NONLINEAR-REGRESSION FLOW MODEL OF THE GULF COAST AQUIFER SYSTEMS IN THE SOUTH-CENTRAL UNITED STATES

By Logan K. Kuiper

U.S. GEOLOGICAL SURVEY

Water-Resources Investigations Report 93-4020

A Contribution of the
Regional Aquifer-System Analysis
Program

Austin, Texas
1994

U.S. DEPARTMENT OF THE INTERIOR
BRUCE BABBITT, *Secretary*

U.S. GEOLOGICAL SURVEY
Gordon P. Eaton, *Director*

1994

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CONTENTS

ILLUSTRATIONS

ILLUSTRATIONS--Continued

ILLUSTRATIONS--Continued

TABLES

CONVERSION FACTORS, VERTICAL DATUM, AND ABBREVIATED WATER QUALITY UNITS

Multiply	By	To obtain
	Length	
foot (ft)	0.3048	meter
inch (in.)	25.40	millimeter
mile (mi)	1.609	kilometer
	Area	
square mile (mi^2)	2.590	square kilometer
	Flow	
billion gallons per day (Ggal/d)	43.81	cubic meter per second
cubic foot per day (ft^3/d)	0.02832	cubic meter per day
inch per year (in/yr)	25.4	millimeter per year
	Pressure	
pound per square inch (lb/in^2)	6.895	kilopascal

Sea level: In this report, "sea level" refers to the National Geodetic Vertical Datum of 1929--a geodetic datum derived from a general adjustment of the first-order level nets of the United States and Canada, formerly called Sea Level Datum of 1929.

Chemical concentrations and seawater density are given in metric units. Chemical concentration is given in milligrams per liter (mg/L) and seawater density is given in grams per cubic centimeter (gm/cm^3).

SYMBOLS AND DIMENSIONS

[Number in parentheses refers to the page or illustration where the symbol first appears or where additional clarification may be obtained. Symbols are also defined in the text.]

Symbol	*Dimensions*	*Description*
A	L^2	Cross sectional area (42).
B	-------	Parameter vector (40).
$\hat{B}$	-------	Value for parameter vector B at which S(B) is a minimum (46).
B_i to B_p	-------	Regression parameters.
C_s	1	Specific storage factor (54).
Chi^2	1	Probability distribution function (76).
D	-------	Function (63).
E	-------	Random vector (40).
F	1	F probability distribution function (46).
G	1	Function (63).
K	LT^{-1}	Hydraulic conductivity tensor (18).
K_c	LT^{-1}	Hydraulic conductivity of clay (51).
K_s	LT^{-1}	Hydraulic conductivity of sand (51).
M	1	Number of grid-elements with a specified head (39).
Mu	$ML^{-1}T^{-1}$	Mean weighted residual (76).
N	1	Number of grid-elements dependent upon regression parameters (19).
P	$ML^{-1}T^{-2}$	Pressure (18).
P_c	LT^{-1}	Hydraulic conductivity parameter of clay (96).
P_s	LT^{-1}	Hydraulic conductivity parameter of sand (76).
Q	L^3T^{-1}	Rate of discharge (18).
R	L	Radius (57).
S_s	T^{-1}	Specific storage (18).
$S(B)$	-------	Weighted mean square error (46).
T	-------	Transpose of a matrix (46).
V_c	L^3	Clay fraction of the grid element (51).
X	-------	Matrix (46).
$\hat{X}$	-------	Matrix X evaluated at $B=\hat{B}$ (46)
Y	-------	Random vector (40).
Y_i	-------	Set of observations (40).
Z_i	-------	Random variable (76).
a	-------	Width to vertical thickness ratio (51).
b	L	Grid element thickness (37).
b_1 to b_9	-------	Regression parameters (65).

Symbol	*Dimensions*	*Description*
dd	-------	Function, depth (48, 95).
e	-------	Base of Naperian logarithms, 2.71828 (46).
f(B)	-------	Expectation of Y (40).
g	LT^{-2}	Acceleration due to gravity (18).
h	$ML^{-1}T^{-2}$	Pressure head (18).
h_w	L	Water table altitude (37)
Δh_s	$ML^{-1}T^{-2}$	Head triggering value for subsidence (54).
k	L^2	Intrinsic permeability tensor (18).
n	1	Number of observations (40).
p	1	Number of parameters (46).
ns	1	Number of point observations of head used to formulate pressure head (56).
$\bar{q}$		Specific discharge (18).
r	L	Radius (56).
s	1	Aquifer layer (61).
t	T	Time.
u	1	Number expressing departure from ideal situation (63).
v^2	-------	Variance (76).
x	L	Cartesian coordinate in x-direction
y	L	Cartesian coordinate in y-direction
z	L	Cartesian coordinate in z-direction
α	1	F probability distribution function argument (46).
∂	-------	Partial derivative (18).
μ	$ML^{-1}T^{-1}$	Dynamic viscosity (18).
ω	1	Matrix (40).
ρ	ML^{-3}	Density of fluid (18).
Δ	-------	Change in a quantity.
ϵ	-------	$\epsilon^2\omega^{-1}$ is variance (40, 42).

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ABSTRACT

Multiple-regression methodology was used to calibrate a time-dependent, variable-density ground-water flow model with subsidence of the deep, regional gulf coast aquifer systems in the south-central United States. The methodology was also used to help answer questions concerning model reliability. More than 40 different regression models having 2 to 31 regression parameters are used and detailed results are reported for 12 of the models. More than 3,000 values for grid-element volume-averaged head and volume-averaged hydraulic conductivity are used as observations in the regression models. Calculated prediction interval half widths, though perhaps inaccurate due to a lack of normality of the residuals, are smallest for those models having only four regression parameters. Because of this, and also because the root-mean weighted residual decreases very little with an increase in the number of regression parameters, the models having a small number of parameters are probably the most reliable.

The various models used show considerable overlap between the prediction intervals for shallow head and hydraulic conductivity of sand (K_s). Approximate 95-percent prediction interval half widths for volume-averaged freshwater head exceed 108 feet; for volume-averaged $\log_{10}(K_s)$, they exceed 0.89. All the models produce unreliable predictions of head and ground-water flow in the deeper parts of the aquifer system, including the amount of flow coming from the geopressured zone beneath the aquifer system. Truncating the domain of solution of one of the models to exclude that part of the system having a ground-water density greater than 1.005 grams per cubic centimeter does not appreciably change simulated heads or flow. Also, excluding that part of the system below a depth 3,000 feet below land surface, and setting the density to that of freshwater in the remaining shallow part of the domain of solution, does not appreciably change the results for head and ground-water flow from the model, except for locations close to the truncation surface.

The regression methodology allowed the testing of a wide range of models for the simulation of the aquifer system. It also provided estimates of the accuracy of results and a mechanism to determine sources of model error.

INTRODUCTION

A common approach used to determine the accuracy of a ground-water simulation model is to compare model-computed values and values of the hydraulic parameters used in the model with field observations of physical quantities. Such comparisons form the basis for model calibration, which is that process whereby these parameters are varied to obtain the best possible fit with the observed quantities (Konikow, 1978). Clearly, for such calibration to be possible, it is necessary to establish some criterion to decide whether a particular selection of regression parameters (regression model) gives a better or worse fit than some other selection of regression parameters. Furthermore, for calibrated models to be most useful it is necessary to be able to gage their reliability. Recent advances in regression modeling described by Vecchia and Cooley (1987), Cooley and others (1986), Cooley (1977, 1979, 1982), Neuman (1980), and Yeh and Yoon (1981) treat model calibration as a statistical procedure. These regression procedures provide the necessary fitting criterion and the estimates of model reliability.

The purpose of this report is to illustrate the application of a multiple regression methodology to help answer questions concerning model reliability, to use the method to calibrate a ground-water flow model for a thick regional aquifer system in the Gulf Coastal Plain of the south-central United States, and to develop an improved understanding of flow in the gulf coast aquifer systems.

First, the hydrologic and geologic setting of the gulf coast aquifer systems is described in detail to provide the background needed to understand the modeling effort. A short description of the flow model and the regression procedure for calibration follows. The next section gives formulae to determine the confidence of the model output and the regression parameters used in the model, and a procedure for selecting the model expected to give the best results. Following are sections on: model construction, preparation of observations, application of the model, model error, and analysis of residuals.

This report is one of several presenting the results of the Gulf Coast Regional Aquifer-System Analysis (RASA) study, one of many RASA studies which have been conducted by the United States Geological Survey since 1978, at which time the RASA program was initiated following a congressional mandate (Sun, 1986, p. 2). Professional Paper 1416 consists of several chapters dealing with various aspects of the ground-water flow system in the Gulf Coastal Plain of the south-central United States. Chapter B (Hosman and Weiss, 1991) and C (Weiss, 1992) present physical characteristics of geohydrologic units such as thickness, altitude of the top, and percentage of sand; Chapter D (Ackerman, 1993), E (Ryder and Ardis, in press), H (Martin and Whiteman, in press) and I (Arthur and Taylor, in press) present results of ground-water flow simulations for parts of the gulf coast aquifer systems using a grid element spacing of 5 mi. Chapter F (Williamson, in press) presents results of ground-water flow simulations for the entire gulf coast aquifer systems using grid element spacing of 10 mi. The focus of this report is on regression methodology whereas chapter F of Professional Paper 1416 focuses on the predevelopment (prior to ground-water withdrawal) ground-water flow system and changes due to withdrawal of ground water. Simulations described in chapter F employ hydraulic conductivity estimates from regression analyses described in this report.

DESCRIPTION OF STUDY AREA

The Gulf Coast Regional Aquifer-System Analysis study area is approximately 290,000 mi^2 and consists of part of the states of Arkansas, Florida, Illinois, Texas, Louisiana, Mississippi, Tennessee, Kentucky, Alabama, and Missouri, as well as part of the Gulf of Mexico (fig. 1). The Mississippi River provides the main drainage from the land area. Other important streams are the Rio Grande, Colorado, Arkansas, and Sabine Rivers.

Three aquifer systems have been identified in the study area (Grubb, 1984): The Mississippi embayment aquifer system; its lateral equivalent, the Texas coastal uplands aquifer system; and the coastal lowlands aquifer system (fig. 1). For purposes of this report all three aquifer systems are treated as a unit and are usually referred to as the gulf coast aquifer systems, or simply aquifer system. Comparison among the aquifer systems and detailed discussions of each aquifer system have been presented by Grubb (1987), Williamson and others (1990), Ackerman (1989), Arthur and Taylor (1990), Brahana and Mesko (1988), Martin and Whiteman (1989), and Ryder (1988). The remainder of this section and the Geologic setting section were taken from Williamson and others (1990) with only slight modification.

The land-surface altitude in the study area varies from sea level to more than 800 ft (fig. 2). The dominant feature of the topography is the flat, low-lying Mississippi Alluvial Plain (Fenneman, 1938). The Plain generally lies south and west of the Mississippi embayment structural trough (fig. 3). The topography of the Mississippi embayment is asymmetrical in that the valley lies to the west side of the embayment and the topographically higher hills are mostly to the east side. The Mississippi River, an important feature of the hydrologic system, generally traverses the east side of the valley. The general slope of the land surface is toward the Gulf of Mexico and is incised by large stream valleys that are generally perpendicular to the coastline.

Much of the study area is humid and mean annual precipitation ranges from 20 in. near Mexico to more than 60 in. along the gulf coast of Louisiana, Mississippi, and Alabama. The average mean annual precipitation over the area is 48 in.

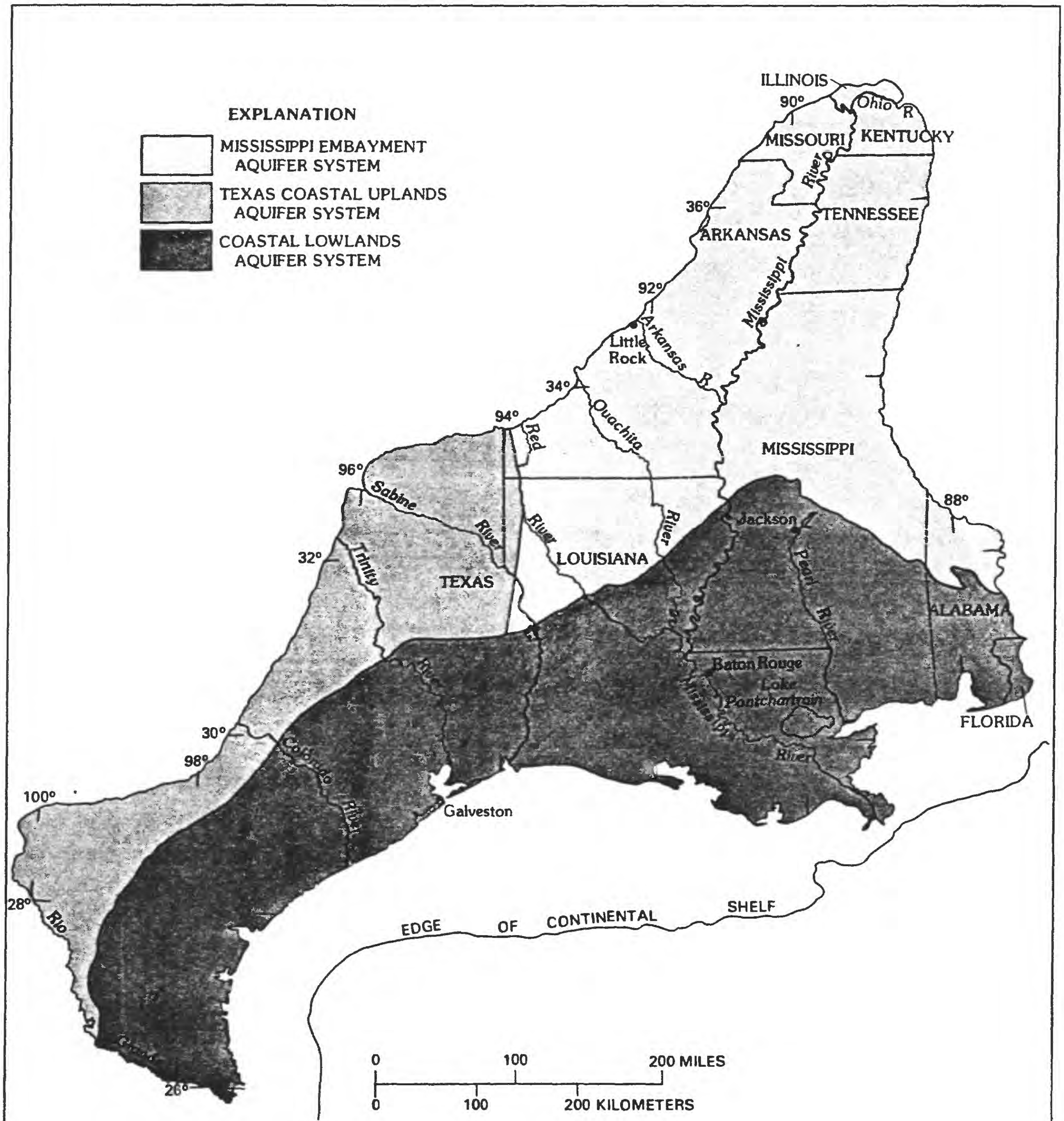

Figure 1.--Location of Gulf Coast Regional Aquifer-System Analysis study area and generalized outcrop of aquifer systems (from Hosman and Weiss, 1991).

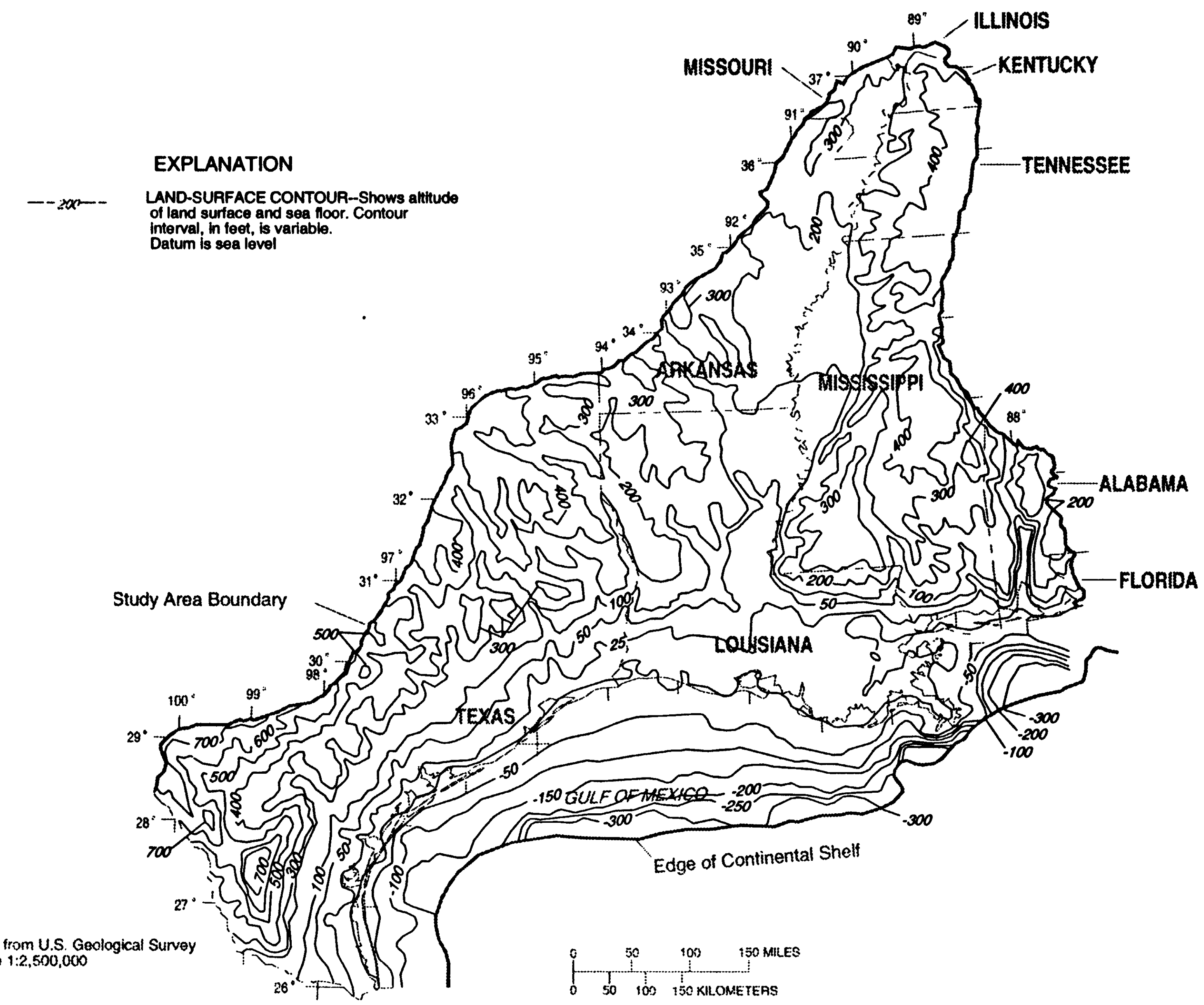

Figure 2. Generalized average land-surface altitude and bathymetry (from Williamson and others, 1990).

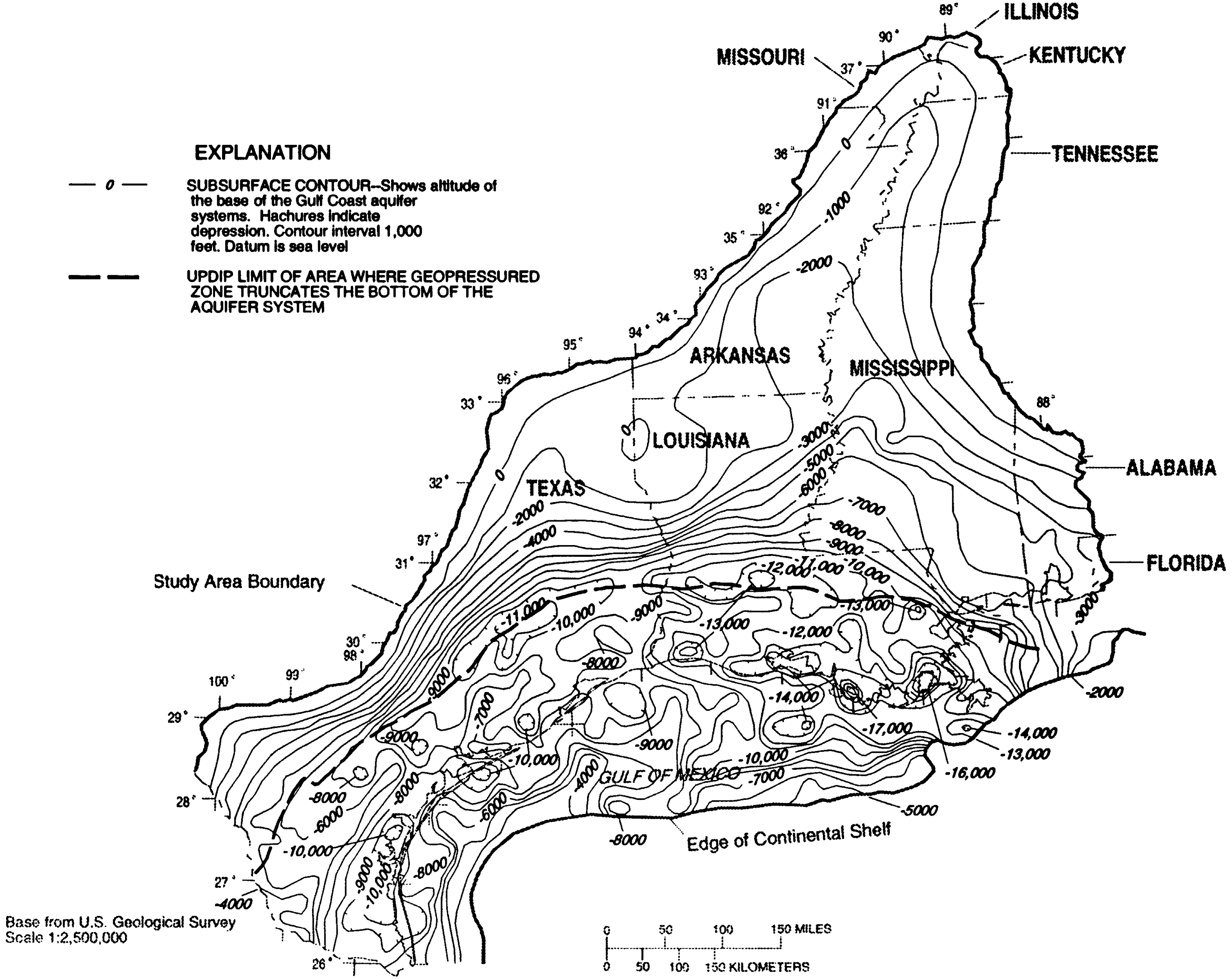

Figure 4. Altitude of the base of the aquifer systems and area of occurrence of geopressure. (From Williamson and others, 1990.)

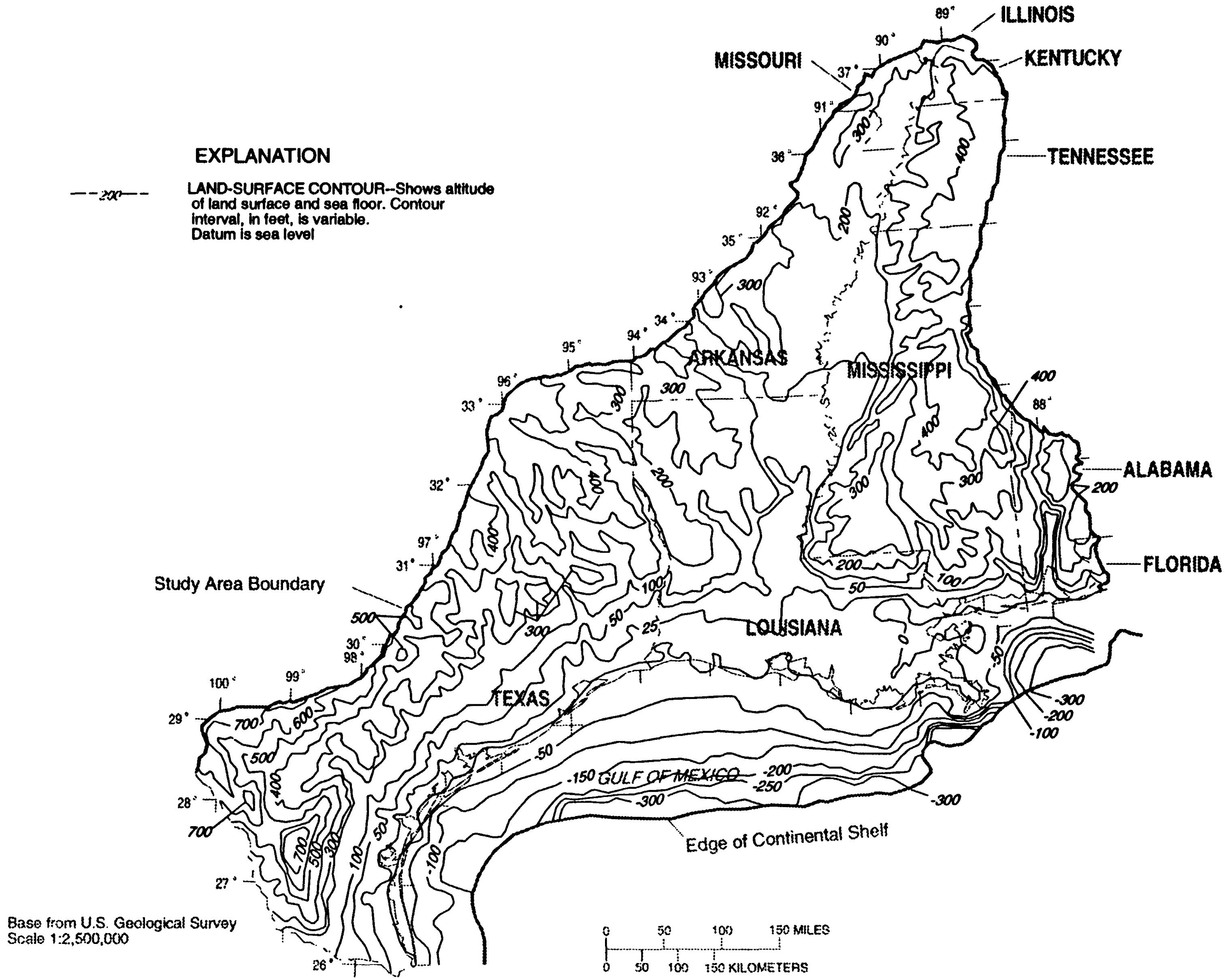

Figure 2. Generalized average land-surface altitude and bathymetry (from Williamson and others, 1990).

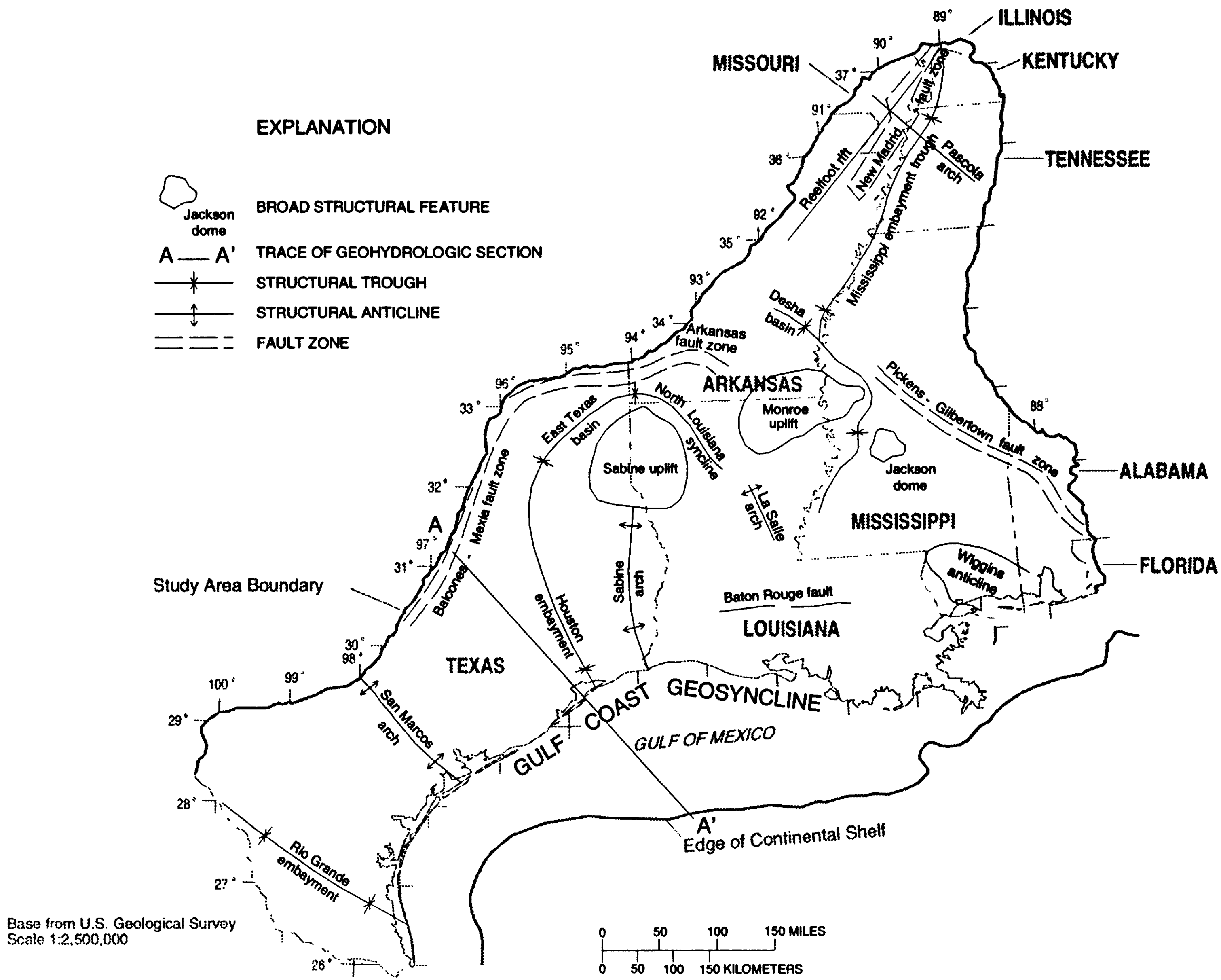

Figure 3. Generalized structural features and location of geohydrologic section (modified from Hosman and Weiss, 1991).

In part of the study area, actual evapotranspiration is limited by the amount of rainfall, which is less than potential evapotranspiration. In south Texas, potential evapotranspiration exceeds rainfall most of the year, especially in the summer and fall; most of the rainfall returns to the atmosphere in a short time. In the eastern part of the area, rainfall substantially exceeds potential evapotranspiration, providing abundant surface-water runoff. The mean annual unit runoff varies from less than 1 in/yr in the southwestern part of the area to more than 20 in/yr in the northern and eastern parts and averages about 15 in/yr (Gebert and others, 1987).

Nearly 10 Ggal/d of ground water was pumped from the gulf coast aquifer systems in 1980 (Mesko and others, 1990). This amount corresponds to approximately 0.9 in/yr spread over the approximately 230,000 mi^2 land portion of the study area. Major areas of irrigation are the Mississippi Alluvial Plain, southwestern Louisiana, and south Texas. More than 60 percent of the municipal and industrial pumpage for public supply and industry is withdrawn from the coastal lowlands aquifer system (fig. 1).

Geologic Setting

The sediments of the gulf coast aquifer systems were deposited during Cenozoic time. Changes in sea level and accompanying transgressions and regressions of the sea caused cyclical sedimentation alternating from predominately continental to predominantly marine environments of deposition. Deposition occurred in fluvial, deltaic, or shallow-marine environments, and the interbedded sequences are composed of sand, silt, and clay, with some gravel, lignite, and limestone. Beds of sediment crop out at the surface in roughly parallel bands that are younger progressively gulfward in a typical offlap sequence. The shifting of facies, both laterally and vertically, resulted in a complex interbedding of sediment types. In general, the more clastic continental deposits have higher permeabilities characteristic of aquifers.

The Gulf Coast geosyncline and the Mississippi embayment, which are the major structural features of the study area (fig. 3), largely control the pattern and thickness of sedimentation (fig. 4). These structural features were present prior to Cenozoic deposition, and were accentuated as the basins subsided and accommodated the increased sediment buildup. Except where affected by local uplifts, the general pattern of sedimentation is one of increasing thickness in a gulfward, downdip direction. Uplift features that affected the deposition patterns are the Sabine uplift, San Marcos arch, Monroe uplift, Pascola arch, Jackson dome, LaSalle arch, and Wiggins uplift. Downwarp features associated with greater sediment buildup are the Desha basin, East Texas basin, Houston embayment, Rio Grande embayment, and Terrebonne embayment.

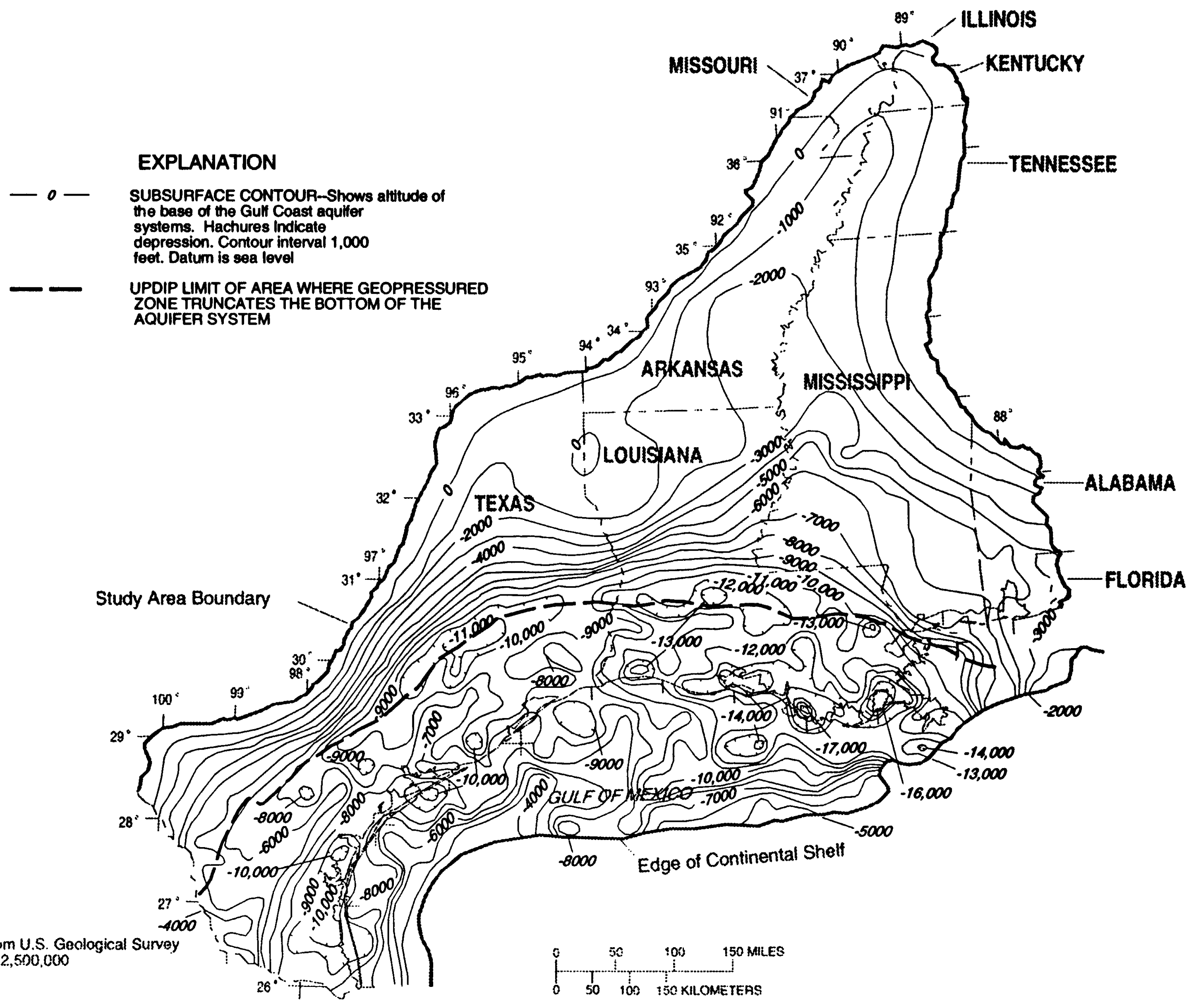

Figure 4. Altitude of the base of the aquifer systems and area of occurrence of geopressure. (From Williamson and others, 1990.)

Faults are common throughout the area, although their effect on regional ground-water movement is not known. In general, fault throws are not great enough to offset the full thickness of hydrologic units described in this report, although individual beds could be offset. Much of the faulting has led to zones of grabens and horsts. Particularly pronounced zones near the perimeter of the Mississippi embayment and the gulf coast geosyncline are the New Madrid fault zone, the Luling-Mexia-Talco fault zone, and the Pickens-Gilbertown fault zone (fig. 3). The Reelfoot rift zone is within the Mississippi embayment and may provide a pathway for upward discharge of ground-water from underlying Paleozoic rocks. Numerous growth faults, which occur contemporaneously with deposition, exist farther gulfward.

Geopressured zones (fig. 4) have fluid pressures significantly greater than normal hydrostatic pressure, and are enclosed by zones of very low permeability. The most probable cause for the development of these abnormally high fluid pressures is restriction of the escape of fluid during sediment compaction, causing pressure buildup and undercompaction of sediments (Fertl, 1976, p. 16).

Salt domes occur throughout the gulf coast aquifer systems, particularly in belts near and roughly parallel to the present-day coastline. The source of the salt domes is the deeply buried Louann Salt of Jurassic age, which has risen as diapirs that penetrate varying amounts of Cenozoic strata. The structural effects of the domes are relatively localized. However, the domes may have a significant effect on water quality due to dissolution of salt by ground water.

The gulf coast aquifer systems were subdivided into 15 hydrogeologic units that correspond to layers in a digital model used for simulation of regional ground-water flow. Five of these are considered to be confining units within which there is almost no horizontal flow. Weiss and Williamson (1985) describe the principles and methodology used for subdividing the aquifer systems into the 15 hydrogeologic units that is based on a combination of lithologic and hydraulic information. Hosman and Weiss (1991) provide a detailed description of the units of the Mississippi embayment and Texas coastal uplands aquifer systems. Weiss (1992) gives a detailed description of the units of the coastal lowlands aquifer system. Table 1, from Williamson and others (1990), gives the relationship and numbering convention for the 15 model layers used in this study to the geology and the previously mapped units in other reports in this series. The 15 model layers are numbered 2 through 11 for permeable layers or aquifers and 13 through 17 for confining units as shown in table 1. In this study, the model layers are referred to by number. The model construction described in the following section gives a detailed description of the geometry of the 15 model layers.

TABLE 1.–*Relation of geologic units, previously defined geohydrologic units, and layers used in regional flow model (from Williamson and others, 1990, table 2)*

[Note: correlations shown here are generalized. Exact relations vary widely from place to place.]

Mississippi embayment and Texas coastal uplands aquifer systems

Geologic Unit: System	Geologic Unit: Series	Geologic Unit: Group	Geohydrologic units defined by previous studies	Gulf Coast Regional Aquifer-System Analysis: Model layer number	Gulf Coast Regional Aquifer-System Analysis: Geohydrologic units
Quaternary	Pleistocene and Holocene		Mississippi River Valley alluvial aquifer (Boswell and others, 1968)	11	Mississippi River Valley alluvial aquifer *
Tertiary	Eocene and Oligocene	Jackson and Vicksburg		15	Vicksburg-Jackson confining unit 1/
Tertiary	Eocene	Claiborne	Cockfield aquifer system (Payne, 1970) Cockfield Formation (Hosman and others, 1968)	6	Upper Claiborne aquifer
Tertiary	Eocene	Claiborne	Cook Mountain Formation	14	Middle Claiborne confining unit
Tertiary	Eocene	Claiborne	Sparta hydraulic system (Payne, 1968) Sparta Sand (Hosman and others, 1968) Memphis aquifer (Hosman and others, 1968) (layers 4 and 5)	5	Middle Claiborne aquifer
Tertiary	Eocene	Claiborne	Cane River Formation	13	Lower Claiborne confining unit
Tertiary	Eocene	Claiborne	Carrizo and Meridian Sand aquifer (Payne, 1975) Carrizo Sand and Meridian-upper Wilcox aquifer (Hosman and others, 1968)	4	Lower Claiborne upper Wilcox aquifer
Tertiary	Eocene	Wilcox	Wilcox Group (Hosman and others, 1968)	3	Middle Wilcox aquifer
Tertiary	Eocene / Paleocene	Wilcox	Lower Wilcox aquifer (Hosman and others, 1968)	2	Lower Wilcox aquifer*
Tertiary	Paleocene	Midway		12	Midway confining unit 1/

1/ The Midway confining unit was referred to as the coastal uplands confining unit and the Vicksburg-Jackson confining unit was referred to as the coastal lowlands confining unit by Grubb, (1984, p. 11).

* Not present in the Texas coastal uplands aquifer system.

TABLE 1.—*Relation of geologic units, previously defined geohydrologic units, and layers used in regional flow model (from Williamson and others, 1990, table 2)—Continued*

Coastal lowlands aquifer system

Geologic Unit			Geohydrologic units defined by previous studies	Gulf Coast Regional Aquifer-System Analysis	
System	Series	Group		Model layer number	Geohydrologic units
Quaternary	Pleistocene and Holocene		Upper Chicot aquifer (Jorgensen, 1975) Chicot aquifer (Meyer and Carr, 1979)	11	Permeable zone A (Holocene-upper Pleistocene deposits)
				10	Permeable zone B (Lower Pleistocene-upper Pliocene deposits)
Tertiary	Pliocene		Evangeline aquifer (Whitfield, 1975) (Meyer & Carr, 1979)	9	Permeable zone C (Lower Pliocene-upper Miocene deposits)
	Miocene		'2,000-foot' sand of the Baton Rouge area (Torak and Whiteman, 1982) Jasper aquifer (Whitfield, 1975)	17	Zone D confining unit
				8	Permeable zone D (Middle Miocene deposits)
				16	Zone E confining unit
	Eocene and Oligocene	Jackson and Vicksburg		7	Permeable zone E (Lower Miocene-upper Oligocene deposits)
				15	Vicksburg-Jackson confining unit 1/

1/ The Midway confining unit was referred to as the coastal uplands confining unit and the Vicksburg-Jackson confining unit was referred to as the coastal lowlands confining unit by Grubb, (1984, p. 11).

Ground-Water Flow

The essential feature of the regional flow system is flow of ground water from upland recharge areas to discharge areas at lower altitudes on land or the sea floor. Differences from this simplified description are caused by the effect of a large volume of dense water that tends to flow beneath fresher water. Except for a small amount of water that may move upward from the geopressured zone into the aquifer system, little flow probably occurs through the aquifer system bottom. There is no lateral flow from beyond the landward boundary of the study area because the sediments that make up the aquifer systems pinch out. The potential for lateral flow exists along the Mexican border and along the north-south boundary of the study area in southern Alabama and western Florida. An arbitrary extension of the aquifer system thicknesses was made into Mexico because no data were available. The flows along the north-south boundary of the study area in southern Alabama and western Florida were ignored because of the minimal permeability of the sediments and the relatively short distance compared to the perimeter of the study area. Recharge from the uplands in south Texas may be limited by the availability of water, considering the fact that potential evapotranspiration exceeds precipitation at some locations. Flows are of course altered by pumping. The areal distribution of pumping from the 10 aquifers, model layers 2-11, for the year 1980 is shown in figure 5. The similarity between the water-table altitude and land-surface altitude is shown by comparing figure 6 with figure 2.

The presence of dense water in the system has a considerable effect upon ground-water flow. A large volume of water having a concentration of dissolved solids greater than seawater is contained within the aquifer systems (fig. 7). Sea water has a concentration of dissolved solids of about 35,000 mg/L and a density of 1.025 gm/cm^3. The estimated depth to water with a concentration of dissolved solids the same as seawater is shown in figure 7. Estimates of dissolved-solids concentrations in ground water were made for about 18,000 sand beds in the study area (Weiss, 1987). About 66 percent of the beds have water with a dissolved-solids concentration greater than 10,000 mg/L (fig. 8). Although not a precise measure of the quantity of water dense enough to effect ground-water flow, the above indicate that much of the volume of water in the gulf coast aquifer systems has a density substantially greater than 1.0 gm/cm^3.

Data Available for Hydraulic Head and Hydraulic Conductivity

More than 600,000 individual measurements of hydraulic head were available. When considered as observations of hydraulic head in a particular well at a particular instant in time, these hydraulic-head measurements are, in general, accurate; except for those cases where errors in recording were made, they usually have less than 0.5 ft of error. For each well, the hydraulic-head measurements were averaged over 2-year intervals to give a representative value of hydraulic head for the well for

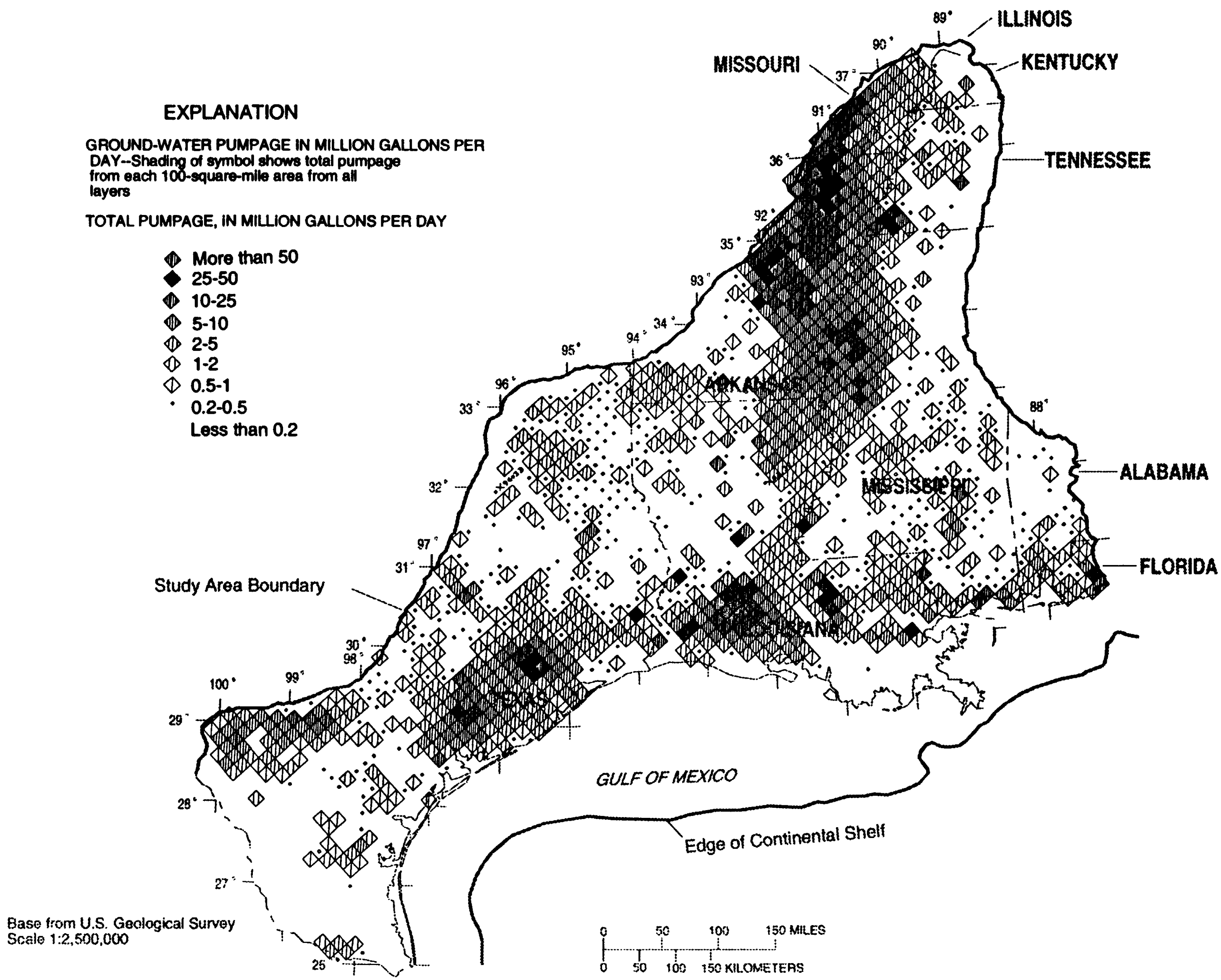

Figure 5. Ground-water pumpage from the gulf coast aquifer systems, 1980. (Modified from Williamson and others, 1990.)

Figure 6. Water-table altitude on shore and equivalent freshwater head at the sea floor offshore. (Modified from Williamson and others, 1990.)

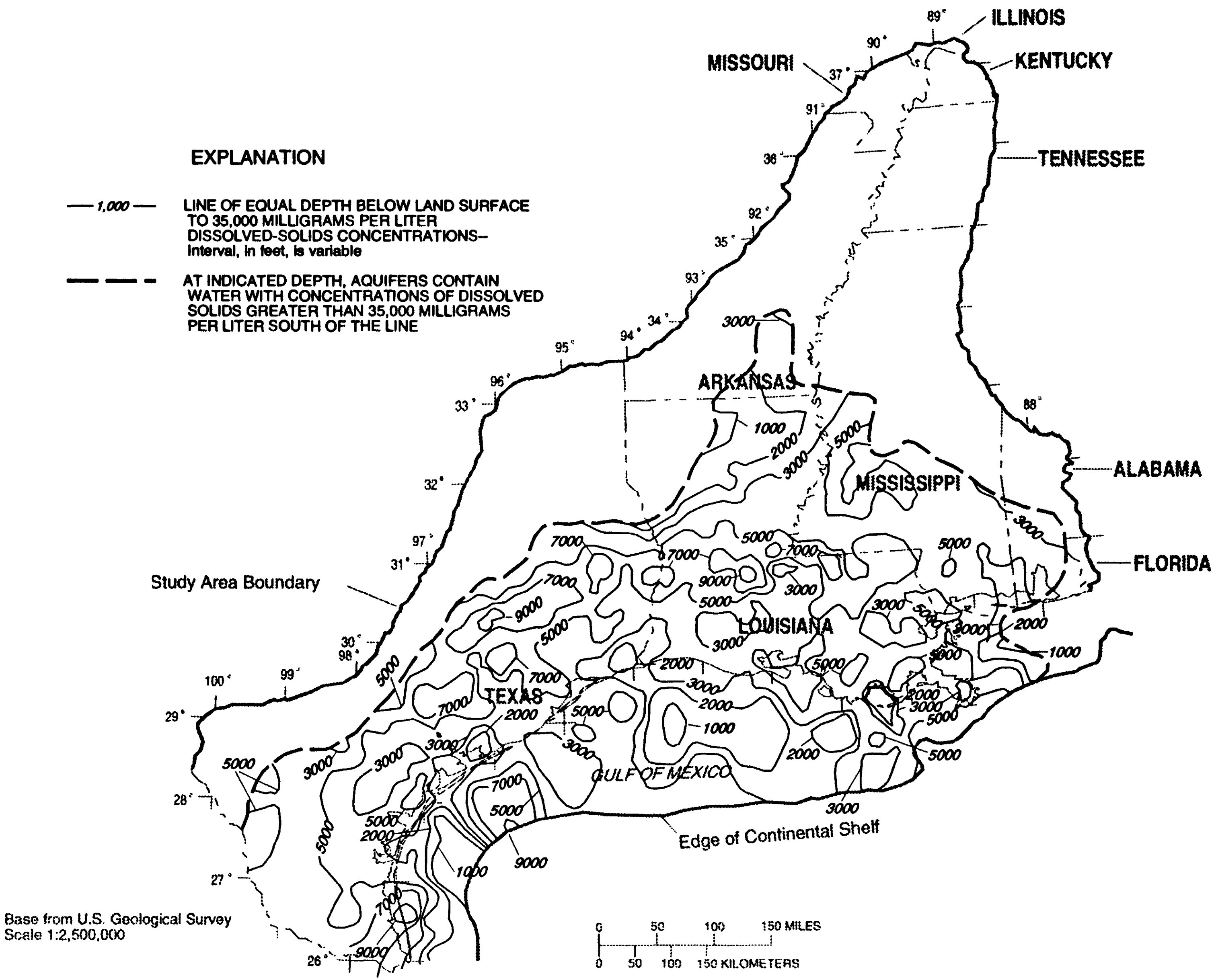

Figure 7. Estimate of depth to water containing 35,000 mg/L dissolved solids.

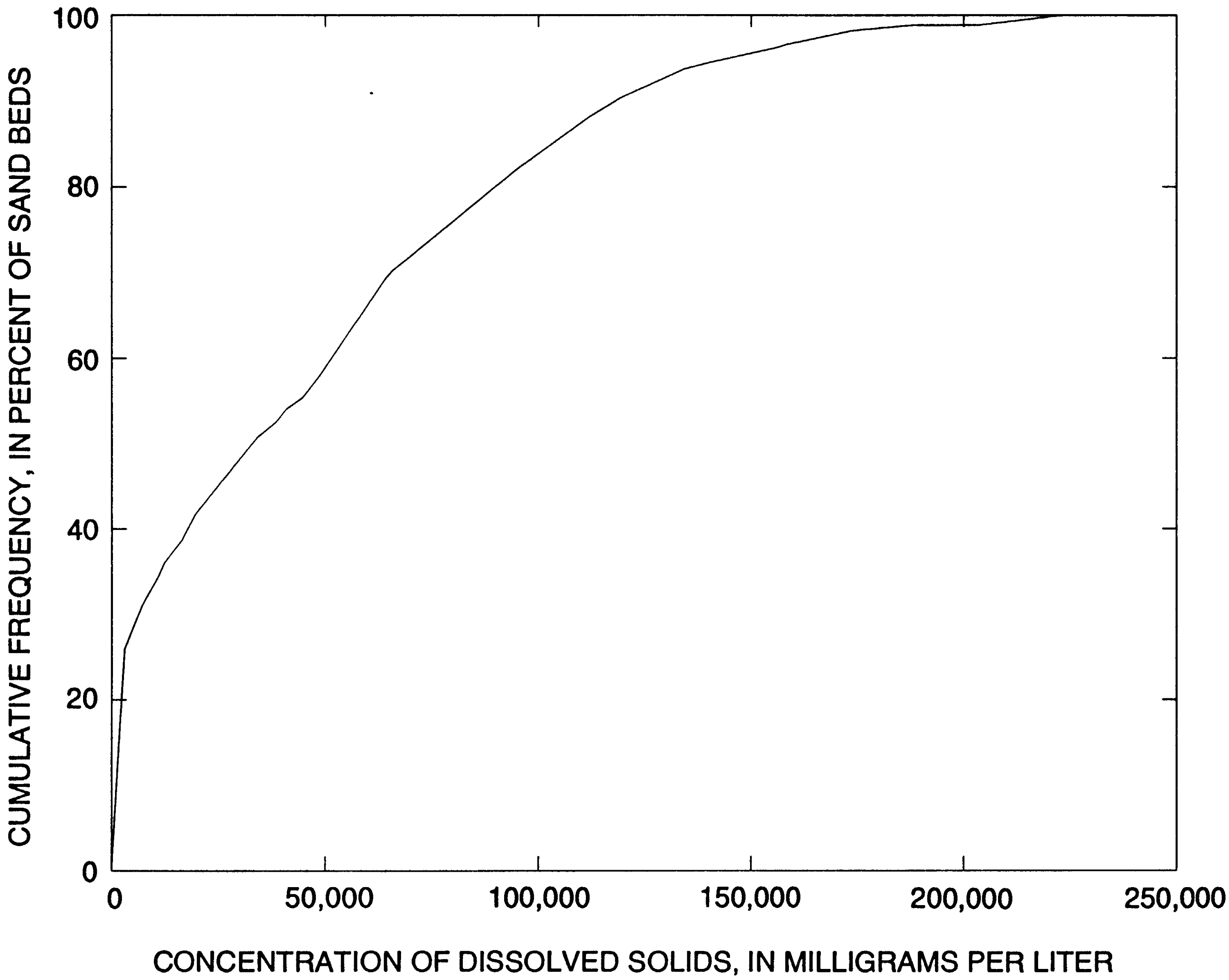

Figure 8. Cumulative frequency distribution of concentration of dissolved solids in 18,000 sand beds in the gulf coast aquifer systems

a given year centered within the 2-year interval. In this manner, the more than 600,000 measurements were reduced to approximately 50,000 time-averaged observations of hydraulic head at individual wells. In addition, there were approximately 15,000 measurements of formation pressure from drill-stem tests. These measurements were used to approximate point-pressure head values. Because of the indirect procedure used, these point pressure head values from drill-stem test can have errors much larger than 0.5 ft. As is generally the case, the numerical-flow model used in this study produces an approximation to grid-element volume-averaged head values that are not equal to values for head at a single point in the aquifer system.

More than 6,700 determinations of hydraulic conductivity were available (Prudic, 1991) from both specific capacity and pumping tests. As with the observations of hydraulic head, most of these determinations of hydraulic conductivity were made using water wells less than 1,000 ft deep.

Because the water-well based data on hydraulic head and hydraulic conductivity were from relatively shallow depths, (and the point pressure-head values from the drill-stem test were quite inaccurate, even though in some cases occurring at considerable depth) the aquifer systems may be characterized as having a paucity of data for all but relatively shallow depths.

NONLINEAR-REGRESSION FLOW MODEL

Ground-water flow in the aquifer systems is assumed to be governed by the general equations for variable-density flow (De Weist, 1969).

The domain of solution of the aquifer systems has as its top that surface consisting of either the water table or land surface or sea bottom, whichever is lowest. This surface is relatively static and is taken as a fixed head surface of the domain of solution. The movement of water outside the domain of solution, such as the flow of water moving downward to the water table, is not simulated. The bottom of the domain of solution is a no-flow boundary when above a geologic unit which is assumed to have no flow. When above the geopressured zone, flow may come from the geopressured zone which is given a specific head.

Along the perimeter, the aquifer system domain of solution thins out and has zero thickness at most locations, except along certain parts of the perimeter offshore in the Gulf of Mexico. Here, nonzero thicknesses are present below the sea floor, but are assumed to allow no flow due to the presence of extremely low permeability clacareous clays.

Specific storage is taken to be constant throughout the entire aquifer system domain of solution. Water density is assumed to be nearly constant in time but to vary with location in the domain of solution.

Hydraulic conductivity varies within the domain of solution. Effective hydraulic conductivity at any location is determined (Desbarats, 1982) from the hydraulic conductivities of the clay and sand at that location and the relative amounts of each of these components as determined from data. The hydraulic conductivities of the clay and sand components are allowed to vary, but use is made of existing data for these conductivities and the manner in which they decrease with depth.

Although the top of the domain of solution has a fixed head, a maximum allowable recharge from that surface is imposed. This maximum value ranges from 2 in/yr to 12 in/yr, decreasing towards the south.

Modeling Methodology and Construction

Ground-water flow in the aquifer systems is assumed to be governed by the following three-dimensional variable-density time-dependent flow equations (Kuiper, 1983). The general vector form of Darcy's equation (De Wiest, 1969; Bear, 1979; Kuiper, 1983), is

$$\bar{q} = -K[\nabla h + (\rho/\rho_0)\nabla z] \qquad (L/T), \qquad (1)$$

Here, $\bar{q}$ is the specific discharge of the fluid, and h is pressure head equal to $P/\rho_0 g$, where ρ_0 = 1 gm/cm^3, g is the acceleration of gravity, and P is pressure. Equivalent freshwater head, equal to (h+z), will be referred to simply as head in the following text. The quantity z is vertical distance measured upward above a datum, $\rho = \rho(x,y,z)$ is the density of the fluid and is assumed to vary with time so slowly that its time dependence is ignored (Kuiper, 1983). Equivalent freshwater head is approximately equal to hydraulic head ($P/\rho g + z$) when fluid density is approximately equal to ρ_0. The hydraulic conductivity tensor K = K(x,y,z) is given by

$$K = k\rho_0 g/\mu \ , \qquad (2)$$

where k is the intrinsic permeability tensor of the porous medium, and μ is the dynamic viscosity of the fluid. The mass balance equation (De Wiest, 1969) is

$$\nabla\bullet(\rho\bar{q}) = -\rho S_s \partial h/\partial t \ -Q \qquad (M/L^3\ T) \ , \qquad (3)$$

where $S_s(x,y,z)$ is specific storage, t is time, and Q(x,y,z,t) is the rate of discharge of fluid per unit volume due to pumping. When equation (1) is substituted for $\bar{q}$ of equation (3), the governing flow equation for h(x,y,z,t) is obtained.

Equation (3), when discretized, forms a matrix equation to be solved for the grid-element volume-averaged heads corresponding to each of the grid elements in the domain of solution. This matrix equation is used to formulate the regression equation for the regression technique (Cooley, 1982). The hydraulic parameters of the aquifer systems, such as specific storage and hydraulic conductivity, and also other unknowns such as specified head or flow on the domain of solution boundary, are specified in some chosen manner by the regression parameters of the regression technique.

The discretization of the domain of solution is defined vertically by the 15 model layers. Grid elements with a constant 10-mi spacing define horizontal discretization. The spacing is constant because of the use of finite-differences which require that grid element rows and columns follow straight lines, and because the regions of the aquifer system that would benefit from small grid element size are numerous and are spread over the aquifer system in an irregular manner. The 10-mi spacing is the smallest allowable, given computer-time constraints.

The total number of grid elements in the domain of solution of the aquifer system is N = 29,345. The grid elements are divided into 10 model layers mentioned earlier and numbered 2 through 11 from bottom to top. Layer 1 is for the geopressured zone. A very thin layer, model layer 12, is located on the top of all of the other layers. This 12th model layer represents the top several feet of the aquifer system. Maximum areal extent and outcrop areas of the hydrogeologic units represented by the model layers are shown in figure 9. The grid element row and column numbers are also shown. Five layers representing confining units mentioned previously and numbered 13 through 17 are interbedded between model layers 4 through 9. The layers representing confining units provide resistance to vertical flow between the adjacent aquifers above and below but do not have any horizontal flow themselves, nor do they store water. The layers that represent confining units do not have grid elements (Kuiper, 1985) and the associated approximating equations do not directly involve head in the confining units. The areal extent of model layers 2 through 11 are shown in figures 10 through 19, and the areal extent of model layers 13 through 17 which represent confining units are shown in figures 20 through 24. The vertical relation of aquifers and confining units across the central part of the study area is shown in figure 25.

Boundary conditions are either specified-flow or specified-head. Specified-flow boundaries are always no-flow boundaries. There are two separate specified-head boundaries, the top of the domain of solution and the geopressured zone part of the bottom of the domain of solution. Each of these two surfaces has two regression parameters associated with it. The specified-boundary head on such a surface is equal to the sum of the product of an initial head distribution, to be described later, with a regression parameter associated with the surface, and an additional second parameter for the surface.

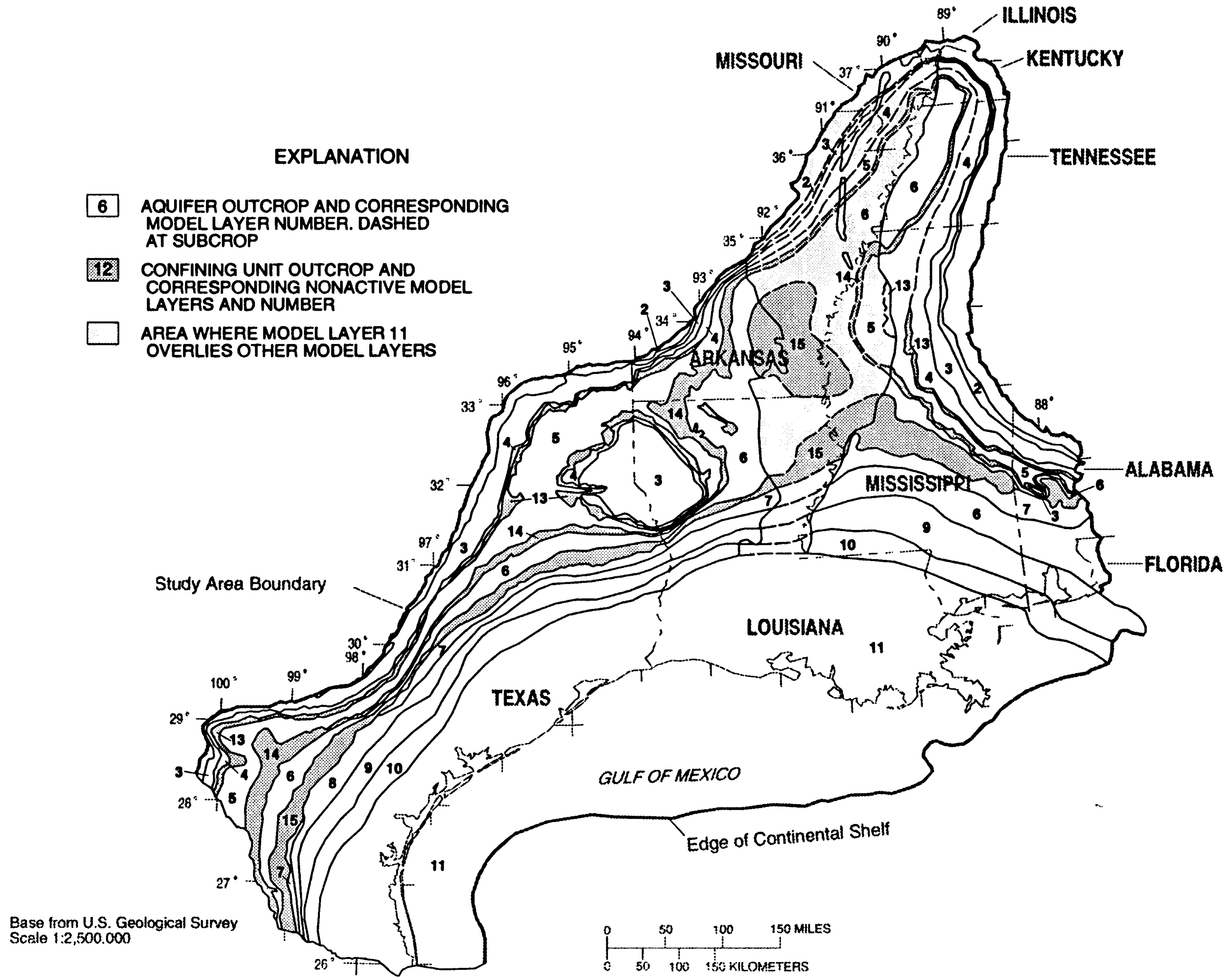

Figure 9. Maximum areal extent of modeled area and outcrop areas of hydrogeologic units represented by model layers (modified from Williamson and others, 1990).

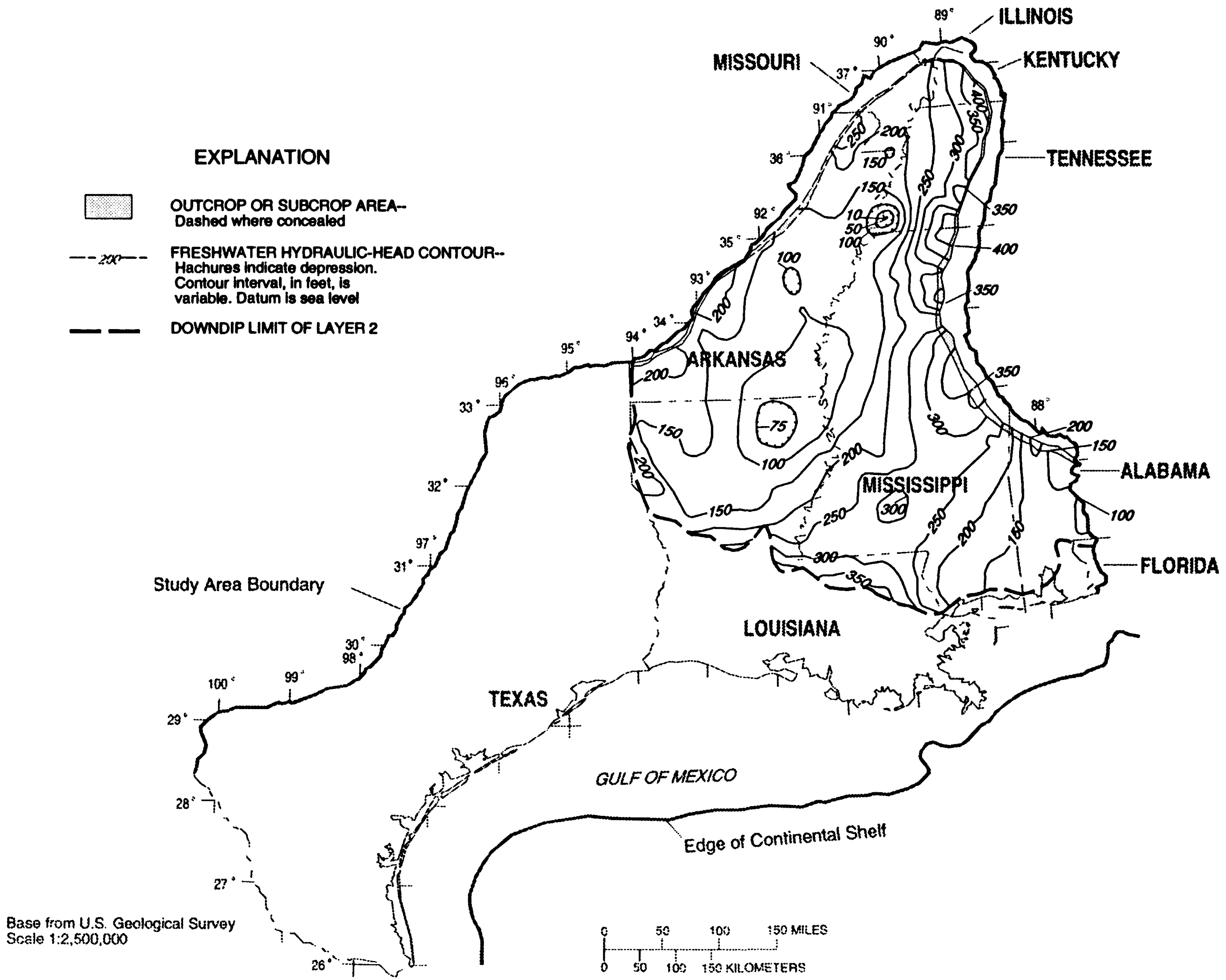

Figure 10. Areal extent of model layer 2 and simulated 1982 equivalent freshwater head from model 4.

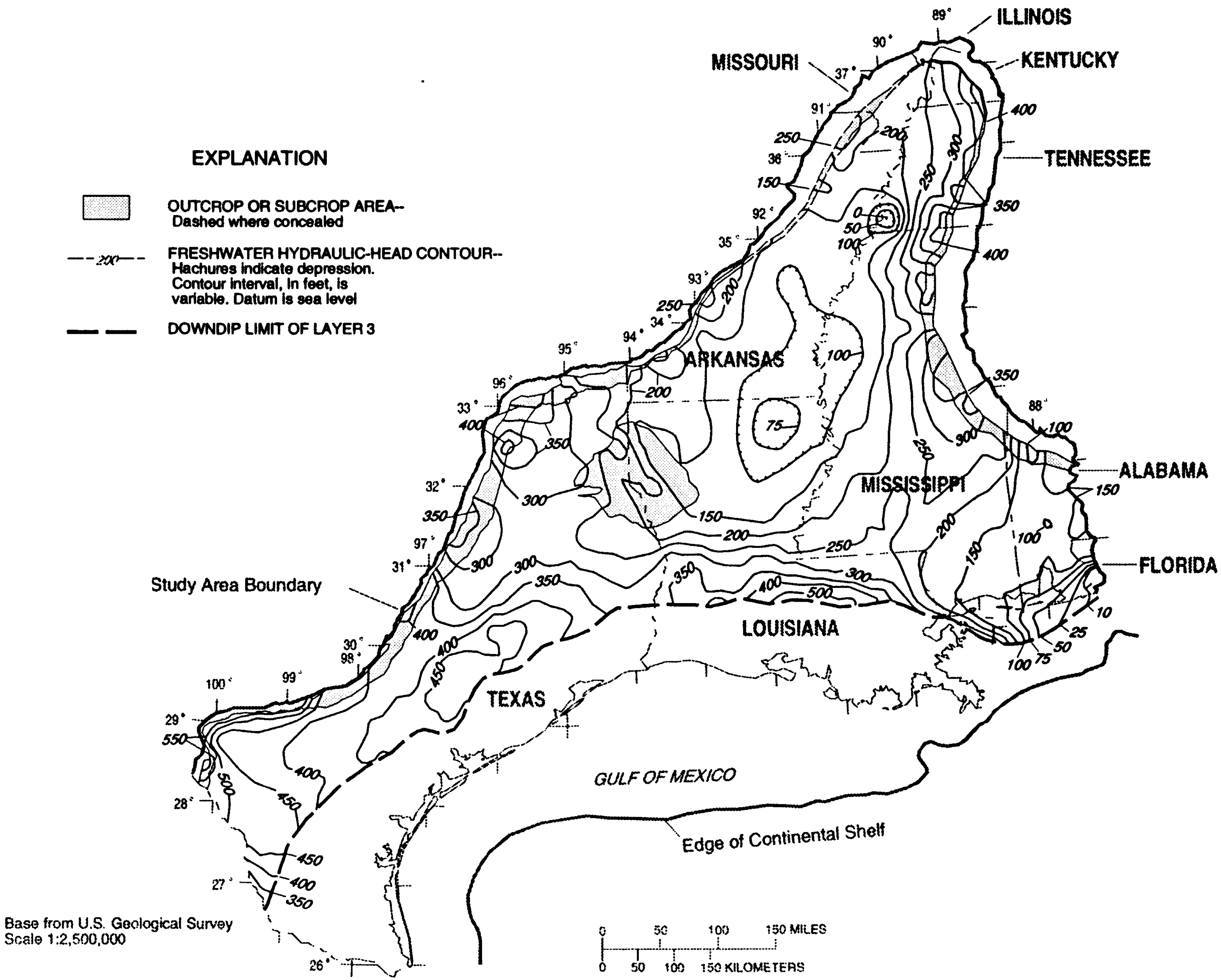

Figure 11. Areal extent of model layer 3 and simulated 1982 equivalent freshwater head from model 4.

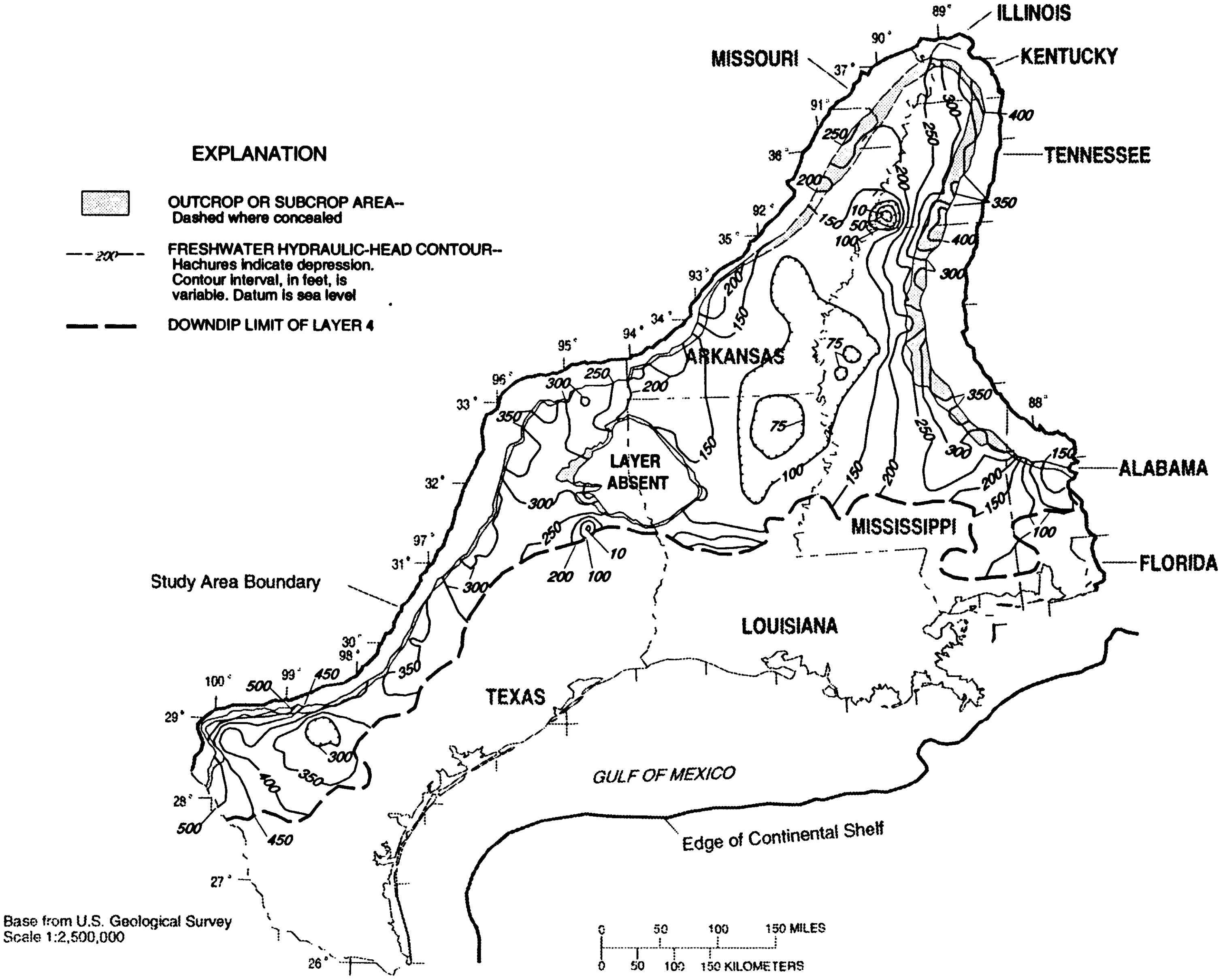

Figure 12. Areal extent of model layer 4 and simulated 1982 equivalent freshwater head from model 4.

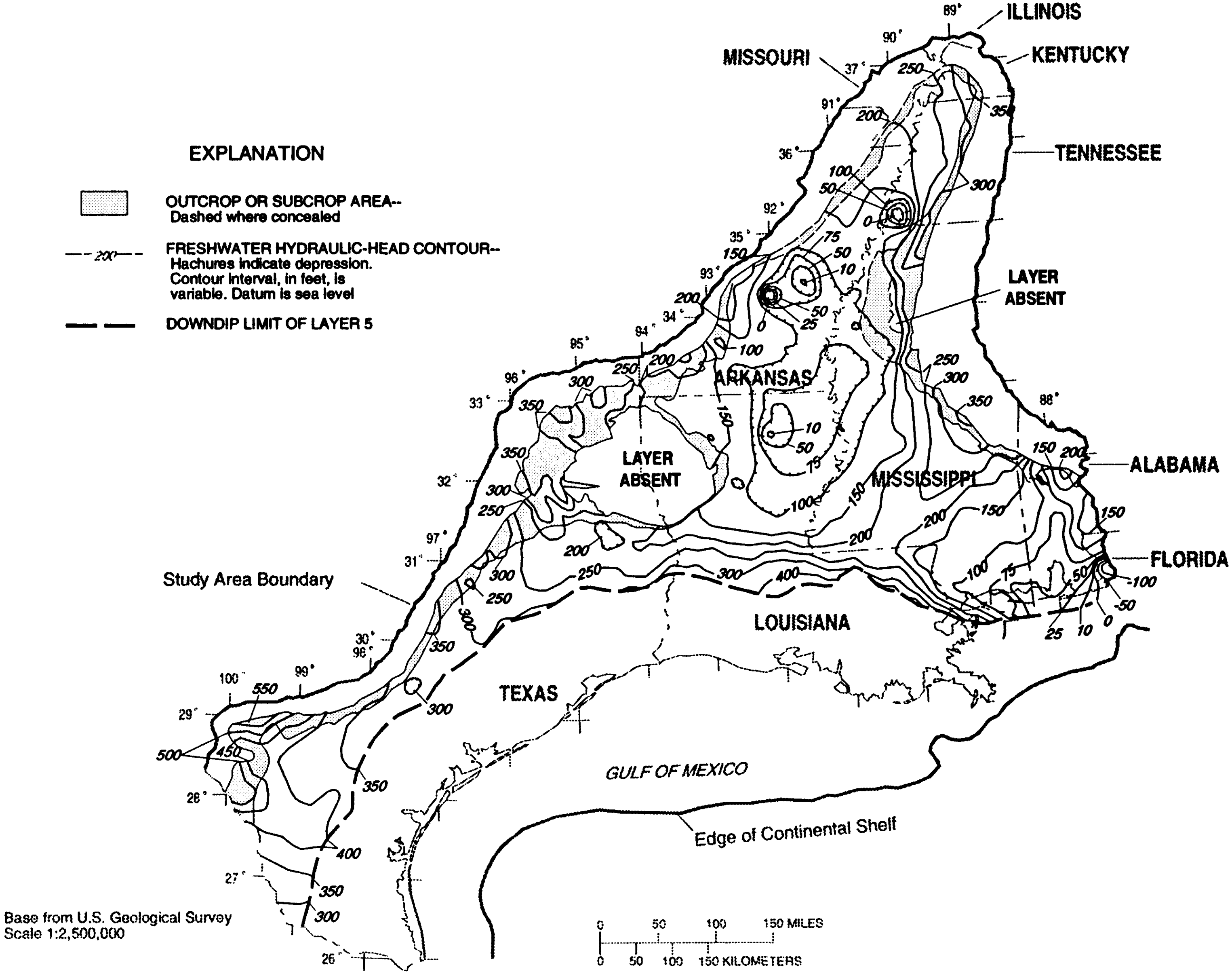

Figure 13. Areal extent of model layer 5 and simulated 1982 equivalent freshwater head from model 4.

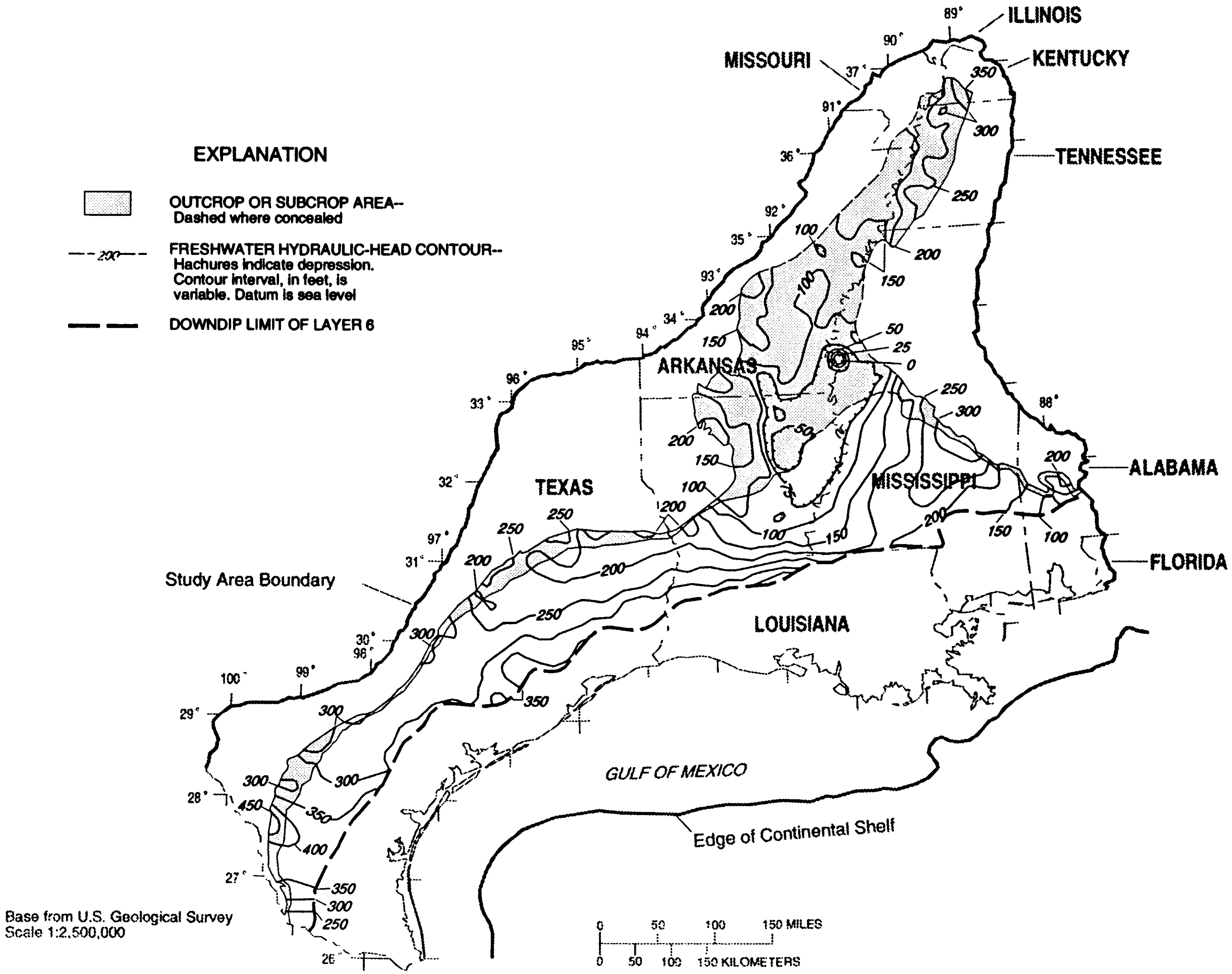

Figure 14. Areal extent of model layer 6 and simulated 1982 equivalent freshwater head from model 4.

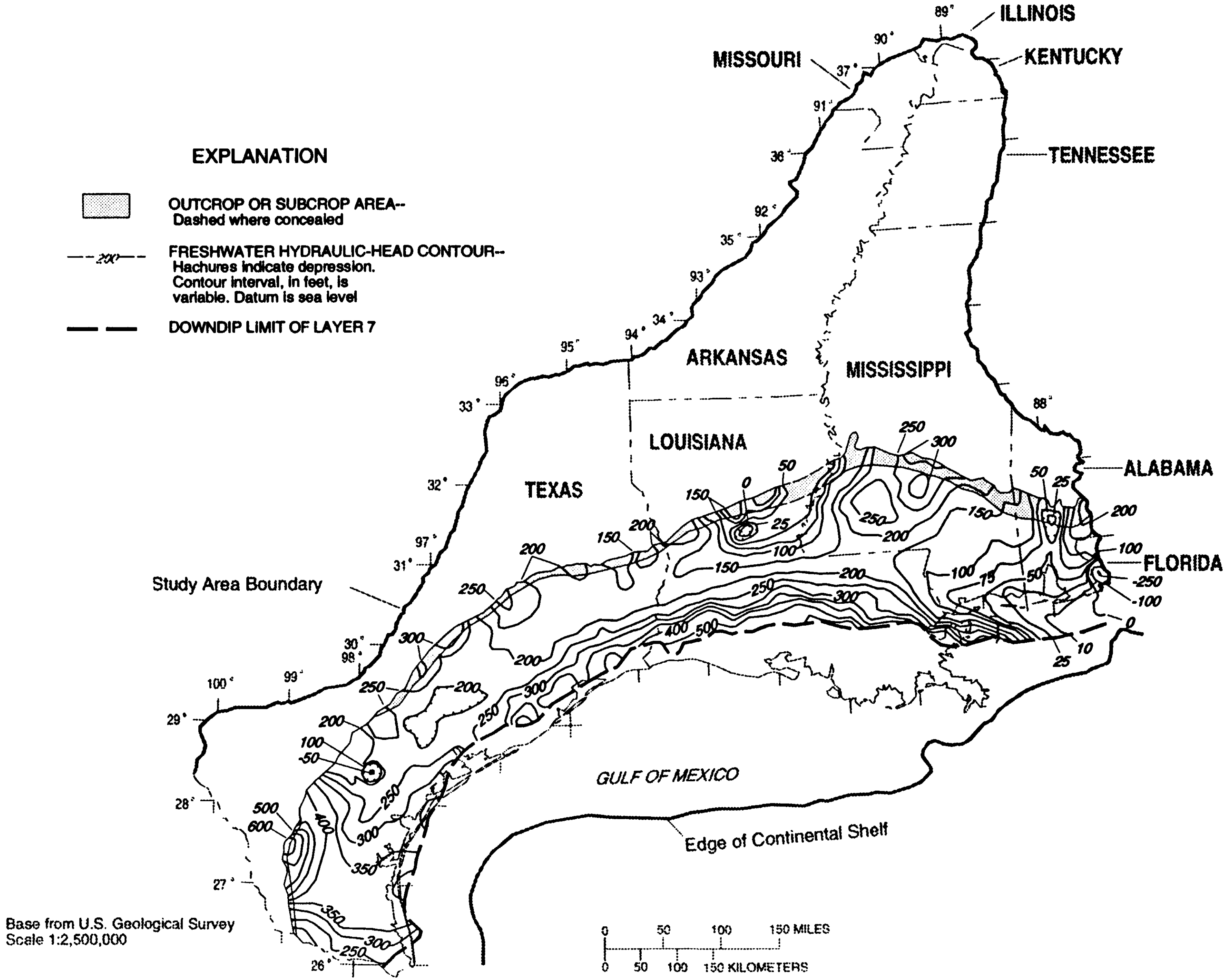

Figure 15. Areal extent of model layer 7 and simulated 1982 equivalent freshwater head from model 4.

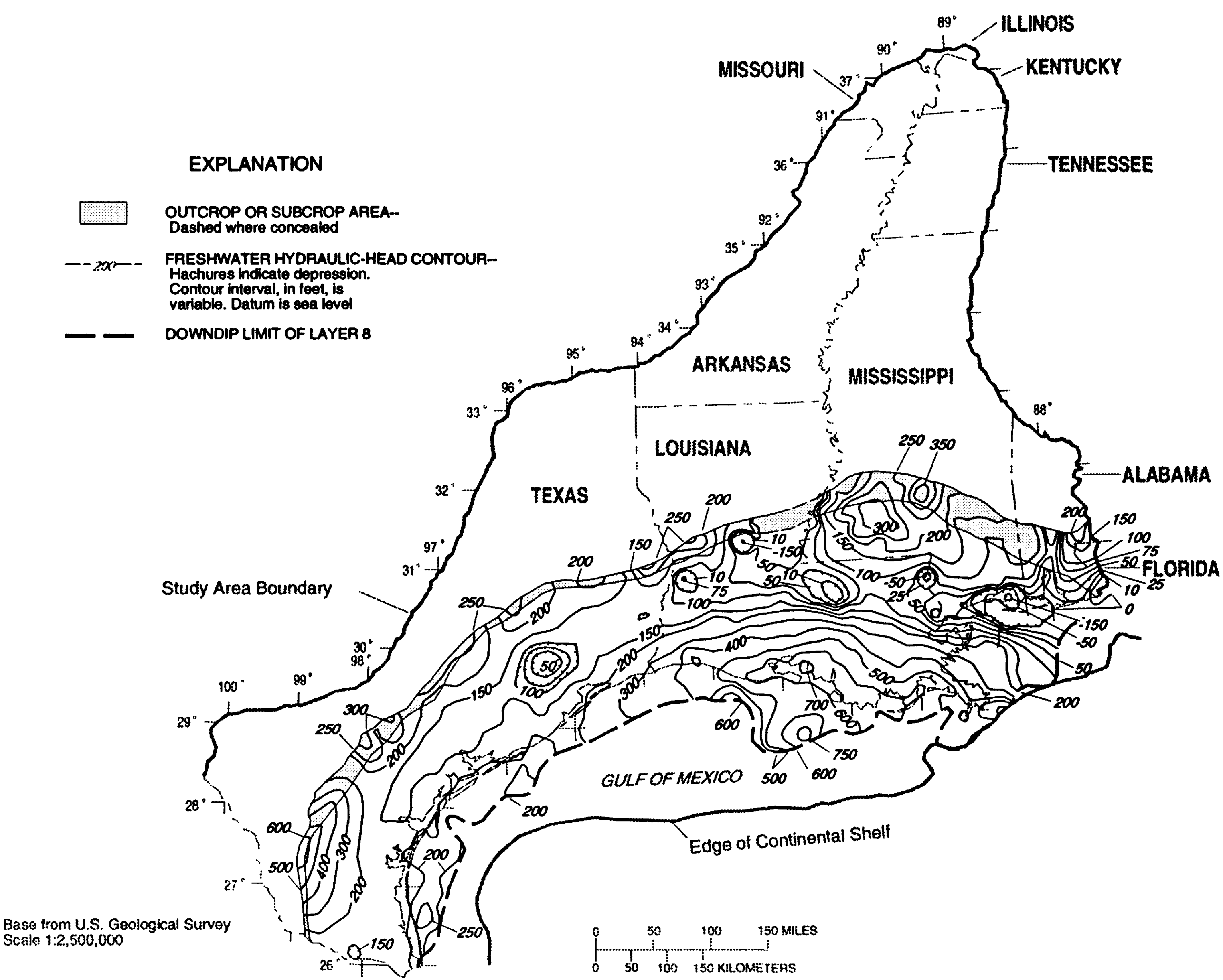

Figure 16. Areal extent of model layer 8 and simulated 1982 equivalent freshwater head from model 4.

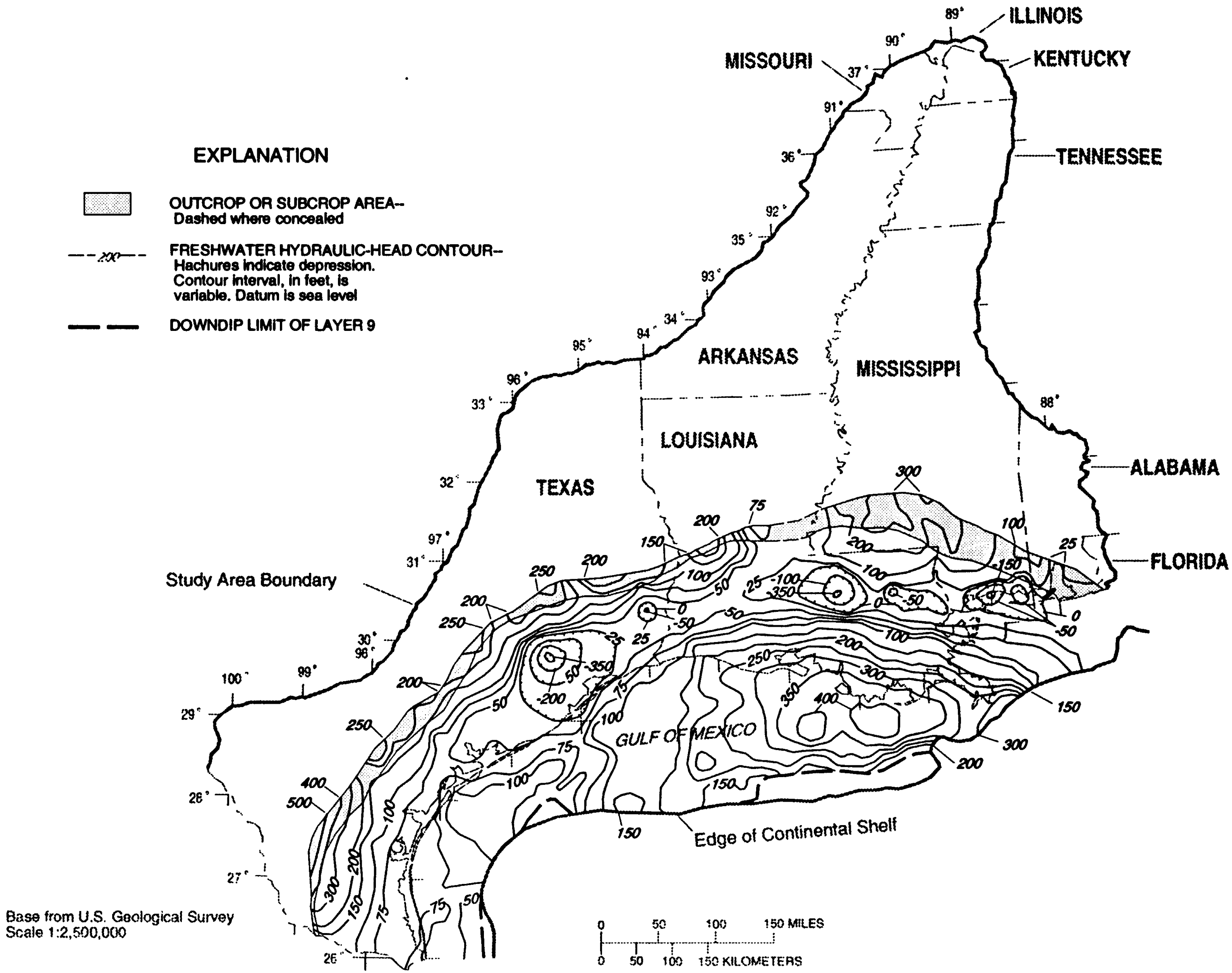

Figure 17. Areal extent of model layer 9 and simulated 1982 equivalent freshwater head from model 4.

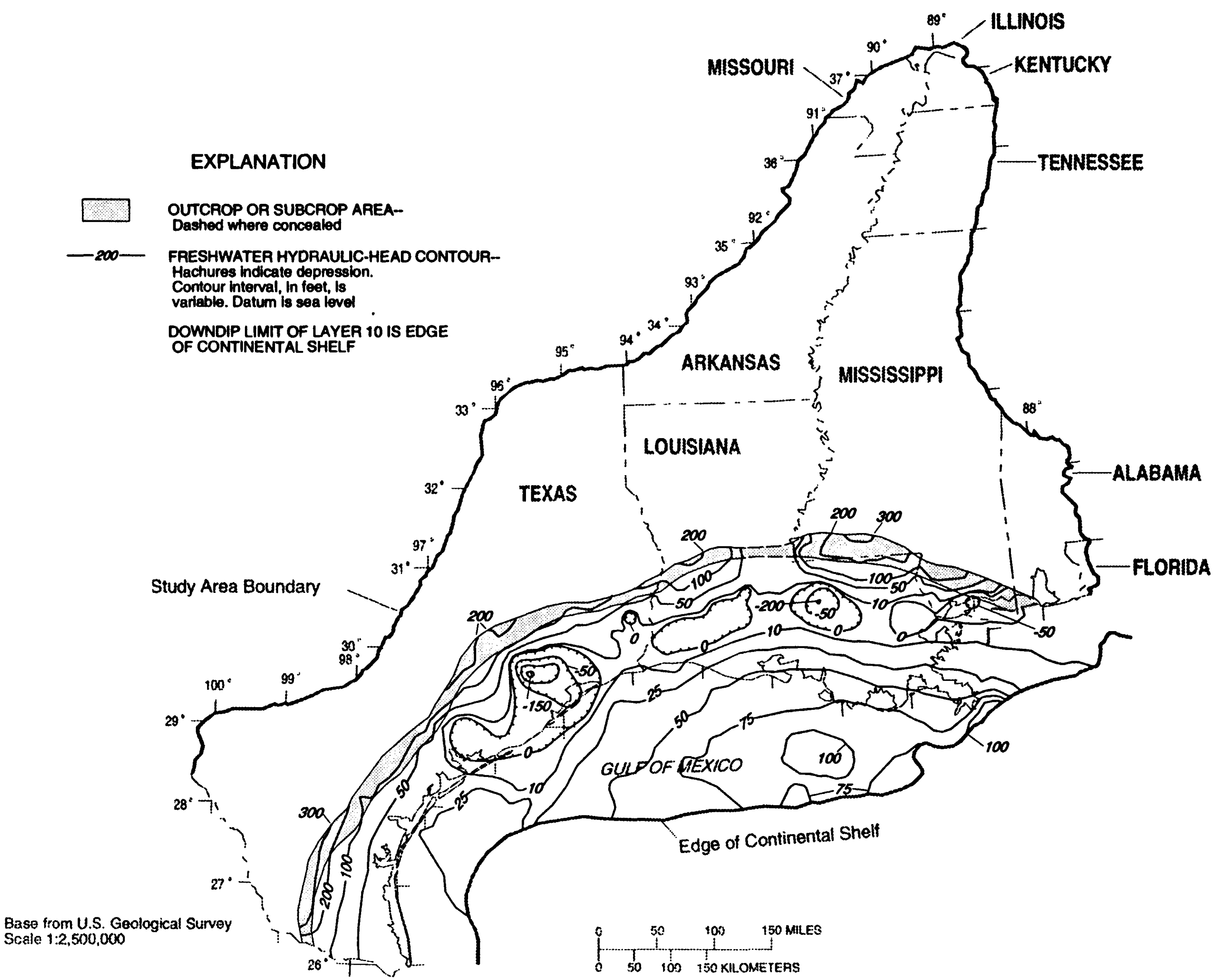

Figure 18. Areal extent of model layer 10 and simulated 1982 equivalent freshwater head from model 4.

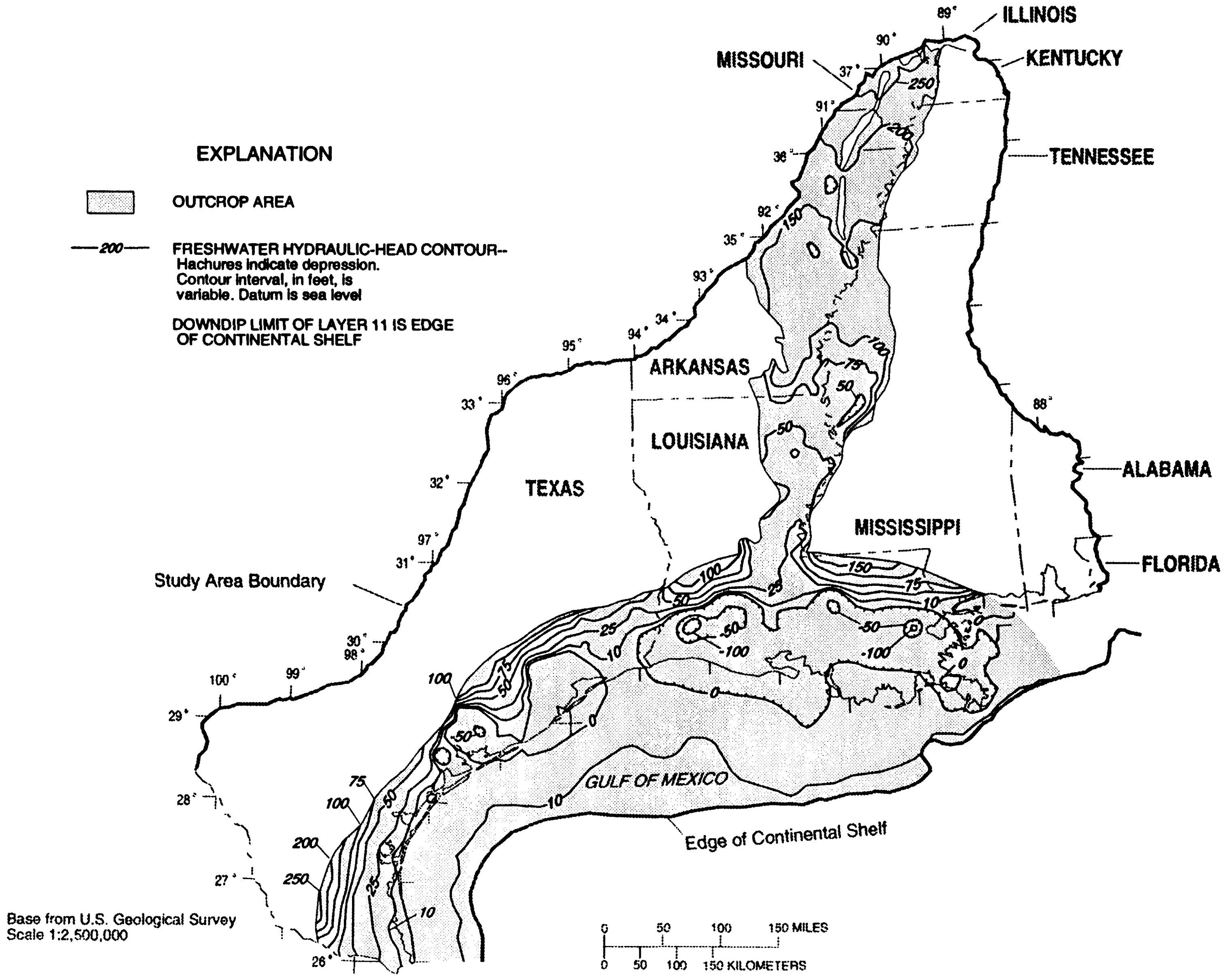

Figure 19. Areal extent of model layer 11 and simulated 1982 equivalent freshwater head from model 4.

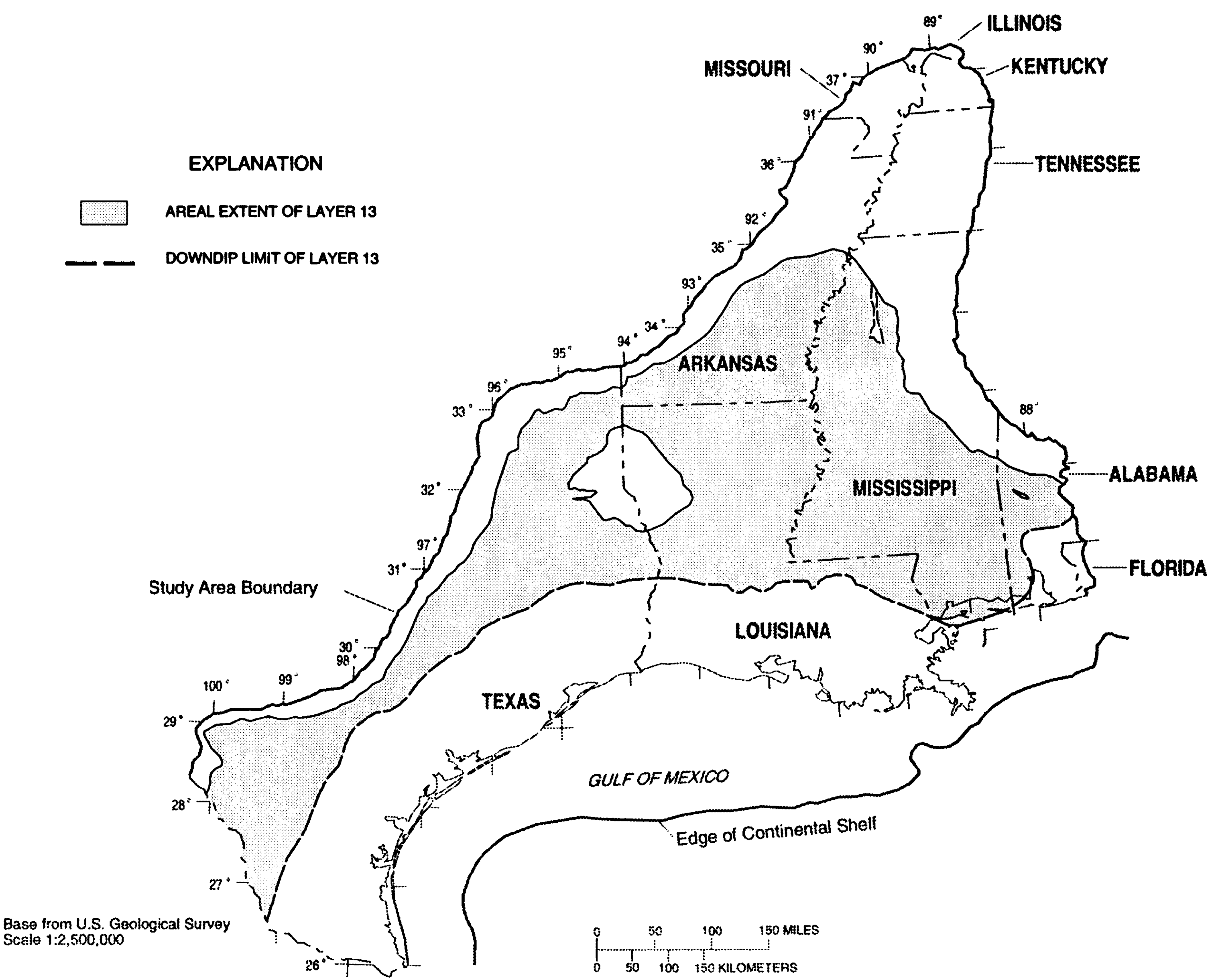

Figure 20. Areal extent of model layer 13. Represents confining unit between model layers 4 and 5.

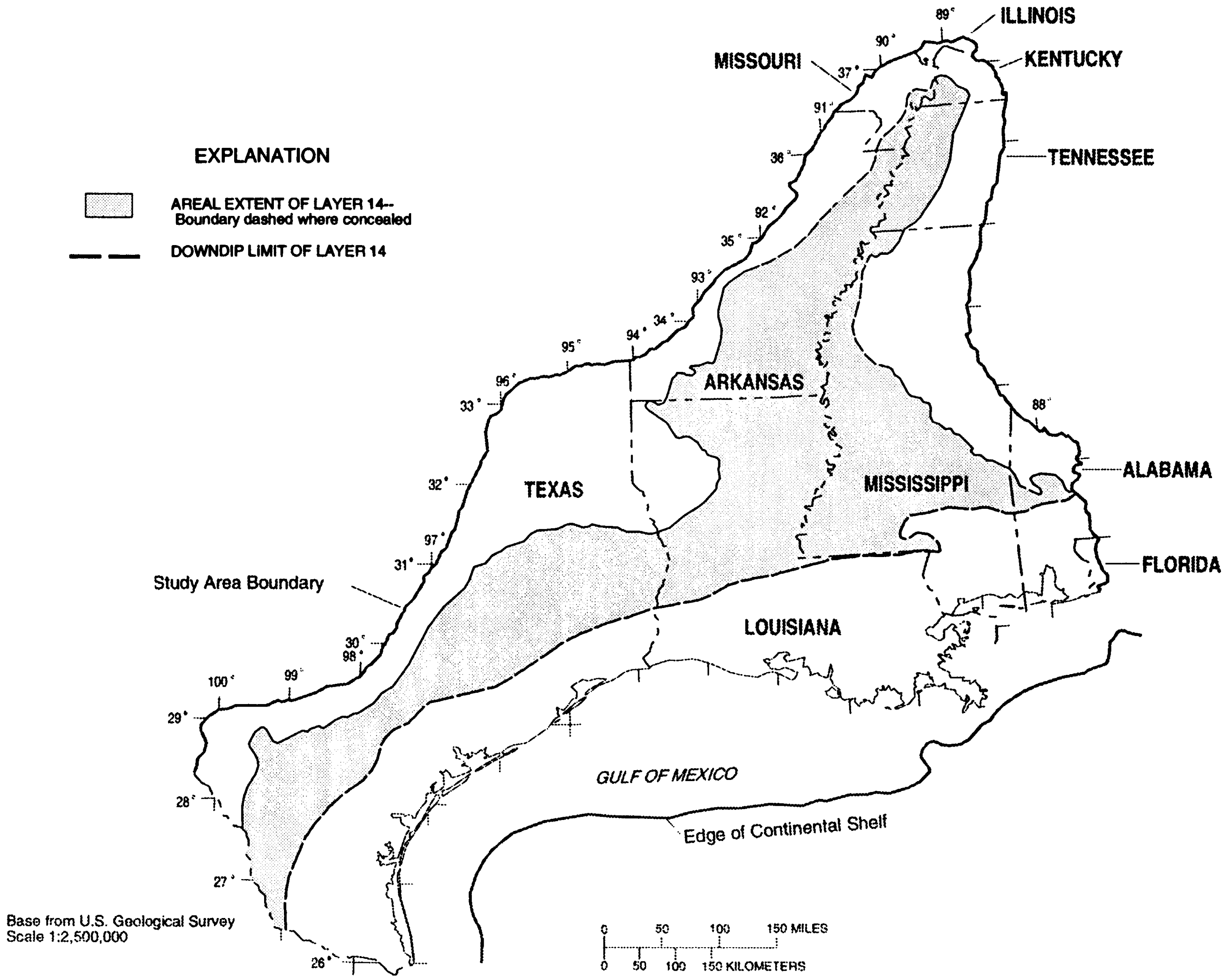

Figure 21. Areal extent of model layer 14. Represents confining unit between model layers 5 and 6.

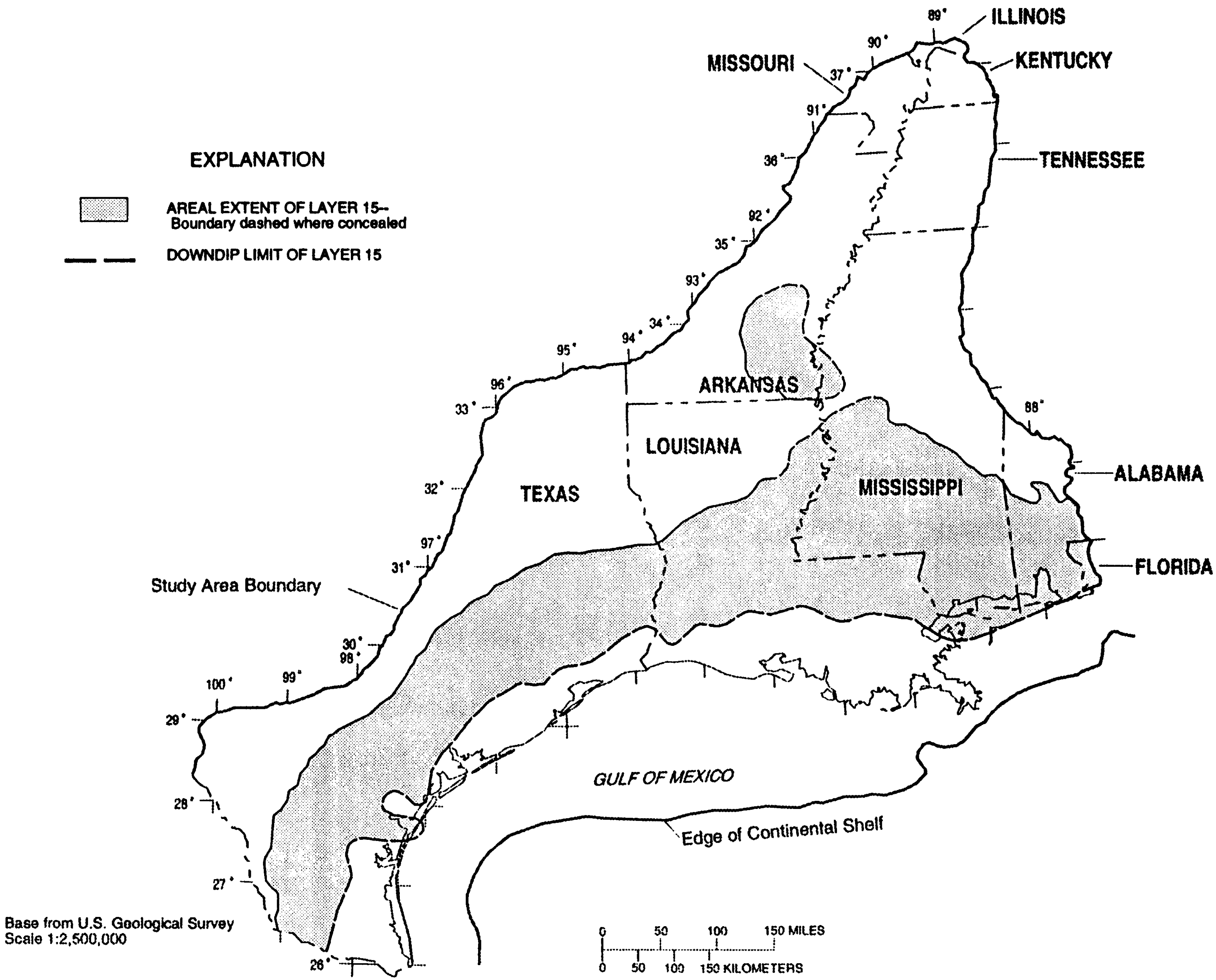

Figure 22. Areal extent of model layer 15. Represents confining unit between model layers 6 and 7

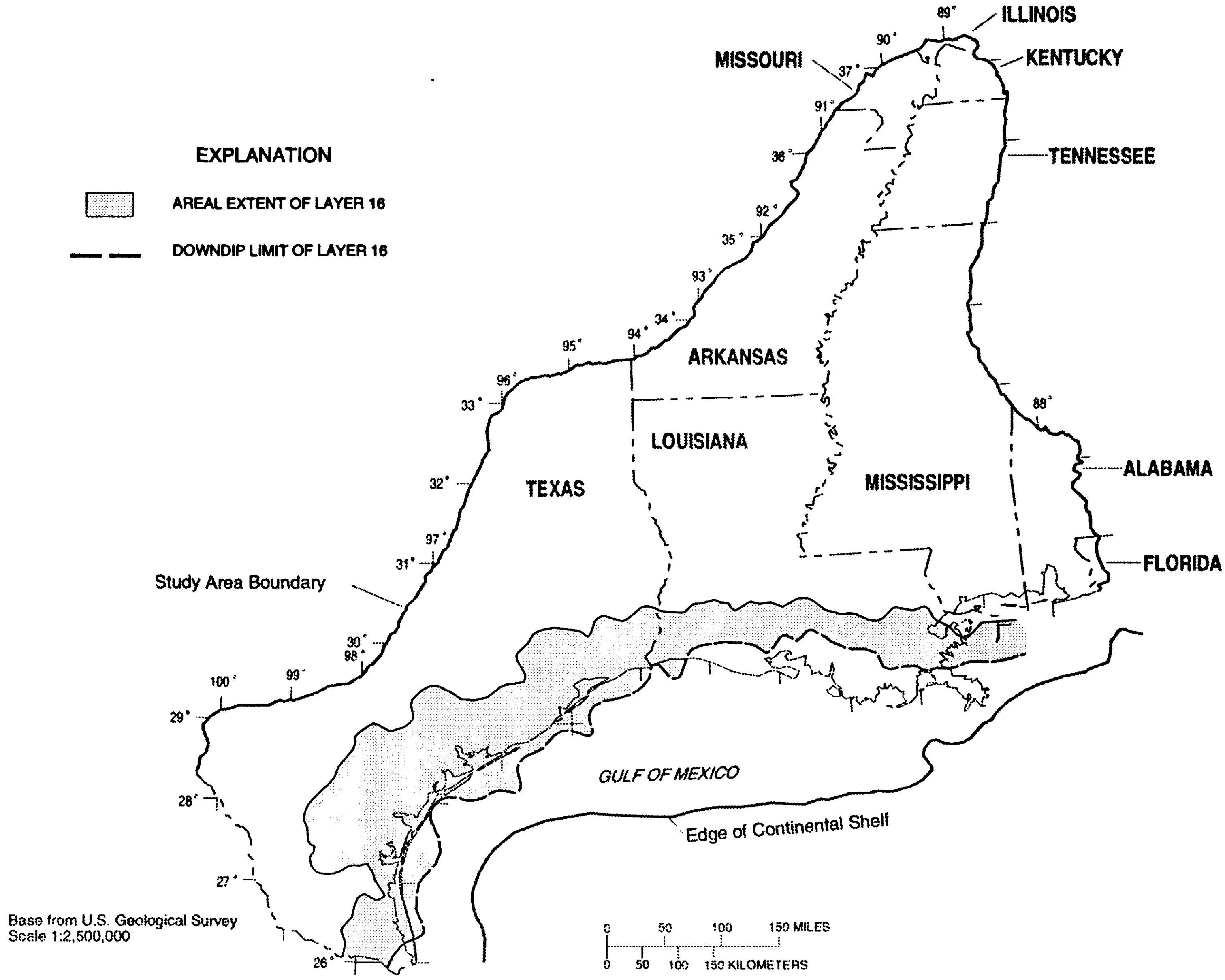

Figure 23. Areal extent of model layer 16. Represents confining unit between model layers 7 and 8.

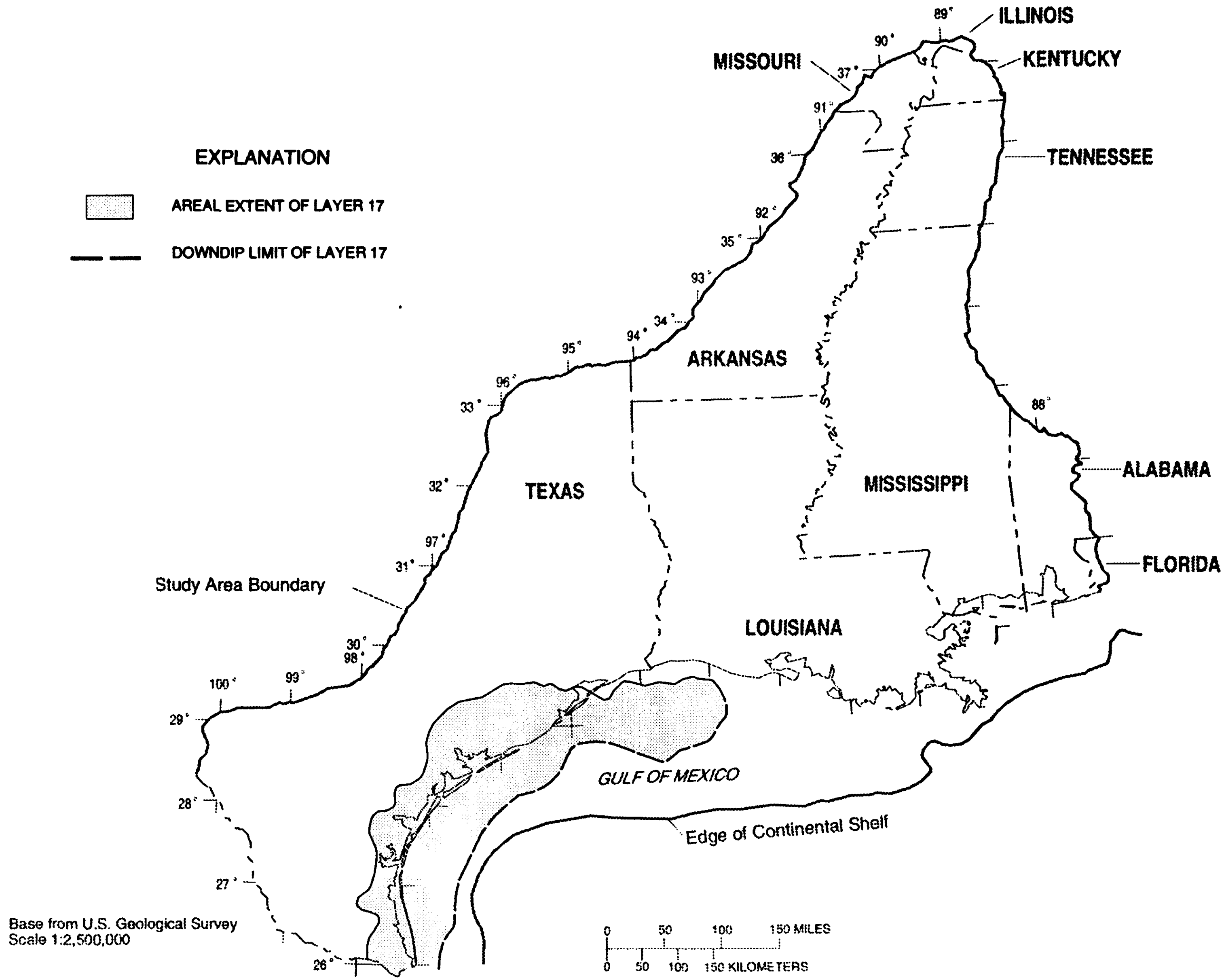

Figure 24. Areal extent of model layer 17. Represents confining unit between model layers 8 and 9.

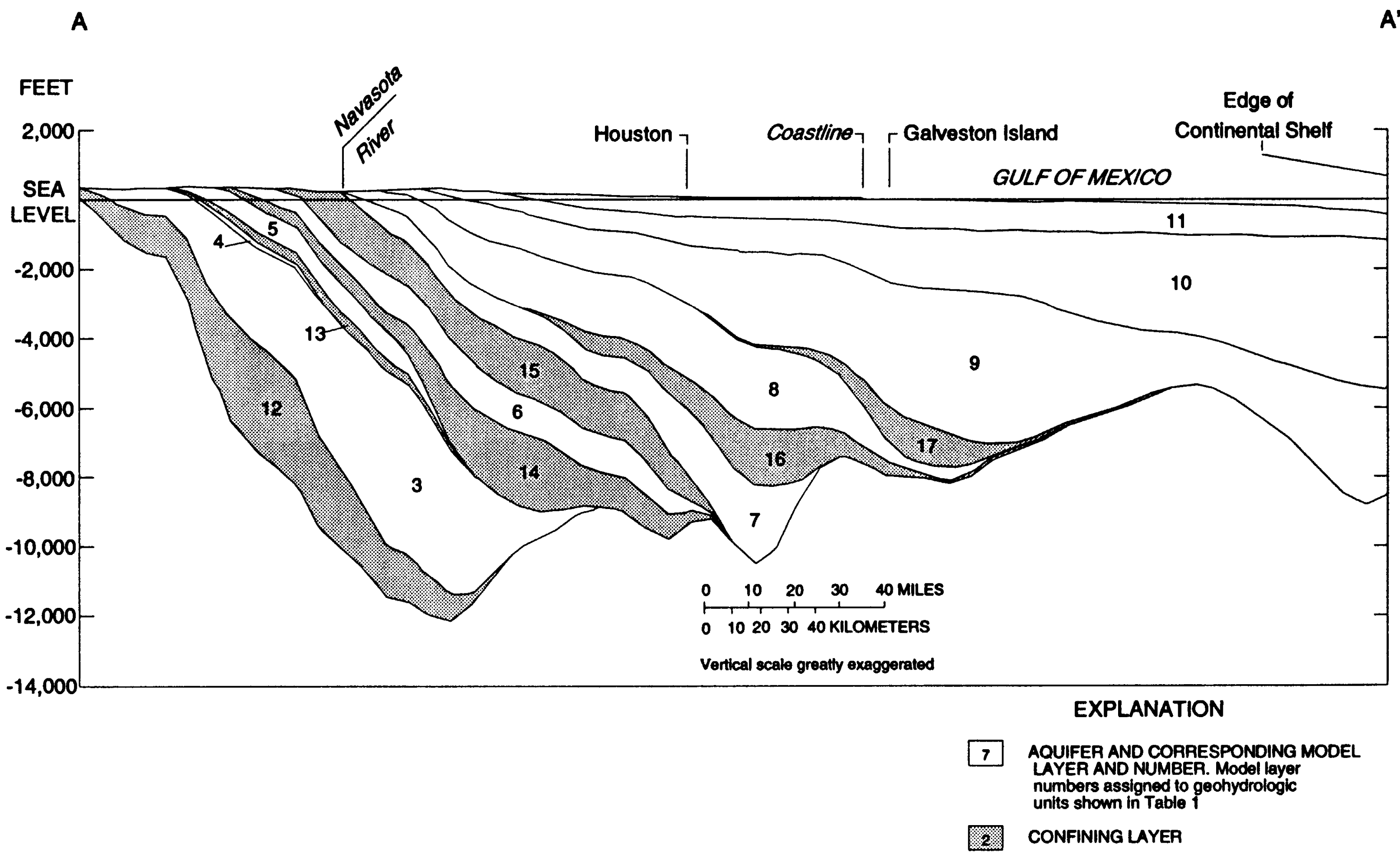

Figure 25. Idealized diagram from central Texas to the edge of the Continental Shelf showing vertical relation of aquifers and confining units (modified from Williamson and others, 1990).

The bottom surface of the aquifer system domain of solution is either in the geopressured zone or on the top of a geologic unit which is assumed to have no flow. The significantly overpressured parts of the geopressured zone beneath the aquifer system in the model are shown in figure 26. The updip extent of the significantly overpressured area is approximately equal to the line where model layer 7 is partially truncated by the geopressured zone as shown by Weiss (1993). Because the geopressured zone presumably extends further offshore than the aquifer system, the offshore limit of the region shown in figure 25 is the edge of the Continental Shelf. A layer of geopressured zone grid elements lies beneath all of the other layers mentioned above and has the areal extent shown in figure 26. The specified geopressured zone head in the geopressured zone layer is equal to the product of 1,000 ft with a regression parameter, plus an additional second regression parameter. For the grid elements in the geopressured zone, the quantity (K_z/b), where K_z is grid element effective vertical hydraulic conductivity and b is grid-element thickness, is equal to the second regression parameter. Note that vertical flow between two grid elements, denoted 1 and 2, is

$$(h_1 - h_2)(\text{area}) \left[\frac{b_1}{(K_1)_z} + \frac{b_2}{(K_2)_z} \right]^{-1} ,$$

whence the choice of (K_z/b) as a regression parameter for the geopressured zone. The two basic properties of a source of fluid adjacent to the domain of solution are the pressure of the source and its ability to actually deliver fluid if permitted by lowering the pressure and increasing the conductivity in the domain of solution adjacent to the fluid source. Because of the way the two parameters determine flow, they relate directly to these two basic properties.

The top of the aquifer system domain of solution is that surface consisting of either the water table, land surface, or sea floor, whichever is lowest. The top model layer, layer 12, has the areal extent shown in figure 10 and overlies the aquifer system. This model layer has a specified head equal to the product of (1) an approximation to the water-table altitude h_w (Williams and Williamson, 1989) when on land, or equivalent freshwater head at the sea floor offshore, with (2) a regression parameter; plus a second regression parameter. The water-table altitude is coincident with the altitude of streams and lakes except when a layer of unsaturated material exists between a lake or stream bottom and the underlying water table. The two parameters function so as to allow for the possibility that the water-table altitude h_w may have an error proportional to h_w (likely assumption) and also a constant error.

The specific storage $S_s = S_s(x,y,z)$ is taken to be constant for the entire aquifer system domain of solution. This single value of S_s is taken to be one regression parameter.

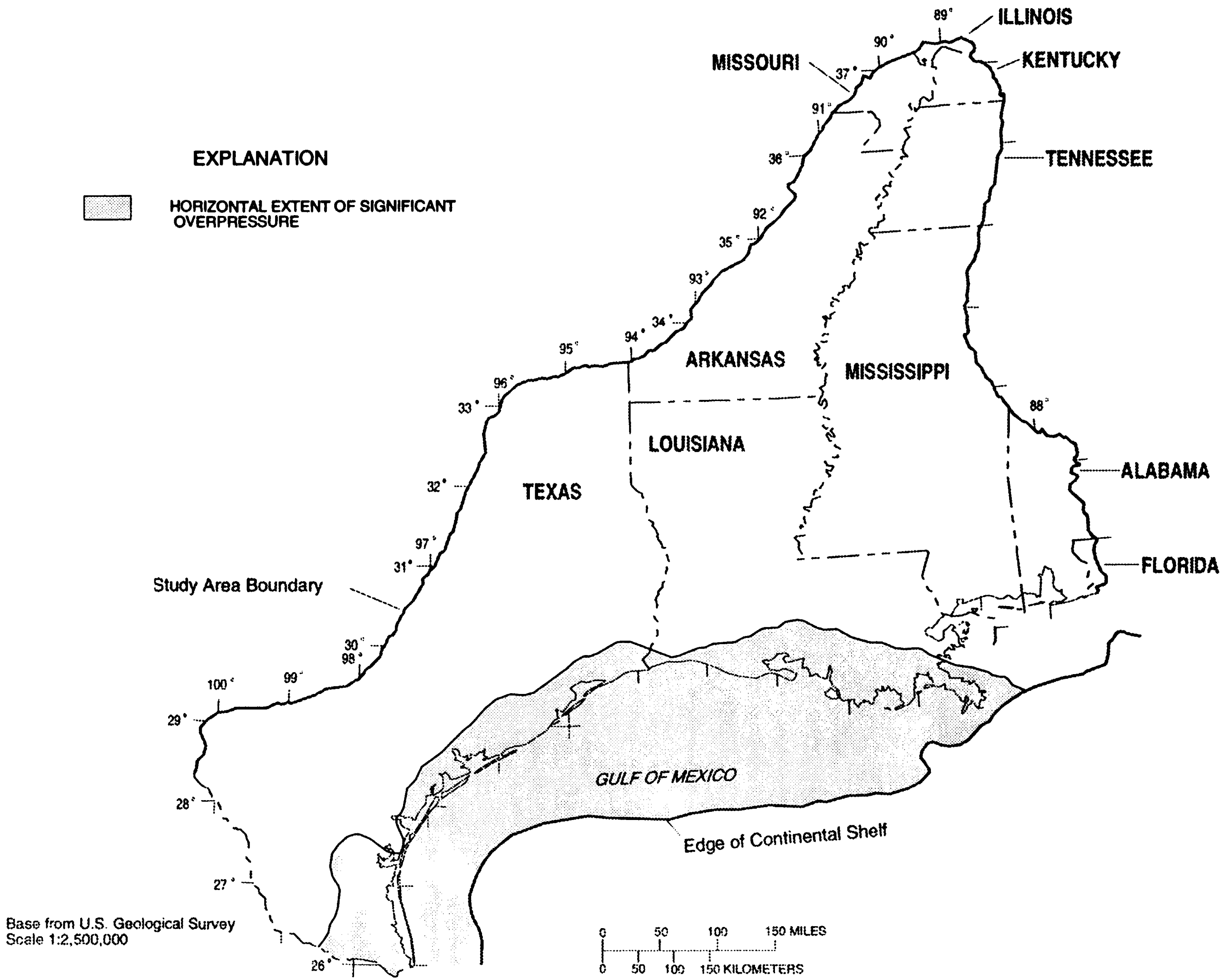

Figure 26. Areal extent of the significantly overpressured parts of the geopressured zone

The density $\rho(x,y,z)$ is given a separate fixed and measured value for each of the N = 29,453 grid elements. Density is assumed to be nearly constant in time. This approximation is valid when grid-element volume-averaged density values change very little during the time span being simulated (Kuiper, 1983; Weiss, 1982; Bennett, 1980). No regression parameters are assigned to $\rho(x,y,z)$. Because pumping is known sufficiently well, no regression parameters are assigned to pumping rate Q(x,y,z,t).

The values for K_x, K_y, and K_z, the principal direction values of the hydraulic conductivity tensor K, are prescribed by subdividing the aquifer system domain of solution into a number of hydraulic conductivity zones, each of which contains many grid elements. For each zone or group of zones, two regression parameters are assigned, one for the hydraulic conductivity of the clay of the medium and one for the sand. Here, as elsewhere, the word clay is intended to mean fine-grained sediment including silt. The hydraulic conductivity of each of the components in each of the grid elements is given by the product of the appropriate regression parameter with a function of the depth of the center of the grid element below ground surface. Both of these functions, one for the clay component and one for the sand component, are described further on and give the rate of decrease of hydraulic conductivity with depth. The clay and sand component conductivity values for each grid element are then combined (Desbarats, 1987) with data on the relative amounts of clay and sand in each grid element to finally arrive at effective hydraulic conductivity values K_z, and $K_x = K_y$ for each of the grid elements. If one wishes to consider the horizontally anisotropic case where $K_x \neq K_y$, a single additional regression parameter is assigned to K_x/K_y, assumed to be constant throughout the domain of solution. A detailed description of the procedure for prescribing K_x, K_y, and K_z, is given in a later section on hydraulic conductivity.

Effective vertical hydraulic conductivity K_z for model layer 12 was adjusted to restrict the amount of recharge that flows into underlying model layers. On the basis of an approximation to precipitation less evaporation considerations the maximum allowable recharge rate was specified to vary linearly from 2 in/yr at row number 80 to 12 in/yr at row number 56, in figure 10. The remaining areas in row numbers greater than 80 and less than 56 were allowed a maximum of 2 and 12 in/yr, respectively. Individual layer 12 grid element values for K_z were reduced from an otherwise constant value of K_z for the layer. Only those few grid elements with a high water-table altitude in southern Texas, and several others near areas of heavy pumping, required adjustment.

The solution of equation (3) above is accomplished by obtaining the finite-difference approximating equations (Kuiper, 1983; Kuiper, 1985) using the implicit approximation for the time derivative. The set of N-M simultaneous equations generated, where N = 29,453 is the total number of grid elements and M is the number with a specified head, is solved at each time step and gives values for the N heads at each of the time steps and at each of the grid elements not located on a specified-head boundary. All of the N heads are regarded as dependent variables depending upon the regression parameters. A measured value of head is assumed to equal computed head, as a function of the regression parameters, plus an error term called the residual. The vector equation expressing these relationships becomes the regression equation for the regression technique (Vecchia and Cooley, 1987).

Quantities other than the heads are also dependent upon the regression parameters. Possibilities include the parameters themselves, values of the hydraulic conductivity in the various zones mentioned, storage depletion as measured by subsidence, ground-water flow rates, and so forth. Thus, the regression equation may include these quantities, also, if they have observed values.

Values for grid-element volume-averaged head were matched for the years 1972 and 1982, and were produced by the numerical flow model by using four time intervals of length 30, 5, 5, and 5 years, respectively. Starting with a steady-state predevelopment zero pumping solution for the year 1937, heads were determined for the years 1967, 1972, 1977, and 1982. For each of these four time intervals, appropriate time-averaged pumping values were used as determined from pumping data. For the three 5-year time intervals 1967-72, 1972-77, and 1977-82, the pumping rates for the years 1970, 1975, and 1980 were used. The pumping rates for these years are close to the average pumping rates during the three time intervals. For the 30-year time interval from 1937 to 1967, an approximation to the average pumping rate during the interval was obtained using the 1970 location of pumped wells, but decreased in accordance with total pumping by layer for the years 1960 and earlier. The total pumping rate by layer for the years 1960, 1970, 1975, and 1980 is shown in figure 27.

Regression Model

The nonlinear regression model (Draper and Smith, 1981; Vecchia and Cooley, 1987) supposes that a set of observations (Y_i, $i = 1,2,\ldots,n$) of the physical system are related to a $p \times 1$ vector of unknown regression parameters B through the stochastic model

$$Y = f(B) + E \ , \tag{4}$$

where Y and E are $n \times 1$ random vectors, and the $n \times 1$ vector f(B) is the expectation of Y. The integer p is the number of regression parameters. The observations Y_i are thought of as a realization of the components of the $n \times 1$ random vector Y. The $n \times 1$ random error vector E is assumed to have the probability distribution

$$E \sim N(0;\epsilon^2\omega^{-1}) \ , \tag{5}$$

where ω is a known $n \times n$ symmetric positive definite matrix and ϵ is a scalar constant. In the text below we shall frequently refer to f(B) as being the numerical flow model. As explained above, it contains quantities other than those coming from the numerical flow model as dependent variables.

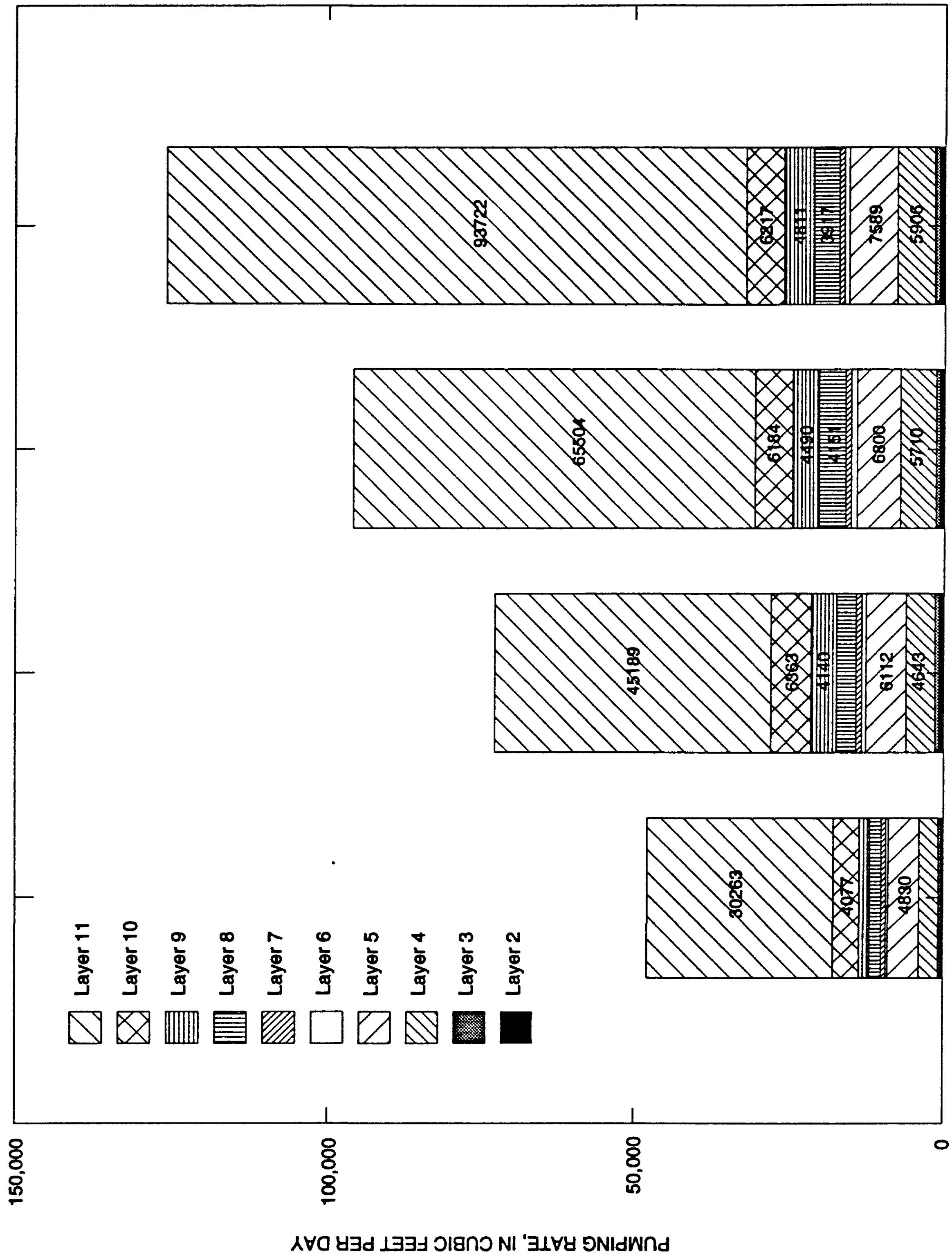

Figure 27. Total pumping rate by layer for the years 1960, 1970, 1975, and 1980.

In the case where n becomes large without limit and a continuum of points exists, such as the continuum of points corresponding to a cartesian coordinate (x,y,z), the vectors Y and E become random fields Y(x,y,z) and E(x,y,z), and the vector f(B) becomes a function f(x,y,z,B). As an example, figure 28 shows a single realization of random field Y(x) as a function of cartesian coordinate x, where realization Y(x) is the volume average of head over a volume extending from the bottom of an aquifer to its top and having a given cross sectional area A. A second example is the volume average of hydraulic conductivity rather than head. The value of realization Y(x) shown is the exact value of the volume average, even though only an approximation to this value could be obtained by measurement. Two different approximations to f(x,B) are shown in figure 28. The first f(x,B), denoted f1(x,B), is coarse compared to f2(x,B) and does not follow Y(x) as closely as f2(x,B). Because f1(x,B) and f2(x,B) only approximate f(x,B), the expectations of E1(x) = Y(x) - f1(x,B) and E2(x) = Y(x) - f2(x,B), though perhaps small, are not zero. Because f1(x,B) follows Y(x) more closely than does f2(x,B), E1(x) has a larger apparent variance than does E2(x). Apparent variance, approximately equal to residual mean square (Draper and Smith, 1981, p. 34) is, in the case of individual observations Y_i, i = 1,...,n, defined to be:

$$\frac{1}{n}\left\{\sum_{i=1}^{n}(Y_i - f_i(B))^2\right\},$$

where n >> 1.

This demonstrates that the apparent variance of E1 = Y-f1(B) is dependent upon the chosen degree to which an approximate f1(x,B) is capable of fitting realization Y, the observations. For a given realization Y, many different sets approximate f(B) and corresponding E can be chosen, each with a different apparent variance. As the chosen f(x,B) departs further from the true f(x,B) the agreement will decrease and apparent variance will increase. Apparent variance values are inflated relative to the true value for variance $\epsilon^2\omega^{-1}$ that would obtain if the true f(x,B) were being used. Criteria for this lack of agreement can sometimes be used (Draper and Smith, 1981, p. 35) to determine if a chosen approximate f(x,B) is reasonably close to the hypothesized true f(x,B).

If measurement error is also present, then E also includes this error component, the size of which depends on the accuracy of the measurements used.

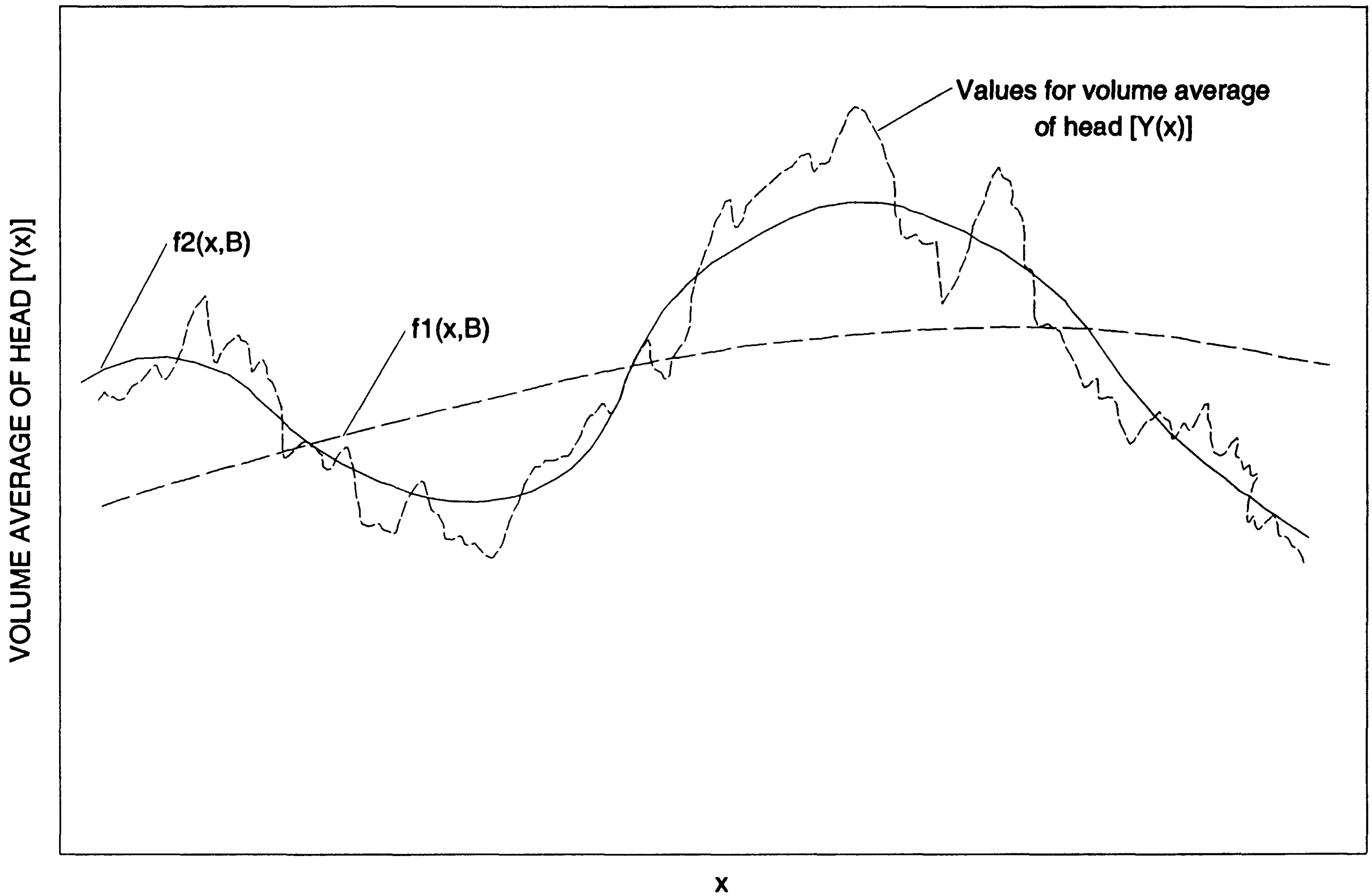

Figure 28. Two potential regression models, f1(x,B) and f2(x,B), for describing the relation between cartesian coordinate (x) and volume average of head [Y(x)]

In this study apparent variance is used to approximate true variance $\epsilon^2\omega^{-1}$, the possibility of this procedure was indicated by Draper and Smith (1981, p. 34). A single approximation ω_j is made for those ω_{ii} corresponding to observations of a given type occuring in a zone j of the domain of solution. Accordingly $\epsilon^2\omega_j$, is approximated by

$$\frac{1}{n_j}\left\{\left[\sum_{i=1}^{n_j}(Y_i - f_i(B))\right]\right\},$$

where n_j (for n_j>> 1) is the number of observations occurring in zone j. The zones j were chosen so that those regions of the domain of solution with like variance were located in the same zone. Different types of observations had different zonations of domain of solution.

In practice, the number of observations is finite, so that in the example in figure 28, f(B) would approximately fit a series of points on the curve Y(x). In this study, when considering observations of head, area A mentioned above is 100 mi^2, the horizontal area of the grid elements of the multilayered numerical flow model used to simulate the system. Where observations of head are available, average values over the 10 mi by 10 mi by layer thickness grid elements have been assigned to the centers of the elements. These grid element head observations are determined, as will be discussed in more detail, from observations of hydraulic head obtained from wells located in or near the grid element. The observed value of head for a given element, as well as the corresponding value for head produced by the numerical flow model, f(B), are considered to be volume averages of head over the grid element. Because of this, substantial measurement error occur in these grid-element volume-averaged heads. The flow model f(B) of this study, is likewise coarse, so that even in the absence of measurement error, f(B) differs substantially from the observations for any choice of B.

Regression modeling requires determining the variance of E. The apparent variance of E, which is an approximation to the true variance of E, depends upon both the degree to which f(B) fits the observations without measurement error and the measurement error of the observations. In some cases, f(B) may be structured with sufficient detail to fit the observations Y_i exactly. For example, in one method for treating hydraulic conductivity observations in this study (the method in model 9 below) the entire aquifer system domain of solution was divided into seven zones. Six of these zones had an observed value for zone volume-averaged hydraulic conductivity as estimated from individual point values of hydraulic conductivity from individual pump tests of wells within the zone. Flow model f(B) was structured to allow seven different values for hydraulic conductivity in the seven zones, and was thus capable of fitting the six observed values for hydraulic conductivity exactly. Accordingly, the variance of E for these six observations was determined using the formula for the variance of an average of quantities, these quantities being the individual aquifer-test values for hydraulic conductivity.

Prediction and Confidence Intervals

The numerical flow model, f(B), of equation (4) should be selected in such a manner that the model is capable of simulating the aquifer system as closely as possible. Many choices are available, not only with regard to the number and type of regression parameters in f(B), but in what manner they are incorporated into the flow model f(B). Some of these choices are mentioned above. As stated previously for example, the specific storage $S_s(x,y,z)$ in f(B) has a single value throughout the domain of solution and is assigned only one regression parameter, but fluid density $\rho(x,y,z)$ and pumping Q(x,y,z,t) are assigned no regression parameters. Hydraulic conductivity and specified boundary heads are assigned several regression parameters.

With regard to simulation, the confidence that one has in the head and flow values produced by the flow model is of primary importance, simply because head and flow measurements are of primary interest. Predictions of hydraulic conductivity and specific storage are really of interest only because of the way they affect head and flow. As such, they are of secondary importance. However, when the flow model f(B) is used to predict head and flow values in the future, or when the regression equation is extrapolated beyond the vicinity of the observations, then the confidence of the regression parameters might seem to be important because of the fact that head and flow depend upon the parameters and no observations of head and flow are available for comparison. Unfortunately, even if the regression parameters of f(B) are known well, some heads and flow rates may still have considerable error if f(B) has not been constructed in a suitable manner. For example, suppose f(B) has a single hydraulic conductivity value for the entire domain of solution, even though the hydraulic conductivity is variable. This single value may be thought of as the average hydraulic conductivity in the domain of solution. Even if it is shown that this so-called average hydraulic conductivity is known quite well, the heads and flow may still be known only very poorly because using a constant hydraulic conductivity is an inadequate approximation to the variable hydraulic conductivity distribution present. Very likely, the best information available on the confidence of heads and flow outside of the vicinity of the observations is the confidence of heads and flow within the vicinity of the observations. In conclusion, the confidence that one has in head and flow values in the vicinity of the observations is of primary interest in evaluating the ability of the flow model to simulate the aquifer system, past, present, and future. As will be shown later, it is sometimes possible to evaluate this confidence.

Equation (75) of Vecchia and Cooley (1987) gives an approximate prediction interval for Y_k:

$$Y_k^e = f_k(\hat{B}) \pm d_{1-\alpha}[\hat{X}_k(\hat{X}^T\omega\hat{X})^{-1}\hat{X}_k^T + \omega_k^{-1}]^{1/2} , \qquad (6)$$

where k may correspond to a location at which there is no observation. Here

$$d_{1-\alpha}^2 = S(\hat{B})[(p+1)/(n-p)]F_\alpha(p+1,n-p) ,$$

where as mentioned previously, the number of observations n is the length of the vector Y, and the number of regression parameters p is the length of the vector B. The upper α percentile of the F distribution with p+1 and n- p degrees of freedom is denoted by $\partial_\alpha(p+1,n-p)$. The value of the vector of regression parameters B at which the sum of squared weighted residuals $S(B) = [Y-f(B)]^T\omega[Y-f(B)]$ is a minimum, is denoted by $\hat{B}$. The T superscript denotes the transpose of a matrix. The sensitivity matrix X is given by $X_{ij} = [\partial f_i(B)/\partial B_j]$, and matrix X evaluated at $B = \hat{B}$ is denoted $\hat{X}$. The jth component of the 1 x p vector X_k is X_{kj}. $\epsilon^2\omega_k^{-1}$ is the variance of $F_k = Y_k - F_k(B)$. If k is not an observation number, then ω does not have to be diagonal and ω_k is not an element of ω. In this case there is no correlation between Y_k and Y_i, one of the n observations of vector Y of equation 4. The interval given by equation 6 is a $(1-\alpha)100$-percent prediction interval for Y_k, the kth member of the n x 1 random vector Y. Thus, if the model is nearly linear, the probability is $1-\alpha$ that $Y_k \leq Y_k^e(+)$ where $Y_k^e(-)$ and $Y_k^e(+)$ are the two values of Y_k given in equation 6. If the model is nonlinear, the probability $1-\alpha$ is approximate. As noted above k need not be contained within the observed set of quantities Y_i. Thus, for example, equation 6 may be used for values of the head at locations where no observation occurs, provided that one is able to find a value for ω_k. If k corresponds to some distant future observation, then ω_k cannot easily be determined. Replacing $d_{1-\alpha}$ with $(S(\hat{B})/(n-p))^{1/2}$ in equation (6) causes the term to the right of the ± sign, when squared, to give the variance of Y_k.

Equation (45) of Vecchia and Cooley (1987) gives an approximate confidence interval for the regression parameter B_i:

$$B_i^e = \hat{B}_i \pm d_{1-\alpha}[i^T(\hat{X}^T\omega\hat{X})^{-1}i]^{1/2} , \qquad (7)$$

Here $d_{1-\alpha} = S(\hat{B})[p/(n-p)]F_\alpha(p,n-p)$ where $S(\hat{B})$ and $F_\alpha(p,n-p)$ have been defined above. The p x 1 vector i is a vector whose only nonzero component is the ith, which has the value unity. The two B_i^e extreme values given in equation (7) determine a $(1-\alpha)100$-percent confidence interval for B_i, the ith member of the p x 1 vector B. Replacing $d_{1-\alpha}$ with $(S(\hat{B})/(n-p))^{1/2}$ in equation (7) causes the term to the right of the ± sign, when squared, to give the variance of B_i.

Equations (6) and (7) may be useful in choosing between the many possible choices for f(B). The smaller the intervals in equations (6) and (7) are, the better the choice for f(B) is considered to be. Especially important is the choice for the number of regression parameters p, because of the distinct way that the two intervals increase with p. A comparison of over 40 different choices for f(B) with p varying from 2 to over 31 is presented in the application and results section.

Equations (6) and (7) require evaluations of S(B), X, and f(B) at $B = \hat{B}$. S(B) may be approximated by

$$S(B) = \sum_{i=1}^{n} (Y_i - f_i(B))^2 \, \omega_i \;, \tag{8}$$

Approximate equation (8) has used $\omega_i = \omega_{ii}$ and $\omega_{ij} = 0$ for $i \neq j$, in the exact expression for S(B) given previously. The minimization S(B) in equation (8) in the p members of B was carried out by the Levenberg-Marquardt algorithm (Gill and others, 1981) for the minimization of the sums of squares. This algorithm is iterative and finds successively smaller values of S(B) at a series of successive choices of B selected by the algorithm. During this iterative process it is interesting to evaluate to the left side of inequality (7) of Vecchia and Cooley (1987):

$$\frac{[Y-f(B)]^T\omega^{1/2}P\omega^{1/2}[Y-f(B)]}{[Y-f(B)]^T\omega^{1/2}(I-P)\omega^{1/2}[Y-f(B)]} \leq (p/n-p)F_\alpha(p,n-p) \;, \tag{9}$$

where $P = \omega^{1/2}X(X^T\omega X)^{-1}X^T\omega^{1/2}$.

Inequality (9) expresses a $(1-\alpha)100$-percent exact confidence region for B. As S(B) decreases with the successive choices for B, the left-hand side of equation (9) also commonly decreases. The contours of S(B) and the contours of the left-hand side of (9) as a function of B, coincide quite closely (Donaldson and Schnabel, 1987; Sundararaj, 1978; Wallace and Grant, 1977). Equation (9) with the equality sign may be used to solve for $\alpha = \alpha_m$, with $B = B_m$, where m is the Levenberg-Marquardt iteration number. With this definition of α_m, the successive values for B, B_m m = 1,2,3,... lie somewhere on the boundary of the $(1-\alpha_m)$ 100-percent confidence region for B. For successively larger values of m, the left-hand side of equation (9), evaluated at $B = B_m$, decreases to zero as does $F_{\alpha m}(p,n-p)$, and at the same time α_m increases to unity from $\alpha_0>0$.

Depth Dependence of Hydraulic Conductivity

The hydraulic conductivity of the porous medium in the domain of solution of the aquifer system, which is composed of interbedded fine-grained and coarse-grained sediments, usually decreases with depth below land surface.

The hydraulic conductivity of the coarse-grained sediments or sands, usually decreases with depth due to decreasing porosity, but increases with depth due to higher temperature and the resulting decrease in fluid viscosity. The net outcome of these factors, however, is usually decreasing hydraulic conductivity with depth. The function $10^{-0.8dd}$, where dd is measured in kilometers, was used to express the rate of the decrease of the hydraulic conductivity of sand with depth dd (Loucks and others, 1986; Lake and Carroll, 1986). Loucks and others (1986) presented data (fig. 29) on the hydraulic conductivity of sand from Tertiary deposits of the Texas Gulf Coastal Plain. The curve (constant) $10^{-0.8dd}$, which is an approximation, is also shown. The exponent of this curve was assigned a regression parameter, but because of the shallow depth of most head observations, the parameter had such a large confidence interval that it was decided that the model would be more accurate if the curve $10^{-0.8dd}$ was assumed.

The hydraulic conductivity of the fine-grained sediments, or clays, in the domain of solution of the aquifer system tends to decrease with depth because of compaction. Figure 30, taken from Neglia (1979), shows the decrease with depth of the hydraulic conductivity of various clay samples. The curve shown in figure 30, (constant)$10^{(-1.167dd+.0833dd^2)}$ where dd is measured in kilometers, was formulated to approximate the measured values shown and was used in this study to give the rate of decrease of hydraulic conductivity of clay with depth. As with the decrease of hydraulic conductivity of sand with depth, a regression parameter included in the exponent had a very large confidence interval and was removed.

The hydraulic conductivity of clay or sand of the fine-grained or coarse-grained sediments in a grid element is found by taking the product of the regression parameter associated with the hydraulic conductivity zone (described below) containing the grid element with the appropriate depth function, where the depth chosen was the depth below land surface of the center of the grid element. The clay and sand components have separate regression parameters, and use the clay and sand depth functions described above respectively. The effective hydraulic conductivity tensor K of the grid element will depend upon the shape, size, and distribution of the sand bodies in the grid element and also on the hydraulic conductivity of these sand bodies and the clay matrix surrounding them. Desbarats (1987) presented a statistically-based procedure for calculating grid element

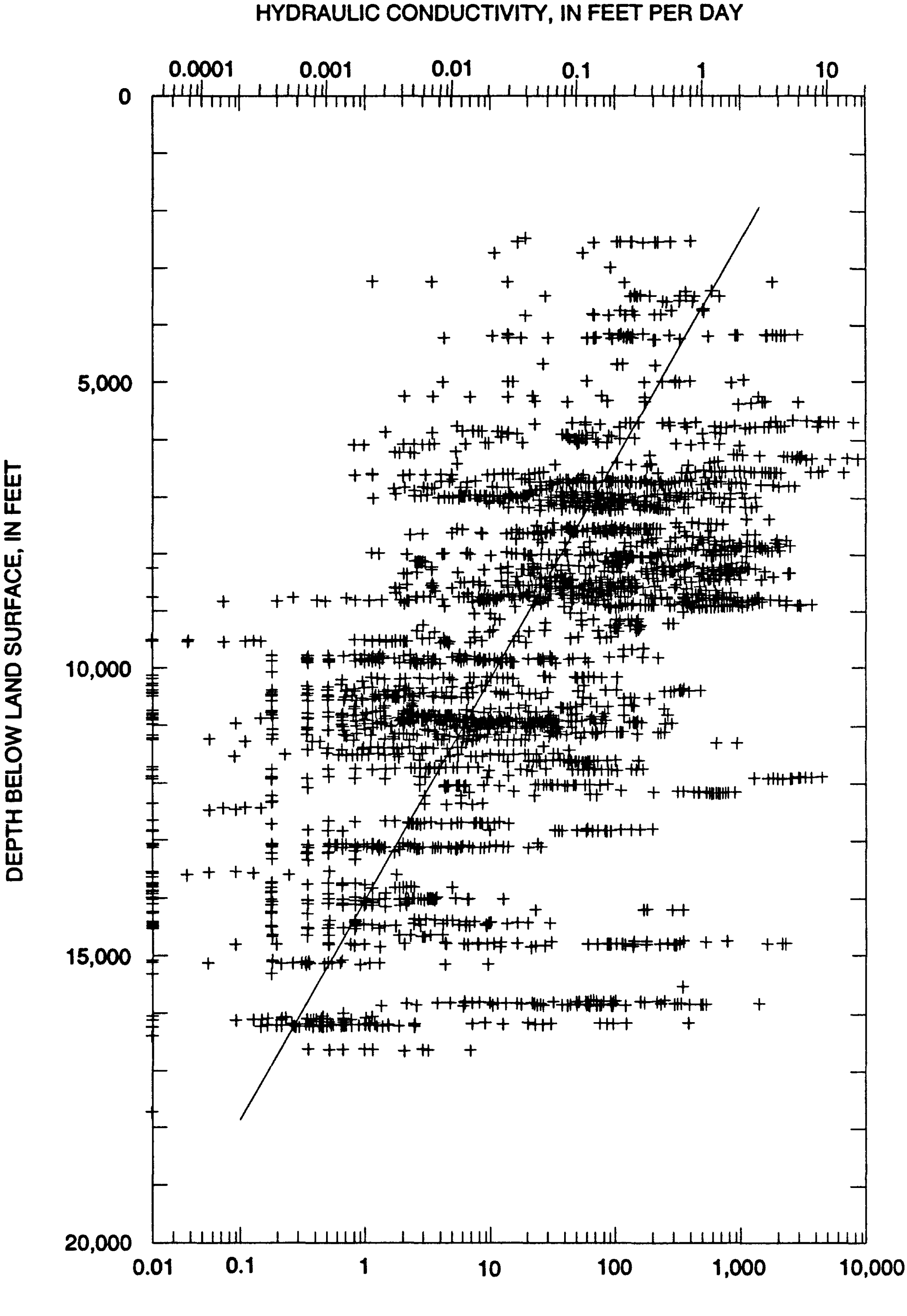

Figure 29. Decrease of hydraulic conductivity of sand with depth. (Modified from Loucks and others, 1986.)

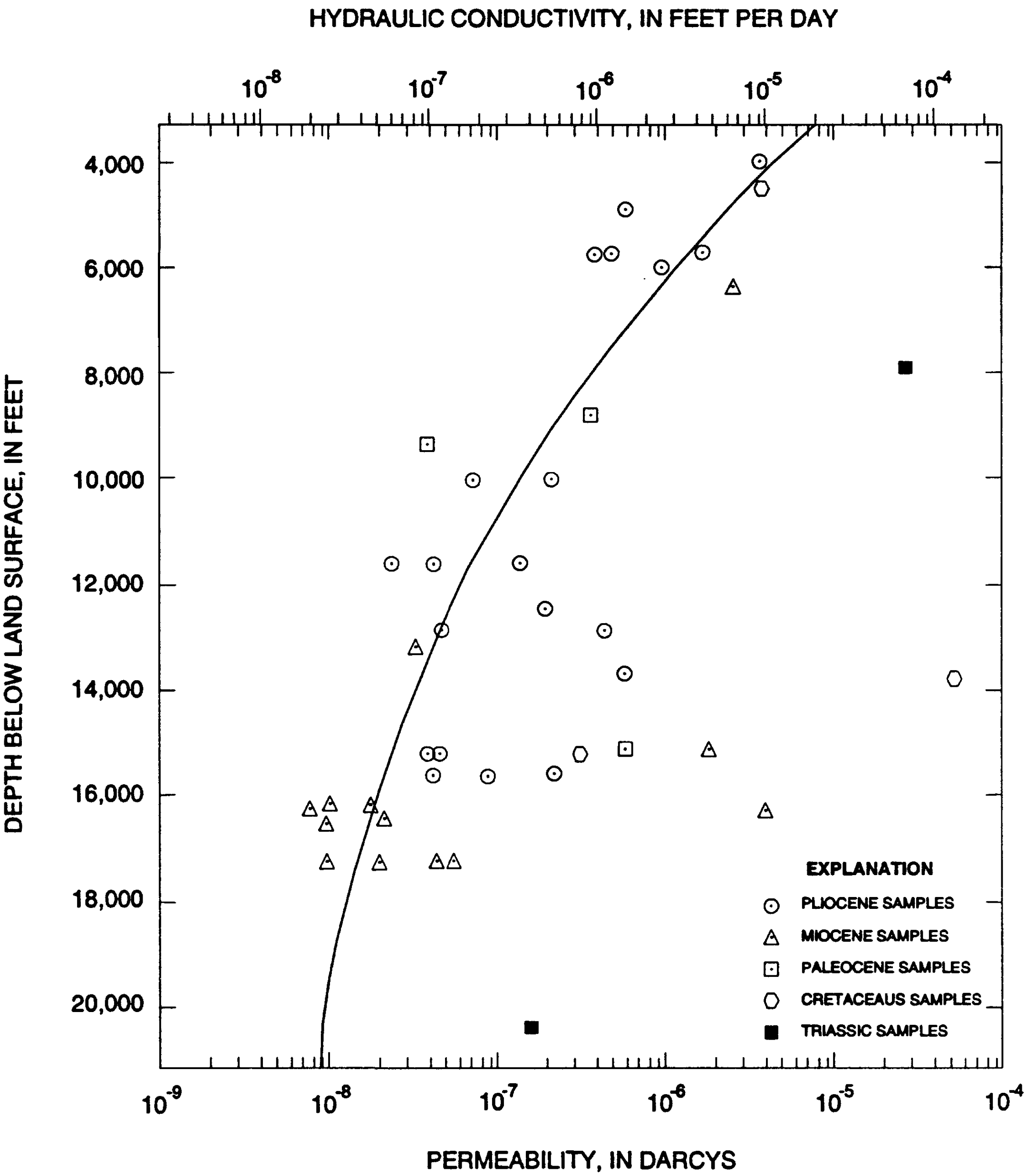

Figure 30. Decrease of hydraulic conductivity of clay with depth. (Modified from Neglia, 1979.)

effective hydraulic conductivity values. Figure 31 shows effective horizontal (K_h), and vertical (K_z) hydraulic conductivity values, as functions of V_c, the clay fraction of the grid element, for the case where $\log_{10}(K_c/K_s) = -4$, where K_c and K_s denote the hydraulic conductivity of the clay and sand components respectively. Then K_h and K_z can be calculated from:

$$\frac{K_z}{K_s} = [K_c/K_s]^{(V_c)^{a^{-1}}} , \tag{10}$$

$$\frac{K_h}{K_s} = [K_c/K_s]^{(V_c)^{a}} . \tag{11}$$

Here, a, related to the width to vertical thickness ratio of the sand bodies, is set equal to 4.0 or to the product of 4.0 and a regression parameter. Horizontal anisotropy is introduced by setting $K_x = K_h$ and by setting K_y equal to the product of K_h with a regression parameter. If anisotropy is not desired, then $K_x = K_y = K_h$ and no additional regression parameter is used.

Hydraulic Conductivity Zones

The hydraulic conductivity zones previously discussed in the Modeling Methodology section are cross referenced by layer and region. The layers are the model layers 2 through 11, top-layer 12, and model layers 13 through 17. The regions are shown in figure 32. Note that the regions specify horizontal position and the layers specify vertical position. The hydraulic conductivity zones are thus specified in three dimensions by cross referencing. A hydraulic conductivity zone does not exist for each of the model layers 2 through 17 and each of the 10 regions because the domain of solution does not extend to all combinations of layers and regions. The function of the hydraulic conductivity zones and the assignment of regression parameters to them allows the model to approximate the heterogeneity of either the clay or sand component in the domain of solution. A particular volume within the domain of solution may consist of several or many adjacent hydraulic conductivity zones. Assigning the same regression parameter, clay or sand whichever the case maybe, to all of the hydraulic conductivity zones within the particular volume causes the hydraulic conductivity of the clay or sand in the volume to be constant except as altered by the depth function for clay as explained previously.

The actual number of regression parameters that may be assigned to the hydraulic conductivity zones is variable. Two regression parameters could, for example, be assigned to all of the zones, the first regression parameter for the clay component in all of the zones and the second for the sand component in all of the zones. Or, for example, 11 regression parameters could be used: 10 for the clay components of the 10 regions and the 11th regression parameter for the sand component of all of the zones,

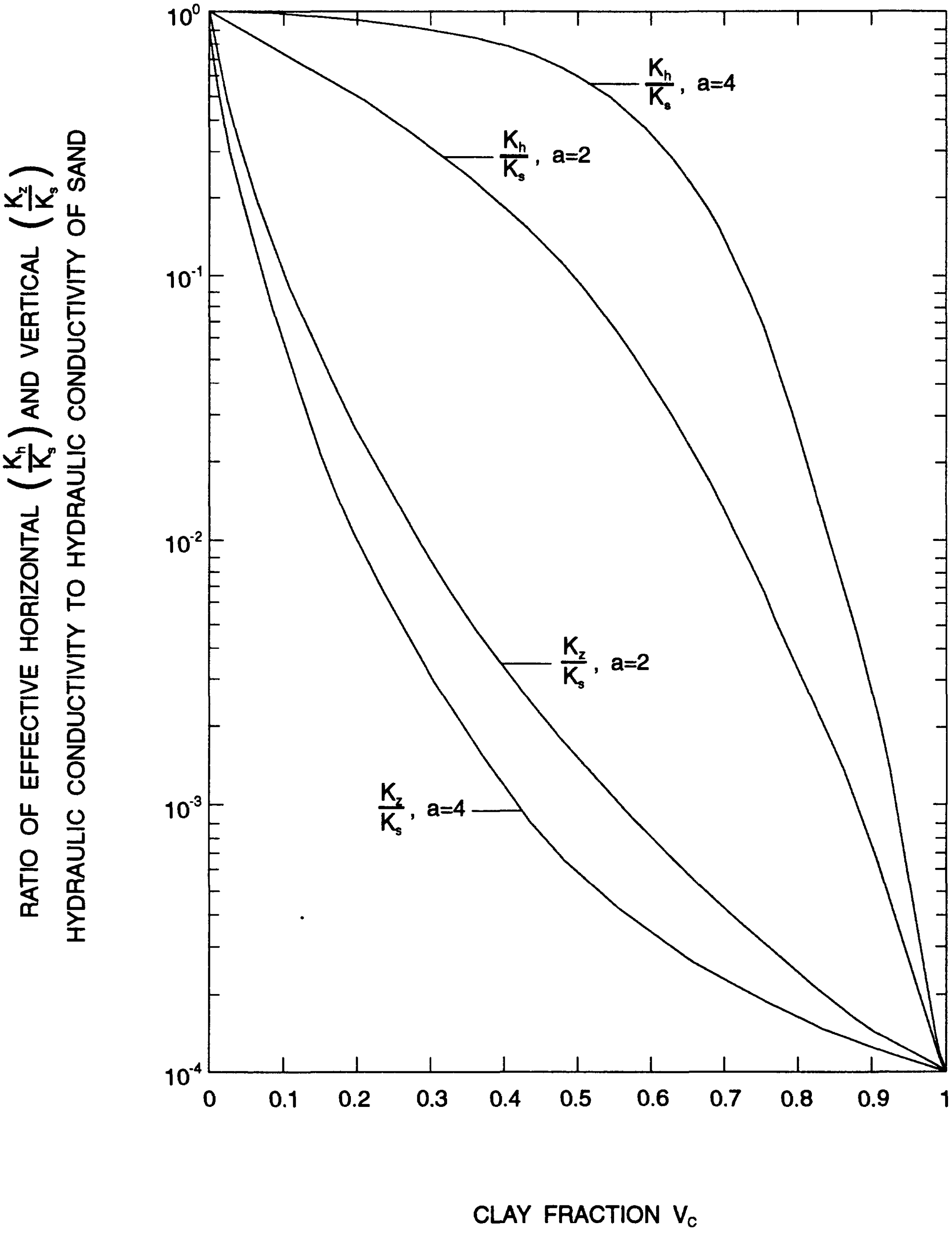

Figure 31. Effective hydraulic conductivity, equations (10) and (11), for $K_s/K_c=10^4$

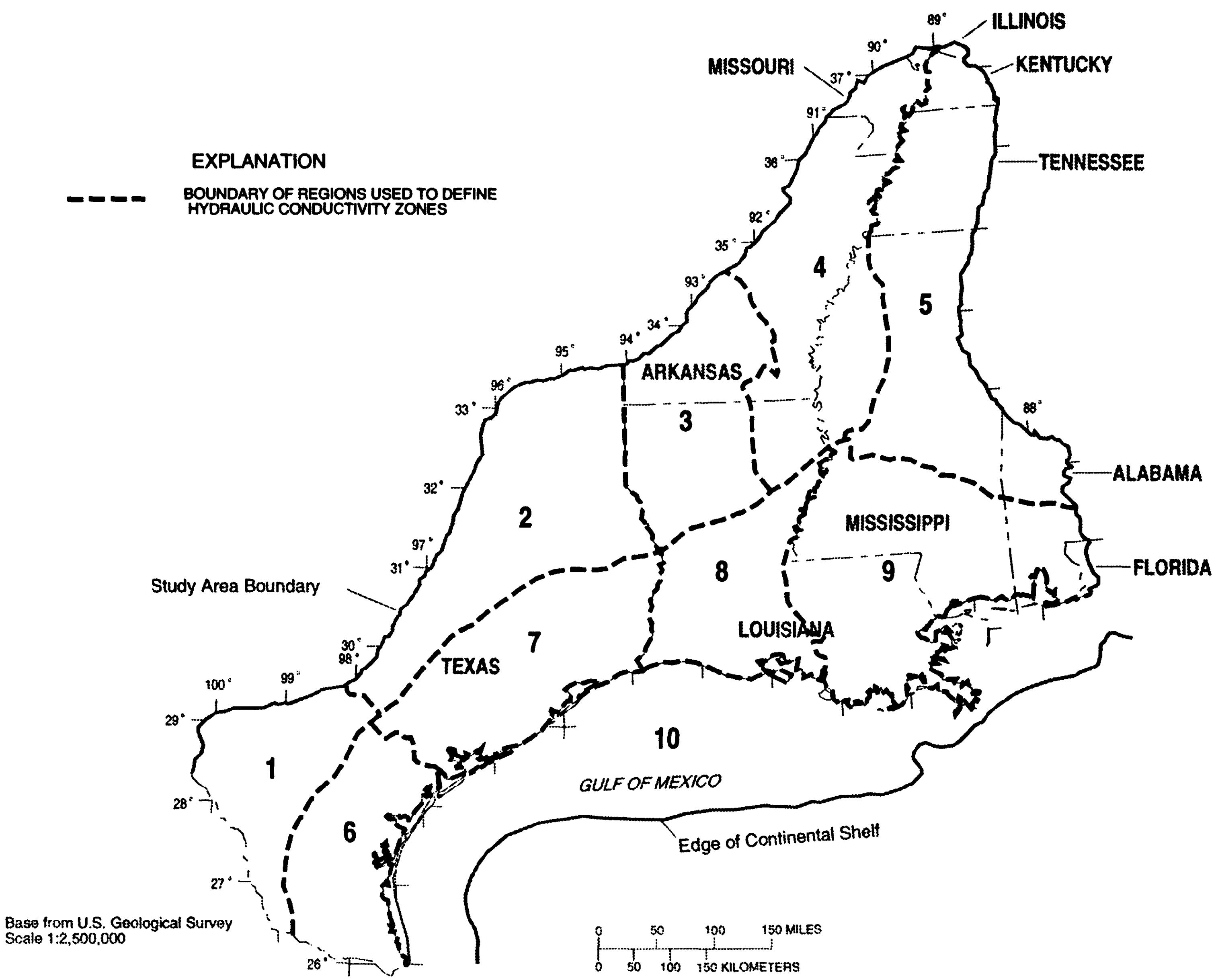

Figure 32. Extent of the 10 regions used to define hydraulic conductivity in the study area. The model layers having hydraulic conductivity data are given in table 2, by region.

and so forth. The application and results section below lists several of the many combinations used. Because the top-layer heads are specified, only the vertical effective hydraulic conductivity in this layer has any effect on the heads. For convenience, the top layer is assigned $V_c = 1$ and this layer has no regression parameters for hydraulic conductivity of sand. Also, the model layers 13 through 17 are mostly clay and, thus, have $V_c = 1$ and have no regression parameters for hydraulic conductivity of sand because little sand is present.

Subsidence

Certain areas of the aquifer system are affected by land subsidence due to ground-water withdrawals. In the numerical flow model, subsidence caused by inelastic compaction of clays was modeled using the procedure of Leake and Prudic (1988). Specific storage is increased from values characteristic of elastic compaction of clays to values characteristic of clays during inelastic compaction and land subsidence. The procedure increases the specific storage by some factor C_s when the grid-element volume-averaged head h declines through values of $h<h_1$, where h_1 is the lowest grid-element volume-averaged head achieved in the recent past. This factor C_s is unity, however, if h has not decreased by more than some triggering value Δh_s in the recent past. Recent past was taken to be all time since 1937, at which time a steady-state numerical flow model with zero pumping defined predevelopment conditions. The value C_s was either set to 40, or to the product of 40 with a regression parameter. In the grid elements of model layers 2 through 6, the subsidence mechanism was not thought to be operative as a result of the absence of materials capable of further compaction, so that C_s was fixed at unity. The triggering value Δh_s was either set equal to 80 ft, or the to product of 80 ft with a regression parameter. The grid area-averaged subsidence for a particular 10-mi by 10-mi grid area located at ground surface is approximately equal to the total volume of the fluid removed from storage in all grid elements lying below the grid area at ground surface, divided by 100 mi^2. This approximation would be nearly exact if water were incompressible. The slight compressibility of water will cause the subsidence to be slightly less than as calculated above, when the volume of the fluid removed is measured at the same temperature and measure of the water at the point of removal.

Preparation of Observations of Head and Hydraulic Conductivity, Y_i

In this study the observations Y_i included observations of head and hydraulic conductivity. The only other possible observations are flow and anthropogenic subsidence. As mentioned previously, recharge into the top of the aquifer system was restricted by adjusting the hydraulic conductivity of layer 12. This mechanism does not, however, involve any observations of flow Y_i. Remaining flow rates, including discharge upward from the aquifer system into layer 12 and also discharge from the geopressured zone, are known poorly so that they would have a very small influence on S(B) because their weights ω_i in equation (8) would be small.

Land subsidence due to ground-water withdrawals can be observed and measured accurately, and measured values are available in locations where substantial subsidence occurs. However, subsidence was not included in the observations Y_i, because of the relatively small areas of those locations where measured values are available.

The point observations of both hydraulic head and hydraulic conductivity were volume-averaged for the grid elements and hydraulic conductivity zones respectively. These volume averages are the observations, Y_i, to which model-simulated values are compared.

Grid Element Volume-Averaged Heads

Approximately 50,000 point observations or measurements of hydraulic head were available. As mentioned previously, these approximately 50,000 values were obtained from time-averaging more than 600,000 individual measurements of hydraulic head in water wells. Accompanying each of these approximately 50,000 point observations was the year, well location, and usually the land-surface altitude. Any of these four items could, however, be in serious error so that the data needed considerable verification. Verification was done partially by persons other than the author and included correlations, such as land-surface altitude versus location, and other procedures. Upon receipt by the author these data were passed through a computer program that looked for anomalies in space and time. If an observation was not in correspondence with five or more adjacent observations in space and time, the observation was deleted. An observation was considered not in correspondence with a single adjacent observation if its hydraulic head was not within 70 ft of the adjacent observation and furthermore, not within 35 ft/mi of distance between the observation and the adjacent observation. Adjacent here means within in the same or an adjoining 10-mi grid element and within 10 years of time. Those point observations of hydraulic head passing verification, about 92 percent, were considered valid and were put into an averaging algorithm described below to form 3,107 grid-element volume-averaged heads.

In addition to the approximately 50,000 point observations of hydraulic head, there were approximately 15,000 measurements of formation pressure from drill-stem tests which were used to approximate point-head values (Lobmeyer, 1985). The resulting point-head values were culled of points in a geopressured condition by eliminating those with pressures outside of the interval of 0.38 to 0.55 (lb/in^2)/ft. The culled point-head data were then used to form grid element averaged heads for those grid elements with 10 or more point heads from drill-stem tests, using a simple unweighted averaging of the culled point heads. The resulting 586 grid-element volume-averaged heads based on drill-stem tests were identified so that they could be distinguished from the grid-element volume-averaged heads based upon point observations of hydraulic head from water wells, because they were located deeper in the aquifer system and were less accurate.

Averaging Algorithm

As mentioned previously, equations (1) and (3) are discretized in order to arrive at finite-difference approximating equations. These approximating equations have N heads associated with the N grid elements in the domain of solution. These grid element heads should be thought of as grid-element volume-averages of head. This volumetric average perhaps should be weighted somewhat in favor of the central portion of the grid element volume. Such an average is not directly observable but must be approximated from point observations of head within the volume of the grid element, or perhaps adjacent grid elements. An averaging algorithm was used to form this average from point observations. Included in the algorithm is a criterion for deciding if there is a sufficient density of point observations of head to justify the determination of a grid-element volume-averaged head. If there is a paucity of point observations, then no grid-element volume-averaged head is calculated, and the grid element does not have an entry for the set of observations $(Y_i, i = 1,2,\ldots,n)$ associated with equation (4). The averaged head calculated by the algorithm is linear in the point observation of head (Gamut, 1986):

$$\hat{h}(t) = \sum_{i=1}^{ns} a_i \hbar_i \quad , \tag{12}$$

Here, $\hat{h}(t)$ is the grid-element volume-averaged head, t is time, and the $\hbar_i$ are point observations of head measured within the time interval $(t-\Delta t, t+\Delta t)$. The integer ns is the number of $\hbar_i$ used to formulate $\hat{h}$ and the a_i are a set of weights which tend to be larger when the head $\hbar_i$ is located closer to the center of the grid element. No consideration is given to the vertical dimension, so that only horizontal distances are considered and the depths of point observations of head within a grid element are disregarded. The justification for this is that vertical dimensions are very small compared to horizontal dimensions and because $\hat{h}$ was formulated from data $\hbar_i$ within the same layer as that of the grid element for which $\hat{h}$ is being sought.

The weights a_i are determined by following these steps:

1) Divide the polar coordinate 0 space surrounding the center of the grid element at which an averaged head is desired into eight octants of 45° each.

2) For each octant, determine the N3 $\hbar_i$ that are at the smallest distances $r_i, i = 1,2,\ldots,N3$, from the center of the grid element, where $r_1 < r_2 < \ldots < r_{N3}$.

N3 is chosen as desired. Eliminate those $\hbar_i$ that are at distances $r>R$, where R is a radius of choice. Note that there may be less than N3 points in the octant with $r<R$. Set all $r_i<R_0$ to R_0, a second radius of choice where $R_0<R$. Let $g_j = r_1$, where j denotes the octant number, $j = 1,2,..,8$. Form the average

$$H_j = \left(\sum_{i=1}^{N3} \hbar_i/r_i^2\right)/\left(\sum_{i=1}^{N3} 1/r_i^2\right) \text{ for each of the octants.}$$

This calculation actually may involve up to N3 values $\hbar_i$. If there are no $\hbar_i$ in the octant with $r<R$, then no H_j is found.

3) Determine $\hat{h} = \left(\sum_{j=1}^{\ell} H_j/g_j^2\right)/\left(\sum_{j=1}^{\ell} 1/g_j^2\right)$, where the summations are over the ℓ octants for which a H_j has been found.

4) The value of $\hat{h}$ found in the previous step is kept for use if

$$\ell \left[\sum_{j=1}^{\ell} 1/r_j^2\right]>C(N3,R,R_0) \quad , \tag{13}$$

where $C(N3,R,R_0)$ is a coefficient of choice.

The use of radius R is to prevent the use of data at very large distances from the grid element center. The use of R_0, in the step where those r_i within an octant are set to R_0 if less than R_0, is to prevent a single point close to the center of the grid element from totally overwhelming the value of H_j for that octant.

The use of $1/r^2$ weighting as opposed to some other power of r requires some justification. In the case of a uniform distribution of data points in two dimensions, a weighting of $1/r$ gives equal weight to points at any radius. This is true because the area between r and $r+\Delta r$ is $2\pi r\Delta r$ which when multiplied by the weighting factor $1/r$ gives $2\pi\Delta r$. Thus data at very large values of radius r ($r <$ radius $< r+\Delta r$) are counted as heavily as those at small r. This is definitely not desired since we seek an average head $\hat{h}$ that is representative of those heads somewhat centrally located within the single grid element for which $\hat{h}$ is being found, not a uniformaly weighted-average head over the entire region. Thus whatever the power n should be in the weighting factor $1/r^n$, it is clear that it should definitely be greater than 1. A value of $n = 2$ was used. This value for n would place twice as much emphasis on points that are twice as close. Several different values for n, all greater than 1.5 were used. For those values of n used, little effect was found on the ability of the model to fit the data as measured by values of mean weighted residual and root mean square weighted residual.

Estimators of the type described above are intended for noisy data with correlation dependent on the separation distance between values of h_i (Gamut, 1986). Such estimators are stable because they interpolate between the values of h_i, so that h must lie between the smallest and largest values of h_i $i = 1,\ldots,s$. Note that h is octally weighted regardless of the number of h_i that may occur in a given octant, provided only that this number is at least 1. N3 was chosen to be 3, $C(N3,R,R_0)$ was $36(10\ \text{mi})^{-2}$, R_0 was 2 mi, and R was 15 mi which is 1.5 times the horizontal dimension of a grid element which was 10 mi. The number of grid-element volume-averaged heads calculated was 3,107: 1,432 for the year 1972, and the remainder of 1,675 for the year 1982, both with $\Delta t = 1$ year. The location of the 1,675 grid-element volume-averaged heads for 1982 is shown in figure 33.

Logarithms of Hydraulic Conductivity of Sand

Grid-element volume-averaged heads, the preparation of which is explained above, form part of the set of n observations Y_i, $i = 1,2,\ldots,n$. The remaining members of Y_i are volume averages of the hydraulic conductivity of sand. The volumes used here are much larger than a single grid element volume and consist of the hydraulic conductivity zones mentioned previously. A total of 33 zones based on the logarithm of the hydraulic conductivity of sand for the sand component of model layers 2 through 11 were constructed by using the layer and region combinations for which hydraulic conductivity data are given in table 2. The point estimates of the logarithm of hydraulic conductivity of sand were derived from aquifer or specific-capacity tests. The geometric mean of these test data were calculated by Prudic (1991) for each layer by region. Preliminary values for these tests, which generally are about 30 percent less than those given by Prudic (1991), were used in this study and are given in table 2. The logarithm of the hydraulic conductivity of sand for a zone is calculated as the simple average of the point observations of the logarithmic values of hydraulic conductivity of sand within the zone, whether determined by aquifer or specific-capacity tests. For one of the models, model 9 below, the 33 zones based on the logarithm of the hydraulic conductivity of sand were combined into six groups, each with a corresponding zone-averaged logarithmic hydraulic conductivity of sand.

It was mentioned previously that the hydraulic conductivity of sand for a grid element j in the numerical model f(B) is given by $k_j = P_j 10^{-0.8d_j}$, where P_j is the hydraulic conductivity parameter for the grid element and d_j is the depth in kilometers of the center of the grid element below ground surface. Thus, when $\log_{10}(k_j) = \log_{10}(P_j) - 0.8d_j$ is averaged over a hydraulic conductivity zone containing m grid elements j but only a single value P for P_j, $\frac{1}{m}\sum_{i=1}^{m}\log_{10}(k_j) = \log_{10}(P) - 0.8d$ is obtained where $d = \frac{1}{m}\sum_{j=1}^{m} d_j$. When viewed as an observation, $\frac{1}{m}\sum_{j=1}\log_{10}(k_j)$ is one of the observations of the logarithms of hydraulic conductivity of sand, Y_i, but when predicted by the model it is the corresponding $f_i(\hat{B})$. The single hydraulic conductivity parameter associated with the hydraulic conductivity zone containing the grid elements $j = 1,2,\ldots,m$ is P.

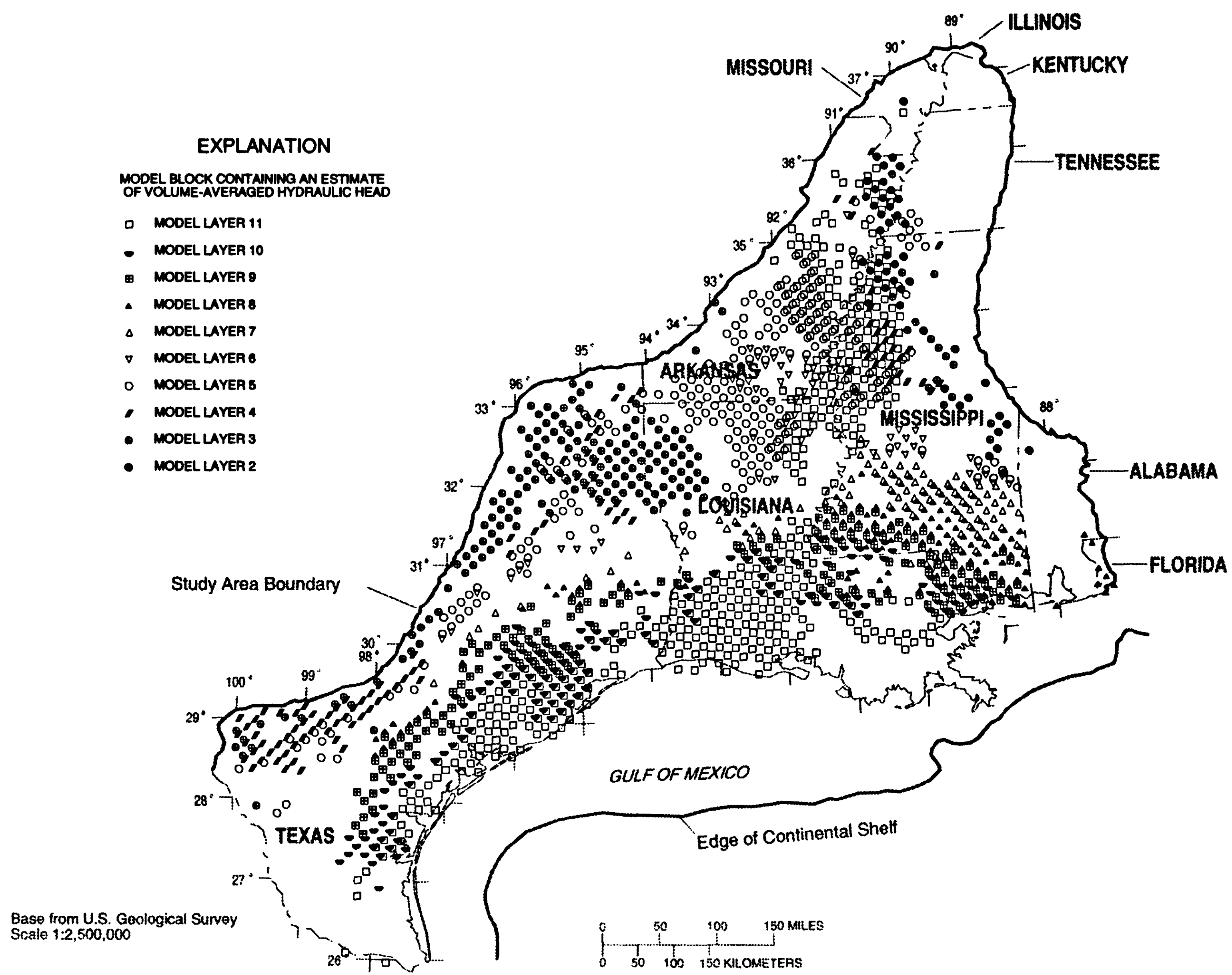

Figure 33. Location of the 1,675 grid-element volume-averaged heads for 1982.

TABLE 2.--*Mean hydraulic conductivity by model layer and region*

[Layer numbers are given in table 1 and region numbers are shown in figure 31. Mean base 10 logarithm of sand hydraulic conductivity is measured in units of feet per day (ft/d); mean $\log_{10}$ hydraulic conductivity of sand is 1.481; average geometric mean hydraulic conductivity of sand is 30.3 ft/d]

Layer	Region	Mean $\log_{10}$ sand hydraulic conductivity	Geometric mean sand hydraulic conductivity (ft/d)
2	5	1.761	57.68
3	2	0.844	6.98
3	3	1.117	13.09
3	5	1.386	24.32
4	1	1.493	31.12
4	2	1.345	22.13
4	5	1.689	48.87
5	2	1.048	11.17
5	3	1.507	32.14
5	4	1.628	42.46
5	5	1.906	80.54
5	9	1.073	11.83
6	2	0.881	7.60
6	3	1.352	22.49
6	4	1.685	48.42
6	5	1.605	40.27
7	8	1.543	34.91
7	9	1.565	36.73
8	7	1.473	29.72
8	8	1.524	33.42
8	9	1.704	50.58
9	6	0.698	4.99
9	7	1.162	14.52
9	8	1.663	46.03
9	9	1.750	56.23
10	6	0.938	8.67
10	7	1.259	18.16
10	8	1.688	48.75
10	9	1.726	53.21
11	4	2.292	195.88
11	7	1.573	37.41
11	8	2.074	118.58
11	9	1.926	84.33

The total number of observations Y_i, $i=1,2,\ldots n$ was either $n = 3{,}107 + 33 = 3{,}140$ or $n = (3{,}107+586) + 33 = 3{,}726$. Here, the number of observations of the logarithms of hydraulic conductivity of sand is 33. The number of grid-element volume-averaged heads from water-wells is 3,107; and the number of grid-element volume-averaged heads from drill-stem tests is 586. For model 9, there are 6 rather than 33 observations of the logarithms of hydraulic conductivity of sand.

Residual Weighting

In equation (5) ω is a known n x n symmetric, positive-definite matrix. The matrix ω is assumed to be diagonal. The minimization of S(B) in equation (8) tends to cause $Y_i-f_i(B)$ to be small when ω_i is large, or equivalently, when the variance $\epsilon^2\omega_i^{-1}$ is small. Thus when the variance of an observable Y_i is small; that is, its value is known well, $\hat{B}$ is found such that the model value $f_i(\hat{B})$ is close to the actual value Y_i. The variance of E_i, the ith member of the n x 1 random vector E, is $\epsilon^2\omega_i^{-1}$ where ϵ is a common variance factor of choice. As explained in the modeling methodology section above, the true variance of E, $\epsilon^2\omega^{-1}$, may be approximated by apparent variance. The ω_i for the 3,107 values of grid-element volume-averaged head were separated into 10 groups, corresponding to the 10 model layers 2 through 11. Thus, $\omega_i = \omega_s$, $i = 1,2\ldots.3{,}107$, where s is the model layer containing the grid element i corresponding to observation Y_i. For $s = 1,2,\ldots,10$, the value of $\epsilon^2\omega_s^{-1}$ was set equal to $\frac{1}{n_s}\sum_{i=1}^{n_s}(Y_i-f_i(\hat{B}))^2$ where n_s is the number of observations in layer s. These ω_s depend upon the number of regression parameters p making up the p x 1 vector of parameters B, and also upon the construction of f(B). The ω_i corresponding to the 586 grid-element volume-averaged heads from drill-stem tests were given a separate single value for $\epsilon^2\omega^{-1}$ equal to $\frac{1}{586}\sum_{i=1}^{586}(Y_i-f_i(\hat{B}))^2$. This value was much larger than the values for $\epsilon^2\omega_s^{-1}$ above.

The $\epsilon^2\omega_i^{-1}$ for the 33 observations of the logarithms of hydraulic conductivities of sand, Y_i, were given a single value. This single value of the $\epsilon^2\omega_i^{-1}$ was calculated as the average of 10 variances. Each of these 10 variances was determined as the variance of those Y_i corresponding to model layer s, about their own mean. The value for $\epsilon^2\omega^{-1}$ obtained in this manner is approximately equal to the value for $\epsilon^2\omega^{-1}$ obtained by determining the variance of all of the 33 observations of the logarithms of the hydraulic conductivities of sand about their own single mean. In model 9, residual weighting for the six observations of the logoarithms of the hydraulic conductivities of sand was done using six different values $\epsilon^2\omega_i^{-1}$ for variance. These variances were smaller than the single value of $\epsilon^2\omega^{-1}$ used for the 33 observations of the logarithms of the hydraulic conductivities of sand, as explained previously in the regression model section.

The above described estimation of the variances $\epsilon^2\omega_i^{-1}$ for grid element volume-averaged head, made use of the values $Y_i-f_i(\hat{B})$. But to find $\hat{B}$ in $f(\hat{B})$ one needs to minimize $S(B) = \sum_{i=1}^{n} (Y_i-f_i(B))^2\omega_i$ which itself depends upon the very weights ω_i which are being sought. This apparent predicament is solved by doing several minimizations of S(B), putting the ω_i from a given iteration into S(B) for the next iteration. For the values ω_i, this process converges sufficienty in only two iterations.

Weighted mean square error $S(B) = \sum_{i=1}^{n} [Y_i-f_i(B)]^2\omega_i$ may be written $\sum_{i=1}^{n} [Y'_i-f'_i(B)]^2\omega'_i$, where $Y'_i= Y_i(\omega_i/\omega'_i)^{1/2}$, $f'_i=f_i(\omega_i/\omega'_i)^{1/2}$, and $\omega'_i = 0(1)$. Thus, it is possible to normalize Y_i and f_i such that, for each of the three classes of observations: grid-element volume-averaged heads, from (1) water-well data and from (2) drill-stem tests, and (3) volume-averaged logarithms of hydraulic conductivity of sand; the weights ω_i are order unity. Normalizing heads from water wells, in conjunction with a selection for ϵ^2, allowed the weights ω_i i=1,2,...,3107 for the grid element volume-averaged heads to satisfy $.4< \omega_i<1.7$ and $(1/3{,}107) \sum_{i=i}^{3107} \omega_i = 1$. In addition, the single weight for the grid-element volume-averaged heads from drill-stem tests and also the single weight for the 33 observations of the logarithms of the hydraulic conductivities of sand are allowed to both be unity.

APPLICATION OF REGRESSION FLOW MODELS AND RESULTS

The principal value of the regression methodology is that it allows one to measure the accuracy of the predictions produced by a model.

The principal use of the regression flow models in this study is to give an indication of the accuracy of the predicted values for grid-element volume- averaged head, and the logarithm of hydraulic conductivity of sand values in the models. Also desired is an indication of the accuracy of the values obtained for the regression parameters. These indications of accuracy are obtained from the prediction interval half widths and confidence interval half widths, available in equations (6) and (7) respectively.

Equation (6) above gives an approximate prediction interval for the kth member of the random vector Y of equation (4). The term in equation (6) containing the sensitivity matrix X was, except for the six observations of the logarthms of the hydraulic conductivities of sand of model 9, always less than 4 percent of ω_k^{-1}. Dropping this term causes equation (6) to become:

$$\omega_k e_k^2 = d_{1-\alpha}^2 = S(\hat{B})D^2(p,n,\alpha) \ , \tag{14}$$

where

$$D^2(p,n,\alpha) = [(p+1)/(n-p)]F_\alpha(p+1,n-p) \ .$$

The probability is approximately (1-A)100 percent that $f_k(\hat{B})-e_k<Y_k<f_k(\hat{B})+e_k$, if $f_k(B)$ is a linear model. This result is exact when $f(B)$ is a linear model, and the $\omega_k^{1/2}e_k$ have a normal distribution. For fixed n, equation (14) shows that $S(\hat{B})$ will have to decrease faster with p than $D^2(p,n,\alpha)$ increases with p in order that the prediction interval half width e_k decrease with increasing p. Figure 34 shows plots of $D^2(p,n,\alpha)$ as a function of p.

Having considered the dependence of $\omega_k e_k^2$ upon p with fixed n, let us now consider its dependence upon n with fixed p. If $S(\hat{B})/(n-p)$ is written as $(1+u)\epsilon^2$, then equation (14) becomes

$$\omega_k e_k^2 = (1+u)\epsilon^2 G(p,n,\alpha) \ , \tag{15}$$

where

$$G = (p+1)F_\alpha(p+1,n-p) \ .$$

When $n\to\infty$ and $n>>p$, it can be shown that $1+u = S(\hat{B})/[(n-p)\epsilon^2]\to 1$ (R.L. Cooley, U.S. Geological Survey, oral commun., 1989), so that when n is large, 1+u changes very little with n. Furthermore, for $n>1,000$ and $p<<n$ as in this study, G changes very little for any increase in n. Thus, $\omega_k e_k^2$ displays no strong dependence upon n.

Description of Models Used

All models have the basic features as given in the section on "Modeling Methodology and Construction." Each model has the same domain of solution. The differences between the models are: the degree and manner by which the domain of solution is partitioned into hydraulic conductivity zones; and the degree to which unknowns related to boundary conditions, the manner of the dependence of effective hydraulic conductivity on clay and hydraulic conductivity of sand, and subsidence, are parameterized. When an unknown is parameterized, an optimal value for the parameter is found by the model. If an unknown value is not parameterized, a fixed default value is used. Thus, models with a larger number of parameters are more flexible but not conceptually more complex.

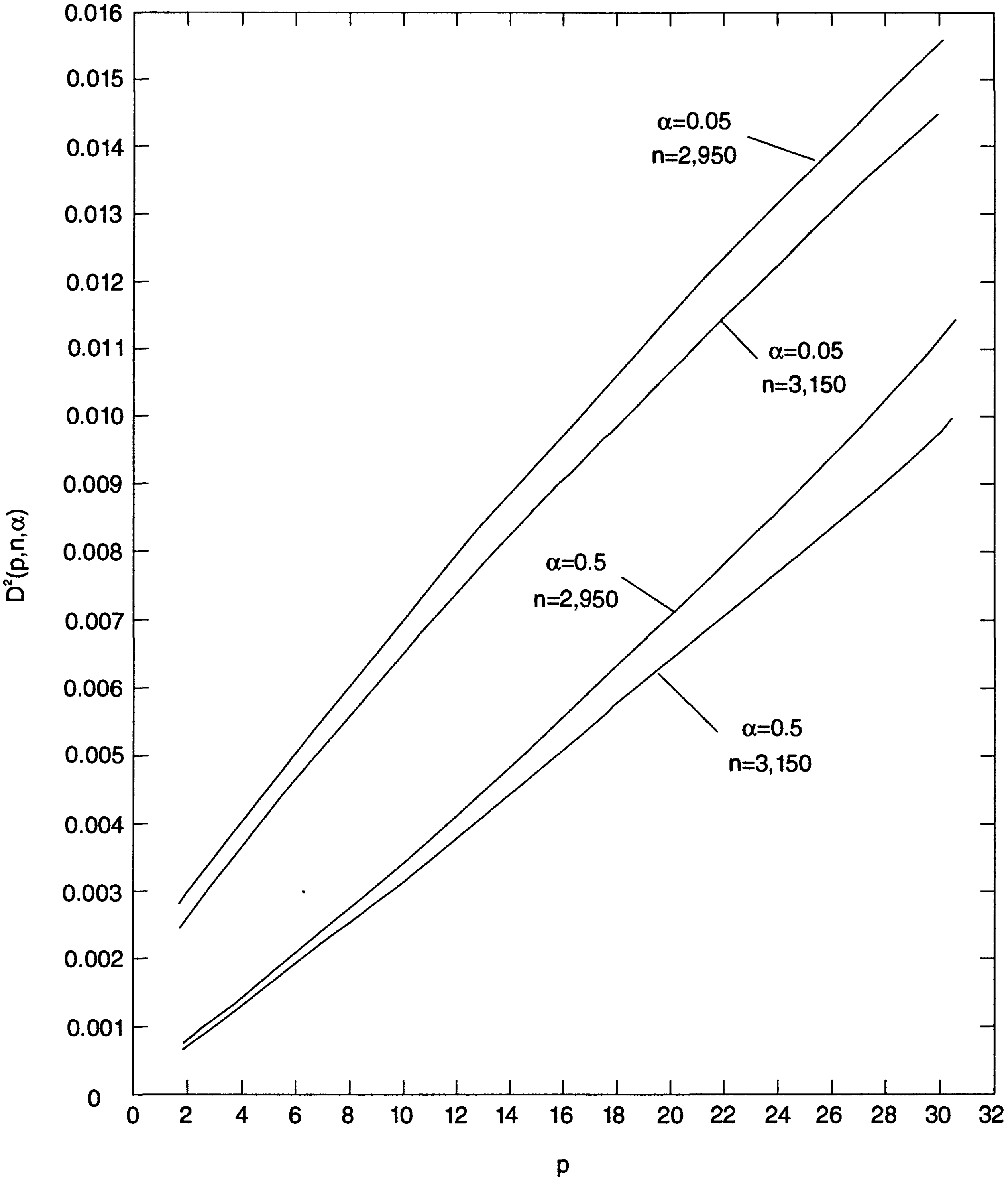

Figure 34. Change in multiplier [$D^2(p,n,\alpha)$] for prediction interval size with number of regression parameters.

Many different numerical flow models f(B), with different numbers of regression parameters p were used. Several of these are shown in table 3. The regression parameters b_i used in this table are defined as follows. The specified boundary head in the top-layer grid elements is equal to $h_w b_1 + b_2$. Here, h_w is water-table altitude or equivalent freshwater head at the sea floor, as discussed previously. The specified boundary head in the geopressured zone, shown in figure 26, is (1,000 ft)b_3. The value of (K_z/b) in the geopressured zone grid elements is equal to b_4. Regression parameter b_5 is equal to .25a. Quantity a was discussed previously for use with equations (10) and (11) to calculate effective hydraulic conductivity. Regression parameter b_6 is K_y/K_x for horizontal anisotropy. Regression parameter b_7 is the parameter which when multiplied by 80 ft gives the subsidence triggering value Vh_s discussed previously. Regression parameter b_8 is the parameter that when multiplied by 40 gives the subsidence specific storage multiplication factor C_s. Regresssion parameter b_9 is specific storage.

In table 3, when B_j occurs in the row starting with b_i, it indicates that b_i is equal to B_j, the jth element of the p x 1 regression parameter vector B. Note that more than one of the b_i may be equal to a single one of the B_j. When no entry occurs in the row b_i, it indicates that the regression parameter b_i is not used and that the default value applies. The default values for the regression parameters are $b_1 = 1$, $b_2 = 0$, $b_3 = 1$, $b_5 = 1$, $b_6 = 1$, $b_7 = 1$, and $b_8 = 1$. When b_4 is not used, (K_z/b) for the geopressured zone grid elements is set equal to either a very large number or zero. When b_4 is very large, the geopressured zone provides no resistance to vertical flow, and all resistance is provided by the model layers lying above the geopressured zone. When b_4 is zero, no flow is allowed from the geopressured zone. In table 3, entry 0 for b_4 denotes that (K_z/b) is zero, "very large" denotes that (K_z/b) is very large. Default values of regression parameters b_i were selected to prevent biasing results in favor of few regression parameter models. Such a bias occurs when results from models with many regression parameters are used to arrive at default values of the parameters in a model with few parameters. For example, if a given model f(B) with many regression parameters determines that b_7 (the regression estimate for $\hat{b}_7$ corresponding to $B = \hat{B}$) should be 1.3, then in another model f(B) with few parameters in which b_7 is not used, the default value of b_7 may not be set to 1.3, and so forth. The single value b_9 of the specific storage for the aquifer domain of solution, is always used and hence has no default value.

The entries in table 3 below row b_9 give the assignment of regression parameters to the hydraulic conductivity zones. For example, model 3 has $B_1 = b_1 = b_3$ and $B_2 = b_9$. Regression parameter B_3 is assigned to the hydraulic conductivity of clay of all those hydraulic conductivity zones contained in model layers 2 through 11, top-layer 12, and also model layers 13 through 17 which represent confining units. Regression parameter B_4 is

TABLE 3.—*Regression parameters for models 1 through 12*

[When B_j occurs in the row starting with b_i, it indicates that b_i is equal to B_j, the j^{th} element of the p x 1 regression parameter vector B. Note that more than one of the b_i may be equal to a single one of the B_j. Shading indicates that combination of models and parameters was not used or was not applicable; do, ditto; VL, is very large]

Sediment		Model 1	2	3	4	5	6	7	8	9	10	11	12
	Number of parameters	2	3	4	4	6	7	8	9	11	18	26	31
	b_1 for head in top layer		B1	B1	B1	B1	B1	B1	B1	B1	B1	B1	B1
	b_2 for head in top layer (feet)												
	b_3 for geopressured head			B1		B2	B1	B1	B2		B1	B1	B1
	b_4 for geopressured head (day^{-1})	0	0	VL	0	B3	VL	VL	B3	0	B2	B2	B2
	b_5 for .25a in equations 10 and 11								B4		B3	B3	B3
	b_6 horizontal anisotropy												
	b_7 for subsidence triggering head								B5		B4	B4	B4
	b_8 for subsidence specific storage								B6				
	b_9 specific storage (feet^{-1})	B1	B2	B2	B2	B4	B2	B2	B7	B2	B5	B5	B5
Clay hydraulic conductivity	Layer 2	B2	B3	B3	B3	B5	B3	B3 and B5	B8	B3	B6		B6
	Layer 3	do	do	do	do	do	do	do	do	do	do		B7
	Layer 4	do	do	do	do	do	do	do	do	do	do		B8
	Layer 5	do	do	do	do	do	do	do	do	do	do		B9
	Layer 6	do	do	do	do	do	do	do	do	do	do		B10
	Layer 7	do	do	do	do	do	B4	do	do	do	do		B11

TABLE 3.—*Regression parameters for models 1 through 12*—Continued

Sediment		Model 1	2	3	4	5	6	7	8	9	10	11	12
Clay hydraulic conductivity	Layer 8	B2	B3	B3	B3	B5	B4	B3 and B5	B8	B3	B6		B12
	Layer 9	do	do	do	do	do	do	do	do	do	do		B13
	Layer 10	do	do	do	do	do	do	do	do	do	do		B14
	Layer 11	do	do	do	do	do	do	do	do	do	do		B15
	Layer 12	do	do	do	do	do	do	do	do	do	B7	B6	B16
	Layers 13-17	do	do	do	do	do	B5	B4 and B6	do	B4	B8		B17-B21
	Region 1							B3 and B4				B7	
	Region 2							do				B8	
	Region 3							do				B9	
	Region 4							do				B10	
	Region 5							do				B11	
	Region 6							B5 and B6				B12	
	Region 7							do				B13	
	Region 8							do				B14	
	Region 9							do				B15	
	Region 10							do				B16	

TABLE 3.—*Regression parameters for models 1 through 12*—Continued

Sediment		Model 1	2	3	4	5	6	7	8	9	10	11	12
Sand hydraulic conductivity	Layer 2	B2	B3	B4	B4	B6	B6		B9	B5-B11	B9		B22
	Layer 3	do	do	do	do	do	do		do	do	B10		B23
	Layer 4	do	do	do	do	do	do		do	do	B11		B24
	Layer 5	do	do	do	do	do	do		do	do	B12		B25
	Layer 6	do	do	do	do	do	do		do	do	B13		B26
	Layer 7	do	do	do	do	do	B7		do	do	B14		B27
	Layer 8	do	do	do	do	do	do		do	do	B15		B28
	Layer 9	do	do	do	do	do	do		do	do	B16		B29
	Layer 10	do	do	do	do	do	do		do	do	B17		B30
	Layer 11	do	do	do	do	do	do		do	do	B18		B31
	Region 1							B7		do		B17	
	Region 2							do		do		B18	
	Region 3							do		do		B19	
	Region 4							do		do		B20	
	Region 5							do		do		B21	
	Region 6							B8		do		B22	
	Region 7							do		do		B23	
	Region 8							do		do		B24	
	Region 9							do		do		B25	
	Region 10							do		do		B26	

assigned to the hydraulic conductivity of sand of model layers 2 through 11. Top-layer 12, and confining units represented by model layers 13 through 17 have no sand and thus need no regression parameter assignment.

Table 3 is self explanatory except for the following. The regions mentioned are those of figure 31, and allow for a horizontal discretization as opposed to vertical discretization using the layers. For hydraulic conductivity of clay, model 7 uses B_3 and B_5 for layers 2 through 11, B_3 in regions 1 through 5 and B_5 in regions 6 through 10. B_4 and B_6 are used for layer 12, B_4 for regions 1 through 5, and B_6 for regions 6 through 10. For hydraulic conductivity of sand, model 9 uses a discretization of the domain of solution into seven zones, selected on the basis of hydrogeologic considerations. Each of these zones correspond to various combinations of the layers and regions. B_5 through B_{11} are used for these seven zones. Model 12 uses B_{17} through B_{21} for layers 13 through 17.

The following is a short description of the models of table 3 in physical terms. The specific storage, the horizontal hydraulic conductivity of sand, and the vertical hydraulic conductivity of clay were allowed to vary within bounds and the values were determined as a result of the regression model and are tabulated in table 4. Other factors such as the head in the geopressured zone, the altitude of the water table, and head decline necessary for the onset of land subsidence were allowed to vary in some models but were fixed in other models.

The value of specific storage for layers 2 through 11 ranged from 4.5×10^{-7} to 8.0×10^{-7}, except that model 12 had a specific storage of 3.0×10^{-8}. The horizontal hydraulic conductivity of sand was uniform at about 30 ft/d for all layers in models 1, 2, 3, 4, 5, and 6. Both models 10 and 12 have different values of horizontal hydraulic conductivity of sand for each layer. The values range from about 8 ft/d for layer 3 in model 10 to 650 ft/d for layer 2 in model 12. Models 10 and 12 are similar in that layers that have large values of horizontal hydraulic conductivity of sand in one model also have large values in the other model. Two values of horizontal hydraulic conductivity of sand were used in model 6; the values are about 73 ft/d for layers 2 through 6 and 25 ft/d for layers 7 through 11. The horizontal hydraulic conductivity of sand varied by geographic region in models 7 and 11. The variability was greater in model 11 with 10 conductivity values ranging from 5 ft/d to 134 ft/d, than in model 7 with 2 values of about 24 ft/d and 68 ft/d. Six values of horizontal hydraulic conductivity of sand were used in model 9; the values range from about 6 ft/d to about 380 ft/d. These values were for combinations of layer and geographic region which, in general, correspond to area and layer combinations of Prudic (1991, p. 28), and the values used correspond closely to the geometric means presented by Prudic (1991).

The vertical hydraulic conductivity of clay was uniform for both the clay fraction within aquifers and for the confining units in models 1 through 5, and model 8; the values range from about 3.6×10^{-4} ft/d to about 30 ft/d. In models 6, 7, 9, 10, and 12 the hydraulic conductivity of

TABLE 4.—*Results from flow models 1 through 12*

[The 95 percent prediction intervals for the observations are obtained from $w_k^{1/2}e_k$, the 95 percent confidence intervals for the parameters with entry a(b) are from ab^{-1} to ab. Shading indicates that combination of models and parameters was not used or was not applicable; do, ditto]

	Model											
	1	2	3	4	5	6	7	8	9	10	11	12
Number of parameters	2	3	4	4	6	7	8	9	11	18	26	31
$w_k^{1/2}e_k$ of equation 14	175	166	141	141	157	162	168	178	167	214	219	243
(1+u) of equation 15	3.17	2.42	1.50	1.49	1.48	1.45	1.43	1.48	1.14	1.29	1.02	1.11
Mean weight residual	33.2	9.23	2.64	2.54	2.53	2.20	1.48	2.34	3.74	2.22	1.70	2.64
Root-mean-square weighted residual	60.5	53.0	41.6	41.5	41.4	40.96	40.72	41.4	36.3	38.6	34.4	35.9
Chi^2	678	413	316	324	326	204	242	321	260	261	164	188
Parameters:												
b_1 for head in top layer		.834 (1.04)	.898 (1.04)	.899 (1.04)	0.900 (1.04)	0.913 (1.05)	.901 (1.05)	0.900 (1.05)	.932 (1.05)	0.913 (1.07)	.920 (1.07)	.911 (1.08)
b_2 for head in top layer (feet)												
b_3 for geopressured head			.898 (1.04)		0.054 (>10^3)	0.913 (1.05)	.901 (1.05)	0.25 (>10^3)		.913 (1.07)	.920 (1.07)	.911 (1.08)
b_4 for geopressurred head (day $^{-1}$)					$1.25x10^{-8}$ (>10^3)			$0.94x10^{-8}$(>10^3)		$.16x10^{-4}$ (>10^3)	$1.5x10^{-5}$ (>10^3)	$.71x10^{-5}$ (>10^3)
b_5 for .25a in equations 10 and 11								1.01 (1.14)		.97 (1.19)	1.09 (1.22)	.88 (1.20)
b_6 horizontal anisotropy												
b_7 for subsidence triggering head								1.31 (1.75)		1.28 (1.92)	2.28 (2.04)	2.75 (39.2)
b_8 for subsidence specific storage								1.06 (6.24)				
b_9 specific storage (feet $^{-1}$)	$.66x10^{-6}$ (>10^3)	$.66x10^{-6}$ (>10^3)	$.45x10^{-6}$ (1.73)	$.45x10^{-6}$ (1.73)	$.47x10^{-6}$ (1.93)	$.46x10^{-6}$ (2.03)	$.54x10^{-6}$ (2.04)	$.54x10^{-6}$ (12.1)	$.51x10^{-6}$ (1.83)	$.68x10^{-6}$ (3.94)	$.80x10^{-6}$(4.70)	$.30x10^{-7}$(6.61)

TABLE 4.—*Results for flow models 1 through 12—Continued*

	Model											
	1	2	3	4	5	6	7	8	9	10	11	12
	Clay hydraulic conductivity parameters (feet per day)											
Layer 2	29.9 (1.63)	29.9 (1.66)	$.365x10^{-3}$ (1.23)	$.362x10^{-3}$ (1.23)	$.361x10^{-3}$ (1.26)	$.12x10^{-3}$ (2.27)	$.47x10^{-3}$ (2.70) $.35x10^{-3}$ (1.33)	$.374x10^{-3}$ (1.53)	$.22x10^{-3}$ (1.32)	$.26x10^{-3}$ (2.03)		$.78x10^{-4}$ (>10^3)
Layer 3	do	do	do	do	do	do	do	do	do	do		$.20x10^{-2}$ (22.3)
Layer 4	do	do	do	do	do	do	do	do	do	do		$.94x10^{-3}$ (>10^3)
Layer 5	do	do	do	do	do	do	do	do	do	do		$.44x10^{-4}$ (51.9)
Layer 6	do	do	do	do	do	do	do	do	do	do		$.22x10^{-1}$ (>10^3)
Layers 7	do	do	do	do	do	$.39x10^{-3}$ (1.33)	do	do	do	do		$.53x10^{-4}$ (4.50)
Layer 8	do	do	do	do	do	$.39x10^{-3}$ (1.33)	do	do	do	do		$.83x10^{-2}$ (26.7)
Layer 9	do	do	do	do	do	do	do	do	do	do		$.29x10^{-2}$ (5.86)
Layer 10	do	do	do	do	do	do	do	do	do	do		2.98 (>10^3)
Layer 11	do	do	do	do	do	do	do	do	do	do		$.11x10^{-3}$ (2.70)
Layer 12	do	do	do	do	do	do	do	do	do	$.98x10^{-3}$ (670)	$.11x10^{-2}$ (>10^3)	$.31x10^{-2}$ (>10^3)
Layers 13-17	do	do	do	do	do	$.22x10^{-4}$ (2.72)	$.29x10^{-4}$ (3.36) $.21x10^{-5}$ (>10^3)	do	$.11x10^{-3}$ (1.67)	$.67x10^{-4}$ (2.19)		$.48x10^{-6}$ (>5) $.16x10^{-3}$
Region 1											$.24x10^{-3}$ (2.76)	
Region 2											$.10x10^{-3}$ (3.23)	
Region 3											$.33x10^{-4}$ (2.28)	
Region 4											$.66x10^{-3}$ (4.34)	
Region 5											$.35x10^{-3}$ (5.41)	
Region 6											$.21x10^{-3}$ (4.75)	
Region 7											$.30x10^{-3}$ (1.88)	
Region 8											$.31x10^{-3}$ (3.53)	
Region 9											$.68x10^{-3}$ (2.43)	
Region 10											$.27x10^{-3}$ (>10^3)	

TABLE 4.—*Results for flow models 1 through 12—Continued*

	Model											
	1	2	3	4	5	6	7	8	9	10	11	12
	Sand hydraulic conductivity parameters (feet per day)											
Layer 2	29.9 (1.63)	29.9 (1.66)	29.9 (1.20)	30.3 (1.19)	30.3 (1.23)	72.7 (1.22)		29.0 (1.31)		266.0 (2.50)		650 (2.88)
Layer 3	do	do	do	do	do	do		do		7.6 (4.27)		8.70 (6.36)
Layer 4	do	do	do	do	do	do		do		75.0 (1.83)		57.6 (2.49)
Layer 5	do	do	do	do	do	do		do		35.8 (1.87)		47.0 (2.32)
Layer 6	do	do	do	do	do	do		do		58.7 (2.83)		58.3(4.38)
Layer 7	do	do	do	do	do	25.0 (1.35)		do		34.3 (4.72)		53.8 (5.10)
Layer 8	do	do	do	do	do	do		do		55.5 (2.95)		125 (3.07)
Layer 9	do	do	do	do	do	do		do		27.2 (2.54)		19.1 (3.77)
Layer 10	do	do	do	do	do	do		do		9.7 (2.18)		11.8 (2.48)
Layer 11	do	do	do	do	do	do		do		49.1 (1.92)		84.9 (2.11)
Region 1							68.2 (1.24)		group 1 383.0 (2.03)		17 (2.37)	
Region 2							do		group 2 115.0 (1.44)		12 (3.07)	
Region 3							do		group 3 94.3 (1.33)		10 (2.63)	
Region 4							do		group 4 30.0 (1.39)		134 (3.49)	
Region 5							do		grou p 5 14.9 (1.51)		67 (2.60)	
Region 6							24.2 (1.37)		group 6 6.53 (1.95)		19 (5.27)	
Region 7							do		group 7 16.4 (235)		18 (1.93)	
Region 8							do				72 (2.67)	
Region 9							do				72 (1.92)	
Region 10							do				5 ($>10^3$)	

clay within the aquifers had a different value than that for the confining units. In model 11 the vertical hydraulic conductivity of clay was uniform throughout each of the 10 geographic areas for clay both within aquifers and confining units. In model 12 a separate value of vertical hydraulic conductivity of clay was used for each aquifer and each confining unit; the values range from 4.8×10^{-7} ft/d to about 3 ft/d.

The estimated water-table altitude was reduced by a factor ranging from 0.834 to 0.932, except that for model 1 the water-table was fixed at the values estimated by Williams and Williamson (1989). The flow of water to and from the water table was controlled by a hydraulic conductivity factor for the top layer 12 and the value was the same as that for the vertical hydraulic conductivity of clay within aquifers for models 1 through 9.

The top of the geopressured zone was assumed to be a no-flow boundary in models 1, 2, 4, and 9. A head of 1,000 ft at the top of the geopressured zone was reduced by a factor in models 3, 5, 6, 7, 8, 10, 11, 12; the reduction factors ranged from 0.054 to 0.92. A parameter analogous to the "vertical conductance" of MacDonald and Harbaugh (1988) for controlling flow from the geopressured zone was determined for models 5, 8, 10, and 11; the values range from 9.4×10^{-9} d^{-1} to 1.6×10^{-5} d^{-1}. The resistance to vertical flow for models 3, 6, 7, and 12 was due to clay in the sediments above the geopressured zone and no "vertical conductance" value was used for the geopressured zone.

The onset of land subsidence began after a head decline of 80 ft from predevelopment in models 1 through 7 and model 9. The head decline necessary for the onset of land subsidence ranged between 102 ft and 220 ft for models 8, 10, 11, and 12.

The coefficient used to adjust hydraulic conductivity and obtain "effective" hydraulic conductivity according to equations 10 and 11 was allowed to vary in models 8, 10, 11, and 12; the values ranged from 0.22 to 0.27.

The results from the flow models f(B) defined in table 3 for $n = 3{,}107 + 33 = 3{,}140$, except for model 9 which has $n = 3{,}107+6 = 3{,}113$, are given in table 4. As mentioned previously, 3,107 is the number of grid-element volume-averaged heads from water wells, and 33 or 6 is the number of observations of the logarithms of the hydraulic conductivities of sand. Models using $n = 3{,}107 + 33 + 586 = 3{,}726$ and including the drill-stem data were also used and have similar results. These models were considered to be somewhat less reliable because of possible bias in the preparation of the drill-stem test data, and are thus not shown. Sources of bias are the proximity of wells to active oil fields and the indirect procedure for arriving at formation pressure from drill-stem tests. The second row of table 4 gives $\omega_k^{1/2}e_k$ of equation (14) for the 95-percent prediction interval for random variable Y_k. The number shown for $\omega_k^{1/2}e_k$ is in feet for Y_k corresponding to the grid-element volume-averaged heads Y_i, $i = 1,2,\ldots,3,107$. As shown previously, these ω_k vary from 0.4 to 1.7. For model layers 2 through 11, the values of ω_s $s = 1,2,\ldots,10$ are: 1.1, 0.5, 0.4, 0.5, 1.1, 0.8, 0.8, 0.5, 0.6, and 1.7. Thus, for layer 2 in the third model, for example, the probability is approximately 95 percent that $f_k(\hat{B})-141(1.1)^{-1/2}<Y_k<f_k(\hat{B})+141(1.1)^{-1/2}$. In other words, the probability is 95 percent that the observed values of grid-element volume-averaged head in layer 2 lie within $\pm 141(1.1)^{-1/2}$ ft of the model predicted values. This prediction interval applies to grid-element volume-averaged heads at or in the vicinity of the grid-element volume-averaged heads in layer 2 (fig. 33). Because all of the grid-element volume-averaged heads from water wells were at relatively shallow depths, this prediction interval does not apply to deep head values.

For the 33 observations of the logarithm of the hydraulic conductivities of sand, Y_i, the number shown for $\omega_k^{1/2}e_k$ needs to be divided by 149 to yield the desired prediction interval half width. The factor 149 is $(\omega_i/\omega'_i)^{1/2}$ of the previous section. For flow model 3, for example, the probability is approximately 95 percent that $\log_{10}(29.9)-0.8dd_k-(141/149)<Y_k<\log_{10}(29.9)-0.8dd_k+(141/149)$, or that $\log_{10}(29.9)-(141/149)<(Y_k+0.8dd_k)<\log_{10}(29.9)+(141/149)$. Here, Y_k is the log of the volume-averaged logarithm of hydraulic conductivity (ft/d) of sand of zone k, one of the 33 zones based on the logarithm of the hydraulic conductivity of sand, or a zone in the vicinity of these zones. The average depth of hydraulic conductivity zone k, measured in kilometers, is dd_k. Each Y_k has the same predicted value $\log_{10}(29.9)-0.8dd_k$. The single hydraulic conductivity of sand parameter for each hydraulic conductivity zone k is 29.9 ft/d. Note that the 33 observed values of $Y_i+0.8dd_i$ shown in table 2 have the average value $1.481 = \log_{10}(30.3)$ and fall within the 95-percent prediction interval $\log_{10}(29.9)\pm(141/149)$. As with head, the 33 Y_i are at relatively shallow depths.

Appearing in the third through sixth rows in table 4 are $(S(B)/(n-p)\epsilon^2) = (1+u)$, the mean weighted residual $(1/n)\sum_{i=1}^{n}[Y_i-f_i(\hat{B})]\omega_i^{1/2}$, the root-mean square-weighted residual $[(1/n)\sum_{i=1}^{n}(Y_i-f_i(\hat{B}))^2\omega_i]^{1/2} = [S(\hat{B})/n]^{1/2}$, and Chi^2 to be discussed in the next section. Also shown in table 4 are the values obtained for the regression parameter estimates (the elements of the p x 1 vector $\hat{B}$), and also the confidence intervals of these values as obtained from equation (7). For example, model 3 in table 4 shows $\hat{b}_9 = \hat{B}_2$, the specific storage, equal to $0.45 \times 10^{-6}ft^{-1}$ (the specific storage for pure water is approximately 1.5×10^{-6} ft^{-1}). The 95-percent confidence interval for $b_9 = B_2$ is from $0.45 \times 10^{-6}(1.73)^{-1}ft^{-1}$ to $0.45 \times 10^{-6}(1.73)ft^{-1}$. As with the other regression parameters, the confidence interval is expressed in multiplicative factor form because the parameter variable used in the minimization routine was the base 10 logarithm of the parameter rather than the parameter itself. The units of $b_1,\ldots,b_9$ are: 1,feet,1,$(day)^{-1}$,1,1,1,1, and $(feet)^{-1}$. The units of the hydraulic conductivity of clay and sand parameters are ft/d.

Choosing the Best Regression Model

Draper and Smith (1981), chapter 6, suggest several methods for selecting the "best" regression equation or model. One procedure is to plot mean square residual as a function of the number of parameters p. Such a plot is shown in their figure 6.1, p. 298. Draper and Smith state that when mean square residual versus p ceases to decrease with increasing p, then an optimal choice for the number of parameters is the value of p where further decline in the mean-square residual is small. Values for root-mean square-weighted residual, $(S(\hat{B})/n)^{\frac{1}{2}}$, are shown in table 4. Note that the values shown for $(S(B)/n)^{\frac{1}{2}}$, and thus also the variances $[S(\hat{B})/(n-p)]\omega_k^{-1}$ of the Y_k (recall that the variance of Y_k is $[S(\hat{B})/(n-p)][\hat{X}_k(\hat{X}^T\omega\hat{X})^{-1}\hat{X}_k^T+\omega_k^{-1}]\doteq[S(\hat{B})/(n-p)]\omega_k^{-1}$), do not decrease appreciably with increasing p. For example, model 4 with p = 4 has a root-mean square-weighted residual of 41.5, but model 12 with almost eight times as many regression parameters has a root-mean square residual of 35.9, only 13 percent less. This would suggest that an optimal choice for p would be quite small and that p = 4 would perhaps be a reasonable choice.

A second trend shown in table 4 is that although the root-mean square-weighted residual generally decreases as the number of regression parameters p used increases, the prediction interval half width e_k increases as p increases if p is greater than 4. In other words, even though the fit of the 3,107 heads and 33 logarithm values of hydraulic conductivity of sand becomes better as the number of regression parameters increases, the certainty that one has in the predicted values for head and hydraulic conductivity decreases with increasing p, if p exceeds 4.

A related trend in table 4 is that the confidence that one has in the values obtained for the regression parameters tends to decrease (the 95-percent confidence intervals increase) as the number of parameters increases above 4. For example, model 3 predicts that the value of regression parameter B_4 for the hydraulic conductivity of sand of model layers 2 through 11 is 29.9 ft/d, and that there is 95-percent confidence that the actual value of B_4 is between 29.9/1.20 ft/d and (29.9)(1.20) ft/d. For model 10 with p = 18, which has the regression parameters B_9 through B_{18} for the hydraulic conductivity of sand for model layers 2 through 11, values for B_9 though B_{18} are not known with the same amount of confidence as in model 3. For example, model 10 has a value for B_{10} of 7.6 ft/d, but the 95-percent confidence interval is from $7.6(4.27)^{-1}$ft/d to (7.6)(4.27) ft/d. Note that model 12, with 31 regression parameters, shows larger confidence intervals for the 10 parameters for the hydraulic conductivity of sand of model layers 2 through 11 than does model 10 with 18 regression parameters.

The above discussions regarding prediction and confidence interval behavior as a function of the number of parameters p tend to lend support to the choice of a small value for p, if values for the prediction and confidence intervals are assumed to be accurate or accurate relative to each other. This assumption would be in question if the residuals were to exhibit a considerable lack of normality. As discussed below, such lack of normality was in fact the case, so that the prediction interval half widths e_k calculated and shown in table 4 may not be accurate. The regression parameter confidence intervals shown may also be inaccurate for the same reason. Thus using the widths of these intervals as a criteria for selecting an optimal f(B) or p may be in question.

Regression parameter variances, mentioned previously but not shown in table 4, increase as p increases. For example, model 4 has 30.3 ft/d for the hydraulic conductivity of sand parameter (P_s). The variation of $\log_{10}(P_s)$ within one standard deviation of $\log_{10}(30.3)$ allows P_s to vary in the range $30.3(1.06)^{-1}$ to 30.3(1.06)ft/d. However, for model 12 layer 8 for example, the range is $125(1.18)^{-1}$ to 1.25(1.18)ft/d. As with prediction and confidence interval half widths, regression parameter variances will be in error if there is a lack of normality.

The above results regarding: the very slow decrease of root-mean square-weighted residual with p greater than 4, the increase of prediction interval half-widths with p greater than 4, and the increase of regression parameter variances with p, lead to the conclusion that a model with a relatively small number of regression parameters would probably be the best choice.

Analysis of Residuals

The weighted residuals for all models used were tested to see if they differed significantly from those drawn from a normal distribution. Deviations from a normal distribution indicate the possible presence of factors that cause residuals to differ from those resulting solely from random fluctuations in the hydraulic conductivity or other properties of the porous medium, and (or) random measurement error. Significant deviations also preclude further model testing based on Chi^2, F, or t distributions. The standard Chi^2 test for normality (Croxton, 1953, p. 282-283) was used, for which the number of classes was approximated using the relation 5 log(n) = 16 (Panofsky and Brier, 1965, p. 4). The variance used for the test is given by

$$v^2 = \frac{1}{n} \sum_{i=1}^{n} (Y_i - f_i(\hat{B}))^2 \omega_i - Mu^2 = S(B) - Mu^2 ,$$

where the mean-weighted residual Mu (shown in table 4) is given by

$$Mu = \frac{1}{n} \sum_{i=1}^{n} (Y_i - f(B))\omega_i^{\frac{1}{2}} .$$

Normality of the weighted residuals would result in $Z_i = \{[Y_i - f_i(B)]\omega_i^{\frac{1}{2}} - Mu\}/v$ being N(0,1). Because of the use of v and also Mu, the number of degrees of freedom for the Chi^2 test is (number of classes) - 3 = 16 - 3 = 13. It may be preferable to test whether Z_i with Mu set equal to zero is ~N(0,1), because the model f(B) should at least be able to produce $f_i(\hat{B})$ such that $Mu^2 \ll v^2$. This approach would give essentially the same results because as table 4 shows, values for Mu^2 were much smaller than v^2.

For 13 degrees of freedom, the hypothesis of normality of the weighted residuals is rejected at the 1-percent significance level for $Chi^2 > 27.7$ (Hogg and Craig, 1965). That normality is rejected in every case by a considerable margin is shown in table 4. Typical results for the number of points falling into the 16 classes of equal probability on the N(0,1) curve show a nonuniform distribution, generally with a sizable number of excess points falling into the far right interval corresponding to the largest positive values for the residual defined as $Y_i - f_i(B)$. For example, model 4 has the following numbers of points in the 16 classes: 131, 153, 185, 229, 248, 288, 276, 261, 184, 197, 179, 163, 134, 98, 109, and 305. A uniform spread indicating normality would have approximately 196 points in each interval.

At one point in the study, the observed grid-element volume-averaged heads Y_i were rejected when, for the original set of 3,140 predicted values $f_i(B)$, they fell in the far right class of the 16 classes. After eliminating these approximately 300 observations, a new B was found. The new Chi^2 value for the new B was dramatically reduced from its original value. When an application of this culling procedure was applied to model 4, the number of observations was reduced by 314 from 3,140 to 2,826, and the value for Chi^2 fell from 324 to 65. The new model f(B) gave different results, because it was based on the new culled set of 2,826 observations. The root-mean square-weighted residual fell from 41.5 to 28.8. The value for $\omega_k^{\frac{1}{2}} e_k$ fell from 141 to 128. The hydraulic conductivity of clay and sand parameters, 0.362×10^{-3} and 30.3 ft/d, changed to 0.407×10^{-3} and 37.3 ft/d. When this culling procedure was applied to model 9 with 11 regression parameters, the value of Chi^2 fell from 260 to 56. Since the Chi^2 values 65 and 56 are greater than 27.7, the hypothesis of normality is rejected at the 1-percent significance level.

The possibility that the logarithms of the hydraulic conductivities of sand and the grid-element volume-averaged heads were incompatible, thus causing large Chi^2 values, was investigated by deleting the use of the logarithms of the hydraulic conductivities of sand as observables and determining the corresponding Chi^2 values. No decrease in Chi^2 values was found so that this possibility is rejected.

The most likely cause of the large Chi^2 values is that the 10-mi grid spacing is very large with respect to the variability of the head. Most of the head data is from shallow depths and is heavily influenced by land-surface altitude. This can be seen from the very close similarity between water-table altitude in figure 6 and land-surface altitude in figure 2. Note that within only a single 10 mi by 10 mi grid, land-surface altitude may have a complex nature and that there may be several hill tops and valleys along a given cross section. It is thus unlikely that the residuals for grid-element volume-averaged head would be approximately statistically normal, which is what is required for small Chi^2 values. More likely, the residual would express some land-surface altitude characteristic. This would be true even if the effective hydraulic conductivity was constant or had an exact log normal distribution.

A model with a 30-mi grid spacing was used early in the study and gave residuals that were considerably larger than those for a 10-mi grid spacing. Although Chi^2 values were not actually determined, the residuals had a pronounced pattern that suggest the model would have had very large Chi^2 values for this reason.

A complicating factor is that head measurements have some tendency to be located near areas of pumping which may be near the edge of a grid element, but the model approximates the somewhat centrally weighted average drawdown that would be produced in a grid element as if all the pumping in the grid element was located at the center of the grid element.

It would be appropriate to use a much smaller grid spacing in those parts of the aquifer system where the head shows considerable variability. This was not done because of the already large number of nodes in the computer model.

Several methods (Draper and Smith, 1981, p. 34-40) are available to investigate lack of fit and the presence of model bias. Bias causes the inflation of what is, herein, called apparent variance with respect to true variance. Plots of residuals $Y_i-f_i(B)$ for grid-element volume-average head were observed for each of the model layers 2 through 11. No discernible pattern or bias was seemingly present using this simple approach to investigate lack of fit. Values for mean residual grid-element volume-averaged head by layers were less than 10 ft in magnitude for each of the layers 2 through 11 for each of the models with four or more parameters. For these same models, values for the mean-weighted residual (table 4) for all of the observations were ususally less than 3 ft.

Other methods available to investigate the presence of lack of fit make use of prior estimates of variance or repeat measurements and are thus not applicable.

Comparison of Models

To show self consistency of the regression approach, it is appropriate that the 12 models in table 4 be compared with respect to their predictions $f_i(\hat{B})$ for the observed quantities Y_i, which are composed of the 3,107 grid-element volume-averaged heads, and the 33 or 6 observations of the logarithm of the hydraulic conductivities of sand. The models should also be compared with respect to their predictions for flow even though there are no prediction intervals for flow.

Models 1 and 2, with two and three regression parameters, respectively, gave flow rates from the upland surface recharge areas to the lower altitude discharge areas on land or the sea floor which were approximately 10 times larger than those of the other models, and also larger than would be expected from precipitation data and infiltration estimates. This might be expected, because the observations did not include flow rates, and these two models allowed only a single hydraulic conductivity for both the clay and sand components in the entire domain of solution. These two models are thus rejected and will not be considered.

The 10 models 3 through 12 gave nearly the same value for the mean weighted residual head. The predicted heads $f_i(\hat{B})$, i = 1,2,...,3,107, averaged by layer, differ between the 10 models by as much as 30 ft for one layer, butless than 5 ft for most of the layers. These values are well within the 95-percent prediction interval half widths e_k, which in table 4 show values of from 141 ft/$(1.7)^{\frac{1}{2}}$ = 108 ft to 243 ft/$(.4)^{\frac{1}{2}}$ = 384 ft. Figures 10 through 19 show the values for head as produced by model 4, the model in table 4 with the smallest value for $\omega_k^{\frac{1}{2}}e_k$, for the year 1982. Predicted heads at deep parts of the aquifer system showed differences of up to 800 ft between the 10 models, 3 through 12.

Results for predictions of the 33 logarithm values of hydraulic conductivity of sand also show good overlap between the prediction intervals of the 10 models. Model 3, with four regression parameters, has the interval $\log_{10}(29.9)-0.8dd_k\pm141/149$ as the 95-percent prediction interval for each of the 33 logarithm values of hydraulic conductivity of sand, each of which has the predicted value of $\log_{10}(29.9)-0.8dd_k$. The other prediction intervals for hydraulic conductivity of sand in the 10 models overlap to a high degree. The most extreme case is that of the predicted value of $\log_{10}(650)-0.8dd_k$ for the 33 logarithm values of hydraulic conductivity of sand in layer 2 in model 12. The prediction interval in this case is $\log_{10}(650)-0.8dd_k\pm243/149$, which is a wide interval that overlaps $\log_{10}(29.9)-0.8dd_k\pm141/149$ of model 3.

The flow rates obtained from the models in table 4 were compared by looking at the total flow across various surfaces cutting across the domain of solution and also by looking at the flow rates across the many individual grid element faces. One of these sets of surfaces separate the layers. Values were obtained for the flow into and out of the top and bottom surfaces of each of the 10 model layers, a total of 40 flow rates. Comparison of these 40 flow rates for models 3 through 11 shows a considerable amount of consistency. Models 1 and 2 are rejected for the reasons given previously. Model 12 with 31 regression parameters gave flow rates considerably different from the other models, probably as a result of instability resulting from having too many parameters, so that flow rates for model 12 are not considered either. Rows 1 through 5 in table 5 show respectively (1) the flow out of the geopressured zone; (2) the recharge into the aquifer system from model layer 12 through the top of the aquifer system; (3) the flow out of the aquifer system into layer 12 through the top of the aquifer system; (4) the net total flow into the aquifer system from layer 12 through the top of the aquifer system; and (5) the total recharge to the aquifer system from both the geopressured zone and model layer 12. The second flow minus the third is equal to the fourth, and the first plus the fourth is equal to the fifth. Rows 6 through 15 in table 5

TABLE 5.-- *1982 Ground-water flow rates for models 3 through 11*

[The number above/below the slash is the flow rate into/out of the tops of the 10 aquifer layers; units are 10^5 cubic feet per day]

	Model								
	3	4	5	6	7	8	9	10	11
Flow out of the geopressured zone	993	0	-18	973	844	9	0	775	539
Recharge into the aquifer system from layer 12 through the top of the aquifer system	13500	13600	13600	13600	13900	13600	13600	13400	13700
Flow out of the aquifer system into layer 12 through the top of the aquifer system	2270	1360	1350	2330	2560	1340	1500	2100	1950
Net total flow into the aquifer system from layer 12 through the top of the aquifer system	11300	12200	12300	11200	11300	12200	12100	11300	11800
Total recharge to the aquifer system from both the geopressured zone and layer 12	12250	12250	12240	12200	12170	12240	12130	12110	12300
Flow rate into/out of top of: layer 2	177/1080	178/86	198/87	175/1060	235/988	181/98	205/114	402/1090	239/686
layer 3	651/467	654/471	653/470	700/544	961/794	645/467	417/269	514/408	516/356
layer 4	1020/404	1020/408	1020/408	864/406	963/492	1020/403	982/389	1140/593	1190/614
layer 5	1490/517	1490/517	1480/515	1140/474	1300/623	1480/511	1110/273	1090/434	1320/497
layer 6	765/286	766/284	765/282	444/283	493/319	760/273	482/219	530/296	801/456
layer 7	203/485	216/138	218/133	174/445	159/390	216/142	160/84	163/380	291/377
layer 8	657/566	676/319	679/309	630/509	515/449	673/318	656/310	726/590	839/559
layer 9	663/669	687/312	691/305	649/614	605/530	684/318	564/240	579/559	773/509
layer 10	949/663	984/321	988/315	949/613	917/534	981/325	851/171	776/445	994/391
layer 11	10100/1190	10100/321	10100/318	9990/1190	10000/1120	10100/321	10600/849	10100/1050	10300/1080

show the flow rates into and out of the tops of the 10 model layers for models 3 through 11. All numbers in table 5 have three significant figures except row 5 which has four. All flow rates shown are for the year 1982, at which time an approximately steady-state situation existed. The pumping rate for 1982 was 0.1265×10^{10} ft^3/d. The difference between this pumping rate and the total recharge to the aquifer system on row 5 is the rate that water is coming from storage. Note that the values for the total recharge to the aquifer system are in each case greater than 95 percent of the 1982 pumping rate, indicating that less than 5 percent is coming from storage, and that a near steady-state condition exists.

Because the possible error of estimates for maximum allowable recharge (discharge) into the aquifer system from model layer 12 are large in comparison with the differences for this flow among models 3 through 11, none of the models seem any more likely to be valid than the others. The reason for this is that the flow rates into the aquifer system from model layer 12 in table 4 are very small compared to flow rates that occur at land surface, such as precipitation, runoff, evapotranspiration, etc. For example, the flow rate out of the aquifer system into layer 12 is 0.136×10^9 ft^3/d for model 4. This water would either evaporate, be transpired, or flow into streams in lowland areas. However, this amount of water spread over 100,000 mi^2, an area approximately a third the size of the study area, is equivalent to a flow rate of only 0.21 in/yr, which is very small compared to evapotranspiration or runoff rates.

The direction and general location of ground-water flow that corresponds in a general way to the flow rates of table 5 are shown in figure 35. Figure 35 represents the actual aquifer system and actual flow locations and directions only in a very general and schematic manner. Actual flow patterns, both in the model and in the field, are far more complex. The recharge (discharge) into the aquifer system from model layer 12 through the top of the aquifer system for each grid element for the year 1982, model 4 is shown in figure 36. Equivalent freshwater drawdown since 1937 for the year 1982, model 4, is shown in figures 37 through 46.

Models 3, 4, 5, and 8 have a similar structure in that they all have a single regression parameter assigned to the hydraulic conductivity of sand for the entire aquifer system, and a single parameter assigned to hydraulic conductivity of clay. All four models also have a regression parameter for b_9, specific storage, and regression parameter b_1, even though model 3 has the same regression parameter B_1 assigned to both b_1 and b_3. The values obtained for the parameter for hydraulic conductivity of sand are 29.9, 30.3, 30.3, and 29.0 ft/d for the four models, respectively. For hydraulic conductivity of clay, the four values are 0.365×10^{-3}, 0.362×10^{-3}, 0.361×10^{-3}, and 0.374×10^{-3} ft/d. The values obtained for the predicted heads $f_i(\hat{B})$ $i = 1,2,\ldots,3,107$ are very close. When averaged by layer, the four models give values for head that differ by less than 1.1 ft. Clearly the four models are very similar in structure and give nearly the same results for hydraulic conductivity of clay and sand and

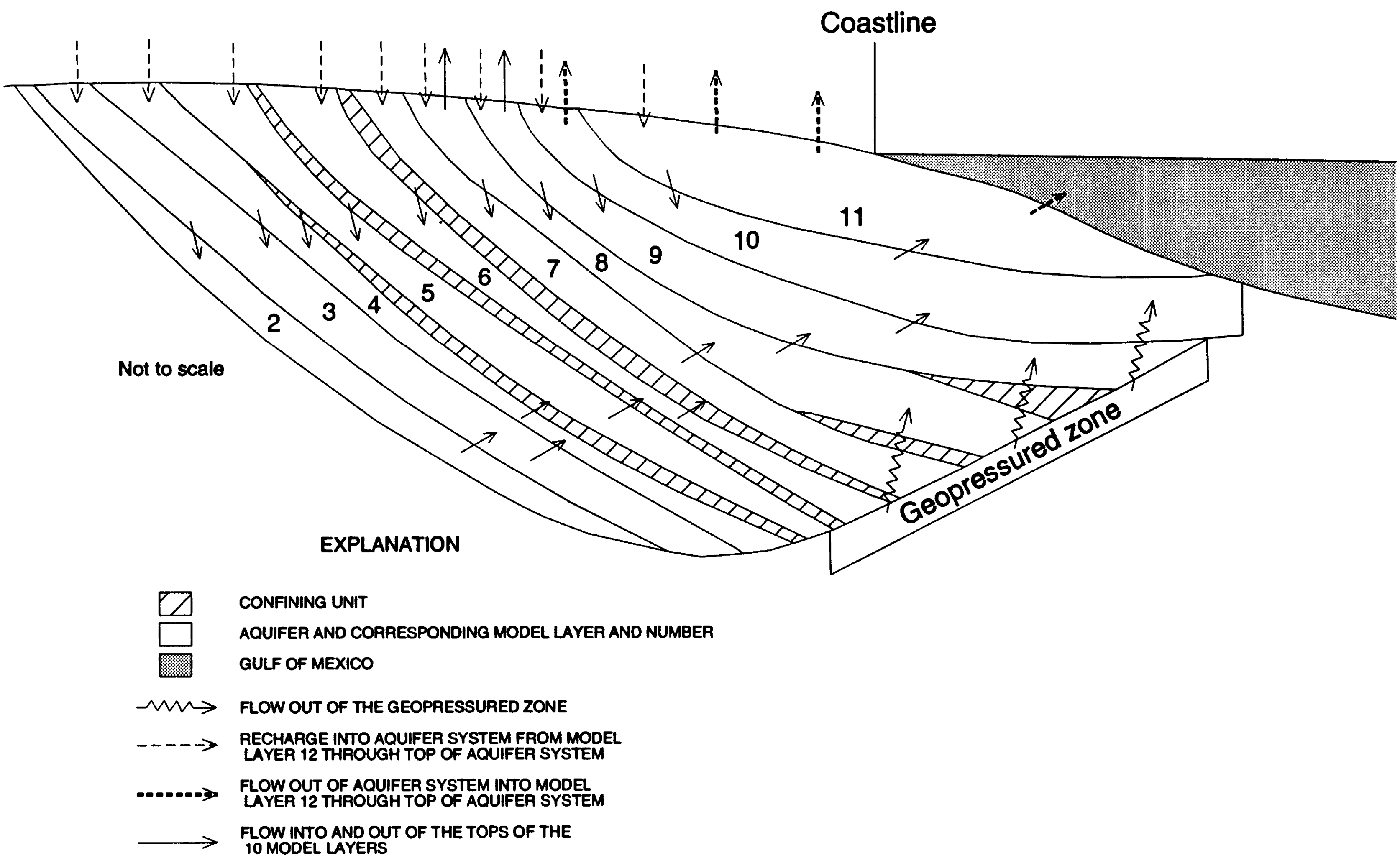

Figure 35. Generalized pattern of ground-water flow.

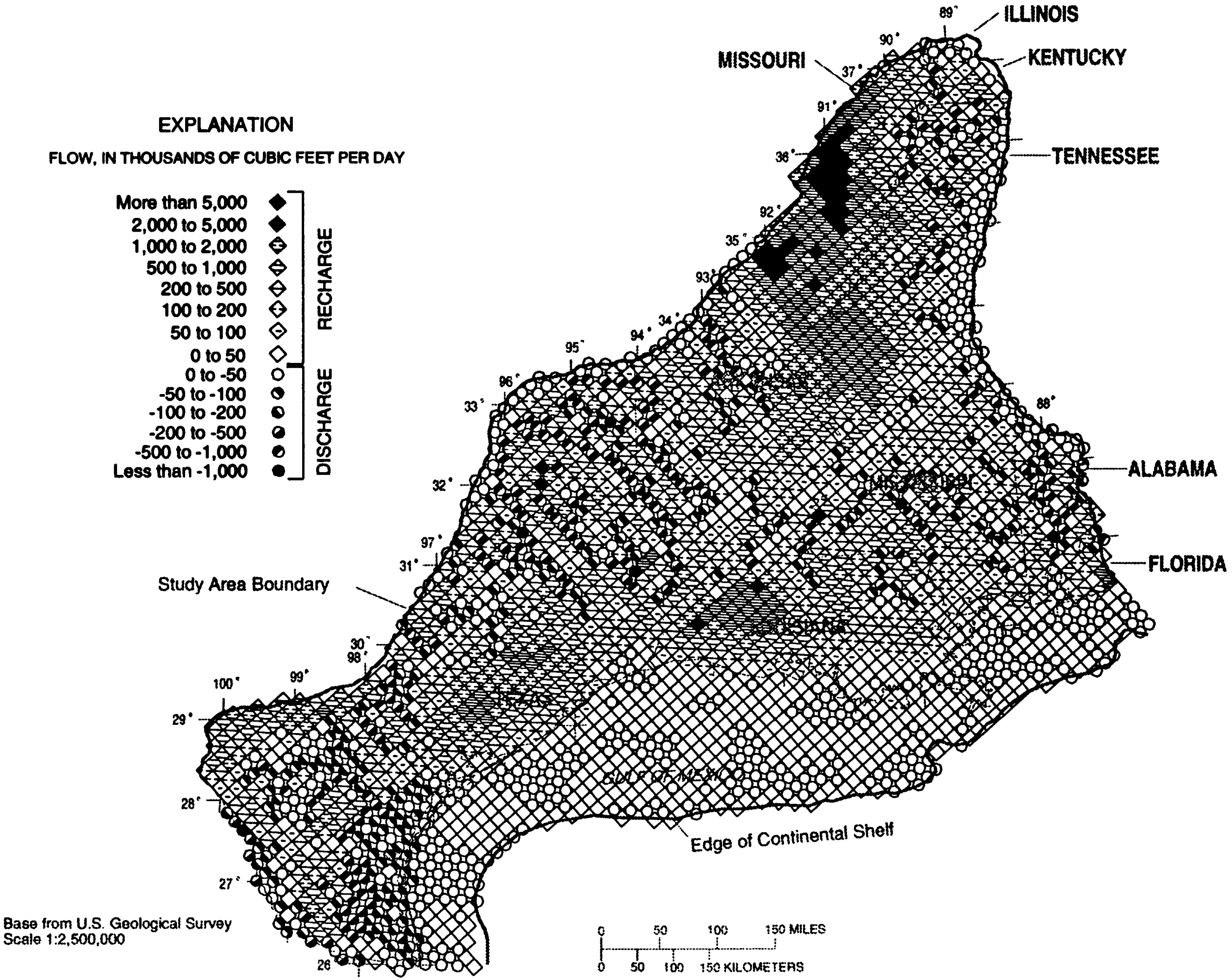

Figure 36. Simulated recharge (discharge) into the top of the aquifer system for 1982, model 4.

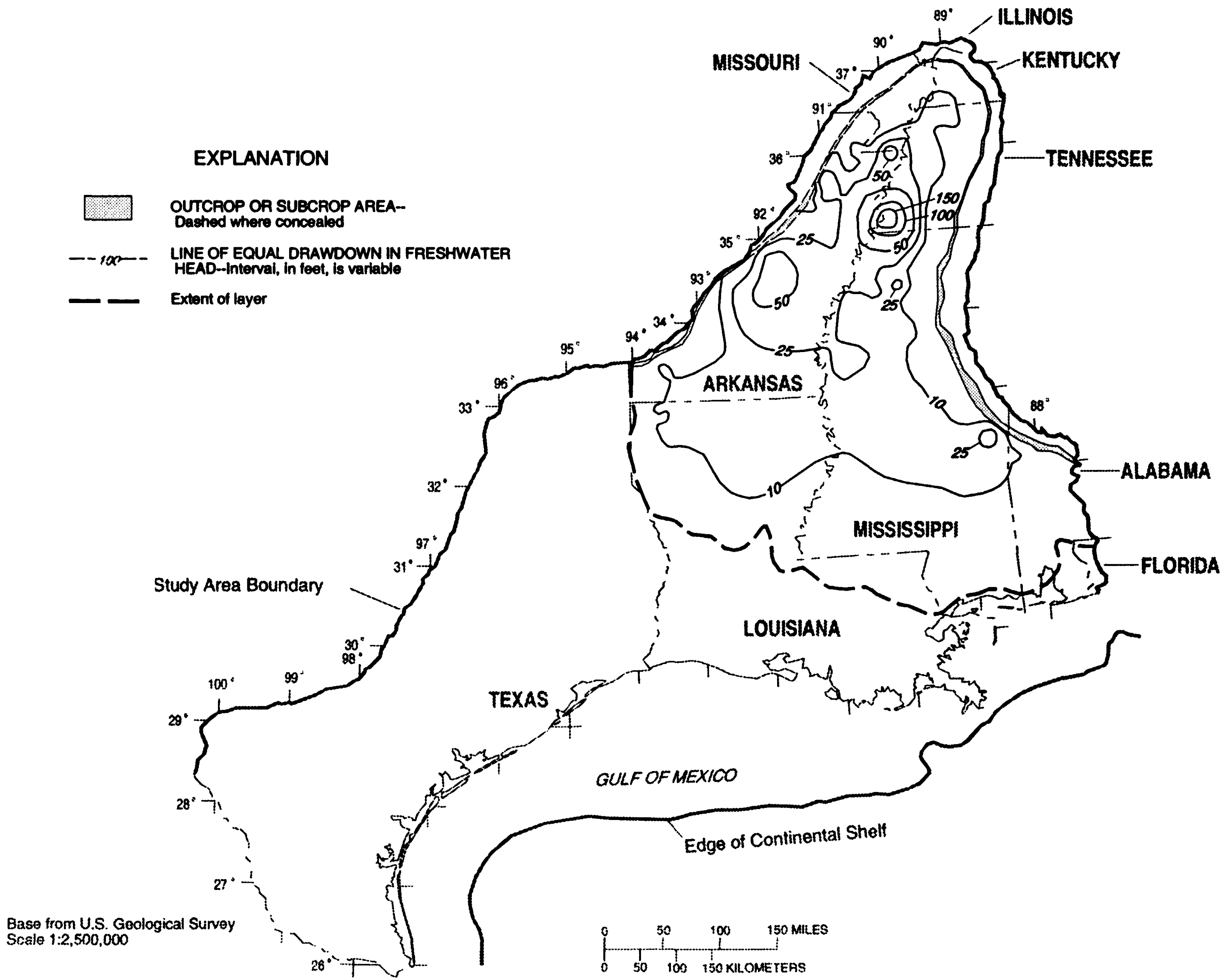

Figure 37. Simulated drawdown of equivalent freshwater head from 1937-82, model 4, model layer 2.

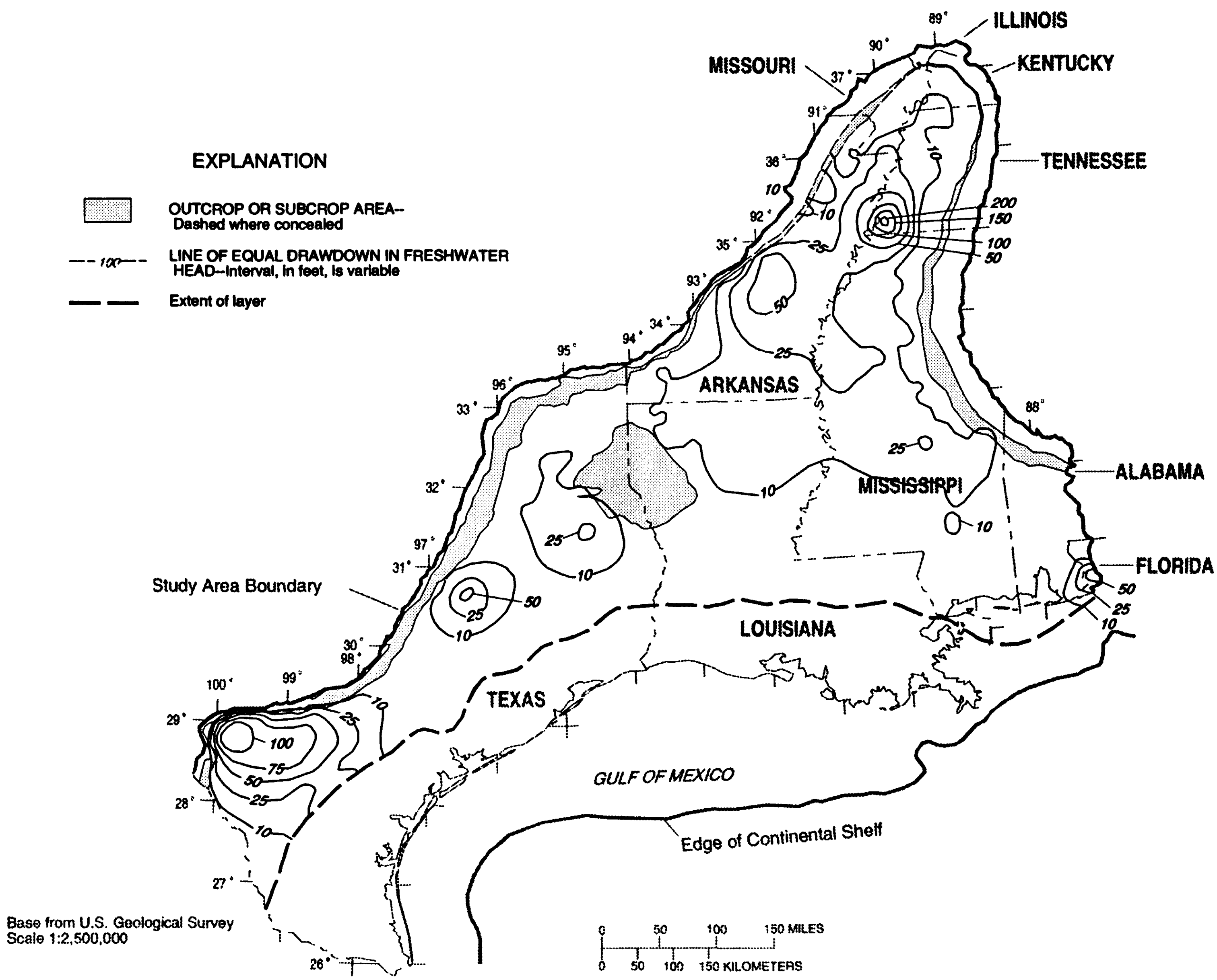

Figure 38. Simulated drawdown of equivalent freshwater head from 1937-82, model 4, model layer 3.

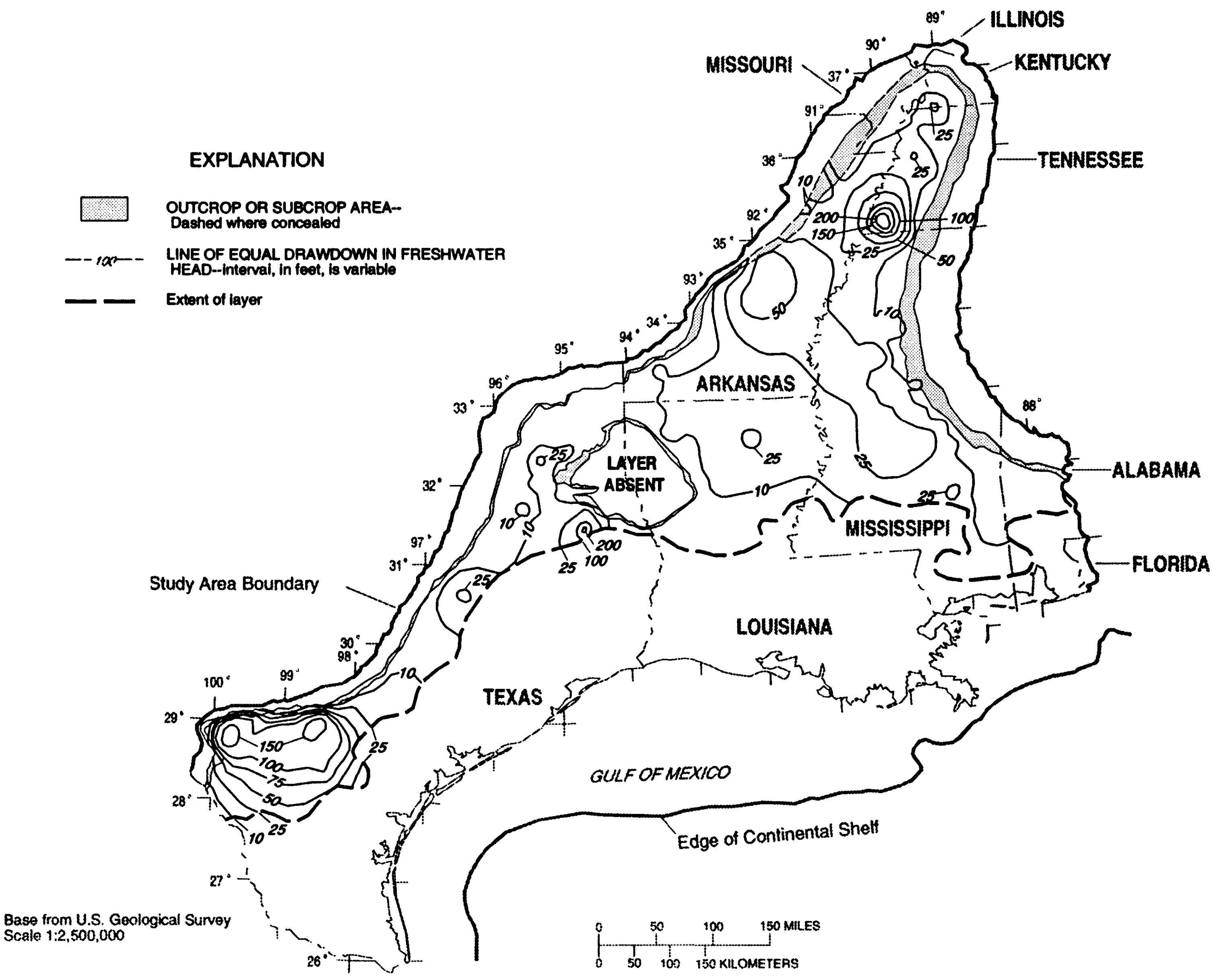

Figure 39. Simulated drawdown of equivalent freshwater head from 1937-82, model 4, model layer 4.

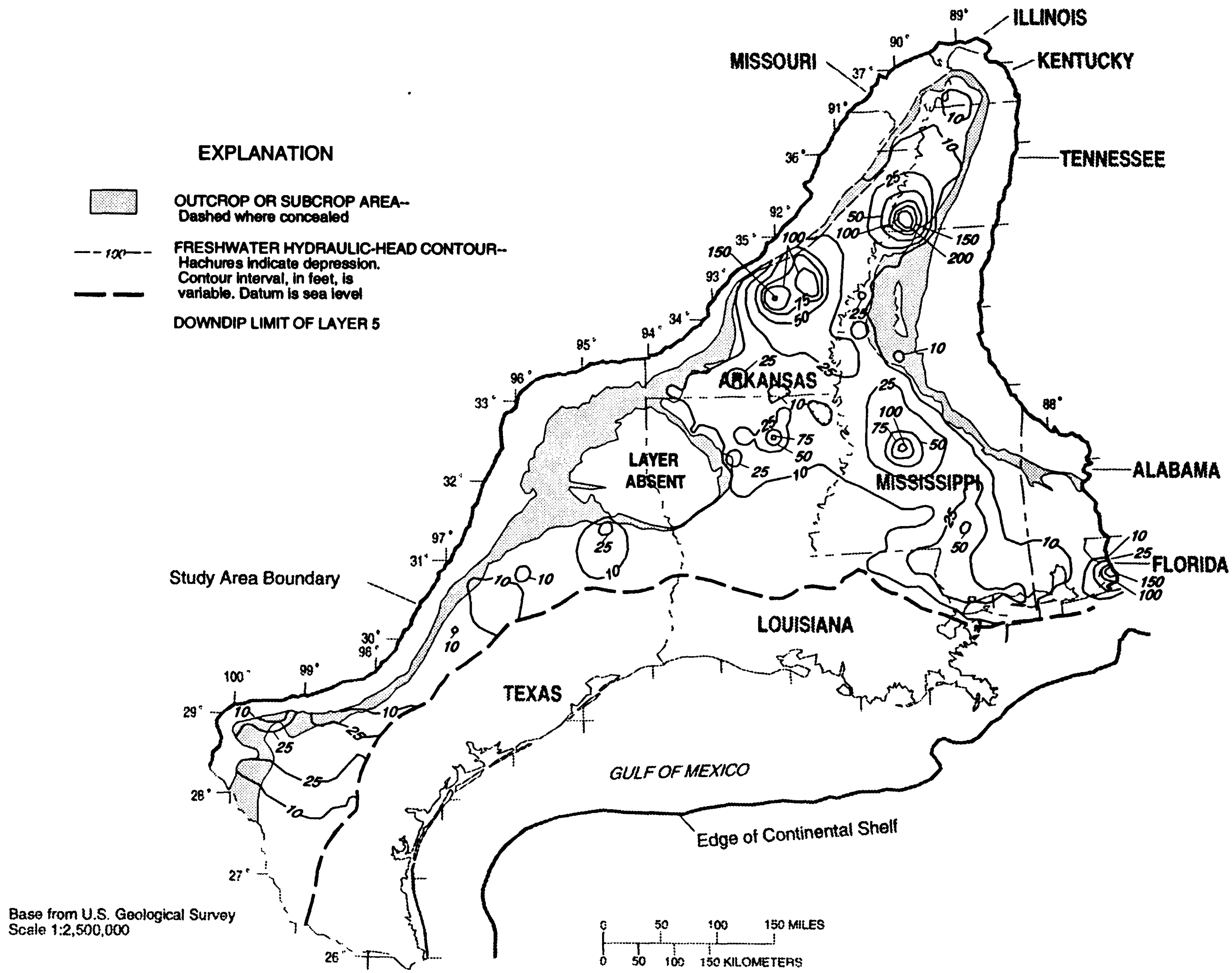

Figure 40. Simulated drawdown of equivalent freshwater head from 1937-82, model 4, model layer 5.

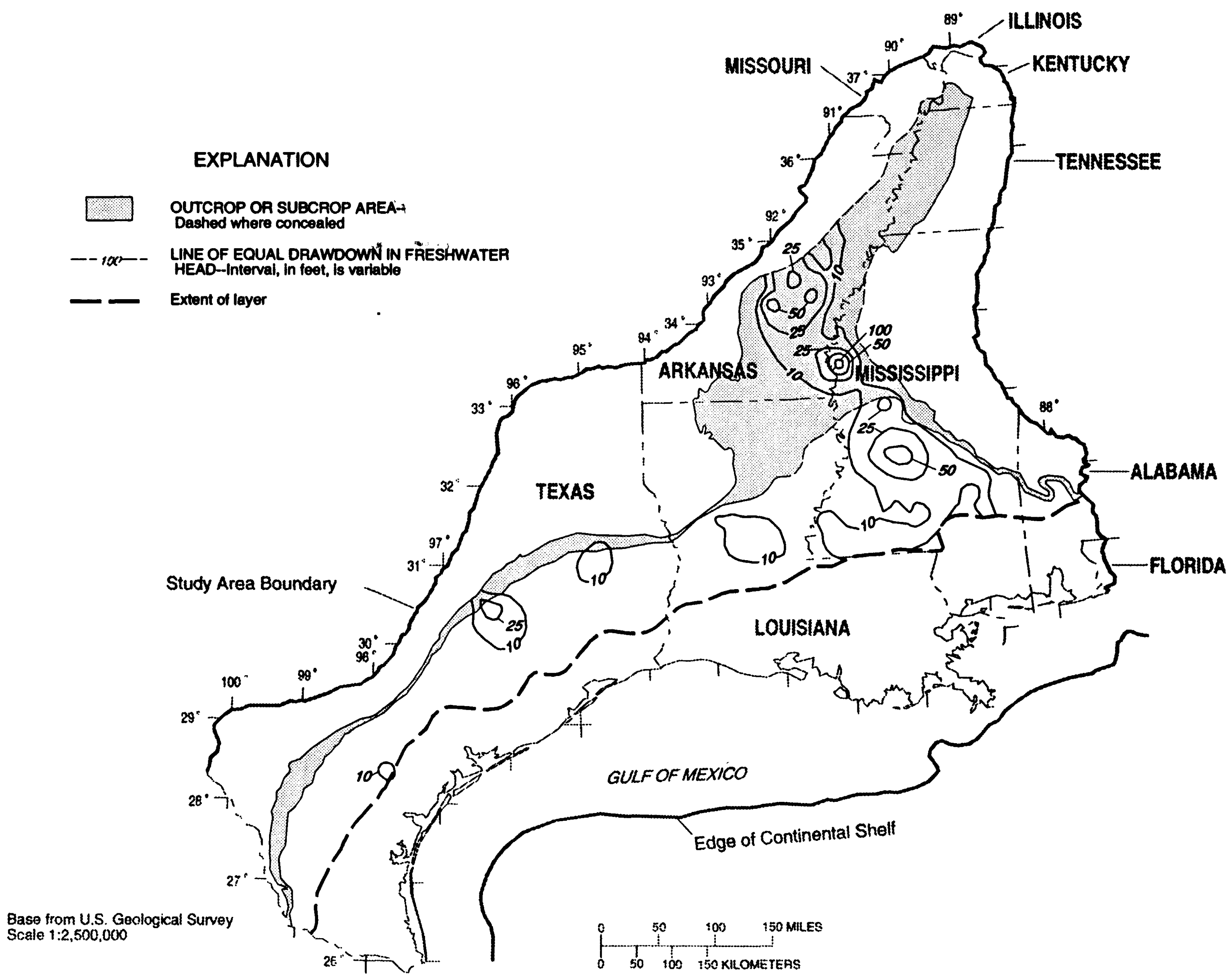

Figure 41. Simulated drawdown of equivalent freshwater head from 1937-82, model 4, model layer 6.

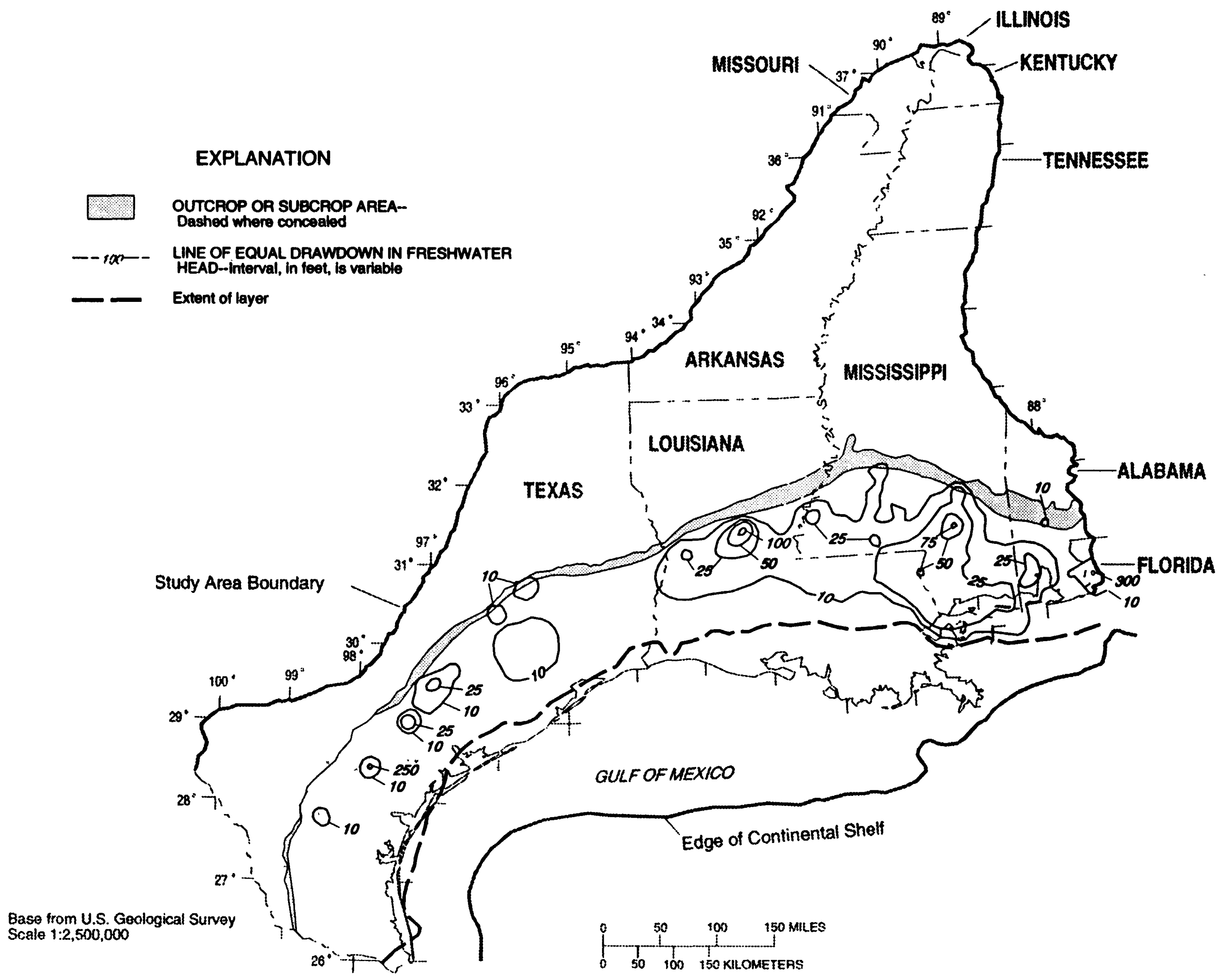

Figure 42. Simulated drawdown of equivalent freshwater head from 1937-82, model 4, model layer 7.

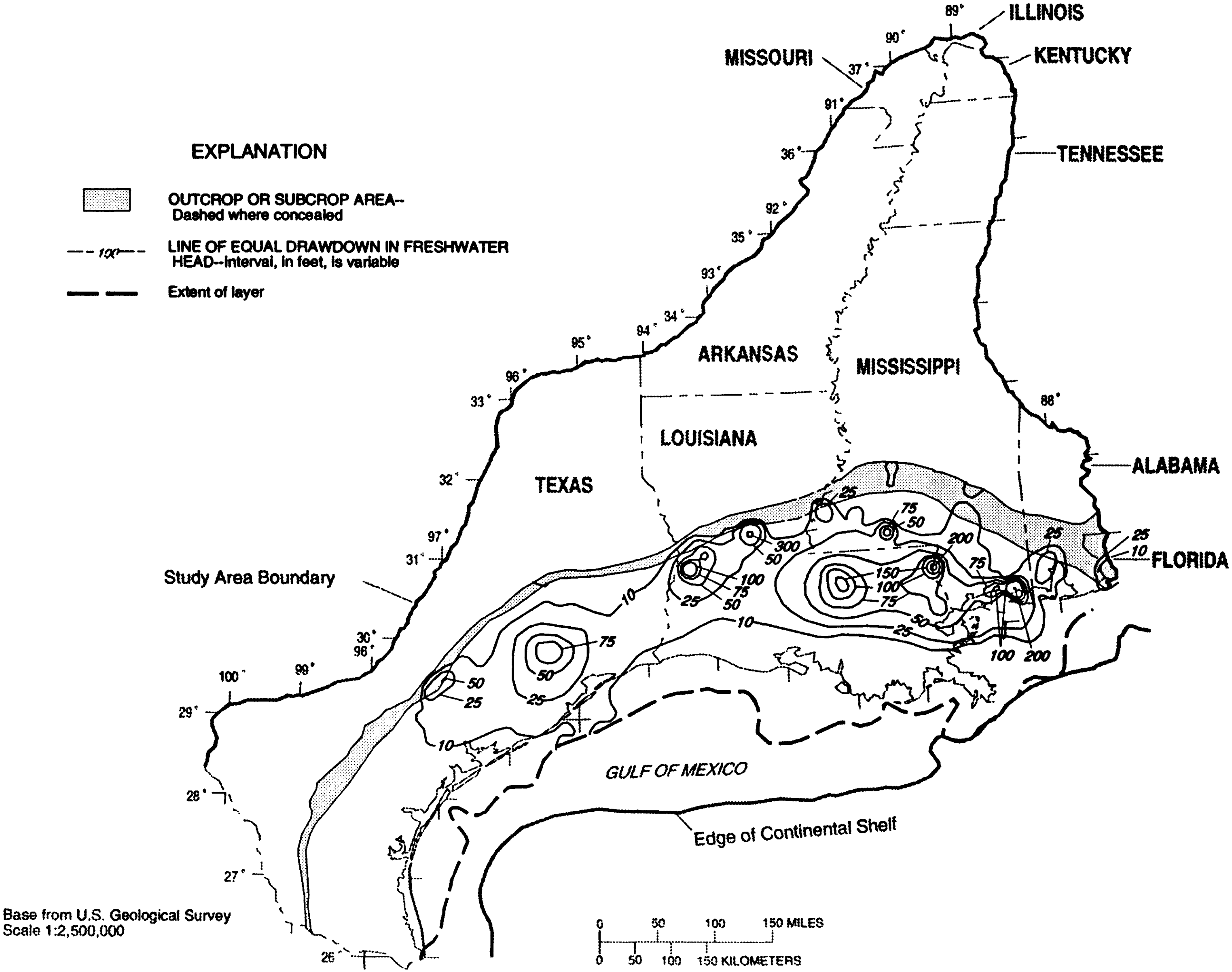

Figure 43. Simulated drawdown of equivalent freshwater head from 1937-82, model 4, model layer 8.

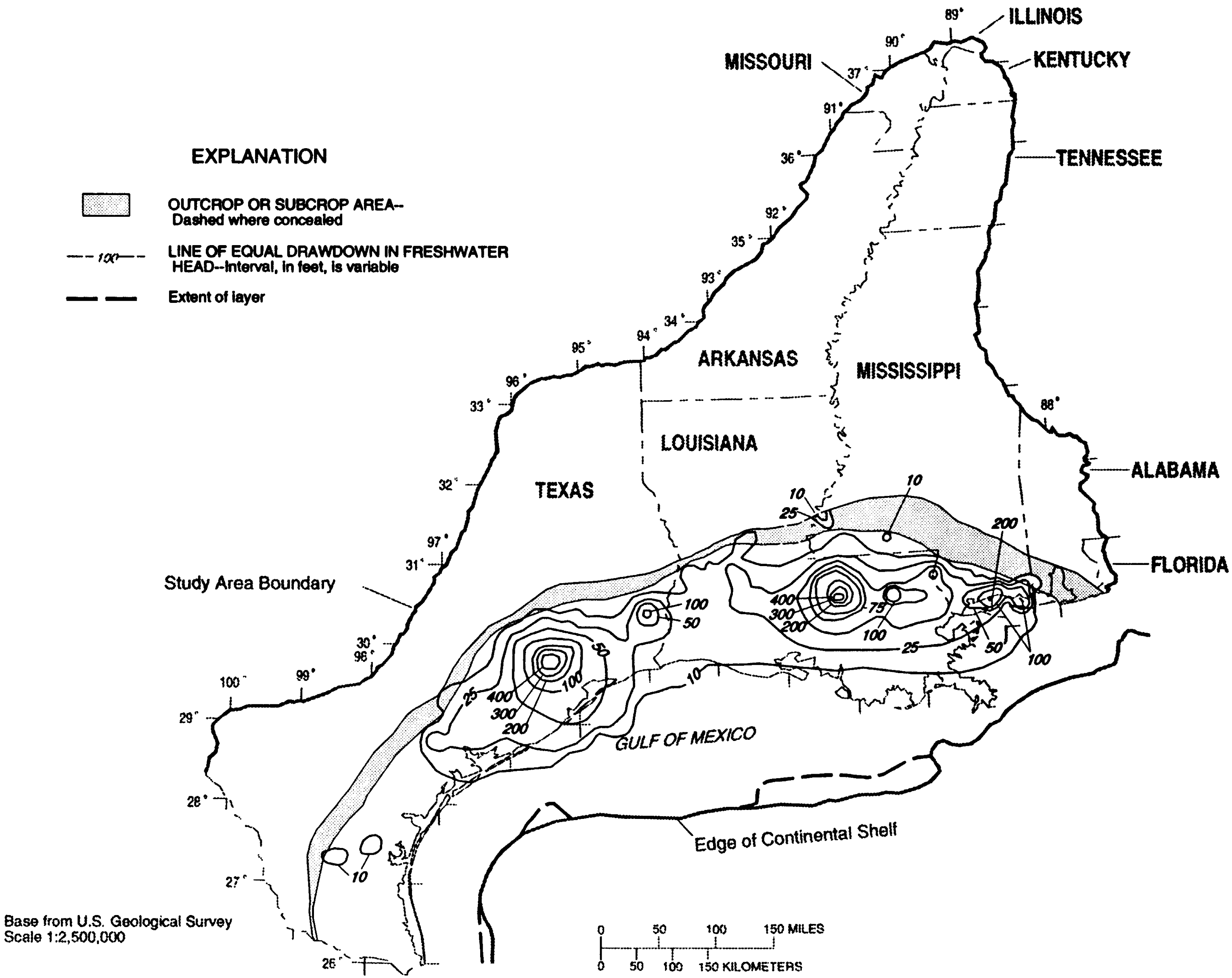

Figure 44. Simulated drawdown of equivalent freshwater head from 1937-82, model 4, model layer 9.

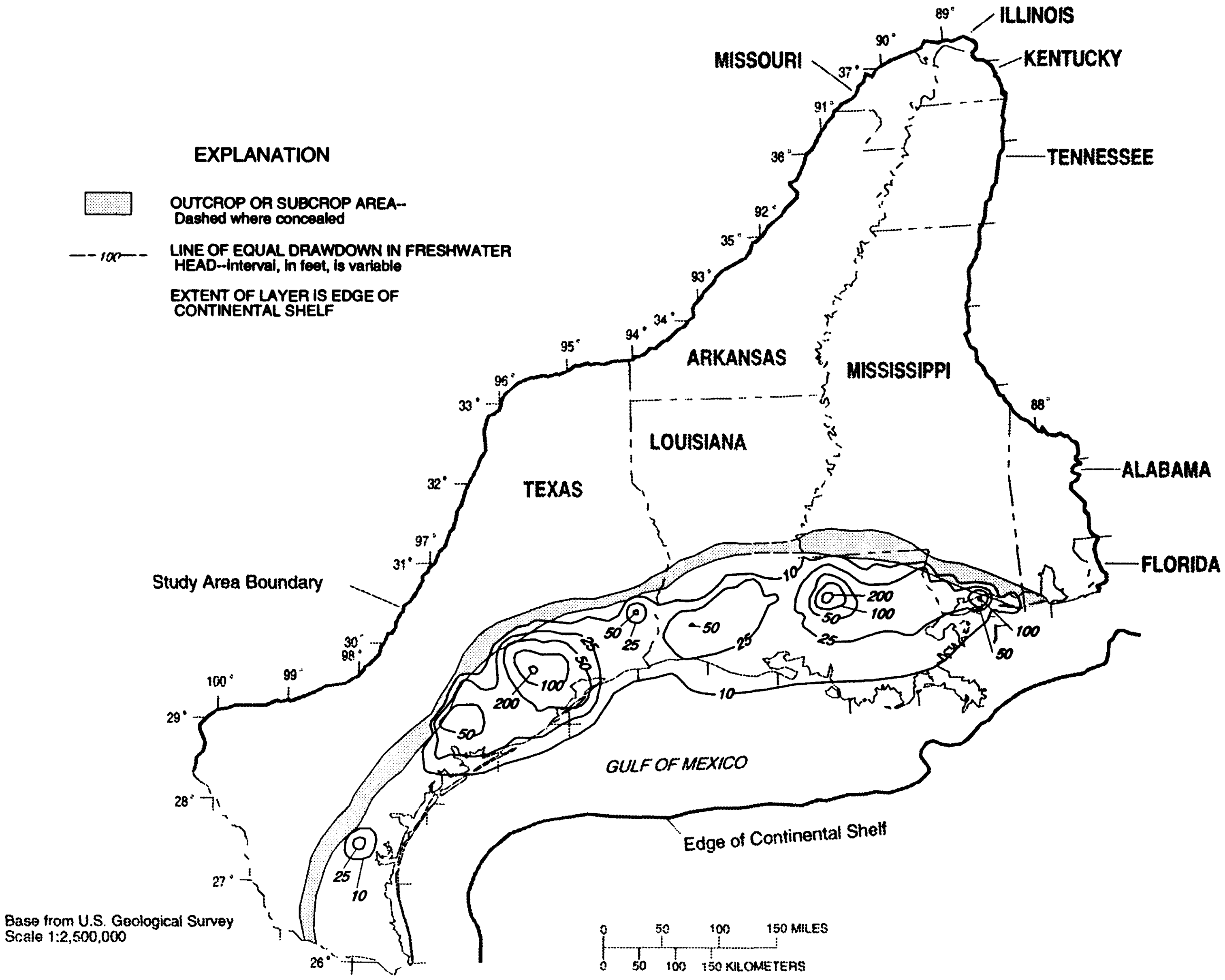

Figure 45. Simulated drawdown of equivalent freshwater head from 1937-82, model 4, model layer 10.

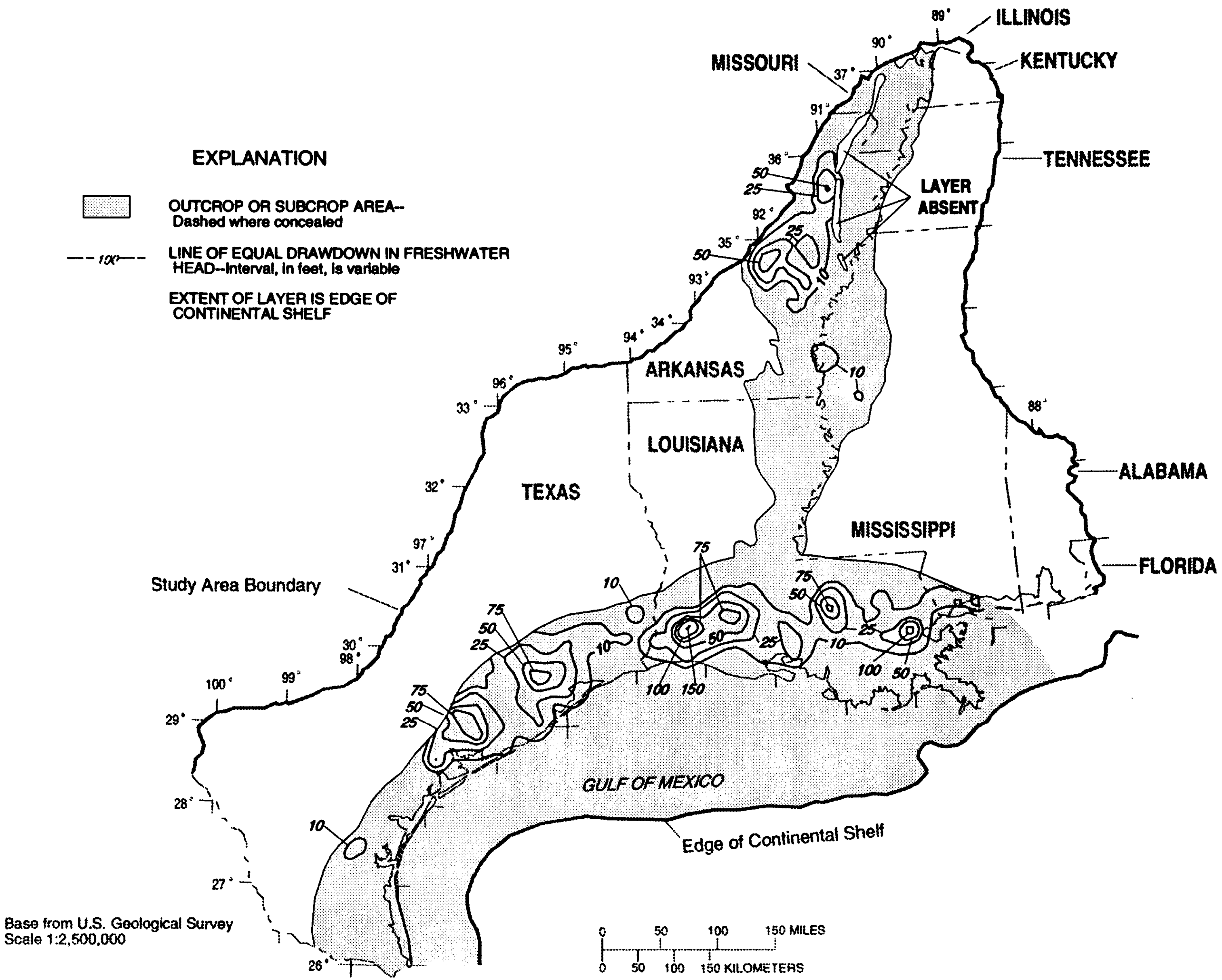

Figure 46. Simulated drawdown of equivalent freshwater head from 1937-82, model 4, model layer 11.

predicted shallow heads $f_i(\hat{B})$, i = 1,2,...,3,107. However, table 5 shows that the flow rates are not the same, the main difference being differing amounts of flow from the geopressured zone passing up through the aquifer system through model layers 7 through 12. Also, heads at depth differ by as much as 800 ft. Note that in table 5, model 3 has a flow of 0.993 x 10^8 ft^3/d from the geopressured zone. Model 4, however, has a zero flow condition because b_4 is set to zero. Models 5 and 8 have almost zero flow because of the values obtained for b_3 and b_4, even though the confidence intervals for b_3 and b_4 are extremely large. A model not shown in tables 4 and 5 and having the same selection of regression parameters as model 5, but using grid-element volume-averaged heads from drill-stem test as well as the 3,107 grid-element volume-averaged heads from water wells, gave a flow from the geopressured zone of 0.544 x 10^8 ft^3/d. Models 6, 7, 9, 10, and 11 have flow from the geopressured zone of from 0 to 0.973 x 10^8 ft^3/d. The conclusion is that the modeling efforts of this study are very approximate with regard to deep flow and heads. This result is not unexpected because almost all of the data for hydraulic head and hydraulic conductivity were taken from shallow depths.

A flow from the geopressured zone as large as that shown by model 3 could exist because order of magnitude calculations show that with a large value of specific storage, the geopressured zone under the aquifer system could maintain an average upward flow of approximately 10^8 ft^3/d if experiencing a rate of drop in head of several thousand feet per 100 million years $(S(\Delta h/\Delta t))$ (volume of geopressured zone) = flow rate: $(0.5/ft)(5x10^3 ft/365x10^8 d)(10^{15} ft^3) = 0.7 \times 10^8 ft^3/d)$. However, as shown above, the models of this study give values of the flow from the geopressured zone of from 0 to 10^8 ft^3/d with no indication of which is more correct. Values of head at depth also vary widely.

Model Error

It has been shown above that most of the various models show agreement regarding prediction of shallow heads and hydraulic conductivity values (the 3,107 grid-element volume-averaged heads and the 33 logarithm values of hydraulic conductivity of sand). The overlapping of the associated prediction intervals has also been demonstrated. It is appropriate to consider the extent to which the predictions of shallow heads and hydraulic conductivity could be in error because of conditions or constructions that are common to all of the models f(B) used.

Before considering model error, it is appropriate to mention that many possible sources of error have already been removed by assigning regression parameters to unknown values such as: b_1 and b_2 for the top surface specified head, b_3 and b_4 for the geopressured zone, b_5 for the way in which effective hydraulic conductivity is determined from the hydraulic conductivity of the sand and clay components, b_6 for anisotropy, b_7 and b_8 for the subsidence mechanism, and b_9 for specific storage. Remaining sources of model error are (1) the evaluation of the density function $\rho(x,y,z)$ from data that may be in error or lacking; (2) the depth dependence of clay and hydraulic conductivity of sand as described previously, specifically the functions used for the rate of decrease with depth; (3) the procedure for obtaining grid element effective hydraulic conductivity values by Desbarats (1987); and (4) the selection of the hydraulic conductivity zones. These four sources of error are considered in order in the following four paragraphs, and results of model alterations to explore these sources of error are summarized in table 6. An additional source of error which is very likely the major cause of the large root-mean square-weighted residuals and the large 95 percent prediction interval half widths e_k (table 4) is the relatively large 10 mi grid spacing, as discussed previously. Unfortunately, no tests with smaller grid spacing were performed because of the large number of grid elements that would be needed. Consequences of the use of 30-mi grid spacing have been discussed previously.

Several models were used in which grid-element volume-averaged density ρ was smoothed between grid elements in the same layer. This smoothing was done by assigning to ρ the average of ρ over nine locations, the neighboring eight grid elements and the center grid element for which the smoothed value of ρ is being sought. Changes in predicted $f_i(\hat{B})$ for observed quantities were very minimal. Using model 4, the maximum change in layer-averaged predicted head was 0.2 ft. The values for the hydraulic conductivity of clay and sand parameters, 0.362 x 10^{-3} and 30.3 ft/d, changed to 0.364 x 10^{-3} and 30.2 ft/d. The value of $\omega_k^{1/2}e_k$ was unchanged at 141, so that the confidence and prediction interval widths were unchanged. However, small local perturbations in head, flow velocity, and flow direction resulted in those parts of the aquifer system with high grid element densities if smoothing caused significant changes in these grid element densities. This would be expected from theoretical considerations regarding the variable-density flow equations (1) and (3). Larger changes in density, other than just smoothing, cause larger changes in predicted shallow head and hydraulic conductivity values. Using model 4, with the density ρ that of freshwater throughout the entire aquifer system, the maximum change in layer-averaged predicted head was 1.3 ft. The values for the hydraulic conductivity of clay and sand parameters, 0.362 x 10^{-3} and 30.3 ft/d in model 4, changed to 0.357 x 10^{-3} and 29.8 ft/d. The value for $\omega_k^{1/2}e_k$ was 141, unchanged. The 15 flow rates from model 4 appearing in table 5 changed by as much as 3 percent. Deep heads, in areas of high density originally, decreased as much as 600 ft due to the absence of the elevated pressure caused by the dense saline water.

As mentioned previously, the rates of decrease with depth of hydraulic conductivity of clay and sand were approximated by the functions $10^{-0.8dd}$ and $10^{-(1.167dd-0.0833dd^2)}$, respectively, where dd is measured in kilometers. Also as mentioned, these functions were assigned an exponent regression parameter in several models, but the confidence intervals for these regression parameters were very large. In accordance with these large confidence intervals, changes in the depth functions affected predicted heads $f_i(\hat{B})$ very little, provided only that the depth functions caused the hydraulic conductivity values of clay and sand to decrease in some substantial manner with depth.

TABLE 6.--*The effect of model alterations on: (1) layer-averaged predicted head,* h_{layer}*; (2) hydraulic conductivity of clay and sand,* P_c *and* P_s*; (3)* $w_k^{1/2}e_k$*; (4) the 15 flows of table 5; and (5) deep head for models 3 through 11 and model 4*

[Model 4 comparisons are on rows 2 and below; unaltered model 4 has P_c= 0.362x10^{-3} feet per day, (ft/d), P_s= 30.3 ft/d, and $w_k^{1/2}e_k$= 141. >, greater than; <, less than; gm/cm^3, grams per cubic centimeter]

Alteration	h_{layer} (ft)	P_c (ft/d)	P_s (ft/d)	$w_k^{1/2}e_k$	15 flows (percent)	Deep head (ft)
Nine models, 3 through 11	< 30 usually < 5	see table 4	see table 4	141-243	large differences see table 5	800
Smoothed density	< 0.2	0.364x10^{-3}	30.2	141	< 1	53
Density of freshwater	< 1.3	0.357x10^{-3}	29.8	141	< 3	600
Depth dependence of hydraulic conductivity	< 2.7	0.335x10^{-3} 0.371x10^{-3}	23.8 32.8	140 142	< 30 usually < 10	60
Effective hydraulic conductivity procedure	< 10 usually < 2	0.1980x10^{-2}	30.2	145	< 70 usually < 30	34
Truncation at density > 1.005 gm/cm^3	< 2.1	0.336x10^{-3}	32.0	142	< 35 usually < 7	no deep heads
Truncation at 3,000 ft below ground surface	< 2.3	0.336x10^{-3}	31.8	142	< 20 usually < 5	no deep heads

Multiplying both of the exponents of the functions $10^{-0.8dd}$ and $10^{-(1.167dd-0.0833dd^2)}$ by the factor 0.5 and also the factor 1.3 in model 4 gave predicted heads $f_i(\hat{B})$ that, when averaged by layer, differed from those of model 4 without alteration by at most 2.7 ft. The values for the hydraulic conductivity of clay and sand parameters, 0.362×10^{-3} and 30.3 ft/d in model 4, changed to 0.335×10^{-3} and 23.8 ft/d and 0.371×10^{-3} and 32.8 ft/d for the factors 0.5 and 1.3, respectively. This type of result is expected from the previously described mechanism relating the hydraulic conductivity of a grid element in the model, the hydraulic conductivity parameter for the grid element, and the depth d of the center of the grid element in kilometers. The value for $\omega_k^{1/2}e_k$ was 140 for factor 0.5 and 142 for the factor 1.3, both very close to the value of 141 in model 4. Thus, the prediction intervals for head and hydraulic conductivity of sand did not change significantly. Several of the 15 flow rates from model 4 appearing in table 5 changed by as much as 30 percent, but changes were usually less than 10 percent. As explained previously, the original hydraulic conductivity rate of decrease with depth functions were chosen to approximate data taken from the literature, and thus presumably should cause the model f(B) to be the most accurate with the exponents unaltered. In the event that this is not correct, the analysis above shows that the results change insignificantly for very substantial changes in the exponents.

An alternative to using the procedure of Desbarats (1987) to obtain grid element effective horizontal and vertical hydraulic conductivity values, is to use the harmonic mean to obtain effective vertical hydraulic conductivity, and to use the arithmetic mean to obtain effective horizontal hydraulic conductivity. This procedure yields effective vertical and horizontal hydraulic conductivity values that correspond to the assumption that the clay in the grid element extends horizontally and continuously across the entire grid element as a single or several uniform layers. It would thus tend to give a larger than actual effective horizontal conductivity and a smaller than actual effective vertical conductivity. The formulas for effective vertical and horizontal hydraulic conductivity, K_z and K_h, for a grid element are:

$$K_z = [(V_c/K_c) + ((1-V_c)/K_s)]^{-1} , \tag{16}$$

and

$$K_h = (V_cK_c + (1-V_c)K_s , \tag{17}$$

where, as in formulas (10) and (11), K_c and K_s denote the hydraulic conductivity of the clay and sand components respectively, and V_c denotes the clay fraction of the grid element. Equations (16) and (17) were used in several of the models in table 4 as a replacement for equations (10) and (11). This was done by the direct use of equations (16) and (17), and also by using replacements for a and a^{-1} in equations (10) and (11), causing these equations to give nearly the same values for K_z and K_h as equations (16) and (17). Use of equations (16) and (17) as opposed to equations (10) and (11) produced changes in the values for the hydraulic conductivity of clay and sand parameters for the various hydraulic conductivity zones. This would be expected from the functionally different manner that K_c and K_s appear in equations (10) and (11), as compared to the way they appear in equations (16) and (17). By use of equations (16) and (17) in model 4 gave predicted heads $f_i(B)$, which when averaged by layer differed by 10 ft at most and generally less than 2.0 ft from predicted heads obtained using equations (10) and (11). However, the values for the hydraulic conductivity of clay and sand parameters, 0.362×10^{-3} and 30.3 ft/d in model 4, changed to 0.198×10^{-2} and 30.2 ft/d, respectively. Due to the difference between equations (10) and (11) as compared to equations (16) and (17), effective hydraulic conductivity values K_z and K_h did not generally change by more than 50 percent, despite the substantial change of the hydraulic conductivity of clay parameter value from 0.362×10^{-3} ft/d to 0.198×10^{-2} ft/d. The value for $\omega_k^{1/2}e_k$ was 145, only slightly larger than the value of 141 in model 4, so that the prediction intervals for head and hydraulic conductivity of sand did not change significantly. Several of the 15 model 4 flow rates appearing in table 5 changed by as much as 70 percent, but changes were usually less than 30 percent. The value for Chi^2 did not change appreciably.

The manner of selecting the hydraulic conductivity zones was not changed, except for the use of various choices for the boundaries of the 10 regions in figure 31. However, because the various models had different choices for the assignment of the more than 100 hydraulic conductivity zones to a given set of hydraulic conductivity of clay and sand parameters, model error due to discretization of the aquifer system domain of solution with regard to the values used for hydraulic conductivity was partially eliminated. This is true only if a sufficiently large number of different chosen assignments of hydraulic conductivity zones was used. Since more than 40 different assignments were used (seven of which are shown in table 4), model error due to discretization is probably largely eliminated. This should be interpreted relative to the differences in results noted in the previous section.

Model Truncation

Some studies of aquifer systems with saline water at depth use a constant freshwater density model and truncate the domain of solution at or near that depth where the density begins to increase substantially, treating this truncation surface as a no-flow boundary. This approach may be seen as an approximation to the method of this study which, by the use of a variable-density model, models both the shallow freshwater system and the underlying saline water system as a total system. Obviously the truncation of the deep saline part of the aquifer system precludes the modeling of this deep part. The hope, however, is that in spite of truncation, the results for the remaining shallow part of the aquifer system are reasonably accurate.

With the truncation approaches described below, predicted shallow head and hydraulic conductivity were similar to values obtained by the variable density model. Flow from the geopressured zone in some of the models is shown in table 5. This flow, if it occurs, passes up through the no-flow truncation surface of the truncated system. Thus, flow at the depth of the truncation surface differs substantially between the two approaches because, when modeling the full system, flow occurs across the truncation surface which is a no-flow boundary for the truncated system. Heads at depth cannot be compared between the two approaches, because the truncated aquifer system does not have any deep heads. Heads just above the truncation surface were found to be fairly close. Heads adjacent to the truncation surface and in areas of heavy pumping differed the most.

Model 4 was used with a truncation surface defined as that surface at which grid element density first exceeds 1.005 gm/cm^3 when proceeding downward from the top grid element at each horizontal location. This truncation reduced the number of observed grid-element volume-averaged heads Y_i from 3,107 to 3,067, because 50 heads Y_i were located below the truncation surface. Density in the grid elements above the truncation surface was set to that of freshwater. The maximum change in layer-averaged predicted head was 2.1 ft. The values for the hydraulic conductivity of clay and sand parameters, 0.362 x 10^{-3} and 30.3 ft/d in model 4, changed to 0.336 x 10^{-3} and 32 ft/d. The value for $\omega_k^{1/2}e_k$ was 142. The 15 flow rates from model 4 appearing in table 5 changed by as much as 35 percent, but changes were usually less than 7 percent. The maximum change in the 3,067 predicted heads was 49 ft. This occurred at a grid element located adjacent to the truncation surface that had a drawdown of almost 400 ft due to heavy pumping. Most of the changes in the 3,067 predicted heads were less than 5 ft. Flows in the area of the grid elements with heavy pumping were different in the truncated model due to the proximity of the no-flow truncation surface.

Model 4 was also used with a truncation surface at 3,000 ft below land surface. This truncation decreased the number of grid-element volume-averaged heads Y_i to 3,087. Density was set to that of freshwater. The maximum change in layer-averaged predicted head was 2.3 ft. The values for the hydraulic conductivity of clay and sand parameters, 0.362 x 10^{-3} and 30.3 ft/d in model 4, changed to 0.336 x 10^{-3} and 31.8 ft/d. The value for $\omega_k^{1/2}e_k$ was 142. The 15 flow rates from model 4 appearing in table 5 changed by as much as 20 percent, but changes were usually less than 5 percent.

Note that with both methods of truncation, layer-averaged predicted head does not change by more than 2.3 ft. This small change occurs in spite of the fact that some of the layers have a reduced number of heads Y_i, so that the average for the layer is over a reduced set of those Y_i that were in the layer before truncation. The 15 flow rates in table 5 show change, as mentioned, but some of this change occurs because the extent of the truncated layers has decreased, in some cases by a considerable amount. Because flow decreases substantially with depth due to decreasing hydraulic conductivity of clay and sand, the truncation of the deeper part of a layer may have a relatively small effect on the total flow going into or out of its top surface.

Changes that occur in layer-averaged predicted head and flow due to the truncation of model 4 are less than those that occur between the nine models in table 5, all without truncation (table 6). These results with the truncation of model 4 indicate that in general little predictive ability is lost by the use of a truncated model because neither the full nor truncated models are able to make any useful predictions for deep head and flow. The only exception to this would be that the full model is more accurate for the prediction of head and flow near the truncation surface, particularly if there is heavy pumping nearby.

SUMMARY AND CONCLUSIONS

SUMMARY

The major results of the study are:

1. Of the 12 models evaluated in this report, the model with the smallest prediction interval half widths, model 4, had only four regression parameters. With all models, the residuals failed the Chi^2 test for normality at the 1-percent significance level, possibly as a result of the large 10-mi grid spacing used.

2. The dominant factor controlling shallow hydraulic heads in the simulated aquifer system is the specified head along the top surface of the aquifer system, which consists of the water-table altitude and equivalent freshwater head at the sea floor. Because the water-table altitude surface is usually not far below land surface and has a very similar shape, a dominant factor controlling shallow heads is land-surface altitude. These shallow heads are also affected by pumping. Heads at depth may be affected by flow from the geopressured zone. However, the models used show almost no ability to predict deep heads and flow, including flow from the geopressured zone.

   The various models used show considerable overlap among the prediction intervals for shallow head and hydraulic conductivity of sand. The 95-percent prediction interval half widths for grid-element volume-averaged head all exceed 108 ft, and those for volume-averaged $\log_{10}$ hydraulic conductivity of sand all exceed 0.94(141/149) = 0.89.

3. The essential feature of the flow system is the flow of water from upland surface recharge areas to discharge areas at lower altitude on land or the sea floor. No prediction intervals were obtained for flow. The variability from model to model in values obtained for shallow flow varies greatly. The flow across some surfaces is not known to within a factor of 10. For other surfaces, the flow rates are known to within a smaller factor. There is little certainty about the flow across many of the individual grid element surfaces, particularly those that have relatively small flow.

Because the possible error of estimate for maximum allowable recharge (discharge) into the aquifer system from model layer 12 is large in comparison with the differences for this flow among models 3 through 11, none of the models seem any more likely to be valid than another. The reason for this is that the flow rates into the aquifer system from model layer 12 are very small compared to flow rates in the hydrologic budget, such as precipitation, runoff, evapotranspiration, etc. Estimates for the flow rate from the geopressured zone are sufficiently approximate that they also provide no basis for selecting any one of the models 3 through 11 as being better than any of the others.

4. Truncating the domain of solution of one of the models below a certain ground-water density or depth below land surface and setting the density to that of freshwater in the remaining shallow part of the domain of solution did not appreciably change results for hydraulic head and flow produced by the model, except for locations close to the truncation surface.

5. The regression methodology allowed the testing of a very wide range of models for the simulation of the aquifer system. The time saved by being able to find the optimal selection of the regression parameters for a given model in only a single computer run was used to formulate and test many different types of models and procedures.

   The regression methodology also provided estimates of the accuracy of results, in the form of prediction and confidence intervals. These accuracy estimates point out the limitations of the predictive ability of a model, and are thus very valuable. Testing, to determine the most likely sources of model error, led to the conclusion that the 10- mi grid element spacing was quite large relative to the variablility of hydraulic head and is thus the probable cause of significant error.

CONCLUSIONS

The predictive ability of the models used was quite low in many aspects. The models showed almost no ability to predict deep heads and flow, including flow from the geopressured zone. Regression analysis shows that even the best of the models used had a rather poor ability to accurately predict head in any of the layers. Evidence of this is the 95-percent prediction interval half width of 108 ft mentioned previously. For predicting hydraulic conductivity of sand, the 95-percent prediction interval half widths for $\log_{10}$ hydraulic conductivity of sand all exceeded 0.89, thus hydraulic conductivity of sand could not be predicted to within a factor of almost 8 ($10^{.89}$). These results show the considerable inaccuracy of the models chosen and perhaps the inaccuracy of any model based on the same data and relatively large 10-mi grid spacing. The very important contribution of regression analysis is showing the limitations of the predictive ability of the models used. A less thorough study probably would have left the impression that the model or models used were significantly more accurate than they actually were.

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ATTACHMENTS

Introduction

The computer code used to find the p vector $\hat{B}$ which minimizes the weighted mean square error $S(B) = \frac{1}{n} \sum_{i=1}^{n} (Y_i - f_i(B))^2 \omega_i$, consists of two parts. The first part is taken from Durbin (1983). The second part is Kuiper's variable-density ground-water flow model, VARDEN (Kuiper 1985), which is used to evaluate the $f_i(B)$. The first part consists mainly of a section called MAIN in the code which chooses a sequence of parameter vectors B_{st}, s = 1,2,...,t = 1,2,...,p, which are used to find a sequence B_s, s = 1,2,..., which converges to $\hat{B}$. This process is an implementation of the Levenberg-Marquardt method for the minimization of a sum of squares. The remainder of the first part is a linear-system solver called SOLEQU, which is used as a subroutine to MAIN. MAIN uses Kuiper's (1985) variable density ground-water flow model VARDEN, by calling MODEL when an evaluation of f(B) is required for B equal to one of the vectors B_{st} or B_s which MAIN has chosen. In turn, MODEL calls the linear-system solvers PCG, and SIP.

MODEL, PCG, and SIP make up VARDEN and have been explained in detail by Kuiper (1985). MAIN from Durbin (1983) and MODEL from Kuiper (1985) were substantially modified for this application and the modifications are explained below. Some of the modifications consist of inserted code used to calculate prediction and confidence intervals as given by equations (6) and (7) in the prediction and confidence interval section of this report. Several slight modifications were made to PCG and SIP and they also are explained below. SOLEQU was not modified. The FORTRAN code for MAIN, MODEL, SOLEQU, PCG, and SIP are listed below in their respective sections. The sections for MAIN and MODEL contain a listing of meaningful variables. Dummy variables used only to implement a calculation are not listed. The manner of selection of the sequences B_{st} and B_s as dictated by the Leven-Marquardt procedure is explained in the next section.

The Levenberg-Marquardt Method

The Levenberg-Marquardt method (Durbin, 1983; Gill, Murray, and Wright, 1981) for the minimization of a sum of squares $S(B) = \sum_{i=1}^{n} (Y_i - f_i(B))2\omega_i$, is:

$$(X^T\omega X + \lambda I)\Delta B = X^T\omega(Y - f(B)) \tag{18}$$

Here X(B) is the sensitivity matrix defined as $X_{ij} = \partial f_i/\partial B_j$, i = 1,2,...,n, j = 1,2,...,p, where n is the number of observations Y_i and p is the number of parameters. B is the p vector of parameters, I is the identity matrix, and vector ΔB is the change in B. $\lambda(B)$ is equal to $(\sum_{i=1}^{n} r_i^2)^{1/2}$/RLEVM, where n vector r = Y-f(B) and RLEVM is the "maximum Levenberg radius." Weights in S(B) are denoted by ω_i.

Equation (18) is applied iteratively, starting with some initial B of choice and corresponding ΔB from (18). For the next iteration B is replaced with $B + \Delta B$, which value is then used in (18) in X(B), $\lambda(B)$, and f(B) to produce a new ΔB, etc. The successive values of the vector B are labeled B_s where s is the iteration count. In order to evaluate X(B) it is necessary to find the n x p derivatives $\partial f_i/\partial B_j$. These are found n at a time from p evaluations of f(B) at $B = B_{st}$, $t = 1,2,\ldots$ p and also f(B) at $B = B_s$. The derivative $\partial f_i/\partial B_j$ at $B = B_s$ is approximated by

$$\partial f_i/\partial B_j = (f_i(B_{st}) - f_i(B_s))/((B_{st})_j - (B_s)_j)$$

where

$$(B_{st})_j - (B_s)_j = (TST)(B_s)_j,\ (B_{st})_\ell = (B_s)_\ell \text{ for } \ell \neq j,$$

and TST is called the "perturbation factor." Having evaluated $X(B_s)$by this means, equation (18) is used to find $B_{s+1} = B_s + \Delta B$.

It is readily shown from equation (18) that $\Sigma(\Delta B)_j^2$ cannot exceed $(RLEVM)^2$. Thus RLEVM can be chosen to limit the size of ΔB. In general RLEVM should be chosen large enough that ΔB is larger than (TST)B, but not so large that the number of iterations needed for B_j, $j = 1,2,\ldots,$ to arrive at a suitable approximation to $\hat{B}$ is caused to increase. TST should be chosen as small as possible before deterioration in the accuracy of the derivatives of f(B) with respect to B occurs. The parameters B_j should be scaled in f(B) so that approximately equal percentage changes occur in S(B) corresponding to equal percentage change in the individual members B_j of vector B.

MAIN

The function of MAIN is to implement the Levenberg-Marquardt method. It calls subroutine MODEL when it needs f(B) for some chosen value of B as dictated by the Levenberg-Marquardt method. MODEL is actually called by either a call to MODEL1 or MODEL2; MODEL1 being used only for an initial call which reads in data needed for the evaluation of f(B) for any value of B.

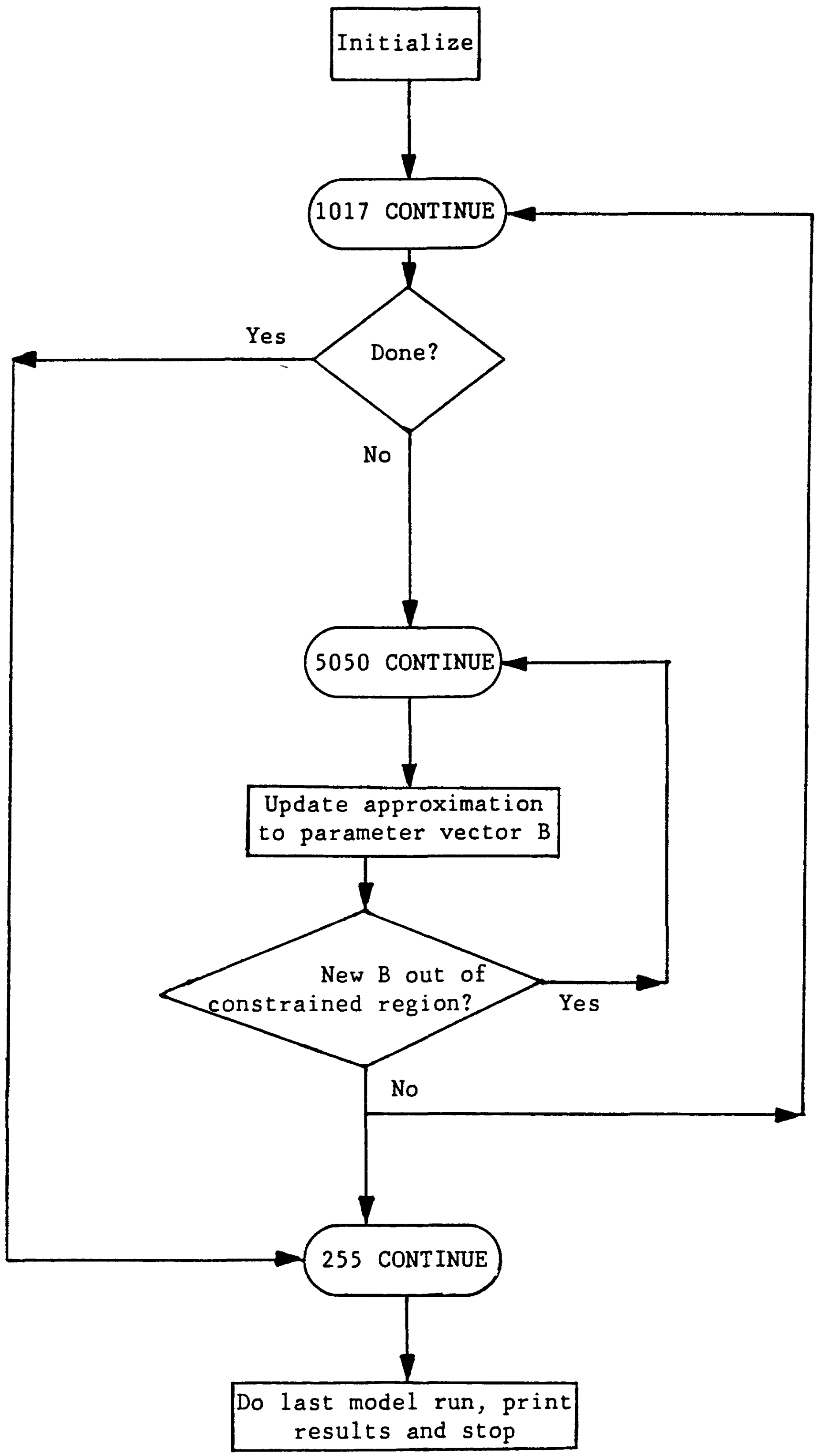
Function f(B) is the ground-water flow model VARDEN. The values for $f_i(B)$, $i = 1,2,\ldots,n$ are the flow model predictions for the observations Y_i, many of which are grid-element volume-averaged heads. Model f(B) requires the specification of the sand and clay component of hydraulic conductivities for each of the grid elements. The mapping of the p values for grid-element sand and clay component hydraulic conductivity and other parameters of f(B) is accomplished by the following procedure. PARAMX(I)=PARMO(I)*PARAM(IPARAM(I)) I = 1,2,...,IPARX, where IPARX is the number of global parameters, the total number of parameters needed for model f(B) including many thousand grid-element sand and clay hydraulic conductivity values. PARAM is a p (p=IPAR) vector equal to the parameter vector B. IPARAM(I), I = 1,2,...,IPARX has the values 1 through p and maps I to an associated parameter number IPARAM(I). PARAMO(I) is a multiplicative factor. PARAMX(I) is used within the model f(B). In the case of the sand and clay component hydraulic conductivities, the PARAMX(I) are used to arrive at multiplicative factors which are multiplied by read-in values for sand and clay component hydraulic conductivity.

Explanation of MAIN Listing

COMMENT	LINE NUMBERS WITH MAN PREFIX
Read in the number of columns, rows, layers and intervening confining beds in the modeled area: NI10, NJ10, NK10, NK4.	700
Read in: Number of global parameters Number of active parameters Maximum number of iterations Perturbation factor Maximum error change Maximum Levenberg radius IPARX, IPAR, JITU, TST, PCHL, RLEVM.	830
Read in IPARAM.	990-1170
Read in PARAMO.	1180-1480
Do initial model call (MODEL1).	1550-1710
If JITU=0 go to 255 CONTINUE for final model run, print out, and stop.	1730
This is the final run, print, and stop section addressed by 255 CONTINUE.	1750-2060
Set initial conditions (JIT = 1).	2090-2400
1017 CONTINUE.	2420
Set JIT = JIT+1.	2430
Calculate new PRARMX corresponding to new PARAM and call MODEL2.	2440-2520

COMMENT	LINE NUMBERS WITH MAN PREFIX
Calculate SB=S(B) and root mean square weighted residual PZ $\equiv (n^{-1}\ S(B))^{1/2}$. Print out PZ, new estimates of the parameters B_j, j = 1,2,...,p (PARAM(I),I = 1,2,...,IPAR), the change in these parameters from the previous iteration (MU(I), I = 1,2,...,IPAR), and $\partial(PZ)/\partial B_j$ j=1,2,...,p (GRAP(I), I=1,2, ...IPAR).	2530-2700
If: 1) The iteration number JIT is greater than the "maximum number of iterations" JITU, or 2) the change in the root mean square weighted residual PZ was less than PCHL, the "maximum error change," go to 255 CONTINUE for eventual stop.	2800-2890
Do numerical gradient computations. The p = IPAR values for the PARAM vector used are the B_{st} mentioned previously. Here s = JIT the iteration number and t = 1,2,...,p = IPAR. GRSS is the sensitivity matrix appearing in equations (6) and (7) in the text. PARO is used to scale PARAM.	2980-3150
Get B2 = $X^T\omega(Y-f(B)$ on the right hand side of equation (18) above. GRP is $\nabla S(B)$ here.	3170-3240
Do regression pack.	3260-3840
5050 CONTINUE.	3900
Get A2 = $(X^T\omega X + \lambda I)$, the matrix on the left hand side of equation (18) above.	3910-4030
Solve $(A2)\Delta B = (B2)$ for MU = ΔB.	4050-4090
If new B is out of imposed constraining region, go to 5050 CONTINUE for a recalculation of ΔB. This recalculation will have those members of new B which were placed out of their respective contraint values.	4120-4640
Calculate $\partial(PZ)/\partial B_t$ t = 1,2,...p = IPAR (GRP(I), I + 1,2,...IPAR).	4660-4690
Go to 1017.	

Flow Chart of MAIN



Definition of Program Variables

EPS	$Y_i - f_i(B)$.
GRSS	Sensitivity matrix X in equations (6) and (7) above in body of report.
GRP	$\partial S(B)/\partial B_t$ or $\partial(PZ)/\partial B_t$.
IOUT	If IOUT = 1, MODEL1 or MODEL2 prints out certain results, if IOUT = 0, no printout occurs.
IPAR	Number of active parameters
IPARAM(I)	Maps global parameter numbers used by model f(B) to set of p active parameters of parameter vector B.
IPARX	Number of global parameters.
JIT	Iteration number (JIT = 5).
JITU	Maximum number of iterations.
MU	Change ΔB in the parameter vector B.
NI10	Number of columns in the modeled area.
NJ10	Number of rows in the modeled area.
NK10	Number of layers in the modeled area.
NK4	Number of intervening confining beds.
NWLMT	Number of observations Y_i
PARAMO(I)	Multiplicative factor in PARAMX(I) = PARAMO(I)*PARAM(IPARAM(I)) I = 1,2, . . . IPARX
PARAMX(I)	Global parameters used in model f(B)
PARO	Scale factor for parameters in parameter vector B
PCHL	Maximum error change
PDIV	Equal to NWLMT
PZ	$(\frac{1}{n} S(B))^{1/2}$, root-mean-square-weighted residual

RLEVM Maximum Levenberg radius

SB $S(B) = \frac{1}{n} \sum_{i=1}^{n} (Y_i - f_i(B))^2$

TST Perturbation factor.

WLC Calculated values $f_i(B)$ for observations Y_i

WLM Measured values for observations Y_i

```
      IMPLICIT REAL*8(A-H,O-Z)
      COMMON WS,HMAX,RELX1,RELX2,COEF,ERR,XX10,DELT,ER5,SRZ,SUMRZ
     1,ERRSV,XX10SV,JIT,NIJ10,NI11,NJ11,NK11,NNN,NSKP1
     2,NSKP2,ITMAX,ICNT,IEVP,IWR1,NW1,NW2,NW3,N320,NUM4
     3,L9,LENGTH,NK1115,IT01,IT15,ICRO,DDK,BBK
     4,NI10,NJ10,NK10,NI12,NJ12,NK12,SVIJ,VV40,SV35
      COMMON /XX/ XX
      COMMON /DT/ DT
      COMMON /VV/ VV
      COMMON /E2/ E2
      COMMON /F2/ F2
      COMMON /G2/ G2
      COMMON /YQ/ YQ
      COMMON /NT/ NT
      COMMON /DD/ DD
      COMMON /BB/ BB
      COMMON /ZZ/ ZZ
      COMMON /XXS/ XXS
C     COMMON /ALN/ ALN
C     COMMON /XXE/ XXE
C     COMMON /SV/ SV
      COMMON /HL/ HL
      COMMON /LB/ LB
      COMMON /MHD/ MHD
      COMMON /C/ DXI,DYJ,N325,ISOR,IPDD
     1,NPINT,DT0,TOT,TFAC,NINT,IPRNT,IWRT,LFLOW,NK4,LLRO,IWRTXX,
     2ILZ2,MAQ1,NU1,NU2,NU3,LFLO,IPH
     3,LBB,SUMF,SUNF,SQ2,SG2,SYQ,SVV,SUMFM,SUNFM
      COMMON /B/ COL,COU,PARAM,PAR0,MU,W,EPS,WLM,LWLC,WLC,A2,B2,GRP
     1,CONA,CONB,SCALE
     2,NBND1,NBND2,ICON,NND,NWLM,NWLM1,NWLMT,IOUT
      COMMON /GRSS/ GRSS
      COMMON /XXSTR/ XXSTR
      COMMON /XKZZ/ XKZZ
      COMMON /LRO/ LRO
      COMMON /YQ1/ YQ1
      COMMON /LZ2/ LZ2
      COMMON /PARAMX/ PARAMX
      COMMON /PARAM0/ PARAM0
      COMMON /IPARAM/ IPARAM
      COMMON /YQ2/ YQ2
      REAL*4 YQ2(65078)
      DIMENSION A3(50,50)
      REAL*4COL(50),COU(50),PARAM(50),PAR0(50),MU(50),W(3303),EPS(3303),
     1   WLM(3303),LWLC(3303),WLC(3303),A2(50,50),B2(50),GRP(50),
     2   CONA(50,50),CONB(50),SCALE(50),GRSS(3303,5),
     3   PARAMX(159872),PARAM0(159872)
      INTEGER*4 NBND1(50),NBND2(50),ICON(50),
     1   NND(3303)
      INTEGER*2 IPARAM(159872)
      REAL*4 DD(65078),BB(65078),ZZ(65078),YQ(65078),
     1XXS(65078),Q2(300),DDK(50),BBK(50),NT(94658),HL(65078)
     2,DXI(250),DYJ(250),XXSTR(65078),YQ1(65078,3)
     3,XKZZ(65078)
```

```
      INTEGER*2 LRO(94658),IPH(50),LFLO(4),J7(5),K7(5)                  MAN0550
     1,NI(300),NJ(300),NK(300),MAQ1(50),NU1(3),NU2(3),NU3(3)            MAN0560
      DIMENSION DT(65078),E2(65078),F2(65078),G2(65078),VV(65078)       MAN0570
      DIMENSION XX(65078)                                               MAN0580
     1,WS(10),MHD(65078),LZ2(65078),LB(94658),LBB(5918)                 MAN0590
     2,AOC(4),BOC(4),COC(4),DOC(4)                                      MAN0600
     3,SUMF(3,250),SUNF(3,250),SQ2(3,250),SG2(3,250),SYQ(3,250)         MAN0610
     4,SVV(3,250),SUMFM(3,250),SUNFM(3,250)                             MAN0620
     *,IGR6(12,12)                                                      MAN0630
      DATA PARAM/.9874,1.022,.8725,1.083,1.004,.7712,.9917,1.061,       MAN0640
     *1.017,.9249,1.041,.8042,.9301,1.180,.9482,.9572,                  MAN0650
     *.9964,1.037,1.186,.9108,.9988,.9775,.9000,1.027,                  MAN0660
     *1.002,.9558,                                                      MAN0670
     *1.,1.,1.,1.,1.,1.,1.,1.,1.,1.,1.,1.,1.,                           MAN0680
     *1.,1.,1.,1.,1.,1.,1.,1.,1.,1.,1./                                 MAN0690
      READ(5,99) NI10,NJ10,NK10,NK4                                     MAN0700
      NI11=NI10+1                                                       MAN0710
      NJ11=NJ10+1                                                       MAN0720
      NK11=NK10+1                                                       MAN0730
      NIJ10=NI10*NJ10                                                   MAN0740
      NIJK10=NIJ10*NK10                                                 MAN0750
      N320=NIJK10+1                                                     MAN0760
      N325=NIJ10*(NK10+NK4)+1                                           MAN0770
      NK15=NK11+NK4                                                     MAN0780
C     NUMBER OF GLOBAL PARAMETERS, NUMBER OF ACTIVE PARAMETERS,         MAN0790
C        PERTURBATION FACTOR, MAXIMUM ERROR CHANGE, AND                 MAN0800
C        MAXIMUM LEVENBERG RADIUS                                       MAN0810
C                                                                       MAN0820
      READ(5,940) IPARX,IPAR,JITU,TST,PCHL,RLEVM                        MAN0830
      IPARX=N325+N320+2*NK10-1+10                                       MAN0840
      IPAR=4                                                            MAN0850
      WRITE(6,941) IPARX,IPAR,JITU,TST,PCHL,RLEVM                       MAN0860
C                                                                       MAN0870
 940  FORMAT(3I6,3F12.0)                                                MAN0880
 941  FORMAT(1H0,10X,'SERACH DEFINITION'/1H ,10X,17('-')/               MAN0890
     1   1H0,10X,'NUMBER OF GLOBAL PARAMETERS',9X,I7/                   MAN0900
     2   1H ,10X,'NUMBER OF ACTIVE PARAMETERS',9X,I3/                   MAN0910
     3   1H ,10X,'MAXIMUM NUMBER OF ITERATIONS',8X,I3/                  MAN0920
     4   1H ,10X,'PERTURBATION FACTOR',11X,1PE9.3/                      MAN0930
     5   1H ,10X,'MAXIMUM ERROR CHANGE',10X,1PE9.3/                     MAN0940
     6   1H ,10X,'MAXIMUM LEVENBERG RADIUS',6X,1PE9.3)                  MAN0950
C                                                                       MAN0960
C     GLOBAL PARAMETER SET AND ASSIGNMENT TO ACTIVE PARAMETERS          MAN0970
C                                                                       MAN0980
C  *******READ IPARAM                                                   MAN0990
      NK1115=NK15                                                       MAN1000
      IT15=1                                                            MAN1010
      ICRO=0                                                            MAN1020
      CALL RDWRT                                                        MAN1030
      DO 209 IJ=2,N325                                                  MAN1040
209   IPARAM(IJ)=1*NT(IJ)                                               MAN1050
      IPARAM(1)=0                                                       MAN1060
      NK1115=NK11                                                       MAN1070
      CALL RDWRT                                                        MAN1080
      DO 211 IJ=2,N320                                                  MAN1090
```

```
      I7=IJ+N325-1
211   IPARAM(I7)=1*NT(IJ)
      READ(5,99) (IPARAM(I7+I),I=1,NK10)
      WRITE(6,99) (IPARAM(I7+I),I=1,NK10)
      I8=I7+NK10
      READ(5,99) (IPARAM(I8+I),I=1,NK10)
      WRITE(6,99) (IPARAM(I8+I),I=1,NK10)
99    FORMAT(16I5)
C     READ PARAMO
      NK1115=NK15
      CALL RDWRT
      DO 309 IJ=2,N325
309   PARAM0(IJ)=NT(IJ)
      PARAM0(1)=0
      NK1115=NK11
      CALL RDWRT
      DO 311 IJ=2,N320
      I7=IJ+N325-1
311   PARAM0(I7)=NT(IJ)
      READ(5,98) (PARAM0(I7+I),I=1,NK10)
      WRITE(6,97) (PARAM0(I7+I),I=1,NK10)
      I8=I7+NK10
      READ(5,98) (PARAM0(I8+I),I=1,NK10)
      WRITE(6,97) (PARAM0(I8+I),I=1,NK10)
98    FORMAT(10G8.0)
97    FORMAT(10D12.3)
C  *************
      I1=IPARX-9
      DO 147 IL=I1,IPARX
      IPARAM(IL)=0
147   PARAM0(IL)=1
      PARAM0(I1+2)=.5900
      PARAM0(I1+7)=.8996
      IPARAM(I1+2)=1
      IPARAM(I1+7)=2
C   ***
      WRITE(6,99) (IPARAM(IL),IL=I1,IPARX)
      WRITE(6,97) (PARAM0(IL),IL=I1,IPARX)
C  **************
C     ACTIVE PARAMETER VALUES, UPPER BOUNDS, AND LOWER BOUNDS
      DO 834 I=1,IPAR
      COL(I)= 01
      COU(I)=100
      PARAM(I)=1
834   CONTINUE
C     Determine PARAMX and do initial model call
      DO 210 I=1,IPARX
      X=PARAM(IPARAM(I))
      IF(IPARAM(I).EQ.0) X=1
      PARAMX(I)=PARAM0(I)*X
 210  CONTINUE
      WRITE(6,946)
      WRITE(6,947) (I,PARAM(I),COL(I),COU(I),I=1,IPAR)
C
 945  FORMAT(6X,3F12.0)
```

```
 946  FORMAT(1H0,10X,'ACTIVE PARAMETER SET'/1H ,10X,20('-')/
     1   1H0,10X,'ACTIVE',11X,'INITIAL',10X,'LOWER',10X,'UPPER'/
     2   1H ,10X,'PARAMETER',8X,'VALUE',12X,'BOUND',10X,'BOUND'/)
 947  FORMAT((1H ,10X,I9,3F15.4))
      JIT=-1
      IOUT=0
      CALL MODEL1
C       STOP
      IF(JITU.NE.0) GO TO 256
C     PRINT FINAL MODEL RUN
 255  CONTINUE
      DO 251 I=1,IPARX
      X=PARAM(IPARAM(I))
      IF(IPARAM(I).EQ.0) X=1
      PARAMX(I)=PARAM0(I)*X
 251  CONTINUE
      IOUT=1
      CALL MODEL2
C
C     WATER-LEVEL RESIDUALS
C
      DO 656 I=1,NWLMT
      EPS(I)=WLM(I)-WLC(I)
 656  CONTINUE
C
      WRITE(6,966)
C     WRITE(6,967) (NND(I),WLM(I),WLC(I),EPS(I),I=1,NWLMT)
      PDIV=NWLMT
      P=0.0
      DO 2566 IWL=1,NWLMT
      P=P+EPS(IWL)**2*W(IWL)
 2566 CONTINUE
      PZ=DSQRT(P/PDIV)
      WRITE(6,9566) PZ
C
 966  FORMAT(1H0,10X,'WATER-LEVEL RESIDUALS'/1H ,10X,21('-')/
     1   1H0,10X,'STEP',3X,'NODE',3X,'MEASURED',3X,'COMPURED',
     2   3X,'RESIDUAL'/)
 967  FORMAT((1H ,10X,I4,3X,I4,3F11.1))
9566  FORMAT(1H0,10X,'STANDARD ERROR OF ESTIMATE',1X,1PE10.3)
C
      STOP
 256  CONTINUE
C
C     SET INITIAL CONDITIONS
C
      FLAGA=0.
      IPARC=IPAR
      GDIV=IPAR
      PDIV=NWLMT
      PZP=0.
      PZPT=0.
      JIT=0.
      RLEV=0.0
      SCMIN=0.0
```

```
      IC=0
      DO 8000 I=1,IPAR
      MU(I)=0.0
      GRP(I)=0.0
      NBND1(I)=0
      NBND2(I)=0
 8000 CONTINUE
C
      WRITE(6,960)
C
 960  FORMAT(1H1,10X,'PARAMETER SEARCH RESULTS'/1H ,10X,24('-'))
C
C     INITIAL PARAMETERS
C
      DO 419 J=1,IPAR
      PARO(J)=PARAM(J)
      IF(COL(J).LE.1.0E-08.OR.COU(J).LE.1.0E-08) WRITE(6,130)
 130  FORMAT(4(/),40X,'BOTH CONSTRAINTS MUST BE GREATER THAN 1.E-8')
      IF(PARO(J).GE.(COU(J)-1.E-5)) NBND1(J)=1
      IF(PARO(J).LE.(COL(J)+1.E-5)) NBND1(J)=-1
 419  CONTINUE
C
 1017 CONTINUE
      JIT=JIT+1
      DO 250 I=1,IPARX
      X=PARAM(IPARAM(I))
      IF(IPARAM(I).EQ.0) X=1
      PARAMX(I)=PARAMO(I)*X
 250  CONTINUE
      IOUT=0
      WRITE(6,999) (PARAM(I),I=1,IPAR)
999   FORMAT(10E12.5)
      CALL MODEL2
      P=0.
      DO 7115 IWL=1,NWLMT
      LWLC(IWL)=WLC(IWL)
      EPS(IWL)=WLM(IWL)-WLC(IWL)
      P=P+EPS(IWL)**2*W(IWL)
 7115 CONTINUE
      SB=P
      PZ=DSQRT(P/PDIV)
C
C     PRINT-OUT OF SEARCH RESULTS
C
      WRITE(6,7602) JIT,PZ,RLEV,SCMIN
 7602 FORMAT(1HO),10X,'ITERATION',I4/1H,10X,13('-')/
     1       1HO,10X,'STANDARD ERROR OF ESTIMATE',1X,1PE10.3/1H ,10X,
     2'LEVENBERG RADIUS',11X,1PE10.3/1H ,10X,'MINIMUM SCALING FACTOR',
     3  5X,1PE10.3/1HO,10X,2('PARAMETER',3X,'ESTIMATE',7X,'CHANGE',5X,
     4'GRADIENT',5X)/)
      WRITE(6,7607)(I,PARAM(I),MU(I),GRP(I),I=1,IPAR)
      IF(IC.EQ.0) GO TO 8003
      WRITE(6,8002)
      WRITE(6,8001) (ICON(IC),IC=1,ICMAX)
 8003 CONTINUE
```

```
C
 8002 FORMAT(1H0,10X,'CONSTRAINED PARAMETERS'/)
 8001 FORMAT(1H ,5X,10I8)
 7607 FORMAT((1H ,10X,2(I5,5X, 1PE10.3,3X,1PE10.3,3X,1PE10.3,4X)))
C
C     CONVERGENCE AND ITERATION TESTS
C
      IF(JIT.GE.JITU) GO TO 6260
      PCH=ABS(PZ-PZP)
      IF(PCH.LT.PCHL) GO TO 6274
      PZP=PZ
      GO TO 1305
 6260 WRITE(6,6261) JITU
 6261 FORMAT(1H0,10X,'NUMBER OF ITERATIONS GREATER THAN ',I5)
      GO TO 255
 6274 WRITE(6,6277) PCHL
 6277 FORMAT(1H0,10X,'OBJECTIVE FUNCTION CHANGES LESS THAN',F8.4)
      GO TO 255
C
C     BEGINNING OF NEW ITERATION
C
 1305 CONTINUE
C
C     NUMERICAL GRADIENT COMPUTATION
C
      DO 1732 J=1,IPAR
      DP=TST*PARAM(J)
      PARAM(J)=PARAM(J)+DP
C
      DO 252 I=1,IPARX
      X=PARAM(IPARAM(I))
      IF(IPARAM(I).EQ.0) X=1
      PARAMX(I)=PARAM0(I)*X
 252  CONTINUE
      IOUT=0
      CALL MODEL2
C
      PARAM(J)=PARAM(J)-DP
      DO 1730 IWL=1,NWLMT
 1730 GRSS(IWL,J)=(WLC(IWL)-LWLC(IWL))*PAR0(J)/DP
 1732 CONTINUE
C
C     COMPUTATION OF A2 AND B2
C
      DO 1430 J=1,IPAR
      SUM=0
      DO 1427 IWL=1,NWLMT
 1427 SUM=SUM+GRSS(IWL,J)*EPS(IWL)*W(IWL)
      B2(J)=SUM
 1430 GRP(J)=-2.*SUM
      WRITE(6,999) (B2(J),J=1,IPAR)
C    ###########Regression Insert Pack###
C     Get left hand side of inequality (9) in text
      DO 1675 JR=1,IPAR
      DO 1650 JS=1,IPAR
```

```
      SUM5=0.                                                           MAN3300
      DO 1640 IWL=1,NWLMT                                               MAN3310
 1640 SUM5=SUM5+GRSS(IWL,JR)*GRSS(IWL,JS)*W(IWL)                        MAN3320
 1650 A3(JR,JS)=SUM5                                                    MAN3330
1675  CONTINUE                                                          MAN3340
C     DO 619 JJ=1,IPAR                                                  MAN3350
C619   WRITE(6,999) (A3(JJ,J),J=1,IPAR)                                 MAN3360
      DO 625 J1=1,IPAR                                                  MAN3370
      DO 625 J2=1,IPAR                                                  MAN3380
625   A2(J2,J1)=A3(J2,J1)                                               MAN3390
      CALL SOLEQU (A2,B2,IPAR)                                          MAN3400
      WRITE(6,999) (B2(J),J=1,IPAR)                                     MAN3410
      SUM=0.                                                            MAN3420
      DO 620 J=1,IPAR                                                   MAN3430
620   SUM=SUM+GRP(J)*B2(J)                                              MAN3440
      SUM=SUM*(-.5)                                                     MAN3450
      X=SUM/(SB-SUM)                                                    MAN3460
      WRITE(6,2999) 99,99,X,SB,SUM                                      MAN3470
C     Here, X is left hand side of inequality (9) in text               MAN3480
2999  FORMAT(2I8,6D16.9)                                                MAN3490
C     Take a look at several of the terms just preceding omega sub k    MAN3500
C     in equation (6) in the text to see how large they are relative    MAN3510
C     to omega sub k                                                    MAN3511
      DO 693 I10=1,30                                                   MAN3520
      IWL=100*I10                                                       MAN3530
      IF(I10.EQ.27) IWL=3122                                            MAN3540
      IF(I10.EQ.28) IWL=3138                                            MAN3550
      IF(I10.EQ.29) IWL=3159                                            MAN3560
      IF(I10.EQ.30) IWL=3196                                            MAN3570
      DO 694 J=1,IPAR                                                   MAN3580
694   B2(J)=GRSS(IWL,J)                                                 MAN3590
      DO 626 J1=1,IPAR                                                  MAN3600
      DO 626 J2=1,IPAR                                                  MAN3610
626   A2(J2,J1)=A3(J2,J1)                                               MAN3620
      CALL SOLEQU(A2,B2,IPAR)                                           MAN3630
      SUM=0                                                             MAN3640
      DO 695 J=1,IPAR                                                   MAN3650
695   SUM=SUM+GRSS(IWL,J)*B2(J)                                         MAN3660
      WRITE(6,2999) I10,NND(IWL),SUM                                    MAN3670
C     SUM is one of the terms just preceding omega sub k                MAN3680
693   CONTINUE                                                          MAN3690
C     Evaluate the term in the square root of equation (7) of the text  MAN3700
      DO 697 JS=1,IPAR                                                  MAN3710
      DO 696 J=1,IPAR                                                   MAN3720
696   B2(J)=0                                                           MAN3730
      B2(JS)=1                                                          MAN3740
      DO 627 J1=1,IPAR                                                  MAN3750
      DO 627 J2=1,IPAR                                                  MAN3760
627   A2(J2,J1)=A3(J2,J1)                                               MAN3770
      CALL SOLEQU(A2,B2,IPAR)                                           MAN3780
      ANS1=DSQRT(B2(JS)*SB)*PARO(JS)                                    MAN3790
      ANS=ANS1*DSQRT(X)                                                 MAN3800
      WRITE(6,2999) JS,JS,ANS,ANS1,B2(JS),PARO(JS)                      MAN3810
C     B2(JS) is the term                                                MAN3820
697   CONTINUE                                                          MAN3830
```

```
C    ###########################End of Regression Insert Pack###
      DO 5070 J=1,IPAR
      CONB(J)=1.
      DO 5070 K=1,IPAR
      SCALE(J)=1.
 5070 CONA(J,K)=1.
 5050 CONTINUE
      SUM7=0.
      DO 1558 J=1,IPAR
      B2(J)=-.5*GRP(J)*CONB(J)
 1558 SUM7=SUM7+B2(J)*B2(J)
      RLEV=RLEVM
      FLEV=DSQRT(SUM7)/RLEV
      DO 1575 JR=1,IPAR
      DO 1550 JS=1,IPAR
      SUM5=0.
      DO 1540 IWL=1,NWLMT
 1540 SUM5=SUM5+GRSS(IWL,JR)*GRSS(IWL,JS)*W(IWL)
 1550 A2(JR,JS)=SUM5*CONA(JR,JS)
 1575 A2(JR,JR)=A2(JR,JR)+FLEV
C
C     SOLUTION OF THE SYSTEM OF LINEAR EQUATIONS
C
      CALL SOLEQU (A2,B2,IPARC)
      DO 1570 J=1,IPAR
 1570 MU(J)=B2(J)*PAR0(J)*CONB(J)
C     WRITE(6,97) (MU(I),I=1,IPAR)
C
C     COMPUTATION OF BOUNDARY SET
C
      DO 5100 J=1,IPAR
      THRR=PARAM(J)+MU(J)
      IF(THRR.GE.COU(J)) GO TO 5120
      IF(THRR.LE.COL(J)) GO TO 5140
      NBND2(J)=0
      GO TO 5100
 5120 NBND2(J)=1
      GO TO 5100
 5140 NBND2(J)=-1
 5100 CONTINUE
C
C     SELECTION OF DECISION VARIABLES
C
      IC=0
      IF(FLAGA) 5160,5160,5170
 5160 DO 5200 J=1,IPAR
      IF(NBND2(J).EQ.0) GO TO 5200
      IF(NBND2(J).NE.NBND1(J)) GO TO 5200
      CONB(J)=0.
      FLAGA=1.
      DO 5210 JJ=1,IPAR
      CONA(J,JJ)=0.
 5210 CONA(JJ,J)=0.
      CONA(J,J)=1.
      IC=IC+1
```

```
      ICON(IC)=J                                                        MAN4390
      WRITE(6,97) FLAGA,(CONB(I),I=1,IPAR)                              MAN4400
 5200 CONTINUE                                                          MAN4410
      ICMAX=IC                                                          MAN4420
      GO TO 5250                                                        MAN4430
 5170 FLAGA=0.                                                          MAN4440
 5250 IF(FLAGA) 5260,5260,5050                                          MAN4450
5260  CONTINUE                                                          MAN4460
C                                                                       MAN4470
C     UPDATE OF BOUNDARY SET                                            MAN4480
C                                                                       MAN4490
      DO 5400 J=1,IPAR                                                  MAN4500
      THRR=PARAM(J)+MU(J)                                               MAN4510
C     WRITE(6,97) MU(J),THRR                                            MAN4520
 5430 IF(THRR.GE.(COU(J))) GO TO 5450                                   MAN4530
      IF(THRR.LE.(COL(J))) GO TO 5460                                   MAN4540
      NBND2(J)=0                                                        MAN4550
      GO TO 5420                                                        MAN4560
 5450 NBND2(J)=1                                                        MAN4570
      THRR=COU(J)                                                       MAN4580
      GO TO 5420                                                        MAN4590
 5460 NBND2(J)=-1                                                       MAN4600
      THRR=COL(J)                                                       MAN4610
 5420 CONTINUE                                                          MAN4620
      PARAM(J)=THRR                                                     MAN4630
 5400 CONTINUE                                                          MAN4640
C     Calculate gradient of PZ                                          MAN4650
      DO 5500 J=1,IPAR                                                  MAN4660
      NBND1(J)=NBND2(J)                                                 MAN4670
      GRP(J)=GRP(J)/(2.*PZ*PDIV*PARO(J))                                MAN4680
 5500 CONTINUE                                                          MAN4690
      WRITE(6,99) (NBND1(I),I=1,IPAR)                                   MAN4700
      GO TO 1017                                                        MAN4710
      END                                                               MAN4720
```

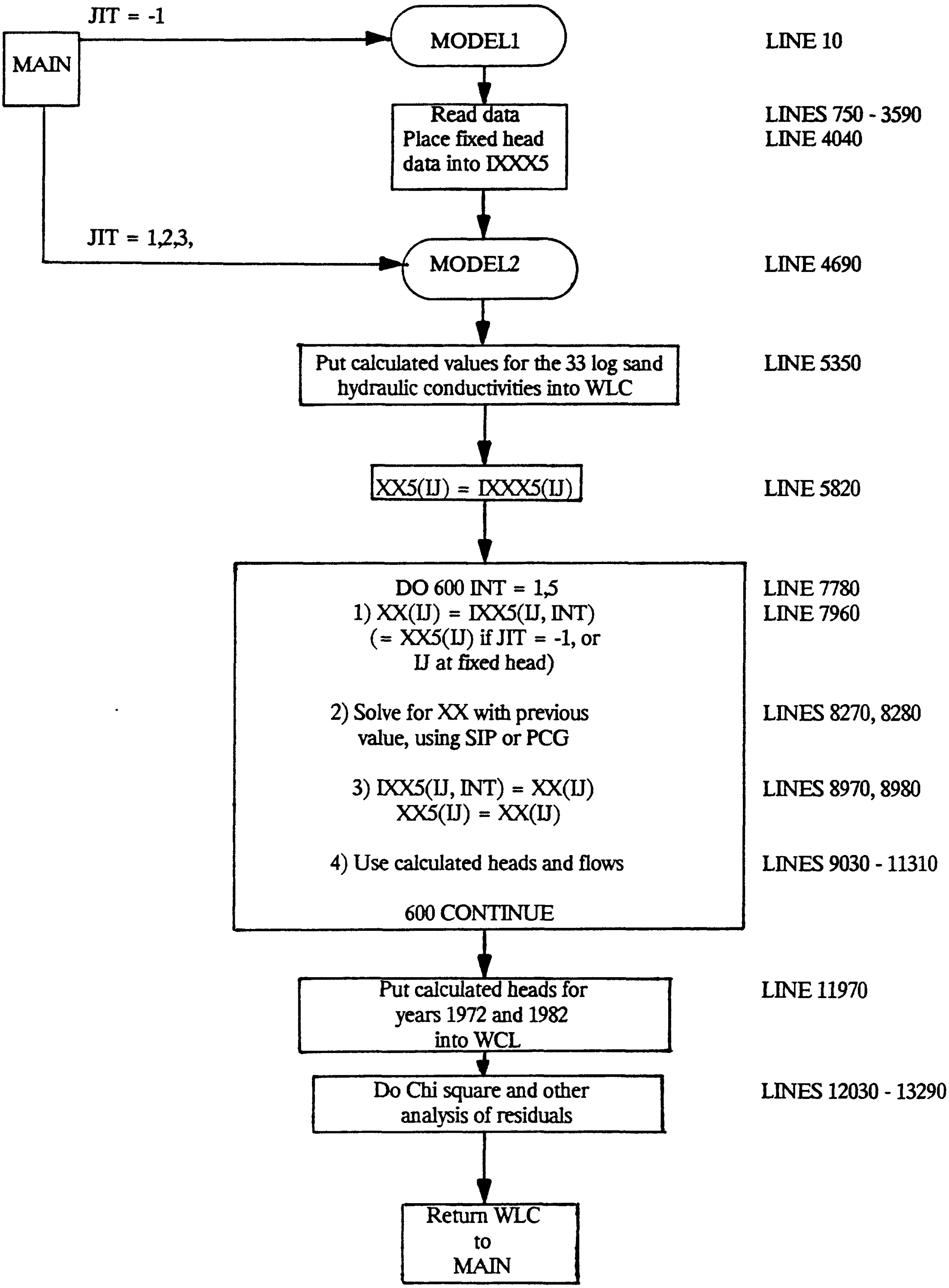
MODEL

The basic function of MODEL is to evaluate f(B) given a value of B from MAIN. MAIN passes a value of JIT = -1 when going to MODEL1, and values of JIT = 1,2,3,..., when going to MODEL2. The approximations $f_i(B)$ to the observations Y_i include heads for the years 1972 and 1982 as well as 33 $f_i(B)$ corresponding to the 33 log sand hydraulic conductivity observations. Each time MODEL is called it finds heads for five different years corresponding to INT = 1,2,3,4,5. INT = 3 corresponds to 1972 and INT = 5 corresponds to 1982. Convergence of SIP or PCG is faster when good starting values for head (= XX) are used. Thus the values of XX(IJ) found for a given INT and JIT are saved in IXX5(IJ, INT) to be used as starting values for XX(IJ) at JIT = JIT + 1. XX5 is used as the value of XX at INT (t) when finding XX at INT + 1 ($t+\Delta t$), corresponding to the next time step.

IOUT = 1 is passed to MODEL by MAIN when various printouts are desired. When IOUT = 0 this printout does not occur

The most important changes of VARDEN necessary to produce MODEL are: (1) The emplacement of the global parameters PARAMX into the calculation of the quantities DD, BB, and ZZ, the elements of the matrix the inversion of which gives f(B); (2) implementation of the functions giving the rate of decrease with depth of sand and clay hydraulic conductivity combined with the use of equations (10) and (11) in the text and giving effective hydraulic conductivities. The first of these changes appears at lines MD06040 through MD06130 of the listing of MODEL. The second occurs at lines MD06140 through MD06440.

Essential Features of Model



Definition of Program Variables

ALFA — Parameter b_5 in text.

BBK — See FCNT.

DDK — See FCNT.

DD,BB,ZZ — Elements of the matrix, the inversion of which gives f(B), the solution to the model with parameters B.

DEL — Array combining the five time intervals.

DELT — Time interval.

DXI,DYJ — DXI(I) is the I direction horizontal dimension of the grid elements in column I. DYJ(J) is the J direction horizontal dimension of the grid elements in row J.

ERR, XX10 — Iteration terminates when

$$\max_{\text{(over IJ)}} \left| XX^{ICNT} - XX^{(ICNT-1)} \right| +$$

$$\max_{\text{(over IJ)}} \left| XX^{(ICNT-1)} - XX^{(ICNT-2)} \right| < ERR,$$

the maximum residual error

$$\max_{\text{(over IJ)}} \left| [M]\, XX^{ICNT} - YQ \right| < XX10, \text{ or } ICNT \geq ITMAX.$$

FC — Factors used to approximate pumping rates for first time interval.

FCNT,NT, DDK,BBK — K_{xx}(IJ) = (NT(IJ) X (DDK(K)),
K_{yy}(IJ) = (NT(IJ) X (BBK(K)),
where K_{xx}(IJ), K_{yy}(IJ) denote the x, y components of K, the hydraulic conductivity for grid element IJ. NT is used as a dummy variable for various data sets.

GP5 — Multiplier for K_z/b for geopressure zone.

HL	HL(IJ) is used for subsidence. It is the lowest value that XX(IJ) = head has achieved yet for any of the previous or the current time step.
HLS	HLS is subsidence at ground surface.
HMAX	SIP parameter β' (Trescott and Larson, 1977).
I	Positional locator along a row, also column number.
ICT	In SIP, a repeating counter for iteration parameter number.
IGP	IGP(J) traces I along boundary of geopressured zone.
IJ	Single subscript replacement for I,J,K corresponding to grid element I,J,K.
IJKM1	Replacement for I,J,K-1, or for I,J,K+1 when the K direction is reversed in SIP.
IJKP1	Replacement for I,J,K+1, or for I,J,K-1 when the K direction is reversed in SIP.
IJLB	IJ for confining bed grid elements.
IJM1K	Replacement for I,J-1,K, or for I,J+1,K when the J direction is reversed in SIP.
IJP1K	Replacement for I,J+1,K, or for I,J-1,K when the J direction is reversed in SIP.
IM1JK	Single subscript replacement for I-1,J,K.
IP1JK	Single subscript replacement for I+1,J,K.
IOUT	If IOUT = 1, MODEL prints various quantities. If IOUT = 0, it does not.
IP1JK	Single subscript replacement for I+1,J,K.
IPH(K), K=2 NK11	Set IPH(K) = 1 if you want: hydraulic head or freshwater head, flow rate data, or drawdown, for layer K. Set IPH(K) = 0 if you do not want these quantities for layer K. The layers are numbered K = 2 through K = NK11 = NK10+1.

IPR,JPR	At I = IPR(I10), J = JPR(I10) all of the layers present in region I10 (fig. 31) occur, I10 = 1,2,...,10.
ISOR	ISOR = 0.
ITMAX	Maximum number of iterations allowed.
IWRT NU1(I), I=1,3 NU2(I) I=1,3), NU3(I), I=1,3	Set IWRT to 1 if you want to watch the convergence of freshwater head h' = XX, at the three locations: I = NU1(1), J = NU1(2), K = NU1(3) I = NU2(1), J = NU2(2), K = NU2(3) and I = NU3(1), J = NU3(2), K = NU3(3).
IXX5	Storage value for starting values for the solution finding XX, head.
IXXX5	Storage array for fixed (and other) head values.
J	Positional locator along a column, also row number.
JIT	Iteration number. Called s in previous text in this attachment.
K	Layer number.
KOUT	The total number of confining beds between layer 2 and layer K. Only one effective confining bed is allowed between any two layers. If there are more than two confining beds, they are combined into one effective confining bed.
L9	In SIP, L9 is: 1 for J direction reversal, 2 for J and K direction reversal, and 3 for K direction reversal.
LB	LB(IJ) is the vertical dimension of grid element IJ.
LBB	Elevation of the base of the lowermost layer, K = 2.

LENGTH	Number of iteration parameters in SIP.
LFLOW LFLOW LFLO(1) LFLO(2) LFLO(3) LFLO(4)	Four types of flow-rate data are available (1) the flow rate out of each grid element in the negative I, J, and K directions, (2) the flow rate out of fixed head (MHD = 0) grid elements, (3) head dependent discharge flow rates from grid elements having such discharge, and (4) total flow rate budgets for sets of grid elements having the same I, the same J, and the same K. All flow rates are in units of mass divided by ρ_0 = 1 gm/cm^3 per unit time (L^3/T) (see the section on "Units" in the text). Set LFLOW = 1 if you want any flow-rate data, set LFLOW = 0 if no flow rate data is desired. Set LFLO(i) = 0 if you do not want type i flow rate data. For i = 1, set LFLO(1) = 1, 2, 3, for type 1 flow rate data in the negative I,J, and K directions respectively. If LFLO(1) = 4 all directions are given. Set LFLO(2) = 1 if you want type 2 flow rate data. Set LFLO(3) = 1 if you want type 3 flow rate data. Set LFLO(4) = 1, 2, 3, for the budgets of sets of grid elements having the same I, J, and K respectively. If LFLO(4) = 4 budgets are given for I, J, and K. Corresponding to the integers 1, 2, and 3 in the first column of output the second column gives I, J, and K respectively.
LRO	LRO(IJ) = (ρ'/ρ_0). $\rho = \rho' + \rho_0$ is the water density for grid element IJ. LRO(IJLB) = (ρ'/ρ_0), where $\rho = \rho' + \rho_0$ is the water density for confining bed grid element IJLB. Values given are truncated after the fifth decimal place.
LZ2	LZ2(IJ) = M, where the integer M divided by 10 is the elevation of node point IJ located at the center of grid element IJ.
MAQ1	MAQ1(K) = 0 except when a confining bed lies between layers K-1 and K, in which case MAQ1(K) = 1.
MHD	Holds read-in values for fixed heads initially. After these are read into IXXX5, MHD(IJ) is = 0,1,2, corresponding to: a regular active grid element, fixed element, and an element outside of the domain of solution, respectively.
NI10	Number of columns in the modeled area.

NINT	Number of time intervals, NINT = 5.
NJ10	Number of rows in the modeled area.
NK10	Number of layers in the modeled area.
NK4	Number of intervening confining beds.
NND	NND(I) is equal to the IJ location of observation I.
NT	See FCNT.
NU1	See IWRT.
NU2	See IWRT.
NU3	See IWRT.
NWLM	Number of observations of grid-element volume-averaged head for years 1972 and 1982. Equal to 1,432 + 1,675 = 3,107.
NWLM1	Number of observations of grid element volume averaged head for year 1972 only. Equal to 1,432.
P1	Parameter b_4 in text.
P2	Parameter b_9 in text.
P3	Parameter b_7 in text.
P7	Parameter b_8 in text.
P8	Parameter b_3 in text.
P9	Parameter b_7 in text.
SV	SV specifies specific storage S_s.
SV35	P2*SV35 is specific storage for all grid elements (except as modified by subsidence which multiplies the value by VV40 for the clay component).
VV40	VV40 = P7*40. VV40*SV35*P2 is specific storage for clay component in subsidence condition.

W	W(I) is the weight of observation Y_i in the expression of S(B).
WLC	WLC(IJ) is the calculated value $f_i(B)$ produced by the model.
WLM	WLM(I) is the measured value Y_i of observation number i.
WMAX	In SIP, (XYFC)*(XY) is used for $(1-\alpha_{max})$ (Weinstein and others, 1969) when WMAX = 0. When WMAX = 0, $(1-\alpha_{max})$ = WMAX is used.
XTV	Chi square test data.
XX	Freshwater head = (pressure head h) + z.
XX10	Error step for the residual b - Ax in the solvers PCG and SIP.
XXS	The value of XX for the previous time step.
XXSTR	XXSTR(IJ) is percent sand in grid element IJ.
YQ	Contains pumping rates and also psuedo sources prior to each solution by PCG or SIP.
YQ1	YQ1(IJ,I3) I3 = 1,2,3 has pumping rates for the years 1970, 1975, and 1980, respectively.

```
      SUBROUTINE MODEL1                                                 MD00010
       IMPLICIT REAL*8 (A-H,O-Z)                                        MD00020
       COMMON WS,HMAX,RELX1,RELX2,COEF,ERR,XX10,DELT,ER5,SRZ,SUMRZ      MD00030
     1,ERRSV,XX10SV,JIT,NIJ10,NI11,NJ11,NK11,NNN,NSKP1                  MD00040
     2,NSKP2,ITMAX,ICNT,IEVP,IWR1,NW1,NW2,NW3,N320,NUM4                 MD00050
     3,L9,LENGTH,NK1115,IT01,IT15,ICRO,DDK,BBK                          MD00060
     4,NI10,NJ10,NK10,NI12,NJ12,NK12,SVIJ,VV40,SV35                     MD00070
       COMMON /XX/ XX                                                   MD00080
       COMMON /DT/ DT                                                   MD00090
       COMMON /VV/ VV                                                   MD00100
       COMMON /E2/ E2                                                   MD00110
       COMMON /F2/ F2                                                   MD00120
       COMMON /G2/ G2                                                   MD00130
       COMMON /YQ/ YQ                                                   MD00140
       COMMON /NT/ NT                                                   MD00150
       COMMON /DD/ DD                                                   MD00160
       COMMON /BB/ BB                                                   MD00170
       COMMON /ZZ/ ZZ                                                   MD00180
       COMMON /XXS/ XXS                                                 MD00190
C      COMMON /ALN/ ALN                                                 MD00200
C      COMMON /XXE/ XXE                                                 MD00210
C      COMMON /SV/ SV                                                   MD00220
       COMMON /HL/ HL                                                   MD00230
       COMMON /LB/ LB                                                   MD00240
       COMMON /MHD/ MHD                                                 MD00250
       COMMON /C/ DXI,DYJ,N325,ISOR,IPDD                                MD00260
     1,NPINT,DT0,TOT,TFAC,NINT,IPRNT,IWRT,LFLOW,NK4,LLRO,IWRTXX,        MD00270
     2ILZ2,MAQ1,NU1,NU2,NU3,LFLO,IPH                                    MD00280
     3,LBB,SUMF,SUNF,SQ2,SG2,SYQ,SVV,SUMFM,SUNFM                        MD00290
       COMMON /B/ COL,COU,PARAM,PAR0,MU,W,EPS,WLM,LWLC,WLC,A2,B2,GRP    MD00300
     1,CONA,CONB,SCALE                                                  MD00310
     2,NBND1,NBND2,ICON,NND,NWLM,NWLM1,NWLMT,IOUT                       MD00320
       COMMON /GRSS/ GRSS                                               MD00330
       COMMON /XXSTR/ XXSTR                                             MD00340
       COMMON /XKZZ/ XKZZ                                               MD00350
       COMMON /LRO/ LRO                                                 MD00360
       COMMON /YQ1/ YQ1                                                 MD00370
       COMMON /LZ2/ LZ2 ·                                               MD00380
       COMMON /PARAMX/ PARAMX                                           MD00390
       COMMON /PARAM0/ PARAM0                                           MD00400
       COMMON /IPARAM/ IPARAM                                           MD00410
       COMMON /YQ2/ YQ2                                                 MD00420
C    *****EXTRAS****                                                    MD00430
       COMMON /FC/ FC,DEL,ISO,OS4,GP5,LFLOA,IPHA                        MD00440
       COMMON /IXX5/ IXX5                                               MD00450
       COMMON /IXXXS/ IXXXS                                             MD00460
       COMMON /HLS/ HLS                                                 MD00470
       COMMON /IPR/ IPR,JPR,X3150,X493,IIII,XTV                         MD00480
       DIMENSION LA(40),IA(40),XL1(40),XTV(20),IX7(20)                  MD00490
       INTEGER*2 IXXXS(65078),IXX5(65078,5),LFLOA(4),IPHA(50)           MD00500
       INTEGER*2 IGP(120),IPR(10),JPR(10)                               MD00510
       REAL*4 DPT(4),XSS(4),XSH(4),HLS(5918)                            MD00520
C ********                                                              MD00530
       REAL*4 YQ2(65078)                                                MD00540
       REAL*4COL(50),COU(50),PARAM(50),PAR0(50),MU(50),W(3303),EPS(3303),MD00550
```

```
     1   WLM(3303),LWLC(3303),WLC(3303),A2(50,50),B2(50),GRP(50),
     2   CONA(50,50),CONB(50),SCALE(50),GRSS(3303,5),
     3   PARAMX(159872),PARAM0(159872)
      INTEGER*4 NBND1(50),NBND2(50),ICON(50),
     1   NND(3303)
      INTEGER*2 IPARAM(159872)
      REAL*4 DD(65078),BB(65078),ZZ(65078),YQ(65078),
     1XXS(65078),Q2(300),DDK(50),BBK(50),NT(94658),HL(65078)
     2,DXI(250),DYJ(250),XXSTR(65078),YQ1(65078,3)
     3,XKZZ(65078)
      INTEGER*2 LRO(94658),IPH(50),LFLO(4),I7(5),J7(5),K7(5)
     1,NI(300),NJ(300),NK(300),MAQ1(50),NU1(3),NU2(3),NU3(3)
      DIMENSION DT(65078),E2(65078),F2(65078),G2(65078),VV(65078)
      DIMENSION XX(65078)
     1,WS(10),MHD(65078),LZ2(65078),LB(94658),LBB(5918)
     2,AOC(4),BOC(4),COC(4),DOC(4)
     3,SUMF(3,250),SUNF(3,250),SQ2(3,250),SG2(3,250),SYQ(3,250)
     4,SVV(3,250),SUMFM(3,250),SUNFM(3,250),FC(5,15),DEL(5)
     *,IC7(65)
      PI=3.1415926
      DATA DEL/1.D40,10950.D0,1825.D0,1825.D0,1825.D0/
      DATA IPR/12,16,18,23,28,27,20,28,35,30/
      DATA JPR/90,56,41,36,29,94,66,44,38,60/
      DATA XTV/-1.D20,-1.53413,-1.15035,-.88715,-.67449,-.48877,
     *-.31864,-.15731,.0,.0,.0,.0,.0,.0,.0,.0,.0,.0,.0,.0/
      DO 719 I=10,17
719   XTV(I)=-XTV(18-I)
      WRITE(6,3555) (XTV(I),I=1,17)
      WRITE(6,3555) (DEL(I),I=1,5)
      DO 10  I=1,50
10    MAQ1(I)=0
      READ(5,8877) ((FC(I,J),J=1,10),I=1,2)
      FC(1,11)=1
      FC(2,11)=1
8877  FORMAT(10F8.0)
      WRITE(6,8877) ((FC(I,J),J=1,10),I=1,2)
      READ(5,2003) (IGP(I),I=2,103)
      WRITE(6,2003) (IGP(I),I=2,103)
      OS4=1.E-25
      GP5=5.E-30
      VV40=40
      WRITE(6,9022)
      WRITE(6,3555) VV40,OS4,GP5
      L9=2
      LENGTH=5
      XYFC=1
      WMAX=0
      READ(5,2000) ISOR
      IF(ISOR.GE.0) READ(5,2000) ITMAX,ERR,XX10,HMAX
      ISOR=0
      ITMAX=70
      ERR=.001
      XX10=8
C     ERR=.0002
C     XX10=1
```

```
      HMAX=1.0
      READ(5,2020) NI10,NJ10,NK10,NK4
      READ(5,2003) IWRT,(NU1(I),I=1,3),(NU2(I),I=1,3),(NU3(I),I=1,3)
      IWRT=0
      READ(5,2003) LFLOW,(LFLO(I),I=1,4),(LFLOA(I),I=1,4)
      NK11=NK10+1
      READ(5,2003) (IPH(K),K=2,NK11)
      READ(5,2003) (IPHA(K),K=2,NK11)
      NUM4=ISOR-1
      NUM4=3
      NI11=NI10+1
      NJ11=NJ10+1
      NK15=NK11+NK4
      NI12=NI10+2
      NJ12=NJ10+2
      NK12=NK10+2
      NIJ10=NI10*NJ10
      NIJK10=NIJ10*NK10
      NNN=NIJK10+2
      N315=NIJ10+1
      N320=NIJK10+1
      N325=NIJ10*(NK10+NK4)+1
      NW1=NU1(1)+NI10*(NU1(2)-2)+NIJ10*(NU1(3)-2)
      NW2=NU2(1)+NI10*(NU2(2)-2)+NIJ10*(NU2(3)-2)
      NW3=NU3(1)+NI10*(NU3(2)-2)+NIJ10*(NU3(3)-2)
C  READ IN AND PRINT ARRAYS MHD,LRO,LBB,LB,XX,SV,XXSTR, XKZZ, FCNT,
C                             DDK,BBK,NT,MAQ1,DXI,DYJ,YQ1
C  GROUP II
      WRITE(6,5000)
      NK1115=NK11
      IT15=1
      ICRO=0
      CALL RDWRT
      DO 20  IJ=2,N320
20    MHD(IJ)=NT(IJ)*(10)+.1
      WRITE(6,5002)
      NK1115=NK15
      CALL RDWRT
      DO 90  IJ=2,N325
90    LRO(IJ)=NT(IJ)*1.D+5+.1
      WRITE(6,5003)
      NK1115=2
      CALL RDWRT
      DO 100 IJ=2,N315
100   LBB(IJ)=NT(IJ)
      WRITE(6,5004)
      NK1115=NK15
      IT15=10
      CALL RDWRT
      DO 110 IJ=2,N325
110   LB(IJ)=NT(IJ)
      LB(1)=1
      IT15=1
C  ****** Insert LB=1 for geopressured zone *****
      DO 219 J=2,NJ11
```

```
      IGPJ=IGP(J)                                                       MD01660
      DO 217 I=2,NI11                                                   MD01670
      IF=I+NI10*(J-2)                                                   MD01680
C  %%%%%%% If LB<10 at any location, set to zero and adjust LBB         MD01690
C     accordingly                                                       MD01691
      LL=0                                                              MD01700
      DO 216 K=2,NK10                                                   MD01710
      IJ=IF+(K-2)*NIJ10                                                 MD01720
      LBIJ=LB(IJ)                                                       MD01730
      IF(LBIJ.GE.10) GO TO 215                                          MD01740
      LL=LL+LBIJ                                                        MD01750
      LB(IJ)=0                                                          MD01760
215   CONTINUE                                                          MD01770
216   CONTINUE                                                          MD01780
      LBB(IF)=LBB(IF)+LL                                                MD01790
C  %%%%%%%% LB<10 to zero done                                          MD01800
      LB1=LB(IF)                                                        MD01810
      I2236=22*I+36*J-2349                                              MD01820
      IF(I2236.LE.0) GO TO 218                                          MD01830
      IF(LB(IF+NIJ10*9).EQ.0) GO TO 218                                 MD01840
      IF(I.LT.IGPJ) GO TO 218                                           MD01850
C     LB1=1                                                             MD01860
218   LB(IF)=LB1                                                        MD01870
C218   CONTINUE                                                         MD01880
217   CONTINUE                                                          MD01890
C     WRITE(6,3004) J,(LB(I+NI10*(J-2)),I=2,NI11)                       MD01900
219   CONTINUE                                                          MD01910
C  ******** LB=1 for geopressured zone done *****                       MD01920
C     Read in observations Yi(=WLM) of grid element volume averaged     MD01930
C     head for the years 1970 and 1980. Number them consecutively       MD01931
C     using I9 as shown. NWLM is the total number of these. NND is      MD01940
C     the IT location of each observation                               MD01950
C    ************                                                       MD01960
      NK11 15=NK11                                                      MD01970
      IT15=10                                                           MD01980
      CALL RDWRT                                                        MD01990
      DO 246 I=1,3303                                                   MD02000
      WLM(I)=0                                                          MD02010
      W(I)=0                                                            MD02020
      WLC(I)=0                                                          MD02030
246   NND(I)=0                                                          MD02040
      I9=0                                                              MD02050
      DO 247 IJ=2,N320                                                  MD02060
      XNTIJ=NT(IJ)                                                      MD02070
      IF((XNTIJ*LB(IJ)).EQ.0.) GO TO 247                                MD02080
      I9=I9+1                                                           MD02090
      NND(I9)=IJ                                                        MD02100
      WLM(I9)=XNTIJ                                                     MD02110
247   CONTINUE                                                          MD02120
      NWLM1=I9                                                          MD02130
      CALL RDWRT                                                        MD02140
      IT15=1                                                            MD02150
      DO 274 IJ=2,N320                                                  MD02160
      XNTIJ=NT(IJ)                                                      MD02170
      IF((XNTIJ*LB(IJ)).EQ.0.) GO TO 274                                MD02180
```

```
      I9=I9+1
      NND(I9)=IJ
      WLM(I9)=XNTIJ
274   CONTINUE
      NWLM=I9
      NWLMT=NWLM+100
C  ******* Done getting grid element value averaged head
C     observations *******
C     Set weights for gerah observations
      CALL RDWRT
      DO 248 I=1,NWLM
      IJ=NND(I)
248   W(I)=(39.093*39.093)/NT(IJ)**2
      WRITE(6,952)
      DO 249 I=1,NWLM
      IF (W(I).LE.0.0) W(I)=1.0
 249  CONTINUE
C     WRITE(6,953) (NND(I),WLM(I),W(I),I=1,20)
      WRITE(6,2020) NWLM
C
 953  FORMAT((1H ,10X,I4,3X,I4,F12.1,F10.4))
C  ***** Set weights done ****
C     Determine observations and corresponding weights for the 33
C     zone averaged hydraulic conductivities
C     GO TO 876
      N33=33
      SUM=0
      LAA=2
      Y1=0
      Y2=0
      NL=0
      DO 125 I=1,N33
      READ(5,2777) LA(I),IA(I),M1,V1,V2,M2,V3,V4,M3,XL1(I)
      WRITE(6,2777) LA(I),IA(I),66,XL1(I)
      IF((LA(I).EQ.LAA).AND.(I.NE.33)) GO TO 1266
      SUM=SUM+(Y2-Y1*Y1/NL)/NL
      NL=0
      Y1=0
      Y2=0
      LAA=LAA+1
1266  CONTINUE
      X=XL1(I)
      Y2=Y2+X*X
      Y1=Y1+X
      NL=NL+1
125   CONTINUE
      AV65=SUM/10
      X493=39.093/DSQRT(AV65)
      DO 126 I=1,N33
      II=NWLM+(LA(I)-2)*10+IA(I)
      W(II)=1
      WLM(II)=X493*(XL1(I))
      WRITE(6,3 001) I,II,XL1(I),W(II),WLM(II),AV65,X493
3001  FORMAT(2I7,7D12.3)
126   CONTINUE
```

```
2777  FORMAT(2I4,3(I4,2F8.3))
C876   CONTINUE
C     ***** Done with 33 observations and weights *****
      WRITE(6,5052)
      NK1115=NK11
      CALL RDWRT
      DO 150  IJ=2,N320
150   XX(IJ)=NT(IJ)*10
      XX(1)=0
      XX(NNN)=0
      WRITE(6,5054)
      CALL RDWRT
C     DO 160  IJ=2,N320
C160   SV(IJ)=NT(IJ)*1
      SV35=.3D-5
      WRITE(6,3555) SV35
      WRITE(6,9954)
      CALL RDWRT
      DO 1601  IJ=2,N320
1601  XXSTR(IJ)=NT(IJ)
      WRITE(6,5060)
      CALL RDWRT
      DO 166 IJ=2,N320
166   XKZZ(IJ)=NT(IJ)
C     Read pumping rate for 3 time periods 1970, 1975, and 1980
C     into YQ1
      READ(5,2000) NINT
      WRITE(6,5053)
      NK1115=NK11
      DO 1642 13=1,3
      CALL RDWRT
      DO 1681 IJ=2,N320
1681  YQ1(IJ,I3)=1*NT(IJ)
1642  CONTINUE
      WRITE(6,5005)
      NK1115=NK15
      IT15=2
      CALL RDWRT
C  ***** REMOVE PINCHED OUT NODES WHEN APPROPRIATE *****
      DO 163 I=2,NI11
      DO 163 J=2,NJ11
      IJIJ=I+NI10*(J-2)
      DO 163 K=2,NK11
      IJ=IJIJ+NIJ10*(K-2)
      IF(LB(IJ).NE.0) GO TO 165
      IF((K.EQ.2).OR.(K.EQ.NK11)) GO TO 164
      IT5=0
      KM1=K-1
      DO 161 K1=2,KM1
      IJ1=IJIJ+NIJ10*(K1-2)
161   IF(LB(IJ1).NE.0) IT5=IT5+1
      IT6=0
      KP1=K+1
      DO 162 K2=KP1,NK11
      IJ2=IJIJ+NIJ10*(K2-2)
```

```
162   IF(LB(IJ2).NE.0) IT6=IT6+1
      IF((IT5*IT6).GT.0) GO TO 165
164   NT(IJ)=0
165   CONTINUE
      IF(NT(IJ).EQ.0) XKZZ(IJ)=0
      IF((IKZZ.NE.0).AND.(XKZZ(IJ).EQ.0)) NT(IJ)=0
163   CONTINUE
C  ****** DONE Pinched out nodes DONE ******
C  GROUP III
      IF(NK4.EQ.0) GO TO 170
      READ(5,2003) (MAQ1(K),K=2,NK11)
      WRITE(6,5010)
      WRITE(6,4002) (K,MAQ1(K),K=2,NK11)
170   CONTINUE
      READ(5,2000) FDXI,FDYJ,MDXI1,MDYJ1,MD2
      IF(MDXI1.EQ.1) READ(5,2000) (DXI(I),I=2,NI11)
      IF(MDYJ1.EQ.1) READ(5,2000) (DYJ(J),J=2,NJ11)
      IF(MDXI1.EQ.1) GO TO 190
      DO 180 I=2,NI11
180   DXI(I)=FDXI
190   IF(MDYJ1.EQ.1) GO TO 210
      DO 200 J=2,NJ11
200   DYJ(J)=FDYJ
210   IF(MD2.NE.0) GO TO 220
      WRITE(6,5001)
      WRITE(6,3010) (DXI(I),I=2,NI11)
      WRITE(6,3010) (DYJ(I),I=2,NJ11)
220   CONTINUE
      DXI(1)=1
      DYJ(1)=1
      DXI(NI12)=1
      DYJ(NJ12)=1
C  READ IN AND PRINTING OF ARRAYS NOW COMPLETE
C  INITIALIZE VARIOUS ARRAYS TO ZERO
      DO 230  IJ=1,NNN
      HL(IJ)=0
      DD(IJ)=0
      BB(IJ)=0
      ZZ(IJ)=0
      DT(IJ)=0
      E2(IJ)=0
      F2(IJ)=0
      G2(IJ)=0
      YQ2(IJ)=0
230   VV(IJ)=0
C  DETERMINE LZ2
      KOUT=0
      DO 280 K=2,NK11
      KOUT=KOUT+MAQ1(K)
      DO 280 J=2,NJ11
      DO 280 I=2,NI11
      I2236=22*I+36*J-2349
      DXY=DXI(I)*DYJ(J)
      IJF=I+NI10*(J-2)
      IJ=IJF+NIJ10*(K-2)
```

```
      LBIJ=LB(IJ)
      KLB=NK11+KOUT
      IJLB=IJF+NIJ10*(KLB-2)
      DTIJ=LRO(IJ)*1.0D-5
C     SV(IJ)=SV(IJ)*DXY*LBIJ
C  DETERMINE LZ2
      IJKM1=IJ-NIJ10
      IF(K.EQ.2) GO TO 240
      LZ2(IJ)=LZ2(IJKM1)+5*(LBIJ+LB(IJKM1))+10*MAQ1(K)*LB(IJLB)
      GO TO 250
240   LZ2(IJ)=LBIJ*5+10*(LBB(IJ)+MAQ1(K)*LB(IJLB))
250   CONTINUE
C     Place layer 12 head from MHD and also head of zero for
C     geopressured zone into IXXX5 for storage.
C     Set MHD=0,1,2 for active, fixed, and out of domain
C     elements respectively
      XXMHD=XX(IJ)
      MHDIJ=MHD(IJ)
      IF(K.EQ.NK11) XXMHD=MHDIJ
      LZ2IJ=LZ2(IJ)
      X=((XXMHD-LZ2IJ)*(DTIJ+1)+LZ2IJ)*.1D0
      IF((K.EQ.2).AND.(I2236.GT.0)) X=3000
      IF(NT(IJ).EQ.0.0) X=0.0
      IXXXS(IJ)=X+.5
      IX=0
      IF((K.EQ.2).AND.(I2236.GT.0)) IX=1
      IF(K.EQ.NK11) IX=1
      IF(NT(IJ).EQ.0.0) IX=2
      MHD(IJ)=IX
C  PUT PUMPING RATES to zero if out of domain of solution
      DO 1643 I3=1,3
      X=YQ1(IJ,I3)
      IF(IX.EQ.2) X=0
      YQ1(IJ,I3)=X
1643  CONTINUE
280   CONTINUE
C  ****** DO removal (W(I)=0) OF WLM'S that lie outside of domain of
C     solution *****
      ISM9=0
      DO 643 I=1,NWLM
      IJ=NND(I)
      IF(.NOT.(MHD(IJ).GE.1)) GO TO 644
      ISM9=ISM9+1
      W(I)=0
      WRITE(6,1003) ISM9,IJ,MHD(IJ),777.,WLM(I)
644   CONTINUE
643   CONTINUE
      DO 895 I10=1,10
      I2464=IPR(I10)+(JPR(I10)-2)*NI10
      DO 993 I=1,10
      IJ=I2464+(I-1)*NIJ10
      II3=I+1
      I8=NWLM+(II3-2)*10+I10
      IF(MHD(IJ).GE.1) W(I8)=0
      IF(MHD(IJ).GE.1) WRITE(6,2020) I8,I10,I,IJ,MHD(IJ)
```

```
993   CONTINUE                                                    MD04360
895   CONTINUE                                                    MD04370
C     STOP                                                        MD04380
C  ***** DONE *******                                             MD04390
      LZ2(1)=0                                                    MD04400
      SVIJ=SV35*DXY                                               MD04410
C     Put elevation of top layer 12 into LBB for later use, print MD04420
      WRITE(6,8002)                                               MD04430
      WRITE(6,6000)                                               MD04440
      WRITE(6,4002) NK11                                          MD04450
      DO 300 J=2,NJ11                                             MD04460
      DO 290 I=2,NI11                                             MD04470
      IJF=I+NI10*(J-2)                                            MD04480
      IJ=IJF+NIJ10*(NK11-2)                                       MD04490
      X=(LZ2(IJ)*.1D0+LB(IJ)*.5D0+.1)                             MD04500
      LBB(IJF)=X                                                  MD04510
290   CONTINUE                                                    MD04520
300   CONTINUE                                                    MD04530
      DO 301 J=2,NJ11                                             MD04540
      I5=NI10*(J-2)                                               MD04550
C     WRITE(6,3004) J,(LBB(I5+I),I=2,NI11)                        MD04560
301   CONTINUE                                                    MD04570
C     STOP                                                        MD04580
C     Set up several variables which are to be used in subsequent MD04590
C     calls to model2                                             MD04600
      N98=N320+N325-2                                             MD04610
      N99=N98+NK10                                                MD04620
      ERRSV=ERR                                                   MD04630
      XX10SV=XX10                                                 MD04640
      X3150=NWLM+N33                                              MD04650
      IIII=-1                                                     MD04660
C                                                                 MD04670
C                                                                 MD04680
      ENTRY MODEL2                                                MD04690
C                                                                 MD04700
C                                                                 MD04710
      I92=60                                                      MD04720
      J92=74                                                      MD04730
      R42=37*37                                                   MD04740
      N324=N325-1                                                 MD04750
C     Set error criteria for solution dependent upon iteration    MD04760
C     count JIT                                                   MD04761
      XL5=1.0*JIT                                                 MD04770
      IF(XL5.GT.9.) XL5=9.                                        MD04780
      ERR=(19.-2.*XL5)*ERRSV                                      MD04790
      XX10=(19.-2.*XL5)*XX10SV                                    MD04800
C     Error criteria ERR and XX10 now set                         MD04810
      N111=N99+NK11                                               MD04820
      N121=N111+7                                                 MD04830
C  ^^                                                             MD04840
      IF(IOUT.EQ.0) GO TO 780                                     MD04850
      WRITE(6,9030)                                               MD04860
      DO 792 I=1,27                                               MD04870
      IJ=I*NIJ10-100                                              MD04880
792   WRITE(6,4502) PARAMX(IJ)                                    MD04890
```

```
      WRITE(6,3555) (PARAMX(N111+I),I=1,8)                            MD04900
780   CONTINUE                                                        MD04910
C  ^^ For IJ=2,N121(N121=IPARX-3,N111=IPARX-10)                       MD04920
C     Change PARAMX(IJ) to EXP[(PARAMX(IJ)-1)*LS]                     MD04930
C     LS=5 for sand and clay hydraulic conductivities                 MD04940
      DO 312 IJ=2,N121                                                MD04950
      L5=5                                                            MD04960
      IF(IJ.GE.(N111+2)) L5=2                                         MD04970
312   PARAMX(IJ)=10**((PARAMX(IJ)-1)*L5)                              MD04980
C     Change done                                                     MD04990
C     Assign those PARAMX(IJ) for IJ>N111 to specific parameters      MD05000
      P1=PARAMX(N111+1)                                               MD05010
      P2=PARAMX(N111+3)                                               MD05020
      P3=PARAMX(N111+4)                                               MD05030
      ALFA=PARAMX(N111+5)                                             MD05040
      P7=PARAMX(N111+6)                                               MD05050
      P8=PARAMX(N111+7)                                               MD05060
      P9=PARAMX(N111+8)                                               MD05070
      VV40=P7*40                                                      MD05080
C     Write out PARAMX,XKZZ,NT, and resulting effective               MD05090
C     conductivities and their logarithms for depths of 600 and       MD05100
C     6000 ft for each of the 10 regions of figure 31 in the text.    MD05110
C     Determine calculated values WLC for observations corresponding  MD05120
C     to the 33 log sand hydraulic conductivities at line MD05350     MD05130
C   ^^^                                                               MD05140
      DO 795 I10=1,10                                                 MD05150
      I2464=IPR(I10)+(JPR(I10)-2)*NI10                                MD05160
      IF(IOUT.EQ.1) WRITE(6,9031) I10                                 MD05170
      DO 793 I=1,27                                                   MD05180
      IJ=I2464+(I-1)*NIJ10                                            MD05190
      PP=PARAMX(IJ)                                                   MD05200
      IJ32=IJ-N324                                                    MD05210
      IF(I.GE.17) W8=XKZZ(IJ32)*(LB(IJ32)/(LB(IJ32)+1.D-6))           MD05220
      IF(I.LT.17) W8=NT(IJ)    *(LB(IJ)  /(LB(IJ)  +1.D-6))           MD05230
      X=PP*W8                                                         MD05240
      II=I+1                                                          MD05250
      II3=II                                                          MD05260
      IF(I.GE.17) II=II-16                                            MD05270
      X1=DLOG10(X+1.D-20)                                             MD05280
C  ^^                                                                 MD05290
C     DPTI4=(LBB(I2464)-LZ2(IJ))/6000.                                MD05300
C     IF(DPTI4.LT.0.) DPTI4=0                                         MD05310
C     XFAC=10**(-DPTI4/6.72)                                          MD05320
C     X2=DLOG10(X*XFAC+1.D-20)                                        MD05330
C  ^^^^                                                               MD05340
      IF(II3.LE.11) WLC(NWLM+(II3-2)*10+I10)=X1*X493                  MD05350
      IF(IOUT.EQ.1) WRITE(6,4501) II,PP,X,X1                          MD05360
793   CONTINUE                                                        MD05370
      IF(IOUT.EQ.0) GO TO 795                                         MD05380
      WRITE(6,3555) P1,P6,P2,P3,ALFA,P7,P8,P9                         MD05390
      DO 794 I44=1,2                                                  MD05400
      DO 794 I=1,11                                                   MD05410
      IJ=I2464+(I-1)*NIJ10                                            MD05420
      XSS(1)=NT(IJ)*PARAMX(IJ)                                        MD05430
      XSH(1)=XKZZ(IJ)*PARAMX(IJ+N324)                                 MD05440
```

```
      DPTI4=.67
      IF(I44.EQ.2) DPTI4=10
      X=.1829*DPTI4
      FACSH=1*10**((.08333*X*X-1.1677*X)*1.0)
      FACSS=1*10**(-DPTI4/6.72)
      XSH(1)=XSH(1)*FACSH
      XSS(1)=XSS(1)*FACSS
      X=1-.5
      AL25=ALFA*4
      ALP5=1./AL25
      C0 =XSS(1)*(XSH(1)/(XSS(1)+1.D-20))**((X                         )**AL25)
      C01=XSS(1)*(XSH(1)/(XSS(1)+1.D-20))**((X                         )**ALP5)
      X0=DLOG10(C0+1.D-20)
      X01=DLOG10(C01+1.D-20)
      WRITE(6,4501) I+1,C0,C01,X0,X01
794   CONTINUE
795   CONTINUE
C     Write out DONE
      I3684=NWLM+1
      I3783=NWLM+100
      IF(JIT.EQ.-1) WRITE(6,4502) (WLC(I),I=I3684,I3783)
C   ^^^
      KOUT=0
      DO 322 K=2,NK11
      MAQ1K=MAQ1(K)
      KOUT=KOUT+MAQ1K
      KLB=NK11+KOUT
      DO 321 J=2,NJ11
      DO 320 I=2,NI11
      I2236=22*I+36*J-2349
      IJF=I+NI10*(J-2)
      IJ=IJF+NIJ10*(K-2)
C  ++++++++ FIXED HEAD TO XXS ++++++++
      I5=IXXXS(IJ)
C     X=I5*P9+20-P8
      X=I5*P9
      IF(I5.GT.(3000-5)) X=P8*3000
      XXS(IJ)=X
C  +++++++++++++++++++++++++++++++++++++++
      IJLB=IJF+NIJ10*(KLB-2)
      IM1JK=IJ-1
      IJM1K=IJ-NI10
      IJKM1=IJ-NIJ10
      IF(IJM1K.LT.1) IJM1K=1
      IF(IJKM1.LT.1) IJKM1=1
      DXII=DXI(I)
      DYJJ=DYJ(J)
      DXIM1=DXI(I-1)
      DYJM1=DYJ(J-1)
      B0=LB(IJ)
      LBIM1=LB(IM1JK)
      LBIJM=LB(IJM1K)
      BDD=(LBIM1*DXII+B0*DXIM1)/(DXII+DXIM1)
      BBB=(LBIJM*DYJJ+B0*DYJM1)/(DYJJ+DYJM1)
      WDD=LBIM1*B0
```

```
      WBB=LBIJM*BO                                                      MD06000
      IF(WDD.EQ.0.0) BDD=0                                              MD06010
      IF(WBB.EQ.0.0 ) BBB=0                                             MD06020
      B4=LB(IJKM1)                                                      MD06030
C    ***** Use PARAMX as multicative factor for sand (XSS) and          MD06040
C      clay (XSH) hydraulic conductivities *****                        MD06050
      XSS(1)=NT(IJ)*PARAMX(IJ)                                          MD06060
      XSS(2)=NT(IM1JK)*PARAMX(IM1JK)                                    MD06070
      XSS(3)=NT(IJM1K)*PARAMX(IJM1K)                                    MD06080
      XSS(4)=NT(IJKM1)*PARAMX(IJKM1)                                    MD06090
      XSH(1)=XKZZ(IJ)*PARAMX(IJ+N324)                                   MD06100
      XSH(2)=XKZZ(IM1JK)*PARAMX(IM1JK+N324)                             MD06110
      XSH(3)=XKZZ(IJM1K)*PARAMX(IJM1K+N324)                             MD06120
      XSH(4)=XKZZ(IJKM1)*PARAMX(IJKM1+N324)                             MD06130
C    ****** Multiply these factors (XSJ and XSH) by decrease of         MD06140
C      sand and clay hydraulic conductivity with depth functions        MD06150
C      from text *******                                                MD06151
      LBBO=LBB(IJF)                                                     MD06160
      DPT(1)=(LBBO-LZ2(IJ)*.1)/600.                                     MD06170
      DPT(2)=(LBB(IJF-1   )-LZ2(IM1JK)*.1)/600.                         MD06180
      IJF8=IJF-NI10                                                     MD06190
      IF(IJF8.LT.1) IJF8=1                                              MD06200
      DPT(3)=(LBB(IJF8)-LZ2(IJM1K)*.1)/600.                             MD06210
      DPT(4)=(LBBO-LZ2(IJKM1)*.1)/600.                                  MD06220
      FACA=0                                                            MD06230
      DO 897 I4=1,4                                                     MD06240
      DPTI4=DPT(I4)                                                     MD06250
      IF(DPTI4.LT.0.) DPTI4=0                                           MD06260
      IF(DPTI4.GT.100.) DPTI4=100                                       MD06270
      X=.1829*DPTI4                                                     MD06280
      FACSH=1*10**((.08333*X*X-1.1677*X)*1.0)                           MD06290
      FACSS=1*10**(-DPTI4/6.72)                                         MD06300
      XSH(I4)=XSH(I4)*FACSH                                             MD06310
      XSS(I4)=XSS(I4)*FACSS                                             MD06320
      IF((I4.EQ.1).OR.(I4.EQ.4)) FACA=FACA+FACSH*.5                     MD06330
897   CONTINUE                                                          MD06340
C    **** Use equations (10) and (11) of text to arrive at effective    MD06350
C      conductivities. XXSTR is percentage sand=1-clay function ***     MD06360
      X=1-XXSTR(IJ)                                                     MD06370
      AL25=ALFA*4                                                       MD06380
      ALP5=1./AL25                                                      MD06390
      CO =XSS(1)*(XSH(1)/(XSS(1)+1.D-20))**((X            )**AL25)      MD06400
      CD =XSS(2)*(XSH(2)/(XSS(2)+1.D-20))**((1-XXSTR(IM1JK))**AL25)     MD06410
      CB =XSS(3)*(XSH(3)/(XSS(3)+1.D-20))**((1-XXSTR(IJM1K))**AL25)     MD06420
      CO1=XSS(1)*(XSH(1)/(XSS(1)+1.D-20))**((X            )**ALP5)      MD06430
      CZ1=XSS(4)*(XSH(4)/(XSS(4)+1.D-20))**((1-XXSTR(IJKM1))**ALP5)     MD06440
C    UUUUUUUUUUUUUUUUU                                                  MD06450
C      Introduce parpameter P1 (b4 in text)                             MD06460
      IF((K.EQ.3).AND.(I2236.GT.0)) CZ1=LB(IJKM1)*P1*GP5                MD06470
      IF(K.NE.NK11) GO TO 1123                                          MD06480
1123  CONTINUE                                                          MD06490
319   CONTINUE                                                          MD06500
      IF(I.EQ.2) CD=0                                                   MD06510
      IF(J.EQ.2) CB=0                                                   MD06520
      IF(K.EQ.2) CZ1=0                                                  MD06530
```

```
      DD(IJ)=-DYJJ*2*DDK(K)*PARAMX(N98+K)*BDD*CD*C0/(DXII*CD+            MD06540
     1DXIM1*C0+1.D-30)                                                   MD06550
      BB(IJ)=-DXII*2*BBK(K)*PARAMX(N99+K)*BBB*CB*C0/(DYJJ*CB+            MD06560
     1DYJM1*C0+1.D-30)                                                   MD06570
      CC=FACA*NT(IJLB)*PARAMX(IJLB)                                      MD06580
      IF(MAQ1K.EQ.0) CC=1                                                MD06590
      ALP=.5D0*(B0*CC*CZ1+B4*CC*C01)+MAQ1K*LB(IJLB)*C01*CZ1              MD06600
      CCC=C01*CC*CZ1                                                     MD06610
      ZZIJ=1.D0                                                          MD06620
      IF(ALP.NE.0.0) ZZIJ=CCC/ALP                                        MD06630
      IF(CCC.EQ.0.0) ZZIJ=0                                              MD06640
      ZZ(IJ)=-DXI(I)*DYJ(J)*ZZIJ                                         MD06650
320   CONTINUE                                                           MD06660
321   CONTINUE                                                           MD06670
322   CONTINUE                                                           MD06680
C  THE QUANTITIES DD, BB, AND ZZ, ARE NOW  DETERMINED.                   MD06690
C     Calculate pseudo sources and put into YQ2                          MD06700
      SUX=0                                                              MD06710
      NCNT=0                                                             MD06720
      NI2=NI10*NI10                                                      MD06730
      NJ2=NJ10*NJ10                                                      MD06740
      NK2=NK10*NK10                                                      MD06750
      KOUT=0                                                             MD06760
      DO 350 K=2,NK11                                                    MD06770
      KOUT=KOUT+MAQ1(K)                                                  MD06780
      KLB=NK11+KOUT                                                      MD06790
      DO 350 J=2,NJ11                                                    MD06800
      DO 340 I=2,NI11                                                    MD06810
      IJF=I+NI10*(J-2)                                                   MD06820
      IJ=IJF+NIJ10*(K-2)                                                 MD06830
      IF(MHD(IJ).GE.1) GO TO 340                                         MD06840
      NCNT=NCNT+1                                                        MD06850
      IJLB=IJF+NIJ10*(KLB-2)                                             MD06860
      IJLB1=IJLB+NIJ10                                                   MD06870
      IP1JK=IJ+1                                                         MD06880
      IM1JK=IJ-1                                                         MD06890
      IJP1K=IJ+NI10                                                      MD06900
      IJM1K=IJ-NI10                                                      MD06910
      IJKP1=IJ+NIJ10                                                     MD06920
      IJKM1=IJ-NIJ10                                                     MD06930
      IF(IJM1K.LT.1) IJM1K=1                                             MD06940
      IF(IJKM1.LT.1) IJKM1=1                                             MD06950
      IF(IJP1K.GT.NNN) IJP1K=NNN                                         MD06960
      IF(IJKP1.GT.NNN) IJKP1=NNN                                         MD06970
      IF(IJLB1.GT.N325) IJLB1=N325                                       MD06980
      DD1=DD(IJ)                                                         MD06990
      DD2=DD(IP1JK)                                                      MD07000
      BB1=BB(IJ)                                                         MD07010
      BB2=BB(IJP1K)                                                      MD07020
      ZZ1=ZZ(IJ)                                                         MD07030
      ZZ2=ZZ(IJKP1)                                                      MD07040
C  ^^^^                                                                  MD07050
      IF((WMAX.NE.0.0).OR.(ISOR.NE.0.0)) GO TO 806                       MD07060
      D1=(BB1+BB2+ZZ1+ZZ2)/(DD1+DD2-1.D-30)                              MD07070
      B1=(DD1+DD2+ZZ1+ZZ2)/(BB1+BB2-1.D-30)                              MD07080
```

```
      Z1=(DD1+DD2+BB1+BB2)/(ZZ1+ZZ2-1.D-30)
      IF(D1.GT.1.D+9) D1=0
      IF(B1.GT.1.D+9) B1=0
      IF(Z1.GT.1.D+9) Z1=0
      D1=1/(NI2*(1+D1))
      B1=1/(NJ2*(1+B1))
      Z1=1/(NK2*(1+Z1))
      X=D1
      IF(B1.LT.X) X=B1
      IF(Z1.LT.X) X=Z1
      SUX=SUX+X
806   CONTINUE
C  ^^^^
      B0=LB(IJ)
      B4=LB(IJKM1)
      B5=LB(IJKP1)
      BT0=LRO(IJ)*1.D-5
      BT4=LRO(IJKM1)*1.D-5
      BT5=LRO(IJKP1)*1.D-5
      BTDD=LRO(IM1JK)*1.D-5
      BTFF=LRO(IP1JK)*1.D-5
      BTBB=LRO(IJM1K)*1.D-5
      BTHH=LRO(IJP1K)*1.D-5
      NB1=LB(IJLB)
      NB2=LB(IJLB1)
C       FB1=NB1/(NB1+.01)
C       FB2=NB2/(NB2+.01)
      X=(.5D0*(BT0*B0+BT4*B4)+MAQ1(K)*(LRO(IJLB)*1.0D-5)*NB1)*ZZ1
     1-(.5D0*(BT0*B0+BT5*B5)+MAQ1(K+1)*(LRO(IJLB1)*1.0D-5)*NB2)*ZZ2
C     3+(ZZ2*FB2*MAQ1(K+1)*(BT5-BT0)+ZZ1*FB1*MAQ1(K)*(BT4-BT0))*OS4
      LZ0=LZ2(IJ)
      DXII=DXI(I)
      DXIM1=DXI(I-1)
      DXIP1=DXI(I+1)
      DYJJ=DYJ(J)
      DYJM1=DYJ(J-1)
      DYJP1=DYJ(J+1)
      BT0X=BT0*DXII
      BT0Y=BT0*DYJJ
      YQ2(IJ)=X-(((BTDD*DXIM1+BT0X)/(DXIM1+DXII))*DD1*(LZ2(IM1JK)-LZ0)
     1         +((BTFF*DXIP1+BT0X)/(DXIP1+DXII))*DD2*(LZ2(IP1JK)-LZ0)
     2         +((BTBB*DYJM1+BT0Y)/(DYJM1+DYJJ))*BB1*(LZ2(IJM1K)-LZ0)
     3         +((BTHH*DYJP1+BT0Y)/(DYJP1+DYJJ))*BB2*(LZ2(IJP1K)-LZ0)
     4)*.1D0
340   CONTINUE
350   CONTINUE
C  YQ2 NOW DETERMINED
C     Get iteration parameters for SIP subroutine
      IF(ISOR.NE.0) GO TO 390
      XY=.5D0*PI*PI*SUX/NCNT
C  XY IS NOW DETERMINED
      XY=XYFC*XY
      IF(WMAX.NE.0.0) XY=WMAX
C  DETERMINE ITERATION PARAMETERS WS(I)
      DO 360 I=1,LENGTH
```

```
360   WS(I)=1-XY**((I-1.)/(LENGTH-1))
C     WRITE(6,5056)
C     WRITE(6,5006)
C     WRITE(6,3555) (WS(I),I=1,LENGTH)
390   CONTINUE
C     Have coefficients
      IF(JIT.EQ.-1) WRITE(6,1003) 99,ISOR,ITMAX,ERR,XX10
     1,HMAX,XYFC,WMAX
      DO 599 IJ=2,N315
599   HLS(IJ)=0
      NINT=5
C     Begin main loop of pgn.
C     There are 5 time intervals INT=1,2,...5.
C     DELT is time interval
      DO 600 INT=1,5
      INT2=INT-2
      IF(INT2.LT.1) INT2=1
      DELT=DEL(INT)
      DELT=DELT/P2
      SUMYQ=0
      DO 231 IDBZ=1,3
      DO 231 N=1,250
231   SYQ(IDBZ,N)=0
C     Put pumping for this time interval into YQ
      DO 331 K=2,NK11
      K1=K-1
      DO 330 J=2,NJ11
      DO 329 I=2,NI11
      IJ=I+NI10*(J-2)+NIJ10*(K-2)
C   *** GOOD FIRST GUESS AND FIXED HEAD TO XX ***
      X=IXX5(IJ,INT)*.1
      IF((JIT.EQ.-1).OR.(MHD(IJ).EQ.1)) X=XXS(IJ)
      XX(IJ)=X
C  ***************
      XDEN=FC(2,K1)
      X=XDEN
      IF(INT.EQ.1) X=0
      IF(INT.EQ.2) X=FC(1,K1)*.79
      FCX=X/XDEN
      XQ=YQ1(IJ,INT2)*FCX
      IF(IOUT.EQ.0) GO TO 701
      SYQ(1,I)=SYQ(1,I)+XQ
      SYQ(2,J)=SYQ(2,J)+XQ
      SYQ(3,K)=SYQ(3,K)+XQ
      SUMYQ=SUMYQ+XQ
701   CONTINUE
329   YQ(IJ)=XQ+YQ2(IJ)
330   CONTINUE
331   CONTINUE
C     YQ now has pumping including pseudo sources
      I01=1
      IWR1=IWRT*I01
      LFLO1=LFLOW*I01
      IF(IWR1.NE.1) GO TO 420
      WRITE(6,5007)
```

```
      WRITE(6,5072) NU1(1),NU2(1),NU3(1)
      WRITE(6,5073) NU1(2),NU2(2),NU3(2)
      WRITE(6,5074) NU1(3),NU2(3),NU3(3)
420   CONTINUE
C  BEGIN SIP,OR PCG ITERATIONS.
C     Call PCG for firsit time interval,
C     SIP for second through fifth intervals
      ISO=ISO+1
      IF(INT.GT.1) CALL SIP
      IF(INT.EQ.1) CALL PCG
      WRITE(6,1003) JIT,INT,ICNT,ER5,SRZ,SUMRZ,HMAX,XX(9583),XX(30904)
C  @@@@@@@ GET SUBSIDENCE STORAGE (RATE) @@@@@@
      IF(IOUT.EQ.0) GO TO 673
      WRITE(6,9020)
      DO 670 K=2,NK11
      SUMT=0
      SUMS=0
      DO 671 I=2,NI11
      DO 671 J=2,NJ11
      IJ=I+NI10*(J-2)+NIJ10*(K-2)
      IF(MHD(IJ).GE.1) GO TO 672
      XXIJ=XX(IJ)
      X=HL(IJ)-XXIJ
      SM=XXS(IJ)-HL(IJ)
      S1=X
      SS=0
      IF(X.LE.0) GO TO 675
      S1=X*VV40*(1-XXSTR(IJ))
      SS=S1
675   CONTINUE
      SUMT=SUMT+(SM+S1)*LB(IJ)
      SUMS=SUMS+SS*LB(IJ)
672   CONTINUE
671   CONTINUE
      SUMT=SUMT*SVIJ/DELT
      SUMS=SUMS*SVIJ/DELT
      WRITE(6,4501) K,SUMS,SUMT
670   CONTINUE
C &&&&&&&&&&&&&&&&&&&&
      DO 771 I=2,NI11
      DO 771 J=2,NJ11
C     RXY=1/(DXI(I)*DYJ(J))
      IJJ=I+NI10*(J-2)
      SUM=0
      DO 770 K=2,NK11
      IJ=IJJ+NIJ10*(K-2)
      IF(MHD(IJ).GE.1) GO TO 772
      XXIJ=XX(IJ)
      X=HL(IJ)-XXIJ
      SM=XXS(IJ)-HL(IJ)
      S1=X
      SS=0
      IF(X.LE.0) GO TO 775
      S1=X*VV40*(1-XXSTR(IJ))
      SS=S1
```

```
775   CONTINUE
      SUM=SUM+(SM+S1)*LB(IJ)
772   CONTINUE
770   CONTINUE
      IF(INT.NE.1) HLS(IJJ)=HLS(IJJ)+SUM*SV35*P2
771   CONTINUE
673   CONTINUE
C @@@@@@@@@@@@@@@ End subsidence @@@@@@@@@@@@@@@
C     Save XX in IXX5(IJ,INT) for use as the starting solution
C     for XX at the next iteration (JIT+1), same time interval INT.
C     Also save XX in XXS for use in PCG & SIP as the starting value
C     for XX at the beginning of the next time interval (INT+1)
C     Calculate HL(IJ), the lowest value for head yet achieved
C     at location IJ
      DO 430 K=2,NK11
      DO 430 J=2,NJ11
      DO 430 I=2,NI11
      IJ=I+NI10*(J-2)+NIJ10*(K-2)
      XXIJ=XX(IJ)
      HLIJ=HL(IJ)
      IF(XXIJ.LT.HLIJ) HL(IJ)=XXIJ
      P380=P3*80
      IF(K.LE.6) P380=1000
      IF(INT.EQ.1) HL(IJ)=XXIJ-P380
      IXX5(IJ,INT)=XXIJ*10.+.5
430   XXS(IJ)=XXIJ
C     Done with IXX5,XX5,HLIJ
      IF((JIT.EQ.-1).AND.(INT.EQ.5)) GO TO 7197
      IF((LFLO1*IOUT).EQ.0) GO TO 560
7197  CONTINUE
C  DETERMINE  WATER FLOW RATES
C     Much of this is in VARDEN and deals with calculation of various
C     flow rates.  See WRIR 84-4302, p. B-6, Kuiper
      DO 440 IJ=1,NNN
440   G2(IJ)=0
      DO 511 IDBZ=1,3
      DO 450 N=1,250
      SUMFM(IDBZ,N)=0
      SUNFM(IDBZ,N)=0
      SUNF(IDBZ,N)=0
450   SUMF(IDBZ,N)=0
      IRITE=0
      L1=LFLO(1)
      IF((L1.EQ.IDBZ).OR.(L1.EQ.4)) IRITE=1
      IF(((INT-3).NE.2).OR.(IOUT.NE.1)) IRITE=0
      IF(IRITE.NE.1) GO TO 460
      IF(IDBZ.EQ.1) WRITE(6,5101)
      IF(IDBZ.EQ.2) WRITE(6,5102)
      IF(IDBZ.EQ.3) WRITE(6,5103)
      WRITE(6,6000)
460   CONTINUE
      IRITEA=0
      L1=LFLOA(1)
      IF((L1.EQ.IDBZ).OR.(L1.EQ.4)) IRITEA=1
      IF(((IABS(INT-3).NE.2).OR.(IOUT.NE.1))) IRITEA=0
```

```
      IF(IRITEA.NE.1) GO TO 461                                         MD09280
C     IF(IDBZ.EQ.1) WRITE(7,5101)                                       MD09290
C     IF(IDBZ.EQ.2) WRITE(7,5102)                                       MD09300
C     IF(IDBZ.EQ.3) WRITE(7,5103)                                       MD09310
C     WRITE(7,6000)                                                     MD09320
461   CONTINUE                                                          MD09330
      KOUT=0                                                            MD09340
      DO 511 K=2,NK11                                                   MD09350
      KOUT=KOUT+MAQ1(K)                                                 MD09360
      KLB=NK11+KOUT                                                     MD09370
      IF((IRITE.EQ.1).AND.(IPH(K).GE.0)) WRITE(6,4002) K                MD09380
C     IF((IRITEA.EQ.1).AND.(IPHA(K).EQ.1)) WRITE(7,4002) K              MD09390
      I88=0                                                             MD09400
      SM9=0                                                             MD09410
      DO 510 J=2,NJ11                                                   MD09420
      I88=I88+1                                                         MD09430
      IF(I88.EQ.11) I88=1                                               MD09440
      IJMI=NI10*(J-2)+NIJ10*(K-2)                                       MD09450
      IJLBMI=NI10*(J-2)+NIJ10*(KLB-2)                                   MD09460
      DO 500 I=2,NI11                                                   MD09470
      IJ=IJMI+I                                                         MD09480
      IJLB=IJLBMI+I                                                     MD09490
      XX0=XX(IJ)                                                        MD09500
      BT0=LRO(IJ)*1.0D-5                                                MD09510
      LZ0=LZ2(IJ)                                                       MD09520
      IF(IDBZ.NE.1) GO TO 470                                           MD09530
      DBZ=DD(IJ)                                                        MD09540
      IF(I.EQ.2) GO TO 490                                              MD09550
      IM1JK=IJ-1                                                        MD09560
      XXI=XX(IM1JK)                                                     MD09570
      BTDD=LRO(IM1JK)*1.0D-5                                            MD09580
      DXII=DXI(I)                                                       MD09590
      DXIM1=DXI(I-1)                                                    MD09600
      BT0X=BT0*DXII                                                     MD09610
      FLOW=((BTDD*DXIM1+BT0X)/(DXIM1+DXII))*(LZ2(IM1JK)-LZ0)*.1D0       MD09620
      GO TO 490                                                         MD09630
470   IF(IDBZ.NE.2) GO TO 480                                           MD09640
      DBZ=BB(IJ)                                                        MD09650
      IF(J.EQ.2) GO TO 490                                              MD09660
      IJM1K=IJ-NI10                                                     MD09670
      XXI=XX(IJM1K)                                                     MD09680
      BTBB=LRO(IJM1K)*1.0D-5                                            MD09690
      DYJJ=DYJ(J)                                                       MD09700
      DYJM1=DYJ(J-1)                                                    MD09710
      BT0Y=BT0*DYJJ                                                     MD09720
      FLOW=((BTBB*DYJM1+BT0Y)/DYJM1+DYJJ))*(LZ2(IJM1K)-LZ0)*.1D0        MD09730
      GO TO 490                                                         MD09740
480   DBZ=ZZ(IJ)                                                        MD09750
      IF(K.EQ.2) GO TO 490                                              MD09760
      IJKM1=IJ-NIJ10                                                    MD09770
      XXI=XX(IJKM1)                                                     MD09780
      B0=LB(IJ)                                                         MD09790
      B4=LB(IJKM1)                                                      MD09800
      BT4=LRO(IJKM1)*1.0D-5                                             MD09810
      NB1=LB(IJLB)                                                      MD09820
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      FB1=NB1/(NB1+.01)                                                 MD09830
      FLOW=-(.5D0*(BT0*B0+BT4*B4)+MAQ1(K)*(LRO(IJLB)*1.0D-5)*NB1)       MD09840
C     1-FB1*MAQ1(K)*(BT4-BT0)*OS4                                       MD09850
490   X=DBZ*(XXI-XX0+FLOW)                                              MD09860
      IM=1                                                              MD09870
      IF(IDBZ.EQ.2) IM=NI10                                             MD09880
      IF(IDBZ.EQ.3) IM=NIJ10                                            MD09890
      IJM=IJ-IM                                                         MD09900
      IF(IJM.LT.1) IJM=1                                                MD09910
      G2(IJM)=G2(IJM)-X                                                 MD09920
C &&&&&&&&&&& Adjust Z component of hydraulic conductivity              MD09930
C     in top layer 12 if flow through is in excess. This is done        MD09940
C     for JIT=-1 only.                                                  MD09941
C     Subsequent iterations JIT=1,2,... use the reduced value of the    MD09950
C     Z component of hydraulic conductivity for layer 12                MD09960
      IF(.NOT.((JIT.EQ.-1).AND.(INT.EQ.5))) GO TO 762                   MD09970
      IF((IDBZ.NE.3).OR.(K.NE.NK11)) GO TO 762                          MD09980
      X600=1*600                                                        MD09990
      X100=1*100                                                        MD10000
      T=X600-((X600-X100)/30)*(J-50)                                    MD10010
      IF(T.GT.X600) T=X600                                              MD10020
      IF(T.LT.X100) T=X100                                              MD10030
      T=T*1.D4                                                          MD10040
      IF(X.LT.T) GO TO 762                                              MD10050
C     X77=.5*T/X                                                        MD10060
      X77=.6/(X*10/T-9)                                                 MD10070
C     ZZ(IJ)=ZZ(IJ)*X77                                                 MD10080
C     NT(IJ)=NT(IJ)*X77                                                 MD10090
      XKZZ(IJ)=XKZZ(IJ)*X77                                             MD10100
      IJ66=I+(J-2)*NI10                                                 MD10110
      DO 837 KK=11,2,-1                                                 MD10120
      IJ1=(KK-2)*NIJ10+IJ66                                             MD10130
      IF(LB(IJ1).EQ.0) GO TO 838                                        MD10140
C     IF(KK.NE.11) ZZ(IJ1+NIJ10)=ZZ(IJ1+NIJ10)*X77                      MD10150
C     XKZZ(IJ1)=XKZZ(IJ1)*X77                                           MD10160
      GO TO 839                                                         MD10170
838   CONTINUE                                                          MD10180
837   CONTINUE                                                          MD10190
839   CONTINUE                                                          MD10200
762   CONTINUE                                                          MD10210
C  $$$ GET FLOW ACROSS GULF COAST$$$$$                                  MD10220
      DI=I-I92                                                          MD10230
      DJ=J-J92                                                          MD10240
      R2=DI*DI+DJ*DJ                                                    MD10250
      IF(IDBZ.EQ.1) DI=DI-1                                             MD10260
      IF(IDBZ.EQ.2) DJ=DJ-1                                             MD10270
      R3=DI*DI+DJ*DJ                                                    MD10280
      IF((R3.GT.R42).AND.(R2.LE.R42)) SM9=SM9+X                         MD10290
      IF((R2.GT.R42).AND.(R3.LE.R42)) SM9=SM9-X                         MD10300
C   $$$$$$$$$$$                                                         MD10310
C  &&&&&&&&&&&&&                                                        MD10320
      DTI=X                                                             MD10330
      IF(LB(IJ).EQ.0) DTI=-1.E+38                                       MD10340
      DT(I)=DTI                                                         MD10350
      IJK=I                                                             MD10360
```

```
      IF(IDBZ.EQ.2) IJK=J                                               MD10370
      IF(IDBZ.EQ.3) IJK=K                                               MD10380
      IF(LB(IJ).EQ.0) GO TO 491                                         MD10390
      IF(X.GT.0) SUMF(IDBZ,IJK)=SUMF(IDBZ,IJK)+X                        MD10400
      IF(X.LT.0) SUNF(IDBZ,IJK)=SUNF(IDBZ,IJK)-X                        MD10410
491   IF(LB(IJM).EQ.0) GO TO 492                                        MD10420
      IF(X.GT.0) SUMFM(IDBZ,IJK)=SUMFM(IDBZ,IJK)+X                      MD10430
      IF(X.LT.0) SUNFM(IDBZ,IJK)=SUNFM(IDBZ,IJK)-X                      MD10440
492   CONTINUE                                                          MD10450
      G2(IJ)=G2(IJ)+X                                                   MD10460
500   CONTINUE                                                          MD10470
C     IF(I88.NE.-1) GO TO 934                                           MD10480
      IF(.NOT.(((J.LE.69).AND.(J.GE.62)).OR.((J.LE.19).AND.(J.GE.15)))) MD10490
     *GO TO 934                                                         MD10500
      IF((IRITE.EQ.1).AND.(IPH(K).EQ.1)) WRITE(6,4500) J,(DT(I),I=2     MD10510
     1,NI11)                                                            MD10520
934   CONTINUE                                                          MD10530
C     IF((IRITEA.EQ.1).AND.(IPHA(K).EQ.1)) WRITE(7,5500) J-1,           MD10540
C    1(DT(I),I=2,NI11)                                                  MD10550
510   CONTINUE                                                          MD10560
      IF(IOUT.EQ.1) WRITE(6,1003) IDBZ,K,5555,SM9                       MD10570
511   CONTINUE                                                          MD10580
      IF((LFLO1*IOUT).EQ.0) GO TO 560                                   MD10590
      IF(LFLO(2).EQ.1) WRITE(6,8010)                                    MD10600
      I5=0                                                              MD10610
      SUMG2=0                                                           MD10620
      DO 549 IDBZ=1,3                                                   MD10630
      DO 549 N=1,250                                                    MD10640
549   SG2(IDBZ,N)=0                                                     MD10650
      DO 550 K=2,NK11                                                   MD10660
      DO 550 J=2,NJ11                                                   MD10670
      DO 550 I=2,NI11                                                   MD10680
      IJ=I+NI10*(J-2)+NIJ10*(K-2)                                       MD10690
      IF(MHD(IJ).NE.1) GO TO 540                                        MD10700
      G2IJ=G2(IJ)                                                       MD10710
      I5=I5+1                                                           MD10720
      DT(I5)=G2IJ                                                       MD10730
      I7(I5)=I                                                          MD10740
      J7(I5)=J                                                          MD10750
      K7(I5)=K                                                          MD10760
      SG2(1,I)=SG2(1,I)+G2IJ                                            MD10770
      SG2(2,J)=SG2(2,J)+G2IJ                                            MD10780
      SG2(3,K)=SG2(3,K)+G21J                                            MD10790
      SUMG2=SUMG2+G2IJ                                                  MD10800
540   IF(I5.NE.5) GO TO 550                                             MD10810
      I5=0                                                              MD10820
      IF(LFLO(2).EQ.1) WRITE(6,3560) (I7(L),J7(L),K7(L),DT(L),L=1,5)    MD10830
550   CONTINUE                                                          MD10840
      IF((LFLO(2).EQ.1).AND.(I5.GE.1))                                  MD10850
     1WRITE(6,3560) (I7(L),J7(L),K7(L),DT(L),L=1,I5)                    MD10860
      IF(LFLO(3).EQ.1) WRITE(6,9010)                                    MD10870
      I5=0                                                              MD10880
      SUMVV=0                                                           MD10890
      DO 649 IDBZ=1,3                                                   MD10900
      DO 649 N=1,250                                                    MD10910
```

```
649   SVV(IDBZ,N)=0
      DO 650 K=2,NK11
      DO 650 J=2,NJ11
      DO 650 I=2,NI11
      IJ=I+NI10*(J-2)+NIJ10*(K-2)
C     X=XX(IJ)-XXE(IJ)
C     VVIJ=X*ALN(IJ)
      IF(.NOT.((IEVP.EQ.1).AND.(VVIJ.GT.0))) GO TO 641
      I5=I5+1
      DT(I5)=VVIJ
      I7(I5)=I
      J7(I5)=J
      K7(I5)=K
      SVV(1,I)=SVV(1,I)+VVIJ
      SVV(2,J)=SVV(2,J)+VVIJ
      SVV(3,K)=SVV(3,K)+VVIJ
      SUMVV=SUMVV+VVIJ
641   IF(I5.NE.5) GO TO 650
      I5=0
      IF(LFLO(3).EQ.1) WRITE(6,3560) (I7(L),J7(L),K7(L),DT(L),L=1,5)
650   CONTINUE
      IF((LFLO(3).EQ.1).AND.(I5.GE.1))
     1WRITE(6,3560) (I7(L),J7(L),K7(L),DT(L),L=1,I5)
      WRITE(6,8011) SUMYQ,SUMG2,SUMVV
      IF(LFLO(4).EQ.0) GO TO 530
      WRITE(6,7001)
      IDBZ1=LFLO(4)
      IDBZ3=IDBZ1
      IF(IDBZ1.GE.4) IDBZ1=1
      IF(IDBZ3.GE.4) IDBZ3=3
      DO 520 IDBZ=IDBZ1,IDBZ3
      N20=NI11
      IF(IDBZ.EQ.2) N20=NJ11
      IF(IDBZ.EQ.3) N20=NK11
      DO 519 N=2,N20
      NP1=N+1
519   WRITE(6,8012) IDBZ,N,SYQ(IDBZ,N),SG2(IDBZ,N),SVV(IDBZ,N)
     1,SUNF(IDBZ,N),SUMFM(IDBZ,NP1),SUMF(IDBZ,N),SUNFM(IDBZ,NP1)
520   CONTINUE
530   CONTINUE
560   IF((1*IOUT).EQ.0) GO TO 590
C     End of flow rate section
C     Printout heads in various formats and media.  Change things
C     in this section to choose media, which heads, etc.
      WRITE(6,5008)
      WRITE(6,6000)
C     IF(IABS(INT-3).GE.0) WRITE(8,5008)
C     IF(IABS(INT-3).GE.0) WRITE(8,6000)
C      WRITE(10,5009)
C      WRITE(10,6000)
C  DETERMINE HYDRAULIC HEAD FROM XX = H PRIME = (PRESSURE HEAD H) + Z
      XSM=0
      DO 580 K=2,NK11
C     IF(IABS(INT-3).GE.0) WRITE(8,4002) K
C      WRITE(10,4002) K
```

```
      XXSM=0
      WRITE(6,2020) 1,1,0,K
      DO 579 J=2,NJ11
      IJMI=NI10*(J-2)+NIJ10*(K-2)
      DO 570 I=2,NI11
      IJ=IJMI+I
      XZ210=LZ2(IJ)*.1D0
      DTI=(1/(LRO(IJ)*1.0D-5+1))*(XX(IJ)-XZ210)+XZ210
      IF(MHD(IJ).EQ.2) DTI=0
      X9=1
      IF(LB(IJ).EQ.0) X9=0
      XXF=XX(IJ)*X9
      XXH=DTI*X9
      IF(K.NE.NK11) XSM=XSM+XXF
      IF(K.NE.NK11) XXSM=XXSM+XXF
      E2(I)=XXF
      E2(2)=J
      IF(XXF.EQ.0) XXF=-1.E+38
      IF(XXH.EQ.0) XXH=-1.E+38
      G2(I)=XXF
570   DT(I)=XXH
      IF((INT.EQ.5).AND.(IPH(K).GE.1)) WRITE(6,1008) (E2(I),
     *I=2,NI11)
C       IF(IWRTXX.LE.1) WRITE(10,1001) J-1,(DT(I),I=2,NI11)
C     IF(IABS(INT-3).GE.0) WRITE(8,5500) J-1,(G2(I),I=2,NI11)
1008  FORMAT(20F6.0)
579   CONTINUE
      WRITE(6,3555) XXSM
580   CONTINUE
      WRITE(6,3555) XSM
C     End of head printout
590   CONTINUE
600   CONTINUE
C     End of MAIN loop
C   ********* PRINT OUT TOTAL SUBSIDENCE *******
      IF(IOUT.EQ.0) GO TO 664
      WRITE(6,9021)
      DO 779 J=2,NJ11
      IJMI=NI10*(J-2)
C     WRITE(6,1001) J,(HLS(IJMI+I),I=2,NI11)
779   CONTINUE
664   CONTINUE
C     ******** DONE WITH SUBSIDENCE PRINT OUT ******
C     Put calculated heads into WLC=f(B) for use by MAIN
      DO 642 I=1,NWLM
      IJ=NND(I)
      XZ210=LZ2(IJ)*.1D0
C642   WLC(I)=(1/(LRO(IJ)*1.0D-5+1))*(XX(IJ)-XZ210)+XZ210
      IF(I.LE.NWLM1) XXX=IXX5(IJ,3)*.1
      IF(I.GT.NWLM1) XXX=XX(IJ)
642   WLC(I)=XXX
C     Read into WLC done
      IF(IOUT.EQ.0) GO TO 736
C     Insert pack for observation of residual errors
C     Y-f(B)=WLM(IJ)-WLC(IJ)
```

```
C     Various formats occur: Average residual error by layer and        MD12010
C     by total model. Root mean square residual error by layer and      MD12011
C     total model                                                       MD12020
      K40=1                                                             MD12030
      K40M1=0                                                           MD12040
      CUM=0                                                             MD12050
      CCUM=0                                                            MD12060
      DO 737 I2=1,2                                                     MD12070
      K41=0                                                             MD12080
      SSUM=0                                                            MD12090
      TSUM=0                                                            MD12100
      SWLMT=0                                                           MD12110
      WRITE(6,9998)                                                     MD12120
      DO 735 K=2,NK11                                                   MD12130
      K39=0                                                             MD12140
      PSUM=0                                                            MD12150
      PSUM1=0                                                           MD12160
      SWLM=0                                                            MD12170
      WRITE(6,4003) K                                                   MD12180
      DO 734 J=2,NJ11                                                   MD12190
      DO 733 I=2,NI11                                                   MD12200
      IJ=I+NI10*(J-2)+NIJ10*(K-2)                                       MD12210
      DT(I)=0                                                           MD12220
      IF(IJ.NE.NND(K40)) GO TO 732                                      MD12230
      XXX=WLC(K40)                                                      MD12240
      WLM40=WLM(K40)                                                    MD12250
      W40=W(K40)                                                        MD12260
      K40=K40+1                                                         MD12270
      IF(W40.LT.1.D-20) GO TO 732                                       MD12280
      SWLMT=SWLMT+WLM40                                                 MD12290
      SWLM =SWLM +WLM40                                                 MD12300
      X=XXX-WLM40                                                       MD12310
      DT(I)=X                                                           MD12320
      K39=K39+1                                                         MD12330
      K40M1=K40M1+1                                                     MD12340
      K41=K41+1                                                         MD12350
      SSUM=SSUM+X*X                                                     MD12360
      TSUM=TSUM+X*1                                                     MD12370
      PSUM=PSUM+X*1                                                     MD12380
      CUM=CUM+X*1                                                       MD12390
      PSUM1=PSUM1+X*X                                                   MD12400
      CCUM=CCUM+X*X                                                     MD12410
732   CONTINUE                                                          MD12420
733   CONTINUE                                                          MD12430
C     IF(I2.EQ.2) WRITE(6,1007) J,(DT(I),I=2,NI11)                      MD12440
734   CONTINUE                                                          MD12450
      PSUM=PSUM/(K39+1.D-20)                                            MD12460
      PSUM1=DSQRT(PSUM1/(K39+1.D-20))                                   MD12470
      SWLM=SWLM/(K39+1.D-20)                                            MD12480
      WRITE(6,4501) K39,PSUM,PSUM1,SWLM                                 MD12490
735   CONTINUE                                                          MD12500
      SSUM=DSQRT(SSUM/(K41))                                            MD12510
      TSUM=TSUM/(K41)                                                   MD12520
      SWLMT=SWLMT/(K41)                                                 MD12530
      WRITE(6,1003) K40M1,66,K41,TSUM,SSUM,SWLMT                        MD12540
```

```
737   CONTINUE
      CUM=CUM/K40M1
      CCUM=DSQRT(CCUM/K40M1)
      WRITE(6,1003) 66,66,66,CUM,CCUM
C     End of residual error pack
C   *************** DO CHI square test for normalcy of residuals
C     Discarding of outlying data lying outside of certain
C     normalcy bounds is optimal, at JIT=K4
C     Subsequent values of JIT will use reduced data set with
C     others discarded
736   CONTINUE
      K4=1
      IF(JIT.EQ.K4) IIII=IIII+1
      J8=JIT+IIII
      WRITE(6,2020) 5555,5555,K4,JIT,IIII,J8
      IF(.NOT.((J8.EQ.K4).OR.(IOUT.NE.0))) GO TO 1849
      DO 1839 IW=0,1
      CUM=0
      CCUM=0
      DO 1836 K40=1,NWLMT
      X=WLC(K40)-WLM(K40)
      W40=W(K40)
      IJ=NND(K40)
      IF(W40.LE.1.D-20) GO TO 1836
      X=X*DSQRT(W40)
      CUM=CUM+X
      CCUM=CCUM+X*X
1836  CONTINUE
      CUM=CUM/X3150
      CCUM=DSQRT(CCUM/X3150)
      DO 1837 I=1,16
1837  IX7(I)=0
      RHO1=DSQRT(CCUM*CCUM-CUM*CUM)
      F2242=(RHO1/(RHO1S+1.D-40))
      RHO1S=RHO1
      LR=0
      DO 1838 K40=1,NWLMT
      W40=W(K40)
      IF(W40.LE.1.D-20) GO TO 1939
      X=WLC(K40)-WLM(K40)
      X=(X-CUM)/RHO1
      IF((X.LT.1.54).AND.(X.GT.-3.4)) GO TO 1950
      IF(K40.GT.NWLM) GO TO 1950
      LR=LR+1
      WRITE(6,3011) LR,K40,NND(K40),888,X,WLC(K40),WLM(K40)
3011  FORMAT(4I8,6D11.3)
      IF((J8.EQ.K4).AND.(IW.EQ.0)) W(K40)=0
1950  CONTINUE
      DO 1792 II=1,16
      XR=XTV(II+1)
      XL=XTV(II)
      IF((XL.LT.X).AND.(X.LE.XR)) IX7(II)=IX7(II)+1
1792  CONTINUE
1939  CONTINUE
1838  CONTINUE
```

```
      XNP=X3150/16.
      SUM=0
      DO 893 I=1,16
      IXI=IX7(I)
      X=IXI-XNP
893   SUM=SUM+X*X
      SUM=SUM/XNP
      WRITE(6,2020) (IX7(I),I=1,16)
      WRITE(6,3555) SUM,CUM,CCUM,RHO1,XNP,X3150
      IF((J8.EQ.K4).AND.(IW.EQ.0)) X3150=X3150-LR
      WRITE(6,1003) 3333,3333,LR,X3150
1839  CONTINUE
      IF(J8.NE.K4) GO TO 1952
      X493=X493*F2242
      NWLM3=NWLM+1
      DO 1951 NN=NWLM3,NWLMT
      WLC(NN)=WLC(NN)*F2242
1951  WLM(NN)=WLM(NN)*F2242
1952  CONTINUE
      WRITE(6,1003) 3333,3333,LR,X3150,F2242,X493
1849  CONTINUE
1001  FORMAT(I12,20F6.1/(12X,20F6.1))
1003  FORMAT(3I10,7D12.4)
1004  FORMAT(12X,15F8.1)
1007  FORMAT(I12,20F6.0/(12X,20F6.0))
2000  FORMAT(8G10.0)
2003  FORMAT(20I4)
2007  FORMAT(4(3I3,D11.3))
2020  FORMAT(8I10)
3004  FORMAT(I12,20I6/(12X,20I6))
3005  FORMAT(22I6)
3007  FORMAT(4(I12,2I4,D11.3))
3010  FORMAT(' ',12F10.1)
3555  FORMAT(8D15.7)
3560  FORMAT(' ',5(3I4,D12.3))
4002  FORMAT(I6,I12)
4003  FORMAT('1',5I6)
4500  FORMAT(I12,10D12.3/(12X,10D12.3))
5500  FORMAT(I12,13D9.3/(12X,13D9.3))
4501  FORMAT(I12,9D12.3)
4502  FORMAT(10D10.3)
4997  FORMAT('1',44X,'LENGTH IN FEET OF WATER COLUMN IN WELL BORE')
4998  FORMAT('1',64X,'MOLALITY')
4999  FORMAT('1',53X,'TEMPERATURE DEGREES CENTIGRADE')
5000  FORMAT('1',52X,'FIXED HYDRAULIC HEAD ')
5001  FORMAT('1',49X,'X AND Y DIMENSIONS OF GRID ELEMENTS')
5002  FORMAT('1',41X,'(DENSITY OF WATER RHO, IN GRAMS PER CUBIC CENTIMET
     1ER)-1.0 ')
5003  FORMAT('1',59X,'BASE ELEVATION')
5004  FORMAT('1',52X,'Z DIMENSION OF GRID ELEMENTS')
5005  FORMAT('1',54X,'HYDRAULIC CONDUCTIVITY')
5052  FORMAT('1',56X,'INITIAL HYDRAULIC HEAD')
5053  FORMAT('0',53X,'RECHARGE RATE YQ  (L*L*L/T)')
5054  FORMAT('1',57X,'SPECIFIC STORAGE')
5055  FORMAT('1',53X,'RECHARGE RATES Q2    (L*L*L/T)')
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5056  FORMAT('1',51X,'SIP CONVERGENCE PARAMETERS CHOSEN'//              MD13640
     18X,'L9',5X,'ITMAX',4X,'LENGTH',14X,'HMAX',14X,'XYFC',             MD13650
     215X,'ERR',14X,'WMAX',14X,'XX10')                                  MD13660
5057  FORMAT('1',51X,'SOR CONVERGENCE PARAMETERS CHOSEN'//              MD13670
     15X,'NSKP1',5X,'NSKP2',5X,'ITMAX',13X,'RELX1',13X,'RELX2',         MD13680
     214X,'COEF',15X,'ERR',14X,'XX10')                                  MD13690
5058  FORMAT('1',51X,'PCG(',I1,') CONVERGENCE PARAMETERS CHOSEN'//      MD13700
     17X,'ITMAX',9X,'ERR',8X,'XX10')                                    MD13710
5060  FORMAT('1',50X,'Z COMPONENT OF HYDRAULIC CONDUCTIVITY')           MD13720
5006  FORMAT('0',58X,'SIP COEFFICIENTS')                                MD13730
5007  FORMAT('0',56X,'WATCHING CONVERGENCE'//25X,'I,J,K, IS THE LOCATIONMD13740
     1 AT WHICH THE MAXIMUM CHANGE IN XX OCCURED.'/25X,'MAXIMUM RESIDUALMD13750
     2 ERROR  = THE MAXIMUM OVER ALL THE GRID ELEMENTS OF THE'/25X,'DIFFMD13760
     3ERENCE BETWEEN THE WATER FLOW RATE INTO AND OUT OF EACH GRID ELEMEMD13770
     4NT.')                                                             MD13780
5074  FORMAT(7X,' I    J    K',9X,'XX(I,J,K)',9X,'XX(I,J,K)',13X,'ERROR'MD13790
     1,3(12X,'K=',I4))                                                  MD13800
5072  FORMAT('0',72X,3(6X,'XX AT I=',I4))                               MD13810
5073  FORMAT(46X,'CHANGE IN',6X,'MAX RESIDUAL',3(12X,'J=',I4))          MD13820
5075  FORMAT('0','ITERATIONS USED =',I3,                                MD13830
     1'    MAXIMUM CHANGE IN XX BETWEEN LAST 2 ITERATIONS =',D11.3/     MD13840
     2' MAXIMUM RESIDUAL ERROR FOR GRID ELEMENTS NOT HAVING FIXED HYDRAUMD13850
     3LIC HEAD =',D11.3,'     TOTAL =',D11.3)                           MD13860
5008  FORMAT('1',59X,'FRESH WATER HEAD')                                MD13870
5009  FORMAT('1',59X,'WATER WELL HEAD')                                 MD13880
5010  FORMAT('1',37X,                                                   MD13890
     1'LAYERS UNDERLAIN BY A CONFINING BED HAVE 1, OTHER LAYERS 0')     MD13900
5101  FORMAT('1',46X,'WATER FLOW RATE IN NEGATIVE I DIRECTION')         MD13910
5102  FORMAT('1',46X,'WATER FLOW RATE IN NEGATIVE J DIRECTION')         MD13920
5103  FORMAT('1',46X,'WATER FLOW RATE OUT GRID ELEMENT BOTTOM')         MD13930
6000  FORMAT('0','LAYER   ROW')                                         MD13940
7000  FORMAT('1','TIME INTERVAL NUMBER',I3,'    DURATION =',D11.4,'     MD13950
     1   TOTAL ELAPSED TIME =',D11.4)                                   MD13960
7001 FORMAT('0',31X,'FLOW FROM',1X,'HEAD DEPENDENT',8X,'FLOW IN'        MD13970
     1,8X,'FLOW IN',7X,'FLOW OUT',7X,'FLOW OUT'                         MD13980
     2/18X,'RECHARGE',4X,'FIXED HEADS',6X,'DISCHARGE',9X,'BOTTOM'       MD13990
     3,12X,'TOP',9X,'BOTTOM',12X,'TOP'//)                               MD14000
8000  FORMAT('1',25X,'THE ELEVATION OF THE HYDRAULIC HEAD (MEASURED RELAMD14010
     1TIVE TO THE CENTER OF'/26X,'THE GRID ELEMENT) AT WHICH HEAD DEPENDMD14020
     2ANT DISCHARGE BEGINS')                                            MD14030
8001  FORMAT('1',20X,'(RATE OF LOSS (L/T))/(((HYDRAULIC HEAD)-(ELEVATIONMD14040
     1 AT WHICH DISCHARGE BEGINS)) (L))')                               MD14050
8002  FORMAT ('1',36X,'THE ELEVATION OF THE TOP OF THE UPPERMOST GRID ELMD14060
     1EMENT LAYER')                                                     MD14070
8003  FORMAT('1','PUMPING INTERVAL=',I5)                                MD14080
8004  FORMAT('1',45X,'DRAWDOWN DURING THIS PUMPING INTERVAL')           MD14090
8010  FORMAT('1',30X,'WATER FLOW RATES OUT OF FIXED HEAD GRID ELEMENTS')MD14100
8011  FORMAT('1',50X,'MASS FLOW BUDGETS'//5X                            MD14110
     1,'TOTAL RECHARGE TO MODELED REGION=',D13.4/5X                     MD14120
     2,'TOTAL FLOW FROM FIXED HEAD GRID ELEMENTS INTO MODELED REGION='  MD14130
     3,D13.4/5X,'TOTAL HEAD DEPENDANT DISCHARGE FROM MODELED REGION'    MD14140
     4,D13.4)                                                           MD14150
8012  FORMAT(' ',2I5,7D15.3)                                            MD14160
9010  FORMAT('1',50X,'HEAD DEPENDANT DISCHARGE RATES')                  MD14170
9011  FORMAT('0',4X,'TOTAL YQ RECHARGE RATE TO MODELED REGION=',D13.4/  MD14180
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     15X,'TOTAL Q2 RECHARGE RATE TO MODELED REGION=',D13.4/            MD14190
     25X,'TOTAL RECHARGE RATE TO MODELED REGION=',D13.4)               MD14200
9954  FORMAT('0',45X,'PERCENT SAND')                                   MD14210
9998  FORMAT('0',45X,'ERROR, CALCULATED MINUS MEASURED FRESH WATER HD') MD14220
9020  FORMAT('1','RATE OF RELEASE OF WATER FROM STORAGE BY LAYER,',1X,' MD14230
     1SUBSIDENCE ONLY, TOTAL')                                          MD14240
9021  FORMAT('1','TOTAL SUBSIDENCE FROM ALL LAYERS FOR THIS PUMPING INTEMD14250
     1RVAL')                                                            MD14260
9022  FORMAT('1',9X,'SUBSIDENCE SPECIFIC STORAGE MULTIPLICATION FACTOR')MD14270
9023  FORMAT('0','DEPTH BELOW INITIAL FRESH WATER HEAD AT WHICH SUBSIDENMD14280
     1CE BEGINS IS',D10.1)                                              MD14290
9030  FORMAT('1','PARAMETERS IN LOG FORM-FOR NEW PARAMO')               MD14300
9031  FORMAT('0','PARAMETERS ACTUAL VALUE',I10)                         MD14310
      RETURN                                                            MD14320
      END                                                               MD14330
      SUBROUTINE RDWRT                                                  MD14340
      IMPLICIT REAL*8 (A-H,O-Z)                                         MD14350
      COMMON WS,HMAX,RELX1,RELX2,COEF,ERR,XX10,DELT,ER5,SRZ,SUMRZ       MD14360
     1,ERRSV,XX10SV,JIT,NIJ10,NI11,NJ11,NK11,NNN,NSKP1                  MD14370
     2,NSKP2,ITMAX,ICNT,IEVP,IWR1,NW1,NW2,NW3,N320,NUM4                 MD14380
     3,L9,LENGTH,NK1115,IT01,IT15,ICRO,DDK,BBK                          MD14390
     4,NI10,NJ10,NK10,NI12,NJ12,NK12,SVIJ,VV40,SV35                     MD14400
      COMMON /XX/ XX                                                    MD14410
      COMMON /DT/ DT                                                    MD14420
      COMMON /VV/ VV                                                    MD14430
      COMMON /E2/ E2                                                    MD14440
      COMMON /F2/ F2                                                    MD14450
      COMMON /G2/ G2                                                    MD14460
      COMMON /YQ/ YQ                                                    MD14470
      COMMON /NT/ NT                                                    MD14480
      COMMON /DD/ DD                                                    MD14490
      COMMON /BB/ BB                                                    MD14500
      COMMON /ZZ/ ZZ                                                    MD14510
      COMMON /XXS/ XXS                                                  MD14520
C     COMMON /ALN/ ALN                                                  MD14530
C     COMMON /XXE/ XXE                                                  MD14540
C     COMMON /SV/ SV                                                    MD14550
      COMMON /HL/ HL                                                    MD14560
      COMMON /LB/ LB                                                    MD14570
      COMMON /MHD/ MHD                                                  MD14580
      REAL*4 DD(65078),BB(65078),ZZ(65078),XXS(65078),                  MD14590
     1YQ(65078),DDK(50),BBK(50),NT(94658),HL(65078)                     MD14600
      DIMENSION DT(65078),E2(65078),F2(65078),G2(65078),VV(65078)       MD14610
      DIMENSION XX(65078)                                               MD14620
     1,WS(10),MHD(65078),LB(94658)                                      MD14630
     2,IF1(10),IF2(10),NNT(250)                                         MD14640
      READ(5,1000) II,(IF1(I1),I1=1,9),(IF2(I2),I2=1,10)                MD14650
      WRITE(6,6000)                                                     MD14660
      DO 161 K=2,NK1115                                                 MD14670
      READ(5,2000) FCNTK,IVAR,IPRN,DDK(K),BBK(K)                        MD14680
      IF(IPRN.EQ.1) GO TO 158                                           MD14690
      IF(IT15.EQ.10) WRITE(6,7000) K,FCNTK                              MD14700
      IF(IT15.EQ.1) WRITE(6,1002) K,FCNTK                               MD14710
      IF(IT15.EQ.2) WRITE(6,1002) K,FCNTK,DDK(K),BBK(K)                 MD14720
158   DO 161 J=2,NJ11                                                   MD14730
```

```
      IJMI=NI10*(J-2)+NIJ10*(K-2)                                       MD14740
      IF(IVAR.EQ.1) READ(5,IF1) (NT(IJMI+I),I=2,NI11)                   MD14750
      DO 159 I=2,NI11                                                   MD14760
      IJ=IJMI+I                                                         MD14770
      X=1                                                               MD14780
      IF(IVAR.EQ.1) X=NT(IJ)                                            MD14790
159   NT(IJ)=X                                                          MD14800
      IF(ICRO.EQ.0) GO TO 1605                                          MD14810
      DO 13 I=2,NI11                                                    MD14820
      IJ=IJMI+I                                                         MD14830
      LBIJ=LB(IJ)                                                       MD14840
      IF(LBIJ.NE.-30000) NT(IJ)=LBIJ*1.D-5/FCNTK                        MD14850
13    CONTINUE                                                          MD14860
1605  IF(.NOT.((IPRN.EQ.0).AND.(IVAR.EQ.1))) GO TO 41                   MD14870
      DO 40 I=2,NI11                                                    MD14880
40    NNT(I)=NT(IJMI+I)                                                 MD14890
      IF(II.EQ.0) WRITE(6,IF2) J,(NNT(I),I=2,NI11)                      MD14900
      IF(II.EQ.1) WRITE(6,IF2) J,(NT(IJMI+I),I=2,NI11)                  MD14910
41    CONTINUE                                                          MD14920
      DO 42 I=2,NI11                                                    MD14930
      IJ=IJMI+I                                                         MD14940
42    NT(IJ)=NT(IJ)*FCNTK                                               MD14950
161   CONTINUE                                                          MD14960
1000  FORMAT(I4,19A4)                                                   MD14970
1002  FORMAT(I6,6D18.4)                                                 MD14980
2000  FORMAT(8G10.0)                                                    MD14990
4000  FORMAT(I6,I10)                                                    MD15000
6000  FORMAT('0','LAYER   ROW')                                         MD15010
7000  FORMAT('1',I6,6D18.4)                                             MD15020
      RETURN                                                            MD15030
      END                                                               MD15040
```

```
      SUBROUTINE SOLEQU (A,X,N)                                         SLQ0010
C                                                                       SLQ0020
C     THIS SUBROUTINE SOLVES A SET OF SYMMETRIC LINEAR EQUATIONS        SLQ0030
C                                                                       SLQ0040
      REAL*4A(50,50),X(50)                                              SLQ0050
      NM=N-1                                                            SLQ0060
      DO 85 I=1,NM                                                      SLQ0070
      IP=I+1                                                            SLQ0080
      DO 25 J=IP,N                                                      SLQ0090
      IF(ABS(A(I,I)).LT.1.E-8) GO TO 200                                SLQ0100
      A(I,J)=A(I,J)/A(I,I)                                              SLQ0110
 25   CONTINUE                                                          SLQ0120
      DO 50 J=IP,N                                                      SLQ0130
      DO 45 K=J,N                                                       SLQ0140
      IF(A(I,J)) 35,50,35                                               SLQ0150
 35   A(K,J)=A(K,J)-A(K,I)*A(I,J)                                       SLQ0160
      A(J,K)=A(K,J)                                                     SLQ0170
 45   CONTINUE                                                          SLQ0180
 50   CONTINUE                                                          SLQ0190
      X(I)=X(I)/A(I,I)                                                  SLQ0200
      DO 75 K=IP,N                                                      SLQ0210
      X(K)=X(K)-A(K,I)*X(I)                                             SLQ0220
 75   CONTINUE                                                          SLQ0230
 85   CONTINUE                                                          SLQ0240
      IF(ABS(A(N,N)).LT.1.E-8) GO TO 202                                SLQ0250
      X(N)=X(N)/A(N,N)                                                  SLQ0260
 100  DO 125 II=1,NM                                                    SLQ0270
      I=N-II+1                                                          SLQ0280
      DO 125 K=I,N                                                      SLQ0290
      X(I-1)=X(I-1)-A(I-1,K)*X(K)                                       SLQ0300
 125  CONTINUE                                                          SLQ0310
      AD=0.                                                             SLQ0320
      GO TO 130                                                         SLQ0330
 200  WRITE(6,1)                                                        SLQ0340
      WRITE(6,300) I, A(I,I)                                            SLQ0350
 300  FORMAT(//,40X,I5,E20.6)                                           SLQ0360
      GO TO 130                                                         SLQ0370
 202  WRITE(6,1)                                                        SLQ0380
      WRITE(6,300) N, A(N,N)                                            SLQ0390
      AD=2.                                                             SLQ0400
 130  RETURN                                                            SLQ0410
 1    FORMAT(//,8X,'BAD INVERSE, ZERO ON DIAGONAL')                     SLQ0420
      END                                                               SLQ0430
```

```
      SUBROUTINE PCG
      IMPLICIT REAL*8 (A-H,O-Z)
      COMMON WS,HMAX,RELX1,RELX2,COEF,ERR,XX10,DELT,ER5,SRZ,SUMRZ
     1,ERRSV,XX10SV,JIT,NIJ10,NI11,NJ11,NK11,NNN,NSKP1
     2,NSKP2,ITMAX,ICNT,IEVP,IWR1,NW1,NW2,NW3,N320,NUM4
     3,L9,LENGTH,NK1115,IT01,IT15,ICRO,DDK,BBK
     4,NI10,NJ10,NK10,NI12,NJ12,NK12,SVIJ,VV40,SV35
      COMMON /XX/ XX
      COMMON /DT/ DT
      COMMON /VV/ VV
      COMMON /E2/ E2
      COMMON /F2/ F2
      COMMON /G2/ G2
      COMMON /YQ/ YQ
      COMMON /NT/ NT
      COMMON /DD/ DD
      COMMON /BB/ BB
      COMMON /ZZ/ ZZ
      COMMON /XXS/ XXS
C     COMMON /ALN/ ALN
C     COMMON /XXE/ XXE
C     COMMON /SV/ SV
      COMMON /HL/ HL
      COMMON /LB/ LB
      COMMON /MHD/ MHD
      COMMON /E22/ E22
      COMMON /D2S/ D2S
      COMMON /XXSTR/ XXSTR
C  ^^^
      COMMON /XXSS/ XXSS
      REAL*4 XXSS(65078)
C  ^^^^
      DIMENSION E22(65078),D2S(65078)
      REAL*4 DD(65078),BB(65078),ZZ(65078),XXS(65078),
     1YQ(65078),DDK(50),BBK(50),NT(94658),HL(65078)
     2,XXSTR(65078)
      DIMENSION DT(65078),E2(65078),F2(65078),G2(65078),VV(65078)
      DIMENSION XX(65078)
     1,WS(10),MHD(65078),LB(94658)
C  IF YOU WANT TO SAVE STORAGE, REMOVE D2S AND E22 FROM THE DIMENSION
C  STATEMENT AND CARDS "76 CONTINUE" THROUGH "56 CONTINUE", AND "86
C  CONTINUE" THROUGH "662 CONTINUE". THIS REMOVES PCG METHODS SIPCG
C  AND SFPCG.
      ICNT=0
      DO 999 IC=1,ITMAX
      DO 1 IJ=1,NNN
      DT(IJ)=0
      E2(IJ)=0
      F2(IJ)=0
      G2(IJ)=0
1     VV(IJ)=0
C     Modification to PCG from VARDEN due to subsidence. Replaces
C     DO 16 and DO 2 loops immediately following this modification
      DO 998 IJ=2,N320
      XXIJ=XX(IJ)
```

```
      HLIJ=HL(IJ)
      XXSIJ=XXS(IJ)
      E2(IJ)=XXIJ
      SVIJX=SVIJ*LB(IJ)
      SVDT=SVIJX/DELT
      SVDT40=SVDT*VV40*(1-XXSTR(IJ))
      SVDT1=SVDT
      IF(XXSIJ.LT.HLIJ) SVDT1=SVDT40
      SVDT2=SVDT40
      IF(XXIJ.GT.HLIJ) SVDT2=SVDT
      SHODT=SVDT1*(XXSIJ-HLIJ)+SVDT2*HLIJ
      DELE=0
      IF(MHD(IJ).GE.1) GO TO 997
      DELE=SVDT2
      VV(IJ)=YQ(IJ)+SHODT
997   CONTINUE
      XXSS(IJ)=DELE
998   CONTINUE
C     End of modification
C      DO 16 IJ=2,N320
C      E2(IJ)=XX(IJ)
C      IF(MHD(IJ).GE.1) GO TO 16
C      SVIJX=SVIJ*LB(IJ)
C      DELY=(SVIJX/DELT)*XXS(IJ)
C     IF((IEVP.EQ.1).AND.(XX(IJ).GT.XXE(IJ))) DELY=DELY+ALN(IJ)*XXE(IJ)
C      VV(IJ)=YQ(IJ)+DELY
C16    CONTINUE
C      DO 2 IJ=2,N320
C      DELE=0
C      IF(MHD(IJ).GE.1) GO TO 19
C      SVIJX=SVIJ*LB(IJ)
C      DELE=SVIJX/DELT
C     IF((IEVP.EQ.1).AND.(XX(IJ).GT.XXE(IJ))) DELE=DELE+ALN(IJ)
C19    XXS(IJ)=DELE
C2     CONTINUE
      GO TO (71,71,71,74,74,74),NUM4
71    CONTINUE
      Y1=0
      Z1=0
      DO 51 IJ=2,N320
      IF(.NOT.(MHD(IJ).GE.1)) GO TO 777
      Y1=0
      Z1=0
      GO TO 51
777   CONTINUE
      IJP1K=IJ+NI10
      IJM1K=IJ-NI10
      IJKP1=IJ+NIJ10
      IJKM1=IJ-NIJ10
      IF(IJP1K.GT.NNN) IJP1K=NNN
      IF(IJKP1.GT.NNN) IJKP1=NNN
      IF(IJM1K.LT.1) IJM1K=1
      IF(IJKM1.LT.1) IJKM1=1
      X=DD(IJ)
      Y=BB(IJ)
```

```
      Z=ZZ(IJ)                                                          PCG1110
      EEIJ=-(X+Y+Z+DD(IJ+1)+BB(IJP1K)+ZZ(IJKP1))+XXSS(IJ)               PCG1120
      IF(MHD(IJ-1 ).GE.1) X=0                                           PCG1130
      IF(MHD(IJM1K).GE.1) Y=0                                           PCG1140
      IF(MHD(IJKM1).GE.1) Z=0                                           PCG1150
      Y1Z1=0                                                            PCG1160
      XP=X                                                              PCG1170
      YP=Y                                                              PCG1180
      F2X=F2(IJ-1)                                                      PCG1190
      F2Y=F2(IJM1K)                                                     PCG1200
      F2Z=F2(IJKM1)                                                     PCG1210
      IF(NUM4.EQ.1) GO TO 20                                            PCG1220
      JP1=IJM1K+1                                                       PCG1230
      KP1=IJKM1+1                                                       PCG1240
      IJMMN=IJ-(NIJ10-NI10)                                             PCG1250
      IF(IJMMN.LT.1) IJMMN=1                                            PCG1260
      WY1=0                                                             PCG1270
      WZ1=0                                                             PCG1280
      IF(F2Y.NE.0.0) WY1=Y/F2Y                                          PCG1290
      IF(F2Z.NE.0.0) WZ1=Z/F2Z                                          PCG1300
      XP=X-(WY1*Y1+WZ1*Z1)                                              PCG1310
      G2(IJ)=XP                                                         PCG1320
      YP=Y-WZ1*D2S(IJM1K)                                               PCG1330
      E22(IJ)=YP                                                        PCG1340
      Y1=-WY1*G2(JP1)                                                   PCG1350
      Z1=-WZ1*G2(KP1)                                                   PCG1360
      ZB=-WZ1*E22(IJMMN)                                                PCG1370
      D2S(IJ)=ZB                                                        PCG1380
      F21=F2(JP1)                                                       PCG1390
      F22=F2(KP1)                                                       PCG1400
      F23=F2(IJMMN)                                                     PCG1410
      P1=0                                                              PCG1420
      P2=0                                                              PCG1430
      P3=0                                                              PCG1440
      IF(F21.NE.0.0) P1=Y1*Y1/F21                                       PCG1450
      IF(F22.NE.0.0) P2=Z1*Z1/F22                                       PCG1460
      IF(F23.NE.0.0) P3=ZB*ZB/F23                                       PCG1470
      Y1Z1=P1+P2+P3                                                     PCG1480
20    CONTINUE                                                          PCG1490
      XF=0                                                              PCG1500
      YF=0                                                              PCG1510
      ZF=0                                                              PCG1520
      IF(F2X.NE.0.0) XF=XP*XP/F2X                                       PCG1530
      IF(F2Y.NE.0.0) YF=YP*YP/F2Y                                       PCG1540
      IF(F2Z.NE.0.0) ZF=Z*Z/F2Z                                         PCG1550
      F2(IJ)=EEIJ-(XF+YF+ZF+Y1Z1)                                       PCG1560
51    CONTINUE                                                          PCG1570
      GO TO 80                                                          PCG1580
74    CONTINUE                                                          PCG1590
      DO 54 IJ=2,N320                                                   PCG1600
      IF(MHD(IJ).GE.1) GO TO 54                                         PCG1610
      IJP1K=IJ+NI10                                                     PCG1620
      IJKP1=IJ+NIJ10                                                    PCG1630
      IF(IJP1K.GT.NNN) IJP1K=NNN                                        PCG1640
      IF(IJKP1.GT.NNN) IJKP1=NNN                                        PCG1650
```

```
      EEIJ= (-(DD(IJ)+BB(IJ)+ZZ(IJ)+DD(IJ+1)
     1+BB(IJP1K)+ZZ(IJKP1))+XXSS(IJ))
      IF(NUM4.LE.5) GO TO 539
      X=DD(IJ)
      F2X=F2(IJ-1)
      XF=0
      IF(F2X.NE.0.0) XF=X*X/F2X
      F2(IJ)=EEIJ-XF
      GO TO 54
539   F2(IJ)=EEIJ
54    CONTINUE
80    CONTINUE
      SPR=1.D-50
      ER5S=100
      L78=2.5*ER5S
C     L78=8
      DO 100 ITER=1,L78
      ICNT=ICNT+1
      IF(ICNT.EQ.ITMAX) GO TO 202
      SPP=0
      DO 3 IJ=2,N320
      IF(MHD(IJ).GE.1) GO TO 3
      IP1JK=IJ+1
      IJP1K=IJ+NI10
      IJM1K=IJ-NI10
      IJKP1=IJ+NIJ10
      IJKM1=IJ-NIJ10
      IF(IJP1K.GT.NNN) IJP1K=NNN
      IF(IJKP1.GT.NNN) IJKP1=NNN
      IF(IJM1K.LT.1) IJM1K=1
      IF(IJKM1.LT.1) IJKM1=1
      DDIJ=DD(IJ)
      DDIP1=DD(IP1JK)
      BBIJ=BB(IJ)
      BBIJP=BB(IJP1K)
      ZZIJ=ZZ(IJ)
      ZZIJK=ZZ(IJKP1)
      E2IJ=E2(IJ)
      DTIJ=(-(DDIJ+DDIP1+BBIJ+BBIJP+ZZIJ+ZZIJK)+XXSS(IJ))*E2IJ
     1+DDIJ*E2( IJ-1)+DDIP1*E2(IP1JK)
     2+BBIJ*E2(IJM1K)+BBIJP*E2(IJP1K)
     3+ZZIJ*E2(IJKM1)+ZZIJK*E2(IJKP1)
      DT(IJ)=DTIJ
      SPP=SPP+E2IJ*DTIJ
3     CONTINUE
      A1=SPR/(SPP+1.D-70)
      A2=A1
      IF(ITER.GT.1) GO TO 35
      A1=0
      A2=1.
35    SRZ=0
      SUMRZ=0
      ER5=0
      DO 4 K=2,NK11
      DO 4 J=2,NJ11
```

```
      DO 4 I=2,NI11
      IJ=I+NI10*(J-2)+NIJ10*(K-2)
      IF(MHD(IJ).GE.1) GO TO 4
      DX=A1*E2(IJ)
      X=XX(IJ)+DX
      XX(IJ)=X
      ADX=DABS(DX)
      IF(ADX.LT.ER5) GO TO 111
      IMX=I
      JMX=J
      KMX=K
      XXPMX=X
      ER5=ADX
111   CONTINUE
      X=VV(IJ)-A2*DT(IJ)
      SUMRZ=SUMRZ+X
      DSR=DABS(X)
      IF(DSR.GT.SRZ) SRZ=DSR
      VV(IJ)=X
4     CONTINUE
C     GO TO 202
      IF(IWR1.EQ.1) WRITE(6,1000) IC,ICNT,IMX,JMX,KMX,XXPMX,ER5,SRZ,
     1XX(NW1),XX(NW2),XX(NW3)
C     IF((ER5+ER5S).LT.ERR) GO TO 202
C    C OUT NEXT FOR NONLINEAR
      IF(SRZ.LT.XX10) GO TO 202
      IF((SRZ.LT.XX10).AND.(ITER.EQ.1)) GO TO 202
      ER5S=ER5
      SPRS=SPR
      SPR=0
      GO TO (81,81,81,87,83,85),NUM4
81    CONTINUE
      DO 10 IJ=2,N320
      IF(MHD(IJ).GE.1) GO TO 10
      IJM1K=IJ-NI10
      IJKM1=IJ-NIJ10
      IF(IJM1K.LT.1) IJM1K=1
      IF(IJKM1.LT.1) IJKM1=1
      B6=0
      Z6=0
      DDIJ=DD(IJ)
      BBIJ=BB(IJ)
      IF(NUM4.EQ.1) GO TO 21
      DDIJ=G2(IJ)
      BBIJ=E22(IJ)
      JP1=IJ-NI10+1
      KP1=IJ-NIJ10+1
      IF(JP1.LT.1) JP1=1
      IF(KP1.LT.1) KP1=1
      IJMMN=IJ-(NIJ10-NI10)
      IF(IJMMN.LT.1) IJMMN=1
      B6=0
      Z6=0
      F2J=F2(IJM1K)
      F2K=F2(IJKM1)
```

```
      IF(F2J.NE.0.D0) B6=DT(JP1)*G2(JP1)/F2J
      IF(F2K.NE.0.D0) Z6=(DT(KP1)*G2(KP1)+DT(IJMMN)*E22(IJMMN))/F2K
21    CONTINUE
      DT(IJ)=(VV(IJ)-DDIJ*DT(IJ-1)-BBIJ*(DT(IJM1K)-B6)
     1-ZZ(IJ)*(DT(IJKM1)-Z6))/F2(IJ)
10    CONTINUE
      DO 11 IJB=2,N320
      IJ=N320+2-IJB
      IF(MHD(IJ).GE.1) GO TO 11
      IP1JK=IJ+1
      IJP1K=IJ+NI10
      IJKP1=IJ+NIJ10
      IF(IJP1K.GT.NNN) IJP1K=NNN
      IF(IJKP1.GT.NNN) IJKP1=NNN
      XAD=0
      DDD=DD(IP1JK)
      BBB=BB(IJP1K)
      IF(NUM4.EQ.1) GO TO 22
      JM1=IJ+NI10-1
      KM1=IJ+NIJ10-1
      IF(JM1.GT.NNN) JM1=NNN
      IF(KM1.GT.NNN) KM1=NNN
      IJM1K=IJ-NI10
      IJPMN=IJ+(NIJ10-NI10)
      IF(IJM1K.LT.1) IJM1K=1
      IF(IJPMN.GT.NNN) IJPMN=NNN
      DDD=G2(IP1JK)
      BBB=E22(IJP1K)
      XAD1=0
      XAD2=0
      F2I=F2(IJ-1)
      F2J=F2(IJM1K)
      IF(F2I.NE.0.D0) XAD1=-(E22(JM1)*DT(JM1)+ZZ(KM1)*DT(KM1))*
     1G2(IJ)/F2I
      IF(F2J.NE.0.D0) XAD2=-ZZ(IJPMN)*E22(IJ)*DT(IJPMN)/F2J
      XAD=XAD1+XAD2
22    CONTINUE
      DTIJ=DT(IJ)-(DDD*DT(IP1JK)+BBB*DT(IJP1K)+ZZ(IJKP1)
     1*DT(IJKP1)+XAD)/F2(IJ)
      DT(IJ)=DTIJ
      SPR=SPR+DTIJ*VV(IJ)
11    CONTINUE
      GO TO 90
83    CONTINUE
      DO 63 IJ=2,N320
      IF(MHD(IJ).GE.1) GO TO 63
      F2IJ=F2(IJ)
      VVIJ=VV(IJ)
      DTIJ=VVIJ/F2IJ
      DT(IJ)=DTIJ
      SPR=SPR+DTIJ*VVIJ
63    CONTINUE
      GO TO 90
85    CONTINUE
      DO 651 IJ=2,N320
```

```
      IF(MHD(IJ).GE.1) GO TO 651
      DT(IJ)=(VV(IJ)-DD(IJ)*DT(IJ-1))/F2(IJ)
651   CONTINUE
      DO 652 IJB=2,N320
      IJ=N320+2-IJB
      IF(MHD(IJ).GE.1) GO TO 652
      IP1JK=IJ+1
      DTIJ=DT(IJ)-DD(IP1JK)*DT(IP1JK)/F2(IJ)
      DT(IJ)=DTIJ
      SPR=SPR+DTIJ*VV(IJ)
652   CONTINUE
      GO TO 90
C    ****** NEW SSOR METHOD*********
87    CONTINUE
      IF(HMAX.EQ.0.D0) HMAX=1
      DO 771 IJ=2,N320
      IF(MHD(IJ).GE.1) GO TO 771
      IM1JK=IJ-1
      IJM1K=IJ-NI10
      IJKM1=IJ-NIJ10
      IF(IJM1K.LT.1) IJM1K=1
      IF(IJKM1.LT.1) IJKM1=1
      C2S=DD(IJ)
      B2S=BB(IJ)
      A2S=ZZ(IJ)
      DT(IJ)=(VV(IJ)-C2S*DT(IJ-1)-B2S*DT(IJM1K)
     1-A2S*DT(IJKM1))/(F2(IJ)/HMAX)
771   CONTINUE
      DO 772 IJB=2,N320
      IJ=N320+2-IJB
      IF(MHD(IJ).GE.1) GO TO 772
      IP1JK=IJ+1
      IJP1K=IJ+NI10
      IJKP1=IJ+NIJ10
      IF(IJP1K.GT.NNN) IJP1K=NNN
      IF(IJKP1.GT.NNN) IJKP1=NNN
      DTIJ=DT(IJ)
     1-(DD(IP1JK)*DT(IP1JK)+BB(IJP1K)*DT(IJP1K)
     2+ZZ(IJKP1)*DT(IJKP1))/F2(IJ)
      DT(IJ)=DTIJ*HMAX
772   CONTINUE
      DO 773 IJ=2,N320
      DTIJ=DT(IJ)*(2-HMAX)/HMAX
      SPR=SPR+DTIJ*VV(IJ)
773   DT(IJ)=DTIJ
C    ******END SSOR METHOD*********
90    CONTINUE
      B6=SPR/SPRS
      IF(ITER.EQ.1) B6=0
      DO 5 IJ=2,N320
      E2IJ=DT(IJ)+B6*E2(IJ)
      IF(MHD(IJ).GE.1) E2IJ=0
      E2(IJ)=E2IJ
5     CONTINUE
100   CONTINUE
```

```
999   CONTINUE                                                         PCG3860
202   CONTINUE                                                         PCG3870
1000  FORMAT(' ',2I3,3I5,6D18.7)                                       PCG3880
      RETURN                                                           PCG3890
      END                                                              PCG3900
```

```
      SUBROUTINE SIP
      IMPLICIT REAL*8 (A-H,O-Z)
      COMMON WS,HMAX,RELX1,RELX2,COEF,ERR,XX10,DELT,ER5,SRZ,SUMRZ
     1,ERRSV,XX10SV,JIT,NIJ10,NI11,NJ11,NK11,NNN,NSKP1
     2,NSKP2,ITMAX,ICNT,IEVP,IWR1,NW1,NW2,NW3,N320,NUM4
     3,L9,LENGTH,NK1115,IT01,IT15,ICRO,DDK,BBK
     4,NI10,NJ10,NK10,NI12,NJ12,NK12,SVIJ,VV40,SV35
      COMMON /XX/ XX
      COMMON /DT/ DT
      COMMON /VV/ VV
      COMMON /E2/ E2
      COMMON /F2/ F2
      COMMON /G2/ G2
      COMMON /YQ/ YQ
      COMMON /NT/ NT
      COMMON /DD/ DD
      COMMON /BB/ BB
      COMMON /ZZ/ ZZ
      COMMON /XXS/ XXS
C     COMMON /ALN/ ALN
C     COMMON /XXE/ XXE
C     COMMON /SV/ SV
      COMMON /HL/ HL
      COMMON /LB/ LB
      COMMON /MHD/ MHD
      COMMON /E22/ E22
      COMMON /D2S/ D2S
      COMMON /XXSTR/ XXSTR
      DIMENSION E22(65078),D2S(65078)
      REAL*4 DD(65078),BB(65078),ZZ(65078),XXS(65078),
     1YQ(65078),DDK(50),BBK(50),NT(94658),HL(65078)
     2,XXSTR(65078)
      DIMENSION DT(65078),E2(65078),F2(65078),G2(65078),VV(65078)
      DIMENSION XX(65078)
     1,WS(10),MHD(65078),LB(94658)
      DO 889 IJ=1,NNN
      E2(IJ)=0
      F2(IJ)=0
      G2(IJ)=0
      VV(IJ)=0
889   DT(IJ)=0
      ICNT=0
      ICT=0
      ER5S=100
      DO 501 ITERO=1,60
      DO 499 I33=1,2
      I3=I33
      I4=1
      IF(L9.EQ.2) I4=I3
      IF(L9.LT.3) GO TO 195
      I3=1
      I4=I33
195   CONTINUE
      ICT=ICT+1
      IF(ICT.EQ.(LENGTH+1)) ICT=1
```

```
      ICNT=ICNT+1
      IF(ICNT.EQ.ITMAX) GO TO 202
      W=WS(ICT)
      JJM1=2*I3-3
      JJP1=-JJM1
      KKM1=2*I4-3
      KKP1=-KKM1
C  ACCOMPLISH EQUATIONS (10) AND (14) BY WEINSTEIN [1969]
      SRZ=0
      SUMRZ=0
      DO 100 KB=2,NK11
      ISF1=0
      IF((KB.GE.3).AND.(KB.LE.NK10)) ISF1=1
      DO 100 JB=2,NJ11
      ISF2=0
      IF((JB.GE.3).AND.(JB.LE.NJ10)) ISF2=1
      J=JB
      K=KB
      IF(I3.EQ.2) J=NJ12+1-JB
      IF(I4.EQ.2) K=NK12+1-KB
      DO 50 I=2,NI11
      IJF=I+NI10*(J-2)
      IJ=IJF+NIJ10*(K-2)
      IF(MHD(IJ).GE.1) GO TO 49
      IM1JK=IJ-1
      IP1JK=IJ+1
      IJM1K=IJ+JJM1*NI10
      IJP1K=IJ+JJP1*NI10
      IJP2K=IJ+NI10
      IJKM1=IJ+KKM1*NIJ10
      IJKP1=IJ+KKP1*NIJ10
      IJKP2=IJ+NIJ10
      IF(ISF1.EQ.1) GO TO 91
      IF(IJKM1.LT.1) IJKM1=1
      IF(IJKM1.GT.NNN) IJKM1=NNN
      IF(IJKP1.LT.1) IJKP1=1
      IF(IJKP1.GT.NNN) IJKP1=NNN
      IF(IJKP2.GT.NNN) IJKP2=NNN
      IF(ISF2.EQ.1) GO TO 91
      IF(IJM1K.LT.1) IJM1K=1
      IF(IJM1K.GT.NNN) IJM1K=NNN
      IF(IJP1K.LT.1) IJP1K=1
      IF(IJP1K.GT.NNN) IJP1K=NNN
      IF(IJP2K.GT.NNN) IJP2K=NNN
91    CONTINUE
      XXIJ=XX(IJ)
      HLIJ=HL(IJ)
      XXSIJ=XXS(IJ)
      Z=ZZ(IJ)
      B=BB(IJ)
      D=DD(IJ)
      F=DD(IP1JK)
      H=BB(IJP2K)
      S=ZZ(IJKP2)
      IF(I3.EQ.1) GO TO 42
```

```
      BS=B                                                              SIP1110
      B=H                                                               SIP1120
      H=BS                                                              SIP1130
42    CONTINUE                                                          SIP1140
      IF(I4.EQ.1) GO TO 43                                              SIP1150
      ZS=Z                                                              SIP1160
      Z=S                                                               SIP1170
      S=ZS                                                              SIP1180
43    CONTINUE                                                          SIP1190
C     Due to subsidence                                                 SIP1200
C     Next 8 lines modified from SIP from VARDEN                        SIP1210
C     SVDT2 replaces SVDT used before in equation for E                 SIP1220
      SVIJX=SVIJ*LB(IJ)                                                 SIP1230
      SVDT=SVIJX/DELT                                                   SIP1240
      SVDT40=SVDT*VV40*(1-XXSTR(IJ))                                    SIP1250
      SVDT1=SVDT                                                        SIP1260
      IF(XXSIJ.LT.HLIJ) SVDT1=SVDT40                                    SIP1270
      SVDT2=SVDT40                                                      SIP1280
      IF(XXIJ.GT.HLIJ) SVDT2=SVDT                                       SIP1290
      E=-(Z+B+D+F+H+S)+SVDT2                                            SIP1300
C     DH1=XXSIJ-HLIJ                                                    SIP1310
C     DH2=HLIJ-XXIJ                                                     SIP1320
C     IF(SVDT1.NE.SVDT2) SVDTA=(DH1*SVDT1+DH2*SVDT2)/(DH1+DH2)          SIP1330
C     IF(SVDT1.EQ.SVDT2) SVDTA=SVDT1                                    SIP1340
C     E=-(Z+B+D+F+H+S)+SVDTA                                            SIP1350
      LEV=0                                                             SIP1360
C     IF((IEVP.EQ.1).AND.(XXIJ.GT.XXE(IJ))) LEV=1                       SIP1370
      IF(LEV.NE.1) GO TO 44                                             SIP1380
C     ALNI=ALN(IJ)                                                      SIP1390
      E=E+ALNI                                                          SIP1400
44    CONTINUE                                                          SIP1410
      E2I=E2(IM1JK)                                                     SIP1420
      E2J=E2(IJM1K)                                                     SIP1430
      E2K=E2(IJKM1)                                                     SIP1440
      F2I=F2(IM1JK)                                                     SIP1450
      F2J=F2(IJM1K)                                                     SIP1460
      F2K=F2(IJKM1)                                                     SIP1470
      G2I=G2(IM1JK)                                                     SIP1480
      G2J=G2(IJM1K)                                                     SIP1490
      G2K=G2(IJKM1)                                                     SIP1500
      A2=Z/(1+W*(E2K+F2K))                                              SIP1510
      B2=B/(1+W*(G2J+E2J))                                              SIP1520
      C2=D/(1+W*(G2I+F2I))                                              SIP1530
      AC=W*(A2*E2K+B2*E2J)                                              SIP1540
      TG=W*(A2*F2K+C2*F2I)                                              SIP1550
      WU=W*(C2*G2I+B2*G2J)                                              SIP1560
      D2=E+(AC+TG+WU)-C2*E2I-B2*F2J-A2*G2K                              SIP1570
      D2=1/D2                                                           SIP1580
      E2(IJ)=D2*(F-AC)                                                  SIP1590
      F2(IJ)=D2*(H-TG)                                                  SIP1600
      G2(IJ)=D2*(S-WU)                                                  SIP1610
      RZ=YQ(IJ)-(XX(IM1JK)*D+XX(IP1JK)*F+XX(IJM1K)*B+XX(IJP1K)*H        SIP1620
     1+XX(IJKM1)*Z+XX(IJKP1)*S+XXIJ*E)                                  SIP1630
C     Next line added to SIP from VARDEN                                SIP1640
     2+SVDT1*(XXSIJ-HLIJ)+SVDT2*HLIJ                                    SIP1650
```

```
C     Next line was used by SIP from VARDEN
C    2+SVDTA*XXSIJ
C     Next line was used by SIP from VARDEN
C     IF(LEV.EQ.1) RZ=RZ+ALNI*XXE(IJ)
      SUMRZ=SUMRZ+RZ
      DSR=DABS(RZ)
      IF(DSR.GT.SRZ) SRZ=DSR
      VV(IJ)=D2*(RZ-A2*VV(IJKM1)-B2*VV(IJM1K)-C2*VV(IM1JK))
49    CONTINUE
50    CONTINUE
100   CONTINUE
C  EQUATIONS (10) AND (14) ARE NOW ACCOMPLISHED
C  ACCOMPLISH EQUATION (15) BY WEINSTEIN [1969]
      ER5=0
      DO 102 KB=2,NK11
      ISF1=0
      IF((KB.GE.3).AND.(KB.LE.NK10)) ISF1=1
      DO 102 JB=2,NJ11
      ISF2=0
      IF((JB.GE.3).AND.(JB.LE.NJ10)) ISF2=1
      J=NJ12+1-JB
      K=NK12+1-KB
      IF(I3.EQ.2) J=JB
      IF(I4.EQ.2) K=KB
      DO 62 IB=2,NI11
      I=NI12+1-IB
      IJ=I+NI10*(J-2)+NIJ10*(K-2)
      IF(MHD(IJ).GE.1) GO TO 61
      IP1JK=IJ+1
      IJP1K=IJ+JJP1*NI10
      IJKP1=IJ+KKP1*NIJ10
      IF(ISF1.EQ.1) GO TO 92
      IF(IJKP1.LT.1) IJKP1=1
      IF(IJKP1.GT.NNN) IJKP1=NNN
      IF(ISF2.EQ.1) GO TO 92
      IF(IJP1K.LT.1) IJP1K=1
      IF(IJP1K.GT.NNN) IJP1K=NNN
92    CONTINUE
      X=VV(IJ)-E2(IJ)*DT(IP1JK)-F2(IJ)*DT(IJP1K)-G2(IJ)*DT(IJKP1)
      DT(IJ)=X
      X=X*HMAX
      XXP=XX(IJ)+X
      XX(IJ)=XXP
      X=DABS(X)
      IF(X.LE.ER5) GO TO 111
      IMX=I
      JMX=J
      KMX=K
      XXPMX=XXP
      ER5=X
111   CONTINUE
61    CONTINUE
62    CONTINUE
102   CONTINUE
C  EQUATION (15) IS NOW ACCOMPLISHED
```

```
      IF(IWR1.EQ.1) WRITE(6,1000)
     1 ICNT,IMX,JMX,KMX,XXPMX,ER5,SRZ,XX(NW1),XX(NW2),XX(NW3)
      IF((ER5+ER5S).LT.ERR) GO TO 202
      IF(SRZ.LT.XX10) GO TO 202
      ER5S=ER5
499   CONTINUE
501   CONTINUE
202   CONTINUE
1000  FORMAT(' ',I3,3I5,6D18.7)
      RETURN
      END
```