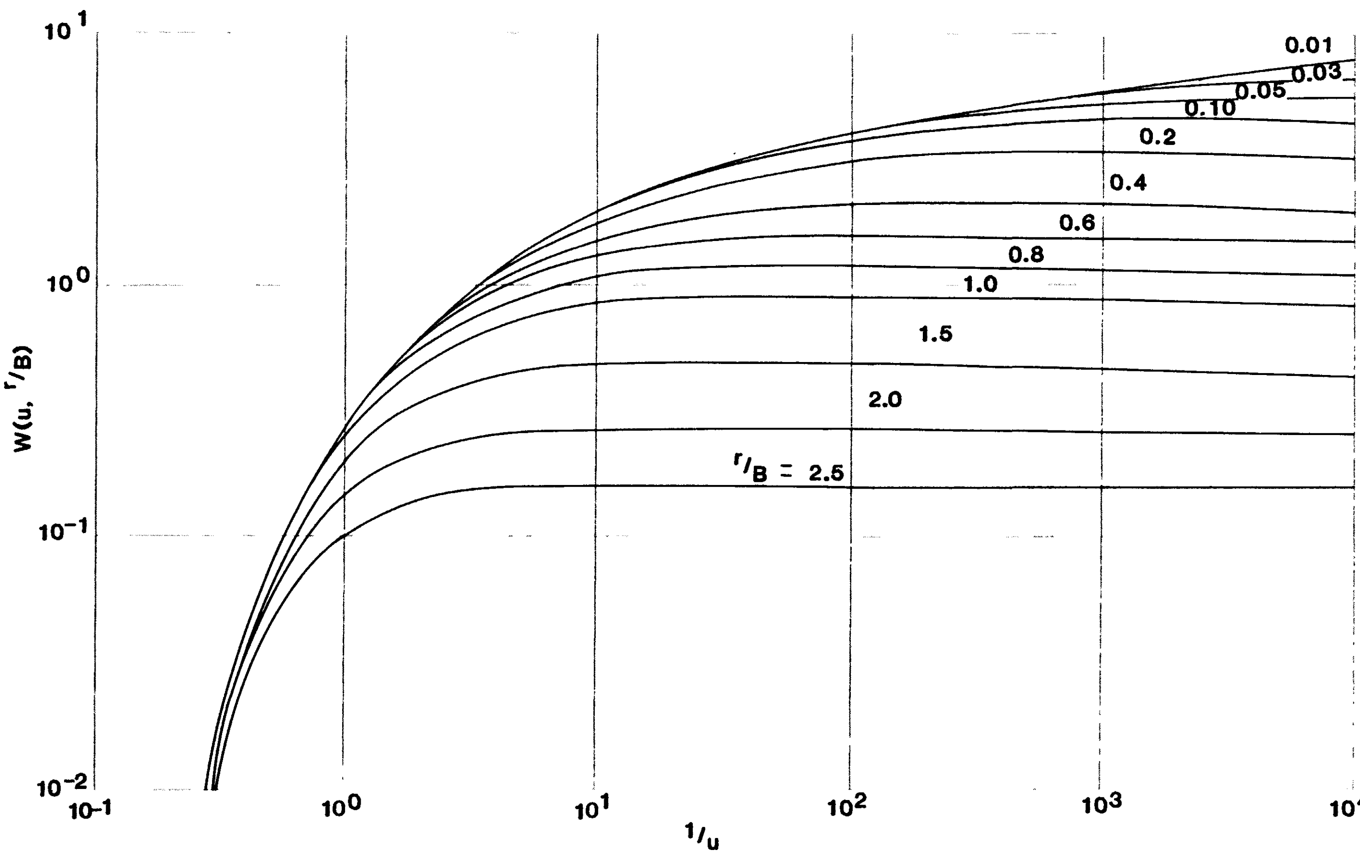


Figure 18.-- Type curve for a leaky, confined aquifer and a partially penetrating well for K_z/K_r : 0.1.

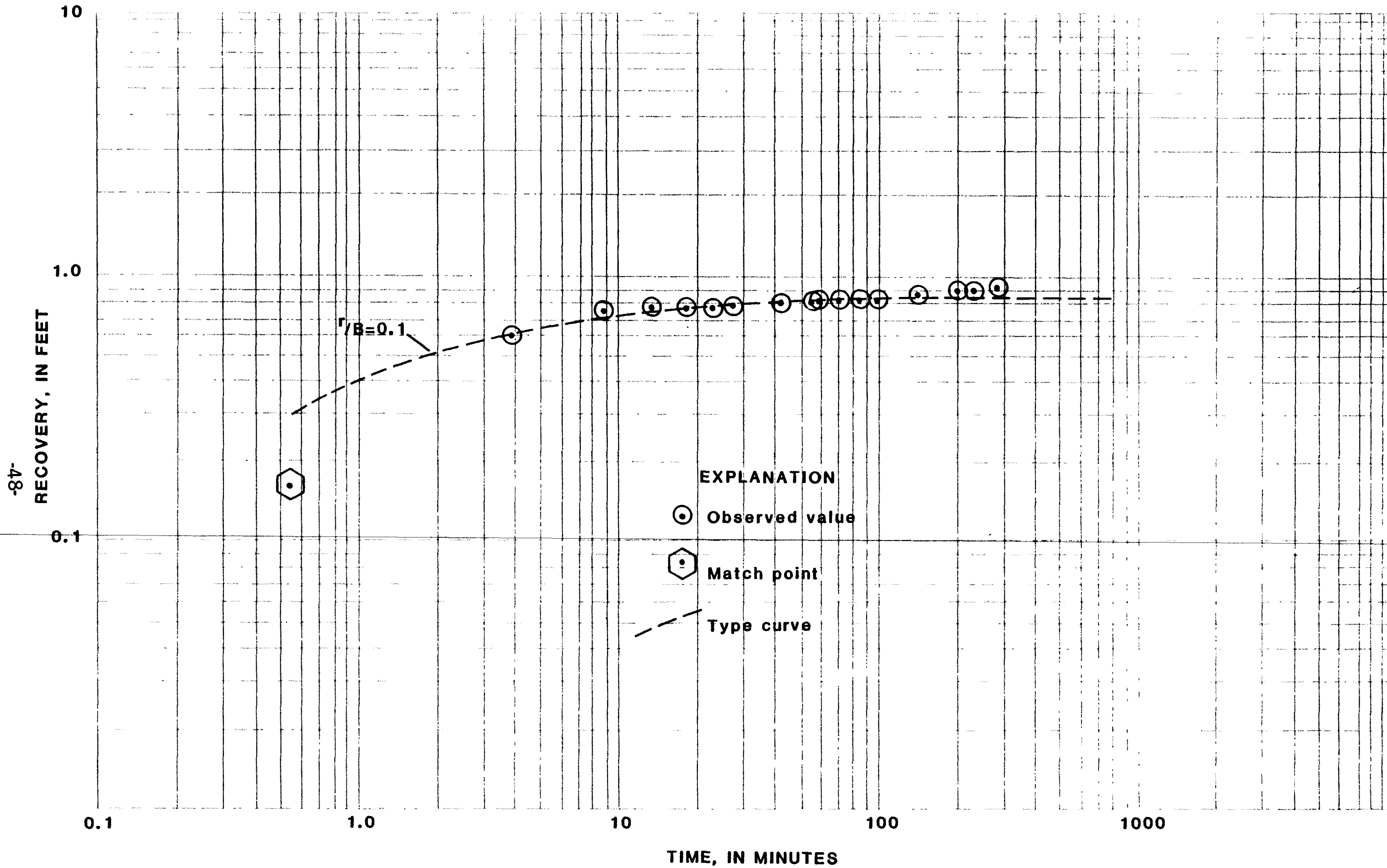


Figure 19.– Relation of recovery to time for observation well I29-5, Howe area, June 27 and 28, 1982, and type curve for a leaky, confined aquifer and partially penetrating well.

Discussion

In this section, the results are more easily discussed in terms of drawdown. However, the reader is reminded that analysis was done by using recovery data.

The transmissivity, 57,445 ft/d, indicates that well I29-3 penetrates an aquifer capable of producing significant quantities of water (Freeze and Cherry 1979, p. 60). The high productivity results from the thickness and the hydraulic conductivity of the aquifer. The value of K_r, 574 ft/d, is near the upper limit reported for sand and gravel (table 3). The high transmissivity is undoubtedly the major reason for the small drawdown (less than 1 ft) in observation well R29-5.

The vertical hydraulic conductivity of the confining bed, K', is within the range of values calculated from drawdown near well I101-1 (table 5) and indicates a higher-than-average permeability for clay (table 3). The relatively high permeability and the fact that the bed is only 4 ft thick result in a high leakage that partially explains why the drawdown curve flattens out so quickly (fig. 19).

At about t=100 minutes (fig. 19), the field-data points begin to deviate upward from the type curve indicating that drawdown was greater than expected. Three factors that could have caused this deviation are (1) a dewatering of the upper source bed with consequent decrease in hydraulic gradient across the confining bed, (2) a decrease in transmissivity encountered as the cone of depression spreads farther from the pumping well, and (3) an increase in the rate of pumping. Existing information is not sufficient to suggest which of these factors might have caused the deviation.

Model of Unconfined Aquifer and Partially Penetrating Well

The three analytical models described thus far involved flow in confined aquifers where pumping induces a hydraulic gradient toward the well by creating drawdown in the potentiometric surface (Freeze and Cherry, 1979, p. 324). Water is released from storage by contraction of the aquifer material and by expansion of water. The aquifer is not dewatered.

Pumping from a well that penetrates an unconfined aquifer results in drawdown in the water table and dewatering of the aquifer material within the cone of depression (fig. 20). Where vertical recharge is negligible, the water table represents a flow line. During pumping, the depression in the water table introduces a vertical component of flow. If the pumping well only partially penetrates the aquifer, additional vertical components of flow are introduced as described in the section "Model of Leaky, Confined Aquifer and Partially Penetrating Well."

In an unconfined aquifer, the drawdown near a pumping well has three recognizable phases. These phases are caused by changes in the way that water is released from storage. Phase 1 begins when pumping begins (fig. 21) and represents water released from storage owing to compaction of the aquifer and expansion of entrapped air (Todd, 1980, p. 134-135). Storage coefficients calculated from this part of the drawdown curve are very similar to those of confined aquifers. Phase 2 begins as dewatering becomes significant, and gravity drainage of the sediments "rains" down on the cone of depression. Gravity drainage, also called delayed yield, forms a flat drawdown pattern similar to later stages of drawdown in a leaky confined aquifer (see fig. 12). Phase 3 approaches an equilibrium between gravity drainage and the rate of drawdown. The shape of the drawdown curve approaches that of the Theis curve. Storage calculated from this part of the curve is specific yield and represents water released primarily from pore spaces.

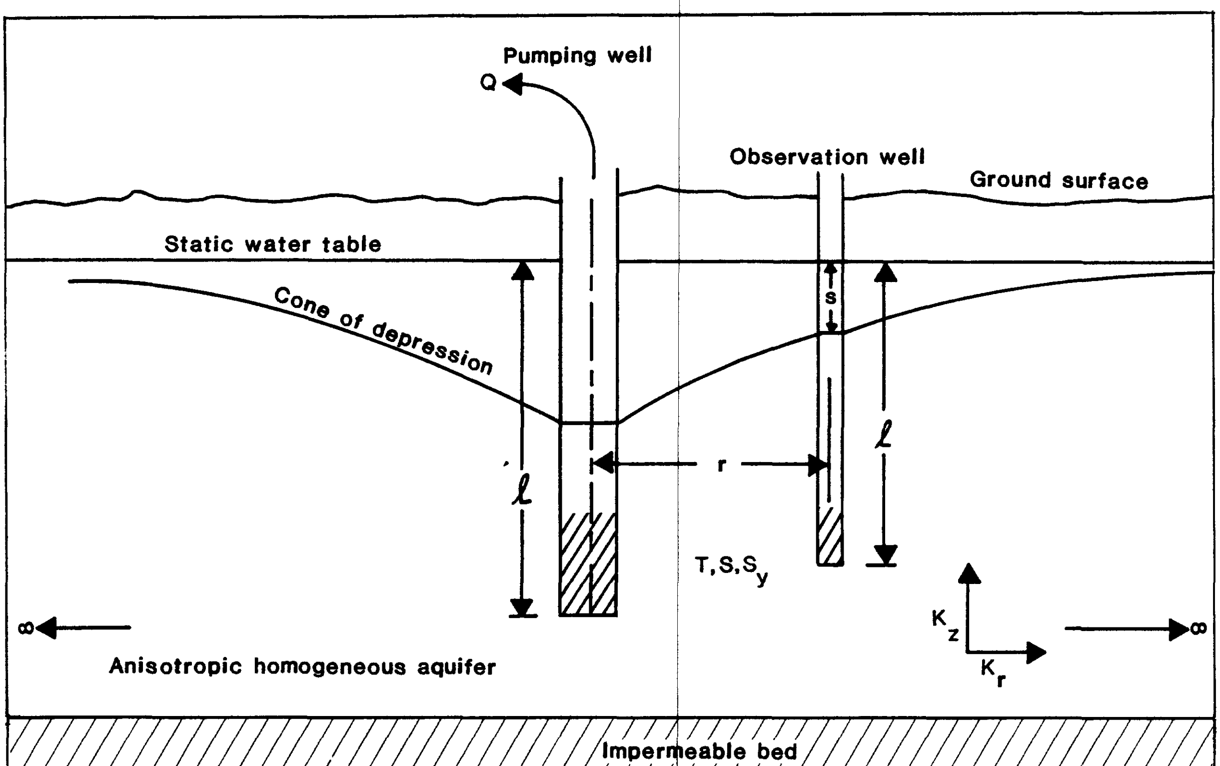


Figure 20.- Section through a pumping well that partially penetrates an unconfined aquifer

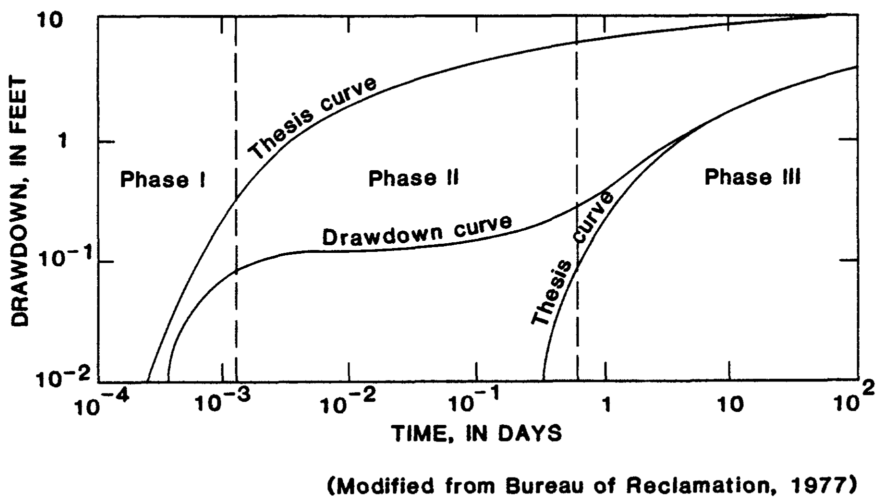


Figure 21.– Relation of drawdown to time near a pumping well that penetrates an unconfined aquifer.

Historically, drawdown in unconfined aquifers was first analyzed by using models designed for confined aquifers. However, this approach was limited by the requirement that the pumping period be long enough for drawdown to reach phase 3, which might be several days. A model developed by Boulton (1963) was the first to reproduce all three segments of the drawdown curve. More recent work by Neuman (1975) improved on Boulton's model by recognizing the vertical components of flow according to Freeze and Cherry (1979, p. 326).

Streltsova (1974) provided a simplified expression for early time-drawdown distribution in an unconfined aquifer tapped by a partially penetrating well. She combined the early time distribution and a late time distribution developed by Neuman (1974).

Conceptualization

The equation that describes drawdown near a pumping well that partially penetrates an unconfined aquifer is presented in the following form by Streltsova (1974):

$$s = \frac{Q}{4\pi T\ 1/b} W(u_A,\ u_B,\ \beta,\ \ell/_b,\ \ell'/_b) \tag{21}$$

where

$W(u_A, u_B, \beta, \ell/_b, \ell'/_b)$ is the well function for an unconfined aquifer and partially penetrating well, dimensionless;

$$\beta = (r/b)\ (K_z/K_r)^{\frac{1}{2}}, \text{ dimensionless;} \tag{22}$$

$u_A = r^2 S/4Tt$ used for small values of time, dimensionless; (23)

$u_B = r^2 S_y/4Tt$ used for large values of time, dimensionless; (24)

S_y is specific yield, dimensionless;

ℓ is the distance from the static water table to the bottom of the well screen of the pumping well, in feet; and

ℓ' is the distance from the static water table to the bottom of the screen of the observation well, in feet (fig. 20).

Assumptions for the Neuman-Streltsova model include those listed as 1 and 2 on p. 20, that listed as 2 on p. 41, and the following three assumptions:

1. The aquifer is bounded above by a water table at atmospheric pressure and below by a horizontal impermeable bed.
2. The transmissivity of the aquifer is constant in time and space.
3. The aquifer is infinite in areal extent.

The Neuman-Streltsova model can, of course, be used for fully penetrating wells. However, Neuman (1975) developed a simpler model to use if the effects of partial penetration are negligible. Equation 4 provides a criterion for testing whether the effects of partial penetration can be neglected based on distance from the pumping well. Using data from unconfined aquifers, Neuman (1974, p. 309) demonstrated that the effects of partial penetration decrease with increasing t. When

$$t >= 10\ S_y r^2/T, \tag{25}$$

the effects of partial penetration are negligible for

$$r > b(K_r/K_z)^{\frac{1}{2}}. \tag{26}$$

If drawdown in the aquifer is more than 10 percent of the original saturated thickness, changes in transmissivity become large enough to violate assumption 2. Jacob (1963) showed that drawdown in an unconfined aquifer can be corrected for the effects of decreasing transmissivity by the equation

$$s' = s - s^2/2b \tag{26.5}$$

where s is observed drawdown, in feet; and
s' is drawdown in an equivalent confined aquifer, in feet.

Assumption 1 specifies that the underlying impermeable bed is horizontal. Sloping aquifers can be analyzed by using methods described by Hantush (1964a, p. 420).

The Theis model can be applied to unconfined aquifers if (1) the effect of partial penetration is assumed to be negligible, and (2) the release of water from storage is assumed to be instantaneous. The second assumption is valid during early and late parts of the pumping period. During the early part of the period (seconds to perhaps several minutes after pumping begins), water is released from storage instantaneously as water level declines (Streltsova, 1974). However, reliable drawdown data during this part of the aquifer test are difficult to collect. Also, Theis analysis of early-time data provides estimates of transmissivity and storage coefficient but not of specific yield or anisotropy.

After a sufficiently long pumping period, the effects of delayed yield become negligible as the rate of gravity drainage approaches the rate of drawdown. Weeks (1969) suggested the following criteria for estimating the time at which delayed yield becomes negligible:

$$t = b\ S_y/K_z,\ \text{if}\ \beta < 0.4 \tag{27}$$

and

$$t = b\ S_y/K_z\ (0.5 + 1.25\ \beta),\ \text{if}\ \beta > 0.4 \tag{28}$$

[Note: Equation 28 was published incorrectly in the original text (Edwin P. Weeks, U.S. Geological Survey, oral commun., 1982).]

Application

The type curves used in the Neuman model are constructed by plotting values of the well function (eq. 21) on the vertical axis and values of $1/u_A$ and $1/u_B$ on the horizontal axis for a range of values of β^2 and specified values of $\ell/b = 0.4$ and $\ell'/b = 0.4$. Streltsova (1974) offers eight tables with several combinations of ℓ/b and ℓ'/b that can be used to construct type curves. She also offers a computer program for generating additional tables.

Each curve is actually a composite of two curves joined asymptotically (Neuman, 1975, p. 330). Curves on the left side of figure 22 are called A curves, and those to the right are called B curves. The A curves are used to analyze early drawdown data and to estimate S. The B curves are used to analyze later drawdown data and to estimate S_y. The two curves approach a horizontal asymptote and have two scales--one for $1/u_A$ and one for $1/u_B$.

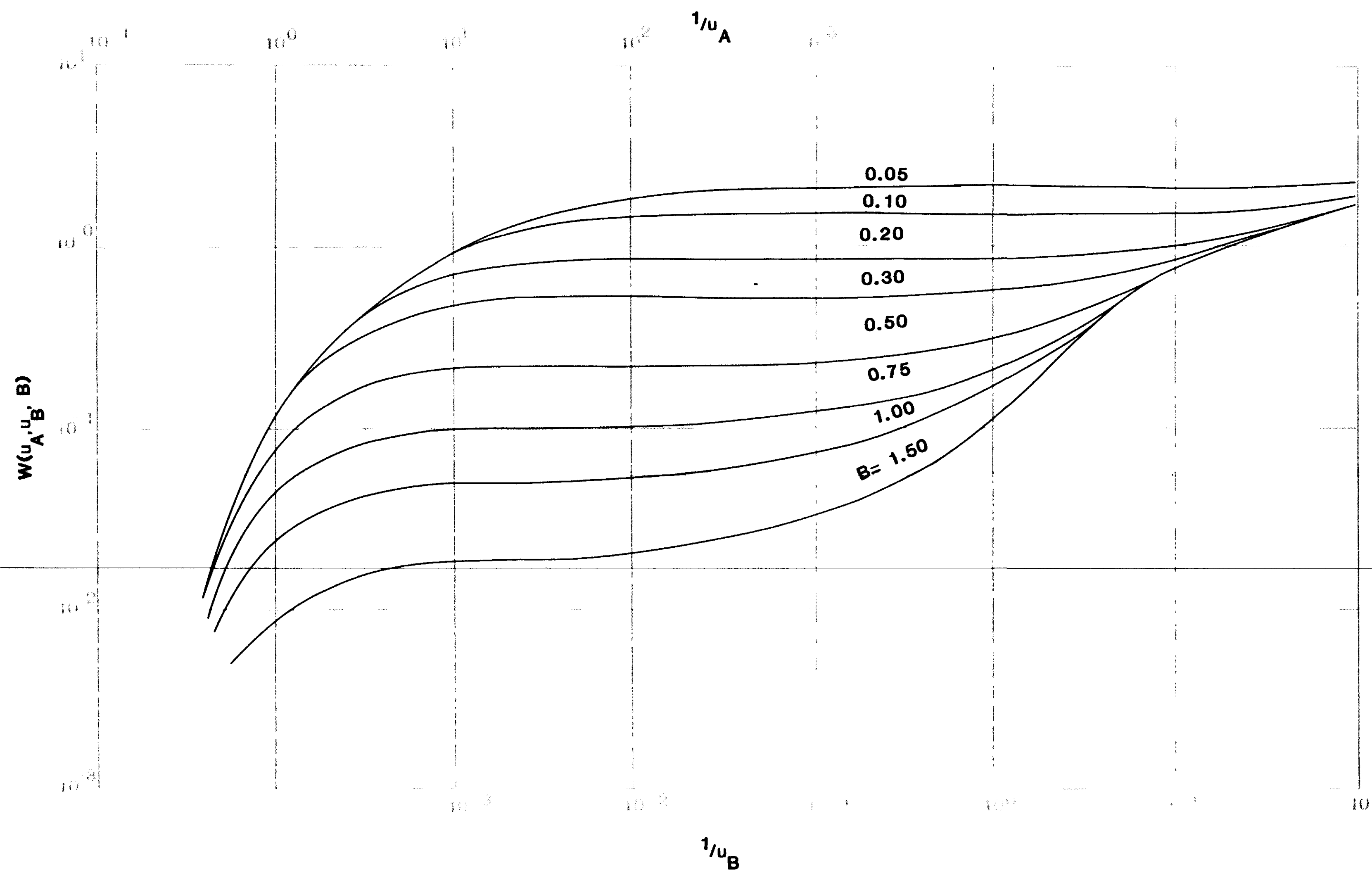


Figure 22.– Type curves for an unconfined aquifer and partially penetrating well.

The following four-step procedure for curve matching was outlined by Prickett (1965, p. 7-9).

1. Plot the field data, s versus t(or t/r^2) on log-log paper.

2. Superimpose the field-data plot on the B type curves keeping axes parallel and match as much of the latest field data to a particular type curve as possible. Choose a convenient match point and note coordinate values for $W(u_B, \beta, \ell/b, \ell'/b)$, $1/u_B$, and s_B, t_B and also the value for β.

3. Superimpose the field-data plot on the A type curves. Keep axes parallel and match as much of the earliest field data to the same type curve (same value of β) used in (2) as possible. Choose a second match point and note coordinate values for $W(u_A, \beta, \ell/b, \ell'/b)$, $1/u_A$, and s_A, t_A.

Thus, in the curve-matching process, the field-data curve is moved only horizontally over the type curve.

4. By rearranging equations 21 and 24, one can calculate transmissivity and specific yield as follows:

$$T = \frac{Q}{4\pi s_B 1/b} W(u_B, \beta, \ell/b, \ell'/b) \tag{29}$$

$$S_y = \frac{4Tt_B u_B}{r^2} . \tag{30}$$

By rearranging equation 23, the storage coefficient is calculated as follows:

$$S = \frac{4Tt_A u_A}{r^2} . \tag{31}$$

An estimate of anisotropy is obtained by rearranging equation 22 as follows:

$$K_z/K_r = \frac{\beta^2 b^2}{r^2} . \tag{32}$$

Example

Well I22-1 in the Howe area (fig. 2) was used to irrigate about 345 acres in 1982. The well is 145 ft in depth and the casing is 14 in. in diameter. The well is gravel packed and has a 31-ft-long screen with a nominal diameter of 12 in. The rate of pumping during irrigation was 281,435 ft^3/d (2.11 Mgal/d). Lithologic information from 11 wells within a 1-mi radius of well I22-1 indicates that well I22-1 produces from a layer of sand and gravel whose average saturated thickness is 165 ft (fig. 23). Several thin, discontinuous clay and silt lenses are present within the sand and gravel but probably do not act as confining beds near well I22-1. The sand and gravel is underlain by a layer of clay on the upper surface of impermeable shale bedrock (Bailey and others, 1985). Based on this information, the author assumed that well I22-1 partially penetrates an unconfined aquifer.

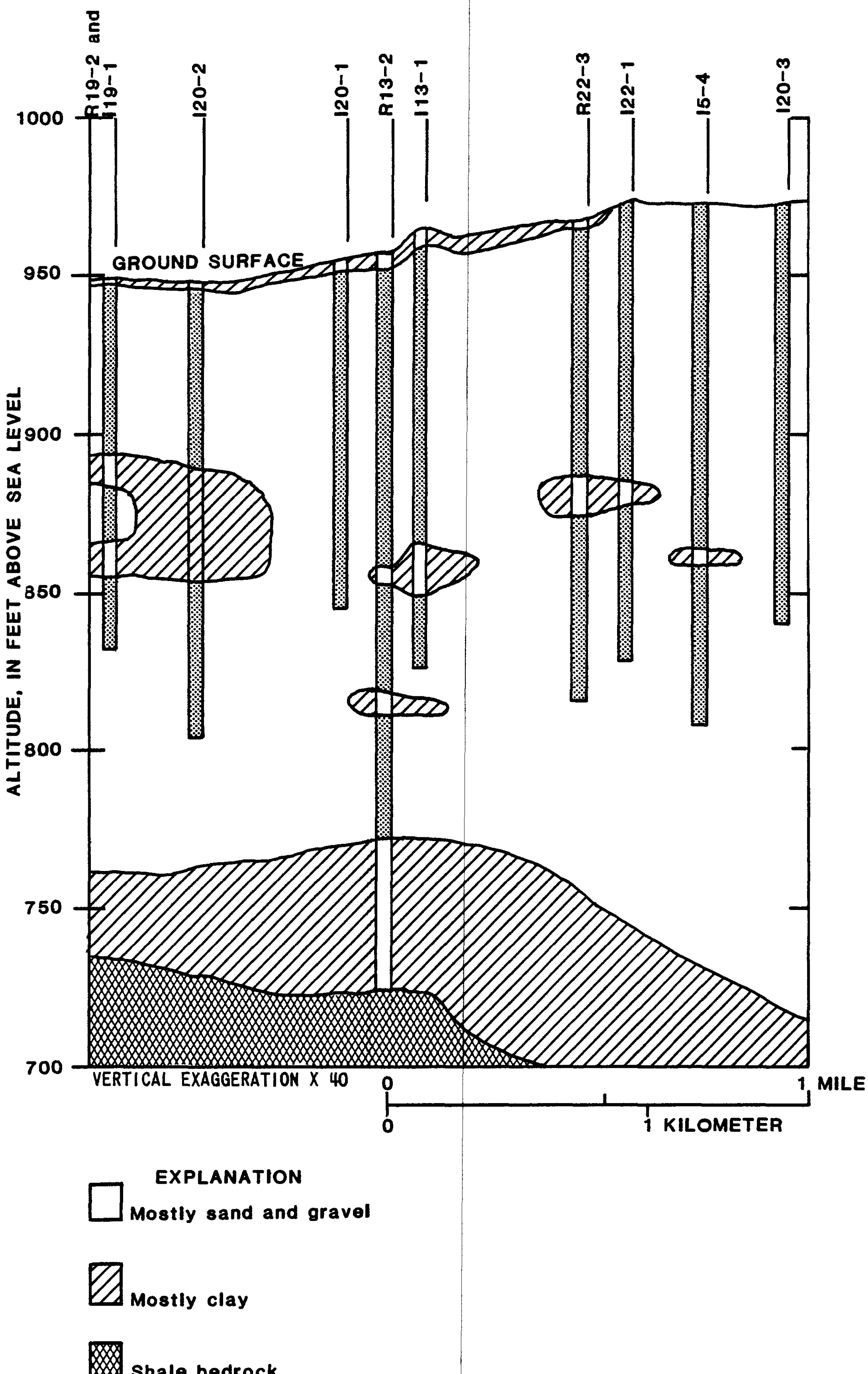


Figure 23.-- Geologic section near well I22-1, Howe area. (See fig. 2)

Well R22-3 is a 4-in-diameter observation well 145.5 ft west of well I22-1. The observation well is cased to the same depth as well I22-1 and was equipped with an automatic water-level recorder.

The type curves that were used to analyze the field data from well R22-3 were constructed by first determining values for ℓ/b and ℓ'/b. The static water level measured during the summer of 1982 was 57 ft below land surface. ℓ and ℓ' were both calculated to be

$$145 \text{ ft} - 57 \text{ ft} = 88 \text{ ft}.$$

Thus,

$$\ell/b = \ell'/b = 88 \text{ ft}/165 \text{ ft} = 0.5$$

Values of the well function for u_A and u_B, for values of β ranging from 1.5 to 0.05 and for $\ell/b = \ell'/b = 0.5$ were obtained from Streltsova (1974, tables 4 and 5). These values were used to plot the type curves in figure 22.

Water levels in well R22-3 from three separate pumping periods were used for analysis. On June 28, 1982, an observer measured drawdown in well R22-3 at short intervals during the first few minutes of pumping well I22-1 (table 7). The pump was then shut off, and after the water level recovered to near static level, drawdown was recorded during a second pumping period. On July 27 and 28, 1982, during 30 hours of pumping well I22-1, water levels in well R22-3 were measured using the automatic water-level recorder. The static water level for the three pumping periods was 57 ft. Plotted together, the drawdowns from the three pumping periods provided a continuous field-data plot (fig. 24) that was used for curve matching.

The type curve corresponding to β=0.2 was selected as the one that most accurately matched the field data. Values of parameters determined from the match points for early and late data are as follows:

Early-time data, $s_A = 0.89$ ft, $t_A/r^2 = 2.2 \times 10^{-6}$ min/ft^2 $= 1.5 \times 10^{-9}$ d/ft^2 $W(u_A, \beta, \ell/b, \ell'/b) = 1.0$, and $u_A = 1.0$.

Late-time data, $s_B = 0.89$ ft, $t_z/r^2 = 1.4 \times 10^{-3}$ min/ft^2 $= 9.7 \times 10^{-7}$ d/ft^2 $W(u_B, \beta, \ell/b, \ell'/b) = 1.0$, and $u_B = 1.0$.

Using equations 29-32,

$$T = \frac{(281{,}435 \text{ ft}^3/\text{d})(1.0)}{4\pi\ (0.89 \text{ ft})\ (0.4)} = 62{,}910 \text{ ft}^2/\text{d},$$

$$S_y = 4(62{,}910 \text{ ft}^2/\text{d})(9.7 \times 10^{-7} \text{ d/ft}^2)(1.0) = 0.24,$$

$$S = 4(62{,}910 \text{ ft}^2/\text{d})(1.5 \times 10^{-9} \text{ d/ft}^2)(1.0) = 3.8 \times 10^{-4}, \text{ and}$$

$$K_z/K_r = \left[\frac{(0.2)(225)}{145.4}\right]^2 = 0.1.$$

Also, from equation 10,

$$K_r = \frac{62,910 \text{ ft}^2/\text{d}}{165 \text{ ft}} = 381 \text{ ft/d, and}$$

$$K_z = 0.1\ (K_r) = 38 \text{ ft/d.}$$

Table 7.--Drawdown of water level in observation well R22-3, Howe area, June 28 and July 27-28, 1982

[t, time since pumping began, in minutes; s, drawdown, in feet; r, distance between pumping and observation wells, in feet]

June 28 Test 1			June 28 Test 2			July 27-28		
t	t/r^2	s	t	t/r^2	s	t	t/r^2	s
0.250	1.18×10^{-5}	0.52	0.083	3.94×10^{-6}	0.21	5.00	2.36×10^{-4}	0.67
.500	2.36×10^{-5}	.60	.166	7.87×10^{-6}	.39	10.0	4.72×10^{-4}	.67
1.75	8.27×10^{-5}	.67	.250	1.18×10^{-5}	.46	15.0	7.09×10^{-4}	.68
2.00	9.45×10^{-5}	.67	.333	1.57×10^{-5}	.54	20.0	9.45×10^{-4}	.70
2.50	1.18×10^{-4}	.67	.416	1.96×10^{-5}	.58	30.0	1.42×10^{-3}	.71
3.50	1.65×10^{-4}	.67	.500	2.36×10^{-5}	.60	40.0	1.89×10^{-3}	.73
5.50	2.60×10^{-4}	.64	.583	2.75×10^{-5}	.63	60.0	2.83×10^{-3}	.76
6.00	2.83×10^{-4}	.64	.667	3.15×10^{-5}	.64	90.0	4.25×10^{-3}	.81
7.50	3.54×10^{-4}	.64	1.00	4.72×10^{-5}	.67	120	5.67×10^{-3}	.85
8.50	4.02×10^{-4}	.66	2.16	1.02×10^{-4}	.65	150	7.09×10^{-3}	.88
10.0	4.72×10^{-4}	.66	2.75	1.30×10^{-4}	.65	225	1.06×10^{-2}	.92
12.0	5.67×10^{-4}	.67	4.41	2.08×10^{-4}	.65	300	1.42×10^{-2}	.96
13.6	6.46×10^{-4}	.67	12.0	5.67×10^{-4}	.67	375	1.77×10^{-2}	.99
			25.0	1.18×10^{-3}	.69	450	2.13×10^{-2}	1.01
			33.0	1.56×10^{-3}	.70	600	2.83×10^{-2}	1.04
			41.0	1.94×10^{-3}	.73	750	3.54×10^{-2}	1.09
						900	4.25×10^{-2}	1.10
						1200	5.67×10^{-2}	1.13
						1500	7.09×10^{-2}	1.19
						1800	8.50×10^{-2}	1.26

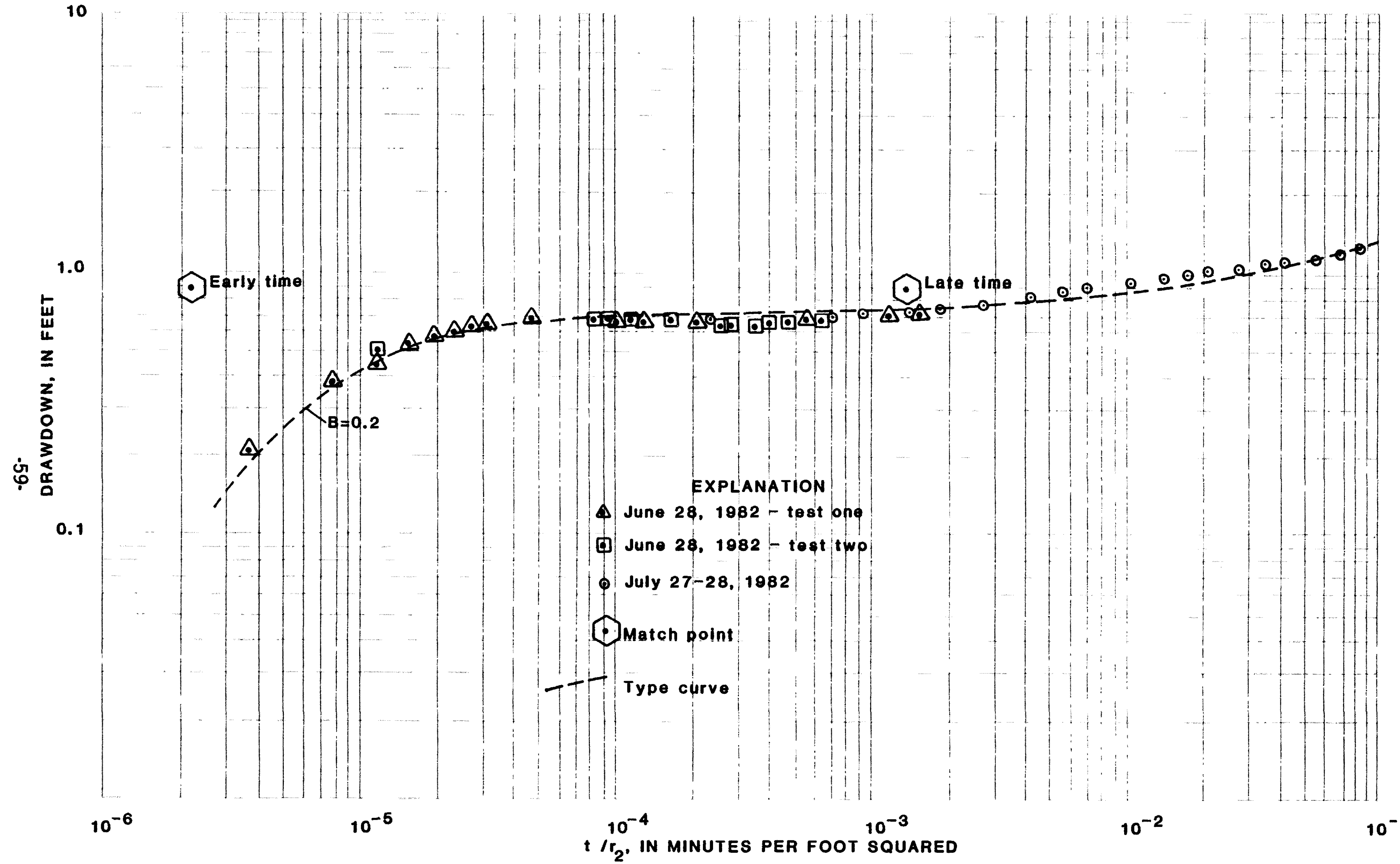


Figure 24.-- The relation of drawdown and time for observation well R22-3, Howe area, June 28 and July 27 and 28, 1982, and type curves for an unconfined aquifer and partially penetrating well.

Discussion

The transmissivity value of nearly 63,000 ft^2/d indicates an aquifer capable of producing significant quantities of water (Freeze and Cherry, 1979, p. 60). The high value results both from the permeable outwash and from the large saturated thickness of the aquifer. High transmissivity explains the minimal drawdown observed in well R22-3--only 1.3 ft after 30 hours of pumping.

The storage coefficient is an order of magnitude greater than the range of 1.5×10^{-5} to 3.1×10^{-5} for sand and gravel (Jumikis, 1962, as reported by Walton, 1970, p. 627). Part of the explanation for the high storage coefficient is the depth of the aquifer. Specific storage, S_s, can be calculated from

$$S_s = S/b = 3.8 \times 10^{-4}/165 \text{ ft} = 2.3 \times 10^{-6}$$

This value of specific storage compares favorably with the "average" value, 10^{-6}, suggested by Lohman (1972, p. 53). The specific yield, 0.24, falls midway between 0.21 and 0.27, the range of average specific yield measured for various sand and gravel samples (Johnson, 1967, p. D1).

The ratio K_z/K_r is a measure of anisotropy. A ratio of 0.1 indicates that the aquifer sediments are 10 times as permeable in the radial (horizontal) direction as in the vertical direction. Anisotropy in water-deposited materials results from two factors. First, individual particles are seldom spherical and tend to be deposited with their flat side down. Second, deposition tends to be in horizontal layers composed of dissimilar materials. Values of K_z/K_r generally range from 0.1 to 0.5 (Morris and Johnson, 1967). Figure 22 illustrates that the lower the value of β, and, hence, the lower the value of K_z/K_r, the more the drawdown pattern resembles that of a leaky, confined aquifer (fig. 18).

The Neuman-Streltsova model (equation 21) is applicable to partially penetrating wells. One can use equation 4 to determine the distance beyond which effects of partial penetration can be neglected. For the previous example,

$$r = 1.5(165 \text{ ft})(10)^{\frac{1}{2}} = 782 \text{ ft}.$$

Within about 800 ft of the pumping well, the effects of partial penetration cannot be neglected. Calculations made with equations 25 and 26 indicate that even after

$$\frac{10\ (0.24)\ (145.5 \text{ ft})^2}{62{,}900 \text{ ft}^2/\text{d}} = 0.81 \text{ d},$$

the effects of partial penetration may be important within

$$r = (165 \text{ ft})\ (10)^{\frac{1}{2}} = 522 \text{ ft}$$

of the pumping well.

Because of the high permeability and the high storage capacity of the outwash aquifer, drawdown was small compared to the saturated thickness. From equation 26.5, the largest correction to drawdown for dewatering would be

$$s^2/2b = \frac{(1.3\ ft)^2}{2(165\ ft)} = 0.005\ ft$$

which was ignored for the analysis.

One can determine whether the effects of gravity drainage can be neglected by using equation 27. If β = 0.2, K_z = 38 ft/d, S_y = 0.24, and b = 165 ft, then,

$$t = \frac{(165\ ft)\ (0.24)}{38\ ft/d} = 1.04\ d\ \text{(about 1,500 minutes)}.$$

Because the actual pumping was 1,800 minutes, the effects of gravity drainage could not be neglected during the latter part of pumping.

Model for Estimating Transmissivity from Specific Capacity

Aquifer tests are generally the most accurate method of calibrating analytical models. However, the tests can be time consuming and expensive. Observation wells may not be available and may be too expensive to install. Thus, the aquifer test is not always practical.

Specific-capacity data from pumping wells can be used to estimate transmissivity in aquifers through a modification of the Theis equation. Specific capacity is the rate of pumping divided by drawdown, Q/s, which is usually available on driller's logs. By using this source of information, the hydrologist can quickly estimate transmissivity at many locations and for many types of aquifers. The method is simpler than analysis of aquifer-test data and does not require field-data plots or type curves. Though subject to many sources of error, specific-capacity data are useful in estimating properties of aquifers if other information is not available.

Conceptualization

The use of specific capacity to estimate transmissivity is based on the Theis equation (eq. 1) and on drawdown measured in the pumping well. If the point of observation is in the pumping well itself, r and u become very small. In this case, equation 1 can be simplified to

$$s = \frac{Q}{4\pi T}\left[\ln\left(\frac{4Tt}{r^2 S}\right)-0.577\right] \qquad (33)$$

(Ferris and others, 1962, p. 99).

In terms of specific capacity, the equation can be written

$$Q/s = \frac{4\pi T}{\ln\ (4Tt/r^2 S)-0.577} \tag{34}$$

Equation 34 can be used to estimate T for a confined aquifer (Brown, 1963) or an unconfined aquifer (Theis, 1963). Equation 34 is based on the following assumptions:

1. Well loss is negligible.
2. The effective well radius is equal to the nominal well radius and is unaffected by drilling and developing of the well.
3. The aquifer is homogeneous, isotropic, and infinite in areal extent.
4. If the aquifer is confined, the confining beds are nonleaky.
5. Water in the aquifer matrix is released instantaneously from storage.

Application

Transmissivity can be estimated from specific capacity of a well by using a three-step procedure.

Step 1.--Observed drawdown in the pumping well is adjusted for effects of partial penetration and (or) dewatering, if necessary.

The equation used to adjust observed drawdown in a pumping well for the effects of partial penetration was derived by Kozeny (1933) and is presented by Butler (1957, p. 160).

$$s_p = s_o \propto \left\{1 + 7 \left[\frac{r_w}{2\propto b}(K_z/K_r)^{\frac{1}{2}}\right]^{\frac{1}{2}} \cos(\propto\pi/2)\right\} \tag{35}$$

where

s_p is drawdown adjusted for partial penetration, in feet;
s_o is observed drawdown, in feet;
$\propto$ is screened interval divided by b, dimensionless;
r_w is well radius, in feet; and
$\pi/2$ = 90 degrees.

Drawdown can be corrected for dewatering in an unconfined aquifer by using equation 26.5. If adjustments for partial penetration and dewatering both are needed, the larger correction should be applied first.

Step 2.--For confined aquifers, the parameter T' is calculated by using the following equation (Brown, 1963):

$$T' = \frac{2.3Q}{4\pi s_a}\left[M - \log_{10}(5 \times 10^3\ S) + \log_{10} t\right] \qquad (36)$$

where T' is uncorrected transmissivity, in feet squared per day;

$$M = -0.25 - \log_{10}(3.74 \times 10^{-9} r^2), \text{ dimensionless; and} \qquad (37)$$

s_a is drawdown adjusted for partial penetration and (or) dewatering, in feet.

The equations have been adjusted to units used in this report. Values of Q, t, and r needed for solving equations 36 and 37 are usually available from driller's logs. s_a is the adjusted drawdown from step 1. If the storage coefficient is not available from other data, several options for estimation are available (see "Discussion," p. 65). Analogous formulas for an unconfined aquifer are presented by Theis (1963).

Step 3.--T is determined from values of T' and Q/s in figure 25.

Example

Specific-capacity data for well I101-1 are used to illustrate the calculations. From information presented on p. 31,

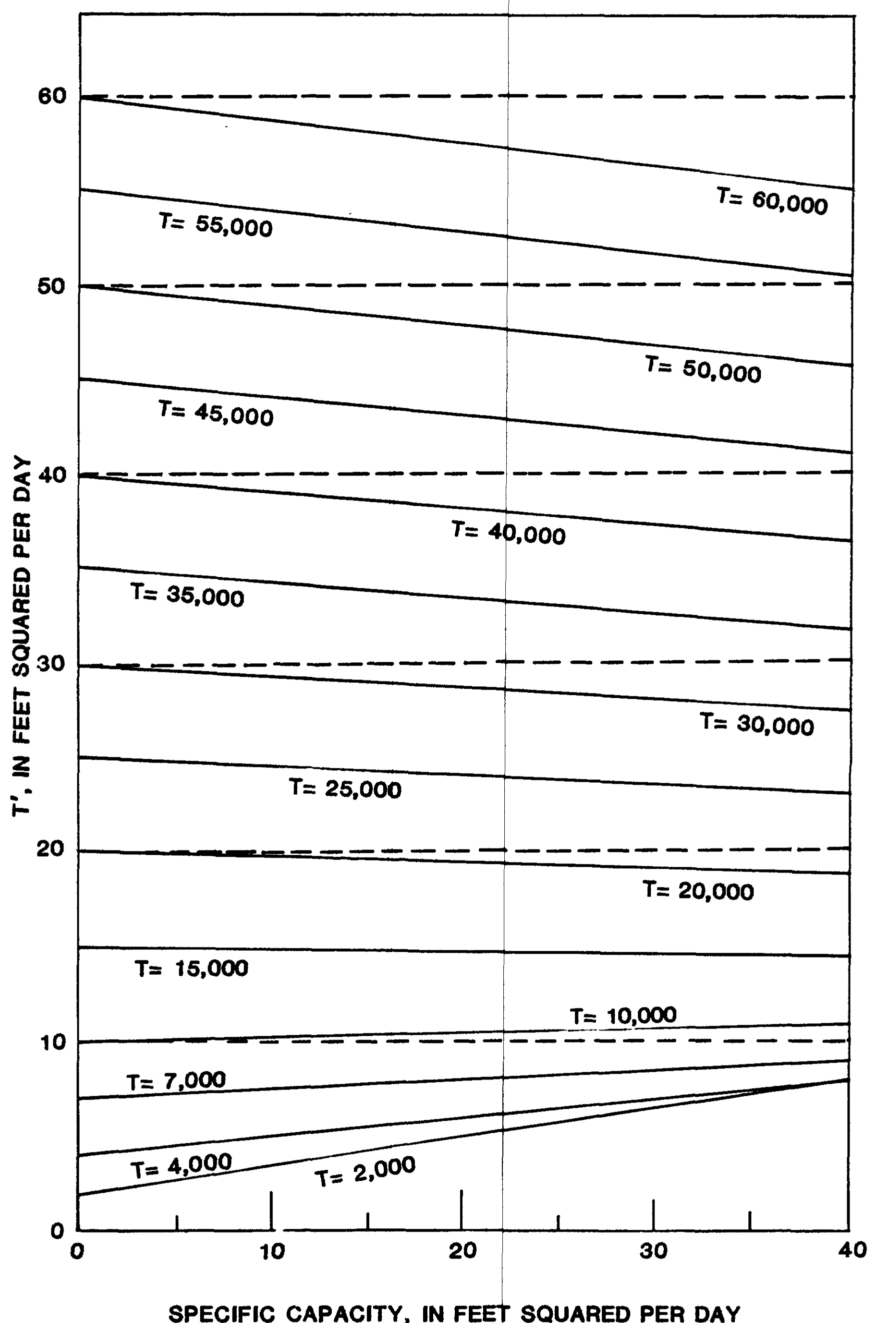
$$r_w = 0.5 \text{ ft},\ b = 31 \text{ ft, and } \alpha = \frac{20 \text{ ft}}{31 \text{ ft}} = 0.65.$$

During development, the well was pumped at 211,750 ft^3/d (1.58 Mgal/d), and drawdown in the well was 51 ft after 1 hour of pumping. K_z/K_r was assumed to be 0.1.

Step 1.--Observed drawdown is corrected for the effects of partial penetration by using equation 35 as follows:

$$s_p = (51 \text{ ft})\ (0.65)\left[1 + 7\left(\frac{0.5 \text{ ft } (0.1)^{\frac{1}{2}}}{2(0.65)\ (31 \text{ ft})}\right)^{\frac{1}{2}} \cos\ (0.65 \times 90)\right] = 40.5 \text{ ft}.$$

Adjustment to drawdown for dewatering is not needed as long as the aquifer remains fully saturated.



(Modified from Theis and others, 1963)

Figure 25.- Curves used for estimating the transmissivity of an aquifer from the specific capacity of a well.

Step 2.--From equation 37,

$$M = -0.25 - \log[3.74 \times 10{-}9(0.5\ ft)^2] = 8.78,$$

and from equation 36,

$$T' = \frac{0.183(211{,}750\ ft^2/d)}{40.5\ ft}\{8.78 - \log[5 \times 10^3(0.0002)] + \log(0.04d)\} = 7{,}080\ ft^2/d$$

Step 3.--If

$$Q/s = \frac{211{,}750\ ft^3/d}{40.5\ ft} = 5{,}228\ ft^2/d$$

and

$$T' = 7{,}080\ ft^2/d,$$

then by using figure 25, the estimate of T is about 6,800 ft^2/d.

Discussion

Specific capacity decreases with time because drawdown increases with time until steady state is approached. For this reason, the duration of pumping for each value of specific capacity should be stated.

There are several potential sources of error associated with estimating transmissivity from specific capacity. The major potential sources result from the effects of partial penetration and well loss. Additional errors are possible from estimates of the well radius and storage coefficient. In most cases, these factors tend to cause low estimates of specific capacity (Walton, 1970, p. 314).

Equation 35 is designed to correct measured drawdown for the effect of partial penetration. The equation is derived for steady-state conditions and is strictly applicable only where the rate of drawdown has slowed to near zero. In observation well R101-2, 36 ft east of well I101-1, 98 percent of the total drawdown after 4 hours of pumping occurred during the first hour--the period of pumping used in estimating the specific capacity of well I101-1. Therefore, the error in calculations attributed to the correction for partial penetration is probably insignificant.

Well loss can be a potentially large source of error in specific capacity calculations. Well loss is caused by turbulent flow through the well screen, well bore, and pump intake. It results in a greater drawdown in the well bore than in the surrounding aquifer and increases with increasing rate of pumping. It is also affected by well construction and aquifer properties. Well loss can be approximated by the following equation (Jacob, 1947):

$$s_1 = D\ Q^2$$

where

s_1 is well loss, in feet;
D is the well-loss constant, in seconds per foot raised to the fifth power; and
Q is the pumping rate in cubic feet per second.

D is calculated by using a step-drawdown test (Jacob, 1947). Step-drawdown data were not available for the irrigation wells discussed in this report.

Estimates of T can be significantly reduced by well loss. For three pumping wells for which specific-capacity and aquifer-test data were available, values of T calculated from specific-capacity data were one-third to one-half the values calculated from aquifer tests (table 8).

Table 8.--Values of transmissivity calculated from aquifer tests and from specific-capacity data

[T, transmissivity, in feet squared per day, rounded to two significant digits; Q/s, specific capacity, in feet squared per day]

		Values calculated from aquifer tests	Values calculated from specific-capacity data		
Location	Well	T	T	Period of pumping (hours)	Q/s
Milford area	I101-1	[1] 12,000	6,800	1	5,228
Howe area	I6-1	13,000	3,800	8	2,676
Howe area	I29-3	57,000	24,000	8	10,974

[1] Average of three values of T (table 5).

Other possible errors in estimates of T based on specific-capacity data result from errors in estimates of S and from assuming that the effective well radius equals the nominal well radius. However, in equations 36 and 37, T' (and thus T) vary with the logarithm of S and r. Thus, the result of large errors in estimates S or r should be small errors in estimates in T (Walton, 1962, p. 12, 13). A tenfold change in the value of S used in the previous example resulted in a 14 percent change in the value of T. A twofold change in the value of r resulted in an 18 percent change in the value of T.

In unconfined aquifers, a significant error in estimates of T based on Q/s can be introduced by delayed yield from storage. Many short duration specific-capacity tests are not long enough to overcome the effects of delayed yield and result in an overestimate of T. By using an "apparent" specific-yield term, Hurr (1966) devised a method of compensating for delayed yield even though drawdown is measured a few minutes after pumping begins. Hurr's method is recommended over Theis' method if the effects of delayed yield might be significant.

Perhaps the simplest method of estimating T from Q/s was suggested by Johnson and Sniegocki (1967). The method is based on the equilibrium equation developed by Thiem (1906),

$$T = \frac{Q \log (r_2/r_1)}{s_1 - s_2}$$

where

r_1 and r_2 are the distances to near and far observation points, in feet, and
s_1 and s_2 are the drawdowns at near and far observation points, in feet.

If r_2 is assumed to be the distance at which drawdown is zero ($s_2 = 0$), the Thiem equation can be rewritten as follows:

$$T = (Q/s_1)\log (r_2/r_1)$$

where s_1 represents drawdown in the pumping well, r_1 is the effective well radius, and Q/s_1 is the specific capacity of the well. Johnson and Sniegocki (1967) suggest that, for an unconfined aquifer, r^2 can be assumed to equal 1,000 ft and, for a confined aquifer, r_2 can be assumed to equal 10,000 ft. Thus, by using this method, estimates of transmissivity can be made quickly (though probably less accurately) from values of specific capacity and well radius only.

Prediction

Solving the prediction (or simulation) problem involves the same analytical equations used to solve the calibration problem; but, instead of solving the equations to obtain values of aquifer properties, the equations are used to calculate (predict) one of four variables (s, r, t, or Q) while assuming values for the other three. Values for T and S as well as values for other parameters in the well function (for example, r/B for a leaky, confined aquifer) must also be known or estimated. Unlike the calibration problem, the prediction problem is said to be well-posed--that is, for a given set of knowns only one value of the unknown is possible. These equations can be solved algebraically and curve matching is not needed.

Single-Well System

This section discusses the effects of pumping a single well on water levels near the well. Four examples of hypothetical questions related to water-management are presented. Solution to the questions involve calculating values of drawdown, pumping rate, time of pumping, and distance from the pumping well. Though the questions are hypothetical, the solutions are based on data collected from a well in the Howe study area. The well selected for this section fully penetrates a nonleaky, confined aquifer. The procedures for answering the questions for wells in other hydrogeologic settings would be the same as the procedures discussed here, except that a different analytical model would be used.

Examples

Well I6-1 in the Howe area (fig. 2) is used for the examples in this section. The following values of aquifer properties are calculated in the section "Model of Nonleaky, Confined Aquifer and Fully Penetrating Well" and are rounded to two significant digits.

$$T = 13{,}000\ \mathrm{ft^2/d}$$
$$S = 0.000043$$

Example 1: Solving for s.--What would be the expected drawdown 1,000 ft away from well I6-1 when the well is pumped continuously at a rate of 110,000 ft^3/d (0.82 Mgal/d) for 30 days?

Drawdown is calculated from equations 2 and 5, which are repeated here for convenience.

$$u = \frac{r^2 S}{4Tt} \qquad (2)$$

$$s = \frac{Q}{4\pi T} W(u) \qquad (5)$$

From equation 2,
$$u = \frac{(1{,}000\ \mathrm{ft})^2 (0.000043)}{4(13{,}000\ \mathrm{ft^2/d})\ (30\ \mathrm{d})} = 2.8 \times 10^{-5}.$$

From table 2 in Ferris and others (1962), the value of W(u) corresponding to $u = 2.8 \times 10^{-5}$ is 9.9. By using equation 5,

$$s = \frac{(110{,}000\ \mathrm{ft^3/d})\ (9.9)}{4\pi\ (13{,}000\ \mathrm{ft^2/d})} = 6.7\ \mathrm{ft}.$$

Example 2: Solving for Q.--At what rate could the well be pumped for 30 days so that the resulting drawdown would not exceed 5 ft of drawdown, 1 mi away from the pumping well? From equation 2

$$u = \frac{(5,280 \text{ ft})^2 \ (0.000043)}{4 \ (13,000 \text{ ft}^2/\text{d}) \ (30 \text{ d})} = 7.68 \times 10^{-4}.$$

The corresponding value of W(u) is 6.6. By rearranging equation 5,

$$Q = \frac{4\pi Ts}{W(u)}$$

$$= \frac{4(3.1416) \ (13,000 \text{ ft}^2/\text{d}) \ (5 \text{ ft})}{6.6}$$

$$= 123,760 \text{ ft}^3/\text{d}.$$

Example 3: Solving for t.--How much time would be required for the well pumping continuously at 110,000 ft^3/d to cause 1 ft of drawdown 1 mi from the well?

By rearranging equation 5,

$$W(u) = \frac{4\pi Ts}{Q} = \frac{4(3.1416)(13,000 \text{ ft}^2/\text{d})(1 \text{ ft})}{110,000 \text{ ft}^3/\text{d}} = 1.49$$

From table 2 in Ferris and others (1962), u = 0.14. By rearranging equation 2,

$$t = \frac{r^2 S}{4Tu} = \frac{(5,280\text{ft})^2 \ (0.000043)}{4(13,000 \text{ ft}^2/\text{d}) \ (0.14)} = 0.16 \text{ d (or about 4 hours)}.$$

Example 4: Solving for r.--After 1 day of pumping at 110,000 ft^3/d, at what distance from the well would drawdown be 1 ft?

As in example 3, W(u) = 1.49 and u = 0.14. By rearranging equation 2,

$$r = \left(\frac{4Ttu}{S}\right)^{\frac{1}{2}} = \left[\frac{4(13,000 \text{ ft}^2/\text{d}) \ (1 \text{ d}) \ (0.14)}{0.000043}\right]^{\frac{1}{2}} = 13,011 \text{ ft} = 2.5 \text{ mi}.$$

Discussion

The results of the prediction analyses are subject to assumptions inherent in the analytical model. These assumptions include uniform aquifer geometry and homogeneity. Based on the limited lithologic information near well I6-1 (fig. 9), the degree of violation in assumptions of aquifer uniformity probably increases as values of r increase. Stated differently, as pumping continues, the cone of depression enlarges, and the likelihood of the aquifer being uniform throughout the area of the cone decreases. Values of T and S from other parts of the Howe area are, on the average, larger than those in the immediate vicinity of well I6-1. Thus, the author believes that the values of the variables calculated in this section are conservative values and that the "effective" values for T and S probably increase as t increases.

The examples in this section are based only on the analytical model for a nonleaky, confined aquifer. All questions answered by these examples could be answered by using other analytical models in the same way by substituting for equation 5, the equation with the appropriate well function for the aquifer and well conditions being modeled. For example, drawdown near a well that partially penetrates an unconfined aquifer can be predicted by substituting equation 21 for equation 5. Values for β, ℓ/b, and ℓ'/b must be known or estimated for the well function in equation 21. Values of u are calculated in the same way that they are for equation 5, except that S_y rather than S is used for most applications. Tables of well functions can generally be found in references describing the particular model. Hantush (1964a) provides tables for several functions that are commonly used in ground-water equations. Many programs are available to calculate functional values by using both programmable calculators and micro-computers. An extensive library of these programs is maintained by the International Groundwater Modeling Center at the Holcomb Research Institute, Butler University, Indianapolis, and by the National Water Well Association, Worthington, Ohio.

Multiple-Well System

When two or more wells are pumping near each other and from the same aquifer, the drawdown of each well is additive to the drawdown(s) of the other wells. That is, the drawdown pattern resulting from one pumping well can be superimposed on the drawdown patterns of the others (fig. 26). Superposition can be used to predict drawdown in multiple-well systems as discussed in this section. Superposition can also be used to predict drawdown near hydrologic boundaries by using image-well theory as discussed in the following section.

Examples

Wells I102-1 and I106-1 (fig. 4) are used in the two examples of this section. The wells are assumed to fully penetrate a confined aquifer with characteristics similar to those described for the aquifer near well I101-1 (see p. 31). In 1982, pumping rates for wells I102-1 and I106-1 were about 196,000 ft^3/d (1.47 Mgal/d) and 106,000 ft^3/d (0.79 Mgal/d). The distance between the wells is about 1,700 ft.

Values of T, S, and K' used in the examples--12,000 ft^2/d, 0.00023, and 0.13 ft/d--are averages of data in table 5 rounded to two significant digits. b' is assumed to be 8 ft, the value used to calculate aquifer properties near well I101-1.

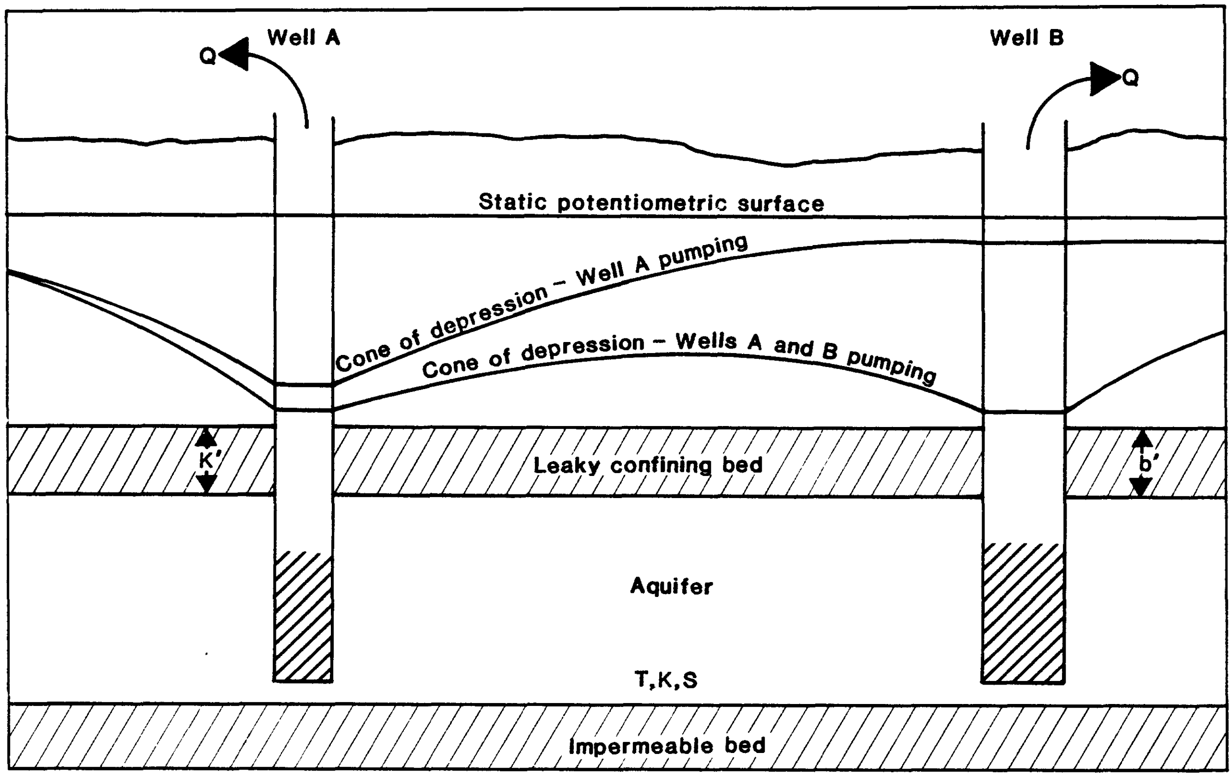


Figure 26.-- Generalized patterns of drawdown near two wells pumping from a confined aquifer.

Example 1.--What is the total drawdown at a point equidistant between the two wells after 12 hours of pumping? For this example, the distance from both wells to the point of calculated drawdown is

$$\frac{1,700 \text{ ft}}{2}, \text{ or}$$

$$r_{I102-1} = r_{I106-1} = 850 \text{ ft.}$$

Thus, total drawdown is the sum of $s_{I-102-1}$ and $s_{I-106-1}$ at $r = 850$ ft. The analytical model that describes drawdown near a pumping well that fully penetrates a leaky, confined aquifer is equation 11, which is repeated here for convenience.

$$s = \frac{Q}{4\pi T} W(u, r/B).$$

Values of u and r/B are needed to determine the corresponding value of W(u, r/B). From equation 2,

$$u = \frac{(850\ ft)^2\ (0.00023)}{4(12{,}000\ ft^2/d)\ (0.5\ d)} = 6.92 \times 10^{-3}$$

and from equation 12,

$$B = \left[\frac{(12{,}000\ ft^2/d)\ (8\ ft)}{0.13\ ft/d}\right]^{\frac{1}{2}} = 859\ ft.$$

Thus, for both wells,

$$r/B = \frac{850\ ft}{859\ ft} = 0.990.$$

For u = 6.92 x 10-3 and r/B = 0.990, W(u, r/B) = 0.842 (Hantush, 1956, table 2). Using equation 11,

$$s_{I102-1} = \frac{(196{,}000\ ft^3/d)\ (0.842)}{4\pi\ (12{,}000\ ft^2/d)} = 1.09\ ft, \text{ and}$$

$$s_{I106-1} = \frac{(106{,}000\ ft^3/d)\ (0.842)}{4\pi\ (12{,}000\ ft^2/d)} = 0.59\ ft.$$

Thus, the predicted total drawdown between the two wells after 2 hours of continuous pumping is

$$1.09\ ft + 0.59\ ft = 1.68\ ft.$$

Though the point at which drawdown is calculated is equidistant from the wells, the drawdowns are not equal because the pumping rates are not equal. Equation 11 demonstrates that drawdown is directly proportional to pumping rate.

The point at which drawdown is calculated need not be equidistant from the pumping wells. Drawdown can be calculated for any location; however, the greater the distance that the location is from the pumping wells, the greater the probability that the assumptions on which the model is based will be violated.

Example 2.--A domestic well 1,200 ft from well I102-1 and 1,000 ft from well I106-1 (fig. 4) produces from the same aquifer as the two irrigation wells. How long could the two irrigation wells pump simultaneously exceeding a 3-ft limitation for drawdown at the domestic well?

A simple way to solve this problem is to calculate total drawdown at the domestic well for several pumping periods and construct a graph of s and t for use as a nomogram. For t = 0.1 day, the calculations for well I102-1 are

$$u_{I102-1} = \frac{(1{,}000\ ft)^2 (0.00023)}{4(12{,}000\ ft^2/d)\ (0.1\ d)} = 0.0479, \text{ and}$$

$$\frac{r_{I-102-1}}{B} = \frac{1{,}000\ ft}{859\ ft} = 1.164.$$

For these values of u and r/B,

$$w(u,r/B) = 0.68 \text{ (Hantush, 1956, table 2), and}$$

$$s_{I102-1} = \frac{(196{,}000\ ft^3/d)\ (0.68)}{4\pi\ (12{,}000\ ft^2/d)} = 0.88\ ft.$$

For well I106-1, with t = 0.1 d,

$$u_{I106-1} = \frac{(1{,}200\ ft)^2\ (0.00023)}{4(12{,}000\ ft^2/d)\ (0.1\ d)} = 0.069$$

$$\frac{r_{I106-1}}{B} = \frac{1{,}200\ ft}{859\ ft} = 1.40.$$

For these values of u, and r/B,

$$W(u,\ r/B) = 0.51 \text{ (Hantush, 1956, table 2), and}$$

$$s_{I106-1} = \frac{(106{,}000\ ft^3/d)\ (0.51)}{4\pi\ (12{,}000\ ft^2/d)} = 0.36\ ft.$$

Total drawdown at the domestic well after about 2.5 hours of pumping of wells I102-1 and I106-1 would be

$$0.88\ ft + 0.36 = 1.24\ ft.$$

Drawdown at the domestic well can be plotted for different values of pumping time. The resulting curve can be used to estimate total drawdown at any time during pumping (fig. 27). On the basis of the curve resulting from pumping rates measured in 1982, the system is at steady state after about 2 hours of pumping, and the 3-ft drawdown limitation at the domestic well would not be exceeded by these pumping rates.

Stabilization of drawdown in only 2 hours is attributed to the high hydraulic conductivity and the thin confining layer above the aquifer. Because of these two factors, water can seep quickly from the source aquifer to the pumped aquifer in response to drawdown in the pumped aquifer. As long as the head in the source aquifer is not lowered significantly, drawdown in the pumped aquifer will remain constant. If the head in the source bed were lowered significantly, the rate of seepage induced through the confining bed would be reduced and drawdown in the pumped aquifer would increase.

Example 3.--At what rates could wells I102-1 and I106-1 be pumped without exceeding the 3-ft limitation for drawdown at the domestic well in example 2?

By varying the pumping rates in equation 11, corresponding values of drawdown can be calculated. Drawdowns at the domestic well for several multiples of the 1982 rates for wells I102-1 and I106-1 are shown in figure 27. At a multiple value of 2, drawdown at the domestic well would not exceed the 3-ft limitation before the system reached equilibrium. At multiple values of 3 and 4, drawdown would exceed the 3-ft limitation after 29 and 13 minutes of pumping. Drawdown corresponding to multiples between those presented can be obtained by linear interpolation. Thus, the combined pumping rates of wells I102-1 and I106-1 could increase by a factor of about 2.4 before predicted drawdown would exceed 3 ft.

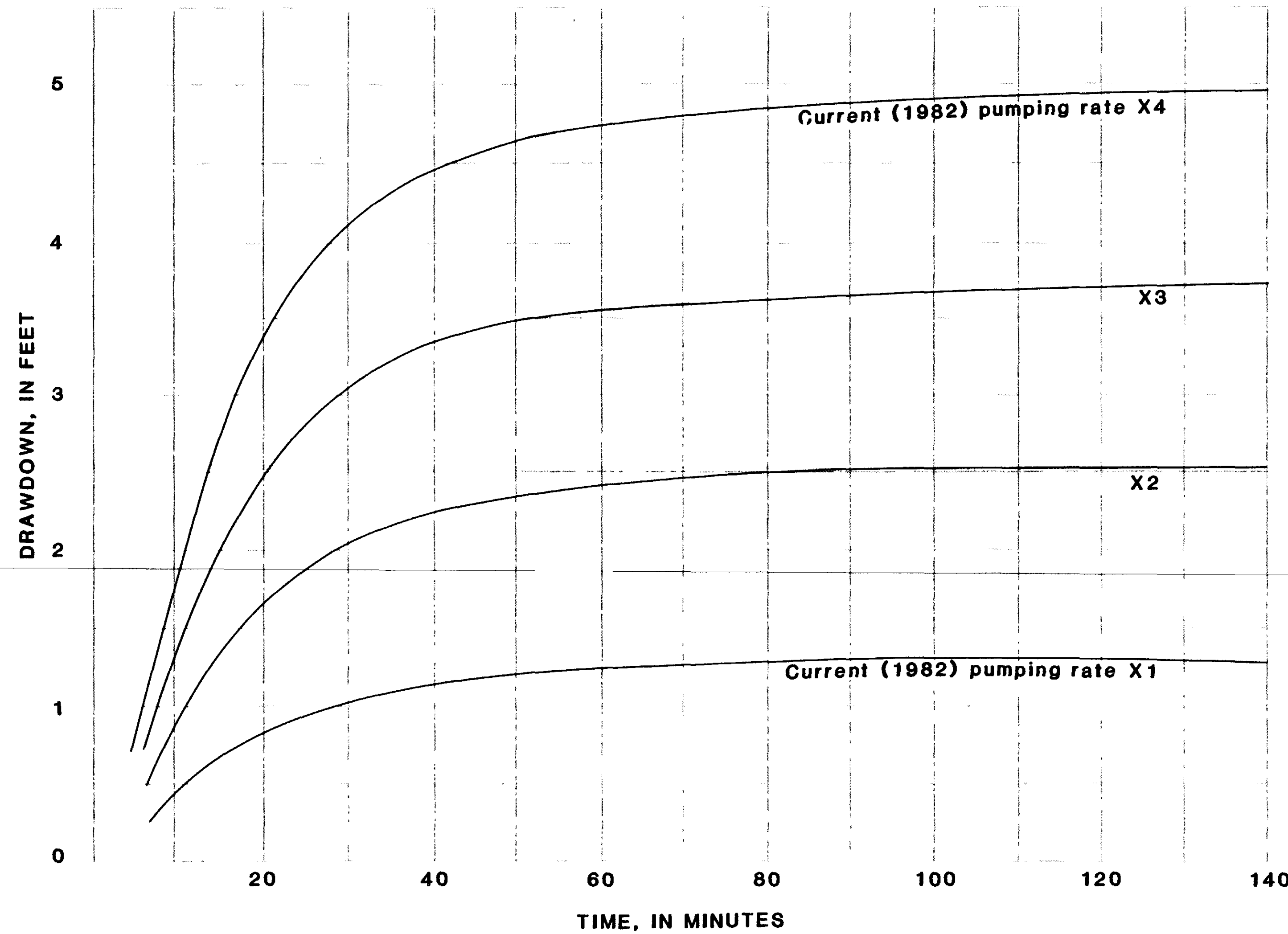


Figure 27.-- Relation of drawdown to time near pumping wells I102-1 and I106-1, Milford area.

Discussion

In the analytical model used in the preceding examples, constant head is assumed in the source aquifer. If drawdown is to be predicted for long periods of pumping, a model that accommodates decreases in head in the source aquifer should be considered. (See Hantush, 1967b, and Neuman and Witherspoon, 1969a.)

For more than two pumping wells, the total drawdown, s_t, at any point is the sum of the drawdowns from the individual wells. For n wells,

$$s_t = s_1 + s_2 + \ldots + s_n. \tag{38}$$

For a recharge well, the sign for values of s in equation 38 would be negative.

Hydrologic Boundaries

In all of the analytical models described thus far, aquifers are assumed to be infinite in areal extent. Though this assumption is valid for many large aquifers and (or) short pumping periods, boundary effects must be included in the analysis for many other situations. Image-well theory provides a method of quantifying the effects of both barrier and recharge boundaries on drawdown near pumping wells. Image-well theory is described by Walton (1970, p. 158) as follows:

> The effect of a barrier boundary on the drawdown in a well, as a result of pumping from another well, is the same as thóugh the aquifer were infinite and a like discharging well were located across the real boundary on a perpendicular thereto and at the same distance from the boundary as the real pumping well. For a recharge boundary, the principle is the same except that the image well is assumed to be discharging to the aquifer instead of pumping from it.

A recharge boundary (stream) and a recharging image well used to simulate the stream are depicted in figure 28.

What follows is an application of image-well theory to an unconfined aquifer with a stream as a recharge boundary. Well I103-1, 350 ft west of Kieffer ditch (fig. 4), fully penetrates a 47-ft thick sand and gravel aquifer in the north-central part of the Milford area. In this part of the area, the ditch is incised into the aquifer and is hydraulically connected to it. Using limited field data and an analytical model for an unconfined aquifer and fully penetrating well (Neuman, 1975), the author calculated values for the following aquifer properties:

$$T = 13,000\ ft^2/d,$$
$$S_y = 0.15,\ \text{and}$$
$$K_z/K_r = 1.0.$$

The pumping rate of well I103-1 was measured with a flowmeter to be 74,000 ft^3/d.

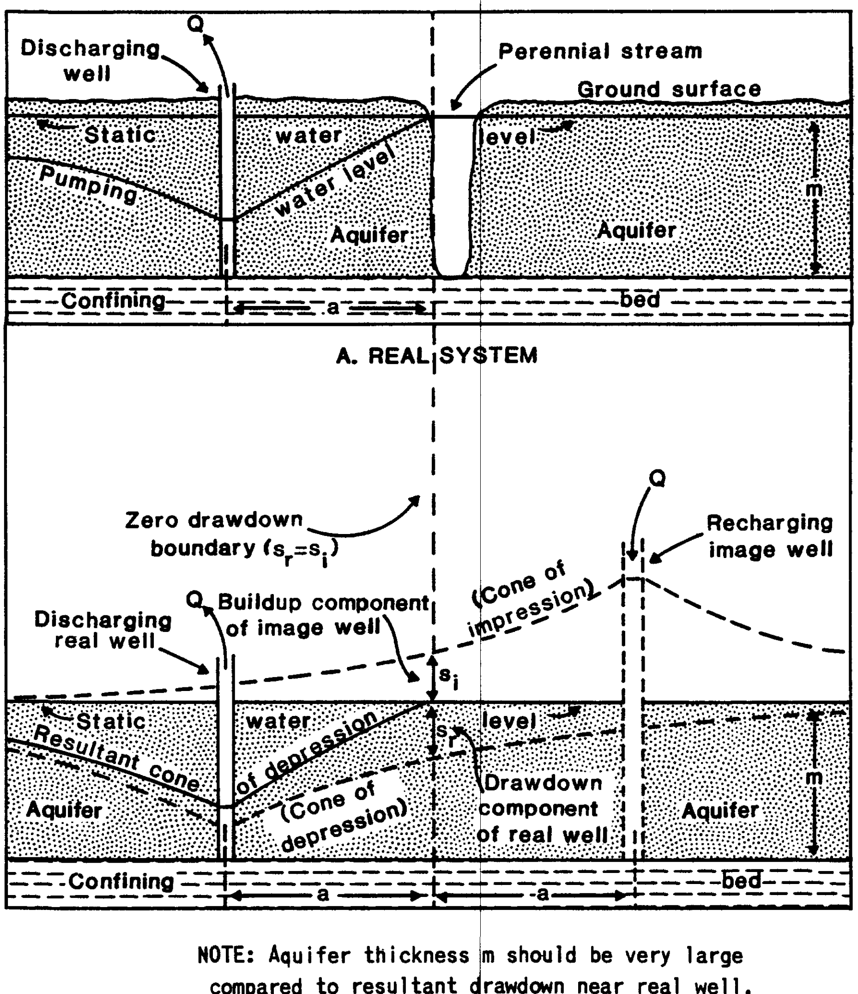


Figure 28.– Idealized sections of a discharging well in a semi-infinite aquifer bounded by a perennial stream (A), and the equivalent hydraulic system in an infinite aquifer (B). (Ferris and others, 1962, p. 146).

Example

What would be the drawdown midway between well I103-1 and Kieffer ditch after 24 hours of pumping?

Figure 29 illustrates how a recharging image well can be used to simulate Kieffer ditch. The image well is placed the same distance from Kieffer ditch as is well I103-1 (350 ft). The recharge rate of the image well equals the pumping rate of well I103-1 (74,000 ft^3/d or 0.55 Mgal/d). At the point midway between well I103-1 and Kieffer ditch, r_{I103-1} = 175 ft and r_{image} = 525 ft.

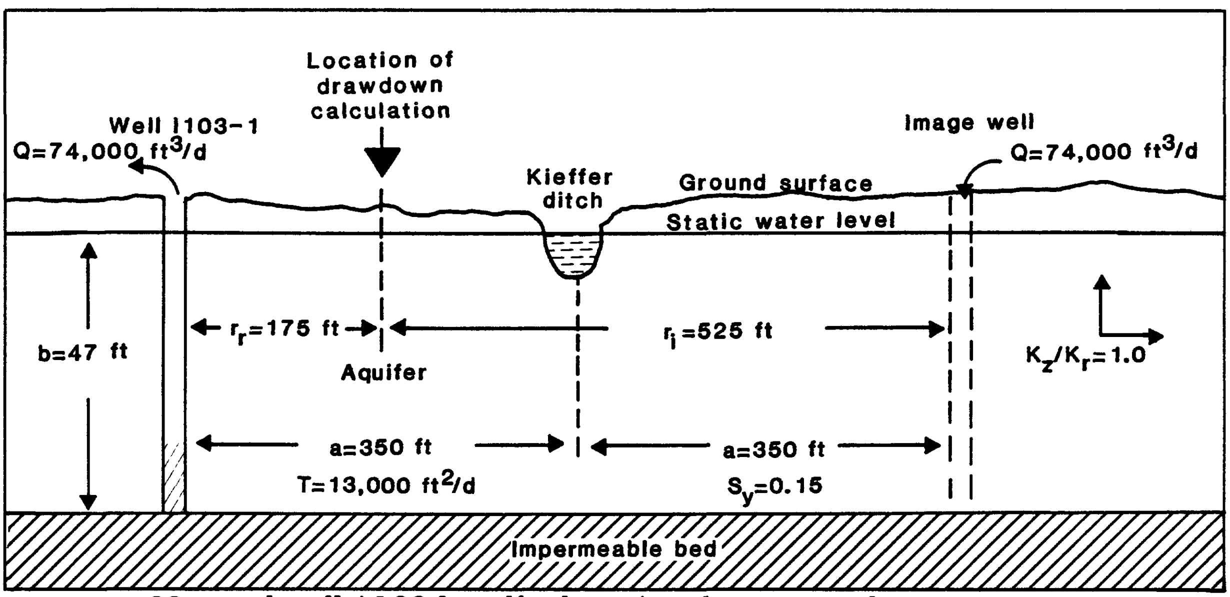


Figure 29.– Real well (I103-1, Milford area) and corresponding image well used to simulate the effect of Kieffer ditch on drawdown after 24 hours of pumping.

The equation for drawdown near a discharging (or recharging) well that fully penetrates an unconfined aquifer is given by Neuman (1975) as

$$s = \frac{Q}{4\pi T} W(u_A, u_B, \beta). \tag{39}$$

[Note: β in this report is $\beta^{\frac{1}{2}}$ in Neuman (1975).] Equation 39 represents a special case of equation 21 in which the terms 1/b and 1'/b for partial penetration are not needed when the well is fully penetrating. For the present example, the pumping period is 24 hours, and, therefore, u_B rather than u_A is the parameter needed for calculating drawdown. u_B for well I103-1 is calculated by using equation 24, as follows:

$$u_B = \frac{(175 \text{ ft})^2(0.15)}{4(13{,}000 \text{ ft}^2/\text{d})(1 \text{ d})} = 0.088.$$

From equation 22,

$$\beta = \frac{175 \text{ ft } (1)^{\frac{1}{2}}}{47 \text{ ft}} = 3.72.$$

From Neuman (1975, table 1b),

$$W(0.088,\ 3.72) = 3.09.$$

Thus, from equation 39, drawdown attributed to pumping if the aquifer were infinite in areal extent would be

$$s = \frac{(74{,}000 \text{ ft}^3/\text{d})\ (3.09)}{4\pi\ (13{,}000 \text{ ft}^2/\text{d})} = 1.40 \text{ ft}.$$

Similar calculations are made for the image well. From equations 24 and 22,

$$u_B = \frac{(525 \text{ ft})^2(0.15)}{4\ (13{,}000 \text{ ft}^2/\text{d})(1 \text{ d})} = 0.80, \text{ and}$$

$$\beta = \frac{525 \text{ ft } (1)^{\frac{1}{2}}}{47 \text{ ft}} = 11.$$

Also,

$$W(0.80,\ 11) = 0.32.$$

[Note: The value for β, 11, is higher than values in the table presented by Neuman (1975). However, as β increases, the type curve in the area where u_B is 0.8 approaches the Theis curve. Thus, the value of W(0.8, 11), 0.32, was interpolated from the Theis curve for u = 0.8.]

Reduction in drawdown caused by Kieffer ditch (image well) is calculated from equation 39 where the minus sign denotes reduction in drawdown.

$$s = -\frac{(74{,}000 \text{ ft}^3/\text{d})\ (0.32)}{4\pi\ (13{,}000 \text{ ft}^2/\text{d})} = -\ 0.14$$

The predicted net drawdown midway between the well and Kieffer ditch would be

$$1.40 \text{ ft} - 0.14 \text{ ft} = 1.26 \text{ ft}.$$

Discussion

A barrier boundary, such as a clay-rich till bordering an outwash system, can also be represented by using an image well. Calculations are made in the same way as for a recharge boundary, except that drawdown for the barrier boundary is added to the drawdown for the well.

Image wells can be used to simulate the effects of several boundaries--constant head and (or) constant flux--within the same system. Ferris and others (1962, p. 154-161) give examples of several multiple-boundary systems. These simulations require more image wells, which result in additional image wells (Freeze and Cherry, 1979, p. 331). A repeating pattern of image wells that extends to infinity is created. However, for practical purposes, the effects of individual image wells need to be added only so long as their effect significantly influences the cumulative effect on drawdown at the point of interest (Ferris and others, 1962, p. 159).

Streamflow Reduction

Initially, when a well is pumped, water is removed from storage in the aquifer, and a cone of depression is formed. All the water removed from storage represents intercepted ground water that would have discharged to a stream or other surface-water body if evapotranspiration and underflow are assumed to be constant and if pumping had not occurred. At a low pumping rate, ground water between the well and stream continues to flow toward the stream but at a reduced rate. If the rate of pumping is high enough, the ground-water gradient between the well and stream can be reversed. In this case, water would be induced to flow from the stream (streambed seepage) to the well (fig. 30). In this report, streamflow reduction includes both the reduction in ground water moving toward the stream and the streambed seepage into the aquifer.

In the discussion and examples that follow, the rate and volume of streamflow reduction during pumping are considered. Also considered are the effects of pumping on streamflow after the pumping is stopped (residual effects) and the effects of intermittent pumping compared to those of continuous pumping.

The procedures in this section are related to pumping wells and streamflow reduction. The same procedures could be used to analyze recharging wells and streamflow accretion. The examples deal with one-well systems. The effects of several pumping wells on streamflow are additive in the same way that the effects on drawdown are additive.

Analysis of streamflow reduction is based on work by Theis (1941), Glover and Balmer (1954), Rorabaugh (1957), Theis and Conover (1963), and Hantush (1964a, 1965). This later work has been summarized and presented in a readily usable format by Jenkins (1970) from which much of this discussion was extracted.

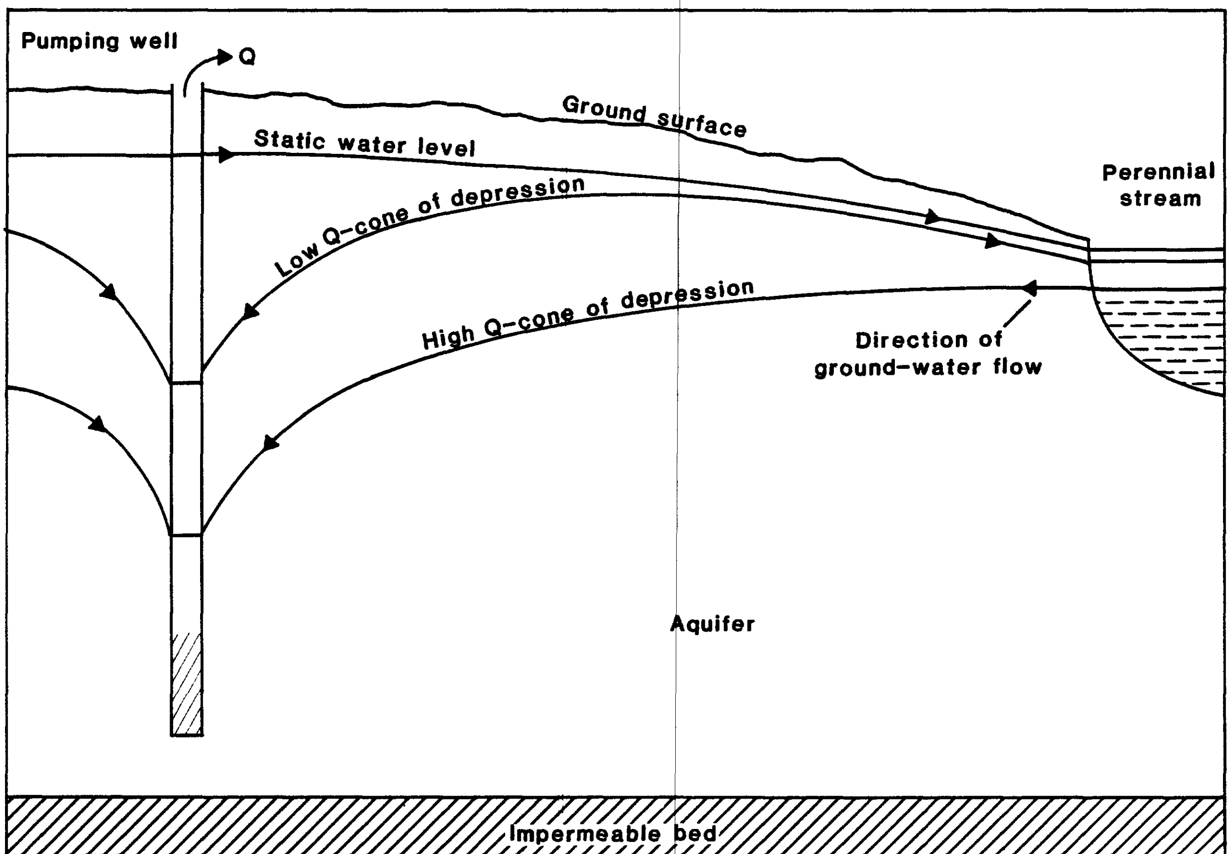


Figure 30.– Water levels in an aquifer near a pumping well and a stream for low and high rates of pumping.

Conceptualization

The two equations used in this section describe the effect of ground-water pumping on streamflow and are written in Jenkins' (1970) notation as follows:

$$q/Q = \text{erfc}\left(\frac{srf}{4t}\right)^{\frac{1}{2}} = 1-\text{erf}\left(\frac{srf}{4t}\right)^{\frac{1}{2}} \qquad (40)$$

and,

$$v/Qt = \left(\frac{srf}{2t} + 1\right) \text{erfc}\left(\frac{srf}{4t}\right)^{\frac{1}{2}} - \left(\frac{srf}{4t}\right)^{\frac{1}{2}} \frac{2}{\pi} \exp\left(-\frac{srf}{4t}\right) \qquad (41)$$

where

q is the rate of streamflow reduction, in cubic feet per day;
Q is the rate of pumping, in cubic feet per day;
erf (x) is the error function of $x = \frac{2}{\pi} \int_0^x e^{-t^2} dt$;
erfc (x) is the complimentary error function of x = 1 - erfx;
srf is the streamflow reduction factor, in days;
t is time since pumping began, in days; and
v is the volume of streamflow reduction, in cubic feet.

The streamflow reduction factor, srf, is defined as

$$srf = a^2 \; T/S \tag{42}$$

where a is the distance between the pumping well and the stream, in feet.

Equation 40 is used to calculate the fraction of the pumping rate supplied by the rate of streamflow reduction. Equation 41 is used to calculate the fraction of the pumping volume supplied by the volume of streamflow reduction. Equations 40 and 41 are based on the following assumptions:

1. Well discharge rate is constant.
2. The diameter of the well is infinitesimal. The well has no storage and is open to the full thickness of the aquifer.
3. The aquifer is homogeneous, isotropic, semi-infinite (infinite in the direction away from the stream) in areal extent, and uniform in thickness.
4. Water in the aquifer is released instantaneously from storage.
5. Transmissivity of the aquifer is constant with time. This implies that drawdown in a water-table aquifer is much less than the total saturated thickness.
6. The stream that forms an aquifer boundary is straight and fully penetrates the aquifer.
7. The temperature of the water in the stream is the same as that of the water in the aquifer.
8. All other fluxes of water to and from the aquifer (for example, areal recharge and underflow) are constant.

The usefulness of this method, as with other analytical methods presented in this report, depends on the user's recognizing departures from ideal conditions. As Jenkins (1970) stated,

> Departure from idealized conditions may cause actual [streamflow reductions] to be either greater or less than the values determined by methods presented in this report. Although the user usually cannot determine the magnitude of these discrepancies, he should, where possible, be aware of the direction the discrepancies take.

For example, in an unconfined aquifer, substantial dewatering would result in a notable decrease in T and would violate assumption 5. For a given value of Q, a decrease in T would result in a decrease in q. A change in the temperature of the water in the stream alters the viscosity of the water and can be an important factor. Streamflow reduction is generally higher in summer than in winter.

Other sources of water to the pumping well, such as reduced evapotranspiration or intercepted ground-water discharge to another surface-water body, will cause a decrease in streamflow reduction. An undetected no-flow boundary would cause an opposite effect.

Application

Jenkins (1970) used equations 40 and 41 to construct three graphs that make the calculation of the rate and the volume streamflow reduction more convenient. Figure 31 is a nomogram for use in calculating the effect of ground-water pumping on the rate and volume of streamflow reduction during the period of pumping only. Curve A provides estimates of q/Q, and curve B provides estimates of v/Qt_p for given values of t_p/srf. Figure 32 is a nomogram for use in calculating the effect of pumping on the rate of streamflow reduction during and after the pumping period. The graph provides estimates of q/Q for given values of $(t_p + t_i)$/srf and t_p/srf. Figure 33 is a nomogram for use in calculating the effect of pumping on the volume of streamflow reduction during and after the pumping period. The graph provides estimates of v/Qsrf for given values of $(t_p + t_i)$/srf and t_p/srf. The curves in figure 31 provide the same results as the curves in figures 32 and 33 if $t_i = 0$ and are somewhat easier to use when only results during the period of pumping are needed.

Examples

Well I103-1 is the pumping well and Kieffer ditch is the stream (fig. 4) used in examples that follow. As mentioned in the previous section, well I103-1 is pumped at a rate of 74,000 ft^3/d (0.55 Mgal/d) and is 350 ft west of Kieffer ditch. Measured values of T and S_y for the unconfined aquifer are 13,000 ft^2/d and 0.15. The drainage area and the average flow of Kieffer ditch near well I103-1 are about 5 mi^2 and 0.7 ft^3/s (Lindgren and others, 1985).

Example 1.--If well I103-1 is pumped continuously for 24 hours, what would be the effect of pumping on the rate and the volume of streamflow reduction (a) at the end of 24 hours and (b) after 50 days? For part a, figure 31 is used.

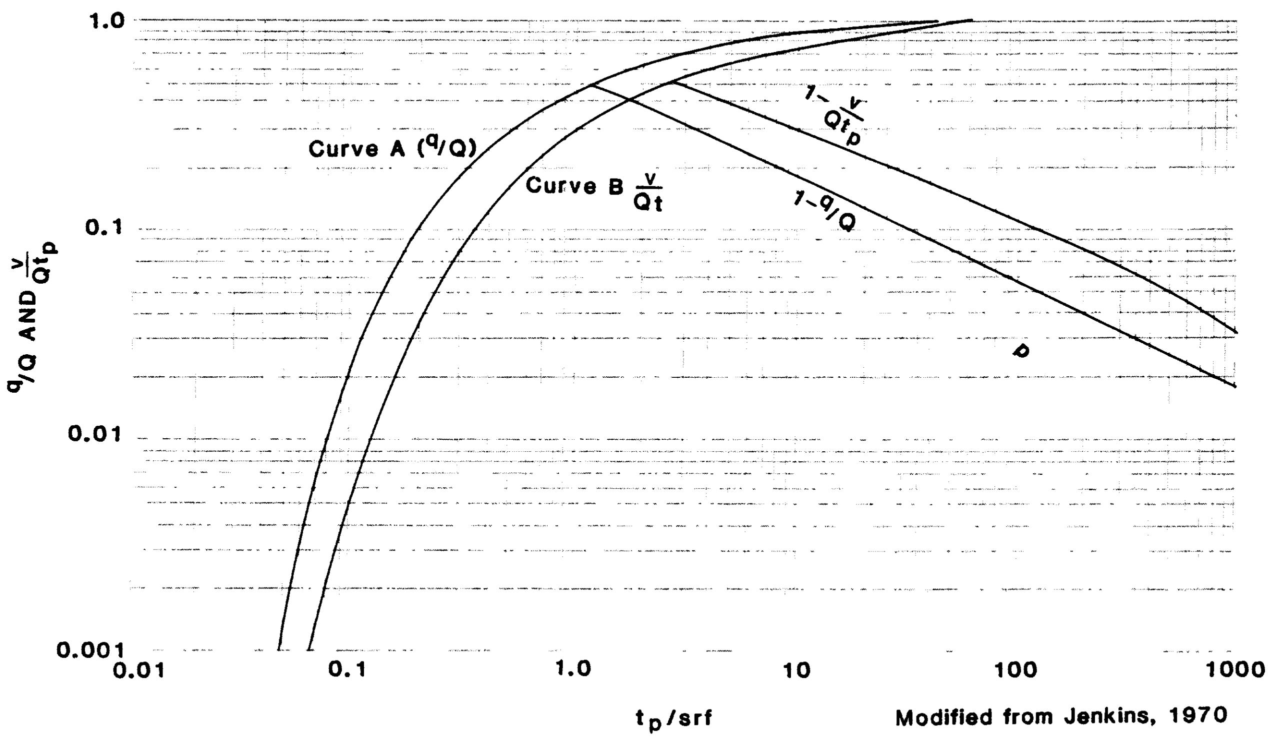


Figure 31.– Curves used to estimate the rate (Curve A) and volume (Curve B) of streamflow reduction during pumping.

If

$$t_p = 1 \text{ d},$$
$$a = 350 \text{ ft},$$
$$T = 13{,}000 \text{ ft}^2/\text{d},$$
$$S_y = 0.15, \text{ and}$$
$$Q = 74{,}000 \text{ ft}^3/\text{d},$$

what are q and v?

From equation 42,

$$srf = \frac{(350 \text{ ft})^2\ (0.15)}{13{,}000 \text{ ft}^2/\text{d}} = 1.41 \text{ d},$$

and

$$t_p/srf = 1 \text{ d}/1.41 \text{ d} = 0.71.$$

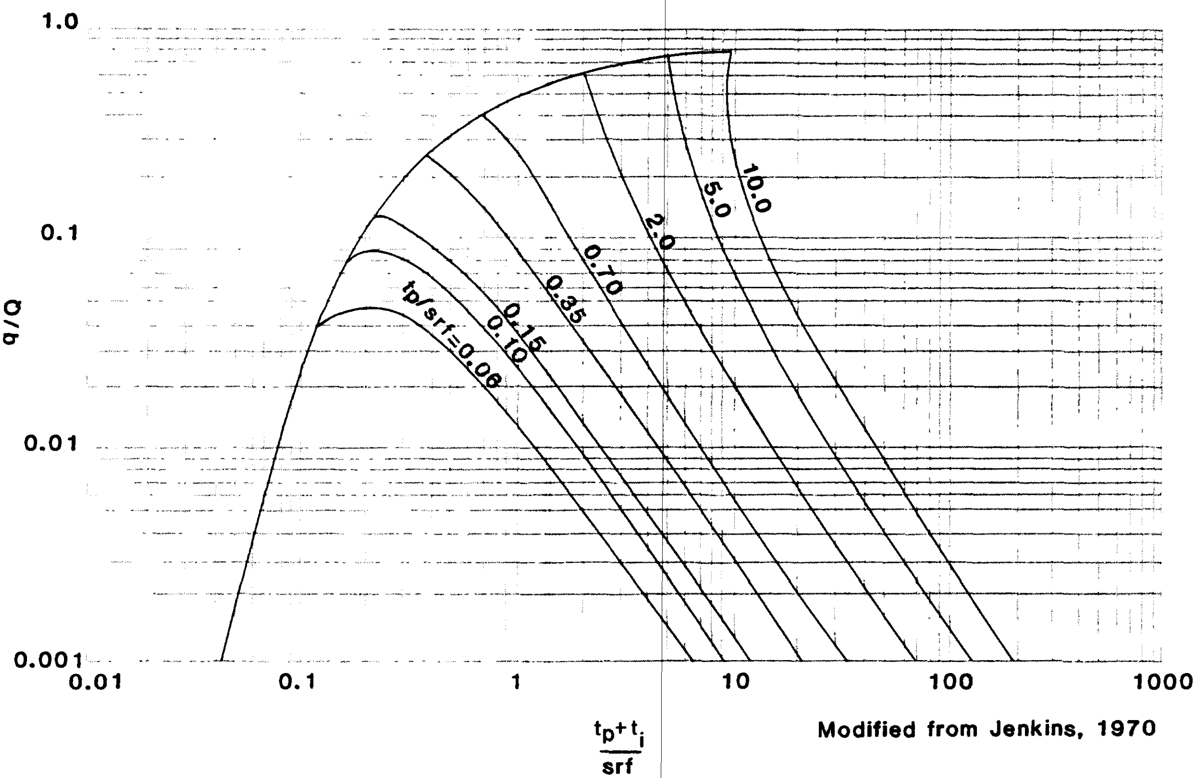


Figure 32.-- Curves used to estimate the rate of streamflow reduction during and after pumping.

From curve A (fig. 31), the value of q/Q corresponding to $t_p/srf = 0.71$ is 0.40, or, at the end of 1 day of pumping, 40 percent of the water being pumped from the well would be coming from streamflow reduction. The rate of streamflow reduction at this time would be

$$q = 0.40\ (74{,}000\ ft^3/d) = 29{,}600\ ft^3/d.$$

The remaining 60 percent or 44,400 ft^3/d would be coming from aquifer storage.

The value of t_p/srf, 0.71, is used in figure 31 to find v. Using curve B, the value of v/Qt_p, corresponding to $t_p/srf = 0.71$, is 0.21, or, of the total amount of water pumped from the well in 1 day, 21 percent was from streamflow reduction. Total pumpage would be

$$Qt_p = (74{,}000\ ft^3/d)(1\ d) = 74{,}000\ ft^3,$$

and the volume of streamflow reduction would be

$$v = 0.21\ (74{,}000\ ft^3) = 15{,}540\ ft^3.$$

The remaining 79 percent or 58,460 ft^3 would be from aquifer storage.

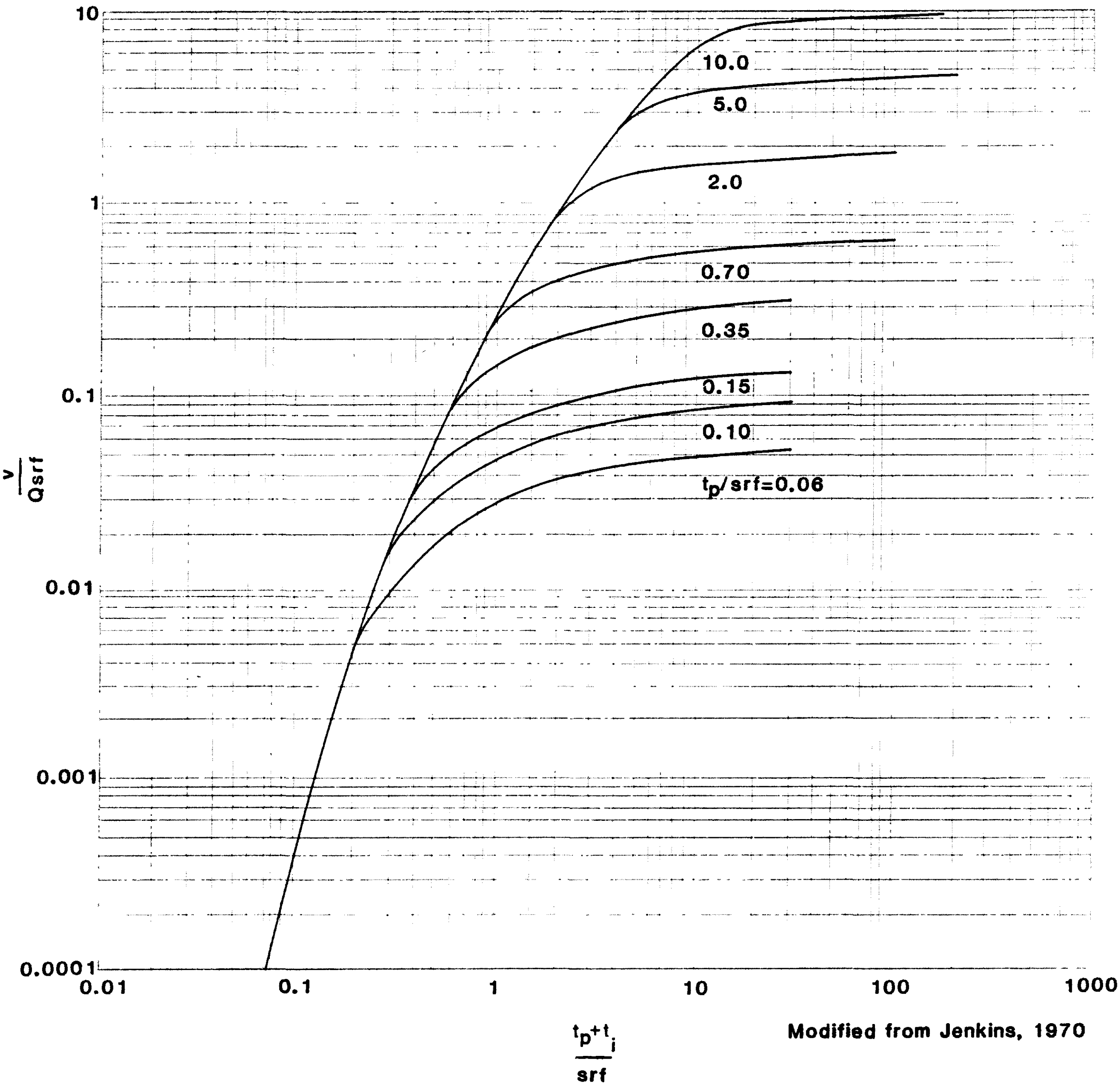


Figure 33.-- Curves used to estimate the volume of streamflow reduction during and after pumping.

Part b is an examination of the residual effects of pumping on streamflow reduction after pumping is stopped. Values of q and v were calculated by using figures 32 and 33 for various times during the 1 day of pumping and during 49 days after pumping. The procedure used to calculate the values of q and v is similar to the procedure described in part a, except that values of $(t_p + t_i)/srf$, in addition to values of t_p/srf, were needed to use the curves in figures 32 and 33. Table 9 is a summary of the calculations.

Values for t_p/srf are not included in the table. During the 1-day pumping period,

$$t_p/\text{srf} = (t_p + t_i)/\text{srf}$$

because $t_i = 0$. After pumping is stopped,

$$t_p/\text{srf} = 1 \text{ d}/1.41 \text{ d} = 0.71$$

for the remaining 49 days.

Table 9.--Summary of computations used to predict the effects of pumping from well I103-1 on streamflow reduction in Kieffer ditch, Milford area

[t_p, time of pumping in days; t_i, time since pumping stopped in days; srf, streamflow reduction factor; q, rate of streamflow reduction in cubic feet per day; Q, rate of pumping in cubic feet per day; v, volume of streamflow reduction in cubic feet]

$t_p + t_i$	$\frac{t_p + t_i}{\text{srf}}$	q/Q*	$q \times 10^3$	v/Qsrf≠	v x 10^{-3}
0.25	0.18	0.10	7.4	0.0050	0.52
.50	.35	.23	17	.035	3.7
.75	.53	.31	23	.078	8.1
1.00	.71	.38	28	.14	15
1.08	.77	.40	30	.17	18
1.25	.89	.32	24	.22	23
1.50	1.1	.27	20	.26	27
1.75	1.2	.19	14	.31	34
2.0	1.4	.14	10	.35	37
2.5	1.8	.11	8.1	.37	39
3.0	2.1	.070	5.2	.41	43
4.0	2.8	.041	3.0	.46	48
5.0	3.6	.030	2.2	.48	50
10	7.1	.010	0.74	.53	55
20	14	.0047	.35	.59	62
30	21	.0020	.15	.61	64
50	35	.0010	.074	.63	66

*Values obtained from figure 32.
≠Values obtained from figure 33.

The predicted rate of streamflow reduction, q, in Kieffer ditch increased quickly during the 1 day of pumping period and decreased quickly after pumping was stopped (fig. 34). The predicted maximum rate of reduction, 30,000 ft^3/d, was reached about 2 hours after pumping was stopped. On day 10 (9 days after pumping was stopped), the rate of reduction was still 740 ft^3/d.

The accumulated volume of streamflow reduction, v, increased rapidly through day 4 after which the slope the curve decrease rapidly. At the end of the 50-day period,

$$\begin{aligned} v &= 66{,}000 \text{ ft}^3, \\ Qt_p &= (74{,}000 \text{ ft}^3/\text{d})\ (1 \text{ d}) = 74{,}000 \text{ ft}^3, \text{ and} \\ v/Qt_p &= 0.89; \end{aligned}$$

in other words, streamflow reduction accounts for 89 percent of the total pumpage. Eventually, v/Qt_p will become equal to unity as v becomes equal to Qt_p. That is, all the water pumped from the well is water that would have been ground-water discharge to the stream had pumping not occurred. The eventual equality $v = Qt_p$ is a direct consequence of assumption 8, because no changes are possible for other terms in the ground-water budget and because all the water removed from storage eventually will be replaced.

Example 2.--If minimum streamflow standards for Kieffer ditch require that pumping from well I103-1 be stopped when streamflow reduction reaches 25,900 ft^3/d, how long could well I103-1 be pumped and what would be the volume of streamflow reduction during this period? What would be the maximum rate of streamflow reduction, q_{max}, and when would it occur?

Given the information in example 1, what are the values of

t_p,

v at t_p,

q_{max}, and

t of q_{max}?

Figure 31 (curve A) and q/Q can be used to find t_p. If

$$q/Q = \frac{25{,}900 \text{ ft}^3/\text{d}}{74{,}000 \text{ ft}^3/\text{d}} = 0.35, \text{ then}$$

the value of t_p/srf corresponding to q/Q = 0.35 is 0.60. Because srf = 1.41,

$$t_p = (0.60)\ (1.41 \text{ d}) = 0.85 \text{ d}$$

or the well could be pumped for about 21 hours.

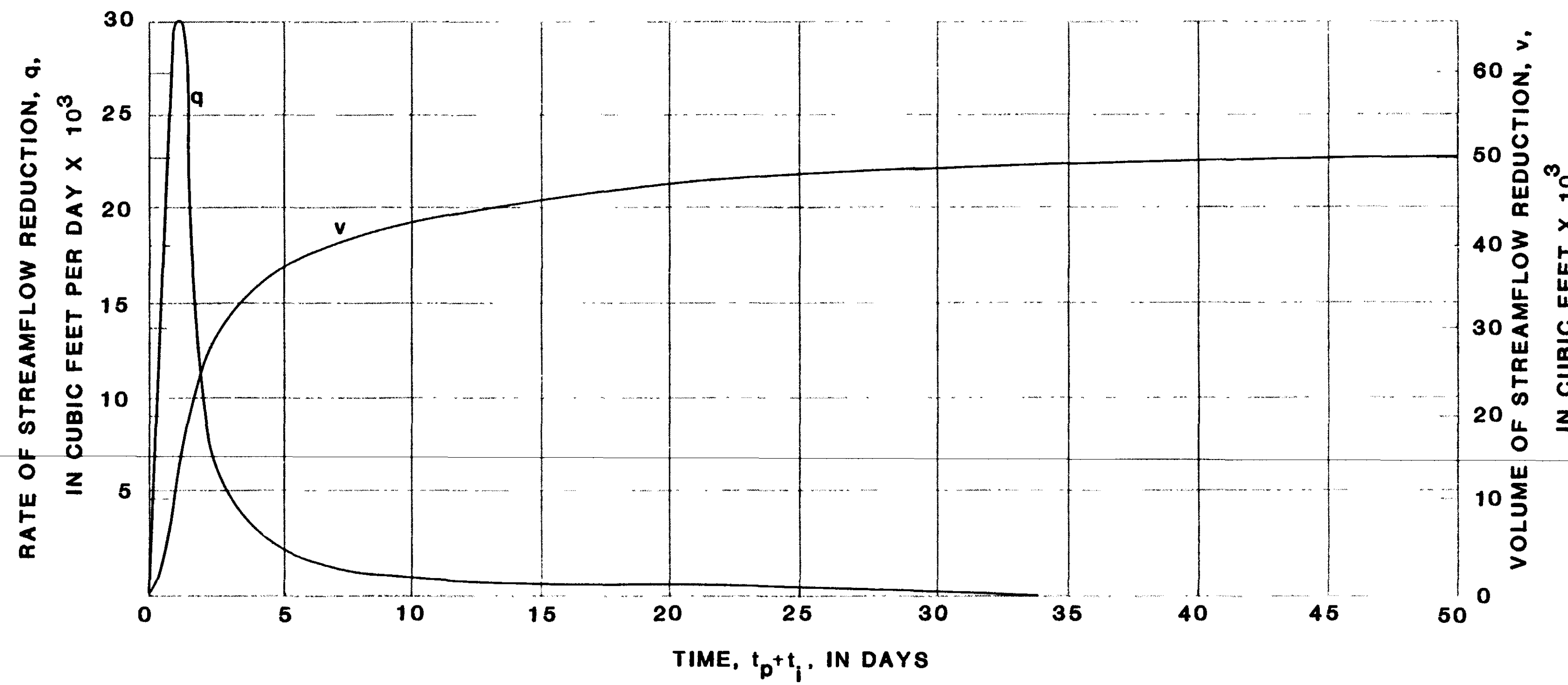


Figure 34.-- Predicted rate and volume of streamflow reduction in Kieffer ditch resulting from 1 day of pumping from well I103-1, Milford area.

If t_p/sdf = 0.60, then, from figure 31 (curve B),

$$v/Qt_p = 0.18$$

which implies that

$$v = (0.18)\ (74{,}000\ ft^3/d)\ (0.85\ d) = 11{,}322\ ft^3.$$

At the end of the pumping period, 11,322 ft^3 of water which normally would have been streamflow would instead be pumped from the well. This volume represents 18 percent of the total well pumpage.

q_{max} can be calculated by first determining the value of q_{max}/Q --the value for q/Q corresponding to the highest part of the curves in figure 32. The highest part of an interpolated curve for t_p/srf = 0.60 is at q_{max}/Q = 0.37 (read from the vertical axis). The corresponding value of $(t_p + t_i)$/sdf would be 0.70 (read from the horizontal axis). The maximum rate of streamflow reduction is

$$q_{max} = 0.37(74{,}000\ ft^3/d) = 27{,}208\ ft^3/d,$$

which occurs at

$$t_p + t_i = 0.70\ (1.41\ d) = 0.98\ d,$$

after pumping was started, or

$$0.98\ d - 0.85\ d = 0.13\ d$$

after pumping was stopped.

Example 3.--Restrictions to streamflow reduction are sometimes based on the total volume of water pumped from a well without regard for when the water is pumped. However, the scheduling of pumping also affects the rate of streamflow reduction.

If the owner of well I103-1 is permitted to pump a total of 1,100,000 ft^3 of water during the irrigation season (90 days), at a pumping rate of 74,000 ft^3/d, then the period of continuous pumping needed would be

$$\frac{1{,}100{,}000\ ft^3}{74{,}000\ ft^3/d} = 15\ d.$$

Alternatively, the same volume could be pumped during three 5-day periods. What would be the difference (a) in the rate of streamflow reduction and (b) in the volume of streamflow reduction during the 90-day irrigation season?

For this example, continuous pumping period begins on day 1 and ends on day 15. The intermittent pumping periods begin on days 1, 31, and 61, and each lasts for 5 days (table 10). The values in table 10 were calculated the same way as those for table 9 except that total q/Q and v/Qsrf at any time during the 90-day period of intermittent pumping are the sums of q/Q and v/Qsrf, respectively, from the three pumping periods.

Table 10.--Summary of computations used to predict the effects of continuous and intermittent pumping from well I103-1 on streamflow reduction in Kieffer ditch

[t_p, time of pumping, in days; t_i, time since pumping stopped, in days; srf, streamflow reduction factor; q, rate of streamflow reduction, in cubic feet per day; Q, rate of pumping, in cubic feet per day; v, volume of streamflow reduction, in cubic feet]

Continuous pumping						Intermittent pumping							
Day 1 - 15 inclusive						Day 1 - 5 inclusive				Day 31 - 35 inclusive			
t_p+t_i	t_p+t_i / srf	q/Q*	v≠ / Qsrf	$q\times10^3$	$v\times10^3$	t_p+t_i	t_p+t_i / srf	q/Q*	v≠ / Qsrf	t_p+t_i	t_p+t_i / srf	q/Q*	v≠ / Qsrf
0	0	0	0	0	0	0	0	0	0	-	-	-	-
5	3.5	.70	1.8	52	190	5	3.5	.70	1.8	-	-	-	-
10	7.1	.80	4.5	59	470	10	7.1	.092	2.3	-	-	-	-
15	10	.81	7.1	60	740	15	10	.039	2.5	-	-	-	-
20	14	.12	7.8	8.9	810	20	14	.023	2.6	-	-	-	-
25	18	.075	8.3	5.5	870	25	18	.016	2.7	-	-	-	-
30	21	.045	8.5	3.3	890	30	21	.012	2.8	0	0	0	0
35	25	.034	8.6	2.5	900	35	25	.0089	2.8	5	3.5	.70	1.8
40	28	.024	8.7	1.8	910	40	28	.0073	2.9	10	7.1	.092	2.3
45	32	.020	8.8	1.5	920	45	32	.0060	2.9	15	10	.039	2.5
50	35	.017	8.9	1.3	930	50	35	.0051	2.9	20	14	.023	2.6
55	39	.015	8.9	1.1	930	55	39	.0043	2.9	25	18	.016	2.7
60	42	.013	9.0	.96	940	60	42	.0038	2.9	30	21	.012	2.8
65	46	.011	9.0	.81	940	65	46	.0034	2.9	35	25	.0089	2.8
70	50	.010	9.0	.74	940	70	50	.0030	3.0	40	28	.0073	2.9
75	53	.0090	9.0	.67	940	75	53	.0027	3.0	45	32	.0060	2.9
80	57	.0080	9.1	.59	950	80	57	.0025	3.0	50	35	.0051	2.9
85	60	.0070	9.1	.52	950	85	60	.0023	3.0	55	39	.0043	2.9
90	64	.0065	9.1	.48	950	90	64	.0022	3.0	60	40	.0038	2.9

*Values obtained from figure 32.
≠Values obtained from figure 33.

Table 10.--Summary of computations used to predict the effects of continuous and intermittent pumping from well I103-1 on streamflow reduction in Kieffer ditch--Continued

Intermittent pumping							
Day 61 - 65 inclusive				Totals			
t_p+t_i	$\frac{t_p+t_i}{srf}$	q/Q*	$\frac{v\neq}{Qsrf}$	q/Q	v/Qsrf	$qx10^3$	$vx10^3$
-	-	-	-	0	0	0	
-	-	-	-	0.70	1.8	52	190
-	-	-	-	.092	2.3	6.8	240
-	-	-	-	.039	2.5	2.3	260
-	-	-	-	.023	2.6	1.7	270
-	-	-	-	.016	2.7	1.2	280
-	-	-	-	.012	2.8	.89	290
-	-	-	-	.71	4.6	52	480
-	-	-	-	.10	5.2	7.4	540
-	-	-	-	.045	5.4	3.3	560
-	-	-	-	.028	5.5	2.1	570
-	-	-	-	.020	5.6	1.5	580
0	0	0	0	.016	5.7	1.2	590
5	3.5	.70	1.8	.71	7.5	52	780
10	7.1	.092	2.3	.10	8.2	7.4	860
15	10	.039	2.5	.048	8.4	3.6	880
20	14	.023	2.6	.031	8.5	2.3	890
25	18	.016	2.7	.023	8.6	1.7	900
30	21	.012	2.8	.018	8.7	1.3	910

*Values obtained from figure 32.
≠Values obtained from figure 33.

A graph of q and v versus $t_p + t_i$ (fig. 35) can be used to compare streamflow reduction at different times for the two pumping schemes. In graph (a), 15 days of continuous pumping can be seen to cause a greater maximum rate of reduction than intermittent pumping causes, but the effect of intermittent pumping continues throughout the 90 days. In both cases, the effect of pumping becomes negligible about 25 days after pumping is stopped. In graph (b), the volume of streamflow reduction approaches a maximum value much sooner with continuous pumping than with intermittent pumping. Toward the end of the irrigation season, the volumes of the two pumping plans approach the same value--that is, the total amount of water pumped from the well. Thus, the rate of streamflow reduction at any time depends on the schedule of pumping, and the volume of streamflow reduction depends only on the volume of pumpage.

Discussion

The preceding examples demonstrate several factors about the relation between ground water and surface water in the Milford area. Water can move quickly through the ground-water system, and pumping a well can affect flow in a nearby stream within hours. The effect of pumping does not stop when pumping stops but might continue for several weeks until the volume of water pumped from the well equals the volume of streamflow reduction. The volume of pumpage determines the volume of streamflow reduction, but the scheduling of pumping influences the timing of streamflow reduction. Intermittent pumping causes a lesser effect for a longer period of time compared to pumping the same volume continuously.

NUMERICAL MODELS

Partial differential equations are available to describe a wide variety of ground-water situations. However, analytical solutions are available for only a few of these equations and may not be helpful in dealing with the complexities of heterogeneous and irregularly shaped aquifers often found in field situations (Freeze and Cherry, 1979, p. 352). Numerical models offer a logical alternative to the more constrained analytical models.

Numerical models provide approximate solutions to partial differential equations by using a set of equations in which time and area are discrete variables. The set of discrete equations is solved by one of two numerical methods--finite-difference method or finite-element method. A description and comparison of the two methods is given by Faust and Mercer (1980).

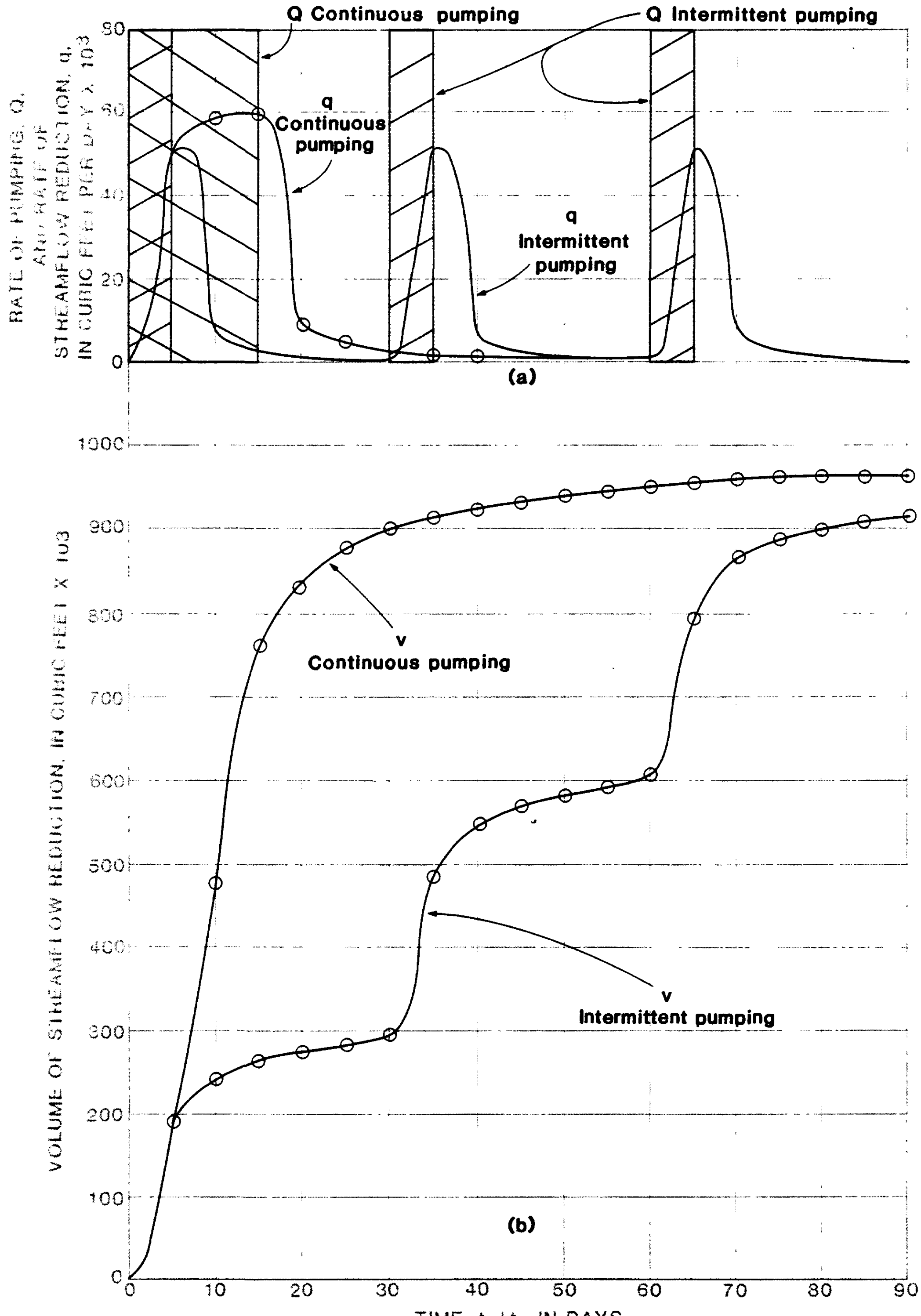


Figure 35.– Predicted rate and volume of streamflow reduction in Kieffer ditch resulting from 15 days of continuous and intermittent pumping from well I103-1,Milford area.

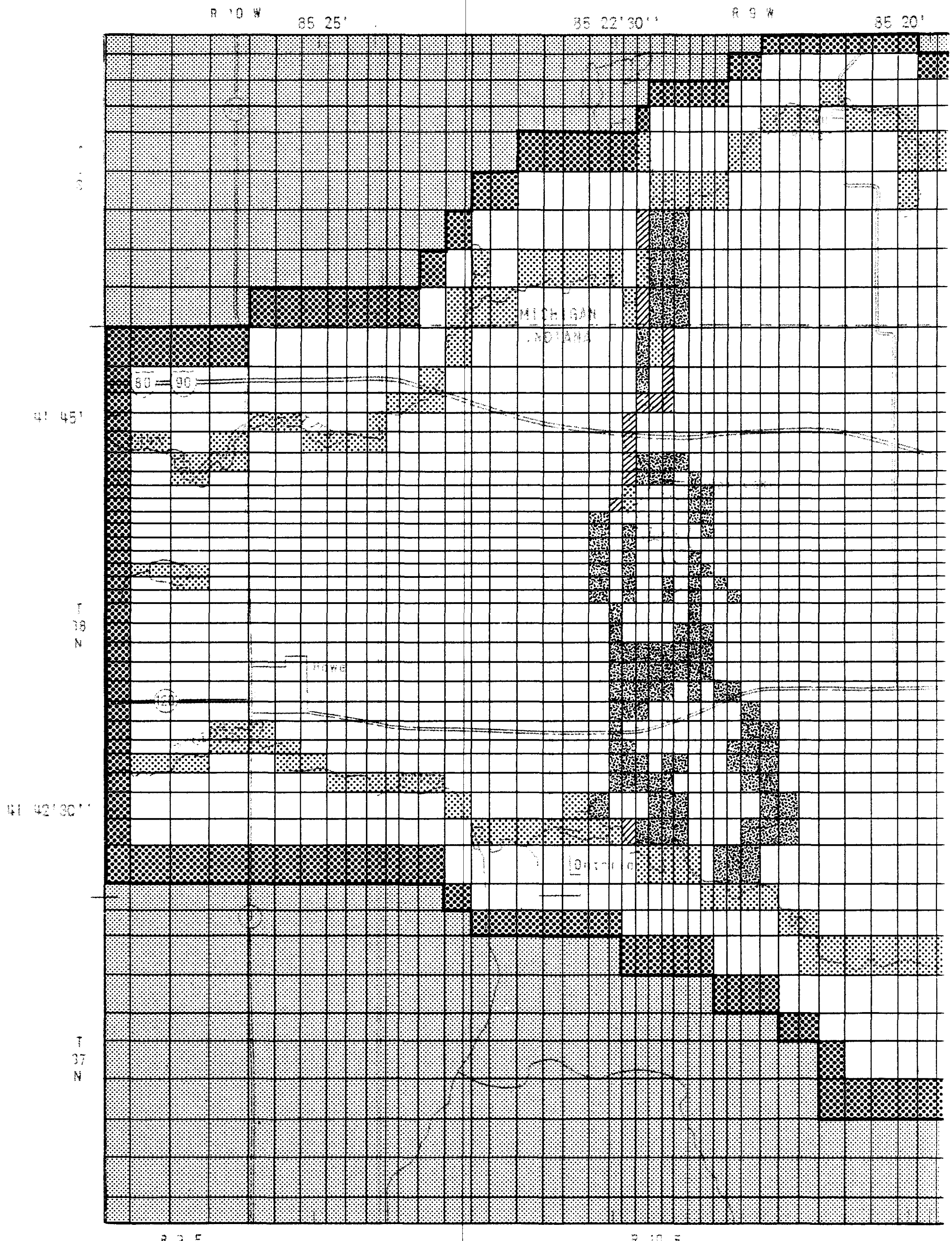
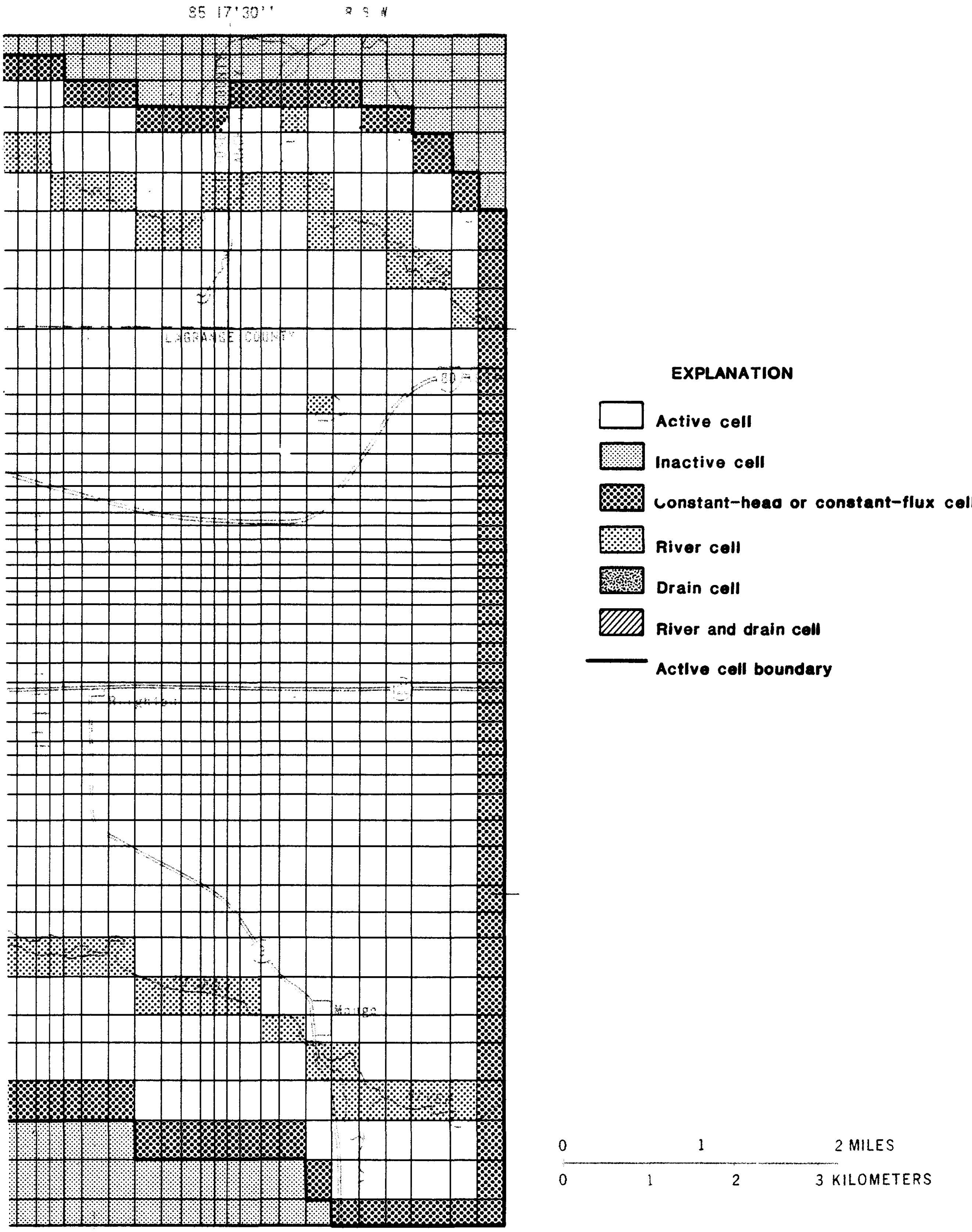


Figure 36.-- Numerical model grid for the Howe area.



Modified from Bailey, Greeman, and Crompton, 1985

The area to be modeled is subdivided by a grid into cells which can differ in size. The center of each cell is called a node, and the hydrogeologic conditions at node are assumed to extend throughout the rest of the cell. A set of flow equations is solved for each node, so that a numerical model is not restricted to one set of hydrogeologic conditions as is the case with an analytical model. Moreover, the spacing of nodes can vary to accommodate differences in available data and (or) accuracy requirements from one area to another. A water budget for each node is also calculated. These qualities provide greatly increased flexibility to numerical models compared to analytical models.

A three-dimensional, finite-difference model developed by McDonald and Harbaugh (1984) was used to simulate the effects of water withdrawals on water resources near Howe (Bailey and others, 1985).

The area of study is 46.5 mi^2 and is intensively irrigated, primarily with ground water. Glacial deposits composed mostly of sand and gravel (fig. 26) as much as 350 ft thick overlay impermeable shale bedrock. Interbedded clay of variable thickness and areal extent is common throughout the area.

The modeled area was subdivided into cells of different dimensions (fig. 36). For example, the cells around Cedar Lake in the middle of the study area were small (500 ft by 500 ft) because of the interest in the effects of pumping in this area. At the boundaries, where less detail was needed, the cells were larger (1,500 ft by 1,500 ft).

Certain constraints were applied to nodes in which differing hydrologic conditions were simulated. For example, drain nodes, which permitted water only to discharge from the ground-water system, where used to simulate the marshes around Cedar Lake. River nodes were used to simulate water either discharging from or recharging to ground water. For calibration of the model, the boundary nodes were constrained by constant heads--that is, the potentiometric head in each boundary node was held constant.

For the assumption of constant-head boundaries to be valid during periods of pumping, the model boundaries were established away from the centers of pumping. The northern and southern boundaries were about 0.5 mi north of Fawn River and south of Pigeon River. The eastern and western boundaries were as far as practical from the high-production wells on either side of Cedar Lake.

The complex glacial geology of the area was simplified vertically into three aquifers (fig. 37). Aquifer 1, the uppermost aquifer, is unconfined throughout most of the area. Aquifer 2 is separated from aquifer 1 by discontinuous clay and is confined throughout most of the area. Aquifer 3 is composed of sand, silt, and clay having reduced permeability near the bedrock surface.

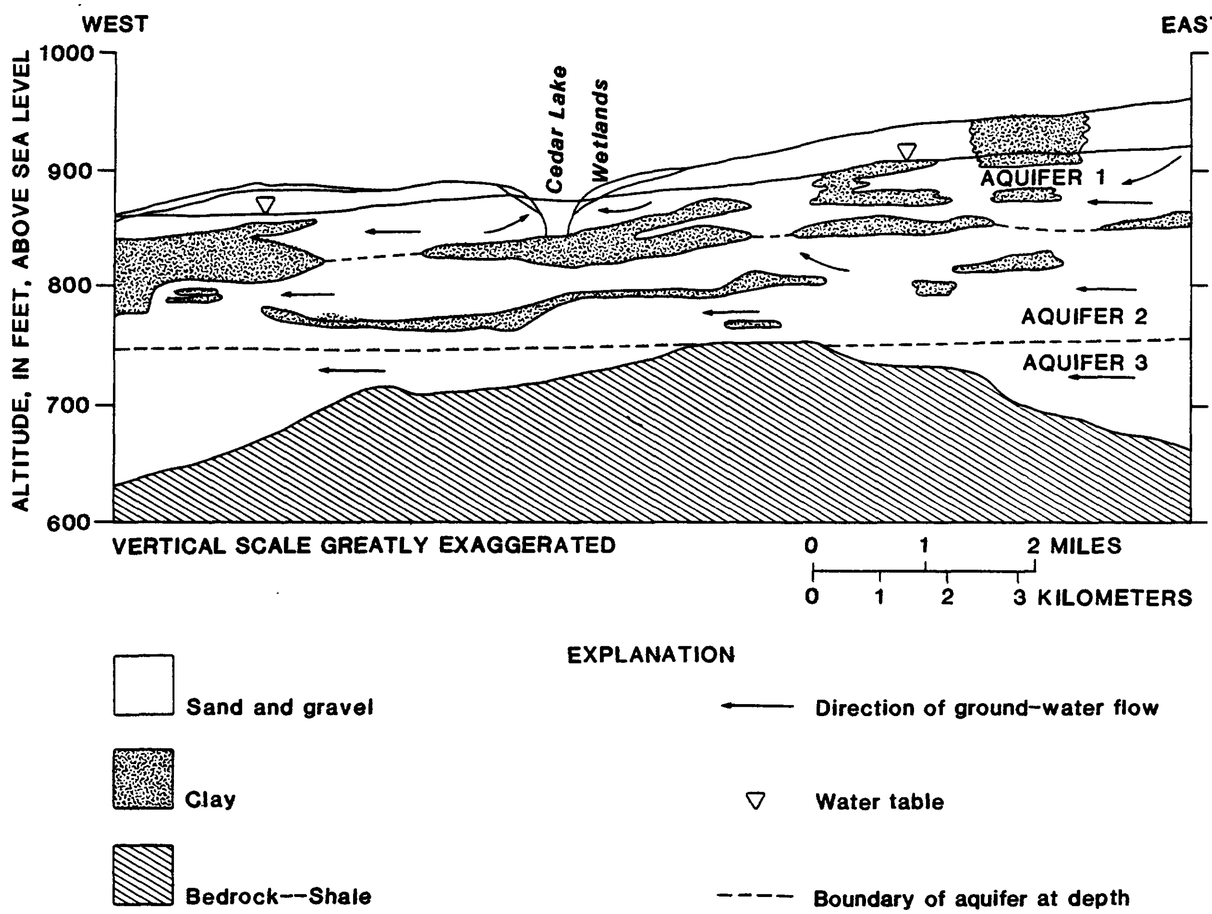


Figure 37.– Generalized geologic section of the Howe area.

Calibration

As with analytical models, numerical models require calibration to estimate the properties of aquifers being modeled. Unlike the analytical models in which only one value each of transmissivity and storage coefficient must be used for the entire aquifer, digital models can be used to account for areal variations in these properties.

If measurements of transmissivity and storage were available for each nodal position, model calibration would be quite simple. In practice, this is almost never the case, and a trial-and-error procedure is used to arrive at acceptable values of aquifer parameters. The calibration procedure (fig. 38) involves the use of initial estimates of aquifer parameters (such as transmissivity) to compute aquifer performance (such as drawdown) and flow components (such as streambed seepage). If the difference between computed and measured aquifer performance is unacceptable, values of the aquifer parameters are adjusted and a different aquifer performance is computed. The process is repeated until the difference between computed and measured performance is acceptably small. Error analysis can be as simple as a root-mean-square analysis (Bailey and Imbrigiotta, 1982, p. 44) or as sophisticated as a formal optimization procedure (Neuman, 1973). Adjustments to values of parameters should be within the range of measured values or at least compatible with values reported for other similar hydrogeologic settings.

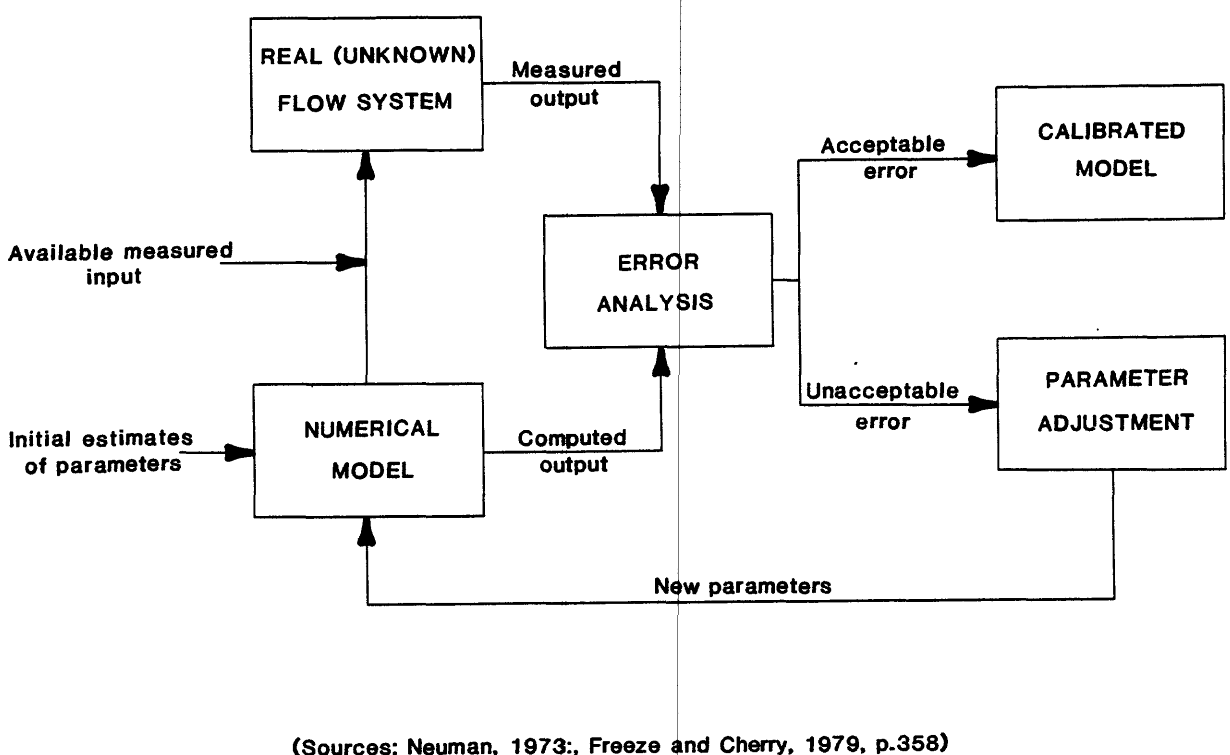


Figure 38.-- Trial-and-error process for calibration of a numerical model.

The Howe model was calibrated by comparing the values of water levels and streambed seepage calculated by the model with values measured in autumn 1982 when water levels were changing very slowly and the hydrologic system was assumed to be in equilibrium (steady state). During calibration, initial values of variables needed for the model were changed to see if the match between calculated and measured water levels and streambed seepage could be improved. Values of recharge, hydraulic conductivity of the aquifers, leakage between aquifers, and hydraulic conductivity of stream beds were varied over reasonable ranges. In general, initial values of these variables provided acceptable matches and were used in the final calibration.

The choice of which combination of input values to use is not always obvious. Two criteria were used to evaluate calibration--water levels and streambed seepage. A set of input values that provided a close match for water levels sometimes provided an unacceptable match for streambed seepage. Another difficulty in deciding on the best combination of values is the fact that various combinations can produce identical outputs; that is, solutions to the algebraic equations are not unique. Sound hydrogeologic judgment, in addition to field data, is needed to choose the input values that best match the real system.

For the Howe model, changes in input values result in larger changes in stream seepage than in water levels. For this reason, stream seepage was given more weight than water levels as a test criterion during the calibration. For the final calibration, calculated seepage to Pigeon River was 38 ft^3/s compared to 41 ft^3/s measured in autumn 1982, whereas the average difference between calculated and measured water levels over the entire area was about 6 ft (Bailey and others, 1985).

Prediction

After the model is calibrated, it can be used to predict how the hydrologic system will react to differing hydrologic stresses. The Howe model was constructed to simulate the effects of increased use of water, primarily for agricultural irrigation. Five hypothetical pumping plans were devised that simulated irrigation in response to several irrigation schemes and amounts of precipitation. These combinations produced four levels of water use during the irrigation season (June, July, and August). Criteria for the five pumping plans are presented in table 11 and are discussed by Bailey and others (1985).

Table 11.--Five hypothetical pumping plans for irrigation simulated by the numerical model for the Howe area

Pumping plan	Condition of precipitation	Application of water[1] (inches)	Irrigated land (acres)
1	above normal	4.0	8,259 (same as 1982)
2	normal	7.2	8,259 (same as 1982)
3	below normal	9.7	8,259 (same as 1982)
4	normal	7.2	22,336 (maximum available)
5	below normal	9.7	22,336 (maximum available)

[1]Water added by irrigation in addition to precipitation.

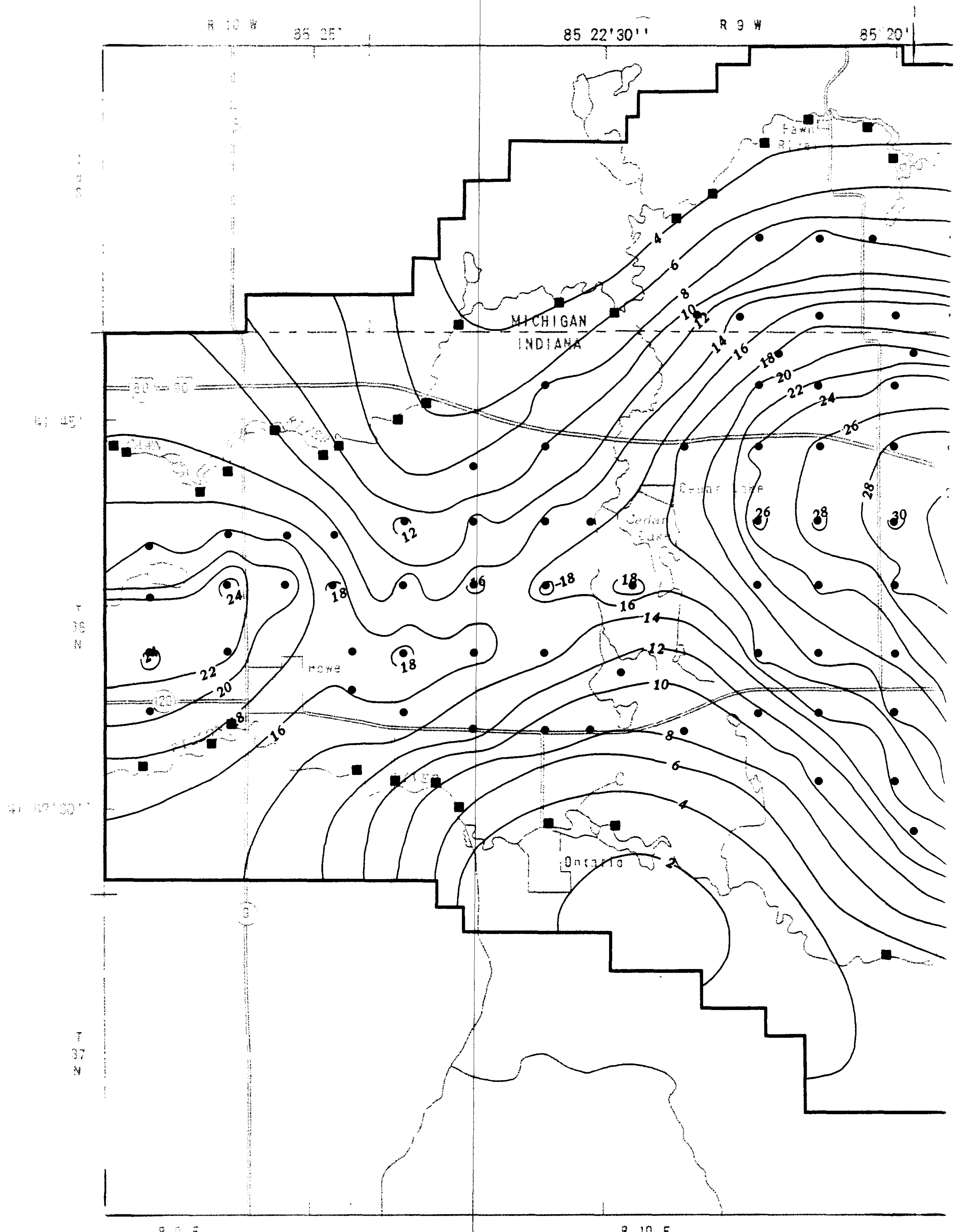
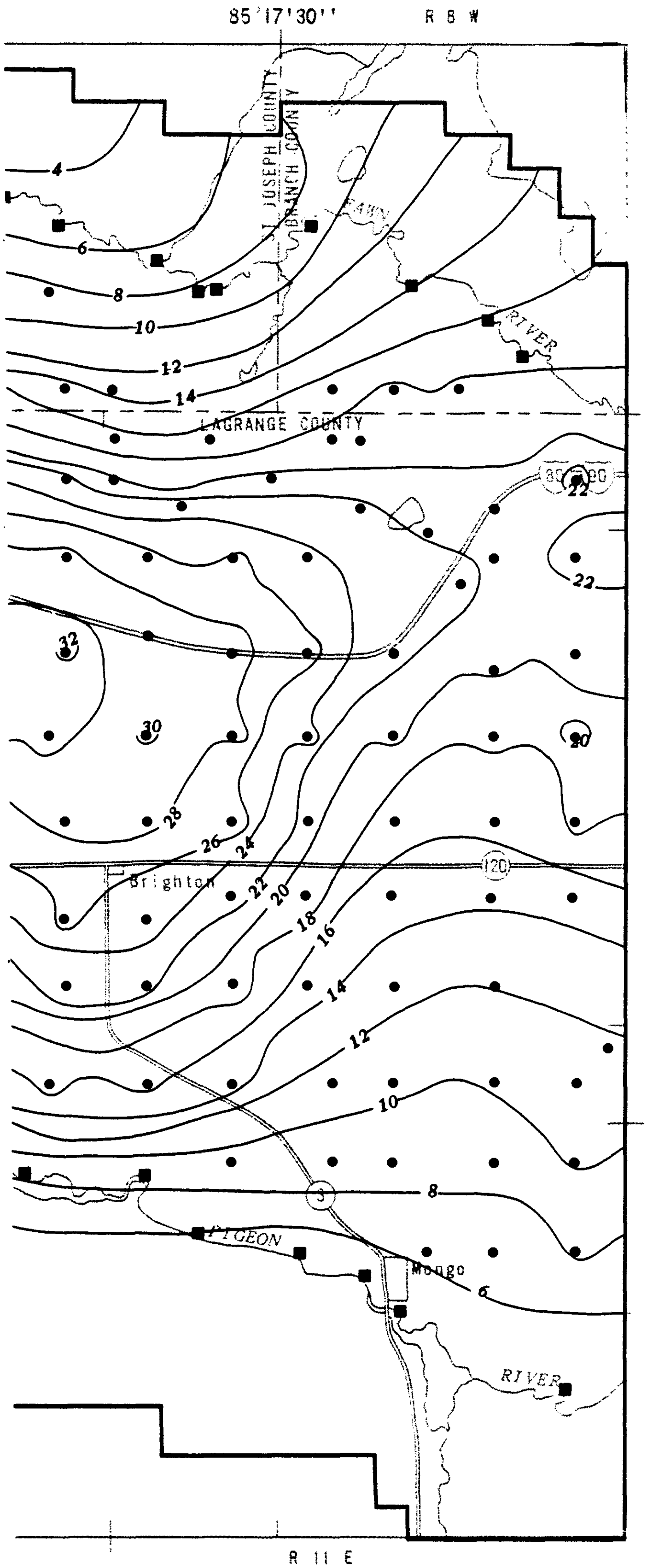


Figure 39.-- Drawdown in aquifer 2 from pumping plan 5 calculated by



Modified from Bailey, Greeman, and Crompton, 1985

EXPLANATION

—2— Line of equal drawdown. Layer 2. Interval 2 feet

● Simulated well

■ Simulated surface-water pump

▬ Active-cell boundary

0 1 2 MILES

0 1 2 3 KILOMETERS

e numerical model for the Howe area.

The computer output from the numerical model is of two types: (1) head distribution and (2) water budget. Values for both types of output can be calculated for each node in the model. Values of potentiometric head can be used to construct contour maps of drawdown for each aquifer. Figure 39 is a map of drawdown in aquifer 2 from pumping plan 5 after 27 days of continuous pumping. This type of map is useful in determining the location and magnitude of drawdown for a given set of pumping conditions.

A ground-water budget for the entire area is obtained by algebraically summing the individual water-budget components for each node. Water budgets for the model calibration and the five pumping plans are presented in table 12. The budgets contain four source terms and four discharge terms. Recharge was assumed to be constant (11.3 in) for each application of the model.

Constant-flux and constant-head boundaries were used for simulation. Constant-flux boundaries allow for only a predetermined amount of water to flow across active boundary nodes. Constant-head boundaries allow for a constant head to be maintained at the active boundary nodes. If ground-water recharge (and discharge) outside the boundaries remains constant, then constant-flux boundary nodes simulate the minimum amount of water that can be induced across boundaries by pumping within the modeled area, and constant-head boundaries simulate the maximum amount of water induced by pumping.

These differences are illustrated by comparing the sources of water for pumping for the two boundary conditions in pumping plan 5 (table 12). For constant-flux boundaries, the flow across the boundary nodes is limited to 58 ft^3/s and most of the water for pumping (60 percent) is supplied by water from storage. However, for constant-head boundaries the pumping induces increases in flow across the boundary nodes, so that for pumping plan 5, water supplied from the boundaries is greater than that supplied from storage.

Examining the two types of boundary conditions also provides insight into the range of effects of pumping on ground-water levels and streamflow. Pumping under constant-flux boundary conditions results in more water being removed from storage and drawdowns being greater compared to removal and drawdown pumping under constant-head conditions. For the five pumping plans, maximum drawdowns for constant-flux boundary simulations were greater than or equal to maximum drawdowns for constant-head boundary simulations, though differences were generally less than 2 ft (Bailey and others, 1985).

Streamflow reduction is also greater for constant-flux boundaries than for constant-head boundaries. Streamflow reduction for a pumping simulation is calculated as the increase in water seeping out of the stream channel plus the decrease in water seeping into the stream channel compared to values calculated for calibration. As an example, streamflow reduction in Pigeon River for pumping plan 5 (constant-flux boundaries) is calculated using information in table 12 as follows:

Streamflow reduction = (11 ft^3/s - 7 ft^3/s) + (85 ft^3/s - 49 ft^3/s) = 40 ft^3/s.

Table 12.--Water budgets from calibration and five hypothetical pumping plans calculated by the numerical model for the Howe area

[ft^3/s, cubic feet per second; CF, constant flux; CH, constant head. Table modified from Bailey and others (1985)]

Model application	Boundary	Sources (ft^3/s)					Discharges (ft^3/s)				
		Storage	Boundary flux	Recharge	Stream seepage	Total water	Boundary flux	Wells	Stream seepage	Drains	Total water
Calibration	CH	0	58	39	7	104	16	2	85	1	104
Pumping plan 1	CF	61	58	39	7	165	17	70	78	2	167
	CH	52	71	39	7	169	17	71	80	2	170
Pumping plan 2	CF	62	58	39	7	166	15	75	75	2	167
	CH	51	73	39	7	170	15	77	77	2	171
Pumping plan 3	CF	60	58	39	7	164	16	74	73	2	165
	CH	49	74	39	7	169	15	78	74	2	169
Pumping plan 4	CF	166	58	39	11	274	17	204	52	2	275
	CH	116	116	39	11	282	11	205	65	2	283
Pumping plan 5	CF	161	58	39	11	269	14	205	49	2	270
	CH	112	120	39	8	279	8	205	65	2	280

For the five pumping plans, streamflow reduction was greater for constant-flux boundaries than for constant-head boundaries (table 13). In plan 5, the difference in streamflow reduction between the two boundary conditions is nearly 100 percent. Thus, constant-flux boundaries simulate the maximum effect of ground-water pumping on both drawdown and streamflow reduction while constant-head boundaries simulate the minimum effect.

Table 13.--Streamflow reduction, in cubic feet per second, by ground-water pumping for five hypothetical pumping plans calculated by the numerical model for the Howe area

Boundary condition	Pumping plan				
	1	2	3	4	5
Constant flux	7	10	12	37	40
Constant head	5	8	11	24	21

Because the water budgets for individual model nodes can be summed algebraically, water budgets for selected areas of the model can be calculated. One application of this technique is to examine the effect of pumping on streamflow reduction in a single stream. This application is particularly useful in assessing whether different pumping plans might reduce flow below an established minimum value.

Net streamflow for Pigeon River (table 14) was calculated by subtracting the streamflow reduction in Pigeon River from the natural flow at the western (downstream) edge of the study area (fig. 2). [Note: For all the pumping plans, irrigated land within 0.5 mi of major stream channels was assumed to be irrigated with water pumped directly from the stream. Streamflow reduction in table 14 includes a reduction in ground-water flow to the stream, an increase in streambed seepage to the underlying aquifer, and water pumped directly from the stream channel.] The net streamflow can be compared to a predetermined value of minimum flow to predict whether the rates of pumping would cause streamflow to be reduced below the minimum flow. The 7-day, 10-year low flow, $Q_{7,10}$ is a minimum flow commonly used for streamflow regulation. $Q_{7,10}$ is the lowest flow that occurs during a continuous 7-day period, on the average, once every 10 years. For Pigeon River, $Q_{7,10}$ at the west edge of the study area was estimated to be 69 ft^3/s (Bailey and others, 1985). Net streamflow in Pigeon River for pumping plan 5, where constant-flux boundaries were assumed, is 118 ft^3/s (table 14) or 1.7 times the $Q_{7,10}$. Under the conditions described for the pumping plans, additional use of ground water is possible without violating of the $Q_{7,10}$ standard for Pigeon River.

Table 14.--Changes in streamflow in Pigeon River attributed to ground-water pumping for five hypothetical pumping plans calculated by the numerical model for the Howe area

[ft^3/s, cubic feet per second; CF, constant flux; CH, constant head]

Pumping plan	Boundary	Natural streamflow[a] (ft^3/s)	Streamflow reduction[b] (ft^3/s)	Streamflow reduction[b] Percent of natural flow	Estimated net streamflow (ft^3/s)
1	CF	156	8	5	148
	CH	156	6	4	150
2	CF	211	10	5	201
	CH	211	9	4	202
3	CF	150	11	7	139
	CH	150	6	4	144
4	CF	211	30	14	181
	CH	211	21	10	190
5	CF	150	32	21	118
	CH	150	24	16	126

[a]Values of natural streamflow are based on measured flow and flow duration analysis described by Bailey and others (1985).
[b]Includes pumping directly from Pigeon River--5 ft^3/s for pumping plans 1, 2, and 3 and 15 ft^3/s for pumping plans 4 and 5.

At the end of the pumping period, the effect of pumping directly from the stream channel on streamflow ends immediately. However, the effect of pumping from wells continues for some time, and the maximum streamflow reduction attributable to ground-water pumping may not be recorded until several days after pumping is stopped. Thus, at the end of the pumping period net streamflow would increase quickly by the amount of pumping from the channel, but net streamflow would not approach natural streamflow until some time later.

COMPARISON OF ANALYTICAL AND NUMERICAL MODELS

For any water-resource evaluation, the decision as to what type of model, if any, to use for a particular application is based on factors such as the type of questions to be answered, the availability of time and money, expertise of the staff, the nature of the hydrologic system, and the required accuracy of results. This section compares characteristics of analytical and numerical models that need to be considered in the decision-making process. Characteristics of the models are compared in terms of methods of simulation, requirements for data, and type of information provided (output). The differences and similarities provide a basis for discussion about applications most appropriate to each type of model. In the final part of the section, methods for determining the accuracy of models are presented.

Constraints placed on a model when it is developed affect how well the model can simulate a given hydrologic system. The differences in constraints on an analytical model compared to those on a numerical model can be demonstrated by observing how the two models might be used to simulate the same hydrologic system.

The complexity of the glacial geology in the basin is illustrated in the generalized block diagram in figure 40. Discontinuous clay beds (tills) are interspersed in a heterogeneous matrix of sand, gravel, and silt (outwash). Perennial streams generally penetrate only part of surficial sand and gravel aquifers. Figure 40 represents part of an aquifer system as it might actually exist.

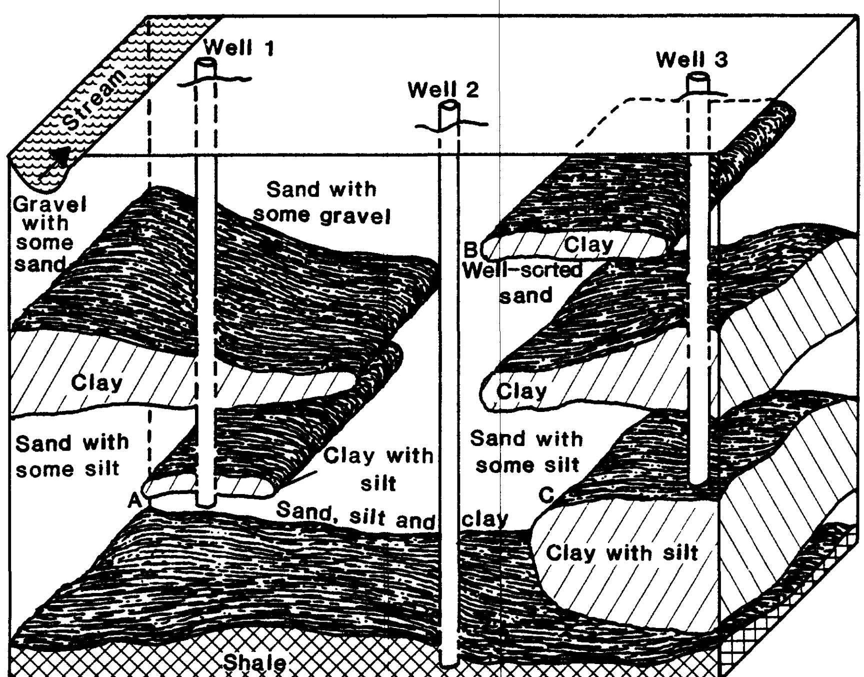


Figure 40.- Generalized lithology of glacial deposits of the type in the St. Joseph River basin, Indiana.

The following discussion describes how this system might be simulated by each type of model. The assumption is made that the only information known to the modeler about the lithology of the area would be from geophysical logs of three wells. Information from the log for well 1 would indicate that the well penetrates layers of gravel with some sand, gray clay, sand with some silt, clay with silt, and sand with some silt. Well 2 is drilled to bedrock. The log for well 2 would show no clay layer but would indicate that the amount of fine material increases with depth. The log for well 3 would show a layer of gray clay at about the same altitude as the gray clay in the log for well 1. Logs for the three wells would also show that the surficial material becomes coarser close to the stream.

An analytical model used for this area might simulate the lithology as presented in figure 41. The discontinuous gray clay could be simulated as a continuous, leaky confining layer with a single average value of K'. Average values for T and S could be used for each of the aquifers above and below the gray clay. The stream, if simulated, would be assumed to penetrate the entire depth of the unconsolidated material. Thus, use of an analytical model would require major simplifications of the lithology for this type of geologic setting.

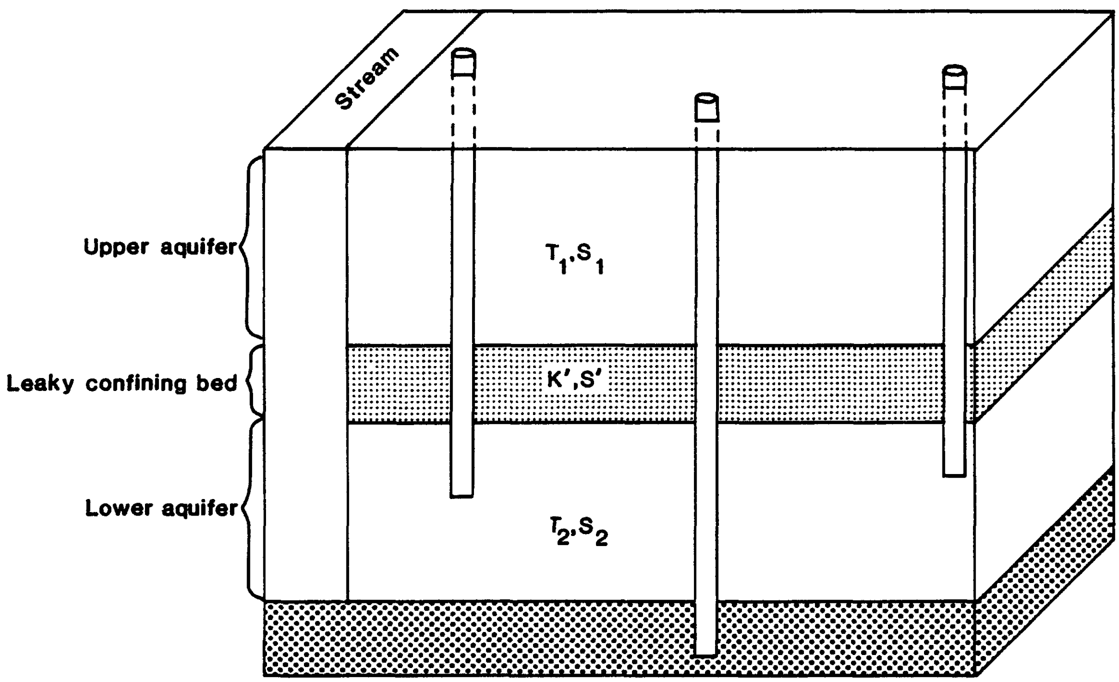


Figure 41.– Lithology of part of a glacial aquifer system as simulated by an analytical model.

For a numerical model, the lithology might be simulated as in figure 42. Greater flexibility in simulating the known clay layers is possible with a numerical model than with an analytical model. Separate values of hydraulic properties would be possible for each node; however, the number of values is limited by the amount of available data used to define the areal variation. For example, three values of T and S obtained from the three wells could be used in aquifer 1 but only one value of T and S would be available for aquifer 3. If the stream were simulated, the modeler could assume it completely penetrated only aquifer 1. The fewer constraints of the numerical model permit a more accurate representation of a complex outwash system than does an analytical model.

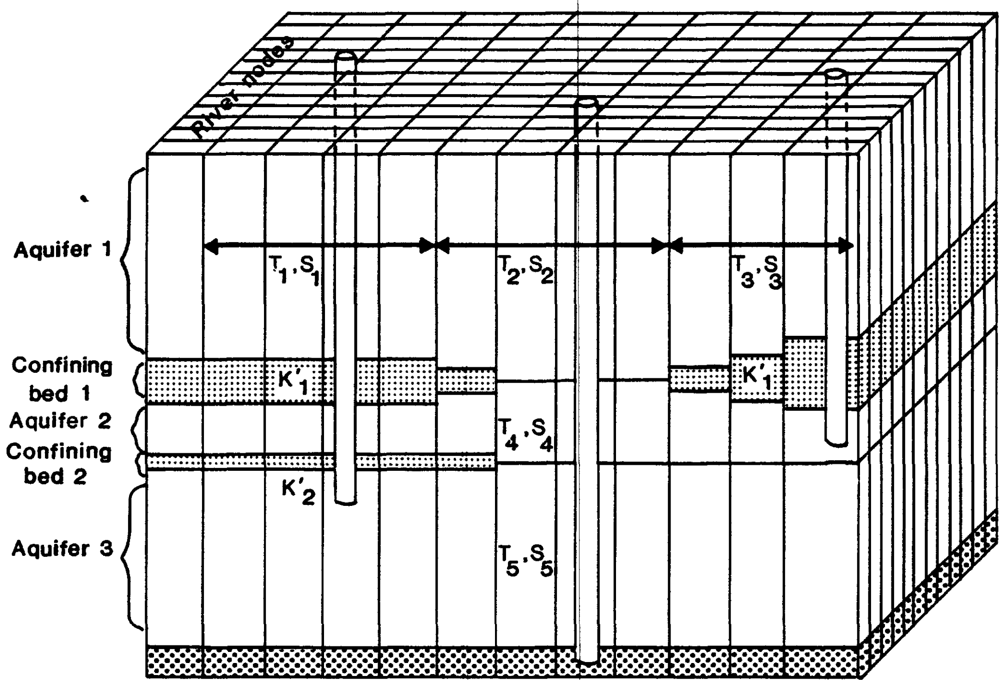


Figure 42.– Lithology of part of a glacial aquifer system as simulated by a numerical model.

For numerical models, errors caused by improperly defining lithology are usually more serious than errors caused by model constraints. Examples of improper lithologic interpretation can be noted by comparing figures 40 and 42. The presence and extent of clay lenses in the outwash would be difficult to discern in all cases. The clay lenses at locations B and C (fig. 40) would not be detected from well-log information. At location A, the extent of the clay is not known and might be assumed (incorrectly) to extend to the left (fig. 42).

Incorrect interpretations of lithology might be detected during calibration if heads computed for certain areas by the model do not compare favorably to those measured in the field. However, for areas where lithology is locally diverse, the amount of head data needed to detect deviations between actual and simulated lithology might be beyond the scope of a regional model. An undetected clay lens could cause local confining conditions resulting in reduced transmissivities and storage coefficients. These factors could cause large discrepancies between actual and predicted heads (Bailey and others, 1985). If improved prediction is required in local areas, additional lithologic and head data would be needed, and the number of nodes for those areas would need to be increased.

DATA REQUIREMENTS

Generally, fewer data are needed for analytical models then for numerical models. Most often, the differences in data requirements result from the differences in application for which the two types of models are best suited. Analytical models are more often used for aquifer-test analysis or for drawdown prediction involving one or a few wells during relatively short periods of time. In these cases, the area affected by pumping is usually no more than several square miles, and the aquifer properties within that area are likely to be homogeneous. Digital models are more often used for regional systems involving many wells and more variation in aquifer characteristics, recharge, water surface altitude, and boundary conditions.

The analytical and the digital models used to evaluate water resources near Howe can be used as an example of the differences in the data used to calibrate the two types of models. The three analytical models for the Howe area were calibrated by using aquifer-test data. Driller's logs for several wells near each pumping well were used to determine lithology and to select an appropriate analytical model. Drawdown data from nearby observation wells were also needed for the analytical models (table 15). The area simulated by the numerical model is considerably larger, and information is required from many more sites to define the lithology and boundary conditions. Initial estimates of aquifer and streambed properties were needed as well as estimates of areal recharge.

The types of data needed for individual analytical and digital models can differ greatly. However, the examples cited point out two differences in data requirements commonly found between the two types of models. First, well characteristics that are used to analyze the effect of well hydraulics on observed drawdown data are incorporated into analytical models, whereas well hydraulics are generally ignored in numerical models. Second, numerical models provide water budgets that can be used to estimate changes in such factors as recharge and underflow; and data to support these estimates are required. These factors are generally assumed to be constant for analytical modeling, and estimates of recharge or underflow are not needed.

Whenever possible, the values of aquifer properties used for calibrating a numerical model should be obtained from analytical methods based on field data whether the data are collected in the study area or in other similar areas. Too often the input for numerical models is not based on observed data so that the output may not adequately represent the area being modeled. Data should "create" models, not the other way around.

Table 15.--Characteristics of data used to calibrate four analytical models for the Howe and Milford areas and one numerical model for the Howe area

Type of data	Analytical models	Numerical model
Area of model(s)	Less than 1 mi^2	65 mi^2
Lithology: thickness and extent of aquifers and confining beds	6 to 11 points of observation	250 points of observation
Water levels in observation wells	Measurements made in at least one observation well; drawdown at various times times during pumping period	One measurement made in each of 51 different wells during non-pumping period
Hydraulic properties of:		
Aquifers and confining beds	None	Initial estimates from three aquifer tests and calculations using specific-capacity data from 24 wells
Streambeds	None	Initial values obtained from literature
Recharge	None	Initial values obtained from literature for four types of surficial material
Streambed seepage	None	One measurement made at each of 24 sites
Well hydraulics	Pumping rate, length and location of well screens, distance between pumping well and observation well	None

TYPE OF OUTPUT

The output from analytical models is in three forms.

(1) Estimation of hydraulic properties--calibration.
(2) Simulation of hydrologic stress--prediction.
(3) Evaluation of well hydraulics.

The calibration of analytical models provides estimates of the hydraulic properties of aquifers and confining beds near the pumping well as described on pages 14-16. These properties include T, S, K_z/K_r, and K'. Only one "average" value of these properties is calculated by the model for each aquifer or confining layer (table 16). When used for simulation, analytical models predict one of the following variables (if the others are known): s, t, r, and Q. Water-budget terms such as recharge and underflow are assumed to be constant so that changes in them cannot be simulated. Evaluation of well hydraulics involves several factors that affect well performance and, thereby, affect drawdown data used in analytical models. Most important among these factors are partial penetration and well loss. The degree of the well's penetration of the aquifer is especially important in highly anisotropic aquifers (Weeks, 1977). Analytical equations which use water levels from piezometers or observation wells are available to compute the effect of partial penetration. (See, for example, the section "Model of Leaky, Confined Aquifer and Partially Penetrating Well".) Drawdown data from partially penetrating wells can also be used to estimate the degree of anisotropy in aquifers (Weeks, 1964, 1969; Mansur and Dietrich, 1965). Well loss from turbulence in the well bore causes greater drawdown than predicted by analytical models, in which laminar flow is assumed. The effects of well loss on drawdown can be evaluated by using step-drawdown tests. For descriptions of methods for conducting step-drawdown tests and for analyzing results, see Jacob (1947), Rorabaugh (1953), and Lennox (1966).

Numerical models can provide estimates of hydraulic properties for each node (table 16). Values of drawdown and all fluxes in the water budget are calculated at each node and at each time step. These values represent discrete "average" values for the entire cell. The hydraulic properties of individual wells are generally not simulated in numerical models. Unlike analytical models, numerical models can be used to generate flow nets that are used in estimating the velocity and volume of ground-water flow in specific areas of aquifers. Flow nets are particularly useful in evaluating the movement of contaminants within the aquifer.

Table 16.--Characteristics of output from analytical and numerical models

Type of output	Analytical model	Numerical model
Hydraulic properties of aquifers and confining beds	One value for each aquifer or confining bed	One value for each node
Drawdown	Calculated as a continuous variable in time and space	Calculated as a discrete variable for each node and time step
Water budget	Assumes most fluxes are constant in time and space	All fluxes can be calculated for each node and time step
Hydraulic properties of wells	Ability to incorporate effects of partial penetration, well loss, and well storage	Well hydraulics are usually ignored
Flow nets	Not applicable	Velocity and volume of flow can be calculated for each node

APPLICATION

Comparisons between analytical and numerical models in terms of methods of simulation, data requirements, and output can be used in establishing the guidelines for appropriate applications of the two types of models (table 17). Analytical models are more appropriate for aquifer-test analysis or prediction over short periods of time, in simple geologic settings, and in local areas. Digital models are more appropriate for long-term, ground-water assessments of large areas with wide ranges of aquifer properties, geometry, boundary conditions, and water-budget fluxes.

These general guidelines should be evaluated as a whole to be useful. For example, it might be desirable to use a numerical model to simulate a simple hydrologic system with only a few wells pumping for short periods of time if the modeler is evaluating the effects of variable recharge on changes in streamflow. An analytical model might be appropriate to obtain initial estimates of regional drawdown when assumptions can be made that the aquifer is homogeneous and recharge is constant. In practice, the choice of model to use involves balancing the questions to be answered with the available resources of time, money, and data and the required level of accuracy.

Table 17.--Guidelines for use of analytical and numerical models

	Analytical model	Numerical model
Area	Small, local (acres to several square miles)	Large, regional (many square miles)
Geology	Simple, homogeneous	Complex, heterogeneous
Time of simulation	Short (hour to several days)	Long (many days to several years)
Number of pumping wells	Few (less than 10)	Many (tens to hundreds)
Boundary conditions	Simple, linear	Complex, variable
Water budget	Recharge and discharge are constant	Recharge and discharge are variable
Well hydraulics	Includes, degree of well penetration, well loss, and well storage	Generally ignored
Aquifer-test analysis	Quickly done if response curves are available for aquifer system	Can be used to generate response curves if analytical model is not available

ACCURACY

The accuracy of a model is a measure of how well it will predict the effect of a hydrologic stress (for example, pumping) on a hydrologic property (for example, water levels) under differing hydrologic conditions. Accuracy is determined through a process of model verification in which a second set of observed data, independent of the set used for calibration, is compared with values calculated by the model. The hydrologic conditions under which the observed data used for calibration and verification were collected, should be different. The greater the difference in the hydrologic conditions, the greater the useful range of the model.

Verification of analytical models is not commonly discussed in literature. However, analytical models can be verified by doing an aquifer test using values of model variables (for example, Q or r) different from the values used to calibrate the model. Curve matching is used to confirm (or reject) the results of the calibrated model. If the verification procedure does not yield acceptable results, the hydrologic system needs to be reevaluated and a more appropriate analytical model (if one exists) should be selected.

Data from three observation wells with three separate values of r were used to calibrate the analytical model for a leaky confined aquifer and fully penetrating well (see p. 31). Curve matching by using the three sets of data is a form of verification. The good agreement between the three sets of results provides an increased level of confidence in the model.

The verification procedure for numerical models involves using a set of measurements, water levels, and stream seepages collected under different hydrologic conditions than those used for calibration. If the match between computed and measured values used for verification is acceptable, the model is considered verified. If not, the model should be recalibrated. The recalibration process requires a reevaluation of boundary conditions, hydraulic properties, aquifer geometry, and (or) fluxes in the ground-water budget and may require the collection of additional field data.

The numerical model for the Howe area could not be verified because only one set of observed water-level measurements and streamflows was available. Thus, simulations using this model cannot be taken as precise predictions of hydrologic response until verification is possible. Rather, the results of the unverified model provide useful indications of what may occur even though the accuracy of the model is unknown (Soukup and others, 1984).

SUMMARY AND CONCLUSIONS

This report provides information about the use of analytical and numerical models for water-resource assessment in the St. Joseph River basin in Indiana. Four analytical models for analysis of aquifer-test data from glacial aquifer systems in two areas of the basin are described in detail. Analytical models that would be useful for other hydrogeologic settings in the basin are cited. Analytical models and numerical models are compared in terms of methods of simulation, data requirements, output, applications, and determination of accuracy.

Drawdown data from aquifer tests were used with analytical models to estimate the hydraulic properties of aquifers and confining beds. Data from nonleaky confined, leaky confined, and unconfined aquifers and from fully and partially penetrating wells were used to illustrate the calibration process for analytical models. The use of specific-capacity data to estimate aquifer transmissivity also is described.

Drawdown data from observation wells were collected at three locations in the Howe area (fig. 2) and from one location in the Milford area (fig. 4). Results of the modeling are indicative of highly permeable aquifer material in both areas. Estimates of hydraulic conductivity for the glacial aquifers ranged from about 280 to 600 ft/d. The thickness and the transmissivity of the aquifers are reduced locally by the presence of clay lenses of variable thickness and areal extent. Transmissivity ranging from about 11,000 to 63,000 ft^2/d is indicative of aquifers capable of producing significant amounts of water (Freeze and Cherry, 1979, p. 60).

The storage coefficient for confined aquifers ranges from 4.3×10^{-5} to 3.8×10^{-4}, and the two values of specific yield calculated for unconfined aquifers are 0.15 and 0.24.

Vertical hydraulic conductivity of the confining beds ranges from 0.05 to 0.21 ft/d. This range, which is high for clay material (Freeze and Cherry, 1977, p. 29), may indicate discontinuities and (or) high sand content in the confining beds.

One estimate of anisotropy, K_z/K_r, for a thick unconfined sand and gravel aquifer in the Howe area was 0.1. Data from a thinner unconfined sand aquifer in the Milford area indicated isotropic conditions. The value of 0.1 for anisotropy is probably representative of most glacial aquifers in the basin.

The use of specific-capacity data to estimate transmissivity is based on analytical methods. These data, which are more easily obtained than aquifer-test data, provide preliminary estimates of aquifer performance. However, these estimates are subject to large errors. Based on data from three pumping wells, estimates of transmissivity from specific-capacity data were one-third to one-half the values calculated from aquifer tests. The lesser values calculated from specific-capacity data are attributed mainly to well loss.

Once calibrated, analytical models can be used to predict aquifer performance in response to hydrologic stress. Several models were used to illustrate how s, r, t, or Q can be predicted if values of the other three variables, as well as T and S, are known or estimated. Predictions were made for aquifers with a single pumping well, multiple pumping wells, and hydrologic boundaries. Predictions of streamflow reduction from ground-water pumping were also illustrated. Results indicate that drawdown and streamflow reduction in response to ground-water pumping are minimal for the productive glacial aquifers in the basin.

Numerical models provide an alternative to analytical models when simulating complex aquifer systems with many pumping wells. Numerical models simulate ground-water flow by using a series of discrete nodes and allow for variations in hydraulic properties and geometry of aquifers.

A three-dimensional, numerical model was used to simulate the effects of ground-water withdrawals in the Howe study area (Bailey and others, 1985). The model was calibrated by using water levels in observation wells and data on ground-water seepage to (and from) streams collected at 24 sites in the autumn of 1982. Five transient pumping plans were used to predict the effect of agricultural irrigation during June, July, and August under several hypothetical irrigation schemes and amounts of precipitation. Results of the simulations indicate that current (1982) irrigation does not adversely affect water supply because of the high transmissivity of the glacial aquifers in the area. These aquifers can support additional growth in seasonal irrigation development (Bailey and others, 1985).

The examples of the analytical and numerical models applied to the St. Joseph River basin were used to compare the two types of models. Comparisons were based on methods of simulation, data requirements, type of results, applications, and determination of accuracy. In general, analytical models are better suited for making preliminary estimates of aquifer properties in local areas with uniform hydrogeology and simple boundary conditions. Predictions of drawdown based on analytical models are usually limited to a few wells for short periods of time in a small area. Numerical models are better suited for evaluating water resources in regional areas with many wells over long periods of time. Numerical models also can be used to examine the components of water budgets. Selection of which type of model to use should be based on the kind of hydrologic questions asked, the complexity and scope of the system to be simulated, the types of data available, and the degree of accuracy desired.

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