Glacier Ice-Volume Modeling and Glacier Volumes On Redoubt Volcano, Alaska

By Dennis C. Trabant *and* Daniel B. Hawkins

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CONVERSION FACTORS, VERTICAL DATUM, AND ABBREVIATIONS

Multiply	By	To Obtain
meter (m)	3.281	foot
kilometer (km)	0.6214	mile
square meter (m^2)	10.76	square foot
square kilometer (km^2)	0.3861	square mile
cubic meter (m^3)	35.31	cubic foot
cubic kilometer (km^3)	0.2399	cubic mile
kilopascal (kPa)	0.1450	pound per square inch
kilogram per cubic meter (kg/m^3)	0.06242	pound per cubic foot

VERTICAL DATUM:

In this report "sea level" refers to the National Geodetic Vertical Datum of 1929 (NGVD of 1929), a geodetic datum derived from a general adjustment of the first-order level nets of both the United States and Canada, formerly called Sea Level Datum of 1929.

ABBREVIATIONS USED IN THIS REPORT:

m/μs, meters per microsecond
MHz, Megahertz
m/s^2, meters per second per second

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ABSTRACT

Assessment of ice volumes and hydrologic hazards on Redoubt Volcano began four months before the 1989-90 eruptions removed 0.29 cubic kilometer of perennial snow and ice from Drift glacier. A volume model was developed for evaluating glacier volumes on Redoubt Volcano. The volume model is based on third-order polynomial simulations of valley cross sections. The third-order polynomial is an interpolation from the valley walls exposed above glacier surfaces and takes advantage of ice-thickness measurements. The fortuitous 1989-90 eruptions removed the ice from a 4.5-kilometer length of Drift glacier, providing a unique opportunity for verification of the volume model. A 2.5-kilometer length was chosen in the denuded glacier valley and the ice volume was measured by digitally comparing two new maps: one derived from the most recent pre-eruption 1979 aerial photographs and the other from post-eruption 1990 aerial photographs. The measured volume in the reference reach was 99 x 10^6 cubic meters, about 1 percent less than was estimated by the volume model. The volume estimate produced by this volume model was much closer to the measured volume than was the volume estimated by other techniques. The verified volume model was used to evaluate the total volume of perennial snow and glacier ice on Redoubt Volcano, which was estimated to be 4.1±0.8 cubic kilometers. Substantial snow and ice covers on volcanoes exacerbate the hydrologic hazards associated with eruptions. The glacier volume on Redoubt Volcano is about 23 times the volume that was present on Mount St. Helens before its 1980 eruption, which generated lahars and floods.

INTRODUCTION

Background

In the wake of the 1980 eruption of Mount St. Helens, Washington, the U.S. Geological Survey accelerated assessments of volcano-related hazards. Assessment priorities were guided by the historical pattern of eruptions and the significance of societal hazards, emphasizing evaluation of the volume of perennial snow and ice on potentially hazardous volcanoes. It is well documented that the most voluminous and catastrophic lahars and floods result from eruptions of glacier-clad volcanoes (Major and Newhall, 1989). At Redoubt Volcano, the eruptive history suggested that an eruption was likely during the next 50 years. Furthermore, an eruption of Redoubt Volcano could affect half of the State's population and produce flooding and lahars threatening down-valley recreation sites, an oil pipeline, and an oil tanker loading facility; it could also produce airborne ash that would be a hazard to local and international air traffic (Till and others, 1993). Assessment of volcano-related hazards at Redoubt Volcano began in August 1989; the 1989-90 eruption of Redoubt Volcano (Miller and Chouet, 1994) began four months later.

Purpose and Scope

The goal of this study was to assess the distribution of both the volume and area of perennial snow and ice on Redoubt Volcano by altitude and general aspect. This report contains (1) a description of the model that was developed to evaluate the perennial snow- and ice-volume distribution; (2) the measured glacier-thickness data; and (3) a tabulation of the glacier-volume and glacier-area distribution on Redoubt Volcano subdivided into the parts that lie in the two principal drainage basins for the area. Glacier snow and ice volumes are important for assessing flood and lahar hazards (Major and Newhall, 1989), glacio-volcano–ground-water interactions related to magma evolution (Mastin, 1995), possible relations to volcanic explosivity and mass failures during eruptions (Newhall and Self, 1982, Till and others, 1993), geohydrologic hazards (Hoblitt and others, 1995; Scott and others, 1995; Wolfe and Pierson, 1995; Scott, 1988), and the potential for outburst floods (Björnsson, 1975). A glacier-volume model that can be universally applied to valley glaciers is needed in the general study of glaciers because the volumes of the glaciers in most mountain systems, as well as the distribution of glacier volume with altitude, are largely unknown. The relation between glacier volume and altitude is needed to predict the long-term consequences of global change, which is expected to drive substantial glacier-volume and sea-level changes (Meier, 1984).

Location and Description

Redoubt Volcano is near the northeastern end of the Aleutian volcanic arc, on the west side of Cook Inlet, about 180 km southwest of Anchorage, Alaska (fig. 1). Before the 1989-90 eruption, the summit caldera was glacier filled. The highest part of the caldera rim reached 3,108 m altitude along the southeastern edge (see cover photo). “Drift glacier” (unofficial name), the largest glacier on Redoubt Volcano, drained the 1.8-km-wide caldera through a breach in the northern part of the rim and descended about 2,550 m to the head of its piedmont lobe at 650 m altitude (fig. 2). The piedmont lobe is about 3.5 km long, 3.7 km wide, and its terminus calves into the Drift River at 300 m altitude. The 1989-90 vent was under Drift glacier at the breach in the caldera rim at about 2,400 m altitude (see cover photo). The 1989-90 eruption beheaded Drift glacier, removing most of the glacier ice between 750 m and 2,500 m altitude (fig. 3). The destruction of Drift glacier by the 1989-90 eruption was analyzed and described by Trabant and others (1994).

ASSESSMENT OF PREVIOUS WORK

Glacier volumes have been estimated by a variety of empirical and physically based methods. The most widely used of these methods were investigated for evaluation of the glacier volumes on Redoubt Volcano. The method recommended by the United Nations Educational, Scientific, and Cultural Organization—International Association of Scientific Hydrology (UNESCO/IASH) (1970) is to estimate the average thickness of each glacier using an empirical relation between the average glacier thickness and glacier surface area for defined glacier "types" within climatic regions. Average thickness estimates were systematized by making area-volume tables. A unique area-volume table was developed for each of three pilot studies used to test the feasibility of the recommended method (UNESCO/IASH, 1970). The discussion of error in each of the pilot studies contains no quantitative conclusions largely because no known volumes were available for comparison. Glacier volumes were not reported in subsequent UNESCO data releases. Additional applications and refinements of the UNESCO/IASH method have been published by Ommanney (1969), Post and others (1971), Østrem and others (1973), Müller and others (1976), and Kotlyakov (1980).

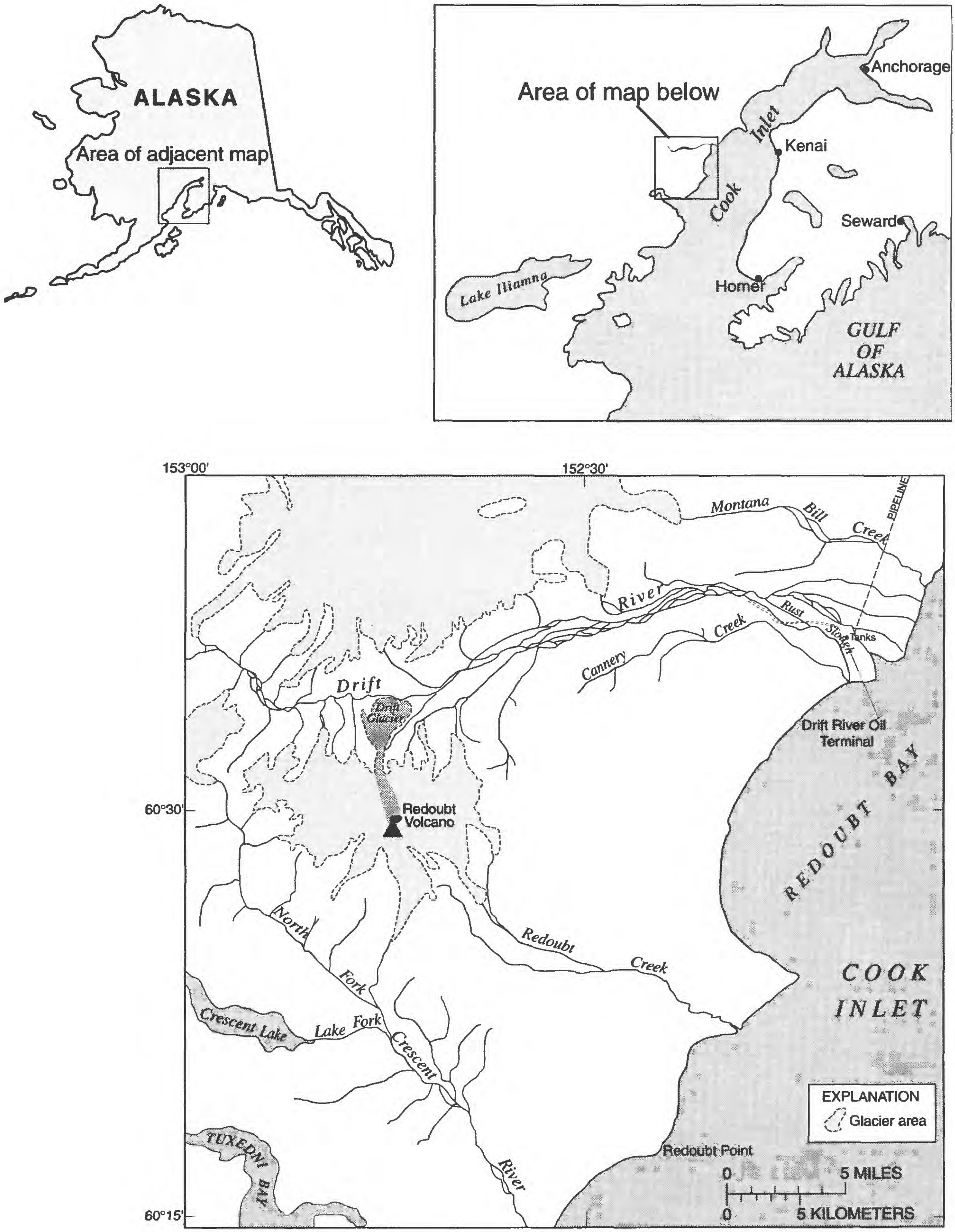


Figure 1. Location of Redoubt Volcano and Drift glacier, and their regional setting.

Figure 2. The upper part of Redoubt Volcano showing Drift glacier before the 1989-90 eruptions. This southward view shows rocks protruding through Drift glacier near the dome which was emplaced near the end of the 1960's, and a dirty avalanche running down the surface of the Drift glacier in the canyon below the 1960's and 1989-90 vents. Photograph date, July 28, 1988.

Other empirical relations have been developed on the basis of glacier surface area. The following equation was developed by Yerasov (1968):

$$V = 0.027S^{1.5} \tag{1}$$

where V is volume in cubic kilometers and S is surface area in square kilometers. This equation is important for its pervasive use in evaluating and comparing glacier volumes in the former Soviet Union, and for its role in the evaluation of global glacier volumes for the "World Atlas of Snow and Ice Resources," soon to be published by the Institute of Geography of the Russian Academy of Sciences, Moscow. Yerasov's formula was combined with the frequency distribution of glacier sizes to estimate the total glacier volume in glacierized regions (Likhacheva and others, 1975; Likhacheva and others, 1980). The volume of Rusty Glacier, Yukon Territory, Canada, has been carefully determined (Collins, 1972). The Yerasov equation overestimated the volume of Rusty Glacier by 1.7 times. The Yerasov relation is used in the comparison of methods below.

Lagarec and Cailleux (1972) analyzed glacier-thickness data from 21 glaciers and presented an empirical equation relating the maximum thickness (h_{max} in meters) to glacier surface area (S in square kilometers):

Figure 3. The upper part of Redoubt Volcano showing the deglaciated canyon where the Drift glacier had been before the 1989-90 eruptions. The dome formed over the 1989-90 vent is at the head of the canyon. Some seasonal snow has already accumulated in the canyon following the last events in the eruption during late April 1990. Photograph date, August 2, 1990.

$$h_{max} = \log_{10} (k_1 + k_2 \log_{10} S) \qquad (2)$$

where constants, $k_1$and k_2 both change for each of four morphological classes of glaciers. Lagarec and Cailleux also determined that the average glacier thickness ($\bar{h}$, in meters) is = 0.4 h_{max}; therefore, glacier volume (V in cubic kilometers) is:

$$V = S(0.04h_{max}/1000) \qquad (3)$$

Collins (1977) tested the Lagarec and Cailleux relation on Rusty Glacier, and found the technique overestimated the volume of Rusty Glacier by 2.9 times. The Lagarec and Cailleux relation is not considered further in this study.

Brückl 's (1970) algorithm requires glacier-thickness data before a volume estimate is possible, thus limiting its general application. Brückl (1973), Müller and others (1976, p. 12), Shih and others (1981, p. 194), and Zhuravlev (1980) used empirical relations between the average glacier thickness and glacier surface area, all similar and of the form:

$$\bar{h} = k_1 + k_2 S^m \qquad (4)$$

where $\bar{h}$ is the average glacier thickness in meters, k_1, k_2, and m are constants, and S is the glacier surface area in square meters. Only the Müller and others (1976) implementation was used for comparison (see below).

Macheret and Zhuravlev (1982, p. 310) fit parabolic cross sections to glacier-thickness data from glaciers on Svalbard Island, to calibrate a relation between glacier volume and surface area:

$$\log_{10}V = k_1 + k_2 \log_{10}S \qquad (5)$$

where V is glacier volume in cubic kilometers and S is glacier surface area in square kilometers. Macheret and Zhuravlev (1982) defined four glacier-type groups and estimated the volumes of 59 glaciers; they also analyzed the ratio of the average glacier thickness ($\bar{h} = V/S$) to the maximum glacier thickness and found a value of 0.51 for two of their glacier types, and 0.29 for the other two glacier-type groups. Differences of less than 8 percent were reported between the estimated volumes and those independently calculated for the three glaciers with the most thickness data. The Macheret and Zhuravlev relation is used in the comparison of methods below.

Development of an ice-flow law (Glen, 1955) provided a physical basis for estimating ice thicknesses from surface slope and assumptions about the channel's shape relative to its width. Nye (1952a) tested a force-balance relation based on the ice-flow law and demonstrated good agreement between the theoretical ice thickness derived from glacier surface slope and the measured longitudinal thickness profile on Unteraar Glacier in Switzerland. Budd and Allison (1975) suggested a continuity approach that combined Nye's adaptation of the flow law for valley glaciers, glacier dimensions, and mass balance gradient with altitude in a nomogram that estimates maximum thickness and ice flux. Paterson (1970) suggested using Nye's (1952b) equation of forces to estimate the maximum glacier thickness at the centerline:

$$\tau = f\rho g h \sin\alpha \qquad (6)$$

where τ is basal shear stress in pascals, f is a dimensionless valley shape factor, ρ is ice density in kilograms per cubic meter, g is the acceleration due to gravity in meters per second per second, h is ice thickness in meters, and α is surface slope in degrees. Paterson (1970), assuming a constant basal shear stress of 100 kPa (1 bar) and an average thickness of a glacier's cross section as two-thirds of the centerline thickness, substituted reasonable values for the other constants, and reduced equation (6) to:

$$\bar{h} \approx 11/\alpha \qquad (7)$$

where average ice thickness, $\bar{h}$, is in meters and the surface slope, α, is in radians. Collins (1977) tested Paterson's simplification on seven glaciers with measured cross sections and found that the estimated cross-sectional average thicknesses differed from the measured thicknesses by an average of 190 percent. This approach has not been pursued in the literature and is not considered further here.

Driedger and Kennard (1984) estimated the average basal shear stresses on 24 glaciers and found a range from 0.3 to 1.6 bars. Furthermore, they found the glacier-averaged basal shear stress values clustered into two groups that are distinguished by glacier size. They concluded that an assumption of a constant basal shear stress is not warranted for glaciers less than 2,600 m in length, but is useful for larger glaciers.

Driedger and Kennard (1984 and 1986a) estimated the volumes of glaciers on four Cascade volcanoes of the western United States by dividing the glaciers into two size classes and calibrating an area-volume relation for the small glaciers (those less than 2,600 m in length):

$$V = 3.93S^{1.124} \quad (8)$$

In this relation, glacier volume (V, in cubic meters) is a function of the planimetric area of the glacier (S, in square meters). For larger glaciers, they developed a power law relation containing variables: planimetric surface area (S, in square meters), surface slope (α in degrees), and basal shear stress (τ, in pascals) to compute volumes within discrete altitude increments:

$$V^* = \tau^*/\rho g \, \Sigma(S^*/\sin\alpha^*) \quad (9)$$

and

$$\tau^* = 2.7x104 \, \Sigma(S^*/\cos\alpha^*)0.106 \quad (10)$$

where "*" denotes incremental values that are calculated for 300-m altitude intervals; ρ is ice density, in kilograms per cubic meter; and g, the acceleration due to gravity, is in meters per second per second. Kennard (1983) demonstrated that an error of 25 percent was appropriate for volume estimates of completely unmeasured glaciers.

Comparison of Previous Methods

Driedger and Kennard (1986b) compared measured volumes from 32 glaciers ranging in size from 0.1 to 11 km^2 with glacier volumes estimated by their own and three other techniques. Glacier-volume estimates produced by the Yerasov (1968) equation were added to the Driedger and Kennard comparison. Considering only the nine glaciers that were not part of their method-development suite, the Driedger and Kennard two-class method of glacier-volume estimation produced volumes that differed from the measured volumes by +25 and -11 percent. The average difference was 8 percent, slightly biased toward overestimating glacier volume. The average difference is the average of the absolute values of the percentage differences between measured and estimated volumes. Implementation of the UNESCO/IASH (1970) estimation technique by Post and others (1971) produced volume estimates for the same nine glaciers and resulted in maximum differences from the measured volumes of +226 and -40 percent, with an average difference of 44 percent, strongly biased toward overestimating the glacier volume. The Müller and others (1976) (eq. 4) method had maximum differences of +14 and -77 percent, with an average difference of 61 percent, strongly underestimating the volumes of the same nine glaciers. The Macheret and Zhuravlev (1982) (eq. 5) volume-estimation method, applied to the same nine glaciers, had maximum differences of +209 and -53 percent, with an average difference of 63 percent, strongly biased toward overestimation. For the same nine glaciers, the Yerasov (1968) (eq. 1) volume estimates had maximum differences from the measured volumes of +54 and -65 percent, with an average difference of 53 percent, slightly biased toward underestimating the volume. The Yerasov equation produced as good or better glacier-volume estimates than any of the predecessors tested except the Driedger and Kennard (1984) method. This comparison shows that for glaciers outside of the development-calibration suites, the two-class Driedger and Kennard (1984) method estimates glacier volumes more accurately and with less tendency to bias the estimates than do the four empirical area-volume algorithms. The limited area for which tuned empirical area-volume relations may be applied has been explained (for example, UNESCO/IASH, 1970) as arising from differences in climate, glacier morphological type, glacier size, and glacier orientation.

Demonstration of the possible superiority of the more physically based estimation technique developed by Driedger and Kennard (1986b) for the "large glacier" class is negated by the innate problem of transferring tuned empirical algorithms to a different set of glaciers. However, the two methods developed by Driedger and Kennard (1984, 1986a) can be compared. Considering again all 32 glaciers, the extreme differences between estimated and measured glacier volumes (+28 and -22 percent) both occurred in the "small" glaciers that were estimated by the simple area-volume relation. The maximum differences between measured and estimated volumes of the larger glaciers, calculated by the more physically based technique using area, surface slope, and basal shear stress, were +25 and -11 percent. For these glaciers, the arithmetic average of the differences was +3 percent, suggesting only the slightest tendency to overestimate glacier volume.

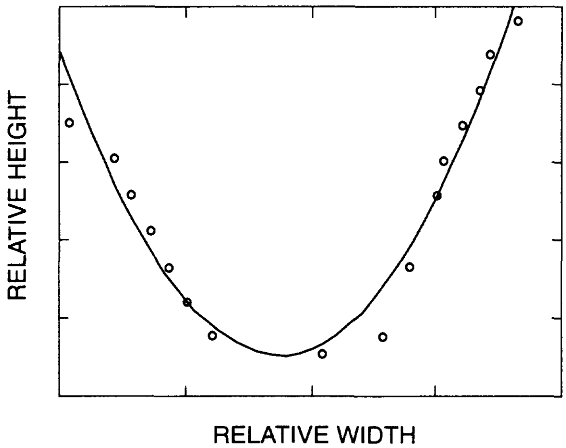
There is no physical basis for expecting more than the most general relation between glacier surface area and volume. The much exploited area-volume relation apparently applies best to "small" glaciers because as both surface area and volume approach zero, the range of possible values for volume decreases rapidly. Furthermore, none of the proposed area-volume algorithms have been widely applied beyond their calibration regions nor to all sizes of glaciers, indicating a lack of broad acceptance. The two-class Driedger and Kennard (1984) method shows the best promise for general application. However, the Driedger and Kennard (1984) method underestimated the volume of a measured part of Drift glacier by 33 percent, suggesting that a more accurate estimation technique would be desirable for assessing the glacier volume on Redoubt Volcano.

GLACIER-VOLUME MODEL

For Redoubt Volcano, glacier volumes were estimated by constructing and analyzing three-dimensional volume models that were augmented by sparse glacier-thickness measurements. A glacier-volume model consists of two surfaces, the exposed glacier surface and the glacier-bed surface. In this study, glacier volume was determined by "surface subtraction," whereby one surface is subtracted from the other, and the difference is the enclosed volume. It is critical in this subtraction that the two surfaces be coincident along the edges of the glacier. The exposed glacier surface is easily derived from maps or compiled aerial photographs. Modeling the glacier-bed surface is the challenge. The original goal of the glacier-bed modeling was to develop a dimensionless valley shape that could be scaled to fit any valley containing a glacier.

The valley-shape analysis began by digitizing the topography of 29 formerly glaciated valleys from thirteen 30-minute (1:63,360) U.S. Geological Survey maps of Alaska. The valleys were chosen because their "U" shapes had not been extensively modified by post-glacial infilling or stream erosion. The valleys ranged in length from 600 m to 19 km, were derived from a broad geographic distribution within Alaska, and represented several valley orientations in each area. The digitized valley contours were re-sampled so that each valley was represented by the same number and relative distribution of data points, thus creating a set of dimensionless valleys that preserved the original height-width relation. The whole valley-shape modeling approach was abandoned when the problem of rigorously modeling curved valleys arose.

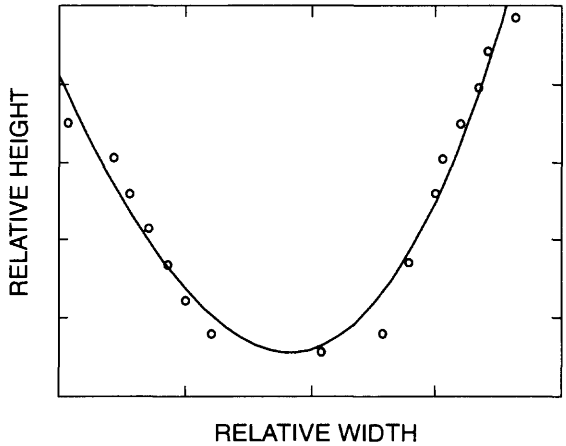
The analysis turned to polynomial fitting of cross sections to the dimensionless valleys (fig. 4). This process parallels many empirical studies that have demonstrated that glacier valleys commonly have approximately parabolic cross sections (Svennson, 1958; Graf, 1970; Doornkamp and King, 1971; Girard, 1976; Aniya and Welch, 1981; Aniya and Naruse, 1985). However, no widely accepted numerical expression, either polynomial or power law, has emerged. This analysis was undertaken to choose an expression that could be used in subsequent volume modeling.



Second-order polynomial equation

Raw r-squared (1-residual/total)	= 0.996573
Corrected r-squared (1-residual/total)	= 0.946926

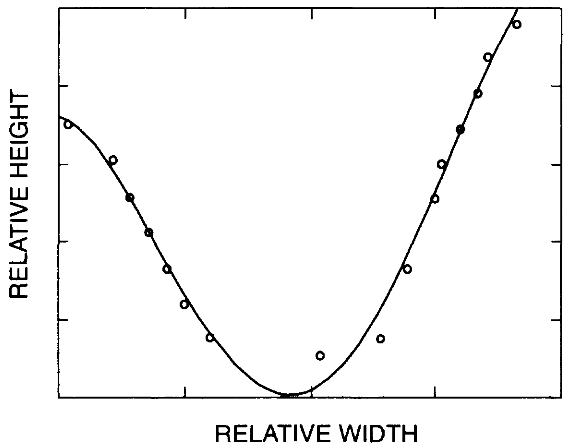
Parameter	Estimate
K1	1833.041227
K2	-84.205333
K3	1.124630



Third-order polynomial equation

Raw r-squared (1-residual/total)	= 0.997030
Corrected r-squared (1-residual/total)	= 0.954009

Parameter	Estimate
K1	1368.990545
K2	-44.536616
K3	0.053097
K4	0.009209



Fourth-order polynomial equation

Raw r-squared (1-residual/total)	= 0.998945
Corrected r-squared (1-residual/total)	= 0.983665

Parameter	Estimate
K1	-2433.551691
K2	403.818697
K3	-18.917547
K4	0.350690
K5	-0.002217

Figure 4. Dimensionless valley cross section fitted by second-, third-, and fourth-order polynomial equations. The change in the variance was not found to be statistically significant. As shown here, the fourth-order fit may overestimate the maximum valley depth.

Second- through fourth-order polynomial fitting of the dimensionless valley cross sections all had correlation coefficients greater than 0.8, with no statistically significant improvement between the second-, third-, and fourth-order fits. The second-order polynomial has the general form:

$$Y = k_1 + k_2X + k_3X^2 \tag{11}$$

where k_1, k_2, and k_3 are constants. As seen in figure 4, second- and third-order fits typically fell short of the observed maximum valley depth, whereas fourth-order fits sometimes overestimated the maximum valley depth.

A valley cross section with the valley partly filled with glacier ice was simulated by removing the lowest one-third of the dimensionless data values. The upper two-thirds of the data set is analogous to the valley-shape information available from maps of valleys that currently contain glaciers. Test fitting of the censored data showed that the third-order polynomial produced a better estimate of maximum valley depth than did second- and fourth-order approximations (table 1). No statistically significant difference was found in the areas of the second- and third-order cross-section simulations. However, the fourth-order cross sections were significantly larger. Therefore, third-order polynomials were used in subsequent cross-section simulations, recognizing that the third-order polynomial simulations likely underestimate maximum valley depth.

Table 1. Simulated maximum dimensionless valley depth as a percentage of the calculated valley depth derived from the complete dimensionless data set

Dimensionless cross-section	Percentage of maximum valley depth
Complete dimensionless data for cross section	100
Simulation	
Second-order fit to the complete dimensionless data set	78
Second-order fit with lower third of the dimensionless data censored	64
Third-order fit with lower third of the dimensionless data censored	80
Fourth-order fit with lower third of the dimensionless data censored	146
Second-order fit, lower third of the dimensionless data censored, plus one thickness measurement	73
Third-order fit, lower third of the dimensionless data censored, plus one thickness measurement	83

For the example data in table 1, underestimation of the maximum glacier thickness, and therefore cross-sectional area, introduces a bias into subsequent volume calculations resulting in an underestimation of volume by as much as 20 percent. This bias can be reduced by incorporating measured glacier thicknesses in the cross-section simulations (table 1), reducing the underestimation bias to no more than 17 percent. Thus, it seems to be important to include glacier-thickness data in as many simulated cross sections as possible when developing input for a glacier-bed-surface model.

In this study, both the exposed glacier surface and the glacier bed are modeled as irregular triangular network grids. Input for both surface models includes the 1979 glacier edge, thus ensuring that the volume calculation will not contain "edge errors." Edge errors can easily skew volume calculations because unrequited points on either surface will be subtracted from either zero or an infinitely large number, introducing large absolute values into the volume integration. Gridded surface representation, manipulation, editing, and volume evaluation were handled by Quicksurf, a digital surface- and volume-modeling application supported by AutoCAD.

Glacier-volume modeling in this report is simply a combination of a broadly accepted glacier-valley cross-sectional shape generalization and an established computer-based surface- and volume-modeling application. As such, the glacier-bed surface is modeled as a computer-based interpolation of a group of cross-sectional forms arrayed in three-dimensional space. The glacier-bed surface is combined with the glacier's exposed upper surface to enclose a volume and the enclosed volume is calculated by the supporting software. If this method proves to accurately assess the volume distribution of a variety of valley glaciers, the approach will find wide use in the study of valley glaciers and will expand the knowledge of the distribution of glacier volumes in mountain systems.

Verification of the Volume Model

The 1989-90 eruption of Redoubt Volcano removed most of Drift glacier between 750 m and 2,500 m altitude (figs. 2, 3, and 5). Therefore, the glacier volume removed from this reach can be directly measured as the difference between the pre- and post-eruption maps. Direct measurement of a glacier volume presents a unique opportunity for verifying the results of glacier-volume modeling.

Special detailed maps with 10-m contour intervals were compiled for this analysis. The pre-eruption map was compiled from August 1979 aerial photographs, which are the most recent pre-eruption mapping-quality aerial photography. The post-eruption map was compiled from May 1990 aerial photographs. The two maps were digitized, and surface models were created for the area inside the 1979 Drift glacier boundary. Both surface models were truncated at the 1979 glacier edge. Truncating the 1990 surface models at the 1979 glacier edge introduced no surface perturbations, verifying that the 1979 glacier edge was mapped correctly at the ice-rock contact.

The reach chosen for comparing measured and simulated volumes is a rather ideal subsection of the deglaciated part of the Drift glacier valley. The reach is 2,500 m long and begins just above the confluence with a tributary glacier at about 1,200 m altitude, and extends to just below a nunatak at about 2,050 m altitude (fig. 5). The reach is ideal because the valley walls are high and continuous and the bed contains no major bedrock perturbations (nunataks) (fig. 5). The high and continuous valley walls aid the simulation of glacier cross sections by providing a relatively large surface above the former glacier for curve fitting.

The "measured" glacier volume for this reach was calculated by truncating the 1979 and 1990 digital map-surface models at the lowest and highest cross sections (fig. 5), and subtracting the recently exposed "glacier-bed" surface (the 1990 map) from the 1979 glacier surface. Subtraction of the two surfaces yields the volume between the two surfaces. Therefore, the glacier ice that was removed from the reach had a measured volume of 99 x 10^6 m^3. This reference volume is judged to have an error of ±5 percent on the basis of this direct comparison of two high quality maps, each prepared by the same experienced cartographer using a 10-m contour interval.

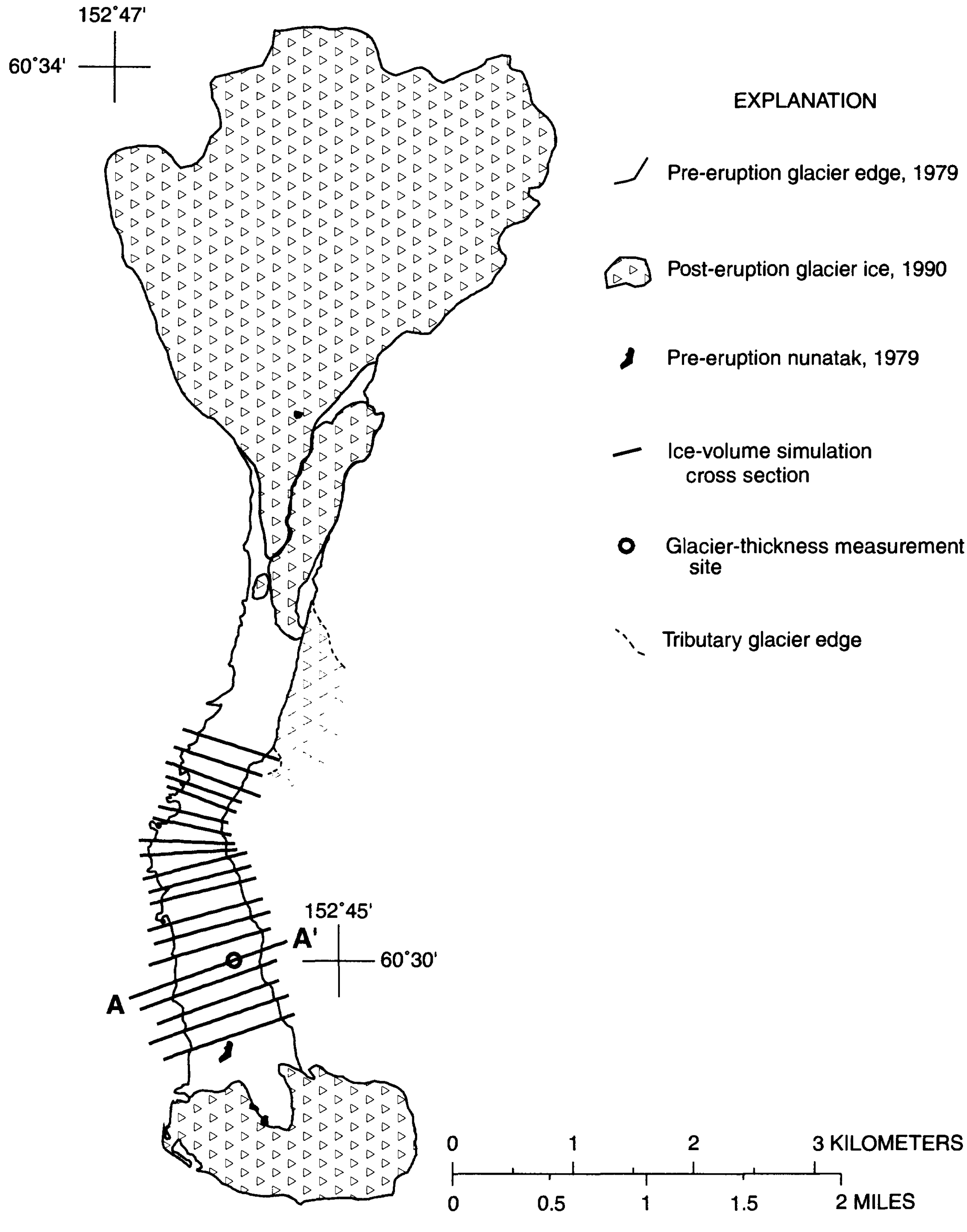


Figure 5. The area that was removed from Drift glacier by the 1989-90 eruptions and the locations of the cross sections in the reference-volume reach used for model verification. Cross section A–A' is shown in figure 6.

The modeled volume for the same reach was created by simulating 20 cross sections (fig. 5), and using them as the input for generating a simulated glacier-bed surface. The cross sections were simulated by fitting the third-order polynomial to the parts of the cross sections that lie on the bedrock valley walls above the 1979 glacier edge. The cross sections were placed approximately parallel to the 50-m glacier-surface contours, on the pre-eruption (1979) map. Cross section A-A′ (fig. 5) contains the only glacier-thickness measurement in the reference-volume reach. Both measured and simulated cross sections at A-A′ are shown in figure 6. The modeled glacier volume was calculated by subtracting the simulated glacier-bed surface from the 1979 (pre-eruption) glacier surface. The modeled volume for the reach was 106 x 10^6 m^3, about 7 percent greater than the measured glacier volume. On the basis of the cross-section fitting discussed above in which third-order fits were found to underestimate maximum cross-section depths, the modeled volume was expected to be as much as 20 percent smaller than the measured volume. The simulated volume is a better approximation of the true volume than was originally expected because the Drift glacier valley is probably more "V"-shaped than is a "mature" glacier valley and cross-sectional area increases as "V"-shaped valleys become "U"-shaped (approximately parabolic). Harbor (1992) found that 10,000 years of glacier erosion are required to produce a mature valley shape. It is unlikely that many glacier valleys on active volcanoes are continuously occupied by glaciers for 10,000 years. This is especially true on Redoubt Volcano where more than 30 eruptions have occurred during the past 10,000 years (Till and others, 1993).

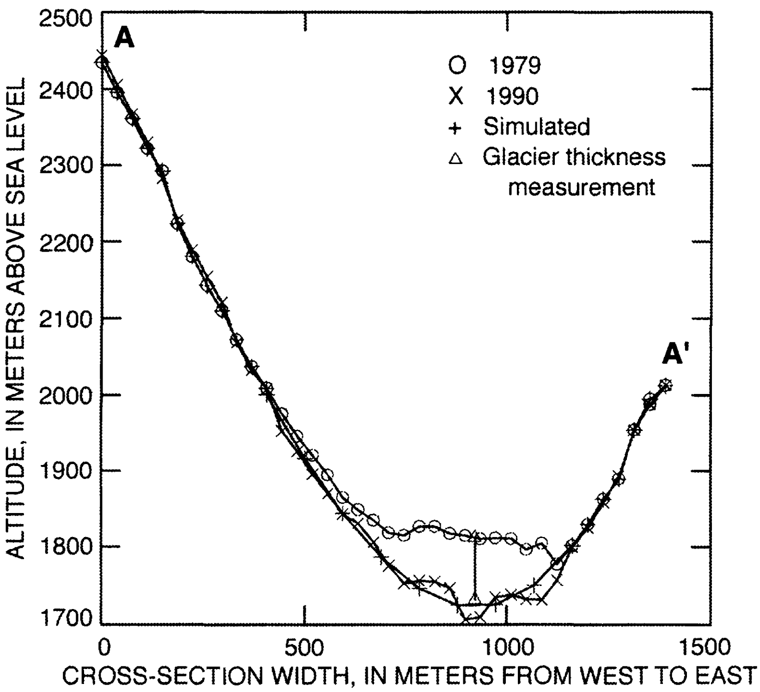


Figure 6. A simulated and two measured cross sections at A–A' (fig. 5) and one glacier-thickness measurement on Drift glacier. Note that the glacier bed (1990 cross section) is not a simple shape comparable to the dimensionless valleys (fig. 4) used for algorithm testing.

The 18 cross sections between the ends of the reach were arranged several ways during the model development phase. An equal longitudinal distance separation first defined the number of cross sections. Then the 18 cross sections were redistricted into equal altitude separations, and into a placement concentrated in the curved parts of the reach and, finally, into an intuitive combination of the altitudinal and curvature distributions. When all of these arrangements produced simulated volumes varying by only a small percentage, it was concluded that an estimated error of ±10 percent was safely conservative.

Application of Volume Modeling

Efficient application of the volume model to all the glaciers on Redoubt Volcano required a reduction in the time-consuming process of cross-section simulation. A sensitivity analysis helped determine how the density of cross sections influences the modeled volume. The analysis used the 20 cross sections from the verification study but was simplified by assuming that all the cross sections were parallel and separated by a distance equal to their midpoint separations in their true orientations (fig. 5). The true areas of the cross sections were preserved so the integration with their separations would result in a volume reasonably close to that derived from surface subtraction. The volume of the 20 parallel cross sections was 110×10^6 m^3, showing reasonable agreement with the modeled volume of 106×10^6 m^3. The average of the volumes calculated using the two end sections and only one intervening section, taken one at a time (18 possible combinations) is 115×10^6 m^3 with 95 percent confidence limits at 122×10^6 m^3 and 108×10^6 m^3. The average of the volumes calculated using the two end sections and any two other sections (153 combinations) was 112×10^6 m^3 with 95 percent confidence limits at 114×10^6 m^3 and 110×10^6 m^3. The average of the volumes calculated using the two end sections and any three other sections (816 combinations) was 110×10^6 m^3 with 95 percent confidence limits at 111×10^6 m^3 and 109×10^6 m^3. The results, of course, converge as the number of cross sections increases. However, using two cross sections in addition to the end sections results in a volume that has a 95 percent confidence limit that includes the volume determined using all 20 cross sections (110×10^6 m^3). The standard deviation of the four cross-section set is 0.14×10^6 m^3, making this density of simulated cross sections a reasonable compromise between the number of cross sections that must be developed and analyzed and the accuracy of the resulting volume.

Reducing the number of simulated cross sections from 20 to 4 (fig. 7) caused problems with the surface modeling. Like a canoe with a loose canvas skin over too few ribs, the reduced input resulted in a simulated bed surface that pinched inward between the controlling cross sections. Basal surface "pinching" was removed by making linear interpolations between the cross sections and adding the linear interpolations to the surface-modeling input. In contrast with increasing the number of simulated cross sections, linear interpolations between sparse cross sections can be made rapidly and incorporated into the three-dimensional modeling environment used for this analysis. Strategic location of the reduced number of cross sections was also necessary to guide the bed simulation around the curve in the reach (fig. 7). As applied, the northernmost three cross sections in figure 7 (lowest altitude) are separated by approximately 250 m altitude; the third and fourth cross sections are separated by about 450 m altitude. The modeled bed surface was created on the basis of the four cross sections, linear interpolations between them, and the 1979 glacier edge (fig. 7). This modeled glacier-bed surface was subtracted from the 1979 (pre-eruption) mapped glacier surface producing a glacier volume estimate for the reach of 100×10^6 m^3. This volume estimate is about 1 percent greater than the measured volume (99×10^6 m^3) derived from map comparisons

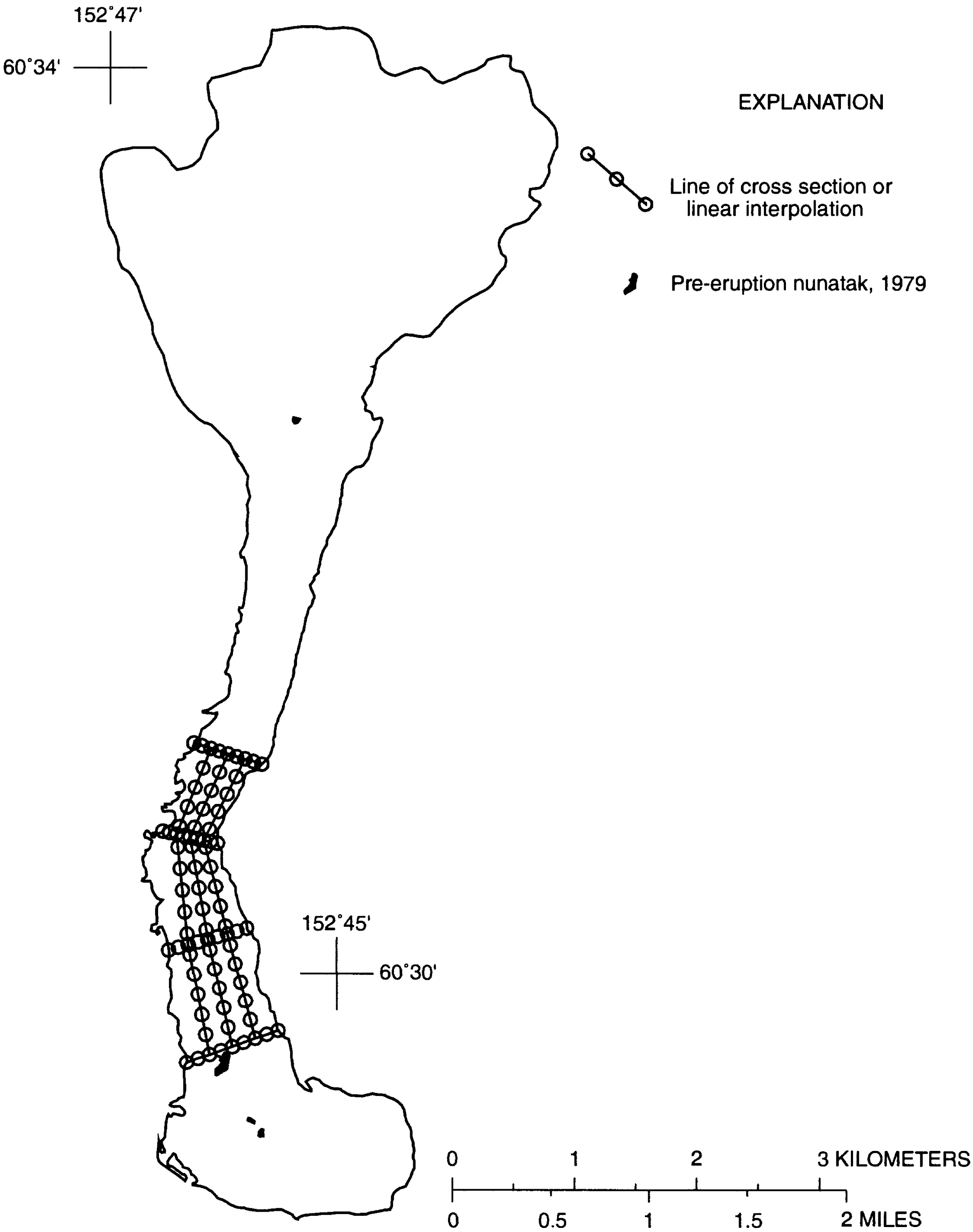


Figure 7. Control points on cross sections and linear interpolations between cross sections used for modeling the glacier-bed surface in the reference-volume reach of Drift glacier.

and about 6 percent smaller than the modeled volume (106×10^6 m^3). This satisfactory result indicates that cross-section simulations separated by 250 m altitude and connected by linear interpolations could be used to create adequate bed-surface models for other glaciers on Redoubt Volcano and that the expected error, for similarly constrained reaches, is not greater than about ±7 percent.

Comparison with Other Volume-Estimation Methods

The results of the volume model are compared to the Driedger and Kennard basal-shear-stress method of glacier-volume estimation (Driedger and Kennard, 1986b) (eq. 8). The Driedger and Kennard (1986b) method produced a volume estimate of 66×10^6 m^3 (the sum of three incremental altitude intervals) for the model development reach of Drift glacier. This estimate is 33 percent smaller than the measured volume of 99×10^6 m^3 for this reach. The unexpectedly large error may be in part due to the reach being about 6 percent short of the 900-m minimum altitude span (minimum of three 300-m altitude intervals) recommended by Kennard (1983) for application of the method. Furthermore, the estimated basal shear stress of 1.2×10^5 pascals (1.2 bars) is probably low for this steep (16 to 23 degrees) reach of Drift glacier where basal sliding may exceed the deformational speed.

GLACIER-THICKNESS MEASUREMENTS AND ERRORS

In support of the glacier-volume modeling, glacier thickness was measured at 46 sites on Redoubt Volcano in August 1989 (fig. 8 and table 2), using a surface-based monopulse ice-radar system similar to those described by Watts and Wright (1981) and Driedger and Kennard (1984, 1986a). The signal frequencies used in this investigation were between 1.2 and 2.0 MHz. A first approximation of glacier thickness was calculated from the separation between the radar transmitter and receiver, wave propagation speed, and the time delay between the arrival of the air-surface wave and the reflected wave, as described by Mayo and Trabant (1982). The first approximation of glacier thickness, *h* (in meters), is given by:

$$h = \frac{1}{2}\sqrt{v_i^2(t_d + S/v_a)^2 - S^2} \qquad (12)$$

where v_i and v_a are the wave-propagation speeds in ice and air respectively (meters per microsecond), S is the separation distance between the transmitter and receiver (meters), and t_d is the delay time (microseconds). Wave-propagation speed is known to vary slightly with frequency, snow and ice density, and ice temperature. However, the propagation speeds were assumed to be constants for this analysis with v_i =168 m/µs and v_a = 299.7 m/µs. Assuming constant wave-propagation speeds introduces less than 1 percent error in glacier-thickness evaluations.

The separation distance, S, between transmitter and receiver was usually determined by surveying from a known instrument station. Eighteen of the separation distances were either paced or measured with a climbing rope because of lost intervisibility with the surveying station or failure of the distance-measuring device. Nevertheless, all separation distances are thought to have errors of less than 10 percent. This has a small influence on the thickness determination. For example, 10 percent error in the separation distance typically results in less than a 2 percent difference in the thickness estimate.

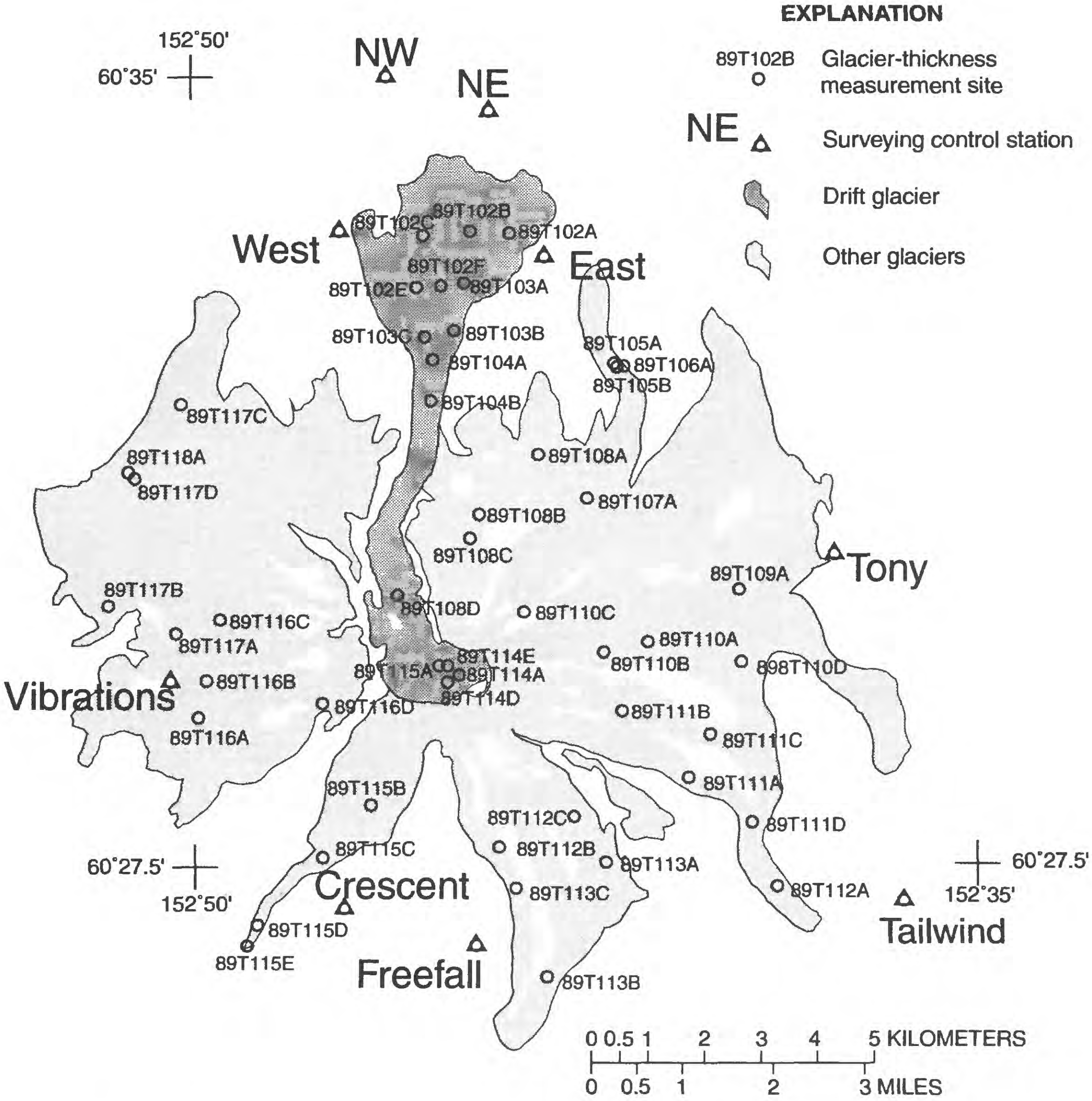


Figure 8. Glacier-thickness measurement sites and surveying-control stations on Redoubt Volcano. The Universal Transverse Mercator location, glacier thickness, and other data are listed by site-identification number in table 2.

Delay time (t_d) errors arise from three sources. The error of the oscilloscope sweep rate (the receiver) is ±4 percent. The error introduced during reading of poor signals is estimated to be ±4 percent and for good signals ±2 percent. In addition, snow and firn overlying glacier ice effectively give a shallow bias to the radar-thickness determinations because the reduced density increases the radar propagation rate. Kennard (1983) estimated that this effect is on the order of -1.25 percent. The combined oscilloscope sweep rate, reading, and density errors result in an estimated error for glacier-thickness measurements of ±6 percent.

Table 2. Glacier-thickness measurement locations, antenna separations, measured delay times, qualitative signal quality, and derived ice thickness

[Map identification locations are shown in figure 8. Unsurveyed locations (in italics) were estimated in the field and marked on field maps. Delay time is reported in microseconds (µsec); "Air" indicates that the thickness was measured using the helicopter altimeter readings observed at the bottom and top of a near-vertical ice cliff. Qualitative signal quality: E, excellent; G, good, P, poor. "NA" appears where the data are not available]

Map ID (fig. 8)	Mid-point location, UTM coordinates (meters)			Transmitter/ receiver separation (meters)	Delay time (µsec)	Signal quality	Ice thickness (meters)
	E	N	Z				
89T102A	514665.0	6713442.4	417.1	62.7	1.20	E	114
89T102B	514022.3	6713486.8	442.1	60.9	1.20	E	114
89T102C	513128.0	6713404.4	447.9	73.0	1.23	E	118
89T102E	513081.4	6712478.8	530.0	79.9	1.43	G	137
89T102F	513476.9	6712531.2	495.2	75	1.00	G	98
89T103A	513885.6	6712582.4	480.9	72	1.40	G	133
89T103B	513697.5	6711744.7	578.2	60	1.30	G	122
89T103C	513191.5	6711654.2	615.4	96.7	1.73	G	165
89T104A	513299.3	6711209.9	673.9	68.3	1.40	G	132
89T104B	513285.7	6710513.4	779.1	70.0	1.50	G	141
89T105A	516562.9	6711122.5	709.5	82.2	1.20	G	117
89T105B	516588.3	6711090.2	718.1	164.4	1.40	G	142
89T106A	516713.3	6711108.4	680.8	100	0.70	G	71
89T107A	156060	6708780	NA	158	0.53	P	41
89T108A	515247.3	6709557.5	1153.0	130	0.70	G	70
89T108B	514156.5	6708492.7	1383.9	Air	Air	Air	30
89T108C	513995.2	6708072.4	1487.6	200	0.63	G	43
89T108D	512867	6707131	NA	80	0.83	G	83
89T109A	518714.8	6707181.8	1270.4	84.0	0.85	G	85
89T110A	517140.8	6706231.1	1573.1	Air	Air	Air	30
89T110B	516361.4	6706046.4	1802.8	100	0.50	G	49
89T110C	514959.3	6706766.0	2237.5	Air	Air	Air	21
89T110D	518784.8	6705827.2	1229.2	123.8	0.90	E	91
89T111A	517840.8	6703859.2	1067.2	74.1	1.60	P	151
89T111B	516652.7	6704975.6	1448.4	100.4	0.80	G	81
89T111C	518249	6704600	NA	56	1.90	G	173
89T111D	518993	6703061	NA	92	1.90	G	180

Table 2. Glacier-thickness measurement locations, antenna separations, measured delay times, qualitative signal quality, and derived ice thickness (Continued)

[Map identification locations are shown in figure 8. Unsurveyed locations (in italics) were estimated in the field and marked on field maps. Delay time is reported in microseconds (µsec); "Air" indicates that the thickness was measured using the helicopter altimeter readings observed at the bottom and top of a near-vertical ice cliff. Qualitative signal quality: E, excellent; G, good, P, poor. "NA" appears where the data are not available]

Map ID (fig. 8)	Mid-point location, UTM coordinates (meters)			Transmitter/ receiver separation (meters)	Delay time (µsec)	Signal quality	Ice thickness (meters)
	E	N	Z				
89T112A	519463.3	6701901.7	630.6	86.6	1.25	E	122
89T112B	514529.2	6702617.0	1570.1	100	0.50	P	49
89T112C	515783.9	6703201.8	1580.1	153.1	0.45	G	26
89T113A	516423.9	6702460.5	1358.9	221.8	0.68	E	44
89T113B	515352.2	6700272.4	844.6	79.0	0.50	E	51
89T113C	514798.8	6701988.6	1330.1	100	0.60	E	60
89T114A	513802	6705641	NA	100	1.75	E	168
89T114D	513597	6705507	NA	100	2.00	G	190
89T114E	513597	6705801	NA	100	1.10	G	110
89T115A	513445	6705801	NA	100	1.38	G	135
89T115B	512224.1	6703286.6	1650.0	129.9	0.65	G	64
89T115C	511402	6702425	NA	120	0.95	G	96
89T115D	510252	6101215	NA	100	1.35	G	132
89T115E	510076	6700852	NA	150	0.49	G	36
89T116A	509204.1	6704870.0	1545.6	125.1	0.46	G	39
89T116B	509292.9	6705534.2	1606.0	134.0	0.50	G	43
89T116C	509534.0	6706663.9	1485.6	154.7	0.66	E	61
89T116D	511394.3	6705132.4	2268.9	Air	Air	Air	64
89T117A	508811.4	6706365.3	1415.5	100	1.20	P	119
89T117B	507571.6	6706835.9	1188.8	103.7	0.50	G	49
89T117C	508890.6	6710408.0	741.2	126.7	1.05	E	106
89T117D	508075.5	6709108.3	913.5	127.7	0.60	E	58
89T118A	507972.4	6709214.3	914.2	197.5	1.13	E	113

An important potential source of error is misattribution of a reflection signal, because no unique characteristic of a reflected signal unambiguously identifies it as a reflection from the glacier bed. The strongest return signals come from the nearest surfaces that are large enough to produce a coherent reflection. Reflections may be produced by nearby bedrock walls, by englacial water or debris layers, as well as by the glacier bed. In the case of multiple reflections, the strongest reflection (usually the shallowest) was assumed to be from the glacier bed, unless nearby measurements or the physical setting strongly suggested otherwise. Examples of physical influences include extreme surface slope, severe and deep crevassing, nearby bedrock exposures, and emergent debris layers in the vicinity. In this study, the influence of the physical setting was minimized by careful site selection in the field. Therefore, all the radar delay times shown in table 2 are confidently assumed to be reflections from the glacier bed. However, because the effective radius of the propagating wave front is large and a reflection comes from the first encounter with a surface capable of producing a coherent reflection, radar thicknesses are always minimum thicknesses for the area.

Vertical thickness is especially important for volume evaluations, which are the product of planimetric area and vertical thickness. Rigorous resolution of vertical thickness from radar data requires a high density of radar soundings and iterative, three-dimensional migration of the reflections (Driedger and Kennard, 1986a). For this study, the expense and field time required for an exhaustive sounding of the glaciers were not justified. When only sparse thickness data are available, vertical glacier thickness is sometimes routinely derived from the first approximation thickness (h, eq. 12) by applying a cosine correction for surface slope (Driedger and Kennard, 1986a). The cosine correction always increases the estimated vertical thickness.

The locus of possible radar reflection points for a specific delay time and antenna separation is an ellipsoid of revolution (spheroid) with the foci at the transmitter and receiver. The first approximation thickness (h, eq. 12) is a half-length of the minor axis of the spheroid. Accepting the cosine-corrected thickness assumes that the reflection comes from the unique point at the end of the minor axis contained in the vertical plane including the foci, and that the reflection surface approximately parallels the glacier surface. However, the return may have come from any part of the spheroid that is at or beneath the glacier surface. The vertical thickness above all other possible reflection points is less than the cosine-corrected thickness. In practice, it is common to assume that reflections come from somewhere beneath the antenna array, that is, an area near the first approximation point. Because of this, the true vertical thickness above the reflection point ranges from a maximum of the cosine-corrected thickness to slightly less than the first approximation thickness and is a function of both the surface and bed slopes.

A cosine correction was not routinely applied to the measurements in this study, for the following reasons: (1) A simple cosine correction for surface slope may be incorrect in sign and is usually a small part of the range of the uncertainty of the measurement (±6 percent). (2) The cosine correction would increase thickness determinations by less than 1 percent below about 500-m altitude on Redoubt Volcano where the glacier-surface slopes are less than 8 degrees. It would increase them by less than 5 percent for the glacier surfaces between 500 and 1,500 m altitude where 80 percent of the glacier volume on Redoubt Volcano is resident, and where surface slopes are less than 18 degrees. The cosine correction is as much as 15 percent only on the steepest slopes, at high altitudes on Redoubt Volcano where less than 20 percent of the glacier volume is resident, and where the glaciers are less than 50 m thick. However, a 15 percent correction applied to a relatively minor thickness results in a negligible correction in absolute value.

GLACIER-VOLUME EVALUATIONS

Application of volume modeling to an entire glacier was first attempted on Drift glacier where part of the volume had been previously determined during model verification and where there were relatively more ice-thickness measurements than elsewhere on Redoubt Volcano. An increased density of ice-thickness measurements helped constrain and verify the modeled volumes.

Drift Glacier Volume

Extending volume modeling to the entire Drift glacier required adaptation of the volume-modeling technique to accommodate parts of the glacier that are not surrounded by valley walls, for example, the piedmont lobe and the summit crater. The bed model for the piedmont lobe was controlled by 10 glacier-thickness measurements (figs. 9 and 10), and guided by observations of a surprisingly flat deglaciated piedmont-lobe bed a few kilometers west. Both the glacier-thickness measurements on Drift glacier and the exposed piedmont-lobe bed suggest that the bed of Drift glacier's piedmont lobe should be flat. Therefore, third-order cross-section simulation was applied only at the edges, and the bulk of the central area was assumed to be generally flat and to conform to the glacier-thickness measurements. The glacier-thickness measurement sites and cross-section locations are shown in figures 9 and 10. The longitudinal profile intersects a nunatak that is exposed near the head of the lobe. The nunatak was simulated by applying the third-order cross-section model to estimate its extent at depth (fig. 10). The most southerly cross section in figure 10 was simulated by the third-order polynomial cross-section fitting method based on the valley-wall profiles (shown extending beyond the glacier edge). The nearby radar glacier-thickness determination was not used in the cross section simulation (fig. 10). The agreement of the simulated and radar-measured bed altitudes near the southern end of the piedmont lobe (fig. 10) is additional confirmation of the fundamental accuracy of the simulated cross sections controlled by valley walls. The 1979 glacier surface is also shown at the southern cross section. The glacier-bed model (fig. 10) for the piedmont lobe was created by extrapolation from the longitudinal profile, the four transverse cross sections, one third-order cross section, and the 1979 glacier edge.

The glacier-bed model for the summit crater was controlled by three simulated cross sections through four glacier-thickness measurement sites (fig. 9). The third-order cross-section simulations were constrained to agree with the glacier-thickness determinations.

The pre-eruptive (1979) volume of Drift glacier was determined to be 0.98 km^3, with an error of ±10 percent. The error was assigned on the basis of the resampling analysis of four cross sections in the reference-volume reach of Drift glacier. Subtracting the 1990 from the 1979 mapped surfaces for the entire Drift glacier revealed that the 1989-90 eruptions removed a total of 0.29 km^3 ±5 percent of the perennial snow and glacier ice (about 30 percent of the total volume of Drift glacier), including 99×10^6 m^3 in the reference-volume reach.

Glacier Volumes on Redoubt Volcano

Evaluation of the total perennial snow and ice volume on Redoubt Volcano was accomplished by extension of the techniques applied on Drift glacier, except that fewer thickness-sounding data were available and fewer valley walls were exposed for fitting third-order polynomial curves in the process of simulating glacier-bed surfaces. Glacier-bed cross sections were simulated at approximately 250-m altitude intervals, connected by linear interpolations along the bed surface, and com-

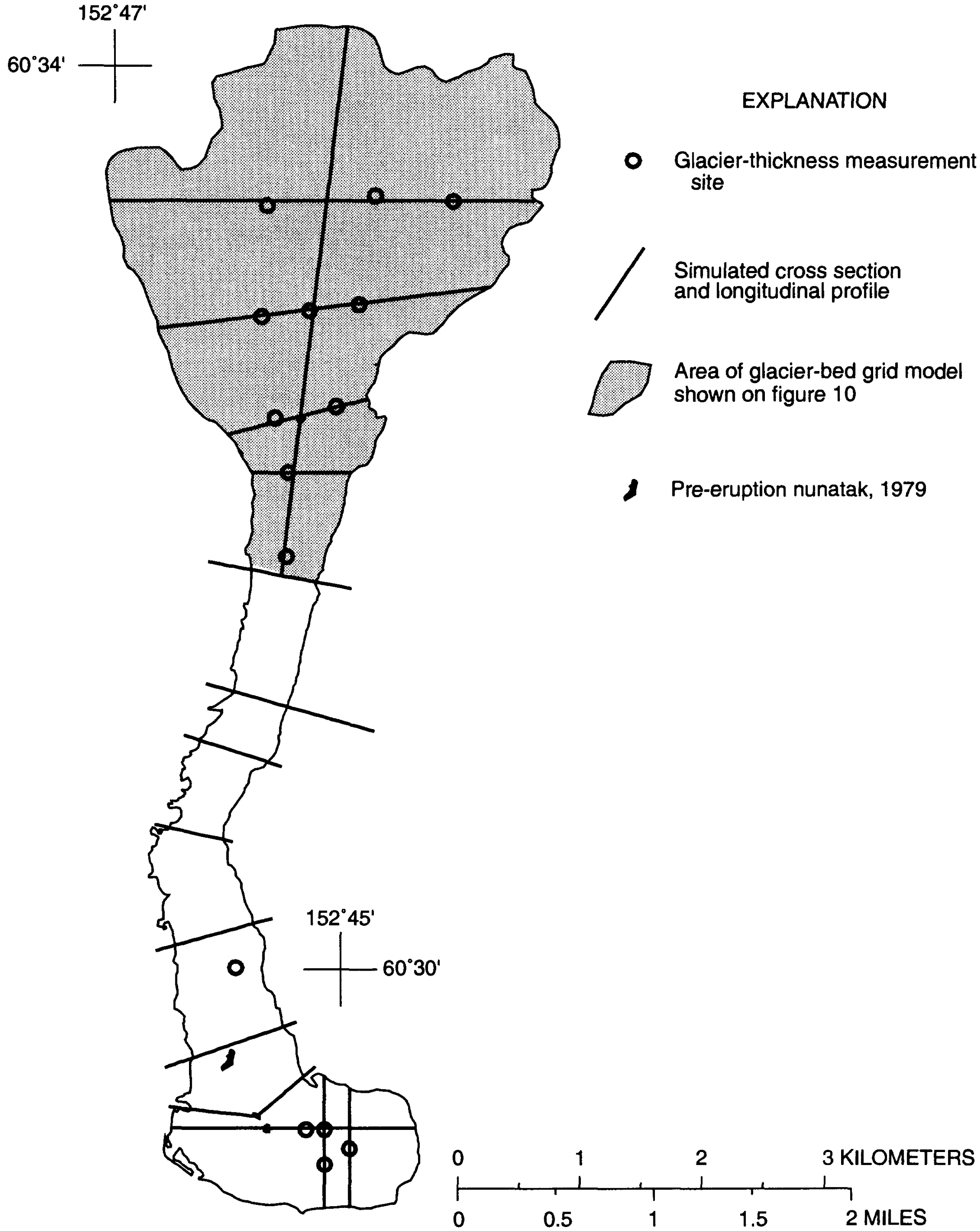


Figure 9. Ice-thickness measurement sites and simulated profiles and cross sections used for evaluation of the ice volume of Drift glacier.

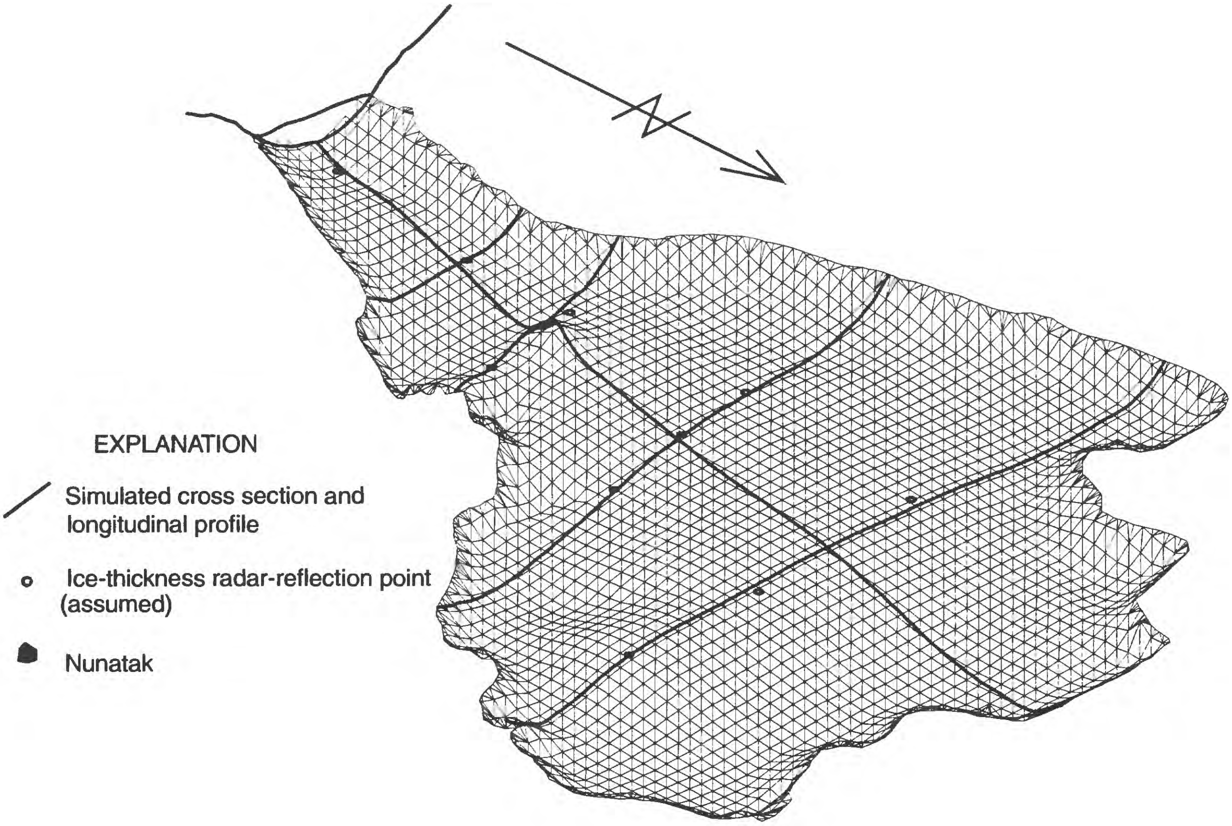


Figure 10. Glacier-bed grid model of the piedmont lobe of Drift glacier. Shown are the ten glacier-thickness measurement points, the longitudinal profile, and the five cross sections used to control the bed-surface modeling.

bined with glacier edges to model glacier-bed surfaces. The modeled glacier-bed surfaces were subtracted from the mapped glacier surfaces to determine glacier volumes. The paucity of controlling data required that more estimates be made. This is especially true on the steep slopes above 1,500 m altitude. Increased estimation inevitably increases the uncertainty of the result, and by an unknown amount. The total perennial snow and ice volume on Redoubt Volcano, 4.1 km^3, is assigned a error of ±20 percent, doubling the error assigned after the resampling analysis of the reference-volume reach of Drift glacier. This volume is comparable to the 4.41 km^3 glacier volume reported for Mount Rainier, Washington (Driedger and Kennard, 1984 and 1986a,b) and almost 23 times more glacier volume than was on Mount St. Helens (Brugman and Meier, 1981) before its 1980 eruptions.

Perennial snow and ice volumes and surface areas were evaluated for each glacier on Redoubt Volcano and accumulated in 500-m altitude intervals (fig. 11). The volumes are subdivided into the two river drainages for the area—the Drift River drainage to the north, and the Crescent River drainage to the south—and summarized for the entire massif (fig. 12 and table 3).

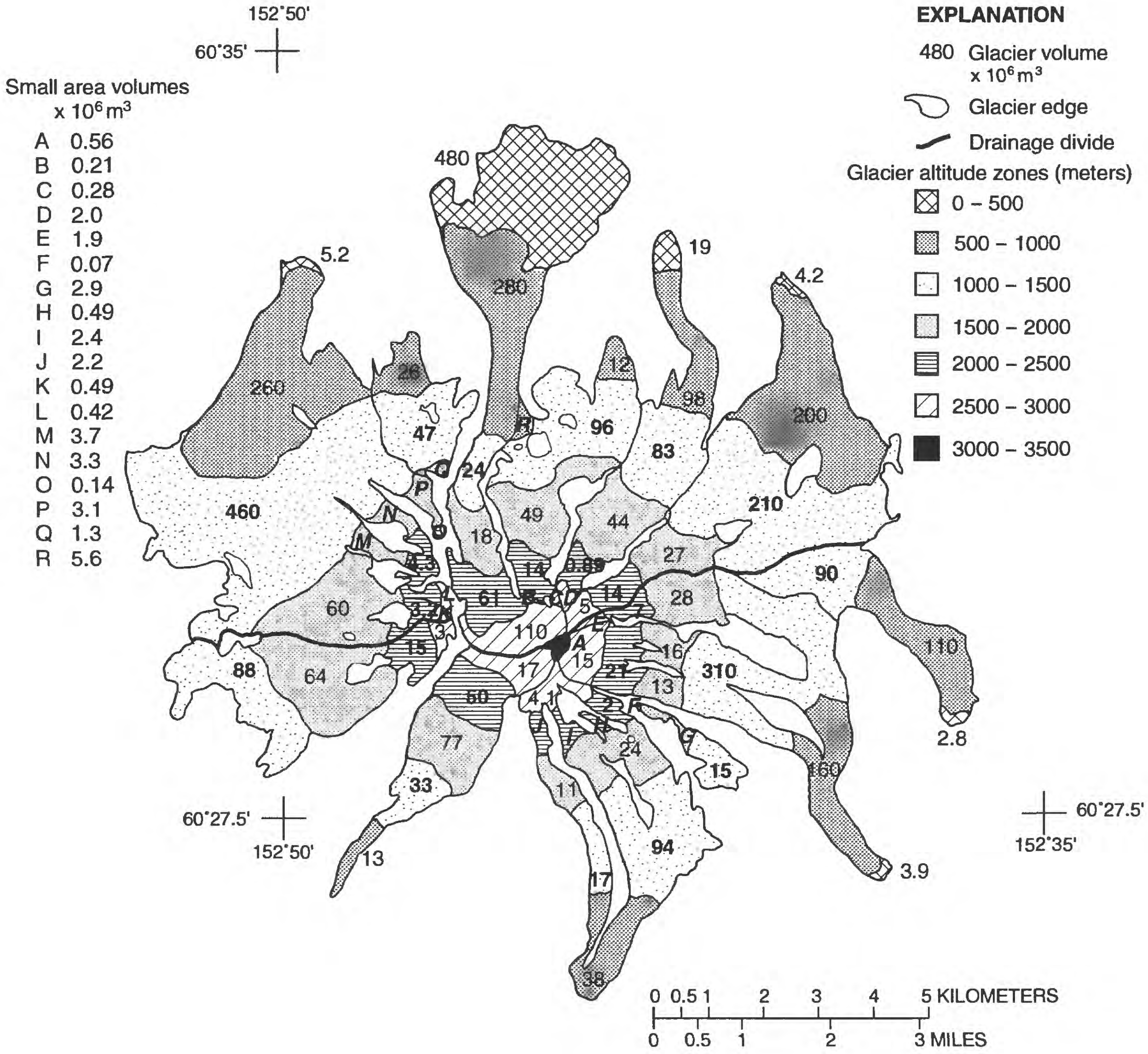


Figure 11. Glacier volumes arrayed by 500-m glacier-altitude sub-areas on Redoubt Volcano.

The volume of perennial snow and ice varies seasonally by the amount lost by melting at low altitudes and the amount gained as seasonal snow. Because the glaciers on Redoubt Volcano are in near dynamic equilibrium with climate, the mass lost by melting at low altitudes is approximately equal to the amount of seasonal snow converted to perennial snow and ice at high elevations. Equilibrium shape is maintained by glacier flow, which redistributes the excess accumulation at high altitudes to the areas of excess melting at low altitudes, so on a year-to-year basis, the distribution of glacier volume with altitude changes slowly.

The volume of seasonal snow on Redoubt Volcano is estimated to be about 0.20 km^3 on the basis of snow depths measured at the ice-thickness sounding sites and subsequent depth and den-

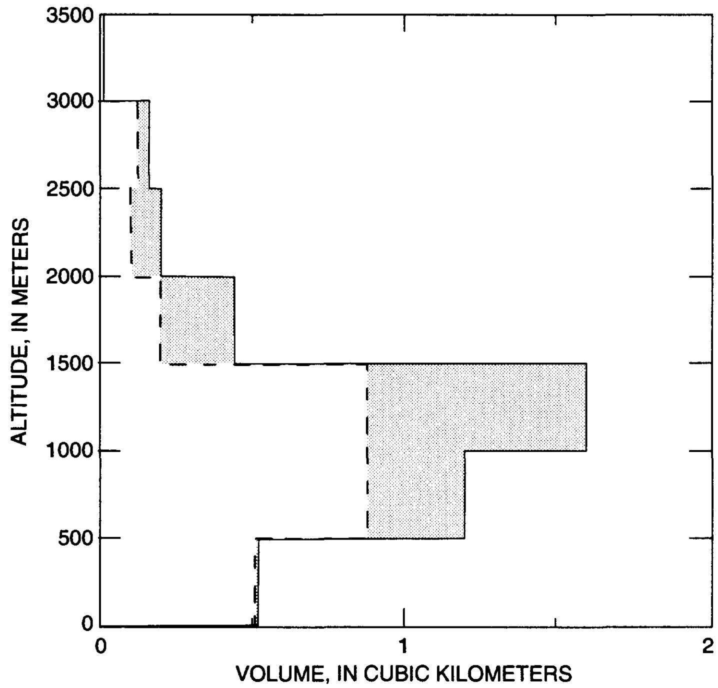


Figure 12. Volume-altitude distribution on Redoubt Volcano. The solid line is the volume-altitude distribution for all of the glaciers on Redoubt Volcano. The area to the left of the dashed line is the glacier-volume distribution that is part of the Drift River drainage. The area between the dashed and solid lines (shaded) is the glacier-volume distribution that is part of the Crescent River drainage.

sity measurements made during the springs of 1990 through 1993. The volume of seasonal snow is an important variable in terms of hydrologic hazards, especially the seasonal snow in valleys that may be incorporated into eruption-generated flows (Trabant and others, 1994).

It is widely recognized that the most voluminous and catastrophic lahars, debris flows, and floods result from eruptions of volcanoes mantled by substantial snow and ice covers (Major and Newhall, 1989). Hydrologic and mass-failure hazards associated with the Cascade Volcanoes have been especially well documented (Scott, 1988; Scott and others, 1995; Wolfe and Pierson, 1995; Hoblitt and others, 1995). The 1980 eruption of Mount St. Helens generated devastating mudflows and lahars, and emphasized the need for information about the volume and distribution of snow and ice on volcanoes (Driedger and Kennard, 1986a,b). However, the relation between the volume distribution of perennial snow and ice and volcanic explosivity (Newhall and Self, 1982), lahars, and flood hazards is poorly defined in the scientific literature. Nevertheless, the mere presence of glaciers on volcanoes portends outburst floods (Björnsson, 1975) and increased volcanic explosivity (Mastin, 1995). Furthermore, icefalls and severely crevassed ice surfaces are common features on volcanoes because of the steepness of the sides of the volcanic edifice. Highly fractured ice surfaces increased the lahar and flood volumes on Redoubt Volcano during the 1989-90 eruptions (Trabant and others, 1994).

Table 3. Glacier areas and volumes on Redoubt Volcano

[m^2, square meters; km^3, cubic kilometers]

Altitude interval (meters)	Drift River glacier volume (km^3)	Crescent River glacier volume (km^3)	Redoubt glacier area ($x10^6$ m^2)	Redoubt glacier volume (km^3)
0-500	0.51	0.0007	6.1	0.52
500-1000	0.88	0.32	23	1.20
1000-1500	0.88	0.69	39	1.60
1500-2000	0.20	0.24	17	0.44
2000-2500	0.10	0.10	7.1	0.20
2500-3000	0.12	0.041	4.5	0.16
3000-3500	0.0001	0.0005	0.08	0.0006
Total	2.69	1.40	97	4.10

SUMMARY AND CONCLUSIONS

The cross-section simulation–volume-modeling approach applied to this assessment of glacier volumes was labor- and computer-intensive and the results have been assigned an error that is only slightly smaller than was assigned to the less demanding procedures developed by Driedger and Kennard (1986b). However, the volume estimated by the Driedger and Kennard method for the reference reach of Drift glacier was significantly less accurate than the modeled volume. The inaccuracy of the Driedger and Kennard method indicates that a volume model may be a more robust tool for volume analysis. However, more verification is needed.

The volume model in this report is a computer-based interpolation (surface-model generation) of a widely accepted valley cross-sectional form. This approach is expected to be broadly applicable to valley glaciers. If so, the approach will augment the study of valley glacier systems because the distribution of glacier volume with aspect and altitude is poorly known. Improved knowledge of the glacier-volume distribution with altitude and aspect would be useful for studies of glacier flow, hazard analysis, and for predicting the long-term consequences of global change, under whose influence substantial glacier-volume changes are expected.

While applying the volume model, at least in the two cases where valley-wall control for cross-section simulation was good, the simulated cross sections were not altered by addition of ice-thickness measurements. When using this volume model, ice-thickness data are necessary only where valley walls do not extend far enough above the glacier surface to provide a basis for simulating the cross-section shape. Availability of improved software and the possibility of customized programming for cross-section simulation promise to significantly reduce the amount of manual manipulation required for producing volume-modeling products.

The volume of perennial snow and ice on Redoubt Volcano is 4.1±0.8 km^3. This is similar to the 4.41 km^3 glacier volume reported for Mount Rainier, Washington (Driedger and Kennard, 1984 and 1986a,b) and almost 23 times more glacier volume than was on Mount St. Helens (Brugman and Meier, 1981) before its 1980 eruptions. The geohydrologic hazards arising from the 1980 eruption of Mount St. Helens are well documented. Projecting similar hazards in the major drainages surrounding Redoubt Volcano and other glacier clad volcanoes is reasonable.

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