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### CONVERSION FACTORS, VERTICAL DATUM, AND ABBREVIATED WATER QUALITY UNITS

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<td>pound per square inch (lb/in²)</td>
<td>6.895</td>
<td>kilopascal</td>
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**Sea level**: In this report, "sea level" refers to the National Geodetic Vertical Datum of 1929—a geodetic datum derived from a general adjustment of the first-order level nets of the United States and Canada, formerly called Sea Level Datum of 1929.

Chemical concentrations and seawater density are given in metric units. Chemical concentration is given in milligrams per liter (mg/L) and seawater density is given in grams per cubic centimeter (gm/cm³).
SYMBOLS AND DIMENSIONS

[Number in parentheses refers to the page or illustration where the symbol first appears or where additional clarification may be obtained. Symbols are also defined in the text.]

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<tr>
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<td>ε^2ω^{-1} is variance (40, 42).</td>
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NONLINEAR-REGRESSION FLOW MODEL OF THE GULF COAST AQUIFER SYSTEMS IN THE SOUTH-CENTRAL UNITED STATES

By

Logan K. Kuiper

ABSTRACT

Multiple-regression methodology was used to calibrate a time-dependent, variable-density ground-water flow model with subsidence of the deep, regional gulf coast aquifer systems in the south-central United States. The methodology was also used to help answer questions concerning model reliability. More than 40 different regression models having 2 to 31 regression parameters are used and detailed results are reported for 12 of the models. More than 3,000 values for grid-element volume-averaged head and volume-averaged hydraulic conductivity are used as observations in the regression models. Calculated prediction interval half widths, though perhaps inaccurate due to a lack of normality of the residuals, are smallest for those models having only four regression parameters. Because of this, and also because the root-mean weighted residual decreases very little with an increase in the number of regression parameters, the models having a small number of parameters are probably the most reliable.

The various models used show considerable overlap between the prediction intervals for shallow head and hydraulic conductivity of sand ($K_s$). Approximate 95-percent prediction interval half widths for volume-averaged freshwater head exceed 108 feet; for volume-averaged $\log_{10}(K_s)$, they exceed 0.89. All the models produce unreliable predictions of head and ground-water flow in the deeper parts of the aquifer system, including the amount of flow coming from the geopressed zone beneath the aquifer system. Truncating the domain of solution of one of the models to exclude that part of the system having a ground-water density greater than 1.005 grams per cubic centimeter does not appreciably change simulated heads or flow. Also, excluding that part of the system below a depth 3,000 feet below land surface, and setting the density to that of freshwater in the remaining shallow part of the domain of solution, does not appreciably change the results for head and ground-water flow from the model, except for locations close to the truncation surface.

The regression methodology allowed the testing of a wide range of models for the simulation of the aquifer system. It also provided estimates of the accuracy of results and a mechanism to determine sources of model error.
INTRODUCTION

A common approach used to determine the accuracy of a ground-water simulation model is to compare model-computed values and values of the hydraulic parameters used in the model with field observations of physical quantities. Such comparisons form the basis for model calibration, which is that process whereby these parameters are varied to obtain the best possible fit with the observed quantities (Konikow, 1978). Clearly, for such calibration to be possible, it is necessary to establish some criterion to decide whether a particular selection of regression parameters (regression model) gives a better or worse fit than some other selection of regression parameters. Furthermore, for calibrated models to be most useful it is necessary to be able to gage their reliability. Recent advances in regression modeling described by Vecchia and Cooley (1987), Cooley and others (1986), Cooley (1977, 1979, 1982), Neuman (1980), and Yeh and Yoon (1981) treat model calibration as a statistical procedure. These regression procedures provide the necessary fitting criterion and the estimates of model reliability.

The purpose of this report is to illustrate the application of a multiple regression methodology to help answer questions concerning model reliability, to use the method to calibrate a ground-water flow model for a thick regional aquifer system in the Gulf Coastal Plain of the south-central United States, and to develop an improved understanding of flow in the gulf coast aquifer systems.

First, the hydrologic and geologic setting of the gulf coast aquifer systems is described in detail to provide the background needed to understand the modeling effort. A short description of the flow model and the regression procedure for calibration follows. The next section gives formulae to determine the confidence of the model output and the regression parameters used in the model, and a procedure for selecting the model expected to give the best results. Following are sections on: model construction, preparation of observations, application of the model, model error, and analysis of residuals.

This report is one of several presenting the results of the Gulf Coast Regional Aquifer-System Analysis (RASA) study, one of many RASA studies which have been conducted by the United States Geological Survey since 1978, at which time the RASA program was initiated following a congressional mandate (Sun, 1986, p. 2). Professional Paper 1416 consists of several chapters dealing with various aspects of the ground-water flow system in the Gulf Coastal Plain of the south-central United States. Chapter B (Hosman and Weiss, 1991) and C (Weiss, 1992) present physical characteristics of geohydrologic units such as thickness, altitude of the top, and percentage of sand; Chapter D (Ackerman, 1993), E (Ryder and Ardis, in press), H (Martin and Whiteman, in press) and I (Arthur and Taylor, in press) present results of ground-water flow simulations for parts of the gulf coast aquifer systems using a grid element spacing of 5 mi. Chapter F (Williamson, in press) presents results of ground-water
flow simulations for the entire gulf coast aquifer systems using grid element spacing of 10 mi. The focus of this report is on regression methodology whereas chapter F of Professional Paper 1416 focuses on the predevelopment (prior to ground-water withdrawal) ground-water flow system and changes due to withdrawal of ground water. Simulations described in chapter F employ hydraulic conductivity estimates from regression analyses described in this report.

**DESCRIPTION OF STUDY AREA**

The Gulf Coast Regional Aquifer-System Analysis study area is approximately 290,000 mi² and consists of part of the states of Arkansas, Florida, Illinois, Texas, Louisiana, Mississippi, Tennessee, Kentucky, Alabama, and Missouri, as well as part of the Gulf of Mexico (fig. 1). The Mississippi River provides the main drainage from the land area. Other important streams are the Rio Grande, Colorado, Arkansas, and Sabine Rivers.

Three aquifer systems have been identified in the study area (Grubb, 1984): The Mississippi embayment aquifer system; its lateral equivalent, the Texas coastal uplands aquifer system; and the coastal lowlands aquifer system (fig. 1). For purposes of this report all three aquifer systems are treated as a unit and are usually referred to as the gulf coast aquifer systems, or simply aquifer system. Comparison among the aquifer systems and detailed discussions of each aquifer system have been presented by Grubb (1987), Williamson and others (1990), Ackerman (1989), Arthur and Taylor (1990), Brahana and Mesko (1988), Martin and Whiteman (1989), and Ryder (1988). The remainder of this section and the Geologic setting section were taken from Williamson and others (1990) with only slight modification.

The land-surface altitude in the study area varies from sea level to more than 800 ft (fig. 2). The dominant feature of the topography is the flat, low-lying Mississippi Alluvial Plain (Fenneman, 1938). The Plain generally lies south and west of the Mississippi embayment structural trough (fig. 3). The topography of the Mississippi embayment is asymmetrical in that the valley lies to the west side of the embayment and the topographically higher hills are mostly to the east side. The Mississippi River, an important feature of the hydrologic system, generally traverses the east side of the valley. The general slope of the land surface is toward the Gulf of Mexico and is incised by large stream valleys that are generally perpendicular to the coastline.

Much of the study area is humid and mean annual precipitation ranges from 20 in. near Mexico to more than 60 in. along the gulf coast of Louisiana, Mississippi, and Alabama. The average mean annual precipitation over the area is 48 in.
Figure 1.—Location of Gulf Coast Regional Aquifer-System Analysis study area and generalized outcrop of aquifer systems (from Hosman and Weiss, 1991).
Figure 2. Generalized average land-surface altitude and bathymetry (from Williamson and others, 1990).
Figure 4. Altitude of the base of the aquifer systems and area of occurrence of geopressure. (From Williamson and others, 1990.)
Figure 2. Generalized average land-surface altitude and bathymetry (from Williamson and others, 1990).
Figure 3. Generalized structural features and location of geohydrologic section (modified from Hosman and Weiss, 1991).
In part of the study area, actual evapotranspiration is limited by the amount of rainfall, which is less than potential evapotranspiration. In south Texas, potential evapotranspiration exceeds rainfall most of the year, especially in the summer and fall; most of the rainfall returns to the atmosphere in a short time. In the eastern part of the area, rainfall substantially exceeds potential evapotranspiration, providing abundant surface-water runoff. The mean annual unit runoff varies from less than 1 in/yr in the southwestern part of the area to more than 20 in/yr in the northern and eastern parts and averages about 15 in/yr (Gebert and others, 1987).

Nearly 10 Ggal/d of ground water was pumped from the gulf coast aquifer systems in 1980 (Mesko and others, 1990). This amount corresponds to approximately 0.9 in/yr spread over the approximately 230,000 mi² land portion of the study area. Major areas of irrigation are the Mississippi Alluvial Plain, southwestern Louisiana, and south Texas. More than 60 percent of the municipal and industrial pumpage for public supply and industry is withdrawn from the coastal lowlands aquifer system (fig. 1).

**Geologic Setting**

The sediments of the gulf coast aquifer systems were deposited during Cenozoic time. Changes in sea level and accompanying transgressions and regressions of the sea caused cyclical sedimentation alternating from predominately continental to predominantly marine environments of deposition. Deposition occurred in fluvial, deltaic, or shallow-marine environments, and the interbedded sequences are composed of sand, silt, and clay, with some gravel, lignite, and limestone. Beds of sediment crop out at the surface in roughly parallel bands that are younger progressively gulfward in a typical offlap sequence. The shifting of facies, both laterally and vertically, resulted in a complex interbedding of sediment types. In general, the more clastic continental deposits have higher permeabilities characteristic of aquifers.

The Gulf Coast geosyncline and the Mississippi embayment, which are the major structural features of the study area (fig. 3), largely control the pattern and thickness of sedimentation (fig. 4). These structural features were present prior to Cenozoic deposition, and were accentuated as the basins subsided and accommodated the increased sediment buildup. Except where affected by local uplifts, the general pattern of sedimentation is one of increasing thickness in a gulfward, downdip direction. Uplift features that affected the deposition patterns are the Sabine uplift, San Marcos arch, Monroe uplift, Pascola arch, Jackson dome, LaSalle arch, and Wiggins uplift. Downwarp features associated with greater sediment buildup are the Desha basin, East Texas basin, Houston embayment, Rio Grande embayment, and Terrebonne embayment.
EXPLANATION

SUBSURFACE CONTOUR—Shows altitude of the base of the Gulf Coast aquifer systems. Hachures indicate depression. Contour interval 1,000 feet. Datum is sea level

UPDIP LIMIT OF AREA WHERE GEOPRESSED ZONE TRUNCATES THE BOTTOM OF THE AQUIFER SYSTEM

Figure 4. Altitude of the base of the aquifer systems and area of occurrence of geopressure. (From Williamson and others, 1990.)
Faults are common throughout the area, although their effect on regional ground-water movement is not known. In general, fault throws are not great enough to offset the full thickness of hydrologic units described in this report, although individual beds could be offset. Much of the faulting has led to zones of grabens and horsts. Particularly pronounced zones near the perimeter of the Mississippi embayment and the gulf coast geosyncline are the New Madrid fault zone, the Luling-Mexia-Talco fault zone, and the Pickens-Gilbertown fault zone (fig. 3). The Reelfoot rift zone is within the Mississippi embayment and may provide a pathway for upward discharge of ground-water from underlying Paleozoic rocks. Numerous growth faults, which occur contemporaneously with deposition, exist farther gulfward.

Geopressed zones (fig. 4) have fluid pressures significantly greater than normal hydrostatic pressure, and are enclosed by zones of very low permeability. The most probable cause for the development of these abnormally high fluid pressures is restriction of the escape of fluid during sediment compaction, causing pressure buildup and undercompaction of sediments (Fertl, 1976, p. 16).

Salt domes occur throughout the gulf coast aquifer systems, particularly in belts near and roughly parallel to the present-day coastline. The source of the salt domes is the deeply buried Louann Salt of Jurassic age, which has risen as diapirs that penetrate varying amounts of Cenozoic strata. The structural effects of the domes are relatively localized. However, the domes may have a significant effect on water quality due to dissolution of salt by ground water.

The gulf coast aquifer systems were subdivided into 15 hydrogeologic units that correspond to layers in a digital model used for simulation of regional ground-water flow. Five of these are considered to be confining units within which there is almost no horizontal flow. Weiss and Williamson (1985) describe the principles and methodology used for subdividing the aquifer systems into the 15 hydrogeologic units that is based on a combination of lithologic and hydraulic information. Hosman and Weiss (1991) provide a detailed description of the units of the Mississippi embayment and Texas coastal uplands aquifer systems. Weiss (1992) gives a detailed description of the units of the coastal lowlands aquifer system. Table 1, from Williamson and others (1990), gives the relationship and numbering convention for the 15 model layers used in this study to the geology and the previously mapped units in other reports in this series. The 15 model layers are numbered 2 through 11 for permeable layers or aquifers and 13 through 17 for confining units as shown in table 1. In this study, the model layers are referred to by number. The model construction described in the following section gives a detailed description of the geometry of the 15 model layers.
TABLE 1.—Relation of geologic units, previously defined geohydrologic units, and layers used in regional flow model (from Williamson and others, 1990, table 2)

[Note: correlations shown here are generalized. Exact relations vary widely from place to place.]

Mississippi embayment and Texas coastal uplands aquifer systems

<table>
<thead>
<tr>
<th>Geologic Unit System</th>
<th>Geohydrologic units defined by previous studies</th>
<th>Gulf Coast Regional Aquifer-System Analysis</th>
</tr>
</thead>
<tbody>
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<td>Model layer number</td>
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[1/ The Midway confining unit was referred to as the coastal uplands confining unit and the Vicksburg-Jackson confining unit was referred to as the coastal lowlands confining unit by Grubb, (1984, p. 11).]

* Not present in the Texas coastal uplands aquifer system.
TABLE 1.—Relation of geologic units, previously defined geohydrologic units, and layers used in regional flow model (from Williamson and others, 1990, table 2)—Continued

*Coastal lowlands aquifer system*

<table>
<thead>
<tr>
<th>Geologic Unit</th>
<th>Geohydrologic units defined by previous studies</th>
<th>Gulf Coast Regional Aquifer-System Analysis</th>
</tr>
</thead>
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<td>Series</td>
<td>Group</td>
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<td>Pleistocene and Holocene</td>
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</table>

1/ The Midway confining unit was referred to as the coastal uplands confining unit and the Vicksburg-Jackson confining unit was referred to as the coastal lowlands confining unit by Grubb, (1984, p. 11).
Ground-Water Flow

The essential feature of the regional flow system is flow of ground water from upland recharge areas to discharge areas at lower altitudes on land or the sea floor. Differences from this simplified description are caused by the effect of a large volume of dense water that tends to flow beneath fresher water. Except for a small amount of water that may move upward from the geopressed zone into the aquifer system, little flow probably occurs through the aquifer system bottom. There is no lateral flow from beyond the landward boundary of the study area because the sediments that make up the aquifer systems pinch out. The potential for lateral flow exists along the Mexican border and along the north-south boundary of the study area in southern Alabama and western Florida. An arbitrary extension of the aquifer system thicknesses was made into Mexico because no data were available. The flows along the north-south boundary of the study area in southern Alabama and western Florida were ignored because of the minimal permeability of the sediments and the relatively short distance compared to the perimeter of the study area. Recharge from the uplands in south Texas may be limited by the availability of water, considering the fact that potential evapotranspiration exceeds precipitation at some locations. Flows are of course altered by pumping. The areal distribution of pumping from the 10 aquifers, model layers 2-11, for the year 1980 is shown in figure 5. The similarity between the water-table altitude and land-surface altitude is shown by comparing figure 6 with figure 2.

The presence of dense water in the system has a considerable effect upon ground-water flow. A large volume of water having a concentration of dissolved solids greater than seawater is contained within the aquifer systems (fig. 7). Sea water has a concentration of dissolved solids of about 35,000 mg/L and a density of 1.025 gm/cm³. The estimated depth to water with a concentration of dissolved solids the same as seawater is shown in figure 7. Estimates of dissolved-solids concentrations in ground water were made for about 18,000 sand beds in the study area (Weiss, 1987). About 66 percent of the beds have water with a dissolved-solids concentration greater than 10,000 mg/L (fig. 8). Although not a precise measure of the quantity of water dense enough to effect ground-water flow, the above indicate that much of the volume of water in the gulf coast aquifer systems has a density substantially greater than 1.0 gm/cm³.

Data Available for Hydraulic Head and Hydraulic Conductivity

More than 600,000 individual measurements of hydraulic head were available. When considered as observations of hydraulic head in a particular well at a particular instant in time, these hydraulic-head measurements are, in general, accurate; except for those cases where errors in recording were made, they usually have less than 0.5 ft of error. For each well, the hydraulic-head measurements were averaged over 2-year intervals to give a representative value of hydraulic head for the well for
EXPLANATION
GROUND-WATER PUMPAGE IN MILLION GALLONS PER DAY—Shading of symbol shows total pumpage from each 100-square-mile area from all layers.

TOTAL PUMPAGE, IN MILLION GALLONS PER DAY

- More than 50
- 25-50
- 10-25
- 5-10
- 2-5
- 1-2
- 0.5-1
- 0.2-0.5
- Less than 0.2

Figure 5. Ground-water pumpage from the gulf coast aquifer systems, 1980. (Modified from Williamson and others, 1990.)
Figure 6. Water-table altitude on shore and equivalent freshwater head at the sea floor offshore. (Modified from Williamson and others, 1990.)
Figure 7. Estimate of depth to water containing 35,000 mg/L dissolved solids.
Figure 8. Cumulative frequency distribution of concentration of dissolved solids in 18,000 sand beds in the gulf coast aquifer systems.
a given year centered within the 2-year interval. In this manner, the more
than 600,000 measurements were reduced to approximately 50,000 time-
averaged observations of hydraulic head at individual wells. In addition,
there were approximately 15,000 measurements of formation pressure from
drill-stem tests. These measurements were used to approximate point-
pressure head values. Because of the indirect procedure used, these point
pressure head values from drill-stem test can have errors much larger than
0.5 ft. As is generally the case, the numerical-flow model used in this
study produces an approximation to grid-element volume-averaged head values
that are not equal to values for head at a single point in the aquifer
system.

More than 6,700 determinations of hydraulic conductivity were available
(Prudic, 1991) from both specific capacity and pumping tests. As with the
observations of hydraulic head, most of these determinations of hydraulic
conductivity were made using water wells less than 1,000 ft deep.

Because the water-well based data on hydraulic head and hydraulic
conductivity were from relatively shallow depths, (and the point pressure-
head values from the drill-stem test were quite inaccurate, even though in
some cases occurring at considerable depth) the aquifer systems may be
characterized as having a paucity of data for all but relatively shallow
depths.

**NONLINEAR-REGRESSION FLOW MODEL**

Ground-water flow in the aquifer systems is assumed to be governed by
the general equations for variable-density flow (De Weist, 1969).

The domain of solution of the aquifer systems has as its top that
surface consisting of either the water table or land surface or sea bottom,
whichever is lowest. This surface is relatively static and is taken as a
fixed head surface of the domain of solution. The movement of water
outside the domain of solution, such as the flow of water moving downward
to the water table, is not simulated. The bottom of the domain of solution
is a no-flow boundary when above a geologic unit which is assumed to have
no flow. When above the geopressured zone, flow may come from the
geopressured zone which is given a specific head.

Along the perimeter, the aquifer system domain of solution thins out
and has zero thickness at most locations, except along certain parts of the
perimeter offshore in the Gulf of Mexico. Here, nonzero thicknesses are
present below the sea floor, but are assumed to allow no flow due to the
presence of extremely low permeability clacareous clays.

Specific storage is taken to be constant throughout the entire aquifer
system domain of solution. Water density is assumed to be nearly constant
in time but to vary with location in the domain of solution.
Hydraulic conductivity varies within the domain of solution. Effective hydraulic conductivity at any location is determined (Desbarats, 1982) from the hydraulic conductivities of the clay and sand at that location and the relative amounts of each of these components as determined from data. The hydraulic conductivities of the clay and sand components are allowed to vary, but use is made of existing data for these conductivities and the manner in which they decrease with depth.

Although the top of the domain of solution has a fixed head, a maximum allowable recharge from that surface is imposed. This maximum value ranges from 2 in/yr to 12 in/yr, decreasing towards the south.

Modeling Methodology and Construction

Ground-water flow in the aquifer systems is assumed to be governed by the following three-dimensional variable-density time-dependent flow equations (Kuiper, 1983). The general vector form of Darcy's equation (De Wiest, 1969; Bear, 1979; Kuiper, 1983), is

$$ q = -K[\nabla h + (\rho/\rho_0)\nabla z] \quad (L/T), \quad (1) $$

Here, $q$ is the specific discharge of the fluid, and $h$ is pressure head equal to $P/\rho_0 g$, where $\rho_0 = 1 \text{ gm/cm}^3$, $g$ is the acceleration of gravity, and $P$ is pressure. Equivalent freshwater head, equal to $(h+z)$, will be referred to simply as head in the following text. The quantity $z$ is vertical distance measured upward above a datum, $\rho = \rho(x,y,z)$ is the density of the fluid and is assumed to vary with time so slowly that its time dependence is ignored (Kuiper, 1983). Equivalent freshwater head is approximately equal to hydraulic head $(P/\rho_0 g + z)$ when fluid density is approximately equal to $\rho_0$. The hydraulic conductivity tensor $K = K(x,y,z)$ is given by

$$ K = k\rho_0 g/\mu, \quad (2) $$

where $k$ is the intrinsic permeability tensor of the porous medium, and $\mu$ is the dynamic viscosity of the fluid. The mass balance equation (De Wiest, 1969) is

$$ \nabla \cdot (\rho q) = -\rho S_s \partial h/\partial t - Q \quad (M/L^3\ T), \quad (3) $$

where $S_s(x,y,z)$ is specific storage, $t$ is time, and $Q(x,y,z,t)$ is the rate of discharge of fluid per unit volume due to pumping. When equation (1) is substituted for $q$ of equation (3), the governing flow equation for $h(x,y,z,t)$ is obtained.
Equation (3), when discretized, forms a matrix equation to be solved for the grid-element volume-averaged heads corresponding to each of the grid elements in the domain of solution. This matrix equation is used to formulate the regression equation for the regression technique (Cooley, 1982). The hydraulic parameters of the aquifer systems, such as specific storage and hydraulic conductivity, and also other unknowns such as specified head or flow on the domain of solution boundary, are specified in some chosen manner by the regression parameters of the regression technique.

The discretization of the domain of solution is defined vertically by the 15 model layers. Grid elements with a constant 10-mi spacing define horizontal discretization. The spacing is constant because of the use of finite-differences which require that grid element rows and columns follow straight lines, and because the regions of the aquifer system that would benefit from small grid element size are numerous and are spread over the aquifer system in an irregular manner. The 10-mi spacing is the smallest allowable, given computer-time constraints.

The total number of grid elements in the domain of solution of the aquifer system is \( N = 29,345 \). The grid elements are divided into 10 model layers mentioned earlier and numbered 2 through 11 from bottom to top. Layer 1 is for the geopressured zone. A very thin layer, model layer 12, is located on the top of all of the other layers. This 12th model layer represents the top several feet of the aquifer system. Maximum areal extent and outcrop areas of the hydrogeologic units represented by the model layers are shown in figure 9. The grid element row and column numbers are also shown. Five layers representing confining units mentioned previously and numbered 13 through 17 are interbedded between model layers 4 through 9. The layers representing confining units provide resistance to vertical flow between the adjacent aquifers above and below but do not have any horizontal flow themselves, nor do they store water. The layers that represent confining units do not have grid elements (Kuiper, 1985) and the associated approximating equations do not directly involve head in the confining units. The areal extent of model layers 2 through 11 are shown in figures 10 through 19, and the areal extent of model layers 13 through 17 which represent confining units are shown in figures 20 through 24. The vertical relation of aquifers and confining units across the central part of the study area is shown in figure 25.

Boundary conditions are either specified-flow or specified-head. Specified-flow boundaries are always no-flow boundaries. There are two separate specified-head boundaries, the top of the domain of solution and the geopressured zone part of the bottom of the domain of solution. Each of these two surfaces has two regression parameters associated with it. The specified-boundary head on such a surface is equal to the sum of the product of an initial head distribution, to be described later, with a regression parameter associated with the surface, and an additional second parameter for the surface.
Figure 9. Maximum areal extent of modeled area and outcrop areas of hydrogeologic units represented by model layers (modified from Williamson and others, 1990).
EXPLANATION

OUTCROP OR SUBCROP AREA--
Dashed where concealed

FRESHWATER HYDRAULIC-HEAD CONTOUR--
Hachures indicate depression.
Contour interval, in feet, is variable. Datum is sea level

DOWNDIP LIMIT OF LAYER 2

Figure 10. Areal extent of model layer 2 and simulated 1982 equivalent freshwater head from model 4.
Figure 11. Areal extent of model layer 3 and simulated 1982 equivalent freshwater head from model 4.
Figure 12. Areal extent of model layer 4 and simulated 1982 equivalent freshwater head from model 4.
Figure 13. Areal extent of model layer 5 and simulated 1982 equivalent freshwater head from model 4.
Figure 14. Areal extent of model layer 6 and simulated 1982 equivalent freshwater head from model 4.
Figure 15. Areal extent of model layer 7 and simulated 1982 equivalent freshwater head from model 4.
Figure 16. Areal extent of model layer 8 and simulated 1982 equivalent freshwater head from model 4.
Figure 17. Areal extent of model layer 9 and simulated 1982 equivalent freshwater head from model 4.
Figure 18. Areal extent of model layer 10 and simulated 1982 equivalent freshwater head from model 4.
Figure 19. Areal extent of model layer 11 and simulated 1982 equivalent freshwater head from model 4.
Figure 20. Areal extent of model layer 13. Represents confining unit between model layers 4 and 5.
Figure 21. Areal extent of model layer 14. Represents confining unit between model layers 5 and 6.
Figure 22. Areal extent of model layer 15. Represents confining unit between model layers 6 and 7.
Figure 23. Areal extent of model layer 16. Represents confining unit between model layers 7 and 8.
Figure 24. Areal extent of model layer 17. Represents confining unit between model layers 8 and 9.
Figure 25. Idealized diagram from central Texas to the edge of the Continental Shelf showing vertical relation of aquifers and confining units (modified from Williamson and others, 1990).
The bottom surface of the aquifer system domain of solution is either in the geopressed zone or on the top of a geologic unit which is assumed to have no flow. The significantly overpressured parts of the geopressed zone beneath the aquifer system in the model are shown in figure 26. The updip extent of the significantly overpressured area is approximately equal to the line where model layer 7 is partially truncated by the geopressed zone as shown by Weiss (1993). Because the geopressed zone presumably extends further offshore than the aquifer system, the offshore limit of the region shown in figure 25 is the edge of the Continental Shelf. A layer of geopressed zone grid elements lies beneath all of the other layers mentioned above and has the areal extent shown in figure 26. The specified geopressed zone head in the geopressed zone layer is equal to the product of 1,000 ft with a regression parameter, plus an additional second regression parameter. For the grid elements in the geopressed zone, the quantity $(K_2/b)$, where $K_2$ is grid element effective vertical hydraulic conductivity and $b$ is grid-element thickness, is equal to the second regression parameter. Note that vertical flow between two grid elements, denoted 1 and 2, is

$$(h_1 - h_2)(\text{area}) \left[ \frac{b_1}{(K_1)_z} + \frac{b_2}{(K_2)_z} \right]^{-1},$$

whence the choice of $(K_2/b)$ as a regression parameter for the geopressed zone. The two basic properties of a source of fluid adjacent to the domain of solution are the pressure of the source and its ability to actually deliver fluid if permitted by lowering the pressure and increasing the conductivity in the domain of solution adjacent to the fluid source. Because of the way the two parameters determine flow, they relate directly to these two basic properties.

The top of the aquifer system domain of solution is that surface consisting of either the water table, land surface, or sea floor, whichever is lowest. The top model layer, layer 12, has the areal extent shown in figure 10 and overlies the aquifer system. This model layer has a specified head equal to the product of (1) an approximation to the water-table altitude $h_w$ (Williams and Williamson, 1989) when on land, or equivalent freshwater head at the sea floor offshore, with (2) a regression parameter; plus a second regression parameter. The water-table altitude is coincident with the altitude of streams and lakes except when a layer of unsaturated material exists between a lake or stream bottom and the underlying water table. The two parameters function so as to allow for the possibility that the water-table altitude $h_w$ may have an error proportional to $h_w$ (likely assumption) and also a constant error.

The specific storage $S_s = S_s(x,y,z)$ is taken to be constant for the entire aquifer system domain of solution. This single value of $S_s$ is taken to be one regression parameter.
Figure 26. Areal extent of the significantly overpressured parts of the geopressured zone
The density $\rho(x,y,z)$ is given a separate fixed and measured value for each of the $N = 29,453$ grid elements. Density is assumed to be nearly constant in time. This approximation is valid when grid-element volume-averaged density values change very little during the time span being simulated (Kuiper, 1983; Weiss, 1982; Bennett, 1980). No regression parameters are assigned to $\rho(x,y,z)$. Because pumping is known sufficiently well, no regression parameters are assigned to pumping rate $Q(x,y,z,t)$.

The values for $K_x$, $K_y$, and $K_z$, the principal direction values of the hydraulic conductivity tensor $K$, are prescribed by subdividing the aquifer system domain of solution into a number of hydraulic conductivity zones, each of which contains many grid elements. For each zone or group of zones, two regression parameters are assigned, one for the hydraulic conductivity of the clay of the medium and one for the sand. Here, as elsewhere, the word clay is intended to mean fine-grained sediment including silt. The hydraulic conductivity of each of the components in each of the grid elements is given by the product of the appropriate regression parameter with a function of the depth of the center of the grid element below ground surface. Both of these functions, one for the clay component and one for the sand component, are described further on and give the rate of decrease of hydraulic conductivity with depth. The clay and sand component conductivity values for each grid element are then combined (Desbarats, 1987) with data on the relative amounts of clay and sand in each grid element to finally arrive at effective hydraulic conductivity values $K_z$, $K_x = K_y$ for each of the grid elements. If one wishes to consider the horizontally anisotropic case where $K_x = K_y$, a single additional regression parameter is assigned to $K_z/K_y$, assumed to be constant throughout the domain of solution. A detailed description of the procedure for prescribing $K_x$, $K_y$, and $K_z$, is given in a later section on hydraulic conductivity.

Effective vertical hydraulic conductivity $K_z$ for model layer 12 was adjusted to restrict the amount of recharge that flows into underlying model layers. On the basis of an approximation to precipitation less evaporation considerations the maximum allowable recharge rate was specified to vary linearly from 2 in/yr at row number 80 to 12 in/yr at row number 56, in figure 10. The remaining areas in row numbers greater than 80 and less than 56 were allowed a maximum of 2 and 12 in/yr, respectively. Individual layer 12 grid element values for $K_z$ were reduced from an otherwise constant value of $K_z$ for the layer. Only those few grid elements with a high water-table altitude in southern Texas, and several others near areas of heavy pumping, required adjustment.

The solution of equation (3) above is accomplished by obtaining the finite-difference approximating equations (Kuiper, 1983; Kuiper, 1985) using the implicit approximation for the time derivative. The set of $N-M$ simultaneous equations generated, where $N = 29,453$ is the total number of grid elements and $M$ is the number with a specified head, is solved at each time step and gives values for the $N$ heads at each of the time steps and at each of the grid elements not located on a specified-head boundary. All of the $N$ heads are regarded as dependent variables depending upon the
regression parameters. A measured value of head is assumed to equal computed head, as a function of the regression parameters, plus an error term called the residual. The vector equation expressing these relationships becomes the regression equation for the regression technique (Vecchia and Cooley, 1987).

Quantities other than the heads are also dependent upon the regression parameters. Possibilities include the parameters themselves, values of the hydraulic conductivity in the various zones mentioned, storage depletion as measured by subsidence, ground-water flow rates, and so forth. Thus, the regression equation may include these quantities, also, if they have observed values.

Values for grid-element volume-averaged head were matched for the years 1972 and 1982, and were produced by the numerical flow model by using four time intervals of length 30, 5, 5, and 5 years, respectively. Starting with a steady-state predevelopment zero pumping solution for the year 1937, heads were determined for the years 1967, 1972, 1977, and 1982. For each of these four time intervals, appropriate time-averaged pumping values were used as determined from pumping data. For the three 5-year time intervals 1967-72, 1972-77, and 1977-82, the pumping rates for the years 1970, 1975, and 1980 were used. The pumping rates for these years are close to the average pumping rates during the three time intervals. For the 30-year time interval from 1937 to 1967, an approximation to the average pumping rate during the interval was obtained using the 1970 location of pumped wells, but decreased in accordance with total pumping by layer for the years 1960 and earlier. The total pumping rate by layer for the years 1960, 1970, 1975, and 1980 is shown in figure 27.

Regression Model

The nonlinear regression model (Draper and Smith, 1981; Vecchia and Cooley, 1987) supposes that a set of observations \( (Y_i, i = 1, 2, \ldots, n) \) of the physical system are related to a \( p \times 1 \) vector of unknown regression parameters \( B \) through the stochastic model

\[ Y = f(B) + E, \quad (4) \]

where \( Y \) and \( E \) are \( n \times 1 \) random vectors, and the \( n \times 1 \) vector \( f(B) \) is the expectation of \( Y \). The integer \( p \) is the number of regression parameters. The observations \( Y_i \) are thought of as a realization of the components of the \( n \times 1 \) random vector \( Y \). The \( n \times 1 \) random error vector \( E \) is assumed to have the probability distribution

\[ E \sim N(0; \epsilon^2 \omega^{-1}), \quad (5) \]

where \( \omega \) is a known \( n \times n \) symmetric positive definite matrix and \( \epsilon \) is a scalar constant. In the text below we shall frequently refer to \( f(B) \) as being the numerical flow model. As explained above, it contains quantities
other than those coming from the numerical flow model as dependent variables.

Figure 27. Total pumping rate by layer for the years 1960, 1970, 1975, and 1980.
In the case where \( n \) becomes large without limit and a continuum of points exists, such as the continuum of points corresponding to a cartesian coordinate \((x,y,z)\), the vectors \( Y \) and \( E \) become random fields \( Y(x,y,z) \) and \( E(x,y,z) \), and the vector \( f(B) \) becomes a function \( f(x,y,z,B) \). As an example, figure 28 shows a single realization of random field \( Y(x) \) as a function of cartesian coordinate \( x \), where realization \( Y(x) \) is the volume average of head over a volume extending from the bottom of an aquifer to its top and having a given cross sectional area \( A \). A second example is the volume average of hydraulic conductivity rather than head. The value of realization \( Y(x) \) shown is the exact value of the volume average, even though only an approximation to this value could be obtained by measurement. Two different approximations to \( f(x,B) \) are shown in figure 28. The first \( f(x,B) \), denoted \( f_1(x,B) \), is coarse compared to \( f_2(x,B) \) and does not follow \( Y(x) \) as closely as \( f_2(x,B) \). Because \( f_1(x,B) \) and \( f_2(x,B) \) only approximate \( f(x,B) \), the expectations of \( E_1(x) = Y(x) - f_1(x,B) \) and \( E_2(x) = Y(x) - f_2(x,B) \), though perhaps small, are not zero. Because \( f_1(x,B) \) follows \( Y(x) \) more closely than does \( f_2(x,B) \), \( E_1(x) \) has a larger apparent variance than does \( E_2(x) \). Apparent variance, approximately equal to residual mean square (Draper and Smith, 1981, p. 34) is, in the case of individual observations \( Y_i, i = 1, \ldots, n \), defined to be:

\[
\frac{1}{n} \left[ \frac{1}{n} \sum_{i=1}^{n} (Y_i - f_i(B))^2 \right],
\]

where \( n \gg 1 \).

This demonstrates that the apparent variance of \( E_1 = Y-f_1(B) \) is dependent upon the chosen degree to which an approximate \( f_1(x,B) \) is capable of fitting realization \( Y \), the observations. For a given realization \( Y \), many different sets approximate \( f(B) \) and corresponding \( E \) can be chosen, each with a different apparent variance. As the chosen \( f(x,B) \) departs further from the true \( f(x,B) \) the agreement will decrease and apparent variance will increase. Apparent variance values are inflated relative to the true value for variance \( \varepsilon^2 \omega^{-1} \) that would obtain if the true \( f(x,B) \) were being used. Criteria for this lack of agreement can sometimes be used (Draper and Smith, 1981, p. 35) to determine if a chosen approximate \( f(x,B) \) is reasonably close to the hypothesized true \( f(x,B) \).

If measurement error is also present, then \( E \) also includes this error component, the size of which depends on the accuracy of the measurements used.
Values for volume average of head \([Y(x)]\)

Figure 28. Two potential regression models, \(f_1(x,B)\) and \(f_2(x,B)\), for describing the relation between cartesian coordinate \((x)\) and volume average of head \([Y(x)]\)
In this study apparent variance is used to approximate true variance \( \epsilon^2 \omega^{-1} \), the possibility of this procedure was indicated by Draper and Smith (1981, p. 34). A single approximation \( \omega_j \) is made for those \( \omega_{ij} \) corresponding to observations of a given type occurring in a zone \( j \) of the domain of solution. Accordingly \( \epsilon^2 \omega_j \), is approximated by

\[
\frac{1}{n_j} \left\{ \sum_{i=1}^{n_j} (Y_i - f_i(B)) \right\}^2,
\]

where \( n_j \) (for \( n_j > 1 \)) is the number of observations occurring in zone \( j \). The zones \( j \) were chosen so that those regions of the domain of solution with like variance were located in the same zone. Different types of observations had different zonations of domain of solution.

In practice, the number of observations is finite, so that in the example in figure 28, \( f(B) \) would approximately fit a series of points on the curve \( Y(x) \). In this study, when considering observations of head, area \( A \) mentioned above is 100 mi\(^2\), the horizontal area of the grid elements of the multilayered numerical flow model used to simulate the system. Where observations of head are available, average values over the 10 mi by 10 mi by layer thickness grid elements have been assigned to the centers of the elements. These grid element head observations are determined, as will be discussed in more detail, from observations of hydraulic head obtained from wells located in or near the grid element. The observed value of head for a given element, as well as the corresponding value for head produced by the numerical flow model, \( f(B) \), are considered to be volume averages of head over the grid element. Because of this, substantial measurement error occur in these grid-element volume-averaged heads. The flow model \( f(B) \) of this study, is likewise coarse, so that even in the absence of measurement error, \( f(B) \) differs substantially from the observations for any choice of \( B \).

Regression modeling requires determining the variance of \( E \). The apparent variance of \( E \), which is an approximation to the true variance of \( E \), depends upon both the degree to which \( f(B) \) fits the observations without measurement error and the measurement error of the observations. In some cases, \( f(B) \) may be structured with sufficient detail to fit the observations \( Y_i \) exactly. For example, in one method for treating hydraulic conductivity observations in this study (the method in model 9 below) the entire aquifer system domain of solution was divided into seven zones. Six of these zones had an observed value for zone volume-averaged hydraulic conductivity as estimated from individual point values of hydraulic conductivity from individual pump tests of wells within the zone. Flow model \( f(B) \) was structured to allow seven different values for hydraulic conductivity in the seven zones, and was thus capable of fitting the six observed values for hydraulic conductivity exactly. Accordingly, the variance of \( E \) for these six observations was determined using the formula for the variance of an average of quantities, these quantities being the individual aquifer-test values for hydraulic conductivity.
Prediction and Confidence Intervals

The numerical flow model, \( f(B) \), of equation (4) should be selected in such a manner that the model is capable of simulating the aquifer system as closely as possible. Many choices are available, not only with regard to the number and type of regression parameters in \( f(B) \), but in what manner they are incorporated into the flow model \( f(B) \). Some of these choices are mentioned above. As stated previously for example, the specific storage \( S_s(x,y,z) \) in \( f(B) \) has a single value throughout the domain of solution and is assigned only one regression parameter, but fluid density \( \rho(x,y,z) \) and pumping \( Q(x,y,z,t) \) are assigned no regression parameters. Hydraulic conductivity and specified boundary heads are assigned several regression parameters.

With regard to simulation, the confidence that one has in the head and flow values produced by the flow model is of primary importance, simply because head and flow measurements are of primary interest. Predictions of hydraulic conductivity and specific storage are really of interest only because of the way they affect head and flow. As such, they are of secondary importance. However, when the flow model \( f(B) \) is used to predict head and flow values in the future, or when the regression equation is extrapolated beyond the vicinity of the observations, then the confidence of the regression parameters might seem to be important because of the fact that head and flow depend upon the parameters and no observations of head and flow are available for comparison. Unfortunately, even if the regression parameters of \( f(B) \) are known well, some heads and flow rates may still have considerable error if \( f(B) \) has not been constructed in a suitable manner. For example, suppose \( f(B) \) has a single hydraulic conductivity value for the entire domain of solution, even though the hydraulic conductivity is variable. This single value may be thought of as the average hydraulic conductivity in the domain of solution. Even if it is shown that this so-called average hydraulic conductivity is known quite well, the heads and flow may still be known only very poorly because using a constant hydraulic conductivity is an inadequate approximation to the variable hydraulic conductivity distribution present. Very likely, the best information available on the confidence of heads and flow outside of the vicinity of the observations is the confidence of heads and flow within the vicinity of the observations. In conclusion, the confidence that one has in head and flow values in the vicinity of the observations is of primary interest in evaluating the ability of the flow model to simulate the aquifer system, past, present, and future. As will be shown later, it is sometimes possible to evaluate this confidence.
Equation (75) of Vecchia and Cooley (1987) gives an approximate prediction interval for \( Y_k \):

\[
Y_k^e = f_k(\hat{B}) \pm d_{1-\alpha}\sqrt{\hat{F}_k^{-1}(\hat{X}_k^T\omega\hat{X}_k^{-1}\hat{X}_k^T + \omega^{-1})^{1/2}}, \tag{6}
\]

where \( k \) may correspond to a location at which there is no observation. Here

\[
d_{1-\alpha}^2 = S(\hat{B})[(p+1)/(n-p)]F_{x}(p+1,n-p),
\]

where as mentioned previously, the number of observations \( n \) is the length of the vector \( Y \), and the number of regression parameters \( p \) is the length of the vector \( B \). The upper \( \alpha \) percentile of the F distribution with \( p+1 \) and \( n-p \) degrees of freedom is denoted by \( d_{\alpha}(p+1,n-p) \). The value of the vector of regression parameters \( B \) at which the sum of squared weighted residuals

\[
S(B) = [Y - f(B)]^T \omega [Y - f(B)]
\]

is a minimum, is denoted by \( \hat{B} \). The \( T \) superscript denotes the transpose of a matrix. The sensitivity matrix \( X \) is given by

\[
X_{ij} = \frac{\partial f_i(B)}{\partial B_j},
\]

and matrix \( X \) evaluated at \( B = \hat{B} \) is denoted \( \hat{X} \). The jth component of the \( 1 \times p \) vector \( X_k \) is \( \hat{X}_{k,j} \). \( \epsilon^2\omega^{-1} \) is the variance of \( F_k = Y_k - f_k(\hat{B}) \). If \( k \) is not an observation number, then \( \omega \) does not have to be diagonal and \( \omega_k \) is not an element of \( \omega \). In this case there is no correlation between \( Y_k \) and \( Y_i \), one of the \( n \) observations of vector \( Y \) of equation 4. The interval given by equation 6 is a \((1-\alpha)100\%\) prediction interval for \( Y_k \), the kth member of the \( n \times 1 \) random vector \( Y \). Thus, if the model is nearly linear, the probability is \( 1-\alpha \) that

\[
Y_k \leq Y_k^e(+)
\]

where \( Y_k^e(-) \) and \( Y_k^e(+) \) are the two values of \( Y_k \) given in equation 6. If the model is nonlinear, the probability \( 1-\alpha \) is approximate. As noted above \( k \) need not be contained within the observed set of quantities \( Y_i \). Thus, for example, equation 6 may be used for values of the head at locations where no observation occurs, provided that one is able to find a value for \( \omega \). If \( k \) corresponds to some distant future observation, then \( \omega_k \) cannot easily be determined. Replacing \( d_{1-\alpha} \) with \( (S(\hat{B})/(n-p))^{1/2} \) in equation (6) causes the term to the right of the sign, when squared, to give the variance of \( Y_k \).

Equation (45) of Vecchia and Cooley (1987) gives an approximate confidence interval for the regression parameter \( B_i \):

\[
B_i^e = \hat{B}_i \pm d_{1-\alpha}(i^T(\hat{X}_i^T\omega\hat{X}_i)^{-1}i)^{1/2}, \tag{7}
\]

Here \( d_{1-\alpha} = S(\hat{B})[p/(n-p)]F_{x}(p,n-p) \) where \( S(\hat{B}) \) and \( F_{x}(p,n-p) \) have been defined above. The \( p \times 1 \) vector \( i \) is a vector whose only nonzero component is the ith, which has the value unity. The two \( B_i^e \) extreme values given in equation (7) determine a \((1-\alpha)100\%\) confidence interval for \( B_i \), the ith member of the \( p \times 1 \) vector \( B \). Replacing \( d_{1-\alpha} \) with \( (S(\hat{B})/(n-p))^{1/2} \) in equation (7) causes the term to the right of the sign, when squared, to give the variance of \( B_i \).
Equations (6) and (7) may be useful in choosing between the many possible choices for f(B). The smaller the intervals in equations (6) and (7) are, the better the choice for f(B) is considered to be. Especially important is the choice for the number of regression parameters p, because of the distinct way that the two intervals increase with p. A comparison of over 40 different choices for f(B) with p varying from 2 to over 31 is presented in the application and results section.

Equations (6) and (7) require evaluations of S(B), X, and f(B) at B = \hat{B}. S(B) may be approximated by

\[ S(B) = \sum_{i=1}^{n} (Y_i - f_i(B))^2 \omega_i, \quad (8) \]

Approximate equation (8) has used \omega_i = \omega_{ii} and \omega_{ij} = 0 for i \neq j, in the exact expression for S(B) given previously. The minimization S(B) in equation (8) in the p members of B was carried out by the Levenberg-Marquardt algorithm (Gill and others, 1981) for the minimization of the sums of squares. This algorithm is iterative and finds successively smaller values of S(B) at a series of successive choices of B selected by the algorithm. During this iterative process it is interesting to evaluate to the left side of inequality (7) of Vecchia and Cooley (1987):

\[ \frac{[Y-f(B)]^T \omega^{1/2} P \omega^{1/2} [Y-f(B)]}{[Y-f(B)]^T \omega^{1/2} (I-P) \omega^{1/2} [Y-f(B)]} \leq \frac{(p/n-p)F_{\alpha}(p,n-p)}{\omega_i}, \quad (9) \]

where \( P = \omega^{1/2} X(X^T \omega X)^{-1} X^T \omega^{1/2} \).

Inequality (9) expresses a (1-\alpha)100-percent exact confidence region for B. As S(B) decreases with the successive choices for B, the left-hand side of equation (9) also commonly decreases. The contours of S(B) and the contours of the left-hand side of (9) as a function of B, coincide quite closely (Donaldson and Schnabel, 1987; Sundararaj, 1978; Wallace and Grant, 1977). Equation (9) with the equality sign may be used to solve for \( \alpha = \alpha_m \), with B = B_m, where m is the Levenberg-Marquardt iteration number. With this definition of \( \alpha_m \), the successive values for B, B_m m = 1,2,3,... lie somewhere on the boundary of the (1-\alpha_m) 100-percent confidence region for B. For successively larger values of m, the left-hand side of equation (9), evaluated at B = B_m, decreases to zero as does \( F_{\alpha_m}(p,n-p) \), and at the same time \( \alpha_m \) increases to unity from \( \alpha_0 > 0 \).
Depth Dependence of Hydraulic Conductivity

The hydraulic conductivity of the porous medium in the domain of solution of the aquifer system, which is composed of interbedded fine-grained and coarse-grained sediments, usually decreases with depth below land surface.

The hydraulic conductivity of the coarse-grained sediments or sands, usually decreases with depth due to decreasing porosity, but increases with depth due to higher temperature and the resulting decrease in fluid viscosity. The net outcome of these factors, however, is usually decreasing hydraulic conductivity with depth. The function $10^{-0.8d}$, where $d$ is measured in kilometers, was used to express the rate of the decrease of the hydraulic conductivity of sand with depth $d$ (Loucks and others, 1986; Lake and Carroll, 1986). Loucks and others (1986) presented data (fig. 29) on the hydraulic conductivity of sand from Tertiary deposits of the Texas Gulf Coastal Plain. The curve (constant) $10^{-0.8d}$, which is an approximation, is also shown. The exponent of this curve was assigned a regression parameter, but because of the shallow depth of most head observations, the parameter had such a large confidence interval that it was decided that the model would be more accurate if the curve $10^{-0.8d}$ was assumed.

The hydraulic conductivity of the fine-grained sediments, or clays, in the domain of solution of the aquifer system tends to decrease with depth because of compaction. Figure 30, taken from Neglia (1979), shows the decrease with depth of the hydraulic conductivity of various clay samples. The curve shown in figure 30, (constant)$10^{-1.167d + 0.083d^2}$ where $d$ is measured in kilometers, was formulated to approximate the measured values shown and was used in this study to give the rate of decrease of hydraulic conductivity of clay with depth. As with the decrease of hydraulic conductivity of sand with depth, a regression parameter included in the exponent had a very large confidence interval and was removed.

The hydraulic conductivity of clay or sand of the fine-grained or coarse-grained sediments in a grid element is found by taking the product of the regression parameter associated with the hydraulic conductivity zone (described below) containing the grid element with the appropriate depth function, where the depth chosen was the depth below land surface of the center of the grid element. The clay and sand components have separate regression parameters, and use the clay and sand depth functions described above respectively. The effective hydraulic conductivity tensor $K$ of the grid element will depend upon the shape, size, and distribution of the sand bodies in the grid element and also on the hydraulic conductivity of these sand bodies and the clay matrix surrounding them. Desbarats (1987) presented a statistically-based procedure for calculating grid element
Figure 29. Decrease of hydraulic conductivity of sand with depth. (Modified from Loucks and others, 1986.)
Figure 30. Decrease of hydraulic conductivity of clay with depth. (Modified from Neglia, 1979.)
effective hydraulic conductivity values. Figure 31 shows effective horizontal ($K_h$), and vertical ($K_z$) hydraulic conductivity values, as functions of $V_c$, the clay fraction of the grid element, for the case where $\log_{10}(K_c/K_s) = -4$, where $K_c$ and $K_s$ denote the hydraulic conductivity of the clay and sand components respectively. Then $K_h$ and $K_z$ can be calculated from:

$$K_h = \left[\frac{K_c}{K_s}\right]^{V_c a^{-1}}$$

$$K_z = \left[\frac{K_c}{K_s}\right]^{V_c a}$$

Here, $a$, related to the width to vertical thickness ratio of the sand bodies, is set equal to 4.0 or to the product of 4.0 and a regression parameter. Horizontal anisotropy is introduced by setting $K_x = K_h$ and by setting $K_y$ equal to the product of $K_h$ with a regression parameter. If anisotropy is not desired, then $K_x = K_y = K_h$ and no additional regression parameter is used.

Hydraulic Conductivity Zones

The hydraulic conductivity zones previously discussed in the Modeling Methodology section are cross referenced by layer and region. The layers are the model layers 2 through 11, top-layer 12, and model layers 13 through 17. The regions are shown in figure 32. Note that the regions specify horizontal position and the layers specify vertical position. The hydraulic conductivity zones are thus specified in three dimensions by cross referencing. A hydraulic conductivity zone does not exist for each of the model layers 2 through 17 and each of the 10 regions because the domain of solution does not extend to all combinations of layers and regions. The function of the hydraulic conductivity zones and the assignment of regression parameters to them allows the model to approximate the heterogeneity of either the clay or sand component in the domain of solution. A particular volume within the domain of solution may consist of several or many adjacent hydraulic conductivity zones. Assigning the same regression parameter, clay or sand whichever the case maybe, to all of the hydraulic conductivity zones within the particular volume causes the hydraulic conductivity of the clay or sand in the volume to be constant except as altered by the depth function for clay as explained previously.

The actual number of regression parameters that may be assigned to the hydraulic conductivity zones is variable. Two regression parameters could, for example, be assigned to all of the zones, the first regression parameter for the clay component in all of the zones and the second for the sand component in all of the zones. Or, for example, 11 regression parameters could be used: 10 for the clay components of the 10 regions and the 11th regression parameter for the sand component of all of the zones,
Figure 31. Effective hydraulic conductivity, equations (10) and (11), for $K_x/K_c=10^a$
Figure 32. Extent of the 10 regions used to define hydraulic conductivity in the study area. The model layers having hydraulic conductivity data are given in table 2, by region.
and so forth. The application and results section below lists several of the many combinations used. Because the top-layer heads are specified, only the vertical effective hydraulic conductivity in this layer has any effect on the heads. For convenience, the top layer is assigned $V_c = 1$ and this layer has no regression parameters for hydraulic conductivity of sand. Also, the model layers 13 through 17 are mostly clay and, thus, have $V_c = 1$ and have no regression parameters for hydraulic conductivity of sand because little sand is present.

Subsidence

Certain areas of the aquifer system are affected by land subsidence due to ground-water withdrawals. In the numerical flow model, subsidence caused by inelastic compaction of clays was modeled using the procedure of Leake and Prudic (1988). Specific storage is increased from values characteristic of elastic compaction of clays to values characteristic of clays during inelastic compaction and land subsidence. The procedure increases the specific storage by some factor $C_s$ when the grid-element volume-averaged head $h$ declines through values of $h < h_i$, where $h_i$ is the lowest grid-element volume-averaged head achieved in the recent past. This factor $C_s$ is unity, however, if $h$ has not decreased by more than some triggering value $\Delta h_s$ in the recent past. Recent past was taken to be all time since 1937, at which time a steady-state numerical flow model with zero pumping defined predevelopment conditions. The value $C_s$ was either set to 40, or to the product of 40 with a regression parameter. In the grid elements of model layers 2 through 6, the subsidence mechanism was not thought to be operative as a result of the absence of materials capable of further compaction, so that $C_s$ was fixed at unity. The triggering value $\Delta h_s$ was either set equal to 80 ft, or the to product of 80 ft with a regression parameter. The grid area-averaged subsidence for a particular 10-mi by 10-mi grid area located at ground surface is approximately equal to the total volume of the fluid removed from storage in all grid elements lying below the grid area at ground surface, divided by 100 mi$^2$. This approximation would be nearly exact if water were incompressible. The slight compressibility of water will cause the subsidence to be slightly less than as calculated above, when the volume of the fluid removed is measured at the same temperature and measure of the water at the point of removal.

Preparation of Observations of Head and Hydraulic Conductivity, $Y_i$

In this study the observations $Y_i$ included observations of head and hydraulic conductivity. The only other possible observations are flow and anthropogenic subsidence. As mentioned previously, recharge into the top of the aquifer system was restricted by adjusting the hydraulic conductivity of layer 12. This mechanism does not, however, involve any observations of flow $Y_i$. Remaining flow rates, including discharge upward from the aquifer system into layer 12 and also discharge from the geopressed zone, are known poorly so that they would have a very small influence on $S(B)$ because their weights $\omega_i$ in equation (8) would be small.
Land subsidence due to ground-water withdrawals can be observed and measured accurately, and measured values are available in locations where substantial subsidence occurs. However, subsidence was not included in the observations $Y_i$, because of the relatively small areas of those locations where measured values are available.

The point observations of both hydraulic head and hydraulic conductivity were volume-averaged for the grid elements and hydraulic conductivity zones respectively. These volume averages are the observations, $Y_i$, to which model-simulated values are compared.

**Grid Element Volume-Averaged Heads**

Approximately 50,000 point observations or measurements of hydraulic head were available. As mentioned previously, these approximately 50,000 values were obtained from time-averaging more than 600,000 individual measurements of hydraulic head in water wells. Accompanying each of these approximately 50,000 point observations was the year, well location, and usually the land-surface altitude. Any of these four items could, however, be in serious error so that the data needed considerable verification. Verification was done partially by persons other than the author and included correlations, such as land-surface altitude versus location, and other procedures. Upon receipt by the author these data were passed through a computer program that looked for anomalies in space and time. If an observation was not in correspondence with five or more adjacent observations in space and time, the observation was deleted. An observation was considered not in correspondence with a single adjacent observation if its hydraulic head was not within 70 ft of the adjacent observation and furthermore, not within 35 ft/mi of distance between the observation and the adjacent observation. Adjacent here means within in the same or an adjoining 10-mi grid element and within 10 years of time. Those point observations of hydraulic head passing verification, about 92 percent, were considered valid and were put into an averaging algorithm described below to form 3,107 grid-element volume-averaged heads.

In addition to the approximately 50,000 point observations of hydraulic head, there were approximately 15,000 measurements of formation pressure from drill-stem tests which were used to approximate point-head values (Lobmeyer, 1985). The resulting point-head values were culled of points in a geopressed condition by eliminating those with pressures outside of the interval of 0.38 to 0.55 (lb/in$^2$)/ft. The culled point-head data were then used to form grid element averaged heads for those grid elements with 10 or more point heads from drill-stem tests, using a simple unweighted averaging of the culled point heads. The resulting 586 grid-element volume-averaged heads based on drill-stem tests were identified so that they could be distinguished from the grid-element volume-averaged heads based upon point observations of hydraulic head from water wells, because they were located deeper in the aquifer system and were less accurate.
Averaging Algorithm

As mentioned previously, equations (1) and (3) are discretized in order to arrive at finite-difference approximating equations. These approximating equations have N heads associated with the N grid elements in the domain of solution. These grid element heads should be thought of as grid-element volume-averages of head. This volumetric average perhaps should be weighted somewhat in favor of the central portion of the grid element volume. Such an average is not directly observable but must be approximated from point observations of head within the volume of the grid element, or perhaps adjacent grid elements. An averaging algorithm was used to form this average from point observations. Included in the algorithm is a criterion for deciding if there is a sufficient density of point observations of head to justify the determination of a grid-element volume-averaged head. If there is a paucity of point observations, then no grid-element volume-averaged head is calculated, and the grid element does not have an entry for the set of observations \((Y_i, i = 1,2,\ldots,n)\) associated with equation (4). The averaged head calculated by the algorithm is linear in the point observation of head (Gamut, 1986):

\[
\hat{h}(t) = \sum_{i=1}^{\text{ns}} a_i \hat{h}_i,
\]

Here, \(\hat{h}(t)\) is the grid-element volume-averaged head, \(t\) is time, and the \(\hat{h}_i\) are point observations of head measured within the time interval \((t-\Delta t,t+\Delta t)\). The integer \(\text{ns}\) is the number of \(\hat{h}_i\) used to formulate \(\hat{h}\) and the \(a_i\) are a set of weights which tend to be larger when the head \(\hat{h}_i\) is located closer to the center of the grid element. No consideration is given to the vertical dimension, so that only horizontal distances are considered and the depths of point observations of head within a grid element are disregarded. The justification for this is that vertical dimensions are very small compared to horizontal dimensions and because \(\hat{h}\) was formulated from data \(\hat{h}_i\) within the same layer as that of the grid element for which \(\hat{h}\) is being sought.

The weights \(a_i\) are determined by following these steps:

1) Divide the polar coordinate \(\theta\) space surrounding the center of the grid element at which an averaged head is desired into eight octants of 45° each.

2) For each octant, determine the \(N_3 \hat{h}_i\) that are at the smallest distances \(r_i, i = 1,2,\ldots,N_3\), from the center of the grid element, where \(r_1 < r_2 < \ldots < r_{N_3}\).
N3 is chosen as desired. Eliminate those $h_i$ that are at distances $r > R$, where $R$ is a radius of choice. Note that there may be less than N3 points in the octant with $r < R$. Set all $r_i < R_0$ to $R_0$, a second radius of choice where $R_0 < R$. Let $g_j = r_1^j$, where $j$ denotes the octant number, $j = 1, 2, \ldots, 8$. Form the average

$$H_j = \frac{\sum_{i=1}^{N3} h_i/r_i^2}{\sum_{i=1}^{N3} 1/r_i^2}$$

for each of the octants. This calculation actually may involve up to N3 values $h_i$. If there are no $h_i$ in the octant with $r < R$, then no $H_j$ is found.

3) Determine $\hat{h} = \frac{\sum_{j=1}^{\ell} H_j/g_j^2}{\sum_{j=1}^{\ell} 1/g_j^2}$, where the summations are over the $\ell$ octants for which a $H_j$ has been found.

4) The value of $\hat{h}$ found in the previous step is kept for use if

$$\ell \left[ \sum_{j=1}^{\ell} 1/r_j^2 \right] > C(N3, R, R_0),$$

where $C(N3, R, R_0)$ is a coefficient of choice.

The use of radius $R$ is to prevent the use of data at very large distances from the grid element center. The use of $R_0$, in the step where those $r_i$ within an octant are set to $R_0$ if less than $R_0$, is to prevent a single point close to the center of the grid element from totally overwhelming the value of $H$, for that octant.

The use of $1/r^2$ weighting as opposed to some other power of $r$ requires some justification. In the case of a uniform distribution of data points in two dimensions, a weighting of $1/r$ gives equal weight to points at any radius. This is true because the area between $r$ and $r + \Delta r$ is $2\pi r \Delta r$ which when multiplied by the weighting factor $1/r$ gives $2\pi r \Delta r$. Thus data at very large values of radius $r$ ($r < radius < r + \Delta r$) are counted as heavily as those at small $r$. This is definitely not desired since we seek an average head $\hat{h}$ that is representative of those heads somewhat centrally located within the single grid element for which $\hat{h}$ is being found, not a uniformly weighted-average head over the entire region. Thus whatever the power $n$ should be in the weighting factor $1/r^n$, it is clear that it should definitely be greater than 1. A value of $n = 2$ was used. This value for $n$ would place twice as much emphasis on points that are twice as close. Several different values for $n$, all greater than 1.5 were used. For those values of $n$ used, little effect was found on the ability of the model to fit the data as measured by values of mean weighted residual and root mean square weighted residual.
Estimators of the type described above are intended for noisy data with correlation dependent on the separation distance between values of $h_i$ (Gamut, 1986). Such estimators are stable because they interpolate between the values of $h_i$, so that $h$ must lie between the smallest and largest values of $h_i \ i = 1, \ldots, s$. Note that $h$ is octally weighted regardless of the number of $h_i$ that may occur in a given octant, provided only that this number is at least 1. N3 was chosen to be 3, $C(N3,R,R_0)$ was $36(10 \text{ mi})^{-2}$, $R_0$ was 2 mi, and $R$ was 15 mi which is 1.5 times the horizontal dimension of a grid element which was 10 mi. The number of grid-element volume-averaged heads calculated was 3,107: 1,432 for the year 1972, and the remainder of 1,675 for the year 1982, both with $\Delta t = 1$ year. The location of the 1,675 grid-element volume-averaged heads for 1982 is shown in figure 33.

Logarithms of Hydraulic Conductivity of Sand

Grid-element volume-averaged heads, the preparation of which is explained above, form part of the set of n observations $Y_i; i = 1, 2, \ldots, n$. The remaining members of $Y_i$ are volume averages of the hydraulic conductivity of sand. The volumes used here are much larger than a single grid element volume and consist of the hydraulic conductivity zones mentioned previously. A total of 33 zones based on the logarithm of the hydraulic conductivity of sand for the sand component of model layers 2 through 11 were constructed by using the layer and region combinations for which hydraulic conductivity data are given in table 2. The point estimates of the logarithm of hydraulic conductivity of sand were derived from aquifer or specific-capacity tests. The geometric mean of these test data were calculated by Prudic (1991) for each layer by region. Preliminary values for these tests, which generally are about 30 percent less than those given by Prudic (1991), were used in this study and are given in table 2. The logarithm of the hydraulic conductivity of sand for a zone is calculated as the simple average of the point observations of the logarithmic values of hydraulic conductivity of sand within the zone, whether determined by aquifer or specific-capacity tests. For one of the models, model 9 below, the 33 zones based on the logarithm of the hydraulic conductivity of sand were combined into six groups, each with a corresponding zone-averaged logarithmic hydraulic conductivity of sand.

It was mentioned previously that the hydraulic conductivity of sand for a grid element $j$ in the numerical model $f(B)$ is given by $k_j = P_j10^{-0.8d_j}$, where $P_j$ is the hydraulic conductivity parameter for the grid element and $d_j$ is the depth in kilometers of the center of the grid element below ground surface. Thus, when $\log_{10}(k_j) = \log_{10}(P_j)-0.8d_j$ is averaged over a hydraulic conductivity zone containing $m$ grid elements $j$ but only a single value $P$ for

$$P_j, \ \frac{1}{m} \sum_{i=1}^{m} \log_{10}(k_j)=\log_{10}(P)-0.8d$$

is obtained where $d = \frac{1}{m} \sum_{j=1}^{m} d_j$. When viewed as an observation, $\frac{1}{m} \sum_{j=1}^{m} \log_{10}(k_j)$ is one of the observations of the logarithms of hydraulic conductivity of sand, $Y_i$, but when predicted by the model it is the corresponding $f_i(B)$. The single hydraulic conductivity parameter associated with the hydraulic conductivity zone containing the grid elements $j = 1, 2, \ldots, m$ is $P$. 58
Figure 33. Location of the 1,675 grid-element volume-averaged heads for 1982.
TABLE 2.—Mean hydraulic conductivity by model layer and region

[Layer numbers are given in table 1 and region numbers are shown in figure 31. Mean base 10 logarithm of sand hydraulic conductivity is measured in units of feet per day (ft/d); mean log_{10} hydraulic conductivity of sand is 1.481; average geometric mean hydraulic conductivity of sand is 30.3 ft/d]

<table>
<thead>
<tr>
<th>Layer</th>
<th>Region</th>
<th>Mean $\log_{10}$ sand hydraulic conductivity</th>
<th>Geometric mean sand hydraulic conductivity (ft/d)</th>
</tr>
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<tr>
<td>11</td>
<td>9</td>
<td>1.926</td>
<td>84.33</td>
</tr>
</tbody>
</table>
The total number of observations \( Y_i, i=1,2,\ldots,n \) was either 
\[ n = 3,107 + 33 = 3,140 \] 
or 
\[ n = (3,107+586) + 33 = 3,726. \]
Here, the number of observations of the logarithms of hydraulic conductivity of sand is 33. The number of grid-element volume-averaged heads from water-wells is 3,107; and the number of grid-element volume-averaged heads from drill-stem tests is 586. For model 9, there are 6 rather than 33 observations of the logarithms of hydraulic conductivity of sand.

**Residual Weighting**

In equation (5) \( \omega \) is a known \( n \times n \) symmetric, positive-definite matrix. The matrix \( \omega \) is assumed to be diagonal. The minimization of \( S(B) \) in equation (8) tends to cause \( Y_i-f_i(B) \) to be small when \( \omega_i \) is large, or equivalently, when the variance \( \epsilon^2 \omega_i^{-1} \) is small. Thus when the variance of an observable \( Y_i \) is small, that is, its value is known well, \( \hat{B} \) is found such that the model value \( f_i(\hat{B}) \) is close to the actual value \( Y_i \). The variance of \( E_i \), the \( i \)th member of the \( n \times 1 \) random vector \( E \), is \( \epsilon^2 \omega_i^{-1} \) where \( \epsilon \) is a common variance factor of choice. As explained in the modeling methodology section above, the true variance of \( E_i \), \( \epsilon^2 \omega_i^{-1} \), may be approximated by apparent variance. The \( \omega_i \) for the 3,107 values of grid-element volume-averaged head were separated into 10 groups, corresponding to the 10 model layers 2 through 11. Thus, \( \omega_i = \omega_s \), \( i = 1,2,\ldots,3,107 \), where \( s \) is the model layer containing the grid element \( i \) corresponding to observation \( Y_i \).

For \( s = 1,2,\ldots,10 \), the value of \( \epsilon^2 \omega_s^{-1} \) was set equal to \( \frac{1}{n_s} \sum_{i=1}^{n_s} (Y_i-f_i(\hat{B}))^2 \)

where \( n_s \) is the number of observations in layer \( s \). These \( \omega_s \) depend upon the number of regression parameters \( p \) making up the \( p \times 1 \) vector of parameters \( B \), and also upon the construction of \( f(B) \). The \( \omega_i \) corresponding to the 586 grid-element volume-averaged heads from drill-stem tests were given a separate single value for \( \epsilon^2 \omega^{-1} \) equal to \( \frac{1}{586} \sum_{i=1}^{586} (Y_i-f_i(\hat{B}))^2 \). This value was much larger than the values for \( \epsilon^2 \omega_s^{-1} \) above.

The \( \epsilon^2 \omega_i^{-1} \) for the 33 observations of the logarithms of hydraulic conductivities of sand, \( Y_i \), were given a single value. This single value of the \( \epsilon^2 \omega_i^{-1} \) was calculated as the average of 10 variances. Each of these 10 variances was determined as the variance of those \( Y_i \) corresponding to model layer \( s \), about their own mean. The value for \( \epsilon^2 \omega_i^{-1} \) obtained in this manner is approximately equal to the value for \( \epsilon^2 \omega^{-1} \) obtained by determining the variance of all of the 33 observations of the logarithms of the hydraulic conductivities of sand about their own single mean. In model 9, residual weighting for the six observations of the logarithms of the hydraulic conductivities of sand was done using six different values \( \epsilon^2 \omega_i^{-1} \) for variance. These variances were smaller than the single value of \( \epsilon^2 \omega^{-1} \) used for the 33 observations of the logarithms of the hydraulic conductivities of sand, as explained previously in the regression model section.
The above described estimation of the variances $\epsilon^2 \omega_i^{-1}$ for grid element volume-averaged head, made use of the values $Y_i - f_i(\hat{B})$. But to find $\hat{B}$ in $f(\hat{B})$ one needs to minimize $S(B) = \sum_{i=1}^{n} (Y_i - f_i(B))^2 \omega_i$ which itself depends upon the very weights $\omega_i$ which are being sought. This apparent predicament is solved by doing several minimizations of $S(B)$, putting the $\omega_i$ from a given iteration into $S(B)$ for the next iteration. For the values $\omega_i$, this process converges sufficiently in only two iterations.

Weighted mean square error $S(B) = \sum_{i=1}^{n} [Y_i - f_i(B)]^2 \omega_i$ may be written

$$\sum_{i=1}^{n} [Y_i - f_i(B)]^2 \omega_i, \text{ where } Y_i = Y_i(\omega_1/\omega_i)^{1/2}, f_i = f_i(\omega_1/\omega_i)^{1/2}, \text{ and } \omega_i = 0(1).$$

Thus, it is possible to normalize $Y_i$ and $f_i$ such that, for each of the three classes of observations: grid-element volume-averaged heads, from (1) water-well data and from (2) drill-stem tests, and (3) volume-averaged logarithms of hydraulic conductivity of sand; the weights $\omega_i$ are order unity. Normalizing heads from water wells, in conjunction with a selection for $\epsilon^2$, allowed the weights $\omega_i$ for the grid element volume-averaged heads to satisfy $0.4 < \omega_i < 1.7$ and $(1/3,107) \sum_{i=1}^{3107} \omega_i = 1$.

In addition, the single weight for the grid-element volume-averaged heads from drill-stem tests and also the single weight for the 33 observations of the logarithms of the hydraulic conductivities of sand are allowed to both be unity.

**APPLICATION OF REGRESSION FLOW MODELS AND RESULTS**

The principal value of the regression methodology is that it allows one to measure the accuracy of the predictions produced by a model.

The principal use of the regression flow models in this study is to give an indication of the accuracy of the predicted values for grid-element volume-averaged head, and the logarithm of hydraulic conductivity of sand values in the models. Also desired is an indication of the accuracy of the values obtained for the regression parameters. These indications of accuracy are obtained from the prediction interval half widths and confidence interval half widths, available in equations (6) and (7) respectively.

Equation (6) above gives an approximate prediction interval for the kth member of the random vector $Y$ of equation (4). The term in equation (6) containing the sensitivity matrix $X$ was, except for the six observations of
the logarithms of the hydraulic conductivities of sand of model 9, always less than 4 percent of $\omega_k^{-1}$. Dropping this term causes equation (6) to become:

$$\omega_k e_k^2 = d_1 - \omega_k^2 = S(\hat{B})D^2(p,n,\alpha),$$  \hspace{1cm} (14)

where

$$D^2(p,n,\alpha) = [(p+1)/(n-p)]F_\alpha(p+1,n-p).$$

The probability is approximately (1-$A$)100 percent that $f_k(\hat{B}) - e_k < Y_k < f_k(\hat{B}) + e_k$, if $f_k(\hat{B})$ is a linear model. This result is exact when $f(\hat{B})$ is a linear model, and the $\omega_k^{1/2}e_k$ have a normal distribution. For fixed $n$, equation (14) shows that $S(\hat{B})$ will have to decrease faster with $p$ than $D^2(p,n,\alpha)$ increases with $p$ in order that the prediction interval half width $e_k$ decrease with increasing $p$. Figure 34 shows plots of $D^2(p,n,\alpha)$ as a function of $p$.

Having considered the dependence of $\omega_k e_k^2$ upon $p$ with fixed $n$, let us now consider its dependence upon $n$ with fixed $p$. If $S(\hat{B})/(n-p)$ is written as $(1+u)e_2^2$, then equation (14) becomes

$$\omega_k e_k^2 = (1+u)e_2^2G(p,n,\alpha),$$  \hspace{1cm} (15)

where

$$G = (p+1)F_\alpha(p+1,n-p).$$

When $n \rightarrow \infty$ and $n \gg p$, it can be shown that $1+u = S(\hat{B})/[(n-p)e_2^2]-1$ (R.L. Cooley, U.S. Geological Survey, oral commun., 1989), so that when $n$ is large, $1+u$ changes very little with $n$. Furthermore, for $n > 1,000$ and $p \ll n$ as in this study, $G$ changes very little for any increase in $n$. Thus, $\omega_k e_k^2$ displays no strong dependence upon $n$.

**Description of Models Used**

All models have the basic features as given in the section on "Modeling Methodology and Construction." Each model has the same domain of solution. The differences between the models are: the degree and manner by which the domain of solution is partitioned into hydraulic conductivity zones; and the degree to which unknowns related to boundary conditions, the manner of the dependence of effective hydraulic conductivity on clay and hydraulic conductivity of sand, and subsidence, are parameterized. When an unknown is parameterized, an optimal value for the parameter is found by the model. If an unknown value is not parameterized, a fixed default value is used. Thus, models with a larger number of parameters are more flexible but not conceptually more complex.
Figure 34. Change in multiplier $[D^2(p, n, \alpha)]$ for prediction interval size with number of regression parameters.
Many different numerical flow models $f(B)$, with different numbers of regression parameters $p$ were used. Several of these are shown in Table 3. The regression parameters $b_i$ used in this table are defined as follows. The specified boundary head in the top-layer grid elements is equal to $h_{ub} + b_2$. Here, $h_{ub}$ is water-table altitude or equivalent freshwater head at the sea floor, as discussed previously. The specified boundary head in the geopressed zone, shown in Figure 26, is $1,000 \text{ ft} b_2$. The value of $(K_z/b)$ in the geopressed zone grid elements is equal to $b_4$. Regression parameter $b_5$ is equal to $.25a$. Quantity $a$ was discussed previously for use with equations (10) and (11) to calculate effective hydraulic conductivity. Regression parameter $b_6$ is $K_y/K_x$ for horizontal anisotropy. Regression parameter $b_7$ is the parameter when multiplied by 80 ft gives the subsidence triggering value $V_{h_a}$ discussed previously. Regression parameter $b_8$ is the parameter that when multiplied by 40 gives the subsidence specific storage multiplication factor $C_s$. Regresssion parameter $b_9$ is specific storage.

In Table 3, when $B_j$ occurs in the row starting with $b_i$, it indicates that $b_i$ is equal to $B_j$, the jth element of the $p \times 1$ regression parameter vector $B$. Note that more than one of the $b_i$ may be equal to a single one of the $B_j$. When no entry occurs in the row $b_i$, it indicates that the regression parameter $b_i$ is not used and that the default value applies. The default values for the regression parameters are $b_1 = 1$, $b_2 = 0$, $b_3 = 1$, $b_5 = 1$, $b_6 = 1$, $b_7 = 1$, and $b_8 = 1$. When $b_4$ is not used, $(K_z/b)$ for the geopressed zone grid elements is set equal to either a very large number or zero. When $b_4$ is very large, the geopressed zone provides no resistance to vertical flow, and all resistance is provided by the model layers lying above the geopressed zone. When $b_4$ is zero, no flow is allowed from the geopressed zone. In Table 3, entry 0 for $b_4$ denotes that $(K_z/b)$ is zero, "very large" denotes that $(K_z/b)$ is very large. Default values of regression parameters $b_i$ were selected to prevent biasing results in favor of few regression parameter models. Such a bias occurs when results from models with many regression parameters are used to arrive at default values of the parameters in a model with few parameters. For example, if a given model $f(B)$ with many regression parameters determines that $b_7$ (the regression estimate for $b_7$ corresponding to $B = \hat{B}$) should be 1.3, then in another model $f(B)$ with few parameters in which $b_7$ is not used, the default value of $b_7$ may not be set to 1.3, and so forth. The single value $b_9$ of the specific storage for the aquifer domain of solution, is always used and hence has no default value.

The entries in Table 3 below row $b_9$ give the assignment of regression parameters to the hydraulic conductivity zones. For example, model 3 has $B_1 = b_1 = b_3$ and $B_2 = b_9$. Regression parameter $B_3$ is assigned to the hydraulic conductivity of clay of all those hydraulic conductivity zones contained in model layers 2 through 11, top-layer 12, and also model layers 13 through 17 which represent confining units. Regression parameter $B_4$ is
### TABLE 3.—Regression parameters for models 1 through 12

(When \( B_j \) occurs in the row starting with \( b_j \), it indicates that \( b_j \) is equal to \( B_j \), the \( j^{th} \) element of the \( p \times 1 \) regression parameter vector \( B \). Note that more than one of the \( b_j \) may be equal to a single one of the \( B_j \).

Shading indicates that combination of models and parameters was not used or was not applicable; do, ditto; VL, is very large.)

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Clay hydraulic conductivity

| Region 1          |     |     |     |     |     |     |     |     |     |     |     | B3 and B4 |
| Region 2          |     |     |     |     |     |     |     |     |     |     |     | B7 |
| Region 3          |     |     |     |     |     |     |     |     |     |     |     | B8 |
| Region 4          |     |     |     |     |     |     |     |     |     |     |     | B9 |
| Region 5          |     |     |     |     |     |     |     |     |     |     |     | B10 |
| Region 6          |     |     |     |     |     |     |     |     |     |     |     | B11 |
| Region 7          |     |     |     |     |     |     |     |     |     |     |     |       |
| Region 8          |     |     |     |     |     |     |     |     |     |     |     |       |
| Region 9          |     |     |     |     |     |     |     |     |     |     |     |       |
| Region 10         |     |     |     |     |     |     |     |     |     |     |     |       |
## TABLE 3.—Regression parameters for models 1 through 12—Continued

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assigned to the hydraulic conductivity of sand of model layers 2 through 11. Top-layer 12, and confining units represented by model layers 13 through 17 have no sand and thus need no regression parameter assignment.

Table 3 is self explanatory except for the following. The regions mentioned are those of figure 31, and allow for a horizontal discretization as opposed to vertical discretization using the layers. For hydraulic conductivity of clay, model 7 uses $B_3$ and $B_5$ for layers 2 through 11, $B_3$ in regions 1 through 5 and $B_5$ in regions 6 through 10. $B_4$ and $B_6$ are used for layer 12, $B_4$ for regions 1 through 5, and $B_6$ for regions 6 through 10. For hydraulic conductivity of sand, model 9 uses a discretization of the domain of solution into seven zones, selected on the basis of hydrogeologic considerations. Each of these zones correspond to various combinations of the layers and regions. $B_5$ through $B_{11}$ are used for these seven zones. Model 12 uses $B_{17}$ through $B_{21}$ for layers 13 through 17.

The following is a short description of the models of table 3 in physical terms. The specific storage, the horizontal hydraulic conductivity of sand, and the vertical hydraulic conductivity of clay were allowed to vary within bounds and the values were determined as a result of the regression model and are tabulated in table 4. Other factors such as the head in the geopressed zone, the altitude of the water table, and head decline necessary for the onset of land subsidence were allowed to vary in some models but were fixed in other models.

The value of specific storage for layers 2 through 11 ranged from $4.5 \times 10^{-7}$ to $8.0 \times 10^{-7}$, except that model 12 had a specific storage of $3.0 \times 10^{-8}$. The horizontal hydraulic conductivity of sand was uniform at about 30 ft/d for all layers in models 1, 2, 3, 4, 5, and 6. Both models 10 and 12 have different values of horizontal hydraulic conductivity of sand for each layer. The values range from about 8 ft/d for layer 3 in model 10 to 650 ft/d for layer 2 in model 12. Models 10 and 12 are similar in that layers that have large values of horizontal hydraulic conductivity of sand in one model also have large values in the other model. Two values of horizontal hydraulic conductivity of sand were used in model 6; the values are about 73 ft/d for layers 2 through 6 and 25 ft/d for layers 7 through 11. The horizontal hydraulic conductivity of sand varied by geographic region in models 7 and 11. The variability was greater in model 11 with 10 conductivity values ranging from 5 ft/d to 134 ft/d, than in model 7 with 2 values of about 24 ft/d and 68 ft/d. Six values of horizontal hydraulic conductivity of sand were used in model 9; the values range from about 6 ft/d to about 380 ft/d. These values were for combinations of layer and geographic region which, in general, correspond to area and layer combinations of Prudic (1991, p. 28), and the values used correspond closely to the geometric means presented by Prudic (1991).

The vertical hydraulic conductivity of clay was uniform for both the clay fraction within aquifers and for the confining units in models 1 through 5, and model 8; the values range from about $3.6 \times 10^{-4}$ ft/d to about 30 ft/d. In models 6, 7, 9, 10, and 12 the hydraulic conductivity of
TABLE 4.—Results from flow models 1 through 12

[The 95 percent prediction intervals for the observations are obtained from $v_{obs}^{1/2}$, the 95 percent confidence intervals for the parameters with entry a(b) are from ab⁻¹ to ab. Shading indicates that combination of models and parameters was not used or was not applicable; do, ditto]

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Parameters:

- $b_y$ for head in top layer
- $b_y$ for head in top layer (feet)
- $b_y$ for geopressured head
- $b_y$ for geopressured head (days⁻¹)
- $b_y$ for 25a in equations 10 and 11
- $b_y$ horizontal anisotropy
- $b_y$ for subsidence triggering head
- $b_y$ for subsidence specific storage
- $b_y$ specific storage (feet⁻¹)

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<td>0.898 (1.04)</td>
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<td>0.900 (1.04)</td>
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<td>0.901 (1.05)</td>
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<td>$b_y$ for head in top layer (feet)</td>
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<td>1.28 (1.92)</td>
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TABLE 4.—Results for flow models 1 through 12—Continued

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Sand hydraulic conductivity parameters
(Feet per day)

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<th>Region 4</th>
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clay within the aquifers had a different value than that for the confining units. In model 11 the vertical hydraulic conductivity of clay was uniform throughout each of the 10 geographic areas for clay both within aquifers and confining units. In model 12 a separate value of vertical hydraulic conductivity of clay was used for each aquifer and each confining unit; the values range from $4.8 \times 10^{-7}$ ft/d to about 3 ft/d.

The estimated water-table altitude was reduced by a factor ranging from 0.834 to 0.932, except that for model 1 the water-table was fixed at the values estimated by Williams and Williamson (1989). The flow of water to and from the water table was controlled by a hydraulic conductivity factor for the top layer and the value was the same as that for the vertical hydraulic conductivity of clay within aquifers for models 1 through 9.

The top of the geopressed zone was assumed to be a no-flow boundary in models 1, 2, 4, and 9. A head of 1,000 ft at the top of the geopressed zone was reduced by a factor in models 3, 5, 6, 7, 8, 10, 11, 12; the reduction factors ranged from 0.054 to 0.92. A parameter analogous to the "vertical conductance" of MacDonald and Harbaugh (1988) for controlling flow from the geopressed zone was determined for models 5, 8, 10, and 11; the values range from $9.4 \times 10^{-9}$ d$^{-1}$ to $1.6 \times 10^{-5}$ d$^{-1}$. The resistance to vertical flow for models 3, 6, 7, and 12 was due to clay in the sediments above the geopressed zone and no "vertical conductance" value was used for the geopressed zone.

The onset of land subsidence began after a head decline of 80 ft from predevelopment in models 1 through 7 and model 9. The head decline necessary for the onset of land subsidence ranged between 102 ft and 220 ft for models 8, 10, 11, and 12.

The coefficient used to adjust hydraulic conductivity and obtain "effective" hydraulic conductivity according to equations 10 and 11 was allowed to vary in models 8, 10, 11, and 12; the values ranged from 0.22 to 0.27.

The results from the flow models $f(B)$ defined in table 3 for $n = 3,107 + 33 = 3,140$, except for model 9 which has $n = 3,107 + 6 = 3,113$, are given in table 4. As mentioned previously, 3,107 is the number of grid-element volume-averaged heads from water wells, and 33 or 6 is the number of observations of the logarithms of the hydraulic conductivities of sand. Models using $n = 3,107 + 33 + 586 = 3,726$ and including the drill-stem data were also used and have similar results. These models were considered to be somewhat less reliable because of possible bias in the preparation of the drill-stem test data, and are thus not shown. Sources of bias are the proximity of wells to active oil fields and the indirect procedure for arriving at formation pressure from drill-stem tests. The second row of table 4 gives $\omega_k^{1/2}e_k$ of equation (14) for the 95-percent prediction interval for random variable $Y_k$. The number shown for $\omega_k^{1/2}e_k$ is
For the 33 observations of the logarithm of the hydraulic conductivities of sand, $Y_i$, the number shown for $\omega_k^{1/2}e_k$ needs to be divided by 149 to yield the desired prediction interval half width. The factor 149 is $(\omega_1'/\omega'^1)^{1/2}$ of the previous section. For flow model 3, for example, the probability is approximately 95 percent that $\log_{10}(29.9) - 0.8dd_k - (141/149) < Y_k < \log_{10}(29.9) - 0.8dd_k + (141/149)$, or that $\log_{10}(29.9) - (141/149) < (Y_k + 0.8dd_k) < \log_{10}(29.9) + (141/149)$. Here, $Y_k$ is the log of the volume-averaged logarithm of hydraulic conductivity (ft/d) of sand of zone $k$, one of the 33 zones based on the logarithm of the hydraulic conductivity of sand, or a zone in the vicinity of these zones. The average depth of hydraulic conductivity zone $k$, measured in kilometers, is $dd_k$. Each $Y_k$ has the same predicted value $\log_{10}(29.9) - 0.8dd_k$. The single hydraulic conductivity of sand parameter for each hydraulic conductivity zone $k$ is 29.9 ft/d. Note that the 33 observed values of $Y_k + 0.8dd_k$ shown in table 2 have the average value $1.481 = \log_{10}(30.3)$ and fall within the 95-percent prediction interval $\log_{10}(29.9) \pm (141/149)$. As with head, the 33 $Y_k$ are at relatively shallow depths.

Appearing in the third through sixth rows in table 4 are $(S(B)/(n-p)\epsilon^2) = (1+u)$, the mean weighted residual $(1/n) \sum^{n}_{i=1} [Y_i - f_i(\hat{B})]\omega_i^{1/2}$, the root-mean square-weighted residual $[(1/n) \sum^{n}_{i=1} (Y_i - f_i(\hat{B}))^2\omega_i]^{1/2} = [S(\hat{B})/n]^{1/2}$, and Chi$^2$ to be discussed in the next section. Also shown in table 4 are the values obtained for the regression parameter estimates (the elements of the $p \times 1$ vector $\hat{B}$), and also the confidence intervals of these values as obtained from equation (7). For example, model 3 in table 4 shows $\hat{b}_9 = \hat{B}_2$, the specific storage, equal to $0.45 \times 10^{-6}$ ft$^{-1}$ (the specific storage for pure water is approximately $1.5 \times 10^{-6}$ ft$^{-1}$). The 95-percent confidence interval for $b_9 = B_2$ is from $0.45 \times 10^{-6}(1.73)$ ft$^{-1}$ to $0.45 \times 10^{-6}(1.73)$ ft$^{-1}$. As with the other regression parameters, the confidence interval is expressed in multiplicative factor form because the parameter variable used in the minimization routine was the base 10 logarithm of the parameter rather than the parameter itself. The units of $b_1, \ldots, b_9$ are: 1, feet, 1, (day)$^{-1}$, 1, 1, 1, 1, and (feet)$^{-1}$. The units of the hydraulic conductivity of clay and sand parameters are ft/d.
Choosing the Best Regression Model

Draper and Smith (1981), chapter 6, suggest several methods for selecting the "best" regression equation or model. One procedure is to plot mean square residual as a function of the number of parameters p. Such a plot is shown in their figure 6.1, p. 298. Draper and Smith state that when mean square residual versus p ceases to decrease with increasing p, then an optimal choice for the number of parameters is the value of p where further decline in the mean-square residual is small. Values for root-mean square-weighted residual, \( (S(B)/n)^{1/2} \), are shown in table 4. Note that the values shown for \( (S(B)/n)^{1/2} \), and thus also the variances \( [S(B)/(n-p)]\omega^{-1} \) of the \( Y_k \) (recall that the variance of \( Y_k \) is \( [S(B)/(n-p)][X_k(X^T\omega X)^{-1}X_k^T+\omega^{-1}][S(B)/(n-p)]\omega^{-1} \), do not decrease appreciably with increasing p. For example, model 4 with p = 4 has a root-mean square-weighted residual of 41.5, but model 12 with almost eight times as many regression parameters has a root-mean square residual of 35.9, only 13 percent less. This would suggest that an optimal choice for p would be quite small and that p = 4 would perhaps be a reasonable choice.

A second trend shown in table 4 is that although the root-mean square-weighted residual generally decreases as the number of regression parameters p used increases, the prediction interval half width \( e_k \) increases as p increases if p is greater than 4. In other words, even though the fit of the 3,107 heads and 33 logarithm values of hydraulic conductivity of sand becomes better as the number of regression parameters increases, the certainty that one has in the predicted values for head and hydraulic conductivity decreases with increasing p, if p exceeds 4.

A related trend in table 4 is that the confidence that one has in the values obtained for the regression parameters tends to decrease (the 95-percent confidence intervals increase) as the number of parameters increases above 4. For example, model 3 predicts that the value of regression parameter \( B_4 \) for the hydraulic conductivity of sand of model layers 2 through 11 is 29.9 ft/d, and that there is 95-percent confidence that the actual value of \( B_4 \) is between 29.9/1.20 ft/d and (29.9)(1.20) ft/d. For model 10 with p = 18, which has the regression parameters \( B_9 \) through \( B_{18} \) for the hydraulic conductivity of sand for model layers 2 through 11, values for \( B_9 \) through \( B_{18} \) are not known with the same amount of confidence as in model 3. For example, model 10 has a value for \( B_{10} \) of 7.6 ft/d, but the 95-percent confidence interval is from 7.6(4.27)^{-1} ft/d to (7.6)(4.27) ft/d. Note that model 12, with 31 regression parameters, shows larger confidence intervals for the 10 parameters for the hydraulic conductivity of sand of model layers 2 through 11 than does model 10 with 18 regression parameters.

The above discussions regarding prediction and confidence interval behavior as a function of the number of parameters p tend to lend support to the choice of a small value for p, if values for the prediction and confidence intervals are assumed to be accurate or accurate relative to each other. This assumption would be in question if the residuals were to exhibit a considerable lack of normality. As discussed below, such lack of

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normality was in fact the case, so that the prediction interval half widths $e_k$ calculated and shown in table 4 may not be accurate. The regression parameter confidence intervals shown may also be inaccurate for the same reason. Thus using the widths of these intervals as a criteria for selecting an optimal $f(B)$ or $p$ may be in question.

Regression parameter variances, mentioned previously but not shown in table 4, increase as $p$ increases. For example, model 4 has 30.3 ft/d for the hydraulic conductivity of sand parameter ($P_s$). The variation of $\log_{10}(P_s)$ within one standard deviation of $\log_{10}(30.3)$ allows $P_s$ to vary in the range 30.3(1.06)$^{-1}$ to 30.3(1.06)ft/d. However, for model 12 layer 8 for example, the range is 125(1.18)$^{-1}$ to 1.25(1.18)ft/d. As with prediction and confidence interval half widths, regression parameter variances will be in error if there is a lack of normality.

The above results regarding: the very slow decrease of root-mean square-weighted residual with $p$ greater than 4, the increase of prediction interval half-widths with $p$ greater than 4, and the increase of regression parameter variances with $p$, lead to the conclusion that a model with a relatively small number of regression parameters would probably be the best choice.

**Analysis of Residuals**

The weighted residuals for all models used were tested to see if they differed significantly from those drawn from a normal distribution. Deviations from a normal distribution indicate the possible presence of factors that cause residuals to differ from those resulting solely from random fluctuations in the hydraulic conductivity or other properties of the porous medium, and (or) random measurement error. Significant deviations also preclude further model testing based on Chi$^2$, F, or t distributions. The standard Chi$^2$ test for normality (Croxton, 1953, p. 282-283) was used, for which the number of classes was approximated using the relation $5 \log(n) = 16$ (Panofsky and Brier, 1965, p. 4). The variance used for the test is given by

$$v^2 = \frac{1}{n} \sum_{i=1}^{n} (Y_i - f_i(B))^2 \sigma_i - \mu^2 = S(B) - \mu^2,$$

where the mean-weighted residual $\mu$ (shown in table 4) is given by

$$\mu = \frac{1}{n} \sum_{i=1}^{n} (Y_i - f(B)) \omega_i^{\frac{1}{2}}.$$

Normality of the weighted residuals would result in $Z_i = [(Y_i - f_i(B))\omega_i^{\frac{1}{2}} - \mu]/v$ being N(0,1). Because of the use of $v$ and also $\mu$, the number of degrees of freedom for the Chi$^2$ test is (number of classes) - 3 = 16 - 3 = 13. It may be preferable to test whether $Z_i$ with $\mu$ set equal to zero is $\sim N(0,1)$, because the model $f(B)$ should at least be able to produce $f_i(B)$ such that $\mu^2 < < v^2$. This approach would give essentially the same results because as table 4 shows, values for $\mu^2$ were much smaller than $v^2$. 

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For 13 degrees of freedom, the hypothesis of normality of the weighted residuals is rejected at the 1-percent significance level for \( \chi^2 > 27.7 \) (Hogg and Craig, 1965). That normality is rejected in every case by a considerable margin is shown in table 4. Typical results for the number of points falling into the 16 classes of equal probability on the \( N(0,1) \) curve show a nonuniform distribution, generally with a sizable number of excess points falling into the far right interval corresponding to the largest positive values for the residual defined as \( Y_i - f_i(B) \). For example, model 4 has the following numbers of points in the 16 classes: 131, 153, 185, 229, 248, 288, 276, 261, 184, 197, 179, 163, 134, 98, 109, and 305. A uniform spread indicating normality would have approximately 196 points in each interval.

At one point in the study, the observed grid-element volume-averaged heads \( Y_i \) were rejected when, for the original set of 3,140 predicted values \( f_i(B) \), they fell in the far right class of the 16 classes. After eliminating these approximately 300 observations, a new \( B \) was found. The new \( \chi^2 \) value for the new \( B \) was dramatically reduced from its original value. When an application of this culling procedure was applied to model 4, the number of observations was reduced by 314 from 3,140 to 2,826, and the value for \( \chi^2 \) fell from 324 to 65. The new model \( f(B) \) gave different results, because it was based on the new culled set of 2,826 observations. The root-mean square-weighted residual fell from 41.5 to 28.8. The value for \( \omega \) fell from 141 to 128. The hydraulic conductivity of clay and sand parameters, \( 0.362 \times 10^{-3} \) and 30.3 ft/d, changed to \( 0.407 \times 10^{-3} \) and 37.3 ft/d. When this culling procedure was applied to model 9 with 11 regression parameters, the value of \( \chi^2 \) fell from 260 to 56. Since the \( \chi^2 \) values 65 and 56 are greater than 27.7, the hypothesis of normality is rejected at the 1-percent significance level.

The possibility that the logarithms of the hydraulic conductivities of sand and the grid-element volume-averaged heads were incompatible, thus causing large \( \chi^2 \) values, was investigated by deleting the use of the logarithms of the hydraulic conductivities of sand as observables and determining the corresponding \( \chi^2 \) values. No decrease in \( \chi^2 \) values was found so that this possibility is rejected.

The most likely cause of the large \( \chi^2 \) values is that the 10-mi grid spacing is very large with respect to the variability of the head. Most of the head data is from shallow depths and is heavily influenced by land-surface altitude. This can be seen from the very close similarity between water-table altitude in figure 6 and land-surface altitude in figure 2. Note that within only a single 10 mi by 10 mi grid, land-surface altitude may have a complex nature and that there may be several hill tops and valleys along a given cross section. It is thus unlikely that the residuals for grid-element volume-averaged head would be approximately statistically normal, which is what is required for small \( \chi^2 \) values. More likely, the residual would express some land-surface altitude characteristic. This would be true even if the effective hydraulic conductivity was constant or had an exact log normal distribution.
A model with a 30-mi grid spacing was used early in the study and gave residuals that were considerably larger than those for a 10-mi grid spacing. Although Chi² values were not actually determined, the residuals had a pronounced pattern that suggest the model would have had very large Chi² values for this reason.

A complicating factor is that head measurements have some tendency to be located near areas of pumping which may be near the edge of a grid element, but the model approximates the somewhat centrally weighted average drawdown that would be produced in a grid element as if all the pumping in the grid element was located at the center of the grid element.

It would be appropriate to use a much smaller grid spacing in those parts of the aquifer system where the head shows considerable variability. This was not done because of the already large number of nodes in the computer model.

Several methods (Draper and Smith, 1981, p. 34-40) are available to investigate lack of fit and the presence of model bias. Bias causes the inflation of what is, herein, called apparent variance with respect to true variance. Plots of residuals \( Y_j - \hat{f}_j(B) \) for grid-element volume-average head were observed for each of the model layers 2 through 11. No discernible pattern or bias was seemingly present using this simple approach to investigate lack of fit. Values for mean residual grid-element volume-averaged head by layers were less than 10 ft in magnitude for each of the layers 2 through 11 for each of the models with four or more parameters. For these same models, values for the mean-weighted residual (table 4) for all of the observations were usually less than 3 ft.

Other methods available to investigate the presence of lack of fit make use of prior estimates of variance or repeat measurements and are thus not applicable.

**Comparison of Models**

To show self consistency of the regression approach, it is appropriate that the 12 models in table 4 be compared with respect to their predictions \( f_i(B) \) for the observed quantities \( Y_i \), which are composed of the 3,107 grid-element volume-averaged heads, and the 33 or 6 observations of the logarithm of the hydraulic conductivities of sand. The models should also be compared with respect to their predictions for flow even though there are no prediction intervals for flow.

Models 1 and 2, with two and three regression parameters, respectively, gave flow rates from the upland surface recharge areas to the lower altitude discharge areas on land or the sea floor which were approximately 10 times larger than those of the other models, and also larger than would be expected from precipitation data and infiltration estimates. This might be
expected, because the observations did not include flow rates, and these two models allowed only a single hydraulic conductivity for both the clay and sand components in the entire domain of solution. These two models are thus rejected and will not be considered.

The 10 models 3 through 12 gave nearly the same value for the mean weighted residual head. The predicted heads $f_i(\hat{B})$, $i = 1, 2, ..., 107$, averaged by layer, differ between the 10 models by as much as 30 ft for one layer, but less than 5 ft for most of the layers. These values are well within the 95-percent prediction interval half widths $e_k$, which in table 4 show values of from 141 ft/$(1.7)^{1/2} = 108$ ft to 243 ft/$(.4)^{1/2} = 384$ ft. Figures 10 through 19 show the values for head as produced by model 4, the model in table 4 with the smallest value for $e_k$, for the year 1982. Predicted heads at deep parts of the aquifer system showed differences of up to 800 ft between the 10 models, 3 through 12.

Results for predictions of the 33 logarithm values of hydraulic conductivity of sand also show good overlap between the prediction intervals of the 10 models. Model 3, with four regression parameters, has the interval $\log_{10}(29.9) - 0.8dd_k + 141/149$ as the 95-percent prediction interval for each of the 33 logarithm values of hydraulic conductivity of sand, each of which has the predicted value of $\log_{10}(29.9) - 0.8dd_k$. The other prediction intervals for hydraulic conductivity of sand in the 10 models overlap to a high degree. The most extreme case is that of the predicted value of $\log_{10}(650) - 0.8dd_k$ for the 33 logarithm values of hydraulic conductivity of sand in layer 2 in model 12. The prediction interval in this case is $\log_{10}(650) - 0.8dd_k + 243/149$, which is a wide interval that overlaps $\log_{10}(29.9) - 0.8dd_k + 141/149$ of model 3.

The flow rates obtained from the models in table 4 were compared by looking at the total flow across various surfaces cutting across the domain of solution and also by looking at the flow rates across the many individual grid element faces. One of these sets of surfaces separate the layers. Values were obtained for the flow into and out of the top and bottom surfaces of each of the 10 model layers, a total of 40 flow rates. Comparison of these 40 flow rates for models 3 through 11 shows a considerable amount of consistency. Models 1 and 2 are rejected for the reasons given previously. Model 12 with 31 regression parameters gave flow rates considerably different from the other models, probably as a result of instability resulting from having too many parameters, so that flow rates for model 12 are not considered either. Rows 1 through 5 in table 5 show respectively (1) the flow out of the geopressed zone; (2) the recharge into the aquifer system from model layer 12 through the top of the aquifer system; (3) the flow out of the aquifer system into layer 12 through the top of the aquifer system; (4) the net total flow into the aquifer system from layer 12 through the top of the aquifer system; and (5) the total recharge to the aquifer system from both the geopressed zone and model layer 12. The second flow minus the third is equal to the fourth, and the first plus the fourth is equal to the fifth. Rows 6 through 15 in table 5
TABLE 5.-- 1982 Ground-water flow rates for models 3 through 11

[The number above/below the slash is the flow rate into/out of the tops of the 10 aquifer layers; units are $10^6$ cubic feet per day]

| Model | Flow out of the geopressed zone | Recharge into the aquifer system from layer 12 through the top of the aquifer system | Flow out of the aquifer system into layer 12 through the top of the aquifer system | Net total flow into the aquifer system from layer 12 through the top of the aquifer system | Total recharge to the aquifer system from both the geopressed zone and layer 12 | Flow rate into/out of top of layer 2 | Flow rate into/out of top of layer 3 | Flow rate into/out of top of layer 4 | Flow rate into/out of top of layer 5 | Flow rate into/out of top of layer 6 | Flow rate into/out of top of layer 7 | Flow rate into/out of top of layer 8 | Flow rate into/out of top of layer 9 | Flow rate into/out of top of layer 10 | Flow rate into/out of top of layer 11 |
|-------|--------------------------------|------------------------------------------|----------------------------------------|----------------------------------------|------------------------------------------|----------------------------------|----------------------------------|----------------------------------|----------------------------------|----------------------------------|----------------------------------|----------------------------------|----------------------------------|----------------------------------|----------------------------------|----------------------------------|
| 3     | 993                            | 0                                        | -18                                    | 973                                   | 844                                      | 9                                | 0                                | 775                              | 539                              | 177/1080                        | 178/86                           | 198/87                           | 175/1060                        | 235/988                         | 181/98                         | 205/114                         | 402/1090                        | 239/686                         |
| 5     | 13500                          | 13600                                    | 13600                                  | 13600                                  | 13900                                    | 13600                            | 13600                            | 13400                            | 13700                            |                                   |                                   |                                   |                                   |                                   |                                   |                                   |                                   |
| 6     | 2270                           | 1360                                     | 13500                                  | 2330                                   | 2560                                     | 13400                            | 1500                             | 2100                             | 1950                             |                                   |                                   |                                   |                                   |                                   |                                   |                                   |                                   |
| 7     | 11300                          | 12200                                    | 12300                                  | 11200                                  | 11300                                    | 12200                            | 12100                            | 11300                            | 11800                            |                                   |                                   |                                   |                                   |                                   |                                   |                                   |                                   |
| 8     | 12250                          | 12250                                    | 12240                                  | 12200                                  | 12170                                    | 12240                            | 12130                            | 12110                            | 12300                            |                                   |                                   |                                   |                                   |                                   |                                   |                                   |                                   |
| 9     | 177/1080                       | 178/86                                   | 198/87                                 | 175/1060                               | 235/988                                  | 181/98                           | 205/114                         | 402/1090                        | 239/686                         |                                   |                                   |                                   |                                   |                                   |                                   |                                   |                                   |
| 10    | 651/467                        | 654/471                                  | 653/470                                | 700/544                               | 961/794                                  | 645/467                          | 417/269                         | 514/408                         | 516/356                         |                                   |                                   |                                   |                                   |                                   |                                   |                                   |                                   |
| 11    | 13500                          | 13600                                    | 13600                                  | 13600                                  | 13900                                    | 13600                            | 13600                            | 13400                            | 13700                            |                                   |                                   |                                   |                                   |                                   |                                   |                                   |                                   |
show the flow rates into and out of the tops of the 10 model layers for models 3 through 11. All numbers in table 5 have three significant figures except row 5 which has four. All flow rates shown are for the year 1982, at which time an approximately steady-state situation existed. The pumping rate for 1982 was $0.1265 \times 10^{10}$ ft$^3$/d. The difference between this pumping rate and the total recharge to the aquifer system on row 5 is the rate that water is coming from storage. Note that the values for the total recharge to the aquifer system are in each case greater than 95 percent of the 1982 pumping rate, indicating that less than 5 percent is coming from storage, and that a near steady-state condition exists.

Because the possible error of estimates for maximum allowable recharge (discharge) into the aquifer system from model layer 12 are large in comparison with the differences for this flow among models 3 through 11, none of the models seem any more likely to be valid than the others. The reason for this is that the flow rates into the aquifer system from model layer 12 in table 4 are very small compared to flow rates that occur at land surface, such as precipitation, runoff, evapotranspiration, etc. For example, the flow rate out of the aquifer system into layer 12 is $0.136 \times 10^9$ ft$^3$/d for model 4. This water would either evaporate, be transpired, or flow into streams in lowland areas. However, this amount of water spread over 100,000 mi$^2$, an area approximately a third the size of the study area, is equivalent to a flow rate of only 0.21 in/yr, which is very small compared to evapotranspiration or runoff rates.

The direction and general location of ground-water flow that corresponds in a general way to the flow rates of table 5 are shown in figure 35. Figure 35 represents the actual aquifer system and actual flow locations and directions only in a very general and schematic manner. Actual flow patterns, both in the model and in the field, are far more complex. The recharge (discharge) into the aquifer system from model layer 12 through the top of the aquifer system for each grid element for the year 1982, model 4 is shown in figure 36. Equivalent freshwater drawdown since 1937 for the year 1982, model 4, is shown in figures 37 through 46.

Models 3, 4, 5, and 8 have a similar structure in that they all have a single regression parameter assigned to the hydraulic conductivity of sand for the entire aquifer system, and a single parameter assigned to hydraulic conductivity of clay. All four models also have a regression parameter for $b_2$, specific storage, and regression parameter $b_4$, even though model 3 has the same regression parameter $B_4$ assigned to both $b_1$ and $b_3$. The values obtained for the parameter for hydraulic conductivity of sand are 29.9, 30.3, 30.3, and 29.0 ft/d for the four models, respectively. For hydraulic conductivity of clay, the four values are $0.365 \times 10^{-3}$, $0.362 \times 10^{-3}$, $0.361 \times 10^{-3}$, and $0.374 \times 10^{-3}$ ft/d. The values obtained for the predicted heads $f_i(\hat{B})$ $i = 1,2,\ldots,3,107$ are very close. When averaged by layer, the four models give values for head that differ by less than 1.1 ft. Clearly the four models are very similar in structure and give nearly the same results for hydraulic conductivity of clay and sand and
Figure 35. Generalized pattern of ground-water flow.
Figure 36. Simulated recharge (discharge) into the top of the aquifer system for 1982, model 4.
Figure 37. Simulated drawdown of equivalent freshwater head from 1937-82, model 4, model layer 2.
Figure 38. Simulated drawdown of equivalent freshwater head from 1937-82, model 4, model layer 3.
Figure 39. Simulated drawdown of equivalent freshwater head from 1937-82, model 4, model layer 4.
Figure 40. Simulated drawdown of equivalent freshwater head from 1937-82, model 4, model layer 5.
Figure 41. Simulated drawdown of equivalent freshwater head from 1937-82, model 4, model layer 6.
Figure 42. Simulated drawdown of equivalent freshwater head from 1937-82, model 4, model layer 7.
Figure 43. Simulated drawdown of equivalent freshwater head from 1937-82, model 4, model layer 8.
Figure 44. Simulated drawdown of equivalent freshwater head from 1937-82, model 4, model layer 9.
Figure 45. Simulated drawdown of equivalent freshwater head from 1937-82, model 4, model layer 10.
Figure 46. Simulated drawdown of equivalent freshwater head from 1937-82, model 4, model layer 11.
predicted shallow heads $f_i(B)$, $i = 1, 2, \ldots, 3, 107$. However, table 5 shows that the flow rates are not the same, the main difference being differing amounts of flow from the geopressed zone passing up through the aquifer system through model layers 7 through 12. Also, heads at depth differ by as much as 800 ft. Note that in table 5, model 3 has a flow of $0.993 \times 10^8$ ft\(^3\)/d from the geopressed zone. Model 4, however, has a zero flow condition because $b_4$ is set to zero. Models 5 and 8 have almost zero flow because of the values obtained for $b_3$ and $b_4$, even though the confidence intervals for $b_3$ and $b_4$ are extremely large. A model not shown in tables 4 and 5 and having the same selection of regression parameters as model 5, but using grid-element volume-averaged heads from drill-stem test as well as the 3,107 grid-element volume-averaged heads from water wells, gave a flow from the geopressed zone of $0.544 \times 10^8$ ft\(^3\)/d. Models 6, 7, 9, 10, and 11 have flow from the geopressed zone of from 0 to $0.973 \times 10^8$ ft\(^3\)/d. The conclusion is that the modeling efforts of this study are very approximate with regard to deep flow and heads. This result is not unexpected because almost all of the data for hydraulic head and hydraulic conductivity were taken from shallow depths.

A flow from the geopressed zone as large as that shown by model 3 could exist because order of magnitude calculations show that with a large value of specific storage, the geopressed zone under the aquifer system could maintain an average upward flow of approximately $10^8$ ft\(^3\)/d if experiencing a rate of drop in head of several thousand feet per 100 million years $(S(\Delta h/\Delta t))$ (volume of geopressed zone) = flow rate: $(0.5/ft)(5 \times 10^3 ft/365 \times 10^8 d)(10^{15} ft^3) = 0.7 \times 10^8 ft^3/d$. However, as shown above, the models of this study give values of the flow from the geopressed zone of from 0 to $10^8$ ft\(^3\)/d with no indication of which is more correct. Values of head at depth also vary widely.

**Model Error**

It has been shown above that most of the various models show agreement regarding prediction of shallow heads and hydraulic conductivity values (the 3,107 grid-element volume-averaged heads and the 33 logarithm values of hydraulic conductivity of sand). The overlapping of the associated prediction intervals has also been demonstrated. It is appropriate to consider the extent to which the predictions of shallow heads and hydraulic conductivity could be in error because of conditions or constructions that are common to all of the models $f(B)$ used.

Before considering model error, it is appropriate to mention that many possible sources of error have already been removed by assigning regression parameters to unknown values such as: $b_1$ and $b_2$ for the top surface specified head, $b_3$ and $b_4$ for the geopressed zone, $b_5$ for the way in which effective hydraulic conductivity is determined from the hydraulic conductivity of the sand and clay components, $b_6$ for anisotropy, $b_7$ and $b_8$ for the subsidence mechanism, and $b_9$ for specific storage. Remaining sources of model error are (1) the evaluation of the density function $\rho(x, y, z)$ from data that may be in error or lacking; (2) the depth dependence of clay and
hydraulic conductivity of sand as described previously, specifically the functions used for the rate of decrease with depth; (3) the procedure for obtaining grid element effective hydraulic conductivity values by Desbarats (1987); and (4) the selection of the hydraulic conductivity zones. These four sources of error are considered in order in the following four paragraphs, and results of model alterations to explore these sources of error are summarized in table 6. An additional source of error which is very likely the major cause of the large root-mean square-weighted residuals and the large 95 percent prediction interval half widths $e_k$ (table 4) is the relatively large 10 mi grid spacing, as discussed previously. Unfortunately, no tests with smaller grid spacing were performed because of the large number of grid elements that would be needed. Consequences of the use of 30-mi grid spacing have been discussed previously.

Several models were used in which grid-element volume-averaged density $\rho$ was smoothed between grid elements in the same layer. This smoothing was done by assigning to $\rho$ the average of $\rho$ over nine locations, the neighboring eight grid elements and the center grid element for which the smoothed value of $\rho$ is being sought. Changes in predicted $f^i(\hat{B})$ for observed quantities were very minimal. Using model 4, the maximum change in layer-averaged predicted head was 0.2 ft. The values for the hydraulic conductivity of clay and sand parameters, $0.362 \times 10^{-3}$ and 30.3 ft/d, changed to $0.364 \times 10^{-3}$ and 30.2 ft/d. The value of $\omega^{1/2}e_k$ was unchanged at 141, so that the confidence and prediction interval widths were unchanged. However, small local perturbations in head, flow velocity, and flow direction resulted in those parts of the aquifer system with high grid element densities if smoothing caused significant changes in these grid element densities. This would be expected from theoretical considerations regarding the variable-density flow equations (1) and (3). Larger changes in density, other than just smoothing, cause larger changes in predicted shallow head and hydraulic conductivity values. Using model 4, with the density $\rho$ that of freshwater throughout the entire aquifer system, the maximum change in layer-averaged predicted head was 1.3 ft. The values for the hydraulic conductivity of clay and sand parameters, $0.362 \times 10^{-3}$ and 30.3 ft/d in model 4, changed to $0.357 \times 10^{-3}$ and 29.8 ft/d. The value for $\omega^{1/2}e_k$ was 141, unchanged. The 15 flow rates from model 4 appearing in table 5 changed by as much as 3 percent. Deep heads, in areas of high density originally, decreased as much as 600 ft due to the absence of the elevated pressure caused by the dense saline water.

As mentioned previously, the rates of decrease with depth of hydraulic conductivity of clay and sand were approximated by the functions $10^{-0.8dd}$ and $10^{-1.167dd-0.0833dd^2}$, respectively, where $dd$ is measured in kilometers. Also as mentioned, these functions were assigned an exponent regression parameter in several models, but the confidence intervals for these regression parameters were very large. In accordance with these large confidence intervals, changes in the depth functions affected predicted heads $f^i(\hat{B})$ very little, provided only that the depth functions caused the hydraulic conductivity values of clay and sand to decrease in some substantial manner with depth.

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TABLE 6.—The effect of model alterations on: (1) layer-averaged predicted head, $h_{layer}$; (2) hydraulic conductivity of clay and sand, $P_c$ and $P_s$; (3) $w_k^{1/2} e_k$; (4) the 15 flows of table 5; and (5) deep head for models 3 through 11 and model 4

(Model 4 comparisons are on rows 2 and below; unaltered model 4 has $P_c = 0.362 \times 10^{-3}$ feet per day, (ft/d), $P_s = 30.3$ ft/d, and $w_k^{1/2} e_k = 141$. >, greater than; <, less than; gm/cm$^3$, grams per cubic centimeter)

<table>
<thead>
<tr>
<th>Alteration</th>
<th>$h_{layer}$ (ft)</th>
<th>$P_c$ (ft/d)</th>
<th>$P_s$ (ft/d)</th>
<th>$w_k^{1/2} e_k$</th>
<th>15 flows (percent)</th>
<th>Deep head (ft)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nine models, 3 through 11</td>
<td>&lt; 30 usually &lt; 5</td>
<td>see table 4</td>
<td>see table 4</td>
<td>141-243</td>
<td>large differences</td>
<td>800</td>
</tr>
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<td>Smoothed density</td>
<td>&lt; 0.2</td>
<td>0.364x$10^{-3}$</td>
<td>30.2</td>
<td>141</td>
<td>&lt; 1</td>
<td>53</td>
</tr>
<tr>
<td>Density of freshwater</td>
<td>&lt; 1.3</td>
<td>0.357x$10^{-3}$</td>
<td>29.8</td>
<td>141</td>
<td>&lt; 3</td>
<td>600</td>
</tr>
<tr>
<td>Depth dependence of hydraulic conductivity</td>
<td>&lt; 2.7</td>
<td>0.335x$10^{-3}$</td>
<td>23.8</td>
<td>140</td>
<td>&lt; 30</td>
<td>60</td>
</tr>
<tr>
<td>Effective hydraulic conductivity procedure</td>
<td>&lt; 10 usually &lt; 2</td>
<td>0.1980x$10^{-2}$</td>
<td>30.2</td>
<td>145</td>
<td>&lt; 70</td>
<td>34</td>
</tr>
<tr>
<td>Truncation at density &gt; 1.005 gm/cm$^3$</td>
<td>&lt; 2.1</td>
<td>0.336x$10^{-3}$</td>
<td>32.0</td>
<td>142</td>
<td>&lt; 35</td>
<td>no deep heads</td>
</tr>
<tr>
<td>Truncation at 3,000 ft below ground surface</td>
<td>&lt; 2.3</td>
<td>0.336x$10^{-3}$</td>
<td>31.8</td>
<td>142</td>
<td>&lt; 20</td>
<td>no deep heads</td>
</tr>
</tbody>
</table>
Multiplying both of the exponents of the functions $10^{-0.8d}$ and $10^{-(1.167d-0.0833d^2)}$ by the factor 0.5 and also the factor 1.3 in model 4 gave predicted heads $h_i(B)$ that, when averaged by layer, differed from those of model 4 without alteration by at most 2.7 ft. The values for the hydraulic conductivity of clay and sand parameters, $0.362 \times 10^{-3}$ and 30.3 ft/d in model 4, changed to $0.335 \times 10^{-3}$ and 23.8 ft/d and $0.371 \times 10^{-3}$ and 32.8 ft/d for the factors 0.5 and 1.3, respectively. This type of result is expected from the previously described mechanism relating the hydraulic conductivity of a grid element in the model, the hydraulic conductivity parameter for the grid element, and the depth $d$ of the center of the grid element in kilometers. The value for $\omega_{k^{1/2}}$ was 140 for factor 0.5 and 142 for the factor 1.3, both very close to the value of 141 in model 4. Thus, the prediction intervals for head and hydraulic conductivity of sand did not change significantly. Several of the 15 flow rates from model 4 appearing in table 5 changed by as much as 30 percent, but changes were usually less than 10 percent. As explained previously, the original hydraulic conductivity rate of decrease with depth functions were chosen to approximate data taken from the literature, and thus presumably should cause the model $f(B)$ to be the most accurate with the exponents unaltered. In the event that this is not correct, the analysis above shows that the results change insignificantly for very substantial changes in the exponents.

An alternative to using the procedure of Desbarats (1987) to obtain grid element effective horizontal and vertical hydraulic conductivity values, is to use the harmonic mean to obtain effective vertical hydraulic conductivity, and to use the arithmetic mean to obtain effective horizontal hydraulic conductivity. This procedure yields effective vertical and horizontal hydraulic conductivity values that correspond to the assumption that the clay in the grid element extends horizontally and continuously across the entire grid element as a single or several uniform layers. It would thus tend to give a larger than actual effective horizontal conductivity and a smaller than actual effective vertical conductivity. The formulas for effective vertical and horizontal hydraulic conductivity, $K_z$ and $K_h$, for a grid element are:

$$K_z = \left(\frac{V_c}{K_c} + \frac{(1-V_c)}{K_s}\right)^{-1}, \tag{16}$$

and

$$K_h = \frac{V_cK_c + (1-V_c)K_s}{K_c}, \tag{17}$$

where, as in formulas (10) and (11), $K_c$ and $K_s$ denote the hydraulic conductivity of the clay and sand components respectively, and $V_c$ denotes the clay fraction of the grid element. Equations (16) and (17) were used in several of the models in table 4 as a replacement for equations (10) and (11). This was done by the direct use of equations (16) and (17), and also by using replacements for $a$ and $a^{-1}$ in equations (10) and (11), causing these equations to give nearly the same values for $K_z$ and $K_h$ as equations (16) and (17). Use of equations (16) and (17) as opposed to equations (10) and (11)
produced changes in the values for the hydraulic conductivity of clay and sand parameters for the various hydraulic conductivity zones. This would be expected from the functionally different manner that $K_c$ and $K_s$ appear in equations (10) and (11), as compared to the way they appear in equations (16) and (17). By use of equations (16) and (17) in model 4 gave predicted heads $f_i(B)$, which when averaged by layer differed by 10 ft at most and generally less than 2.0 ft from predicted heads obtained using equations (10) and (11). However, the values for the hydraulic conductivity of clay and sand parameters, $0.362 \times 10^{-3}$ and 30.3 ft/d in model 4, changed to $0.198 \times 10^{-2}$ and 30.2 ft/d, respectively. Due to the difference between equations (10) and (11) as compared to equations (16) and (17), effective hydraulic conductivity values $K_z$ and $K_h$ did not generally change by more than 50 percent, despite the substantial change of the hydraulic conductivity of clay parameter value from $0.362 \times 10^{-3}$ ft/d to $0.198 \times 10^{-2}$ ft/d. The value for $\omega_k^{1/2} e_k$ was 145, only slightly larger than the value of 141 in model 4, so that the prediction intervals for head and hydraulic conductivity of sand did not change significantly. Several of the 15 model 4 flow rates appearing in table 5 changed by as much as 70 percent, but changes were usually less than 30 percent. The value for Chi$^2$ did not change appreciably.

The manner of selecting the hydraulic conductivity zones was not changed, except for the use of various choices for the boundaries of the 10 regions in figure 31. However, because the various models had different choices for the assignment of the more than 100 hydraulic conductivity zones to a given set of hydraulic conductivity of clay and sand parameters, model error due to discretization of the aquifer system domain of solution with regard to the values used for hydraulic conductivity was partially eliminated. This is true only if a sufficiently large number of different chosen assignments of hydraulic conductivity zones was used. Since more than 40 different assignments were used (seven of which are shown in table 4), model error due to discretization is probably largely eliminated. This should be interpreted relative to the differences in results noted in the previous section.

**Model Truncation**

Some studies of aquifer systems with saline water at depth use a constant freshwater density model and truncate the domain of solution at or near that depth where the density begins to increase substantially, treating this truncation surface as a no-flow boundary. This approach may be seen as an approximation to the method of this study which, by the use of a variable-density model, models both the shallow freshwater system and the underlying saline water system as a total system. Obviously the truncation of the deep saline part of the aquifer system precludes the modeling of this deep part. The hope, however, is that in spite of truncation, the results for the remaining shallow part of the aquifer system are reasonably accurate.

With the truncation approaches described below, predicted shallow head and hydraulic conductivity were similar to values obtained by the variable density model. Flow from the geopressed zone in some of the models is shown in table 5. This flow, if it occurs, passes up through the no-flow truncation surface of the truncated system. Thus, flow at the depth of the
truncation surface differs substantially between the two approaches because, when modeling the full system, flow occurs across the truncation surface which is a no-flow boundary for the truncated system. Heads at depth cannot be compared between the two approaches, because the truncated aquifer system does not have any deep heads. Heads just above the truncation surface were found to be fairly close. Heads adjacent to the truncation surface and in areas of heavy pumping differed the most.

Model 4 was used with a truncation surface defined as that surface at which grid element density first exceeds 1.005 gm/cm$^3$ when proceeding downward from the top grid element at each horizontal location. This truncation reduced the number of observed grid-element volume-averaged heads $Y_i$ from 3,107 to 3,067, because 50 heads $Y_i$ were located below the truncation surface. Density in the grid elements above the truncation surface was set to that of freshwater. The maximum change in layer-averaged predicted head was 2.1 ft. The values for the hydraulic conductivity of clay and sand parameters, $0.362 \times 10^{-3}$ and 30.3 ft/d in model 4, changed to $0.336 \times 10^{-3}$ and 32 ft/d. The value for $\omega_k^{1/2}e_k$ was 142. The 15 flow rates from model 4 appearing in table 5 changed by as much as 35 percent, but changes were usually less than 7 percent. The maximum change in the 3,067 predicted heads was 49 ft. This occurred at a grid element located adjacent to the truncation surface that had a drawdown of almost 400 ft due to heavy pumping. Most of the changes in the 3,067 predicted heads were less than 5 ft. Flows in the area of the grid elements with heavy pumping were different in the truncated model due to the proximity of the no-flow truncation surface.

Model 4 was also used with a truncation surface at 3,000 ft below land surface. This truncation decreased the number of grid-element volume-averaged heads $Y_i$ to 3,087. Density was set to that of freshwater. The maximum change in layer-averaged predicted head was 2.3 ft. The values for the hydraulic conductivity of clay and sand parameters, $0.362 \times 10^{-3}$ and 30.3 ft/d in model 4, changed to $0.336 \times 10^{-3}$ and 31.8 ft/d. The value for $\omega_k^{1/2}e_k$ was 142. The 15 flow rates from model 4 appearing in table 5 changed by as much as 20 percent, but changes were usually less than 5 percent.

Note that with both methods of truncation, layer-averaged predicted head does not change by more than 2.3 ft. This small change occurs in spite of the fact that some of the layers have a reduced number of heads $Y_i$, so that the average for the layer is over a reduced set of those $Y_i$ that were in the layer before truncation. The 15 flow rates in table 5 show change, as mentioned, but some of this change occurs because the extent of the truncated layers has decreased, in some cases by a considerable amount. Because flow decreases substantially with depth due to decreasing hydraulic conductivity of clay and sand, the truncation of the deeper part of a layer may have a relatively small effect on the total flow going into or out of its top surface.
Changes that occur in layer-averaged predicted head and flow due to the truncation of model 4 are less than those that occur between the nine models in table 5, all without truncation (table 6). These results with the truncation of model 4 indicate that in general little predictive ability is lost by the use of a truncated model because neither the full nor truncated models are able to make any useful predictions for deep head and flow. The only exception to this would be that the full model is more accurate for the prediction of head and flow near the truncation surface, particularly if there is heavy pumping nearby.

**SUMMARY AND CONCLUSIONS**

**SUMMARY**

The major results of the study are:

1. Of the 12 models evaluated in this report, the model with the smallest prediction interval half widths, model 4, had only four regression parameters. With all models, the residuals failed the Chi² test for normality at the 1-percent significance level, possibly as a result of the large 10-mi grid spacing used.

2. The dominant factor controlling shallow hydraulic heads in the simulated aquifer system is the specified head along the top surface of the aquifer system, which consists of the water-table altitude and equivalent freshwater head at the sea floor. Because the water-table altitude surface is usually not far below land surface and has a very similar shape, a dominant factor controlling shallow heads is land-surface altitude. These shallow heads are also affected by pumping. Heads at depth may be affected by flow from the geopressed zone. However, the models used show almost no ability to predict deep heads and flow, including flow from the geopressed zone.

The various models used show considerable overlap among the prediction intervals for shallow head and hydraulic conductivity of sand. The 95-percent prediction interval half widths for grid-element volume-averaged head all exceed 108 ft, and those for volume-averaged log₁₀ hydraulic conductivity of sand all exceed 0.94(141/149) = 0.89.

3. The essential feature of the flow system is the flow of water from upland surface recharge areas to discharge areas at lower altitude on land or the sea floor. No prediction intervals were obtained for flow. The variability from model to model in values obtained for shallow flow varies greatly. The flow across some surfaces is not known to within a factor of 10. For other surfaces, the flow rates are known to within a smaller factor. There is little certainty about the flow across many of the individual grid element surfaces, particularly those that have relatively small flow.
Because the possible error of estimate for maximum allowable recharge (discharge) into the aquifer system from model layer 12 is large in comparison with the differences for this flow among models 3 through 11, none of the models seem any more likely to be valid than another. The reason for this is that the flow rates into the aquifer system from model layer 12 are very small compared to flow rates in the hydrologic budget, such as precipitation, runoff, evapotranspiration, etc. Estimates for the flow rate from the geopressed zone are sufficiently approximate that they also provide no basis for selecting any one of the models 3 through 11 as being better than any of the others.

4. Truncating the domain of solution of one of the models below a certain ground-water density or depth below land surface and setting the density to that of freshwater in the remaining shallow part of the domain of solution did not appreciably change results for hydraulic head and flow produced by the model, except for locations close to the truncation surface.

5. The regression methodology allowed the testing of a very wide range of models for the simulation of the aquifer system. The time saved by being able to find the optimal selection of the regression parameters for a given model in only a single computer run was used to formulate and test many different types of models and procedures.

The regression methodology also provided estimates of the accuracy of results, in the form of prediction and confidence intervals. These accuracy estimates point out the limitations of the predictive ability of a model, and are thus very valuable. Testing, to determine the most likely sources of model error, led to the conclusion that the 10-mi grid element spacing was quite large relative to the variability of hydraulic head and is thus the probable cause of significant error.

CONCLUSIONS

The predictive ability of the models used was quite low in many aspects. The models showed almost no ability to predict deep heads and flow, including flow from the geopressed zone. Regression analysis shows that even the best of the models used had a rather poor ability to accurately predict head in any of the layers. Evidence of this is the 95-percent prediction interval half width of 108 ft mentioned previously. For predicting hydraulic conductivity of sand, the 95-percent prediction interval half widths for $\log_{10}$ hydraulic conductivity of sand all exceeded 0.89, thus hydraulic conductivity of sand could not be predicted to within a factor of almost 8 ($10^{0.89}$). These results show the considerable inaccuracy of the models chosen and perhaps the inaccuracy of any model based on the same data and relatively large 10-mi grid spacing. The very important contribution of regression analysis is showing the limitations of the predictive ability of the models used. A less thorough study probably would have left the impression that the model or models used were significantly more accurate than they actually were.
REFERENCES CITED

Bennett, G.D., 1980, Research and model development related to the regional aquifer-system analysis program [abs]: American Geophysical Union Eos Transactions, v. 61, no. 46, p. 951.
    —1982, Incorporation of prior information on parameters into nonlinear regression ground-water flow models: 1, Theory: Water Resources Research, v. 18, no. 4, p. 965-976.


ATTACHMENTS
Introduction

The computer code used to find the p vector \( \hat{B} \) which minimizes the weighted mean square error

\[
S(B) = \frac{1}{n} \sum_{i=1}^{n} (Y_i - f_i(B))^2 \omega_i,
\]

consists of two parts. The first part is taken from Durbin (1983). The second part is Kuiper’s variable-density ground-water flow model, VARDEN (Kuiper 1985), which is used to evaluate the \( f_i(B) \). The first part consists mainly of a section called MAIN in the code which chooses a sequence of parameter vectors \( B_{st}, \ s = 1,2,\ldots, t = 1,2,\ldots, p \), which are used to find a sequence \( B_s, \ s = 1,2,\ldots \), which converges to \( \hat{B} \). This process is an implementation of the Levenberg-Marquardt method for the minimization of a sum of squares. The remainder of the first part is a linear-system solver called SOLEQU, which is used as a subroutine to MAIN. MAIN uses Kuiper’s (1985) variable density ground-water flow model VARDEN, by calling MODEL when an evaluation of \( f(B) \) is required for \( B \) equal to one of the vectors \( B_{st} \) or \( B_s \) which MAIN has chosen. In turn, MODEL calls the linear-system solvers PCG, and SIP.

MODEL, PCG, and SIP make up VARDEN and have been explained in detail by Kuiper (1985). MAIN from Durbin (1983) and MODEL from Kuiper (1985) were substantially modified for this application and the modifications are explained below. Some of the modifications consist of inserted code used to calculate prediction and confidence intervals as given by equations (6) and (7) in the prediction and confidence interval section of this report. Several slight modifications were made to PCG and SIP and they also are explained below. SOLEQU was not modified. The FORTRAN code for MAIN, MODEL, SOLEQU, PCG, and SIP are listed below in their respective sections. The sections for MAIN and MODEL contain a listing of meaningful variables. Dummy variables used only to implement a calculation are not listed. The manner of selection of the sequences \( B_{st} \) and \( B_s \) as dictated by the Leven-Marquardt procedure is explained in the next section.

The Levenberg-Marquardt Method

The Levenberg-Marquardt method (Durbin, 1983; Gill, Murray, and Wright, 1981) for the minimization of a sum of squares

\[
S(B) = \sum_{i=1}^{n} (Y_i - f_i(B))^2 \omega_i,
\]

is:

\[
X^T \omega X + \lambda I) \Delta B = X \omega (Y - f(B)) \tag{18}
\]

Here \( X(B) \) is the sensitivity matrix defined as \( x_{ij} = \delta f_i / \delta B_j \), \( i = 1,2,\ldots,n \), \( j = 1,2,\ldots, p \), where \( n \) is the number of observations \( Y_i \) and \( p \) is the number of parameters. \( B \) is the p vector of parameters, \( I \) is the identity matrix, \( \lambda(B) \) is equal to \( (\sum r_i^2)^{1/2}/\text{RLEV} \),\( \omega_i \) weights in \( S(B) \) are denoted by \( \omega_i \).
Equation (18) is applied iteratively, starting with some initial \( B \) of choice and corresponding \( \Delta B \) from (18). For the next iteration \( B \) is replaced with \( B + \Delta B \), which value is then used in (18) in \( X(B) \), \( \lambda(B) \), and \( f(B) \) to produce a new \( \Delta B \), etc. The successive values of the vector \( B \) are labeled \( B_s \) where \( s \) is the iteration count. In order to evaluate \( X(B) \) it is necessary to find the \( n \times p \) derivatives \( \frac{\partial f_i}{\partial B_j} \). These are found \( n \) at a time from \( p \) evaluations of \( f(B) \) at \( B = B_{st} \), \( t = 1,2,\ldots p \) and also \( f(B) \) at \( B = B_s \). The derivative \( \frac{\partial f_i}{\partial B_j} \) at \( B = B_s \) is approximated by

\[
\frac{\partial f_i}{\partial B_j} = \frac{f_i(B_{st}) - f_i(B_s)}{(B_{st})_j - (B_s)_j}
\]

where

\[
(B_{st})_j - (B_s)_j = (TST)(B_s)_j, \quad (B_{st})_l = (B_s)_l \quad \text{for} \ l \neq j,
\]

and TST is called the "perturbation factor." Having evaluated \( X(B_s) \) by this means, equation (18) is used to find \( B_{s+1} = B_s + \Delta B \).

It is readily shown from equation (18) that \( \Sigma(\Delta B)_j^2 \) cannot exceed \( (RLEVM)^2 \). Thus RLEVM can be chosen to limit the size of \( \Delta B \). In general RLEVM should be chosen large enough that \( \Delta B \) is larger than \( (TST)B \), but not so large that the number of iterations needed for \( B_j, j = 1,2,\ldots, \) to arrive at a suitable approximation to \( B \) is caused to increase. TST should be chosen as small as possible before deterioration in the accuracy of the derivatives of \( f(B) \) with respect to \( B \) occurs. The parameters \( B_j \) should be scaled in \( f(B) \) so that approximately equal percentage changes occur in \( S(B) \) corresponding to equal percentage change in the individual members \( B_j \) of vector \( B \).

**MAIN**

The function of **MAIN** is to implement the Levenberg-Marquardt method. It calls subroutine **MODEL** when it needs \( f(B) \) for some chosen value of \( B \) as dictated by the Levenberg-Marquardt method. **MODEL** is actually called by either a call to **MODEL1** or **MODEL2**; **MODEL1** being used only for an initial call which reads in data needed for the evaluation of \( f(B) \) for any value of \( B \).

Function \( f(B) \) is the ground-water flow model VARDEN. The values for \( f_i(B), i = 1,2,\ldots,n \) are the flow model predictions for the observations \( Y_i \), many of which are grid-element volume-averaged heads. Model \( f(B) \) requires the specification of the sand and clay component of hydraulic conductivities for each of the grid elements. The mapping of the \( p \) values for grid-element sand and clay component hydraulic conductivity and other parameters of \( f(B) \) is accomplished by the following procedure.

\[
\text{PARAMX}(I) = \text{PARMO}(I) \times \text{PARAM(IPARM}(I)) \quad I = 1,2,\ldots,\text{IPARX},
\]

where IPARX is the number of global parameters, the total number of parameters needed for model \( f(B) \) including many thousand grid-element sand and clay hydraulic conductivity values. **PARAM** is a \( p \) (\( p = \text{IPAR} \)) vector equal to the parameter vector \( B \). **IPARM(I), I = 1,2,\ldots,\text{IPARX}** has the values 1 through \( p \) and maps \( I \) to an associated parameter number **IPARM(I)**. **PARMO(I)** is a multiplicative factor. **PARAMX(I)** is used within the model \( f(B) \). In the case of the sand and clay component hydraulic conductivities, the **PARAMX(I)** are used to arrive at multiplicative factors which are multiplied by read-in values for sand and clay component hydraulic conductivity.
Explanation of MAIN Listing

Read in the number of columns, rows, layers and intervening confining beds in the modeled area: NI10, NJ10, NK10, NK4.

Read in: Number of global parameters
Number of active parameters
Maximum number of iterations
Perturbation factor
Maximum error change
Maximum Levenberg radius
IPARX, IPAR, JITU, TST, PCHL, RLEVM.

Read in IPARAM.

Read in PARAMO.

Do initial model call (MODEL1).

If JITU=0 go to 255 CONTINUE for final model run, print out, and stop.

This is the final run, print, and stop section addressed by 255 CONTINUE.

Set initial conditions (JIT = 1).

1017 CONTINUE.

Set JIT = JIT+1.

Calculate new PRARMX corresponding to new PARAM and call MODEL2.
Calculate SB=S(B) and root mean square weighted residual PZ = (n⁻¹ S(B))¹/². Print out PZ, new estimates of the parameters B_j, j = 1,2,...,p (PARAM(I), I = 1,2,...,IPAR), the change in these parameters from the previous iteration (MU(I), I = 1,2,...,IPAR), and ∂(PZ)/∂B_j j=1,2,...,p (GRAP(I), I=1,2,...,IPAR).

If: 1) The iteration number JIT is greater than the "maximum number of iterations" JITU, or 2) the change in the root mean square weighted residual PZ was less than PCHL, the "maximum error change," go to 255 CONTINUE for eventual stop.

Do numerical gradient computations. The p = IPAR values for the PARAM vector used are the B_t mentioned previously. Here s = JIT the iteration number and t = 1,2,...,p = IPAR. GRSS is the sensitivity matrix appearing in equations (6) and (7) in the text. PARO is used to scale PARAM.

Get B2 = Xᵀω(Y-f(B)) on the right hand side of equation (18) above. GRP is ∇S(B) here.

Do regression pack.

5050 CONTINUE.

Get A2 = (XᵀωX + λI), the matrix on the left hand side of equation (18) above.

Solve (A2)ΔB = (B2) for MU = ΔB.

If new B is out of imposed constraining region, go to 5050 CONTINUE for a recalculation of ΔB. This recalculation will have those members of new B which were placed out of their respective constraint values.

Calculate ∂(PZ)/∂B_t t = 1,2,...p = IPAR (GRP(I), I = 1,2,...,IPAR).

Go to 1017.
Flow Chart of MAIN

Initialize

1017 CONTINUE

Yes

Done?

No

5050 CONTINUE

Update approximation to parameter vector B

New B out of constrained region?

Yes

No

255 CONTINUE

Do last model run, print results and stop
**Definition of Program Variables**

**EPS**  
$Y_i - f_i(B)$.

**GRSS**  
Sensitivity matrix $X$ in equations (6) and (7) above in body of report.

**GRP**  
$\partial S(B)/\partial B_t$ or $\partial (PZ)/\partial B_t$.

**IOUT**  
If IOUT = 1, MODEL1 or MODEL2 prints out certain results, if IOUT = 0, no printout occurs.

**IPAR**  
Number of active parameters

**IPARAM(I)**  
Maps global parameter numbers used by model $f(B)$ to set of $p$ active parameters of parameter vector $B$.

**IPARX**  
Number of global parameters.

**JIT**  
Iteration number (JIT = 5).

**JITU**  
Maximum number of iterations.

**MU**  
Change $\Delta B$ in the parameter vector $B$.

**NI10**  
Number of columns in the modeled area.

**NJ10**  
Number of rows in the modeled area.

**NK10**  
Number of layers in the modeled area.

**NK4**  
Number of intervening confining beds.

**NWLMT**  
Number of observations $Y_i$

**PARAMO(I)**  
Multiplicative factor in $\text{PARAMX}(I) = \text{PARAMO}(I) \times \text{PARAM(IPARAM(I))}$  
$I = 1, 2, \ldots, \text{IPARX}$

**PARAMX(I)**  
Global parameters used in model $f(B)$

**PARO**  
Scale factor for parameters in parameter vector $B$

**PCHL**  
Maximum error change

**PDIV**  
Equal to NWLMT

**PZ**  
$\left( \frac{1}{n} S(B) \right)^{1/2}$, root-mean-square-weighted residual
RLEVM Maximum Levenberg radius

\[ S(B) = \frac{1}{n} \sum_{i=1}^{n} (Y_i - f_i(B))^2 \]

TST Perturbation factor.

WLC Calculated values \( f_i(B) \) for observations \( Y_i \)

WLM Measured values for observations \( Y_i \)
IMPLICIT REAL*8(A-H,O-Z)
COMMON WS, HMAX, RELX1, RELX2, COEF, ERR, XX10, DELT, ER5, SRZ, SUMRZ
1, ERRSV, XX10SV, JIT, NJ110, NJ11, NJ11, NK11, NNN, NSKP1
2, NSKP2, ITMAX, ICNT, IEVP, IWR1, NW1, NW2, NW3, N320, NUM4
3, L9, LENGTH, NK115, IT01, IT15, ICRO, DDK, BBK
4, NJ10, NJ10, NK10, NJ12, NJ12, NK12, SVIJ, VV40, SV35
COMMON /XX/ XX
COMMON /DT/ DT
COMMON /W/ W
COMMON /E2/ E2
COMMON /F2/ F2
COMMON /G2/ G2
COMMON /YQ/ YQ
COMMON /NT/ NT
COMMON /DD/ DD
COMMON /BB/ BB
COMMON /ZZ/ ZZ
COMMON /XXS/ XXS
C COMMON /ALN/ ALN
C COMMON /XXE/ XXE
C COMMON /SV/ SV
COMMON /HL/ HL
COMMON /LB/ LB
COMMON /MHD/ MHD
COMMON /C/ DXI, DYJ, N325, ISOR, IPDD
1, NPPNT, DTO, TOT, TFAC, NINT, IPRNT, IWR1, LFLOW, NK4, LLRO, IWRXX,
2, L220, MAQ1, NU1, NU2, NU3, LFLO, IPH
3, LBB, SUMF, SUNF, SQ2, SG2, SYQ, SVV, SUMF, SUNFM
COMMON /B/ COL, COU, PARAM, PARO, MU, W, EPS, WLM, LWLC, WLC, A2, B2, GRP
1, CONA, CONB, SCALE
2, NBND1, NBND2, ICON, NND, NWLM, NWLMT, IOUT
COMMON /GRSS/ GRSS
COMMON /XXSTR/ XXSTR
COMMON /XZKZ/ XZKZ
COMMON /LRO/ LRO
COMMON /YQ1/ YQ1
COMMON /LZ2/ LZ2
COMMON /PARAMX/ PARAMX
COMMON /PARAM0/ PARAM0
COMMON /IPARAM/ IPARAM
COMMON /YQ2/ YQ2
REAL*4 YQ2(65078)
DIMENSION A3(50, 50)
REAL*4COL(50), COU(50), PARAM(50), PAR0(50), MU(50), W(3303), EPS(3303)
1, WLM(3303), LWLC(3303), WLC(3303), A2(50, 50), B2(50), GRP(50),
2, CONA(50, 50), CONB(50), SCALE(50), GRSS(3303, 5)
3, PARAMX(159872), PARAM0(159872)
INTEGER*4 NBND1(50), NBND2(50), ICON(50),
1, NND(3303)
INTEGER*2 IPARAM(159872)
REAL*4 DD(65078), BB(65078), ZZ(65078), YQ(65078),
1XXS(65078), Q2(300), DDK(50), BBK(50), NT(94658), HL(65078)
2, DXI(250), DYJ(250), XXSTR(65078), YQ1(65078, 3)
3, XKZ(65078)
INTEGER*2 LRO(94658),IPH(50),LFL0(4),J7(5),K7(5)
1,NI(300),NJ(300),NK(300),MAQ1(50),NU1(3),NU2(3),NU3(3)
DIMENSION DT(65078),E2(65078),F2(65078),G2(65078),V2(65078)
DIMENSION XX(65078)
1,WS(10),MHD(65078),LZ2(65078),LB(94658),LBB(5918)
2,AOC(4),BOC(4),COC(4),DOC(4)
3,SUMF(3,250),SUNF(3,250),SQ2(3,250),SG2(3,250),SYQ(3,250)
4,SVV(3,250),SUMFM(3,250),SUNFM(3,250)
*,IGR6(12,12)
DATA PARAM/.9874,1.022,.8725,1.083,1.004,.7712,.9917,1.061,
*1.017,.9249,1.041,.8042,.9301,1.180,.9482,.9572,
*.9964,1.037,1.186,.9108,.9988,.9775,.9000,1.027,
*1.002,.9558,
*1.,1.,1.,1.,1.,1.,1.,1.,1.,1.,1.,1.,1.,
*,1.,1.,1.,1.,1.,1.,1.,1.,1.,1./
READ(5,99) NI11,NJ11,NK11,NK4
NI12=NI11+1
NJ12=NJ11+1
NK12=NK11+1
NIJ11=N110*NJ10
NIJK11=N110*NK10
N320=NIJK11+1
N325=NIJ11*(NK11+NK4)+1
NK15=NK11+NK4
C NUMBER OF GLOBAL PARAMETERS, NUMBER OF ACTIVE PARAMETERS,
C PERTURBATION FACTOR, MAXIMUM ERROR CHANGE, AND
C MAXIMUM LEVENBERG RADIUS
READ(5,940) IPARX,IPAR,JITU,TST,PCHL,RLEVM
IPARX=N325+N320+2*NK10-1+10
IPAR=4
WRITE(6,941) IPARX,IPAR,JITU,TST,PCHL,RLEVM
*
940 FORMAT(3I6,3F12.0)
941 FORMAT(1H0,10X,'SERACH DEFINITION'/1H ,10X,17('(''),''/
1 1H0,10X,'NUMBER OF GLOBAL PARAMETERS',9X,I7/
2 1H ,10X,'NUMBER OF ACTIVE PARAMETERS',9X,I3/
3 1H ,10X,'MAXIMUM NUMBER OF ITERATIONS',8X,I3/
4 1H ,10X,'PERTURBATION FACTOR',11X,1PE9.3/
5 1H ,10X,'MAXIMUM ERROR CHANGE',10X,1PE9.3/
6 1H ,10X,'MAXIMUM LEVENBERG RADIUS',6X,1PE9.3)
C
GLOBAL PARAMETER SET AND ASSIGNMENT TO ACTIVE PARAMETERS
C
C *****READ IPARAM
NK1115=NK15
IT15=1
ICRO=0
CALL RDWRT
DO 209 IJ=2,N325
209 IPARAM(IJ)=1*NT(IJ)
IPARAM(1)=0
NK1115=NK11
CALL RDWRT
DO 211 IJ=2,N320
I7=IJ+N325-1

211 IPARAM(I7)=1*NT(IJ)
READ(5,99) (IPARAM(I7+I),I=1,NK10)
WRITE(6,99) (IPARAM(I7+I),I=1,NK10)
I8=I7+NK10
READ(5,99) (IPARAM(I8+I),I=1,NK10)
WRITE(6,99) (IPARAM(I8+I),I=1,NK10)

99 FORMAT(16I5)
C READ PARAMO
NK1115=NK15
CALL RDWRT
DO 309 IJ=2,N325
309 PARAMO(IJ)=NT(IJ)
PARAMO(1)=0
NK1115=NK11
CALL RDWRT
DO 311 IJ=2,N320
I7=IJ+N325-1

311 PARAMO(I7)=NT(IJ)
READ(5,98) (PARAMO(I7+I),I=1,NK10)
WRITE(6,97) (PARAMO(I7+I),I=1,NK10)
I8=I7+NK10
READ(5,98) (PARAMO(I8+I),I=1,NK10)
WRITE(6,97) (PARAMO(I8+I),I=1,NK10)

98 FORMAT(10G8.0)
97 FORMAT(10D12.3)
C ***************
I=1,NK10)
DO 147 IL=I1,IPARX
IPARAM(IL)=0
147 PARAMO(IL)=1
PARAMO(IL+2)=.5900
PARAMO(IL+7)=.8996
IPARAM(IL+2)=1
IPARAM(IL+7)=2
C ***
WRITE(6,99) (IPARAM(IL),IL=I1,IPARX)
WRITE(6,97) (PARAMO(IL),IL=I1,IPARX)
C ***************
C ACTIVE PARAMETER VALUES, UPPER BOUNDS, AND LOWER BOUNDS
DO 834 I=1,IPAR
COL(I)= 01
COU(I)=100
PARAM(I)=1
834 CONTINUE
C Determine PARAMX and do initial model call
DO 210 I=1,IPARX
X=PARAM(IPARAM(I))
IF(IPARAM(I).EQ.0) X=1
PARAMX(I)=PARAMO(I)*X
210 CONTINUE
WRITE(6,946)
WRITE(6,947) (I,PARNM(I),COL(I),COU(I),I=1,IPAR)
C
945 FORMAT(6X,3F12.0)
946 FORMAT(1H0,10X,'ACTIVE PARAMETER SET'/1H ,10X,20('-')/    MAN1650
1  1H0,10X,'ACTIVE',11X,'INITIAL',10X,'LOWER',10X,'UPPER'/    MAN1660
2  1H ,10X,'PARAMETER',8X,'VALUE',12X,'BOUND',10X,'BOUND'/    MAN1670
947 FORMAT((1H ,10X,I9,3F15.4))                                  MAN1680
JIT=-1                                                        MAN1690
IOUT=0                                                        MAN1700
CALL MODEL1                                                  MAN1710
C STOP                                                         MAN1720
IF(JITU.NE.0) GO TO 256                                       MAN1730
C PRINT FINAL MODEL RUN                                       MAN1740
255 CONTINUE                                                  MAN1750
DO 251 I=1,IPARX                                              MAN1760
X=PARAM(IPARAM(I))                                           MAN1770
IF(IPARAM(I).EQ.0) X=1                                       MAN1780
PARAMX(I)=PARAMO(I)*X                                        MAN1790
251 CONTINUE                                                  MAN1800
IOUT=1                                                        MAN1810
CALL MODEL2                                                  MAN1820
C C WATER-LEVEL RESIDUALS                                     MAN1830
C DO 656 I=1,NWLMT                                             MAN1840
EPS(I)=WLM(I)-WLC(I)                                          MAN1850
656 CONTINUE                                                  MAN1860
C WRITE(6,966)                                                MAN1870
C WRITE(6,967) (NND(I),WLM(I),WLC(I),EPS(I),I=1,NWLMT)        MAN1880
PDIV=NWLMT                                                   MAN1890
P=0.0                                                        MAN1900
DO 2566 IWL=1,NWLMT                                           MAN1910
P=P+EPS(IWL)**2*W(IWL)                                       MAN1920
2566 CONTINUE                                                MAN1930
PZ=DSQRT(P/PDIV)                                             MAN1940
WRITE(6,9566) PZ                                             MAN1950
C 966 FORMAT(1H0,10X,'WATER-LEVEL RESIDUALS'//1H ,10X,21('-')/  MAN2000
1  1H0,10X,'STEP',3X,'NODE',3X,'MEASURED',3X,'COMPARED',      MAN2010
2  3X,'RESIDUAL')                                             MAN2020
967 FORMAT((1H ,10X,I4,3X,I4,3F11.1))                         MAN2030
9566 FORMAT(1H0,10X,'STANDARD ERROR OF ESTIMATE',1X,1PE10.3)   MAN2040
C C SET INITIAL CONDITIONS                                    MAN2050
C FLAGA=0.                                                    MAN2100
IPARC=IPAR                                                   MAN2110
GDIV=IPAR                                                   MAN2120
PDIV=NWLMT                                                   MAN2130
PZP=0.                                                       MAN2140
PZPT=0.                                                      MAN2150
JIT=0.                                                       MAN2160
RLEV=0.0                                                     MAN2170
SCMIN=0.0                                                    MAN2180
C 256 CONTINUE                                                MAN2190
C C
IC=0
DO 8000 I=1,IPAR
   MU(I)=0.0
   GRP(I)=0.0
   NBND1(I)=0
   NBND2(I)=0
8000 CONTINUE
*
WRITE(6,960)
*
960 FORMAT(1H1,10X,'PARAMETER SEARCH RESULTS'/1H ,10X,24('-'))
*
J IT=JIT+1
DO 250 I=1,IPARX
   X=PARAM(IPARAM(I))
   IF(IPARAM(I).EQ.O) X=1
   PARAMXCI)=PARAMO(I)*X
250 CONTINUE
IOUT=0
WRITE(6,999) (PARAM(I),1=1,IPAR)
999 FORMAT(10E12.5)
CALL MODEL2
P=0.
DO 7115 IWL=1,NWLMT
   LWLC(IWL)=WLC(IWL)
   EPS(IWL)=WLM(IWL)-WLC(IWL)
   P=P+EPS(IWL)**2*W(IWL)
7115 CONTINUE
SB=P
PZ=DSQRT(P/PDIV)

PRINT-OUT OF SEARCH RESULTS
WRITE(6,7602) JIT,PZ,RLEV,SCMIN
7602 FORMAT(1HO,10X,'ITERATION',14/1H,10X,13('-')/)
   1 1HQ,10X,'STANDARD ERROR OF ESTIMATE',1X,1PE10.3/1H ,10X,
   2 'LEVENBERG RADIUS',11X,1PE10.3/1H ,10X,'MINIMUM SCALING FACTOR',
   3 5X,1PE10.3/1HQ,10X,2('PARAMETER',3X,'ESTIMATE',7X,'CHANGE',5X,
   4 'GRADIENT',5X)/
   WRITE(6,7607)(I,PARAM(I),MU(I),GRP(I),I=1,IPAR)
   IF(IC.EQ.O) GO TO 8003
   WRITE(6,8002)
   WRITE(6,8001) (ICON(IC),IC=1,ICMAX)
8003 CONTINUE
C
8002 FORMAT(1H0,10X,'CONSTRAINED PARAMETERS'/)  
8001 FORMAT(1H ,5X,10I8)  
7607 FORMAT((1H ,10X,2(I5,5X, 1PE10.3,3X,1PE10.3,3X,1PE10.3,4X)))
C
C CONVERGENCE AND ITERATION TESTS
C
IF(JIT.GE.JITU) GO TO 6260
PCH=ABS(PZ-PZP)
IF(PCH.LT.PCHL) GO TO 6274
PZP=PZ
GO TO 1305
6260 WRITE(6,6261) JITU
6261 FORMAT(1HO,1OX/NUMBER OF ITERATIONS GREATER THAN ',15)
GO TO 255
6274 WRITE(6,6277) PCHL
6277 FORMAT(1HO,1OX /OBJECTIVE FUNCTION CHANGES LESS THAN',F8.4)
GO TO 255
C
1305 CONTINUE
C
BEGINNING OF NEW ITERATION
C
1305 CONTINUE
C
NUMERICAL GRADIENT COMPUTATION
C
DO 1732 J=1,IPAR
DP=TST*PARAM(J)
PARAM(J)=PARAM(J)+DP
C
DO 252 I=1,IPARX
X=PARAM(IPARAM(I))
IF(IPARAM(I).EQ.0) X=1
PARAMX(I)=PARAMO(I)*X
252 CONTINUE
IOUT=0
CALL MODEL2
C
PARAM(J)=PARAM(J)-DP
DO 1730 IWL=1,NWLMT
1730 GRSS(IWL,J)=(WLC(IWL)-LWLC(IWL))*PARO(J)/DP
1732 CONTINUE
C
COMPUTATION OF A2 AND B2
C
DO 1430 J=1,IPAR
SUM=0
DO 1427 IWL=1,NWLMT
1427 SUM=SUM+GRSS(IWL,J)*EPS(IWL)*W(IWL)
B2(J)=SUM
1430 GRP(J)=-2.*SUM
WRITE(6,999) (B2(J),J=1,IPAR)
C
#MHHHHHHHHHHHHHHHHHHHHHFOEGGREGATION INSERT PACK###
C
GET LEFT HAND SIDE OF INEQUALITY (9) IN TEXT
DO 1675 JR=1,IPAR
DO 1650 JS=1,IPAR

SUM5=0.
DO 1640 IWL=1,NWLMT
1640 SUM5=SUM5+GRSS(IWL,JR)*GRSS(IWL,JS)*W(IWL)
1650 A3(JR,JS)=SUM5
1670 CONTINUE
C DO 619 JJ=1,IPAR
C 619 WRITE(6,999) (A3(JJ,J),J=1,IPAR)
DO 625 J1=1,IPAR
DO 625 J2=1,IPAR
625 A2(J2,J1)=A3(J2,J1)
CALL SOLEQU(A2,B2,IPAR)
WRITE(6,999) (B2(J),J=1,IPAR)
SUM=0.
DO 620 J=1,IPAR
620 SUM=SUM+GRP(J)*B2(J)
SUM=SUM*(-.5)
X=SUM/(SB-SUM)
WRITE(6,2999) 99,99,X,SB,SUM
C Here, X is left hand side of inequality (9) in text
2999 FORMAT(2I8,6D16.9)
C Take a look at several of the terms just preceding omega sub k
C in equation (6) in the text to see how large they are relative
C to omega sub k
DO 693 I10=1,30
IWL=100*I10
IF(I10.EQ.27) IWL=3122
IF(I10.EQ.28) IWL=3138
IF(I10.EQ.29) IWL=3159
IF(I10.EQ.30) IWL=3196
DO 694 J=1,IPAR
694 B2(J)=GRSS(IWL,J)
DO 626 J1=1,IPAR
DO 626 J2=1,IPAR
626 A2(J2,J1)=A3(J2,J1)
CALL SOLEQU(A2,B2,IPAR)
SUM=0
DO 695 J=1,IPAR
695 SUM=SUM+GRSS(IWL,J)*B2(J)
WRITE(6,2999) I10,NND(IWL),SUM
C SUM is one of the terms just preceding omega sub k
693 CONTINUE
C Evaluate the term in the square root of equation (7) of the text
DO 697 JS=1,IPAR
DO 696 J=1,IPAR
696 B2(J)=0
B2(JS)=1
DO 627 J1=1,IPAR
DO 627 J2=1,IPAR
627 A2(J2,J1)=A3(J2,J1)
CALL SOLEQU(A2,B2,IPAR)
ANS1=DSQRT(B2(JS)*SB)*PARO(JS)
ANS=ANS1*DSQRT(X)
WRITE(6,2999) JS,JS,ANS,ANS1,B2(JS),PARO(JS)
C B2(JS) is the term
697 CONTINUE
C     #End of Regression Insert Pack#
      DO 5070 J=1,IPAR
         CONB(J)=1.
      DO 5070 K=1,IPAR
         SCALE(J)=1.
      5070 CONA(J,K)=1.
      5050 CONTINUE
      SUM7=0.
      DO 1558 J=1,IPAR
         B2(J)=-.5*GRP(J)*CONB(J)
      1558 SUM7=SUM7+B2(J)*B2(J)
      RLEV=RLEVM
      FLEV=DSQRT(SUM7)/RLEV
      DO 1575 JR=1,IPAR
         DO 1550 JS=1,IPAR
            SUM5=0.
            DO 1540 IWL=1,NWLMT
            1540 SUM5=SUM5+GRSS(IWL,JR)*GRSS(IWL,JS)*W(IWL)
            1550 A2(JR,JS)=SUM5*CONA(JR,JS)
            1575 A2(JR,JR)=A2(JR,JR)+FLEV
   C     SOLUTION OF THE SYSTEM OF LINEAR EQUATIONS
      CALL SOLEQU (A2,B2,IPARC)
      DO 1570 J=1,IPAR
         MU(J)=B2(J)*PARO(J)*CONB(J)
      1570 WRITE(6,97) (MU(I),I=1,IPAR)
   C     COMPUTATION OF BOUNDARY SET
      DO 5100 J=1,IPAR
         THRR=PARAM(J)+MU(J)
         IF(THRR.GE.COU(J)) GO TO 5120
         IF(THRR.LE.COL(J)) GO TO 5140
         NBND2(J)=0.
         GO TO 5100
      5120 NBND2(J)=1.
         GO TO 5100
      5140 NBND2(J)=-1.
      5100 CONTINUE
   C     SELECTION OF DECISION VARIABLES
      IC=0
      IF(FLAGA) 5160,5160,5170
      5160 DO 5200 J=1,IPAR
         IF(NBND2(J).EQ.0) GO TO 5200
         IF(NBND2(J).NE.NBND1(J)) GO TO 5200
         CONB(J)=0.
         FLAGA=1.
         DO 5210 JJ=1,IPAR
            CONA(J,JJ)=0.
      5210 CONA(JJ,J)=0.
      IC=IC+1
ICON(IC)=J
WRITE(6,97) FLAGA,(CONB(I),I=1,IPAR)
5200 CONTINUE
ICMAX=IC
GO TO 5250
5170 FLAGA=0.
5250 IF(FLAGA) 5260,5260,5050
5260 CONTINUE
C
C UPDATE OF BOUNDARY SET
C
DO 5400 J=1,IPAR
THRR=PARAM(J)+MU(J)
WRITE(6,97) MU(J),THRR
IF(THRR.GE.(COU(J))) GO TO 5450
IF(THRR.LE.(COL(J))) GO TO 5460
NBND2(J)=0
GO TO 5420
NBND2(J)=1
THRR=COU(J)
GO TO 5420
NBND2(J)=-1
THRR=COL(J)
CONTINUE
PARAM(J)=THRR
CONTINUE
C Calculate gradient of PZ
DO 5500 J=1,IPAR
NBND1(J)=NBND2(J)
GRP(J)=GRP(J)/(2.*PZ*PDIV*PAR0(J))
5500 CONTINUE
WRITE(6,99) (NBNDKI),!=!,IPAR)
GO TO 1017
END
MODEL

The basic function of MODEL is to evaluate $f(B)$ given a value of $B$ from MAIN. MAIN passes a value of $JIT = -1$ when going to MODEL1, and values of $JIT = 1,2,3,...$, when going to MODEL2. The approximations $f_i(B)$ to the observations $Y_i$ include heads for the years 1972 and 1982 as well as 33 $f_i(B)$ corresponding to the 33 log sand hydraulic conductivity observations. Each time MODEL is called it finds heads for five different years corresponding to $INT = 1,2,3,4,5$. $INT = 3$ corresponds to 1972 and $INT = 5$ corresponds to 1982. Convergence of SIP or PCG is faster when good starting values for head (= XX) are used. Thus the values of XX(IJ) found for a given INT and JIT are saved in IXX5(IJ, INT) to be used as starting values for XX(IJ) at JIT = JIT + 1. XX5 is used as the value of XX at INT (t) when finding XX at INT + 1 (t+Δt), corresponding to the next time step.

IOUT = 1 is passed to MODEL by MAIN when various printouts are desired. When IOUT = 0 this printout does not occur.

The most important changes of VARDEN necessary to produce MODEL are:

1. The emplacement of the global parameters PARAMX into the calculation of the quantities DD, BB, and ZZ, the elements of the matrix the inversion of which gives $f(B)$;
2. Implementation of the functions giving the rate of decrease with depth of sand and clay hydraulic conductivity combined with the use of equations (10) and (11) in the text and giving effective hydraulic conductivities. The first of these changes appears at lines MD06040 through MD06130 of the listing of MODEL. The second occurs at lines MD06140 through MD06440.
Essential Features of Model

JIT = -1

MAIN

JIT = 1,2,3,

MODEL1

Read data
Place fixed head data into DXXX5

MODEL2

Put calculated values for the 33 log sand hydraulic conductivities into WLC

XX5(IJ) = DXXX5(IJ)

DO 600 INT = 1,5
1) XX(IJ) = DXXX5(IJ, INT)
   (=XX5(IJ) if JIT = -1, or
   IJ at fixed head)

   2) Solve for XX with previous value, using SIP or PCG

   3) DXXX5(IJ, INT) = XX(IJ)
      XX5(IJ) = XX(IJ)

   4) Use calculated heads and flows

600 CONTINUE

Put calculated heads for years 1972 and 1982 into WCL

Do Chi square and other analysis of residuals

Return WLC to MAIN

LINE 10
LINES 750 - 3590
LINE 4040
LINE 4690
LINE 5350
LINE 5820
LINE 7780
LINE 7960
LINES 8270, 8280
LINES 8970, 8980
LINES 9030 - 11310
LINE 11970
LINES 12030 - 13290
Definition of Program Variables

ALFA  Parameter $b_5$ in text.

BBK   See FCNT.

DDK   See FCNT.

DD, BB, ZZ  Elements of the matrix, the inversion of which gives $f(B)$, the solution to the model with parameters $B$.

DEL   Array combining the five time intervals.

DELT  Time interval.

DXI, DYJ  DXI(I) is the $I$ direction horizontal dimension of the grid elements in column $I$. DYJ(J) is the $J$ direction horizontal dimension of the grid elements in row $J$.

ERR, XX10  Iteration terminates when

$$\max_{(\text{over } IJ)} |XX^{ICNT-1} - XX^{ICNT-2}| < ERR,$$

the maximum residual error

$$\max_{(\text{over } IJ)} |M[XX^{ICNT-YQ}| < XX10, \text{ or } ICNT \geq ITMAX.$$

FC   Factors used to approximate pumping rates for first time interval.

FCNT, NT, DDK, BBK

$K_{xx}(IJ) = (NT(IJ) \times (DDK(K)),$

$K_{yy}(IJ) = (NT(IJ) \times (BBK(K)),$

where $K_{xx}(IJ), K_{yy}(IJ)$ denote the $x, y$ components of $K$, the hydraulic conductivity for grid element $IJ$. NT is used as a dummy variable for various data sets.

GP5   Multiplier for $K_z/b$ for geopressure zone.

124
HL
HL(IJ) is used for subsidence. It is the lowest value that XX(IJ) = head has achieved yet for any of the previous or the current time step.

HLS
HLS is subsidence at ground surface.

HMAX
SIP parameter $\beta'$ (Trescott and Larson, 1977).

I
Positional locator along a row, also column number.

ICT
In SIP, a repeating counter for iteration parameter number.

IGP
IGP(J) traces I along boundary of geopressed zone.

IJ
Single subscript replacement for I,J,K corresponding to grid element I,J,K.

IJKM1
Replacement for I,J,K-1, or for I,J,K+1 when the K direction is reversed in SIP.

IJKP1
Replacement for I,J,K+1, or for I,J,K-1 when the K direction is reversed in SIP.

IJLB
IJ for confining bed grid elements.

IJM1K
Replacement for I,J-1,K, or for I,J+1,K when the J direction is reversed in SIP.

IJP1K
Replacement for I,J+1,K, or for I,J-1,K when the J direction is reversed in SIP.

IM1JK
Single subscript replacement for I-1,J,K.

IP1JK
Single subscript replacement for I+1,J,K.

IOUT
If IOUT = 1, MODEL prints various quantities. If IOUT = 0, it does not.

IP1JK
Single subscript replacement for I+1,J,K.

IPH(K), K=2
Set IPH(K) = 1 if you want: hydraulic head or freshwater head, flow rate data, or drawdown, for layer K. Set

NK11
IPH(K) = 0 if you do not want these quantities for layer K. The layers are numbered K = 2 through K = NK11 = NK10+1.
**IPR, JPR**  
At I = IPR(I10), J = JPR(I10) all of the layers present in region I10 (fig. 31) occur, I10 = 1,2,...,10.

**ISOR**  
ISOR = 0.

**ITMAX**  
Maximum number of iterations allowed.

**IWRT**  
Set IWRT to 1 if you want to watch the convergence of freshwater head h' = XX, at the three locations: I = NU1(1), J = NU1(2), K = NU1(3) I = NU2(1), J = NU2(2), K = NU2(3) and I = NU3(1), J = NU3(2), K = NU3(3).

**NU1(I), I=1,3**  
Storage value for starting values for the solution finding XX, head.

**NU2(I), I=1,3**  
Storage array for fixed (and other) head values.

**NU3(I), I=1,3**  
Positional locator along a column, also row number.

**JIT**  
Iteration number. Called s in previous text in this attachment.

**K**  
Layer number.

**KOUT**  
The total number of confining beds between layer 2 and layer K. Only one effective confining bed is allowed between any two layers. If there are more than two confining beds, they are combined into one effective confining bed.

**L9**  
In SIP, L9 is: 1 for J direction reversal, 2 for J and K direction reversal, and 3 for K direction reversal.

**LB**  
LB(IJ) is the vertical dimension of grid element IJ.

**LBB**  
Elevation of the base of the lowermost layer, K = 2.
Number of iteration parameters in SIP. Four types of flow-rate data are available: (1) the flow rate out of each grid element in the negative I, J, and K directions, (2) the flow rate out of fixed head (MHD = 0) grid elements, (3) head discharge dependent discharge flow rates from grid elements having such discharge, and (4) total flow rate budgets for sets of grid elements having the same I, the same J, and the same K. All flow rates are in units of mass divided by \( \rho_0 = 1 \text{ gm/cm}^3 \) per unit time (\( L^3/T \)) (see the section on "Units" in the text).

Set LFLOW = 1 if you want any flow-rate data, set LFLOW = 0 if no flow rate data is desired. Set LFLO(i) = 0 if you do not want type i flow rate data. For i = 1, set LFLO(1) = 1, 2, 3, for type 1 flow rate data in the negative I, J, and K directions respectively. If LFLO(1) = 4 all directions are given.

Set LFLO(2) = 1 if you want type 2 flow rate data. Set LFLO(3) = 1 if you want type 3 flow rate data. Set LFLO(4) = 1, 2, 3, for the budgets of sets of grid elements having the same I, J, and K respectively. If LFLO(4) = 4 budgets are given for I, J, and K. Corresponding to the integers 1, 2, and 3 in the first column of output the second column gives I, J, and K respectively.

\[
\text{LRO} = (\rho'/\rho_0) \quad \text{where} \quad \rho = \rho' + \rho_0 \text{ is the water density for grid element IJ. LRO(IJLB) = (\rho'/\rho_0')}, \text{where} \quad \rho = \rho' + \rho_0' \text{ is the water density for confining bed grid element IJLB. Values given are truncated after the fifth decimal place.}
\]

\[
\text{LZ2} = M, \text{where the integer M divided by 10 is the elevation of node point IJ located at the center of grid element IJ.}
\]

MAQ1(K) = 0 except when a confining bed lies between layers K-1 and K, in which case MAQ1(K) = 1.

MAQ1(K) holds read-in values for fixed heads initially. After these are read into IXXX5, MAQ1(K) is = 0,1,2, corresponding to: a regular active grid element, fixed element, and an element outside of the domain of solution, respectively.

Number of columns in the modeled area.
NINT Number of time intervals, NINT = 5.
NJ10 Number of rows in the modeled area.
NK10 Number of layers in the modeled area.
NK4 Number of intervening confining beds.
NND NND(I) is equal to the IJ location of observation I.
NT See FCNT.
NU1 See IWRT.
NU2 See IWRT.
NU3 See IWRT.
NWLM Number of observations of grid-element volume-averaged head for years 1972 and 1982. Equal to 1,432 + 1,675 = 3,107.
NWLM1 Number of observations of grid element volume averaged head for year 1972 only. Equal to 1,432.
P1 Parameter $b_4$ in text.
P2 Parameter $b_9$ in text.
P3 Parameter $b_7$ in text.
P7 Parameter $b_8$ in text.
P8 Parameter $b_3$ in text.
P9 Parameter $b_7$ in text.
SV SV specifies specific storage $S_s$.
SV35 $P_2*SV35$ is specific storage for all grid elements (except as modified by subsidence which multiplies the value by $VV40$ for the clay component).
VV40 $VV40 = P_7*40$. $VV40*SV35*P_2$ is specific storage for clay component in subsidence condition.
W(I) is the weight of observation $Y_i$ in the expression of $S(B)$.

WLC(IJ) is the calculated value $f_i(B)$ produced by the model.

WLM(I) is the measured value $Y_i$ of observation number $i$.

WMAX In SIP, $(XYFC)*(XY)$ is used for $(1-\alpha_{max})$ (Weinstein and others, 1969) when $WMAX = 0$. When $WMAX = 0$, $(1-\alpha_{max}) = WMAX$ is used.

XTV Chi square test data.

XX Freshwater head = (pressure head $h$) + $z$.

XX10 Error step for the residual $b - Ax$ in the solvers PCG and SIP.

XXS The value of XX for the previous time step.

XXSTR XXSTR(IJ) is percent sand in grid element IJ.

YQ Contains pumping rates and also psuedo sources prior to each solution by PCG or SIP.

YQ1 YQ1(IJ,I3) $I3 = 1,2,3$ has pumping rates for the years 1970, 1975, and 1980, respectively.
SUBROUTINE MODEL1
IMPLICIT REAL*8 (A-H,O-Z)
COMMON WS,HMAX,RELX1,RELX2,COEF,ERR,XX10,DELT,ER5,SRZ,SUMRZ
1,ERRSV,XX10SV,JIT,NJ10,N11,NJ11,NK11,NNN,NSKP1
2,NSKP2,ITMAX,ICNT,IEVP,IWR1,NW1,NW2,NW3,N320,NUM4
3,L9,LENGTH,NK1115,IT01,IT15,ICRO,DDK,BBK
4,N10,NJ10,NK10,N11,NJ12,NK12,SVIJ,VV40,SV35
COMMON /XX/ XX
COMMON /DT/ DT
COMMON /W/ W
COMMON /E2/ E2
COMMON /F2/ F2
COMMON /G2/ G2
COMMON /YQ/ YQ
COMMON /NT/ NT
COMMON /DD/ DD
COMMON /BB/ BB
COMMON /Z/ ZZ
COMMON /XXS/ XXS
COMMON /ALN/ ALN
C COMMON /XXE/ XXE
C COMMON /SV/ SV
C COMMON /HL/ HL
COMMON /LB/ LB
COMMON /MHD/ MHD
COMMON /C/ DXI,DYJ,N325,ISOR,IPDD
1,NPINT,DTO,TOT,TFAC,NINT,IPRNT,IWR,LFLOW,NUM,LLRO,IWRXX,
21L22,MAQ1,NU1,NU2,NU3,LFLO,IP
3,LBB,SUMF,SUNF,Q2,SG2,SYQ,SVV,SUMFM,SUNFM
COMMON /B/ COL,COU,PARAM,PARO,MU,W,EPS,WLM,LWLC,WLC,A2,B2,GRP
1,CONA,CONB,SCALE
2,NBND1,NBND2,ICON,NND,WLM1,WLMT,IOU
COMMON /GRSS/ GRSS
COMMON /XXSTR/ XXSTR
COMMON /XKZZ/ XKZZ
COMMON /LRO/ LRO
COMMON /YQ1/ YQ1
COMMON /LZ2/ LZ2 .
COMMON /PARAMX/ PARAMX
COMMON /PARAMO/ PARAMO
COMMON /IPARAM/ IPARAM
COMMON /YQ2/ YQ2
C *****EXTRAS****
COMMON /FC/ FC,DEL,ISO,OS4,G5,LFOA,IPHA
COMMON /IIXS/ IIXS
COMMON /IIXX5/ IIXX5
COMMON /IIXXXS/ IIXXXS
COMMON /HLS/ HLS
COMMON /IPR/ IPR,JPR,X3150,X493,III,XTV
DIMENSION LA(40),IA(40),XL1(40),XTV(20),IX7(20)
INTEGER*2 IXX5S(65078),IXX5(65078,5),LFOA(4),IPHA(50)
INTEGER*2 IGP(120),IPR(10),JPR(10)
REAL*4 DPT(4),XSS(4),XSH(4),HLS(5918)
C **********
REAL*4 YQ2(65078)
REAL*4 COL(50),COU(50),PARAM(50),PARO(50),MU(50),W(3303),EPS(3303),MD00550
INTEGER** NBND1 ( 50 ) , NBND2 ( 50 ) , ICON( 50 ) , NND(3303)
INTEGER*2 IPARAM( 159872)
REALM DD( 65078), BB( 65078), ZZ( 65078), YQ( 65078),
XXS( 65078), Q2( 300), DDK( 50), BBK( 50), NT(94658), HL(65078)
 INTEGER*2 LRO(94658) , IPH(50) , LFLO(4) , 17(5) , J7( 5) , K7( 5)
 INTEGER*2 MAQ1(50), NU1(3), NU2(3), NU3(3)
 DIMENSION DT( 65078), E2( 65078), F2( 65078), G2( 65078), W( 65078)
 DIMENSION XX(65078)
 INTEGER** WS( 10), MHD( 65078), LZ2(65078), LB(94658), LBB(5918)
 INTEGER*2 AOC(4), BOC(4), COC(4), DOC(4)
 REAL** SUMF(3,250), SUNF(3,250), SQ2(3,250), SYQ(3,250)
 REAL** SVV(3,250), SUMFM(3,250), SUNFM(3,250), FC(5,15), DEL(5)
*, IC7(65)
PI=3. 1415926
DATA DEL/1 . D40 , 10950 . DO , 1825 . DO , 1825 . DO , 1825 . DO/
DATA IPR/12,16,18,23,28,27,20,28,35,30/
DATA JPR/90,56,41,36,29,94,66,44,38,60/
DATA XTV/-1.D20,- 1.53413, -1.15035, -.88715, -. 31864,-. 15731,. 0 , . 0, . 0, .0, .0, .0, .0, .0/ DO 719 1=10,17
XTV(I)=-XTV(18-I)
WRITE(6,3555) (XTV(I) , 1=1 , 17)
WRITE(6,3555) (DEL(I) , 1=1 , 5)
DO 10 1=1,50
MAQ1(I)=0
READ(5,8877) ((FC(I,J),J=1,10),I=1,2)
FC(1,11)=1
FC(2,11)=1
8877 FORMAT(10F8.0)
WRITE(6,8877) ((FC(I,J),J=1,10),I=1,2)
READ(5,2003) (IGP(I),I=2,103)
WRITE(6,2003) (IGP(I),I=2,103)
OS4=1.E-25
GP5=5.E-30
V40=40
WRITE(6,9022)
WRITE(6,3555) VV40,OS4,GP5
L9=2
LENGTH=5
XYFC=1
WMAX=0
READ(5,2000) ISOR
IF(ISOR.GE.0) READ(5,2000) ITMAX, ERR, XX10, HMAX
ISOR=0
ITMAX=70
ERR=.001
XX10=8
C ERR=.0002
C XX10=1
HMAX=1.0

READ(5,2020) NI10,NJ10,NK10,NK4
READ(5,2003) IWRT,(NU1(I),I=1,3),(NU2(I),I=1,3),(NU3(I),I=1,3) IWRT=0
READ(5,2003) LFLOW,(LFLO(I),I=1,4),(LFLOA(I),I=1,4)
NK11=NI10+1
READ(5,2003) (IPH(K),K=2,NK11)
READ(5,2003) (IPHA(K),K=2,NK11)
NUM4=ISOR-1
NUM4=3
NI11=NI10+1
NJ11=NJ10+1
NK15=NI11+NK4
NI12=NI10+2
NJ12=NJ10+2
NK12=NIK10+2
NIJ10=NI10*NJ10
NIJK10=NIJ10*NK10
NNN=NIJK10+2
N315=NIJ10+1
N320=NIK10+1
N325=NIJ10*(NK10+4)+1
NW1=NU1(I)+NI11*(NU1(2)-2)+NIJ10*(NU1(3)-2)
NW2=NU2(I)+NI11*(NU2(2)-2)+NIJ10*(NU2(3)-2)
NW3=NU3(I)+NI11*(NU3(2)-2)+NIJ10*(NU3(3)-2)

C READ IN AND PRINT ARRAYS MHD,LRO,LBB,XX,SV,XXSTR, XKZZ, FCNT, DDK,BBK,NT,MAQ1,DXI,DYJ,YQ1
C GROUP II
WRITE(6,5000)
NK1115=NIK10
IT15=1
ICRO=0
CALL RDWRT
DO 20 IJ=2,N320
20 MHD(IJ)=NT(IJ)*(10)+.1
WRITE(6,5002)
NK1115=NIK10
CALL RDWRT
DO 90 IJ=2,N325
90 LRO(IJ)=NT(IJ)*1.D+5+.1
WRITE(6,5003)
NK1115=2
CALL RDWRT
DO 100 IJ=2,N315
100 LBB(IJ)=NT(IJ)
WRITE(6,5004)
NK1115=NIK10
IT15=10
CALL RDWRT
DO 110 IJ=2,N325
110 LB(IJ)=NT(IJ)
LB(1)=1
IT15=1

C ******** Insert LB=1 for geopressured zone *****
DO 219 J=2,NJ11

132
IGPJ=IGP(J)

DO 217 I=2,NI11
IF=I+NI10*(J-2)
C %%%%%%% If LB<10 at any location, set to zero and adjust LBB accordingly
LL=0
DO 216 K=2,NK10
IJ=IF+(K-2)*NIJ10
LBIJ=LB(IJ)
IF(LBIJ.GE.10) GO TO 215
LL=LL+LBIJ
LB(IJ)=0
215 CONTINUE
216 CONTINUE
LBB(IF)=LBB(IF)+LL
C %%%%%%% LB<10 to zero done
LB1=LB(IF)
I2236=22*I+36*J-2349
IF(LB1.GE.10) GO TO 218
IF(I2236.LE.O) GO TO 218
IF(LB(IF+NIJ10*9).EQ.O) GO TO 218
IF(I.LT.IGPJ) GO TO 218
C LB1=1
218 LB(IF)=LB1
218 CONTINUE
217 CONTINUE
C WRITE(6,3004) J,(LB(I+NI10*(J-2)),I=2,NI11)
219 CONTINUE
C ******** LB=1 for geopressed zone done *****
C Read in observations Yi(=WLM) of grid element volume averaged head for the years 1970 and 1980. Number them consecutively using I9 as shown. NWLM is the total number of these. NND is the IT location of each observation
C **************
NK11=15=NK11
IT15=10
CALL RDWRT
DO 246 I=1,3303
WLM(I)=0
W(I)=0
WLC(I)=0
NND(I)=0
NWLM1=19
I9=0
DO 247 IJ=2,N320
XNTIJ=NT(IJ)
IF((XNTIJ*LB(IJ)).EQ.0.) GO TO 247
I9=I9+1
NND(I9)=IJ
WLM(I9)=XNTIJ
246 CONTINUE
247 CONTINUE
NWLM1=I9
CALL RDWRT
IT15=1
DO 274 IJ=2,N320
XNTIJ=NT(IJ)
IF((XNTIJ*LB(IJ)).EQ.0.) GO TO 274
274 CONTINUE
I9=I9+1
NND(I9)=IJ
WLM(I9)=XNTIJ

274 CONTINUE
NWLM=I9
NWLMT=NWLM+100

C ******** Done getting grid element value averaged head
C observations ********
C Set weights for gerah observations
CALL RDWRT
DO 248 I=1,NWLM
IJ=NND(I)
248 W(I)=(39.093*39.093)/NT(IJ)**2
WRITE(6,952)
DO 249 I=1,NWLM
IF (W(I).LE.O.O) W(I)=1.0
CONTINUE
WRITE(6,953) (NND(I),WLM(I),W(I), I=1,20)
WRITE (6, 2020) NWLM
FORMAT((1H , 10X, 14, 3X, I4,F12 . 1 ,F10 .4) )
*** Set weights done ****

Determine observations and corresponding weights for the 33
zone averaged hydraulic conductivities
GO TO 876
N33=33
SUM=0
LAA=2
Y1=0
Y2=0
NL=0
DO 125 I=1,N33
READ(5,2777) LA(I),IA(I),M1,V1,V2,M2,V3,V4,M3,XL1(I)
WRITE(6,2777) LA(I),IA(I),66,XL1(I)
IF((LA(I).EQ.LAA).AND.(I.NE.33)) GO TO 1266
SUM=SUM+ ( Y2 - Y1*Y1/NL ) /NL
Y1=0
Y2=0
LAA=LAA+1
CONTINUE
X=XL1(I)
Y2=Y2+X*X
Y1=Y1+X
NL=NL+1
CONTINUE
AV65=SUM/10
X493=39.093/DSQRT(AV65)
DO 126 I=1,N33
II=NWLM+(LA(I)-2)*10+1A(I)
W(II)=1
WLM(II)=X493*(XL1(I))
WRITE(6, 3 001) I,II,XL1(I),W(II),WLM(II),AV65,X493
3001 FORMAT(2I7,7D12.3)
126 CONTINUE
2777  FORMAT(2I4,3(I4,2F8.3))  MD02730
C876  CONTINUE  MD02740
C  ***** Done with 33 observations and weights *****  MD02750
    WRITE(6,5052)  MD02760
    NK1115=NK11  MD02770
    CALL RDWRT  MD02780
    DO 150  IJ=2,N320  MD02790
150   XX(IJ)=NT(IJ)*10  MD02800
    XX(1)=0  MD02810
    XX(NNN)=0  MD02820
    WRITE(6,5054)  MD02830
    CALL RDWRT  MD02840
C    DO 160  IJ=2,N320  MD02850
C160   SV(IJ)=NT(IJ)*1  MD02860
   SV35=.3D-5  MD02870
    WRITE(6,3555) SV35  MD02880
    WRITE(6,9954)  MD02890
    CALL RDWRT  MD02900
    DO 1601  IJ=2,N320  MD02910
1601  XXSTR(IJ)=NT(IJ)  MD02920
    WRITE(6,5060)  MD02930
    CALL RDWRT  MD02940
    DO 166  IJ=2,N320  MD02950
166   XKZZ(IJ)=NT(IJ)  MD02960
C    Read pumping rate for 3 time periods 1970, 1975, and 1980  MD02970
C into YQ1  MD02980
    READ(5,2000) NINT  MD02990
    WRITE(6,5053)  MD03000
    NK1115=NK11  MD03010
    DO 1642  I3=1,3  MD03020
    CALL RDWRT  MD03030
    DO 1681  IJ=2,N320  MD03040
1681  YQ1(IJ,I3)=1*NT(IJ)  MD03050
1642  CONTINUE  MD03060
    WRITE(6,5005)  MD03070
    NK1115=NK15  MD03080
    IT15=2  MD03090
    CALL RDWRT  MD03100
C  ***** REMOVE PINCHED OUT NODES WHEN APPROPRIATE *****  MD03110
    DO 163  I=2,NI11  MD03120
    DO 163  J=2,NJ11  MD03130
    IJJJ=I+NI10*(J-2)  MD03140
    DO 163  K=2,NK11  MD03150
    IJ=IJJJ+NIJ10*(K-2)  MD03160
    IF(LB(IJ).NE.0) GO TO 165  MD03170
    IF((K.EQ.2).OR.(K.EQ.NK1D)) GO TO 164  MD03180
    IT5=0  MD03190
    KM1=K-1  MD03200
    DO 161  K1=2,KM1  MD03210
    IJJJ=IJJJ+NIJ10*(K1-2)  MD03220
161   IF(LB(IJJJ).NE.0) IT5=IT5+1  MD03230
    IT6=0  MD03240
    KP1=K+1  MD03250
    DO 162  K2=KP1,NK11  MD03260
    IJ2=IJJJ+NIJ10*(K2-2)
162 IF(LB(IJ2).NE.0) IT6=IT6+1
163 CONTINUE
164 NT(IJ)=0
165 CONTINUE
166 IF((IT5*IT6).GT.0) GO TO 165
167 NT(IJ)=0
168 CONTINUE
C ******* DONE Pinched out nodes DONE ******
C GROUP III  
170 CONTINUE
171 IF(NK4.EQ.0) GO TO 170
172 READ(5,2003) (MAQ1(K),K=2,NK11)
173 WRITE(6,5010)
174 WRITE(6,4002) (K,MAQ1(K),K=2,NK11)
175 CONTINUE
176 READ(5,2000) FDXI,FDYJ,MDXI1,MDYJ1,MD2  
177 IF(MDXI1.EQ.1) READ(5,2000) (DXI(I),I=2,NI11)
178 IF(MDXI1.EQ.1) GO TO 190  
179 DO 180 I=2,NI11
180 DXI(I)=FDXI
190 IF(MDYJ1.EQ.1) GO TO 210
200 J=2,NJ11
201 DO 200 J=2,NJ11
202 DYJ(J)=FDYJ
210 IF(MD2.NE.0) GO TO 220
211 WRITE(6,5001)
212 WRITE(6,3010) (DXI(I),I=2,NI11)
213 WRITE(6,3010) (DYJ(I),I=2,NJ11)
220 CONTINUE
221 DXI(1)=1
222 DYJ(1)=1
223 DXI(NI12)=1
224 DYJ(NJ12)=1
C READ IN AND PRINTING OF ARRAYS NOW COMPLETE
C INITIALIZE VARIOUS ARRAYS TO ZERO
230 DO 230 IJ=1,NNN
231 HL(IJ)=0
232 DD(IJ)=0
233 BB(IJ)=0
234 ZZ(IJ)=0
235 DT(IJ)=0
236 E2(IJ)=0
237 F2(IJ)=0
238 G2(IJ)=0
239 YQ2(IJ)=0
240 W(IJ)=0
241 KOUT=0
242 DO 280 K=2,NK11
243 KOUT=KOUT+MAQ1(K)
244 DO 280 J=2,NJ11
245 DO 280 I=2,NI11
246 I236=22*I+36*J-2349
247 DXY=DXI(I)*DYJ(J)
248 IIF=I+NI10*(J-2)
249 IJ=IJF+NIJ10*(K-2)
C DETERMINE L22  
250 KOUT=0
251 DO 280 K=2,NK11
252 KOUT=KOUT+MAQ1(K)
253 DO 280 J=2,NJ11
254 DO 280 I=2,NI11
255 I236=22*I+36*J-2349
256 DXY=DXI(I)*DYJ(J)
257 IIF=I+NI10*(J-2)
258 IJ=IJF+NIJ10*(K-2)
LBIJ=LB(IJ)
KLB=NIK11+KOUT
IJLB=IJF+NIJ10*(KLB-2)
DTIJ=LRO(IJ)*1.0D-5
C SV(IJ)=SV(IJ)*DXY*LBIJ
C DETERMINE LZ2
IJKM1=IJ-NIJ10
IF(K.EQ.2) GO TO 240
LZ2(IJ)=LZ2(IJKM1)+5*(LBIJ+LB(IJKM1))+10*MAQ1(K)*LB(IJLB)
GO TO 250
240 LZ2(IJ)=LBIJ*5+10*(LBB(IJ)+MAQ1(K)*LB(IJLB))
250 CONTINUE
C Place layer 12 head from MHD and also head of zero for geopressed zone into IXXX5 for storage.
C Set MHD=0,1,2 for active, fixed, and out of domain elements respectively
XXMHD=XX(IJ)
MHDIJ=MHD(IJ)
IF(K.EQ.NK11) XXMHD=MHDIJ
LZ2IJ=LZ2(IJ)
X=((XXMHD-LZ2IJ)*(DTIJ+1)+LZ2IJ)*.1DO
IF((K.EQ.2).AND.(I2236.GT.O)) X=3000
IF(NTdJ).EQ.O.O) X=0.0
IXXXSdJ)=X+.5
IX=0
IF((K.EQ.2).AND.(I2236.GT.O)) IX=1
IF(K.EQ.NK11) IX=1
IF(NT(IJ).EQ.0.0) IX=2
MHD(IJ)=IX
C PUT PUMPING RATES to zero if out of domain of solution
DO 1643 I3=1,3
X=YQ1(IJ,I3)
IF(IX.EQ.2) X=0
YQ1(IJ,I3)=X
1643 CONTINUE
280 CONTINUE
C ********* DO removal (W(I)=0) OF WLM'S that lie outside of domain of solution
ISM9=0
DO 643 I=1,NWLM
IJ=NND(I)
IF(.NOT.(MHD(IJ).GE.1)) GO TO 644
ISM9=ISM9+1
W(I)=0
WRITE(6,1003) ISM9, IJ, MHD(IJ), 777., WLM(I)
644 CONTINUE
643 CONTINUE
C ********** DO removal (W(I)=0) OF WLM'S that lie outside of domain of solution
DO 895 I10=1,10
I2464=IPR(I10)+(JPR(I10)-2)*NI10
DO 993 I=1,10
IJ=I2464+(I-1)*NIJ10
I3=I+1
I8=NWLM+(I13-2)*10+I10
IF(MHD(IJ).GE.1) W(I8)=0
IF(MHD(IJ).GE.1) WRITE(6,2020) I8, I10, I, IJ, MHD(IJ)
CONTINUE
CONTINUE
STOP
****** DONE ******
LZ2(1)=0
SVIJ=SV35*DXY
C Put elevation of top layer 12 into LBB for later use, print
WRITE(6,8002)
WRITE(6,6000)
WRITE(6,4002) NK11
DO 300 J=2,NJ11
DO 290 I=2,NI11
IJF=I+NI10*(J-2)
IJ=1JF+NIJ10*(NK11-2)
X=(LZ2(IJ)*.1DO+LB(IJ)*.5DO+.1)
LBB(IJF)=X
CONTINUE
CONTINUE
DO 301 J=2,NJ11
I5=NI10*(J-2)
WRITE(6,3004) J, (LBB(I5+l) , 1=2 ,NI11)
CONTINUE
STOP
C Set up several variables which are to be used in subsequent
C calls to model2
N98=N320+N325-2
N99=N98+NK10
ERRSV=ERR
XX10SV=XX10
X3150=NWLM+N33
IIII=-1
C
ENTRY MODEL2
C
I92=60
J92=74
R42=37*37
N324=N325-1
C Set error criteria for solution dependent upon iteration
C count JIT
XL5=1.0*JIT
IF(XL5.GT.9.) XL5=9.
ERR=(19.-2.*XL5)*ERRSV
XX10=(19.-2.*XL5)*XX10SV
C Error criteria ERR and XX10 now set
N111=N99+NK11
N121=N111+7
C
IF(IOUT.EQ.0) GO TO 780
WRITE(6,9030)
DO 792 I=1,27
IJ=I*NIJ10-100
WRITE(6,4502) PARAMX(U)
792
WRITE(6,3555) (PARAMX(N111+I),I=1,8)  MD04900
780 CONTINUE  MD04910
\* For IJ=2,N121(N121=IPARX-3,N111=IPARX-10)  MD04920
\* Change PARAMX(IJ) to EXP[(PARAMX(IJ)-1)*LS]  MD04930
C LS=5 for sand and clay hydraulic conductivities  MD04940
DO 312 IJ=2,N121  MD04950
L5=5  MD04960
IF(IJ.GE.(N111+2)) L5=2  MD04970
312 PARAMX(IJ)=10**((PARAMX(IJ)-1)*L5)  MD04980
C Change done  MD04990
C Assign those PARAMX(IJ) for IJ>N111 to specific parameters  MD05000
P1=PARAMX(N111+1)  MD05010
P2=PARAMX(N111+3)  MD05020
P3=PARAMX(N111+4)  MD05030
ALFA=PARAMX(N111+5)  MD05040
P7=PARAMX(N111+6)  MD05050
P8=PARAMX(N111+7)  MD05060
P9=PARAMX(N111+8)  MD05070
V40=P7*40  MD05080
C Write out PARAMX,XKZZ,NT, and resulting effective  MD05090
C conductivities and their logarithms for depths of 600 and  MD05100
C 6000 ft for each of the 10 regions of figure 31 in the text.  MD05110
C Determine calculated values WLC for observations corresponding  MD05120
C to the 33 log sand hydraulic conductivities at line MD05350  MD05130
C \*\*\*  MD05140
DO 795 I10=1,10  MD05150
I2464=IPR(I10)+(JPR(I10)-2)*NI10  MD05160
IF(IOUT.EQ.1) WRITE(6,9031) I10  MD05170
DO 793 I=1,27  MD05180
IJ=I2464+(I-1)*NIJ10  MD05190
PP=PARAMX(IJ)  MD05200
IJ32=IJ-N324  MD05210
IF(I.GE.17) W8=XKZZ(IJ32)*(LB(IJ32)/(LB(IJ)+1.D-6))  MD05220
IF(I.LT.17) W8=NT(IJ)*(LB(IJ)/(LB(IJ)+1.D-6))  MD05230
X=PP*W8  MD05240
II=I+1  MD05250
IJ3=II  MD05260
IF(I.GE.17) II=II-16  MD05270
X1=DLOG10(X+1.D-20)  MD05280
C \*\*\*  MD05290
C DPTI4=(LBB(I2464)-LZ2(IJJ))/6000.  MD05300
C IF(DPTI4.LT.0.) DPTI4=0  MD05310
C XFAC=10**(-DPTI4/6.72)  MD05320
C X2=DLOG10(X*XFAC+1.D-20)  MD05330
C \*\*\*  MD05340
IF(I.GE.13) WLC(NWLM+(II3-2)*10+I10)=X1*X493  MD05350
IF(IOUT.EQ.1) WRITE(6,4501) II,PP,X,X1  MD05360
793 CONTINUE  MD05370
IF(IOUT.EQ.0) GO TO 795  MD05380
WRITE(6,3555) P1,P6,P2,P3,ALFA,P7,P8,P9  MD05390
DO 794 I44=1,2  MD05400
DO 794 I=1,11  MD05410
IJ=I2464+(I-1)*NIJ10  MD05420
XSS(I)=NT(IJ)*PARAMX(IJ)  MD05430
XSH(I)=XKZZ(IJ)*PARAMX(IJ+N324)  MD05440
DPTI4=.67
IF(I44.EQ.2) DPTI4=10
X=.1829*DPTI4
FACSH=1*10**((.08333*X*X-1.1677*X)*1.0)
FACSS=1*10**(-DPTI4/6.72)
XSH(1)=XSH(1)*FACSH
XSS(1)=XSS(1)*FACSS
X=1-.5
AL25=ALFA*4
ALP5=1./AL25
CO =XSS(1)*((XSH(1)/(XSS(1)+1.D-20))*((X)**AL25)
C01=XSS(1)*((XSH(1)/(XSS(1)+1.D-20))*((X)**ALP5)
X0=DLOG10(C0+1.D-20)
X01=DLOG10(C01+1.D-20)
WRITE(6,4501) I+1,CO,C01,XO,X01
CONTINUE
CONTINUE
Write out DONE
I3684=NWLM+1
I3783=NWLM+100
IF(JIT.EQ.-1) WRITE(6,4502) (WLC(I),1=13684,13783)
KOUT=0
DO 322 K=2,NK11
MAQ1K=MAQ1(K)
KOUT=KOUT+MAQ1K
KLB=NK11+KOUT
DO 321 J=2,NJ11
DO 320 I=2,NI11
IJF=I+NI10*(J-2)
IJ=IJF+NIJ10*(K-2)
+++++ FIXED HEAD TO XXS ++++++
I5=IXXXS(IJ)
X=I5*P9+20-P8
X=I5*P9
IF(15.GT.(3000-5)) X=P8*3000
XXS(IJ)=X
+++++ fixed head to xx ++++++
IJLB=I5+F1*N1J10*(KLB-2)
IJMK=IJ-1
IJMK=IJ-N10
IJKM1=IJK-N1J10
IF(IJMK.LT.1) IJMK=1
IF(IJKM1.LT.1) IJKM1=1
DXII=DXI(I)
DYJJ=DYJ(J)
DXIM1=DXI(I-1)
DYJM1=DYJ(J-1)
B0=LB(IJ)
LB1M=LB(IJMK)
LBIJ=LB(IJMK)
BDD=(LB1M*DXII+B0*DXIM1)/(DXII+DXIM1)
BBB=(LB1JM*DYJJ+B0*DYJM1)/(DYJJ+DYJM1)
WDD=LB1M*B0
WBB=LBIJM*B0
IF(WDD.EQ.0.0) BDD=0
IF(WBB.EQ.0.0 ) BBB=0
B4=LB(IJKM1)

C ***** Use PARAMX as multivatrophic factor for sand (XSS) and
C clay (XSH) hydraulic conductivities *****
XSS(1)=NT(IJ)*PARAMX(IJ)
XSS(2)=NT(IM1JK)*PARAMX(IM1JK)
XSS(3)=NT(IJM1K)*PARAMX(IIJK1M)
XSS(4)=NT(IJKM1)*PARAMX(IJKM1)
XSH(1)=XKZZ(IJ)*PARAMX(IJ+N324)
XSH(2)=XKZZ(IM1JK)*PARAMX(IM1JK+N324)
XSH(3)=XKZZ(IJM1K)*PARAMX(IJM1K+N324)
XSH(4)=XKZZ(IJKM1)*PARAMX(IJKM1+N324)

C ****** Multiply these factors (XSJ and XSH) by decrease of
C sand and clay hydraulic conductivity with depth functions
C from text *******
LBBO=LBB(IJF)
DPT(1)=(LBBO-LZ2(IJ)*.1)/600.
DPT(2)=(LBB(IJF-1 )-LZ2(IM1JK)*.D/600.
IJF8=IJF-N10
IF(IJF8.LT.l) IJF8=1
DPT(3)=(LBB(IJF8)-LZ2(IJM1K)*.1)/600.
DPT(4) = (LBBO-LZ2(IJKM1)*. D/600.

FACA=0
DO 897 I4=1,4
DPTI4=DPT(I4)
IF(DPTI4.LT.O.) DPTI4=0
IF(DPTI4.GT.100.) DPTI4=100
X=.1829*DPTI4
FACSH=l*10**((.08333*X*X-1.1677*X)*1.0)
FACSS=(1.10**(-DPTI4/6.72)
XSH(I4)=XSH(I4)*FACSH
XSS(I4)=XSS(I4)*FACSS
IF((I4.EQ.1).OR.(I4.EQ.4)) FACA=FACA+FACSH*.5
897 CONTINUE

C **** Use equations (10) and (11) of text to arrive at effective
C conductivities. XXSTR is percentage sand=l-clay function ***
X=1-XXSTR(IJ)
AL25=ALFA*4
ALP5=1./AL25
CO =XSS(1)*(XSH(1)/(XSS(1)+1.D-20))**(X
CD =XSS(2)*(XSH(2)/(XSS(2)+1.D-20))**((1-XXSTR(1IM1JK))**AL25)
CB =XSS(3)*(XSH(3)/(XSS(3)+1.D-20))**((1-XXSTR(IJM1K))**AL25)
CA1=XSS(1)*(XSH(1)/(XSS(1)+1.D-20))**(X
CZ1=XSS(4)*(XSH(4)/(XSS(4)+1.D-20))**((1-XXSTR(IJKM1))**ALP5)

C Introduce parameter P1 (b4 in text)
IF(K.EQ.3).AND.(I2236.GT.O)) CZ1=LB(IJKM1)*P1*GP5
IF(K.NE.NK11) GO TO 1123
1123 CONTINUE
319 CONTINUE
IF(I.EQ.2) CD=0
IF(J.EQ.2) CB=0
IF(K.EQ.2) CZ1=0

C UUUUUUUUUUUUUUUUUU

C Introduce parameter P1 (b4 in text)
IF(K.EQ.3).AND.(I2236.GT.O)) CZ1=LB(IJKM1)*P1*GP5
IF(K.NE.NK11) GO TO 1123
1123 CONTINUE
319 CONTINUE
IF(I.EQ.2) CD=0
IF(J.EQ.2) CB=0
IF(K.EQ.2) CZ1=0

C UUUUUUUUUUUUUUUUUU
\[
\begin{align*}
DD(IJ) &= -\text{DYJJ}^2 \times \text{DDK}(K) \times \text{PARAMX}(N98+K) \times \text{BDD} \times \text{C0}/(\text{DXII} \times \text{CD} + 1.D-30) \\
BB(IJ) &= -\text{DXII}^2 \times \text{BBK}(K) \times \text{PARAMX}(N99+K) \times \text{BBB} \times \text{C0}/(\text{DYJJ} \times \text{CB} + 1.D-30) \\
CC &= \text{FACA} \times \text{NT}(IJLB) \times \text{PARAMX}(IJLB) \\
\text{IF(MAQIK.EQ.0)} &= \text{CC}=1 \\
\text{ALP} &= .5 \times \text{DO} \times (\text{BO} \times \text{CC} \times \text{CZ1} + \text{B4} \times \text{CC} \times \text{C01}) + \text{MAQ1K} \times \text{LB}(IJLB) \times \text{C01} \times \text{CZ1} \\
\text{CCC} &= \text{C01} \times \text{CC} \times \text{CZ1} \\
\text{ZZIJ} &= 1.D0 \\
\text{IF(ALP.NE.0.0)} &= \text{ZZIJ}=\text{CCC}/\text{ALP} \\
\text{IF(CCC.EQ.0.0)} &= \text{ZZIJ}=0 \\
\text{ZZ}(IJ) &= -\text{DXI}(I) \times \text{DYJ}(J) \times \text{ZZIJ} \\
\end{align*}
\]

C THE QUANTITIES DD, BB, AND ZZ, ARE NOW DETERMINED.

C Calculate pseudo sources and put into YQ2

SUX=0
NCNT=0
NI2=NI10*NI10
NJ2=NJ10*NJ10
NK2=NK10*NK10
KOUT=0
DO 350 K=2,NK11
KOUT=KOUT+MAQ1(K)
KLB=NK1H-KOUT
DO 350 J=2,NJ11
DO 340 I=2,NI11
IJF=I+NI10*(J-2)
IJ=IJF+NIJ10*(K-2)
IF(MHD(IJ).GE.1) GO TO 340
NCNT=NCNT+1
IJLB=IJF+NIJ10*(KLB-2)
IJLB1=IJLB+NIJ10
IP1JK=IJ+1
IM1JK=IJ-1
IJP1K=IJ+NI10
IJM1K=IJ-NI10
IJKP1=IJ+NIJ10
IJKM1=IJ-NIJ10
IF(IJM1K.LT.1) IJM1K=1
IF(IJKM1.LT.1) IJKM1=1
IF(IJP1K.GT.NNN) IJP1K=NNN
IF(IJKP1.GT.NNN) IJKP1=NNN
IF(IJLB1.GT.N325) IJLB1=N325
DD1=DD(IJ)
DD2=DD(IP1JK)
BB1=BB(IJ)
BB2=BB(IJP1K)
ZZ1=ZZ(IJ)
ZZ2=ZZ(IJKP1)

C **

IF((WWAX.NE.0.0).OR.(ISOR.NE.0.0)) GO TO 806
D1=(BB1+BB2+ZZ1+ZZ2)/(DD1+DD2-1.D-30)
B1=(DD1+DD2+ZZ1+ZZ2)/(BB1+BB2-1.D-30)
Z1=(DD1+DD2+BB1+BB2)/(ZZ1+ZZ2-1.D-30)  
IF(B1.GT.1.D+9) B1=0  
IF(Z1.GT.1.D+9) Z1=0  
D1=1/(N1*(1+D1))  
B1=1/(N1*(1+B1))  
Z1=1/(N1*(1+Z1))  
X=D1  
IF(B1.LT.X) X=B1  
IF(Z1.LT.X) X=Z1  
SUX=SUX+X  
806 CONTINUE

C  
B0=LB(IJ)  
B4=LB(IJKM1)  
B5=LB(IJKP1)  
BTO=LRO(IJ)*1.D-5  
BT4=LRO(IJKM1)*1.D-5  
BT5=LRO(IJKP1)*1.D-5  
BTDD=LRO(IJM1JK)*1.D-5  
BTFF=LRO(IP1JK)*1.D-5  
BTBB=LRO(IJM1K)*1.D-5  
BTHH=LRO(IJP1K)*1.D-5  
NB1=LB(IJLB)  
NB2=LB(IJLB1)  
C  
FB1=NB1/(NB1+.01)  
FB2=NB2/(NB2+.01)  
X=(.5DO*(BTO*B0+BT4*B4)+MAQ1(K)*(LRO(IJLB)*1.0D-5)*NB1)*ZZ1  
1-(.5DO*(BTO*B0+BT5*B5)+MAQ1(K+1)*(LRO(IJLB1)*1.0D-5)*NB2)*ZZ2  
C  
3+(ZZ2*FB2*MAQ1(K+1)*(BT5-BTO)+ZZ1*FB1*MAQ1(K)*(BT4-BTO))*OS4  
LZO=LZ2(IJ)  
DXII=DXI(I)  
DXIM1=DXI(I-1)  
DXIP1=DXI(I+1)  
DYJJ=DYJ(J)  
DYJM1=DYJ(J-1)  
DYJP1=DYJ(J+1)  
BT0X=BTO*DXII  
BT0Y=BT0*DYJJ  
YQ2(IJ)=X-(((BTDD*DXIM1+BTOX)/(DXIM1+DXII))*DD1*(LZ2(IJM1K)-LZO)  
1 +((BTFF*DXIP1+BTOX)/(DXIP1+DXII))*DD2*(LZ2(IJP1K)-LZO)  
2 +((BTBB*DYJM1+BT0Y)/(DYJM1+DYJJ))*BB1*(LZ2(IJM1K)-LZO)  
3 +((BTHH*DYJP1+BT0Y)/(DYJP1+DYJJ))*BB2*(LZ2(IJP1K)-LZO)  
4)*.1DO  
340 CONTINUE  
350 CONTINUE

C YQ2 NOW DETERMINED  
C Get iteration parameters for SIP subroutine  
IF(ISOR.NE.0) GO TO 390  
XY=.5DO*PI*PI*SUX/NCNT  
C XY IS NOW DETERMINED  
XY=XYFC*XY  
IF(WMAX.NE.0.O) XY=WMAX  
C DETERMINE ITERATION PARAMETERS WS(I)  
DO 360 I=1,LENGTH  
360 CONTINUE

143
360  WS(I)=1-XY**( (I-1.)/(LENGTH-1) )
C WRITE(6,5056) MD07640
C WRITE(6,5006) MD07650
C WRITE(6,3555) (WS(I),I=1,LENGTH) MD07670
390 CONTINUE MD07680
C Have coefficients
IF(JIT.EQ.-1) WRITE(6,1003) 99,ISOR,ITMAX,ERR,XX10
1,HMAX,XYFC,WMAX MD07700
DO 599 IJ=2,N315 MD07710
599 HLS(IJ)=0 MD07720
NINT=5 MD07740
C Begin main loop of pgn.
C There are 5 time intervals INT=1,2,...5.
C DELT is time interval
DO 600 INT=1,5 MD07750
600 INT2=INT-2 MD07760
IF(INT2.LT.1) INT2=1 MD07770
DELT=DEL(INT) MD07780
DELT=DELT/P2 MD07790
SUMYQ=0 MD07800
DO 231 IDBZ=1,3 MD07810
DO 231 N=1,250 MD07820
231 SYQ(IDBZ,N)=0 MD07830
C Put pumping for this time interval into YQ
DO 331 K=2,NK11 MD07840
K1=K-1 MD07850
DO 330 J=2,NJ11 MD07860
DO 329 I=2,NI11 MD07870
329 IJ=I+NI10*(J-2)+NIJ10*(K-2)
C *** GOOD FIRST GUESS AND FIXED HEAD TO XX ***
X=IXX5(IJ,INT)*.1 MD07900
IF((JIT.EQ.-1).OR.(MHD(IJ).EQ.1)) X=XXS(IJ) MD07910
XX(IJ)=X MD07920
X=IXX5(IJ,INT)*.1 MD07930
IF((JIT.EQ.-1).OR.(MHD(IJ).EQ.1)) X=XXS(IJ) MD07940
XX(IJ)=X MD07950
XX(IJ)=X MD07960
C ******************
XDEN=FC(2,K1) MD07970
X=XDEN MD07980
IF(INT.EQ.1) X=0 MD07990
IF(INT.EQ.2) X=FC(1,K1)*.79 MD08000
FCX=X/XDEN MD08010
XQ=YQ1(IJ,INT2)*FCX MD08020
IF(IOUT.EQ.0) GO TO 701 MD08030
SYQ(1,I)=SYQ(1,I)+XQ MD08040
SYQ(2,J)=SYQ(2,J)+XQ MD08050
SYQ(3,K)=SYQ(3,K)+XQ MD08060
SUMYQ=SUMYQ+XQ MD08070
SYQ(IJ)=XQ+YQ2(IJ) MD08080
701 CONTINUE MD08090
329 YQ(IJ)=XQ+YQ2(IJ) MD08100
330 CONTINUE MD08110
331 CONTINUE MD08120
C YQ now has pumping including pseudo sources
I01=1 MD08130
IWR1=IWR*T*I01 MD08140
LFL01=LFLOW*I01 MD08150
IF(IWR1.NE.1) GO TO 420 MD08160
WRITE(6,5007) MD08170
WRITE(6,5072) NU1(1),NU2(1),NU3(1)  MD08190
WRITE(6,5073) NU1(2),NU2(2),NU3(2)  MD08200
WRITE(6,5074) NU1(3),NU2(3),NU3(3)  MD08210
420 CONTINUE  MD08220
C BEGIN SIP,OR PCG ITERATIONS.  MD08230
C Call PCG for first time interval,  MD08240
C SIP for second through fifth intervals  MD08250
ISO=ISO+1  MD08260
IF(INT.GT.1) CALL SIP  MD08270
IF(INT.EQ.1) CALL PCG  MD08280
WRITE(6,1003) JIT,INT,ICNT,ER5,SRZ,SUMRZ,HMAX,XX(9583),XX(30904)  MD08290
C @@@@@@ GET SUBSIDENCE STORAGE (RATE) @@@@@@  MD08300
IF(IOUT.EQ.0) GO TO 673  MD08310
WRITE(6,9020)  MD08320
DO 670 K=2,NK11  MD08330
SUMT=0  MD08340
SUMS=0  MD08350
DO 671 I=2,NI11  MD08360
DO 671 J=2,NJ11  MD08370
IJ=I+NI10*(J-2)+NIJ10*(K-2)  MD08380
IF(MHD(IJ).GE.1) GO TO 672  MD08390
XXIJ=XX(IJ)  MD08400
X=HL(IJ)-XXIJ  MD08410
SM=XXS(IJ)-HL(IJ)  MD08420
S1=X  MD08430
SS=0  MD08440
IF(X.LE.0) GO TO 675  MD08450
S1=X*VV40*(1-XXSTR(IJ))  MD08460
SS=S1  MD08470
675 CONTINUE  MD08480
SUMT=SUMT+(SM+S1)*LB(IJ)  MD08490
SUMS=SUMS+SS*LB(IJ)  MD08500
672 CONTINUE  MD08510
671 CONTINUE  MD08520
SUMT=SUMT*SVIJ/DELT  MD08530
SUMS=SUMS*SVIJ/DELT  MD08540
WRITE(6,4501) K,SUMS,SUMT  MD08550
670 CONTINUE  MD08560
C &&&&&&&&&&&&&&&&&&&&  MD08570
DO 771 I=2,NI11  MD08580
DO 771 J=2,NJ11  MD08590
C RXY=1/(DXI(I)*DYJ(J))  MD08600
IJJ=I+NI10*(J-2)  MD08610
SUM=0  MD08620
DO 770 K=2,NK11  MD08630
IJ=IJJ+NIJ10*(K-2)  MD08640
IF(MHD(IJ).GE.1) GO TO 772  MD08650
XXIJ=XX(IJ)  MD08660
X=HL(IJ)-XXIJ  MD08670
SM=XXS(IJ)-HL(IJ)  MD08680
S1=X  MD08690
SS=0  MD08700
IF(X.LE.0) GO TO 775  MD08710
S1=X*VV40*(1-XXSTR(IJ))  MD08720
SS=S1  MD08730
CONTINUE
SUM=SUM+(SM+S1)*LB(IJ)
CONTINUE
IF(INT.NE.1) HLS(IJJ)=HLS(IJJ)+SUM*SV35*P2
CONTINUE
IF(INT.NE.1) HLS(IJJ)=HLS(IJJ)+SUM*SV35*P2
CONTINUE
C @@@@@@@@@@@@@(3(3 End subsidence @@@@@@@@@@@@@@@@@
C Save XX in IXX5(IJJ,INT) for use as the starting solution
C for XX at the next iteration (JIT+1), same time interval INT.
C Also save XX in XXS for use in PCG & SIP as the starting value
C for XX at the beginning of the next time interval (INT+1)
C Calculate HL(IJ), the lowest value for head yet achieved
C at location IJ
DO 430 K=2,NK11
DO 430 J=2,NJ11
DO 430 I=2,NI11
IJJ=I+NI10*(J-2)+NIJ10*(K-2)
XXIJ=XX(IJJ)
HLIJ=HL(IJJ)
IF(XXIJ.LT.HLIJ) HL(IJJ)=XXIJ
P380=P3*80
IF(K.LE.6) P380=1000
IF(INT.EQ.I) HL(IJJ)=XXIJ-P380
IXX5(IJJ,INT)=XXIJ*10.+.5
430 XXS(IJJ)=XXIJ
C Done with IXX5,XX5,HLIJ
IF((JIT.EQ.-1).AND.(INT.EQ.5)) GO TO 7197
IF((LFL01*IOUT).EQ.0) GO TO 560
7197 CONTINUE
C DETERMINE WATER FLOW RATES
C Much of this is in VARDEN and deals with calculation of various
C flow rates. See WRIR 84-4302, p. B-6, Kuiper
DO 440 IJ=1,NNN
G2(IJ)=0
DO 511 IDBZ=1,3
DO 450 N=1,250
SUMFM(IDBZ,N)=0
SUNFM(IDBZ,N)=0
SUNF(IDBZ,N)=0
450 SUMF(IDBZ,N)=0
IRITE=0
L1=FL0(1)
IF((L1.EQ.IDBZ).OR.(L1.EQ.4)) IRITE=1
IF(((INT-3).NE.2).OR.(IOUT.NE.1)) IRITE=0
IF(IRITE.NE.1) GO TO 460
IF(IDBZ.EQ.1) WRITE(6,5101)
IF(IDBZ.EQ.2) WRITE(6,5102)
IF(IDBZ.EQ.3) WRITE(6,5103)
WRITE(6,6000)
460 CONTINUE
IRITEA=0
L1=FL0A(1)
IF((L1.EQ.IDBZ).OR.(L1.EQ.4)) IRITEA=1
IF(((IABS(INT-3).NE.2).OR.(IOUT.NE.1))) IRITEA=0
IF(IRITEA.NE.1) GO TO 461

C IF(IDBZ.EQ.1) WRITE(7,5101)
C IF(IDBZ.EQ.2) WRITE(7,5102)
C IF(IDBZ.EQ.3) WRITE(7,5103)
C WRITE(7,6000)

461 CONTINUE

KOUT=0
DO 511 K=2,NK11
KOUT=KOUT+MAQ1(K)
KLB=NK11+KOUT
IF((IRITE.EQ.1).AND.(IPH(K).GE.0)) WRITE(6,A002) K
C IF((IRITEA.EQ.1).AND.(IPHA(K).EQ.1)) WRITE(7,A002) K
188=0
SM9=0
DO 510 J=2,NJ11
188=188+1
IF(I88.EQ.11) I88=1
IJMI=NI10*(J-2)+NIJ10*(K-2)
IJLBMI=NI10*(J-2)+NIJ10*(KLB-2)
DO 500 I=2,NI11
IJ=IJMI+I
IJLB=IJLBMI+I
XXO=XX(IJ)
BTO=LRO(IJ)*1.0D-5
LZO=LZ2(IJ)
IF(IDBZ.NE.1) GO TO 470
DBZ=DD(IJ)
IF(I.EQ.2) GO TO A90
IM1JK=IJ-1
XXI=XX(IM1JK)
BTDD=LRO(IM1JK)*1.0D-5
DXII=DXI(I)
DXIM1=DXI(I-1)
BTOX=BTO*DXII
FLOW=((BTDD*DXIM1+BTOX)/(DXIM1+DXII))*(LZ2(IM1JK)-LZO)*.1DO
GO TO A90

A70 IF(IDBZ.NE.2) GO TO A80
DBZ=BB(IJ)
IF(J.EQ.2) GO TO A90
IJM1K=IJ-NI10
XXI=XX(IJM1K)
BTBB=LRO(IJM1K)*1.0D-5
DYJJ=DYJ(J)
DYJM1=DYJ(J-1)
BT0Y=BT0*DYJJ
FLOW=((BTBB*DYJM1+BTOY)/DYJM1+DYJJ))*(LZ2(IJM1K)-LZO)*.1DO
GO TO A90

A80 DBZ=ZZ(IJ)
IF(K.EQ.2) GO TO 490
IJKM1=IJ-NIJI10
XXI=XX(IJKM1)
B0=LB(IJ)
B4=LB(IJKM1)
BT4=LRO(IJKM1)*1.0D-5
NB1=LB(IJLB)

GO TO 470
FB1=NB1/(NB1+.01)
FLOW=-(.5D0*(BT0*B0+BT4*B4)+MAQ1(K)*(LRO(IJLB)*1.0D-5)*NB1)
C 1-FB1*MAQ1(K)*(BT4-BT0)*OS4
490 X=DBZ*(XXI-XXO+FLOW)
IM=1
IF(IDBZ.EQ.2) IM=NI10
IF(IDBZ.EQ.3) IM=NIJ10
IJM=IJ-IM
G2(IJM)=G2(IJM)-X
C &&&&&&&&&&&& Adjust Z component of hydraulic conductivity
C in top layer 12 if flow through is in excess. This is done
C for JIT=-1 only.
C Z component of hydraulic conductivity for layer 12
C Subsequent iterations JIT=1,2,... use the reduced value of the
C &&&&&&&&&&&&&
IF(.NOT.((JIT.EQ.-1).AND.(INT.EQ.5)» GO TO 762
IF((IDBZ.NE.3).OR.(K.NE.NK11)) GO TO 762
X600=1*600
X100=1*100
T=X600-((X600-X100)/30)*(J-50)
IF(T.GT.X600) T=X600
IF(T.LT.X100) T=X100
T=T*1.D4
IF(X.LT.T) GO TO 762
C X77=.5*T/X
X77=.6/(X*10/T-9)
C ZZ(IJ)=ZZ(IJ)*X77
C NT(IJ)=NT(IJ)*X77
C XKZZ(IJ)=XKZZ(IJ)*X77
C IJ66=I+(J-2)*NI10
DO 837 KK=11,2,-1
IJ1=(KK-2)*NIJ10+IJ66
IF(LB(IJ).EQ.O) GO TO 838
C IF(KK.NE.11) ZZdJl+NU10)=ZZdJl+NIJ10)*X77
C XKZZ(IJ1)=XKZZ(IJ1)*X77
GO TO 839
838 CONTINUE
837 CONTINUE
839 CONTINUE
762 CONTINUE
C $$$ GET FLOW ACROSS GULF COAST$$$$$
DI=I-I92
DJ=J-J92
R2=DI*DI+DJ*DJ
IF(IDBZ.EQ.1) DI=DI-1
IF(IDBZ.EQ.2) DJ=DJ-1
R3=DI*DI+DJ*DJ
IF((R3.GT.R42).AND.(R2.LE.R42)) SM9=SM9+X
IF((R2.GT.R42).AND.(R3.LE.R42)) SM9=SM9-X
C $$$$$$$$$$$$
C &&&&&&&&&&&&
DTI=X
IF(LB(IJ).EQ.0) DTI=-1.E+38
DT(I)=DTI
IJK=I
C $$$$$$$$$$$$
C &&&&&&&&&&&&
148
IF(IDBZ.EQ.2) IJK=J MD10370
IF(IDBZ.EQ.3) IJK=K MD10380
IF(LB(IJ).EQ.0) GO TO 491 MD10390
IF(X.GT.0) SUMF(IDBZ,IJK)=SUMF(IDBZ,IJK)+X MD10400
IF(X.LT.0) SUNF(IDBZ,IJK)=SUNF(IDBZ,IJK)-X MD10410
491 IF(LBM(IJ)).EQ.0) GO TO 492 MD10420
IF(X.GT.0) SUMFM(IDBZ,IJK)=SUMFM(IDBZ,IJK)+X MD10430
IF(X.LT.0) SUNFM(IDBZ,IJK)=SUNFM(IDBZ,IJK)-X MD10440
492 CONTINUE MD10450
G2(IJ)=G2(IJ)+X MD10460
500 CONTINUE MD10470
C IF(188.NE.-1) GO TO 934 MD10480
IF(.NOT.(((J.LE.69).AND.(J.GE.62)).OR.((J.LE.19).AND.(J.GE.15)))) MD10490
*GO TO 934 MD10500
IF(1RTEQ.1).AND.(1PH(I).EQ.1)) WRITE(6,4500) J,(DT(I),I=2 MD10510
1,NI1) MD10520
934 CONTINUE MD10530
C IF(1RITEA.EQ.1).AND.(1PHA(I).EQ.1)) WRITE(7,5500) J-1, MD10540
C 1(DT(I),I=-2,NI11) MD10550
510 CONTINUE MD10560
C IF(IOUT.EQ.1) WRITE(6,1003) IDBZ,K,5555,SM9 MD10570
511 CONTINUE MD10580
C IF((LFLO1*IOUT).EQ.0) GO TO 560 MD10590
C IF(LFLO2.EQ.1) WRITE(6,8010) MD10600
I5=0 MD10610
SUMG2=0 MD10620
DO 549 IDBZ=1,3 MD10630
DO 549 N=1,250 MD10640
549 SG2(IDBZ,N)=0 MD10650
DO 550 K=2,NK11 MD10660
DO 550 J=2,NJ11 MD10670
DO 550 I=2,NI11 MD10680
IJ=I+NI10*(J-2)+NIJ10*(K-2) MD10690
IF(MHD(IJ).NE.1) GO TO 540 MD10700
G2IJ=G2(IJ) MD10710
I5=I5+1 MD10720
DT(I5)=G2IJ MD10730
I7(I5)=I MD10740
J7(I5)=J MD10750
K7(I5)=K MD10760
SG2(I,I)=SG2(I,I)+G2IJ MD10770
SG2(J,J)=SG2(J,J)+G2IJ MD10780
SG2(K,K)=SG2(K,K)+G2IJ MD10790
SUMG2=SUMG2+G2IJ MD10800
540 IF(I5.NE.5) GO TO 550 MD10810
I5=0 MD10820
C IF((LFLO2.EQ.1).AND.(I5.GE.1)) MD10830
C WRITE(6,3560) (I7(L),J7(L),K7(L),DT(L),L=1,5) MD10840
550 CONTINUE MD10850
C IF((LFLO2.EQ.1).AND.(I5.GE.1)) MD10860
C WRITE(6,3560) (I7(L),J7(L),K7(L),DT(L),L=1,5) MD10870
C I5=0 MD10880
SUMVV=0 MD10890
DO 649 IDBZ=1,3 MD10900
DO 649 N=1,250 MD10910

149
649     SVV(IDBZ,N)=0
     DO 650 K=2,NK11
     DO 650 J=2,NJ11
     DO 650 I=2,NI11
     IJ=I+NI10*(J-2)+NIJ10*(K-2)
     C     X=XX(IJ)-XXE(IJ)
     C     VVIJ=X*ALN(IJ)
     IF(.NOT.((IEVP.EQ.1).AND.(VVIJ.GT.0))) GO TO 641
     I5=I5+1
     DT(I5)=VVIJ
     I7(I5)=I
     J7(I5)=J
     K7(I5)=K
     SVV(1,I)=SVV(1,I)+VVIJ
     SVV(2,J)=SVV(2,J)+VVIJ
     SVV(3,K)=SVV(3,K)+VVIJ
     SUMV=SUMV+VVIJ
     641  IF(I5.NE.5) GO TO 650
     I5=0
     IF(LFLO(3).EQ.1) WRITE(6,3560) (I7(L),J7(L),K7(L),DT(L),L=1,5)
     CONTINUE
     IF((LFLO(3).EQ.1).AND.(I5.GE.1))
     WRITE(6,3560) (I7(L),J7(L),K7(L),DT(L),L=1,15)
     WRITE(6,8011) SUMYQ,SUMG2,SUMW
     IF(LFLO(4).EQ.0) GO TO 530
     WRITE(6,7001)
     IDBZ1=LFLO(4)
     IDBZ3=IDBZ1
     IF(IDBZ1.GE.4) IDBZ1=1
     IF(IDBZ3.GE.4) IDBZ3=3
     DO 520 IDBZ=IDBZ1,IDBZ3
     N20=NI11
     IF(IDBZ.EQ.2) N20=NJ11
     IF(IDBZ.EQ.3) N20=NK11
     DO 519 N=2,N20
     NP1=N+1
     519  WRITE(6,8012) IDBZ,N,SYQ(IDBZ,N),SG2(IDBZ,N),SVV(IDBZ,N)
     1,SUNF(IDBZ,N),SUMFM(IDBZ,NP1),SUMF(IDBZ,N),SUNFM(IDBZ,NP1)
     520  CONTINUE
     530  CONTINUE
     560  IF((1*IOUT).EQ.0) GO TO 590
     C  End of flow rate section
     C  Printout heads in various formats and media. Change things in this section to choose media, which heads, etc.
     WRITE(6,5008)
     WRITE(6,6000)
     C  IF(IABS(INT-3).GE.0) WRITE(8,5008)
     C  IF(IABS(INT-3).GE.0) WRITE(8,6000)
     C  WRITE(10,5009)
     C  WRITE(10,6000)
     C  DETERMINE HYDRAULIC HEAD FROM XX = H PRIME = (PRESSURE HEAD H) + Z
     XSM=0
     DO 580 K=2,NK11
     C  IF(IABS(INT-3).GE.0) WRITE(8,4002) K
     C  WRITE(10,4002) K

150
XXSM=0
WRITE(6,2020) 1,1,0,K
DO 579 J=2,NJ11
IJMI=NI10*(J-2)+NIJ10*(K-2)
DO 570 I=2,NI11
IJ=IJMI+I
XZ210=LZ2(IJ)*.1DO
DTI=(l/(LRO(U)*1.0D-5+l))*(XX(IJ)-XZ210)+XZ210
IF(MHD(IJ).EQ.2) DTI=0
X9=1
IF(LB(IJ).EQ.0) X9=0
XXF=XX(IJ)*X9
XXH=DTI*X9
IF(K.NE.NK11) XSM=XSM+XXF
IF(K.NE.NK11) XXSM=XXSM+XXF
E2(I)=XXF
E2(2)=J
IF(XXF.EQ.0) XXF=-1.E+38
IF(XXH.EQ.0) XXH=-1.E+38
G2(I)=XXF
570 DT(I)=XXH
IF((INT.EQ.5).AND.(IPH(K).GE.1)) WRITE(6,1008) (E2(I),*I=2,NI11)
C IF(IWRTXX.LE.1) WRITE(10,1001) J-1,(DT(I),I=2,NI11)
C IF(IABS(INT-3).GE.0) WRITE(8,5500) J-1,(G2(I),I=2,NI11)
1008 FORMAT(20F6.0)
579 CONTINUE
WRITE(6,3555) XXSM
580 CONTINUE
WRITE(6,3555) XSM
C End of head printout
590 CONTINUE
600 CONTINUE
C End of MAIN loop
C ********** PRINT OUT TOTAL SUBSIDENCE **********
IF(IOUT.EQ.0) GO TO 664
WRITE(6,9021)
DO 779 J=2,NJ11
IJMI=NI10*(J-2)
WRITE(6,1001) J,(HLS(IJMI+I),I=2,NI11)
779 CONTINUE
C ********** DONE WITH SUBSIDENCE PRINT OUT ********
C Put calculated heads into WLC=f(B) for use by MAIN
DO 642 I=1,NWLM
IJ=NND(I)
XZ210=LZ2(IJ)*.1DO
C642 WLC(I)=(1/(LRO(IJ)*1.0D-5+l))*(XX(IJ)-XZ210)+XZ210
IF(I.LE.NWLM1) XXX=IXX5(IJ,3)*.1
IF(I.GT.NWLM1) XXX=XX(IJ)
642 WLC(I)=XXX
C Read into WLC done
IF(IOUT.EQ.0) GO TO 736
C Insert pack for observation of residual errors
C Y-f(B)=WLM(IJ)-WLC(IJ)
C Various formats occur: Average residual error by layer and MD12010 by total model. Root mean square residual error by layer and MD12011 total model MD12020
K40=1 MD12030
K40M1=0 MD12040
CUM=0 MD12050
CCUM=0 MD12060
DO 737 I2=1,2 MD12070
K41=0 MD12080
SSUM=0 MD12090
TSUM=0 MD12100
SWLMT=0 MD12110
WRITE(6,9998) MD12120
DO 735 K=2,NK11 MD12130
K39=0 MD12140
PSUM=0 MD12150
PSUM1=0 MD12160
SWLM=0 MD12170
WRITE(6,4003) K MD12180
DO 734 J=2,NJ11 MD12190
DO 733 I=2,NI11 MD12200
IJ=I+NI10*(J-2)+NIJ10*(K-2) MD12210
DT(I)=0 MD12220
IF(IJ.NE.NND(K40)) GO TO 732 MD12230
XXX=WLC(K40) MD12240
WLM40=WLM(K40) MD12250
W40=W(K40) MD12260
K40=K40+1 MD12270
IF(W40.LT.1.D-20) GO TO 732 MD12280
SWLMT=SWLMT+WLM40 MD12290
SWLM =SWLM +WLM40 MD12300
X=XXX-WLM40 MD12310
DT(I)=X MD12320
K39=K39+1 MD12330
K40M1=K40M1+1 MD12340
K41=K41+1 MD12350
SSUM=SSUM+X*X MD12360
TSUM=TSUM+X*1 MD12370
PSUM=PSUM+X*1 MD12380
CUM=CUM+X*1 MD12390
PSUM1=PSUM1+X*X MD12400
CCUM=CCUM+X*X MD12410
732 CONTINUE MD12420
733 CONTINUE MD12430
C IF(I2.EQ.2) WRITE(6,1007) J,(DT(I),I=2,NI11) MD12440
734 CONTINUE MD12450
PSUM=PSUM/(K39+1.D-20) MD12460
PSUM1=DSQRT(PSUM1/(K39+1.D-20)) MD12470
SWLM=SWLM/(K39+1.D-20) MD12480
WRITE(6,4501) K39,PSUM,PSUM1,SWLM MD12490
735 CONTINUE MD12500
SSUM=DSQRT(SSUM/(K41)) MD12510
TSUM=TSUM/(K41) MD12520
SWLMT=SWLMT/(K41) MD12530
WRITE(6,1003) K40M1,66,K41,TSUM,SSUM,SWLMT MD12540
CONTINUE
CUM=CUM/K40M1
CCUM=DSQRT(CCUM/K40M1)
WRITE(6,1003) 66,66,66,CUM,CCUM
C End of residual error pack
C ************** DO CHI square test for normalcy of residuals
C Discarding of outlying data lying outside of certain
C normalcy bounds is optimal, at JIT=K4
C Subsequent values of JIT will use reduced data set with
C others discarded
CONTINUE
K4=1
IF(JIT.EQ.K4) IIII=III+1
J8=JIT+IIII
WRITE(6,2020) 5555,5555,K4,JIT,IIII,J8
IF(.NOT.((J8.EQ.K4).OR.(IOUT.NE.0))) GO TO 1849
DO 1839 IW=0,1
CUM=0
CCUM=0
DO 1836 K40=1,NWLM
X=WLC(K40)-WLM(K40)
W40=W(K40)
IJ=NND(K40)
IF(W40.LE.1.D-20) GO TO 1836
X=X*DSQRT(W40)
CUM=CUM+X
CCUK=CCUM+X*X
1836 CONTINUE
CUM=CUM/X3150
CCUM=DSQRT(CCUM/X3150)
DO 1837 I=1,16
IX7(I)=0
Rh01=DSQRT(CCUM*CUM-CUM*CUM)
F2242=(Rh01/(Rh01S+1.D-40))
Rh01S=Rh01
LR=0
DO 1838 K40=1,NWLM
X=WLC(K40)-WLM(K40)
X=(X-CUM)/Rh01
IF((X.LT.1.54).AND.(X.GT.-3.4)) GO TO 1950
IF(K40.GT.NWLM) GO TO 1950
LR=LR+1
WRITE(6,3011) LR,K40,NND(K40),888,X,WLC(K40),WLM(K40)
1838 CONTINUE
3011 FORMAT(4I8,6D11.3)
1950 CONTINUE
DO 1792 I=1,16
XR=XTV(II+1)
XL=XTV(II)
IF((XL.LT.X).AND.(X.LE.XR)) IX7(II)=IX7(II)+1
XNP=X3150/16.
SUM=0
DO 893 I=1,16
IXI=IX7(I)
X=IXI-XNP
893 SUM=SUM+X*X
SUM=SUM/XNP
WRITE(6,2020) (IX7(I), I=1,16)
WRITE(6,3555) SUM,CUM,CCUM,RH01,XNP,X3150
IF((J8.EQ.KA).AND.(IW.EQ.0)) X3150=X3150-LR
WRITE(6,1003) 3333,3333,LR,X3150
1839 CONTINUE
IF(J8.NE.KA) GO TO 1952
XA93=XA93*F22A2
NWLM3=NWLM+1
DO 1951 NN=NWLM3,NWLMT
WLC(NN)=WLC(NN)*F22A2
1951 WLM(NN)=WLM(NN)*F22A2
1952 CONTINUE
WRITE(6,1003) 3333,3333,LR,X3150,F2242,X493
18A9 CONTINUE
1001 FORMAT(I12,20F6.1/(12X,20F6.D))
1003 FORMAT(3110,7D12.4)
1004 FORMAT(12X,15F8.1)
1007 FORMAT(I12,20F6.0/(12X,20F6.0))
2000 FORMAT(8G10.0)
2007 FORMAT(4(313,D11.3))
2020 FORMAT(8I10)
3004 FORMAT(I12,20I6/(12X,20I6))
3005 FORMAT(22I6)
3007 FORMAT(4(I12,214,D11.3))
3010 FORMAT(1,12F10.1)
3555 FORMAT(8D15.7)
3560 FORMAT(1,5(314,D12.3))
4002 FORMAT(I6,I12)
4003 FORMAT(1',516)
4500 FORMAT(I12,10D12.3/(12X,10D12.3))
5500 FORMAT(I12,13D9.3/(12X,13D9.3))
4501 FORMAT(I12,9D12.3)
4502 FORMAT(10D10.3)
4997 FORMAT(1',44X,'LENGTH IN FEET OF WATER COLUMN IN WELL BORE')
4998 FORMAT(1',64X,'MOLALITY')
4999 FORMAT(1',53X,'TEMPERATURE DEGREES CENTIGRADE')
5000 FORMAT(1',52X,'FIXED HYDRAULIC HEAD')
5001 FORMAT(1',49X,'X AND Y DIMENSIONS OF GRID ELEMENTS')
5002 FORMAT(1',41X,'(DENSITY OF WATER RHO, IN GRAMS PER CUBIC CENTIMETER)-1.0')
5003 FORMAT(1',59X,'BASE ELEVATION')
5004 FORMAT(1',52X,'Z DIMENSION OF GRID ELEMENTS')
5005 FORMAT(1',54X,'HYDRAULIC CONDUCTIVITY')
5052 FORMAT(1',56X,'INITIAL HYDRAULIC HEAD')
5053 FORMAT(0',53X,'RECHARGE RATE YQ (L*L*L/T)')
5054 FORMAT(1',57X,'SPECIFIC STORAGE')
5055 FORMAT(1',53X,'RECHARGE RATES Q2 (L*L*L/T)')
15X,'TOTAL Q2 RECHARGE RATE TO MODELED REGION=',D13.4/M14190
25X,'TOTAL RECHARGE RATE TO MODELED REGION=',D13.4/M14200
9954 FORMAT('0',45X,'PERCENT SAND') M14210
9998 FORMAT('0',45X,'ERROR, CALCULATED MINUS MEASURED FRESH WATER HD') M14220
9020 FORMAT('1','RATE OF RELEASE OF WATER FROM STORAGE BY LAYER',',1X,' M14230
1SUBSIDENCE ONLY, TOTAL') M14240
9021 FORMAT('1','TOTAL SUBSIDENCE FROM ALL LAYERS FOR THIS PUMPING INTM14250
1RVAL') M14260
9022 FORMAT('1','SUBSIDENCE SPECIFIC STORAGE MULTIPLICATION FACTOR')MD14270
9023 FORMAT('0','DEPTH BELOW INITIAL FRESH WATER HEAD AT WHICH SUBSIDEMD14280
1CE BEGINS IS',D10.1) M14290
9030 FORMAT('0','PARAMETERS IN LOG FORM-FOR NEW PARAMO') M14300
9031 FORMAT('0','PARAMETERS ACTUAL VALUE',I10) M14310
RETURN M14320
END M14330
SUBROUTINE RDWRT M14340
   IMPLICIT REAL*8 (A-H,O-Z) M14350
   COMMON WS,HMAX,RELX1,RELX2,COEF,ERR,XX10,DELT,ER5,SRZ,SUMRZ M14360
1,ERRSV,XX10SV,JIT,NIJ10,NI11,NJ11,NK11,NNN,NSKP1 M14370
2,NSKP2,ITMAX,IEVP,IWR1,NW1,NW2,NW3,N320,NUM4 M14380
3,L9,LENGTH,NK1115,IT01,IT15,ICRO,DDK,BBK M14390
4,N110,NK10,NII10,N112,NK12,NVIJ,VD40,SV35 M14400
   COMMON /XX/ XX M14410
   COMMON /DT/ DT M14420
   COMMON /VV/ VV M14430
   COMMON /E2/ E2 M14440
   COMMON /F2/ F2 M14450
   COMMON /G2/ G2 M14460
   COMMON /YQ/ YQ M14470
   COMMON /NT/ NT M14480
   COMMON /DD/ DD M14490
   COMMON /BB/ BB M14500
   COMMON /ZZ/ ZZ M14510
   COMMON /XXS/ XXS M14520
C COMMON /ALN/ ALN M14530
C COMMON /XXE/ XXE M14540
C COMMON /SV/ SV M14550
   COMMON /HL/ HL M14560
   COMMON /LB/ LB M14570
   COMMON /MHD/ MHD M14580
   REAL*4 DD(65078),BB(65078),ZZ(65078),XXS(65078), M14590
1YYQ(65078),DDK(50),BBK(50),NT(94658),HL(65078) M14600
   DIMENSION DT(65078),E2(65078),F2(65078),G2(65078),MHD(65078) M14610
   DIMENSION XXS(65078) M14620
1,WS(10),MHD(65078),LB(94658) M14630
2,IF1(10),IF2(10),NNT(250) M14640
   READ(5,1000) II,(IF1(II),II=1,9),(IF2(II),II=1,10) M14650
   WRITE(6,6000) M14660
   DO 161 K=2,NK1115 M14670
   READ(5,2000) FCNTK,IVAR,IPRN,DDK(K),BBK(K) M14680
   IF(IPRN.EQ.1) GO TO 158 M14690
   IF(IT15.EQ.10) WRITE(6,7000) K,FCNTK M14700
   IF(IT15.EQ.1) WRITE(6,1002) K,FCNTK M14710
   IF(IT15.EQ.2) WRITE(6,1002) K,FCNTK,DDK(K),BBK(K) M14720
158 DO 161 J=2,NJ11 M14730
IJMI=NI10*(J-2)+NIJ10*(K-2)
IF(IVAR.EQ.1) READ(5,IF1) (NT(IJMI+I),I=2,NI11)
DO 159 I=2,NI11
IJ=IJMI+I
X=1
IF(IVAR.EQ.1) X=NT(IJ)
159 NT(IJ)=X
IF(ICRO.EQ.0) GO TO 1605
DO 13 I=2,NI11
IJ=IJMI+I
LBIJ=LB(IJ)
IF(LBIJ.NE.-30000) NT(IJ)=LBIJ*1.D-5/FCNTK
13 CONTINUE
1605 IF(.NOT.((IPRN.EQ.0).AND.(IVAR.EQ.1))) GO TO 41
DO 40 I=2,NI11
40 NNT(I)=NT(IJMI+I)
IF(II.EQ.0) WRITE(6,IF2) J,(NNT(I),I=2,NI11)
IF(II.EQ.1) WRITE(6,IF2) J,(NT(IJMI+I),I=2,NI11)
41 CONTINUE
DO 42 I=2,NI11
42 NT(IJ)=NT(IJ)*FCNTK
161 CONTINUE
1000 FORMAT(I4,19A4)
1002 FORMAT(I6,6D18.4)
2000 FORMAT(8G10.0)
4000 FORMAT(I6,110)
6000 FORMAT(0,'LAYER ROW')
7000 FORMAT(1',I6,6D18.4)
RETURN
END
SUBROUTINE SOLEQU (A,X,N)

C THIS SUBROUTINE SOLVES A SET OF SYMMETRIC LINEAR EQUATIONS

C

REAL*4A(50,50),X(50)

NM=N-1

DO 85 I=1,NM

IP=I+1

DO 25 J=IP,N

IF(ABS(A(I,I)).LT.1.E-8) GO TO 200

A(I,J)=A(I,J)/A(I,I)

CONTINUE

25 CONTINUE

DO 50 J=IP,N

DO 45 K=J,N

IF(A(I,J)) 35,50,35

A(K,J)=A(K,J)-A(K,I)*A(I,J)

A(J,K)=A(K,J)

45 CONTINUE

CONTINUE

50 CONTINUE

X(I)=X(I)/A(I,I)

DO 75 K=IP,N

X(K)=X(K)-A(K,I)*X(I)

75 CONTINUE

CONTINUE

IF(ABS(A(N,N)).LT.1.E-8) GO TO 202

X(N)=X(N)/A(N,N)

CONTINUE

85 CONTINUE

AD=0.

GO TO 130

100 DO 125 II=1,NM

I=N-II+1

DO 125 K=I,N

X(I-1)=X(I-1)-A(I-1,K)*X(K)

125 CONTINUE

AD=0.

GO TO 130

200 WRITE(6,1)

WRITE(6,300) I, A(I,I)

300 FORMAT(//,40X,I5,E20.6)

GO TO 130

202 WRITE(6,1)

WRITE(6,300) N, A(N,N)

AD=2.

RETURN

1 FORMAT(//,8X,'BAD INVERSE, ZERO ON DIAGONAL')

END
SUBROUTINE PCG
IMPLICIT REAL*8 (A-H, O-Z)
COMMON WS, HMAX, RELX1, RELX2, COEF, ERR, X10, DELT, ER5, SRZ, SUMRZ
1, ERRSV, X10SV, JIT, NJ10, NI11, NJ11, NK11, NNN, NSKP1
2, NSKP2, ITMAX, ICNT, IEVP, IWR1, NW1, NW2, NW3, N320, NUM4
3, L9, LENGTH, NK1115, IT01, IT15, IRCO, DDK, BBK
4, NI10, NJ10, NK10, NI12, NJ12, NK12, SVIJ, VV40, SV35
COMMON /XX/ XX
COMMON /DT/ DT
COMMON /W/ W
COMMON /E2/ E2
COMMON /F2/ F2
COMMON /G2/ G2
COMMON /YQ/ YQ
COMMON /NT/ NT
COMMON /DD/ DD
COMMON /BB/ BB
COMMON /ZZ/ ZZ
COMMON /XXS/ XXS
COMMON /ALN/ ALN
COMMON /XXE/ XXE
COMMON /SV/ SV
COMMON /HL/ HL
COMMON /LB/ LB
COMMON /MHD/ MHD
COMMON /E22/ E22
COMMON /D2S/ D2S
COMMON /XXSTR/ XXSTR
DIMENSION E22(65078), D2S(65078)
REAL*4 XXSS(65078)
C IF YOU WANT TO SAVE STORAGE, REMOVE D2S AND E22 FROM THE DIMENSION
C STATEMENT AND CARDS "76 CONTINUE" THROUGH "56 CONTINUE", AND "86
C CONTINUE" THROUGH "662 CONTINUE". THIS REMOVES PCG METHODS SIPCG
C AND SFPCG.
ICNT=0
DO 999 IC=1, ITMAX
DO 1 IJ=1, NNN
DT(IJ)=0
E2(IJ)=0
F2(IJ)=0
G2(IJ)=0
1 VV(IJ)=0
C Modification to PCG from VARDEN due to subsidence. Replaces
C DO 16 and DO 2 loops immediately following this modification
C DO 998 IJ=2, N320
XXIJ=XX(IJ)
C Common /XXS/ XXSS
REAL*4 XXSS(65078)
C Dimension E22(65078), D2S(65078)
REAL*4 DD(65078), BB(65078), ZZ(65078), XXS(65078),
1 YQ(65078), DDK(50), BBK(50), NT(94658), HL(65078)
2, XXSTR(65078)
DIMENSION DT(65078), E2(65078), F2(65078), G2(65078), VV(65078)
DIMENSION XX(65078)
1, WS(10), MHD(65078), LB(94658)
C IF YOU WANT TO SAVE STORAGE, REMOVE D2S AND E22 FROM THE DIMENSION
C STATEMENT AND CARDS "76 CONTINUE" THROUGH "56 CONTINUE", AND "86
C CONTINUE" THROUGH "662 CONTINUE". THIS REMOVES PCG METHODS SIPCG
C AND SFPCG.
ICNT=0
DO 999 IC=1, ITMAX
DO 1 IJ=1, NNN
DT(IJ)=0
E2(IJ)=0
F2(IJ)=0
G2(IJ)=0
1 VV(IJ)=0
C Modification to PCG from VARDEN due to subsidence. Replaces
C DO 16 and DO 2 loops immediately following this modification
C DO 998 IJ=2, N320
XXIJ=XX(IJ)
HLIJ=HL(IJ)
XXSIJ=XXS(IJ)
E2(IJ)=XXIJ
SVIJX=SVIJ*LB(IJ)
SVDT=SVIJX/DELT
SVDT40=SVDT*V40*(1-XXSTR(IJ))
SVDT1=SVDT
IF(XXSIJ.LT.HLIJ) SVDT1=SVDT40
SVDT2=SVDT40
IF(XXIJ.GT.HLIJ) SVDT2=SVDT
SH0DT=SVDT1*(XXSIJ-HLIJ)+SVDT2*HLIJ
DELE=0
IF(MHD(IJ).GE.1) GO TO 997
DELE=SVDT2
V0(IJ)=YQ(IJ)+SH0DT
997 CONTINUE
XXSS(IJ)=DELE
998 CONTINUE
C End of modification
C DO 16 IJ=2,N320
C E2(IJ)=XX(IJ)
C IF(MHD(IJ).GE.1) GO TO 16
C SVIJX=SVIJ*LB(IJ)
C DELY=(SVIJX/DELT)*XXS(IJ)
C IF((IEVP.EQ.1).AND.(XX(IJ).GT.XXE(IJ))) W(IJ)=YQ(IJ)+DELY
C16 CONTINUE
C DO 2 IJ=2,N320
C DELE=0
C IF(MHD(IJ).GE.1) GO TO 19
C SVIJX=SVIJ*LB(IJ)
C DELE=SVIJX/DELT
C IF((IEVP.EQ.1).AND.(XX(IJ).GT.XXE(IJ))) DELE=DELE+ALN(IJ)
C19 XXS(IJ)=DELE
C2 CONTINUE
GO TO (71,71,71,74,74,74,74),NUM4
71 CONTINUE
Y1=0
Z1=0
DO 51 IJ=2,N320
IF(.NOT.(MHD(IJ).GE.1)) GO TO 777
Y1=0
Z1=0
GO TO 51
777 CONTINUE
IJP1K=IJ+NI10
IJM1K=IJ-NI10
IJKP1=IJ+NIJ10
IJKM1=IJ-NIJ10
IF(IJP1K.GT.NNN) IJP1K=NNN
IF(IJKP1.GT.NNN) IJKP1=NNN
IF(IJM1K.LT.1) IJM1K=1
IF(IJKM1.LT.1) IJKM1=1
X=DD(IJ)
Y=BB(IJ)
\[ Z = ZZ(IJ) \]

\[ EEIJ = - (X + Y + Z + DD(IJ + 1) + BB(IJP1K) + ZZ(IJKP1)) + XXSS(IJ) \]

\[
\begin{align*}
&\text{IF}(\text{MHD}(IJ-1).\text{GE}.1) \ X = 0 \\
&\text{IF}(\text{MHD}(IJM1).\text{GE}.1) \ Y = 0 \\
&\text{IF}(\text{MHD}(IJKM1).\text{GE}.1) \ Z = 0 \\
&Y1Z1 = 0 \\
&XP = X \\
&YP = Y \\
&F2X = F2(IJ-1) \\
&F2Y = F2(IJM1K) \\
&F2Z = F2(IJKM1) \\
&\text{IF}(\text{NUM4}.\text{EQ}.1) \ \text{GO TO} \ 20 \\
&\text{JP1} = IJM1K + 1 \\
&\text{KP1} = IJKM1 + 1 \\
&IJMMN = IJ - (\text{NIJ10} - \text{NI10}) \\
&\text{IF}(\text{IJMMN}.\text{LT}.1) \ \text{IJMMN} = 1 \\
&WY1 = 0 \\
&WZ1 = 0 \\
&\text{IF}(F2Y.\text{NE}.0.0) \ WY1 = Y/F2Y \\
&\text{IF}(F2Z.\text{NE}.0.0) \ WZ1 = Z/F2Z \\
&XP = X - (WY1 * Y1 + WZ1 * Z1) \\
&G2(IJ) = XP \\
&YP = Y - WZ1 * D2S(IJM1K) \\
&E22(IJ) = YP \\
&Y1 = -WY1 * G2(JP1) \\
&Z1 = -WZ1 * G2(KP1) \\
&ZB = -WZ1 * E22(IJMMN) \\
&D2S(IJ) = ZB \\
&F21 = F2(JP1) \\
&F22 = F2(KP1) \\
&F23 = F2(IJMMN) \\
&P1 = 0 \\
&P2 = 0 \\
&P3 = 0 \\
&\text{IF}(F21.\text{NE}.0.0) \ P1 = Y1 * Y1/F21 \\
&\text{IF}(F22.\text{NE}.0.0) \ P2 = Z1 * Z1/F22 \\
&\text{IF}(F23.\text{NE}.0.0) \ P3 = ZB * ZB/F23 \\
&Y1Z1 = P1 + P2 + P3 \\
&\text{CONTINUE} \\

\text{20 CONTINUE} \\
&XF = 0 \\
&YF = 0 \\
&ZF = 0 \\
&\text{IF}(F2X.\text{NE}.0.0) \ XF = XP * XP/F2X \\
&\text{IF}(F2Y.\text{NE}.0.0) \ YF = YP * YP/F2Y \\
&\text{IF}(F2Z.\text{NE}.0.0) \ ZF = Z * Z/F2Z \\
&F2(IJ) = EEIJ - (XF + YF + ZF + Y1Z1) \\
&\text{CONTINUE} \\
&\text{GO TO} \ 80 \\

\text{51 CONTINUE} \\
&\text{DO 54 IJ = 2, N320} \\
&\text{IF}(\text{MHD}(IJ).\text{GE}.1) \ \text{GO TO} \ 54 \\
&\text{IJP1K} = IJ + \text{NI10} \\
&\text{IJKP1} = IJ + \text{NI110} \\
&\text{IF}(\text{IJP1K}.\text{GT}.\text{NNN}) \ \text{IJP1K} = \text{NNN} \\
&\text{IF}(\text{IJKP1}.\text{GT}.\text{NNN}) \ \text{IJKP1} = \text{NNN} \\

\text{74 CONTINUE} \\
&\text{DO 54 IJ = 2, N320} \\
&\text{IF}(\text{MHD}(IJ).\text{GE}.1) \ \text{GO TO} \ 54 \\
&\text{IJP1K} = IJ + \text{NI10} \\
&\text{IJKP1} = IJ + \text{NI110} \\
&\text{IF}(\text{IJP1K}.\text{GT}.\text{NNN}) \ \text{IJP1K} = \text{NNN} \\
&\text{IF}(\text{IJKP1}.\text{GT}.\text{NNN}) \ \text{IJKP1} = \text{NNN}
EEIJ = \((-\text{DD}(IJ)+\text{BB}(IJ)+\text{ZZ}(IJ)+\text{DD}(IJ+1)\)
+\text{BB}(IJ\text{JP}1)+\text{ZZ}(IJ\text{KP}1)+\text{XXSS}(IJ))\)

IF(NUM4.LE.5) GO TO 539

X = DD(IJ)
F2X = F2(IJ-1)

IF(F2X.NE.0.0) XF = X*X/F2X

F2(IJ) = EEEJ - XF
GO TO 54

539 F2(IJ) = EEEJ

54 CONTINUE

80 CONTINUE

SPR = 1.D-50
ER5S = 100
L78 = 2.5*ER5S

DO 100 ITER = 1, L78

ICNT = ICNT + 1
IF(ICNT.EQ.ITMAX) GO TO 202

SPP = 0

DO 3 IJ = 2, N320

IF(MHD(IJ).GE.1) GO TO 3

IIJK1 = IJ + NI10
IJM1K = IJ - NI10
IJKP1 = IJ + NJ10
IJKM1 = IJ - NJ10

IF(IJKP1.GT.NNN) IJMP1 = NNN
IF(IJKP1.GT.NNN) IJKP1 = NNN
IF(IJM1K.LT.I) IJMK1 = 1
IF(IJMK1.LT.I) IJMK1 = 1

DDIJ = DD(IJ)
DDMP1 = DD(IJ1K)
BBIJ = BB(IJ)
BBMP1 = BB(IJ1K)
ZZIJ = ZZ(IJ)
ZZJK1 = ZZ(IJKP1)

E2IJ = E2(IJ)

DTIJ = \((-\text{DDIJ}+\text{DDIP1}+\text{BBIJ}+\text{BBMP1}+\text{ZZIJ}+\text{ZZJK1}+\text{XXSS}(IJ))\)*E2IJ

1 +\text{DDIJ}*E2(IJ-1) +\text{DDMP1}*E2(IJ1K)
2 +\text{BBIJ}*E2(IJM1K) +\text{BBMP1}*E2(IJ1K)
3 +\text{ZZIJ}*E2(IJKM1) +\text{ZZJK1}*E2(IJKP1)

DT(IJ) = DTIJ

SPP = SPP + E2IJ*DTIJ

3 CONTINUE

A1 = SPR/(SPP + 1.D-70)
A2 = A1

IF(ITER.GT.1) GO TO 35

A1 = 0
A2 = 1.

35 SRZ = 0
SUMRZ = 0
ER5 = 0

DO 4 K = 2, NK11
DO 4 J = 2, NJ11

35 SRZ = 0
SUMRZ = 0
ER5 = 0

DO 4 = K, NK11
DO 4 = J, NJ11

162
DO 4 I=2,NI11
   IJ=I+NI10*(J-2)+NIJ10*(K-2)
   IF(MHD(IJ).GE.1) GO TO 4
   DX=A1*E2(IJ)
   X=XX(IJ)+DX
   XX(IJ)=X
   ADX=DABS(DX)
   IF(ADX.LT.ER5) GO TO 111
   IMX=I
   JMX=J
   KMX=K
   XXPMX=X
   ER5=ADX
111 CONTINUE
   X=VV(IJ)-A2*DT(IJ)
   SUMRZ=SUMRZ+X
   DSR=DABS(X)
   IF(DSR.GT.SRZ) SRZ=DSR
   VV(IJ)=X
4 CONTINUE
   C GO TO 202
   IF(IWR1.EQ.1) WRITE(6,1000) IC,ICNT,IMX,JMX,KMX,XXPMX,ER5,SRZ,
   1IXX(NW1),XX(NW2),XX(NW3)
   C IF((ER5+ER5S).LT.ERR) GO TO 202
   C OUT NEXT FOR NONLINEAR
   IF(SRZ.LT.XXIO) GO TO 202
   IF((SRZ.LT.XX10).AND.(ITER.EQ.D) GO TO 202
   ER5S=ER5
   SPRS=SPR
   SPR=0
   GO TO (81,81,81,87,83,85),NUM4
81 CONTINUE
   DO 10 IJ=2,N320
   IF(MHD(IJ).GE.1) GO TO 10
   IJM1K=IJ-NI10
   IJKM1=IJ-NIJ10
   IF(IJM1K.LT.1) IJM1K=1
   IF(IJKM1.LT.1) IJKM1=1
   B6=0
   Z6=0
   DDIJ=DD(IJ)
   BB1J=BB(IJ)
   IF(NUM4.EQ.1) GO TO 21
   DDIJ=G2(IJ)
   BB1J=E22(IJ)
   JP1=IJ-NI10+1
   KP1=IJ-NIJ10+1
   IF(JP1.LT.1) JP1=1
   IF(KP1.LT.1) KP1=1
   IJMMN=IJ-(NIJ10-NI10)
   IF(IJMMN.LT.1) IJMMN=1
   B6=0
   Z6=0
   F2J=F2(IJM1K)
   F2K=F2(IJKM1)
IF(F2J.NE.0.DO) B6=DT(JP1)*G2(JP1)/F2J
IF(F2K.NE.0.DO) Z6=(DT(KP1)*G2(KP1)+DT(IJMMN)*E22(IJMMN))/F2K
21 CONTINUE
 DT(IJ)=(VY(IJ)-DDIJ*DT(IJ-1)-BBIJ*(DT(IJKM1)-B6)
1-ZZ(IJ)*(DT(IJKP1)-Z6))/F2(IJ)
10 CONTINUE
 DO 11 IJB=2,N320
 IJ=N320+2-IJB
 IF(MHD(IJ).GE.1) GO TO 11
 IPIJK=IJ+1
 IJP1K=IJ+NI10
 IJKP1=IJ+NIJ10
 IF(IJPIK.GT.NNN) IJP1K=NNN
 IF(IJKP1.GT.NNN) IJKP1=NNN
 XAD=0
 DDD=DD(IP1JK)
 BBB=BB(IJP1K)
 IF(NUMA.EQ.1) GO TO 22
 JMJ1=IJ+NI10-1
 KMK1=IJ+NIJ10-1
 IF(JMJ1.GT.NNN) JMJ1=NNN
 IF(KMK1.GT.NNN) KMK1=NNN
 XAD1=0
 XAD2=0
 F2I=F2(IJ-1)
 F2J=F2(IJM1K)
 IF(F2I.NE.0.DO) XAD1=-(E22(JM1)*DT(JM1)+ZZ(KM1)*DT(KM1))* G2(IJ)/F2I
 IF(F2J.NE.0.DO) XAD2=-ZZ(IJPMN)*E22(IJ)*DT(IJPMN)/F2J
 XAD=XAD1+XAD2
22 CONTINUE
 DTIJ=DT(IJ)-(DDD*DT(IP1JK)+BBB*DT(IJP1K)+ZZ(IJKP1)*DT(IJKP1)+XAD)/F2(IJ)
 DT(IJ)=DTIJ
 SPR=SPR+DTIJ*WdJ
11 CONTINUE
 GO TO 90
83 CONTINUE
 DO 63 IJ=2,N320
 IF(MHD(IJ).GE.1) GO TO 63
 F2IJ=F2(IJ)
 VVij=VV(IJ)
 DTIJ=VVij/F2IJ
 DT(IJ)=DTIJ
 SPR=SPR+DTIJ*VVij
63 CONTINUE
 GO TO 90
85 CONTINUE
 DO 651 IJ=2,N320
IF(MHD(IJ).GE.1) GO TO 651

DT(IJ)=(VV(IJ)-DD(IJ)*DT(IJ-1))/F2(IJ)

651 CONTINUE

DO 652 IJB=2,N320
IJB=N320+2-IJB
IF(MHD(IJ).GE.1) GO TO 652
I1JK=IJ+1
DTIJ=DT(IJ)-DD(I1JK)*DT(I1JK)/F2(IJ)
DT(IJ)=DTIJ
SPR=SPR+DTIJ*VV(IJ)

652 CONTINUE

GO TO 90

C ****** NEW SSOR METHOD********

87 CONTINUE

IF(HMAX.EQ.0.DO) HMAX=1

DO 771 IJ=2,N320
IJ=N320+2-IJB
IF(MHD(IJ).GE.1) GO TO 771
IM1JK=IJ-1
IJMK=IJ-N10
IJKM1=IJ-NIJ10
IF(IJM1K.LT.1) IJM1K=1
IF(IJKM1.LT.1) IJKM1=1
C2S=DD(IJ)
B2S=BB(IJ)
A2S=ZZ(IJ)
DT(IJ)=(W(IJ)-C2S*DT(IJ-1)-B2S*DT(IJM1K)-A2S*DT(IJKM1))/(F2(IJ)/HMAX)

771 CONTINUE

DO 772 IJB=2,N320
IJB=N320+2-IJB
IF(MHD(IJ).GE.1) GO TO 772
IP1JK=IJ+1
IJP1K=IJ+NI10
IJKP1=IJ+NIJ10
IF(IJP1K.LT.NNN) IJP1K=NNN
IF(IJKP1.LT.NNN) IJKP1=NNN
DTIJ=DT(IJ)
1-(DD(I1P1JK)*DT(I1P1JK)+B2S*DT(IJM1K))-1-A2S*DT(IJKM1))/(F2(IJ)/HMAX)

772 CONTINUE

DO 773 IJ=2,N320
IJ=N320+2-IJB
IF(MHD(IJ).GE.1) GO TO 773
DTIJ=DT(IJ)=DTIJ*HMAX

773 CONTINUE

C ****** END SSOR METHOD********

90 CONTINUE

B6=SPR/SPRS
IF(ITER.EQ.1) B6=0

DO 5 IJ=2,N320
E2IJ=DT(IJ)+B6*E2(IJ)
E2(IJ)=E2IJ

5 CONTINUE

100 CONTINUE
999  CONTINUE
202  CONTINUE
1000  FORMAT(' ', 2I3, 3I5, 6D18.7)
      RETURN
      END
SUBROUTINE SIP
IMPLICIT REAL*8 (A-H,O-Z)
COMMON WS,HMAX,RELX1,RELX2,COEF,ERR,XX10,DELT,ER5,SRZ,SUMRZ
1,ERRSV,XX10SV,JIT,NI10,NI11,NJ11,NK11,NNN,NSKP1
2,NSKP2,ITMAX,ICNT,IEVP,IWR1,NW1,NW2,NW3,N320,NUM4
3,L9,LENGTH,NK1115,IT01,IT15,ICRO,DDK,BBK
4,NI10,NJ10,NK10,NI12,NJ12,NK12,SVIJ,VV40,SV35
COMMON /XX/ XX
COMMON /DT/ DT
COMMON /W/ W
COMMON /E2/ E2
COMMON /F2/ F2
COMMON /G2/ G2
COMMON /YQ/ YQ
COMMON /NT/ NT
COMMON /DD/ DD
COMMON /BB/ BB
COMMON /ZZ/ ZZ
COMMON /XXS/ XXS
DIMENSION E22(65078),D2S(65078)
REAL*4 DD(65078),BB(65078),ZZ(65078),XXS(65078),
1YQ(65078),DDK(50),BBK(50),NT(94658),HL(65078)
2,XXSTR(65078)
DIMENSION DT(65078),E2(65078),F2(65078),G2(65078),VV(65078)
DIMENSION XX(65078)
1,WS(10),MHD(65078),LB(94658)
DO 889 IJ=1,NNN
E2(IJ)=0
F2(IJ)=0
G2(IJ)=0
VV(IJ)=0
889 DT(IJ)=0
ICNT=0
ICT=0
ER5S=100
DO 501 ITER0=1,60
DO 499 I33=1,2
I3=I33
I4=1
IF(L9.EQ.2) I4=I3
IF(L9.LT.3) GO TO 195
I3=1
I4=I33
195 CONTINUE
ICT=ICT+1
IF(ICT.EQ.(LENGTH+1)) ICT=1
ICNT=ICNT+1
IF(ICNT.EQ.ITMAX) GO TO 202
W=WS(ICT)
JJM1=2*I3-3
JJP1=-JJM1
KKM1=2*I4-3
KKP1=-KKM1
C ACCOMPLISH EQUATIONS (10) AND (14) BY WEINSTEIN [1969]
SRZ=0
SUMRZ=0
DO 100 KB=2,NK11
ISF1=0
IF((KB.GE.3).AND.(KB.LE.NK10)) ISF1=1
DO 100 JB=2,NJ11
ISF2=0
IF((JB.GE.3).AND.(JB.LE.NJ10)) ISF2=1
J=JB
K=KB
IF(I3.EQ.2) J=NJ12+1-JB
IF(I4.EQ.2) K=NK12+1-KB
DO 50 I=2,NI11
IJF=I+NI10*(J-2)
IJ=IJF+NIJ10*(K-2)
IF(MHD(IJ).GE.l) GO TO 49
IM1JK=IJ-1
IP1JK=IJ+1
IJM1K=IJ+JJM1*NI10
IJP1K=IJ+JJP1*NI10
IJP2K=IJ+NI10
IJKM1=IJ+KKM1*NIJ10
IJKP1=IJ+KKP1*NIJ10
IJKP2=IJ+NIJ10
IF(ISF1.EQ.1) GO TO 91
IF(IJKM1.LT.I) IJKM1=1
IF(IJKM1.GT.NNN) IJKM1=NNN
IF(IJKP1.LT.I) IJKP1=1
IF(IJKP1.GT.NNN) IJKP1=NNN
IF(IJKP2.GT.NNN) IJKP2=NNN
IF(ISF2.EQ.1) GO TO 91
IF(IJM1K.LT.I) IJM1K=1
IF(IJM1K.GT.NNN) IJM1K=NNN
IF(IJP1K.LT.I) IJP1K=1
IF(IJP1K.GT.NNN) IJP1K=NNN
IF(IJP2K.GT.NNN) IJP2K=NNN
CONTINUE
XXIJ=XX(IJ)
HLIJ=HL(IJ)
XXS=XXS(IJ)
Z=ZZ(IJ)
B=BB(IJ)
D=DD(IJ)
F=DD(IP1JK)
H=BB(IJP2K)
S=ZZ(IJKP2)
IF(I3.EQ.l) GO TO 42
BS=B
B=H
H=BS

42 CONTINUE
IF(I4.EQ.1) GO TO 43
ZS=Z
Z=S
S=ZS

43 CONTINUE
C Due to subsidence
C Next 8 lines modified from SIP from VARDEN
C SVDT2 replaces SVDT used before in equation for E
SVIJX=SVIJ*LB(IJ)
SVDT=SVIJX/DELT
SVDT40=SVDT*W40*(1-XXSTR(IJ))
SVDT1=SVDT
IF(XXSIJ.LT.HLIJ) SVDT1=SVDT40
SVDT2=SVDT40
IF(XXIJ.GT.HLIJ) SVDT2=SVDT
E=-(Z+B+D+F+H+S)+SVDT2
C DH1=XXSIJ-HLIJ
C DH2=HLIJ-XXIJ
C IF(SVDT1.NE.SVDT2) SVDTA=(DH1*SVDT1+DH2*SVDT2)/(DH1+DH2)
C IF(SVDT1.EQ.SVDT2) SVDTA=SVDT1
C E=--(Z+B+D+F+H+S)+SVDTA
LEV=0
C IF((IEVP.EQ.1).AND.(XXIJ.GT.XXE(IJ))) LEV=1
IF(LEV.NE.1) GO TO 44
C ALNI=ALN(IJ)
E=E+ALNI

44 CONTINUE
E2I=E2(IJM1K)
E2J=E2(IJKM1)
E2K=E2(IJKM1)
F2I=F2(IM1JK)
F2J=F2(IJKM1)
F2K=F2(IJKM1)
G2I=G2(IM1JK)
G2J=G2(IJKM1)
G2K=G2(IJKM1)
A2=Z/(1+W*(E2K+F2K))
B2=B/(1+W*(G2J+E2J))
C2=D/(1+W*(G2I+F2I))
AC=W*(A2+E2K+B2+E2J)
TG=W*(A2+F2K+G2*I2F)
D2=1/D2
E2(IJ)=D2*(F-AC)
F2(IJ)=D2*(H-TG)
G2(IJ)=D2*(S-WU)
R2=YQ(IJ)-(XX(IM1JK)*D+XX(IP1JK)*F+XX(IJM1K)*B+XX(IJP1K)*H
1+XX(IJKM1)*Z+XX(IJKP1)*S+XXIJ*X(2-SVDT1*XXSIJ-HLIJ)+SVDT2*HLIJ
C Next line was used by SIP from VARDEN
C 2+SVDTA*XXSIJ
C Next line was used by SIP from VARDEN
C IF(LEV.EQ.1) RZ=RZ+ALNI*XXE(IJ)
SUMRZ=SUMRZ+RZ
DSR=DABS(RZ)
IF(DSR.GT.SRZ) SRZ=DSR
VV(IJ)=D2*(RZ-A2*VV(IJKM1)-B2*VV(IJM1K)-C2*VV(IM1JK))
49 CONTINUE
50 CONTINUE
100 CONTINUE
C EQUATIONS (10) AND (14) ARE NOW ACCOMPLISHED
C ACCOMPLISH EQUATION (15) BY WEINSTEIN [1969]
ER5=0
DO 102 KB=2,NK11
ISF1=0
IF((KB.GE.3).AND.(KB.LE.NK10)) ISF1=1
DO 102 JB=2,NJ11
ISF2=0
IF((JB.GE.3).AND.(JB.LE.NJ10)) ISF2=1
J=NJ12+1-JB
K=NK12+1-KB
IF(I3.EQ.2) J=JB
IF(I4.EQ.2) K=KB
DO 62 IB=2,NI11
I=NI12+1-IB
IJ=I+NI10*(J-2)+NIJ10*(K-2)
IF(MHD(IJ).GE.1) GO TO 61
IP1JK=IJ+1
IJP1K=IJ+JJP1*NI10
IJKP1=IJ+KKP1*NIJ10
IF(ISF1.EQ.1) GO TO 92
IF(IJKP1.LT.1) IJKP1=1
IF(IJKP1.GT.NNN) IJKP1=NNN
IF(ISF2.EQ.1) GO TO 92
IF(IJP1K.LT.1) IJP1K=1
IF(IJP1K.GT.NNN) IJP1K=NNN
92 CONTINUE
X=VV(IJ)-E2(IJ)*DT(IP1JK)-F2(IJ)*DT(IJP1K)-G2(IJ)*DT(IJKP1)
DT(IJ)=X
X=X*HMAX
XXP=XX(IJ)+X
XX(IJ)=XXP
X=DABS(X)
IF(X.LE.ER5) GO TO 111
IMX=I
JMX=J
KMX=K
XXPMX=XXP
ER5=X
111 CONTINUE
61 CONTINUE
62 CONTINUE
102 CONTINUE
C EQUATION (15) IS NOW ACCOMPLISHED
IF(IWR1.EQ.1) WRITE(6,1000)  
1 ICNT,IMX,JMX,KMX,XXPMX,ER5,SRZ,XX(NW1),XX(NW2),XX(NW3)  
IF((ER5+ER5S).LT.ERR) GO TO 202  
IF(SRZ.LT.XXIO) GO TO 202  
ER5S=ER5  
499 CONTINUE  
501 CONTINUE  
202 CONTINUE  
1000 FORMAT(' ',I3,3I5,6D18.7)  
RETURN  
END