

In cooperation with the
MONTANA DEPARTMENT OF NATURAL RESOURCES AND CONSERVATION

Characteristics of Extreme Storms in Montana and Methods for Constructing Synthetic Storm Hyetographs

Water-Resources Investigations Report 98-4100



**U.S. Department of the Interior
U.S. Geological Survey**

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By Charles Parrett

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U.S. Department of the Interior

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CONVERSION FACTORS

Multiply	By	To obtain
inch	25.4	millimeter
square mile	2.59	square kilometer

Abbreviations:

PMP	Probable Maximum Precipitation
NWS	National Weather Service

DEFINITIONS

Because of the complex nature of storms, some terms commonly used to define storm characteristics or statistics may be confusing or may have different meanings in different contexts. Meteorological or statistical terms, as used in this report, that may be confusing are explicitly defined below. Other terms are defined in the text or are considered to have relatively clear meanings as used in standard texts.

Annual precipitation maximum--The largest precipitation amount for a specified duration of precipitation (2-, 6-, or 24-hours) for each year of precipitation record at a site. Thus, an annual precipitation maximum for a 2-hour duration is the largest precipitation amount for any contiguous 2-hour period during that year.

Depth-duration curve--A mass curve that shows cumulative precipitation depth for increasing incremental duration. Depth-duration curves are constructed by determining the maximum precipitation depths for each incremental duration such that the maximum depth for the previous incremental duration is included. For example, the depth for the 2-hour incremental duration would be the maximum depth for any 2-hour period that also includes the maximum 1-hour depth. Plotting the maximum depth on the ordinate and incremental duration on the abscissa for each successive incremental duration results in a generally concave-upward curve. If the precipitation data are made dimensionless by dividing the depths for each incremental duration by the total depth for the storm duration, the resulting dimensionless depth-duration curve is particularly useful for storm-to-storm comparisons.

Duration--In the broadest sense, a interval of time during a storm within which precipitation amounts are compiled. Throughout this report, several terms containing the word "duration" are used. Each has a specific meaning as given below.

Incremental durations--Incremental time periods within the total storm duration during which precipitation depths are compiled. For example, a 24-hour storm (total duration = 72 hours) typically has precipitation depths compiled for 1, 2, 3, 6, 9, 12, 18, 24, 36, 48, 60, and 72 hours.

Kernel duration--An incremental duration that is used as a basis for calculating dimensionless precipitation depths for other incremental durations in the construction of probabilistic depth-duration curves. The kernel duration is selected somewhat arbitrarily, depending upon whether peak discharge or flood volume is the most important runoff characteristic being modeled. If peak discharge is the most important, maximum precipitation depth for some incremental duration less than the storm duration is considered likely to produce the maximum peak discharge, and the kernel duration is selected to be 30 minutes, 2 hours, or 6 hours for storm durations of 2-, 6-, and 24-hours, respectively. Flood volume is likely to be the most important runoff characteristic only for 24-hour storms, and the kernel duration for maximum flood volume production was selected to be 48 hours. Precipitation depths for all incremental durations other than the kernel duration are based on regression relations with depths for the kernel duration.

Storm duration--The length of time from beginning to end of storm. For purposes of this report, all storms are categorized as 2-, 6-, or 24-hour duration storms. Storms were initially placed into one of three categories based upon the exceedance probability of the total storm depth for each duration. Thus, for example, if the exceedance probability for total storm depth was less for the 2-hour duration than for either the 6-hour or 24-hour duration, the storm was initially categorized as a 2-hour duration storm. Storms were further examined to determine whether the actual periods of significant precipitation were reasonably close to the durations initially selected. If not, storms were then moved to the duration category closest to the actual period of significant precipitation.

Total storm duration--A period of precipitation activity, including the storm duration, that is 3 times longer than the storm duration. Use of a precipitation period longer than the storm duration enables periods of precipitation that occur before and after the storm of interest to be included in rainfall-runoff modeling. The storm duration of interest, either 2-, 6-, or 24-hours, within the total storm

duration is also termed independent duration. All incremental durations within the total storm duration, except for the storm duration, are termed dependent durations.

Exceedance probability--The probability that the total precipitation depth at any given site will be exceeded during any annual period. A total precipitation depth having an exceedance probability of 0.01 has only a 1 in 100, or 1.0 percent, chance of being exceeded in any year.

Extreme storm--For purposes of this report, a storm whose total precipitation depth has an exceedance probability of about 0.10 or less. This exceedance probability, while somewhat arbitrary, was selected to provide a reasonable balance between the competing needs, (1) to have a large number of storms for statistical analysis in the database, and (2) to have only very large, rare storms most like those required for dam-safety-analysis purposes in the database.

Macro-pattern--One of 12 different storm types depending upon the relative depths of precipitation that occur during each third of the total storm duration.

Probabilistic depth-duration curve--A dimensionless depth-duration curve based on exceedance probability. A probabilistic depth-duration curve is constructed by using regression relations to calculate dimensionless depths for various incremental durations from dimensionless depths for a specific, kernel duration. Dimensionless depths for the kernel duration for various exceedance probabilities are determined from application of the Beta distribution to actual storm data.

Recurrence interval--The reciprocal of exceedance probability and the average length of time in years between exceedances of a given storm depth. The recurrence interval for a storm depth having an exceedance probability of 0.01 is 100 years.

Storm hyetograph--A bar graph that shows the precipitation that occurs during each time increment of a storm.

Total storm depth--Precipitation amount for a specified duration of precipitation (2-, 6-, or 24-hours) for a specific storm. For most storms in the data base of storms for Montana, the total storm depth was the annual precipitation maximum for that duration. In general, the terms storm depth and precipitation depth are synonymous.

Trisector--Three successive equal time periods within the total storm duration. For a total storm duration of 72 hours (independent duration equals 24 hours), for example, the first trisector consists of the first 24 hour period.

Characteristics of Extreme Storms in Montana and Methods for Constructing Synthetic Storm Hyetographs

By Charles Parrett

Abstract

Data from 188 large storms in Montana and 2 in northern Wyoming were used to analyze the temporal characteristics of 2-hour, 6-hour and 24-hour duration (independent duration) storms in Montana and to develop methods for constructing synthetic storm hyetographs. Storm data were included in the data base if the precipitation depths had an exceedance probability of about 0.10 or less. Data were screened to ensure that each storm was spatially and temporally independent and grouped into three homogeneous regions that previously were delineated on the basis of physiography and climate. For each storm in the data base, storm depths were determined for various incremental durations within a time period of precipitation activity three times longer than the storm duration under study.

To enable the construction of synthetic storm hyetographs, various temporal characteristics were analyzed for each storm. First, storms were grouped into one of twelve possible patterns depending upon the distribution of precipitation depth in each third of the total storm duration. Second, the time from beginning of each storm to the time of maximum incremental precipitation depth, termed time-to-peak, was measured. A third temporal characteristic that was determined for each storm was the sequencing pattern of the three largest, adjacent increments of precipitation depth including the peak incremental depth. A final temporal characteristic that was measured and analyzed was the pattern of precipitation occurring in each successive 6-hour block of the 24-hour independent duration. Each 24-hour storm thus was classified into 1 of 24 possible storm patterns.

Maximum precipitation depths for various incremental durations less than the total duration were calculated for each storm. The depths for the incremental durations were divided by the precipitation depth for the independent duration to produce dimensionless depth-duration data.

Dimensionless depth-duration data were grouped by independent duration and by region, and the Beta

distribution was used to determine dimensionless depths for exceedance probabilities from 0.1 to 0.9 for various dependent durations up to the total storm duration. The Beta distribution also was fit to the time-to-peak data, and estimates of time-to-peak for exceedance probabilities ranging from 0.1 to 0.9 were made for each independent duration within each region. Ordinary least-squares regression was used to develop relations between dimensionless depths for a key short duration related to modeling peak discharge, termed the kernel duration, and dimensionless depths for all dependent durations for each independent duration in each region. The regression relations were used, together with the probabilistic dimensionless depth data for the kernel duration, to develop dimensionless depths for exceedance probabilities from 0.1 to 0.9 for the dependent durations for each independent duration within each region.

For use in rainfall-runoff modeling where runoff volume is the primary consideration rather than peak discharge, probabilistic, dimensionless depth-duration data for 24-hour storms were developed from a 48-hour kernel duration rather than a 6-hour kernel duration. Accordingly, regression relations were developed between dimensionless depths for the 48-hour duration and every other dependent duration for 24-hour duration storms in each region. The regression relations were used, together with the probabilistic dimensionless depth data for the 48-hour kernel duration, to calculate dimensionless depths for exceedance probabilities from 0.1 to 0.9 for the dependent durations for each independent duration within each region.

Methods for constructing synthetic storm hyetographs by combining dimensionless, probabilistic depth-duration data with a depth-area adjustment factor, various temporal characteristics, and precipitation depth for the independent duration for a specified recurrence interval were described. Hyetographs for modeling peak discharge under "median-value" conditions are based on depth-duration data and time-to-peak values having a 0.5 exceedance probability and typical temporal characteristics. Hyetographs for

peak-discharge modeling under "design-purpose" conditions are based on depth-duration data and time-to-peak values having a 0.2 exceedance probability and rarer, larger peak-discharge-producing temporal characteristics.

INTRODUCTION

The design of spillways for new dams or the evaluation of existing spillways for dam-safety investigations requires the use of rainfall-runoff models to simulate the flood runoff from very large, rare storms. Where the risk to human life would be large in the event of dam failure, spillways are commonly designed to safely pass the flood runoff from the Probable Maximum Precipitation (PMP) storm under "worst-case" antecedent-moisture and infiltration conditions. Determination of the PMP storm is fairly complex and is based on the extrapolation of data from the largest storms known to have occurred in broadly defined regions of the country that are presumed to be meteorologically similar. Although procedures for estimating PMP storms are well-documented and in wide use, estimates of PMP total depth and its temporal distribution (storm hyetograph) may be controversial, particularly for sites lacking evidence of large historic storms and floods. The controversy is difficult to address because PMP total depth is not based on exceedance probability and thus not comparable from site to site. In addition, methods commonly used to distribute the PMP total depth over the storm duration are based on maximization procedures and may not reflect temporal characteristics of actual storms in the region under study. Without knowing the probability of exceedance of the PMP storm and whether the temporal distribution of the precipitation is reasonable for the area under study, accurate assessment of risk is not possible.

Purpose and Scope

To allow dam-safety and design engineers to better evaluate risk of dam failure and establish a more consistent basis for spillway design, a two-phase study was undertaken by the U.S. Geological Survey in Montana, in cooperation with the Dam Safety Section of the Montana Department of Natural Resources and Conservation. The objective of the first phase was to provide methods for estimating precipitation depth-frequency relations for sites in Montana. To that end, a regional analysis of annual precipitation maxima

resulted in a method for estimation of total precipitation depths for exceedance probabilities as low as 0.0002 (Parrett, 1997). This method, based on previous work in Washington State (Schaefer, 1990), enables dam-safety and design engineers to use a probability-based alternative to the PMP design approach.

The objectives of the second phase of the cooperative study were to (1) analyze the temporal characteristics of large storms in Montana, and (2) develop methods for constructing probabilistic synthetic storm hyetographs that reflect temporal characteristics of observed storms in the study area. The purpose of this report is to describe results of the second phase of the cooperative study. The analyses were conducted for three homogeneous regions within which precipitation characteristics were considered to be reasonably uniform and distinct.

Acknowledgments

Melvin G. Schaefer, Washington Department of Ecology, whose earlier work in Washington formed the basis for the Montana study, provided timely and helpful advice and encouragement to the author regarding data analysis and presentation. His assistance also was invaluable and is gratefully acknowledged.

Dave R. Johnson, U.S. Geological Survey, assisted with data compilation and analysis throughout the project. When the voluminous data were most daunting, Dave was able to quickly develop innovative computer routines for efficient data management. His computer expertise and insightful commentary were invaluable to the author and are greatly appreciated.

DATA BASE OF STORMS

Storm characteristics were analyzed using data from 188 large storms at 84 sites in Montana and 2 storms at 1 site in northern Wyoming. All data were from National Weather Service (NWS) recording precipitation stations that were used in the first phase of the study. Storms that produced precipitation in the form of snow were not included in the data base, because snow typically does not produce immediate runoff. Some large winter storms were included if temperature data indicated that rain was more likely than snow or if streamflow data indicated that runoff from the storms was significant.

Data were categorized into three storm durations (2-, 6-, and 24-hour) so that synthetic storm hyeto-

graphs could be developed for a wide range of basin sizes. Short-duration storms that cover smaller areas than longer duration storms generally are appropriate for rainfall-runoff modeling on small basins, whereas long-duration storms that cover large areas are more appropriate for modeling rainfall-runoff on large basins. In addition, the appropriate storm duration for rainfall-runoff modeling depends somewhat on the primary purpose of the modeling. If peak discharge is the primary runoff component, precipitation intensity may be more important than precipitation depth. For many reservoir projects, both peak discharge and flood runoff volume are important, and both precipitation intensity and depth need to be considered. To fully evaluate the runoff effects, several candidate storms representing various duration, intensities, and depth may need to be investigated.

A recorded storm was considered for inclusion in the data base only if the annual at-site exceedance probability of the total storm depth, based on the regional method developed during the first phase of the study, was about 0.10 or less. An exceedance probability of 0.10, although arbitrary, helped ensure that the analysis of temporal characteristics would be based on data from a reasonably large number of relatively large storms.

To ensure that the data base included only spatially independent storms, those having the same date of occurrence were examined, and only the storm having the smallest exceedance probability (rarest event) was retained in the data base. Where storms on the same date were at widely separated stations, they were subjectively considered to be independent storms and both were included. Because storm duration is related to basin size for rainfall-runoff modeling purposes, a storm was used for only one duration at a site even though it may have produced storm depths with small exceedance probabilities for more than one duration. Generally, a storm with depths having small exceedance probabilities for more than one duration was used for the duration that most closely matched the actual storm length. If the actual storm length was roughly midway between two of the three durations selected for study, it was assigned to the duration whose storm depth had the smallest exceedance probability. Overall, the data base of storms was considered to consist of relatively large storms that were both spatially and temporally independent. The precipitation stations, storm dates, and pertinent data are listed in table 23 at the

back of the report, and the location of each precipitation station is shown on figure 1.

REGIONAL AND SEASONAL STORM CHARACTERISTICS

The first phase of the study determined that the State could be divided into three generally homogeneous regions (fig. 1) for the purpose of estimating precipitation depth for various recurrence intervals. Homogeneous regions in Montana were delineated by Parrett (1997) primarily on the basis of physiography and climate. Thus, Region 1 contains the mountainous portions of western Montana that are primarily affected by storms moving east from the Pacific Ocean. Region 2 contains a relatively narrow band of mountainous terrain that forms the eastern edge of the Rocky Mountains and two small, isolated mountain ranges in central Montana. This region also is affected by storms from the Pacific. In addition, the mountains in this region form a prominent orographic barrier to large general storms that arise in and move generally north and west from the Gulf of Mexico. Some of the largest general storms and resultant floods in Montana have been in Region 2 as a result of the orographic lifting of the large, warm and moist air masses from the Gulf. Region 3, the large area of eastern Montana plains, also receives storm moisture from both the Pacific Ocean and the Gulf of Mexico. Region 3 is distinct from Regions 1 and 2 because of its lack of orographic barriers. Because these regional differences in topography and storm moisture source were considered likely to affect temporal characteristics as well as precipitation depths, the three regions were considered to be homogeneous for the purpose of determining temporal characteristics of large storms and developing regional depth-duration curves and synthetic storm hyetographs. This assumption generally was confirmed by differences observed in the depth-duration data in each region. The distribution of selected storms by duration and region is shown in table 1. The single precipitation station in Wyoming was considered to be in Region 3 for purposes of analysis.

Seasonality of 2-hour Storms

Storms having 2-hour durations, typically characterized as thunderstorms, most often are isolated convective storms that are likely to form anywhere in

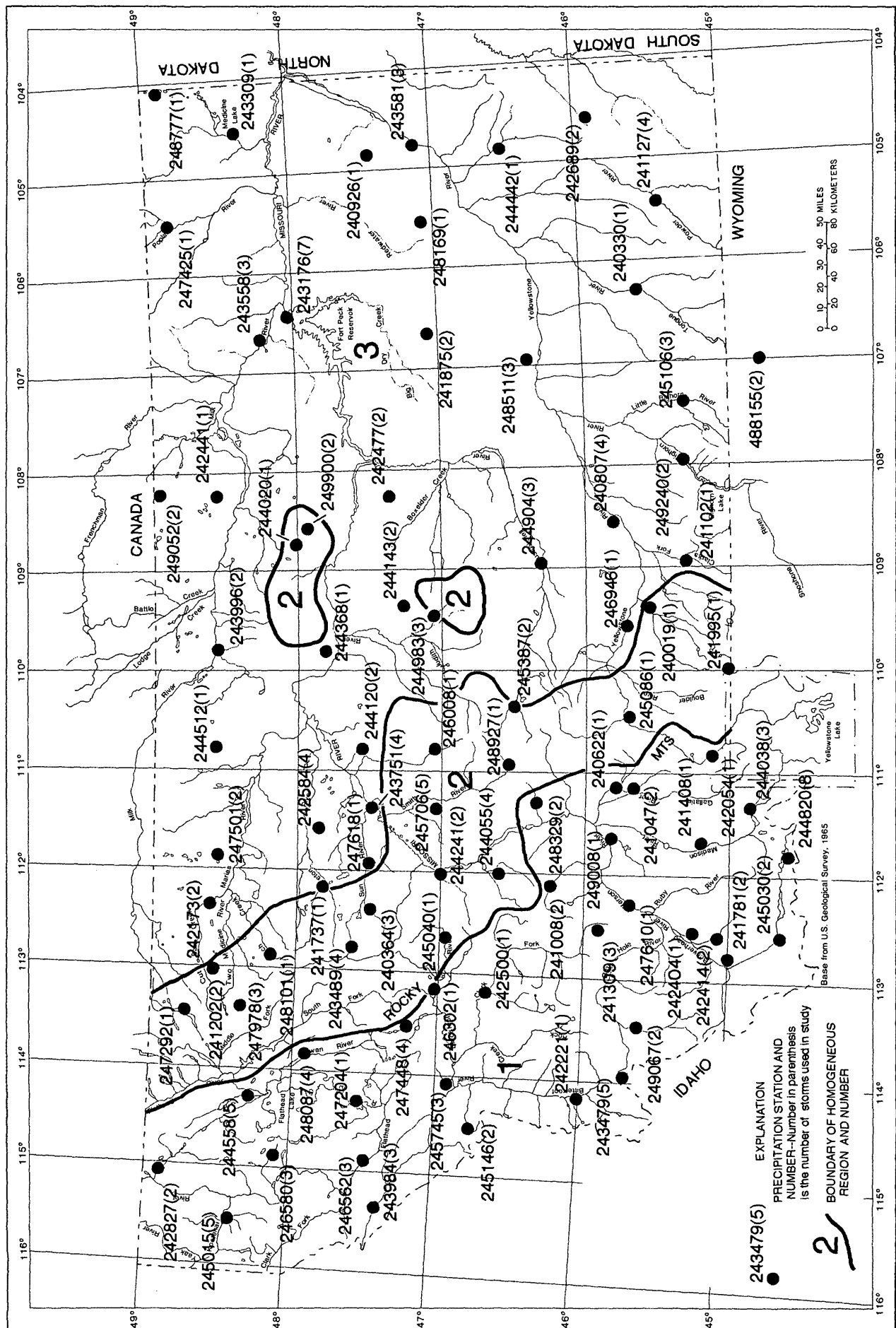


Figure 1. Location of precipitation stations, station number, number of storms used at each station, and regional boundaries, Montana and northern Wyoming.

Table 1. Number of storms analyzed by duration and region and number of precipitation stations by region, Montana and northern Wyoming

Region	Number of storms having indicated duration, in hours			Total	Number of precipitation stations
	2	6	24		
1	36	20	21	77	30
2	11	11	14	36	18
3	30	13	34	77	37
Total	77	44	69	190	85

Montana. Figure 2A shows the distribution of 2-hour duration storms by month for all regions of Montana. As indicated by figure 2A, 2-hour duration storms are most common during the warmer periods of spring and summer (May through September), although they occasionally occur during early spring (March) in the western half of the State (Regions 1 and 2). As shown in table 1, the number of 2-hour storms in the data base is largest in Region 1 and smallest in Region 2, the smallest region. The distribution of 2-hour storms among regions is about the same as the distribution of precipitation stations among regions (table 1), indicating that 2-hour storms generally are evenly distributed throughout the State.

Seasonality of 6-hour Storms

Storms having 6-hour durations may be the result of isolated, convective storm activity that is particularly long-lasting or the result of large general storm systems moving through Montana from the Pacific Ocean or the Gulf of Mexico. As a result, 6-hour duration storms may have characteristics of either 2-hour storms or 24-hour storms. Figure 2B, which illustrates the distribution by month and region, indicates that 6-hour storms also are most common during spring and summer. In the western half of the State, however, the percentage of total storms that occur during fall and winter (October through February) was somewhat larger than for 2-hour duration storms. As shown in table 1, only 13 storms in the 6-hour duration data base were in Region 3 in eastern Montana. Thus, 6-hour duration storms, which may be a complex, mixed population of both convective and general storms, are significantly more common in the mountainous areas of Montana than in the plains.

Seasonality of 24-hour Storms

Storms having 24-hour durations typically are general storms that commonly occur over large areas as they move through Montana. These storms may arise in either the Pacific Ocean or the Gulf of Mexico. Storms from the Pacific Ocean, which commonly occur from late fall to early spring (November through March) have a greater effect on western Montana (Region 1) than on central or eastern Montana (Regions 2 and 3). Accordingly, as shown in figure 2C, Region 1 has a greater percentage of November through March storms in the 24-hour data base than either Region 2 or 3. As described in the previous report on the regionalizing of annual precipitation maxima (Parrett, 1997), April through October storms from the Gulf occasionally produce very large precipitation depths when they are uplifted over the eastern edge of the Rocky Mountains (Region 2). At the same time, the eastern edge of the Rocky Mountains seems to protect Region 1 from large storms from the Gulf, so that maximum 24-hour storm depths in Region 1 are substantially smaller than those in either Regions 2 or 3. Region 3 commonly experiences general storms from the Gulf, but the lack of orographic barriers results in maximum storm depths that are less than in Region 2. As indicated in figure 2C, large 24-hour storms in Regions 2 and 3 are most likely in May and June.

The seasonal distribution of storms can affect rainfall-runoff modeling in two important ways. First, base flows may vary seasonally. For example, large storms in late spring during snowmelt runoff may result in substantially larger peak flows than similar large storms that occur when streamflow typically is low. Second, the infiltration characteristics of a basin may have large seasonal variation. A large storm in early spring when ground generally is frozen may result in larger peak flows than a similar storm later in the year when substantial portions of total rainfall may infiltrate

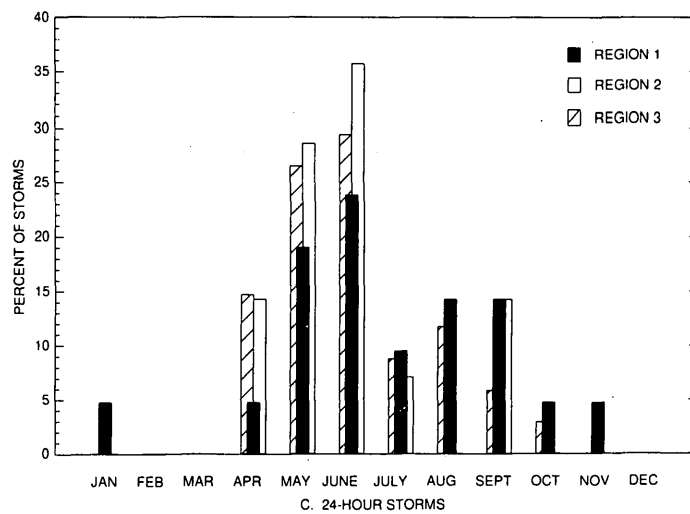
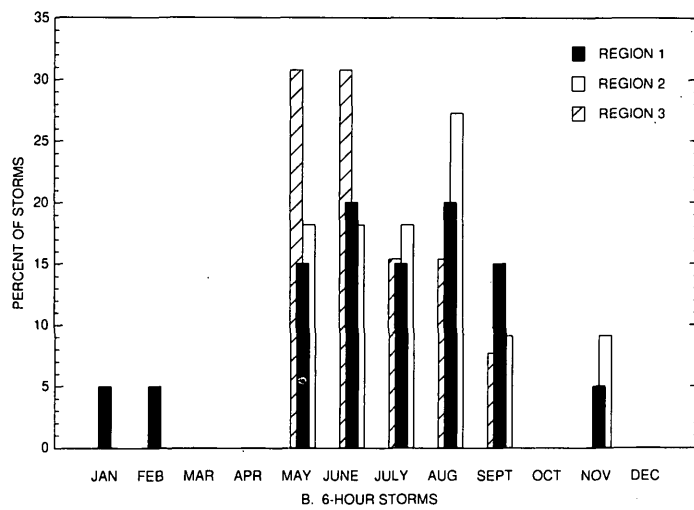
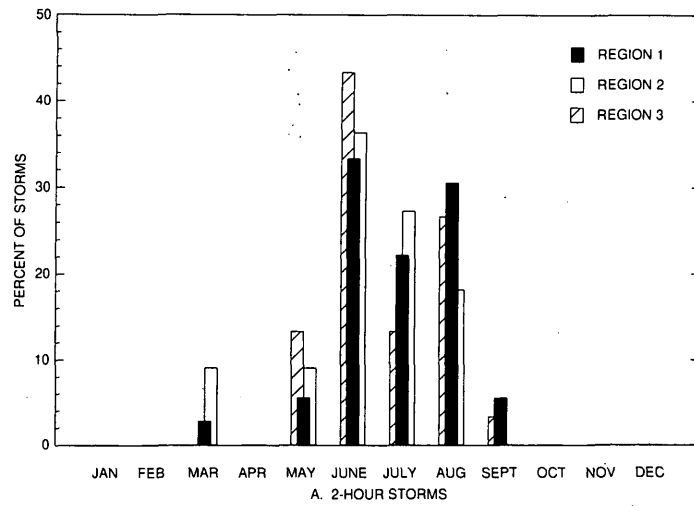


Figure 2. Storm distribution by month, Montana and northern Wyoming.

rather than produce runoff. Accordingly, seasonal variations in storm occurrence need to be considered in the application of design storms used for rainfall-runoff modeling.

TEMPORAL CHARACTERISTICS

Storms are inherently complex with widely varying patterns of precipitation depth that occur throughout the time periods used to categorize the storms. Various terms that are used to describe the temporal and probabilistic characteristics of storms in this report are explained in the "Definitions" section of the report. Terms not included in the "Definitions" section are considered to be relatively clear in meaning, either from the report context or from standardized texts.

Because storm intensity may change rapidly during any storm, analysis of storm characteristics requires storm-depth data at incremental time periods short enough to accurately describe the storm hyetograph throughout the storm duration. For 24-hour duration storms, storm depths at hourly time increments generally are considered adequate for hyetograph definition. Analysis of storms having shorter durations requires storm depths at proportionately shorter time increments for adequate hyetograph definition. Thus, for storms having 2-hour and 6-hour durations, analysis requires depths at 5-minute and 15-minute intervals, respectively. Most recording rain gages currently operated by NWS compile precipitation depth at 15-minute intervals. Only a few NWS stations compile precipitation depth at 5-minute intervals. Data from these few stations were used to develop general relations between 5- and 15-minute and 10- and 15-minute depths.

Determination of Total Storm Duration and Macro-pattern Type

To analyze the temporal characteristics of each storm in the data base, beginning and ending times based on the time increments described previously needed to be uniformly determined. In addition, because runoff from a storm of some specific duration might be enhanced by precipitation occurring before or after the specific duration, a time period of precipitation activity somewhat longer than the storm duration was used. Following the method used by Schaefer (1989) in Washington State, the longer period of precipitation activity, termed total storm duration, was

arbitrarily selected as three times the storm duration. To clarify differences in duration, the 2-, 6-, and 24-hour durations were termed independent durations. They are durations for which precipitation depth-frequency information was previously developed in the first phase of the study. Thus, a total storm duration of 6 hours was examined for a storm having an independent duration of 2 hours, a total storm duration of 18 hours was examined for a storm having an independent duration of 6 hours, and a total storm duration of 72 hours was examined for a storm having an independent duration of 24 hours. The total storm duration was determined such that: (1) the beginning time increment for the total storm duration had precipitation, (2) the total duration completely contained the independent duration of the storm under consideration, and (3) the total duration contained the maximum depth of precipitation for any comparable time period containing the independent duration. For example, for a storm with an independent duration of 2 hours, the total duration had to: (1) begin during a 5-minute period of precipitation, (2) fully contain the 2-hour interval of maximum storm depth, and (3) be the 6-hour time interval having the maximum precipitation depth of any 6-hour interval containing the 2-hour independent duration.

Once the total storm duration was determined for each storm in the data base, it was divided into equal thirds, termed "trisectors" by Schaefer (1989), so that general patterns of precipitation distribution throughout the total storm duration could be determined. These patterns, termed "macro-patterns" by Schaefer (1989), helped to characterize and group storms for further temporal analysis. As shown in figure 3, storms were classified into 12 different macro-patterns depending upon the relative storm depth in each trisector and whether or not precipitation was continuous throughout the total storm duration. For each macro-pattern, the trisectors are given numbers from 1 (largest) to 3 (smallest) that indicate the relative depth in each trisector. For example, macro-pattern 2 has a trisector sequence of 132 which indicates that the first trisector contains the largest storm depth, the second trisector contains the third largest depth, and the third trisector contains the second largest depth. An example storm hyetograph for one of the storms in the data base having a 24-hour independent duration (fig. 4) shows the total storm duration and the distribution of storm depth in each trisector. Thus, comparison of the hyetograph in figure 4 with the macro-patterns in figure 3 indicates that the example storm has a macro-pattern

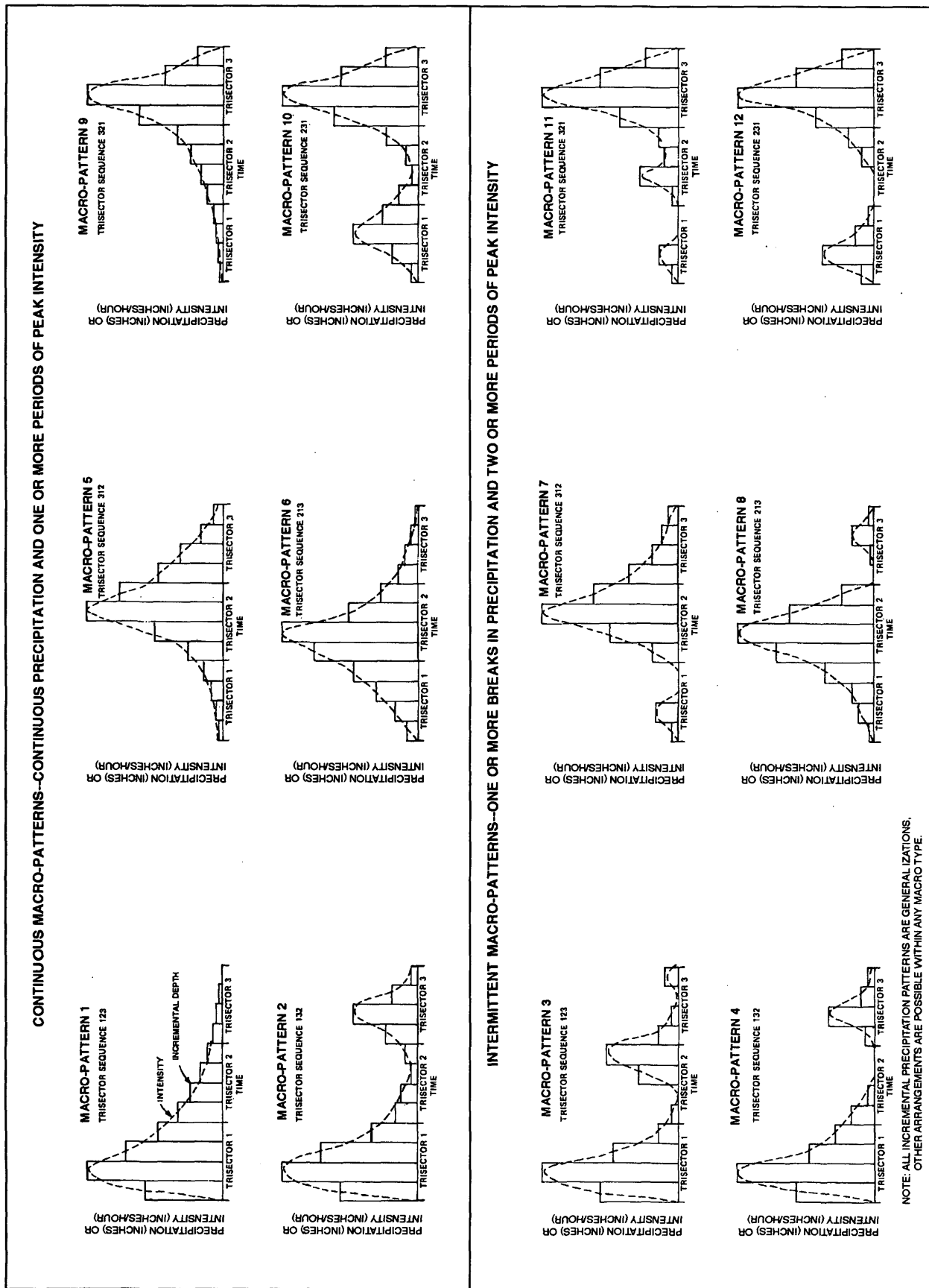


Figure 3. Different storm macro-patterns (modified from Schaefer, 1989).

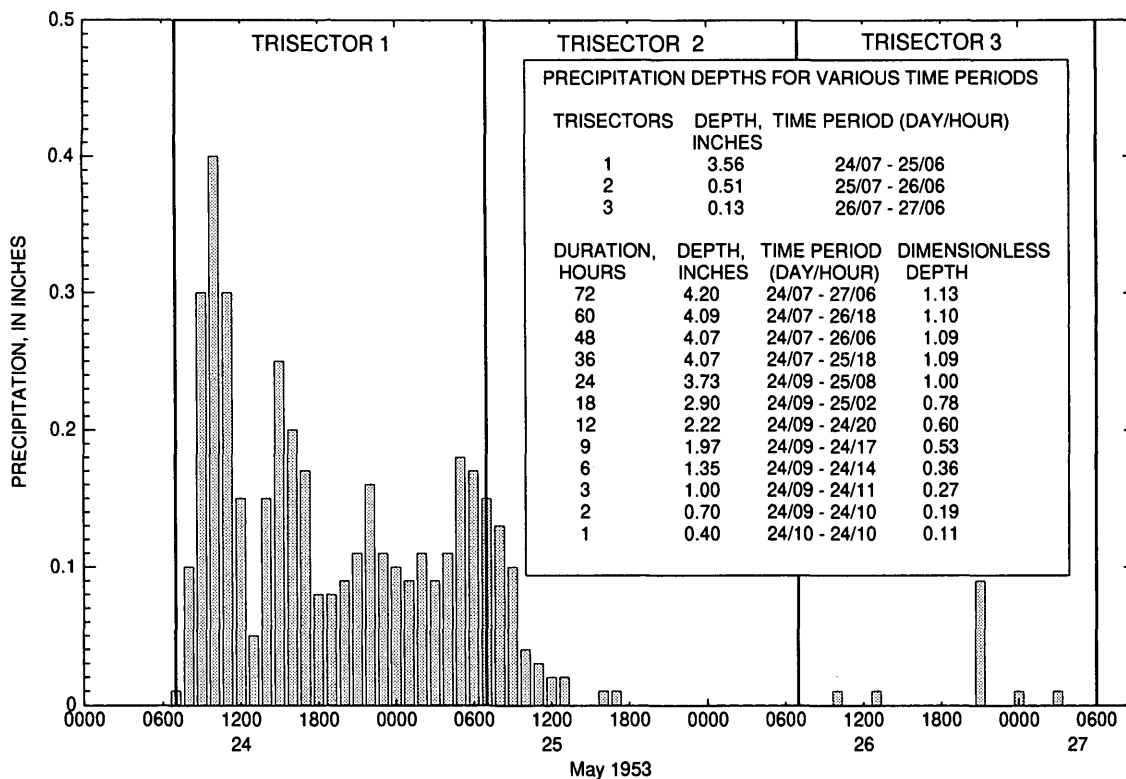


Figure 4. Hyetograph for storm of May 24, 1953 at station 244020, Hays, Montana, Region 2, 24-hour independent duration.

3 (intermittent precipitation, trisector sequence 123). All storms in the data base were thus classified and the distribution of macro-pattern by region and by duration is shown in figure 5. As indicated by figure 5, macro-pattern 1 was the most common for all regions for the 2- and 6-hour duration storms. For both Regions 2 and 3, macro-pattern 1 was also the most common pattern for 24-hour duration storms, whereas macro-pattern 3 was the most common for Region 1. Macro-patterns 1 and 3 are similar in that the trisector sequence for both is 123. The distinction between macro-patterns 1 and 3 sometimes was subtle and somewhat arbitrary because of the occurrence of very short periods of no precipitation during otherwise continuous periods of precipitation. Figure 5 also indicates that 2-hour duration storms were much more likely to have the same macro-pattern than either 6- or 24-hour duration storms.

Timing of Peak Precipitation Intensity

The temporal characteristic of a storm that probably has the greatest effect on peak discharge is the time from the beginning of the storm to the time of peak precipitation intensity. For this study, this "time-to-peak" variable was measured for each storm from the time at

the beginning of total storm duration to the time of maximum incremental storm depth. The variability in time-to-peak among regions for each independent duration is illustrated by boxplots in figure 6. The probabilistic nature of time-to-peak and the effects on synthetic storm construction will be discussed in a later section of this report.

Sequencing Pattern of High-Intensity Incremental Storm Depths

A temporal characteristic of a large storm that is related to the time-to-peak and that may also have an effect on peak discharge is the sequencing of the three largest, adjacent increments of storm depth that includes the peak incremental storm depth. The characterization of these three largest increments is similar to that for storm macro-patterns. For example, the three largest increments of storm depth that includes the peak value for a hypothetical storm shown in figure 7A are the two increments preceding the peak value and the peak value. For this example, the first of the three increments is the smallest, the second increment is the next largest, and the third increment (peak value) is the largest. This sequencing of increments from the smallest to largest in order is characterized as a 321 high-intensity storm pattern. Figure 7B shows a storm

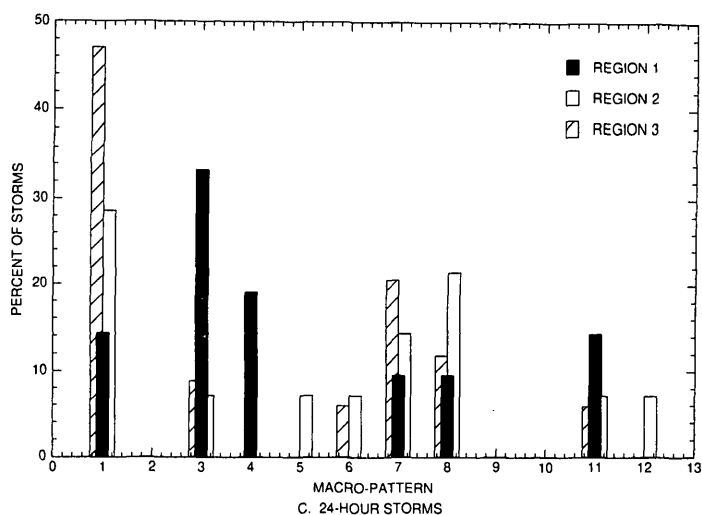
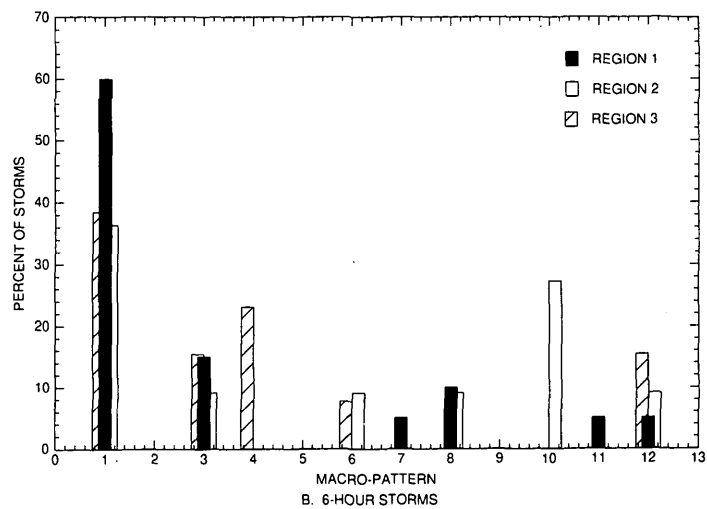
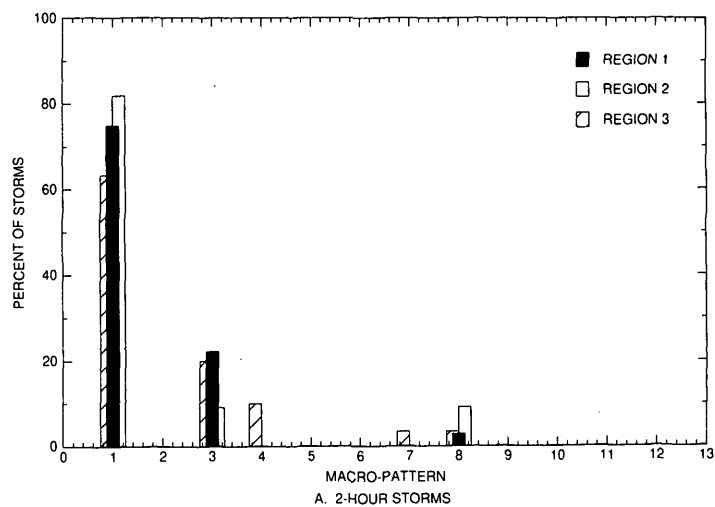


Figure 5. Storm distribution by macro-pattern type, Montana and northern Wyoming.

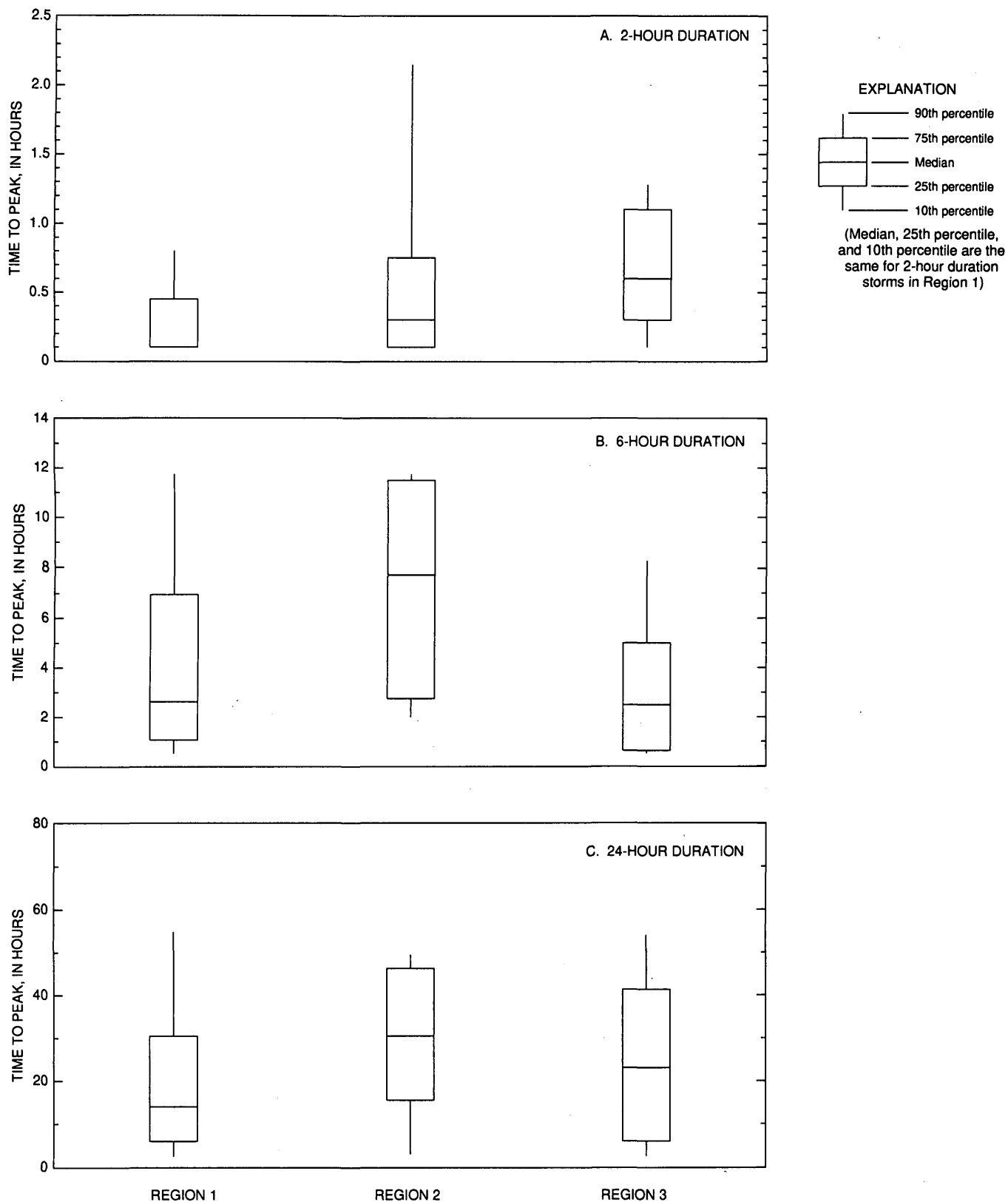
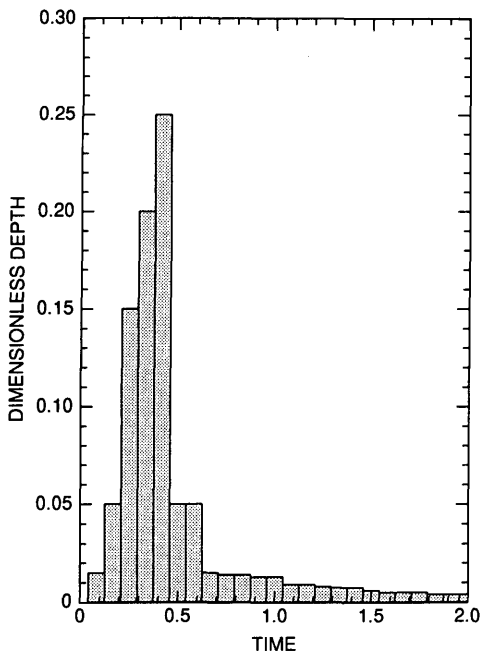
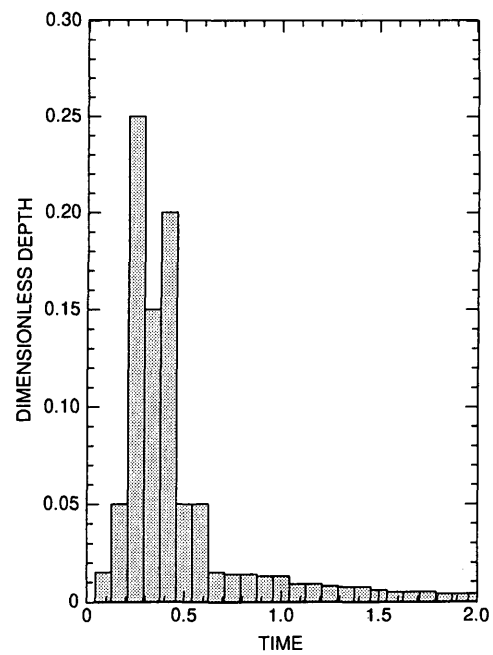


Figure 6. Variability in time-to-peak precipitation intensity, Montana and northern Wyoming.



A. HIGH-INTENSITY PATTERN = 321



B. HIGH-INTENSITY PATTERN = 132

Figure 7. Hypothetical storm hyetographs with different high-intensity patterns.

with the peak value followed by the third largest increment followed by the second largest increment; this sequencing of high-intensity storm segments is characterized as a 132 high-intensity storm pattern. Following this method for characterization of high-intensity incremental storm depths results in the following 6 possible patterns: 123, 132, 213, 231, 312, and 321. On this basis, the high-intensity incremental storm depths were characterized for each 6-hour and 24-hour storm in the data base, and the results are shown in table 2. Characterization of high-intensity incremental storm depths for 2-hour storms was based on data from 28 storms at 9 sites where 5-minute incremental depth data were available. Inasmuch as most of these 28 storms had 123 high-intensity storm patterns, the 123 pattern was used for all 2-hour storms in the data base.

The effects of the sequencing of high-intensity storm segments are not so clearly related to peak discharge as the timing of the peak precipitation intensity. Thus, the probabilistic nature and effects of the various high-intensity storm patterns on peak discharge will be considered in a general way in a later section describing synthetic dimensionless storm hyetograph construction.

24-Hour Storm Patterns

For 2-hour and 6-hour storms, the only two temporal characteristics that were considered to have a significant effect on runoff peaks were time-to-peak and the high-intensity storm pattern. For the longer duration 24-hour storms, Schaefer (1989) also considered the precipitation pattern of the entire 24-hour independent duration to have a potentially significant effect on peak discharge. Accordingly, he developed a 24-hour storm pattern classification by summing the precipitation for each of the four 6-hour periods comprising the 24-hour duration. Each successive 6-hour period was then assigned a ranking from 1 (largest) to 4 (smallest) based on the relative total precipitation during that period. For example, if the first 6-hour period had the smallest precipitation total, the second period had the largest total, the third period had the second largest total, and the last period had the third largest total, the 24-hour storm pattern would be classified as a 4123. Thus, there are 24 possible 24-hour storm patterns, and the relative effects of many on peak discharge are subtle and probably not discernible. On the whole, however, the 24-hour storm patterns having the two largest precipitation periods adjacent to each other and occurring near the end of the storm would reasonably be expected to produce larger peak discharge than other

Table 2. High-intensity storm patterns for 6-hour and 24-hour storms in Montana and northern Wyoming

[Percentages for all patterns may not add to 100 due to independent rounding]

High-intensity pattern	Region 1				Region 2				Region 3			
	6-hour		24-hour		6-hour		24-hour		6-hour		24-hour	
	No. of storms	Per-centage of storms	No. of storms	Per-centage of storms	No. of storms	Per-centage of storms	No. of storms	Per-centage of storms	No. of storms	Per-centage of storms	No. of storms	Per-centage of storms
123	1	5	6	29	4	36	5	36	2	15	11	32
132	0	0	2	9	2	18	2	14	0	0	1	3
213	5	25	2	9	0	0	4	29	2	15	9	26
231	1	5	2	9	1	9	0	0	0	0	3	9
312	3	15	3	14	1	9	2	14	4	31	4	12
321	10	50	6	29	3	27	1	7	5	38	6	18

patterns. On this basis, such 24-hour storm patterns as 4321 and 3421 can be expected to produce higher run-off peaks than 1234, 1243, or 1324. Each 24-hour storm in the data base for Montana was thus classified and the data grouped depending on which 6-hour period had the largest precipitation amount. These results, together with the percentage of storms having the two largest precipitation periods adjacent to each other and the most common pattern in each region, are shown in table 3.

As shown in table 3, more than half the storms in Regions 1 and 3 had the largest precipitation amount in either the first or second period. In Region 2, more than half the storms had the largest precipitation amount in either the second or third period. In addition, more than half the storms in Regions 2 and 3 and more than one-third in Region 1 had the two periods of largest precipitation amount adjacent to each other.

Dimensionless Depths and Dependent Durations

After general classification of each storm on the basis of macro-pattern type, incremental precipitation depths were made dimensionless by dividing each by the total storm depth for the independent duration under study. For a 24-hour duration storm, for example, each hourly storm depth for the 72 hours in the total storm duration was divided by the maximum 24-hour storm depth. Maximum dimensionless depths for selected incremental durations less than or equal to the total storm duration were then compiled. For 2-hour duration storms, dimensionless depths were compiled for the following incremental durations: 5-minute, 10-minute, 15-minute, 30-minute, 45-minute, 60-minute, 90-minute, 2-hour, 3-hour, 4-hour, 5-hour, and 6-hour. For 6-hour duration storms, dimensionless depths were compiled for the following incremental durations: 15-minute, 30-minute, 45-minute, 1-hour, 2-hour, 3-hour, 6-hour, 9-hour, 12-hour, 15-hour, and 18-hour. For 24-

Table 3. 24-hour storm patterns for Montana and northern Wyoming

[Pattern group includes all patterns having the peak 6-hour period of precipitation first (1...), second (.1...), third (...1.), or fourth (...1.). Symbol: --, not applicable]

Pattern group ¹	Region 1		Region 2		Region 3	
	Number of storms	Percent of storms	Number of storms	Percent of storms	Number of storms	Percent of storms
1...	6	29	2	14	15	44
.1..	7	33	4	29	6	18
..1.	4	19	6	43	9	26
...1	4	19	2	14	4	12
Two periods of largest precipitation adjacent	--	34	--	71	--	56

¹Most common pattern in each region: Region 1, 4123 and 1432; Region 2, 4213; and Region 3, 1234.

hour duration storms, dimensionless depths likewise were compiled for the following incremental durations: 1-hour, 2-hour, 3-hour, 6-hour, 9-hour, 12-hour, 18-hour, 24-hour, 36-hour, 48-hour, 60-hour, and 72-hour. These selected incremental durations, except for the independent duration, are termed dependent durations because the dimensionless depths for each are based upon the independent duration precipitation depth. For ease in compiling and reading tables of data for 2- and 6-hour duration storms, all incremental durations in this report are expressed in terms of decimal hours rather than minutes. Thus, 5-minute and 10-minute durations are hereafter expressed to 3 significant figures as 0.0833-hour and 0.167-hour. Fifteen-minute, 30-minute, 45-minute, 60-minute, and 90-minute durations are expressed to 2 significant figures as 0.25-hour, 0.50-hour, 0.75-hour, 1.0-hour, and 1.5-hour durations, respectively.

Dimensionless Depth-Duration Curves

As shown in the table inset for the example storm of 24-hour independent duration on figure 4, the dimensionless depths are less than 1 for dependent durations less than 24 hours and are greater than 1 for

dependent durations greater than 24 hours. The dimensionless depths thus may be plotted against duration to produce a mass curve of precipitation, commonly termed a depth-duration curve, that shows the cumulative dimensionless precipitation for increasing duration. The dimensionless depth-duration curve for the example storm shown in figure 4 is shown in figure 8. Given the procedures used to plot dimensionless depth-duration curves, the slopes of the curves generally decrease with increasing duration as indicated by figure 8. For storms that have periods of uniformly increasing precipitation followed by periods of uniformly decreasing precipitation, the dimensionless depth-duration curves are smooth curves with continuously decreasing slopes. Because precipitation may occur sporadically during the course of a storm, however, the slope of a depth-duration curve, as indicated on figure 8, does not necessarily decrease smoothly and uniformly throughout. Unlike a storm hyetograph, which shows the incremental precipitation for each time increment from beginning to end of the storm and thus can have an infinite variety of shapes, the depth-duration curve always has the same general shape which facilitates storm to storm comparisons.

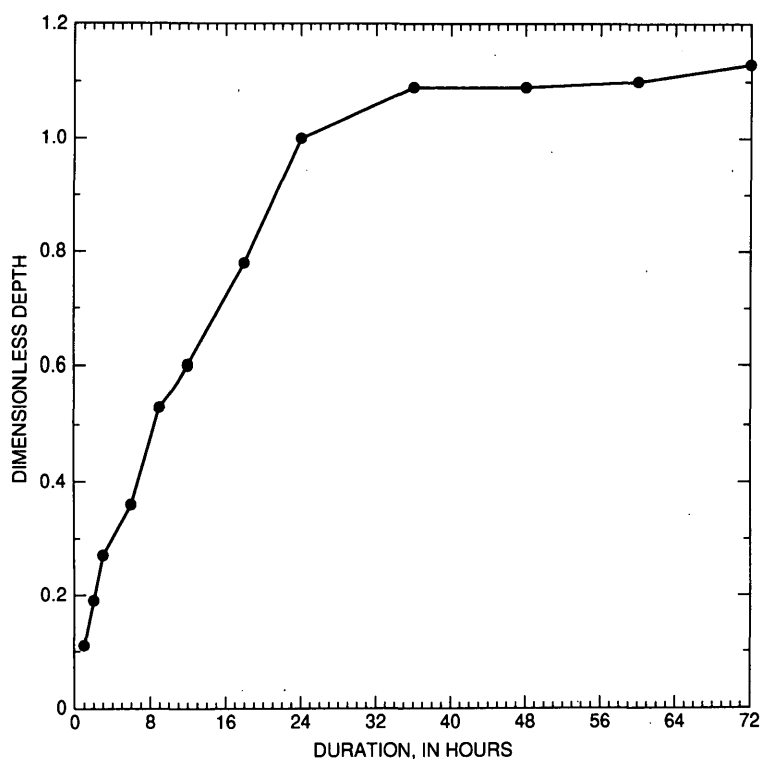


Figure 8. Dimensionless depth-duration curve for storm of May 24, 1953 at station 244020, Hays, Montana, Region 2.

PROBABILISTIC CHARACTERISTICS OF PRECIPITATION DEPTHS

The temporal distribution of precipitation at a given site can be viewed as a stochastic process. The magnitude and sequencing of incremental precipitation amounts within any storm constitute a time series of incremental precipitation. In that time series, the relation between successive incremental precipitation amounts can be described as having both deterministic and random components. A stochastic model could be developed and used to generate a large number of synthetic storms having the statistical characteristics of observed large storms. Such a stochastic model would need to be complex, however, and need to preserve both the deterministic and random elements of the precipitation process.

The approach taken here is to use a simpler method, based only on the correlation among incremental precipitation amounts, to develop probabilistic depth-duration curves that preserve some of the important statistical characteristics of observed storms. Specifically, this simpler method (1) preserves the cumulative precipitation depth for the independent duration, (2) preserves the variability observed for a key dependent duration (kernel duration) that is deemed important for rainfall-runoff modeling, and (3) provides expected values of cumulative precipitation for all other dependent durations given the magnitude of precipitation for the kernel duration.

Magnitude-Frequency Characteristics of Dimensionless Depths

Although storms can be compared qualitatively by examination of individual dimensionless depth-duration curves, qualitative comparisons do not pro-

vide information about storm-depth probabilities for the dependent durations. To provide probabilistic information, dimensionless depth data are grouped by independent storm duration (2-, 6-, and 24-hours) within each region, and a probability distribution fit to the data for each dependent duration within the total storm duration. Because all dimensionless storm depth values for the independent duration are always equal to 1, the probability distribution was not fitted to the independent-duration data. For all 2-hour duration storms in Region 1, for example, the probability distribution was separately fitted to the dimensionless depths for the following durations: 0.0833-hour, 0.167-hour, 0.25-hour, 0.50-hour, 0.75-hour, 1.0-hour, 1.5-hour, 3-hour, 4-hour, 5-hour, and 6-hour.

Following the method of Schaefer (1989), the Beta distribution was used for determination of the exceedance probabilities of dimensionless storm depths for this study. As described by Schaefer (1989) and Benjamin and Cornell (1970), the Beta distribution is a flexible distribution that can take on a wide variety of shapes; it is particularly well suited for describing random variables that have fixed lower and upper bounds.

For dimensionless storm depths, the lower and upper bounds are based on the characteristics of depth-duration curves and vary in a stair-step pattern (Schaefer, 1989). For dimensionless depths less than 1, the lower bound varies from 0 to 0.5 while the upper bound is fixed at 1. For dimensionless depths greater than 1, the lower bound is fixed at 1 while the upper bound varies from 2 to 3.

The probability density function for the Beta distribution is:

$$f(x) = \frac{\Gamma(\alpha + \beta)}{\Gamma(\alpha)\Gamma(\beta)} [(\delta_2 - \delta_1)^{1-\alpha-\beta}] (x - \delta_1)^{\alpha-1} (\delta_2 - x)^{\beta-1} \quad (1)$$

where

$f(x)$ is the probability density function;
 δ_1 and δ_2 are the lower and upper bounds, respectively;
 α and β are parameters of the distribution; and
 $\Gamma()$ is the Gamma function.

The parameters of the Beta distribution are related to the mean, variance, and skew coefficient of the distribution as follows:

$$\mu_x = \delta_1 + (\delta_2 - \delta_1) \left[\frac{\alpha}{(\alpha + \beta)} \right] \quad (2)$$

$$\sigma_x^2 = (\delta_2 - \delta_1)^2 \left[\frac{\alpha\beta}{(\alpha + \beta)^2 (\alpha + \beta + 1)} \right] \quad (3)$$

$$\gamma_x = \frac{\alpha}{\sigma_x^3 (\alpha + \beta)} \left[\frac{(\alpha + 2)(\alpha + 1)}{(\alpha + \beta + 2)(\alpha + \beta + 1)} - \frac{3\alpha(\alpha + 1)}{(\alpha + \beta)(\alpha + \beta + 1)} + \frac{2\alpha^2}{(\alpha + \beta)^2} \right] \quad (4)$$

where

μ_x , σ_x^2 , and γ_x are the mean, variance, and skew coefficient of the Beta distribution, and the other symbols are as previously defined.

The mean, variance, and skew coefficient were computed from the dimensionless depth data for each dependent duration in each region using the method of moments, and the distribution parameters were calculated using equations 2, 3, and 4. Quantile estimates for dimensionless depths for exceedance probabilities ranging from 0.9 to 0.1 then were obtained by numeri-

cally integrating equation 1. Exceedance probabilities less than 0.1 were not used because of the relatively small number of storms for each independent duration within each region. Results from the application of the Beta distribution to dimensionless depths for various dependent durations are shown graphically by duration and by region in figures 9 through 11.

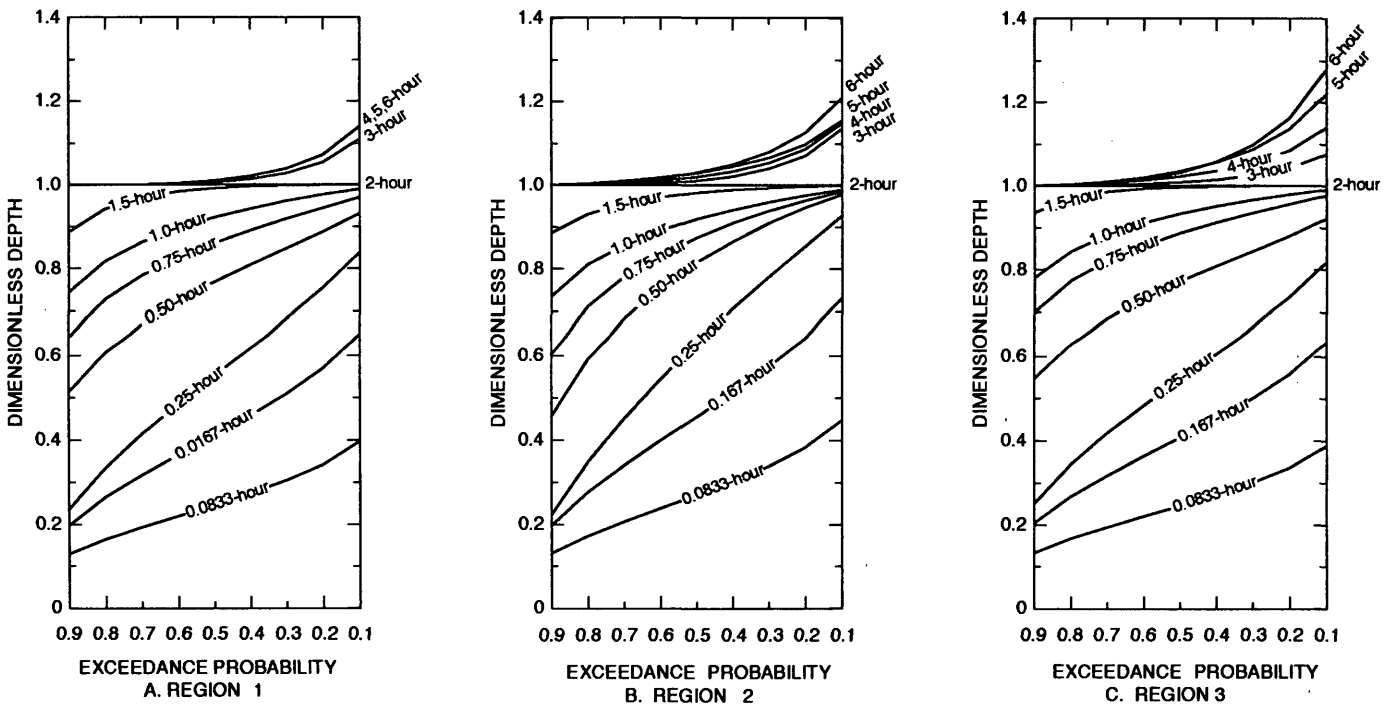


Figure 9. Magnitude-frequency characteristics of storms having independent duration of 2 hours, Montana and northern Wyoming.

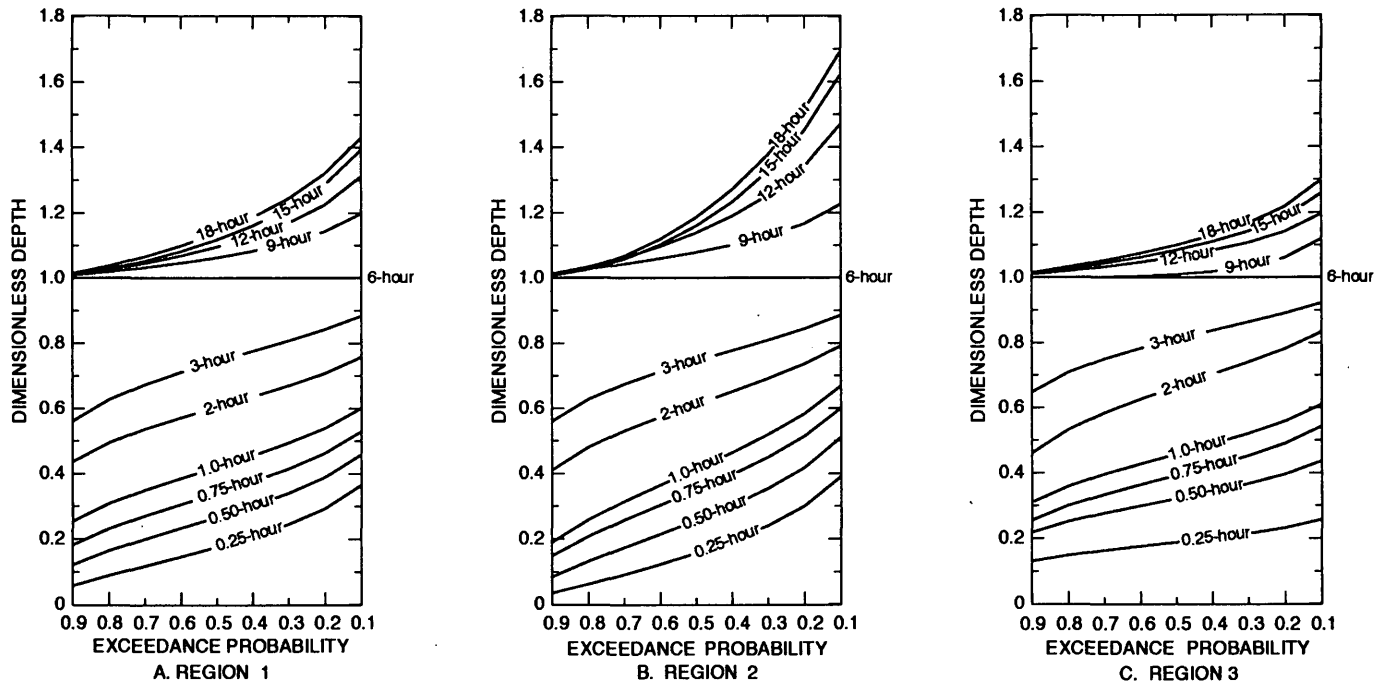


Figure 10. Magnitude-frequency characteristics of storms having independent duration of 6 hours, Montana and northern Wyoming.

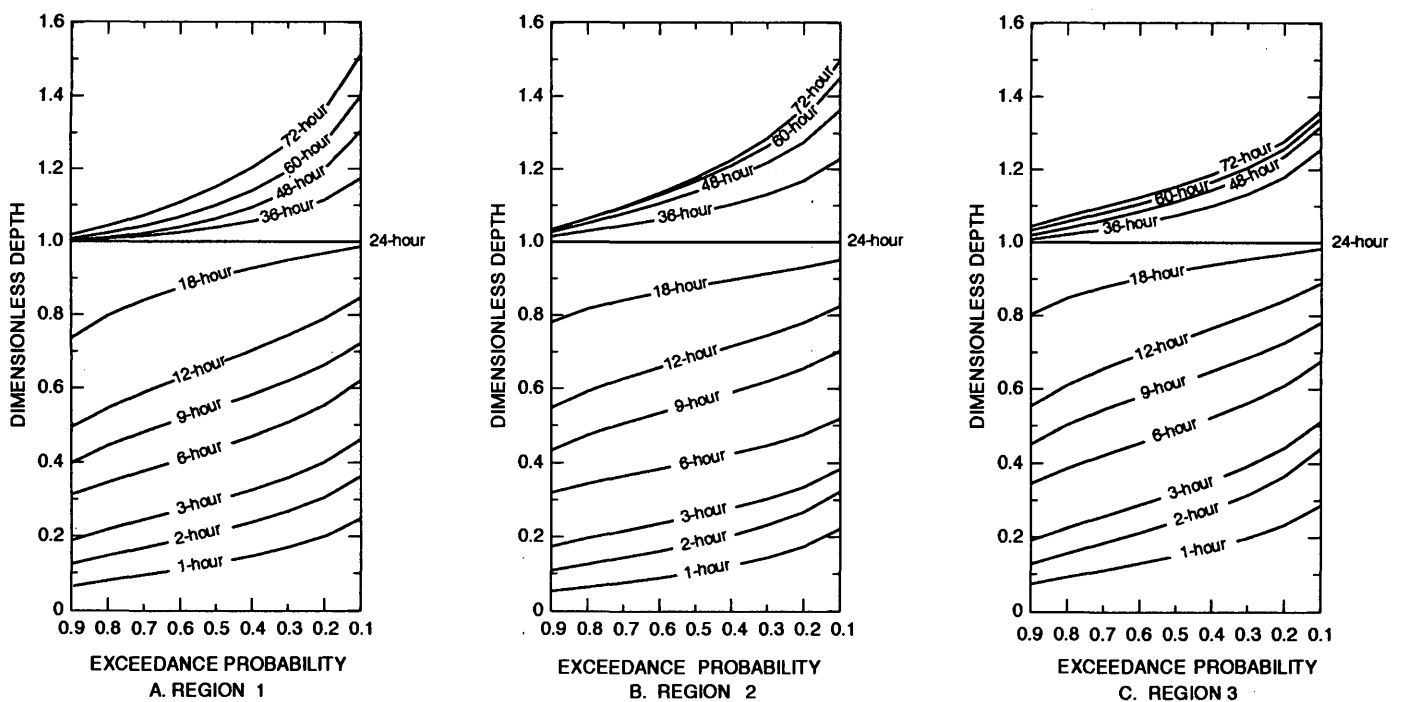


Figure 11. Magnitude-frequency characteristics of storms having independent duration of 24 hours, Montana and northern Wyoming.

For any given dependent duration up to 2 hours, figure 9 indicates similarity in the shapes of the magnitude-frequency curves for the 2-hour independent duration. For dependent durations greater than 2 hours, Region 3 has the greatest variability and Region 1 has the least variability.

For storms having 6-hour independent durations, figure 10 indicates that Region 2 has the greatest variability and Region 3 has the least variability. As previously noted, 6-hour storms are generally rarer in the plains (Region 3) than in the mountainous areas (Regions 1 and 2).

Figure 11 indicates that, for storms having 24-hour independent durations, and dependent durations up to about 9 hours, Region 3 has the greatest variability. For dependent durations greater than about 9 hours, Regions 1 and 2 have about the same variability and are more variable than Region 3. For these longer durations, the orographic effects of the mountains in Regions 1 and 2 evidently result in greater relative depths of precipitation in the tails of the depth-duration curves (smaller exceedance probabilities).

Correlation of Dimensionless-Depth Data

Although the curves in figures 9 through 11 indicate exceedance probabilities for dimensionless-depth data for each dependent duration, they cannot be used directly to develop synthetic storms because the probability distribution was independently fit to the data for each duration. Thus, a storm having a dimensionless depth for a particular dependent duration that has an exceedance probability of 0.1 may have depths for other dependent durations with exceedance probabilities far different from 0.1. Expressed in a different way, an almost infinite number of storms with widely varying hyetograph shapes could have the same dimensionless depth for any particular dependent duration. Nevertheless, within a given storm, depths for various dependent durations tend to be correlated, and this correlation provides a basis for using the probabilistic depth-duration data in figures 9 through 11 to develop probabilistic depth-duration curves and synthetic storm hyetographs.

To examine the degree of correlation among dimensionless-depth data for the various dependent durations, the Pearson correlation coefficient was calculated between dimensionless-depth data pairs for every dependent duration within the 2-hour, 6-hour, and 24-hour independent durations within each region.

The resulting matrices of correlation coefficients are displayed in tables 4 through 6. As the data in tables 4 through 6 indicate, dimensionless depths are strongly correlated for dependent durations that are close in magnitude and less strongly correlated for dependent durations that are relatively different in magnitude. This finding seems reasonable given that a large recorded storm depth for a 1-hour duration, for example, likely means that the depth for the 2-hour duration is also large, whereas the depth for the 24-hour duration may or may not be large.

Kernel Durations and Regression Analyses

To make use of the correlation structure shown in tables 4 through 6, the method described by Schaefer (1989) was used. As discussed by Schaefer, the peak discharge is most often the critical feature of the runoff hydrograph for most rainfall-modeling purposes. The peak discharge, in turn, depends largely on the higher-intensity precipitation that occurs during some duration shorter than that of the independent duration. Because the shorter duration is deemed the most critical to production of peak discharge, it is termed a kernel duration. The duration of high-intensity precipitation previously considered by Schaefer (1989) to be most conducive to the production of high peak discharge was 1/4 to 1/3 of the independent duration as shown in table 7. The same kernel durations used by Schaefer for modeling peak discharge were used for this study.

The correlation between pairs of dimensionless depths for various durations can be used to estimate dimensionless depths for all other dependent durations from the dimensionless depth for the kernel duration. Ordinary least-squares regression was used to develop relations between dimensionless depths for the kernel duration and dimensionless depths for all other dependent durations for each of the three independent durations within each region. The general form of the regression relation can be expressed as follows:

$$dd_{dur} = a + (b \times dd_{kernel_{dur}}) \quad (5)$$

where

- dd_{dur} is the dimensionless depth for some dependent duration, dur ,
- a and b are the regression constant and coefficient, respectively, and
- $dd_{kernel_{dur}}$ is the dimensionless depth for the kernel duration.

Table 4. Correlation matrices for dimensionless-depth data for independent durations of 2, 6, and 24 hours in Region 1, Montana

Dependent duration (hours)	2-hour independent duration										
	Dependent duration (hours)										
	0.0833	0.167	0.25	.50	0.75	1.0	1.5	3	4	5	6
0.0833	1.0	1.0	1.0	0.81	0.66	0.49	0.23	-0.11	-0.15	-0.15	-0.15
.167	1.0	1.0	1.0	.8	.66	.48	.23	-.11	-.14	-.14	-.14
.25	1.0	1.0	1.0	.8	.66	.48	.23	-.1	-.14	-.14	-.14
.50	.81	.8	.8	1.0	.88	.69	.31	-.26	-.27	-.27	-.27
.75	.66	.66	.66	.88	1.0	.85	.42	-.3	-.3	-.3	-.3
1.0	.49	.48	.48	.69	.85	1.0	.49	-.4	-.35	-.35	-.35
1.5	.23	.23	.23	.31	.42	.49	1.0	-.53	-.49	-.49	-.49
3	-.11	-.11	-.1	-.26	-.3	-.4	-.53	1.0	.94	.94	.94
4	-.15	-.14	-.14	-.27	-.3	-.35	-.49	.94	1.0	1.0	1.0
5	-.15	-.14	-.14	-.27	-.3	-.35	-.49	.94	1.0	1.0	1.0
6	-.15	-.14	-.14	-.27	-.3	-.35	-.49	.94	1.0	1.0	1.0

Dependent duration (hours)	6-hour independent duration									
	Dependent duration (hours)									
	0.25	0.50	0.75	1.0	2	3	9	12	15	18
0.25	1.0	0.93	0.88	0.89	0.7	0.43	-0.07	0.11	0.28	0.16
.50	.93	1.0	.97	.91	.8	.55	-.16	.03	.19	.03
.75	.88	.97	1.0	.92	.8	.54	-.24	-.08	.09	-.05
1.0	.89	.91	.92	1.0	.82	.47	-.22	-.14	-.02	-.16
2	.7	.8	.8	.82	1.0	.79	-.28	-.28	-.19	-.33
3	.43	.55	.54	.47	.79	1.0	-.34	-.32	-.23	-.34
9	-.07	-.16	-.24	-.22	-.28	-.34	1.0	.81	.7	.63
12	.11	.03	-.08	-.14	-.28	-.32	.81	1.0	.94	.87
15	.28	.19	.09	-.02	-.19	-.23	.7	.94	1.0	.95
18	.16	.03	-.05	-.16	-.33	-.34	.63	.87	.95	1.0

Dependent duration (hours)	24-hour independent duration										
	Dependent duration (hours)										
	1	2	3	6	9	12	18	36	48	60	72
1	1.0	.96	.90	.57	.29	.32	.38	-.19	-.23	-.12	-.09
2	.96	1.0	.95	.68	.39	.38	.48	-.25	-.21	-.09	-.07
3	.90	.95	1.0	.80	.52	.46	.49	-.31	-.23	-.13	-.05
6	.57	.68	.80	1.0	.87	.69	.49	-.49	-.21	-.11	.02
9	.29	.39	.52	.87	1.0	.90	.56	-.48	-.09	-.03	.09
12	.32	.38	.46	.69	.90	1.0	.74	-.41	-.06	-.02	.06
18	.38	.48	.49	.49	.56	.74	1.0	-.35	-.15	-.16	-.16
36	-.19	-.25	-.31	-.49	-.48	-.41	-.35	1.0	.71	.68	.57
48	-.23	-.21	-.23	-.21	-.09	-.06	-.15	.71	1.0	.93	.89
60	-.12	-.09	-.13	-.11	-.03	-.02	-.16	.68	.93	1.0	.95
72	-.09	-.07	-.05	.02	.09	.06	-.16	.57	.89	.95	1.0

Table 5. Correlation matrices for dimensionless-depth data for independent durations of 2, 6, and 24 hours in Region 2, Montana

Dependent duration (hours)	2-hour independent duration										
	Dependent duration (hours)										
	0.0833	0.167	0.25	.50	0.75	1.0	1.5	3	4	5	6
0.0833	1.0	1.0	1.0	0.88	0.88	0.87	0.7	0.17	0.05	0.0	0.16
.167	1.0	1.0	1.0	.88	.88	.87	.69	.17	.06	.0	.17
.25	1.0	1.0	1.0	.88	.88	.87	.69	.17	.06	.0	.17
.50	.88	.88	.88	1.0	.95	.83	.65	.17	.03	-.02	.13
.75	.88	.88	.88	.95	1.0	.95	.71	.16	-.02	-.05	.09
1.0	.87	.87	.87	.83	.95	1.0	.81	.09	-.11	-.14	.02
1.5	.7	.69	.69	.65	.71	.81	1.0	-.23	-.4	-.46	-.29
3	.17	.17	.17	.17	.16	.09	-.23	1.0	.96	.93	.95
4	.05	.06	.06	.03	-.02	-.11	-.4	.96	1.0	.96	.96
5	.0	.0	.0	-.02	-.05	-.14	-.46	.93	.96	1.0	.98
6	.16	.17	.17	.13	.09	.02	-.29	.95	.96	.98	1.0

Dependent duration (hours)	6-hour independent duration									
	Dependent duration (hours)									
	0.25	0.50	0.75	1.0	2	3	9	12	15	18
0.25	1.0	0.96	0.93	0.92	0.88	0.82	-.75	-.75	-.72	-.75
.50	.96	1.0	.99	.99	.91	.86	-.76	-.78	-.74	-.77
.75	.93	.99	1.0	.99	.91	.88	-.74	-.76	-.72	-.75
1.0	.92	.99	.99	1.0	.93	.9	-.79	-.78	-.75	-.77
2	.88	.91	.91	.93	1.0	.91	-.82	-.76	-.69	-.74
3	.82	.86	.88	.9	.91	1.0	-.76	-.82	-.78	-.81
9	-.75	-.76	-.74	-.79	-.82	-.76	1.0	.83	.76	.82
12	-.75	-.78	-.76	-.78	-.76	-.82	.83	1.0	.98	.98
15	-.72	-.74	-.72	-.75	-.69	-.78	.76	.98	1.0	.99
18	-.75	-.77	-.75	-.77	-.74	-.81	.82	.98	.99	1.0

Dependent duration (hours)	24-hour independent duration										
	Dependent duration (hours)										
	1	2	3	6	9	12	18	36	48	60	72
1	1.0	.97	.90	.33	-.07	-.11	-.09	-.26	-.25	-.26	-.26
2	.97	1.0	.97	.43	.06	.04	.05	-.38	-.30	-.32	-.33
3	.90	.97	1.0	.57	.21	.18	.21	-.54	-.41	-.41	-.41
6	.33	.43	.57	1.0	.78	.75	.63	-.64	-.40	-.43	-.43
9	-.07	.06	.21	.78	1.0	.95	.74	-.50	-.06	-.14	-.16
12	-.11	.04	.18	.75	.95	1.0	.86	-.58	-.16	-.26	-.29
18	-.09	.05	.21	.63	.74	.86	1.0	-.72	-.39	-.40	-.40
36	-.26	-.38	-.54	-.64	-.50	-.58	-.72	1.0	.83	.83	.80
48	-.25	-.30	-.41	-.40	-.06	-.16	-.39	.83	1.0	.95	.90
60	-.26	-.32	-.41	-.43	-.14	-.26	-.40	.83	.95	1.0	.99
72	-.26	-.33	-.41	-.43	-.16	-.29	-.40	.80	.90	.99	1.0

Table 6. Correlation matrices for dimensionless-depth data for independent durations of 2, 6, and 24 hours in Region 3, Montana and northern Wyoming

2-hour independent duration											
Dependent duration (hours)	Dependent duration (hours)										
	0.0833	0.167	0.25	.50	0.75	1.0	1.5	3	4	5	6
0.0833	1.0	1.0	1.0	0.85	0.6	0.51	0.46	-0.21	0.09	-0.01	-0.1
.167	1.0	1.0	1.0	.85	.6	.51	.46	-.2	.09	-.01	-.1
.25	1.0	1.0	1.0	.85	.6	.51	.46	-.2	.09	-.01	-.1
.50	.85	.85	.85	1.0	.87	.77	.7	-.31	-.05	-.14	-.26
.75	.6	.6	.6	.87	1.0	.94	.76	-.41	-.18	-.27	-.39
1.0	.51	.51	.51	.77	.94	1.0	.81	-.46	-.2	-.28	-.4
1.5	.46	.46	.46	.7	.76	.81	1.0	-.56	-.27	-.36	-.51
3	-.21	-.2	-.2	-.31	-.41	-.46	-.56	1.0	.79	.73	.68
4	.09	.09	.09	-.05	-.18	-.2	-.27	.79	1.0	.88	.74
5	-.01	-.01	-.01	-.14	-.27	-.28	-.36	.73	.88	1.0	.95
6	-.1	-.1	-.1	-.26	-.39	-.4	-.51	.68	.74	.95	1.0

6-hour independent duration											
Dependent duration (hours)	Dependent duration (hours)										
	0.25	0.50	0.75	1.0	2	3	9	12	15	18	
0.25	1.0	0.87	0.7	0.59	0.38	0.34	-0.67	-0.62	-0.43	-0.33	
.50	.87	1.0	.94	.88	.62	.6	-.61	-.61	-.41	-.38	
.75	.7	.94	1.0	.98	.77	.76	-.45	-.52	-.34	-.32	
1.0	.59	.88	.98	1.0	.79	.8	-.34	-.41	-.23	-.24	
2	.38	.62	.77	.79	1.0	.96	-.06	-.13	.04	.04	
3	.34	.6	.76	.8	.96	1.0	-.02	-.08	.04	.0	
9	-.67	-.61	-.45	-.34	-.06	-.02	1.0	.8	.7	.58	
12	-.62	-.61	-.52	-.41	-.13	-.08	.8	1.0	.91	.82	
15	-.43	-.41	-.34	-.23	.04	.04	.7	.91	1.0	.96	
18	-.33	-.38	-.32	-.24	.04	.0	.58	.82	.96	1.0	

24-hour independent duration											
Dependent duration (hours)	Dependent duration (hours)										
	1	2	3	6	9	12	18	36	48	60	72
1	1.0	.96	.92	.79	.60	.54	.41	-.33	-.27	-.22	-.17
2	.96	1.0	.97	.81	.64	.61	.46	-.29	-.27	-.23	-.18
3	.92	.97	1.0	.86	.69	.65	.48	-.30	-.27	-.23	-.19
6	.79	.81	.86	1.0	.88	.76	.52	-.33	-.33	-.26	-.22
9	.60	.64	.69	.88	1.0	.90	.50	-.23	-.29	-.20	-.16
12	.54	.61	.65	.76	.90	1.0	.61	-.36	-.43	-.36	-.35
18	.41	.46	.48	.52	.50	.61	1.0	-.61	-.69	-.63	-.57
36	-.33	-.29	-.30	-.33	-.23	-.36	-.61	1.0	.88	.83	.78
48	-.27	-.27	-.27	-.33	-.29	-.43	-.69	.88	1.0	.96	.92
60	-.22	-.23	-.23	-.26	-.20	-.36	-.63	.83	.96	1.0	.97
72	-.17	-.18	-.19	-.22	-.16	-.35	-.57	.78	.92	.97	1.0

Table 7. Relation between duration of high-intensity portion of storm and independent duration

Independent duration (hours)	Duration of high-intensity portion (hours)
2	0.50
6	2
24	6

The results of the regression analysis, including the regression constant, regression coefficient, and standard error, are shown in tables 8 through 10. To ensure consistency in subsequent calculations, the regression constants and coefficients and all dimensionless depths calculated from the regression equations are shown to 3 decimal places. In general, the smaller the standard error, which is a measure of the scatter about a regression line, the more reliable is the regression relation. As indicated by the results in tables 8 through 10, the regression relations generally are more reliable for dependent durations that are closest to the kernel duration and are less reliable for dependent durations farthest from the kernel duration. The relatively large standard errors are an indication that the random components of the stochastic storm process are relatively large. Overall, the regression relations provide a reasonable means for estimating precipitation amount given the occurrence of a particular amount for the kernel duration and provide a reasonable basis for the construction of probabilistic depth-duration curves.

Estimation of 0.0833- and 0.167-hour Duration Data

Precipitation depth data, recorded as a continuous trace on a recorder chart for each storm, were obtained from the NWS (Thomas Ross, written commun., 1996). Data for 28 large (exceedance probability less than 0.125 for 2-hour depth), short-duration (total storm length less than 6 hours) storms at nine stations were used to develop general relations between maximum depths for 0.0833- and 0.25-hour durations and for 0.167- and 0.25-hour durations. To ensure that general relations for only a few storms could be considered generally applicable to 2-hour duration storms throughout the area, the 28 storms were selected so that each region had at least 7 storms from at least 2 different stations. Because the recorder charts represent raw data that may have been adjusted slightly during later

compilation by the NWS, not all storm data used for estimation of 0.0833- and 0.167-hour data were used for subsequent analyses.

The selected storm dates, locations, and pertinent characteristics are shown in table 11. On the basis of graphical plots, the ratios of 0.0833- to 0.25-hour storm depth and 0.167- to 0.25-hour storm depth were not strongly correlated with exceedance probability or with region. Accordingly, the mean values of the ratios shown in table 11 were considered to be reasonably representative of ratios expected for any large storm throughout the area and were used to estimate 0.0833- and 0.167-hour dimensionless depths for all storms in the data base having durations of 2 hours.

Probabilistic Depth-Duration Curves

Probabilistic dimensionless-depth data used to develop the curves shown in figures 9-11 are shown in table 12 for just the kernel durations. The values in table 12 were used in equation 5 with appropriate values of the regression constant and coefficient to calculate dimensionless depths for all dependent durations. For an independent duration of 2 hours in Region 2, for example, the dimensionless depth for the 0.50-hour kernel duration having an exceedance probability of 0.1 is determined from table 12 to be 0.978. To estimate the dimensionless depth for a dependent duration of 1.0 hour that has the same 0.1 exceedance probability in Region 2, the regression constant and coefficient are obtained from table 9 and used together with the dimensionless depth for the kernel duration in equation 5 as follows:

$$dd_{1.0} = 0.549 + (0.443 \times 0.978)$$

$$dd_{1.0} = 0.982$$

where

$dd_{1.0}$ is the dimensionless depth for a duration of 1.0 hour.

Table 8. Results of regression analysis relating dimensionless depths for kernel duration to dependent durations, Region 1, Montana

Dependent duration (hours)	2-hour independent duration, 0.50-hour kernel duration		
	Regression constant	Regression coefficient	Standard error
0.0833	0.129	0.519	0.06
.167	-.213	.851	.10
.25	-.282	1.113	.14
.75	.297	.719	.06
1.0	.559	.446	.08
1.5	.872	.124	.06
3	1.110	-.101	.06
4	1.138	-.127	.08
5	1.138	-.127	.08
6	1.138	-.127	.08

Dependent duration (hours)	6-hour independent duration, 2-hour kernel duration		
	Regression constant	Regression coefficient	Standard error
0.25	-0.215	0.685	0.09
.50	-.229	.848	.08
.75	-.169	.863	.08
1.0	-.106	.883	.08
3	.251	.801	.08
9	1.196	-.182	.08
12	1.299	-.278	.12
15	1.308	-.240	.16
18	1.449	-.441	.16

Dependent duration (hours)	24-hour independent duration, 6-hour kernel duration		
	Regression constant	Regression coefficient	Standard error
1	-0.019	0.364	0.07
2	-.015	.542	.07
3	-.014	.722	.07
9	.148	.897	.06
12	.321	.764	.10
18	.696	.405	.09
36	1.215	-.326	.07
48	1.223	-.241	.14
60	1.233	-.162	.18
72	1.200	.034	.22

Table 9. Results of regression analysis relating dimensionless depths for kernel duration to dependent durations, Region 2, Montana

Dependent duration (hours)	2-hour independent duration, 0.50-hour kernel duration		
	Regression constant	Regression coefficient	Standard error
0.0833	-0.121	0.527	0.06
.167	-.200	.866	.10
.25	-.252	1.117	.13
.75	.277	.725	.05
1.0	.549	.443	.07
1.5	.820	.182	.05
3	.996	.062	.08
4	1.043	.011	.08
5	1.064	-.008	.08
6	1.025	.061	.11

Dependent duration (hours)	6-hour independent duration, 2-hour kernel duration		
	Regression constant	Regression coefficient	Standard error
0.25	-0.334	0.861	0.07
.50	-.340	1.022	.07
.75	-.286	1.072	.08
1.0	-.275	1.151	.07
3	.254	.790	.06
9	1.406	-.501	.06
12	1.777	-.962	.13
15	1.937	-1.150	.19
18	2.081	-1.337	.19

Dependent duration (hours)	24-hour independent duration, 6-hour kernel duration		
	Regression constant	Regression coefficient	Standard error
1	-0.002	0.305	0.07
2	.004	.477	.08
3	.017	.612	.07
9	.141	1.030	.07
12	.275	.997	.07
18	.651	.535	.05
36	1.404	-.726	.07
48	1.458	-.699	.13
60	1.609	-.965	.17
72	1.666	-1.062	.19

Table 10. Results of regression analysis relating dimensionless depths for kernel duration to dependent durations, Region 3, Montana and northern Wyoming

Dependent duration (hours)	2-hour independent duration, 0.50-hour kernel duration		
	Regression constant	Regression coefficient	Standard error
0.0833	-0.179	0.577	0.05
.167	-.297	.948	.09
.25	-.383	1.229	.11
.75	.349	.680	.06
1.0	.545	.481	.06
1.5	.826	.205	.03
3	1.093	-.089	.04
4	1.068	-.024	.07
5	1.153	-.100	.11
6	1.275	-.243	.13

Dependent duration (hours)	6-hour independent duration, 2-hour kernel duration		
	Regression constant	Regression coefficient	Standard error
0.25	0.103	0.134	0.05
.50	.084	.366	.07
.75	.005	.598	.08
1.0	.044	.636	.08
3	.320	.727	.03
9	1.057	-.029	.07
12	1.134	-.073	.08
15	1.094	.029	.11
18	1.112	.030	.13

Dependent duration (hours)	24-hour independent duration, 6-hour kernel duration		
	Regression constant	Regression coefficient	Standard error
1	-0.106	0.546	0.05
2	-.132	.795	.07
3	-.090	.855	.07
9	.176	.876	.06
12	.344	.762	.08
18	.755	.299	.06
36	1.248	-.281	.10
48	1.304	-.319	.12
60	1.297	-.264	.13
72	1.294	-.224	.13

Table 11. Continuous-record precipitation data used to estimate 0.0833- and 0.167-hour duration depths, Montana

Date	Station	Region	(0.0833/0.25) hour ratio	(0.167/0.25) hour ratio	Exceedance probability
08/03/76	Kalispell WSO, AP	1	0.383	0.702	0.011
06/29/82	Kalispell WSO, AP	1	.452	.714	<.001
07/05/90	Kalispell WSO, AP	1	.49	.78	.072
06/01/56	Missoula WSO, AP	1	.5	.875	.100
06/11/58	Missoula WSO, AP	1	.4	.77	.005
08/10/63	Missoula WSO, AP	1	.389	.75	.078
08/20/81	Missoula WSO, AP	1	.805	.854	.028
07/21/87	Missoula WSO, AP	1	.412	.794	.008
07/16/78	Bozeman 6 W Exp Farm	2	.606	.972	.040
08/22/83	Bozeman 6 W Exp Farm	2	.658	.863	.057
06/30/86	Bozeman 6 W Exp Farm	2	.615	.923	.086
07/10/55	Helena WSO, AP	2	.5	.8	.070
07/03/75	Helena WSO, AP	2	.627	.932	.077
06/18/79	Helena WSO, AP	2	.385	.641	.015
08/21/83	Helena WSO, AP	2	.613	.813	.010
07/04/75	Billings WSO, AP	3	.421	.71	.038
06/16/82	Billings WSO, AP	3	.569	.754	.072
07/01/75	Glasgow WSO, AP	3	.42	.718	.016
08/02/85	Glasgow WSO, AP	3	.354	.687	.011
08/09/50	Great Falls WSCMO, AP	3	.403	.726	.024
05/23/80	Great Falls WSCMO, AP	3	.343	.821	.083
05/28/90	Great Falls WSCMO, AP	3	.415	.698	.044
06/20/91	Great Falls WSCMO, AP	3	.451	.775	.095
08/23/65	Havre WSO, AP	3	.361	.67	.014
07/16/86	Havre WSO, AP	3	.602	.878	.100
08/05/54	Winnett 8 ESE	3	.356	.689	.013
05/21/72	Winnett 8 ESE	3	.365	.683	.125
08/12/91	Winnett 8 ESE	3	.36	.68	.004
Mean			.47	.77	

Table 12. Dimensionless depths for indicated kernel durations ($dd_{kernel\ dur}$) and exceedance probabilities, Montana and northern Wyoming

Region	Kernel duration (hours)	Exceedance probability								
		0.9	0.8	0.7	0.6	0.5	0.4	0.3	0.2	0.1
1	0.50	0.515	0.607	0.671	0.723	0.768	0.809	0.849	0.888	0.931
2	.50	.460	.592	.684	.756	.816	.866	.910	.947	.978
3	.50	.549	.630	.687	.733	.773	.809	.845	.881	.922
1	2	.437	.494	.536	.571	.604	.636	.670	.707	.756
2	2	.411	.480	.530	.573	.612	.651	.691	.735	.790
3	2	.461	.532	.583	.625	.664	.702	.740	.781	.832
1	6	.313	.346	.377	.406	.437	.470	.508	.554	.620
2	6	.320	.344	.365	.384	.403	.424	.447	.476	.519
3	6	.346	.387	.422	.455	.489	.524	.563	.609	.674

In a similar manner, dimensionless depths for other dependent durations having an exceedance probability of 0.1 are calculated. These estimated depths, together with the constant dimensionless depth of 1.000 for the independent duration of 2 hours, constitute dimensionless depth-duration data that can be used to plot a depth-duration curve for Region 1 for an independent duration of 2 hours having an exceedance probability of 0.1 (fig. 12). As indicated by figure 12,

the curve based on the regression equations does not have a continuously decreasing slope throughout. As previously discussed, depth-duration curves for storm hyetographs that have smoothly rising and falling precipitation periods have continuously decreasing slopes. Thus, to ensure that a design storm hyetograph developed from a depth-duration curve based on regression data in table 8 will have the desired characteristics of smoothly rising and falling precipitation periods, the

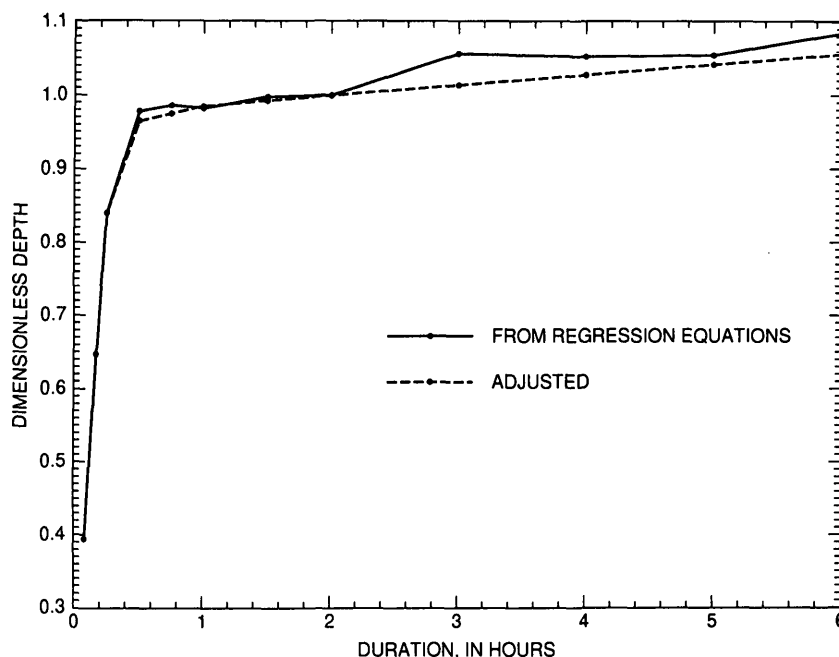


Figure 12. Dimensionless depth-duration curve for independent duration of 2 hours and for exceedance probability of 0.1 in Region 2, Montana.

depth-duration curve needs to be smoothed or adjusted to have a continuously decreasing slope (fig. 12). Because the regression relations generally are least reliable for dependent durations farthest from the kernel duration, the adjustments made to the curve in figure 12 generally were made for durations farthest from 0.50 hour.

Similar solutions of the regression relation (equation 5) and subsequent smoothing of the depth-duration curves for other exceedance probabilities and for other independent duration data in other regions result in the probabilistic depth-duration data shown in tables 13 through 15. To illustrate the general relation among depth-duration curves having different exceedance probabilities, data from tables 13 through 15 were used to draw depth-duration curves having exceedance probabilities of 0.1, 0.5, and 0.9 for each independent duration within each region in figures 13 through 15. A consistent trend exhibited by figures 13 through 15 is that, for a given independent duration and region, the curve with the lowest exceedance probability has the steepest slope for dependent durations up to the independent duration and the flattest slope for dependent durations greater than the independent duration. Conversely, the curve with the highest exceedance probability has the flattest slope for dependent durations up to the independent duration and the steepest slope for dependent durations greater than the independent duration. Thus, the rarer, higher-intensity storms have characteristic depth-duration curves with large dimensionless depths for short durations and very flat slopes after the independent duration, indicating that little additional precipitation occurs after that time.

The reverse is true for commonly occurring, low-intensity storms which have depth-duration curves with relatively small dimensionless depths for short durations and relatively steep slopes after the independent duration, indicating that significant precipitation occurs after that time. Ironically, storm depth for the total storm duration is greater for the commonly occurring storm with the larger exceedance probability than for the rarer storm with the smaller exceedance probability. Small exceedance probability in this context, however, is associated with the storm that produces the larger, rare peak discharge rather than with the storm with the greatest depth for a total storm duration. As will be discussed later, flood volume may be the primary consideration for rainfall-runoff modeling on some larger watersheds rather than peak discharge. If so, total precipitation depth is a more important storm

characteristic than precipitation intensity, and depth-duration curves for 24-hour duration storms based on dimensionless depths for a kernel duration longer than the independent duration will be developed.

Probabilistic Depth-Duration Data for 24-Hour Storms Based on 48-Hour Kernel Duration

For some rainfall-runoff modeling studies of larger watersheds where the use of 24-hour duration storms is required, flood volume rather than peak discharge may be the primary consideration. In this context, a depth-duration curve having a low exceedance probability (less than 0.5) would be expected to have a greater total storm depth and a resultant larger runoff volume than a storm having a larger exceedance probability. As previously discussed, however, probabilistic depth-duration curves based on a 6-hour kernel duration have smaller total storm depths for low exceedance probabilities than for large exceedance probabilities. Consequently, for modeling studies using flood volume as the primary consideration, probabilistic depth-duration curves need to be based on a kernel duration greater than 24 hours. In accordance with Schaefer's (1989) previous work in Washington, 48 hours was selected as a reasonable kernel duration for that purpose, and ordinary least-squares regression was used to develop relations between dimensionless depths for the 48-hour duration and dimensionless depths for all dependent durations for 24-hour storms in each region. The results of the regression analyses based on a 48-hour kernel are shown in table 16.

The regression results in table 16 were used together with the 48-hour dimensionless depths for exceedance probabilities from 0.1 to 0.9 (table 17) to calculate probabilistic dimensionless depth-duration data for each region (table 18). As was the case for dimensionless depth-duration data developed on the basis of a 6-hour kernel duration, the data developed from the 48-hour kernel duration were smoothed to ensure that dimensionless depth-duration curves with uniformly decreasing slopes would result. Selected data from table 18 for exceedance probabilities of 0.9, 0.5, and 0.1 are displayed as depth-duration curves in figure 16. As shown in figure 16, the depth-duration curve for an exceedance probability of 0.1 has the greatest total storm depth, whereas the depth-duration curve for an exceedance probability of 0.9 has the smallest total storm depth.

Table 13. Dimensionless depths for various durations and exceedance probabilities, Region 1, Montana

2-hour independent duration 0.50-hour kernel duration									
Dependent duration (hours)	Exceedance probability								
	0.9	0.8	0.7	0.6	0.5	0.4	0.3	0.2	0.1
0.0833	0.139	0.186	0.219	0.246	0.270	0.291	0.312	0.332	0.354
.167	.225	.304	.358	.403	.441	.476	.510	.543	.580
.25	.291	.393	.465	.522	.573	.618	.663	.706	.754
.50	.515	.607	.671	.723	.768	.809	.849	.888	.931
.75	.667	.733	.779	.816	.849	.878	.907	.935	.966
1.0	.789	.830	.859	.882	.902	.920	.938	.956	.975
1.5	.935	.947	.955	.961	.967	.972	.977	.982	.987
2	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
3	1.058	1.048	1.042	1.037	1.032	1.028	1.024	1.020	1.016
4	1.073	1.062	1.053	1.047	1.041	1.036	1.031	1.026	1.021
5	1.073	1.062	1.053	1.047	1.041	1.036	1.031	1.026	1.021
6	1.073	1.062	1.053	1.047	1.041	1.036	1.031	1.026	1.021

6-hour independent duration 2-hour kernel duration									
Dependent duration (hours)	Exceedance probability								
	0.9	0.8	0.7	0.6	0.5	0.4	0.3	0.2	0.1
0.25	0.084	0.123	0.152	0.176	0.199	0.221	0.244	0.269	0.303
.50	.152	.196	.230	.259	.283	.310	.339	.371	.412
.75	.218	.264	.300	.330	.356	.384	.414	.445	.487
1.0	.273	.331	.368	.399	.428	.456	.486	.519	.562
2	.437	.494	.536	.571	.604	.636	.670	.707	.756
3	.601	.647	.680	.708	.735	.760	.787	.817	.856
6	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
9	1.116	1.106	1.098	1.092	1.086	1.080	1.074	1.067	1.058
12	1.177	1.161	1.150	1.140	1.131	1.122	1.112	1.103	1.096
15	1.203	1.189	1.179	1.170	1.162	1.155	1.147	1.138	1.126
18	1.228	1.217	1.208	1.197	1.182	1.168	1.153	1.145	1.132

24-hour independent duration 6-hour kernel duration									
Dependent duration (hours)	Exceedance probability								
	0.9	0.8	0.7	0.6	0.5	0.4	0.3	0.2	0.1
1	0.094	0.106	0.118	0.128	0.140	0.152	0.165	0.182	0.206
2	.138	.165	.189	.205	.222	.240	.260	.285	.321
3	.180	.220	.250	.275	.301	.325	.352	.386	.433
6	.313	.346	.377	.406	.437	.470	.508	.554	.620
9	.440	.469	.490	.520	.550	.580	.610	.650	.710
12	.561	.586	.609	.632	.655	.680	.709	.745	.795
18	.822	.836	.845	.855	.865	.880	.901	.920	.947
24	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
36	1.113	1.102	1.092	1.083	1.073	1.062	1.054	1.045	1.040
48	1.147	1.139	1.132	1.125	1.117	1.109	1.100	1.089	1.080
60	1.182	1.175	1.172	1.167	1.160	1.150	1.140	1.130	1.120
72	1.211	1.205	1.203	1.201	1.200	1.190	1.180	1.170	1.160

Table 14. Dimensionless depths for various durations and exceedance probabilities, Region 2, Montana

2-hour independent duration 0.50-hour kernel duration									
Dependent duration (hours)	Exceedance probability								
	0.9	0.8	0.7	0.6	0.5	0.4	0.3	0.2	0.1
0.0833	0.121	0.191	0.239	0.277	0.309	0.335	0.359	0.378	0.394
.167	.199	.313	.393	.455	.507	.550	.588	.621	.647
.25	.262	.409	.512	.592	.660	.715	.765	.806	.840
.50	.460	.592	.684	.756	.816	.866	.910	.947	.965
.75	.611	.706	.773	.825	.869	.905	.937	.960	.975
1.0	.752	.811	.851	.883	.910	.932	.952	.970	.985
1.5	.904	.928	.945	.958	.969	.978	.986	.990	.993
2	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
3	1.023	1.021	1.020	1.019	1.018	1.017	1.016	1.016	1.014
4	1.042	1.039	1.037	1.036	1.036	1.035	1.034	1.032	1.028
5	1.060	1.056	1.054	1.053	1.052	1.051	1.050	1.047	1.042
6	1.072	1.071	1.070	1.068	1.067	1.066	1.064	1.062	1.056

6-hour independent duration 2-hour kernel duration									
Dependent duration (hours)	Exceedance probability								
	0.9	0.8	0.7	0.6	0.5	0.4	0.3	0.2	0.1
0.25	0.020	0.079	0.122	0.159	0.193	0.227	0.261	0.299	0.346
.50	.090	.160	.208	.246	.286	.326	.367	.412	.468
.75	.154	.228	.282	.328	.370	.412	.454	.502	.561
1.0	.210	.277	.335	.384	.429	.474	.520	.571	.634
2	.411	.480	.530	.573	.612	.651	.691	.735	.790
3	.578	.633	.672	.706	.737	.768	.799	.834	.878
6	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
9	1.200	1.166	1.140	1.119	1.099	1.080	1.060	1.038	1.010
12	1.382	1.315	1.267	1.226	1.188	1.151	1.112	1.070	1.017
15	1.464	1.385	1.328	1.278	1.233	1.189	1.143	1.092	1.029
18	1.531	1.439	1.372	1.314	1.262	1.210	1.157	1.098	1.032

24-hour independent duration 6-hour kernel duration									
Dependent duration (hours)	Exceedance probability								
	0.9	0.8	0.7	0.6	0.5	0.4	0.3	0.2	0.1
1	0.070	0.094	0.109	0.115	0.121	0.127	0.134	0.143	0.156
2	.120	.144	.163	.188	.197	.207	.218	.232	.252
3	.170	.193	.214	.232	.250	.276	.290	.308	.334
6	.320	.344	.365	.384	.403	.424	.447	.476	.519
9	.470	.495	.516	.536	.556	.577	.601	.631	.675
12	.594	.618	.639	.658	.677	.698	.720	.749	.792
18	.822	.835	.846	.856	.866	.877	.890	.905	.928
24	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
36	1.172	1.154	1.139	1.125	1.111	1.096	1.080	1.064	1.048
48	1.235	1.218	1.203	1.190	1.177	1.162	1.146	1.126	1.096
60	1.300	1.277	1.257	1.239	1.220	1.200	1.178	1.150	1.108
72	1.326	1.300	1.278	1.258	1.238	1.215	1.191	1.160	1.114

Table 15. Dimensionless depths for various durations and exceedance probabilities, Region 3, Montana and northern Wyoming

2-hour independent duration 0.50-hour kernel duration									
Dependent duration (hours)	Exceedance probability								
	0.9	0.8	0.7	0.6	0.5	0.4	0.3	0.2	0.1
0.0883	0.138	0.185	0.217	0.244	0.267	0.288	0.309	0.329	0.353
.167	.223	.300	.354	.398	.436	.470	.504	.538	.577
.25	.291	.391	.461	.518	.567	.611	.655	.699	.750
.50	.549	.630	.687	.733	.773	.809	.845	.881	.922
.75	.722	.777	.816	.847	.875	.899	.924	.948	.976
1.0	.809	.848	.876	.898	.917	.935	.952	.969	.988
1.5	.938	.955	.967	.980	.975	.980	.985	.990	.994
2	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
3	1.044	1.037	1.032	1.028	1.025	1.021	1.018	1.014	1.011
4	1.065	1.061	1.058	1.055	1.050	1.042	1.036	1.028	1.026
5	1.084	1.082	1.080	1.079	1.075	1.062	1.053	1.041	1.039
6	1.102	1.101	1.100	1.097	1.087	1.078	1.069	1.054	1.050

6-hour independent duration 2-hour kernel duration									
Dependent duration (hours)	Exceedance probability								
	0.9	0.8	0.7	0.6	0.5	0.4	0.3	0.2	0.1
0.25	0.165	0.174	0.181	0.187	0.192	0.197	0.202	0.208	0.215
.50	.250	.275	.295	.311	.328	.341	.355	.370	.389
.75	.295	.330	.360	.380	.401	.424	.447	.471	.502
1.0	.330	.380	.415	.442	.466	.491	.515	.541	.573
2	.461	.532	.583	.625	.664	.702	.740	.781	.832
3	.596	.707	.744	.774	.803	.830	.858	.888	.925
6	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
9	1.060	1.050	1.048	1.046	1.044	1.043	1.041	1.040	1.038
12	1.100	1.095	1.091	1.088	1.085	1.083	1.080	1.077	1.073
15	1.127	1.122	1.118	1.115	1.111	1.108	1.105	1.102	1.100
18	1.152	1.147	1.143	1.140	1.136	1.133	1.130	1.126	1.122

24-hour independent duration 6-hour kernel duration									
Dependent duration (hours)	Exceedance probability								
	0.9	0.8	0.7	0.6	0.5	0.4	0.3	0.2	0.1
1	0.083	0.105	0.125	0.143	0.161	0.180	0.202	0.227	0.262
2	.143	.176	.203	.230	.257	.285	.316	.352	.404
3	.206	.241	.271	.299	.328	.358	.391	.430	.486
6	.346	.387	.422	.455	.489	.524	.563	.609	.674
9	.480	.516	.546	.575	.605	.636	.670	.710	.767
12	.607	.638	.665	.690	.716	.743	.773	.808	.857
18	.859	.871	.881	.891	.901	.912	.924	.937	.957
24	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
36	1.150	1.139	1.129	1.120	1.110	1.100	1.089	1.076	1.058
48	1.194	1.180	1.169	1.159	1.148	1.137	1.124	1.110	1.089
60	1.206	1.195	1.186	1.177	1.168	1.159	1.149	1.136	1.119
72	1.217	1.208	1.200	1.192	1.185	1.177	1.168	1.158	1.143

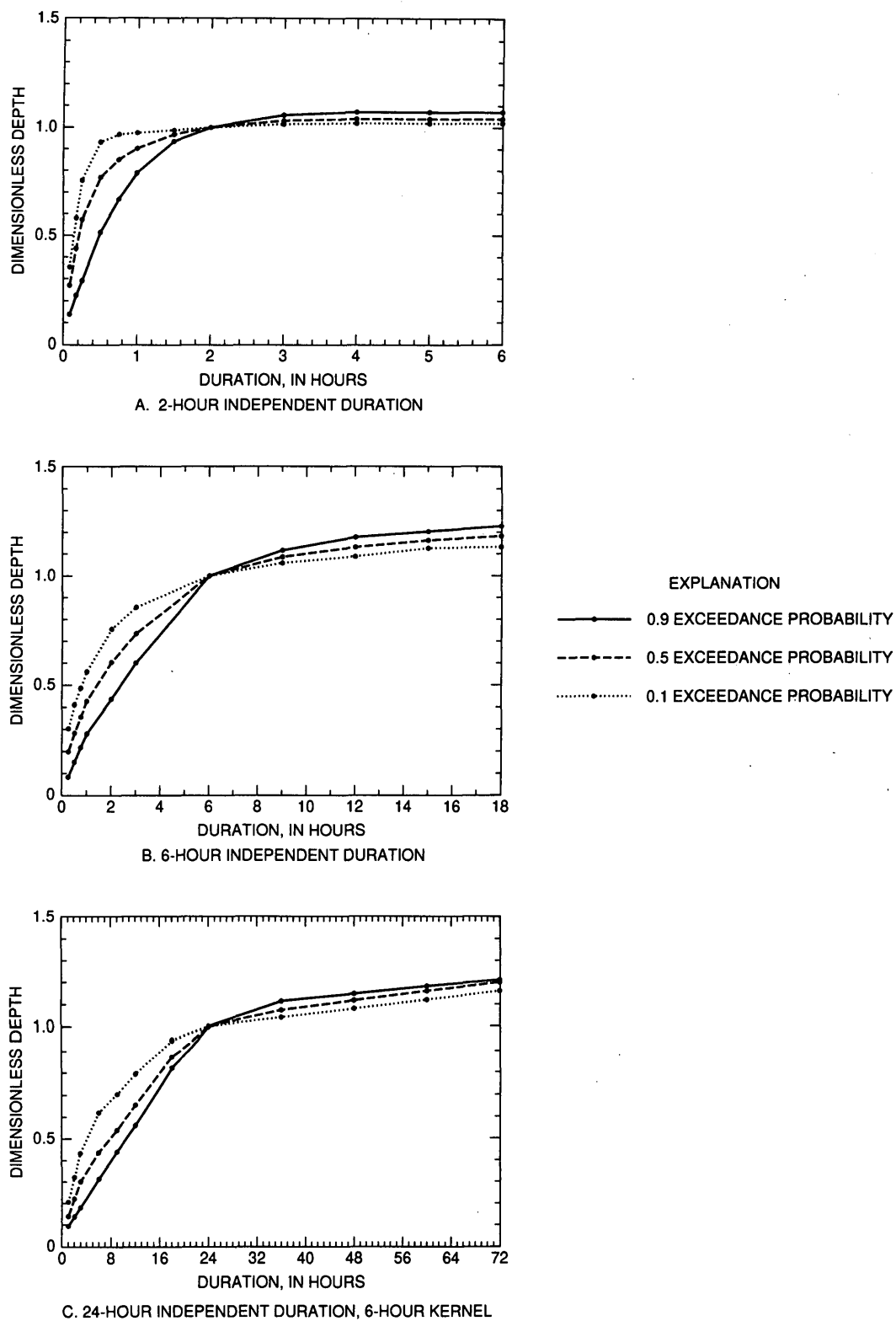


Figure 13. Probabilistic depth-duration curves, Region 1, Montana.

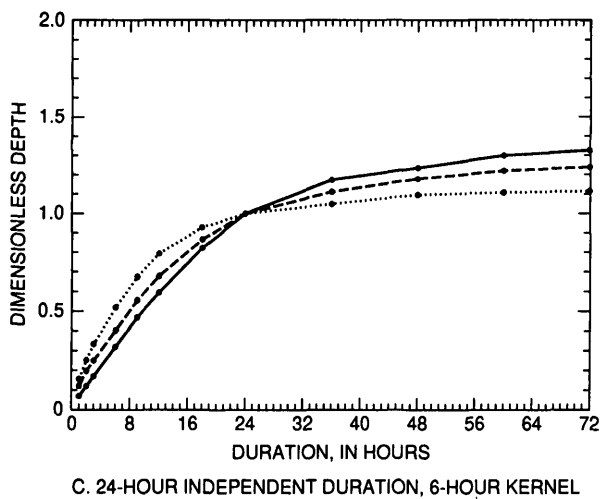
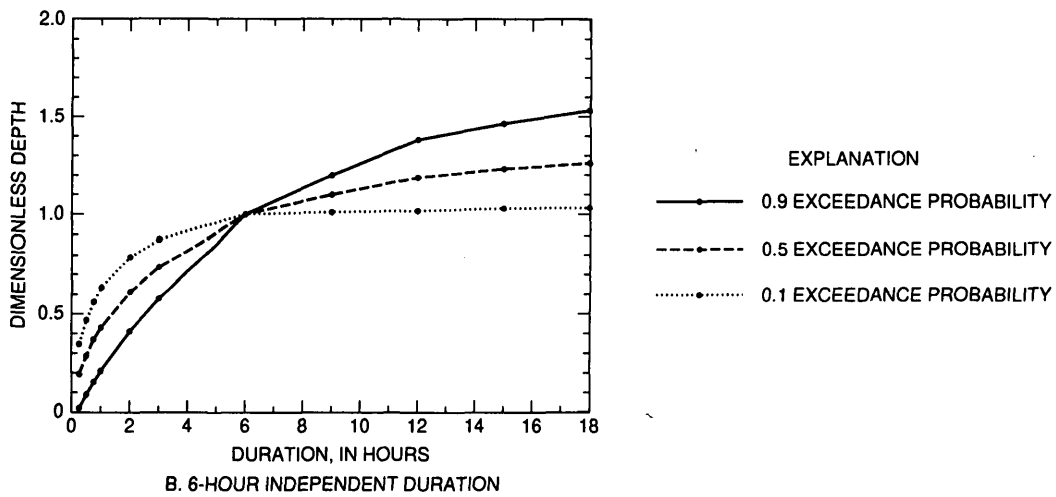
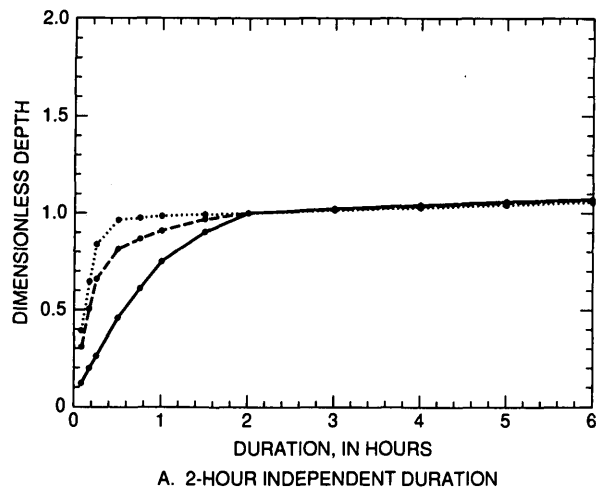
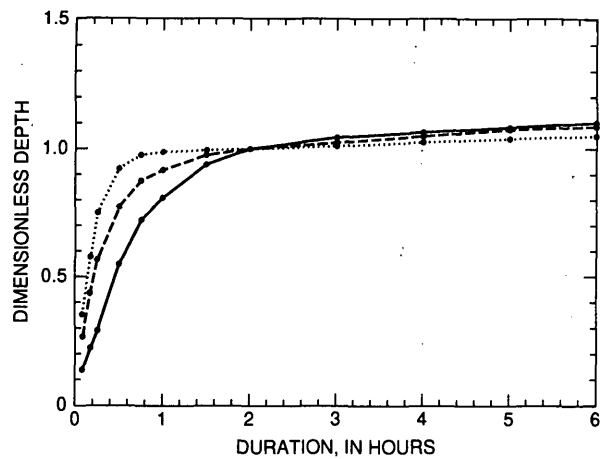
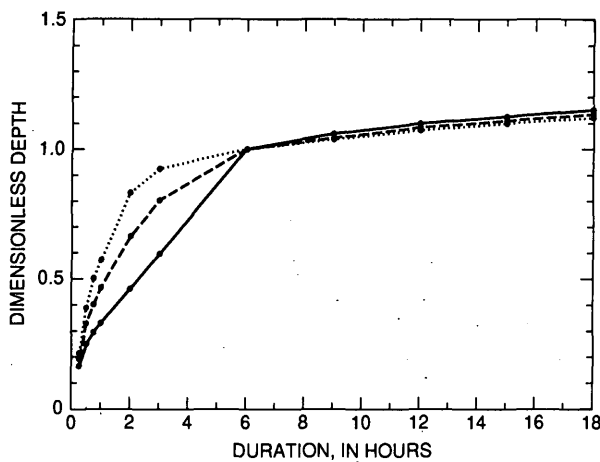


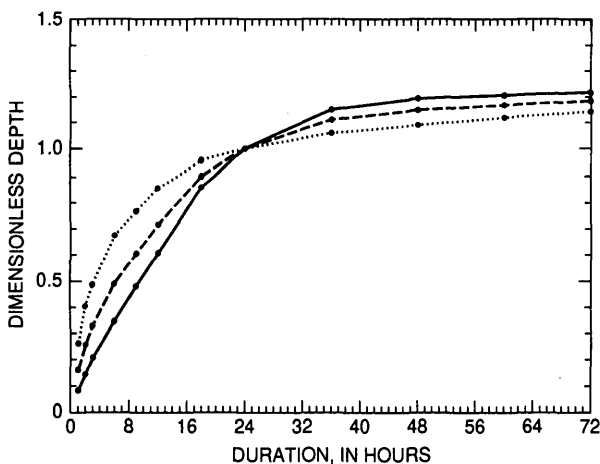
Figure 14. Probabilistic depth-duration curves, Region 2, Montana.



A. 2-HOUR INDEPENDENT DURATION



B. 6-HOUR INDEPENDENT DURATION



C. 24-HOUR INDEPENDENT DURATION, 6-HOUR KERNEL

EXPLANATION

- 0.9 EXCEEDANCE PROBABILITY
- - -●- - 0.5 EXCEEDANCE PROBABILITY
-●..... 0.1 EXCEEDANCE PROBABILITY

Figure 15. Probabilistic depth-duration curves, Region 3, Montana and northern Wyoming.

Table 16. Results of regression analysis relating dimensionless depths for 48-hour kernel duration to dependent durations for 24-hour storms, Montana and northern Wyoming

Region 1			
Dependent duration (hours)	Regression constant	Regression coefficient	Standard error
1	0.293	-0.132	0.08
2	.395	-.148	.10
3	.518	-.184	.11
9	.666	-.191	.12
12	.648	-.083	.13
18	.733	-.058	.14
36	1.003	-.111	.10
48	.595	.424	.06
60	-.204	1.225	.07
72	-.354	1.410	.10

Region 2			
Dependent duration (hours)	Regression constant	Regression coefficient	Standard error
1	0.280	-0.133	0.07
2	.430	-.195	.09
3	.567	-.254	.08
9	.686	-.233	.08
12	.617	-.044	.11
18	.828	-.121	.11
36	1.093	-.189	.07
48	.461	.550	.05
60	-.223	1.226	.06
72	-.289	1.296	.09

Region 3			
Dependent duration (hours)	Regression constant	Regression coefficient	Standard error
1	0.387	-0.191	0.08
2	.578	-.272	.12
3	.656	-.278	.12
9	.882	-.333	.12
12	.952	-.295	.12
18	1.225	-.436	.12
36	1.366	-.403	.05
48	.237	.761	.05
60	.016	1.004	.04
72	.071	.971	.05

Table 17. Dimensionless depths for 48-hour kernel duration for various exceedance probabilities, Montana and northern Wyoming

Region	Kernel duration (hours)	Exceedance probability								
		0.9	0.8	0.7	0.6	0.5	0.4	0.3	0.2	0.1
1	48	1.003	1.010	1.022	1.040	1.063	1.094	1.137	1.199	1.302
2	48	1.027	1.052	1.078	1.106	1.138	1.174	1.218	1.274	1.361
3	48	1.019	1.039	1.061	1.085	1.112	1.144	1.183	1.236	1.317

Table 18. Dimensionless depths and exceedance probabilities for 24-hour storms having 48-hour kernel duration, Montana and northern Wyoming

Region 1									
Dependent duration (hours)	Exceedance probability								
	0.9	0.8	0.7	0.6	0.5	0.4	0.3	0.2	0.1
1	0.160	0.159	0.158	0.155	0.152	0.148	0.143	0.134	0.121
2	.247	.246	.244	.242	.238	.234	.227	.218	.203
3	.334	.332	.330	.327	.323	.317	.309	.298	.279
6	.475	.474	.471	.468	.463	.457	.449	.437	.418
9	.576	.576	.574	.572	.571	.565	.555	.548	.540
12	.674	.674	.673	.672	.671	.669	.660	.659	.657
18	.872	.872	.872	.872	.870	.870	.870	.870	.868
24	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
36	1.002	1.008	1.013	1.020	1.040	1.050	1.077	1.103	1.147
48	1.003	1.010	1.022	1.040	1.063	1.094	1.137	1.199	1.302
60	1.004	1.012	1.031	1.060	1.086	1.136	1.189	1.264	1.391
72	1.005	1.014	1.040	1.080	1.109	1.178	1.241	1.329	1.473

Region 2									
Dependent duration (hours)	Exceedance probability								
	0.9	0.8	0.7	0.6	0.5	0.4	0.3	0.2	0.1
1	0.143	0.139	0.136	0.132	0.128	0.123	0.117	0.110	0.098
2	.229	.224	.219	.214	.208	.201	.192	.181	.164
3	.306	.299	.293	.286	.277	.268	.257	.243	.221
6	.446	.440	.434	.428	.420	.412	.402	.389	.368
9	.572	.568	.564	.560	.557	.554	.550	.529	.508
12	.704	.701	.698	.694	.690	.686	.681	.668	.648
18	.898	.894	.889	.884	.877	.871	.862	.852	.835
24	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
36	1.014	1.030	1.054	1.069	1.087	1.106	1.131	1.162	1.209
48	1.027	1.052	1.078	1.106	1.138	1.174	1.218	1.274	1.361
60	1.035	1.066	1.098	1.132	1.171	1.216	1.270	1.338	1.445
72	1.042	1.074	1.108	1.144	1.186	1.232	1.289	1.362	1.475

Region 3									
Dependent duration (hours)	Exceedance probability								
	0.9	0.8	0.7	0.6	0.5	0.4	0.3	0.2	0.1
1	0.192	0.188	0.184	0.179	0.174	0.168	0.160	0.150	0.135
2	.300	.295	.289	.282	.275	.266	.256	.241	.219
3	.373	.367	.361	.354	.347	.338	.327	.312	.290
6	.543	.536	.529	.521	.512	.501	.488	.470	.443
9	.652	.646	.640	.633	.625	.615	.604	.588	.564
12	.780	.771	.762	.751	.739	.725	.708	.685	.650
18	.955	.947	.938	.929	.918	.905	.889	.868	.835
24	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
36	1.010	1.025	1.044	1.062	1.082	1.107	1.136	1.177	1.238
48	1.019	1.039	1.061	1.085	1.112	1.144	1.183	1.236	1.317
60	1.028	1.053	1.078	1.106	1.133	1.165	1.204	1.257	1.339
72	1.037	1.067	1.095	1.125	1.151	1.182	1.220	1.271	1.350

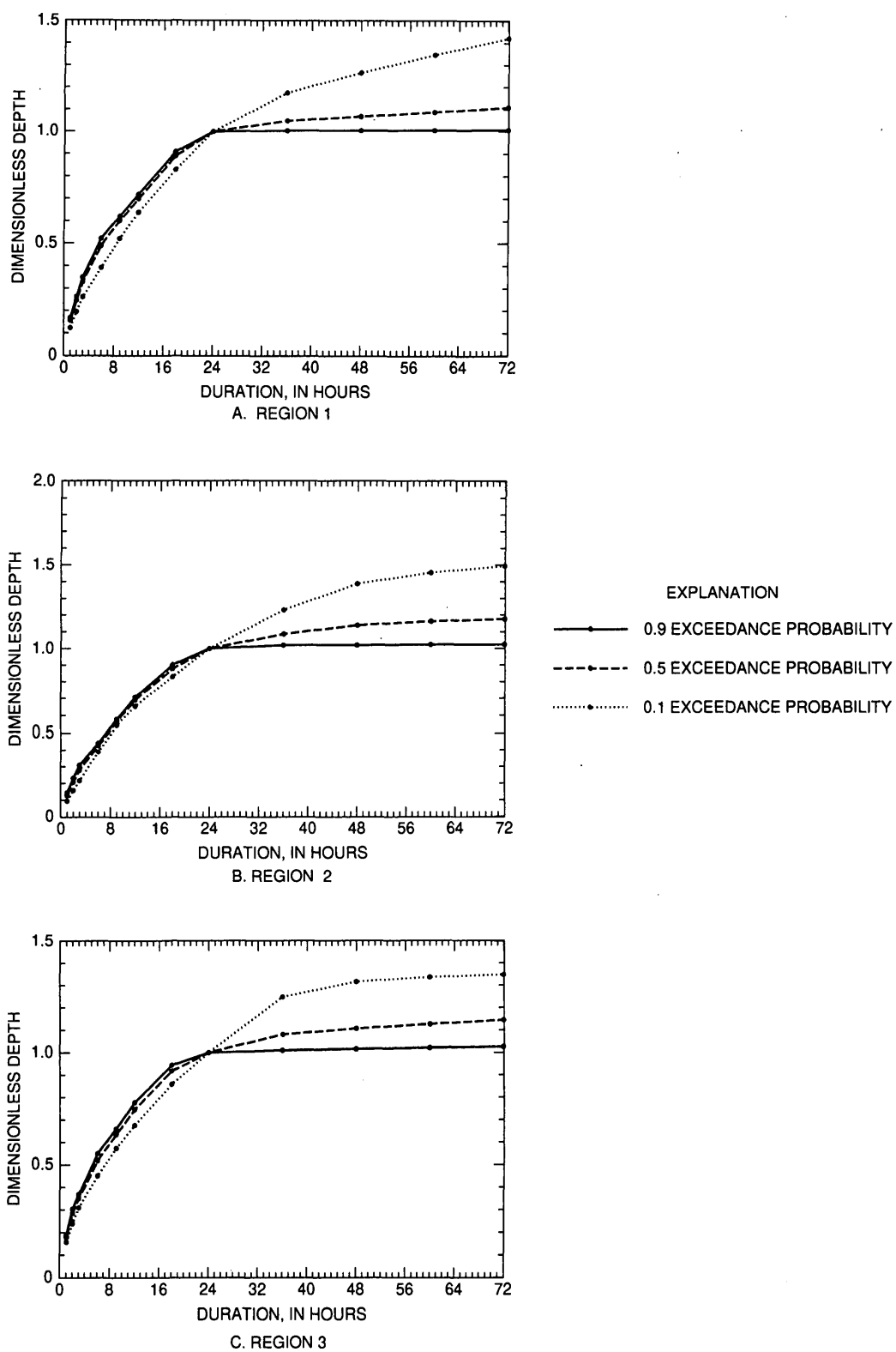


Figure 16. Probabilistic depth-duration curves for 24-hour storms, 48-hour kernel duration, Montana and northern Wyoming.

Probabilistic Time-to-Peak Data

To analyze the probabilistic nature of the time-to-peak variable, its effect on runoff needs to be considered. Because the longer the time-to-peak, the larger the peak discharge with all other runoff conditions being equal, the exceedance probability for time-to-peak can be considered in the same context as dimensionless depth for the kernel duration. In that context, the larger, rarer values produce larger values of peak discharge. Therefore, the Beta distribution (Benjamin and Cornell, 1970) was fitted to the time-to-peak data for each independent duration within each region. For each application of the Beta distribution, the lower bound of the distribution was set to 0, and the upper bound was set to the total storm duration. Estimates of time-to-peak for exceedance probabilities ranging from 0.9 to 0.1 are shown in table 19. Data in table 19 are shown to the nearest 0.0833-hour time increment for 2-hour duration storms, to the nearest 0.25-hour time increment for 6-hour duration storms, and to the nearest 1-hour increment for 24-hour storms.

CONSTRUCTION OF SYNTHETIC STORM HYETOGRAPHS

The probabilistic depth-duration data in tables 13 through 15 and table 18, together with probabilistic time-to-peak data and general data for other temporal

storm characteristics, provide the basis for construction of synthetic storm hyetographs. Before hyetograph construction is described through the use of some example applications, the relation between temporal characteristics of storms and their effects on runoff needs to be considered. In addition, the use of depth-area adjustment curves to adjust point precipitation depths for each incremental duration also needs to be considered.

Exceedance Probability Considerations

Use of the probabilistic dimensionless depth-duration data and probabilistic temporal storm characteristics to construct synthetic storm hyetographs requires some consideration of the relation between precipitation depth for the independent duration and the temporal characteristics of storms and their relative effects on runoff. For example, precipitation depth for the independent duration has a clear and direct relation to both peak discharge and flood volume. On this basis, the first phase of this study (Parrett, 1997) was directed toward the determination of site-specific precipitation magnitude-frequency curves for various independent durations with the obvious implication that a precipitation depth having a low exceedance probability would produce a runoff peak or volume having a low exceedance probability. Thus, the most important consideration in developing synthetic storms for rainfall-runoff

Table 19. Time-to-peak precipitation for various exceedance probabilities, Montana and northern Wyoming

Exceedance probability	Time-to-peak precipitation, in hours, for indicated independent duration, in hours, and region								
	2			6			24		
	Region			Region			Region		
	1	2	3	1	2	3	1	2	3
0.9	0.0833	0.0833	0.0833	0.25	2	0.25	2	8	4
.8	.0833	.0833	.167	.75	3	.75	5	13	7
.7	.0833	.0833	.25	1.25	4.25	1	8	18	12
.6	.0833	.0833	.417	2	5.25	1.5	12	22	17
.5	.167	.25	.50	3	6.5	2.25	17	27	22
.4	.25	.333	.667	4	7.5	3	24	32	28
.3	.417	.50	.917	5.5	9	4	31	37	35
.2	.75	.833	1.167	7.5	10.25	5	40	44	43
.1	1.167	1.333	1.583	10	12.25	7	52	52	53

modeling purposes is precipitation depth for the independent duration and its associated exceedance probability. As discussed by Parrett (1997), selection of a suitable exceedance probability for precipitation depth is a risk-based decision where criteria are usually set by regulatory agencies.

Given a suitable exceedance probability for independent duration precipitation depth, the relation between precipitation depth and the temporal storm characteristics needs to be further considered. Temporal characteristics have great variability, and there is wide latitude in using observed temporal characteristics to construct synthetic storms.

To simplify the problem of exceedance probability specification, a reasonable approach for most rainfall-runoff modeling applications might be to select the dimensionless storm depths and timing characteristics most likely to occur for any large storm depth. On this basis, selection of the median or 0.5 exceedance probability for both the dimensionless depth for the kernel duration and the time-to-peak and use of the most commonly occurring values for high-intensity storm pattern, macro-storm pattern, and 24-hour storm pattern can be considered to constitute the most likely temporal characteristics for a large storm depth having any exceedance probability. Although the rainfall-runoff process is complex and the exceedance probability of a runoff peak or volume is affected by many factors in addition to storm depth and temporal characteristics, the use of commonly occurring or median values for temporal characteristics is considered to provide the best assurance that exceedance probabilities for runoff are most closely related to exceedance probabilities for the independent duration storm depth. Thus, with all other factors that affect runoff held equal, a precipitation depth having a 0.01 exceedance probability is considered more likely to produce a runoff peak having an exceedance probability close to 0.01 if the storm's temporal characteristics are median values than if they are rarer, more extreme characteristics.

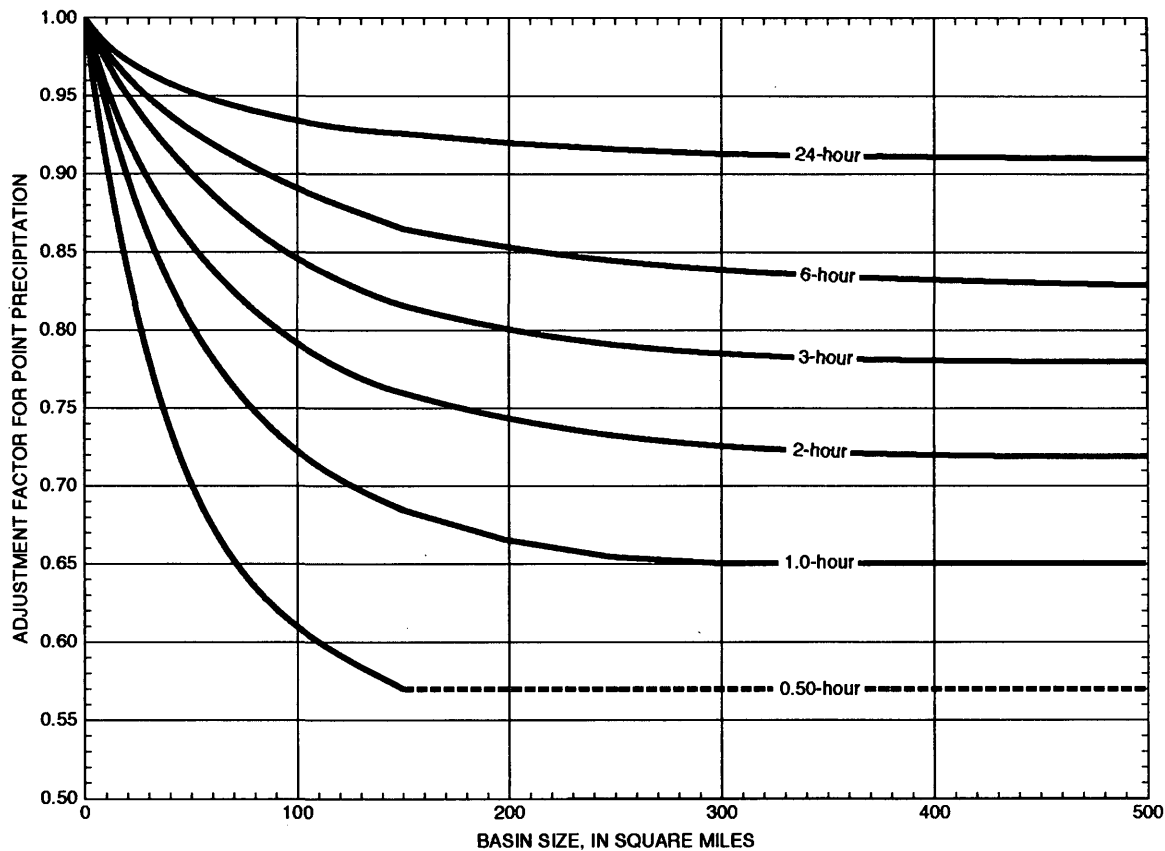
Although the use of median temporal characteristics is considered to provide reasonable synthetic storm data for most rainfall-runoff modeling purposes, there may be times when more conservative temporal characteristics are required. Conservative in this context means that the selected temporal characteristics will result in a larger peak discharge or a larger flood volume. Applications where more conservative temporal characteristics might be desirable include the design of

new spillway structures where some additional margin of safety may routinely be warranted, or for modification of existing structures that are located in environmentally sensitive areas. Thus, for these applications, the exceedance probability for both the dimensionless depth for the kernel duration and the time-to-peak needs to be less than 0.5, and values for high-intensity storm pattern, macro-storm type, and 24-hour storm pattern need to be selected so as to produce higher runoff peaks or volumes than values that more commonly occur. The required degree of conservatism for a particular purpose again needs to be administratively determined. For illustrative purposes in this report, the dimensionless depth for the kernel duration and the time-to-peak having an exceedance probability of 0.2 and values for high-intensity storm pattern, macro-storm type, and 24-hour storm pattern that occurred only about 1 or 2 times out of 10 were arbitrarily considered to provide a reasonable degree of conservatism for design purposes in example problems. Example synthetic storm hyetographs developed in this report on the basis of these conservative temporal storm characteristics are hereafter referred to as "design-purpose" storms, and example synthetic storm hyetographs developed on the basis of median values of temporal storm characteristics are hereafter referred to as "median-value" storms.

Depth-Area Adjustment Curves

As described by Parrett (1997), estimates of at-site total precipitation depth need to be adjusted downward using depth-area adjustment curves to provide basin average values. In addition to the adjustment for total precipitation depth, precipitation depths for incremental durations also need adjustment. Because published depth-area adjustment curves by Miller and others (1973) resulted in the smallest downward adjustment of point precipitation values, they were suggested by Parrett (1997) for the adjustment of total precipitation depth and are also suggested for the adjustment of precipitation depths for incremental storm durations. A more complete version of the depth-area adjustment curves than that shown by Parrett (1997) is shown in figure 17.

Because Miller and others (1973) showed no depth-area adjustment for durations greater than 24 hours, none is used for this study. The shortest incremental duration used by Miller and others (1973) for the construction of depth-area adjustment curves was



Modified from Miller and others (1973)

Figure 17. Depth-area adjustment curves for Montana.

0.5 hour. To adjust point precipitation for incremental durations less than 0.5 hour, use of the adjustment curve for 0.5 hour is considered to provide reasonable, but conservatively large estimates. In addition, the largest area used by Miller and others (1973) for the 0.5-hour adjustment was 150 square miles. Use of the value shown for 150 square miles for larger areas (dashed line in figure 17) also is considered to be conservatively reasonable. Depth-area adjustments vary with incremental duration; consequently, the curves in figure 17 need to be applied to dimensionless depth-duration data used to construct synthetic hyetographs rather than the hyetographs themselves. Use of the curves in figure 17 to adjust point precipitation will be illustrated in the Application Examples section of the report.

Application Examples

To construct a specific synthetic storm hyetograph for use in rainfall-runoff modeling studies, a calculated precipitation depth determined from methods described by Parrett (1997) is used together with appropriate dimensionless probabilistic depth-duration data and temporal characteristics and a depth-area adjustment factor. As described earlier, the recurrence interval of the precipitation depth is the primary factor that determines the overall exceedance probability of the storm and needs to be determined administratively. Once an appropriate recurrence interval for precipitation depth has been selected and storm depth has been determined, the appropriate exceedance probability for the temporal distribution of the precipitation depth needs to be determined. Once again, the determination

of appropriate exceedance probability is administrative and beyond the scope of this report. Nevertheless, based on previous work by Schaefer (1989) in Washington State, use of two levels of exceedance probability for temporal distribution, one based on typical conditions (0.5 exceedance probability) and one based on a more extreme condition (0.2 exceedance probability) is suggested. The following examples are intended to illustrate how methods developed by Parrett (1997) are combined with methods described in this study to produce synthetic storm hyetographs.

Example 1: "Median-value" example hyetograph for 6-hour storm, Region 1, Montana.

As described in Example 1 of the previous report by Parrett (1997, p. 30), a storm depth having a recurrence interval of 100 years has been determined for a 6-hour storm in Region 1 in Montana to be 1.7 inches. The estimate is to be used in a rainfall-runoff model for a 5-square mile basin in southwestern Montana where the desired result is peak discharge having a recurrence interval of 100 years. Based on the assumption that average or typical infiltration and storm characteristics will, on average, produce a runoff peak with a recur-

rence interval comparable to that for storm depth, a "median-value" hyetograph for a 6-hour storm in Region 1 will be constructed based on probabilistic dimensionless depth-duration data in table 13. Although the basin is so small that the depth-area adjustment factor for total storm depth for a 6-hour duration storm is close to 1.00 (0.98) and might be ignored, the adjustment for shorter incremental durations is more significant and needs to be considered. Accordingly, figure 17 will be used to adjust depth-duration data for this example.

To construct the dimensionless storm hyetograph, probabilistic depth-duration data for a 6-hour duration storm for Region 1 (table 13) are first arrayed in the first two columns of a work table as shown in table 20. Depth-area adjustment factors for each incremental duration are either directly selected or interpolated from figure 17 and placed in column 3 of the work table. Each row in column 2 is then multiplied by the appropriate depth-area adjustment factor in column 3 to produce adjusted dimensionless depths as shown in column 4. Because columns 1 and 4 represent cumulative values of time and dimensionless depth, incremental values for each are computed by subtracting each cumulative value from the next cumulative value as

Table 20. Work table for construction of 6-hour duration, "median-value," synthetic hyetograph for 5-square mile basin in Region 1, Montana

[--, not applicable]

Column 1	Column 2	Column 3	Column 4	Column 5	Column 6	Column 7	Column 8	Column 9
Cumulative incremental duration, hours	Cumulative dimensionless depth	Depth-area adjustment factor	Cumulative adjusted dimensionless depth	Incremental time, hours	Incremental dimensionless depth	Number of 0.25-hour time periods per incremental time	Dimensionless depth per 0.25-hour period	Depth per 0.25-hour period, inch
0.00	0.000	--	0.000	--	--	--	--	--
.25	.199	0.94	.187	0.25	0.187	1	0.1870	0.3179
.50	.283	.94	.266	.25	.079	1	.0790	.1343
.75	.356	.96	.342	.25	.076	1	.0760	.1292
1.0	.428	.97	.415	.25	.073	1	.0730	.1241
2	.604	.97	.586	1	.171	4	.0428	.0728
3	.735	.98	.720	1	.134	4	.0335	.0570
6	1.000	.98	.980	3	.260	12	.0217	.0369
9	1.086	.99	1.075	3	.095	12	.0079	.0134
12	1.131	.99	1.120	3	.045	12	.0038	.0065
15	1.162	.99	1.150	3	.030	12	.0025	.0042
18	1.182	.99	1.170	3	.020	12	.0017	.0029

shown in columns 5 and 6 of table 20. Columns 7 and 8 are the number of 0.25-hour time periods in each block of incremental time (column 5 divided by 0.25) and the dimensionless depth per each 0.25-hour time period (column 6 divided by column 7), respectively. The last column (9) in table 20 is the incremental storm depth, in inches, and is the dimensionless value in each row of column 8 times the total at-site storm depth of 1.7 inches. Column 9 thus represents each value of depth to be plotted on the storm hyetograph, and column 7 indicates the number of 0.25-hour time periods having that value of depth. The hyetograph will have 4 values of depth equal to 0.0728 inch, for example. Data in columns 8 and 9 are shown to 4 decimal places to ensure that the sum of the 72 time periods of precipitation depth will equal the cumulative total at the end of column 4 times 1.7 inches.

A hyetograph is constructed by locating calculated depths for each 0.25-hour increment along the x-axis, starting with the largest depth and proceeding, in order, to the smallest. Information provided by time-to-peak, high-intensity storm pattern, and storm macro-pattern is used to locate depths as follows. First, the location of the largest incremental storm depth, 0.3179 inch, is determined by the time-to-peak value having an

exceedance probability of 0.5. From table 19, this time-to-peak value for 6-hour storms in Region 1 is determined to be 3 hours, and the incremental storm depth of 0.3179 is placed at time 3-hour on the x-axis of the hyetograph (fig. 18). The next two largest depth increments, 0.1343 and 0.1292 inch, are located in accordance with the most common high-intensity storm pattern for 6-hour storms in Region 1 (321) as determined from table 2. Thus, depths 0.1292 inch and 0.1343 inch are placed at times 2.5 and 2.75 hours, respectively.

The location of subsequent incremental depths is somewhat subjective. The only criterion for the location of all remaining depth increments on the hyetograph is that an appropriate storm macro-pattern result. From figure 5, macro-pattern 1, with a trisector sequence of 123, is the most common macro-pattern for 6-hour storms in Region 1. Location of the three largest precipitation depths in the first trisector has assured that this trisector will have the greatest depth. Thus, the location of all remaining incremental depths on the hyetograph needs to be done so that the cumulative depth for the second 6-hour period is greater than that for the last 6-hour period. From column 9 of table 20, the next largest incremental depth is shown to be

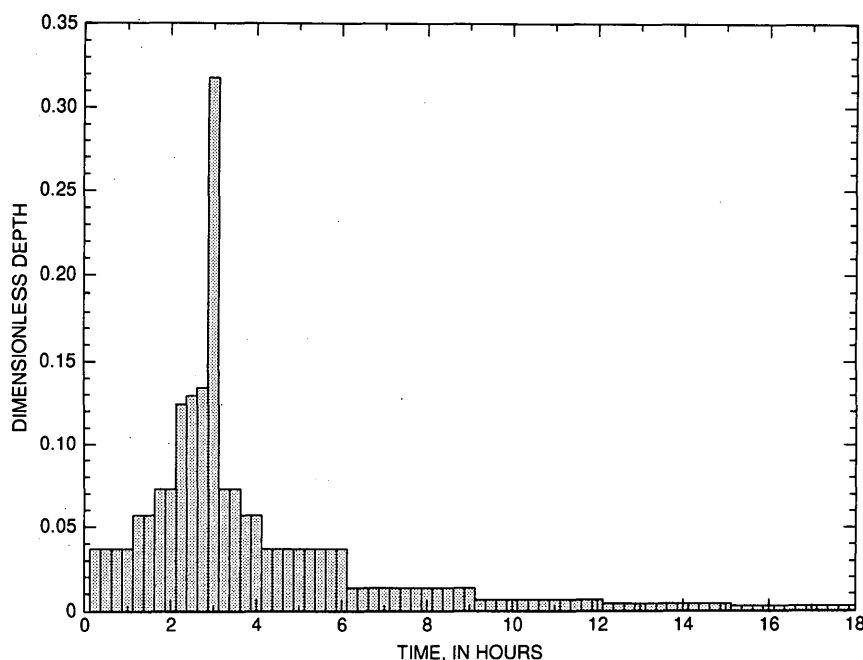


Figure 18. "Median-value" example hyetograph for 6-hour storm, 5-square mile basin, Region 1, Montana.

0.1241 inch. This incremental depth was arbitrarily located just before the three largest depth increments at time 2.25 hours. The next largest depth increment from column 9 of table 20 is 0.0728 inch, and column 7 of table 20 shows that four time periods have this value. Two values of 0.0728 inch are thus located before previously located depths at times 1.75- and 2-hours, and two values of 0.0728 inch are located after previously located depths at times 3.25- and 3.5-hours. Likewise, the next four values of incremental depth shown in column 9 of table 20 (0.0570) are similarly located with two values before all values previously located and two values after those previously located. The same pattern, with half of each group of equal incremental depths placed before and half placed after all depths previously located, is used to locate remaining values of incremental depth until all time periods in the first trisector have been filled. Thereafter, all depths in column 9 are located after all previously located values.

The resultant synthetic storm hyetograph is shown on figure 18. This hyetograph is considered to be a reasonable estimate of the most likely temporal storm pattern for an at-site 6-hour storm depth of 1.7 inches in Region 1 based on the available storm data.

Because the storm hyetograph will be used to calculate peak discharge under typical infiltration conditions, the seasonality of 6-hour storms in Region 1 is not significant. If, however, a conservative, "design-purpose" storm had been required for analysis; a more conservative approach to infiltration characteristics might also have been required. In that instance, the likelihood of a large 6-hour storm occurring in winter, when ground is frozen and infiltration thus minimized, needs to be considered. As shown in figure 3, about 15 percent of the 6-hour storms in Region 1 were winter storms (November through February), and the use of minimal infiltration conditions during a winter storm may indeed be warranted for conservative rainfall-runoff modeling applications.

Example 2: "Design-purpose" example hyetograph for 24-hour storm, 6-hour kernel duration, Montana.

From Example 2 in the previous report by Parrett (1997, p. 30), a basin-average at-site precipitation depth for a 24-hour storm having a 2,000-year recurrence interval was required for a 150-square mile basin partly located in Regions 2 and 3 in Central Montana. For this example it is further assumed that a "design-

purpose" storm hyetograph is required for modeling peak discharge for spillway-design purposes. Because part of the large basin is located in Region 2 and part in Region 3, some hydrologic judgment about the appropriate basin average at-site precipitation depth, dimensionless depth-duration data, and temporal characteristics is required.

One approach is to consider the portion of total basin in each region as a separate subbasin to which a rainfall-runoff model is separately applied. Each runoff hydrograph could then be hydraulically routed and combined to produce the final hydrograph at the basin outlet. Although this approach is conceptually reasonable, it is based on a clear and distinct difference in meteorological conditions in the two subbasins that may be more theoretical than real. As discussed by Parrett (1997), the regional boundaries are somewhat arbitrary, and changes in meteorologic and hydrologic conditions from one region to another are probably more gradual and subtle than the abrupt changes implied by fixed boundary lines.

On that basis, use of dimensionless depth-duration data and temporal characteristics for just one region is considered to be a better approach, and data for Region 2 or for Region 3 needs to be selected for hyetograph construction. Because it is not readily apparent which region will produce the largest peak discharge from rainfall-runoff modeling, a conservative approach would be to use data from both in separate applications and make the final selection on the basis of whichever one produces the largest peak discharge. For purposes of this example, construction of a synthetic hydrograph based on data for Region 2 will be discussed. From Example 2 in the previous report (Parrett, 1997, p. 31), the basin average at-site precipitation depth calculated using the dimensionless precipitation-frequency curve for Region 2 was 7.4 inches.

As with the previous example, a work table is used to demonstrate the construction of a synthetic hyetograph. Because a design-purpose hyetograph is required, probabilistic depth-duration data for a 24-hour storm having an exceedance probability of 0.2 for Region 2 (table 14) are placed in the first two columns of table 21. Depth-area adjustment factors are read directly or interpolated from figure 17 for a drainage area of 150 square miles and placed in column 3 of the work table, and adjusted values for dimensionless depth are calculated by multiplication of values in column 2 by values in column 3. Incremental time (column 5) and incremental dimensionless depth (column

Table 21. Work table for construction of a 24-hour duration, “design-purpose,” synthetic hyetograph (6-hour kernel duration) for a 150-square mile basin in Region 2, Montana

[--, not applicable]

Column 1	Column 2	Column 3	Column 4	Column 5	Column 6	Column 7	Column 8	Column 9
Cumulative incremental duration, hours	Cumulative dimensionless depth	Depth-area adjustment factor	Cumulative adjusted dimensionless depth	Incremental time, hours	Incremental dimensionless depth	Number of 1-hour time periods per incremental time	Dimensionless depth per 1-hour period	Depth per 1-hour period, inch
0	0	--	0.000	--	--	--	--	--
1	.143	0.735	.105	1	0.105	1	0.1050	0.7770
2	.232	.76	.176	1	.071	1	.0770	.5698
3	.308	.82	.253	1	.077	1	.0710	.5254
6	.476	.865	.412	3	.159	3	.0530	.3922
9	.631	.88	.555	3	.143	3	.0477	.3530
12	.749	.895	.670	3	.115	3	.0383	.2834
18	.905	.91	.824	6	.154	6	.0257	.1902
24	1.000	.925	.925	6	.101	6	.0168	.1243
36	1.064	1.000	1.064	12	.139	12	.0116	.0858
48	1.126	1.000	1.126	12	.062	12	.0052	.0385
60	1.150	1.000	1.150	12	.024	12	.0020	.0148
72	1.160	1.000	1.160	12	.010	12	.0008	.0059

6) are calculated as before by subtracting succeeding values of incremental duration (column 1) and adjusted dimensionless depth (column 4). As a result of the varying depth-area adjustment factor, the third value of incremental dimensionless depth in column 6 is slightly larger than the second value. To ensure that calculated values of dimensionless depth per 1-hour period (column 8) are arrayed from largest to smallest, the location of the second and third values of incremental depth in column 8 are switched from their locations as calculated for column 6. Because the value of incremental time for a 24-hour hyetograph is 1 hour, the number of 1-hour periods per incremental time in column 7 is the same as incremental time in column 5. Precipitation depths for each 1-hour period of the hyetograph (column 9) are calculated by multiplying each dimensionless value in column 8 by the basin average at-site precipitation depth of 7.4 inches.

Location of each precipitation depth shown in column 9 of table 21 is based on temporal characteristics for rarer, “design-purpose” conditions in Region 2. Thus, the time-to-peak for a 24-hour storm in Region 2 having a 0.2 exceedance probability is determined from table 19 to be 44 hours, and the largest value of precipitation depth in column 9 of table 21 (0.7770 inch) is located at hour 44 on the hyetograph (figure

19). Given that high-intensity storm patterns with the largest value at the end of the high-intensity period are considered to provide the largest peak runoff, the high-intensity pattern selected for this example is 321. Thus, the second and third largest values in column 9 (0.5698 and 0.5254 inch) are located at hours 43 and 42, respectively, on the hyetograph.

Placement of the remaining values of depth shown in column 9, although somewhat arbitrary, is dependent upon 24-hour storm pattern and storm macro-pattern. Selection of a suitable 24-hour storm pattern also is based on the hypothesis that larger 6-hour periods of precipitation located at the end of the 24-hour period will result in larger values of peak runoff. On this basis, the 24-hour pattern selected for this example is 4321. To ensure that this pattern is followed, the next three values of precipitation depth (0.3922 inch) in column 9 are placed before all previously located values on the hyetograph at hours 39 through 41. The next three values of 0.3530 inch are similarly located at hours 36 through 38, and the next three values of 0.2834 inch are located at hours 33 through 35. Likewise, the next six values of 0.1902 inch are located at hours 27 through 32 and the next six values of 0.1243 inch are located at hours 21 through 26. The 24-hour period from hour 21 through hour 44

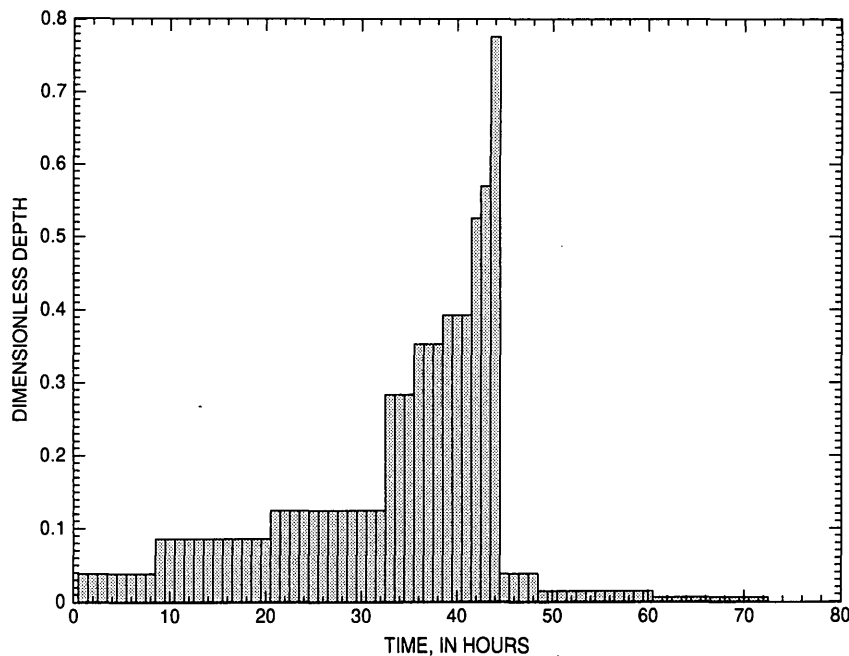


Figure 19. "Design-purpose" example hyetograph for 24-hour storm, 6-hour kernel duration, 150-square mile basin, Region 2, Montana.

thus constitutes the 24-hour period of maximum precipitation depth having the desired 4321 pattern.

The final temporal characteristic to be considered in hyetograph construction is the storm macro-pattern. Given the time-to-peak, high-intensity pattern, and 24-hour storm pattern selected for use for this example, acceptable macro-patterns for this example are those with the largest 24-hour depth in the second trisector (macro-patterns 5, 6, 7, and 8 in figure 3). Because continuous macro-patterns generally are more desirable for rainfall-runoff modeling than intermittent patterns and because both the high-intensity and 24-hour storm patterns have the second largest value preceding the first, macro-pattern 6, with a trisector sequence 213, is selected. The remaining values of precipitation depth in column 9 of table 21 need only be located on the hyetograph to ensure that the depth in the first trisector is greater than the depth in the third. On that basis, the 12 values of 0.0858 inch in column 9 are located before all previously placed hyetograph values at hours 9 through 20. Eight of the next 12 values of

0.0385 inch are located at hours 1 through 8, and the remaining 4 values of 0.0385 inch are located after the peak 1-hour period at hours 45 through 48. The remaining 24 hours of the total 72-hour storm period are filled from hours 49 through 60 with values of 0.0148 inch and from hours 61 through 72 with values of 0.0059 inch. The completed hyetograph is shown in figure 19.

Because this example is for design purposes, the effects of storm seasonality on infiltration conditions need to be considered as discussed in the previous example. In this instance, however, figure 2 indicates that 24-hour duration winter storms are highly unlikely in either Region 2 or 3. Thus, frozen ground and its resultant low infiltration characteristic is not considered to be a factor. On the other hand, given that 24-hour storms in Regions 2 and 3 are most likely to occur during spring and early summer (April through June), high baseflows as a result of snowmelt runoff need to be considered in final application of the rainfall-runoff model.

Example 3: “Design-purpose” example hyetograph for 24-hour storm, 48-hour kernel duration, Region 3, Montana and Northern Wyoming.

A 24-hour average at-site storm depth for a 1,000-year recurrence interval for a 300-square mile basin in eastern Montana (Region 3) has been determined to be 6.5 inches. A reservoir at the basin outlet requires substantial modification, and flood volume rather than peak discharge is the primary rainfall-runoff modeling consideration. Accordingly, a 24-hour “design-purpose” synthetic hyetograph is required for rainfall-runoff modeling and will be based on probabilistic dimensionless depth-duration data for a 48-hour kernel duration in Region 3 (table 18). As in previous examples, a 9-column work table based on appropriate dimensionless depth-duration data, depth-area adjustment factors, and average at-site storm depth is used to calculate precipitation depths for synthetic hyetograph construction (table 22).

For hyetograph construction where flood volume rather than peak discharge is the primary consideration, selection of temporal storm characteristics such as time-to-peak, high-intensity pattern, 24-hour storm pattern, and macro-pattern is not critical because those characteristics affect only peak discharge. Accordingly, for both “design-purpose” and “median-value” conditions, typical temporal characteristics are appropriate for construction of synthetic hyetographs for

flood volume modeling purposes. Thus, the largest value of precipitation depth in column 9 of table 22 (0.6370 inch) is located on the hyetograph at hour 22, the time-to-peak value having a 0.5 exceedance probability for 24-hour storms in Region 3 (table 19). The most common high-intensity pattern for 24-hour storms in Region 3 is 123, and the second and third largest values of precipitation depth in column 9 (0.5070 and 0.4485 inch) of table 22 are located at hours 23 and 24, respectively. The most common 24-hour storm pattern for Region 3 is 1234. To ensure that the example hyetograph has this pattern, the next three largest values of depth in column 9 (0.3250 inch) are placed after all previously located values at hours 25 through 27. These values are followed by the next three largest values (0.2275 inch) at hours 28 through 30, the next three largest values (0.2015 inch) at hours 31 through 33, the next six largest values (0.1937 inch) at hours 34 through 39, and the next six largest values (0.1547 inch) at hours 40 through 45. These depth locations on the hyetograph result in a 1234 24-hour storm pattern and also result in a macro-pattern with the largest precipitation depth in the second trisector. As a result, macro-pattern 6, with a trisector sequence of 213, is arbitrarily selected for this example, and the remaining depths in column 9 are located to ensure that the first trisector has a greater depth than the third. Accordingly, the next 12 values of depth in column 9 (0.1417 inch) are placed before all previously located

Table 22. Work table for construction of a 24-hour duration, “design-purpose,” synthetic hyetograph (48-hour kernel duration) for a 300-square mile basin in Region 3, Montana and northern Wyoming

[--, not applicable]

Column 1	Column 2	Column 3	Column 4	Column 5	Column 6	Column 7	Column 8	Column 9
Incremental duration, hours	Dimensionless depth	Depth-area adjustment factor	Adjusted dimensionless depth	Incremental time, hours	Incremental dimensionless depth	Number of 1-hour time periods per incremental time	Dimensionless depth per 1-hour period	Depth per 1-hour period, inch
0	0.000	--	0.000	--	--	--	--	--
1	.150	0.65	.098	1	0.098	1	0.0980	0.6370
2	.241	.73	.176	1	.078	1	.0780	.5070
3	.312	.785	.245	1	.069	1	.0690	.4485
6	.470	.84	.395	3	.150	3	.0500	.3250
9	.588	.85	.500	3	.105	3	.0350	.2275
12	.685	.865	.593	3	.093	3	.0310	.2015
18	.868	.89	.772	6	.179	6	.0298	.1937
24	1.000	.915	.915	6	.143	6	.0238	.1547
36	1.177	1.000	1.177	12	.262	12	.0218	.1417
48	1.236	1.000	1.236	12	.059	12	.0049	.0318
60	1.257	1.000	1.257	12	.021	12	.0018	.0117
72	1.271	1.000	1.271	12	.014	12	.0012	.0078

values on the hyetograph at hours 10 through 21. Likewise, 9 of the next 12 values (0.0318 inch) are placed at hours 1 through 9, and all remaining depth values are placed after all previously located values. The resultant hyetograph for this example application is shown in figure 20.

As in the previous example, the effects of storm seasonality on infiltration conditions need to be considered for design-purpose applications. As before, figure 2 indicates that 24-hour duration winter storms are highly unlikely in Region 3. Thus, frozen ground and its resultant low infiltration characteristic is not a factor, but high baseflows as a result of snowmelt runoff need to be considered in final application of the rainfall-runoff model.

SUMMARY

A cooperative study to (1) analyze the temporal characteristics of large storms in Montana, and (2) develop methods for using data from the temporal analysis together with storm depth estimates from methods described in an earlier study to construct synthetic

storm hyetographs based on exceedance probability was undertaken. Data from 188 large storms in Montana and 2 in northern Wyoming where the exceedance probability of the storm depth was about 0.10 or less were used to analyze the temporal characteristics of 2-hour, 6-hour, and 24-hour duration storms in Montana. In general, storms in the data base were used for only one station and for only one duration to ensure spatial and temporal independence. Data were grouped into three homogeneous regions that previously were delineated on the basis of physiography and climate. Seasonal and regional characteristics of storms were generally described. Recorded data generally were available only for 0.25-hour time intervals for most stations. To enable the accurate delineation of hyetographs for short-duration (2-hour) storms, data from 28 storms at 9 continuous-record precipitation stations were used to develop general relations between 0.0833-hour and 0.25-hour duration data and between 0.167-hour and 0.25-hour duration data. For each storm in the data base, storm depths were determined for various time increments within a time period of precipitation activity three times longer than the storm

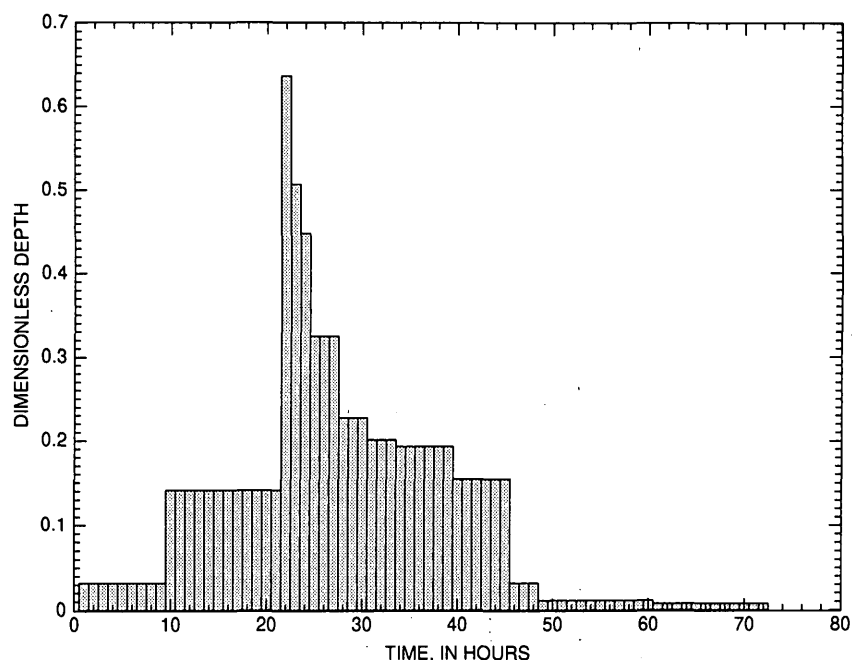


Figure 20. "Design-purpose" example hyetograph for 24-hour storm, 48-hour kernel duration, 300-square mile basin, Region 3, Montana and northern Wyoming.

duration under study. This longer period of precipitation activity was termed total storm duration, while the shorter duration of interest (2-, 6-, or 24-hours) was termed independent duration.

Storms were categorized into 12 possible macro-patterns, depending on the relative storm depth in each third of the total storm duration. Various other temporal characteristics considered to have an effect on peak discharge also were determined from the storm data. First, the time from beginning of each storm to the time of maximum incremental storm depth, termed time-to-peak, was calculated. Another temporal characteristic that was determined for each storm was the sequencing of the three largest, adjacent increments of storm depth including the peak incremental storm depth. On this basis, each 6-hour and 24-hour storm was grouped into one of six possible sequencing patterns, termed the high-intensity storm patterns. Because incremental depths for 0.0833-hour and 0.167-hour durations were estimated for 2-hour duration storms on the basis of 28 storms at 9 sites, data from those 28 storms were used to characterize the high-intensity storm pattern for all 2-hour duration storms. A final temporal characteristic that was considered to have a potentially significant effect on peak discharge only from long-duration, 24-hour storms was the pattern of precipitation occurring in each successive 6-hour block of the 24-hour independent duration. Each 24-hour storm thus was classified into 1 of 24 possible 24-hour storm patterns.

Maximum storm depths for various durations less than the total duration were calculated for each storm and divided by the storm depth for the independent duration to produce dimensionless depth-duration data. The various durations for which the dimensionless depths were calculated, besides the independent duration, were termed dependent durations.

Dimensionless depth-duration data were grouped by independent duration and by region, and the Beta distribution was used to determine dimensionless depths for exceedance probabilities from 0.1 to 0.9 for various dependent durations. A high-intensity portion of storm precipitation shorter than the independent duration that is considered most conducive to generation of peak discharge was defined for each independent duration. This shorter duration, termed a kernel duration for peak discharge modeling purposes, was determined to be 0.5 hour for an independent duration of 2 hours, 2 hours for an independent duration of 6 hours, and 6 hours for an independent duration of 24 hours. Ordinary least-squares regression was used to

develop relations between dimensionless depths for the kernel duration and dimensionless depths for all dependent durations for each independent duration in each region. The regression relations were used, together with the probabilistic dimensionless depth data for the kernel duration, to calculate dimensionless depths for exceedance probabilities from 0.1 to 0.9 for the dependent durations for each independent duration within each region. The resultant tables of probabilistic, dimensionless depth-duration data were smoothed to ensure that the data would produce dimensionless depth-duration curves with uniformly decreasing slopes.

The temporal characteristic considered to have the greatest effect on peak discharge in rainfall-runoff modeling applications was time-to-peak. Accordingly, the Beta distribution was fit to the time-to-peak data, and estimates of time-to-peak for exceedance probabilities ranging from 0.1 to 0.9 were made for each independent duration within each region.

For use in rainfall-runoff modeling where runoff volume is the primary consideration rather than peak discharge, probabilistic, dimensionless depth-duration data for 24-hour storms were developed from a 48-hour kernel duration rather than a 6-hour kernel duration. Accordingly, regression relations were developed between dimensionless depths for the 48-hour duration and every other dependent duration for 24-hour duration storms in each region. The regression relations were used, together with the probabilistic dimensionless depth data for the 48-hour kernel duration, to calculate dimensionless depths for exceedance probabilities from 0.1 to 0.9 for the dependent durations for each independent duration within each region.

Three example applications requiring synthetic hyetographs for rainfall-runoff modeling were described. For each application, dimensionless depth-duration data for either a "median-value" (0.5 exceedance probability) or "design-purpose" (0.2 exceedance probability) condition were first adjusted using a depth-area adjustment factor for basin size. Adjusted values were then multiplied by precipitation depth for the independent duration as determined by methods described in the first phase of the study (Parrett, 1997). Finally, the resultant depth-duration data were arranged in accordance with either "median-value" or "design-purpose" temporal characteristics (time-to-peak, high-intensity pattern, 24-hour storm pattern, and macro-pattern) to produce a synthetic storm hyetograph.

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Table 23. Precipitation data for analysis of temporal storm characteristics, Montana and northern Wyoming

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Station no.	Station name	Latitude, in decimal degrees	Longitude, in decimal degrees	Region	Date of storm	Duration (hours)	Depth for indicated duration (inches)	Exceedance probability
240019	Absarokee, Montana	45.5500	109.3833	2	08-01-1976	6	2.20	0.032
240330	Ashland Ranger Station, Montana	45.6000	106.2667	3	06-23-1991	6	1.70	.085
240364	Augusta, Montana	47.4833	112.3833	2	05-15-1981	2	1.80	.015
240364	Augusta, Montana	47.4833	112.3833	2	06-03-1958	24	3.24	.030
240364	Augusta, Montana	47.4833	112.3833	2	06-14-1962	24	2.64	.073
240622	Belgrade AP, Montana	45.7833	111.1500	1	08-28-1964	24	1.84	.052
240807	Billings WSO AP, Montana	45.8000	108.5333	3	04-27-1978	24	3.15	.020
240807	Billings WSO AP, Montana	45.8000	108.5333	3	05-21-1952	24	3.15	.037
240807	Billings WSO AP, Montana	45.8000	108.5333	3	06-16-1982	2	1.06	.072
240807	Billings WSO AP, Montana	45.8000	108.5333	3	07-04-1975	2	1.44	.038
240926	Bloomfield 6 E, Montana	47.4167	104.8000	3	06-10-1967	24	3.05	.037
241008	Boulder St School, Montana	46.2333	112.1167	1	06-21-1991	2	1.20	.013
241008	Boulder St School, Montana	46.2333	112.1167	1	08-19-1990	2	1.50	.006
241047	Bozeman 6 W Exp Farm, Montana	45.6667	111.1500	1	06-30-1986	2	.73	.086
241047	Bozeman 6 W Exp Farm, Montana	45.6667	111.1500	1	07-16-1978	2	.79	.057
241102	Bridger, Montana	45.3000	108.9167	3	04-25-1964	24	2.65	.044
241127	Broadus, Montana	45.4333	105.4000	3	06-17-1944	24	3.64	.008
241127	Broadus, Montana	45.4333	105.4000	3	06-19-1980	2	1.90	.050
241127	Broadus, Montana	45.4333	105.4000	3	07-03-1974	6	2.30	.021
241127	Broadus, Montana	45.4333	105.4000	3	10-01-1971	24	2.74	.037
241202	Browning, Montana	48.5667	113.0167	2	05-29-1985	24	4.00	.019
241202	Browning, Montana	48.5667	113.0167	2	08-18-1980	6	1.70	.066
241309	Butte 8 S, Montana	45.9000	112.5500	1	06-01-1985	6	1.10	.060
241309	Butte 8 S, Montana	45.9000	112.5500	1	07-08-1980	2	1.30	.009
241309	Butte 8 S, Montana	45.9000	112.5500	1	08-23-1976	6	1.00	.092
241408	Cameron, Montana	45.2000	111.6833	1	05-25-1976	6	1.20	.052
241737	Choteau Airport, Montana	47.8167	112.1667	3	06-15-1948	24	2.49	.067
241781	Clark Canyon Dam, Montana	45.0000	112.7833	1	04-24-1971	24	1.75	.041
241781	Clark Canyon Dam, Montana	45.0000	112.7833	1	05-16-1987	24	1.80	.035
241875	Cohagen, Montana	47.0500	106.6167	3	09-09-1961	24	2.97	.024
241875	Cohagen, Montana	47.0500	106.6167	3	09-18-1988	6	1.80	0.050
241995	Cooke City 2 W, Montana	45.0167	109.9667	2	11-07-1980	6	1.10	.095
242054	Corwin Springs 1 NNW, Montana	45.1333	11.8167	1	10-30-1964	24	1.70	.040
242173	Cut Bank FAA AP, Montana	48.6000	112.3667	3	06-04-1986	2	.93	.056
242173	Cut Bank FAA AP, Montana	48.6000	112.3667	3	06-20-1991	2	1.20	.030
242221	Darby, Montana	46.0167	114.1667	1	09-07-1973	6	1.00	.070
242404	Dillon Airport, Montana	45.2500	112.5500	1	06-24-1958	24	1.96	.023
242414	Dillon 9 SSE, Montana	45.0833	112.6000	1	05-10-1981	24	1.80	.063
242414	Dillon 9 SSE, Montana	45.0833	112.6000	1	08-20-1983	2	.90	.026
242441	Dodson 11 N, Montana	48.5500	108.2000	3	07-09-1983	24	4.02	.004

Table 23. Precipitation data for analysis of temporal storm characteristics, Montana and northern Wyoming (Continued)

Station no.	Station name	Latitude, in decimal degrees	Longitude, in decimal degrees	Region	Date of storm	Duration (hours)	Depth for indicated duration (inches)	Exceedance probability
242477	Dovetail, Montana	47.3500	108.2500	3	06-13-1943	24	5.50	.001
242477	Dovetail, Montana	47.3500	108.2500	3	07-13-1948	24	2.67	.050
242500	Drummond Aviation, Montana	46.6667	113.1500	1	05-24-1985	2	.80	.061
242584	Dutton 6 E, Montana	47.8500	111.5833	3	05-25-1987	6	1.50	.092
242584	Dutton 6 E, Montana	47.8500	111.5833	3	08-01-1991	2	1.30	.044
242584	Dutton 6 E, Montana	47.8500	111.5833	3	08-20-1975	6	1.80	.042
242584	Dutton 6 E, Montana	47.8500	111.5833	3	08-24-1989	24	3.10	.030
242689	Ekalaka, Montana	45.8833	104.5333	3	05-03-1955	24	3.40	.025
242689	Ekalaka, Montana	45.8833	104.5333	3	05-27-1956	24	2.68	.077
242827	Eureka R S, Montana	48.9000	115.0667	1	08-14-1989	2	1.00	.023
242827	Eureka R S, Montana	48.9000	115.0667	1	09-11-1985	24	1.50	.066
243176	Fort Peck PWR PL, Montana	48.0167	106.4000	3	05-05-1965	24	3.91	.015
243176	Fort Peck PWR PL, Montana	48.0167	106.4000	3	06-08-1976	2	2.00	.031
243176	Fort Peck PWR PL, Montana	48.0167	106.4000	3	06-17-1964	24	5.26	.002
243176	Fort Peck PWR PL, Montana	48.0167	106.4000	3	06-20-1974	2	2.30	.017
243176	Fort Peck PWR PL, Montana	48.0167	106.4000	3	07-13-1981	2	2.60	.012
243176	Fort Peck PWR PL, Montana	48.0167	106.4000	3	07-17-1983	2	1.50	.090
243176	Fort Peck PWR PL, Montana	48.0167	106.4000	3	08-20-1966	24	2.89	.066
243309	Froid, Montana	48.3333	104.5000	3	06-07-1950	24	2.43	.055
243479	Gibbons Pass, Montana	45.7000	113.9500	1	02-24-1986	6	2.30	.004
243479	Gibbons Pass, Montana	45.7000	113.9500	1	05-19-1974	24	2.32	.077
243479	Gibbons Pass, Montana	45.7000	113.9500	1	06-26-1988	2	.80	.095
243479	Gibbons Pass, Montana	45.7000	113.9500	1	08-13-1978	6	1.30	.107
243479	Gibbons Pass, Montana	45.7000	113.9500	1	09-12-1988	2	1.00	.038
243489	Gibson Dam, Montana	47.6000	112.7667	2	05-18-1991	6	3.00	.009
243489	Gibson Dam, Montana	47.6000	112.7667	2	05-24-1962	24	3.68	.035
243489	Gibson Dam, Montana	47.6000	112.7667	2	06-27-1983	2	1.40	.023
243489	Gibson Dam, Montana	47.6000	112.7667	2	08-30-1971	6	2.70	.015
243558	Glasgow WSO AP, Montana	48.2167	106.6167	3	07-01-1975	2	2.31	.016
243558	Glasgow WSO AP, Montana	48.2167	106.6167	3	07-13-1962	24	3.98	.010
243558	Glasgow WSO AP, Montana	48.2167	106.6167	3	08-02-1985	2	2.00	.011
243581	Glendive, Montana	47.1000	104.7167	3	06-27-1990	2	1.60	.086
243581	Glendive, Montana	47.1000	104.7167	3	08-15-1980	24	4.00	.019
243581	Glendive, Montana	47.1000	104.7167	3	08-30-1989	2	2.60	.016
243751	Great Falls WSCMO AP, Montana	47.4833	111.3667	3	05-28-1990	2	1.41	.044
243751	Great Falls WSCMO AP, Montana	47.4833	111.3667	3	06-06-1976	2	1.22	.076
243751	Great Falls WSCMO AP, Montana	47.4833	111.3667	3	06-20-1991	2	1.14	.095
243751	Great Falls WSCMO AP, Montana	47.4833	111.3667	3	08-09-1950	2	1.67	.024
243984	Haugan 3 E (Deborgia), Montana	47.3833	115.3500	1	01-08-1983	6	1.50	.038
243984	Haugan 3 E (Deborgia), Montana	47.3833	115.3500	1	06-10-1989	2	.80	.052

Table 23. Precipitation data for analysis of temporal storm characteristics, Montana and northern Wyoming (Continued)

Station no.	Station name	Latitude, in decimal degrees	Longitude, in decimal degrees	Region	Date of storm	Duration (hours)	Depth for indicated duration (inches)	Exceedance probability
243984	Haugan 3 E (Deborgia), Montana	47.3833	115.3500	1	08-23-1979	2	.70	.088
243996	Havre WSO AP, Montana	48.5500	109.7667	3	07-16-1986	2	1.06	.100
243996	Havre WSO AP, Montana	48.5500	109.7667	3	08-23-1965	2	1.76	.014
244020	Hays, Montana	48.0000	108.7000	2	05-24-1953	24	3.73	.009
244038	Hebgen Dam, Montana	44.8667	111.3333	1	03-29-1982	2	1.20	.025
244038	Hebgen Dam, Montana	44.8667	111.3333	1	09-22-1973	2	.80	.095
244038	Hebgen Dam, Montana	44.8667	111.3333	1	09-26-1979	6	1.10	.100
244055	Helena WSO AP, Montana	46.6000	112.0000	2	06-18-1979	2	1.47	.015
244055	Helena WSO AP, Montana	46.6000	112.0000	2	07-03-1975	2	.96	.077
244055	Helena WSO AP, Montana	46.6000	112.0000	2	07-10-1955	2	.98	.070
244055	Helena WSO AP, Montana	46.6000	112.0000	2	08-21-1983	2	1.60	.010
244120	Highwood, Montana	47.5500	110.7833	3	05-17-1959	24	3.51	.034
244120	Highwood, Montana	47.5500	110.7833	3	06-01-1954	24	3.33	.044
244143	Hilger, Montana	47.2500	109.3500	3	05-24-1990	2	1.70	.031
244143	Hilger, Montana	47.2500	109.3500	3	06-21-1991	2	2.40	.005
244241	Holter Dam, Montana	47.0000	112.0167	2	04-29-1951	24	2.24	.066
244241	Holter Dam, Montana	47.0000	112.0167	2	06-15-1989	2	.90	.075
244368	Iliad, Montana	47.8000	109.7833	3	08-14-1968	24	4.08	.009
244442	Ismay, Montana	46.5000	104.8000	3	09-02-1973	24	3.15	.023
244512	Joplin, Montana	48.5667	110.7667	3	09-11-1985	2	1.00	.090
244558	Kalispell WSO AP, Montana	48.3000	114.2667	1	06-03-1966	24	2.35	.016
244558	Kalispell WSO AP, Montana	48.3000	114.2667	1	06-15-1956	24	1.97	.045
244558	Kalispell WSO AP, Montana	48.3000	114.2667	1	06-29-1982	2	2.65	<.001
244558	Kalispell WSO AP, Montana	48.3000	114.2667	1	07-05-1990	2	.87	.072
244558	Kalispell WSO AP, Montana	48.3000	114.2667	1	08-03-1976	2	1.57	.011
244820	Lakeview, Montana	44.6000	111.8000	1	06-08-1977	2	.90	.078
244820	Lakeview, Montana	44.6000	111.8000	1	06-10-1990	24	2.20	.072
244820	Lakeview, Montana	44.6000	111.8000	1	06-29-1973	2	1.10	.051
244820	Lakeview, Montana	44.6000	111.8000	1	07-09-1991	2	1.10	.035
244820	Lakeview, Montana	44.6000	111.8000	1	07-10-1987	6	1.50	.044
244820	Lakeview, Montana	44.6000	111.8000	1	07-17-1987	24	2.20	.072
244820	Lakeview, Montana	44.6000	111.8000	1	09-06-1976	6	1.30	.086
244820	Lakeview, Montana	44.6000	111.8000	1	11-12-1973	6	1.60	.031
244904	Lavina, Montana	46.3000	108.9333	3	05-24-1991	2	1.10	.073
244904	Lavina, Montana	46.3000	108.9333	3	06-05-1991	6	1.60	.064
244904	Lavina, Montana	46.3000	108.9333	3	06-06-1967	24	4.88	.001
244983	Lewistown 2 SW, Montana	47.0500	109.4500	3	04-19-1973	24	4.20	.009
244983	Lewistown 2 SW, Montana	47.0500	109.4500	3	05-04-1986	6	1.60	.077
244983	Lewistown 2 SW, Montana	47.0500	109.4500	3	05-20-1962	24	3.65	.021
245015	Libby 1 NE R S, Montana	48.4000	115.5333	1	06-21-1984	6	1.50	.017

Table 23. Precipitation data for analysis of temporal storm characteristics, Montana and northern Wyoming (Continued)

Station no.	Station name	Latitude, in decimal degrees	Longitude, in decimal degrees	Region	Date of storm	Duration (hours)	Depth for indicated duration (inches)	Exceedance probability
245015	Libby 1 NE R S, Montana	48.4000	115.5333	1	07-23-1990	6	1.90	.004
245015	Libby 1 NE R S, Montana	48.4000	115.5333	1	07-30-1987	2	.90	.034
245015	Libby 1 NE R S, Montana	48.4000	115.5333	1	08-14-1981	2	.90	.034
245015	Libby 1 NE R S, Montana	48.4000	115.5333	1	11-24-1990	24	2.10	.028
245030	Lima, Montana	44.6500	112.5833	1	08-11-1989	2	.70	.095
245030	Lima, Montana	44.6500	112.5833	1	08-21-1986	6	.90	.100
245040	Lincoln R S, Montana	46.9500	112.6500	2	04-24-1975	24	2.60	.040
245086	Livingston FAA AP, Montana	45.7000	111.4500	2	09-21-1984	6	1.24	.080
245106	Lodge Grass, Montana	45.3000	107.3667	3	05-17-1978	24	4.60	.004
245106	Lodge Grass, Montana	45.3000	107.3667	3	06-03-1986	6	1.60	.076
245106	Lodge Grass, Montana	45.3000	107.3667	3	06-14-1967	24	3.29	.028
245146	Lolo Hot Springs 2 NE, Montana	46.7500	114.5167	1	06-24-1978	6	1.10	.079
245146	Lolo Hot Springs 2 NE, Montana	46.7500	114.5167	1	08-20-1984	24	3.50	.002
245387	Martinsdale 3 NNW, Montana	46.5000	110.3333	2	06-09-1987	2	1.10	.044
245387	Martinsdale 3 NNW, Montana	46.5000	110.3333	2	06-19-1979	6	1.30	.084
245706	Millegan, Montana	47.0333	111.3667	2	06-02-1953	24	4.17	.010
245706	Millegan, Montana	47.0333	111.3667	2	06-14-1980	6	2.10	.020
245706	Millegan, Montana	47.0333	111.3667	2	07-12-1989	2	1.40	.031
245706	Millegan, Montana	47.0333	111.3667	2	08-04-1981	2	1.20	.042
245706	Millegan, Montana	47.0333	111.3667	2	09-11-1978	24	2.80	.062
245745	Missoula WSO AP, Montana	46.9167	114.0833	1	06-11-1958	2	1.48	.005
245745	Missoula WSO AP, Montana	46.9167	114.0833	1	08-10-1963	2	.75	.078
245745	Missoula WSO AP, Montana	46.9167	114.0833	1	08-20-1981	2	.97	.028
246008	Neihart 8 NNW, Montana	47.0500	110.7833	2	07-04-1988	6	1.90	.037
246302	Ovando, Montana	47.0167	113.1333	1	06-20-1974	2	1.50	.005
246562	Plains R S, Montana	47.4667	114.8833	1	06-23-1955	24	1.75	.047
246562	Plains R S, Montana	47.4667	114.8833	1	07-09-1986	2	.70	.084
246562	Plains R S, Montana	47.4667	114.8833	1	08-14-1987	2	.70	.084
246580	Pleasant Valley 4 SE, Montana	48.1000	114.8667	1	01-15-1974	24	2.66	.013
246580	Pleasant Valley 4 SE, Montana	48.1000	114.8667	1	07-24-1990	6	1.20	.062
246580	Pleasant Valley 4 SE, Montana	48.1000	114.8667	1	08-15-1985	2	.90	.031
246946	Reedpoint, Montana	45.7167	109.5500	3	06-15-1957	24	2.73	.050
247204	Round Butte 1 NNE, Montana	47.5333	114.2833	1	07-02-1956	24	2.01	.052
247292	Saint Mary, Montana	48.7500	113.4333	2	07-21-1987	24	4.50	.017
247425	Scobey 3 N, Montana	48.8333	105.4333	3	06-29-1988	6	2.00	.044
247448	Seeley Lake R S, Montana	47.2167	113.5167	1	05-09-1980	6	1.50	.030
247448	Seeley Lake R S, Montana	47.2167	113.5167	1	06-12-1982	2	.90	.055
247448	Seeley Lake R S, Montana	47.2167	113.5167	1	06-13-1986	6	1.60	.020
247448	Seeley Lake R S, Montana	47.2167	113.5167	1	06-27-1986	2	1.00	.035
247501	Shelby AP, Montana	48.5500	111.8667	3	05-20-1981	6	2.50	.006

Table 23. Precipitation data for analysis of temporal storm characteristics, Montana and northern Wyoming (Continued)

Station no.	Station name	Latitude, in decimal degrees	Longitude, in decimal degrees	Region	Date of storm	Duration (hours)	Depth for indicated duration (inches)	Exceedance probability
247501	Shelby AP, Montana	48.5500	111.8667	3	08-05-1976	2	1.10	.084
247610	Silver Star, Montana	45.6833	112.2833	1	07-31-1990	2	2.20	<.001
247618	Simms, Montana	47.5000	111.9167	3	05-08-1983	24	2.70	.048
247978	Summit, Montana	48.3167	113.3500	2	03-22-1987	2	.80	.099
247978	Summit, Montana	48.3167	113.3500	2	06-07-1964	24	7.37	.001
247978	Summit, Montana	48.3167	113.3500	2	06-18-1975	24	4.60	.012
248087	Swan Lake, Montana	47.9167	113.8333	1	05-23-1981	2	1.00	.033
248087	Swan Lake, Montana	47.9167	113.8333	1	07-13-1982	2	.90	.051
248087	Swan Lake, Montana	47.9167	113.8333	1	08-23-1965	24	3.04	.020
248087	Swan Lake, Montana	47.9167	113.8333	1	08-31-1979	6	1.50	.062
248101	Swift Dam, Montana	48.1667	112.8500	2	05-30-1985	6	1.60	.102
248169	Terry 21 NNW, Montana	47.0667	105.5000	3	06-29-1991	2	1.60	.064
248329	Townsend 12 ENE, Montana	46.3500	111.2833	1	05-20-1991	6	1.10	.097
248329	Townsend 12 ENE, Montana	46.3500	111.2833	1	05-24-1980	24	2.60	.017
248511	Vananda 5 ESE, Montana	46.3833	106.9000	3	05-19-1987	2	1.60	.036
248511	Vananda 5 ESE, Montana	46.3833	106.9000	3	06-19-1991	2	2.10	.011
248511	Vananda 5 ESE, Montana	46.3833	106.9000	3	08-31-1986	6	3.70	.001
248777	Westby, Montana	48.8667	104.0500	3	06-11-1976	24	3.85	.009
248927	White Sulphur Springs, Montana	46.5333	110.9167	2	07-02-1987	6	2.20	.009
249008	Willow Creek, Montana	45.8167	111.6500	1	09-11-1976	24	2.20	.023
249052	Winnett 8 ESE, Montana	48.9500	108.1833	3	08-05-1954	2	2.04	.013
249052	Winnett 8 ESE, Montana	48.9500	108.1833	3	08-12-1991	2	2.63	.004
249067	Wisdom, Montana	45.6167	113.4500	1	06-10-1976	2	.60	.093
249067	Wisdom, Montana	45.6167	113.4500	1	09-07-1970	24	1.80	.022
249240	Yellowtail Dam, Montana	45.3167	107.9333	3	05-06-1984	6	1.70	.107
249240	Yellowtail Dam, Montana	45.3167	107.9333	3	07-25-1981	6	3.10	.005
249900	Zortman, Montana	47.9167	108.5333	2	05-06-1988	24	5.90	.012
249900	Zortman, Montana	47.9167	108.5333	2	09-24-1986	24	5.50	.017
488155	Sheridan WB AP, Wyoming	44.7600	106.9600	3	04-03-1955	24	3.06	.042
488155	Sheridan WB AP, Wyoming	44.7600	106.9600	3	04-27-1963	24	3.24	.032

