

EFFECT OF GRID SIZE ON DIGITAL SIMULATION OF GROUND-WATER  
FLOW IN THE SOUTHERN HIGH PLAINS OF TEXAS AND NEW MEXICO

By Richard R. Luckey and Diane M. Stephens

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### CONVERSION TABLE

For use of readers who prefer to use metric (International System) units, conversion factors for inch-pound units used in this report are listed below:

<i>Multiply inch-pound unit</i>	<i>By</i>	<i>To obtain metric unit</i>
acre-foot	$1.233 \times 10^{-3}$	cubic hectometer
cubic foot per second (ft <sup>3</sup> /s)	$2.832 \times 10^{-2}$	cubic meter per second
foot (ft)	$3.048 \times 10^{-1}$	meter
foot per day (ft/d)	$3.048 \times 10^{-1}$	meter per day
inch	$2.540 \times 10^{+1}$	millimeter
inch per year (in/yr)	$2.540 \times 10^{+1}$	millimeter per year
mile (mi)	1.609	kilometer
square mile (mi <sup>2</sup> )	2.589	square kilometer

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ABSTRACT

Three models of the aquifer in the southern High Plains were compared to determine the effect of grid size on simulated water levels. The first model, calibrated prior to this study, had 10-mile grid spacing. The mean difference between the simulated and measured predevelopment water levels in this model was +0.22 foot with a standard deviation of 41.6 feet. For 1980 water levels, the mean difference was +0.28 foot with a standard deviation of 25.8 feet.

The second model, calibrated during this study independently of the first model, had 5-mile grid spacing. The mean difference between the simulated and measured predevelopment water levels was -0.01 foot with a standard deviation of 44.4 feet. For 1980 water levels, the mean difference was +8.22 feet with a standard deviation of 27.9 feet.

The results from the first and second models were compared. The standard deviation of the differences in simulated water levels was 19.0 feet for the predevelopment period and 21.8 feet for 1980. There appeared to be no hydrologic significance to the pattern of the differences.

A third model, constructed by aggregating the data from the second model, had 10-mile grid spacing. The mean difference in simulated predevelopment water levels between the second and third models was +0.86 foot with a standard deviation of 8.9 feet. For the 1980 water levels, the mean difference between the models was +0.39 foot with a standard deviation of 4.4-feet.

The study found that the same hydrologic conclusions would have been reached had 5-mile grid spacing or 10-mile grid spacing been used. It was further concluded that the difference in simulated water levels between models with 5-mile grid spacing or 10-mile grid spacing was five to six times smaller than the differences between the simulated and measured water levels.

## INTRODUCTION

The High Plains aquifer underlies about 174,000 mi<sup>2</sup> of the central Great Plains in parts of Colorado, Kansas, Nebraska, New Mexico, Oklahoma, South Dakota, Texas, and Wyoming. More than 20 percent of the irrigated land in the United States overlies the High Plains aquifer, and about 30 percent of the ground water pumped in the United States during 1980 was from the High Plains aquifer (Gutentag and others, 1984, p.7). From predevelopment (the 1930's) to 1980, over 400 million acre-feet of water have been pumped from the High Plains aquifer (U.S. Geological Survey, 1984, p. 40-41). This pumpage has caused water-level declines that exceeded 10 ft in over 50,000 mi<sup>2</sup> and exceeded 100 ft in about 3,000 mi<sup>2</sup> (Luckey and others, 1981).

The U.S. Geological Survey began a study of the High Plains regional aquifer in 1978 (Weeks, 1978). One of the major objectives of the study was to develop computer models to simulate the aquifer system. These models were used to calculate future water levels in response to continued ground-water use.

For the purpose of computer simulation, the High Plains was divided into three parts (fig. 1). The southern High Plains included 29,000 mi<sup>2</sup> south of about 35° latitude, the central High Plains included 48,500 mi<sup>2</sup> between about 35° and 39° latitude, and the northern High Plains included 96,500 mi<sup>2</sup> north of about 39° latitude. Each of the three parts of the High Plains was simulated separately with a two-dimensional finite-difference model (Trescott and others, 1976) using a regular network of nodes that were spaced 10 mi apart in both the north-south and the east-west directions. There were 303 active nodes in the southern High Plains model, 513 active nodes in the central High Plains model, and 943 active nodes in the northern High Plains model. There was one common node between the southern and central High Plains models and five common nodes between the central and northern High Plains models.

The models were calibrated before they were used to project future water levels. The calibration was done in two phases: (1) A predevelopment-period calibration that simulated the system prior to large-scale irrigation development, and (2) a development-period calibration that simulated the effects of irrigation development on the aquifer system. The calibration consisted of adjusting selected simulated hydrologic properties until the models adequately simulated the historical water levels, water-level changes, base flow to rivers, and cross-boundary flow.

A complete discussion of the geohydrologic setting of the High Plains aquifer is given by Gutentag and others (1984). Detailed versions of the maps in that report provided the input data for the models. The details of model calibration are reported by Luckey and others (1986). A summary of the results of the calibration are repeated in this report in the sections on the coarse-grid model.

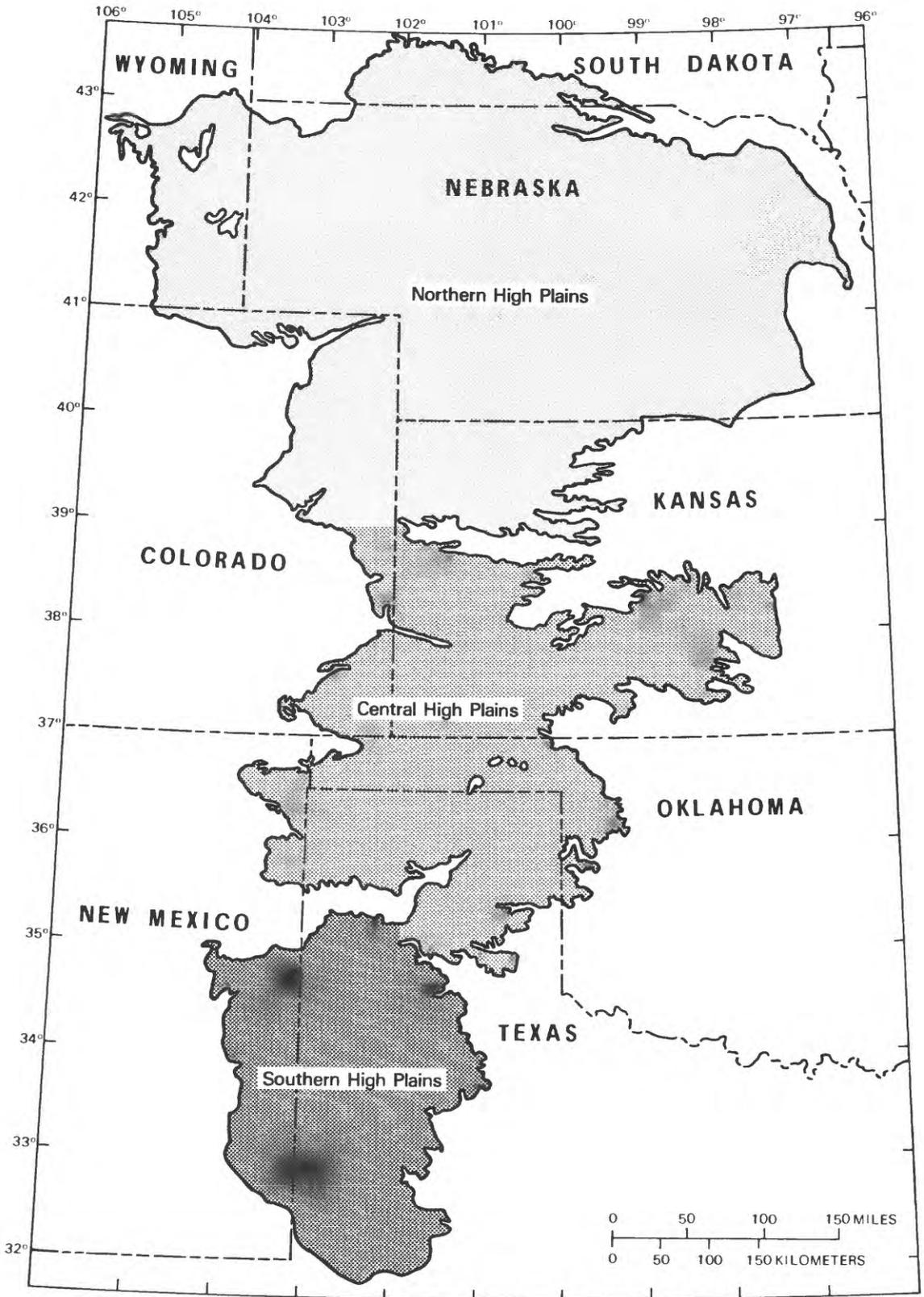


Figure 1.--Location of High Plains aquifer and division of aquifer for simulation.

## PURPOSE AND SCOPE

This report discusses tests that were conducted to examine the effects of differences in grid spacing on the models of the High Plains aquifer. These tests were conducted to answer two questions: (1) Would a finer model grid have led to a different understanding of the hydrologic system; and (2) how much would the computed water levels (and hence water-level changes) have changed if the model grid were made finer.

The southern High Plains was selected for these tests because it is the smallest of the three parts of the High Plains but has had the largest amount of irrigation development for the longest time. Irrigation development began in the southern High Plains in the 1930's, and by 1980 approximately one-half of all water pumped from the High Plains aquifer had been pumped in the southern High Plains. This is despite the fact that the southern High Plains occupies only 17 percent of the total area of the High Plains and contains only 6 percent of the drainable water in storage in the aquifer.

This study attempted to test the new models independently of the previous model (Luckey and others, 1986). However, the insights and conclusions reached in calibrating the previous model undoubtedly had an influence on the tests that were performed in this study.

This study examined different models of the same area constructed by using the same data and two different grid spacings. Hence, this study noted differences in models that were due to both construction and grid size, construction only, and grid size only. All differences are important in answering the two questions posed above.

## APPROACH

The first new model of the southern High Plains was constructed using four times as many nodes as there were in the original model. In this report, the original model, with 100 mi<sup>2</sup> grids (10-mi node spacing) is called the *coarse-grid model*. The new model, with 25 mi<sup>2</sup> grids (5-mi node spacing) is called the *fine-grid model*. A grid is a rectangular block of a model over which aquifer properties are assumed to be uniform. A node is the point at the center of a grid at which the water level is computed.

A new grid was drawn for the *fine-grid model*. The grid was drawn such that each grid in the *coarse-grid model* was represented by four grids in the *fine-grid model*. This allowed the results of the *fine-grid model* to be aggregated upward and compared to the results from the *coarse-grid model*. The *fine-grid model* could more closely follow the boundary of the aquifer so there were not always four nodes in the *fine-grid model* corresponding to each node in the *coarse-grid model*. In a few places, grids in the *fine-grid model* were outside the *coarse-grid model*. There were 1,201 active nodes in the *fine-grid model* and 303 active nodes in the *coarse-grid model*.

The data used (see Luckey and others, 1986, p. 3-7) for the *fine-grid model* were generated from the same original maps that were used to generate the data for the *coarse-grid model*. These data, in addition to boundary conditions, included initial water levels, hydraulic conductivities, altitudes of the base of the aquifer, specific yields, pumpage, and recharge. Pumpage was estimated by using maps of irrigated acreage and crop consumptive use. Recharge was estimated from various maps (Luckey and others, 1986, p. 10-11). All other data were mapped directly (Gutentag and others, 1984). It is important to note that the data for the *fine-grid model* were generated from the original maps and not from the data used in the *coarse-grid model*. As a result, differences in the data (due to visual interpolation) would be expected between the two models, and the different data would result in different results.

Other differences in results between the two models could be expected as a result of the different grid sizes. This difference is called discretization difference in this report. Discretization difference is caused by: (1) Representing aquifer parameters, which are mapped as continuous surfaces, with a series of discrete values; (2) approximating the location of the boundary of the aquifer with a series of lines or points, the location of which is governed by the grid; and (3) substituting finite-difference approximations for the partial derivatives in the ground-water flow equation. The discretization difference between a finite grid and an infinitesimal grid is called the discretization error in this report. It would be impossible to eliminate discretization error, but as the grid spacing becomes progressively smaller, the discretization error should become smaller until it becomes infinitesimal as the grid spacing approaches zero.

Note that the term discretization error is frequently used in numerical analysis to describe the error from only the third source listed above. In that case, discretization error is synonymous with truncation error. However, in this report, discretization error includes all the errors that occur when a complex system is approximated by finite, discrete areas.

The size of the discretization error is problem dependent. For systems with smoothly varying aquifer parameters and regular boundaries, the discretization error is probably smaller than for systems with rapidly changing aquifer parameters or very irregular boundaries. A finer grid obviously can approximate an irregular surface better than a coarser grid.

To estimate the size of the discretization difference (and hence discretization error) in the model of the aquifer in the southern High Plains, another new model, called the *aggregated model*, was constructed. The *aggregated model* could not measure exactly the discretization error, but by comparison with the *fine-grid model*, could give an indication of the size of the error. The *aggregated model* had a grid identical to that of the *coarse-grid model* and simulated parameters as close to those of the *fine-grid model* as the grid would permit. The values were not identical because it was impossible to represent the aquifer-boundary conditions in exactly the same manner using the 25-mi<sup>2</sup> and 100-m<sup>2</sup> grids. The constant-head boundary along the eastern side of the aquifer, which in the model is located at model nodes,

would necessarily be located at different places in the 5-mi and 10-mi grids. The no-flow boundary around the rest of the aquifer, which in the model is located along model grids, could either be coincident in the two models or could be located at different places.

All of the data from the *fine-grid model* were aggregated from 25-mi<sup>2</sup> grids to 100-mi<sup>2</sup> grids by taking the arithmetic average of the values at the *fine-grid-model* nodes that were within the *aggregated-model* grid. *Fine-grid-model* nodes that were outside of the original *coarse-grid model* were ignored. The arithmetic average of the values for the *fine-grid model* became the values used in the *aggregated model*.

The differences between the *coarse-grid model* and the *fine-grid model* included both differences due to grid size and differences due to having constructed the models independently of each other. The differences between the *fine-grid model* and the *aggregated model* would be due only to differences in grid size. The differences between the *coarse-grid model* and the *aggregated model* would be due only to the independent construction of the models.

#### EFFECT OF GRID SIZE ON PREDEVELOPMENT-PERIOD MODEL

The models were first used to simulate the system prior to large-scale irrigation development. These models were calibrated by adjusting hydraulic conductivity estimates in selected areas and recharge estimates throughout the area to obtain the best correspondence between simulated and historical predevelopment water levels and outflow along the eastern boundary (cross-boundary flow). A predevelopment water-level map was constructed using the earliest water-level measurements available in the area. There was only a limited amount of information on predevelopment cross-boundary flow, but what information was available was used to help calibrate the model. After a good correspondence between the shapes of the simulated and measured predevelopment water-level maps was obtained, the models were "fine tuned" by making small adjustments to the average recharge to obtain the minimum mean residual between the simulated and measured water levels.

#### Coarse-Grid Model

The *coarse-grid model* was used to test various predevelopment recharge patterns to see which produced the best simulated water-level map (as defined above). A pattern with very little recharge over most of the High Plains and more recharge concentrated in the northern part of the area produced the best correspondence between the simulated and measured predevelopment water levels. Recharge in this simulation ranged from 0.086 to 1.03 in/yr and averaged 0.13-in/yr. The total recharge for this simulation was 270 ft<sup>3</sup>/s. The average cross-boundary flow was 0.71 ft<sup>3</sup>/s per mi of boundary.

During the *coarse-grid model* calibration, it was determined that the hydraulic conductivity had been over-estimated in areas where the original estimates were greater than 50 ft/d. The hydraulic conductivity, as revised during calibration of the *coarse-grid model*, is presented by Gutentag and others (1984, fig. 10).

The mean difference between the simulated and measured predevelopment water levels at the 303 active nodes in the *coarse-grid model* was +0.22 ft. The largest differences were -113 ft and +99 ft. The standard deviation of the differences was 41.6 ft and the mean of the absolute values of the differences was 31.9 ft. Ninety percent of the absolute differences were less than 70 ft and eighty percent were less than 55 ft. The water levels ranged from more than 5,000 ft to less than 2,600 ft. The saturated thickness ranged from nearly zero to over 200 ft. Figure 2 shows the distribution of the differences between the simulated and measured predevelopment water levels for the *coarse-grid model*.

### Fine-Grid Model

Some of the same recharge patterns that were tested with the *coarse-grid model* also were tested with the *fine-grid model*. These included a uniform recharge over the entire southern High Plains and a distribution that increased recharge from south to north. As was the case with the *coarse-grid model*, these recharge distributions did not provide a satisfactory correspondence between the simulated and measured predevelopment water-level maps.

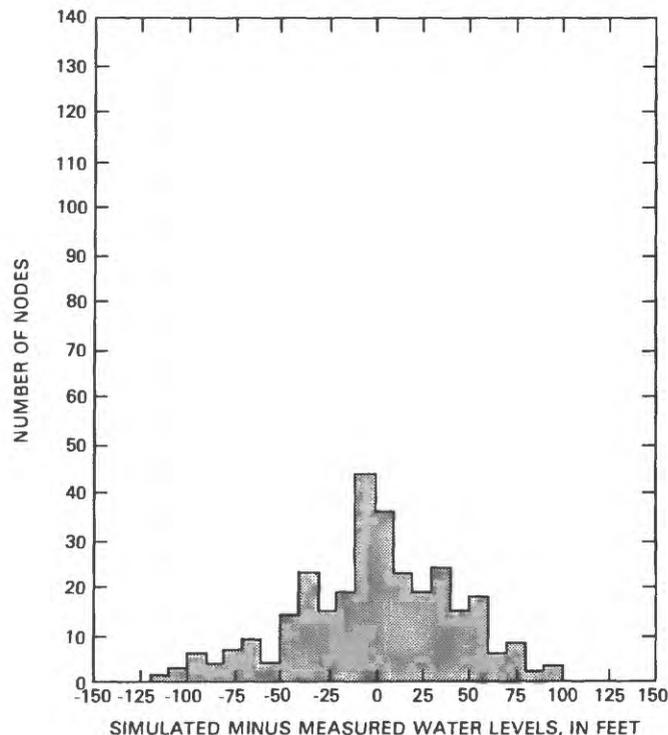


Figure 2.--Differences between simulated and measured predevelopment water levels for the *coarse-grid model*.

The hydraulic conductivity estimates, as revised during calibration of the *coarse-grid model* and presented by Gutentag and others (1984, fig. 10) was not altered during calibration of the *fine-grid model*.

The recharge distribution that resulted in calibration of the *fine-grid model* is shown in figure 3. Recharge is concentrated along the Running Water Draw - White River lineaments (Finch and Wright, 1970). The pattern of recharge is identical to that of the *coarse-grid model* (Luckey and others, 1986, fig. 3), but the values are slightly different. The total recharge was 268 ft<sup>3</sup>/s. The minimum recharge was 0.084 in/yr; the maximum recharge was 1.004 in/yr; and the average recharge was 0.125 in/yr. The average cross-boundary flow was 0.71 ft<sup>3</sup>/s per mi of boundary. The total recharge in the *fine-grid model* was 0.7 percent less than that for the *coarse-grid model* while the average recharge was 4 percent less. The percentages for the total and the average are somewhat different because the sizes of the modeled areas are slightly different between the *coarse-grid model* and the *fine-grid model*.

Wood and Osterkamp (1984) proposed that much of the recharge to the aquifer in the southern High Plains comes from playa lakes. This concept was not tested with the *fine-grid model* as it was not known to the authors at the time that the *coarse-grid model* was calibrated. Because one of the purposes of this study was to determine if different conclusions would have been reached during the original model calibration if a finer grid had been used, it would not have been appropriate to use information not available during calibration of the *coarse-grid model*. The playa lakes are concentrated in the northern part of the southern High Plains and hence, making recharge a function of playa-lake distribution might result in a recharge distribution similar (or better) to the one that was obtained with the *coarse-grid model*.

The simulated and measured predevelopment water levels for the *fine-grid model* are shown in figure 4. There is a reasonable correspondence between the shapes of the contours. The mean difference between the two surfaces at the 1,201 active nodes was -0.01 ft. The minimum difference of -155 ft occurred at about 35°50' latitude, 102°50' longitude; the maximum difference of +119 ft occurred at about 33°40' longitude, 103°40' longitude. The standard deviation of the differences was 44.4 ft and the mean of the absolute values of the differences was 33.4 ft. The calibration statistics for both the *coarse-grid model* and *fine-grid model* for the predevelopment period are summarized in table 1.

A sensitivity analysis was done for the predevelopment-period *fine-grid model* in exactly the same manner as was done for the *coarse-grid model* (Luckey and others, 1986, p. 47-51). The results of the sensitivity analysis for the *fine-grid model* were virtually identical to the results for the *coarse-grid model*. Hence, the results of the sensitivity analysis are not reported here. The similarity of the results of the sensitivity analysis indicated that the *coarse-grid model* and *fine-grid model* behave in the same manner, not only at the calibration point, but away from the calibration point.

The simulated predevelopment water levels from the *fine-grid model* were aggregated by taking the arithmetic average of the simulated water levels at up to four nodes from the *fine-grid model* that corresponded to one node in the *coarse-grid model*. This was done so that a node-by-node comparison could be made between the water levels in the *fine-grid model* and *coarse-grid model*.

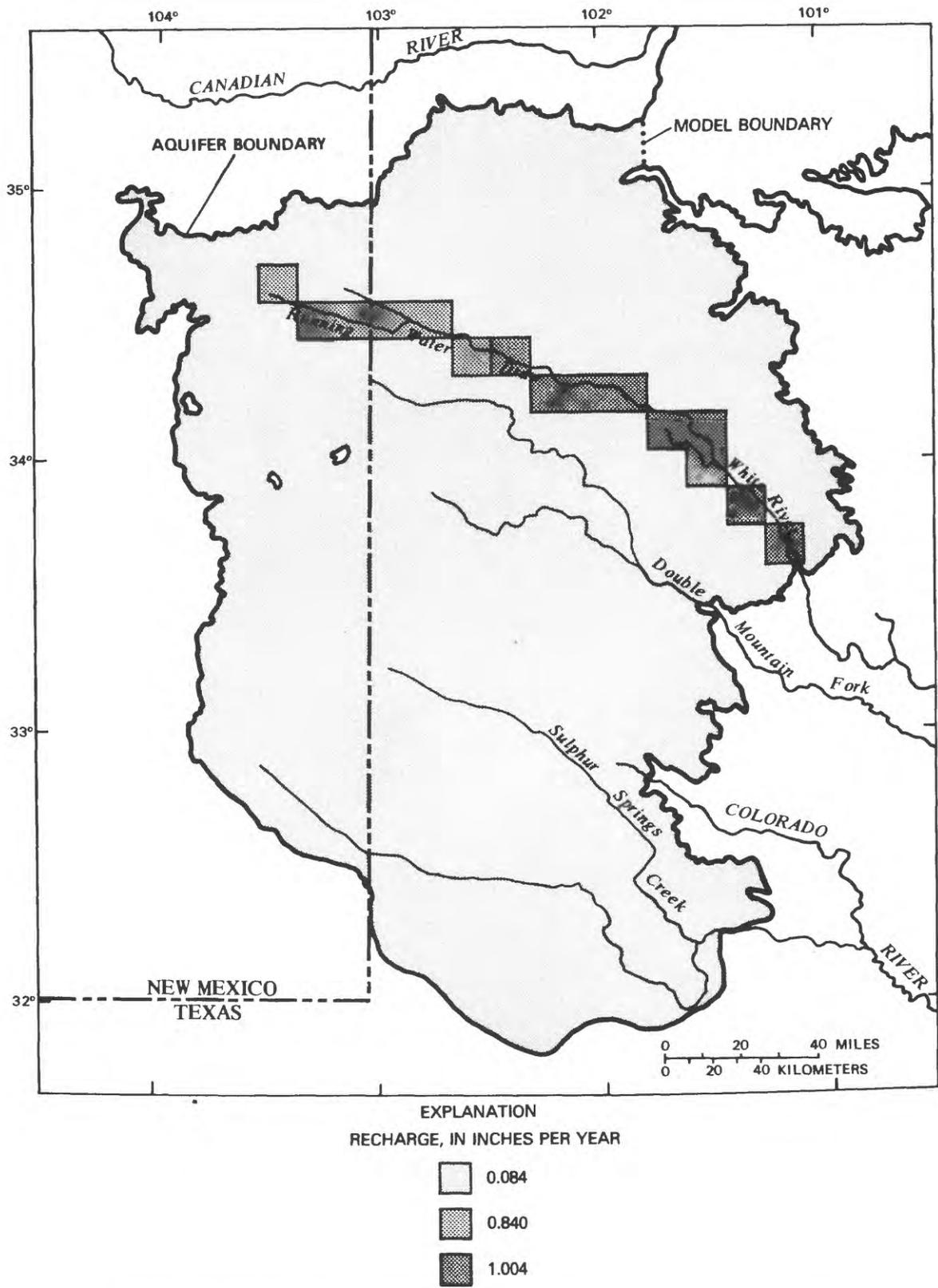
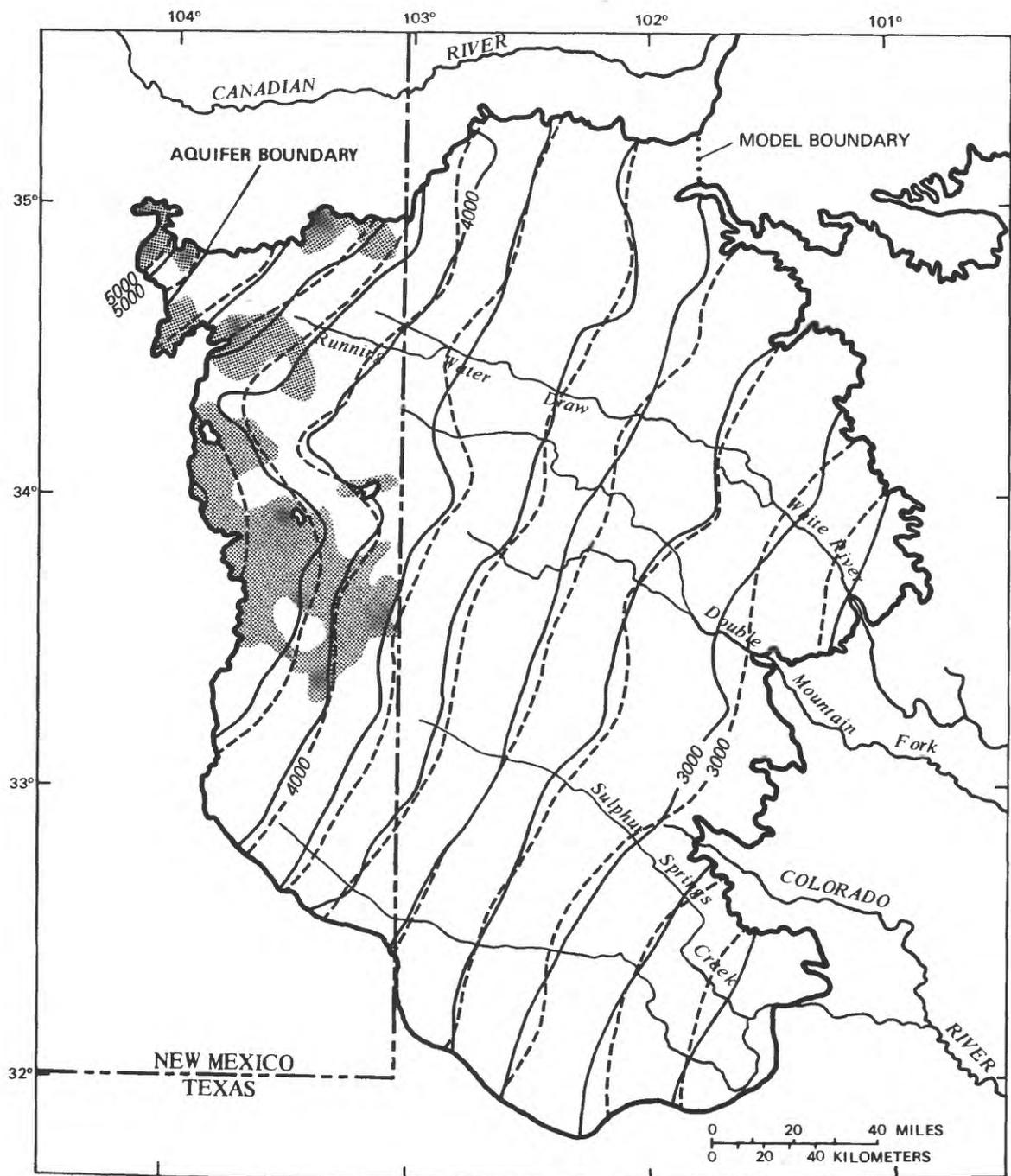


Figure 3.--Estimated predevelopment, long-term average recharge rates for the fine-grid model.



**EXPLANATION**

 AREA OF LITTLE OR NO SATURATED THICKNESS

1980 WATER LEVEL CONTOUR —  
Contour interval is 200 feet. Datum is sea level

--- 4000 --- Measured

— 4000 — Simulated

Figure 4.--Simulated and measured predevelopment water levels for the fine-grid model.

Table 1.--*Model statistics for predevelopment-period calibration of the fine-grid and coarse-grid models*

[Units are as indicated]

Statistic	Fine-grid model	Coarse-grid model
Number of active nodes at calibration	1,201	303
Size of node (square miles)	25	100
Mean difference <sup>1</sup> (feet)	-0.01	+0.22
Standard deviation of differences <sup>1</sup> (feet)	44.4	41.6
Mean of absolute value of differences <sup>1</sup> (feet)	33.4	31.9
Most negative difference <sup>1</sup> (feet)	-155	-113
Most positive difference <sup>1</sup> (feet)	+119	+99

<sup>1</sup>Difference refers to the measured predevelopment water level minus the simulated predevelopment water level.

The mean difference between the simulated predevelopment water levels for the *fine-grid model* and the *coarse-grid model* was -1.09 ft. The standard deviation was 19.0 ft.

The distribution of the differences in simulated predevelopment water levels between the *fine-grid model* and *coarse-grid model* is shown in figure 5. As can be seen, 93 percent of the differences between the models were less than 30 ft and 81 percent of the differences were less than 20 ft. Two nodes had differences of more than 100 ft and five nodes had differences between 50 and 100 ft.

The differences between the simulated water levels from the *fine-grid model* and *coarse-grid model* are small compared to the differences between the simulated and measured predevelopment water levels. This can be seen by comparing figure 2 with figure 5. The standard deviation is a statistical measure of the spread of a set of values. The standard deviation of the differences between the simulated and measured predevelopment water levels was 41.6 ft for the *coarse-grid model* and 44.4 ft for the *fine-grid model*. These standard deviations are more than twice as large as the standard deviation of the differences in simulated predevelopment water levels between the *fine-grid model* and *coarse-grid model* of 19.0 ft.

The areal distribution of the differences between the simulated predevelopment water levels for the *fine-grid model* and *coarse-grid model* is shown in figure 6. There is a large area of positive difference in the north-central part of the southern High Plains. The other positive-difference areas are concentrated along the boundary of the aquifer and in areas of little saturated thickness (and hence sparse hydrologic data). There are two large areas of negative difference in the southern one-third of the southern High Plains. Most of these areas fall within the -10 to -25 ft range. The large

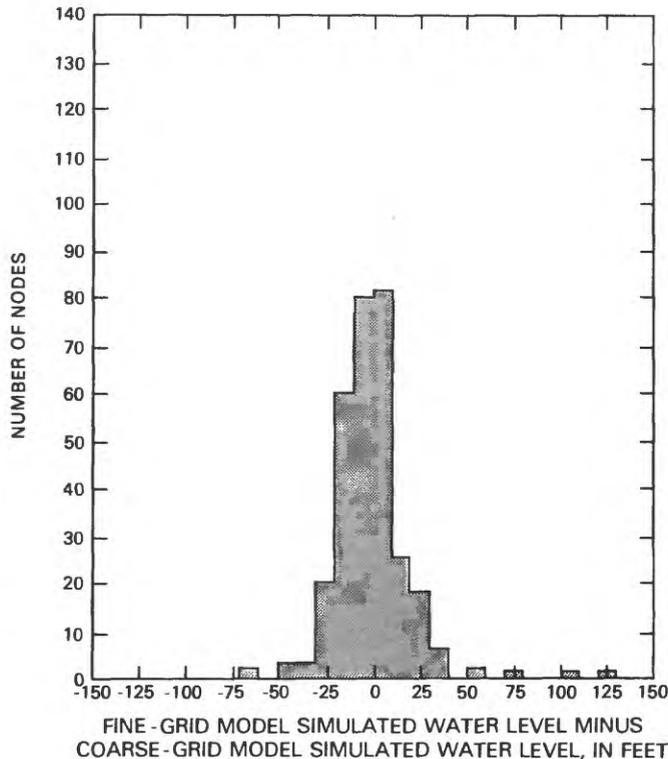
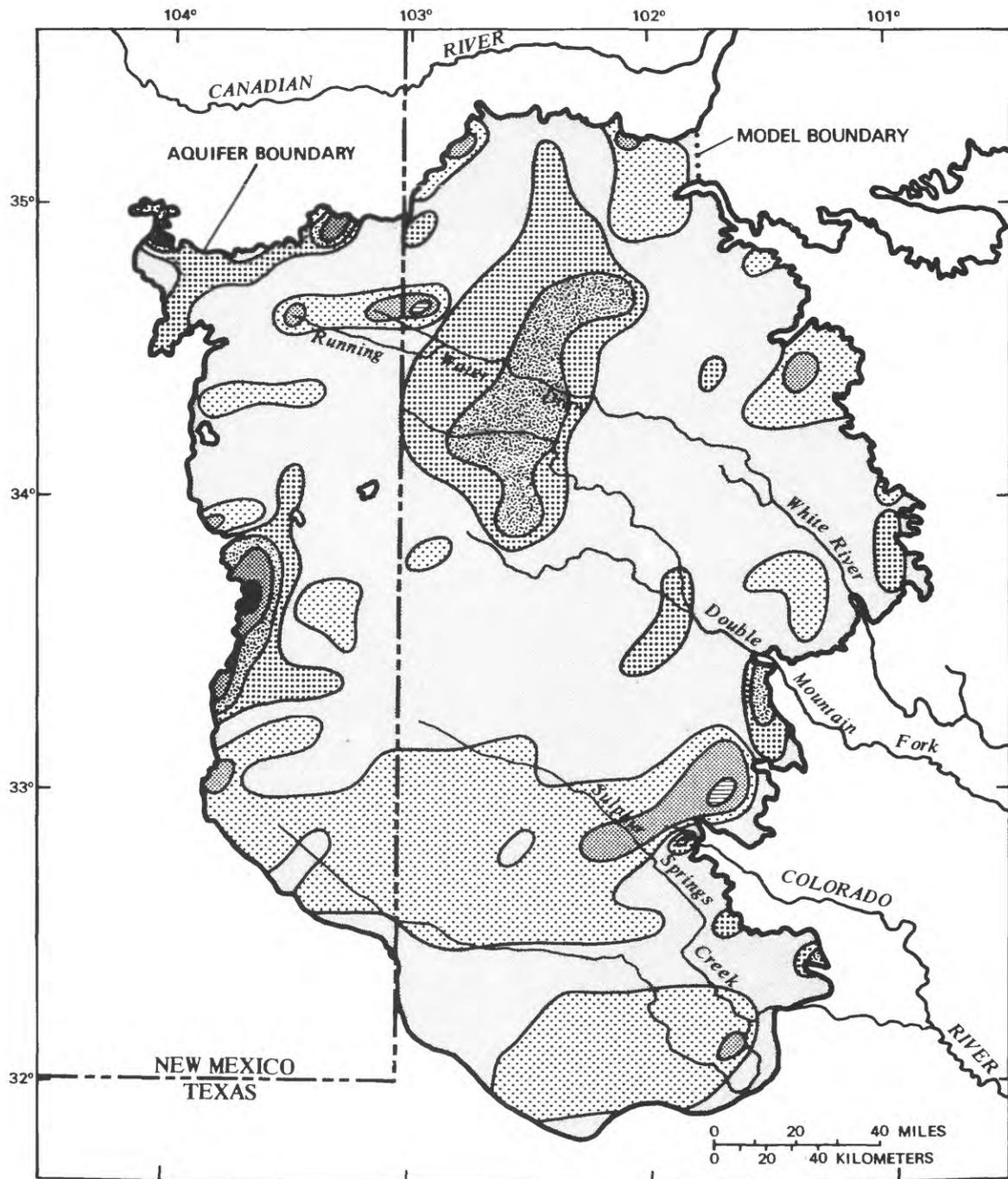


Figure 5.--Differences in simulated predevelopment water levels between the fine-grid and coarse-grid models.

differences tend to be concentrated in areas of sparse data. The pattern of differences does not seem to be related to either the hydraulic conductivity (Gutentag and others, 1984, fig. 10) or the altitude of the base of aquifer (Gutentag and others, 1984, fig. 6). Hence, no hydrologic significance can be attached to the pattern of differences between these two models.

The simulated predevelopment water levels for the *coarse-grid model* and *fine-grid model* for the nodes with the most positive and most negative differences are listed in table 2 and their location is shown in figure 7. These differences are below the fifth percentile and above the ninety-fifth percentile and represent 10 percent of the modeled area. The base of aquifer and the hydraulic conductivity also are shown for these nodes because the simulated predevelopment water levels are sensitive to these two inputs. For some nodes, large differences exist between the model values (for example, the base of aquifer in nodes numbered 28, 29, and 30) but for other nodes the values are similar. These model values are not identical because they were picked independently from each other even though they came from the same original maps. Such differences in visual interpolation are to be expected. When picking values for the *fine-grid model*, the values used in the *coarse-grid model* were not consulted to see whether the values for the two models were similar. After the two sets of values were compared, it generally was difficult to determine which set of values more correctly represented the original maps. The difficulty was caused by the wide range of values possible



EXPLANATION

DIFFERENCES IN SIMULATED WATER LEVELS BETWEEN FINE-GRID AND COARSE-GRID MODELS, IN FEET

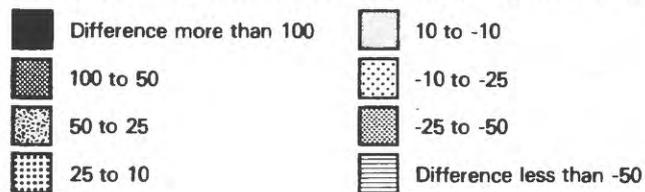
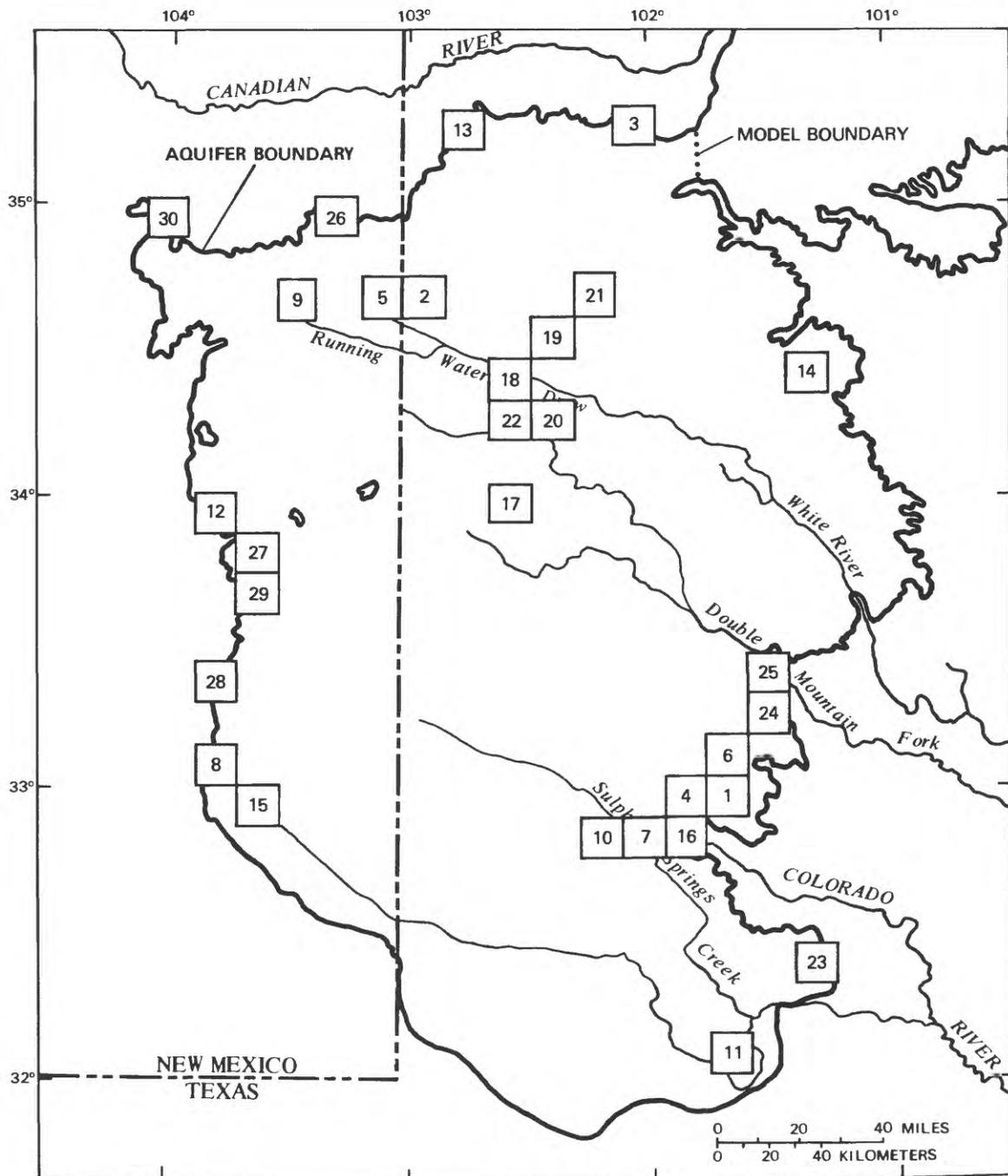


Figure 6.--Areal distribution of differences in simulated predevelopment water levels between the fine-grid and coarse-grid models.



EXPLANATION

16 NODE—Number refers to node in table 2

Figure 7.--Location of nodes with large differences in simulated predevelopment water levels between the fine-grid and coarse-grid models.

Table 2.--Simulated predevelopment water level, altitude of base of aquifer, and hydraulic conductivity for nodes with large differences between fine-grid and coarse-grid models

Map number from figure 7	Node		Predevelopment water level (feet)		Base of aquifer (feet)		Hydraulic conductivity (feet per day)	
	Row	Column	Fine <sup>1</sup>	Coarse <sup>1</sup>	Fine	Coarse	Fine	Coarse
1	18	15	2944	3009	-65.2	2867	50	40
2	6	8	4015	4077	-61.9	4006	182	115
3	2	13	3601	3647	-45.6	3596	212	107
4	18	14	3022	3065	-42.6	2957	58	60
5	6	7	4120	4160	40.6	4095	70	30
6	17	15	2985	3019	-34.3	2908	34	32
7	19	13	3019	3053	-34.1	2925	170	93
8	17	3	4011	4041	-30.2	3994	39	40
9	6	5	4363	4393	-30.0	4317	51	45
10	19	12	3082	3111	-29.6	2976	81	67
11	24	15	2533	2563	-29.1	2470	217	104
12	11	3	4437	4462	-25.5	4360	12	12
13	2	9	4042	4067	-24.5	4012	16	13
14	8	17	3170	3195	-24.5	3091	152	115
15	18	4	3939	3963	-24.0	3927	52	50
16	19	14	2939	2912	27.6	2891	148	91
17	11	10	3669	3641	28.1	3617	55	50
18	8	10	3797	3768	28.3	3502	173	115
19	7	11	3742	3713	29.2	3481	70	60
20	9	11	3702	3671	30.5	3511	168	93
21	6	12	3697	3665	31.4	3646	136	76
22	9	10	3778	3744	33.9	3488	157	115
23	22	17	2509	2475	33.9	2411	190	104
24	16	16	2917	2881	35.9	2832	29	30
25	15	16	2990	2952	37.7	2852	16	20
26	4	6	4555	4495	59.8	4545	41	11
27	12	4	4424	4364	59.9	4357	10	10
28	15	3	4328	4258	70.5	4325	20	20
29	13	4	4422	4314	107.7	4400	11	11
30	4	2	5242	5118	124.6	5208	10	10

<sup>1</sup>Rounded to nearest foot.

<sup>2</sup>Differences are calculated from original data and rounded to the nearest 0.1 foot.

<sup>3</sup>Difference is negative if the fine-grid value is less than the coarse-grid value.

<sup>4</sup>Ratio is less than 1.0 if the fine-grid value is less than the coarse-grid value.

when picking one value to represent 100 mi<sup>2</sup> or four values, each representing 25 mi<sup>2</sup>. The largest differences, especially for the base of aquifer, occurred at nodes along the boundary in places where the gradient of the base of aquifer changes rapidly.

### Aggregated Model

The predevelopment-period *aggregated model* was used to calculate the predevelopment water level using a 100 mi<sup>2</sup> grid. The *aggregated model* had 303 active nodes. Model inputs were not changed during this simulation.

The simulated predevelopment water levels for the *fine-grid model* were compared for those nodes in the *aggregated model* which had four corresponding nodes in the *fine-grid model*. The comparison was made by subtracting the mean of the four *fine-grid-model* water levels from the corresponding *aggregated-model* water level. These differences are an estimate of the discretization difference between 25-mi<sup>2</sup> and 100-mi<sup>2</sup> grids. A histogram of the differences in simulated predevelopment water levels between the *fine-grid model* and *aggregated model* is shown in figure 8. The differences ranged from -35 ft to +36 ft and averaged 0.86 ft for the 278 nodes in the *aggregated model* that had four corresponding nodes in the *fine-grid model*. The standard deviation of the differences was 8.9 ft. Ninety percent of the nodes had differences of less than 15 ft and eighty percent had differences of less than 10 ft. Comparing this histogram to figure 2 shows that the discretization difference between the 25-mi<sup>2</sup> and 100-mi<sup>2</sup> grids is small compared to the difference between the simulated and measured predevelopment water levels. The differences between the *coarse-grid model* and the *fine-grid model* were caused both by differences in model inputs and by discretization difference. This histogram (fig. 8) shows much smaller differences than the differences between the *coarse-grid model* and the *fine-grid model* (fig. 5). This indicates that much of the difference between the *coarse-grid model* and the *fine-grid model* was due to differences in the values picked for the model rather than discretization error.

The areal distribution of the differences in simulated predevelopment water levels between the *fine-grid model* and *aggregated model* is shown in figure 9. Over much of the southern High Plains, the differences between these models were less than 5 ft. All of the differences greater than 20 ft were concentrated along the boundary of the aquifer. This indicates that for these models, a large part of the discretization difference was caused by different approximations of the location of the boundary of the aquifer. There is a fairly large area away from the boundary of the aquifer at about 33° latitude with -5 to -10 ft differences. This area contains three smaller areas of -10 to -20 ft differences. At about 34° latitude, an area of +5 to +10 ft differences away from the boundary contains a smaller area of +10 to +20 ft difference. These areas both correspond to areas where the hydraulic conductivity map (a detailed version of figure 10 in Gutentag and others, 1984) indicated large changes in hydraulic conductivity over fairly small distances. For example, in the northern area, hydraulic conductivity changed from 15 to 75 ft/d in about 10 mi in one place and in the southern area, it changed from 25 to 160 ft/d in about the same distance. This was a

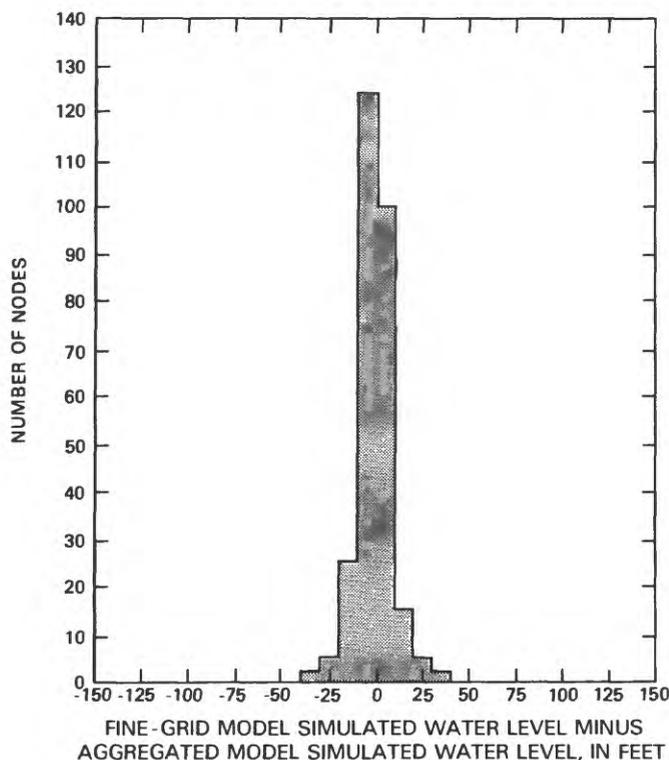


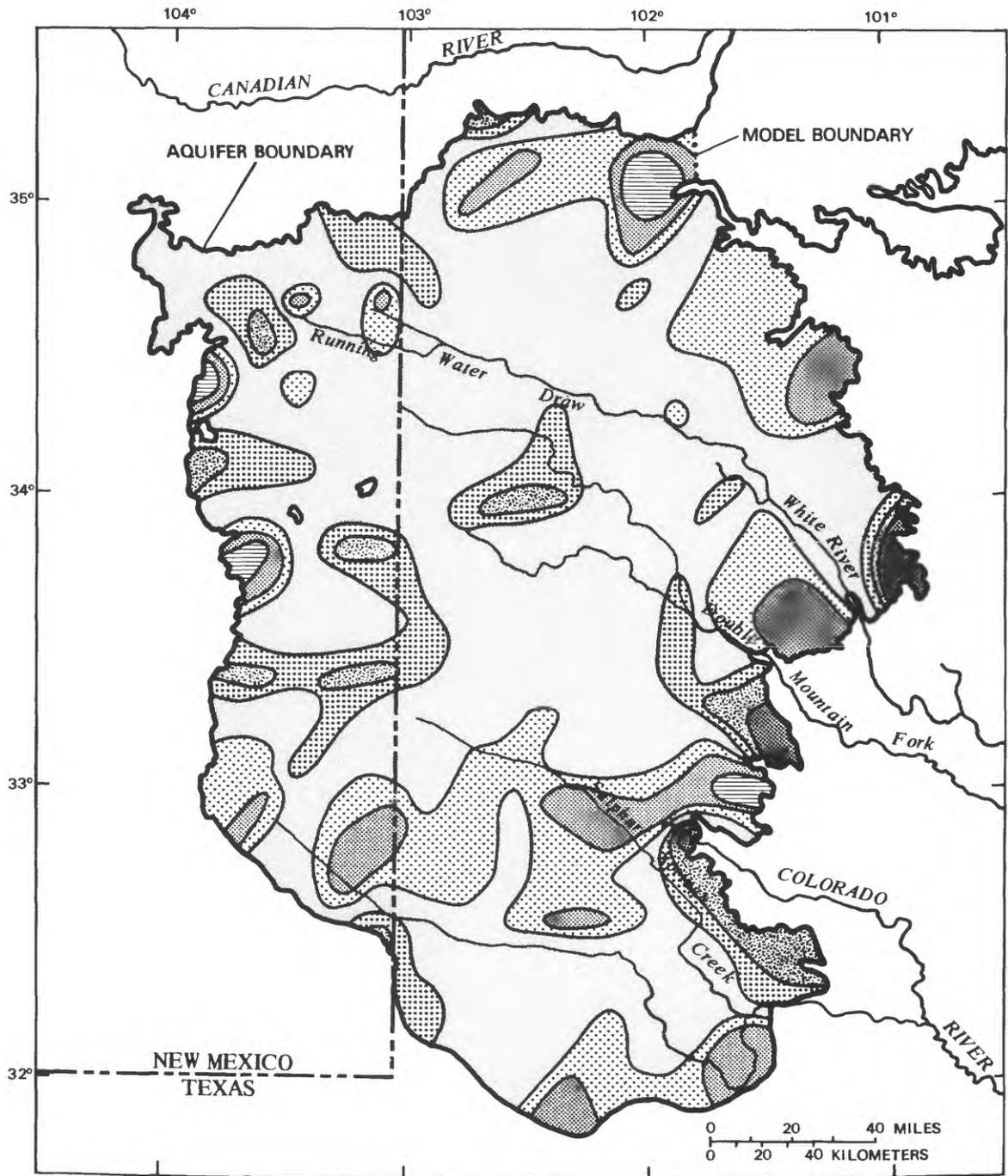
Figure 8.--Differences in simulated predevelopment water levels between the fine-grid and aggregated models.

five- to six-fold change. In these areas, the discretization difference could be due to approximating the ground-water flow equation with a finite-difference equation or representing the hydraulic conductivity with a series of discrete values.

The *aggregated model* also was tested using the geometrically averaged (rather than arithmetically averaged) hydraulic conductivity. The areal distribution of water levels was virtually identical to that obtained using the arithmetic average. The histogram of water-level differences between this model and the *fine-grid model* showed a skew to the right with a mean difference of 3.44 ft. Because of this bias and little difference in distribution of differences between the models, the *aggregated model* used the arithmetically averaged hydraulic conductivity to estimate discretization error.

#### EFFECT OF GRID SIZE ON DEVELOPMENT-PERIOD MODEL

The effects of irrigation development on the aquifer in the southern High Plains was simulated using the models. These models began with the predevelopment (1940) water level and were calibrated by comparing the simulated and measured 1980 water-levels. The primary new stress on the aquifer in the southern High Plains after 1940 was the withdrawal of large



EXPLANATION  
 DIFFERENCES IN SIMULATED WATER LEVELS BETWEEN FINE-GRID AND AGGREGATED MODELS, IN FEET

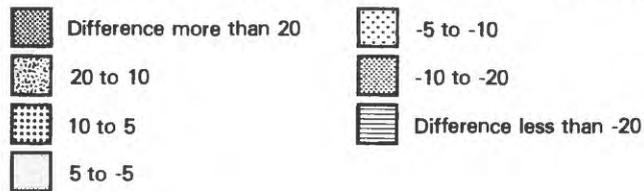


Figure 9.--Areal distribution of differences in simulated predevelopment water levels between the fine-grid and aggregated models.

amounts of water for irrigation. Not all of this water was consumed, but part of it returned to the aquifer as increased recharge due to irrigation (return flow). The amount of return flow was the primary value that was varied during the development-period calibration. Return flow was assumed to be a linear function of the difference between total pumpage and the amount of water that the crops required in excess of precipitation (irrigation requirement, Heimes and Luckey, 1982).

The development-period calibration used all of the data from the predevelopment-period calibration. Additional values needed for the development-period calibration included specific yield and net pumpage. The measured 1980 water level was needed for model calibration.

### Coarse-Grid Model

The development-period calibration for the *coarse-grid model* started with the measured predevelopment (about 1940) water levels and simulated the aquifer system from 1940 to 1980. Return flow from irrigation was varied during calibration until the best correspondence between the simulated and measured 1980 water-level maps was obtained. Calibration was achieved when return flow was adjusted such that net withdrawal (total pumpage minus return flow) was equal to approximately 90 percent of the estimated irrigation requirement. During the 1960-80 part of the development period, 2 in/yr additional recharge was added to all agricultural land as explained by Luckey and others (1986, p. 18).

For the *coarse-grid model*, the mean difference between the simulated and measured 1980 water levels was +0.28 ft. The minimum and maximum differences were -111 ft and +93 ft respectively. The standard deviation of the differences between the simulated and measured 1980 water levels was 25.8 ft, and the mean of the absolute values of the differences was 19.3 ft. Figure 10 shows the distribution of the differences between the simulated and measured 1980 water levels for the *coarse-grid model*.

There were 282 nodes that remained active in the model by 1980; the remaining 21 nodes became inactive in the model during the calibration as they became essentially dewatered between 1940 and 1980. Some of these nodes were within the intensely pumped areas, but many were in areas that had little saturated thickness at the beginning of the simulation period. The total pumpage, return flow, cross-boundary flow, and change in storage for this simulation are summarized in table 3.

### Fine-Grid Model

The measured 1980 water-level data set used to calibrate the *fine-grid model* was different from that used to calibrate the *coarse-grid model*. The data set used to calibrate the *coarse-grid model* had a mean of 3,501 ft while the data set used to calibrate the *fine-grid model* had a mean of 3,492 ft. The 9-ft difference in water levels represents about 25 million acre-feet of water or about 12 percent of total pumpage. The 9-ft difference between these data sets was important because the mean difference between the simulated and measured 1980 water levels was to be the final ("fine-tuned") calibration criterion.

Table 3.--Water budget for development-period calibration of the fine-grid and coarse-grid models

[Units are in millions of acre-feet]

Budget item	Fine-grid model	Coarse-grid model
Outflows:		
Total pumpage	209.55	209.55
Boundary outflow	<u>4.33</u>	<u>4.11</u>
Total	213.88	213.66
Inflows:		
Return flow from irrigation	98.58	93.65
Recharge from precipitation on agricultural land	24.07	23.34
Recharge from precipitation on rangeland	<u>6.32</u>	<u>6.38</u>
Total	128.97	123.37
Decrease in storage	84.91	90.29

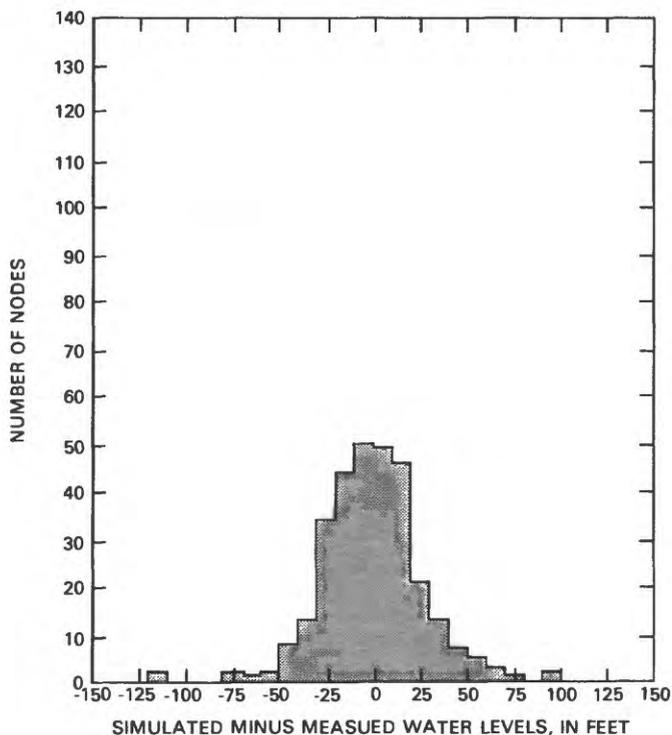


Figure 10.--Differences in simulated and measured 1980 water levels for the coarse-grid model.

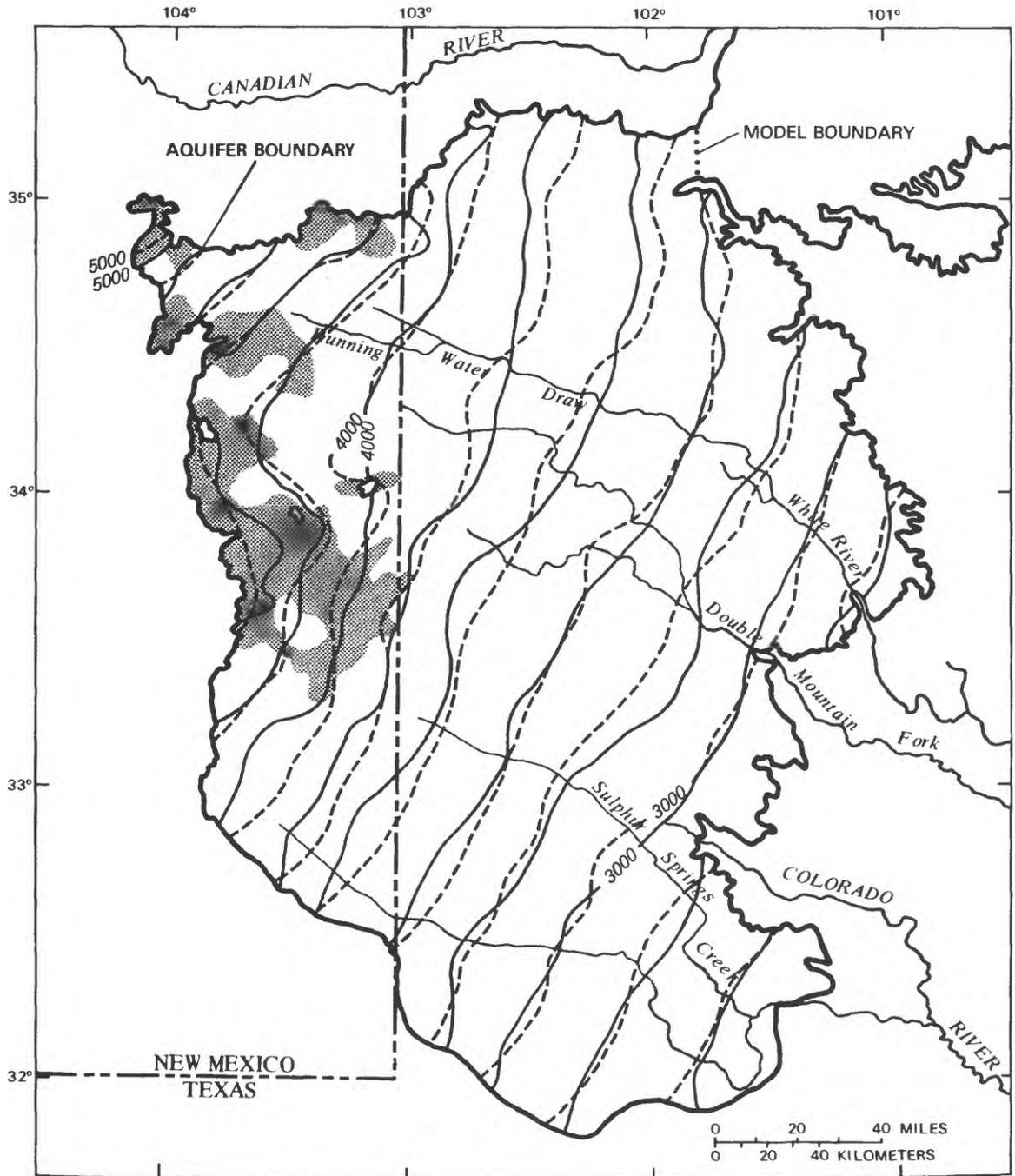
The measured 1980 water-level data sets used to calibrate the *coarse-grid model* and *fine-grid model* were examined to determine which better represented the original map. Over approximately the northern one-half of the area, differences between the two data sets appeared random, but in the southern part of the area, there was a bias of up to 20 ft between the two data sets. Such a bias could result from either the coarse grid or the fine grid being shifted or rotated with respect to the original map. The original map had a 100-ft contour interval and was made at a scale of 1:1,000,000. The 20-ft bias noted above represented only about 0.11 in. shift of the model grid on the original map. Hence, it was virtually impossible to determine which of the two data sets better represented the original map.

If both models were being recalibrated during this study, the authors would have shifted each of the grids one-half of the difference so that they would be coincident. However, the *coarse-grid model* was not recalibrated for this study so the "fine-tuned" calibration criterion was changed from a 0.00 ft residual to 8.07 ft residual. The 8.07 ft residual is the result of a +8.94 ft difference in the measured 1980 water-level data sets and a -0.87 ft difference in the measured 1940 water-level data sets. With an 8.07 ft residual, the *fine-grid model* and *coarse-grid model* would have the same change in ground-water storage from 1940 to 1980 and approximately the same 1980 water level. The change in calibration criteria would overcompensate in the northern part of the area and undercompensate in the southern part. However, this overcompensation and undercompensation would be small compared to the contour interval on the original map.

The model was considered calibrated when return flow was adjusted such that net pumpage was 86 percent of the estimated irrigation requirement. The total pumpage for this simulation was 210 million acre-feet, and return flow was 99 million acre-feet (table 3).

The simulated and measured 1980 water levels for the calibrated *fine-grid model* are shown in figure 11. There is a reasonable correspondence between the shapes of the contours. The mean difference between the two surfaces is 8.22 ft. The "fine-tuned" calibration criterion was 8.07 ft, so this criterion was missed by 0.15 ft. The minimum and maximum differences between the two surfaces was -92 and +166 ft respectively for the 1,201 nodes. The standard deviation of the difference between the simulated and measured 1980 water levels was 27.9 ft, and the mean of the absolute values of the differences was 21.5 ft. The statistics for the development-period calibration of both the *coarse-grid model* and the *fine-grid model* are summarized in table 4.

As was done with the predevelopment period, the simulated 1980 water levels from the *fine-grid model* were aggregated by taking an arithmetic average of the simulated 1980 water levels at (up to four) nodes from the *fine-grid model* that corresponded to one node in the *coarse-grid model*. A node-by-node comparison was made between the simulated 1980 water levels from the *fine-grid model* and the *coarse-grid model*. The mean difference in 1980 water levels from the *fine-grid model* and *coarse-grid model* was 1.43 ft with a standard deviation of 21.8 ft. A histogram of the differences between the



EXPLANATION

 AREA OF LITTLE OR NO SATURATED THICKNESS  
 PREDEVELOPMENT WATER LEVEL CONTOUR —  
 Contour interval is 200 feet. Datum is sea level  
 - - 4000 - - Measured  
 — 4000 — Simulated

Figure 11.--Simulated and measured 1980 water levels for the fine-grid model.

Table 4.--Model statistics for development-period calibration of the fine-grid and coarse-grid models

[Units are as indicated]

Statistic	Fine-grid model	Coarse-grid model
Number of active nodes at calibration	1,201	282
Size of node (square miles)	25	100
Residual <sup>1</sup> from calibration criterion (feet)	+0.15	+0.28
Mean difference <sup>2</sup> (feet)	+8.22	+0.28
Standard deviation of differences <sup>2</sup> (feet)	27.9	25.8
Mean of absolute value of differences <sup>2</sup> (feet)	21.5	19.3
Most negative difference <sup>2</sup> (feet)	-99	-111
Most positive difference <sup>2</sup> (feet)	+160	+93

<sup>1</sup>Residual refers to the mean difference at calibration and the mean difference sought.

<sup>2</sup>Difference refers to the measured 1980 water level minus the simulated 1980 water level.

simulated 1980 water levels from the two models is shown in figure 12. Two nodes had negative differences of more than -50 ft and six nodes had positive differences of more than +50 ft. At 90 percent of the nodes, the absolute values of the differences between the two models were less than 30 ft and at 73 percent of the nodes, the absolute values of the differences were less than 20 ft.

The areal distribution of the differences between the simulated 1980 water levels from the *fine-grid model* and *coarse-grid model* is shown in figure 13. The largest area of negative difference is in the northern part of the southern High Plains while the largest area of positive difference is in the southern part. There appears to be no hydrologic significance to the differences. Comparing the differences in this figure with the differences shown in figure 6, the areas of positive difference in this map roughly correspond to areas of negative difference in the previous map and vice versa. Hence, there does not seem to be any consistent pattern of differences between the *coarse-grid model* and *fine-grid model* for either the predevelopment-period or development-period calibration.

Table 5 lists the simulated 1980 water levels for the *fine-grid model* and *coarse-grid model* for those nodes which had the most positive and most negative differences between the two models. The location of these nodes is shown in figure 14. These nodes are less than the fifth percentile and greater than the ninety-fifth percentile of the differences and represent 10 percent of the model area. The 1940 water level (the initial water level) and the specific yield are also shown for these nodes because the simulated 1980

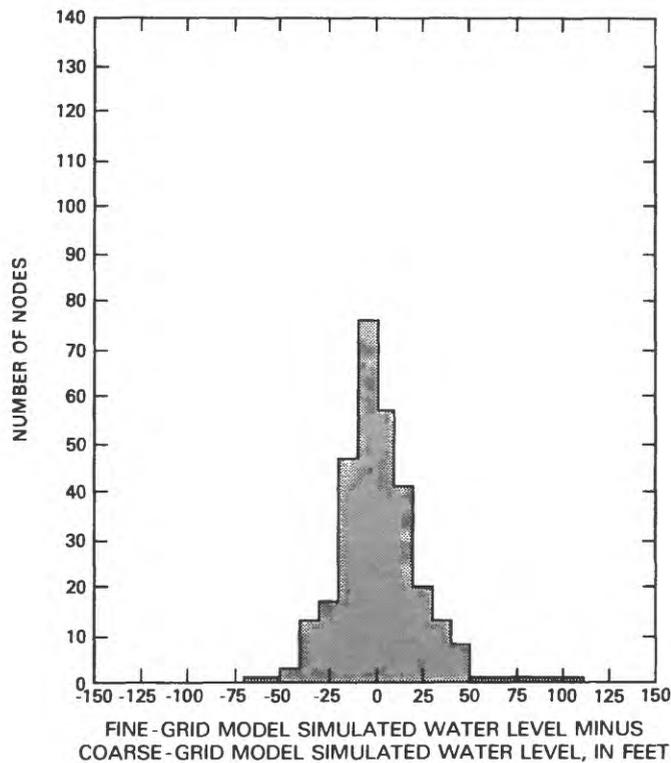
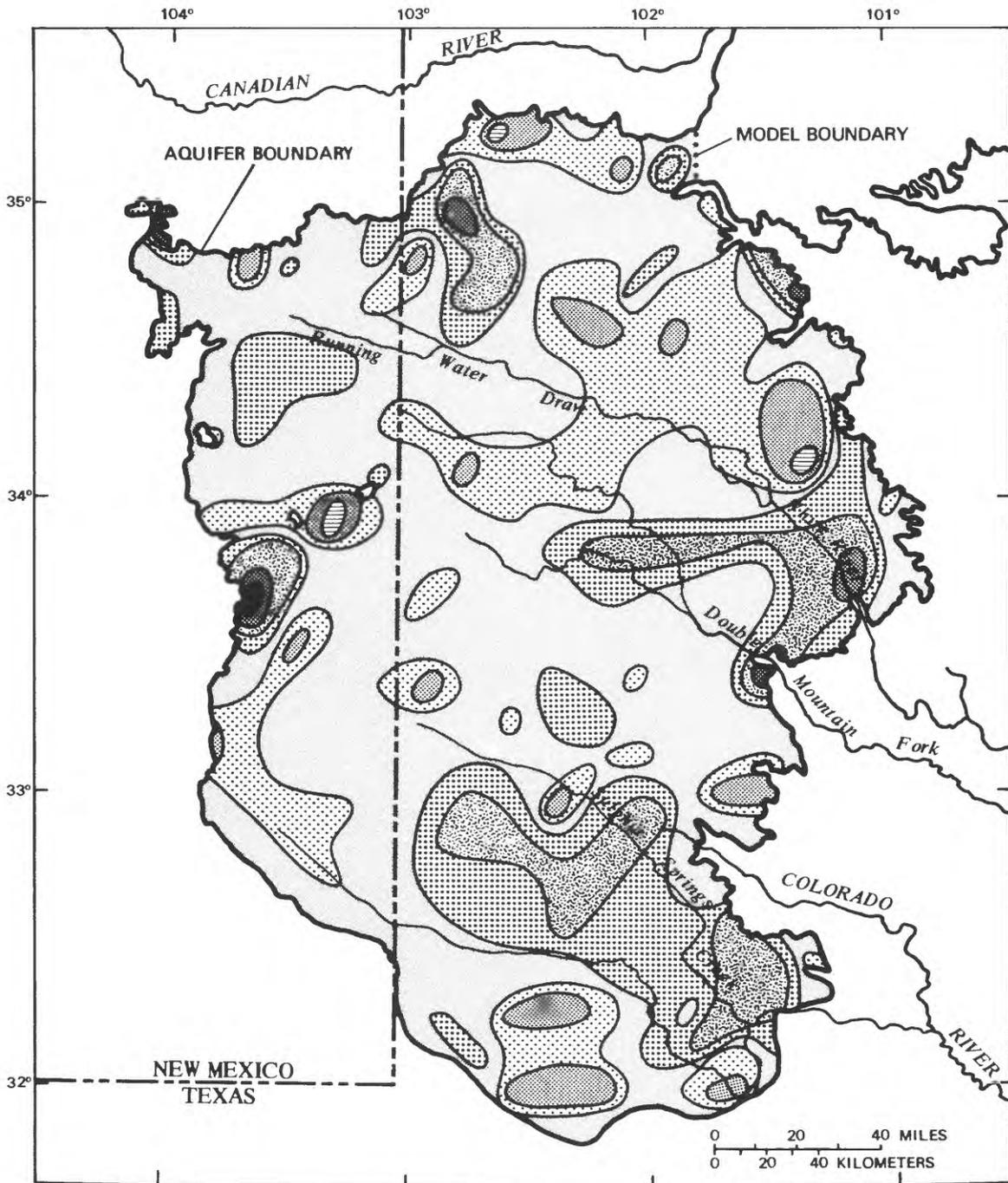


Figure 12.--Differences in simulated 1980 water levels between the fine-grid and coarse-grid models.

water levels are sensitive to these two values. The table shows that, for the most part, the specific yield in the *coarse-grid model* and the *fine-grid model* were similar. The 1940 water level was also similar at most nodes. An exception is the node number 29 in the table. However, this node is at the edge of the model in an area where water level values are extremely difficult to determine.

#### Aggregated Model

After calibration of the development-period *fine-grid model*, the values were aggregated to 100-mi<sup>2</sup> grids. The method of aggregating the data was identical to that used for the predevelopment period. The development-period *aggregated model* was used to simulate the 1980 water level using values as close to those used in the *fine-grid model* as the grid would permit. Because of the slight differences in boundaries between the models, the pumpage in the *aggregated model* was 0.26 percent greater than in the *fine-grid model*. Similarly, total recharge was 3.9 percent greater in the *aggregated model* even though recharge at individual nodes was identical. The mean differences between the simulated and measured 1980 water levels for the *aggregated model* was 8.50 ft compared to 8.22 ft for the *fine-grid model*.



EXPLANATION

DIFFERENCES IN SIMULATED WATER LEVELS BETWEEN FINE-GRID AND COARSE-GRID MODELS, IN FEET

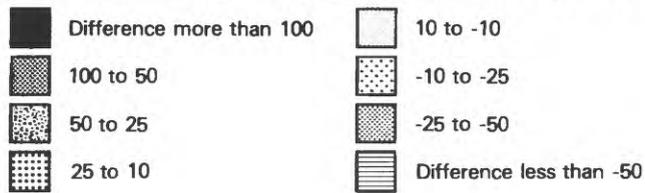


Figure 13.--Areal distribution of differences in simulated 1980 water levels between the fine-grid and coarse-grid models.

Table 5.--Simulated 1980 water level and specific yield for nodes with large differences between fine-grid and coarse-grid models

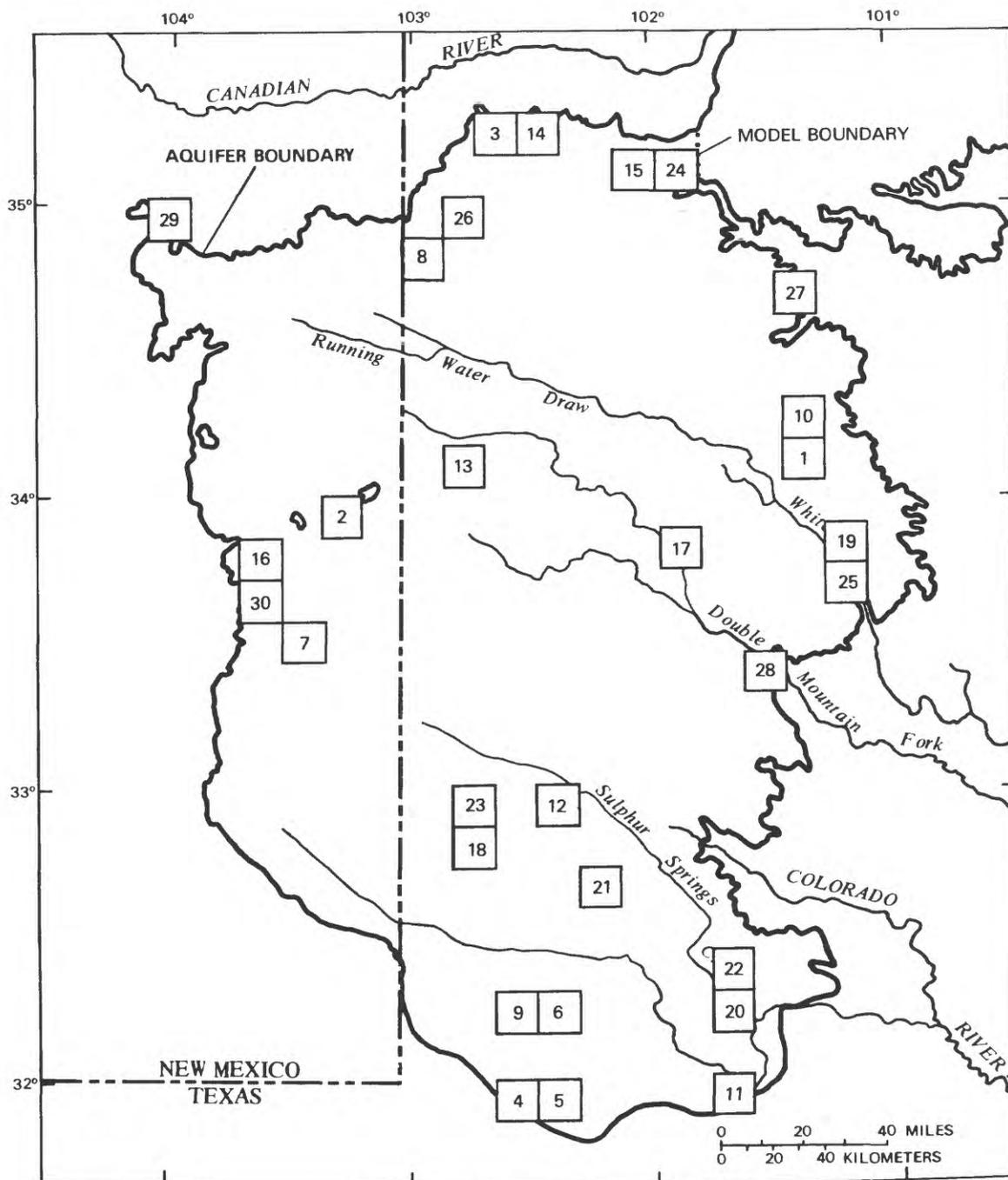
Map number from figure 14	Node		1980 Water level (feet)		1940 Water level (feet)		Specific yield (percent)			
	Row	Column	Fine <sup>1</sup>	Coarse <sup>1</sup>	Fine	Coarse	Fine	Coarse		
1	10	17	3040	3101	-61.1	3132	3123	16	17	0.94
2	11	6	4132	4185	-52.6	4148	4134	14	15	.93
3	2	10	3926	3976	-49.9	3926	3978	14	15	.93
4	25	10	2934	2978	-43.5	2934	2946	17	18	.94
5	25	11	2850	2891	-40.8	2855	2860	16	16	1.00
6	23	11	2933	2972	-39.6	2939	2943	14	12	1.17
7	14	5	4161	4200	-39.4	4158	4154	14	15	.93
8	5	8	4141	4179	-38.8	4150	4147	15	15	1.00
9	23	10	3033	3072	-38.6	3033	3033	15	15	1.00
10	9	17	3061	3098	-36.6	3123	3129	20	20	1.00
11	25	15	2492	2525	-33.4	2495	2502	13	15	.87
12	18	11	3137	3170	-33.2	3173	3176	13	14	.93
13	10	9	3767	3799	-32.6	3801	3800	12	12	1.00
14	2	11	3806	3838	-32.4	3816	3820	15	15	1.00
15	3	13	3585	3616	-31.4	3624	3648	15	15	1.00
16	12	4	4373	4333	39.2	4368	4368	14	15	.93
17	12	14	3169	3129	40.0	3265	3265	10	10	1.00
18	19	9	3350	3307	42.5	3385	3389	14	13	1.08
19	12	18	2866	2821	44.7	2878	2884	17	18	.94
20	23	15	2576	2531	45.1	2564	2568	16	15	1.07
21	20	12	3003	2958	45.2	2985	2985	12	11	1.09
22	22	15	2649	2603	46.5	2642	2643	16	16	1.00
23	18	9	3424	3376	47.5	3461	3463	16	17	.94
24	3	14	3534	3486	47.7	3547	3538	14	16	.88
25	13	18	2815	2761	53.9	2819	2830	11	13	.85
26	4	9	4022	3954	67.4	4021	4011	16	17	.94
27	6	17	3261	3189	72.5	3262	3255	16	18	.89
28	15	16	2990	2910	80.0	2990	2952	12	10	1.20
29	4	2	5222	5130	92.8	5218	5120	14	15	.93
30	13	4	4411	4301	109.3	4411	4404	15	15	1.00

<sup>1</sup>Rounded to nearest foot.

<sup>2</sup>Differences are calculated from original data and rounded to the nearest 0.1 foot.

<sup>3</sup>Difference is negative if the fine-grid value is less than the coarse-grid value.

<sup>4</sup>Ratio is less than 1.0 if the fine-grid value is less than the coarse-grid value.



EXPLANATION

12 NODE—Number refers to node in table 5

Figure 14.--Location of nodes with large differences in simulated 1980 water levels between the fine-grid and coarse-grid models.

The simulated 1980 water levels for the *fine-grid model* and *aggregated model* were compared on a node-by-node basis. A histogram of the differences in simulated 1980 water levels between the models is shown in figure 15. The differences ranged from -18 ft to +17 ft and averaged 0.39 ft. The standard deviation of the differences was 4.4 ft. At 90 percent of the nodes, the differences were less than 6.5 ft and at 80 percent of the nodes, the differences are less than 4.7 ft. This histogram should be compared to the differences between the simulated and measured 1980 water levels for the coarse-grid model (fig. 2). As can be seen in this comparison, the discretization difference between the 100-mi<sup>2</sup> and 25-mi<sup>2</sup> grids is small compared to the differences between the simulated and measured 1980 water levels.

The areal distribution of the differences between the *fine-grid model* and *aggregated model* is shown in figure 16. Over most of the area, the difference between the two models was less than 5 ft. There were some small areas where the differences were larger than 10 ft. Most of these areas were concentrated along the eastern boundary, where the location of the boundary between the two models was different. The area of +5 to +10 ft difference just south of Double Mountain Fork at about 102° longitude is the largest area of

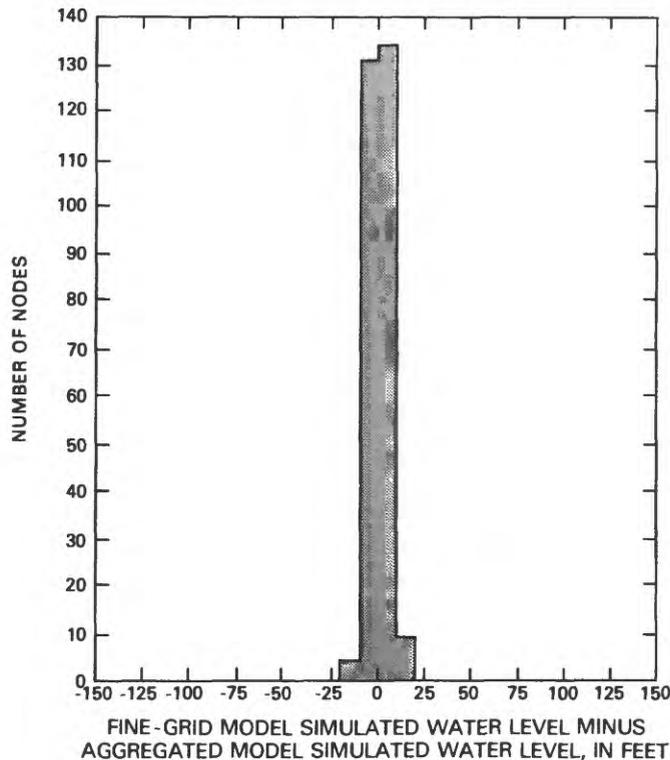
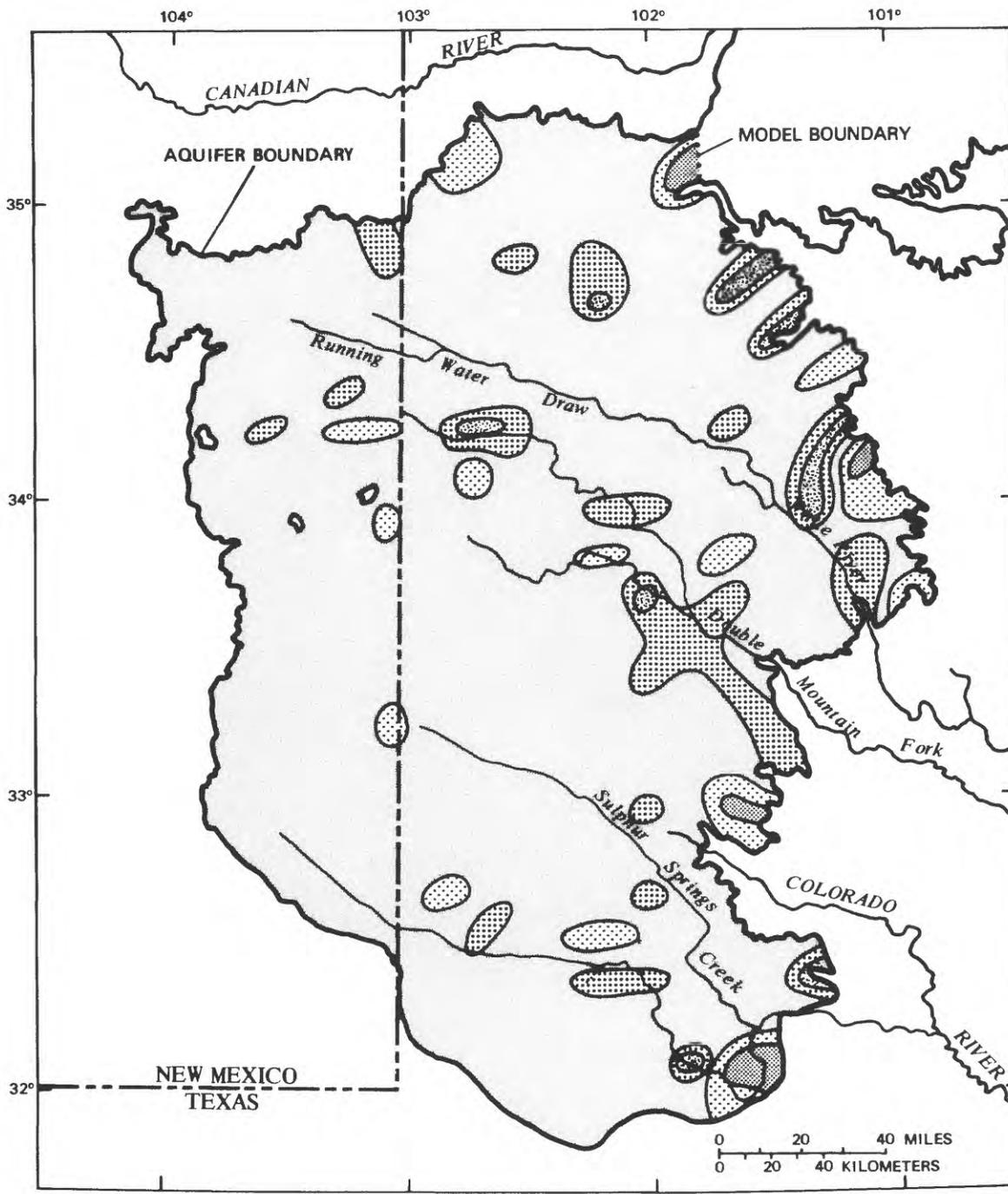


Figure 15.--Differences in simulated 1980 water levels between the fine-grid and aggregated models.



EXPLANATION

DIFFERENCES IN SIMULATED WATER LEVELS BETWEEN FINE-GRID AND AGGREGATED MODELS, IN FEET

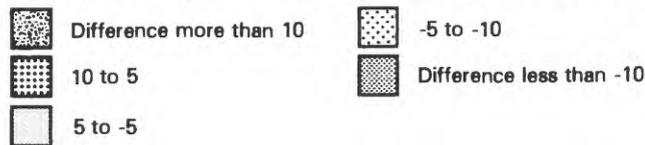


Figure 16.--Areal distribution of differences in simulated 1980 water levels between the fine-grid and aggregated models.

difference. The specific yield in this area is not any more variable than it is across the rest of the southern High Plains. Pumpage in this area is not particularly large, especially when compared to the area further north. Hence, if there is any hydrologic significance to the difference between the *fine-grid model* and *aggregated model* in this area, it is not apparent.

#### SUMMARY AND CONCLUSIONS

Three models of the aquifer in the southern High Plains were used to determine the effect of grid size on the models. The *coarse-grid model* had 10-mi node spacing (100-mi<sup>2</sup> grids). This was the original model that was constructed and calibrated prior to this study. The *fine-grid model* had 5-mi node spacing (25-mi<sup>2</sup> grids). It was constructed and calibrated in this study independently of the *coarse-grid model*. The same original maps were used to generate the data used to construct these two models, but the data differed somewhat between the models because of differences in visual interpolation. The *aggregated model* was constructed by aggregating the data from the *fine-grid model* to 100-mi<sup>2</sup> grids. The data used for the *aggregated model* were as close to those for the *fine-grid model* as the grid would permit.

The distribution of recharge as determined with the predevelopment-period *coarse-grid model* and *fine-grid model* was the same. The total recharge differed by less than 1 percent between the two models. The mean difference in predevelopment water levels was -1.09 ft with a standard deviation of 19.0 ft. The simulated water levels differed between the two models in some areas, but these differences did not seem to have any hydrologic significance.

The development-period *coarse-grid model* was calibrated using a net pumpage (total pumpage minus return flow) that was 90 percent of the estimated irrigation requirement. The development-period *fine-grid model* was calibrated using a net pumpage that was 86 percent of the estimated irrigation requirement. For practical purposes, the difference between these results is not significant. The two models differed in simulated 1980 water levels in some areas but these differences did not seem to have any hydrologic significance.

While calibrating the *fine-grid model*, it was learned that there is far more potential for error in developing the data set that will be used for calibration than the error caused only by using a larger grid size. It is extremely important to have accurate data for the model and if the accuracy of the data (particularly those items to which the model is sensitive) requires a larger than optimal grid spacing, then sacrificing small grid spacing to achieve accurate model inputs would be a good choice for the type of hydrologic system in this study.

Comparison of results from the *coarse-grid model* and *fine-grid model* indicated that the same general hydrologic conclusions would have been reached as a result of model calibration based on data that were available at the time the original model was calibrated whether 25-mi<sup>2</sup> grids or 100-mi<sup>2</sup> grids were used.

The predevelopment-period *aggregated model* calculated water levels very close to those calculated with the *fine-grid model*. The mean difference between the water levels was 0.86 ft with a standard deviation of 8.9 ft. The differences ranged from -35 ft to + 36 ft. Most of the larger differences occurred near the boundary of the aquifer, but some occurred in areas where the hydraulic conductivity changed rapidly over short distances.

The development-period *aggregated model* calculated 1980 water levels that also were very close to those calculated with the *fine-grid model*. The mean difference was 0.39 ft with a standard deviation of 4.4 ft. The differences ranged from -18 to +17 ft. Most of the larger differences occurred near the boundary of the aquifer.

If 25-mi<sup>2</sup> grids had been used in the original model of the aquifer in the southern High Plains instead of 100-mi<sup>2</sup> grids, the same general conclusions about the operation of the hydrologic system would have been reached. Slightly different estimates would have been made for recharge and net pumpage. The differences between the simulated and measured water levels were much larger than the differences between the simulated water levels between models using 25-mi<sup>2</sup> and 100-mi<sup>2</sup> grids. For the predevelopment-period *fine-grid model*, the standard deviation of the difference between the simulated and measured predevelopment water levels was 44.4 ft. The standard deviation of the difference between the simulated predevelopment water levels from the *fine-grid model* and *aggregated model* was 8.9 ft. Similar results were obtained for the 1980 water levels where the respective standard deviations were 27.9 ft and 4.4 ft.

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