ICE IN STREAMS--ITS FORMATION AND EFFECTS ON FLOW

U.S. GEOLOGICAL SURVEY

Water-Resources Investigations Report 86-4209

Prepared in cooperation with

MICHIGAN TECHNOLOGICAL UNIVERSITY

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by H. S. Santeford, G. R. Alger, and J. A. Stark

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CONVERSION FACTORS AND ABBREVIATIONS

For the convenience of readers who may prefer to use metric (International System) units rather than the inch-pound units used in this report, values may be converted by using the following factors:

Multiply inch-pound unit	By	To obtain metric unit
inch (in.)	25.4	millimeter (mm)
foot (ft)	0.3048	meter (m)
mile (mi)	1.609	kilometer (km)
square mile (mi^2)	2.590	square kilometer (km^2)
gallon per minute (gal/min)	6.309×10^{-5}	cubic meter per second (m^3/s)
cubic foot per second (ft^3/s)	0.0283	cubic meter per second (m^3/s)
acre	0.4047	hectare
degree Fahrenheit (°F)	(°F-32)/1.8	degree Celsius (°C)

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ABSTRACT

The formation of an ice cover on an open channel causes a change in the water-surface profile and stage-discharge relation. The extent of the change depends on the nature of the control section and the location of the ice cover relative to the control.

In terms of ice effects, control sections can be classified as elevation control and resistance control. For elevation control, a floating ice cover has no effect on stage or discharge. For resistance control, the stage-discharge relation is a function of section geometry, ice roughness, and buoyant displacement of the ice.

For river reaches with nonuniform flow, the ice-covered, water-surface profile and stage-discharge relation will differ from the open-water condition. Depending on location of the ice cover relative to the control section and the nature of the control itself, the slope of the energy gradeline at all points along the profile can be classified as being either stable or unstable. If stable, the slope of the energy gradeline is independent of ice thickness; the resulting stage is directly related to ice thickness at the gage. This condition is referred to as the period of stable-ice control.

An analytical model developed for this study relates ice-covered profile to open-water profile, and stage to discharge. For the period of stable-ice control, a simple two-step correction relates stage of an ice-covered stream to discharge. For stable-ice control conditions, the difference between predicted and measured discharges were often within ± 5 percent.

INTRODUCTION

Ice cover has long been recognized as a factor in determining winter stage-discharge relations. Through the years, various empirical techniques have been used to "correct" for ice effects. The techniques commonly provide reasonable results when conditions under which they are used are similar to conditions for which the techniques were developed. Many standard techniques were developed, however, with little attention given to actual river conditions.

The Water Survey of Canada conducted an extensive field investigation during the early 1960's (Rosenberg and Pentland, 1966) to determine which techniques worked best and under what conditions. The Water Survey of Canada found that the best technique for a given site depends upon the type of stream and the general climate; no single technique was universally best.

The problem of flow in ice-covered streams received little study during the next 20 years. In the early 1980's, Santeford and Alger (1983) developed a theoretical analysis of the effect of ice cover based not on field data but on basic laws of fluid mechanics as they apply to open-channel flow supplemented with a conventional empiric-resistance equation. Using this technique, the hydraulics of ice-covered rivers were described (Santeford and Alger, 1983, 1984a, 1984b, 1984c, 1984d; Alger and Santeford 1984, 1985). Key points of the technique were that, at the time of freeze-up and breakup, flow had to be both unsteady and nonuniform and that, during most of the period of ice cover, flow had to be sufficiently stable to permit assumption of a steady-state condition--a condition referred to as the period of stable-ice control. Because of the unsteady nature of flow at freeze-up and breakup, different correlation techniques are applied for these periods. Although both unsteady- and steady-state flow can be described by one complex mathematical function, it is much easier to discuss and model them separately. Field studies were made to verify the model as it applies to the period of stable-ice control only; unsteady-state conditions were not verified.

Purpose and Scope

This report presents the results of a study to determine the relation of ice formation and ice cover to river stage and discharge by means of unsteady- and steady-state mathematical models. The models were developed to predict flow in ice-covered streams (Santeford and Alger, 1983). Discharge data were measured at sites on two rivers in Michigan's Upper Peninsula and one in the Lower Peninsula (fig. 1) for use in verifying the models.

Methods of Investigation

Discharges were measured by the U.S. Geological Survey on the Sturgeon River near Sidnaw (04040500)[1], the Sturgeon River near Nahma Junction (04057510), and the Red Cedar River near Williamston (04111379). The sites were chosen because their historical hydrographs indicate definite ice effects. Also, all stage records collected during freeze-up indicate a sharp rise to a crest followed by a moderate decrease in backwater that continued throughout winter.

The Sidnaw site was the primary field station. It was chosen because it contains several distinctly different river reaches. At the Sidnaw site, the primary gage used a stilling well equipped with a strip-chart recorder. Two additional stage recorders at the Sidnaw site used servo-manometer units (bubbler gages) equipped with strip-chart and punch-tape recorders. Strip-chart recorders were temporarily installed in stilling wells at seven additional locations. During freeze-up, a strip-chart recorder was installed at the upper end of the upper reach at the study site.

Discharge measurements were made, using a pygmy current meter, round-the-clock during freeze-up. Measurements were reduced to three each week during steady-state conditions. At the time discharge was measured, ice thickness was measured at selected sites.

Stage data from all three sites were collected and initially processed by the U.S. Geological Survey. Copies of the hourly data and, where applicable, strip-chart records were analyzed by MTU (Michigan Technological University).

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[1] Number is the U.S. Geological Survey gaging-station number.

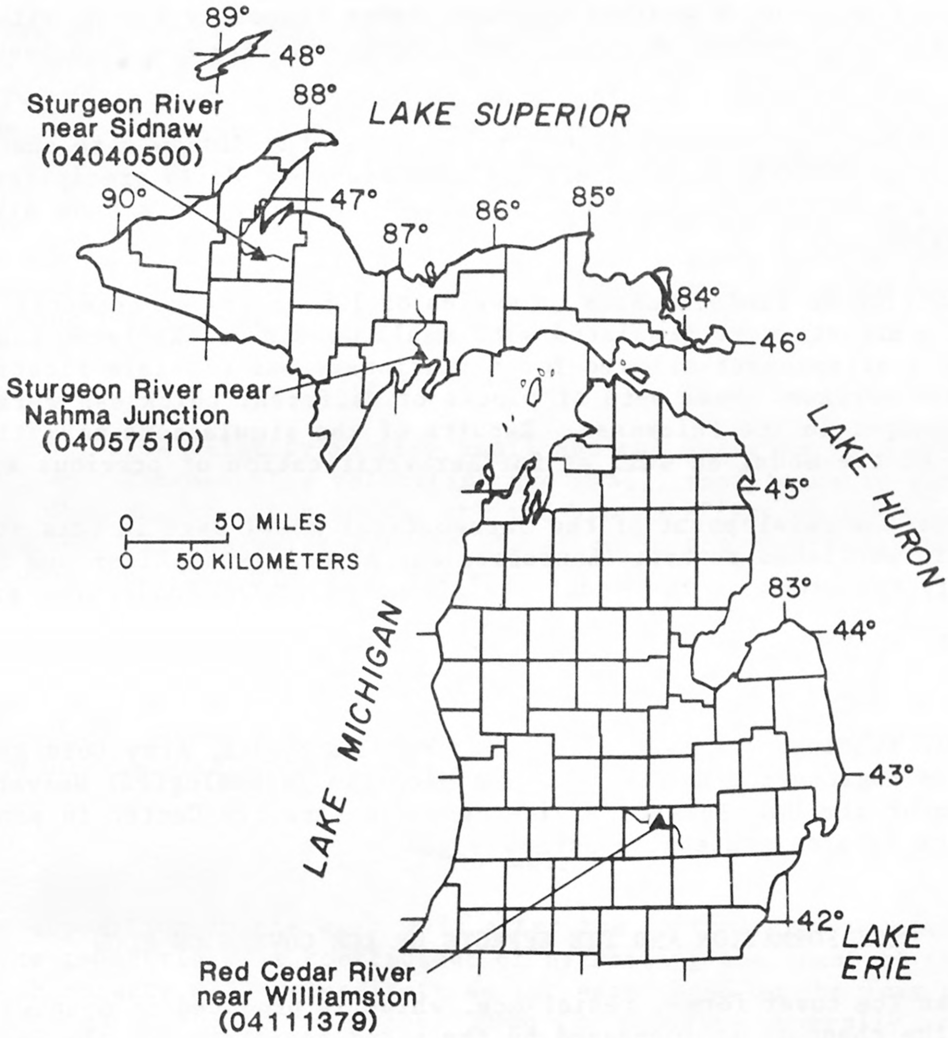


Figure 1.--Location of study areas (numbers in parenthesis are U.S. Geological Survey gaging-station numbers).

All supporting data from the Nahma Junction and Williamston sites were collected by the U.S. Geological Survey and analyzed by MTU.

Beginning in early summer 1984, MTU project staff mapped and surveyed nearly 5,000 ft of river at the Sidnaw site. Control benchmarks were established along the entire reach and detailed river cross sections made. By the onset of freeze-up, a weather station, seven temporary gaging sites, a self-contained lighting system, and temporary living quarters had been installed at the site.

Weather data were obtained from a station at the Sidnaw site and were supplemented with maximum and minimum temperature and daily precipitation data from a station at Kenton (10 mi to the south) and hourly data from Alberta (8 mi to the north).

In addition to field studies, a series of laboratory experiments were made by MTU. Ice effects were simulated with small wooden blocks laced together with string. This arrangement allowed for a stationary but flexible floating cover on the water surface. Four sets of blocks of different thickness were used to simulate changes in ice thickness. Results of the simulations permitted further refinement of the model as well as further verification of previous analyses.

Much of the development of the mathematical model used in this study is described in published reports (Santeford and Alger, 1983; Alger and Santeford, 1984). Only key items of the model are discussed in the following pages.

Acknowledgments

Partial funding of this project was provided by U.S. Army Cold Regions Research and Engineering Laboratory, and Michigan Technological University. The cooperation of the U.S. Forest Service and Ford Forestry Center in providing weather data is also gratefully acknowledged.

ICE FORMATION AND THE EFFECTS OF ICE COVERS ON FLOW

When an ice cover forms, resistance, which is produced in open water primarily by the channel, is increased by the added resistance of the underside of the cover. The total resistance of the channel boundary plus the ice generally is larger than that of open water at the same discharge. This increased resistance decreases velocity, necessitating a greater flow area to pass the same discharge. Numerous research studies have shown that, in most circumstances, the ice cover floats. Thus, there also may be an increase in stage resulting from buoyant displacement of the ice. These two factors--resistance and buoyant displacement--often interact to cause the stage for ice-covered channels to be higher than open-water stage at the same discharge. The condition is commonly described as ice-induced backwater or simply backwater. In many instances, this is a misnomer. For a backwater condition to exist, the slope of the energy gradeline must be flatter than it was in the unaffected condition. However, with an ice-covered channel, the slope of the gradeline is almost always steeper than that of the open-water channel.

The combination of increased stage and slope of energy gradeline, accompanied by decreased discharge, may at first seem paradoxical. A key element in understanding the observed response is the relation of various controls to ice. A "control" may be defined as any feature that determines a stage-discharge relation. In terms of ice effects, a control may be classified as being either an elevation control or resistance control.

Elevation Control and Formation of Ice Covers

Elevation controls include those features that generally act as weirs. Discharge is purely a function of the head on the weir. Elevation controls include a wide variety of natural and manmade features. Although weirs are the most obvious, this category also includes inlets and outlets from lakes or reservoirs, adverse slopes, and, in some instances major channel constrictions.

The reach of river upstream of the weir is termed the pool. An elevation-control reach includes both the pool and the weir. Ice can form in just the pool or in both the pool and weir. It is not likely that ice would form in the weir section and not the pool.

Ice in the Pool

In the pool, flow velocity is very small and the water surface is horizontal or nearly so. Because flow velocities are small, resistance is virtually nonexistent. When an ice cover forms, the ice has no effect on the water-surface elevation. If the water-surface elevation in the pool increases, the head on the weir and discharge also increase.

In a natural river system, a measurable velocity and, therefore, resistance in the reach upstream of the weir are often present. In such cases, the reach is not a true pool, and ice in the upstream section causes an apparent backwater or increased stage. When this condition occurs, the reach is considered to be a multiple-control reach (discussed in a subsequent section).

Ice in the Weir

Any ice deposition in the weir will alter the control section either by decreasing the geometric weir coefficient or by raising the crest of the weir. Either or both effects will necessitate an increased head on the weir to produce the same discharge through the section. Such ice deposits generally are weather dependent and often fluctuate from time to time as weather conditions change. A gage located in the pool will indicate an increased head. No systematic method is available, however, for relating indicated head to changing ice conditions in the weir section.

Resistance Control and Formation of Ice Covers

Resistance controls are features, such as the channel or an ice cover, that produce frictional resistance to flow. This resistance establishes the water-surface elevation. When resistance forces for a reach of channel are exactly equal to gravitational forces, the flow is said to be uniform. The slope of the water surface, channel bottom, and energy gradeline are all parallel; the reach must be very long and straight, and of constant slope, cross-sectional geometry, and unit resistance or roughness. This condition is referred to as channel control. Flow in channel-control reaches is uniform. Although such conditions are not normally found in natural rivers, the concept of a channel control and uniform flow is fundamental to this study.

Ice Cover in Channel-Control Reaches

When an ice cover first forms in a channel-control reach, the air-water boundary that previously existed on the water surface is replaced by the rougher ice-water boundary. Also, the increased size of the boundary, or wetted perimeter, results in an increase in total resistance to the flow. These effects are easily shown by considering Manning's equation applied to two conditions at the same section in a channel. The first condition has an open-water surface; the second has an ice-covered surface. The discharge, Q, is the same for both conditions. Using the subscript "o" for open water and "i" for ice covered:

$$Q_o = \frac{1.486}{n_o} A_o R_o^{2/3} S^{1/2} = \frac{1.486}{n_i} A_i R_i^{2/3} S^{1/2} = Q_i \qquad (1)$$

where:

A is the cross-section area,
R is hydraulic radius
S is the energy-gradeline slope, and
n is a roughness coefficient.

For the ice-covered condition, n_i is the composite roughness coefficient composed of the combined effects of the channel boundary and ice cover. Using mean hydraulic depth, D, which is defined as the cross-section area, A, divided by top width, B--that is, D = A/B--and, for the moment assuming a wide shallow channel, then:

$$A_o = D_o B_o \qquad R_o = \frac{A_o}{P_o} \simeq \frac{D_o B_o}{B_o} \simeq D_o$$

$$A_i = D_i B_i \qquad R_i = \frac{A_i}{P_i} \simeq \frac{D_i B_i}{2B_i} \simeq D_i/2$$

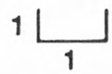
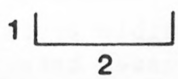
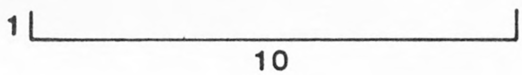
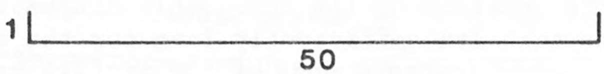
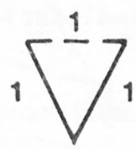
Substituting into equation 1 above and rearranging, then:

$$D_o/D_i = (0.764)(B_i/B_o)^{0.6}(S_i/S_o)^{0.3}(n_o/n_i)^{0.6} \qquad (2)$$

For uniform flow, the slope with and without ice must be the same, that is, $S_i = S_o$.

The coefficient, 0.764, used in equation 2 originated by assuming a wide shallow channel. For different geometries, different coefficients exist, as shown by examples in table 1. Assuming that the channel is basically rectangular in cross section such that B_i B_o, it can be seen that the ratio of mean hydraulic depths, D_o/D_i depends on the section geometry and the ratio of relative roughness, $(n_o/n_i)^{0.6}$. A generally accepted range in values of $(n_o/n_i)^{0.6}$ is from 0.8 to 1.1.

Table 1.--Values of coefficient used in equation 2 for representative cross-section geometries

Geometry (Numbers indicate ratio of boundaries)	Coefficient
	0.891
	.850
	.785
	.764
	.850

Considering different geometric sections and an average value for (n_o/n_i), it can be seen that ice cover will increase the normal depth of uniform flow by 5 to 20 percent. This increase applies to the depth or area available to the flow. If the ice cover has an appreciable thickness, the water level must increase an additional amount because of buoyant displacement.

Ice Cover in Multiple-Control Reaches

In natural rivers, channel slope, unit resistance, and cross-sectional geometry commonly change along the course of flow; these changes define different reaches. The section of channel through which slope, unit resistance, and cross-sectional geometry change from one set of conditions to another is referred to in this study as a multiple-control reach (figs. 2 and 3). In a multiple-control reach, flow is nonuniform and is gradually variable; controls are present at each end of the reach. Depending on the relative nature of the controls, the flow through the reach will either be accelerating or decelerating. When the flow is accelerating, the slope of the water surface and energy gradeline are both greater than the slope of the channel bottom and increase in a downstream direction. The water-surface profile is termed a M-2 profile (drawdown curve) (fig. 2). When flow is decelerating, the slope of the water surface and energy gradeline continually flatten in a downstream direction. The resulting water-surface profile is termed a M-1 profile (backwater curve) (fig. 3). Discussion is limited to subcritical flow because it is generally accepted that ice cover cannot form when flow velocity is greater than 2.5 to 3.0 ft/s.

Depending on the site, a wide range of possible profile configurations could exist. Only selected conditions are discussed here to illustrate the effects of an ice cover. From these examples, one should be able to postulate the anticipated ice effects for any given set of conditions.

M-2 profile (drawdown curve)

The M-2 profile originates from an abrupt change in energy slope with the lower reach being steeper than the upper reach (fig. 2). Several different ice effects can result, depending on the location of the ice. This discussion is limited to multiple-control reaches that are sufficiently long for the entire profile to develop. Both upstream and downstream ends of the profile meet channel-control reaches with uniform flow. Three conditions of ice effects (fig. 4) need to be discussed, as follows:

Condition 1: Ice cover only in the upstream channel-control reach.

When ice forms in the upstream reach, the under-ice normal depth must increase to some value larger than open-water condition. The amount of increase will depend on section geometry as shown in table 1 and equation 2. As the upstream depth increases, the profile will lengthen. A gage located in this reach will show an increased stage. However, a gage in the multiple-control reach will be unaffected by the upstream ice cover.

Condition 2: Ice cover in the upstream channel-control reach and multiple-control reach.

Under this condition the depth at the upstream end of the profile must increase as with Condition 1. However, the lower end of the profile remains fixed at the downstream open-water normal depth. This causes the profile to pivot about the downstream end increasing the slope of the water surface and energy gradeline at all points on the profile.

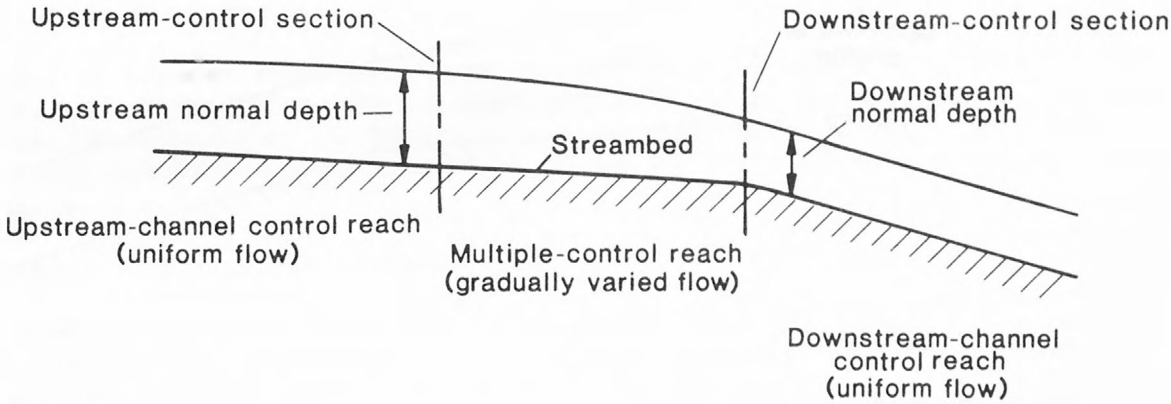


Figure 2.--Reaches and profile for segment of stream having a M-2 profile (drawdown curve) configuration.

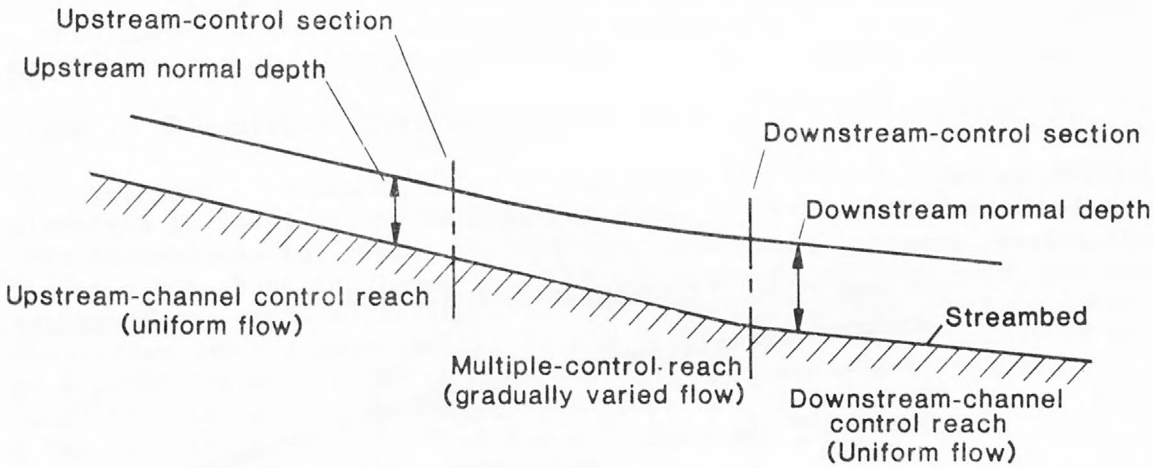


Figure 3.--Reaches and profile for segment of stream having a M-1 profile (backwater curve) configuration.

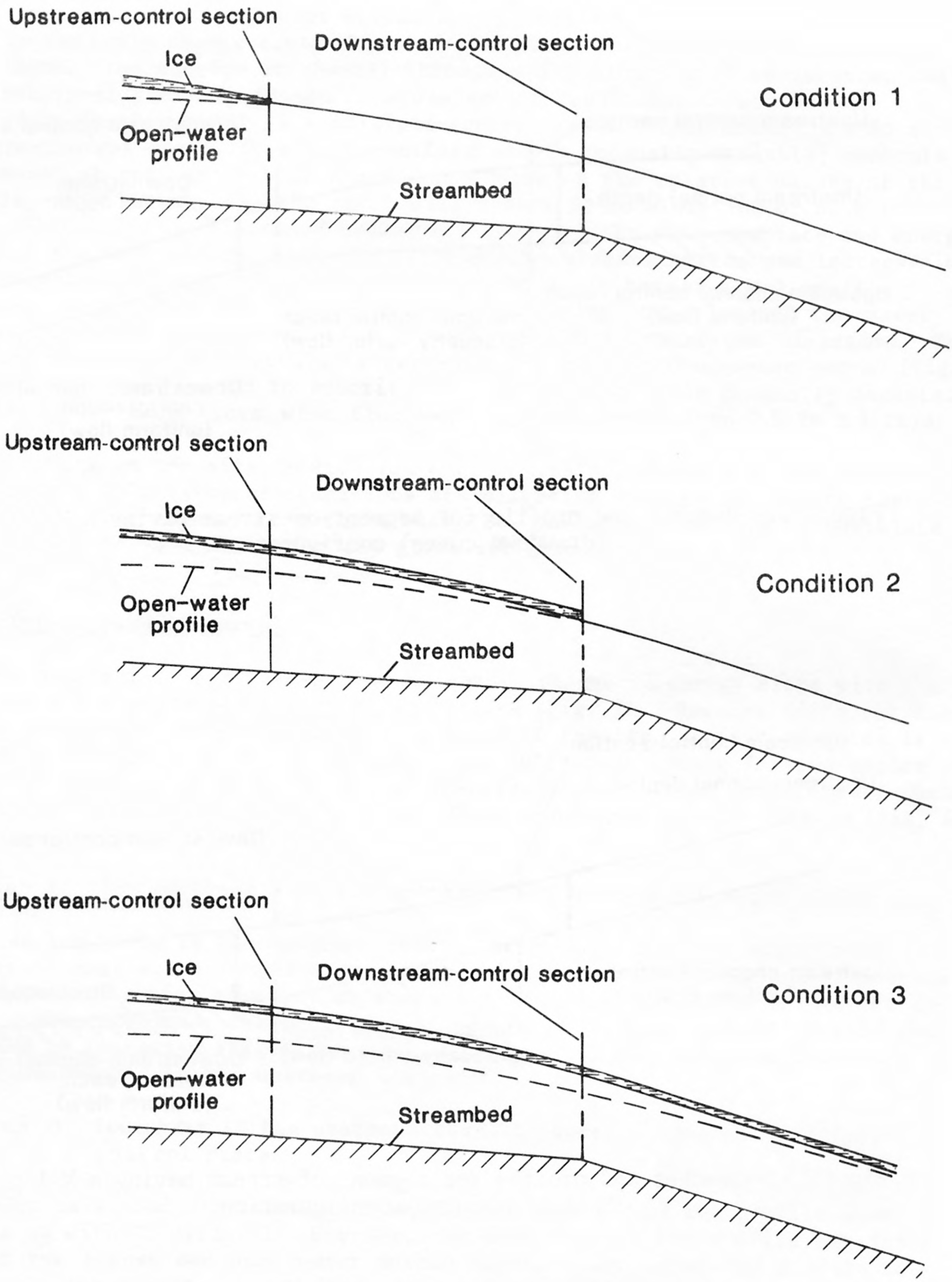


Figure 4.--Effect of ice cover in reaches of stream having a M-2 profile (drawdown curve) configuration.

As the upstream ice thickens, the upper end of the water-surface profile continues to increase. However, the lower end is still fixed at the downstream open-water normal depth. The profile is termed unstable in that, for the same discharge, the energy slope at each point on the profile will change as the ice thickness increases and unit resistance remains constant. For this to occur, the under-ice velocity along the profile must increase as the upstream ice thickness increases. With increasing velocity, the flow area at all points on the profile decreases.

A gage located along this type of profile would have a family of rating curves, each being a function of the ice thickness at the upstream end of the profile. A constant or increasing stage along the profile would imply an increasing discharge. A decreasing stage would imply a constant or decreasing discharge.

Condition 3: Ice in all reaches.

When an ice cover first forms, the depth increases at both ends, as well as along the profile. The amount of increase at any point is a function of geometry as shown in table 1 and equation 2. The resulting profile will generally differ from the open-water profile. As the ice thickens, the stage at all points in the system increases. For a given discharge, the slope of the energy gradeline at any point along the profile is independent of ice thickness. This condition is referred to as the period of stable-ice control.

M-1 profile (backwater curve).

The upstream end of the M-1 profile is set by upstream normal depth of uniform flow. At the downstream end, flow could be affected by elevation control or resistance control. In either case, the flow velocity in the downstream reach is less than in the upstream reach. Thus, it is reasonable to assume that, when ice is present in the multiple-control reach, it is also present in the downstream reach. This may produce one of the following conditions:

Condition 1: Downstream elevation control, ice in all reaches (fig. 5).

The presence or absence of ice at the downstream control has no effect. The elevation is fixed by the head on the weir. When an ice cover forms in the upstream channel-control reach, the upstream depth must increase causing the profile to pivot about the lower end. As the upstream ice thickens, the upstream end of the profile continues to rise, causing further pivoting about the downstream end. A gage located on the profile will have a family of rating curves dependent upon the ice thickness at the upstream end of the profile. This condition is said to be unstable, because the slope of the energy gradeline along the profile is a function of ice thickness and there is no constant relation between depth and discharge.

Condition 2: Downstream resistance control, ice in all reaches (fig. 6).

When an ice cover develops, the depth of flow at all reaches increases. The resulting profile will, in all likelihood, differ from the open-water profile. As the ice thickness throughout the system increases, stages at all locations also increase. The profile is referred to as being stable because, for a given discharge, the slope of the energy gradeline is independent of ice thickness.

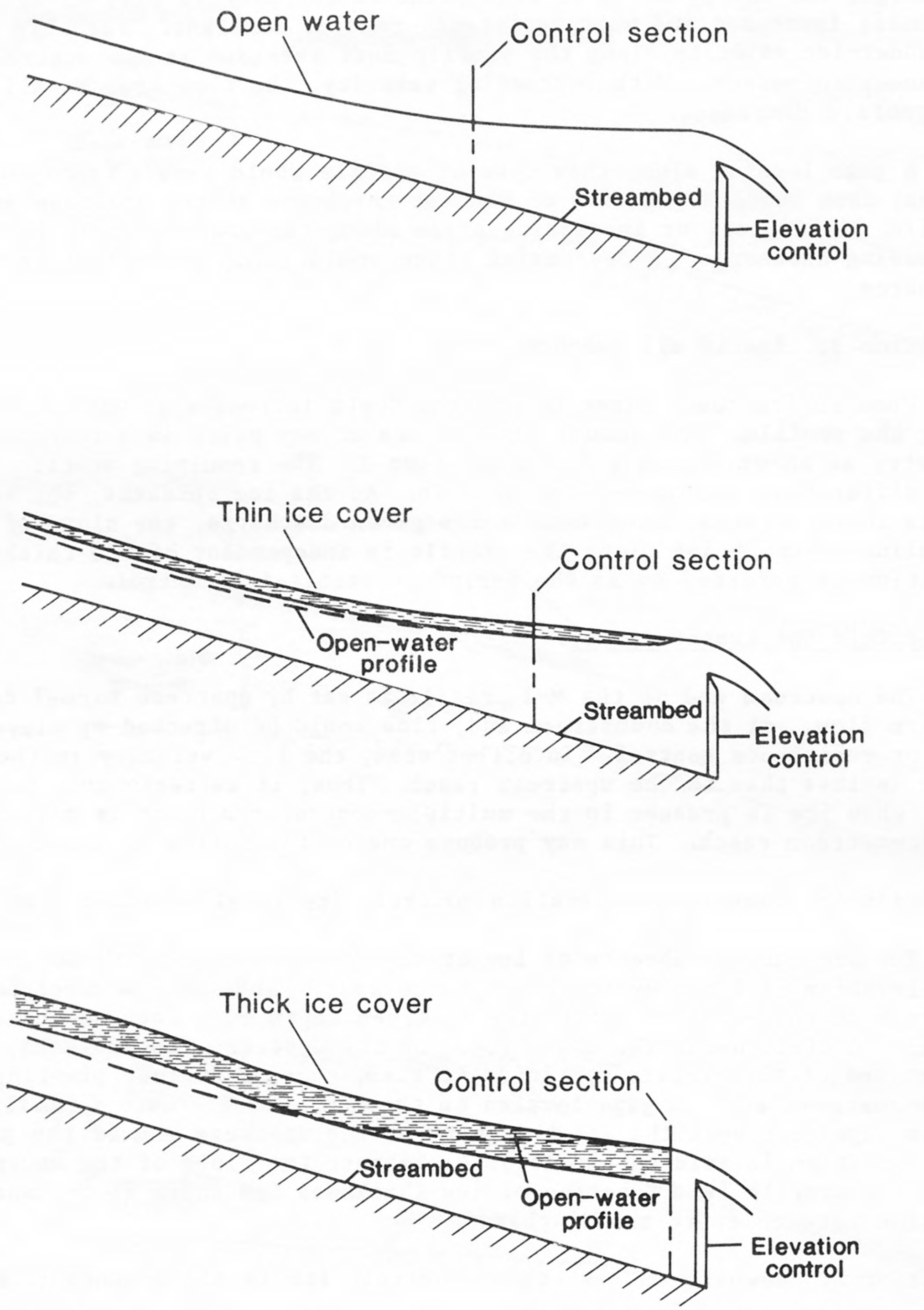


Figure 5.--Effect of ice cover in reaches of stream having a M-1 profile (backwater curve) configuration and downstream elevation control.

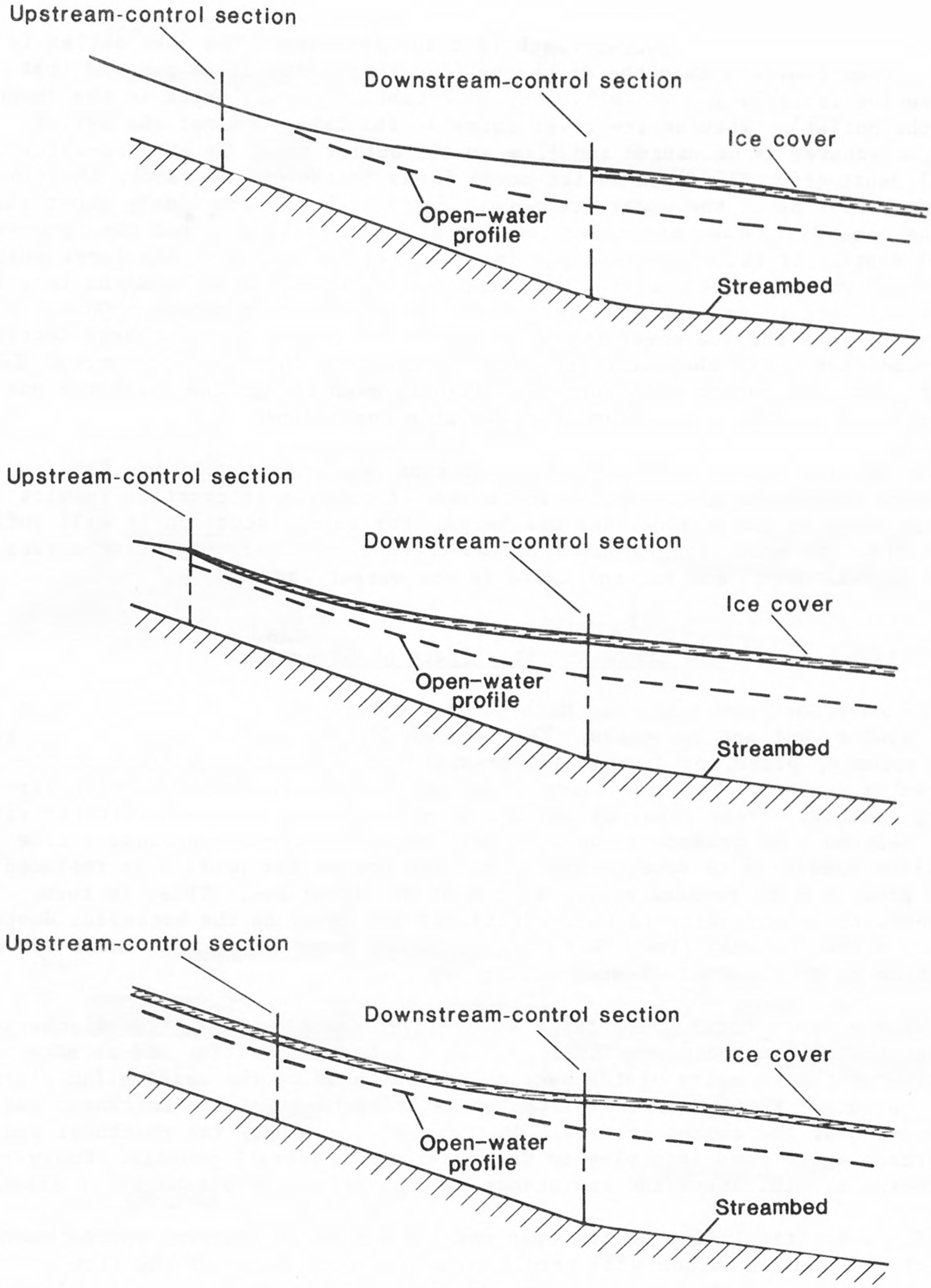


Figure 6.--Effect of thin ice cover in reaches of a stream having a M-1 profile (backwater curve) configuration and downstream resistance control.

Ice Cover in Lake-Outlet Reach

Ice cover in a lake-outlet reach is a special case. The lake outlet is an elevation control and sets the discharge (for simplicity it is assumed that the lake-outlet reach is sufficiently long to establish normal depth in the channel near the outlet). When an ice cover forms in the lake, and not the outlet reach, discharge is unchanged and flow in the outlet reach is at open-water normal depth (fig. 7). When an ice cover forms in the outlet reach, the flow in the reach must be at the under-ice normal depth. It was previously shown that, for the same discharge, the under-ice normal depth is larger than the open-water normal depth. If the same discharge is to be maintained, the lake level must increase. In many cases, winter discharge is relatively small and the lake is relatively large. As such, the lake level appears to be constant. Consequently, when the ice cover forms in the outlet reach, the discharge decreases to a condition where the under-ice depth is equal to the under-ice normal depth. The stage at the outlet will increase slightly even though the discharge has been reduced to only a fraction of open-water conditions.

As the ice in the outlet channel thickens, space available for flow decreases and discharge generally decreases. A complex interaction results between stage in the channel and discharge. For this discussion it will suffice to say that the condition is unstable and there is a family of rating curves based on lake level and ice thickness in the outlet channel.

The Choke and its Effect on Streamflow

In many northern rivers in Michigan ice covers form in some reaches in early winter, but not in others. This allows for the production of frazil ice (fine spicale, plate, or discoid ice crystals in supercooled waters) in the open-water reaches. The frazil ice moves downstream and collects in slower-moving reaches. These deposits can become quite large, encroach into the river cross section, and produce a choked condition. This choke can cause a true backwater condition to develop upstream. The open-water profile is replaced by a M-1 profile with reduced energy slopes at all locations. This, in turn, increases the opportunity to form additional ice cover as the backwater deepens and slows the upstream flow. With time, the ice cover progresses in an upstream direction from slower- to faster-moving sections.

When a choke forms, very large water-level changes occur. The discharge through the choked section is analogous to a submerged orifice and as such is a function of the geometry of the section and the head on the orifice (an elevation control). Therefore, the discharge is independent of ice thickness and roughness near the choked section. Upstream of the choke, ice thickness and resistance again come into play in determining the overall profile. However, at the choke, ice thickness and resistance have no effect on discharge or stage.

For subcritical flow in the open channel a moderate lateral encroachment or rise of the channel bottom will result in a decreased depth as the flow accelerates through the constriction. No change in upstream water levels occurs until the size of the encroachment is large enough to produce critical flow at the reduced section. Laboratory experiments show the response to be the same both with and without the floating cover. The primary factor dictating the formation of a choke is the space available for flow and not the depth to the free-water surface. Thus, depending on discharge and the submerged thickness of

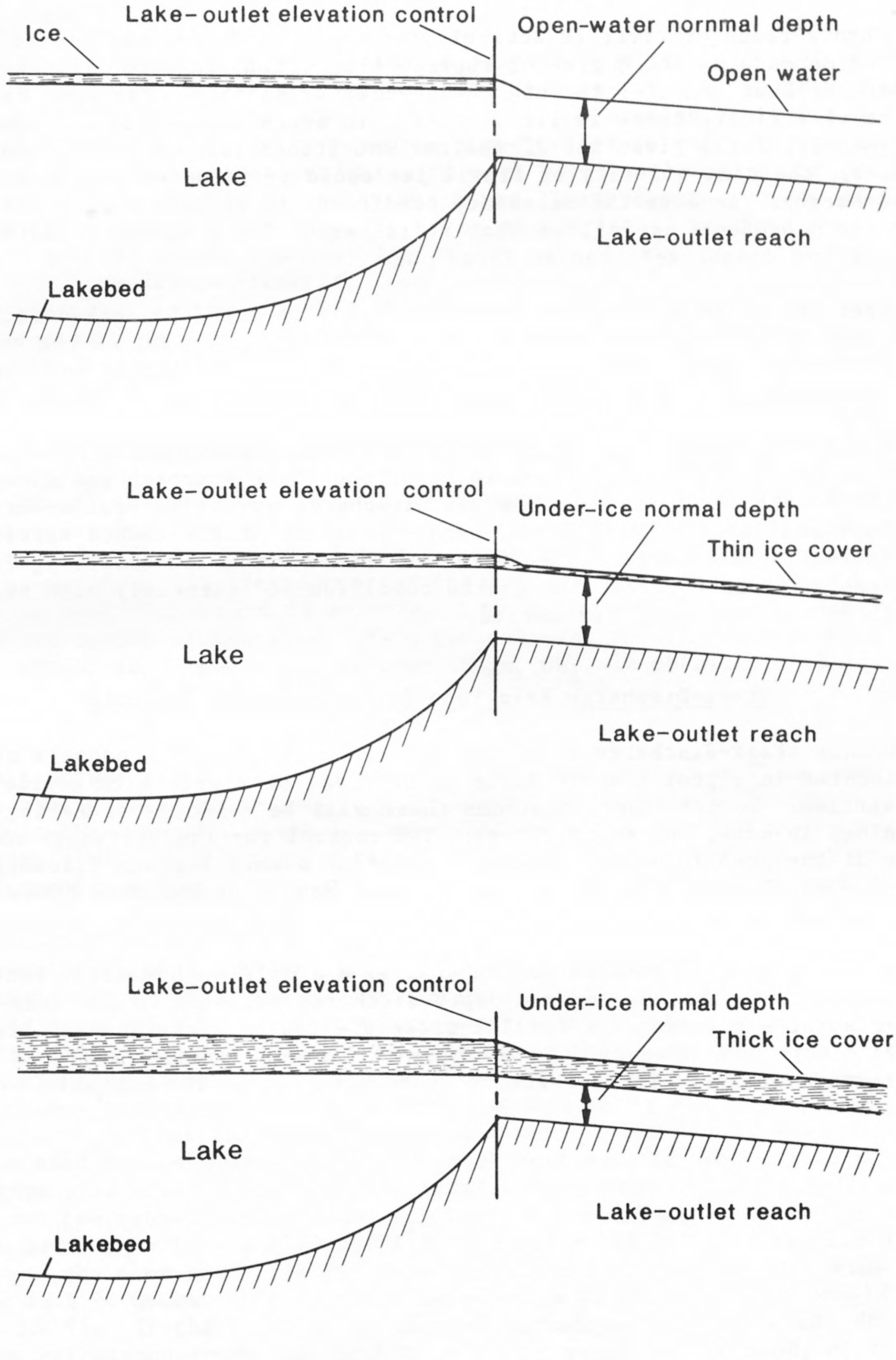


Figure 7.--Effect of ice cover on profiles at the outlet of a lake.

the floating cover, a given degree of channel constriction may or may not result in a choked condition.

When a reach of river is actively forming frazil ice, the amount of ice produced depends on the degree of supercooling which, in turn, is dependent on the rate of heat transfer through the open-water surface. For most fast-moving reaches of a river, there is little change in surface area with discharge. Consequently, for a given set of weather conditions (and therefore heat transfer), the same quantity of frazil ice could be produced over a wide range of discharges. Because the necessary conditions to produce a choke are discharge dependent, it follows that it is easier for a choked condition to occur at low discharges than at large ones.

When the choke occurs, the flow velocities through the choked section are very large even though the discharge may be among the lowest of the year. These high velocities produce excessive scour to both the ice deposit and the stream bed. Conventional sediment-transport equations have no application.

Similarly, stages that occur upstream of the choke often are extreme. When the choke forms, stages in the vicinity of the choke approach and exceed those associated with large floods. However, discharge may be several orders of magnitude smaller. In some cases, high velocities at the choked section along with decreasing discharge may relieve the choked condition resulting in stage decreases. In other cases, the choked condition and extremely high stages exist throughout the entire winter period.

Stage-Discharge Relations for Ice-Covered Channels

Unique stage-discharge relations exist for ice-covered channels only for a gage located in a pool a short distance upstream of a weir with no ice in the weir section. In all other locations there will be a family of rating curves depending, in part, on ice thickness. The control for ice thickness may or may not be at the gage location. A unique relation occurs between discharge and under-ice depth. However, no unique relation can be established between discharge and stage.

When ice-control conditions are stable, a simple mathematical function can be used to relate the ice-covered depth-discharge relation to the open-water stage-discharge relation. A similar procedure can be used for unstable ice-control conditions. However, the analysis is far more complex and must employ a simultaneous analysis for the entire reach affected by the unstable conditions.

APPLICATION OF STAGE-DISCHARGE RELATIONS TO PERIODS OF STABLE-ICE CONTROL

Ice cover changes open-water stage-discharge relations, in most locations, because of increased resistance and buoyant displacement. Although rating curves for ice-covered conditions are generally not available, it is possible to determine discharge by using measured stage values if sufficient additional data are known. The easiest condition to work with is the period of stable-ice control. For this period, the relations among stage, depth, buoyant displacement, and discharge can be defined in terms of conditions at the gage. A stable-ice control exists when the stream has formed it's complete ice cover. For all other conditions--that is, those previously defined as being periods of unstable ice-control--the stage at the gage will be a function of buoyant displacement at some upstream reach which, in turn, results in a change in the slope of the energy gradient at the gage.

Float Depth of the Ice--A Buoyant-Displacement Correction

The actual or true stage of an ice-covered stream can be determined by using the under-ice flow area (A_i), that is, the area under the ice cover which is available for flow. To calculate the under-ice flow area, the term float-depth--the measured difference between the free-water surface and the bottom of the ice--is used. This term is synonymous to buoyant displacement and is applicable to all channel geometries. When float depth, FD, is deducted from recorded stage, GH_w, the actual or true stage, GH_i, is obtained; that is:

$$GH_i = GH_w - FD$$

It is this actual stage that is used to compute the under-ice flow area. For channels having rectangular geometry, the under-ice flow area is determined simply by subtracting the average float depth from the free-water depth and multiplying that figure by the width of the channel. However, for channels having irregular geometries, that procedure will not yield the correct flow area.

The relation of float depth to flow area in channels having irregular geometrics is shown in figure 8. The maximum float depth, y, occurs along the upper-side boundaries of the irregular-shaped channel; float depth diminishes toward the center of the channel. To illustrate this configuration in figure 8, the submerged ice is shown in cross section as two triangular-shaped masses (combined they form a parallelogram having a base of 5y and a height of y). The wetted area (A_w), that is, the part of the channel below the free-water surface, is represented by a simple trapozoidal cross section. If, for figure 8, we assume that y is equal to 5, then the total wetted area is $(10y + 2y)/2 * 2y = 300\ ft^2$ and the submerged-ice area (A_s) is $5y * y = 125\ ft^2$. Thus, the under-ice flow area, $A_i = Aw - As$, is $300\ ft^2 - 125\ ft^2 = 175\ ft^2$. If the ice was removed and the same area, $175\ ft^2$, was available to flow, the GH_i (true stage) would be 12.2 ft ($Area = 10y + 2y^2$). The average FD (float depth) would be 15.0 - 12.2 = 2.8 ft. If the rectangular-channel technique had been used, FD would have been calculated to be $125\ ft^2/10y = 2.5$ ft resulting in a GH_i of 12.5 ft. A stage of 12.5 ft would cause one to falsely conclude that the under-ice flow area was larger than what was actually available.

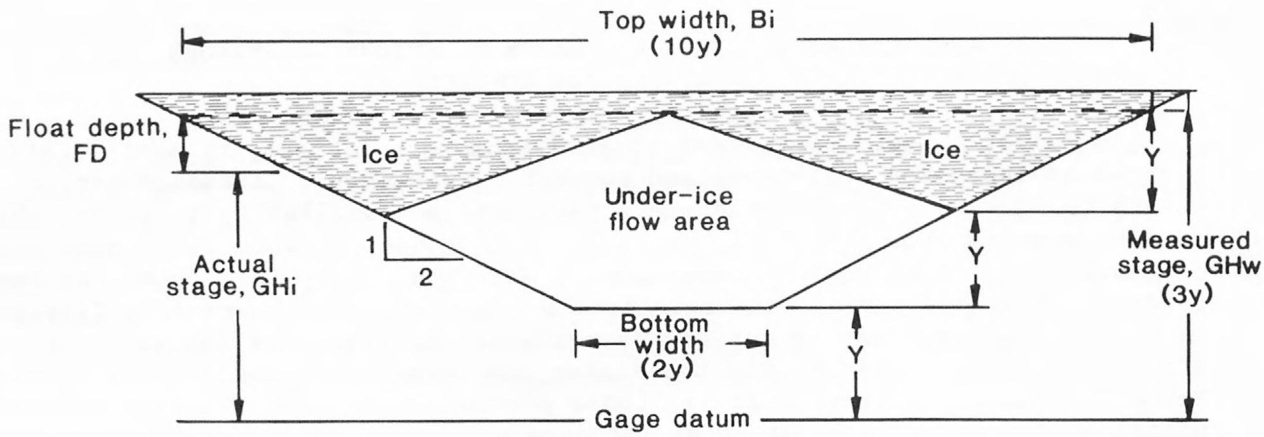


Figure 8.--Relation of average ice thickness to float depth.

The flow area, for any given stage at a given cross section, can be easily determined. At the same section, top width, stage, and discharge, can be measured. When these data are referenced to gage datum and open-water rating curves, the relations among stage (or gage height), area, top width, mean hydraulic depth (flow-area/top width) and discharge can be determined. An example of these relations for a partial set of data for the Sturgeon River near Nahma Junction is shown in table 2. As this tabulation indicates, given any single parameter, the others are also known. All parameters, however, apply only to the location defined by the gage and cross section.

Table 2.--Data for Sturgeon River near Nahma Junction
[Subscript i is used to indicate ice-cover condititons.
Mean hydraulic depth, $D_i = A_i/B_i$]

Actual stage,GHi (feet)	Discharge, Q (cubic feet per second)	Under-ice flow area, Ai (square feet)	Top width, Bi (feet)	Mean hydraulic depth, Di (feet)
4.10	86.0	99.1	60.3	1.64
4.09	84.4	98.4	60.2	1.63
4.08	82.8	97.8	60.2	1.63
4.07	81.2	97.2	60.2	1.62
4.06	79.6	96.6	60.1	1.61
4.05	78.0	96.0	60.1	1.60
4.04	76.4	95.4	60.1	1.59
4.03	74.8	94.8	60.0	1.58
4.02	73.2	94.2	60.0	1.57
4.01	71.6	93.6	59.9	1.56
4.00	70.0	93.0	59.9	1.55

An ice cover causes a change in the size, geometry, and roughness of the boundary. Using Manning's equation, it can be shown that

$$D_o/D_i = (Cg)(B_i/B_o)^{0.6}(S_i/S_o)^{0.3}(n_o/n_i)^{0.6} = Cg(B_i/B_o)^{0.6}(X_i/X_o)^{0.6} \quad (3)$$

where: D is the mean hydraulic depth,
Cg is a constant based on the geometry of the section,
B is the top width of the flow area,
S is the energy-gradeline slope,
n is Manning's coefficient, and
$X = S^{1/2}/n$.

Subscript "i" is for ice-covered conditions; "o" is for open-water conditions.

When a stable condition exists, the ratio (X_i/X_o) is a constant termed the winter-regime coefficient. For the range of discharges and stages normally encountered during the winter period, the variations in top width, B, will be small. Because the ratio (B_i/B_o) in equation 3 is raised to the 0.6 power, it is reasonable to assume the quantity is approximately a constant. Thus, the entire right-hand side of equation 3 can be replaced by a single constant, IAF (ice-adjustment factor),

$$D_o/D_i = \text{constant} = \text{IAF} \quad (4)$$

or

$$D_o = (\text{IAF})D_i$$

The magnitude of IAF will be site specific and can be found through either analytical procedures or field measurements. Only the procedure based on field measurements is discussed here.

For periods of stable-ice control, discharge can be measured at any convenient location near the gage; however, float depth must be obtained at the gage section. Field experience suggests that changes in bottom configuration can occur over a shorter distance than changes of the same magnitude in submerged thickness of the ice. Furthermore, frequent and repeated measurements of ice thickness at the same location often tends to produce an ice bridge at the measuring section that may not represent natural conditions. It is therefore suggested that the actual field procedure consist of measuring submerged ice thickness in the vicinity of the gage and applying the measurements to the gage cross section to determine float depth.

The mean hydraulic depth under ice-covered conditions, D_i, can be obtained by using an actual stage value and site-specific relations, such as shown in table 2 for Nahma Junction. Similarly, the mean hydraulic depth under open-water conditions, D_o, can be obtained by using discharge and the same site-specific relations. The ratio D_i/D_o defines IAF. The following is an example of these determinations:

On February 11, 1985, the U.S. Geological Survey measured discharge, under ice-cover conditions, at the Sturgeon River near Nahma Junction and reported the following:

Discharge (Q) = 83 ft^3/s

Stage (GH_w) = 4.92 ft

Submerged area of the ice (A_s) = 54.3 ft^2

From cross-section data at the gage, the wetted area, A_w, corresponding to a recorded stage, GH_w, of 4.92 ft, is 149.1 ft^2. From this, the under-ice flow area, A_i, is calculated to be:

$$A_i = A_w - A_s$$

$$A_i = 149.1 - 54.3 = 94.8 \text{ ft}^2$$

From table 2, the actual stage, GH_i, corresponding to a flow area of 94.8 ft^2 is 4.03 ft. Deducting actual stage from measured stage gives the float depth, FD; that is:

$$FD = GH_w - GH_i$$

$$FD = 4.92 - 4.03 = 0.89 \text{ ft}$$

The mean hydraulic depth, D_i, corresponding to a flow area of 94.8 ft^2 is 1.58 ft (table 2). However, the open water mean hydraulic depth, D_o, corresponding to a measured discharge of 83 ft^3/s is 1.63 ft. These two mean hydraulic depths can be used to determine the ice adjustment factor, IAF; that is:

$$IAF = D_o/D_i$$

$$IAF = 1.63/1.58 = 1.03$$

For the period between discharge measurements, values of float depth are determined by linear interpolation. The first step in the adjustment process is to determine the actual stage, GH_i, by subtracting float depth from the recorded stage, GH_w. From the relationship between gage height and hydraulic depth, D_i can be determined. The second step of the procedure adjusts the hydraulic mean depth using the ice adjustment factor, i.e., $D_o = IAF * D_i$. From the relationship between hydraulic mean depth and discharge, the adjusted flow rate is determined.

COMPARISON OF MEASURED AND PREDICTED CONDITIONS

Data from three study areas on three streams (fig. 1) were used to verify the model presented above. Data were collected at Sturgeon River near Sidnaw, Sturgeon River near Nahma Junction, and Red Cedar River near Williamston. Additional studies were performed at the Sidnaw site.

Comparisons were made between measured discharges and predicted values based on the instantaneous stage which existed at the time of discharge measurement. As an example of how the model could be used on an operation basis, predicted daily mean discharges were also compared to instantaneous discharges. In many cases this would be an invalid comparison, however, in the three river basins studied, the daily mean discharge under complete ice cover approximates an instantaneous measurement of discharge.

The Sidnaw Site

The Sturgeon River at the Sidnaw site is composed of four distinct reaches (fig. 9). The upper reach has a length of 3.3 miles; however, only the lower 2,000 ft is included in the primary study area. The reach begins as a deep pool (depth greater than 10 ft) just below the river's confluence with Rock River (3.3 miles upstream from Baraga Plains Road). In this reach, the river is 80 ft wide, 6 to 10 ft deep, and has winter flow velocities of 0.15 to 0.35 ft/s. The streambed at the downstream end of the upper reach is formed by bedrock (slate) outcrops which dip in the upstream direction. For this report, this reach of outcrops is referred to as the "adverse-slope reach". Two gages--the upper and middle-- were in the upper reach about 650 and 150 ft, respectively, upstream of the riffle crest. The upper gage is upstream of the adverse slope; whereas, the middle gage is in the adverse-slope reach.

The second reach at the Sidnaw site consists of 700 ft of moderate slope. In the upstream part of this reach the flow channel divides forming a small island. Both branches contain riffle sections (about 30 ft long) that terminate downstream in relatively deep pools. The riffle crest constitues the divide between the upper and second reaches. In the second reach, the river is 80 ft wide, has depths in the order of 1 to 2 ft, and mean velocities of 1 to 1.5 ft/s. A highway bridge and the primary gage are both in this reach. At the lower end of the reach, the river makes a sharp bend westward and has an abrupt change in slope. The slope change forms the control for the second reach.

The third reach--the fastest-moving reach--is 1,500 ft long. In this reach the river drops more than 10 ft and has velocities of 2 to 3 ft/s. Width of the third reach tapers from 100 ft at the upstream end to 65 ft near the downstream end. Large boulders and bedrock outcrops occur in the channel. At low-flow conditions, flow cascades over and around numerous small steps of boulders and bedrock. At the lower end of this reach, the river flows southwestward forming the fourth reach. This fourth reach is 120 ft wide and has numerous shallow spots and large gravel bars.

Bridge construction in the second reach during 1984 necessitated relocating the primary gage. Because construction was completed late (2 weeks prior to freeze-up), an adequate open-water rating curve for the full range of discharges occurring during winter was not obtained. The extremely low-flow conditions that occurred in February 1985 have not been observed with open-water conditions and the current primary gage configuration. Consequently, some comparisons between ice-cover and open-water conditions are not possible at this time.

Freeze-up

Freeze-up occurred at three separate times on the upper reach during the 1984-85 winter. The first in mid-November was accompanied by cold, dry weather. The second in late November and the third in mid-December occurred during

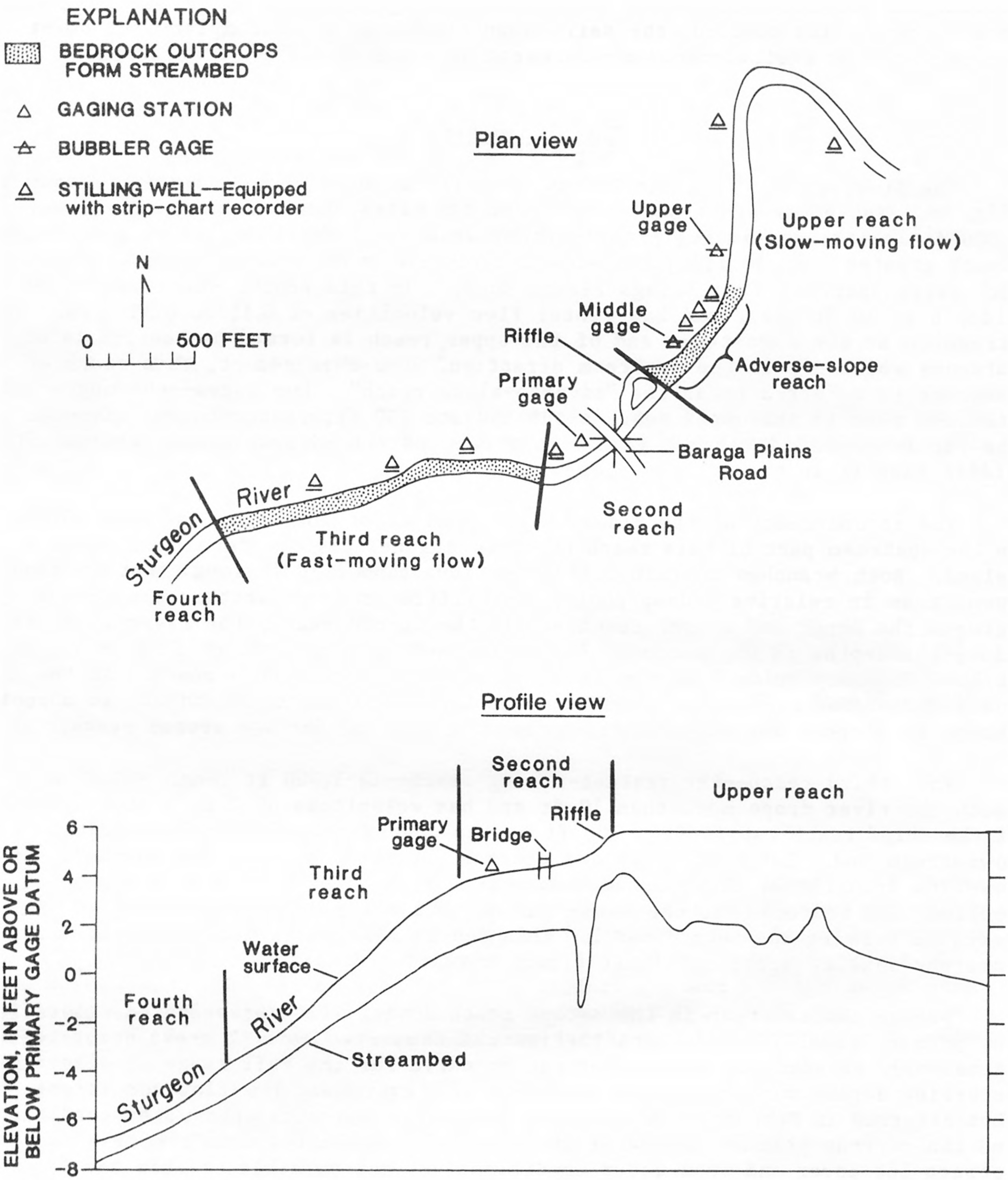


Figure 9.--Map and profile of the Sidnaw site.

periods of somewhat warmer temperatures and snowfall. Freeze-up processes under these different climatic conditions were considerably different.

Under the cold, dry conditions of mid-November, shorefast ice first formed along each bank. Crystalline ice formed on the open water during early morning hours. The ice had a black appearance and, thus, is referred to as "black" ice in this report. At times, large sheets of ice would break free of the shorefast ice and flow downstream. Ice-cover development was primarily by inplace growth of black ice--often in a downstream direction. Ice cover progression in a downstream direction stopped about 700 ft upstream of the riffle section at the downstream end of the upper reach. The trailing edge of the ice was about 50 ft upstream from the upper gage. In the open water downstream from the stationary ice cover, generally 50 to 60 ft downstream, patches of black ice formed on the water surface. As the patches moved downstream, they rapidly increased in surface area yet remained paper thin; during nighttime hours they were readily detected with flood lights. Although formation and growth of these patches of black ice continued for several hours after sunrise, they were never observed in mid-afternoon. When the solid-ice cover finally formed in the downstream part of the upper reach, it consisted of an accumulation of thin, black-ice patches that had stalled on the rocks just upstream of the riffle between the upper and second reaches.

Under the warmer, snowy conditions of late November, shorefast ice first formed along each bank. A nearly constant width of shorefast ice formed along the outside of each meander bend. On the inside of the bend, the width of shorefast ice increased downstream for two-thirds of the distance of the bend. The open-water section between the shorefast ice along each bank formed a long taper or funnel into each bend. Snow falling in the open-water section initiated development of large quantities of slush ice. Because of slow velocities, the slush ice stuck to the shorefast ice along the outside of each meander bend. These deposits thickened from the outside to the inside of each bend. The uniformity of the system was such that the rate of progression on each bend was nearly the same. Finally, sufficient slush was deposited at each bend to bridge the gap, forming a continuous ice cover from bank to bank. The open-water sections between each bend continued to produce slush ice that was subsequently deposited at the next downstream bend. Thus, between bends, the ice cover progression was in an upstream direction, filling the outside of the bend at a faster rate than the inside. For the entire upper reach, downstream parts of the reach froze over at the same time as the upstream parts.

Temperature measurements made in the upper reach during all three freeze-ups indicated a surface temperature of $0^{\circ}C$. Within a few inches of the surface, the water temperature was $0.2^{\circ}C$; it increased to $0.5^{\circ}C$ near the channel bottom. As water flowed over the riffle between the upper and second reaches, mixing occurred, producing water having a more uniform temperature distribution. During freeze-up of the upper reach, no frazil ice or other ice (except for a few inches of very thin shorefast ice along each bank) was noted in the downstream reaches.

The first appreciable ice effects at the primary gage were noted on December 20, 1984. By this date the upper reach was completely ice covered except for a short section just upstream of the riffle. Relatively warm water from the upper reach, coupled with short travel distance between the riffle and the gage, made water at the primary gage the last to freeze. Ice accumulation that eventually affected the gage began well downstream in the fourth reach, backed up through the third reach, and continued to back up until it reached the gage. At the end of freeze-up, ice was 2 to 5 ft thick in the third reach, producing backwater of more than 1 ft in the second reach.

Subsequent analyses have shown that a choked condition routinely occurs at each of four rock outcroppings in the third reach. As the reach became ice covered, frazil production was reduced. This, along with erosion of existing frazil and diminishing inflows to the system, allowed the choke(s) to be relieved somewhat and caused water levels to drop. As levels dropped, the ice became supported on numerous boulders. By mid-January, after water levels had dropped several feet, the ice in some parts of the third reach was no longer in contact with flowing water.

Stage and Discharge at Freeze-up

During the November freeze-up of the upper reach at the Sidnaw site, no ice or ice effects were noted at the primary gage and the gage continued to rate on the open-water rating. Two gages near the downstream end of the upper reach also showed no ice effects. However, discharge showed a dramatic response each night that active freezing occurred. Active freezing generally began about midnight. By 3 a.m., discharge was 60 to 65 pecent of what it had been 3 hours earlier. This condition was observed on each freeze-up. When first observed, it was not certain whether the condition was limited to the upper reach or whether nighttime freezing further upstream was affecting flow to the upper reach. Although frazil ice was a major problem, it was possible on several occasions to obtain adequate measurements. These measurements, and data from a stage recorder placed at the site, confirmed that inflow to the upper reach was reduced by nighttime freezing upstream and that, when active freezing was occurring upstream from a gage location, changes in water level of 0.3 to 0.4 ft occurred in 2 to 3 hours. In all cases, discharge had returned to a value nearly the same as that at midnight by noon the following day (sometimes earlier, depending on weather).

When stage and discharge decreased at the upstream end of the upper reach, one would expect similar decreases, but lagged by travel time (15 to 18 hr) in both stage and discharge at the lower end of the reach. However, for the three freeze-up events, it was not possible to correlate a single drop in stage at the upstream gage with a lagged event at the downstream gage. When a drop in stage and discharge was noted upstream, a similar response at the same time occurred downstream. If there was no response at one gage, there was no response at the others.

The width-to-depth ratio for the three gages at the Sidnaw site ranged from 20:1 to 70:1; the sections of slower-moving flows having the greater depth and smaller ratios. Data in table 1 and equation 2 suggest that, for a given geometry, the sudden addition of an ice cover could reduce discharge 60 to 65 percent of its initial value. The size of the decrease depends on the rate of development of the ice cover. When development in the length of the cover could be observed to occur in terms of miles per hour, the measured reductions in discharge were large. When development was observed to be only a few feet per day, reductions in discharge were too small to measure. With time, the upstream water levels change causing the slope to increase and the discharge to return to near its original value.

Data from recorders at all three study sites indicate that a decrease in stage and discharge occurred downstream when active freezing occurred upstream of a gage.

The upper, middle, and primary gages were all within 1,000 ft of one another. Data indicate that, although the sites were exposed to the same temperature and precipitation conditions, there is no correlation of ice thickness or float depths between sites. For any given stream cross section, distribution of ice can change drastically from one side of the river to the other in 2 to 3 days. Based on available data, it must be concluded that the best estimate of changes in float depth with time is a simple linear interpolation.

Stage-Discharge Relations

It should be possible with the use of the model to show a direct relation between open-water and ice-covered stage-discharge relations and, in turn, to relate the discharge hydrograph to the stage hydrograph.

During January 1985, when a known, stable open-water rating was available for comparison, the system performed exactly as predicted. The relation of measured stage to measured discharge is shown in figure 10. Figure 11 shows adjusted stage related to measured discharge. For values shown, the maximum difference between predicted and measured discharge was 8.6 percent which corresponds to a combined difference in float depth and gage height of 0.03 ft. The range in measured discharges over which stable conditions could be applied was from 50 to 156 ft^3/s. IAF (ice adjustment factor) was found to be 0.98 (mean value).

For comparative purposes it was assumed that discharge measurements were made only on January 4 and 29. For the intervening period only the stage record was available. The IAF's for the two measurements were computed and the average value used. It was assumed that float depth varied linearly during the time period. The results of this analysis are shown in table 3. Also shown in table 3 are actual measured discharges obtained during the time period. Although the comparison between mean daily discharge and instantaneous discharge is not strictly valid, the variation in discharge is slow enough to allow for the comparison.

At the Sidnaw site, an extremely cold period beginning on February 1, 1985, produced record low temperatures for longer than 2 weeks. Nighttime minimum temperatures range from -20^{o} to $-40^{o}F$. Daytime maximums were well below freezing. At the onset of this cold period, discharge dropped below any of the measured values on the flow rating at the new primary gage site. The measured 35 ft^3/s was well below minimum winter discharges that normally range from 60 to 80 ft^3/s. During this extreme cold period, ice thickness upstream of the bridge increased greatly, reaching a thickness of about 1.2 ft. This resulted in greatly increased velocities (more than doubled) and a complete redistribution of flow relative to the center bridge pier. A large scour developed on one side of the pier.

Starting on February 21, the cold period was followed by a period of sunny days and record high temperatures. Although nighttime temperatures remained below freezing, daytime highs during the next several weeks ranged from $40^{o}F$ to $60^{o}F$. Within a few days, increased runoff eroded the ice cover at the primary

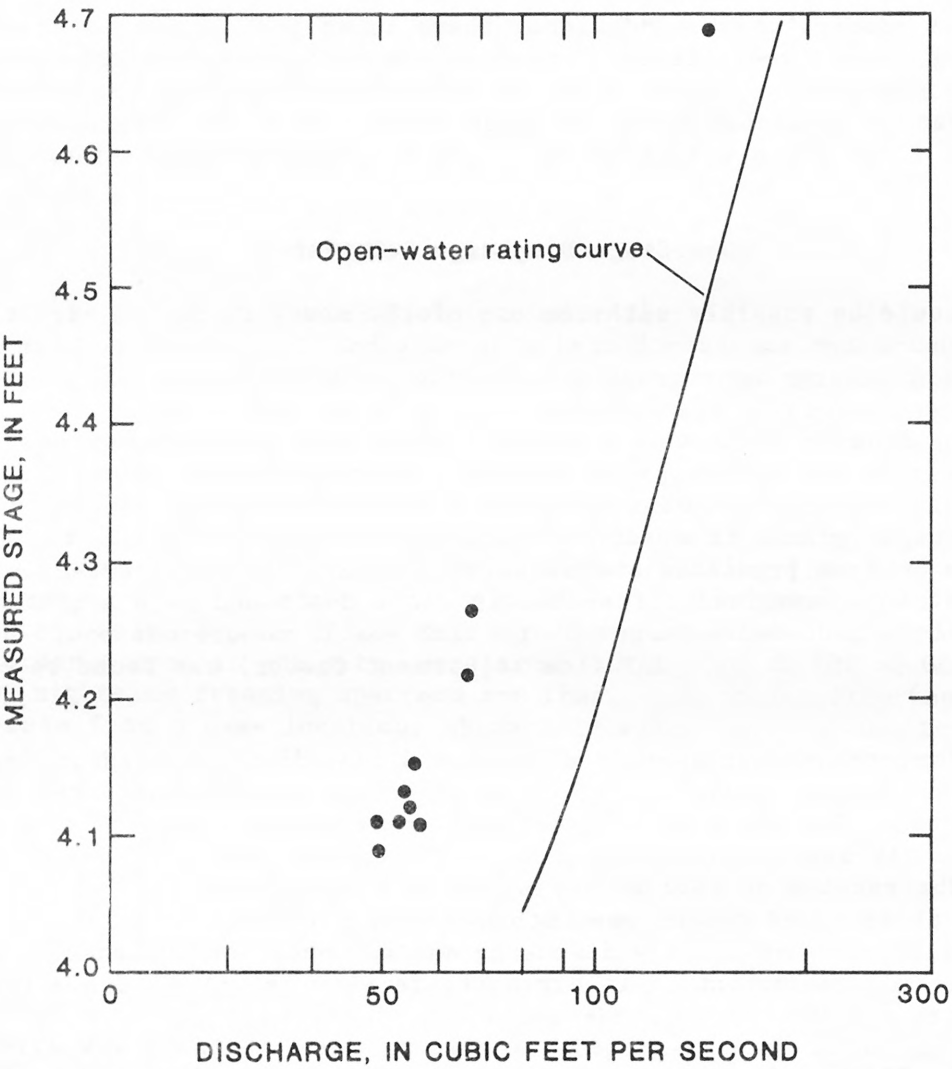


Figure 10.--Semilog plot of measured stage and discharge for January 1985 at the primary gage for the Sturgeon River near Sidnaw.

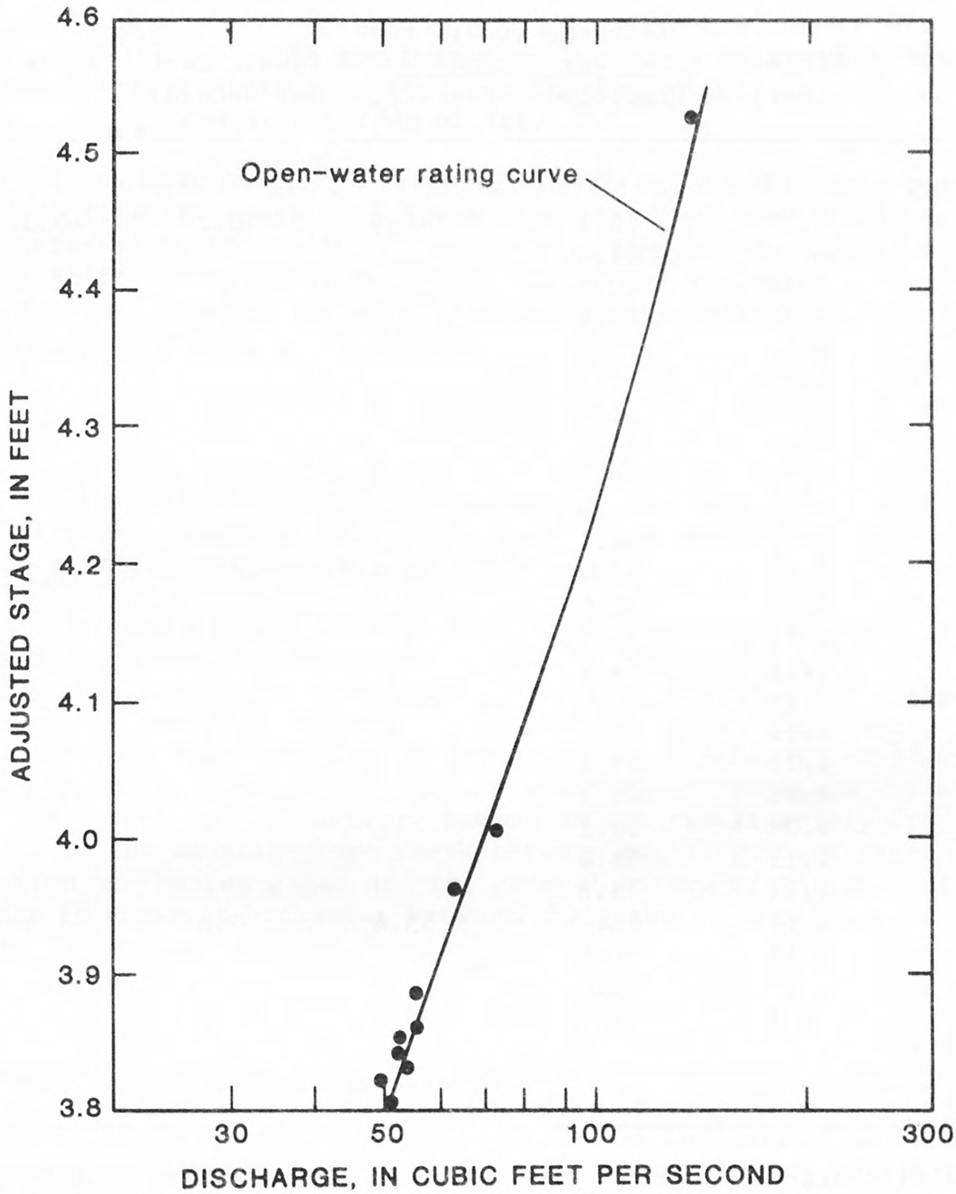


Figure 11.--Semilog plot of adjusted stage and discharge for January 1985 at the primary gage for the Sturgeon River near Sidnaw.

Table 3.--Comparison between predicted and measured discharge during January 1985 for the primary gage at Sidnaw, Michigan

Date	Stage (feet)	Discharge (cubic feet per second) Predicted[a]	Discharge (cubic feet per second) Measured	Error (cubic feet per second)	Error Percent
January, 1985					
4	4.23	66.1	65.9	+0.2	+0.3
5	4.19	61.4			
6	4.18	60.3			
7	4.26	69.4			
8	4.27	70.6	66.7	+3.9	+5.8
9	4.23	65.7			
10	4.19	61.0			
11	4.16	57.0			
12	4.13	54.7	57.2	-2.5	-4.4
13	4.11	52.8			
14	4.10	51.7			
15	4.12	53.6	52.9	+0.7	+1.3
16	4.12	53.6			
17	4.11	52.5			
18	4.13	54.3			
19	4.15	56.2	53.6	+2.6	+4.9
20	4.11	52.2			
21	4.08	49.3			
22	4.08	49.3	50.7	-1.4	-2.8
23	4.09	50.1			
24	4.13	53.8			
25	4.13	53.8			
26	4.14	54.6	55.4	-0.8	-1.4
27	4.12	52.6			
28	4.12	52.6			
29	4.11	51.5	49.7	+1.8	+3.6
			Average error:	0.56	0.9

[a]Predicted discharge is based on an assumption that discharge and float depth were measured only on January 4 and 29. A linear interpolation of float depth was used for all intervening days.

gage to a condition that was not safe to walk on. Within a week, only shorefast ice remained at the gage. Examination of historical data showed that similar events occurred in the past, but normally not until late March or early April. When the ice cover melted in late February it was found that a new deposit of sand had accumulated on the previously well-armored cobblestone bottom at the gage. Stage measurements suggested a backwater of 0.13 to 0.15 ft. By March 12, the reach was completely free of ice.

During the next month, 14 discharge measurements all indicated backwater of 0.13 to 0.15 ft even though no ice was present. Measured discharge for the period increased to 604 ft^3/s. It seems that the new deposit of sand produced a temporary shift in rating at the gage. The flood of record on April 20-21, 1985, completely removed the sand deposit and the rating returned to a condition very close to the provisional rating.

The rating curve for the primary gage at the Sidnaw site is extremely sensitive on the lower end. For stages that occurred during the winter period, a change of 0.01 ft in stage corresponds to a change in discharge of 1.0 to 1.5 ft^3/s (2 to 3 percent of the mean discharge). For low-flow conditions in February 1985, a change of 0.01 ft in float depth could also produce a 3 percent change in predicted discharge.

The upper and middle gages at the Sidnaw site respond differently than does the primary gage. The adverse-slope reach, analogous to a weir section, produces a complex control for these gages. Stage at the upper gage, which is upstream of the adverse-slope reach, is unaffected by float depth. However, ice in the riffle and adverse-slope reach can produce backwater at the upper gage. Stage at the middle gage, which is in the adverse-slope reach, is unaffected by float depth. Here again, however, backwater can result solely from ice in the riffle and in the adverse-slope reach between the riffle and gage. Because of the location of the two gages and the general nature of the control, the difference in apparent backwater between the gages is a measure of resistance caused by ice cover.

Data for the upper and middle gages are shown in table 4. A complete ice cover in the reach between the gages was not observed until January 4, 1985. After this date, float depth of the ice increased, reaching maximum values of 1.67 ft at the upper gage on February 23 and 0.95 ft at the middle gage on February 12. At the time of these maximums, the indicated backwater at the upper and middle gages was 0.20 and 0.17 ft, respectively. Photographs taken during field visits indicate progressive ice encroachment into the riffle until the thaw beginning on February 21. At times when the riffle seemed to be totally bridged over, open water could be seen and heard flowing under a bridge of ice and snow supported by numerous rocks. Although measurements were not taken, an examination of the photographs suggests that the magnitude of backwater is closely correlated to the degree of ice encroachment in the riffle section.

Theory predicts that the difference in apparent backwater between the two gages is a direct measure of head loss caused by the frictional resistance that occurs between the two gages. Head loss should be proportional to the mean velocity head, $v^2/2g$, and, consequently, the square of discharge divided by area. Headloss is also a function of distance between gages, size of contact boundary, acceleration of gravity, and unit resistance. The first three factors

Table 4.--Data for upper and middle gages at the Sidnaw site
[A dash indicates not measured or not available.]

Date	Measured discharge (cubic feet per second)	Upper gage Stage (feet)	Upper gage Float depth (feet)	Upper gage Apparent backwater (feet)	Middle gage Stage (feet)	Middle gage Float depth (feet)	Middle gage Apparent backwater (feet)
1984							
12-8	107	6.04	0.31	0.06	5.99	0.25	0.05
12-13	107	5.98	.32	.01	5.93	.22	.00
12-18	219	6.54	.35	.07	6.43	--[b]	-.01
12-21	156	6.30	.43	.08	6.21	.15	.03
1985							
1-4	66	5.92	.73	.19	5.85	.52	.14
1-8	67	5.89	.75	.15	5.81	.56	.09
1-12	57	5.81	.83	.14	5.75	.60	.10
1-15	52	5.86	.90	.22	5.78	.65	.16
1-19	--	5.85	.97	--	5.77	.68	.14
1-22	51	5.81	1.09	.18	5.75	.67	.15
1-25	55	5.79	--	--	5.72	.69	--
1-29	50	5.77	1.05	.15	5.72	.71	.12
2-3	40	5.78	1.20	.24[a]	5.76	.84	.24[a]
2-5	35	5.79	1.23	.24[a]	5.76	.85	.27[a]
2-7	38	5.79	1.27	.26[a]	5.75	.87	.24[a]
2-9	--	5.71	1.32	--	5.68	.94	--
2-12	34	5.70	1.39	.20[a]	5.65	.95	.17[a]
2-23	48	5.81	1.67	.20	5.74	--[b]	.15
2-27	62	5.85	1.59	.14	5.76	--[b]	.07
3-2	86	5.99	1.47	.14	5.21	--[b]	.06
3-7	84	5.98	1.57	.14	5.89	--[b]	.07
3-9	79	5.93	1.43	.12	5.85	.54	.06
3-10	77	5.99	1.37	.19	5.89	.54	.11
3-12	93	6.04	1.48	.15	5.94	--[b]	.08
3-14	81	5.96	--	.14	5.90	--[b]	.10
3-16	--	5.95	1.41	--	5.87	--[b]	--
3-17	78	5.93	1.49	.12	5.84	--[b]	.05
3-19	81	5.98	--	.16	5.89	--[b]	.08
3-21	80	5.95	1.36	.13	5.87	--[b]	.07
3-24	143	6.37	1.28	.23	6.25	--[b]	.14
1-26	210	6.60	1.03	.17	6.47	.00	.07
3-27	604	7.56	--[b]	--[a]	7.35	.00	--[a]
3-28	519	7.61	--[b]	--[a]	6.33	.00	--[a]

[a] Discharge beyond limits of open-water rating curve.
[b] Ice unsafe for measurements.

are fixed. Therefore, if the difference in apparent backwater is equal to a constant $(\text{discharge/area})^2$, then the unit resistance must also be a constant.

During the period from January 4, 1985, through February 21, 1985, a complete ice cover existed between the two gages. Using the average under-ice flow area for the two gage sections, and a velocity-head constant of 0.48, the head loss for each measurement was computed. Measured and computed head loss are summarized in table 5. Within the limits of measurement, head loss is considered to be equal to the difference in backwater between the two gages. Thus, unit resistance (Manning's coefficient) for the composite section and the ice must also have been constant.

The Nahma Junction Site

Stage-Discharge Relations

Figure 12 is a semilog plot of measured stage-discharge data for Sturgeon River near Nahma Junction. A stage-discharge relation is not readily recognizable. However, using an IAF of 1.03 and measured float depths obtained at the time of each stage-discharge measurement, the data were adjusted for ice cover and replotted (fig. 13). Using measured stage, measured float depths and the average value for IAF, a winter hydrograph was constructed (fig. 14). For periods between measurements, a linear interpolation of float depth by the model was used. A discharge hydrograph based on an open-water rating curve is also shown in figure 14.

The Williamston Site

Stage-Discharge Relations

At the Williamston site, weather and ice-cover conditions at the Red Cedar River gage during the 1984-85 winter were more variable than in the other two study areas. Only six measurements could be obtained for the period of stable-ice control. Measurements were made several hundred feet upstream or downstream from the gage because of open-water conditions in places along the gage section. Measurements of float depth were made at the gage. For two measurements, float depth was not obtained. For the four usable measurements, measured discharges were basically the same. Discharge comparisons are shown in table 6. For measured discharges, an error of 0.05 ft in float depth corresponds to an error of 3.6 ft^3/s (about 7 percent of average measured discharge for January and February 1985).

Table 5.--Comparison between computed and measured head loss between the upper and middle gages at the Sidnaw site

Date	Discharge (cubic feet per second)	Average area (square feet)	Head loss	
			Computed[a] (feet)	Measured (feet)
January 1985				
4	66	202	0.05	0.05
8	67	197	.06	.06
12	57	186	.05	.04
15	53	184	.04	.06
22	51	168	.04	.03
25	55	170	.05	.05
29	50	168	.04	.03
February 1985				
3	40[b]	159	.03	.00
5	35[b]	157	.02	.01
7	38[b]	155	.03	.02
12	34[b]	139	.03	.03
23	48	153	.05	.05
27	62	167	.07	.07

[a] Assuming constant unit resistance.
[b] Discharge below well established rating curve.

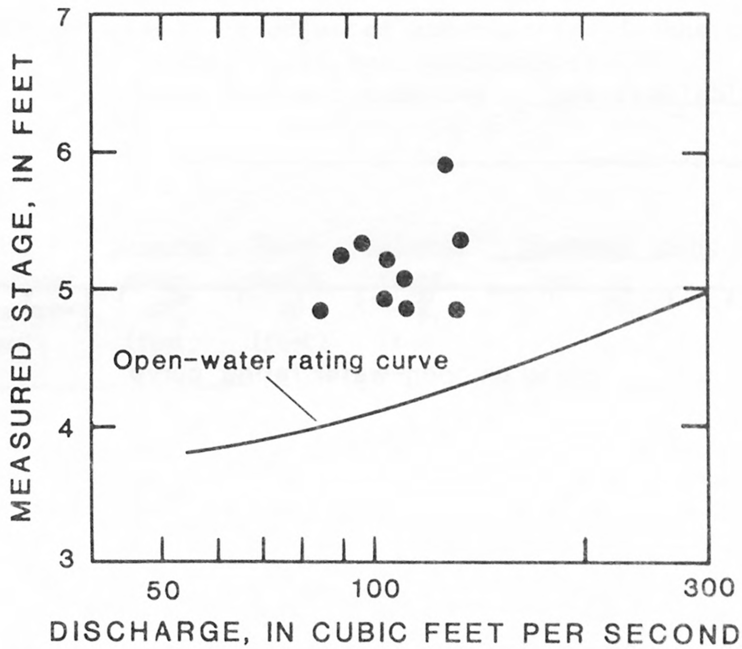


Figure 12.--Semilog plot of measured stage and discharge for the 1984-85 winter at the gage for the Sturgeon River near Nahma Junction.

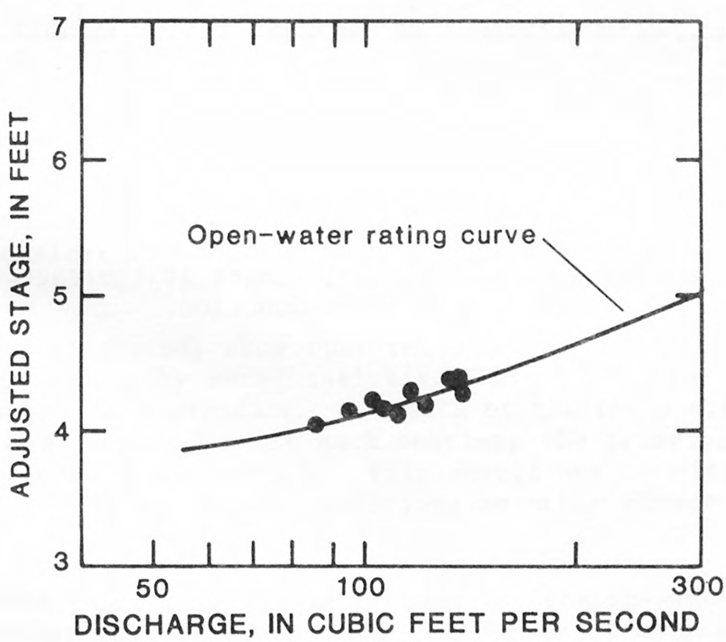


Figure 13.--Semilog plot of adjusted stage and discharge for the 1984-85 winter at the gage for the Sturgeon River near Nahma Junction.

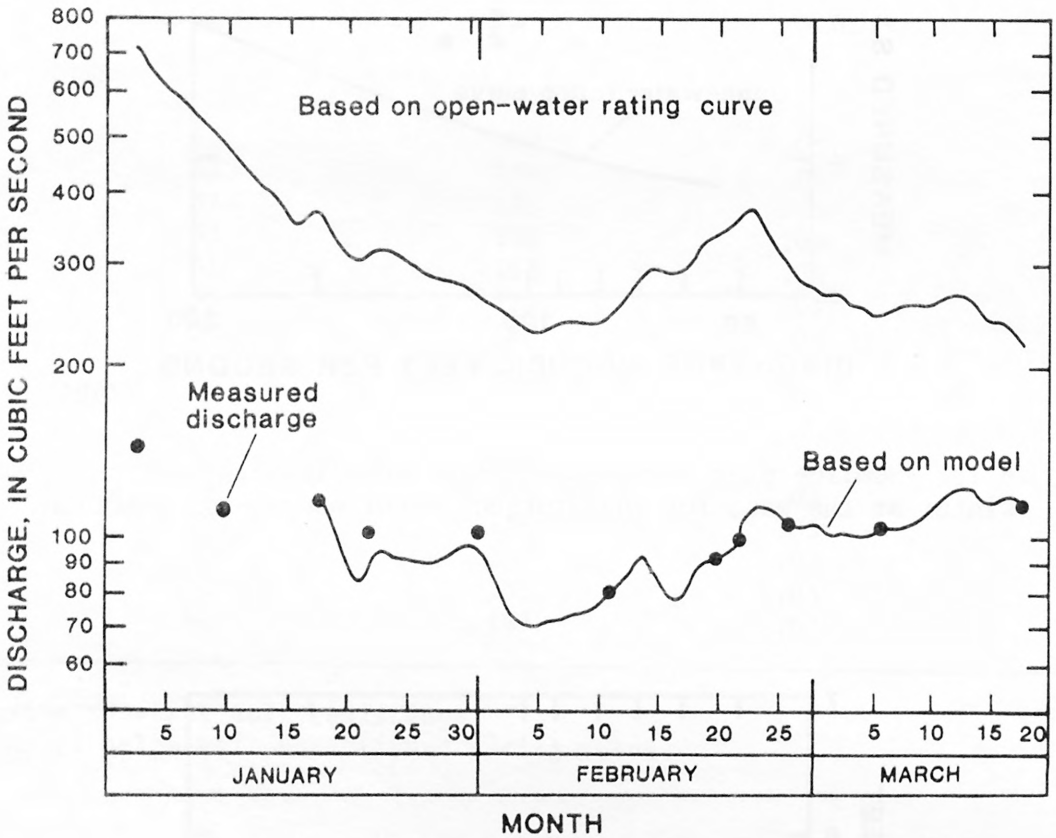


Figure 14.--Hydrographs for the 1984-85 winter at the gage for the Sturgeon River near Nahma Junction.

Table 6.--Comparisons of adjusted and measured discharge for Red Cedar River near Williamston
[A dash indicates data not measured or not available]

Date	Width at gage (feet)	Number of ice thickness measurements	Measured stage, GH_w (feet)	Float depth, FD (feet)	Adjusted stage, GH_i (feet)	Discharge (cubic feet per second) Based on GH_w	Discharge Based on GH_i	Discharge Adjusted	Discharge Measured
1985									
1-22	[a]50	4	3.40	0.40	3.00	84.9	59.2	56.6	53.7
1-25	90	7	3.47	0.53	2.94	89.5	55.4	53.0	53.2
1-31	100	8	3.53	0.56	2.97	93.4	57.3	54.8	53.7
2-6	—	0	3.38	[b]0.63	[b]2.75	83.6	[b]43.5	[b]41.6	44.0
2-13	—	0	3.69	[b]0.70	[b]2.99	104.0	[b]58.5	[b]55.9	45.0
2-21	61	5	3.76	0.77	2.99	108.9	58.5	55.9	56.1

[a]Estimated section width used.
[b]Estimated, based on linear interpolation of float depth.

DISCUSSION

All previous techniques for determining stage-discharge relations for ice-cover conditions have been developed without consideration of the hydraulics existing at a given gage. In development of previous techniques, it was assumed that if a correction for ice cover existed, then it should be universally applicable. In this report, analysis of effects of an ice cover indicates that, for each energy regime, effects of the cover differ. Therefore, a different ice-cover correction is needed for each regime. Also, none of the currently used techniques consider the effects of buoyant displacement of ice; in this report it is shown that buoyant displacement has a pronounced effect on stage.

The results of this study show that the observed response of stage to an ice cover was as predicted by model analysis. There is, however, one aspect of the results which may be misleading. For each of the three stations on the M-2 profile, the IAF was about 1.0. At each station, the principal increase in stage was due to buoyant displacement. This should not be misconstrued, however, as being representative of conditions on other rivers or at different sites in the same river.

Throughout this report, reference is made to "the period of stable-ice control". It is shown that stable-ice control can exist only when the channel is: (1) completely ice covered; and (2) the flow is totally controlled by resistance. Because basic criteria for selecting an open-water gaging site strongly favor a M-2 profile, the latter criterion for a stable-ice control is often met. Discussions of ice effects on various water-surface profiles along with discussions of specific conditions at given gages provide further insight in determining if conditions at a site are favorable for development of stable-ice control. Assuming that basic criteria are met, an examination of the stage hydrograph, in conjunction with local air temperature data, will generally provide sufficient information to determine when the period of stable-ice control begins.

For a gage on a M-2 profile, the freeze-up process begins downstream and progresses upstream past the gage. The developing ice cover produces a backwater condition at the gage prior to development of an ice cover. The resulting freeze-up stage hydrograph is readily discernable from a runoff stage hydrograph because (1) the freeze-up can only occur with sub-freezing temperatures when rainfall and/or snowmelt are not possible, and (2) the crest of freeze-up closely correlates with development of a complete ice cover, a process that also requires sub-freezing temperatures. Thus, any rise in stage on the stage hydrograph during sub-freezing temperatures should be suspected as being ice-induced. The recession portion of a freeze-up stage hydrograph also differs significantly from that of a runoff stage hydrograph. For the M-2 profile at the time of freeze-up, the ice effect will always produce a stage which is greater than that which would have existed at the same discharge for the open-water condition. Since the recession on the freeze-up hydrograph is for a discharge less than that associated with the open-water condition for the indicated stage, the recession coefficients must also be less--that is, the slope of the recession for the freeze-up hydrograph will be less than that for a runoff hydrograph producing the same stages. Using these criteria, it is generally possible to differentiate the freeze-up hydrograph from the runoff hydrograph. The period of stable-ice control is assumed to begin with the recession portion of the freeze-up hydrograph and continue through the initiation of breakup (Santeford and Alger, 1984c).

In some climatic zones it may be possible to have more than one freeze-up and breakup in a single winter season. When this occurs, there can be more than one period of stable-ice control. However, during each freeze-up and breakup a variable relation will exist between stage and discharge.

SUMMARY AND CONCLUSIONS

Theoretical analysis and laboratory experiments were used to determine effects of a floating ice cover on water-surface profiles. In the laboratory, wood blocks laced together with string formed a stationary floating cover representing ice. The results were verified by field measurements made on the Sturgeon River near Sidnaw, the Sturgeon River near Nahma Junction, and the Red Cedar River near Williamston.

With an ice cover, two categories of control are possible--elevation control and resistance control. Elevation controls are associated with weirs and weir-like features. The discharge is solely a function of the head on the weir and as such is independent of ice thickness and roughness. Resistance controls are a function of the size and roughness of the boundary. When an ice cover develops, size, geometry and roughness of the boundary are altered from open-water conditions. Therefore, the under-ice flow area, and thus the depth, are different from the open-water condition.

Control sections dictate depth-discharge relations for entire reaches of a river. When a change occurs at the control, a corresponding change occurs throughout the reach. For resistance controls, a change in ice thickness results in a change in elevation of the free-water surface or stage. This, in turn, propagates a change in stage throughout the entire channel reach. If the changes in ice thickness at the controls results in a change in the slope of the energy gradeline at points along the reach (discharge held constant), the condition is referred to as being unstable. If, however, changes in ice thickness at the controls produces no change in the slope of the energy gradeline along the reach, the condition is stable and referred to as the period of stable-ice control.

For the period of stable-ice control, stage is a function of discharge and ice thickness at the gage section. A two-step procedure, based on a theoretical analysis verified by laboratory experiments, has been developed to relate stage to discharge. Measurements in the three study areas confirm the applicability to field conditions. For stable-ice control conditions, the difference between predicted and measured discharges were often within $\pm$ 5 percent. During an unusually low-flow condition at the Sidnaw site, 7 to 9 percent error between predicted and measured discharges were observed. For the low-flow condition, the open-water rating curve used for comparison had to be extended. A difference of 0.01 ft in either float depth or stage results in nearly a 3 percent error in predicted discharge.

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