# MEAN VELOCITY, LONGITUDINAL DISPERSION, AND REAERATION CHARACTERISTICS OF SELECTED STREAMS IN THE KENTUCKY RIVER BASIN

By Kevin J. Ruhl and James L. Smoot

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CONTENTS

## ILLUSTRATIONS

TABLES

## CONVERSION FACTORS AND ABBREVIATIONS

Values in this report are given in inch-pound or English units. For those who may wish to use metric or International System units, the conversion factors are as follows:

| Multiply | By | To obtain |
|---|---|---|
| | **Length** | |
| foot (ft) | 0.3048 | meter (m) |
| mile (mi) | 1.609 | kilometer (km) |
| foot per mile (ft/mi) | 0.1894 | meter per kilometer (m/km) |
| | **Flow** | |
| foot per second (ft/s) | 0.3048 | meter per second (m/s) |
| foot per day (ft/d) | 0.3048 | meter per day (m/d) |
| square foot per second ($ft^2/s$) | 0.09294 | square meter per second ($m^2/s$) |
| cubic foot per second ($ft^3/s$) | 0.02832 | cubic meter per second ($m^3/s$) |
| mile per hour (mi/h) | 1.609 | kilometer per hour (km/h) |
| gallon per minute (gal/min) | 3.785 | liter per minute (L/min) |
| gallon per minute (gal/min) | 0.003785 | milliliter per minute (mL/min) |
| | **Specific Conductance** | |
| micromhos per centimeter at 25° Celsius (µmhos/cm at 25°C) | 1.000 | microsiemens per centimeter at 25° Celsius (µS/cm at 25°C) |
| | **Temperature** | |
| degree Fahrenheit (°F) | °C = 5/9 x (°F - 32) | degree Celsius (°C) |
| | **Specific Weight** | |
| pound per cubic foot ($lb/ft^3$) | $1.602 \times 10^4$ | milligram per liter (mg/L) |
| pound per cubic foot ($lb/ft^3$) | $1.602 \times 10^7$ | microgram per liter (µg/L) |
| | **Acceleration** | |
| foot per second squared ($ft/s^2$) | 0.3048 | meter per second squared ($m/s^2$) |

# MEAN VELOCITY, LONGITUDINAL DISPERSION, AND REAERATION CHARACTERISTICS OF SELECTED STREAMS IN THE KENTUCKY RIVER BASIN

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## ABSTRACT

The mean velocity, the longitudinal dispersion coefficient, and the reaeration coefficient were determined from nine measurements on five streams in the Kentucky River basin. The values of these parameters were used to evaluate the accuracy of published predictive equations and to develop statistically-based predictive equations which may have applicability to other stream reaches within the same geographic area and with similar characteristics. The five stream reaches had drainage areas ranging from 4.02 to 1,100 square miles and were both pool-and-riffle and channel-controlled reaches. Flow during the studies ranged from low to medium streamflow. The time-of-travel for each dye tracer study was accomplished using standard U.S. Geological Survey procedures. The reaeration coefficient was determined using the hydrocarbon gas-tracer technique, where propane is used as the surrogate tracer gas for oxygen. The determination of these parameter values was accomplished using a simultaneous short duration injection of rhodamine-WT dye and commercial grade propane into the streams.

The estimating equations were developed using multiple linear regression techniques and related to a number of easily measured physical and hydraulic properties of the stream reach. Due to the small number of observations in the data set, only 1- and 2-parameter models are presented. The best mean velocity model shows a coefficient of determination of 0.91 and a root mean square error of 0.073 ft/s. The best 1- and 2-parameter longitudinal dispersion coefficient estimating equations have a coefficient of determination of 0.94 and a root mean square error of 1.25 $ft^2/s$. The best reaeration coefficient estimating equation has a coefficient of determination of 0.99 and a root mean square error of 0.85 $days^{-1}$. Using the measured data, the lowest average absolute error of 22 reaeration coefficient estimating equations is 33 percent.

## INTRODUCTION

Stream reaeration coefficients are necessary input parameters for stream water-quality models. Water-resource managers utilize stream water-quality and dissolved-oxygen models to more effectively estimate the consequences of outside influences on the stream environment and to establish environmental standards and stream waste-load allocations. The models are sensitive to the value of the reaeration coefficient used, and at present, values of the reaeration coefficient are usually estimated from theoretical, empirical, or semi-empirical equations published in the literature. Improper use of empirical or semi-empirical estimating equations may lead to significant errors in the reaeration coefficient estimated for a particular stream reach if input parameter values are outside the range of those from which the equation was developed. Theoretical models tend to be inadequate due to the manner in which model parameters have been related to bulk-flow hydraulic variables.

Several methods of field measuring the reaeration coefficient are available and some are considered more accurate than others. The dissolved-oxygen balance technique involves measuring all of the oxygen sinks and sources except reaeration and subsequently determining the reaeration needed to produce the dissolved oxygen measured at the end of the study reach. This method requires measuring a number of variables before a trial and error procedure of calculating the reaeration coefficient can proceed. The uncertainties associated with measuring these variables could result in a very inaccurate estimate of the reaeration coefficient. The disturbed-equilibrium technique is similar to the dissolved-oxygen balance technique, but some terms in the analysis are eliminated by measuring dissolved oxygen at a different concentration. This different level is achieved by adding sodium sulfite or other chemicals to the stream. The disturbed-equilibrium technique assumes a constant dissolved oxygen and water temperature throughout the measurement. These assumptions could lead to inaccurate measurements because both stream dissolved-oxygen concentration and water temperature may vary diurnally. The addition of sodium sulfite to the stream is also only practical for small streams.

Tracer techniques have also been used to determine the reaeration coefficient. The basis of the technique is the use of a surrogate tracer for oxygen that is involved in the mass-transfer process at the air-water interface without being affected by other chemical or biological processes. Because tracer dispersion and dilution cannot be eliminated, a conservative tracer must be injected along with the surrogate tracer gas for oxygen. The assumptions of the technique are that: (1) the relation between the reaeration coefficient and the tracer-gas desorption coefficient is independent of mixing and temperature conditions; (2) the dispersion-dilution tracer is conservative or that adjustments can be made for minor losses; and (3) the tracer gas and conservative tracer undergo the same dispersion-dilution process with the tracer gas being lost only to the atmosphere through the air-water interface.

Radioactive krypton and tritium have been used as the tracer gas and conservative tracer, respectively. Because of an increase in regulations covering the procurement and use of radioactive substances, the radioactive-tracer method has become impractical to use. The modified or hydrocarbon-gas tracer technique as described by Rathbun and Grant (1978) uses a low-molecular weight hydrocarbon gas as the tracer gas and a fluorescent dye as the conservative tracer. The three assumptions concerning tracer methods remain, and in addition, the dye concentrations must be adjusted for the losses of dye mass at sampling locations. The advantage of the modified or hydrocarbon-gas tracer technique is that no licensing is required to obtain or use the materials when conducting the measurement.

Use of a fluorescent dye as the conservative tracer in these measurements allows for time-of-travel determination. This information, in addition to being necessary for the reaeration coefficient calcuation, can be used to accurately determine mean velocity in the reach. Mean velocity is commonly needed for water-quality models and in solute-transport studies. It is also needed for calculation of the longitudinal dispersion coefficient for the reach. The longitudinal dispersion coefficient is useful for estimating the response or breakthrough curve at some point downstream of an injection of material into the stream.

The analysis of the error associated with the use of predictive equations for estimating mean velocity, the longitudinal dispersion coefficient, and the reaeration coefficient can be made using measured values and hydraulic parameters determined from the measurements. This analysis can be useful in determining the equations that best estimate the measured values for the selected streams.

## Purpose and Scope

The purposes of this report are to: (1) provide an assessment of the accuracy of published predictive equations for estimating mean velocity, the longitudinal dispersion coefficient, and the reaeration coefficient as measured in Kentucky River basin streams; and (2) present some statistically-developed predictive equations which may have applicability to other Kentucky River basin streams and other streams with similar characteristics. The estimating techniques should employ easily measured physical and hydraulic stream characteristics. A total of nine field measurements on five streams provided time-of-travel and dispersion information and information to determine the reaeration coefficient. Values of physical and hydraulic parameters were also determined during each measurement.

Standard time-of-travel techniques used by the U.S. Geological Survey as described by Wilson and others (1984) and Hubbard and others (1982) were applied to each of the measurements. The modified or hydrocarbon-gas tracer technique as described by Rathbun and Grant (1978) was used to determine the stream reaeration coefficient for each of the reaches.

The selected stream reaches reflect pool-and-riffle and channel-control conditions and measurements were made during low to low-to-medium streamflow. The measurements were made between August 1984 and October 1985. Values of a number of physical and hydraulic stream characteristics were determined during each field measurement for inclusion into existing predictive equations and for use in statistical model building. Data on other parameters were collected to determine in a qualitative manner if these parameters were adversely affecting either the dye concentrations or the gas desorption rate coefficient during the measurement.

## Acknowledgments

The authors thank those persons who assisted in the labor intensive data collection and compilation. These include members of the Kentucky Geological Survey and field personnel from the Kentucky District of the U.S. Geological Survey in Louisville, Kentucky. We also thank Janet Hren of the Ohio District of the U.S. Geological Survey for making available much of the equipment used to perform the measurements and additional technical advice. Special thanks are given to Kevin Jackson of the Kentucky District of the U.S. Geological Survey for his assistance in data collection and computation of parameter values.

## Approach

Measurements of time-of-travel and gas desorption were made on five Kentucky River basin streams. Values of other physical and hydraulic stream characteristics were also determined at the time of each measurement. These characteristics included reach length, water-surface slope, mean velocity, mean discharge, mean channel width, mean flow depth, mean cross-sectional area, mean hydraulic radius, Froude number, shear velocity, water temperature, and longitudinal dispersion coefficient. Channel roughness values were computed based on hydraulic properties of the stream reach.

The values for some of these parameters were correlated with the mean velocity, the longitudinal dispersion coefficient, and the reaeration coefficient to define relations. Scatter plots were made for various combinations of response and regressor variables to check for linearity. Linear transformation of the regressor variables were then made when appropriate and regression model-building analysis performed. Due to the minimal number of observations in the data set, only 1- and 2-parameter models are presented.

Other parameters collected during each measurement included specific conductance, suspended-sediment concentration, and wind speed above the water surface. The values collected for these parameters were used in a qualitative manner as an aid in interpretation of results. The limited data preclude the determination of the effects of the suspended-sediment concentration, specific conductivity, and wind speed on the mean velocity, the longitudinal dispersion coefficient, and the reaeration coefficient.

## SITE DESCRIPTION

All field measurements were conducted on streams in the Kentucky River basin. The measurement locations are shown in figure 1. Drainage areas for each site are shown below.

| Study reach | Drainage area in $mi^2$ |
|---|---|
| Glenns Creek near Versailles, Ky. | 4.02 |
| Mill Creek near Manchester, Ky. | 6.20 |
| South Elkhorn Creek near Midway, Ky. | 105 |
| South Fork Kentucky River near Booneville, Ky. | 686 |
| North Fork Kentucky River near Jackson, Ky. | 1,100 |

Glenns Creek and Mill Creek are small pool-and-riffle streams with a high gradient and low mean depth in the measurement reaches. The measurement on Glenns Creek was conducted on August 15, 1984, and the measurement on Mill Creek was conducted on August 29, 1984. The water-surface slopes of the two reaches were 21 and 54 ft/mi, respectively, and the mean depths were 0.34 and

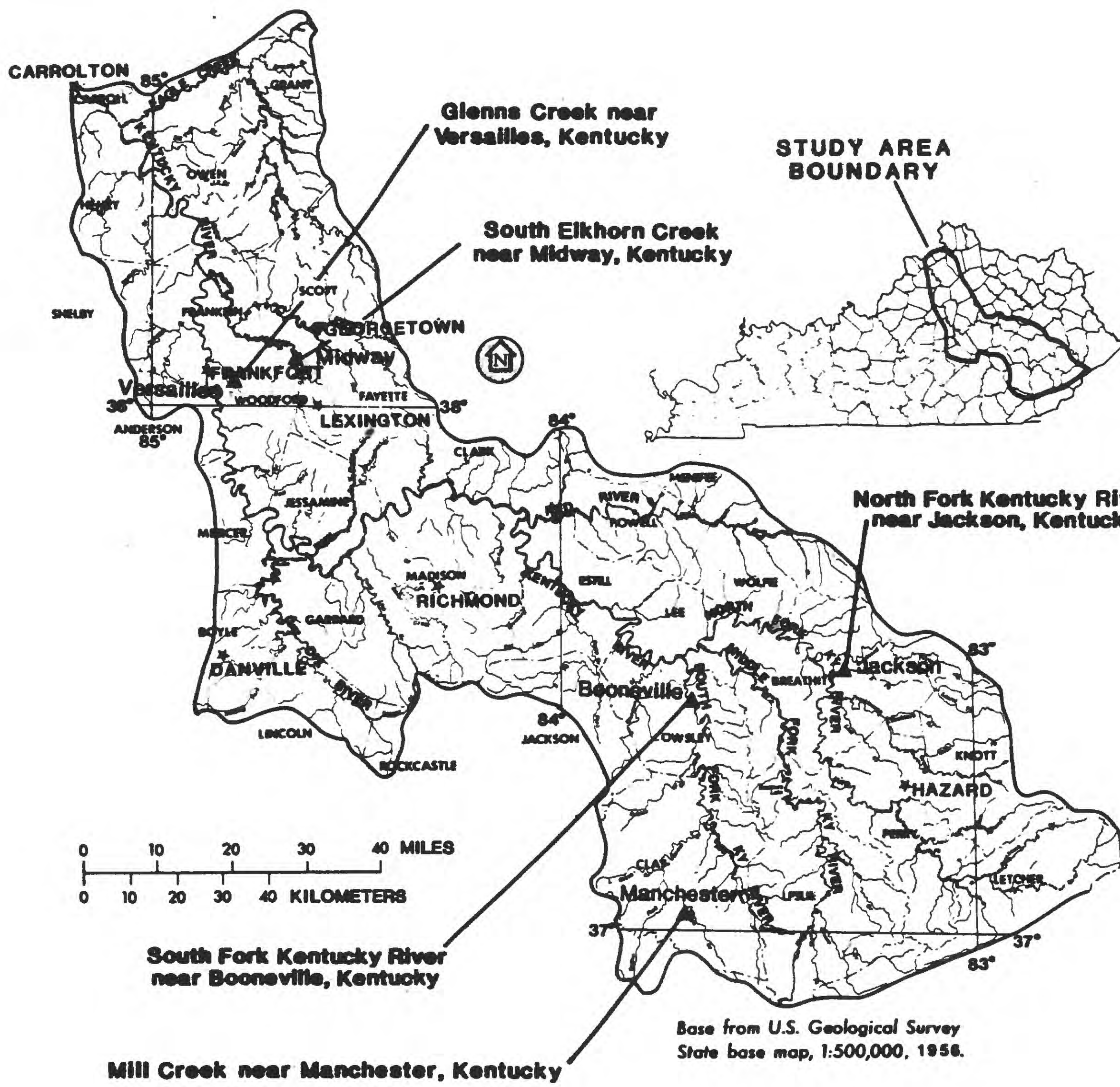



Figure 1.--Location of test reaches in the Kentucky River basin.

0.20 ft, respectively. Only one reach (2 sampling locations) was measured at these sites and only under a single flow condition. Two reaches (3 sampling locations) were planned at Glenns Creek but channel modifications by a local resident at the time of the measurement made the second reach unusable. The location of the study reach, dye and gas injection site, and the two sampling sections are shown in figures 2 and 3. Flow at Glenns Creek was somewhat regulated by the wastewater treatment plant at Versailles, Kentucky. Water discharge at Glenns Creek averaged 1.58 $ft^3/s$ and the discharge at Mill Creek averaged 0.27 $ft^3/s$.

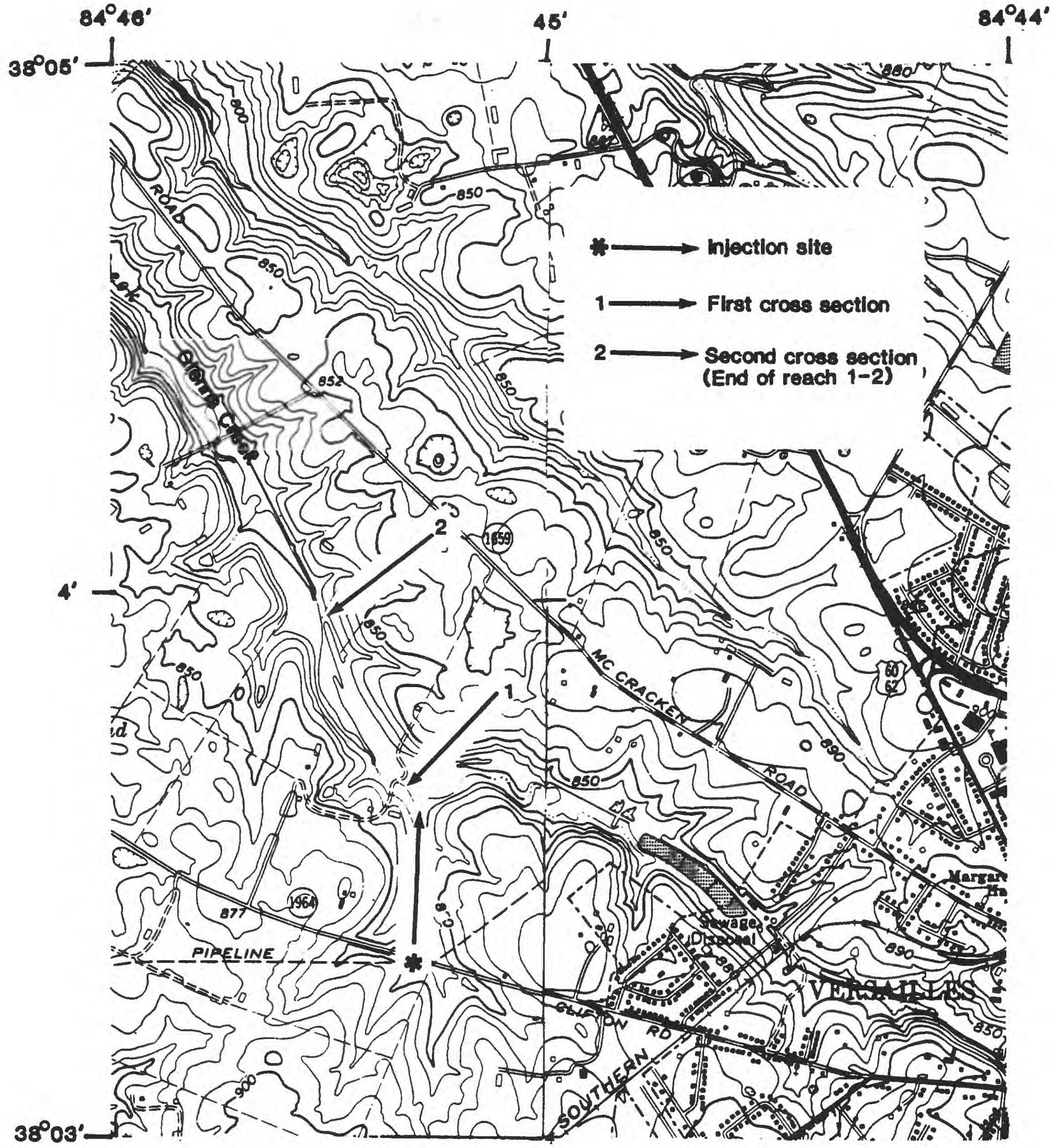
South Elkhorn Creek had a low water-surface slope of 0.70 ft/mi, and the largest mean depth (2.36 ft) of the five reaches. The discharge during the measurement was 37.3 $ft^3/s$, which is moderately low considering that the flow is regulated substantially by diversion and treatment plant outflows from Lexington, Kentucky. One measurement was made at South Elkhorn Creek on August 2, 1984. Locations of the injection site and the upstream and downstream sampling sites are shown in figure 4. The test reach exhibited characteristics of both pool-and-riffle and channel-controlled sections.

The reaches on the North and South Forks of the Kentucky River typified both pool-and-riffle and channel-controlled reaches during the measurements. The South Fork reach was measured on September 6, 1984, and on October 10, 1985. The September 6, 1984, measurement was made at a flow of 95.1 $ft^3/s$ and the October 10, 1985, measurement at 60.2 $ft^3/s$. These discharges are approximately the 70-percent and 80-percent flow duration values (Quinones and others, 1980). The South Fork reach (fig. 5) had a water-surface slope of 0.73 ft/mi and mean depths of 2.15 and 1.80 ft for the two measurements, respectively. The North Fork of the Kentucky River near Jackson, Kentucky, was measured on October 17, 1984, and October 8, 1985, at discharges of approximately 70 and 168 $ft^3/s$, respectively. These discharges are approximately the 85-percent and 70-percent flow duration values. The first measurement consisted of two reaches (fig. 6) having water-surface slopes of 2.9 and 0.72 ft/mi, respectively. The mean depths during the measurements were 1.50 and 1.69 ft, respectively. The measurements on the two reaches were combined to form a third measurement. The water-surface slope for the composite reach was 1.67 ft/mi, and the mean depth was 1.61 ft. The measurement made on October 8, 1985, consisted of the composite of the two reaches. The overall water-surface slope was assumed to be equal to that previously measured, and the mean depth was determined to be 2.08 ft. The flow of the North Fork of the Kentucky River near Jackson is regulated by a control structure on Carr Fork Lake and to some extent, by the City of Jackson waterworks.

## THEORY

### Mean Velocity and Longitudinal Dispersion

The mean streamflow velocity for a reach is an important input parameter to water-quality models. Mean velocity estimates based on discharge measurements taken along a reach and averaged to represent the entire reach tend to overestimate the true value of the mean velocity. This is due to the fact that discharge measurements are more likely made at constricted sections as opposed to pooled sections. A mean velocity estimate in conjunction with longitudinal dispersion can be used in estimating solute-transport

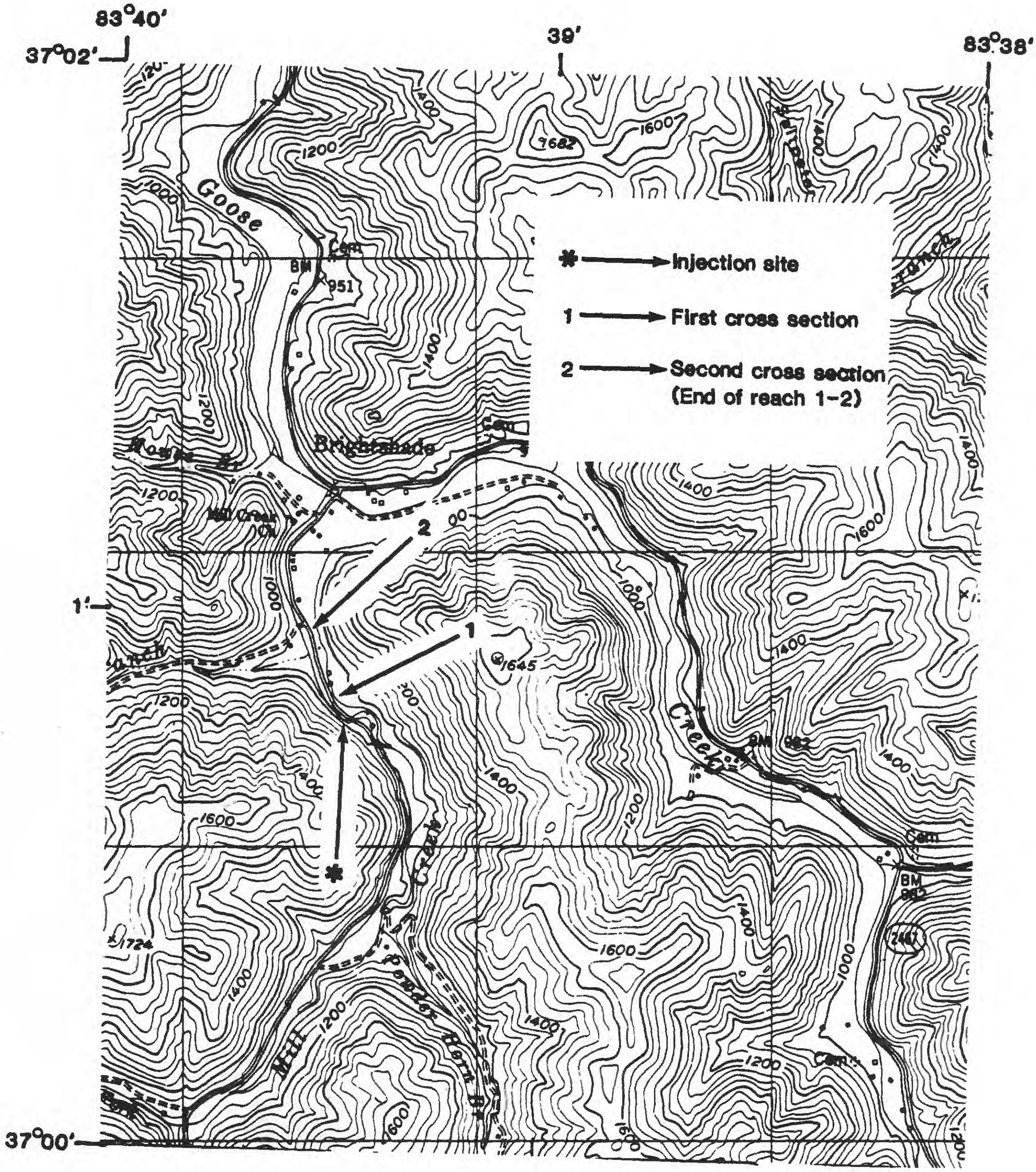



Base from U.S. Geological Survey 1:24,000, Tyrone, 1967, Versailles, 1965. Interim revision as of 1978

0 0.5 MILE
0 0.5 KILOMETER

CONTOUR INTERVAL 10 FEET
NATIONAL GEODETIC VERTICAL DATUM OF 1929

Figure 2.--Test reach on Glenns Creek.

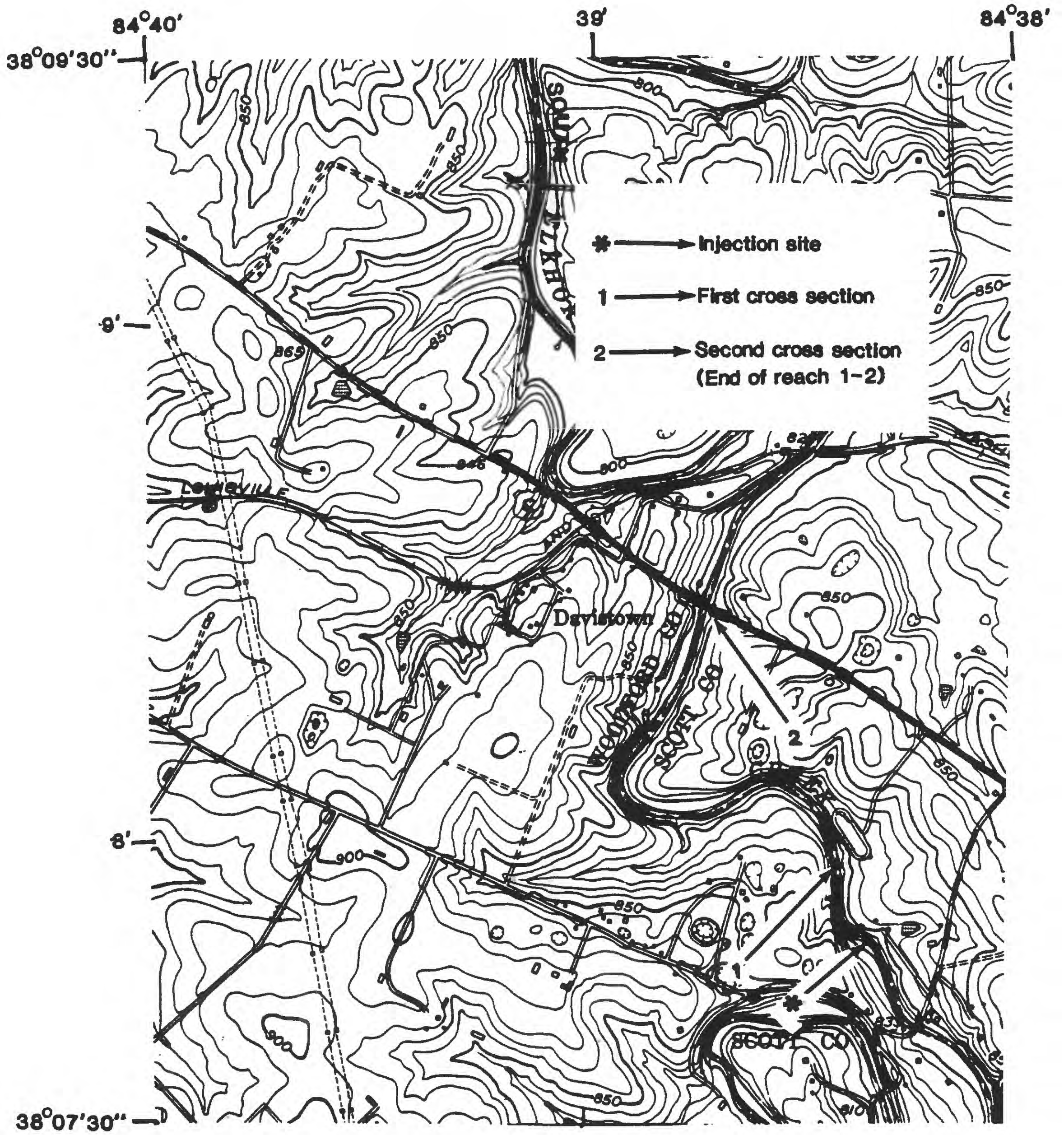



Base from U.S. Geological Survey
1:24,000, Ogle, 1979

0 0.5 MILE
0 0.5 KILOMETER

CONTOUR INTERVAL 40 FEET
NATIONAL GEODETIC VERTICAL DATUM OF 1929

Figure 3.—Test reach on Mill Creek.




Base from U.S. Geological Survey
1:24,000, Midway, 1954

0 0.5 MILE
0 0.5 KILOMETER

CONTOUR INTERVAL 10 FEET
NATIONAL GEODETIC VERTICAL DATUM OF 1929

Figure 4.--Test reach on South Elkhorn Creek.

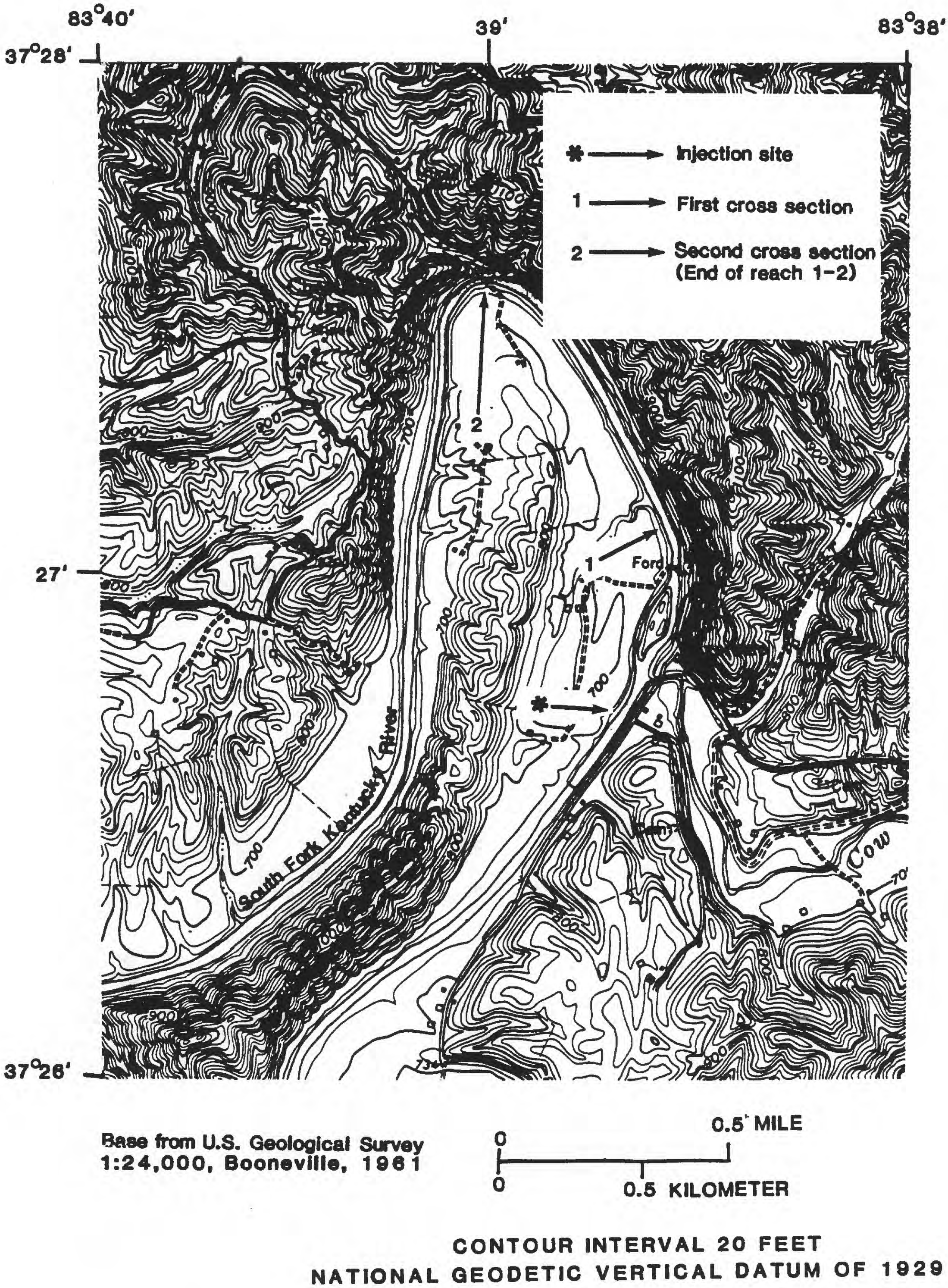
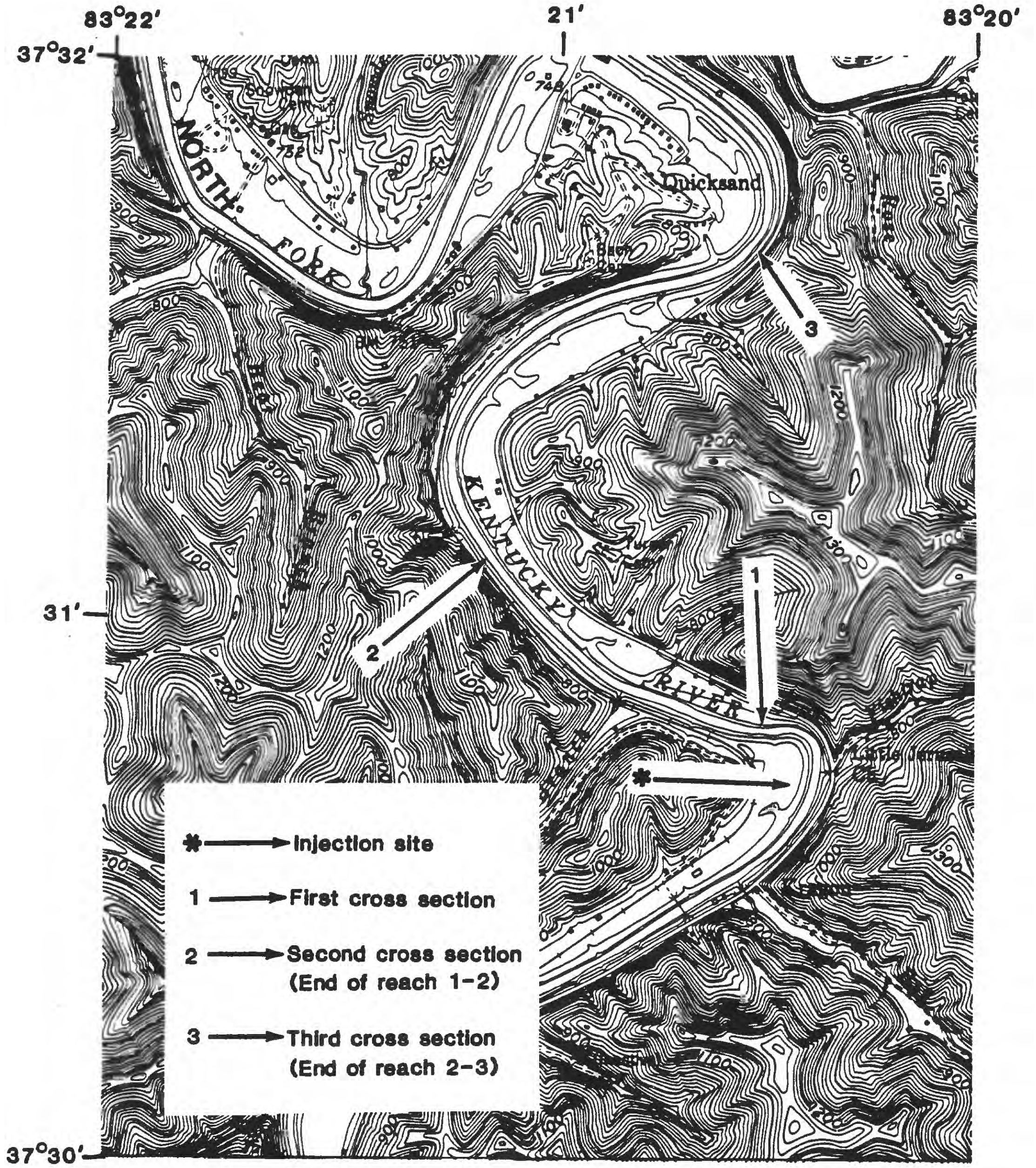



Figure 5.—Test reach on South Fork Kentucky River.




Base from U.S. Geological Survey
1:24,000, Quicksand, 1961

0 0.5 MILE
0 0.5 KILOMETER

CONTOUR INTERVAL 20 FEET
NATIONAL GEODETIC VERTICAL DATUM OF 1929

Figure 6.--Test reach on North Fork Kentucky River.

characteristics. It could then be used to determine the characteristics of a contaminant plume as it moves through a stream reach of interest.

If an estimate of both mean velocity and longitudinal dispersion can be made for a reach, then the response or breakthrough curve resulting from a slug injection of some conservative solute could be estimated for any point in that reach. This estimate usually assumes that the response curve is normally distributed, the channel is uniform, and the discharge is constant throughout the reach resulting in longitudinal dispersion occurring at a constant rate. The longitudinal dispersion coefficient is defined by Fisher (1968) as the rate of change of variance of travel time of a conservative tracer and can be calculated by the following:

$$D = \frac{1}{2} V^2 \frac{\sigma_{t_d}^2 - \sigma_{t_u}^2}{\bar{t}_d - \bar{t}_u} \qquad (1)$$

where $D$ is the longitudinal dispersion coefficient, in $ft^2/s$,
$V$ is the mean reach velocity, in ft/s,
$\sigma_t^2$ is the variance of the concentration-time curve, in seconds squared, and
$\bar{t}$ is the centroid travel time, in seconds.

The subscripts d and u denote the downstream and upstream ends of the reach, respectively. The variance of the concentration-time curve is determined by the method of moments and is expressed by:

$$\sigma_t^2 = \frac{\Sigma t^2 C}{\Sigma C} - t^2 \qquad (2)$$

where $C$ is the dye concentration, in μg/L, and
$t$ is the time from the start of injection for a certain value of dye concentration, in seconds.

Values of longitudinal dispersion were calculated using equation 1 after adjustments for dye loss were made.

## Reaeration Coefficient

Reaeration is the process whereby atmospheric oxygen is transferred to the water from the air through the air-water interface. The difference between the saturation and dissolved oxygen concentrations of the stream drives this process. The absorption of oxygen by a water body may be described by the first-order mass transfer relation:

$$\frac{dC}{dt} = \frac{K_l}{H} (C_s - C) \qquad (3)$$

where $C$ is the concentration of dissolved oxygen, in mg/L
$C_s$ is the saturation concentration of dissolved oxygen at a given temperature, in mg/L,
$H$ is the depth of water, in ft, and
$K_l$ is a proportionality constant defined as the liquid-film mass transfer coefficient, in ft/d.

The reaeration coefficient, $K_2$, is defined as $K_l/H$ and is usually expressed in reciprocal days. Equation 3, therefore, takes the form:

$$\frac{dC}{dt} = K_2(C_s - C) \qquad (4)$$

and assumes a first order process which occurs throughout the depth of water and not only at the water surface (Rathbun and others, 1978). The term $C_s - C$ in equation 4 is called the dissolved-oxygen deficit.

The reaeration coefficients for this study were determined using the hydrocarbon gas-tracer technique as described by Rathbun and Grant (1978) which was adapted from the technique described by Tsivoglou and Neal (1976). The hydrocarbon gas-tracer technique is based on the assumption that a constant relation exists between the absorption coefficient for oxygen and the desorption coefficient for a selected hydrocarbon gas. In stirred-tank experiments where oxygen absorption and hydrocarbon gas desorption were measured simultaneously, it was shown that the absorption of oxygen and the desorption of a hydrocarbon gas from the same water body were independent of mixing conditions and temperature of the water. This linear relation may be expressed by:

$$R = \frac{K_2}{K_t} \qquad (5)$$

where $R$ is the coefficient ratio for oxygen and a hydrocarbon gas,
$K_2$ is the absorption coefficient for oxygen, in $days^{-1}$, and
$K_t$ is the desorption coefficient for a hydrocarbon gas, in $days^{-1}$.

The R value using propane as the hydrocarbon gas was determined to be 1.39 from the stirred-tank experiments (Rathbun and others, 1978).

As described by Rathbun and Grant (1978), the desorption coefficient for a hydrocarbon tracer-gas can be calculated using the ratio of the peak gas and dye concentrations of the upstream and downstream cross sections (called the peak method) and is denoted as:

$$K_t = \frac{1}{t_d \quad t_u} \log_e \frac{(C_t/C_d)_u}{(C_t/C_d)_d} \qquad (6)$$

where $K_t$ is the desorption coefficient, in $days^{-1}$,
$t$ is the time of travel of the peak concentration of the conservative dispersion-dilution tracer, in days,
$C_t$ is the peak concentration of the gas tracer, in µg/L, and
$C_d$ is the peak concentration of the dispersion-dilution tracer, in µg/L.

The subscripts d and u denote the upstream and downstream ends of the reach. The method assumes that the relation in equation 5 is true for natural streams, and that the hydrocarbon gas behaves the same as the dispersion-dilution tracer with the only loss of gas being through the air-water interface. Another assumption is that the dispersion-dilution tracer is conservative or that corrections can be made to account for any losses.

Because most dyes are not completely conservative, sufficient samples need to be collected to define the complete concentration-time curves at each sampling location. Knowing the water discharge corresponding to these curves allows corrections to be made to the dye concentrations. Because the complete dye curve is sampled, the complete gas concentration-time curve can also be defined with little additional effort. This leads to the total weight method which can be described by the relation:

$$K_t = \frac{1}{\bar{t}_d - \bar{t}_u} \log_e \frac{A_u Q_u}{A_d Q_d} \qquad (7)$$

where $\bar{t}$ is the centroid travel time of the gas cloud, in days,
$A$ is the area of the gas-tracer concentration time curve, in µg/L, and
$Q$ is the water discharge, in $ft^2/hr$.

Because the reaeration coefficient changes proportionally with temperature, its value as determined from equation 6 or 7 is adjusted to 20 °C using the equation:

$$K_{20^oC} = K_2(1.0241)^{20-\theta} \qquad (8)$$

where θ is the average water temperature of the water body in degrees Celsius throughout the measurement (Rathbun 1979).

## METHODOLOGY

### Field and Laboratory Techniques

The hydrocarbon gas-tracer technique was used to determine the desorption coefficients for the stream reaches in this study. A hydrocarbon tracer-gas and a dispersion-dilution tracer were injected simultaneously and at constant rates for a specific short period of time. Commercial grade propane was used as the hydrocarbon tracer-gas and 20-percent rhodamine-WT[1] dye was used as the conservative dispersion-dilution tracer. For each study reach the gas and dye were injected at a constant rate for a period ranging from 15 to 45 minutes. The gas was injected using a 20-pound propane tank with a single-stage regulator and a propane rotameter calibrated in liters per minute. Flexible plastic tubing was connected from the rotameter to flat porous tile diffusers placed on the stream bottom at approximately the 50-percent flow line of the stream. The diffusers have a pore size of 2 microns. The rate of gas injection ranged from 2 to 10 L/min for the stream reaches in this study.

The dye was injected for the same period of time as the propane. A quantity of dye was diluted with distilled water so that 40 mL/min of the dye-water mixture could be injected over the sampling period and would provide detectable concentrations at the end of the study reach. The injection rate is arbitrary, but the rate of 40 mL/min used in the study was convenient. The mixture was injected onto the water surface at the same point as the propane using flexible plastic tubing and a laboratory-grade metering pump. The injection rates of gas and dye were monitored continuously throughout the injection period.

The first sampling section was selected such that adequate vertical and lateral mixing of the tracers would occur. Gas and dye samples were collected at intervals sufficient to define the passage of the tracer clouds at the sampling location. Propane samples were collected in 40-milliliter septum vials using dissolved-gas samplers. These septum vials were sealed airtight using teflon disks. One milliliter of 37-percent reagent-grade formaldehyde was added to each sample before they were sealed to prevent biological degradation. Dye samples were collected in 8-dram glass vials and initially analyzed at the site using a field fluorometer. The dye samples were later analyzed under controlled conditions and with all samples at a constant temperature. For selected measurements, three verticals were sampled at the upstream site to ensure complete lateral mixing of the tracers. These verticals were located at the mid-points of thirds of the flow (the 16-, 50-, and 82-percent flow points). At the downstream site, or sites, gas and dye samples were collected at mid-depth on the 50-percent flow vertical. After gas sampling was terminated, an automatic sampler was sometimes used to collect dye samples to completely define the tail portion of the dye cloud. Gas samples were analyzed at the U.S. Geological Survey laboratory in Doraville, Georgia, using a stripping line and cold trap apparatus in conjunction with a gas chromatograph equipped with a flame ionization detector. This procedure is described by Shultz and others (1976), and procedures for the analysis of dye samples are outlined by Hubbard and others (1982) and Wilson and others (1984).

Values of other parameters collected at the time of the measurement included water discharge, water temperature, wind speed above the water surface, specific conductance, and suspended-sediment concentration. Change

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1. Use of brand names is for identification purposes only and does not constitute endorsement by the U.S. Geological Survey.

in water-surface elevation at each of the sampling locations was determined by differential leveling. Cross-section geometry was collected at equally spaced intervals along the reach to define depth, width, and hydraulic radius.

## Data Analysis

Upon completion of all laboratory analyses, dye and gas concentration-time curves were plotted and a curve fitted to the data by linear interpolation between data points. Any extrapolated points needed for the rising and falling limb of the curves were based on the shape of the existing curve. Data points for the tail portion of the dye and gas curves were estimated assuming a logarithmic decay and extended to background concentration for the dye and to zero concentration for the gas. Values of gas concentration in the extrapolated tail portion of the curves were estimated also assuming a desorption coefficient and rate. The dye and gas concentration-time curves for each of the measurements are shown in the Appendix. These figures show the measured dye and gas data points from the analyses prior to adjustments for dye loss. The fitted curves, including the extrapolated portions of the curves, are also shown. Mean streamflow velocity was computed as the reach length divided by the difference between centroid travel times of the dye concentration-time curves at the sampling locations. Longitudinal dispersion was determined using equation 1 and the value of streamflow velocity as described. The variance of the concentration-time curves was computed using equation 2 and adjusted for dye loss.

Both the peak and total weight methods of determining the propane desorption coefficient were used as given by equations 6 and 7. The peak concentrations of dye and gas and the corresponding times were taken from the data or extrapolated values used when necessary for the peak method. The values of the peak dye concentrations were adjusted based on the percent recovery of the mass of dye injected for each of the sampling locations. The percentage of dye recovered was computed as the mass of dye recovered divided by the mass initially injected. The mass of the dye cloud is the area under the dye concentration-time curve multiplied by the water discharge. The area was computed as the concentration of the sample multiplied by the time interval midway between the adjacent samples.

The total weight method uses the mass of the gas concentration-time curve at the sampling locations in conjunction with the corresponding centroid travel times to compute the propane desorption coefficient. If discharge varied during the measurement, the incremental mass was computed to reflect the change, and therefore incorporated into the summation. This was also done when determining dye recovery ratios. Once the propane desorption coefficient was determined using the two methods it was converted to the reaeration coefficient by equation 5 and then standardized to 20 °C using equation 8. Information obtained from the dye and gas traces is presented in table 1. The travel times, in hours after the start of the injection, of the leading edge, peak, centroid, and trailing edge of the dye concentration-time curves are presented as well as the values of the propane desorption and the reaeration coefficient of each measurement standardized to 20 °C. The times of the leading edge, peak, centroid, and trailing edge of the concentration-time curve are defined as follows:

Table 1.--Results of the time-of-travel and reaeration measurements

| Reach | Travel times (hours) | | | | | | | | Mean velocity (feet per second) | Longitudinal dispersion coefficient (square feet per second) | Propane desorption coefficient (days$^{-1}$) | Reaeration coefficient (days$^{-1}$) | Relative measurement error (percent) |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | Leading edge | Peak | Centroid | Trailing edge | Leading edge | Peak | Centroid | Trailing edge | | | | | |
| **Glenns Creek near Versailles, Kentucky — Measurement of 08/15/84** | | | | | | | | | | | | | |
| 1-2 | 0.17 | 0.57 | 0.53 | 1.22 | 1.75 | 2.58 | 3.23 | 7.50 | 0.252 | 41.3 | 12.6 | 17.5 | 3 |
| **Mill Creek near Manchester, Kentucky — Measurement of 08/29/84** | | | | | | | | | | | | | |
| 1-2 | .12 | .35 | .37 | .85 | 1.83 | 2.50 | 3.17 | 8.50 | .093 | 6.33 | 22.4 | 31.1 | 2 |
| **North Fork Kentucky River near Jackson, Kentucky — Measurement of 10/17/84** | | | | | | | | | | | | | |
| 1-2 | .58 | 1.25 | 1.27 | 2.75 | 1.92 | 2.67 | 3.28 | 6.00 | .521 | 116 | 1.36 | 1.89 | 57 |
| 2-3 | 1.92 | 2.67 | 3.28 | 6.00 | 4.17 | 5.33 | 6.35 | 12.0 | .458 | 119 | 1.39 | 1.93 | 100 |
| 1-3 | .58 | 1.25 | 1.27 | 2.75 | 4.17 | 5.33 | 6.35 | 12.0 | .483 | 119 | 1.38 | 1.91 | 40 |
| **Measurement of 10/08/85** | | | | | | | | | | | | | |
| 1-3 | .33 | .58 | .78 | 1.50 | 2.00 | 3.25 | 3.58 | 5.92 | .898 | 188 | 2.44 | 3.39 | 126 |
| **South Elkhorn Creek near Midway, Kentucky — Measurement of 08/02/84** | | | | | | | | | | | | | |
| 1-2 | .75 | 1.50 | 1.56 | 3.22 | 3.92 | 5.50 | 6.33 | 12.3 | .263 | 46.0 | .95 | 1.32 | 77 |
| **South Fork Kentucky River near Booneville, Kentucky — Measurement of 09/06/84** | | | | | | | | | | | | | |
| 1-2 | .33 | 1.00 | 1.03 | 2.33 | 2.00 | 2.67 | 3.10 | 5.67 | .448 | 85.6 | .65 | .90 | 83 |
| **Measurement of 10/10/85** | | | | | | | | | | | | | |
| 1-2 | .58 | 1.05 | 1.22 | 2.83 | 2.58 | 3.17 | 3.95 | 7.75 | .338 | 67.2 | 1.18 | 1.64 | 78 |

leading edge: the time after the start of the injection when the dye or gas is first detectable at the sampling location;
peak: the time after the start of the injection when the dye or gas concentration is at a maximum;
centroid: the time after the start of the injection of the center of the mass of the dye or gas response curve; and
trailing edge: the time after the start of the injection when the dye or gas concentration reaches one percent of the maximum concentration for the response curve.

The last column in table 1 contains the relative error of the reaeration measurement in percent. This value is based on an assumed 5-percent sampling and analysis error in conjuction with the amount of desorption of the gas that occurred relative to the amount of dispersion of the dye that occurred. It is evident from the values that a greater error is associated with those measurements which had low reaeration coefficients. This indicates that much of the decrease in downstream gas concentration for those measurements was due to dispersion and not desorption.

## Reach Characteristics

For each stream reach included in the study, a number of parameters were also determined at each sampling location at the time of the measurement. These included stream discharge, water temperature, water-surface slope, specific conductance, wind speed above the water surface, and suspended-sediment concentration. Stream cross sections were taken at intervals such that approximately 15 to 20 sections per reach were defined. Values of roughness were calculated for the main channel at each cross section from the hydraulic data. Mean depth and Froude number were calculated using streamflow values and channel geometry. A description of each parameter determined during the measurement is given below with the value corresponding to each measurement given in table 2.

(1) Mean discharge (Q) was measured in $ft^3/s$ using the Standard Price AA or Pygmy current meters in conjunction with procedures outlined by Buchanan and Somers (1969).
(2) Reach length (L) was determined in feet by taping the distance between the 15 to 20 cross sections taken along the length of the stream channel.
(3) Mean width (B) was determined in feet by averaging the measured distance between the edge of water at each bank for the 15 to 20 cross sections taken along the measurement reach.
(4) Mean velocity (V) was determined in ft/s as the stream distance between each sampling section divided by the difference in centroid travel times of the dye cloud at each sampling section.
(5) Mean depth (H) was determined in feet as the discharge divided by the mean width and mean velocity for the reach.
(6) Mean cross-sectional area (A) was determined in $ft^2$ as the product of the mean width and mean depth for the reach.
(7) Water-surface slope (S) was measured in ft/ft by differential leveling between the sampling sections. The difference in elevation was divided by the stream reach length. The quantity is assumed to be equal to the average energy gradient.

Table 2.--Reach characteristics

| Reach | Reach length (feet) | Slope (feet per feet) | Mean velocity (feet per second) | Mean discharge (cubic feet per second) | Cross sectional area (square feet) | Mean width (feet) | Mean depth (feet) | Froude number | Mean shear velocity (feet per second | Longitudinal dispersion coefficient (square feet per second) | Mean wind speed (miles per hour) | Specific conductance (microsiemens per centimeter) | Water temperature (degrees Celsius) | Suspended sediment (milligram per liter) | Average roughness coefficient |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Glenns Creek near Versailles, Kentucky Measurement of 08/15/84 | | | | | | | | | | | | | | | |
| 1-2 | 2,450 | 0.00396 | 0.252 | 1.58 | 6.25 | 18.4 | 0.340 | 0.0762 | 0.208 | 41.3 | 0.93 | 720 | 27.6 | 26 | 0.180 |
| Mill Creek near Manchester, Kentucky Measurement of 08/29/84 | | | | | | | | | | | | | | | |
| 1-2 | 936 | .0103 | .093 | .27 | 2.91 | 14.4 | .202 | .0364 | .259 | 6.33 | .82 | 120 | 22.5 | 4 | .558 |
| North Fork Kentucky River near Jackson, Kentucky Measurement of 10/17/84 | | | | | | | | | | | | | | | |
| 1-2 | 3,780 | .000558 | .521 | 72.6 | 139 | 92.7 | 1.50 | .0748 | .164 | 116 | 1.17 | 680 | 18.4 | 5 | .088 |
| 2-3 | 5,060 | .000136 | .458 | 68.2 | 149 | 88.1 | 1.69 | .0622 | .086 | 119 | 1.08 | 700 | 19.1 | 7 | .054 |
| 1-3 | 8,840 | .000317 | .483 | 69.9 | 145 | 90.1 | 1.61 | .0672 | .128 | 119 | 1.11 | 685 | 18.8 | 6 | .076 |
| Measurement of 10/08/85 | | | | | | | | | | | | | | | |
| 1-3 | 8,840 | .000317 | .898 | 168 | 187 | 90.1 | 2.08 | .110 | .146 | 188 | ---- | --- | 13.3 | 18 | .048 |
| South Elkhorn Creek near Midway, Kentucky Measurement of 08/02/84 | | | | | | | | | | | | | | | |
| 1-2 | 4,520 | .000133 | .263 | 37.3 | 142 | 60.1 | 2.36 | .0302 | .100 | 46.0 | 1.26 | 710 | 23.3 | 28 | .116 |
| South Fork Kentucky River near Booneville, Kentucky Measurement of 09/06/84 | | | | | | | | | | | | | | | |
| 1-2 | 3,330 | .000138 | .448 | 95.1 | 212 | 98.9 | 2.15 | .0539 | .098 | 85.6 | 2.20 | 305 | 21.2 | 22 | .065 |
| Measurement of 10/10/85 | | | | | | | | | | | | | | | |
| 1-2 | 3,330 | .000138 | .338 | 60.2 | 178 | 98.9 | 1.80 | .0444 | .089 | 67.2 | ---- | 308 | 16.3 | 5 | .076 |

(8) Froude number (F) is a dimensionless quantity defined as the mean velocity divided by the square root of the product of the acceleration due to gravity and the mean depth.

(9) Hydraulic radius (R) was determined in feet as the mean of the cross-sectional area divided by the wetted perimeter of each of the 15 to 20 measured cross sections within the reach. For the measurements made during this study, hydraulic radius was approximately equal to mean depth in all cases, and mean depth was used in its place.

(10) Mean shear velocity ($u^*$) was determined in ft/s as the square root of the product of the acceleration due to gravity, the hydraulic radius (mean depth for the measurements in this study), and the water-surface slope.

(11) Longitudinal dispersion coefficient (D) was determined in $ft^2/s$ and is defined as the change in variance divided by the change in centroid time of the concentration-time curves at each end of the reach multiplied by one half of the squared mean velocity.

(12) Water temperature (O) was measured in °C using a laboratory grade thermometer.

(13) Specific conductance (SC) was measured in μS/cm at each sampling location.

(14) Mean wind speed (WS) was measured in mi/h using a totalizing anemometer placed above the stream surface. The height of the anemometer above the water surface ranged from 1 to 3 ft.

(15) Suspended-sediment concentration (SS) was determined in mg/L using procedures outlined by Guy and Norman (1970) and Guy (1969).

(16) Manning's roughness coefficient (n) was computed based on the hydraulic properties of the stream reach using Manning's equation.

## ESTIMATING EQUATIONS

### Method of Analysis

The estimating equations were developed using multiple linear regression analysis techniques. The response variables in each of the analyses were mean velocity, longitudinal dispersion coefficient, and reaeration coefficient. The regressor variables used in the analyses included water-surface slope, mean discharge, mean velocity, mean width, mean depth, Froude number, shear velocity, and longitudinal dispersion coefficient. Values of channel roughness coefficient were not used in the analysis due to the unreasonable values obtained for certain study reaches. The determination of these values appears affected by the presence of pools and riffles in the study reach, each of which heavily influence the hydraulic parameters measured. Scatter plots or scatter diagrams were made for various combinations of response and regressor variables. Transformations of the regressor variables were then selected and made. Only 1- and 2-parameter models were considered from the regression analyses due to the limited number of observations (degrees of freedom) in the data set. If the scatter plots indicated that the relation appeared to be inappropriate from a physical aspect (such as increasing slope and decreasing velocity) then a model was not developed for that pair. These types of relations are probably being heavily influenced by other factors which cause such unrealistic trends.

When the intercept for a relation was found to be not significant, a no-intercept model was used. The coefficient of determination for this relation was determined by first using the sum of squares total computed with the mean of the observations and subtracting the sum of squares error associated with the "no intercept" model to obtain the corrected sum of squares regression. The sum of squares regression was divided by the sum of squares total to obtain the coefficient of determination. The coefficient of determination computed in this way for the "no-intercept" model will be called the corrected coefficient of determination.

## Mean Velocity

The mean velocity estimating equations were determined using multiple linear regression analysis techniques. The values of mean velocity were used as the response variable and the values of several other parameters were used as regressor variables. The regressor variables used in the analysis were water-surface slope, mean discharge, mean width, mean depth, Froude number, and shear velocity.

Because the study reaches typified pool-and-riffle reach characteristics more so than channel-controlled reach characteristics, and because of the limited number of observations, all nine observations were grouped to develop a relation of the form by Boning (1974) for pool-and-riffle channels, which is:

$$VP = 0.38\ Q^{0.40}\ S^{0.20} \quad (9)$$

where VP is the velocity corresponding to the peak of the dye cloud, in ft/s,
Q is the water discharge, in $ft^3/s$, and
S is the water-surface slope, in ft/ft.

The model was constructed using a variable which grouped discharge and slope and the corresponding exponents and allowed the intercept to float. The model indicated that the intercept was statistically not significant and the model was then redefined using an intercept of zero. The resulting relation is:

$$V = 0.474\ Q^{0.40}\ S^{0.20} \quad (10)$$

where V is the water velocity in feet per second determined from the centroids of the upstream and downstream dye-response curves. The remaining variables have the same units as given in equation 9. The variables were significant at the 0.01 level and had a corrected coefficient of determination of 0.88. The root mean square error was 0.080 ft/s. The value of the regression coefficient compares favorably with that of Boning's equation and results in estimated values of the velocity that are about 20 percent higher.

Regression analysis was also performed on the logarithmic transforms of the same variables and the intercept was again found to be not significant and the model was redefined without using an intercept. The resulting relation is:

$$V = Q^{0.492}\ S^{0.341} \quad (11)$$

The equation has a corrected coefficient of determination of 0.93 and the root mean square error was 1.19 ft/s. The variance inflation factor increased from 5.8 to 6.8 for the respective models indicating a slight increase in multicollinearity. The variance inflation factor measures the effect of the dependencies among the regressors on the variance of the terms. A large value indicates multicollinearity which may become significant when values exceed 5 or 10 (Montgomery and Peck, 1982). Both variables were significant at the 0.01 level.

The most statistically significant 1-parameter model includes discharge as the regressor variable and has the form:

$$V = 0.146 + 0.004\ Q^{1.0} \tag{12}$$

No transformation was required to linearize the variables. The relation has a coefficient of determination of 0.91 and both terms are significant at the 0.01 level. The root mean square error was 0.073 ft/s. The discharge ranged from 0.27 to 168 $ft^3/s$ for the measurements. No additional 2-parameter models were statistically significant or physically appropriate for use as a predictive tool.

The equations should be used only for the range in values from which the relations were defined. Equation 12 is sensitive to this requirement due to the value of intercept. Variables required as input to the equations should be determined under steady conditions.

## Longitudinal Dispersion Coefficient

The longitudinal dispersion coefficient estimating equations were also determined using multiple linear regression analysis techniques with the value of the longitudinal dispersion coefficient being the response variable. Regressor variables used in the analysis included water-surface slope, mean discharge, mean velocity, mean width, mean depth, Froude number, and shear velocity. Data obtained from a dye trace taken the day before the tracer-gas measurement of October 17, 1984, on the North Fork of the Kentucky River at Jackson, Kentucky, were also included in the analysis.

The best 1- and 2-parameter models were developed using logarithmic transformed values of the variables. The most statistically significant 1-parameter model for estimating the longitudinal dispersion coefficient is:

$$D = 281\ V^{1.481} \tag{13}$$

where D is the longitudinal dispersion coefficient, in $ft^2/s$, and
V is the mean velocity, in ft/s.

The velocity values ranged from 0.09 to 0.90 ft/s for the measurements. The relation has a coefficient of determination of 0.94 and a root mean square error of 1.25 $ft^2/s$. Both variables were significant at the 0.01 level. The next most statistically significant 1-parameter model is:

$$D = 16.2\ Q^{0.413} \tag{14}$$

where Q is the mean water discharge in cubic feet per second in the reach. The discharge ranged from 0.27 to 168 $ft^3/s$ for the measurements. This relation has a coefficient of determination of 0.83 and a root mean square error of 1.45 $ft^2/s$. Both variables were significant at the 0.01 level.

The only statistically significant 2-parameter model has the form:

$$D = 2270\ F^{1.316}\ H^{0.812} \tag{15}$$

where F is the Froude number, and
H is the mean depth, in ft.

The relation has a coefficient of determination of 0.94 and a root mean square error of 1.25 $ft^2/s$. All terms were significant at the 0.01 level. The variance inflation factor for the variables was 1.0, indicating little multicollinearity. The Froude number ranged from 0.03 to 0.11, and the mean depth ranged from 0.20 to 2.36 ft. No other 2-parameter model was statistically appropriate. A number of variables included with velocity produced high values of the coefficient of determination but none of the variables proved to be significant at the 0.15 level.

The data were analyzed to match the form of the McQuivey and Keefer (1974) formula expressed as:

$$D = 0.059\ \frac{Q}{SB} \tag{16}$$

where S is the mean water-surface slope, in ft/ft, and
B is the mean channel width, in ft.

The derived relation has an intercept of 0.020 which results in values of the longitudinal dispersion coefficients 66 percent lower than those predicted by equation 16. The model that was developed from the data set, however, was not statistically significant, and is presented for comparison purposes only.

Even though equation 13 is the most accurate 1-parameter model from a statistical standpoint and is comparable to equation 15 in predictive capability, it is not recommended to use the relation to estimate longitudinal dispersion coefficient if the value of velocity is obtained using equation 12 or some other estimating equation. It may be more appropriate to use equation 14 if an estimate of the longitudinal dispersion coefficient is required when the average reach velocity is not known.

## Reaeration Coefficient

The reaeration coefficient estimating equations were developed using multiple linear regression techniques with the value of reaeration coefficient being the response variable. Regressor variables used in the analyses were mean water-surface slope, mean discharge, mean velocity, mean width, mean depth, Froude number, shear velocity, and longitudinal dispersion coefficient.

Mean depth and mean water-surface slope were the two variables that were the most significant statistically for 1-parameter models. The best 1-parameter model included depth as the regressor variable and has the form:

$$K_2 = -1.737 + 6.601\ H^{-1.0} \qquad (17)$$

where $K_2$ is the reaeration coefficient, in days$^{-1}$, and
$H$ is the mean depth, in ft.

The relation has a coefficient of determination of 0.99 and a root mean square error of 0.85 days$^{-1}$. Both parameters were significant at the 0.01 level. The estimated reaeration coefficient is limited by the value of depth since the intercept is negative. Using the logarithmic transforms of the values, the derived relation is:

$$K_2 = 3.720\ H^{-1.358} \qquad (18)$$

The relation has a coefficient of determination of 0.98 and a mean root square error of 1.22 days$^{-1}$. The variables were significant at the 0.01 level. The value of depth for the measurements used in the analysis ranged from 0.20 to 2.36 ft.

Water-surface slope in feet per foot was also highly significant and the best relation using slope has the form:

$$K_2 = -3.128 + 331.9\ S^{0.5} \qquad (19)$$

The coefficient of determination for the relation was 0.99, the root mean square error was 1.31 days$^{-1}$, and both terms were significant at the 0.01 level. Again the value of the estimated reaeration coefficient is limited by the value of slope used in the relation. The value of slope for the measurements ranged from 0.000133 to 0.0103 ft/ft. Using the logarithmic transforms of the values, the derived relation is:

$$K_2 = 815\ S^{0.733} \qquad (20)$$

The relation has a coefficient of determination of 0.93. The root mean square error was 1.42 days$^{-1}$ and the variables were significant at the 0.01 level. No other parameters were statistically significant when included with either depth or slope and therefore no 2-parameter models are presented. A relation using the form of the O'Connor-Dobbins (1958) relation was also derived to compare the coefficient values. The resulting equation has the form:

$$K_2 = 8.35\ V^{0.5}\ H^{-1.5} \qquad (21)$$

where V is the mean velocity in feet per second. The value of the regression coefficient from the O'Connor-Dobbins' equation is 12.81, resulting in values approximately 35 percent higher than those from equation 21. The derived equation has a corrected coefficient of determination of 0.97 and a root mean square error of 1.86 days$^{-1}$. The model was first derived with an intercept but the intercept proved to be not significant.

All of the models presented are highly significant statistically, but many of the reaeration coefficient observations are in the very low range. Only one observation is in the medium to high range and one value is in the high range. The remaining values are all in the low range. Again, it should be stressed that the estimating equations presented only be used for values which are within the range from which the relations were derived.

An observation with regard to the reaeration coefficient determination where replicate measurements were made, is that for the North Fork Kentucky River measurements the reaeration coefficient increased with increasing discharge but for the South Fork Kentucky River measurements the reaeration coefficient decreased with increasing discharge. The dependence of the reaeration coefficient on discharge is an important question which has not been adequately studied and is not addressed in this report due to the limited number of replicate measurements.

## Limitations and Accuracy

The estimating equations presented in the previous section are based upon the assumption of steady-flow conditions throughout the study reach and that the physical and hydraulic characteristics of the reach did not change appreciably during the measurement. These equations were developed from measured values within a certain range and are not recommended for use if values are outside that range. The values of the parameters obtained from the measurements are given in table 2. The regression analyses for mean velocity and the reaeration coefficient were developed using the nine observations given in table 2. The longitudinal dispersion coefficient regression analysis used the nine observations in table 2, in addition to three observations obtained from a dye study on the North Fork of the Kentucky River prior to the reaeration measurement. Because of the limited number of observations, the use of 2-parameter estimating equations may be questionable.

The 1-parameter mean velocity estimating equation (equation 12) has a coefficient of determination of 0.91. The root mean square error was 0.073 ft/s. A simple yet statistically accurate model is given by equation 11, which has a corrected coefficient of determination of 0.93 and a root mean square error of 1.19 ft/s. Mean water discharge and water-surface slope ranged in value from 0.27 to 168 $ft^3/s$ and 0.000133 to 0.0103 ft/ft, respectively.

The best 1- and 2-parameter longitudinal dispersion coefficient estimating relations are equations 13 and 15. Both equations have a root mean square error of 1.25 $ft^2/s$ and a coefficient of determination of 0.94. It may be inappropriate, however, to use equation 13 if the velocity is estimated using another predictive equation. Mean velocity ranged in value from 0.09 to 0.90 ft/s and depth ranged in value from 0.20 to 2.36 ft.

Equations 17 and 19 are the best 1-parameter reaeration coefficient estimating equations. Equation 17 has a coefficient of determination of 0.99 and a root mean square error of 0.85 $days^{-1}$. Equation 19 also has a coefficient of determination of 0.99 and a root mean square error of 1.31 $days^{-1}$. The relations developed using the logarithmic transformed values of depth and slope resulted in equations 18 and 20 which have root mean square errors of 1.22 and 1.42 days , respectively. The range in values of mean depth and water-surface slope were given previously. The regressor variables and intercepts were all significant at the 0.01 level.

## COMPARISON OF MEASURED VALUES TO PREDICTIVE EQUATIONS

### Mean Velocity and Longitudinal Dispersion

Equations to estimate mean velocity corresponding to the peak concentration of the dye-response curves are presented by Boning (1974). Although the mean velocity estimating equation presented in this report is determined from the difference in centroid travel times of the response curves, the difference between the upstream and downstream peak and centroid travel times is usually not significant. The two equations presented by Boning are for a pool-and-riffle and a channel-controlled reach. The reaches used for the pool-and-riffle equation ranged in water-surface slope from 0.63 to 30.1 ft/mi and the reaches used for the channel-controlled equation ranged in water-surface slope from 0.08 to 14.3 ft/mi.

For pool-and-riffle type reaches, Boning's equation for estimating the velocity to peak concentration, in feet per second, as given previously in equation 9 is:

$$VP = 0.38\ Q^{0.40}\ S^{0.20}$$

where Q is the mean water discharge, in $ft^3/s$, and
S is the water-surface slope, in ft/ft, and assumed to be equal to the energy gradient slope.

For channel-controlled type reaches, Boning's estimating equation for velocity to peak concentration is:

$$VP = 2.69\ Q^{0.26}\ S^{0.28} \qquad (22)$$

The percentage error for each predictive value of mean velocity and longitudinal dispersion coeffficient was determined by using the expression:

$$\text{percentage error} = \frac{(Xeq - Xmeas)}{Xmeas} \times 100 \qquad (23)$$

where Xeq is the value determined from the predictive equation and Xmeas is the value determined from the measurement. The results are shown in table 3. The average error, neglecting the algebraic signs (average absolute error) of the 1-parameter mean velocity estimating equation (eq. 12) is 20 percent. The average absolute error of the Boning equations for the pool-and-riffle and channel-controlled reaches are 20 and 72 percent, respectively. All nine observations were used to evaluate the pool-and-riffle estimating equation, but the Glenns Creek and Mill Creek observations were excluded from the channel-controlled equation evaluation because those observations were outside the range of values from which the equations were developed.

Table 3.--Comparison of measured and predicted values of mean velocity

| Reach | Measured value (feet per second) | Predicted value equation 12 | Percent error | Predicted value equation 9 | Percent error | Predicted value equation 22 | Percent error |
|---|---|---|---|---|---|---|---|
| | | | Glenns Creek near Versailles, Kentucky<br>Measurement of 08/15/84 | | | | |
| 1-2 | 0.252 | 0.153 | -39 | 0.151 | -40 | --- | --- |
| | | | Mill Creek near Manchester, Kentucky<br>Measurement of 08/29/84 | | | | |
| 1-2 | .093 | .147 | 58 | .090 | - 3 | --- | --- |
| | | | North Fork Kentucky River near Jackson, Kentucky<br>Measurement of 10/17/84 | | | | |
| 1-2 | .521 | .458 | -12 | .472 | - 9 | 1.01 | 94 |
| 2-3 | .458 | .439 | - 4 | .348 | -24 | .667 | 46 |
| 1-3 | .483 | .447 | - 7 | .415 | -14 | .850 | 76 |
| | | | Measurement of 10/08/85 | | | | |
| 1-3 | .898 | .868 | - 3 | .589 | -34 | 1.07 | 19 |
| | | | South Elkhorn Creek near Midway, Kentucky<br>Measurement of 08/02/84 | | | | |
| 1-2 | .263 | .306 | 16 | .380 | 44 | .566 | 115 |
| | | | South Fork Kentucky River near Booneville, Kentucky<br>Measurement of 09/06/84 | | | | |
| 1-2 | .448 | .555 | 24 | .397 | -11 | .730 | 63 |
| | | | Measurement of 10/10/85 | | | | |
| 1-2 | .338 | .405 | 20 | .331 | - 2 | .648 | 92 |
| Average Absolute Error | | | 20 | | 20 | | 72 |

An equation to estimate the longitudinal dispersion coefficient was presented by McQuivey and Keefer (1974) and is given by equation 16 as:

$$D = 0.059 \frac{Q}{SB}$$

where D is the longitudinal dispersion coefficient, in $ft^2/s$,
Q is the mean water discharge, in $ft^3/s$,
S is the water-surface slope, in ft/ft, and
B is the mean channel width, in ft.

The average absolute errors of the 1- and 2-parameter longitudinal dispersion coefficient estimating equations are 18 and 17 percent, respectively. The average absolute error associated with the equation by McQuivey and Keefer was 199 percent which did a poor job of estimating the coefficient for the sites with smaller drainage basins. The computed values are shown in table 4.

## Reaeration Coefficient

Many predictive equations, both empirical and semi-empirical, have been developed to estimate the reaeration coefficient for natural streams. In the report by Grant and Skavroneck (1980), error analysis was performed using several predictive equations and the results of tracer measurements on three small streams in Wisconsin. Some of the equations are also used in this report to investigate the error associated with the measurement and prediction of the reaeration coefficient. For each equation the reaeration-rate coefficient is expressed in base e units of reciprocal days and are corrected to 20°C. The following symbols appear in the equations listed:

F = Froude number = $V/\sqrt{gH}$
g = acceleration due to gravity ($ft/s^2$)
H = average hydraulic depth (ft)
$\Delta h$ = change in elevation between the start and end of the study reach (ft)
Q = average streamflow ($ft^3/s$)
q = specific discharge ($ft^3/s)/mi^2$ = streamflow divided by the total drainage area
R = hydraulic radius (ft)
S = slope of the energy gradient (ft/ft)
t = traveltime in the study reach (hr)
u* = average shear velocity (ft/s) = $\sqrt{gRS}$
V = average stream velocity (ft/s)
coth = hyperbolic cotangent angle, in radians

1. Dobbins (1965)

$$K_2 = 116.6 \frac{1 + F^2}{(0.9 + F)^{1.5}} \frac{(VS)^{0.375}}{H} \coth \frac{4.10\,(VS)^{0.125}}{(0.9 + F)^{0.5}}$$

2. O'Connor-Dobbins (1958)

$$K_2 = 12.81\, V^{0.5}\, H^{-1.5}$$

Table 4.--Comparison of measured and predicted values of longitudinal dispersion coefficient

| Reach | Measured values (feet per second) | Predicted value equation 13 | Percent error | Predicted value equation 15 | Percent error | Predicted value equation 16 | Percent error |
|---|---|---|---|---|---|---|---|
| | | | Glenns Creek near Versailles, Kentucky<br>Measurement of 08/15/84 | | | | |
| 1-2 | 41.3 | 36.5 | -12 | 31.9 | -23 | 1.28 | - 97 |
| | | | Mill Creek near Manchester, Kentucky<br>Measurement of 08/29/84 | | | | |
| 1-2 | 6.33 | 8.32 | 31 | 7.91 | 25 | .11 | - 98 |
| | | | North Fork Kentucky River near Jackson, Kentucky<br>Measurement of 10/16/84 | | | | |
| 1-2 | 104 | 105 | 1 | 103 | - 1 | 91 | -12 |
| 2-3 | 74.9 | 107 | 43 | 107 | 43 | 357 | 377 |
| 1-3 | 87.7 | 106 | 21 | 105 | 20 | 167 | 90 |
| | | | North Fork Kentucky River near Jackson, Kentucky<br>Measurement of 10/17/84 | | | | |
| 1-2 | 116 | 107 | - 8 | 104 | -10 | 192 | 66 |
| 2-3 | 119 | 88.4 | -26 | 89.9 | -24 | 343 | 188 |
| 1-3 | 119 | 95.6 | -20 | 95.7 | -20 | 144 | 210 |
| | | | Measurement of 10/08/85 | | | | |
| 1-3 | 188 | 240 | 28 | 225 | 20 | 347 | 85 |
| | | | South Elkhorn Creek near Midway, Kentucky<br>Measurement of 08/02/84 | | | | |
| 1-2 | 46.0 | 38.9 | -15 | 45.6 | - 1 | 275 | 498 |
| | | | South Fork Kentucky River near Booneville, Kentucky<br>Measurement of 09/06/84 | | | | |
| 1-2 | 85.6 | 85.6 | 0 | 90.5 | - 6 | 411 | 380 |
| | | | Measurement of 10/10/85 | | | | |
| 1-2 | 67.2 | 56.4 | -16 | 60.7 | -10 | 260 | 287 |
| Average Absolute Error | | | 18 | | 17 | | 199 |

3. Krenkel-Orlob (1963)

$$K_2 = 234\ (VS)^{0.408}\ H^{-0.66}$$

4. Cadwallader-McDonnell (1969)

$$K_2 = 336.8\ (VS)^{0.5}\ H^{-1}$$

5. Parkhurst-Pomeroy (1972)

$$K_2 = 48.39\ (1 + 0.17\ F^2)\ (VS)^{0.375}\ H^{-1}$$

6. Bennett-Rathbun (1972)

$$K_2 = 106.16\ V^{0.413}\ S^{0.273}\ H^{-1.408}$$

7. Churchill and others (1962)

$$K_2 = 0.03454\ V^{2.695}\ H^{-3.085}\ S^{-0.823}$$

8. Lau (1972)

$$K_2 = 2515\left(\frac{u^*}{V}\right)^{3.0} VH^{-1}$$

9. Thackston-Krenkel (1969)

$$K_2 = 24.94\ (1 + F^{0.5})\ u^*\ H^{-1}$$

10. Langbein-Durum (1967)

$$K_2 = 7.61\ VH^{-1.33}$$

11. Owens and others (1964)

$$K_2 = 23.23\ V^{0.73}\ H^{-1.75}$$

12. Owens and others (1964)

$$K_2 = 21.74\ V^{0.67}\ H^{-1.85}$$

13. Churchill and others II (1962)

$$K_2 = 11.57\ V^{0.969}\ H^{-1.673}$$

14. Isaacs-Gaudy (1968)

$$K_2 = 8.62\ VH^{-1.5}$$

15. Negulescu-Rojanski (1969)

$$K_2 = 10.92\ (V/H)^{0.85}$$

16. Padden-Gloyna (1971)

$$K_2 = 6.87\ V^{0.703}\ H^{-1.054}$$

17. Bansal (1973)

$$K_2 = 4.67\ V^{0.6}\ H^{-1.40}$$

18. Bennett-Rathbun II (1972)

$$K_2 = 20.19\ V^{0.607}\ H^{-1.689}$$

19. Tsivoglou-Neal (1976)

$$K_2 = 1.296\ \frac{\Delta h}{t}$$

20. Foree (written communication, 1977)

$$K_2 = (0.63 + 0.4S^{1.15})\ q^{0.25}$$

if $q > 1.0$, use $q = 1.0$
if $q < 0.05$, use $q = 0.05$

21. Parker and Gay (1986)

$$K_2 = 252.2\ H^{-0.176}\ V^{0.355}\ S^{0.438}$$

22. Smoot (1987)

$$K_2 = 683.8\ V^{0.5325}\ H^{-0.7258}\ S^{0.6236}$$

The percentage error for each predicted value of $K_2$ was determined by using equation 23:

$$\text{percentage error} = \frac{(K_2eq - K_2meas)}{K_2meas} \times 100$$

where $K_2eq$ is the reaeration value calculated using one of the predictive equations and $K_2meas$ is the reaeration value determined from the gas-tracer measurement. The percentage error for each of the predictive equations for each measurement is presented in tables 5 through 10. The average absolute errors and the ranking of the equations according to this error are presented in table 11. The average absolute error is the average error, neglecting the algebraic signs. The 1-parameter estimating equation presented in this study (eq. 17) had an average absolute error of 25 percent. This equation was not included in the ranking since it was developed using the data set. The predictive equation presented by Thackston and Krenkel obtained the lowest average absolute error of the observations which was 33 percent.

Parker and Gay (1986) suggest that mean water-surface slope may be a factor in determining which predictive equation(s) may be most appropriate for estimating the reaeration coefficient. They indicate that a water-surface slope of approximately 0.003 ft/ft defined a breakpoint in predictive capability of equations. This reported phenomenon was not investigated in this report due to the lack of observations in the data set, especially for observations from streams having a water-surface slope greater than 0.003 ft/ft.

## APPLICATIONS OF ESTIMATING TECHNIQUES

The examples presented in the following sections are provided to illustrate the possible practical applications of the estimating equations presented in this report. Use of the predictive equations should be limited to streams having characteristics that are in the range of those from which the models were developed.

### Example 1. Estimating Mean Velocity and the Longitudinal Dispersion Coefficient

An instantaneous spill of 200 liters of a water soluble conservative

Table 5.--Predictive error of 22 reaeration coefficient estimating equations for Glenns Creek near Versailles, August 15, 1984

| | Measurement of 08/15/84<br>Reach 1-2<br>Measured $K_2$ = 17.5 $days^{-1}$ | |
|---|---|---|
| Predictive equations | Predicted $K_2$ ($days^{-1}$) | Percent error |
| Dobbins | 28.2 | 61 |
| O'Conner-Dobbins | 32.4 | 85 |
| Krenkel-Orlob | 28.4 | 62 |
| Cadwallader-McDonnel | 31.3 | 79 |
| Parkhurst-Pomeroy | 10.7 | -39 |
| Bennett-Rathbun I | 60.6 | 246 |
| Churchill and others I | 2.22 | -87 |
| Lau | 16,500. | 94,200 |
| Thackston-Krenkel | 19.5 | 11 |
| Langbein-Durum | 200. | 1040 |
| Owens and others I | 56.0 | 220 |
| Owens and others II | 63.5 | 263 |
| Churchill and others II | 18.5 | 6 |
| Isaacs-Gaudy | 344. | 1870 |
| Negulescu-Rojanski | 8.5 | -51 |
| Padden-Gloyna | 8.1 | -54 |
| Bansal | 9.2 | -47 |
| Bennett-Rathbun II | 54.1 | 209 |
| Tsivoglou-Neal | 4.66 | -73 |
| Foree | .50 | -97 |
| Parker-Gay | 16.6 | -5 |
| Smoot | 22.8 | 30 |
| Equation 17 | 17.7 | 1 |

Table 6.--Predictive error of 22 reaeration coefficient estimating equations for Mill Creek near Manchester, August 29, 1984

| | Measurement of 08/29/84<br>Reach 1-2<br>Measured $K_2$ = 31.1 $days^{-1}$ | |
|---|---|---|
| Predictive equations | Predicted $K_2$ ($days^{-1}$) | Percent error |
| Dobbins | 49.8 | 60 |
| O'Conner-Dobbins | 43.0 | 38 |
| Krenkel-Orlob | 39.4 | 27 |
| Cadwallader-McDonnel | 51.6 | 66 |
| Parkhurst-Pomeroy | 17.7 | -43 |
| Bennett-Rathbun I | 108. | 247 |
| Churchill and others I | .34 | -99 |
| Lau | 167,000. | 537,000 |
| Thackston-Krenkel | 14.7 | -53 |
| Langbein-Durum | 1,510. | 4,760 |
| Owens and others I | 67.3 | 116 |
| Owens and others II | 85.3 | 174 |
| Churchill and others II | 16.8 | -46 |
| Isaacs-Gaudy | 459. | 1,380 |
| Negulescu-Rojanski | 5.64 | -82 |
| Padden-Gloyna | 6.98 | -76 |
| Bansal | 10.5 | -66 |
| Bennett-Rathbun II | 71.1 | 129 |
| Tsivoglou-Neal | 4.48 | -86 |
| Foree | .31 | -99 |
| Parker-Gay | 19.4 | -38 |
| Smoot | 35.5 | 14 |
| Equation 17 | 30.9 | -1 |

Table 7.--Predictive error of 22 reaeration coefficient estimating equations for North Fork Kentucky River near Jackson, October 17, 1984

| | Measurement of 10/17/84 | | | | | |
|---|---|---|---|---|---|---|
| | Reach 1-2 Measured $K_2$ = 1.89 $days^{-1}$ | | Reach 2-3 Measured $K_2$ = 1.93 $days^{-1}$ | | Reach 1-3 Measured $K_2$ = 1.91 $days^{-1}$ | |
| Predictive equations | Predicted $K_2$ ($days^{-1}$) | Percent error | Predicted $K_2$ ($days^{-1}$) | Percent error | Predicted $K_2$ ($days^{-1}$) | Percent error |
| Dobbins | 3.80 | 100 | 1.07 | -45 | 2.68 | 40 |
| O'Connor-Dobbins | 5.03 | 166 | 3.95 | 105 | 4.36 | 128 |
| Krenkel-Orlob | 6.46 | 242 | 3.18 | 65 | 4.74 | 148 |
| Cadwallader-McDonnell | 3.83 | 103 | 1.57 | -19 | 2.59 | 36 |
| Parkhurst-Pomeroy | 1.52 | -20 | .76 | -61 | 1.12 | -41 |
| Bennett-Rathbun I | 5.93 | 214 | 3.23 | 67 | 4.46 | 134 |
| Churchill and others I | .81 | -57 | 1.27 | -34 | .85 | -55 |
| Lau | 100. | 5,190 | 21.5 | 1,010 | 60.2 | 3,050 |
| Thackston-Krenkel | 3.47 | 84 | 1.59 | -18 | 2.50 | 31 |
| Langbein-Durum | 10.6 | 461 | 10.7 | -466 | 10.6 | 455 |
| Owens and others I | 7.10 | 276 | 5.24 | 172 | 5.93 | 210 |
| Owens and others II | 6.63 | 251 | 2.53 | 31 | 5.53 | 190 |
| Churchill and others II | 3.12 | 65 | 2.26 | 17 | 2.58 | 35 |
| Isaacs-Gaudy | 12.5 | 561 | 12.7 | 558 | 12.6 | 560 |
| Negulescu-Rojanski | 4.44 | 135 | 3.60 | 87 | 3.92 | 105 |
| Padden-Gloyna | 2.83 | 50 | 2.28 | 18 | 2.49 | 30 |
| Bansal | 1.79 | -5 | 1.40 | -27 | 1.55 | -19 |
| Bennett-Rathbun II | 8.97 | 375 | 5.18 | 168 | 5.81 | 204 |
| Tsivoglou-Neal | 1.30 | -31 | .24 | -88 | .72 | -62 |
| Foree | .32 | -83 | .31 | -84 | .32 | -83 |
| Parker-Gay | 7.00 | 270 | 3.53 | 83 | 5.26 | 175 |
| Smoot | 3.36 | -81 | 1.19 | -38 | 2.15 | 13 |
| Equation 17 | 2.66 | 41 | 2.17 | 12 | 2.36 | 24 |

Table 8.--Predictive error of 22 reaeration coefficient estimating equations for North Fork Kentucky River near Jackson, October 8, 1985

| | Measurement of 10/08/85 Reach 1-3 Measured $K_2$ = 3.39 $days^{-1}$ | |
|---|---|---|
| Predictive equations | Predicted $K_2$ ($days^{-1}$) | Percent error |
| Dobbins | 2.55 | -25 |
| O'Conner-Dobbins | 4.05 | 19 |
| Krenkel-Orlob | 5.16 | 52 |
| Cadwallader-McDonnel | 2.73 | -19 |
| Parkhurst-Pomeroy | 1.15 | -66 |
| Bennett-Rathbun I | 4.01 | 18 |
| Churchill and others I | 2.04 | -40 |
| Lau | 5.79 | 71 |
| Thackston-Krenkel | 2.33 | -31 |
| Langbein-Durum | 3.32 | -2 |
| Owens and others I | 5.96 | 76 |
| Owens and others II | 5.22 | 54 |
| Churchill and others II | 3.06 | -10 |
| Isaacs-Gaudy | 3.38 | 0 |
| Negulescu-Rojanski | 5.35 | 58 |
| Padden-Gloyna | 2.94 | -13 |
| Bansal | 1.57 | -54 |
| Bennett-Rathbun II | 5.49 | 62 |
| Tsivoglou-Neal | 1.30 | -62 |
| Foree | .39 | -88 |
| Parker-Gay | 6.26 | 85 |
| Smoot | 2.49 | -26 |
| Equation 17 | 1.44 | -58 |

Table 9.--Predictive error of 22 reaeration coefficient estimating equations for South Elkhorn Creek near Midway, August 2, 1984

| | Measurement of 08/02/84<br>Reach 1-2<br>Measured $K_2$ = 1.32 $days^{-1}$ | |
|---|---|---|
| Predictive equations | Predicted $K_2$ ($days^{-1}$) | Percent error |
| Dobbins | 1.02 | 23 |
| O'Conner-Dobbins | 1.81 | 37 |
| Krenkel-Orlob | 2.02 | 53 |
| Cadwallader-McDonnel | .84 | -36 |
| Parkhurst-Pomeroy | .44 | -67 |
| Bennett-Rathbun I | 1.60 | 21 |
| Churchill and others I | .10 | -92 |
| Lau | 223. | 16,800 |
| Thackston-Krenkel | 1.24 | -6 |
| Langbein-Durum | 14.4 | 991 |
| Owens and others I | 1.95 | 48 |
| Owens and others II | 1.81 | 37 |
| Churchill and others II | .75 | -43 |
| Isaacs-Gaudy | 17.6 | 1230 |
| Negulescu-Rojanski | 1.69 | 28 |
| Padden-Gloyna | 1.09 | -17 |
| Bansal | .63 | -52 |
| Bennett-Rathbun II | 2.10 | 59 |
| Tsivoglou-Neal | .12 | -91 |
| Foree | .49 | -63 |
| Parker-Gay | 2.71 | 105 |
| Smoot | .69 | -48 |
| Equation 17 | 1.06 | -20 |

Table 10.--Predictive error of 22 reaeration coefficient estimating equations for South Fork Kentucky River at Booneville, September 6, 1984 and October 10, 1985

| | Measurement of 09/06/84 Reach 1-2 Measured $K_2 = 0.90\ days^{-1}$ | | Measurement of 10/10/85 Reach 1-2 Measured $K_2 = 1.64\ days^{-1}$ | |
|---|---|---|---|---|
| | Predicted $K_2$ ($days^{-1}$) | Percent error | Predicted $K_2$ ($days^{-1}$) | Percent error |
| Dobbins | 1.38 | 53 | 1.48 | -10 |
| O'Conner-Dobbins | 2.72 | 202 | 3.08 | 87 |
| Krenkel-Orlob | 2.71 | 201 | 2.71 | 65 |
| Cadwallader-McDonnell | 1.23 | 37 | 1.28 | -22 |
| Parkhurst-Pomeroy | .59 | -34 | .64 | -61 |
| Bennett-Rathbun I | 2.29 | 154 | 2.62 | 60 |
| Churchill and others I | .56 | -38 | .45 | -73 |
| Lau | 27.1 | 2,910 | 76.5 | 4,560 |
| Thackston-Krenkel | 1.40 | 56 | 1.50 | -9 |
| Langbein-Durum | 8.00 | 789 | 14.7 | 796 |
| Owens and others I | 3.39 | 277 | 3.76 | 129 |
| Owens and others II | 3.08 | 242 | 3.54 | 116 |
| Churchill and others II | 5.31 | 490 | 1.51 | -8 |
| Isaacs-Gaudy | 9.12 | 913 | 18.2 | 1,010 |
| Negulescu-Rojanski | 2.88 | 220 | 2.64 | 61 |
| Padden-Gloyna | 1.74 | 93 | 1.72 | 5 |
| Bansal | .99 | 10 | 1.07 | -35 |
| Bennett-Rathbun II | 3.40 | 278 | 3.87 | 136 |
| Tsivoglou-Neal | .24 | -73 | .24 | -85 |
| Foree | .38 | -58 | .34 | -79 |
| Parker-Gay | 3.38 | 276 | 3.15 | 92 |
| Smoot | 1.00 | 11 | .96 | -42 |
| Equation 17 | 1.33 | 48 | 1.93 | 18 |

Table 11.--Average absolute error analysis of 22 reaeration coefficient estimating equations

| Predictive equations | Average absolute error | Rank |
|---|---|---|
| Dobbins | 46 | 6.5 |
| O'Conner-Dobbins | 96 | 13 |
| Krenkel-Orlob | 102 | 14 |
| Cadwallader-McDonnel | 46 | 6.5 |
| Parkhurst-Pomeroy | 40 | 5 |
| Bennett-Rathbun I | 129 | 16 |
| Churchill and others I | 63 | 8.5 |
| Lau | 73,900 | 22 |
| Thackston-Krenkel | 33 | 1 |
| Langbein-Durum | 1,080 | 21 |
| Owens and others I | 170 | 18 |
| Owens and others II | 151 | 17 |
| Churchill and others II | 80 | 11 |
| Isaacs-Gaudy | 898 | 20 |
| Negulescu-Rojanski | 92 | 12 |
| Padden-Gloyna | 39 | 4 |
| Bansal | 35 | 3 |
| Bennett-Rathbun II | 180 | 19 |
| Tsivoglou-Neal | 72 | 10 |
| Foree | 63 | 8.5 |
| Parker-Gay | 125 | 15 |
| Smoot | 34 | 2 |
| Equation 17 | 25 | --- |

material has occurred at 1300 hours on July 15 at river mile 21.30 of Example Creek. The waterworks intake for a city is located 1.0 mile downstream from the site of the spill. The operators of the water plant would like to estimate what the characteristics of the solute response curve would be as it passes the waterworks intake and when this will occur.

The mean velocity and longitudinal dispersion coefficient can be estimated from equations 12 and 14, respectively. Equation 14, rather than equations 13 or 15, was used to estimate the dispersion coefficient because it may be inappropriate to use an estimate of velocity in the predictive equation for estimating the longitudinal dispersion coefficient. In both equations only discharge is required but some accuracy is being sacrificed by using equation 14. Discharge is measured by current meter methods in the vicinity of the reach such that no significant inflow occurs between the measuring section and the reach. The spill has occurred during a period of steady flow. The discharge was determined to be 45 $ft^3/s$. From equation 12 the mean velocity in the reach is determined to be:

$$V = 0.146 + 0.004\ Q$$
$$= 0.34\ ft/s$$

The longitudinal dispersion coefficient can be estimated from equation 14:

$$D = 16.2\ Q^{0.413}$$
$$= 78\ ft^2/s$$

Bennett and Rathbun (1972) give the equation describing the response of a stream to an instantaneous input of a conservative pollutant. This equation is:

$$C(x,t) = \frac{W_L}{A\ 2\sqrt{\pi Dt}}\ e^{\frac{-(x-Vt)^2}{4Dt}} \qquad (24)$$

Where C is the concentration at distance x and time t relative to the center of the response function, in mg/L,
$W_L$ is the weight of the pollutant spilled, in mg,
A is the mean cross-sectional area of the stream reach, in $ft^2$,
D is the longitudinal dispersion coefficient, in $ft^2/s$, and
V is the mean streamflow velocity, in ft/s.

The first part of the equation expresses the concentration of the peak at some time, t. The exponential portion of the equation is a factor to adjust the peak concentration relative to a distant (x-Vt) away from the center of the response function. The weight of the material spilled is computed as the amount multiplied by the specific weight, 1.12 x $10^9$ ug/L times 200 liters, or 224 x $10^9$ ug (224 x $10^6$ mg). The stream cross-sectional area is computed as the water discharge, 45 $ft^3/s$, divided by the mean streamflow velocity, 0.34 ft/s, or 132 $ft^2$. Substituting these values into equation 24 for a distance x, of 5,280 ft, yields:

$$C(t) = \frac{31.8}{\sqrt{t}} e^{\frac{-(5,280 - 1,224\ t)^2}{1.123 \times 10^6 t}} \quad \text{mg/L}$$

where t is the time after the spill, in hours. This equation gives the concentration at a distance of 5,280 ft below the location of the spill as a function of time (in hours) after the spill.

For the example problem, the time, the cumulative time after the spill in hours, and the value of concentration, in mg/L, are given in table 12. The substance will first arrive at the waterworks intake at approximately 1500 hr on July 15, peak at 1700 at a concentration of 15.5 mg/L, and will return to zero concentration at 0030 hr on July 16. If it were known that the standard for drinking water containing this substance was 10 mg/L, for example, then the time that the concentration is equalled or exceeded could be estimated. It could be estimated either from the tabular or graphical form of the response function data. In this example, the concentration is greater than 10 mg/L from approximately 1615 to 1830 on July 15. Even though the concentration and time values are only estimates, use of this technique will allow the operators of the waterworks to plan a response strategy with regard to the spill. The uncertainty in such estimates would probably make it wise to not withdraw water from about 1500 to 2100 hours on July 15.

Table 12.--Response data for example 1

| Time | Time since spill (h) | Concentration (mg/L) | Time | Time since spill (h) | Concentration (mg/L) |
|---|---|---|---|---|---|
| July 15 | | | | | |
| 1300 | 0 | 0 | 1930 | 6.5 | 4.7 |
| 1330 | .5 | 0 | 2000 | 7 | 3.1 |
| 1400 | 1 | 0 | 2030 | 7.5 | 1.9 |
| 1430 | 1.5 | 0 | 2100 | 8 | 1.2 |
| 1500 | 2 | .6 | 2130 | 8.5 | .7 |
| 1530 | 2.5 | 3.5 | 2200 | 9 | .4 |
| 1600 | 3 | 8.6 | 2230 | 9.5 | .2 |
| 1630 | 3.5 | 13.3 | 2300 | 10 | .1 |
| 1700 | 4 | 15.5 | 2330 | 10.5 | .1 |
| 1730 | 4.5 | 14.9 | 2400 | 11 | 0 |
| 1800 | 5 | 12.6 | July 16 | | |
| 1830 | 5.5 | 9.7 | 0030 | 11.5 | 0 |
| 1900 | 6 | 6.9 | 0100 | 12 | 0 |

## Example 2. Estimating the Reaeration Coefficient

A sewage treatment facility is planned at river mile 15.0 on Pristine Creek. In order to assess whether a site on this creek is suitable or a site on another creek should be investigated, a wasteload allocation determination will be made, and for the determination, an estimate of the reaeration coefficient is required. The condition to be modeled is low flow. Equation 17 and 19 will be used to make the estimate. The equations require mean channel depth and mean water-surface slope of the reach as input values. A reach length of 2,000 ft is selected just downstream of the proposed site to make the determination.

Under steady flow conditions, a stream cross section is taken every 50 ft and 15 to 20 depth readings are made across each section. The mean depth of each cross section is determined and the values averaged over the entire reach. This is done at as low a discharge as possible to the low flow being modeled, and at other times as well to obtain an idea of the variation with discharge of the estimated values.

Elevation reference marks are also established at each end of the 2,000-ft reach and the elevation relative to each mark determined by differential leveling. The elevation of the water surface can be determined by taping down to the water surface from the elevation reference mark. This is done under different flow conditions and for the condition of the lowest discharge possible, again to obtain an estimate of the variation using the estimating techniques.

At the lowest discharge possible, and under steady flow conditions, the mean depth was determined to be 0.80 ft and the change in water-surface elevation was 2.0 ft; therefore the water-surface slope was 0.001 ft/ft. Other parameters necessary as input to the dissolved oxygen model would be collected at the same time. Using 0.80 ft as the mean depth in equation 17 yields a reaeration coefficient of 6.5 $days^{-1}$. Using the value of 0.001 ft/ft for the water-surface slope in equation 19 yields 7.4 $days^{-1}$ as the reaeration coefficient. The conservative value of 6.5 $days^{-1}$ is chosen as the estimated value of reaeration coefficient under those flow conditions. A discharge measurement is made at the time of the determination to relate the reaeration coefficient to a value of streamflow. The value of $K_2$ and the associated parameters could then be input into a dissolved oxygen model to make a determination of wasteload allocation for the site. These determinations are usually done at low-flow conditions because water-quality standards are most likely to be violated at low flow.

## SUMMARY AND CONCLUSIONS

Nine measurements of time-of-travel and the desorption coefficient were made on five streams in the Kentucky River basin from August 1984 to October 1985. From this data, values of the mean velocity, the longitudinal dispersion coefficient, and the reaeration coefficient were determined.

Error analysis was performed comparing the measured values to values obtained from predictive equations contained in the literature. The mean velocity estimating equations for pool-and-riffle and channel-controlled reaches presented by Boning had a 20- and 72-percent average absolute error, respectively. The longitudinal dispersion coefficient estimating equation presented by McQuivey and Keefer had a 199-percent average absolute error from that of the measured values. The reaeration coefficient predictive equation with the lowest percent average absolute error was that of Thackston and Krenkel with a 33-percent error, followed by that of Smoot with a 34-percent error.

Estimating equations were also developed from the measured values which can be used to estimate mean streamflow velocity, the longitudinal dispersion coefficient, and the reaeration coefficient. These equations use easily measured physical and hydraulic stream characteristics. One- and 2-parameter models are presented for use as estimating tools. Due to the limited number of observations (degrees of freedom) in the data set, it was deemed inappropriate to construct models with more than two parameters. Parameters used as regressor variables in the analysis included mean depth, mean width, mean discharge, mean velocity, the longitudinal dispersion coefficient, Froude number, shear velocity, and mean water-surface slope. Relations that appeared physically inappropriate were excluded from the analysis even if the relation appeared to be statistically significant.

The response variables of mean velocity, longitudinal dispersion coefficient, and reaeration coefficient were determined by time-of-travel techniques in conjunction with the hydrocarbon gas-tracer technique. The 1-parameter equation developed to estimate mean velocity has a coefficient of determination of 0.91, and the variables were significant at the 0.01 level. The root mean square error was 0.073 ft/s. The 1- and 2-parameter models developed to estimate the longitudinal dispersion coefficient both have a coefficient of determination of 0.94 and a root mean square error of 1.25 $ft^2/s$. The variables were significant at the 0.01 level. The 1-parameter estimating equations developed to estimate the reaeration coefficient have a coefficient of determination of 0.99. The relation developed using depth has a root mean square error of 0.85 $days^{-1}$ and the relation using slope has a root mean square error of 1.31 $days^{-1}$.

More measurements should be made to upgrade the data set. Measurements on stream reaches with a drainage area from 10 to 100 $mi^2$ need to be made to eliminate the void between high and low values of the reaeration coefficient determined in the study. With the addition of more observations, regression analysis could be broadened to include more regressor variables and more models.

The equations presented in this report are intended to provide a means of estimating the mean velocity, longitudinal dispersion coefficient, and reaeration coefficient for streams in the Kentucky River basin. Although the relations are fairly well defined from a statistical standpoint, they are based on a limited number of observations. If accurate values of mean velocity, longitudinal dispersion coefficient, or reaeration coefficient are required, direct measurements of these parameters should be made at the flow condition desired.

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# APPENDIX

## DYE AND GAS
## CONCENTRATION-TIME CURVES
## FOR THE MEASUREMENTS

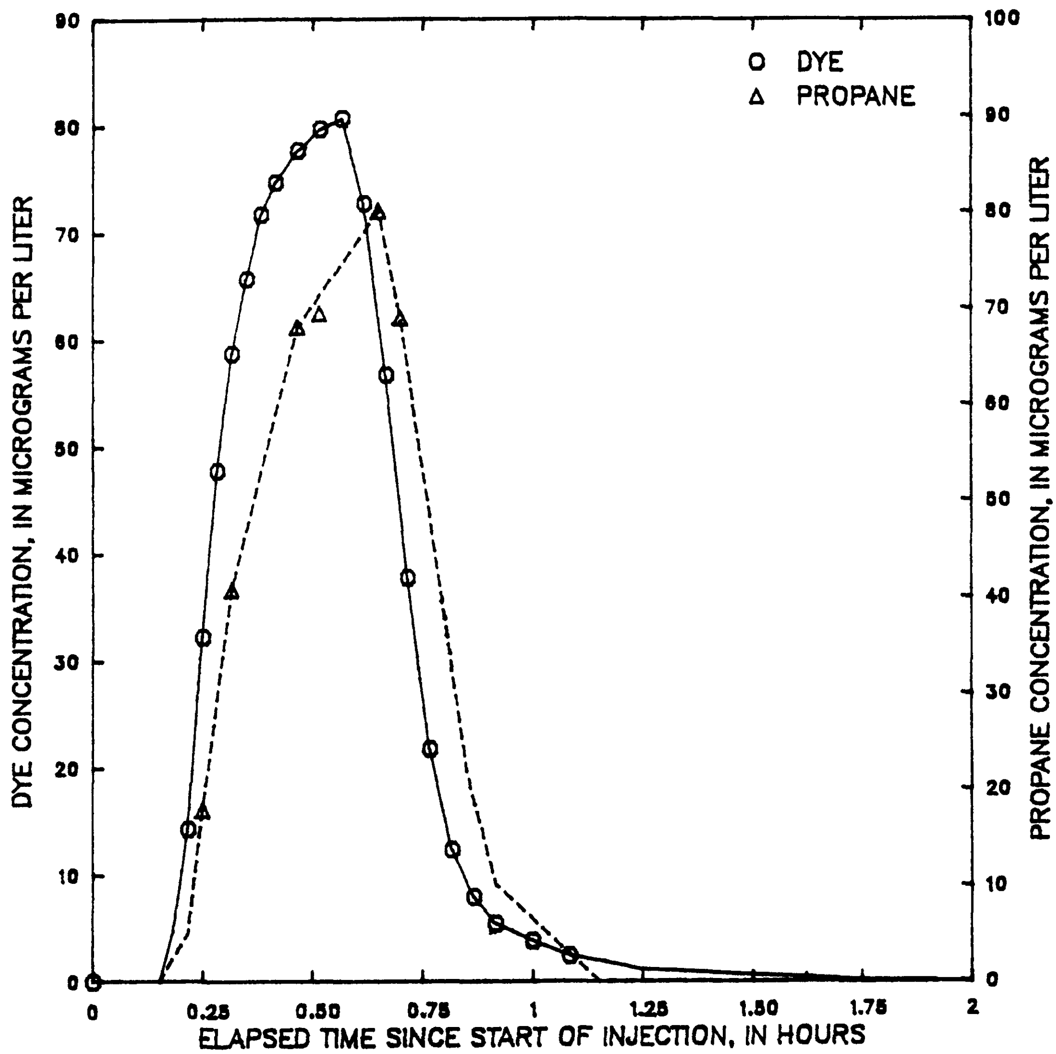


Figure 7.—Reaeration field data for the first cross section on Glenns Creek near Versailles, August 15, 1984.

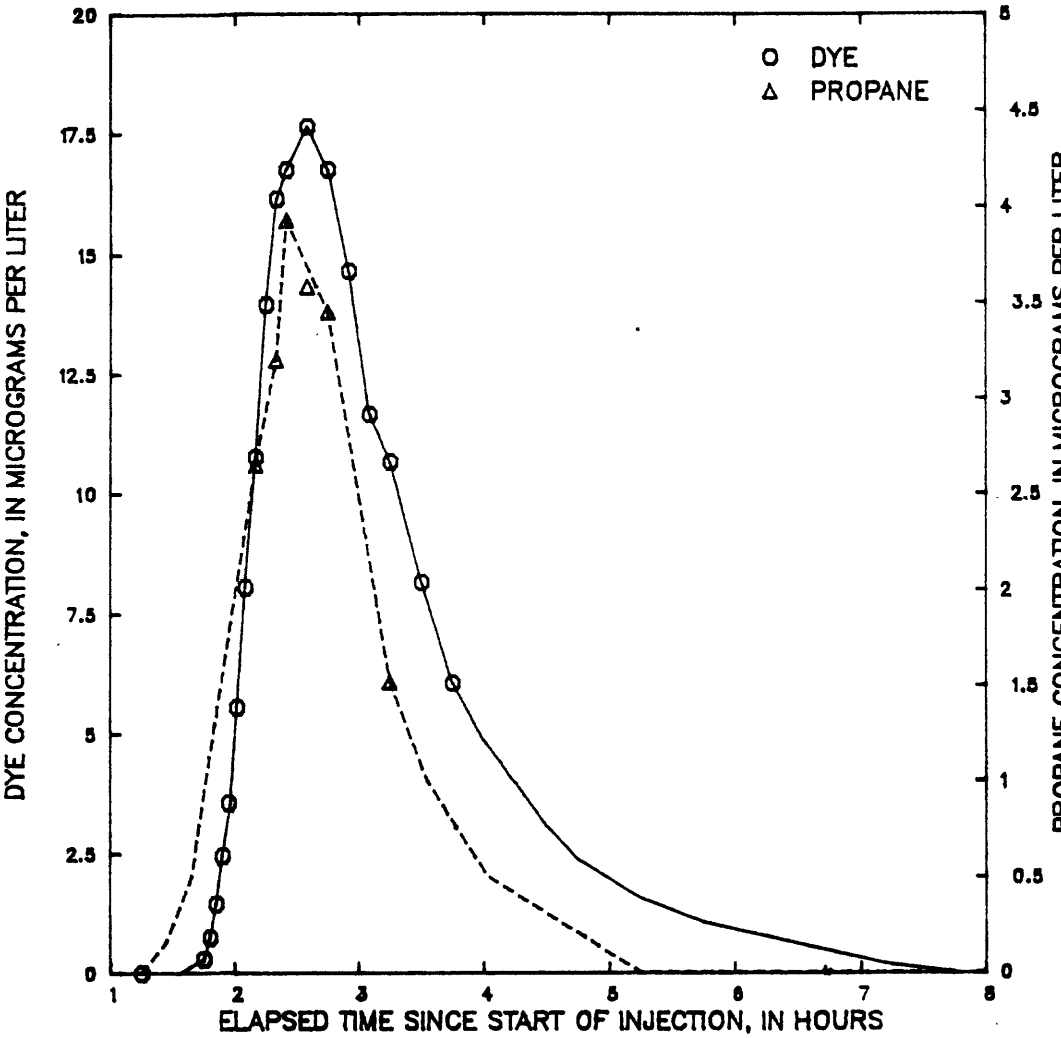


Figure 8.—Reaeration field data for the second cross section on Glenns Creek near Versailles, August 15, 1984.

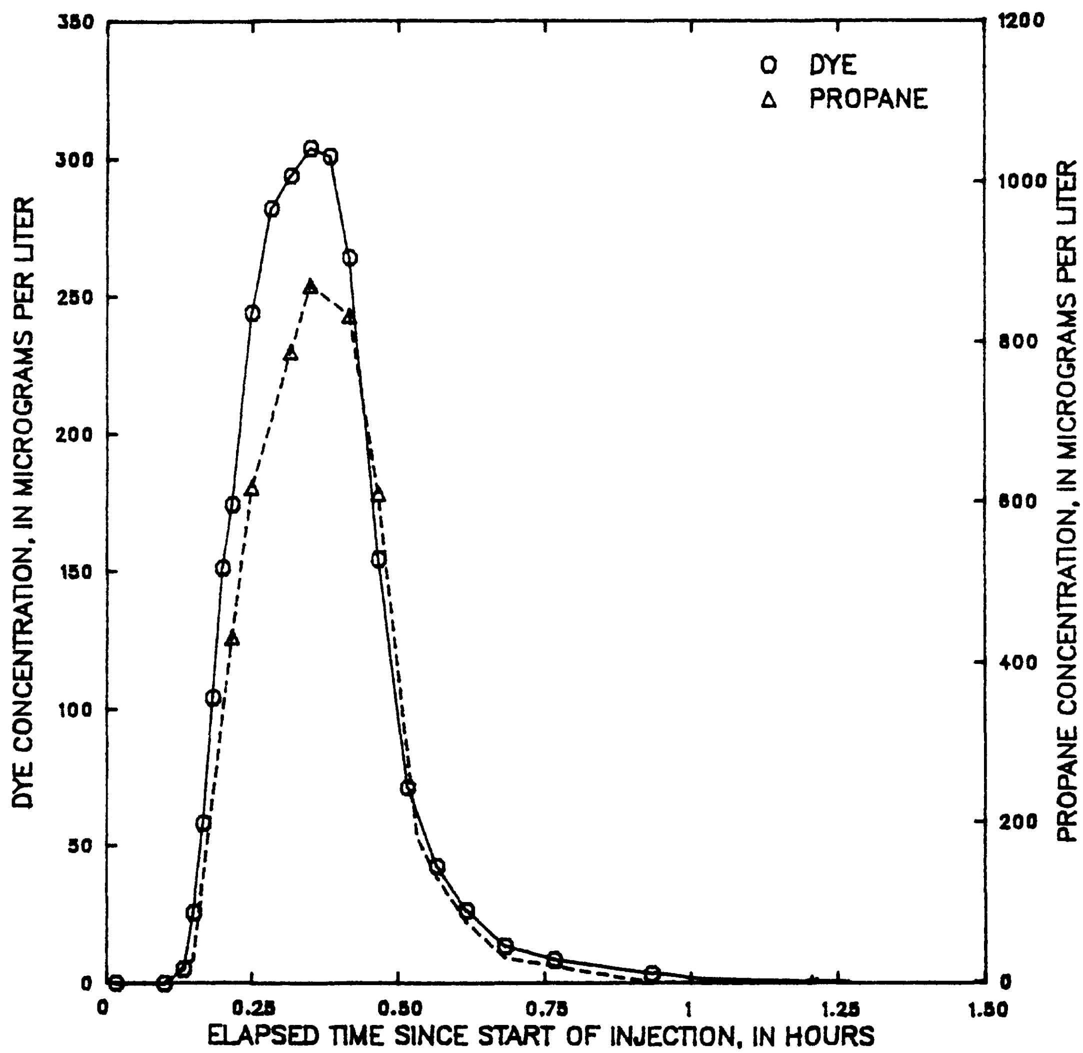


Figure 9.—Reaeration field data for the first cross section on Mill Creek near Manchester, August 29, 1984.

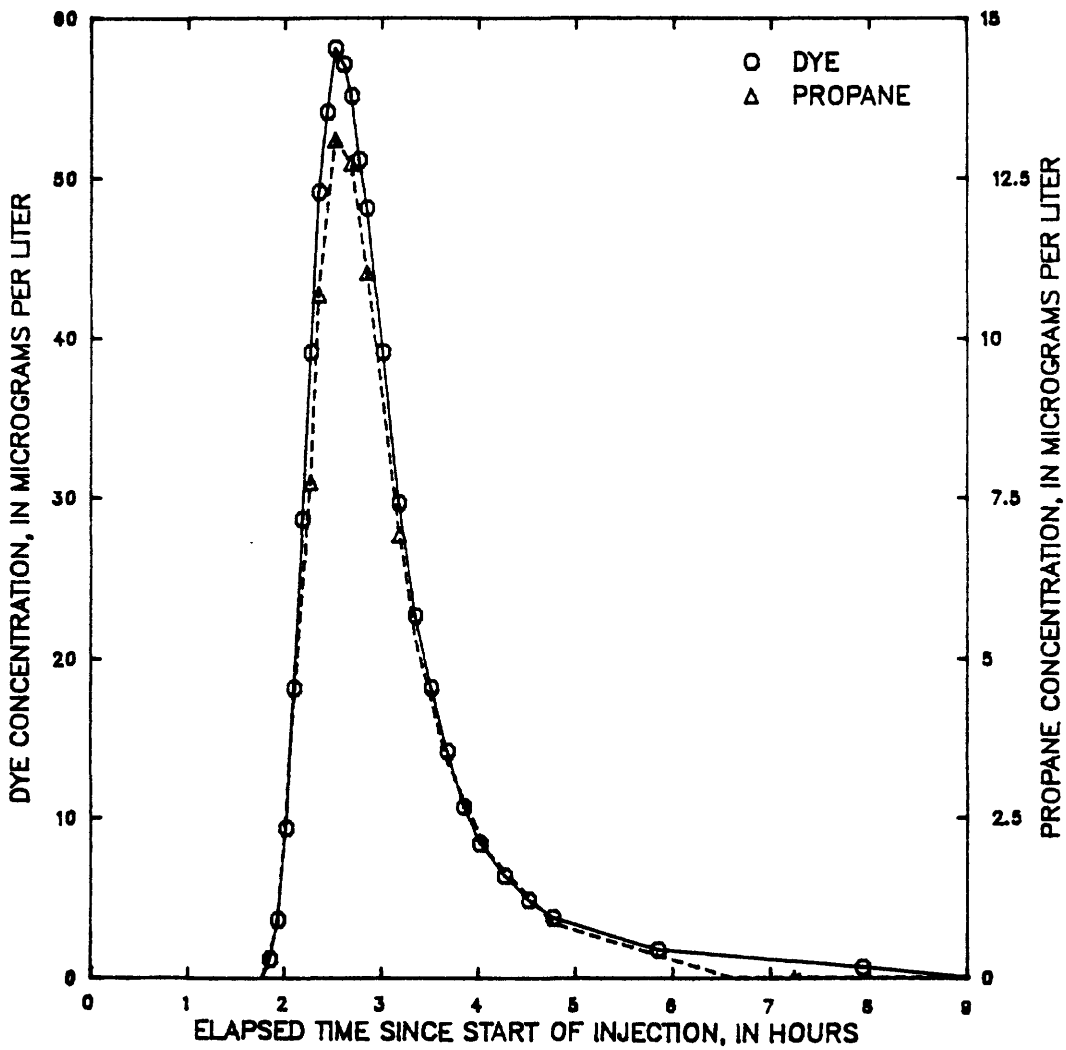


Figure 10.--Reaeration field data for the second cross section on Mill Creek near Manchester, August 29, 1984.

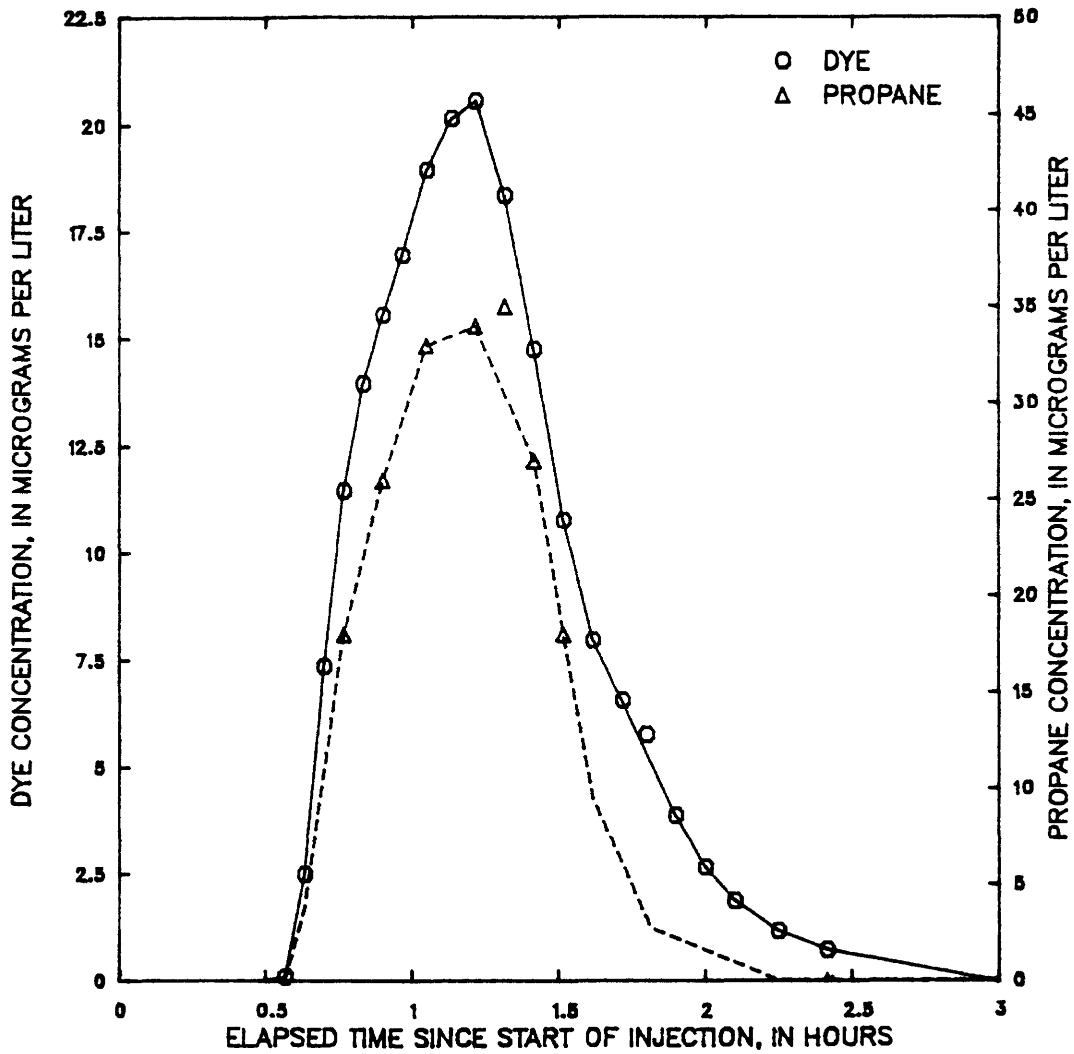


Figure 11.—Reaeration field data for the first cross section on North Fork Kentucky River near Jackson, October 17, 1984.

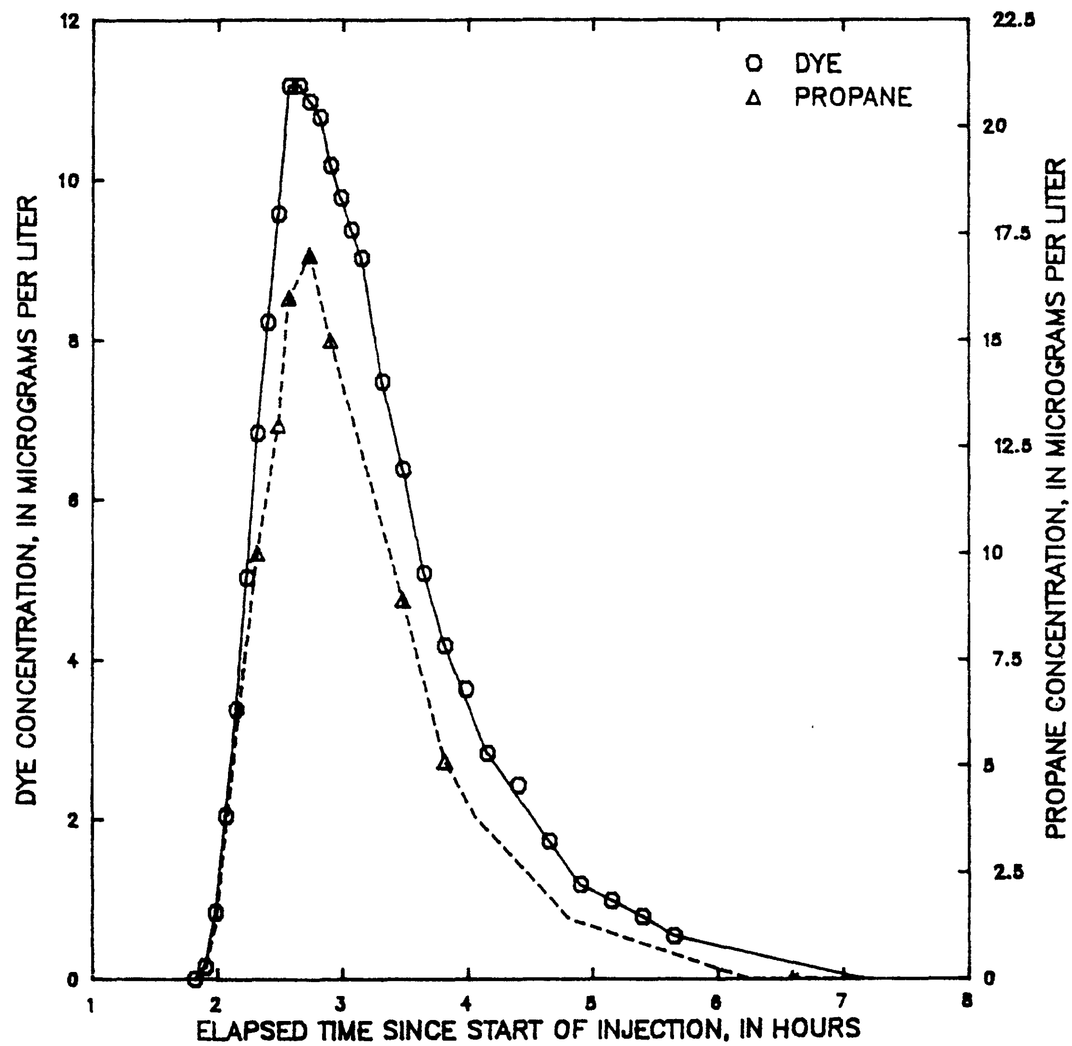


Figure 12.—Reaeration field data for the second cross section on North Fork Kentucky River near Jackson, October 17, 1984.

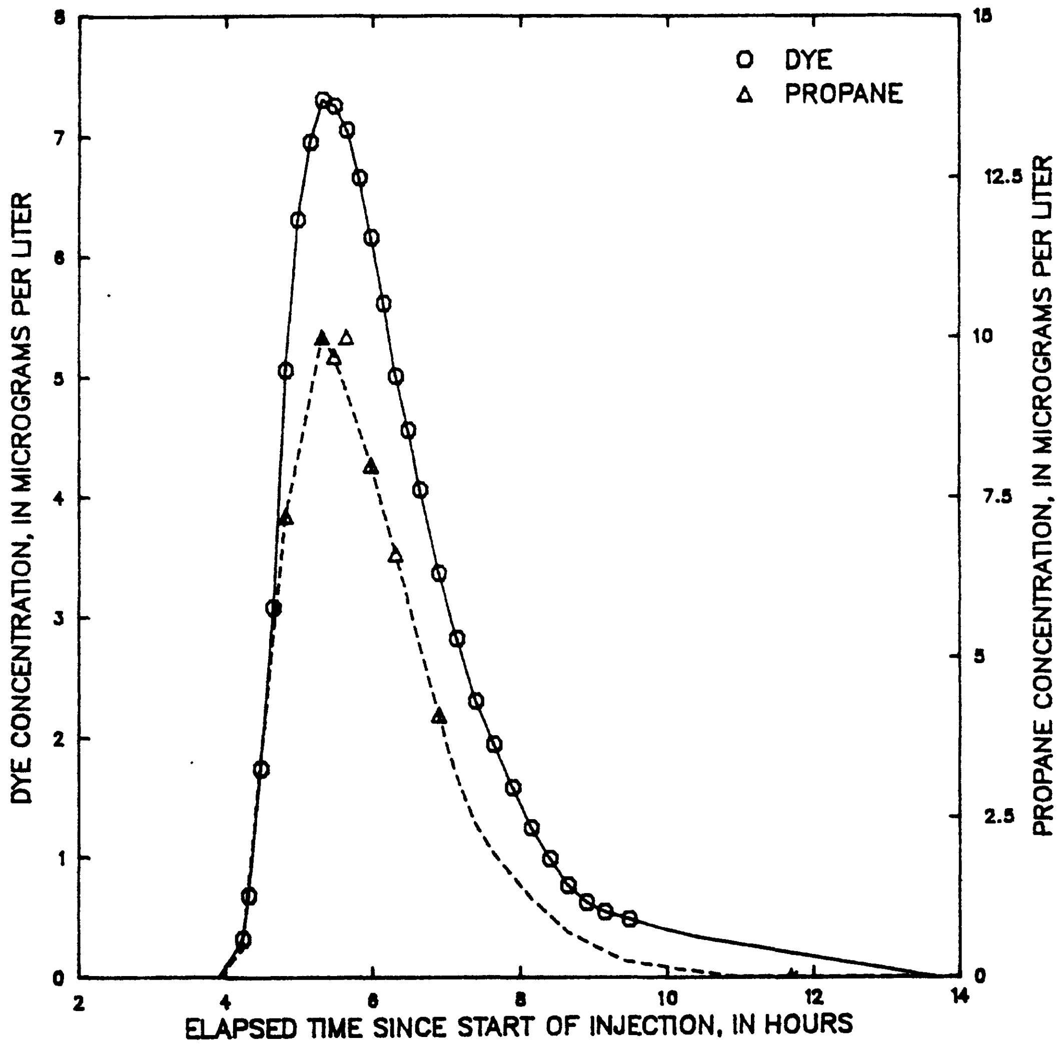


Figure 13.—Reaeration field data for the third cross section on North Fork Kentucky River near Jackson, October 17, 1984.

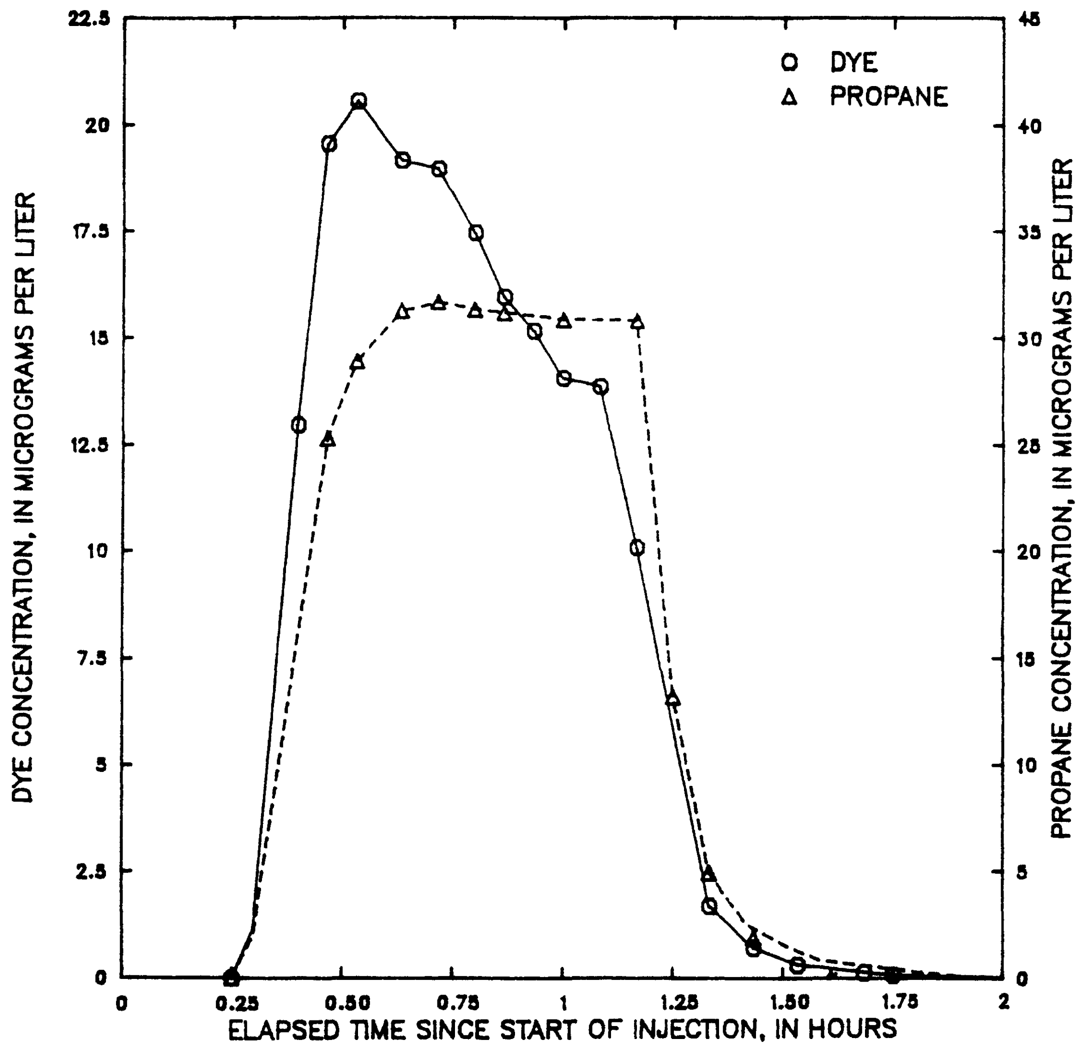


Figure 14.—Reaeration field data for the first cross section on North Fork Kentucky River near Jackson, October 8, 1985.

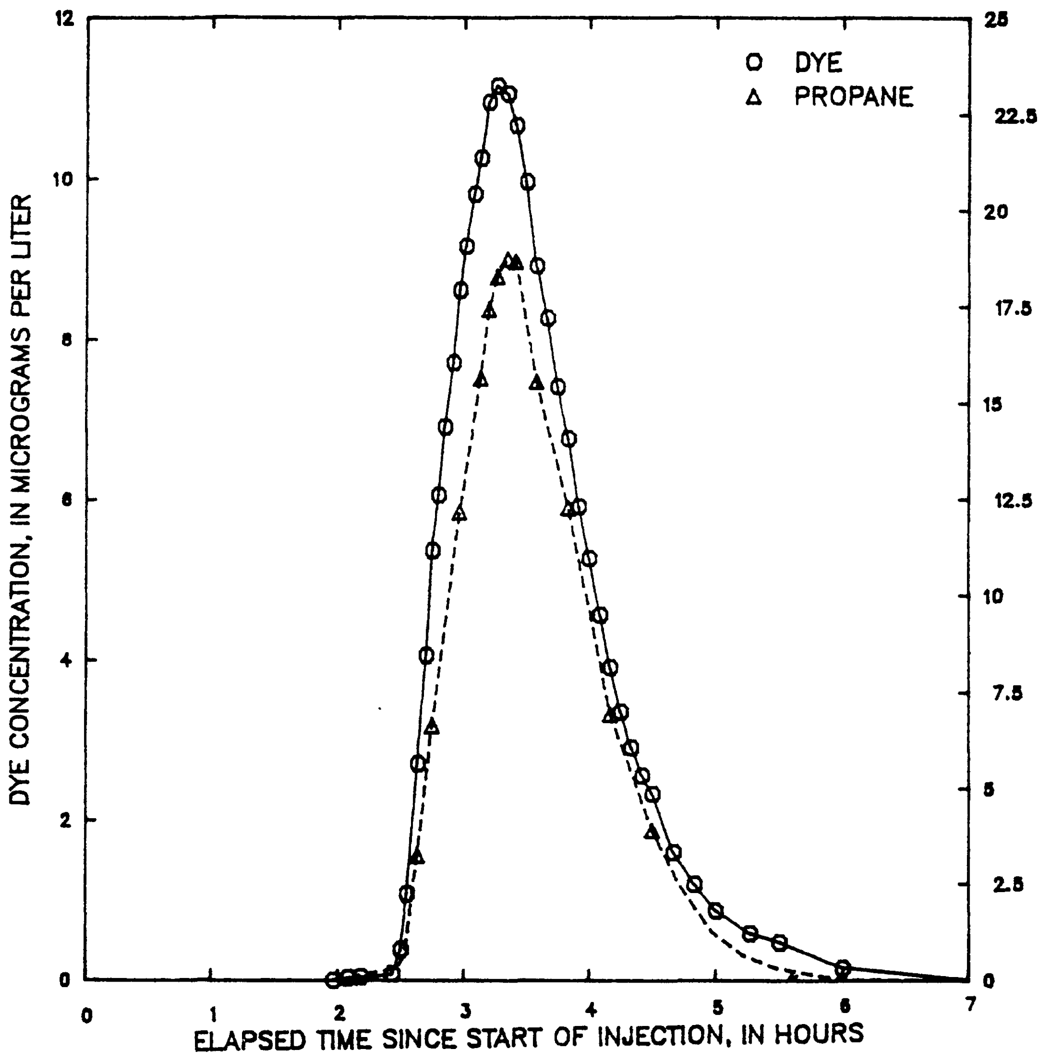


Figure 15.—Reaeration field data for the third cross section on North Fork Kentucky River near Jackson, October 8, 1985.

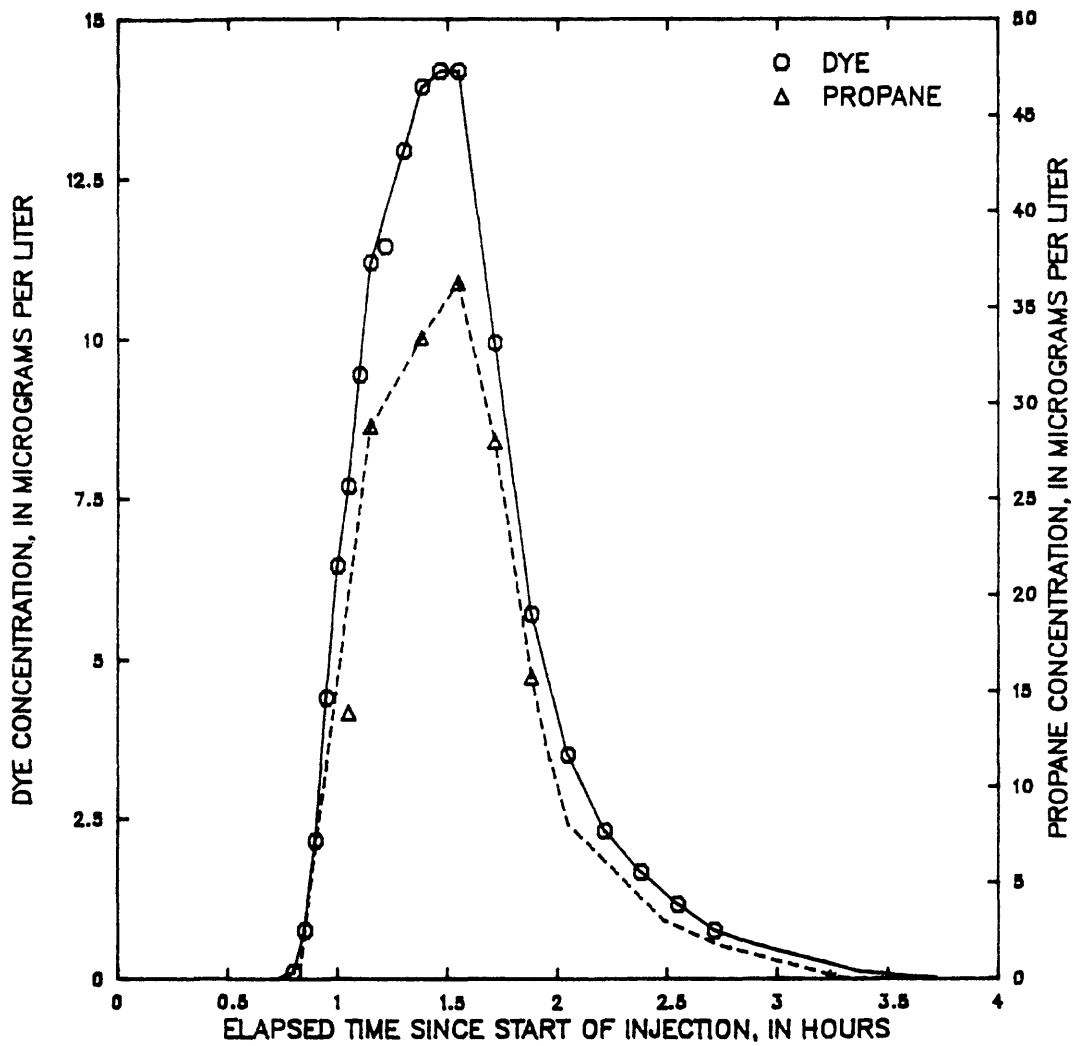


Figure 16.—Reaeration field data for the first cross section on South Elkhorn Creek near Midway, August 2, 1984.

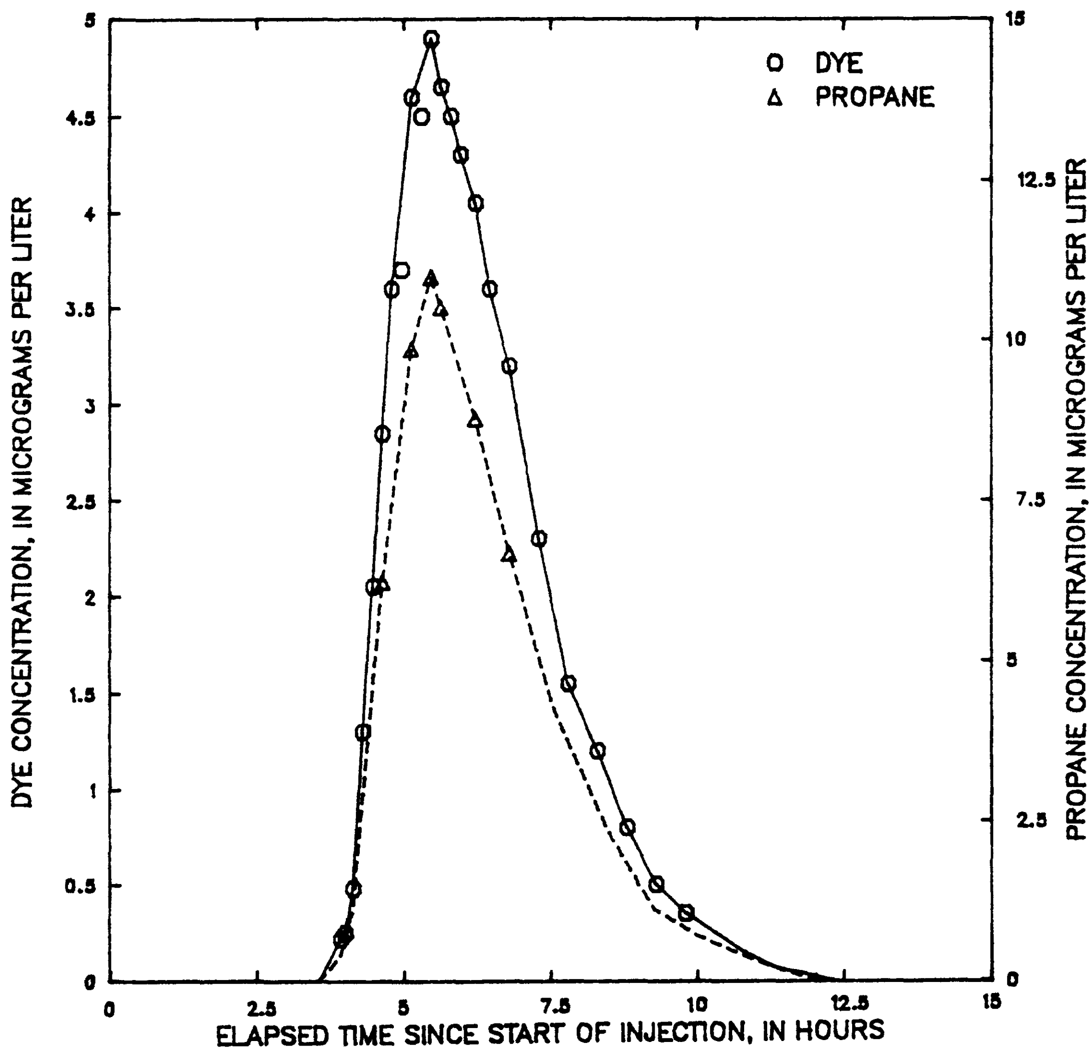


Figure 17.—Reaeration field data for the second cross section on South Elkhorn Creek near Midway, August 2, 1984.

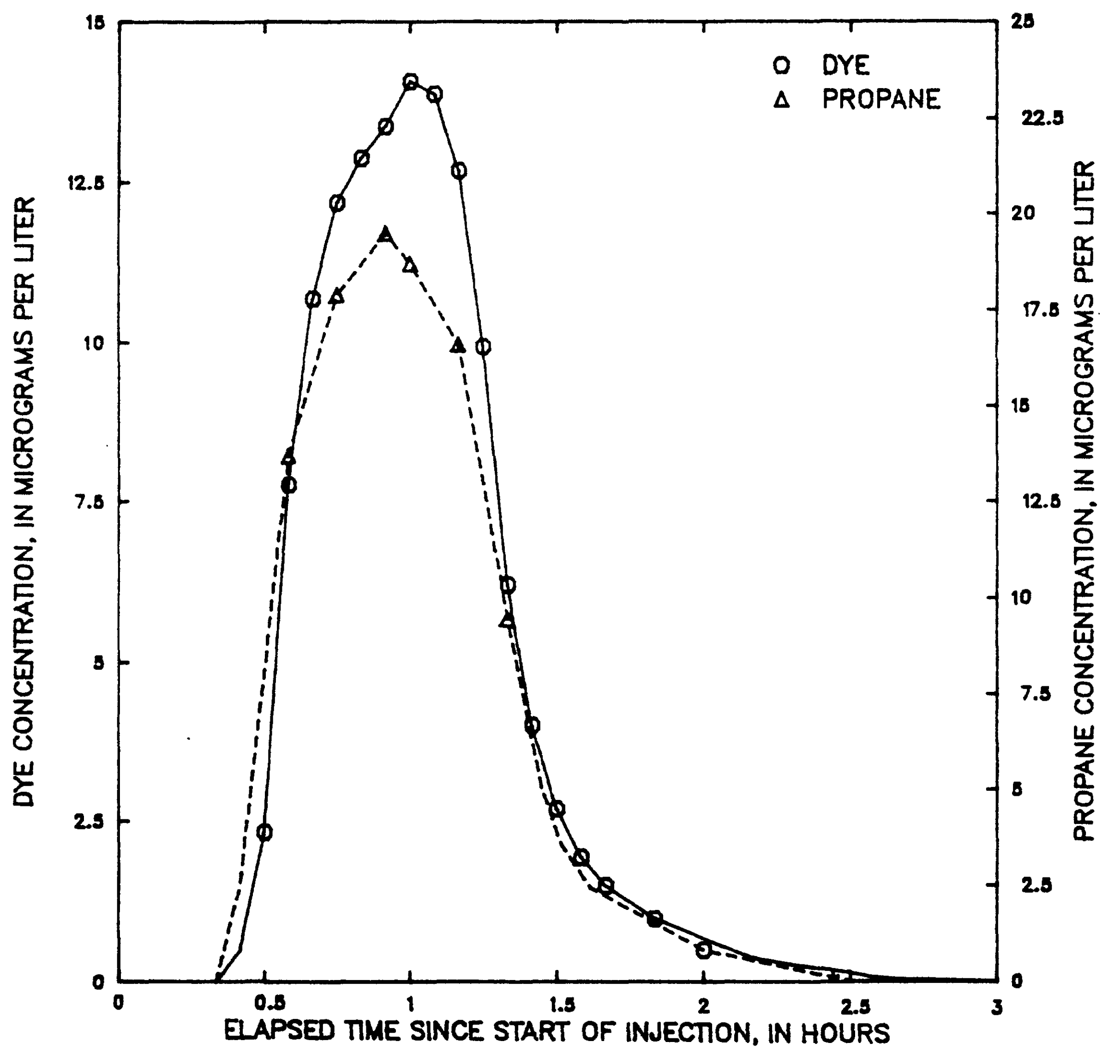


Figure 18.—Reaeration field data for the first cross section on South Fork Kentucky River near Booneville, September 6, 1984.

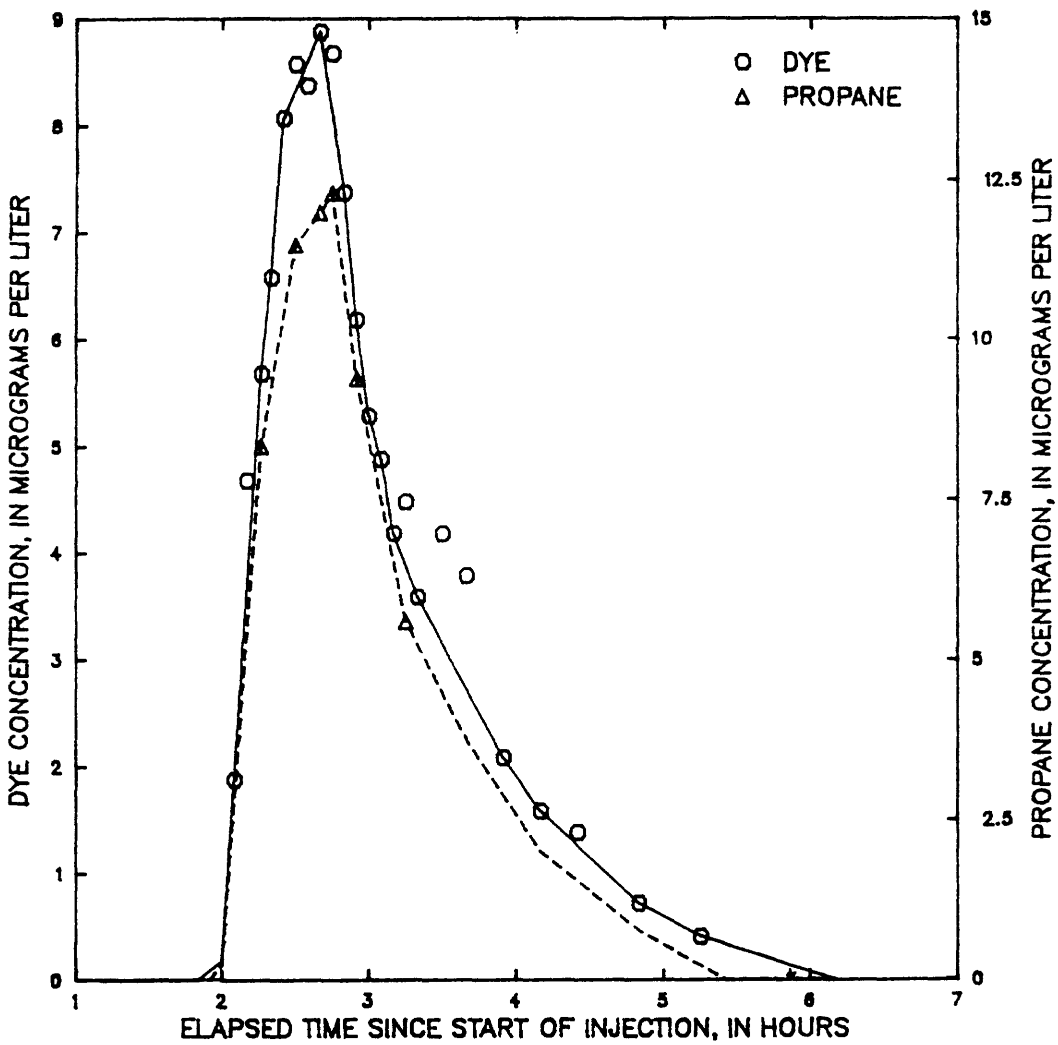


Figure 19.—Reaeration field data for the second cross section on South Fork Kentucky River near Booneville, September 6, 1984.

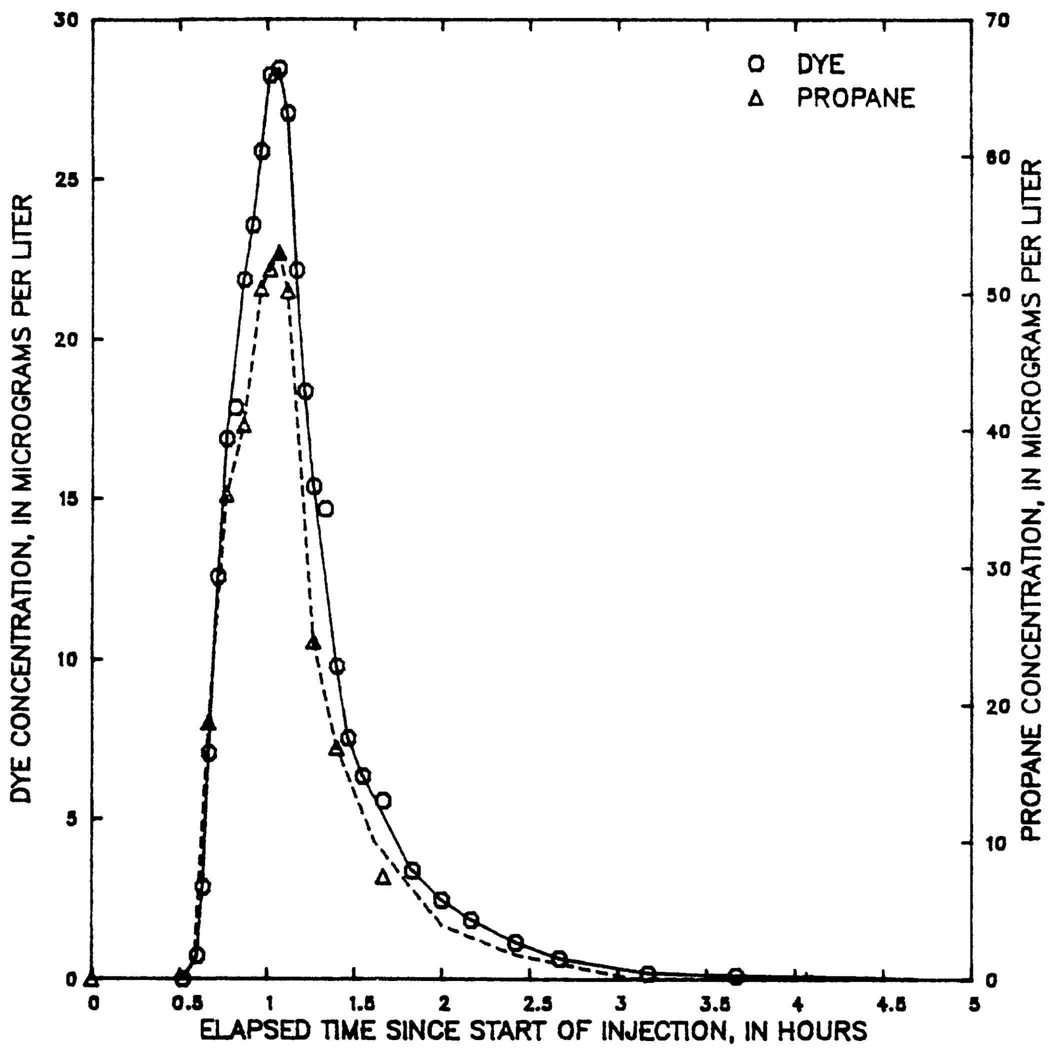


Figure 20.—Reaeration field data for the first cross section on South Fork Kentucky River near Booneville, October 10, 1985.

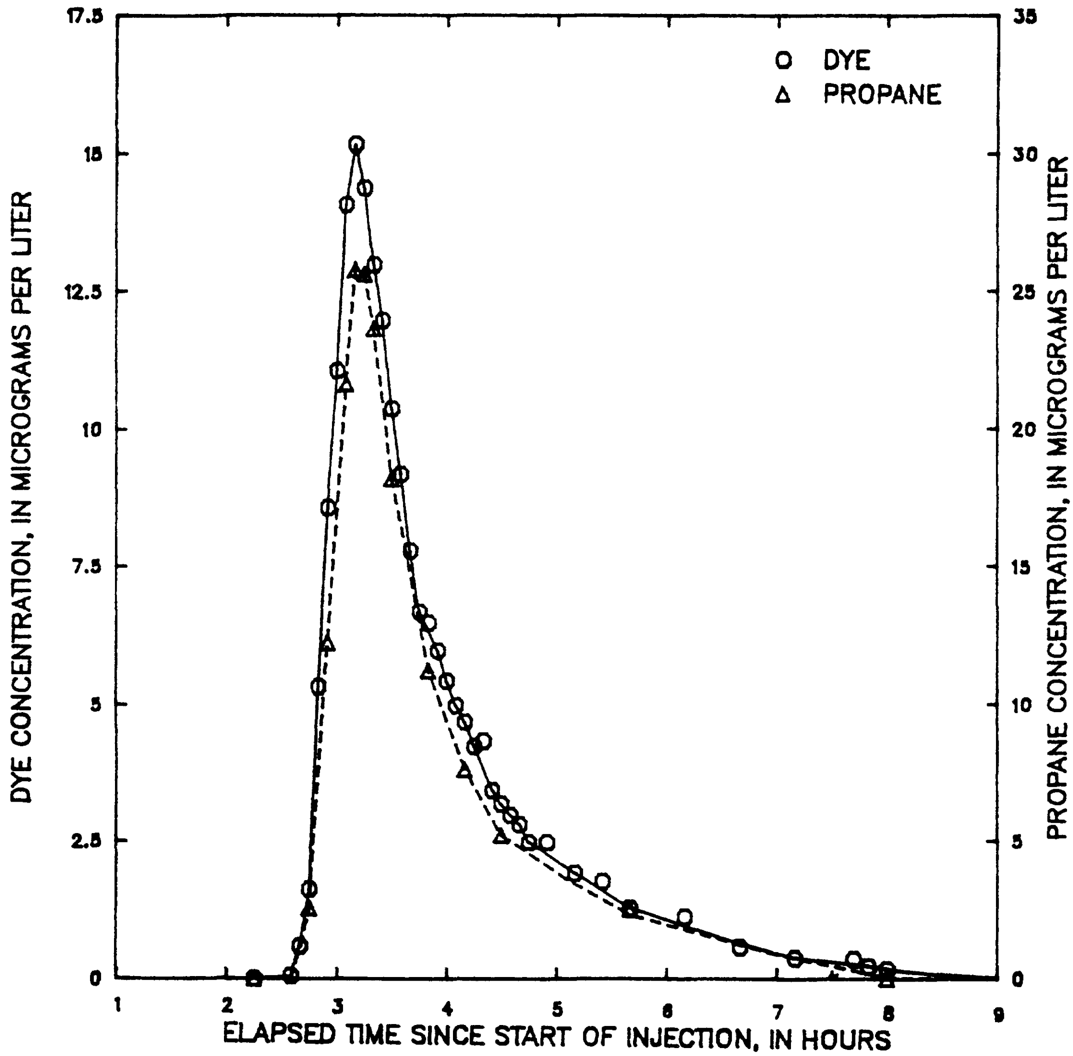


Figure 21.—Reaeration field data for the second cross section on South Fork Kentucky River near Booneville, October 10, 1985.