ESTIMATING THE MAGNITUDE AND FREQUENCY OF LOW FLOWS OF STREAMS IN MASSACHUSETTS

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CONVERSION FACTORS

Multiply	by	To obtain
cubic foot per second (ft^3/s)	0.02832	cubic meter per second
foot (ft)	0.3048	meter
inch (in.)	25.4	millimeter
inch per year (in/yr)	25.4	millimeter per year
mile (mi)	1.609	kilometer
square mile (mi^2)	2.59	square kilometer

Estimating the Magnitude and Frequency of Low Flows of Streams in Massachusetts

By John C. Risley

Abstract

Techniques are presented for estimating 7-day 2-year (7Q2) and 7-day 10-year (7Q10) flows at continuous-record streamflow-gaging stations, partial-record stations, and ungaged sites at streams in Massachusetts. A two-parameter log-normal probability distribution was used to compute 7Q2 and 7Q10 flows at 31 continuous-record streamflow-gaging stations. Graphical and mathematical techniques were used to estimate 7Q10 flows at partial-record stations.

Regionalized regression techniques commonly are used to estimate 7Q2 and 7Q10 flows at ungaged stream sites. The development and application of a low-flow frequency model are presented. The model contains two parameters, mean and standard deviation, which are estimated from two regression equations that use total drainage area and basin relief as independent variables. The coefficients of determination of the mean and standard-deviation regression equations are 0.964 and 0.960. The percentage of standard errors of regression of the equations are 35 and 34 percent, respectively. Except for southeastern Massachusetts, the model is applicable for basins in Massachusetts with a drainage area of 5 to 150 square miles, a slope of less than 4 percent, and an area of stratified drift greater than 4 percent. The model is not applicable to the southeastern part of the State where the geology and topography differ from that in the rest of the State. The model is included in a computer program that can be used to estimate 7Q2 and 7Q10 flows and their 95-percent confidence intervals for a given basin.

INTRODUCTION

Estimates of low flow are used by Federal and State agencies, consultants, local planners, and engineers for determination of waste-load allocations, issuance and (or) renewal of National Pollution Discharge Elimination System (NPDES) permits, and siting of treatment plants and sanitary landfills. Low-flow estimates also are used to make decisions regarding interbasin transfer, withdrawals for water supply, and minimum downstream-release requirements for hydropower, irrigation, and cooling-plant facilities.

Low-flow characteristics may be described by duration or by specific frequency. A flow-duration curve, computed from the streamflow record at a site, shows the percentage of time that any magnitude of streamflow is equaled or exceeded (Searcy, 1959). Low flow commonly is described using the 95, 98, and 99th percentiles of the curve. More commonly, low flow is described as the T-year nonexceedance quantile of the annual minimum D-day-mean flow frequency distribution. This statistic is the minimum D-day-mean streamflow expected to occur on average once during an interval of T-years (Riggs, 1972). The 7-day 2-year (7Q2) and the 7-day 10-year (7Q10) flows are commonly used indexes of low flow in the United States. Annual minimum 7-day mean discharges are usually based on a climatic year (April 1 through March 31).

This study was done by the U.S. Geological Survey (USGS) in cooperation with the Massachusetts Department of Environmental Protection, Division of Water Pollution Control. This report is based on research that included testing selected low-flow frequency distributions (Vogel and Kroll, 1989) and developing a low-flow regression model for ungaged sites (Vogel and Kroll, 1990). The analyses and results discussed in this report update the work performed by Vogel and his colleagues by adding eight continuous-record streamflow-gaging stations to their original data set.

Purpose and Scope

This report presents the techniques for estimating 7Q2 and 7Q10 flows at continuous- and partial-record streamflow-gaging stations and techniques for estimating these values at ungaged stream sites in Massachusetts. Description of the techniques used to make the estimates has been minimized. The reader is encouraged to review the cited references for more detail.

Low-flow statistics were computed at 31 continuous-record streamflow-gaging stations (also referred to as gaged or index stations) by means of graphical and mathematical techniques. Most of the stations were in Massachusetts; however, two were in Rhode Island and one was in Connecticut. For each station, a two-parameter log-normal-probability distribution was computed from the annual minimum 7-day mean discharges for the period of record. The 7Q2 and 7Q10 flows were determined from the fitted distribution curve. A hand-drawn curve through the plotted flows was not significantly different from the mathematical distribution curve.

Graphical and mathematical techniques were used to estimate 7Q10 flows at partial-record stations in Massachusetts. Discharge measurements were made at 11 sites during low-flow periods over several years. The estimated 7Q10 flows were based on the correlation of discharge measurements from each partial-record station and concurrent daily mean discharges of nearby index stations.

Ordinary least-squares regression techniques were used to develop equations for estimating 7Q2 and 7Q10 flows for ungaged sites in Massachusetts. Selected basin characteristics are used as independent variables in the regression equations. The equations were developed from 31 continuous-record streamflow-gaging stations with records ranging from 11 to 78 years. Low-flow estimates computed using the equations were compared with estimates made previously at the 11 partial-record stations.

Physical Setting

Most of Massachusetts is underlain by crystalline metamorphic and igneous rocks. This bedrock layer is mantled by discontinuous glacial till and coarse-grained stratified glacial drift. The till consists of mostly unstratified unsorted clay and commonly is located in basin uplands. The coarse-grained stratified deposits are mostly sorted sand and gravel usually located along the main streambed and the tributaries. Basin relief generally increases in the western two-thirds of the State. The southeastern coastal region is characterized by plains and low hills underlain by a continuous blanket of unconsolidated sediments (Moody and others, 1986). Areal percentages of till decrease from the west to the southeast.

The climate of Massachusetts is temperate. Precipitation averages about 45 in/yr and is fairly uniformly distributed throughout the State and during the year. Annual evaporation from free water surfaces ranges from 26 in. in the west to 28 in. in the east (Moody and others, 1986).

Previous Studies

Low-flow-frequency analysis can be used to compute low flows at gaging stations. Research in low-flow frequency analysis is limited compared to the amount of research that has been done in flood-frequency analysis. Riggs (1972) outlines techniques for low-flow frequency estimation by fitting annual series of low-flow data to the three-parameter Log Pearson Type III distribution. In some studies, the fit of the Log Pearson Type III distribution has been compared with that of other hypothetical distributions by use of various parameter estimation techniques. Discharge data used for the studies were collected at 14 streamflow-gaging stations in the Eastern United States (Matalas, 1963) and at 20 gaging stations in Virginia (Tasker, 1987). Matalas (1963) determined that the three-parameter Weibull and Pearson Type III distributions best fit the low-flow data. Tasker (1987) determined that the Log Pearson Type III distribution best fit the low-flow data. Vogel and Kroll (1989) compared various two- and three-parameter distributions using data from 23 gaging stations in Massachusetts and determined that the low-flow data best fit the two-parameter log-normal distribution (discussed later in this report).

Low flows at partial-record stations are estimated by correlating low-flow measurements made at an ungaged site with concurrent daily mean discharges of a nearby continuous-record index station. Riggs (1972) presented graphical techniques for estimating low flows at partial-record stations. Stedinger and Thomas (1985) showed that low flows determined through the regression of concurrent discharges of the partial-record and index stations contained a bias. Their mathematical technique to remove the bias from the estimate assumes that the population skews of the partial-record and the index stations are identical. Gilroy (1972) and Hirsch (1982) also presented mathematical techniques for estimating low flows at partial-record stations. Stedinger and Thomas (1985) compared and tested five techniques; their technique, discussed in detail in a later section of this report, yielded the lowest bias and error.

Regionalized low-flow regression models, which incorporate basin characteristics as independent variables, have been developed to estimate 7Q2 and

7Q10 flows at ungaged sites in the northeastern United States. Johnson (1970) estimated low flows for sites in Massachusetts, New Hampshire, Rhode Island, and Vermont, using drainage area, mean annual precipitation, and January minimum temperature as independent variables of regression. Tasker (1972) developed a low-flow regression model for southeastern Massachusetts using basin drainage area and a ground-water factor, related to the transmissivity and availability of water in the basin aquifers, as independent variables. Ku and others (1975) developed a low-flow regression model for the Susquehanna River Basin in New York that uses area of sand and gravel and annual mean runoff as independent variables. Cervione and others (1982) developed a low-flow regression model for Connecticut that uses the area of stratified drift and area of till as independent variables. Vogel and Kroll (1990) presented a low-flow regression model for Massachusetts, which uses drainage area and relief as independent variables. An updated version of this model is presented later in this report.

Acknowledgments

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LOW FLOWS IN MASSACHUSETTS

In Massachusetts, the minimum 7-day mean discharge for most streams occurs in August or September, although it also may occur in midwinter during periods of prolonged subfreezing temperatures. August and September also usually have the lowest monthly mean discharge (fig. 1). Some environmental factors affecting occurrence and magnitude of low flows in streams are the discharge of ground water to stream channels, the presence of underflow, surficial geology, and evaporation from lake and swamp areas.

Water available for ground-water recharge is affected by precipitation. Variations in the magnitude and frequency of precipitation during spring and summer can directly affect the magnitude of low flows later in the year. The residence time between spring and summer rainstorms and base flow is a function of basin relief and surficial geology.

The surficial geology of a basin has a direct effect on the base flow. Till and fine-grained stratified-drift deposits are less permeable than coarse-grained stratified-drift deposits. Therefore, infiltration of precipitation is less on till and fine-grained stratified-drift deposits than on coarse-grained stratified-drift deposits. Streams in the basins containing considerable till and (or) fine-grained stratified-drift deposits respond rapidly to precipitation, and most of the precipitation leaves the basin as surface runoff. Basins mantled by considerable areas of coarse-grained stratified-drift deposits retain a greater proportion of the precipitation, which infiltrates the material for later release to streams as base flow.

Vogel and Kroll (1990) point out the effect of basin relief in controlling low-flow variability. Basin relief does not account for as much of the variability as does the size of the drainage area. However, as basin relief increases, the magnitude of low flow increases correspondingly.

FREQUENCY ANALYSIS AT LOW-FLOW CONTINUOUS-RECORD STATIONS

Graphical and mathematical techniques are used to compute low flows at continuous-record streamflow-gaging stations. Prior to using the techniques, several assumptions concerning the annual series of minimum 7-day mean discharge commonly are used. These assumptions include the following:

1. The annual minimum 7-day mean flows are independent random events.
2. The process generating these events is stationary with respect to time.
3. The data sample is representative of the population of all 7-day minimum flows.
4. The data are from a non-mixed population and are not the products of multiple causative processes.
5. There are no measurement or computational errors in the data.

Further elaboration of these assumptions is provided by the Interagency Advisory Committee on Water Data (1982, p. 6-7). The first assumption is critical because gaging records commonly contain patterns of streamflow either below or above average conditions that may persist for several years. The independence of the data can be evaluated by computing the Spearman rank-order correlation coefficient (Iman and Conover, 1983, p. 341).

The second assumption requires a streamflow record that does not contain any significant trends or episodic changes in magnitude. Trends or changes in

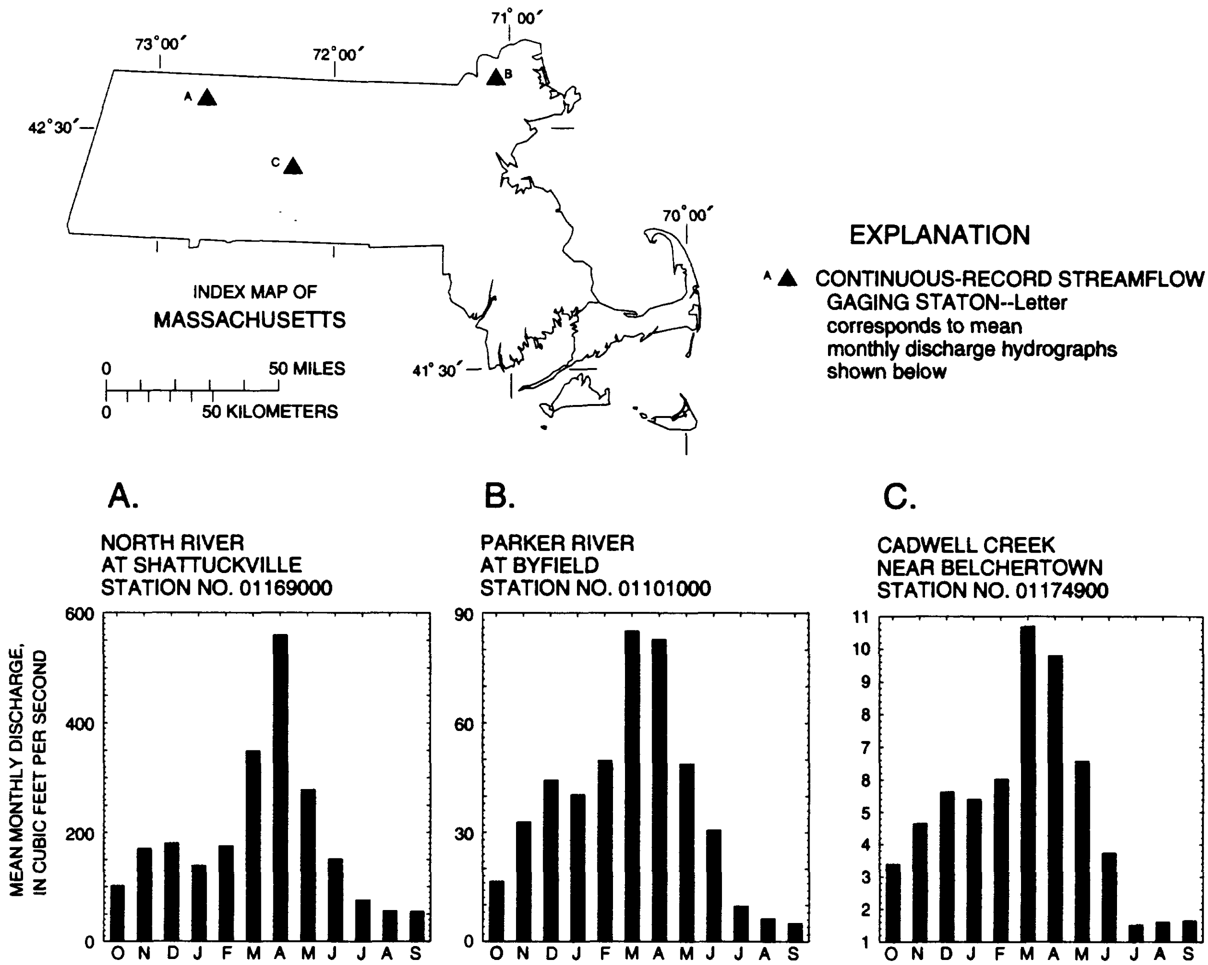


Figure 1. Location of study area and mean monthly discharge data for selected sites in Massachusetts, 1951-80.

magnitude in the data may be the result of streamflow regulation, drainage basin modifications, climate change, or channelization that may have occurred during the period of record. Kendall's tau is a nonparametric test frequently used to evaluate the possible presence of trends in time-series data (Hirsch and others, 1982).

The third assumption requires the sample to be of an adequate size. A minimum of 10 years of record is recommended for low-flow frequency analysis (Riggs, 1972). Shorter streamflow records may not provide sufficient sampling of the variation that may exist in the population.

The fourth assumption is violated if the data samples represent a mixed population. Annual low flows that occur in the same season from year to year are likely to be the result of the same causative climatic process, and from a non-mixed population. Mixed population data can sometimes be recognized by abnormally large skew coefficients.

Despite extensive quality control, the fifth assumption cannot be accomplished with all streamflow records. If the accuracy of a station's record is questionable, the record should be compared with the records of nearby stations.

Graphical Technique

Low-flow frequency estimates can be made for gaged stations by plotting the magnitudes of the annual discharges against their calculated recurrence intervals and drawing a curve through the points. If the data meet the assumptions listed above, the following technique described by Riggs (1972) may be used:

1. The annual series of minimum 7-day mean discharge is ranked in order of ascending magnitude.
2. The recurrence interval for each discharge is computed by the relation

$$T = (n+1)/m \,, \qquad (1)$$

where

T is the recurrence interval, in years,
n is the number of annual events, and
m is the rank order of the annual event.

3. The discharges are plotted on log-normal probability paper.
4. A curve is drawn to fit the data.

Riggs (1972) considers the graphical curve to be the basic frequency curve for annual flows; however, the computer plot should be obtained and the Log Pearson Type III curve used if it is an adequate fit. If not, a graphical interpretation is made on the computer printout.

Aside from the Log Pearson Type III distribution, other distributions, such as the normal, two- and three-parameter log-normal, two- and three-parameter Weibull and Gumbel distributions, may be suitable in low-flow-frequency analysis. Vogel and Kroll (1989) tested and compared the fit of these distributions by computing their probability-plot correlation coefficients. Annual minimum 7-day mean discharges at 23 continuous-record streamflow-gaging stations in Massachusetts were used in the comparison. The two-parameter log-normal distribution had the highest correlation coefficient.

Mathematical Techniques

Probability Distributions

Probability distributions provide a mathematical representation of low-flow-frequency curves. The Log Pearson Type III distribution commonly used in flood-frequency studies also is commonly used for low-flow studies (Riggs, 1972). Haan (1977) describes a technique for using the Log Pearson Type III analysis as follows:

1. The annual 7-day minimum discharges are transformed to base-10 logarithms.
2. The mean, standard deviation, and the coefficient of skewness are computed from the transformed data.
3. The transformed flow of a given recurrence interval is computed by means of the equation:

$$Y_T = \bar{Y} + S_Y K_T \,, \qquad (2)$$

where

Y_T is the logarithm of the estimated flow, for a recurrence interval, T,
$\bar{Y}$ is the mean of the logarithms of the annual flows,
S_Y is the standard deviation of the logarithms of the annual flows, and
K_T is the Log Pearson Type III frequency factor based on the coefficient of skewness and the recurrence interval.

4. The flow for the given recurrence interval is found by taking the antilog of Y_T.

The equation for estimating a D-day, T-year flow statistic using the two-parameter log-normal analysis is

$$Q_{D,T} = exp\left(\hat{\mu}_Y(D) + Z_T\hat{\sigma}_Y(D)\right) \,, \qquad (3)$$

where

$Q_{D,T}$ is the D-day, T-year discharge,
$\hat{\mu}_Y(D)$ is the mean of the natural logarithms of the annual minimum D-day mean discharges,
Z_T is the standard normal random variable corresponding to the T-year event, and
$\hat{\sigma}_Y(D)$ is the standard deviation of the natural logarithms of the annual minimum D-day mean discharges.

For logarithmic distributions, zero values of the minimum 7-day mean discharge do not exist. If zero values are present in the original data set, the frequency curve may be adjusted by applying conditional probability theory, which is based on the same principles as those dealing with flood-frequency analysis (U.S. Interagency Advisory Committee on Water Data, 1982).

Record Augmentation

Hydrologists commonly are faced with the problem of estimating low- and flood-flow frequencies from short-term streamflow records. Techniques have been developed by Fiering (1963), Matalas and Jacobs (1964), and Vogel and Stedinger (1985) to deal specifically with augmenting short-term streamflow records when making these estimates. The annual flow values of a short-term streamflow record must have an adequate cross correlation with annual flows of a long-term streamflow record of a nearby station. Record-augmentation techniques do not actually lengthen short-term records; rather, they provide estimates of the mean and variance of annual flow values for a short-term record station that reflect the long-term period of record of the nearby station. These estimates are then used in a probability distribution, such as the two-parameter log-normal, to estimate discharge frequencies. The streamflow-record-augmentation equations used in the frequency analysis presented in this section are provided in Appendix A.

Stations in Massachusetts

A calculation of 7Q2 and 7Q10 discharges at streamflow-gaging stations in Massachusetts was needed for the development of a statewide low-flow regression model. The following steps were performed in the analysis:

1. More than 70 continuous-record stations in Massachusetts and Rhode Island were reviewed for the analysis. About 50 of these stations are in the report study area in central and western Massachusetts. Nearly all the stations in this area that were not used in the study had flows that were significantly affected by regulation. Regulation included dams, withdrawal of surface and ground water for public supply and industrial use, and return flows from sewage-treatment plants and irrigation. A few of the basins containing small flood-control dams were included in the analysis because the dams are unregulated during the low-flow season in the late summer.

2. Daily streamflow records were retrieved from the USGS National Water Data Storage and Retrieval System (WATSTORE) (Hutchinson, 1975). The Automated Data Processing System (ADAPS; Dempster, 1990) was used to compute the annual minimum 7-day mean discharges for each of the gaging stations for the period of record through 1989. The annual periods were based on a climatic year (April 1 to March 31).

3. Records that contained any zero annual minimum 7-day mean discharges were deleted from the selection. Zero annual values generally occur in basins that have large areas of till, high slopes, and small drainage areas. The regression model developed for this report is not applicable for basins with those characteristics. Thirty-one stations were selected for evaluation (table 1; fig. 2). Zero annual flow only occurred at two of the nonselected stations.

4. Tests for trends and serial correlation using Kendall's tau and the Spearman rank order correlation coefficient were performed on each of the series of annual minimum 7-day mean discharges from the 31 stations. The tests showed no significant trend or serial correlation.

5. A natural logarithmic transformation was made of the annual minimum 7-day mean discharge values and the mean and standard deviation of these values were computed for each record (table 2).

6. Streamflow records for five stations were shorter than 20 years (table 3). The means and standard deviations of the logs for these stations were adjusted using the record augmentation equations shown in Appendix A. Station 01171800 on Bassett Brook near Northampton also had less than 20 years of record, but this record was not augmented because it could not be corrlated satisfactorily with another index station.

7. Estimates of 7Q2 and 7Q10 flow (table 2) were computed for each station using the two-parameter log-normal distribution (eq. 3).

8. The annual minimum 7-day mean discharges for each station were plotted on log-normal-probability paper. The log-normal distribution curve provided an adequate fit of the data. No adjustments to the 7Q2 and 7Q10 discharges determined in step 7 were necessary for any of the discharges.

FREQUENCY ANALYSIS AT LOW-FLOW PARTIAL-RECORD STATIONS

Low-flow partial-record stations are established at stream sites when it is not feasible to construct a continuous-record streamflow-gaging station. When a minimum of eight base-flow measurements have been made at these sites during different flow recessions and over a period of several years (Riggs, 1972), the measured discharges are correlated with concurrent daily mean discharges at a nearby continuous-record streamflow-gaging (index) station. If the correlation is significant, graphical and mathematical techniques can be used to estimate low flows at the low-flow partial-record station based on those of an index station. The graphical technique and the two mathematical techniques discussed in this section are equally acceptable provided the minimum number of base-flow measurements required by the technique are used.

Table 1. Continuous-record streamflow-gaging stations used in the analyses made in this study

Map No. (fig. 2)	Station No.	Station name
1	01096000	Squannacook River near West Groton, Mass.
2	01096910	Boulder Brook at East Bolton, Mass.
3	01097000	Assabet River at Maynard, Mass.
4	01097300	Nashoba Brook near Acton, Mass.
5	01100700	East Meadow River near Haverhill, Mass.
6	01101000	Parker River at Byfield, Mass.
7	01105600	Old Swamp River near South Weymouth, Mass.
8	01106000	Adamsville Brook at Adamsville, R.I.
9	01107000	Dorchester Brook near Brockton, Mass.
10	01109200	West Branch Palmer River near Rehoboth, Mass.
11	01111200	West River below West Hill Dam, near Uxbridge, Mass.
12	01111300	Nipmuc River near Harrisville, R.I.
13	01162500	Priest Brook near Winchendon, Mass.
14	01165500	Moss Brook at Wendell Depot, Mass.
15	01169000	North River at Shattuckville, Mass.
16	01169900	South River near Conway, Mass.
17	01170100	Green River near Colrain, Mass.
18	01171500	Mill River at Northhampton, Mass.
19	01171800	Bassett Brook near Northampton, Mass.
20	01174000	Hop Brook near New Salem, Mass.
21	01174900	Cadwell Creek near Belchertown, Mass.
22	01175670	Sevenmile River near Spencer, Mass.
23	01176000	Quabog River at West Brimfield, Mass.
24	01180000	Sykes Brook at Knightville, Mass.
25	01180500	Middle Branch Westfield River at Goss Heights, Mass.
26	01180800	Walker Brook near Becket Center, Mass.
27	01181000	West Branch Westfield River at Huntington, Mass.
28	01187400	Valley Brook near West Hartland, Conn.
29	01198000	Green River at Great Barrington, Mass.
30	01332000	North Branch Hoosic River, at North Adams, Mass.
31	01333000	Green River at Williamstown, Mass.

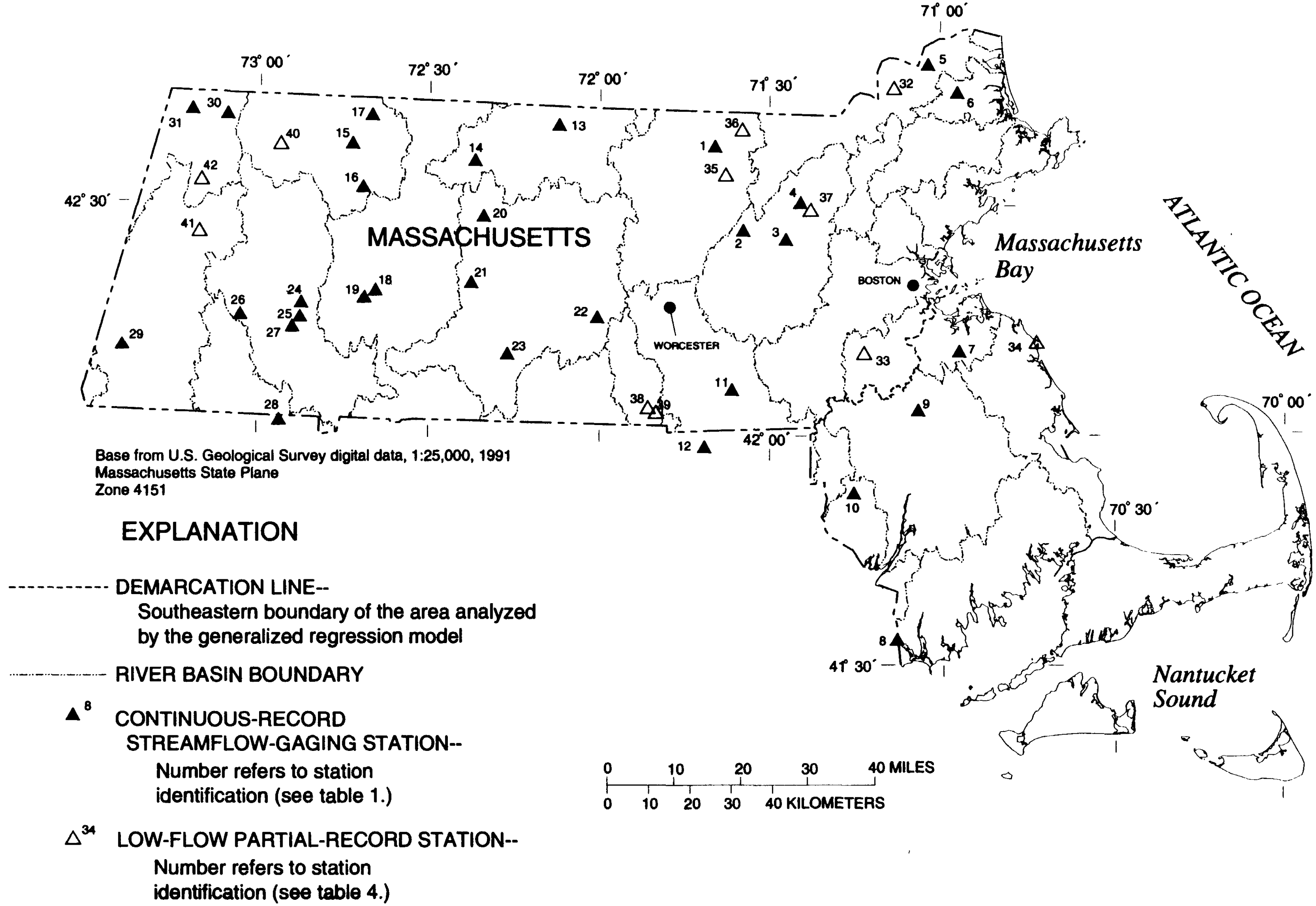


Figure 2. Locations of continuous-record streamflow-gaging stations used in the study.

Table 2. Selected basin characteristics and computed 7-day 2-year, and 7-day 10-year flows of continuous-record streamflow-gaging stations in Massachusetts

[Mean and standard deviation: estimates of the mean and standard deviation of the natural logarithms of the annual minimum 7-day mean discharges. 7Q2 and 7Q10 indicate 7-day 2-year and 7-day 10-year low flow. mi^2, square mile; ft, foot; ft^3/s, cubic foot per second]

Station No.	Period of record (climatic year)	Mean	Standard deviation	Drainage area (mi^2)	Basin relief (ft)	7Q2 (ft^3/s)	7Q10 (ft^3/s)
01096000	1950-89	2.308	0 .433	63.6	1,511	10	6
01096910[1]	1972-84	-2.948	.985	1.61	310	.05	.02
01097000	1942-89	2.612	.866	117	613	13	4.5
01097300	1964-89	-.465	1.003	12.9	310	.63	.17
01100700[1]	1963-74	-1.189	.630	5.48	236	.30	.14
01101000	1946-89	-.273	.999	21.4	330	.76	.21
01105600	1967-89	-.879	.594	4.47	130	.42	.19
01106000	1941-78	-1.762	1.060	7.99	280	.17	.04
01107000[1]	1963-74	-1.905	1.320	4.71	200	.15	.03
01109200[1]	1963-74	-2.281	1.162	4.33	166	.10	.02
01111200	1962-88	1.150	.440	27.8	398	3.2	1.8
01111300	1964-89	-.024	.761	15.9	365	.98	.37
01162500	1919-35, 1937-88	.317	.899	19.2	523	1.4	.43
01165500	1917-83	0.209	.558	12.1	1,108	1.2	.60
01169000	1940-89	2.659	.450	88.9	1,865	14	8.0
01169900	1967-89	1.695	.363	24.1	1,383	5.5	3.4
01170100	1968-89	2.054	.406	41.2	1,967	7.8	4.6
01171500	1939-89	2.264	.386	54.0	1,557	9.6	5.9
01171800	1963-74	-.105	.536	5.56	620	.90	.45
01174000	1948-83	-2.372	1.429	3.39	526	.09	.02
01174900	1962-89	-1.681	.661	2.59	620	.19	.08
01175670	1961-89	-.669	.622	8.69	530	.51	.23
01176000	1913-89	3.415	.609	150	830	30	13
01180000	1946-74	-2.264	.528	1.74	696	.10	.05
01180500	1911-89	1.459	.864	52.8	1,835	4.3	1.4
01180800[1]	1963-77	-1.056	.359	2.95	540	.35	.22
01181000	1936-89	2.405	.517	93.6	1,845	11	5.7
01187400	1941-72	-.694	.625	7.35	890	.50	.16
01198000	1952-71	1.681	.395	51.0	1,375	5.4	3.2
01332000	1932-89	2.082	.366	41.0	2,275	8.0	5.0
01333000	1950-89	2.094	.504	42.6	2,872	8.1	4.3

[1] Mean and standard deviation estimates for these stations were adjusted by streamflow augmentation techniques.

Table 3. Record augmentation for five continuous-record streamflow-gaging stations in Massachusetts with less than 20 years of record

[Mean and variance, values computed from natural logarithms of annual minimum 7-day mean discharges in cubic feet per second]

Short-record station	Index station	Years of record		Mean		
		Concurrent	Nonconcurrent	Short-record stations	Concurrent record at index stations	Nonconcurrent record at index stations
01096910	01096000	12	27	-2.515	2.536	0.750
01100700	01101000	11	32	-1.164	-.186	-.303
01107000	01105600	7	15	-2.047	-.979	-.832
01109200	01105600	7	15	-2.438	-.979	-.832
01180800	01181000	14	39	-1.088	2.347	2.426

Short-record station	Index station	Variance			Correlation coefficient	Augmented estimated	
		Short-record stations	Concurrent record at index stations	Nonconcurrent record at index stations		Mean	Variances
01096910	01096000	0.75	0.116	0.191	0.79	-2.94	0.970
01100700	01101000	.410	1.175	.970	.60	-1.189	.397
01107000	01105600	2.313	.704	.222	.86	-1.905	1.743
01109200	01105600	2.12	.704	.222	.93	-2.281	1.350
01180800	01181000	.135	.286	.266	.84	-1.055	.129

Graphical Technique

The graphical technique can be used for any data set but is especially useful if there are fewer than 10 base-flow measurements or when the relation between base-flow measurements at the ungaged site and the concurrent daily mean discharges at the index station is nonlinear (Riggs, 1972). Daily mean discharges at an index station are plotted along the abscissa (X axis) on log-log paper with concurrent measured discharge values from the ungaged site plotted along the ordinate (Y axis).

Mathematical Techniques

Mathematical techniques can be used to estimate D-day, T-year discharges at low-flow partial-record stations if more than 10 discharge measurements have been recorded. Earlier mathematical estimates of low flows were obtained through a linear regression equation of base-10 logarithms of measured discharge at the low-flow partial-record station and concurrent daily mean discharge at the index station. The D-day, T-year discharge at the index station was then used as the independent variable of regression to compute the low flow at the low-flow partial-record station.

Stedinger and Thomas (1985) found that the regression technique may yield a biased estimate of low flows at the low-flow partial-record station. To remove the bias, the D-day T-year discharge at the low-flow partial-record station was estimated using the equation

$$\hat{Y}_{D,T} = \hat{M}_Y + K_Y \hat{S}_Y , \qquad (4)$$

where

$\hat{Y}_{D,T}$ is the base-10 logarithm of the D-day T-year discharge at the partial-record station,

$\hat{M}_Y$ is the estimated mean of base-10 log transformed annual low flows at the partial-record station; $\hat{M}_Y$ is computed by the equation

$$\hat{M}_Y = a + b\ M_X, \qquad (5)$$

where

a is the intercept of the regression of the base-10 logarithms of the concurrent discharges of the two stations,

b is the slope of the regression of the base-10 logarithms of the concurrent discharges of the two stations, and

M_X is the mean of the base-10 logarithms of the annual low flows at the index station;

K_Y is the Log Pearson Type III frequency factor for recurrence interval T at the partial-record station; and

$\hat{S}_Y$ is the estimated standard deviation of base-10 transformed annual low flows at the partial-record station.

The estimated standard deviation of the transformed annual low flows at the low-flow partial-record station, $\hat{S}_Y$, is computed by the equation

$$\hat{S}_Y = \sqrt{b^2 S_X^2 + S_e^2 \left[1 - \frac{S_X^2}{(L-1)\,(S_{\bar{X}}^2)}\right]}\ , \qquad (6)$$

where

S_X^2 is the variance of base-10 logarithms of annual low flows at the index station,

S_e^2 is the standard error of the regression of the concurrent discharges of the two stations,

L is the number of recorded base-flow measurements, and

$S_{\bar{X}}^2$ is the variance of the base-10 logarithms of the concurrent daily mean discharges at the index station.

When using the Stedinger and Thomas (1985) technique, the skew values of the annual low flows for both stations are assumed equal. Therefore, the Log Pearson Type III frequency factor at the low-flow partial-record station is the same frequency factor that is used for the index station.

Another technique that reduces the bias that would have been introduced using linear regression is the MOVE.1 (Maintenance Of Variance Extension) estimator developed by Hirsch (1982). The MOVE.1 equation to estimate the D-day T-year discharge is

$$\hat{Y}_{D,T} = m_{\bar{Y}} + \frac{S_{\bar{Y}}}{S_{\bar{X}}}(\hat{X}_{D,T} - m_{\bar{X}})\ , \qquad (7)$$

where

$m_{\bar{Y}}$ is the mean of the base-10 logarithms of the discharge measurements at the low-flow partial-record station,

$S_{\bar{Y}}$ is the standard deviation of the base-10 logarithms of the discharge measurements recorded at the low-flow partial-record station,

$\hat{X}_{D,T}$ is the base-10 logarithms of the D-day T-year discharge at the index station, and

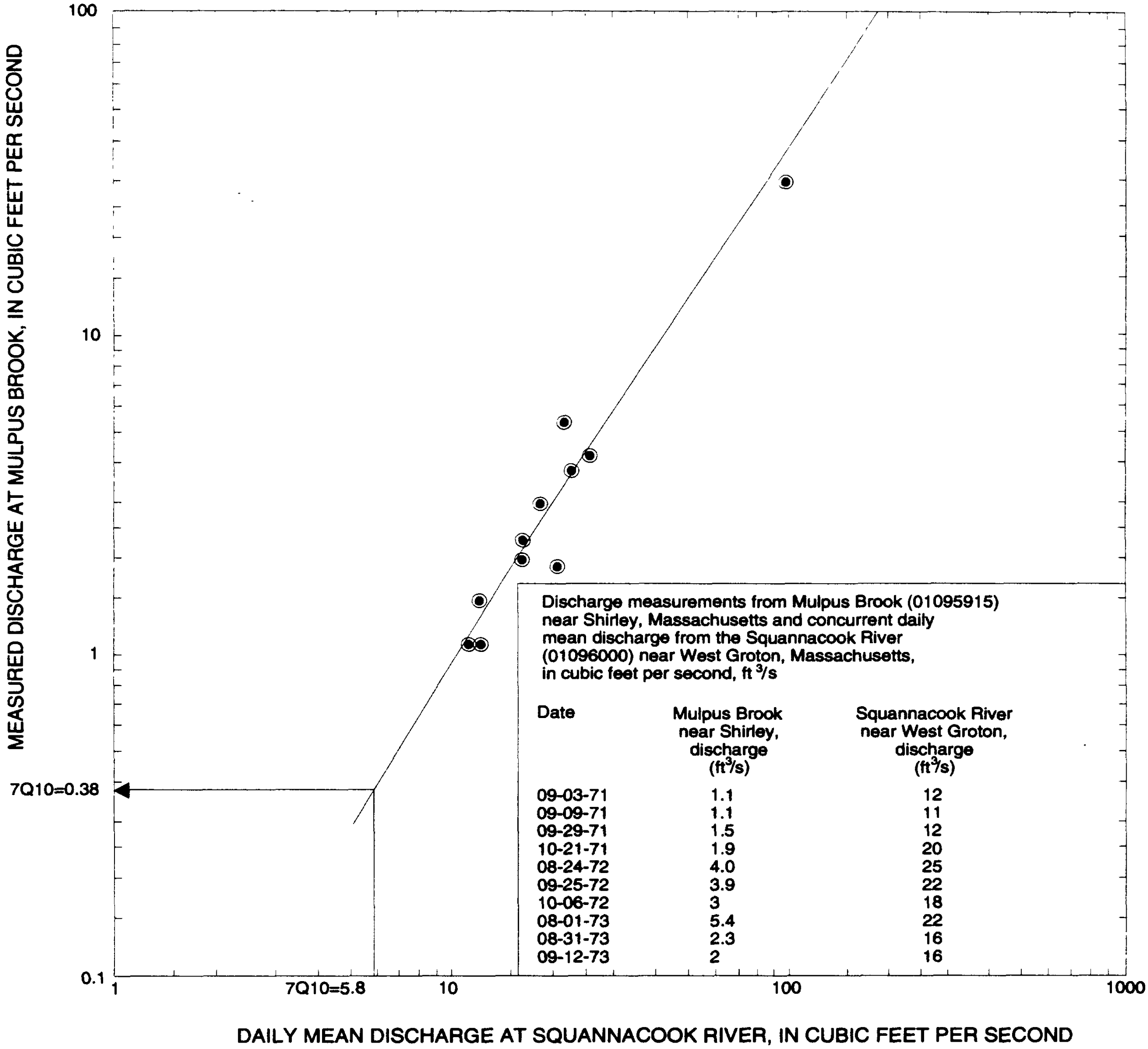
$m_{\bar{X}}$ is the mean of the base-10 logarithms of the concurrent daily discharges at the index station.

Stations in Massachusetts

The relation between discharges at a gaging station on the Squannacook River near West Groton, Massachusetts (01096000) and a partial-record station on Mulpus Brook near Shirley, Massachusetts (01095915) is shown in figure 3. Using a two-parameter log-normal distribution, a flow of 5.8 ft^3/s was computed as the 7Q10 flow for the index station (Squannacook River). The corresponding 7Q10 flow at the Mulpus Brook low-flow partial-record station is estimated from the curve as 0.38 ft^3/s.

Estimating low flows by the graphical technique has limited precision. The estimated 7Q10 flow for station 01095915 could have been 0.40 or 0.36 ft^3/s, depending on the extrapolation of the lower part of the curve. A degree of variability about the line, particularly at the lower end, can be expected because of dissimilarities between the two basins. Although additional data may improve the relation, a point is eventually reached where additional data may only result in additional scatter about the curve.

Data from station 01095915 and the index station 01096000 also were used with the Stedinger and Thomas estimator. The flow data were transformed to



Date	Mulpus Brook near Shirley, discharge (ft^3/s)	Squannacook River near West Groton, discharge (ft^3/s)
09-03-71	1.1	12
09-09-71	1.1	11
09-29-71	1.5	12
10-21-71	1.9	20
08-24-72	4.0	25
09-25-72	3.9	22
10-06-72	3	18
08-01-73	5.4	22
08-31-73	2.3	16
09-12-73	2	16

Figure 3. Relation between discharges at the Squannacook River gaging station near West Groton, Mass., and the Mulpus Brook partial-record station near Shirley, Mass.

base-10 logarithms. The partial-record station discharge measurements were then regressed on concurrent daily mean discharges for the index-station, producing the following values:

Intercept (a): -1.69
Slope (b): 1.67
Correlation coefficient (r): 0.90
Standard error S_e^2: 0.0106502

The annual minimum 7-day mean discharges at the index station were computed from 39 years of streamflow record. The flow values were transformed to base-10 logarithms in order to compute and estimate the following statistics:

Mean of annual log discharges (M_x): 1.0025
Variance of annual log discharges (S^2x): 0.0353
Skew of the annual log discharges: 0.3711
The 10-year Log Pearson Type III frequency factor (K_T): -1.24

The following statistics were computed from the concurrent discharge data:

Variance of the index station log discharges ($S_{\bar{x}}^2$) : 0.016848
Number of concurrent discharge measurements (L): 11

Using these statistics, the components of equation 5 were computed from equations 6 and 7. The Log Pearson Type III frequency factor at the partial-record station (K_Y) was assumed to be equal to (K_T):

Mean component $(\hat{M}_Y)$: -0.0158
Standard deviation component $(\hat{S}_Y)$: 0.3269

Applying equation 4 gave the following results:

Base-10 logarithm of the estimator $(\hat{Y}_{7,10})$: -0.421
7Q10 discharge: 0.38 ft³/s

The base-10 log-transformed data from stations 01095915 and 01096000 also were used as an example of the MOVE.1 estimation technique:

Mean of daily mean discharges at the index station $(M_{\bar{x}})$: 1.241.
Standard deviation of daily mean discharges at the index station $(S_{\bar{x}})$: 0.129.
Mean of the discharge measurements at the low-flow partial-record station $(M_{\bar{y}})$: 0.382.
Standard deviation of discharge measurements at the low-flow partial-record station $(S_{\bar{y}})$: 0.238.
Base-10 logarithm of the 7Q10 discharge at the index station $(\hat{X}_{D,T})$: 0.769.
Base-10 logarithm of the 7Q10 discharge at the low-flow partial-record station $(\hat{Y}_{D,T})$: -0.488.
7Q10 discharge: 0.32 ft³/s.

In addition to the partial-record station on Mulpus Brook near Shirley, Mass., 7Q10 discharge estimates were made for five other partial-record stations in Massachusetts using the graphical, MOVE.1, and Stedinger and Thomas techniques. The locations of these and other partial-record stations are described and shown in table 4 and figure 2. Estimates of 7Q10 discharge made by these three techniques are presented in table 5. The 7Q10 estimates for the six stations were of the same order of magnitude using all three techniques. Table 5 also shows 7Q10 estimates made by the graphical technique for an additional five partial-record stations.

The accepted 7Q10 estimates for each of the 11 partial-record stations listed in table 5 are: (1) those made by the graphical technique for the five stations where no estimates were made using the mathematical techniques and (2) an average of the estimates made by the graphical, MOVE1, and moments techniques for the six other stations.

FREQUENCY ESTIMATION AT LOW-FLOW UNGAGED SITES

The previously described techniques can be used if continuous-record or partial-record discharge data are available and if it is possible to estimate low flows adequately at a site. However, time and cost constraints may preclude the collection of eight or more base-flow measurements, and the ungaged site may not have a streamflow-gaging station nearby with a record that can be significantly correlated with that of the ungaged site. In regions where the geology, topography, and meteorology are homogeneous, it may be possible to estimate low flows at a site by multiplying a known low

Table 4. Partial-record stations used in the study

Map No. (fig. 2)	Station No.	Station name	Number of measurements	Period of record
32	01100660	Bare Meadow Brook at Brookdale Ave. near Methuen, Mass.	7	1973-74
33	01105100	Traphole Brook near Norwood, Mass.	22	1959-68
34	01105830	First Herring Brook near Scituate Center, Mass.	16	1969-71
35	01095915	Mulpus Brook near Shirley, Mass.	12	1971-74
36	01096503	Nissitissit River at Pepperell, Mass.	12	1971-74
37	01097400	Spencer Brook near Concord, Mass.	19	1962-83
38	01124750	Browns Brook near Webster, Mass.	15	1978-83
39	01124800	Sucker Brook near Webster, Mass.	24	1960-83
40	01168300	Cold River near Zoar, Mass.	18	1965-83
41	01180800	Walker Brook near Becket Center, Mass.	14	1962-84
42	01331360	Kitchen Brook at Cheshire, Mass.	18	1965-83

Table 5. Comparison of 7Q10 flow estimates for partial-record stations in Massachusetts

[Graphical technique is described by Riggs (1972). MOVE1 technique is described by Hirsch (1982). Moments technique is described by Stedinger and Thomas (1985). Low-flow regression model is described in this report. ft^3/s, cubic feet per second; --, value not determined]

Partial-record station	Index station	7-day 10-year low flow (ft^3/s)		
		Graphical	MOVE1	Moments
Methuen 01100660	Haverhill 01100700	0.16	--	--
Norwood 01105100	Brockton 01107000	.90	0.85	0.83
Scituate 01105830	S. Weymouth 01105600	.01	--	--
Shirley 01095915	W. Groton 01096000	.38	.32	.38
Pepperell 01096503	W. Groton 01096000	1.3	1.6	2.2
Concord 01097400	Acton 01097300	.02	.02	.01
Webster 01124750	Harrisville 01111300	.01	.06	.05
Webster 01124800	Harrisville 01111300	.03	.03	.02
Zoar 01168300	N. Adams 01332000	1.5	--	--
Becket 01180800	Goss Heights 01180500	.12	--	--
Cheshire 01331360	Williamstown 01333000	.02	--	--

flow by the ratio of the drainage areas of the two sites. However, in New England, this technique may not be usable at many sites due to the heterogeneous nature of the region's glacial geomorphology.

Regional models, whether they be regression-based or physically based, can be a preferable alternative to the drainage-area ratio technique because their parameters are based on information from streamflow-gaging stations in the region and they take into account the physical characteristics of the ungaged basin. An overview of several of the low-flow regression models that have been developed in the northeastern United States was previously described. This section provides a brief overview of the development and application of a low-flow regression model available for use in Massachusetts.

Regression Techniques

Some definitions are provided for improving clarity (Iman and Conover, 1983). A simple linear regression model is

$$y = a + bx \ , \tag{8}$$

where

y is the dependent variable,
a is the intercept parameter,
b is the slope parameter, and
x is the independent variable.

The true values of the parameters, a and b, are unknown. However, assuming that a sample of data is representative of a population, the slope parameter is computed by the equation

$$b = \frac{\sum_{i=1}^{n} y_i x_i - \frac{\left(\sum_{i=1}^{n} y_i\right)\left(\sum_{i=1}^{n} x_i\right)}{n}}{\sum_{i=1}^{n} x_i^2 - \frac{\left(\sum_{i=1}^{n} x_i\right)^2}{n}} . \quad (9)$$

The intercept parameter is computed by the equation

$$a = \bar{y} - b\bar{x}, \quad (10)$$

where

$\bar{y}$ is the sample mean of the dependent variable, y; and

$\bar{x}$ is the sample mean of the independent variable, x.

A multiple linear regression model can be written as

$$y = a + b_1 x_1 + b_2 x_2 + \ldots + b_k x_k , \quad (11)$$

where

k is the number of variables.

Computer statistical packages commonly are used to compute (or estimate) the parameters of a multiple regression equation. Regression techniques are provided by Montgomery and Peck (1982).

Several assumptions are made when using least-squares regression analysis (Iman and Conover, 1983, p. 366-373) :

1. The correct form of the equation is known. If linear regression is used to estimate coefficients of an incorrect model, values predicted by the model will be biased.
2. The expected value or mean of the residuals (the differences between the measured and predicted values) is zero.
3. The residual variance is constant (the variance of the Y does not depend on X).
4. The residuals are normally distributed.
5. The residuals are mutually independent.

Depending on the objectives of a regression analysis, not all of the above assumptions are required. However, all are required to make statistical inferences such as confidence intervals or hypothesis tests.

Least-squares regression assumes that the relation between the regression coefficients and the dependent variables is linear. Logarithmic, exponential, or other transformations can be used to make the data more linear if necessary. Transformation also can decrease the presence of nonconstant variance among the residuals. If nonconstant residual variance cannot be eliminated, weighted least-squares techniques may apply (Montgomery and Peck, 1982). The data are classified into groups of similarity. Weights are computed for each group and are used to weight the dependent variables during the computation.

Generalized least squares (GLS) techniques for parameter estimation were developed in recent years for regional modeling situations where the available streamflow records are of different and widely variable lengths and the concurrent flows at different sites are cross-correlated. The techniques are particularly beneficial when it is necessary to include gaging-station records that are only a few years in length or data that were derived from partial-record stations. Tasker and others (1987) used GLS techniques in regional hydrologic studies for flood-frequency analysis. Vogel and Kroll (1990) were among the first to use GLS techniques in parameter estimation for low-flow regional regression modeling. With data from 23 Massachusetts streamflow-gaging stations, they found that GLS techniques did not provide significant improvements over ordinary least squares (OLS) techniques. The cross correlation between the stations also was not significant enough for GLS to offer any advantage over OLS.

Tasker (1975) presents a low-flow-frequency estimation technique that combines the techniques used in partial-record and regional regression low-flow-frequency estimation. Estimates based on the two techniques for an ungaged site are first made and are then combined into a single estimate. Tasker found that by using this approach the minimum number of base-flow measurements at the site could be reduced from eight to six.

Regression Model Development

The basic form of the low-flow regression model developed by Vogel and Kroll (1990) was updated in this study using data from eight additional continuous-record streamflow-gaging stations. The model uses the two-parameter log-normal distribution (eq. 3) to predict the minimum 7-day mean discharge for a given recurrence interval. Two regression equations were used to estimate the mean and the standard deviation of the distribution. Computed mean and standard deviation values for each of the 31 stations (table 2) were used as the dependent variables in developing the regression equations. Before inclusion in the regression, the values were detransformed using the following equations:

$$\mu_q(D) = \exp(\mu_y(D) + Y_{1/2}\sigma_y^2(D)) , \qquad (12)$$

$$\sigma_q^2(D) = \exp(2\mu_y(D) + \sigma_y^2(D))[\exp(\sigma_y^2(D)) - 1] . \qquad (13)$$

The independent variables of the regression equations included total drainage area in square miles and basin relief in feet for each basin (table 2). Basin relief was defined as the difference in elevation between the basin summit and the streambed at the outlet. Vogel and Kroll (1990) defined the basin summit as the average of the highest peak and the two adjacent peaks on either side of it. The former definition of basin summit was used in the USGS version of the equations because it was easier to determine and was highly correlated with the latter definition when using Massachusetts data.

The USGS version of the Vogel and Kroll regression equations is shown below. The USGS version is based on data from 31 continuous-record streamflow-gaging stations. Drainage area and basin relief were the independent variables of regression in the equation to estimate the mean of the annual minimum 7-day dscharge series as shown by the equation

$$\hat{\mu}_q(D) = 0.0106A^{1.14}H^{0.301} \qquad (14)$$

where

A is drainage area, in square miles, and
H is basin relief, in feet.

The coefficient of determination was 0.964 and the standard error of regression was 35 percent.

Drainage area was used as the independent variable of regression in the equation for estimating the standard deviation of the annual minimum 7-day discharge series as shown by the equation

$$\hat{\sigma}_q(D) = 0.0524A^{1.19} . \qquad (15)$$

The coefficient of determination was 0.960 and the standard error of regression was 34 percent.

The coefficient of determination is the proportion of variation in the dependent variable that is accounted for by the regression equation. A value of 1.00 indicates that the regression equation explains all the variability in the dependent variable. The standard error is a measure of the precision of the regression equation. About 68 percent of the means and standard deviations estimated from the regression equations are within ± 1 standard error of the measured values.

The predicted mean and standard deviation values (eqs. 16 and 17) were transformed. The transformation of the mean value is computed by the equation:

$$\hat{\mu}_y(D) = ln\left[\frac{\hat{\mu}_q(D)}{[1 + (\hat{\sigma}_q(D)/\hat{\mu}_q(D))^2]^{1/2}}\right] . \qquad (16)$$

The transformation for the standard deviation value is computed by the equation

$$\hat{\sigma}_y^2(D) = ln\left[1 + \left(\frac{\hat{\sigma}_q(D)}{\hat{\mu}_q(D)}\right)^2\right] . \qquad (17)$$

The D-day T-year discharge is computed using the two-parameter Log-Normal Distribution (eq. 3).

The standard error of regression from the two regression equations can be combined to compute the standard error of the flow estimate for a given recurrence interval by use of the equation

$$SE(Q_T) = \sqrt{MSE(\hat{\mu}_y) + (Z_T^2 MSE(\hat{\sigma}_y))} , \qquad (18)$$

where

$SE(Q_T)$ is the standard error of the flow estimate in log units,
$MSE(\hat{\mu}_y)$ is the mean squared error of the regression model of the mean, and
$MSE(\hat{\sigma}_y)$ is the mean squared error of the regression model of the standard deviation.

The standard errors of the 7Q2 and 7Q10 discharge estimates are 0.337 and 0.538, in log units, respectively. Converted to Percent Standard Error, these values are 35 and 58, respectively.

A FORTRAN[1] language computer program was written to aid the user of the USGS regression models (eqs. 16 through 17). The user only needs to enter the drainage area and relief. An example of the output of the program using data from the Mulpus Brook partial-record station near Shirley, Massachusetts, is presented in table 6. The code is presented in Appendix B.

[1]Any use of trade, product, or firm names in this publication is for descriptive purposes only and does not constitute endorsement by the U.S. Government.

Table 6. Sample output from the low-flow regression model program

```
LOWFLOW.FOR PROGRAM TO ESTIMATE 7-DAY LOW
FLOWS FOR UNREGULATED BASINS IN WESTERN AND
CENTRAL MASSACHUSETTS. REFERENCE: ESTIMATING
THE MAGNITUDE AND FREQUENCY OF LOW FLOWS OF
STREAMS IN MASSACHUSETTS BY JOHN C. RISLEY,
U.S. GEOLOGICAL SURVEY WATER-RESOURCES
INVESTIGATIONS REPORT 94-4100.

RESULTS:
WATERSHED BASIN: MULPUS BROOK (01095915)

DRAINAGE AREA (SQ. MILES):  15.60
BASIN RELIEF (FEET):  495

MEAN MODEL:
PREDICTED MEAN VALUE (NATURAL LOGSPACE):  0.443
HAT VALUE:  0.040
95% CONFIDENCE INTERVAL:  0.305   0.581
95% PREDICTION INTERVAL:  -0.259   1.145

STANDARD DEVIATION MODEL:
PREDICTED SD VALUE (NATURAL LOGSPACE):  0.319
HAT VALUE:  0.032
95% CONFIDENCE INTERVAL:  0.199   0.439
95% PREDICTION INTERVAL:  -0.360   0.999
****************************************************
PREDICTED 7-DAY 2-YEAR
LOW-FLOW STATISTIC (CUBIC FEET PER SECOND): 1.167
95% CONFIDENCE INTERVAL:  1.009   1.351
95% PREDICTION INTERVAL:   0.573   2.379

PREDICTED 7-DAY 10-YEAR
LOW-FLOW STATISTIC (CUBIC FEET PER SECOND): 0.441
95% CONFIDENCE INTERVAL:  0.376   0.517
95% PREDICTION INTERVAL:   0.213   0.914
****************************************************
```

In addition to computing the regression model predictions of the log-normal distribution parameters and 7Q2 and 7Q10 discharges, the program also computes the 95-percent confidence and prediction intervals. The program computes the "hat" value, which is a measure of how far the values of the independent variables for a site are from the means of the independent variables of the data set used to develop the regression model (Montgomery and Peck, 1982). Usually, if the hat value for a site is two times the mean of the hat values of the model data set, that site is considered an outlier. The program computes the hat value and provides an outlier test of the data for the independent variables entered by the user. If the entered data does not pass the test, an error message is provided and estimates for low-flow statistics are not computed.

A verification of the regression estimators for the mean and standard deviation (eqs. 14 and 15) was made using a prediction sum of squares (PRESS) statistical test (Draper and Smith, 1981, p. 325-327). The PRESS statistic is computed by summing the squared residuals from models defined by sequentially deleting each observation, redefining the regression equation without that observation, then calculating the residual for that observation. Comparable values can be computed for standard error, standard error in percent, and coefficient of determination (table 7). The R-squared value for the mean and standard deviation models decreased only by 0.9 and 0.6 percent, respectively; and the standard error of the two models increased only by 0.037 and 0.026 (values in natural logarithms), respectively.

Regression-Model Limitations

The low-flow regression model described in the previous section can be used on many unregulated ungaged basins in western, central, and northeastern Massachusetts. Before using the model at an ungaged site, the basin's surficial geology and topography need to be evaluated. Small basins containing mostly till material may have extended periods of zero flow in some years. The following guidelines have been suggested for use of the model:

Table 7. Selected results of prediction sum of squares (PRESS) test

Comparisons	Mean model	Standard-deviation model
Sum of squares ($\log_e$)	3.174	3.109
Prediction sum of squares ($\log_e$)	3.913	3.500
Standard error ($\log_e$)	.337	.327
Standard error of prediction ($\log_e$)	.374	.353
Standard error (percent)	35	34
Standard error of prediction (percent)	39	36
R-squared	.964	.960
R-squared of prediction	.955	.954
Cases of data (n)	31	31
Parameters (p)	3	2
Degrees of freedom	28	29

- The basin drainage area should be 5 to 150 mi^2.
- The slope (vertical relief divided by total stream length) should be less than 4 percent.
- The area of stratified drift should exceed 4 percent of the total drainage area.
- The basin is located for use in southeastern Massachusetts (fig. 2).

In the southeast (the coastal region, Cape Cod, Martha's Vineyard, and Nantucket), techniques other than the regression model should be used to estimate low flows. This region has a limited number of unregulated streamflow-gaging records available for model development. The geologic and topographic characteristics of this region also are different from those in the rest of the State. Because of the region's limited relief, drainage basins and aquifer boundaries do not necessarily correspond to each other. An approximate boundary dividing this region from the rest of the State follows the high ground dividing the Charles and Neponset Rivers from the Taunton, Weymouth, and Weir Rivers (fig. 2).

Alternative techniques for finding or estimating low flows in the southeast region of the State are included in the following references: Williams (1968); Brackley and others (1973); Williams and others (1973); Williams and Tasker (1974a); Willey and others (1978); Williams and Tasker (1978). Partial-record techniques may be appropriate at partial-record stations in the region using one or more of the following continuous-record streamflow-gaging stations as index sites: 01105600 on Old Swamp River near South Weymouth, Mass., 01107000 on Dorchester Brook near Brockton, Mass., 01106000 on Adamsville Brook at Adamsville, R.I., and 01109200 on West Branch Palmer River near Rehoboth, Mass. Streams here generally are heavily regulated because of urban and agricultural water use. Gaged and ungaged basins need to be closely scrutinized for evidence of regulation. Tasker's (1972) low-flow regression model may be valid at some sites in the southeast.

CRITERIA FOR CHOOSING LOW-FLOW FREQUENCY ANALYSIS TECHNIQUES

This report presents an overview of techniques used for estimating 7Q2 and 7Q10 discharges on unregulated streams in Massachusetts. The selection of a technique should be based on the availability and limitations of existing data and the time and costs required for the collection of new data. The flow data used in any of the techniques described below must be measured at unregulated basins.

The techniques presented in this report apply to characteristic basins in which zero values of the minimum 7-day mean discharge do not appear in an existing record and would not be expected to occur.

Listed below are suggested guidelines for making estimates of 7Q2 and 7Q10 discharges at a site in Massachusetts and for setting priorities when one or more estimating techniques might be used:

1. Use a combination of graphical and mathematical techniques for frequency analysis described in the "Frequency Analysis at Low-Flow Continuous-Record Stations" section of the report, if continuous-record low-flow data are available at the site.
2. Use a combination of graphical and mathematical techniques described in the "Frequency Analysis at Low-Flow Partial-Record Stations" section of the report, if partial-record flow measurements and corresponding daily mean flows from a nearby index station are available.
3. Use the regional model presented in this report (eqs. 16 and 17) if the stream site is located in western, central, or northeastern Massachusetts. The model relates estimated low flows to the basin surficial area and basin relief. The presentation in Appendix B demonstrates how the FORTRAN language computer program can facilitate the computation work.
4. Use a low-flow profile or seepage-run measurements along a main channel to estimate low flows at intermediate stream sites (Riggs, 1972).
5. Use discharge measurements and records in a region of similar hydrologic characteristics. Estimates may be made by interpolation or by the drainage-area ratio. An alternative technique would be to state the range of a specific low flow, in cubic feet per second per square mile, for nearby sites (Riggs, 1972, 1973).
6. Do not make an estimate if the required data for the options above are not available. As an alternative, base-flow measurements could be made at the requested site to define low-flow characteristics.

SUMMARY

Estimates of 7-day 2-year (7Q2) and 7-day 10-year (7Q10) discharges for streams in Massachusetts are used by Federal, State, and local officials involved with water-resources management. The estimates are used to determine waste-load allocations for sewage-treatment plants, maximum withdrawals for water supply during drought periods, and minimum downstream release requirements for hydropower, irrigation, and cooling-plant facilities. This report presents techniques for estimating 7Q2 and 7Q10 discharges at continuous-record streamflow-gaging stations, partial-record stations, and ungaged stream sites in Massachusetts.

Graphical and mathematical techniques can be used to determine low flows at continuous-record streamflow-gaging stations. The graphical technique is considered the more accurate of the two. The mathematical techniques include both the Log Pearson Type III and the two-parameter log-normal probability distributions. 7Q2 and 7Q10 discharges were determined at 31 continuous-record streamflow-gaging stations in Massachusetts, Rhode Island, and Connecticut. For each station, a two-parameter log-normal probability distribution was computed from the annual minimum 7-day mean discharges of the record. The annual 7-day series were tested for independent random events and trends. The distribution curve and the annual minimum 7-day mean discharges were plotted onto normal probability paper. The 7Q2 and 7Q10 discharges were determined from the distribution. However, adjustments of these values using the graphical information was not required.

The graphical technique for estimating low flows at partial-record stations was used at 11 stream sites in Massachusetts. Two mathematical techniques also were used at 6 of these 11 sites. Over a period of several years, 7 to 240 discharge measurements were made at these sites during low-flow periods. The estimated low flows were based on the cross correlation of discharge measurements from each of the partial-record stations and corresponding daily mean discharges of nearby index stations.

In recent low-flow-frequency studies in the northeastern United States, several regionalized regression models have been developed to that estimate 7Q2 and 7Q10 discharges on the basis of basin characteristics and geology as independent variables. An updated version of a low-flow-regression model was developed for this study with data from 31 unregulated continuous-record streamflow-gaging stations located throughout Massachusetts and Rhode Island. The streamflow records ranged from 11 to 78 years in length. Record augmentation techniques were used on five of the records that were shorter than 20 years. The low-flow regression model contains parameters of mean and standard deviation, and can be used to estimate 7-day mean discharges for various recurrence intervals; both parameters are estimated from separate regression equations. The regression equation for the mean had a coefficient of determination of 0.964 and a standard error of regression of 35 percent. The regression equation for the standard deviation had a coefficient of determination of 0.960 and a standard error of regression of 34 percent. Low-flow estimates computed using the regression equations are compared with estimates made using graphical and mathematical techniques on measured discharge data from six partial-record stations. The estimates for four of the six stations were of the same order of magnitude. It would have been desirable if more partial-record flow data were available in Massachusetts for model verification.

The low-flow-frequency model is valid for use on unregulated basins in western, central, and northeastern Massachusetts. The basin drainage area needs to be 5 to 150 mi^2. Basin slope needs to be less than 4 percent. Area of stratified drift needs to exceed 4 percent. A FORTRAN language computer program was written and included in the report to help users estimate both 7Q2 and 7Q10 discharges and the 95-percent intervals of confidence and prediction.

The following guidelines are listed in order of importance when one or more techniques are suitable for estimating 7Q2 and 7Q10 discharges in Massachusetts:

1. Use a combination of graphical and mathematical techniques for frequency analysis, if continuous-record low-flow data are available.
2. Use a combination of graphical and mathematical techniques, if partial-record low-flow data are available.
3. Use the regional model presented in this report, if the stream site is in western, central, or northeastern Massachusetts.
4. Use a low-flow profile or seepage-run measurements along a main channel to estimate low flows at intermediate stream sites.
5. Use flow measurements and records in a region of similar hydrologic characteristics. Estimates may be made by interpolation or by the drainage-area ratio.
6. Do not make an estimate if the required data for the options above are not available.

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APPENDIX A

EQUATIONS USED FOR STREAMFLOW-RECORD AUGMENTATION

Use of the equations requires the transformation to logarithmic space of the annual values from both the long and short records. Matalas and Jacobs (1964) compute an estimator of the mean using the following equation

$$\hat{\mu}_y = \bar{y}_1 + \delta\ \hat{\beta}\,(\bar{x}_2 - \bar{x}_1)/n_1\ , \tag{19}$$

where

$\hat{\mu}_y$ is the estimator of the mean,
$\bar{y}_1$ is the mean of the shorter record,
δ is a coefficient defined below,
$\hat{\beta}$ is a coefficient defined below,
$\bar{x}_2$ is the mean of the non-concurrent discharges in the longer record,
$\bar{x}_1$ is the mean of the concurrent discharges in the longer record, and
n_1 is the number of concurrent discharges.

The variance estimator is computed by the equation

$$\hat{\sigma}_{y2} = \lambda\left[(n_1 - 1)\,s^2_{y_1} + (n_2 - 1)\,\hat{\beta}^2 s^2_{x_2} + (n_2 - 1)\,\alpha^2\,(1 - p^2)\,s^2_{y_1} + \delta\beta^2\,(\bar{x}_2 - \bar{x}_1)^2\right]\ , \tag{20}$$

where

$\hat{\sigma}_{y2}$ is the variance estimator,
λ is a coefficient defined below,
$s^2_{y_1}$ is the variance of the shorter record,
n_2 is the number of nonconcurrent observations,
$s^2_{x_2}$ is the variance of the nonconcurrent discharges at the longer record,
α^2 is a coefficient defined below,
p^2 is the correlation coefficient of the concurrent discharges.
δ is computed by the equation

$$\delta = (n_1 n_2)/(n_1 + n_2)\ , \tag{21}$$

$\hat{\beta}$ is computed by the equation

$$\hat{\beta} = \frac{\sum_{i=1}^{n_i} (x_i - x_1)\,(y_i - y_1)}{\sum_{i=1}^{n_i} (x_i - x_1)^2}\ , \tag{22}$$

λ is computed by the equation

$$\lambda = 1/(n_1 + n_2 - 1) \text{ , and} \tag{23}$$

α^2 is computed by the equation

$$\alpha^2 = [(n_2(n_1-4)(n_1-1))]/[(n_2-1)(n_1-3)(n_1-2)] . \tag{24}$$

The correlation coefficient is computed by the equation

$$\hat{\rho} = \hat{\beta}\frac{s_{x_1}}{s_{y_1}} , \tag{25}$$

where

s_{x_1} is the standard deviation of the concurrent discharges at the longer record.

Vogel and Stedinger (1985) found that the estimators above could be improved if it was assumed that both series had a bivariate normal population. These modified estimators were used in this study. The estimator of the mean is computed by the equation

$$\hat{\mu}_y{}^{+} = (1-\theta_1)\bar{Y}_1 + \theta_1\hat{\mu}_y , \tag{26}$$

where

θ_1 is a weight coefficient defined below.

The estimator of the variance is computed by the equation

$$\hat{\sigma}_y{}^{+2} = (1-\theta_2)s_{y_1}^2 + \theta_2\hat{\sigma}_y^2 , \tag{27}$$

where

θ_2 is a weight coefficient defined below.

θ_1 is computed by the equation

$$\theta_1 = \frac{(n_1-3)\hat{\rho}^2}{(n_1-4)\hat{\rho}^2+1} \text{ , and} \tag{28}$$

θ_2 is computed by the equation

$$\theta_2 = \frac{(n_1-4)\hat{\rho}^2}{(n_1-8.5)\hat{\rho}^2+4.5} . \tag{29}$$

APPENDIX B

FORTRAN LANGUAGE COMPUTER PROGRAM LISTING FOR THE LOW-FLOW REGRESSION MODEL

U.S. GeoLOGical Survey preliminary
computer program

```
Cc************* U.S. Geological Survey preliminary computer program ************
C*****************************************************************************
C*************          LOWFLOW.FOR version 1.0          *************
C*****************************************************************************
C**                  Language: Fortran 77                    **
C**  Program must be recompiled then link (no system libraries are needed)  **
C**          Prime                                  **
C**          Sun3, Sun4                                **
C**          DG computers                               **
C**  The source code is available from below:                     **
C*::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::
C*===========================================================================
C*   Author/Site,     Date,      Event
C*  ---------------------------------------------------------------------
C*  John C. Risley    5/17/94   USGS-WRD Portland OR  Original Coding
C*
C*===========================================================================
C*::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::
C*
C* Disclaimer:
C        Although this program has been used by the U.S. GeoLOGical Survey,
C        no warranty, EXPressed or implied, is made by the USGS as to the
C        accuracy and functioning of the program and related program
C        material nor shall the fact of distribution constitute any such
C        warranty, and no responsibility is assumed by the USGS in
C        connection therewith.
C*
C*::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::
c
c   LOWFLOW.FOR - PROGRAM TO ESTIMATE 7-DAY LOW FLOWS
c   FOR WESTERN AND CENTRAL MASSACHUSETTS.  REFERENCE:
c   "ESTIMATING THE MAGNITUDE AND FREQUENCY OF LOW FLOWS OF
c   STREAMS IN MASSACHUSETTS" BY JOHN C.RISLEY, U.S. GEOLOGICAL
c   SURVEY WATER-RESOURCES INVESTIGATIONS REPORT 94-4100.
c
c   Declarations
c
      real*8
     &     LXX,TTT,VVV,ZZZ,XXX,YYY,MSE,N,LCI(5),LPI(5),
     &     LYY,QQQ(5),MLCI,MEAN,MUCI,MLPI,MUPI,LH,LDA,
     &     UCI(5),UPI(5),ZT(5)
c
      integer COUNT
C
      character*30 outfile,basinname
c
```

```
c   Output file
c
    outfile='lowflow.out'
    open(27,file=outfile,access='sequential',form='formatted')
    close(27,status='delete')
    open(27,file=outfile,access='sequential',form='formatted')
c
    write(*,*) 'LOWFLOW.FOR - PROGRAM TO ESTIMATE'
    write(*,*) '7-DAY LOW FLOWS FOR UNREGULATED BASINS'
    write(*,*) 'IN WESTERN AND CENTRAL MASSACHUSETTS.  REFERENCE:'
    write(*,*) 'ESTIMATING THE MAGNITUDE AND FREQUENCY OF LOW FLOWS'
    write(*,*) 'OF STREAMS IN MASSACHUSETTS BY JOHN C. RISLEY,'
    write(*,*) 'U.S. GEOLOGICAL SURVEY WATER-RESOURCES INVESTIGATIONS'
    write(*,*) 'REPORT 94-4100'
    write(*,*) ' '
    write(*,*) 'ENTER THE NAME OF BASIN SITE:'
    read (*,'(a)') basinname
    write(*,*) 'ENTER THE TOTAL DRAINAGE AREA IN SQUARE MILES:'
    read (*,*) DA
    write(*,*) 'ENTER RELIEF (DIFFERENCE IN FEET BETTWEEN
   *THE HIGHEST AND'
    write(*,*) 'LOWEST ELEVATIONS OF THE BASIN):'
    read (*,*) H
c
c   COMPUTING THE MEAN ESTIMATE
    LDA = LOG(DA)
    LH = LOG(H)
    LXX = -4.55 + (1.14 * LDA) + (.3 * LH)
    MEAN = EXP(LXX)
c
c   COMPUTING THE STANDARD DEVIATION ESTIMATE
    LYY = -2.95 + (1.19 * LDA)
    SD = EXP(LYY)
c
c   CONVERSION OF THE MEAN ESTIMATE TO LOG SPACE
    XXX = SD / MEAN
    XXX = XXX** 2
    XXX = (1 + XXX)
    XXX = XXX**0.5
    XXX = MEAN / XXX
    XXX = LOG(XXX)
c
c   CONVERSION OF THE STANDARD DEVIATION ESTIMATE TO LOG SPACE
    YYY = SD / MEAN
    YYY = YYY**2
    YYY = LOG(1 + YYY)
    YYY = YYY**0.5
c
c   COMPUTATION OF THE 2-YEAR ZT VALUE
    VVV = (1.0 / 2.0)**0.14
    ZZZ = (1.0 - (1.0 / 2.0))**0.14
    ZT(1) = 4.91*(VVV-ZZZ)
c
```

```
c   COMPUTATION OF THE 10-YEAR ZT VALUE
    VVV = (1.0 / 10.0)**0.14
    ZZZ = (1.0 - (1.0 / 10.0))**0.14
    ZT(2) = 4.91*(VVV-ZZZ)
cc
c   COMPUTING THE 95% CONFIDENCE AND PREDICTION INTERVALS FOR
c   *THE MEAN MODEL
    Q1 = 2.49376 + (LDA * .12169) + (LH * -.42897)
    Q2 = .12169 + (LDA * .03238) + (LH * -.03228)
    Q3 = -.42897 + (LDA * -.03228) + (LH * .07941)
    TT = Q1 + (Q2 * LDA) + (Q3 * LH)
    QQ = .113 * TT
    PP = TT
    IF (PP.GT.0.1935) THEN
      goto 40
    END IF
    QQ = QQ**0.5
    QQ = 2.048 * QQ
    MLCI = LXX - QQ
    MUCI = LXX + QQ
    TT = 1.0 + TT
    TT = TT * .113
    TT = TT**0.5
    QQ = 2.048 * TT
    MLPI = LXX - QQ
    MUPI = LXX + QQ
c
c   COMPUTING THE 95% CONFIDENCE AND PREDICTION INTERVALS FOR
c   *THE STANDARD
c   DEVIATION MODEL
    T = 2.045
    N = 31.0
    MSE = .107
    XM = 2.736
    SXX = 51.93437
    ZZZ = LDA - XM
    ZZZ = ZZZ**2.0
    ZZZ = ZZZ / SXX
    UUU = 1 / N
    ZZZ = UUU + ZZZ
    SS = ZZZ
    IF (SS.GT.0.129) THEN
      goto 40
    END IF
    PZZ = 1 + ZZZ
    ZZZ = MSE * ZZZ
    PZZ = MSE * PZZ
    ZZZ = ZZZ**0.5
    PZZ = PZZ**0.5
    ZZZ = T * ZZZ
    PZZ = T * PZZ
    SDLCI = LYY - ZZZ
```

```
      SDUCI = LYY + ZZZ
      SDLPI = LYY - PZZ
      SDUPI = LYY + PZZ
c
      COUNT = 1
      GOTO 101
 100  COUNT = 2
c
c     COMPUTING THE LOW-FLOW STATISTIC
 101  QQQ(COUNT) = XXX + (ZT(COUNT) * YYY)
      QQQ(COUNT) = EXP(QQQ(COUNT))
c
c     COMPUTING THE 95% CONFIDENCE INTERVALS FOR THE
c    *STATISTIC
      MEAN = EXP(MLCI)
      SD = EXP(SDLCI)
      HHH = SD / MEAN
      HHH = HHH**2.0
      HHH = (1 + HHH)
      HHH = HHH**0.5
      HHH = MEAN / HHH
      HHH = LOG(HHH)
      FFF = SD / MEAN
      FFF = FFF**2.0
      FFF = LOG(1 + FFF)
      FFF = FFF**.5
      CCC = EXP(HHH + (FFF * ZT(COUNT)))
      LCI(COUNT) = CCC
c
      MEAN = EXP(MUCI)
      SD = EXP(SDUCI)
      HHH = SD / MEAN
      HHH = HHH**2.0
      HHH = (1 + HHH)
      HHH = HHH**0.5
      HHH = MEAN / HHH
      HHH = LOG(HHH)
      FFF = SD / MEAN
      FFF = FFF**2.0
      FFF = LOG(1 + FFF)
      FFF = FFF**0.5
      CCC = EXP(HHH + (FFF * ZT(COUNT)))
      UCI(COUNT) = CCC
c
c     COMPUTING THE 95% PREDICTION INTERVALS FOR THE
c    *STATISTIC
      MEAN = EXP(MLPI)
      SD = EXP(SDLPI)
      HHH = SD / MEAN
      HHH = HHH**2.0
      HHH = (1 + HHH)
      HHH = HHH**0.5
      HHH = MEAN / HHH
```

```
      HHH = LOG(HHH)
      FFF = SD / MEAN
      FFF = FFF**2.0
      FFF = LOG(1 + FFF)
      FFF = FFF**0.5
      CCC = EXP(HHH + (FFF * ZT(COUNT)))
      LPI(COUNT) = CCC
c
      MEAN = EXP(MUPI)
      SD = EXP(SDUPI)
      HHH = SD / MEAN
      HHH = HHH**2.0
      HHH = (1 + HHH)
      HHH = HHH**0.5
      HHH = MEAN / HHH
      HHH = LOG(HHH)
      FFF = SD / MEAN
      FFF = FFF**2.0
      FFF = LOG(1 + FFF)
      FFF = FFF**0.5
      CCC = EXP(HHH + (FFF * ZT(COUNT)))
      UPI(COUNT) = CCC
      IF (COUNT.EQ.1) THEN
       GOTO 100
      ELSE
      ENDIF
C     III = TTT
c
c     PRINTING RESULTS TO THE SCREEN
c
      write(*,*) ' '
      write(*,*) 'LOWFLOW.FOR - PROGRAM TO ESTIMATE'
      write(*,*) '7-DAY LOW FLOWS FOR UNREGULATED BASINS'
      write(*,*) 'IN WESTERN AND CENTRAL MASSACHUSETTS.  REFERENCE:'
      write(*,*) 'ESTIMATING THE MAGNITUDE AND FREQUENCY OF LOW FLOWS'
      write(*,*) 'OF STREAMS IN MASSACHUSETTS BY JOHN C. RISLEY,'
      write(*,*) 'U.S. GEOLOGICAL SURVEY WATER-RESOURCES INVESTIGATIONS'
      write(*,*) 'REPORT 94-4100'
      write(*,*) ' '
      write (*,*) 'RESULTS:'
      write (*,'(" WATERSHED BASIN: ",A)'),basinname
      write (*,*) ' '
      write (*,'(" DRAINAGE AREA (SQ. MILES): ",f8.2)'),DA
      write (*,'(" BASIN RELIEF (FEET): ",f6.0)'),H
      write (*,*) ''
      write (*,*) 'MEAN MODEL:'
      write (*,'(" PREDICTED MEAN VALUE (NATURAL ",
     *     "LOG SPACE): ",f8.3)'),LXX
      write (*,'(" HAT VALUE: ",f8.3)'),PP
      write (*,'(" 95% CONFIDENCE INTERVAL: ",f8.3,4x,f8.3)'),
     *     MLCI, MUCI
      write (*,'(" 95% PREDICTION INTERVAL: ",f8.3,4x,f8.3)'),
     *     MLPI, MUPI
```

```
      write (*,*) ' '
      write (*,*) 'STANDARD DEVIATION MODEL:'
      write (*,'(" PREDICTED SD VALUE (NATURAL LOG",
     *      "SPACE): ",f8.3)'),LYY
      write (*,'(" HAT VALUE: ",f8.3)'),SS
      write (*,'(" 95% CONFIDENCE INTERVAL: ",f8.3,4x,f8.3)'),
     *       SDLCI, SDUCI
      write (*,'(" 95% PREDICTION INTERVAL: ",f8.3,4x,f8.3)'),
     *       SDLPI, SDUPI
      write (*,*) ' '
      write (*,*) '**************************************************'
      write (*,*) 'PREDICTED 7-DAY 2-YEAR'
      write (*,'(" LOW-FLOW STATISTIC (CUBIC FEET PER ",
     *      "SECOND): ",f8.3)'),QQQ(1)
      write (*,'(" 95% CONFIDENCE INTERVAL: ",f8.3,4x,f8.3)'),
     *      LCI(1), UCI(1)
      write (*,'(" 95% PREDICTION INTERVAL: ",f8.3,4x,f8.3)'),
     *      LPI(1), UPI(1)
      write (*,*) ' '
      write (*,*)  'PREDICTED 7-DAY 10-YEAR'
      write (*,'(" LOW-FLOW STATISTIC (CUBIC FEET PER ",
     *      "SECOND): ",f8.3)'),QQQ(2)
      write (*,'(" 95% CONFIDENCE INTERVAL: ",f8.3,4x,f8.3)'),
     *      LCI(2), UCI(2)
      write (*,'(" 95% PREDICTION INTERVAL: ",f8.3,4x,f8.3)'),
     *      LPI(2), UPI(2)
      write (*,*) '**************************************************'
c
c
c   PRINTING THE RESULTS TO THE OUTPUT FILE
c
      write(27,*) 'LOWFLOW.FOR - PROGRAM TO ESTIMATE'
      write(27,*) '7-DAY LOW FLOWS FOR UNREGULATED BASINS'
      write(27,*) 'IN WESTERN AND CENTRAL MASSACHUSETTS.  REFERENCE:'
      write(27,*) 'ESTIMATING THE MAGNITUDE AND FREQUENCY OF LOW FLOWS'
      write(27,*) 'OF STREAMS IN MASSACHUSETTS BY JOHN C. RISLEY,'
      write(27,*) 'U.S. GEOLOGICAL SURVEY WATER-RESOURCES'
      write(27,*) 'INVESTIGATIONS REPORT 94-4100'
      write(27,*) ' '
      write (27,*) 'RESULTS:'
      write (27,'(" WATERSHED BASIN: ",A)'),basinname
      write (27,*) ' '
      write (27,'(" DRAINAGE AREA (SQ. MILES): ",f8.2)'),DA
      write (27,'(" BASIN RELIEF (FEET): ",f6.0)'),H
      write (27,*) ' '
      write (27,*) 'MEAN MODEL:'
      write (27,'(" PREDICTED MEAN VALUE (NATURAL ",
     *      "LOG SPACE): ",f8.3)'),LXX
      write (27,'(" HAT VALUE: ",f8.3)'),PP
      write (27,'(" 95% CONFIDENCE INTERVAL: ",f8.3,4x,f8.3)'),
     *       MLCI, MUCI
      write (27,'(" 95% PREDICTION INTERVAL: ",f8.3,4x,f8.3)'),
```

```
     *       MLPI, MUPI
      write (27,*) ' '
      write (27,*) 'STANDARD DEVIATION MODEL:'
      write (27,'(" PREDICTED SD VALUE (NATURAL LOG",
     *       "SPACE): ",f8.3)'),LYY
      write (27,'(" HAT VALUE: ",f8.3)'),SS
      write (27,'(" 95% CONFIDENCE INTERVAL: ",f8.3,4x,f8.3)'),
     *       SDLCI, SDUCI
      write (27,'(" 95% PREDICTION INTERVAL: ",f8.3,4x,f8.3)'),
     *       SDLPI, SDUPI
      write (27,*) ' '
      write (27,*) '*****************************************************'
      write (27,*) 'PREDICTED 7-DAY 2-YEAR'
      write (27,'(" LOW-FLOW STATISTIC (CUBIC FEET PER ",
     *       "SECOND): ",f8.3)'),QQQ(1)
      write (27,'(" 95% CONFIDENCE INTERVAL: ",f8.3,4x,f8.3)'),
     *       LCI(1), UCI(1)
      write (27,'(" 95% PREDICTION INTERVAL: ",f8.3,4x,f8.3)'),
     *       LPI(1), UPI(1)
      write (27,*) ' '
      write (27,*) 'PREDICTED 7-DAY 10-YEAR'
      write (27,'(" LOW-FLOW STATISTIC (CUBIC FEET PER ",
     *       "SECOND): ",f8.3)'),QQQ(2)
      write (27,'(" 95% CONFIDENCE INTERVAL: ",f8.3,4x,f8.3)'),
     *       LCI(2), UCI(2)
      write (27,'(" 95% PREDICTION INTERVAL: ",f8.3,4x,f8.3)'),
     *       LPI(2), UPI(2)
      write (27,*) '*****************************************************'
c
      GOTO 50
40    write(*,*) 'ERROR: INPUT DATA ARE NOT FEASIBLE FOR THE MODEL.'
      write(*,*) 'COMPUTED HAT VALUE OF THE INPUT DATA EXCEEDS 2 TIMES'
      write(*,*) 'THE MEAN OF THE HAT VALUES OF THE ORIGINAL DATA SET.'
c
      write(27,*) 'ERROR: INPUT DATA ARE NOT FEASIBLE FOR THE MODEL.'
      write(27,*) 'COMPUTED HAT VALUE OF THE INPUT DATA EXCEEDS 2 TIMES'
      write(27,*) 'THE MEAN OF THE HAT VALUES OF THE ORIGINAL DATA SET.'
c
50    close(27)
      stop
      end
```