CALIBRATION AND SENSITIVITY ANALYSIS OF A GROUND-WATER FLOW MODEL OF THE COASTAL LOWLANDS AQUIFER SYSTEM IN PARTS OF LOUISIANA, MISSISSIPPI, ALABAMA, AND FLORIDA

By Angel Martin, Jr., and C.D. Whiteman, Jr.

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CONVERSION FACTORS AND ABBREVIATIONS

For the convenience of readers who prefer to use metric (International System) units rather than the inch-pound units used in this report, values may be converted by using the following factors:

Multiply inch-pound units	By	To obtain metric units
foot (ft)	0.3048	meter (m)
foot per mile (ft/mi)	0.1894	meter per kilometer (m/km)
foot squared per day (ft^2/d)	0.09290	meter squared per day (m^2/d)
inch (in.)	25.4	millimeter (mm)
million cubic feet (Mft^3)	28,320	cubic meter (m^3)
million cubic feet per day (Mft^3/d)	0.3278	cubic meter per second (m^3/s)
square mile (mi^2)	2.590	square kilometer (km^2)
mile (mi)	1.609	kilometer (km)

Sea level: In this report "sea level" refers to the National Geodetic Vertical Datum of 1929 (NGVD of 1929)--a geodetic datum derived from a general adjustment of the first-order level nets of both the United States and Canada, formerly called "Sea Level Datum of 1929."

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ABSTRACT

The coastal lowlands aquifer system, consisting of aquifers in sediments of Miocene age and younger in southern Louisiana, Mississippi, and Alabama and in western Florida, is being studied as part of the U.S. Geological Survey's Gulf Coast Regional Aquifer-System Analysis program. This report describes the calibration and sensitivity analysis of a multilayer, finite-difference ground-water flow model developed to quantify flow in the aquifer system.

Initial calibration of the model by trial-and-error was followed by use of a parameter estimation program. Transmissivities of permeable zones within the aquifer system, vertical leakances between the zones, and the storage coefficient of the aquifer system were varied to obtain the best match between model-simulated and measured water levels for the period 1958-82. The mean error, root-mean-square error (RMSE), and standard deviation of the residuals between model-simulated and measured water levels were used to evaluate the progress of calibration, with greatest weight given to minimizing the RMSE.

Best calibration results were obtained in the model layer that represents the uppermost part of the aquifer system. Good results also were obtained in the subsurface part of the rest of the aquifer system where water-level gradients are relatively low and uniform. Calibration of the model is relatively poor in the outcrop areas of the lower part of the aquifer system and near some major pumping centers, where steep and irregular water-level gradients are difficult to simulate at the scale of the model.

Sensitivity analysis of the calibrated steady-state and transient models was performed by varying values of transmissivity, vertical leakance, and storage coefficient; the same parameters were varied during calibration. Changes in RMSE were used as the primary indicator of sensitivity. Near the calibrated values, the model is most sensitive to changes in transmissivity and almost as sensitive to changes in vertical leakance. By layer, the model is most sensitive to changes in the transmissivity of layer 2, which represents the upper part of the aquifer system, and in the vertical leakance between layers 1 and 2, which represents flow between a constant-head upper boundary and the top of the aquifer system. If transmissivity or vertical leakance is changed throughout the model, however, the effects are accentuated in the lower layers because much of the water flowing in these layers passes through and is affected by the overlying layers. The model is relatively insensitive to changes in the coefficient of storage because only a small part of the total flow is derived from storage.

INTRODUCTION

The coastal lowlands aquifer system is being studied as part of the U.S. Geological Survey's Gulf Coast Regional Aquifer-System Analysis (GC RASA) program (Grubb, 1984). The GC RASA program includes a series of investigations that present a regional overview of the hydrogeologic and geochemical conditions in the principal aquifers of the Gulf Coastal Plain. A major objective of this study is to describe ground-water flow within the coastal lowlands aquifer system. A digital flow model (McDonald and Harbaugh, 1988) was the principal tool used to investigate flow in the aquifer system.

Calibration and sensitivity analysis was an integral part of the development of a ground-water flow model of the coastal lowlands aquifer system. Calibration is the process by which model input parameters are adjusted so that model output matches observed conditions to a desired degree of accuracy. Sensitivity analysis involves changing the values of individual model inputs to observe the effects of the changes on model output. If a small change in input results in a large change in output, the model is said to be sensitive to that property. Conversely, if a large change in input produces only a small change in output, the model is insensitive to that property. Sensitivity analysis is useful in evaluating the confidence to be placed in the accuracy of the values of aquifer properties adjusted during the calibration process.

This report describes the calibration and sensitivity analysis of the ground-water flow model used to quantify flow in the coastal lowlands aquifer system. A steady-state version of the model for 1980 was used for calibration of transmissivities of permeable zones and vertical leakances between permeable zones within the aquifer system. A transient version of the model was used to calibrate the storage coefficients of the sands, gravels, silts, and clays that make-up the aquifer system. Steady-state and transient calibrations were based on minimizing the mean error, RMSE, and standard deviation of the residuals between model-simulated and measured water levels. Initial calibration by trial-and-error was followed by use of an optimization program to check and improve calibration.

Sensitivity of the model was determined by comparing water-level residuals produced by the calibrated model with residuals produced by the model with one aquifer property changed. Properties tested for sensitivity were aquifer transmissivity, vertical leakance, and storage coefficient. Results were evaluated on the basis of mean error, RMSE, and standard deviation.

Calibration and sensitivity results are presented as statistical comparisons of model-simulated and measured water levels, plots showing the areal distribution of input parameter values, and hydrographs.

HYDROGEOLOGY

The coastal lowlands aquifer system consists of sediments of Miocene age and younger which occur above the Jackson and Vicksburg Groups of late Eocene and Oligocene age in parts of Louisiana, Mississippi, Alabama, and Florida and adjoining offshore waters (fig. 1). Sand occurring near the top of the underlying Vicksburg Group in some areas was included in the aquifer system.

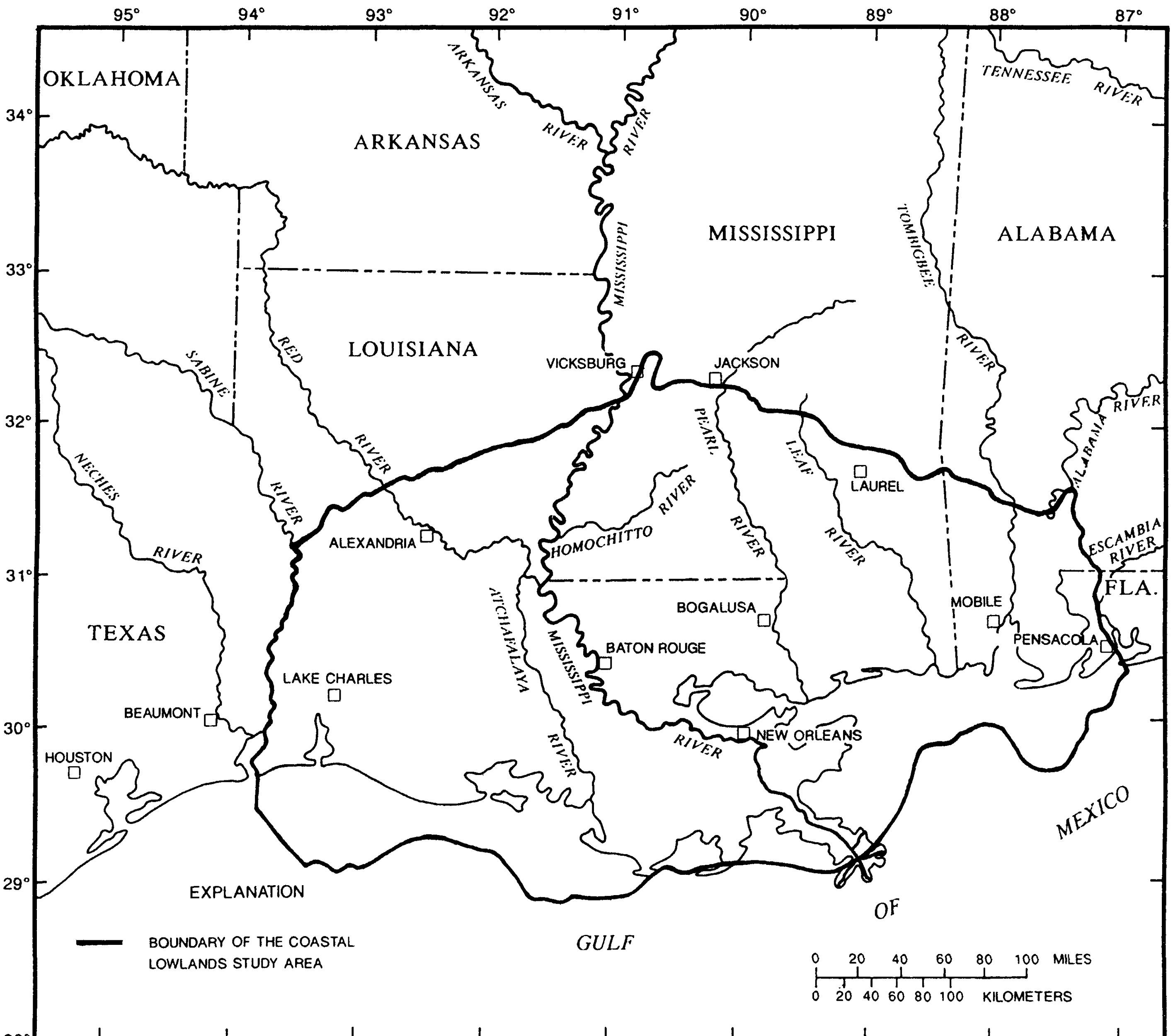

Figure 1.--Location of the study area and major streams.

The aquifer system is characterized by off-lapping, coastward thickening wedges of fluvial, deltaic, and marine sediments of Miocene age and younger (Martin and Whiteman, 1989, p. 3). Deltaic processes have been dominant during deposition of these sediments. Advancing deltaic fronts pushed the shoreline and its associated beach, dune, and lagoonal deposits seaward while blankets of fluvial sediments were deposited on the coastal plain inland, and extensive marine deposits formed offshore. The coastal lowlands aquifer system is underlain by clay, silt, and lime beds of the Jackson and Vicksburg Groups. The Jackson and Vicksburg Groups act as a lower confining unit below the coastal lowlands aquifer system. Flow across the confining unit has a negligible effect on flow in the coastal lowlands aquifer system (Williamson, 1987).

The coastal lowlands aquifer system consists primarily of alternating beds of sand and gravel, silt, and clay. The most extensive sand beds cannot be traced with certainty for more than 30 to 50 mi. Dip of individual sand beds is southerly, ranging from about 10 to 50 ft/mi in the outcrop area and shallow subsurface in the northern part of the study area. Dip increases to the south and with increasing depth to over 100 ft/mi at depths of more than 3,000 ft. Individual clay and silt confining beds are not areally extensive. Gravel is common in the northern part of the study area but becomes finer and less common southward. Grain size of the sand also decreases southward, grading to sandy clay or silt and finally to clay. Martin and Whiteman (1989) describe in detail the hydrogeologic setting and regional flow in the coastal lowlands aquifer system.

Regional ground-water flow in the coastal lowlands aquifer system is primarily from north to south. Figure 2 is a generalized schematic diagram showing predevelopment regional flow. Average annual rainfall ranges from about 48 in. on the northern and western parts of the study area to more than 65 in. on coastal areas of Louisiana, Mississippi, and Alabama. Water entering the aquifer system in upland terrace areas that is not discharged locally to streams or by evapotranspiration moves downward to the regional flow system and then toward discharge areas at lower altitudes in the coastal plain and along major stream valleys. In places, pumping of ground water has altered the natural predevelopment gradients and has initiated recharge in much of the natural discharge area.

Saltwater occurs downdip in the marine and deltaic parts of the aquifer system. Freshwater moving downdip from recharge areas tends to push the saltwater ahead of it, but the downdip movement of saltwater is blocked where the sand beds in the aquifer system pinch-out or are displaced by faulting. Water can move out of the downdip part of the sand beds only by upward leakage through overlying sediments (Martin and Whiteman, 1989, p. 4).

DESIGN OF THE GROUND-WATER FLOW MODEL

The coastal lowlands aquifer system was divided into five permeable zones, A-E as shown in figure 2, in order to use a digital ground-water flow model to investigate the lateral and vertical distribution of flow (Weiss and Williamson, 1985). The massive coastward-thickening wedge of sediments was first divided into zones in intensively-pumped areas (Baton Rouge, Louisiana,

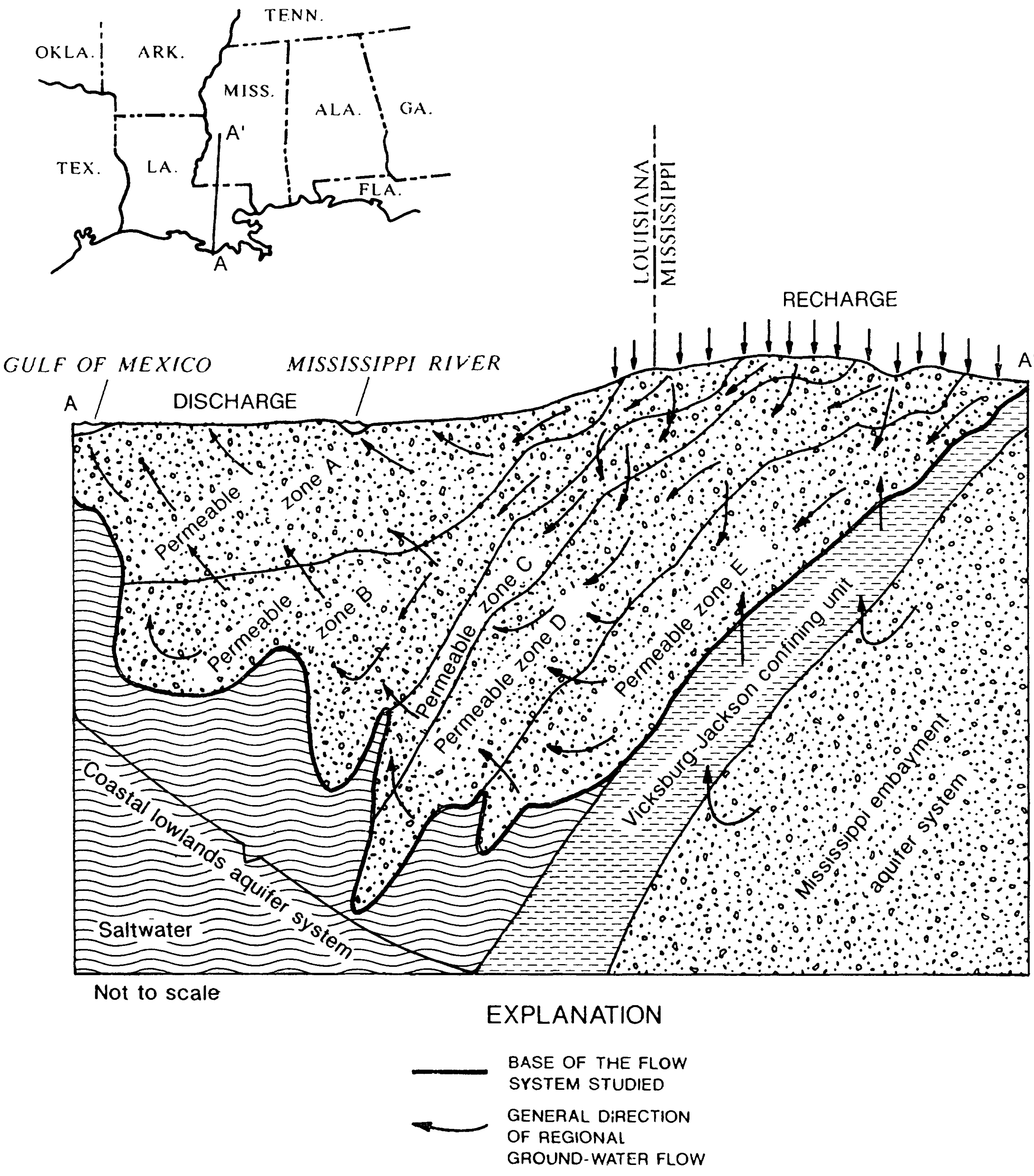

Figure 2.--Generalized hydrogeologic section showing zonation of the aquifer system and direction of regional ground-water flow.

to the east and Houston, Texas, to the west) on the basis of water-level and pumpage information. Emphasis was placed on making the zones thinnest at the top and progressively thicker downward for best resolution in the part of the flow system where most of the freshwater flow occurs. The zones were then extended along the strike of the beds by maintaining the thickness of each zone as the same percentage of the total aquifer system as at the pumping centers. The zones pinch-out in the updip direction simulating the outcrop pattern, where progressively older bands of sediment are exposed (fig. 2). The lower zones pinch-out in the downdip direction reflecting the pinching out of individual sand beds and the rise through the section of saltwater. Each zone pinches out downdip along the line at which all sand beds in the zone contain water with a dissolved-solids concentration greater than 10,000 mg/L (milligrams per liter).

The five permeable zones have been designated, from youngest to oldest, zone A (Holocene-upper Pleistocene deposits), zone B (lower Pleistocene-upper Pliocene deposits), zone C (lower Pliocene-upper Miocene deposits), zone D (middle Miocene deposits), and zone E (lower Miocene-upper Oligocene deposits) (Grubb, 1987, table 1). These permeable zones are defined as hydrogeologic units. The series designations are given as a general indication of relative age, and the zones may contain sediments younger or older than the series age designation.

A finite-difference grid consisting of 78 rows by 70 columns of uniform blocks 5 mi on a side (fig. 3) was constructed for use with the digital flow model. The grid covers an area of 390 mi by 350 mi, or 136,500 mi². The model grid covers an area considerably larger than the study area, which consists of 58,400 mi² inland and 10,100 mi² offshore. Six layers comprise the model (fig. 4). Layers 2-6 represent permeable zones A-E, respectively. Layer 1, the uppermost layer, represents a constant-head upper boundary.

The water level specified for each node of the constant-head upper boundary (layer 1) is the altitude of the water table at the center of that block. Layer 1 acts as a source or sink for all water entering or leaving the simulated flow system through land surface other than that removed by pumpage (fig. 4). All of the lateral model boundaries except the western boundary and a small part of the northern boundary are treated as no-flow boundaries. Along the eastern and most of the northern sides, the no-flow boundaries for layers 2-6 are at the updip limit of the outcrop-subcrop area for each layer. A relatively small amount of water flowing into or out of the northern edge of the model in zone A, in the Mississippi River valley, is accounted for by a specified-flux boundary in model layer 2 (fig. 3). Flows across this boundary, derived from a model of the Mississippi River alluvial aquifer (D.J. Ackerman, U.S. Geological Survey, written commun., 1988), were adjusted at the start of each stress period. The southern boundary for each of these layers is at the line along which water in all sands represented by that layer exceeds a dissolved-solids concentration of 10,000 mg/L.

Analysis of water-level data indicated that flow occurs across the western boundary under natural conditions and in response to pumpage from zone A (model layer 2) in the Lake Charles area in southwestern Louisiana. In model layers 2-6, the western model boundary was formed using general-head-boundary nodes (fig. 3). General-head-boundary nodes permit flow across the boundary

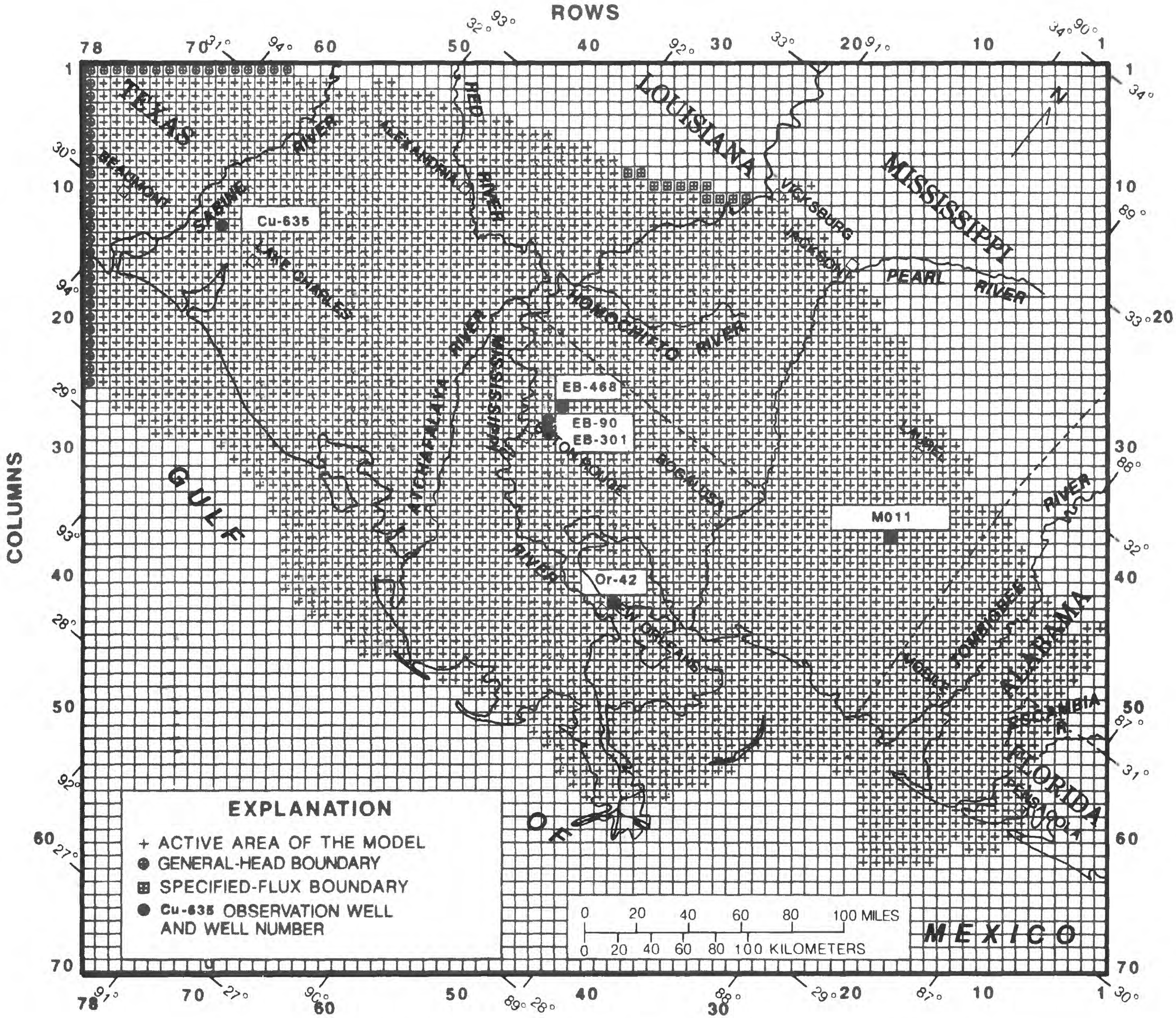

Figure 3.--Finite-difference grid with the active area of the model.

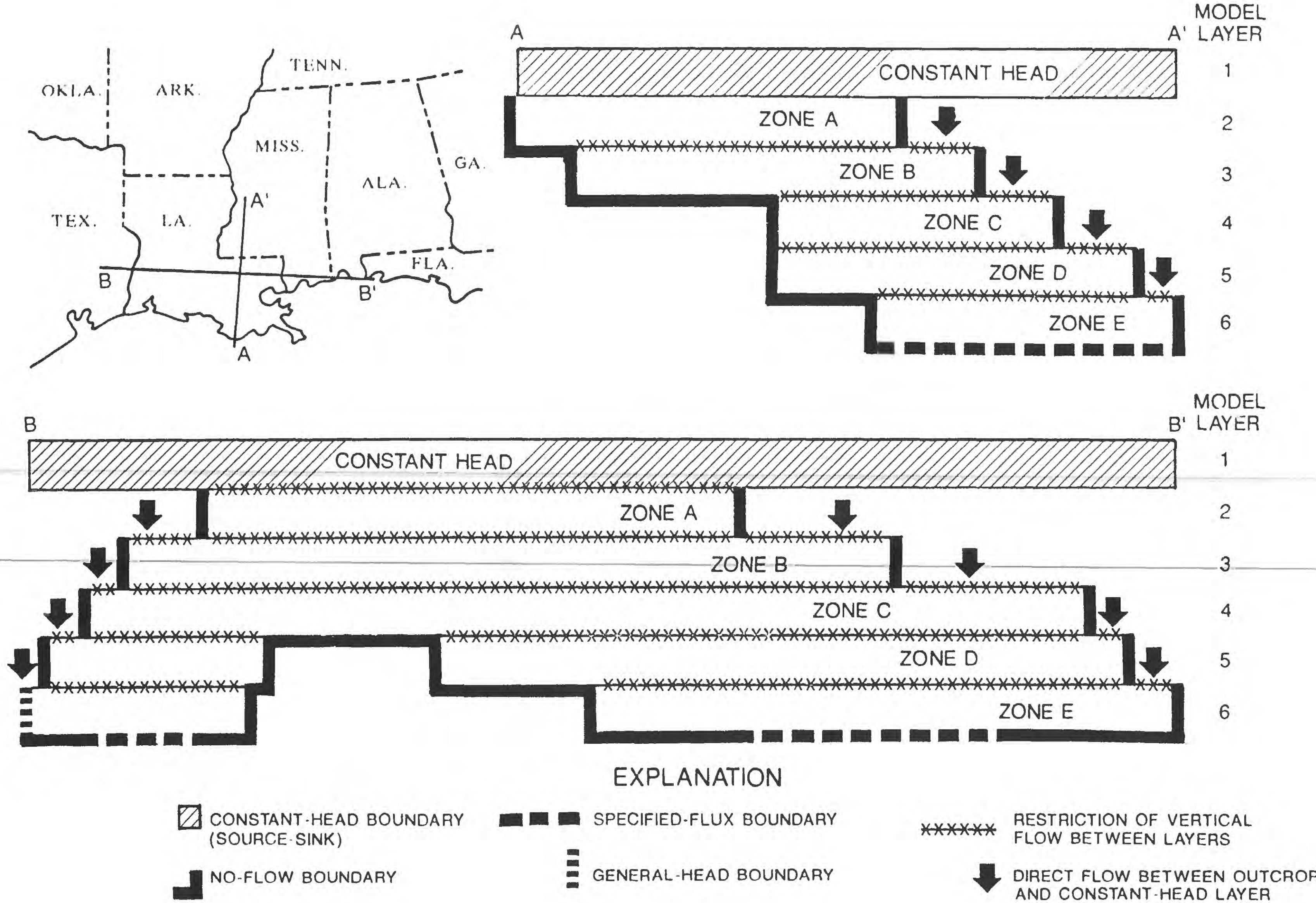

Figure 4.--North-south and east-west schematic sections through the coastal lowlands aquifer system showing representation of the permeable zones as model layers and the simulated boundary conditions.

based on the gradient between model-simulated water levels at the boundary and specified water levels outside of the boundary (McDonald and Harbaugh, 1988, p. 11-1 through 11-27). The specified water levels used in this study were derived from a model of the Texas coastal lowlands aquifer system (P.D. Ryder, U.S. Geological Survey, written commun., 1988) and adjusted at each stress period.

The lower boundary of the model in the northern part of the study area, where freshwater is present throughout the coastal lowlands aquifer system, is at the top of the thick clays of the Vicksburg and Jackson Groups. Water leaking vertically between the underlying Eocene sediments and the coastal lowlands aquifer system is accounted for by a specified-flux boundary in layer 6. The specified fluxes used in this study, adjusted at each stress period, were derived from a model of the underlying Mississippi Embayment aquifer system (J.K. Arthur, U.S. Geological Survey, written commun., 1988). The rate of leakage is small in comparison to the volume of flow in the coastal lowlands aquifer system. To the south, where the lower part of the aquifer system contains saltwater, a no-flow boundary is placed at the bottom of the lowest sand bed containing water with no more than 10,000 mg/L dissolved solids.

Regionally extensive confining units do not occur within the coastal lowlands aquifer system as defined for this study, but large water-level differences do occur vertically within the flow system. In order to simulate the vertical restriction to flow by interbedded clays within the aquifer system, clays in vertically adjacent zones were treated as being equivalent to a single clay bed between the zones (Bredehoeft and Pinder, 1970, p. 884). Leakance between the zones was computed by dividing an average value of vertical hydraulic conductivity for the clays by the total thickness of clay between midpoints of the zones. Computing leakance in this way provides an areal variability in leakance that corresponds to areal variations in the restriction to vertical flow that occur within the aquifer system.

MODEL CALIBRATION

Model calibration consists of adjusting model input parameters, initial conditions, and boundary conditions so that the model simulates the aquifer system to a desired degree of accuracy. The calibration process involves matching water levels, water-level changes, hydraulic gradients, flow rates, volumetric budgets, or a combination of these. The model simulating the coastal lowlands aquifer system was calibrated to 1980 steady-state conditions by matching model-simulated water levels to measured water levels. Although not at true steady state throughout the area in 1980, the rate at which water levels were changing was not considered to be significant in relation to the scale of the aquifer system (Martin and Whiteman, 1989, p. 14). The model was calibrated for transient conditions by matching water levels for the period 1958-87. The mean error, RMSE, and standard deviation of the residuals between model-simulated and measured water levels were used as quantitative comparisons during calibration. The RMSE shows the variation of the residuals about measured water levels. Standard deviation shows the variation of the residuals about the mean of the residuals. The RMSE and standard deviation are defined by:

$$\text{root-mean-square error} = \sqrt{\frac{\sum_{i=1}^{N} (h_i^s - h_i^m)^2}{N}}\text{, and}$$

$$\text{standard deviation} = \sqrt{\frac{\sum_{i=1}^{N} \left[(h_i^s - h_i^m) - \bar{h}\right]^2}{N}},$$

where h^s is the model-simulated water level; h^m is the measured water level; $\bar{h}$ is the mean of the residuals; and N is the number of water-level pairs compared. Due to the relatively coarse lateral and vertical discretization of the aquifer system for modeling, a precise water-level match was not expected.

Flow rates and volumetric water budgets could not be measured with enough accuracy for quantitative comparison with model results because inherent errors in measurement of surface-water flow in the study area may be greater than total flow in the ground-water system. Model-computed flow rates, however, provided qualitative checks on results. The model cannot be considered to be adequately calibrated even though model-simulated and measured water levels may closely match if simulated flows are not reasonable.

Parameters adjusted by trial-and-error and optimization methods during calibration were the transmissivities of the regional zones, the vertical leakances between zones, and the storage coefficients of the zones. Boundary conditions and pumpage were assumed to be correct and were not changed after initial refinements described in Martin and Whiteman (1989). Transmissivity of each zone was calculated as the product of total sand thickness within the zone and an average lateral hydraulic conductivity of the sands and is presented in this report in units of foot squared per day. Vertical leakance between zones was calculated as the average vertical hydraulic conductivity of the clays within and between the zones divided by the total thickness of clay between midpoints of the zones and is given in units of inverse day (day^{-1}). The storage coefficient of each zone was calculated by summing the products of the total thickness of sand times an average specific storage for sand and the total thickness of clay times an average specific storage for clay. Storage coefficient is dimensionless.

Initial adjustments of transmissivity and vertical leakance were made over the full extent of each zone. After preliminary calibration, changes made to improve one part of the model would worsen calibration in other parts of the model. To further refine the calibration, each model layer was divided into areas based on hydrologic distinctions in the corresponding permeable zone, such as the outcrops and areas of intense pumpage.

Steady-State Model

Results of steady-state calibration are shown in terms of mean error, RMSE, and standard deviation of water-level residuals for each model layer and for the entire model in table 1. Values of RMSE and standard deviation are similar because the means of the residuals in all model layers are small. Model layer 2 (zone A), on average, is most accurately simulated in the model. Model layers 3-5 (zones B-D) show increasing values of RMSE, indicating that calibration of the model becomes progressively poorer downward. The RMSE of layer 6 (zone E) is somewhat lower than that of layer 5, but the mean error of layer 6 is higher than that of any other layer.

Table 1.--Results of statistical analysis of water-level residuals of the steady-state calibrated model for 1980 conditions

Model layer	Number of observations	Mean error (feet)	Root-mean-square error (feet)	Standard deviation (feet)
2	349	-0.24	14.75	14.75
3	73	6.59	33.22	32.56
4	164	5.69	40.11	39.70
5	278	.74	56.12	56.12
6	132	17.09	45.84	42.54
All	996	3.81	39.74	39.56

The largest differences between measured and model-simulated water levels occur in the outcrop areas of the lower zones and near major pumping centers (fig. 5). Ground-water gradients are steep over much of the outcrop areas of the lower zones because of relatively large topographic relief. Gradients are also steep near major pumping centers. The finite-difference blocks used in the model are large in relation to the distances over which large water-level changes occur, making it difficult or impossible to accurately simulate water levels in these areas.

Transient Model

The transient simulations use nine stress periods to simulate the period 1898-1987. The first three stress periods, each 20 years in length, simulate the calibration period 1898-1957. Six periods, each 5 years in length, simulate the calibration period 1958-87. Water-level and pumpage data prior to 1958 were too sparse for quantitative use in calibration. Available water-level and pumpage data were used qualitatively for the first three stress periods to adjust pumpage to cause simulated water levels at the end of each period to match available measured levels as closely as possible. This allowed the fourth stress period to begin with transient conditions similar to those present in the aquifer system.

Values of transmissivity and vertical leakance from the steady-state calibration were used as initial values in transient calibration and were not significantly changed as a result of the trial-and-error transient calibration process. Initial values of storage coefficients for the transient simulation were determined using values of specific storage of 1.0×10^{-6} for sand (Lohman, 1972, p. 8) and 4.0×10^{-6} for clay (Ireland and others, 1984, p. 148-149). Storage coefficients were adjusted uniformly throughout the model to achieve the best match between model-simulated and measured water levels.

Hydrographs comparing model-simulated and measured water levels for the period 1958-87 were used throughout transient calibration. In addition to wells with long-term hydrographs, many wells were measured once to a few times during the calibration period. Water-level measurements made near the ends of stress periods 3-9 (1957, 1962, 1967, 1972, 1977, 1982, and 1987) were compared with model-simulated water levels for the ends of the stress periods. Mean error and RMSE of the water-level residuals calculated after each model run during the trial-and-error calibration process were used to guide changes in input parameters for the next model run. Water-level measurements were not available for 1987 for the entire model area at the time of calibration of the transient model, so statistics for stress period 9 (1983-87) were not used in the calibration process.

The model is relatively insensitive to changes in storage coefficient, as discussed later, so calibration was less useful in refining the values of storage coefficient than it was for transmissivity and vertical leakance. Conversely, a broad range of uncertainty in the values of storage coefficient is acceptable because of the insensitivity. Final values of storage coefficients used throughout the aquifer system were one-half of the initial values. Although sensitivity analysis showed that mean error and RMSE could be reduced slightly by lowering the storage-coefficient values to one-tenth of the values actually used, this was not done because the resulting values would be unreasonably low.

Statistical Optimization Program

Following trial-and-error calibration, a statistical optimization program (Durbin, 1983) was applied to the model in an attempt to improve the calibration. This program uses a modified Gauss optimization technique (Wilde and Beightler, 1967, p. 299) based on minimizing an objective function proportional to the RMSE. The program executes the model many times, changing the value of a single parameter for each run. Parameter changes may be made for the entire model, by layer or by areas within layers. After each run, a comparison of water-level changes versus an initial base run is made and the tested parameter is then returned to its former value. Testing of the parameters is continued until each parameter to be optimized has been tested once. The program solves simultaneously for new values of all the parameters to be optimized based on the previous tests. The new parameter values should improve the water-level match. The new values for all of the tested parameters are then used to make a model run that forms the base run for another round of parameter changes. This iterative process is continued until change of the RMSE of the entire model from one iteration to the next is less than a specified level (closure criterion) or until a specified number of iterations is exceeded. The closure criterion allows the optimization program to stop if little improvement in model results occurred as a result of the latest iteration. Specifying a maximum number of iterations prevents the program from running indefinitely if the closure criterion cannot be met.

A total of 27 parameters were initially selected for optimization. These included transmissivity of each layer and vertical leakance between each pair of model layers, with several layers divided into subareas as in the trial-and-error calibration. Storage coefficients were optimized for the model as a whole and not by layer or area. Because execution time of the program is directly proportional to the number of parameters being optimized, parameters were eliminated if no significant changes of the parameters occurred during the first few program iterations.

The optimization program made relatively small changes from trial-and-error calibration results. Figure 6 shows the results from an optimization test in which transmissivities and vertical leakances were optimized by layer and the storage coefficient was optimized for the entire model from trial-and-error calibrated values. Transmissivity values for all layers except 4 and 5 and vertical leakance between all layers except 1 and 2 were slightly higher than for the trial-and-error calibration. The largest changes were for transmissivity of layer 2 (1.1 times the initial value) and vertical leakance between layers 4 and 5 (slightly less than 1.1 times the initial value). Transmissivity for layers 4 and 5 each decreased slightly and storage coefficient increased slightly. Improvement in the RMSE of the entire model was about 0.2 ft, achieved after 15 iterations. The optimization program did not significantly improve the trial-and-error calibration in terms of RMSE, and the tested model input parameters were not significantly changed.

Results of Calibration

The best optimization program results were used in the final version of the model. Values of mean error, RMSE, and standard deviation for each model layer and for the entire model for stress periods 3-8 are shown in table 2. Values of standard deviation generally show the same relation to RMSE as in the steady-state calibration.

As with the steady-state version of the model, layer 2 (zone A) shows the best match and layer 5 (zone D) shows the poorest match of simulated and measured water levels. Significant differences in mean error and RMSE occur for the same model layer in different stress periods. This may be primarily due to the varying number and locations of measured water levels available for comparison.

Negative mean-error values for all layers in stress period 3 indicate that model-simulated water levels generally are lower than measured water levels. One explanation for this might be that simulated pumpage in stress period 3 may have been greater than the actual pumpage. Positive mean-error values for stress periods 4-8 indicate that model-simulated water levels generally are higher than measured water levels.

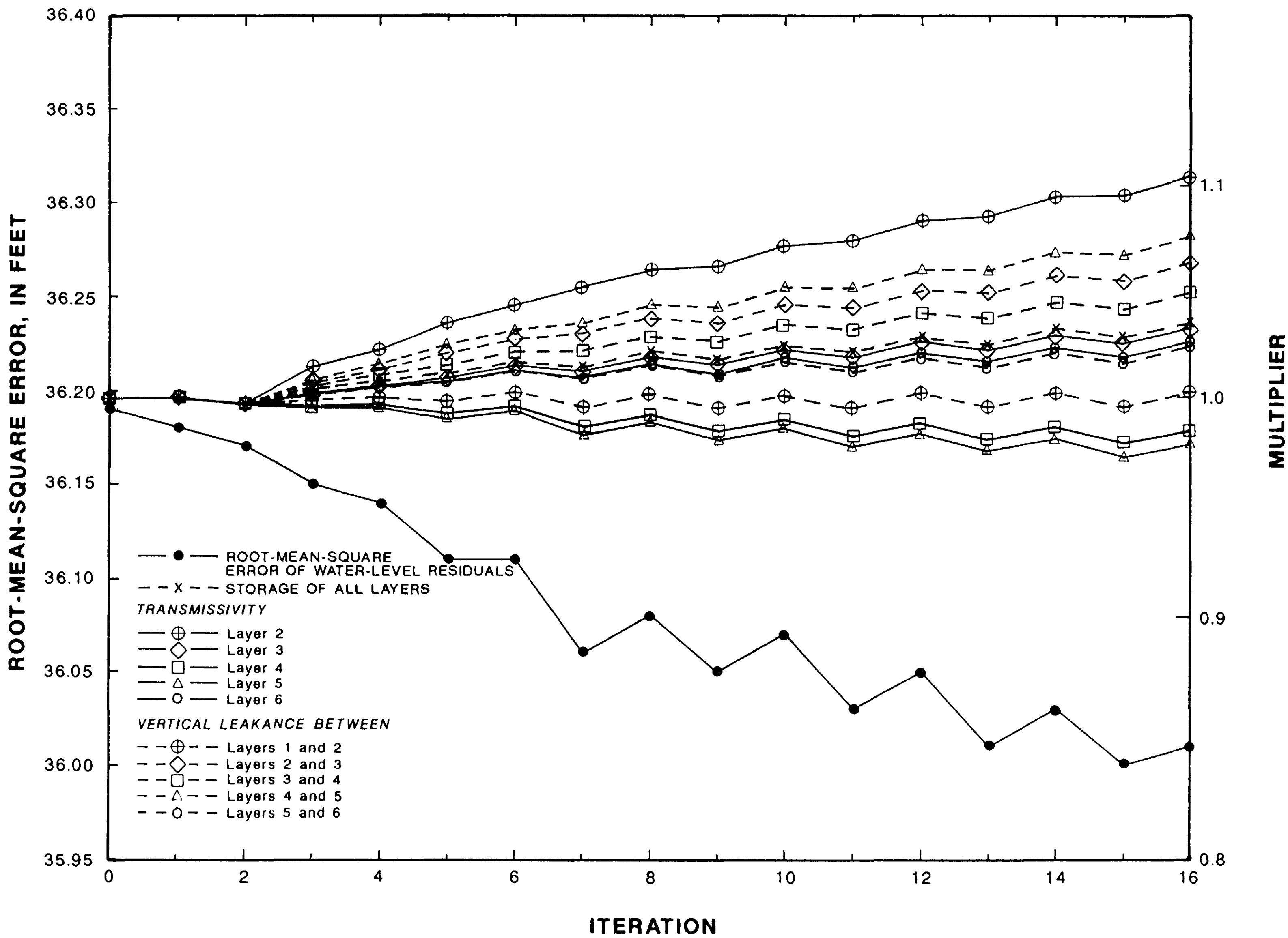

Figure 6.--Results from the optimization program showing changes in parameter values and resulting changes in root-mean-square error of water-level residuals.

Table 2.--Results of statistical analysis of water-level residuals of the transient calibrated model for the period 1957-82

Model layer	Number of observations	Mean error (feet)	Root-mean-square error (feet)	Standard deviation (feet)
		Stress period 3, 1957		
2	301	-0.82	18.27	18.25
3	55	-17.22	33.17	28.36
4	62	-18.00	32.42	26.96
5	70	-.23	44.57	44.57
6	20	-6.00	38.05	37.57
All	508	-4.81	27.86	27.44
		Stress period 4, 1962		
2	344	3.18	18.35	18.08
3	96	7.06	30.52	29.69
4	167	-3.49	32.70	32.51
5	180	16.92	46.54	43.35
6	74	29.45	57.30	49.16
All	861	7.45	34.36	33.54
		Stress period 5, 1967		
2	283	3.87	15.25	14.75
3	121	6.94	27.86	26.98
4	252	.39	34.52	34.51
5	222	12.45	48.08	46.44
6	116	21.09	48.26	43.41
All	994	7.29	35.37	34.61
		Stress period 6, 1972		
2	348	-1.36	19.57	19.52
3	151	3.56	31.31	31.10
4	301	6.80	38.06	37.45
5	365	18.01	53.82	50.71
6	150	18.16	46.86	43.20
All	1,315	8.68	40.00	39.04
		Stress period 7, 1977		
2	348	2.24	17.70	17.56
3	43	4.97	25.38	24.89
4	71	3.65	39.55	39.38
5	64	9.16	37.15	36.00
6	29	27.30	54.47	47.13
All	555	4.74	27.58	27.17

Table 2.--Results of statistical analysis of water-level residuals of the transient calibrated model for the period 1957-82--Continued

Model layer	Number of observations	Mean error (feet)	Root-mean-square error (feet)	Standard deviation (feet)
		Stress period 8, 1982		
2	349	-0.18	14.75	14.75
3	73	7.09	33.40	32.64
4	164	6.74	40.31	39.74
5	278	2.64	55.84	55.77
6	132	19.70	46.82	42.48
All	996	4.92	39.83	39.52
		Overall model, stress periods 3-8, 1957-82		
2	1,973	1.11	17.45	17.30
3	539	3.41	30.49	29.45
4	1,017	1.78	36.54	35.93
5	1,179	11.61	50.90	49.10
6	521	20.39	48.92	43.99
All	5,229	5.77	35.96	34.50

A histogram of the differences between model-simulated and measured water levels for stress periods 3-8 (fig. 7) shows that layer 2 has the best fit of simulated to measured water-level altitudes with 1,615 of 1,973 simulated water levels within 20 ft of the measured water levels. The poorest fit is for layer 5 with 460 of 1,179 simulated water levels differing by more than 40 ft from the measured water levels.

Hydrographs of measured and simulated water levels (figs. 8 and 9) show comparisons between measured and simulated water levels through time at discrete points within the aquifer system. The simulated water levels used in these figures were interpolated to the locations of the measured water levels by distance-weighted averaging of water levels computed at the centers of the grid blocks encompassing the measured level. The model-simulated water levels, which represent the average water level in several grid blocks, do not closely match the measured water levels, but general water-level trends are reproduced. Model calibration, illustrated by figures 8 and 9 and table 2, is satisfactory given the limitations of the model and of the input and comparison data. Further calibration effort would not significantly improve water-level matches.

The areal distribution of calibrated values of transmissivity, vertical leakance, and storage coefficient are shown in figures 10-12. Calibrated transmissivity values (fig. 10) increase downdip in all zones as zone thickness and total sand thickness increase. Maximum transmissivity values generally occur some distance north of the downdip limit of the zones because the thickness of the zones decreases near their downdip limit. Transmissivity

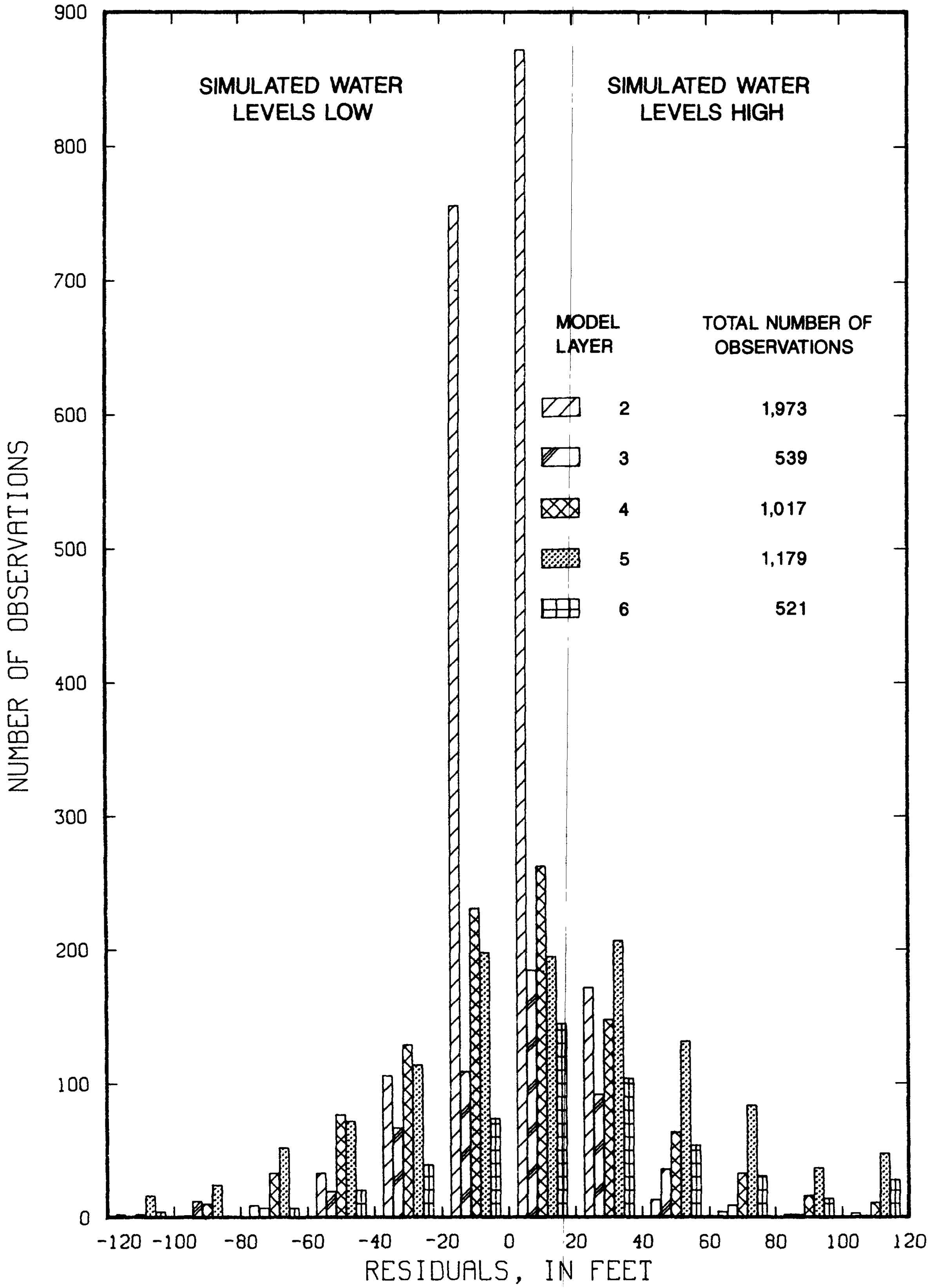

Figure 7.--The distribution of the residuals between model-simulated and measured water levels.

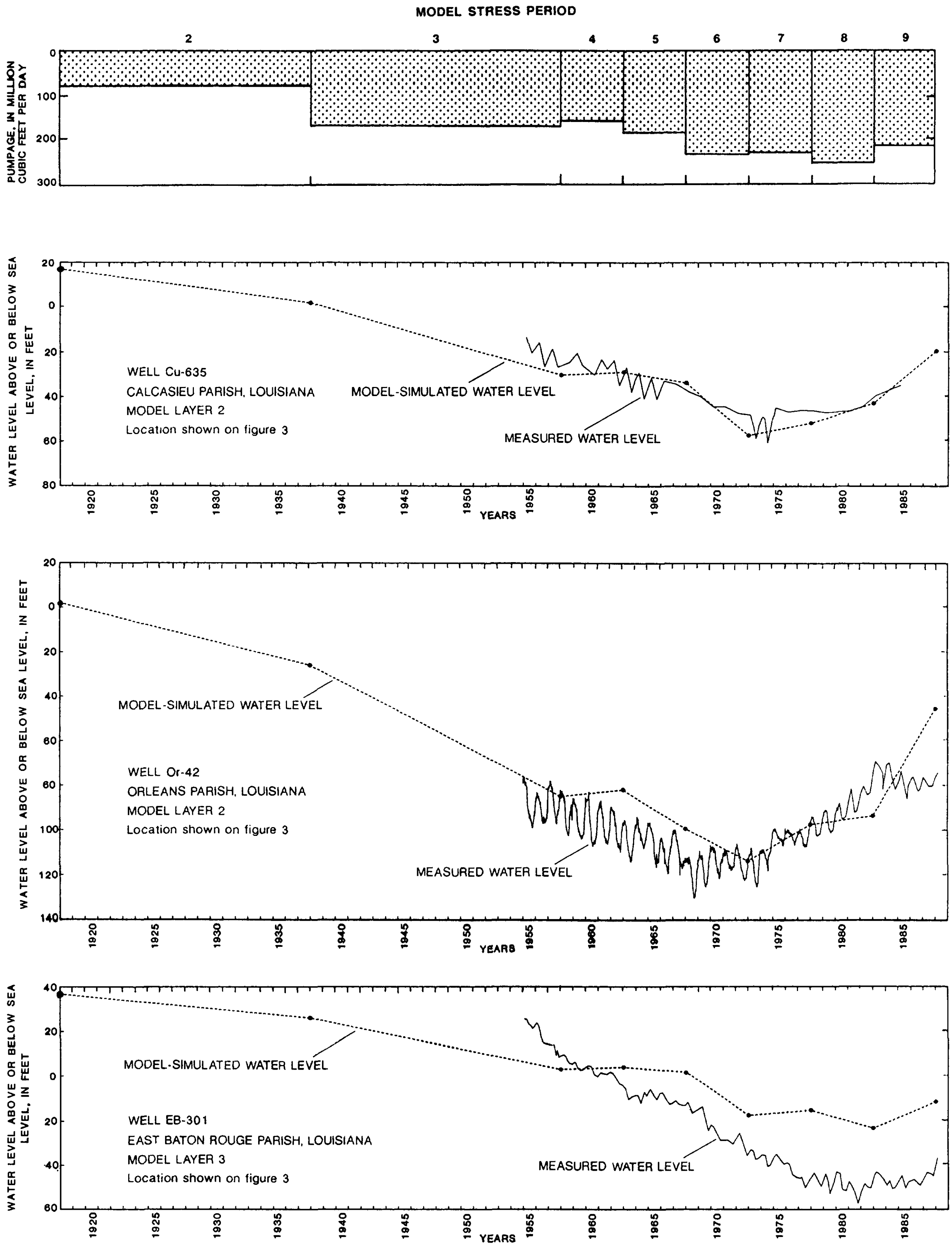

Figure 8.--Comparison of model-simulated and measured water levels for selected wells in model layers 2 and 3.

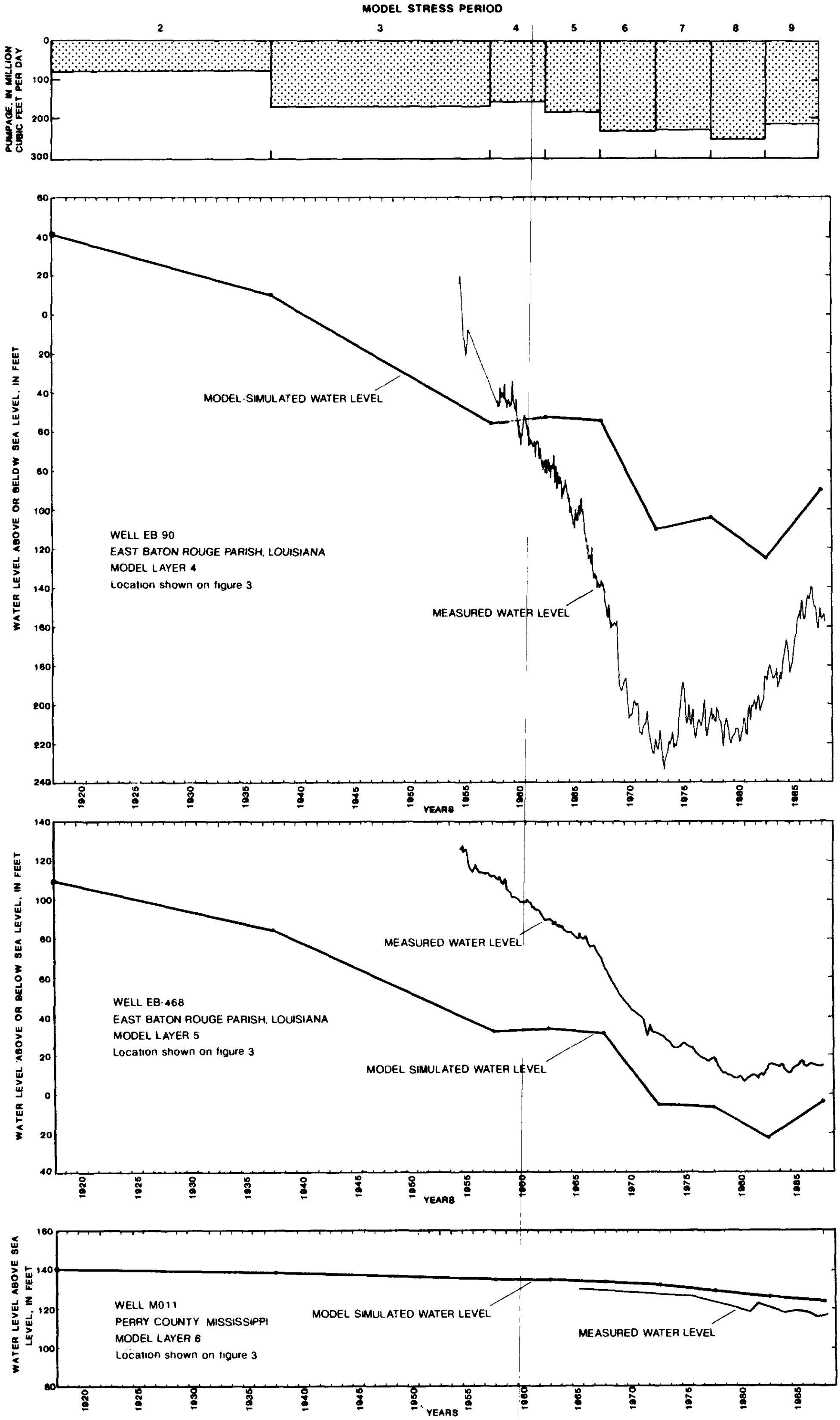

Figure 9.--Comparison of model-simulated and measured water levels for selected wells in model layers 4, 5, and 6.

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distributions generally follow the same pattern as total sand thickness within each zone (Martin and Whiteman, 1989, figs. 14-18). Overall, transmissivities are highest in the upper zones and decrease downward. Transmissivities for large areas in zones A and B are greater than 30,000 ft^2/d, whereas most values in zone E are less than 10,000 ft^2/d.

Calibrated vertical leakance (fig. 11) varies widely within the model area. Values range from less than 10^{-6} day^{-1} to more than 10^{-4} day^{-1}. Vertical leakance is highest in and near the outcrop areas of the permeable zones and generally decreases downdip as zone thickness and total clay thickness within the zones increase--generally the thicker the clay, the lower the vertical leakance. Variations of this pattern occur where values of clay vertical hydraulic conductivity are significantly higher or lower than the regional average for the zone. Overall, vertical leakance decreases in the lower permeable zones. Most values between the constant-head boundary and zone A are greater than 10^{-5} day^{-1}, whereas most values between the lower zones are less than 10^{-5} day^{-1}.

Calibrated storage coefficients range from 1.0×10^{-5} to 5.0×10^{-3} (fig. 12). Values increase downdip as sand and clay thicknesses increase in each zone. Because the specific storage of sand is lower than that of clay, the storage coefficient of a zone varies inversely with the sand percentage of the zone.

SENSITIVITY ANALYSIS

The approach to sensitivity analysis and the presentation of results in the following section were patterned, in part, on a report describing sensitivity analysis of the Southeastern Coastal Plain aquifer system (Pernik, 1987). Results of the sensitivity analyses from other flow-modeling studies covering parts of the study area were used as a guide in changing aquifer properties.

Method of Study

Sensitivity of the model to changes in transmissivity, vertical leakance, and storage coefficient was evaluated using steady-state and transient versions of the calibrated model. Transmissivity and vertical leakance were varied from 0.01 to 100 times calibrated values in the steady-state model. Storage coefficient was varied from 0.1 to 100 times the calibrated value using the transient model. Analyses of water levels, flow rates, and volumetric water budgets during calibration indicate that transmissivity of layer 2 (zone A) and vertical leakance between layers 1 and 2 (the constant-head upper boundary and zone A) were more sensitive than equivalent properties of other layers.

In sensitivity analysis, transmissivity was varied uniformly for all layers through the range of values and independently in layer 2. Similarly, vertical leakance was varied uniformly throughout the model and independently between layers 1 and 2. Storage coefficient was varied uniformly in all layers through the range of values. The mean error and RMSE of the residuals between model-simulated and measured water levels were used to quantify the sensitivity test results. As during calibration, the standard deviation of the water-level residuals was also calculated to show variation of water-level residuals about the mean.

Transmissivity and Vertical Leakance

Model sensitivity to changes in transmissivity and vertical leakance through the range of values for all model layers is shown in tables 3 and 4 and figure 13. Values of RMSE and standard deviation indicate that, in terms of water-level changes, the model is more sensitive to reductions of transmissivity and vertical leakance than to increases in these parameters (tables 3 and 4). Figure 13 shows the effects of varying both transmissivity and vertical leakage throughout the model. The RMSE ranges from less than 40 ft for the calibrated model (parameter multipliers equal to 1.0) to 65 ft when both parameters are increased by two orders of magnitude (factor of 100) to 4,750 ft when both parameters are decreased by two orders of magnitude (factor of 0.01). Within an order of magnitude of the calibrated values, the model is more sensitive to changes in transmissivity than to changes in vertical leakance.

The effects of varying the transmissivity of layer 2 and the vertical leakance between layers 1 and 2 are shown in figure 14. The pattern of changes in RMSE closely resembles the pattern formed when the parameters of all layers are varied (fig. 13) except that the range of changes in RMSE is less. The RMSE increases from the calibrated value of slightly less than 40 ft to 47.8 ft when both parameters are multiplied by 100 and to more than 1,050 ft when both parameters are multiplied by 0.01.

North-south and east-west water-level profiles of individual model layers and the water-level average of all layers showing the effects of varying calibrated values of transmissivity of all layers by a factor of 5 are shown in figures 15 to 18. Higher transmissivity values generally produce lower water-level gradients, with lower water levels in the recharge areas and higher water levels in the discharge areas. Conversely, lower transmissivity values produce higher water-level gradients, with higher water levels in the recharge areas and lower water levels in the discharge areas. This effect is shown most clearly in the north-south profiles (figs. 15 and 16). Varying transmissivity in all layers accentuates water-level changes in the deeper layers because much of the water flowing into or out of the deeper layers passes through and is affected by overlying layers.

In contrast to the sensitivity of the model to decreases in transmissivity shown by water-level changes, total flow circulating within the aquifer system is more affected by increases in transmissivity than by decreases. Increasing calibrated transmissivities of all layers by a factor of 5 increases the total flow circulating within the aquifer system under 1980 conditions by about 63 percent, from 380 to 621 Mft^3/d. Lowering transmissivities to one-fifth the calibrated values reduces total flow in the aquifer system by about 22 percent, from 380 to 297 Mft^3/d.

Table 3.--Results of statistical analysis of water-level residuals showing the sensitivity of the steady-state calibrated model to changes in the transmissivity of all model layers

[Multiplier is the factor by which the calibrated values of transmissivity were varied]

Multi-plier	Model layer	Number of observations	Mean error (feet)	Root-mean-square error (feet)	Standard deviation (feet)
	2	349	-186.69	926.75	909.05
	3	73	-201.77	470.60	428.13
0.01	4	164	-113.34	332.32	313.37
	5	278	-93.17	397.60	387.23
	6	132	-.31	159.60	160.21
	All	996	-124.92	619.70	607.29
	2	349	-28.31	128.58	125.61
	3	73	-39.78	100.10	92.49
	4	164	-33.74	116.32	111.67
.10	5	278	-35.98	155.45	151.50
	6	132	9.93	76.24	75.87
	All	996	-27.07	127.54	124.70
	2	349	-.24	14.75	14.75
	3	73	6.59	33.42	32.56
1	4	164	5.69	40.11	39.70
	5	278	.74	56.12	56.12
	6	132	17.09	45.84	42.54
	All	996	3.81	39.74	39.56
	2	349	24.21	36.09	26.80
	3	73	41.04	71.47	58.92
10	4	164	52.13	84.52	66.74
	5	278	40.73	81.59	70.82
	6	132	36.91	67.94	57.25
	All	996	36.29	66.83	56.15
	2	349	40.23	54.20	36.38
	3	73	43.09	100.74	91.70
100	4	164	59.80	108.84	91.23
	5	278	43.59	97.82	87.72
	6	132	15.05	73.38	72.09
	All	996	41.22	84.20	73.46

Table 4.--Results of statistical analysis of water-level residuals showing the sensitivity of the steady-state calibrated model to changes in vertical leakance between all model layers

[Multiplier is the factor by which the calibrated values of vertical leakance were varied]

Multiplier	Model layer	Number of observations	Mean error (feet)	Root-mean-square error (feet)	Standard deviation (feet)
	2	349	-367.79	432.25	227.42
	3	73	-209.21	258.14	152.29
0.01	4	164	-342.94	373.98	149.64
	5	278	-512.36	544.37	184.25
	6	132	-542.59	549.80	89.13
	All	996	-415.75	465.08	208.55
	2	349	-77.37	101.40	65.63
	3	73	-16.74	57.41	55.29
	4	164	-58.35	85.99	63.36
.10	5	278	-121.81	150.23	88.08
	6	132	-104.75	127.36	72.72
	All	996	-85.88	116.25	78.39
	2	349	-.24	14.75	14.75
	3	73	6.59	33.42	32.56
1	4	164	5.69	40.11	39.70
	5	278	.74	56.12	56.12
	6	132	17.09	45.84	42.54
	All	996	3.81	39.74	39.56
	2	349	22.24	32.27	23.42
	3	73	26.81	41.74	32.22
10	4	164	32.77	56.43	46.07
	5	278	40.28	64.84	50.90
	6	132	57.94	73.98	46.18
	All	996	34.07	53.98	41.89
	2	349	24.77	36.13	26.34
	3	73	27.99	44.81	35.20
100	4	164	35.71	63.65	52.83
	5	278	45.04	70.65	57.50
	6	132	62.67	79.11	48.46
	All	996	37.48	59.10	45.69

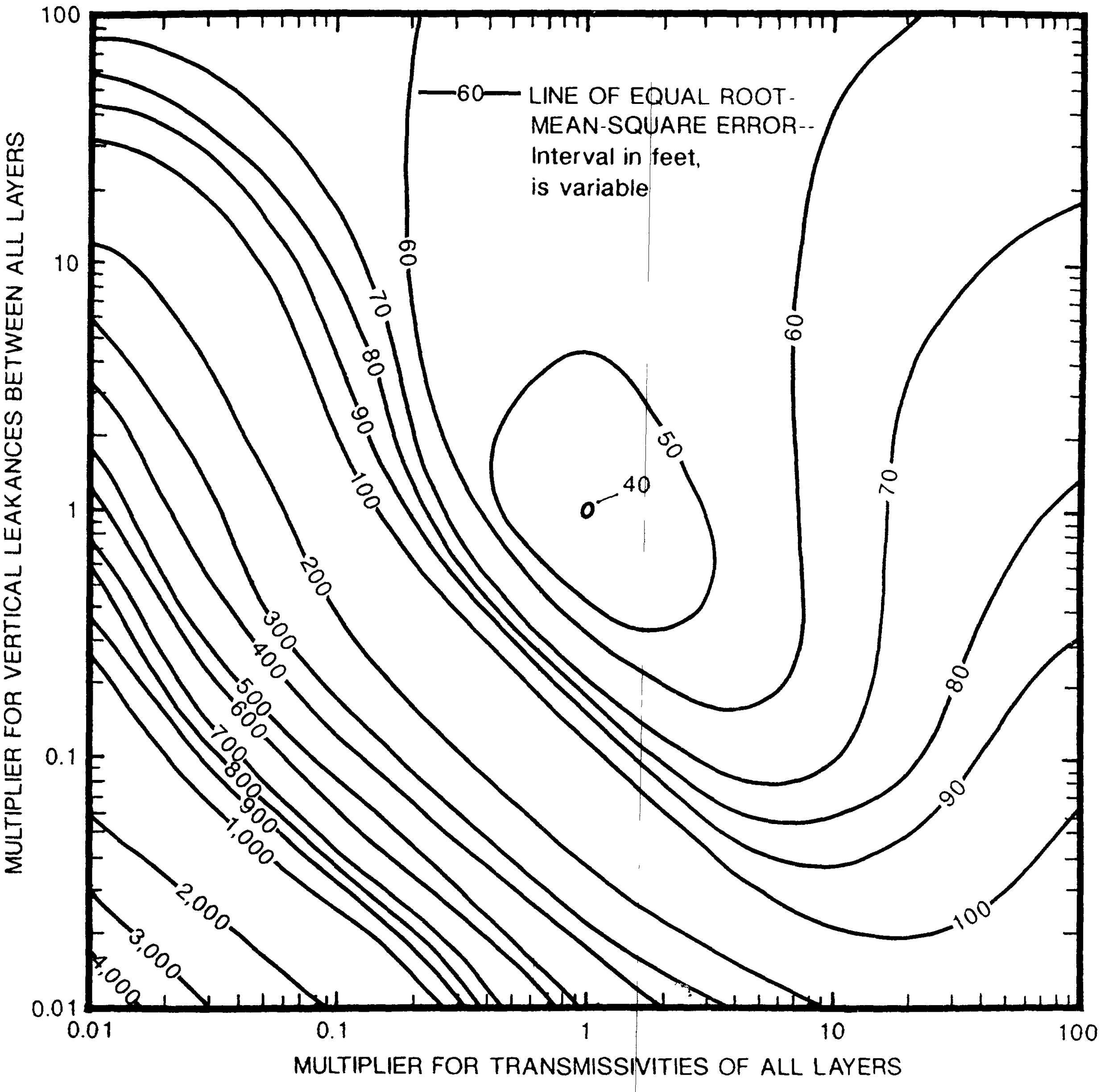

Figure 13.--Root-mean-square errors of water-level residuals resulting from changes in transmissivity and vertical leakance of all model layers.

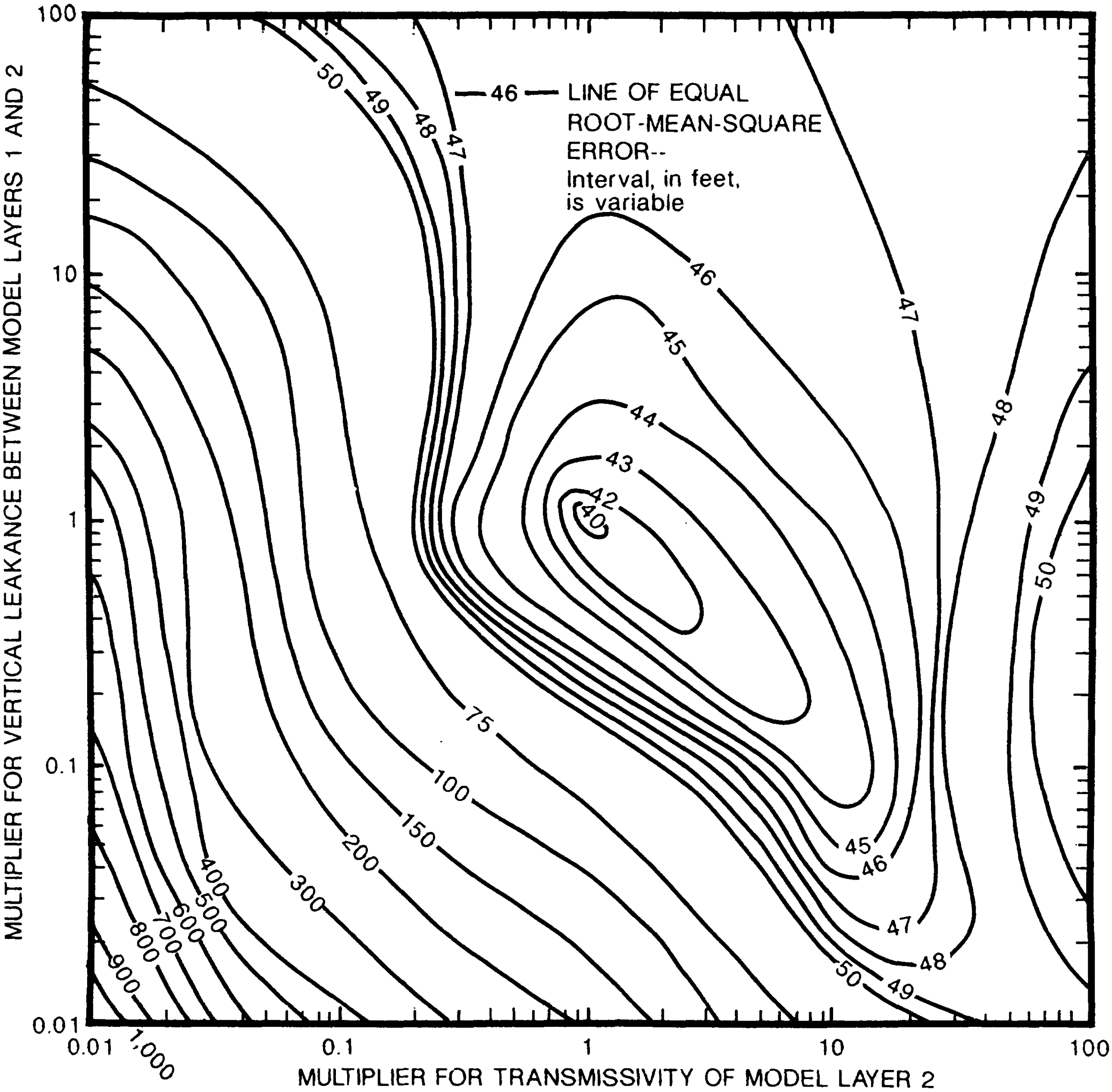

Figure 14.--Root-mean-square errors of water-level residuals resulting from changes in the transmissivity of model layer 2 and the vertical leakance between model layers 1 and 2.

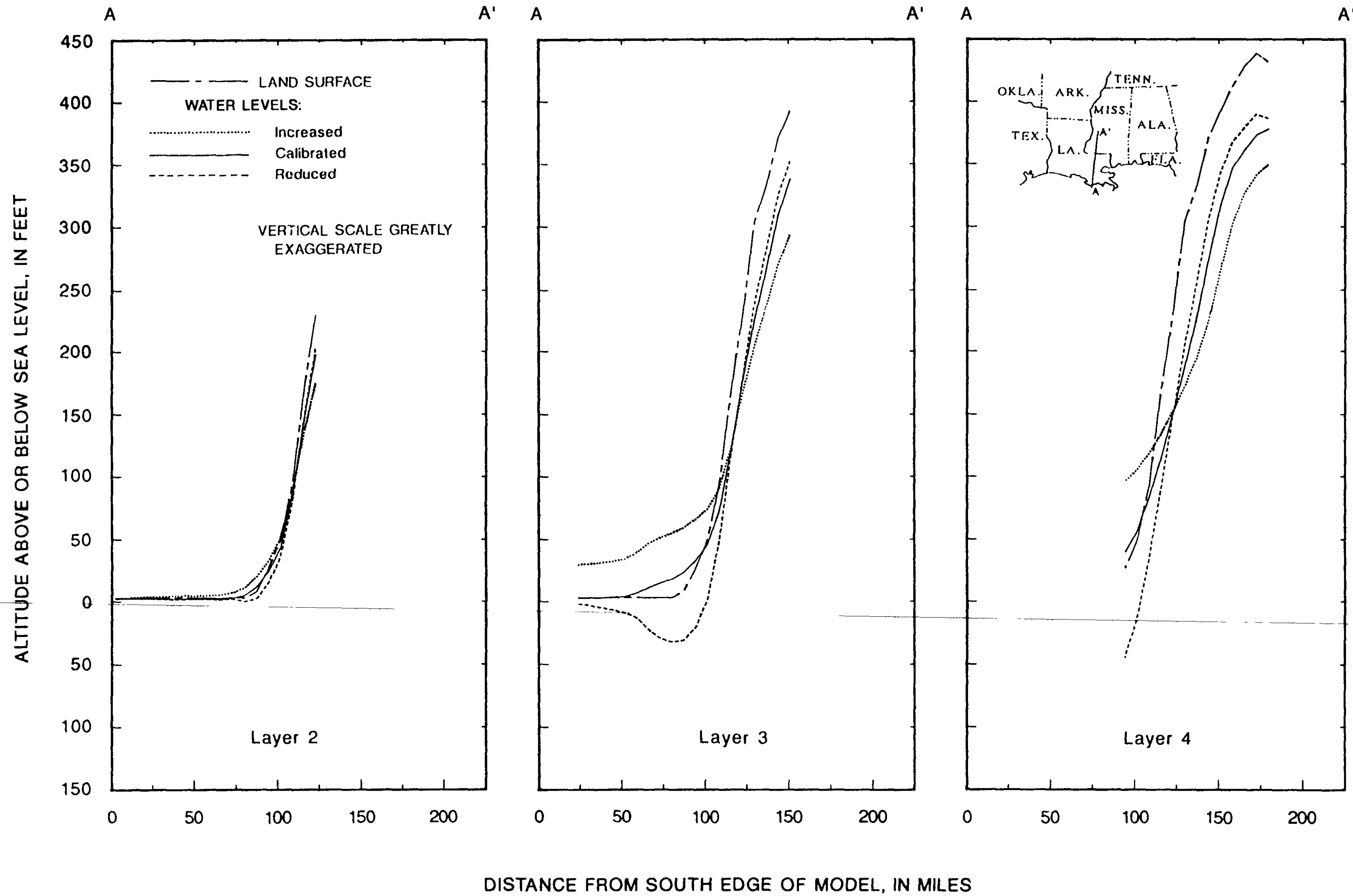

Figure 15.--North-south water-level profiles showing the effect in model layers 2, 3, and 4 of changing the transmissivity of all model layers by factors of 0.2 and 5.

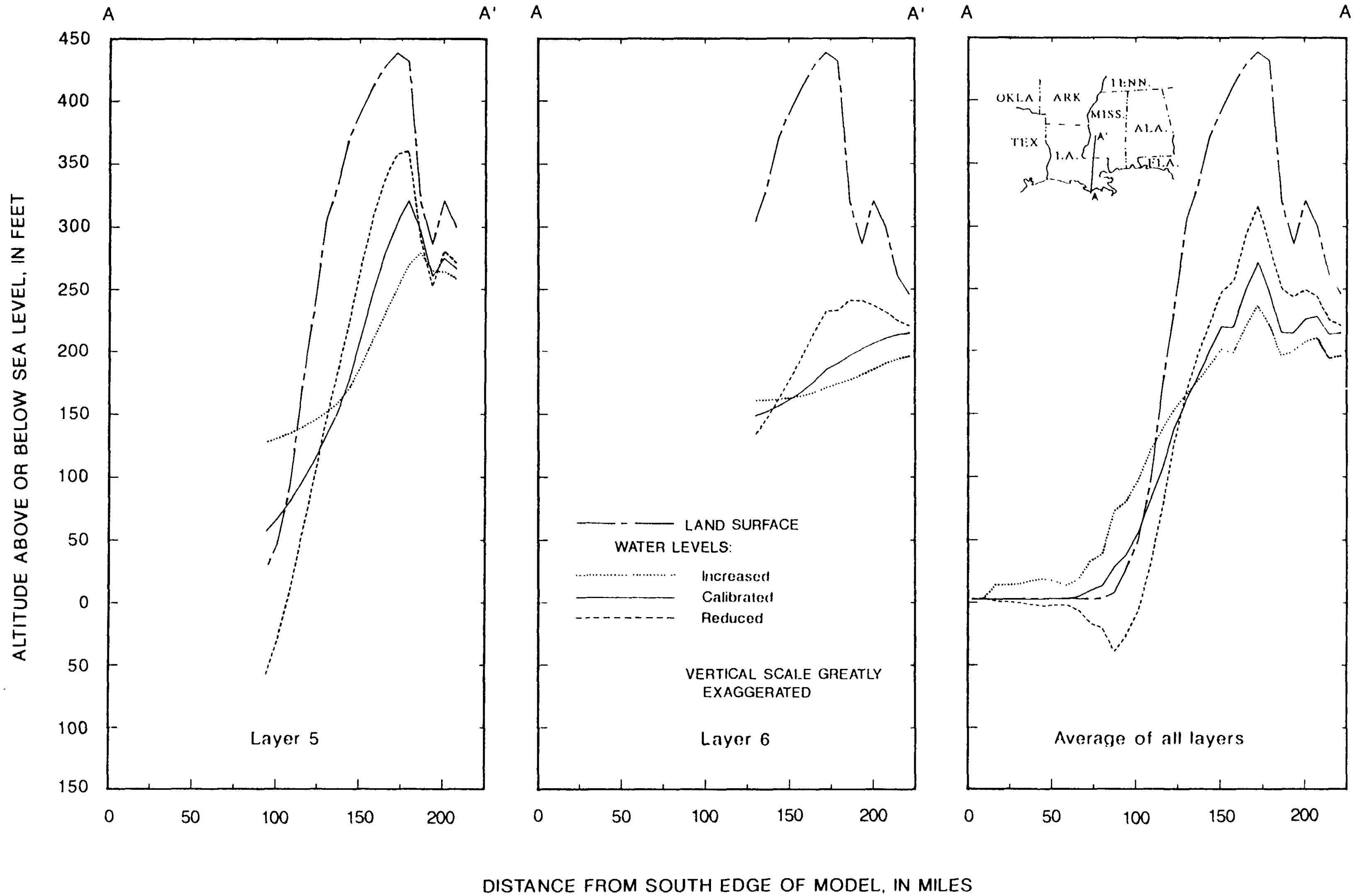

Figure 16.--North-south water-level profiles showing the effect in model layers 5 and 6 and in the average of all layers of changing the transmissivity of all model layers by factors of 0.2 and 5.

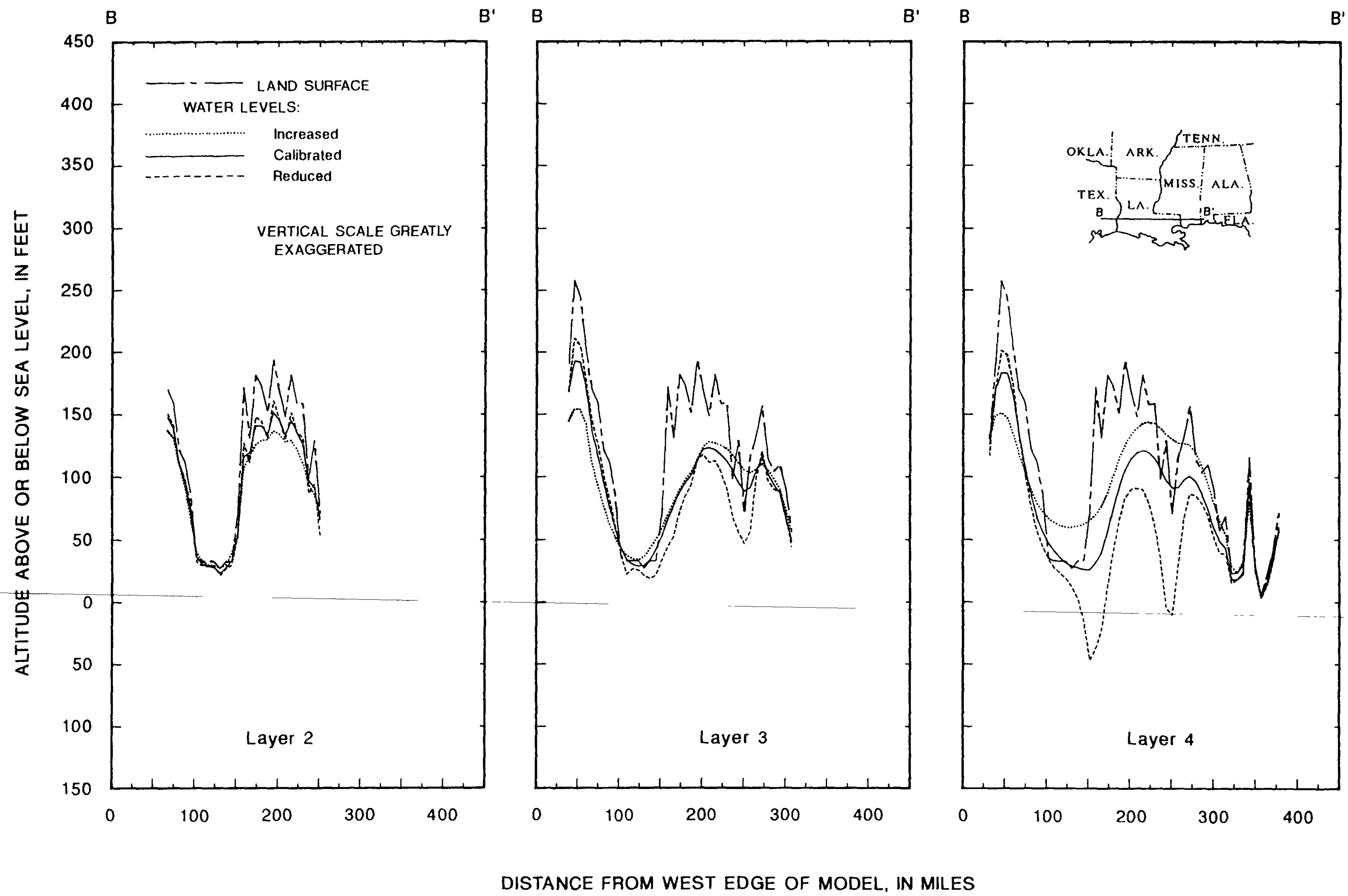

Figure 17.--East-west water-level profiles showing the effect in model layers 2, 3, and 4 of changing the transmissivity of all model layers by factors of 0.2 and 5.

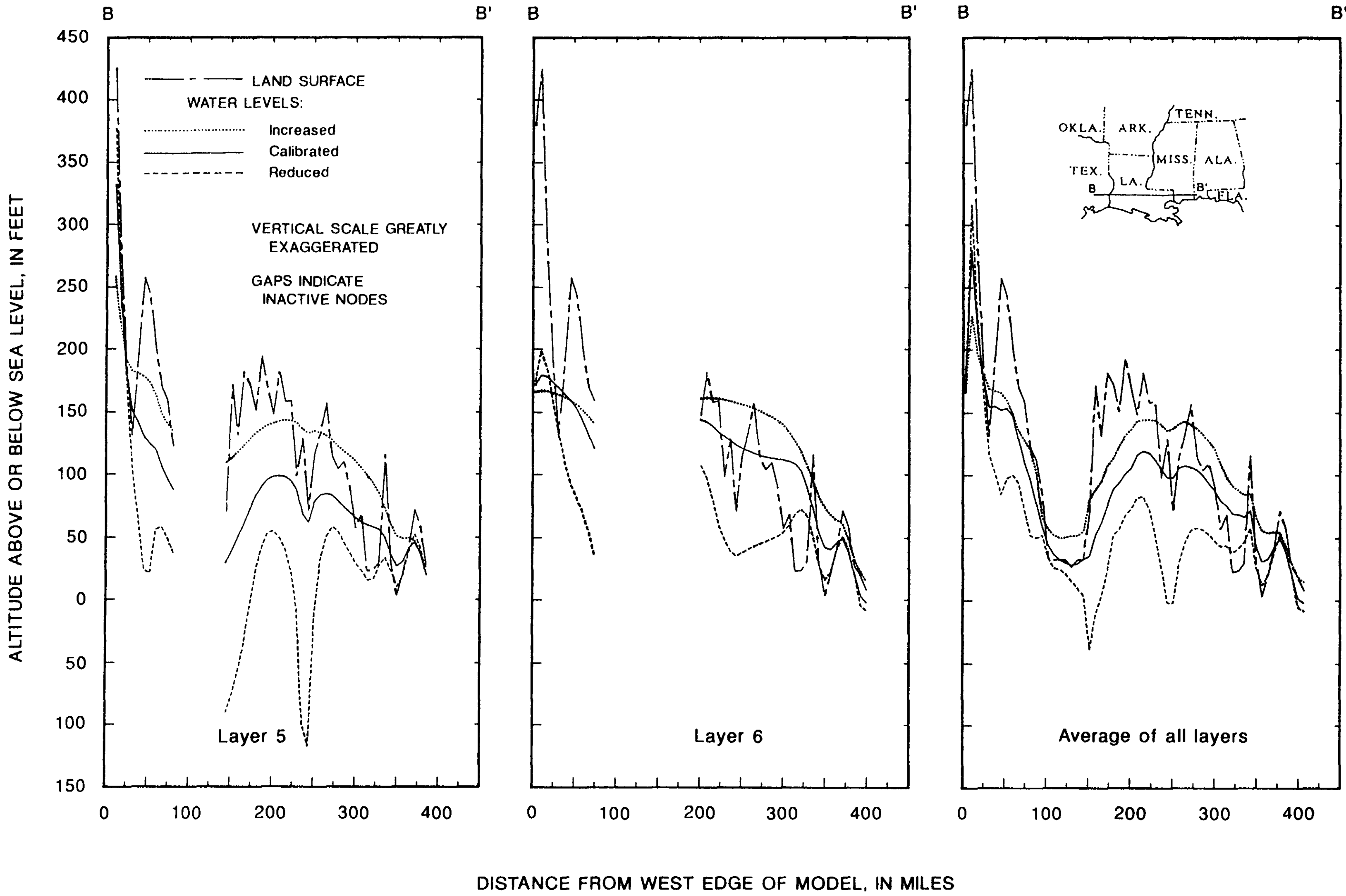

Figure 18.--East-west water-level profiles showing the effect in model layers 5 and 6 and in the average of all layers of changing the transmissivity of all model layers by factors of 0.2 and 5.

Increasing and decreasing only the transmissivity of layer 2 by a factor of 5 produces relatively small changes in the water levels of all layers (figs. 19-22). The effects of these changes are relatively uniform throughout the model and are not accentuated in the deeper layers, as when the transmissivities of all layers are changed.

North-south and east-west water-level profiles showing the results of increasing and decreasing vertical leakance between all layers by a factor of 5 from calibrated values are shown in figures 23-26. In general, these figures show that higher vertical-leakance values increase water levels throughout the aquifer system and lower values decrease water levels. Much of the effect of changing vertical leakances results from the effect these changes have on flow between the aquifer system and the constant-head upper boundary. As with transmissivity, changes in vertical leakances of all layers produce accentuated water-level changes in the deeper layers.

Total flow circulating in the aquifer system is more affected by increases in vertical leakance than by decreases. Increasing the calibrated values of vertical leakage by a factor of 5 increases total flow in the aquifer system by about 47 percent, from 380 to 558 Mft^3/d. Lowering vertical leakances to one-fifth (0.2 times) the calibrated values reduces total flow in the aquifer system by about 22 percent, from 380 to 296 Mft^3/d.

Varying vertical leakance between the constant-head upper boundary and layer 2 from 0.2 to 5 times the calibrated value produces relatively small and uniform changes in water levels throughout the aquifer system (figs. 27-30). These changes are not strongly accentuated in the lower layers.

Storage Coefficient

Model sensitivity to changes in storage coefficient of all model layers is shown in table 5 and figures 31-35. The relative insensitivity of the model to changes in storage coefficient is best shown in figure 31. Large changes in storage-coefficient values result in relatively small changes in RMSE and standard deviation of individual layers and of all layers combined. Table 5 shows that mean error and RMSE could be lowered slightly by reducing the storage-coefficient values to one-tenth of the calibrated values; but, as noted in the section on calibration of the model, the resulting values would have been unreasonably low. The model is more sensitive to increases in the value of the storage coefficient above the calibrated value than to decreases.

Water-level profiles show the results of increasing and decreasing the calibrated value of storage coefficient by a factor of 5 (figs. 32-35). Water-level residuals resulting from changes in storage coefficient are greater in the lower model layers representing deeper zones. This is probably the result of the cumulative effect of water moving into or out of storage in the upper zones that impact the flow distribution in the lower zones.

Water levels rise throughout the aquifer system with increases in the storage coefficient and fall with decreases. More water enters the modeled aquifer system from storage with an increase in storage coefficient, resulting in higher overall water levels. When storage coefficients decrease from calibrated values, water levels must decline throughout the aquifer system to induce more recharge.

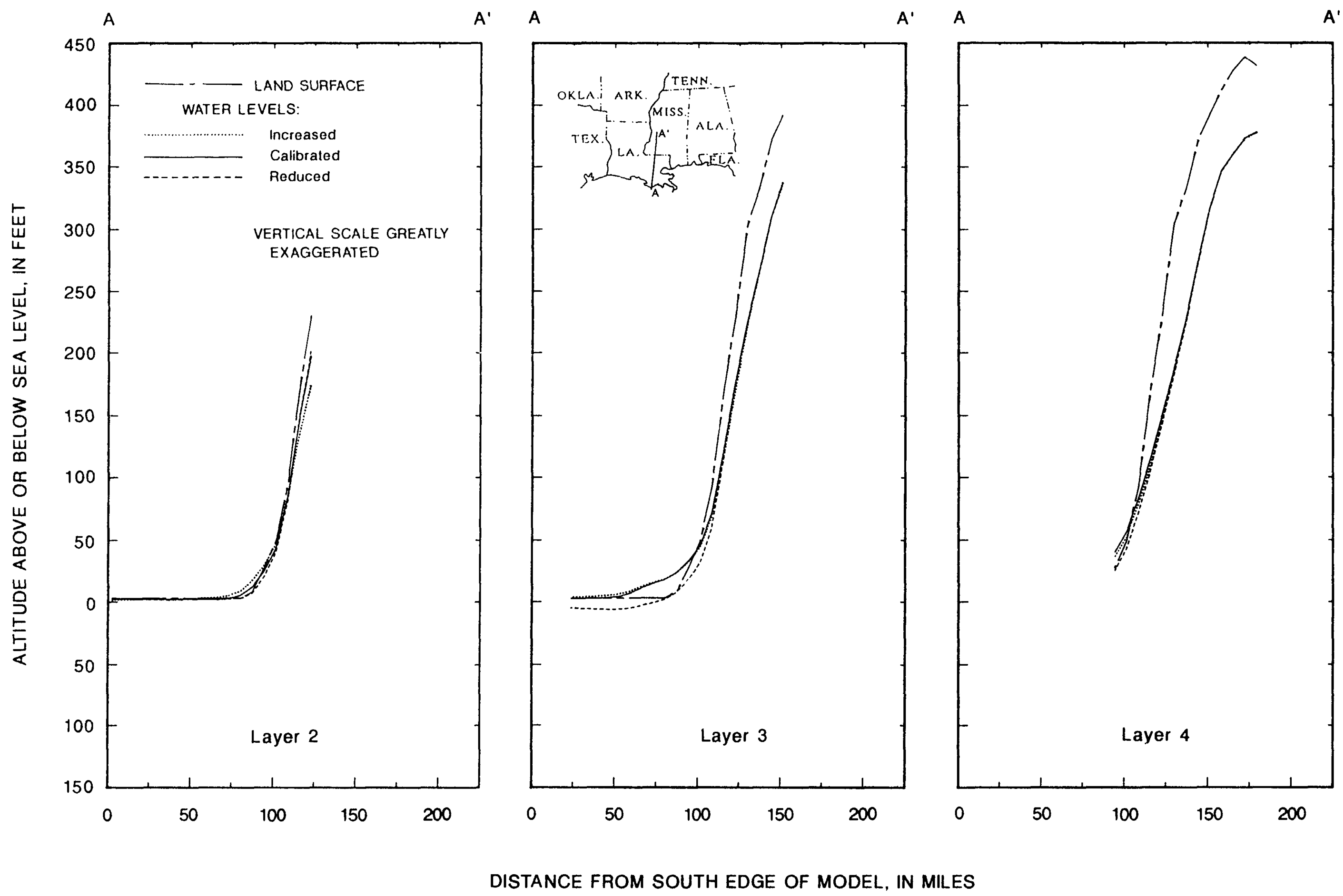

Figure 19.--North-south water-level profiles showing the effect in model layers 2, 3, and 4 of changing the transmissivity of model layer 2 by factors of 0.2 and 5.

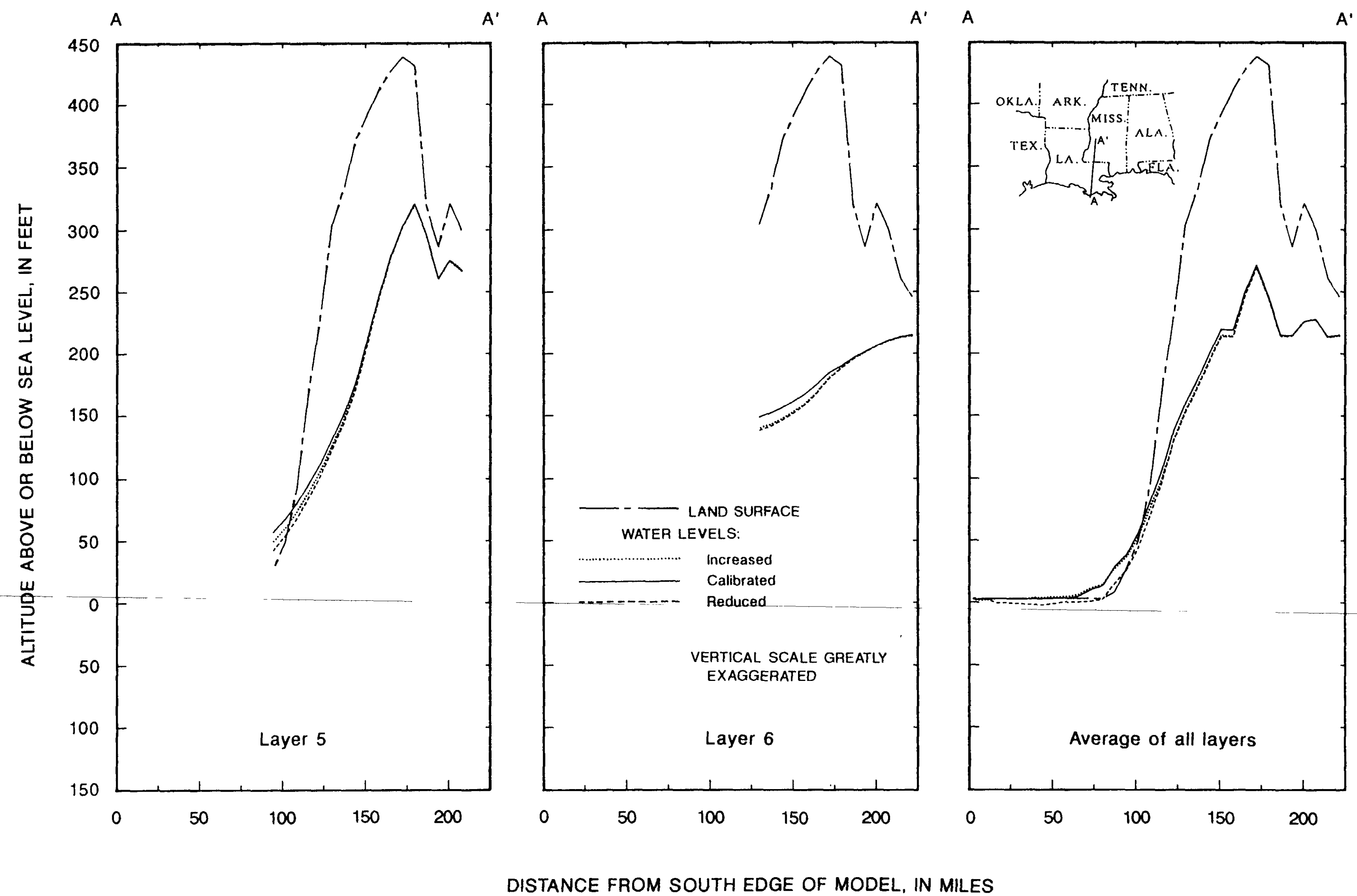

Figure 20.--North-south water-level profiles showing the effect in model layers 5 and 6 and in the average of all layers of changing the transmissivity of model layer 2 by factors of 0.2 and 5.

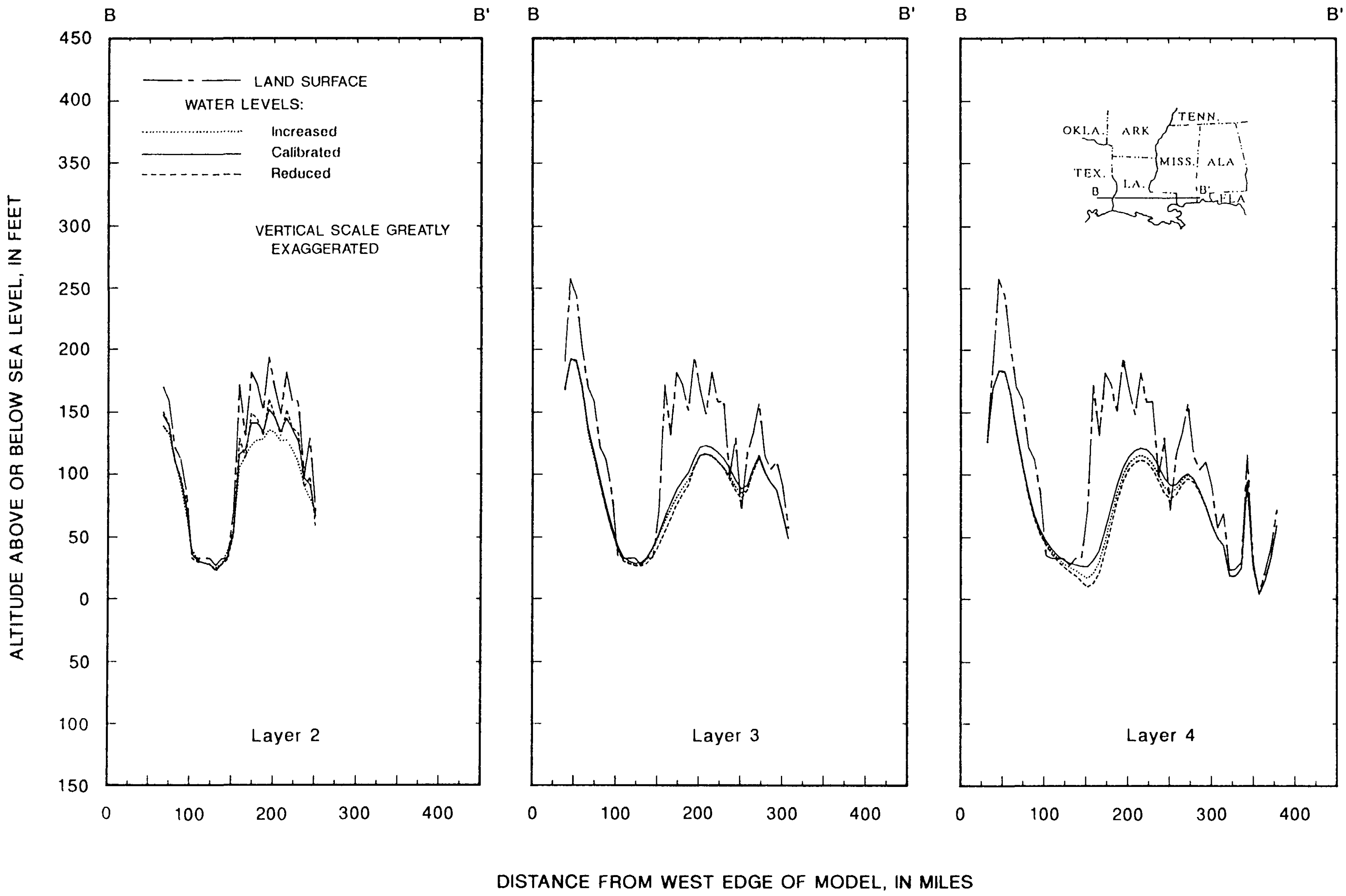

Figure 21.--East-west water-level profiles showing the effect in model layers 2, 3, and 4 of changing the transmissivity of model layer 2 by factors of 0.2 and 5.

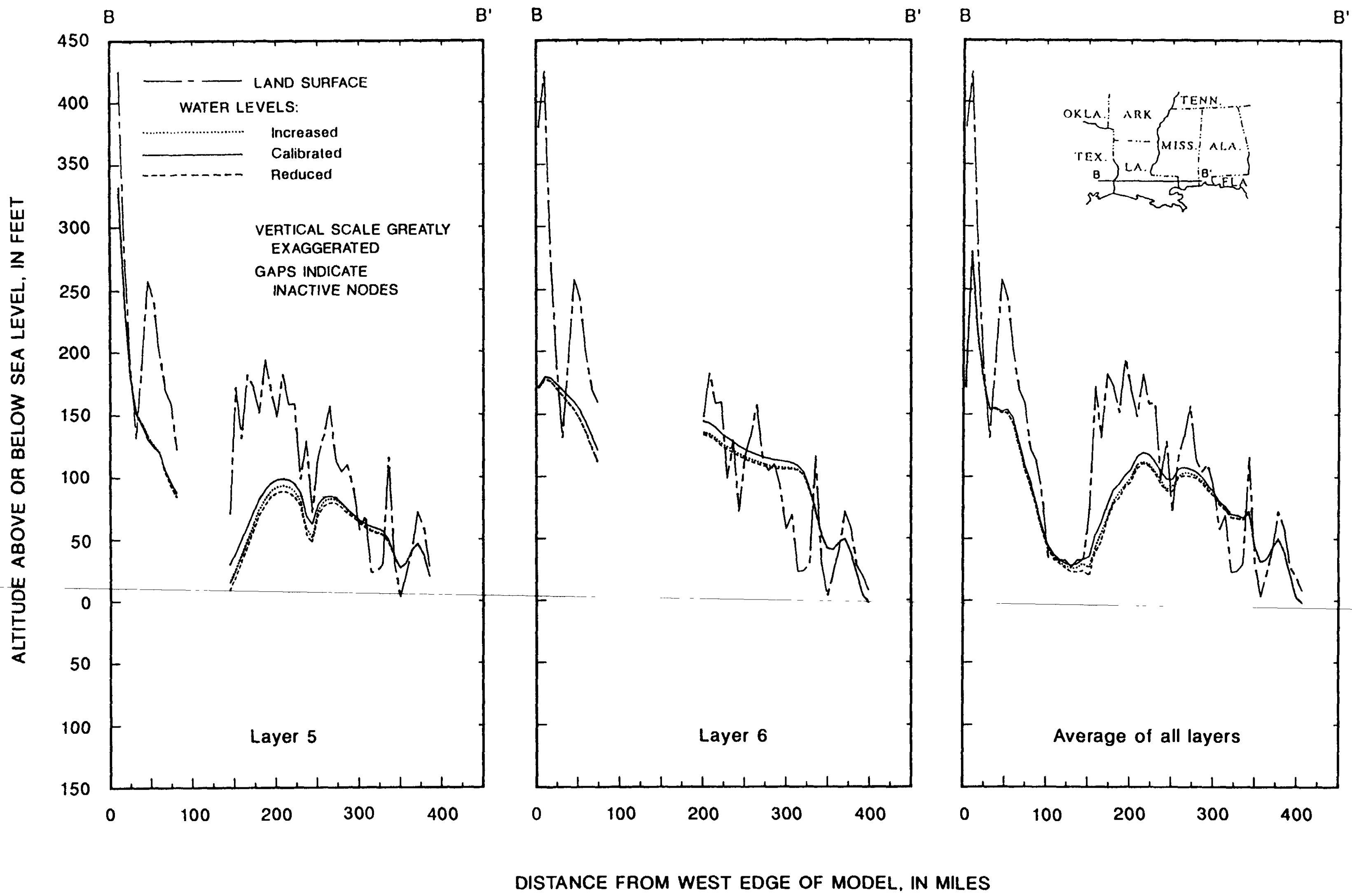

Figure 22.--East-west water-level profiles showing the effect in model layers 5 and 6 and in the average of all layers of changing the transmissivity of model layer 2 by factors of 0.2 and 5.

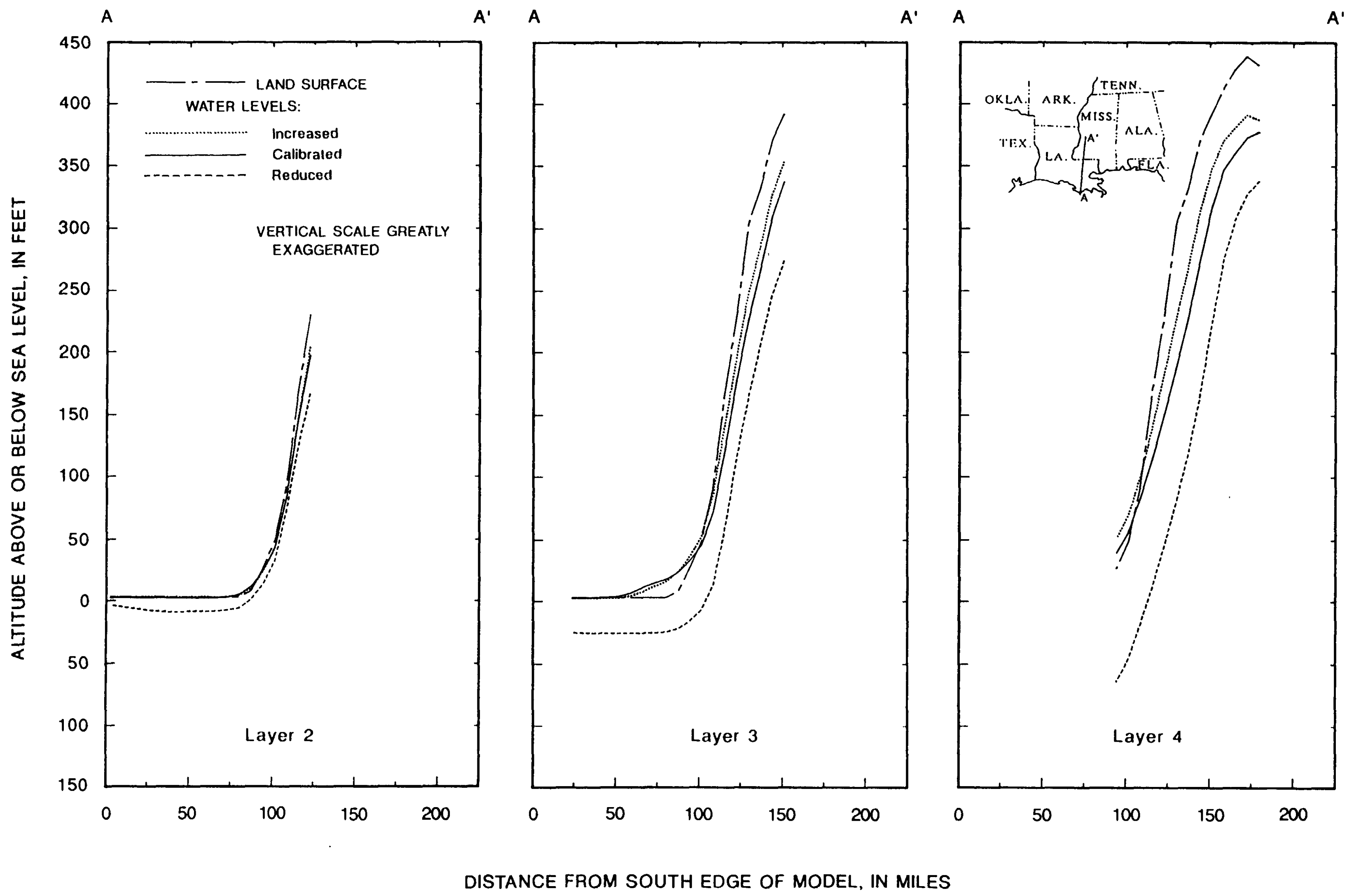

Figure 23.--North-south water-level profiles showing the effect in model layers 2, 3, and 4 of changing vertical leakance between all model layers by factors of 0.2 and 5.

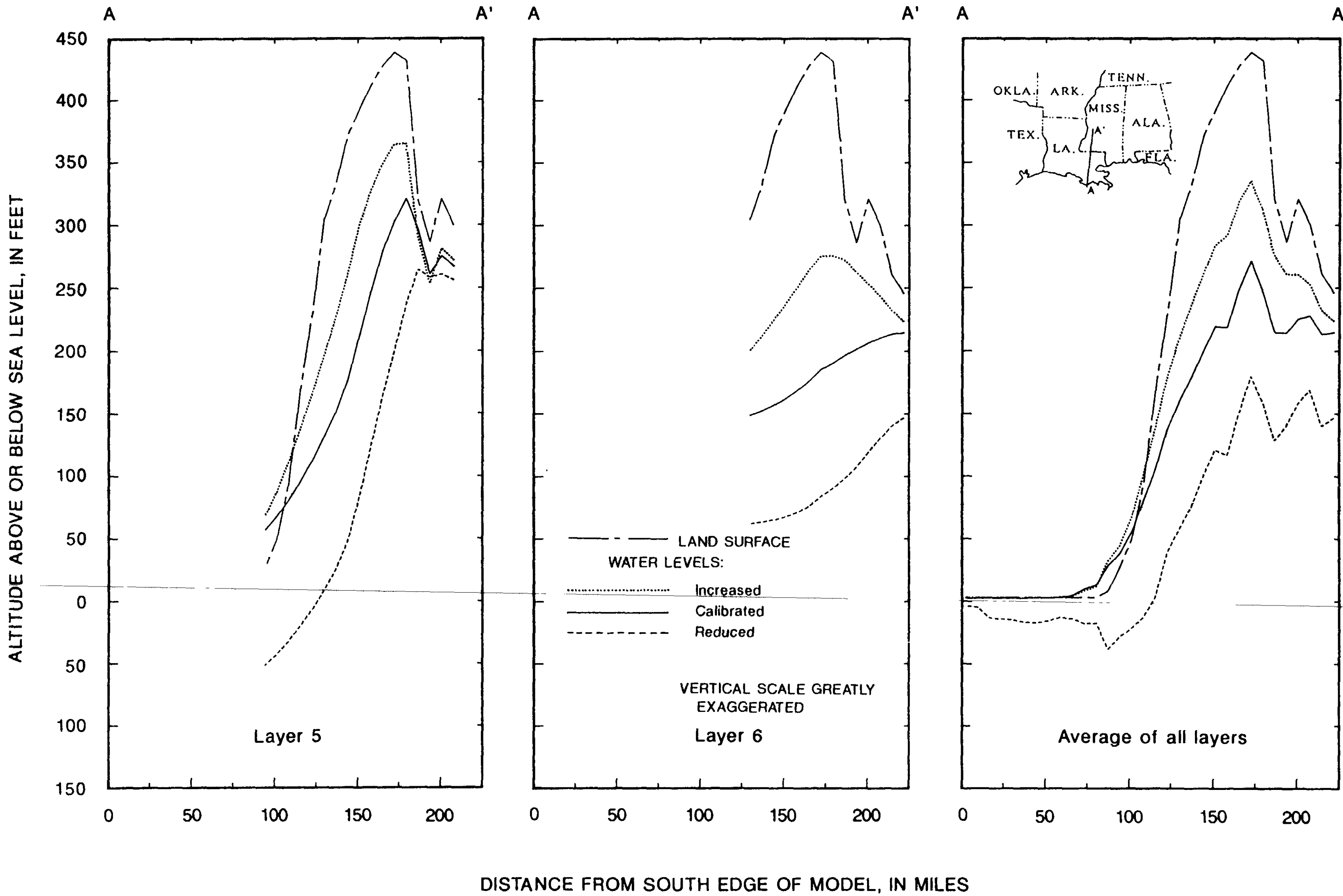

Figure 24.--North-south water-level profiles showing the effect in model layers 5 and 6 and in the average of all layers of changing vertical leakance between all model layers by factors of 0.2 and 5.

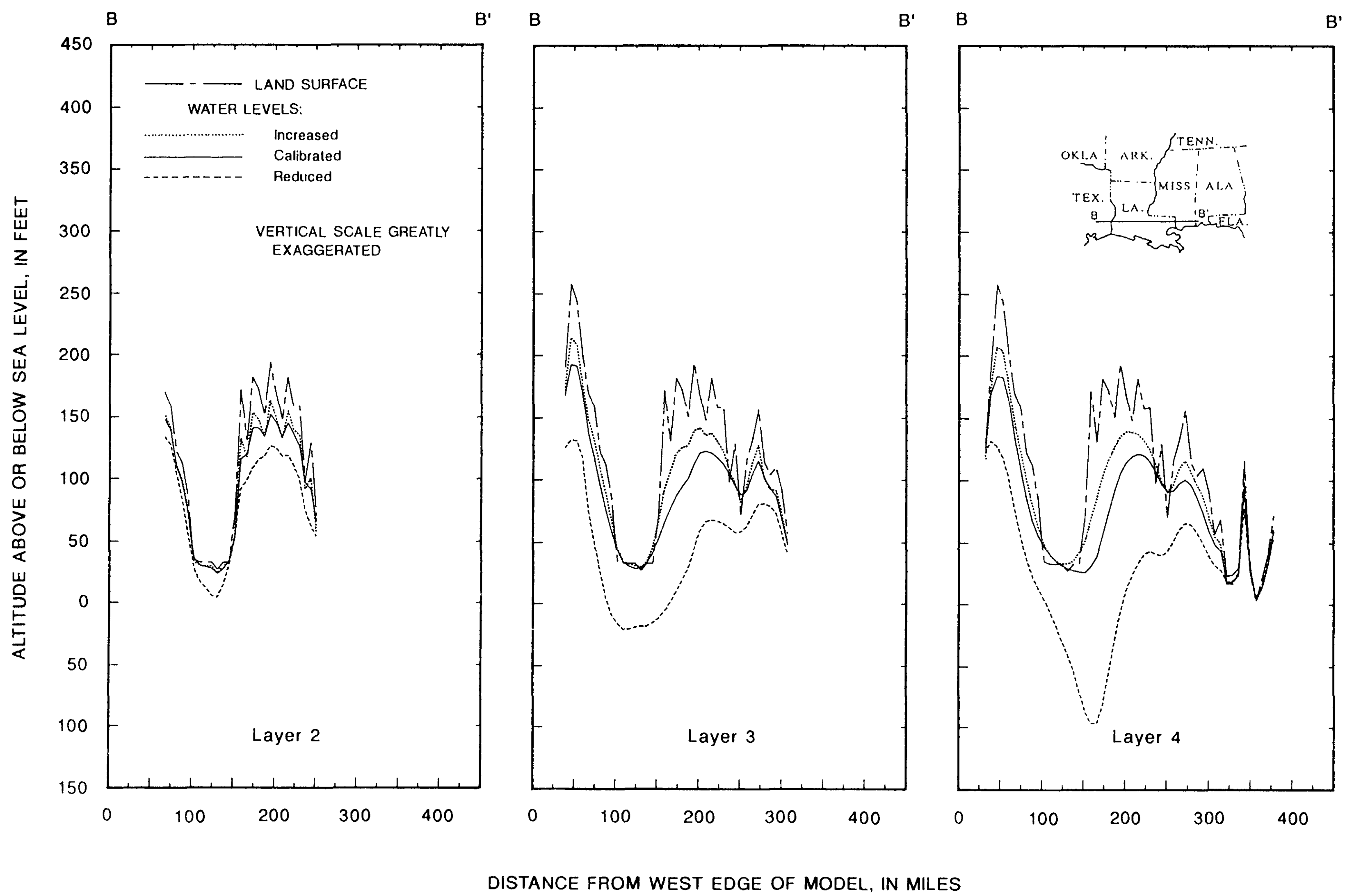

Figure 25.--East-west water-level profiles showing the effect in model layers 2, 3, and 4 of changing vertical leakance between all model layers by factors of 0.2 and 5.

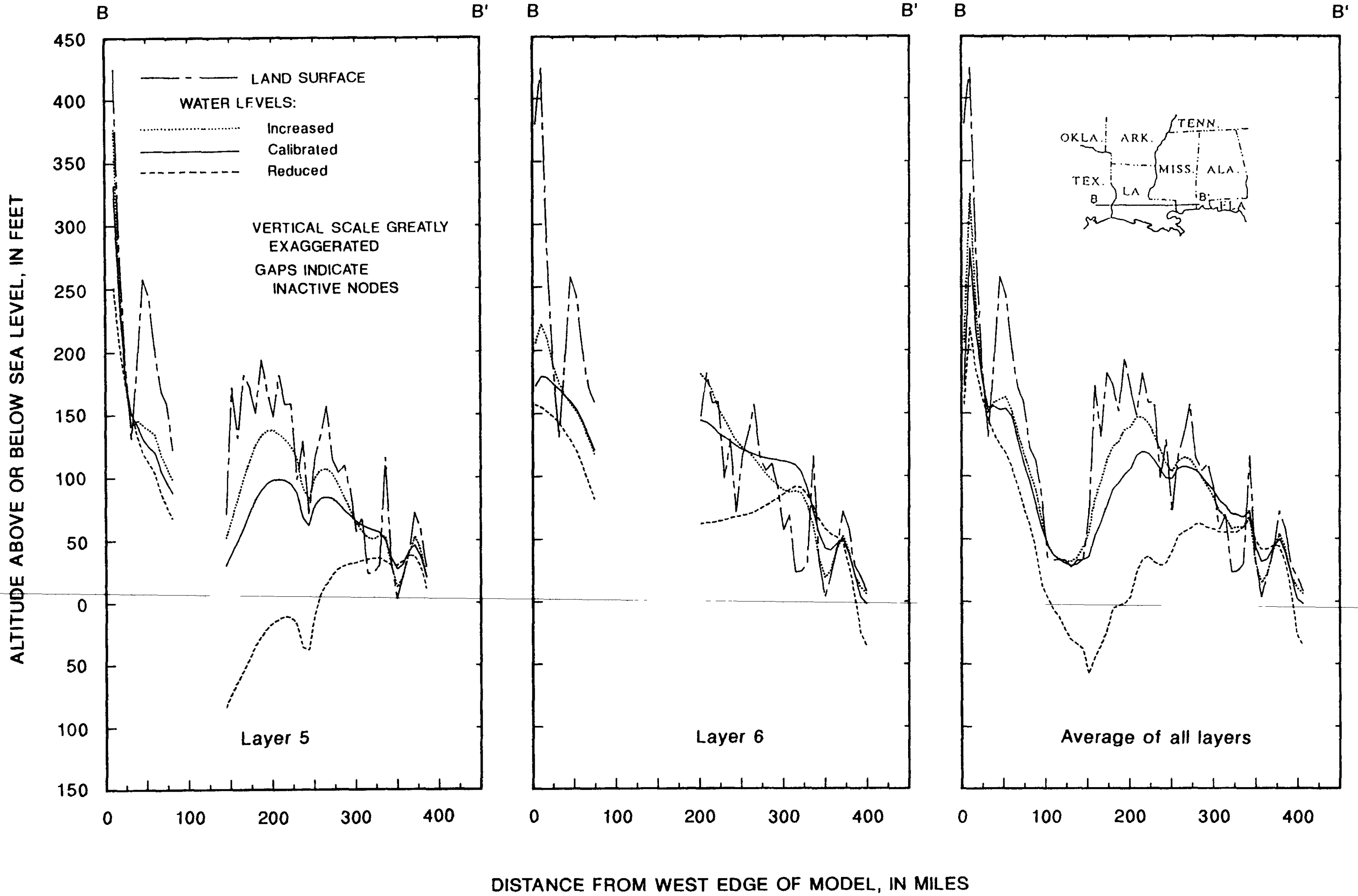

Figure 26.--East-west water-level profiles showing the effect in model layers 5 and 6 and in the average of all layers of changing vertical leakance between all model layers by factors of 0.2 and 5.

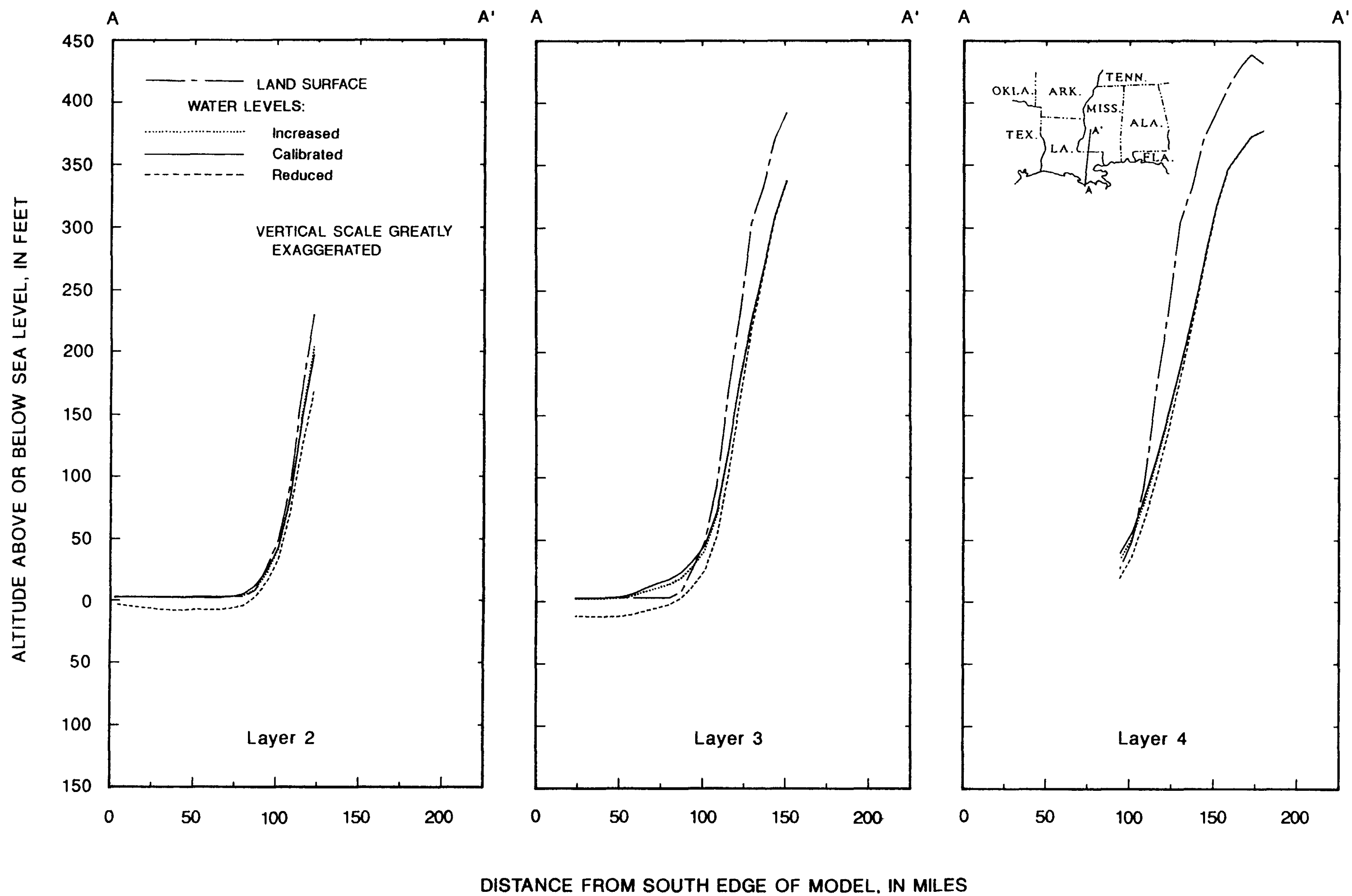

Figure 27.--North-south water-level profiles showing the effect in model layers 2, 3, and 4 of changing vertical leakance between model layers 1 and 2 by factors of 0.2 and 5.

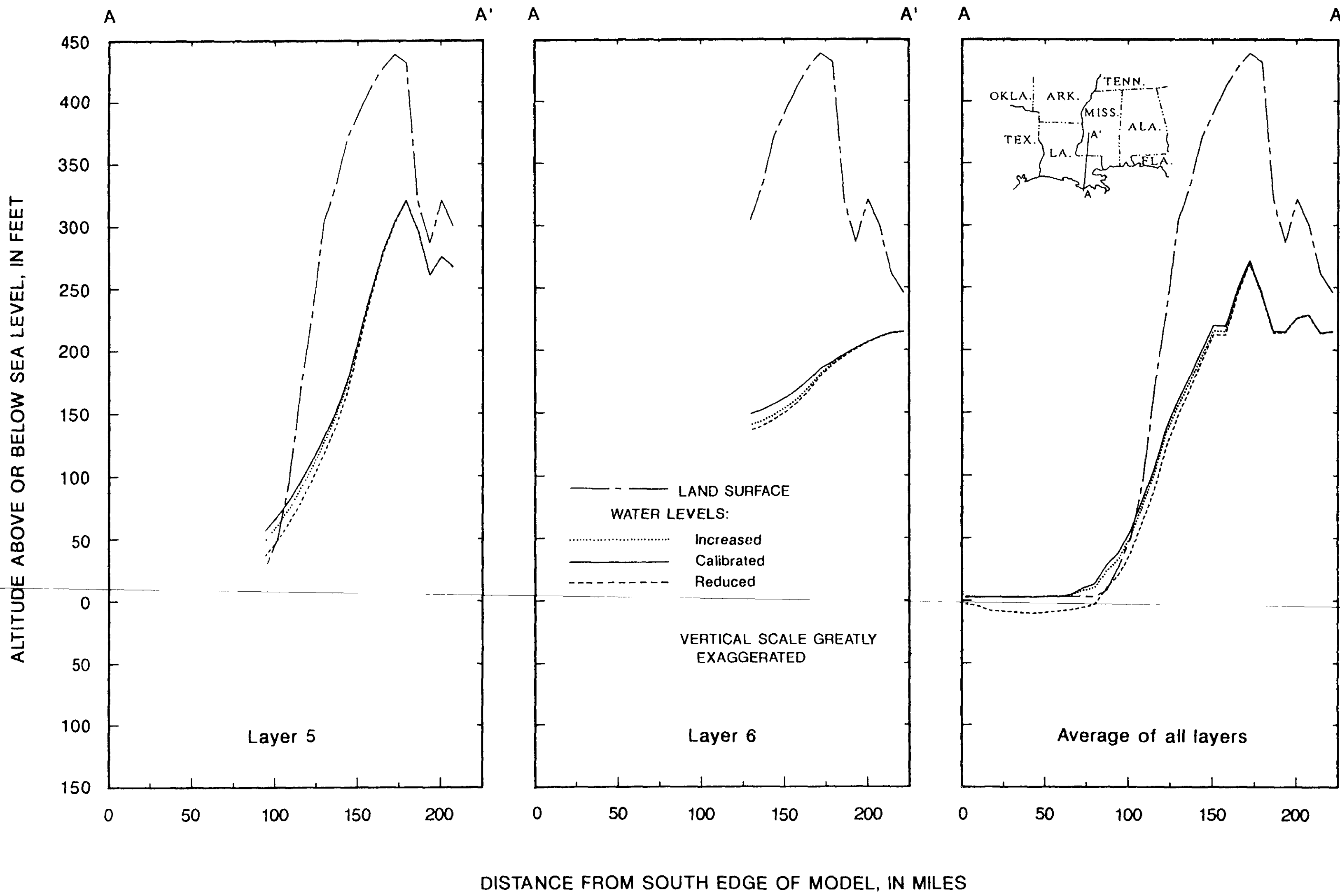

Figure 28.--North-south water-level profiles showing the effect in model layers 5 and 6 and in the average of all layers of changing vertical leakance between model layers 1 and 2 by factors of 0.2 and 5.

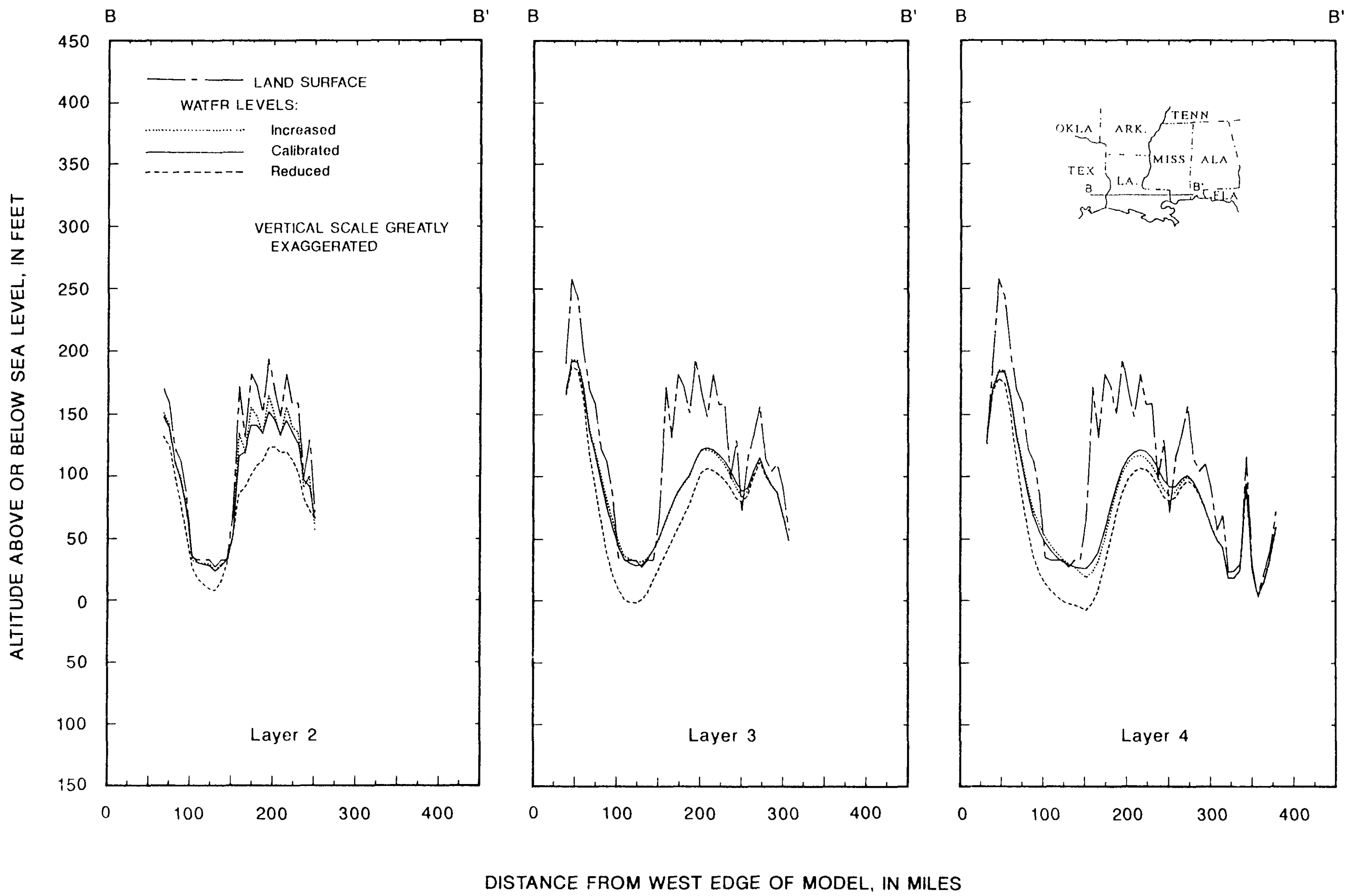

Figure 29.--East-west water-level profiles showing the effect in model layers 2, 3, and 4 of changing vertical leakance between model layers 1 and 2 by factors of 0.2 and 5.

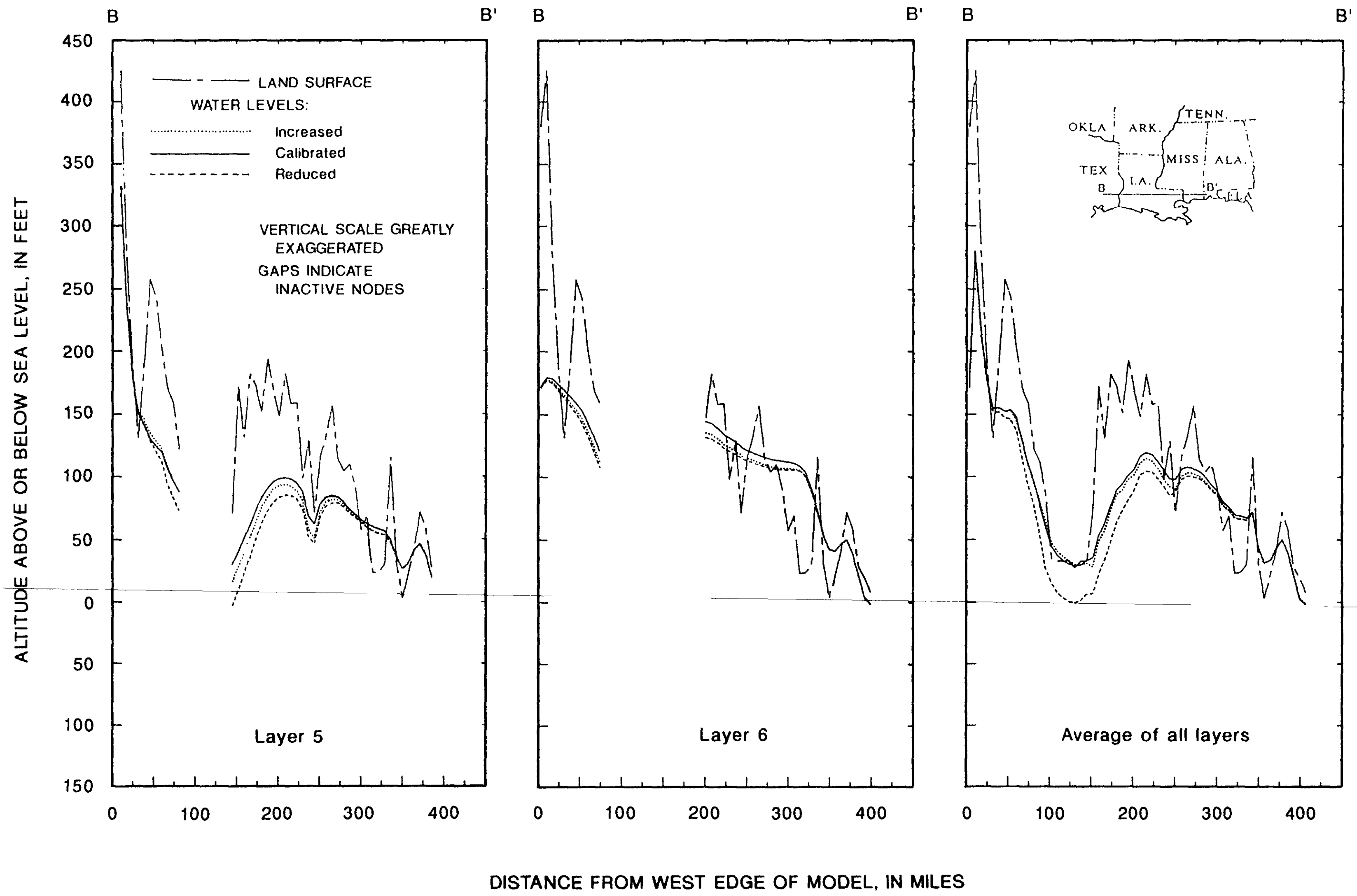

Figure 30.--East-west water-level profiles showing the effect in model layers 5 and 6 and in the average of all layers of changing vertical leakance between model layers 1 and 2 by factors of 0.2 and 5.

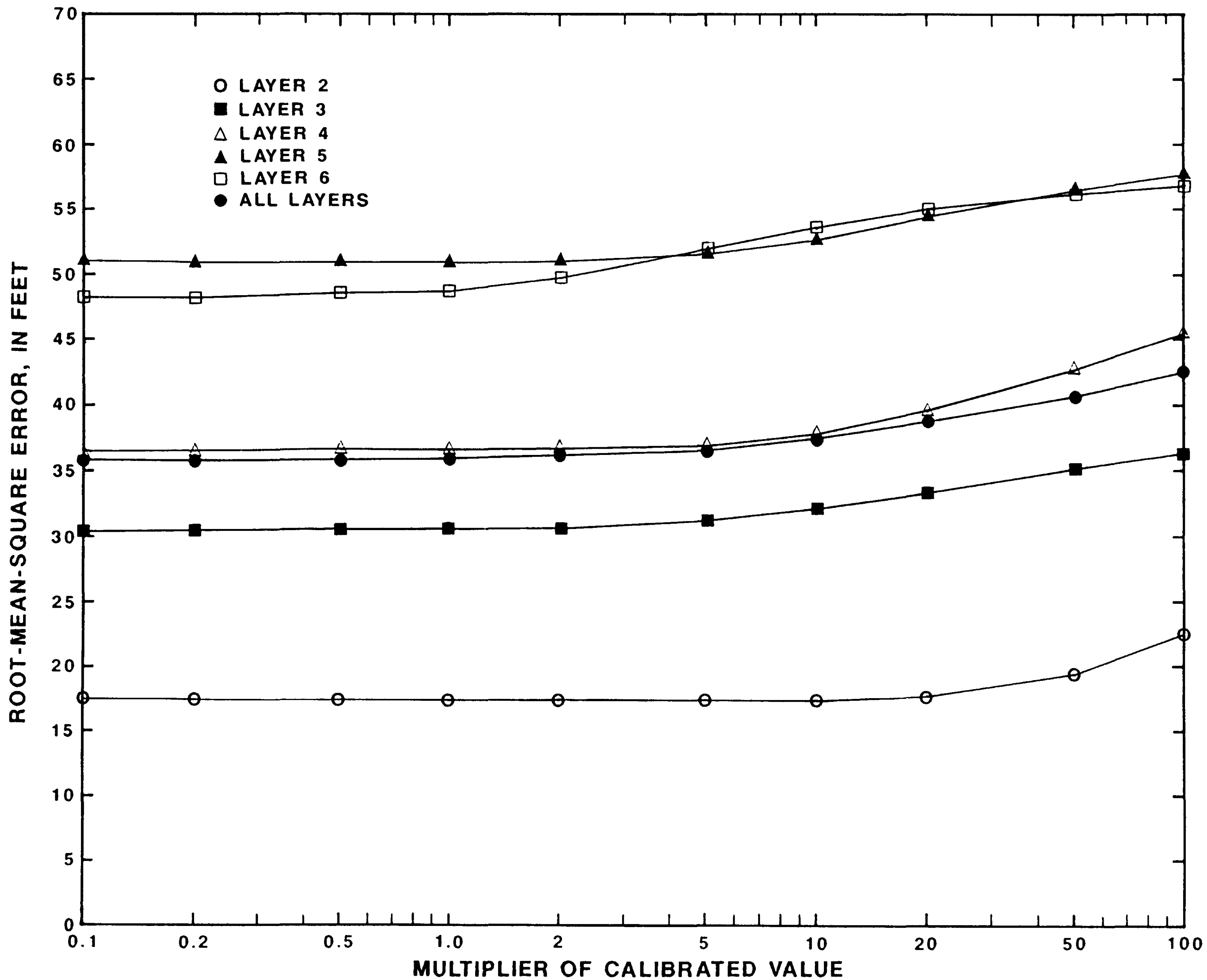

Figure 31.--Root-mean-square errors of water-level residuals resulting from changes in the storage coefficient of all model layers.

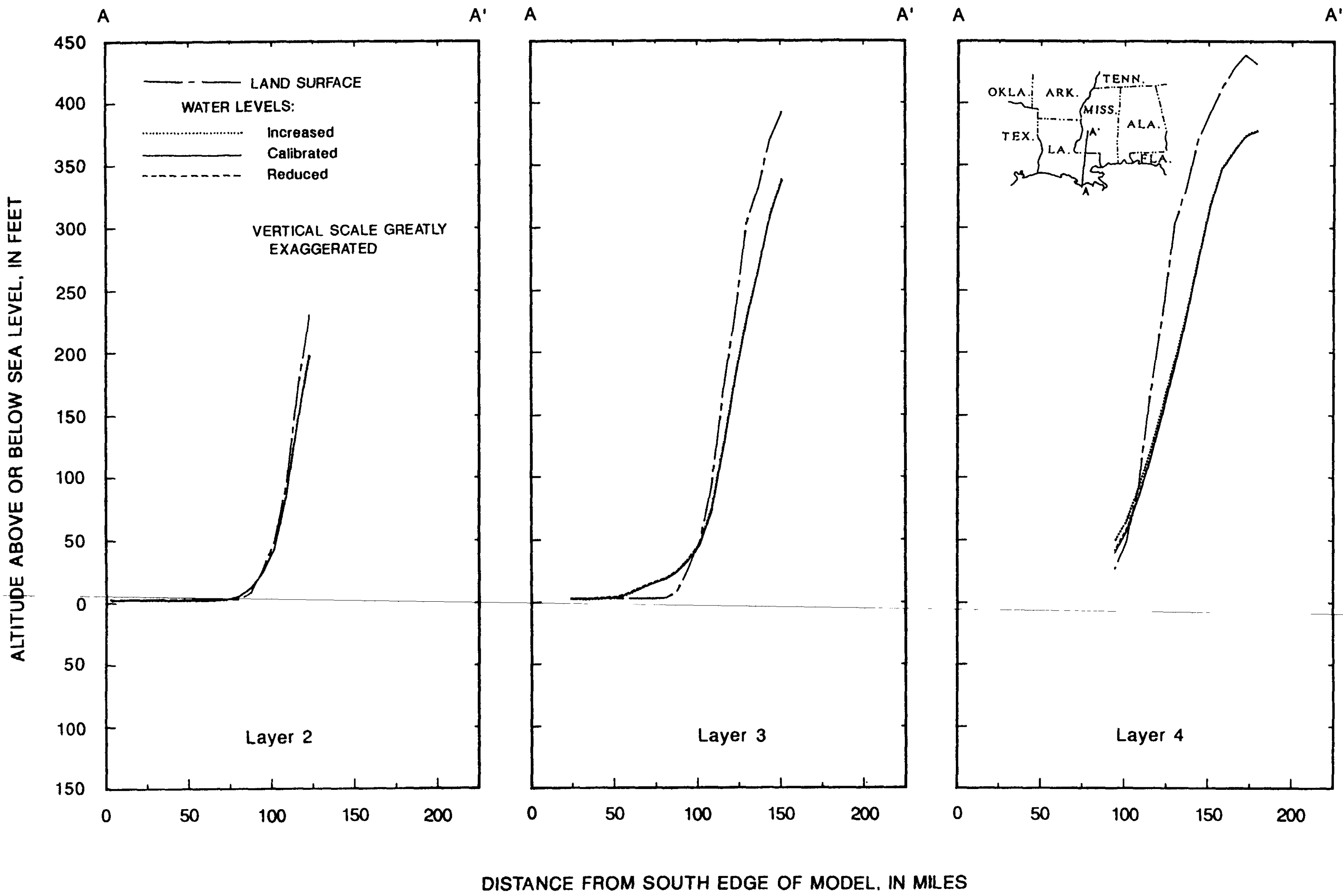

Figure 32.--North-south water-level profiles showing the effect in model layers 2, 3, and 4 of changing the storage coefficient of all model layers by factors of 0.2 and 5.

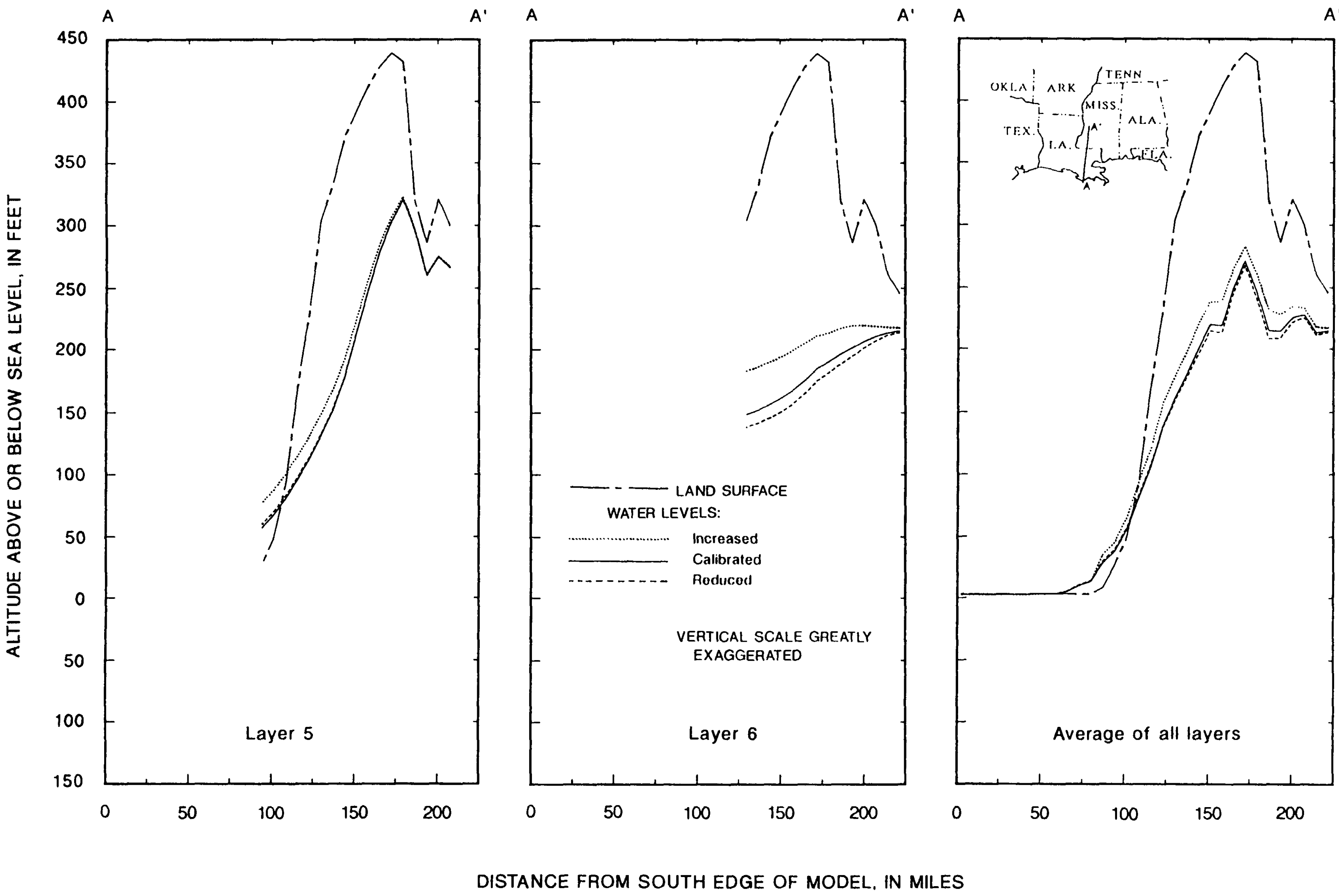

Figure 33.--North-south water-level profiles showing the effect in model layers 5 and 6 and in the average of all layers of changing the storage coefficient of all model layers by factors of 0.2 and 5.

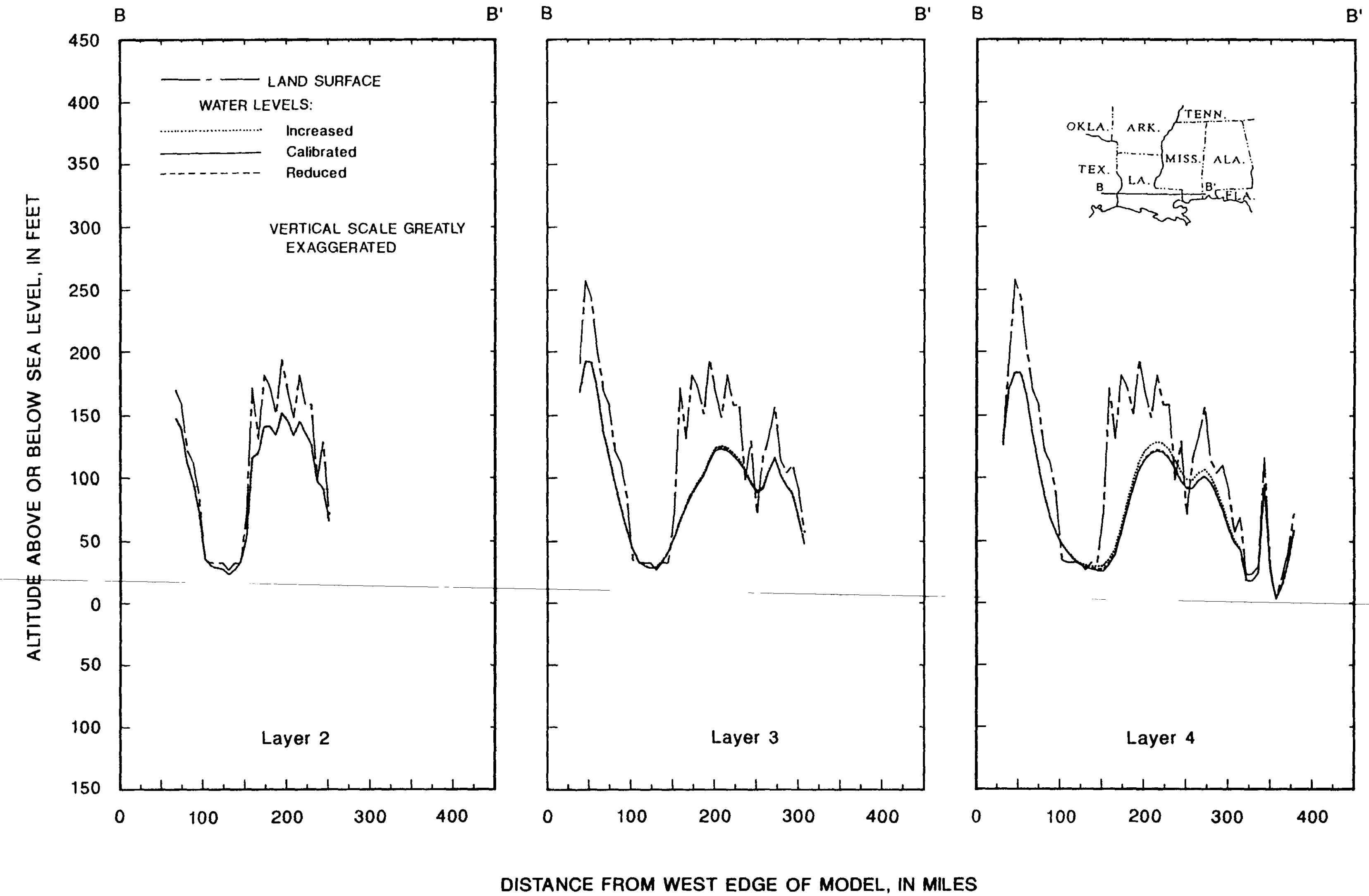

Figure 34.--East-west water-level profiles showing the effect in model layers 2, 3, and 4 of changing the storage coefficient of all model layers by factors of 0.2 and 5.

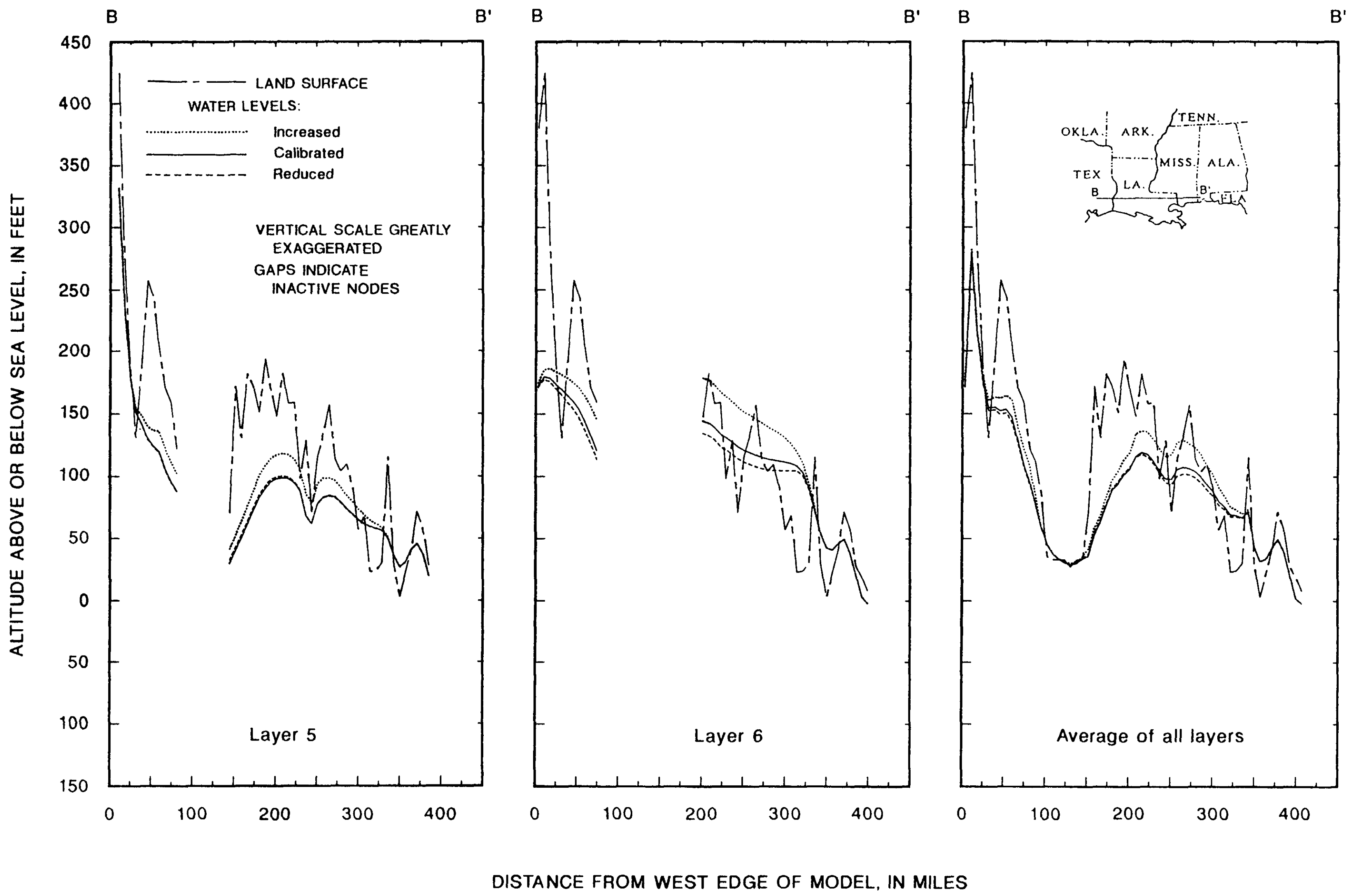

Figure 35.--East-west water-level profiles showing the effect in model layers 5 and 6 and in the average of all layers of changing the storage coefficient of all model layers by factors of 0.2 and 5.

Table 5.--Results of statistical analysis of water-level residuals showing the sensitivity of the transient calibrated model to changes in the storage coefficient of all model layers

[Multiplier is the factor by which the calibrated values of storage coefficient were varied]

Multi-plier	Model layer	Number of observations	Mean error (feet)	Root-mean-square error (feet)	Standard deviation (feet)
	2	1,973	1.01	17.46	17.31
	3	539	2.85	30.35	29.38
0.10	4	1,017	.86	36.48	35.98
	5	1,179	10.32	50.81	49.28
	6	521	18.61	48.10	43.84
	All	5,229	5.02	35.80	34.53
	2	1,973	1.11	17.45	17.30
	3	539	3.41	30.49	29.45
1	4	1,017	1.78	36.54	35.93
	5	1,179	11.61	50.90	49.10
	6	521	20.39	48.92	43.99
	All	5,229	5.77	35.96	34.50
	2	1,973	2.66	17.36	17.07
	3	539	7.91	32.15	30.32
10	4	1,017	8.51	37.85	35.96
	5	1,179	19.76	52.58	48.56
	6	521	26.86	53.42	45.64
	All	5,229	10.61	37.50	34.58
	2	1,973	12.33	21.26	18.81
	3	539	14.55	36.27	32.49
100	4	1,017	18.04	45.36	40.02
	5	1,179	27.89	57.93	50.68
	6	521	31.32	56.90	46.72
	All	5,229	19.07	42.26	36.75

Increasing calibrated storage coefficients of all layers by a factor of 5 increases the amount of water derived from storage for the period 1898-1987 from 110,000 to 433,000 Mft^3. Lowering storage coefficients to one-fifth the calibrated values decreases the amount of water derived from storage for the same time period from 110,000 to 23,000 Mft^3. Although these results show a large difference in the amount of water derived from storage due to changes in storage-coefficient values, this amount of water is only a small part of the total simulated flow in the aquifer system. With storage-coefficient values 5 times the calibrated values, the volume of water derived from storage is about 5 percent of the total simulated flow. With storage-coefficient values one-fifth of calibrated values, the volume of water derived from storage is less than 1 percent of the total simulated flow.

SUMMARY

A six-layer, finite-difference ground-water flow model was developed, calibrated, and tested for sensitivity in order to quantify regional flow in the coastal lowlands aquifer system of Louisiana, Mississippi, Alabama, and Florida. The model was initially calibrated by trial-and-error by varying values of transmissivity of the aquifers, vertical leakances between the aquifers, and the storage coefficients of the aquifer system in order to match model-simulated and measured water levels under steady-state and transient conditions. The mean error, RMSE, and standard deviation of the residuals between model-simulated and measured water levels were used to quantitatively evaluate the progress of calibration.

After trial-and-error calibration, a statistical optimization program was used to see if calibration could be improved. The program improves model calibration by minimizing an objective function which is proportional to the RMSE. The optimization program did not significantly change trial-and-error calibration results.

The model was calibrated best in layer 2, which represents the upper, youngest section of the coastal lowlands aquifer system. Calibration generally worsened in the lower model layers that represent older sections of the aquifer system. Relatively high residual errors in water levels in the lower model layers are largely due to topographic variations in the outcrop areas of the permeable zones represented by these layers. Large topographic variations within a grid block covering 25 mi^2 produce steep, irregular water-level gradients which are difficult to reproduce in the model. Steep water-level gradients and relatively high water-level residuals may also occur near major pumping centers.

Sensitivity analyses of the calibrated steady-state and transient models show the effects of varying values of transmissivity, vertical leakance, and storage coefficients. In terms of mean error, RMSE, and standard deviation, results show that within an order of magnitude of calibrated values the model is most sensitive to changes in transmissivity. The model is more sensitive in terms of water-level change to reductions of transmissivity and vertical leakance than to increases in these parameters. The model is relatively insensitive to changes in storage coefficient over a wide range of values because only a small part of the total flow in the aquifer system is derived from storage.

When values of transmissivity or vertical leakance are changed one layer at a time, results obtained during calibration show that transmissivity of layer 2 and vertical leakance between layer 2 and the constant-head upper boundary are most sensitive. When transmissivity or vertical-leakance values are changed in all layers at the same time, the deeper model layers become more sensitive because flow between the constant-head upper boundary and the deeper layers is affected by the changes made in all intervening layers.

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