PHYSICAL BASIS AND POTENTIAL ESTIMATION TECHNIQUES FOR SOIL EROSION PARAMETERS IN THE PRECIPITATION-RUNOFF MODELING SYSTEM (PRMS)

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William P. Carey and Andrew Simon

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ABSTRACT

Simulation of upland soil erosion by the Precipitation-Runoff Modeling System currently requires the user to estimate two rainfall detachment parameters and three hydraulic detachment parameters. One rainfall detachment parameter can be estimated from rainfall simulator tests. A reformulation of the rainfall detachment equation allows the second parameter to be computed directly.

The three hydraulic detachment parameters consist of one exponent and two coefficients. The initial value of the exponent is generally set equal to 1.5. The two coefficients are functions of the soil's resistance to erosion and one of the two also accounts for sediment delivery processes not simulated in the model. Initial estimates of these parameters can be derived from other modeling studies or from published empirical relations.

INTRODUCTION

Simulation of upland soil erosion by the Precipitation-Runoff Modeling System (PRMS) currently requires the user to estimate two rainfall detachment parameters and three hydraulic detachment parameters (Leavesley and others, 1983). At present, these five parameters must be estimated by calibrating simulated results with observed data collected either on small upland plots or at a downstream gaging station. Neither method provides a physical basis for estimating or transferring parameter values.

To provide a starting point for further research on the physical basis and significance of these parameters, a preliminary assessment of the status of upland soil erosion research and modeling has been conducted. Recent literature on the subject has been reviewed and research personnel who are actively investigating upland soil erosion have been contacted during this study.

The most significant progress in relating measurable soil properties to modeling parameters has come from agricultural research. Studies involving rainfall simulators have shown that some rainfall detachment parameters can be related to drop size distribution, fall velocities, and total mass at impact (Hudson, 1971). Hydraulic detachment parameters have been found to be a function of certain physio-chemical properties of the soil which influence the
material's resistance to erosion. These properties have been found to include particle size, cation-exchange capacity, percentage organic matter, sodium adsorption ratio (SAR), dissolved solids concentration in pore water, and temperature (Partheniades, 1965; Sherard and others, 1972; and Sargunam and others, 1973; Ariathurai and Arulanandan, 1978; Kelly and Gularte, 1981). Site specific factors such as soil density, antecedent moisture, particle orientation, previous stress history, and initial state of compaction have also been found to exert significant influences on the erosion of cohesive soil material (Crissinger, 1966).

The influence of cohesive forces relative to gravitational forces in a soil determines the significance of the physio-chemical properties in parameter evaluation. As yet, no single or combined set of analyses have been shown to yield consistently accurate estimates of parameter values over a wide range of cohesive conditions. The methods presented in this report provide guides to estimating initial parameter values. These values are only estimates and the model user should not hesitate to adjust or replace these initial values in favor of reasonable values that improve model results.

PURPOSE AND SCOPE

The purpose of this study is to begin an investigation of the physical basis of the PRMS soil erosion parameters. Currently available literature was reviewed to define the physical meaning of the parameters, to assess the potential for estimating parameters from physically measurable characteristics, and to make recommendations for model improvements and further research. The recommendations and suggestions are made for the purpose of improving parameter estimates in the current equations and do not necessarily address the computational scheme.

PRMS uses a deterministic, distributed-parameter approach based on erosion mechanics. Investigation of models that use lumped parameters (average values for the characteristics of the entire basin), the Universal Soil Loss Equation, or stochastic approaches to soil erosion is beyond the scope of this project.

STUDY APPROACH

The study consisted of a 3-month effort to review current literature on the status of upland erosion simulation with particular emphasis on the physical significance of model parameters. The literature search was limited to post-1965 material because Masch (1968) published a detailed review of cohesive soil erosion that included literature up to 1965. Computer assisted searches of the AGRICOLA and Water Resources Abstracts data bases were done through the U.S. Geological Survey Library in Reston, Virginia; recent volumes of major journals were searched manually. Information on a number of current studies were obtained by personal contact with Agricultural Research Service researchers at the U.S. Department of Agriculture Sedimentation Laboratory at Oxford, Mississippi.
Although a significant amount of literature was reviewed, this study cannot be classified as comprehensive or exhaustive. One notable deficiency is the lack of foreign references. Although foreign references were not purposely excluded from the computer assisted searches, not many entries appeared. Other possible sources of information that were not explored in detail for this study include unpublished theses and dissertations, and commercial services such as NTIS, INFONET, and Current Contents.

By far the most profitable sources of significant papers have been the references given in the literature and those suggested by researchers in the field. Quite a few of the more relevant papers found through these two sources did not appear in the computer assisted searches.

UPLAND SOIL EROSION

Soil erosion by water from upland areas involves a complex interaction between the processes of detachment and transport and opposing factors that tend to promote soil resistance and retard soil dispersion and movement.

UPLAND MATERIALS

Identification of appropriate physically based parameters and equations to evaluate the detachment process is a function of the nature of the material to be eroded. Certain fundamental characteristics clearly distinguish between two general classifications of soils, cohesive and noncohesive. The differentiation of soils into cohesive and noncohesive is usually accomplished by referring to particle size. However, it should be noted that the term cohesive soil or cohesive material refers to material that is in a cohesive state. The same material may be functionally noncohesive when exposed to a different environment.

Cohesive Soils

Cohesive soils are composed of mixtures of fine silt and clay whose resistance to erosion is related to the electrochemical bond between individual particles. It is the small mass of the particles relative to their large specific area (area per unit particle mass) that determines the importance of physio-chemical forces. These forces can be generated from within the material such as attracting van der Waals forces and electrical forces, or by electrical forces on the particle surface.

Fine soil particles possess a net negative electrical charge due to isomorphous substitution of lower valence cations and from uncompensated valences known as "broken-bonds" on the particle surface. Commonly, adsorption of anions and cations takes place at the edges of the particle, while positive ions (cations) are adsorbed on the surface. Flocculation of fine material is partially attributable to this electrostatic attraction of the positively charged edges to the negatively charged surfaces, plus additional attraction
due to van der Waals forces. However, it is the net negative electric surface charge of the particle that determines the potential for cation adsorption (cation exchange capacity), and the character of the electrochemical bond.

Other types of particle bonding occur in fine soils by hydrogen and cation bonds, bridging, and cementation by chemical compounds such as iron oxides. Of these mechanisms of bonding, the cation type is weaker because it can be broken by water adsorption and swelling.

As a result of the electrical charges, cohesive soil particles will attract ions of opposite charges (counter-ions). The counter-ions of the electrolyte solution exist in a diffused state of decreasing concentration from the particle. This constitutes the "double layer" and an electrically neutral system know as the "clay micelle" (fig. 1). It is the properties of the clay micelle, particularly the double layer, that maintain powerful influences on the erodibility of cohesive material. The strength of the bond represented by the double layer and thus cohesive soil resistance is a function of at least the ionic charge on the particles, the presence and type of electrolytes, mineralogy, temperature, pH, and adsorption potential (Masch, 1968).

**EXPLANATION**

+ Cations
1 Anions

Particle surface

Diffused Double layer

![Figure 1.-- Electrically neutral clay micelle.](image)

**Noncohesive Soils**

In contrast, noncohesive soils are composed of larger particles that resist erosion through gravitational forces. These forces are merely a function of the size, weight, shape, and surface texture of individual grains. Unlike cohesive materials, particle interaction is solely mechanical and is restricted to momentum exchanges occurring from fluid drag, random collisions,
and interlocking support from adjacent grains. Aggregates of cohesive material behave like noncohesive grains. Internally cohesive forces dominate, but between aggregates there is little or no cohesion.

**Resistance**

The significance of the nature of the material in resisting erosion can be illustrated by relating grain diameter to the velocity required to entrain them (critical erosion velocity). Hjulstrom (1935) found that critical erosion velocity is proportional to particle size down to about 50 microns where the trend reverses (fig. 2). Critical erosion velocity and, therefore, resistance to erosion increases for material finer than 50 microns indicating that forces other than the effects of particle size and weight are at work. Additional forces that shape the relation in figure 2 are of a physiochemical nature, and the inflection point empirically shows the approximate boundary for cohesive conditions. The range of critical velocities also increases with decreasing grain size and may attain three orders of magnitude due to potentially diverse electrochemical characteristics of the soil-water system. Critical erosion velocity is used here solely for illustrative purposes. The more widely accepted Shield's Diagram for incipient motion (Vanoni, 1975) also has a concave upward shape with its inflection point indicating the approximate cohesive-noncohesive boundary. However, the relation between particle size and the inflection point is not obvious on the Shield's Diagram.

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**Figure 2.** Relation between grain diameter and critical erosion velocity.
Basically, cohesive soils resist erosion through their electrochemical bonding while coarser noncohesive soils do so by means of their greater weight and size. Therefore silty soils tend to be the most erodible (fig. 2) owing to their lack of both mass and stabilizing cohesive forces. They are subject also to surface sealing which retards infiltration, yielding higher runoff rates and consequently higher detachment rates.

EROSION AND TRANSPORT PROCESSES

The erosive process on upland areas involves the detachment and transport of particles by rainfall and runoff. These processes can be subdivided into (1) detachment by raindrop impact and flowing water and (2) transport by raindrop splash and surface runoff.

To properly conceptualize detachment and transport processes, three separate source areas are identified. They are (1) interrill areas, those areas of the land surface between runoff channels; (2) rills, definable concentrations of flow that have eroded small channels; and (3) gullies, eroded channels that constitute major surface drainage features (Foster and Meyer, 1975; Meyer and others, 1976; Meyer and Harmon, 1981). The contribution of each source area relative to the total erosion from a slope is partly a function of slope length and the distance downslope from the divide (Meyer, 1979, and fig. 3). Rill erosion resulting predominantly from soil detachment and (or) entrainment by surface runoff does not occur at the top of slopes due to insufficient erosive forces, but may reach "major rates" further downslope (Meyer and others, 1976). This is in keeping with Horton's (1945) belt of no erosion and Schumm's (1956) constant of channel maintenance where a minimum distance downslope or contributing drainage area is required for rills to develop. Interrill erosion in contrast, is relatively uniform over an upland surface area since the variance in rainfall impact (the dominant detachment process) is considered negligible (Lattanzi and others, 1974). Soil erosion from gullies involving the dynamics of in-channel sediment transport is beyond the scope of this study and will not be discussed in detail.
For the most part, detachment of soil particles on the interrill areas of slopes occurs by raindrop impact. In addition to detaching primary particles, energy from rainfall on an unprotected soil surface enhances erosion (1) by breaking down aggregated particles making the material easier to transport, (2) by promoting surface seals that reduce infiltration and therefore increase runoff, (3) by moving particles downslope or directly into rills by splash, and (4) by promoting turbulence in runoff that increases transport capacity. When a soil is covered at the surface either by vegetation or mulches, interrill erosion can be virtually eliminated (Lattanzi and others, 1974).

The following is a description by Meyer, Foster, and Romkens (1975) of the affect of slope steepness on erosion of short interrill areas.

Considerable erosion occurs even when the soil surface is level, but the increase with slope steepness over a broad range of steepnesses is relatively small. Erosion only doubled for a steepness change from 2 to 20 pct as compared to the nearly 20-fold increase predicted. Above 20 pct, the erosion rate tended to level off, as also shown by Foster and Martin (1969) for some conditions. A logical explanation is simply that the rate of soil detachment by rainfall changes slowly as slope steepness increases and that this largely governs the rate of soil loss. A slight increase in detachment rate probably occurs as the raindrops strike at a greater and greater angle, but this effect should not cause a major change in total splash detachment. Some particles and aggregates probably cannot be transported at small slope steepnesses, but most apparently can. The transport capacity of interrill flow is believed to increase rapidly as slope steepens; hence the availability of detached soil must be the primary limiting factor.

Soil detachment by rainfall impact is a time-dependent function that peaks rapidly and then decays exponentially to a steady-state rate (Moldenhauer and Koswara, 1968). However, Foster and Meyer (1975) indicate that the detachment rate may increase with time. This time-dependency and rate variability is most likely a function of changing rainfall intensities, time dependent differences between soils, and changing soil characteristics as the storm progresses. Resistance to erosion varies over the course of a storm through varied wetting conditions, the removal of particles, and the formation of surface seals. It generally is accepted that interrill detachment is approximately proportional to the rainfall intensity, squared (Moldenhauer and Long, 1964; Meyer and Wischmeier, 1969; Bubenzer and Jones, 1971; Meyer, 1981).

Detachment by the shear of flowing water on interrill areas is small and often neglected because "the thin-film has not reached a tractive force sufficient to detach soil particles and (or) the transport capability of
Interrill flow is fulfilled with sediment detached by rainfall" (Meyer and others, 1976). Studies of the relative contribution of detachment by interrill flow indicate that this process supplies little material in comparison to the amount detached by raindrop impact (Meyer, Foster, and Romkens, 1975; Mutchler and Young, 1975).

**Interrill Transport**

Interrill erosion is a relatively uniform removal of soil that is difficult to detect visually owing to the lack of formation of even small topographic expressions (rills). Net downslope movement occurs by raindrop splash and by transport via a thin film of runoff (sheet flow). The interrill areas are probably the only locations on an upland surface where true sheet flow occurs. The ability of this nonconcentrated surface runoff to transport raindrop-detached soil is increased by turbulence in the flow caused by continued raindrop impacts. Of course, the initiation of runoff is a time-dependent relation that is a function of the rainfall intensity and duration, antecedent soil moisture, the infiltration rate of the soil, and the surface storage capacity (ponding).

Soil erosion can be limited by either the availability of detached soil particles or by the transport capacity of runoff. On gently sloping interrill areas, it is suggested that sheet flow cannot transport all the material supplied to it by rainfall detachment (Meyer and Wischmeier, 1969; Rowlinson and Martin, 1971). This relation is probably due to the small tractive forces of interrill flow. On steeper slopes, the limiting factor becomes the availability of material (Foster and Meyer, 1975). In the presence of rainfall, Meyer, Foster, and Romkens (1975) state that "the transport capacity of interrill flow is greater and most of the soil detached on interrill areas is transported by the flow." The transport capacity of interrill runoff is influenced by slope length (obtained from rill density), rainfall intensity and how it compares with the infiltration rate, slope steepness (microtopography), and soil transportability (particle size and specific gravity).

**Rill Detachment**

Detachment and entrainment of soil particles in rills occurs as the tractive force of concentrated-runoff overcomes the soil's resistance to erosion. Local topographic irregularities concentrate flow, and rills are formed. Rills do not develop on upland slopes until flow concentrates and a sufficient shear stress is reached at some distance downslope.

Rills occupy a relatively small percentage of the upland surface yet increase rapidly in number and length with greater runoff rates. Rilling is directly related to slope steepness and progresses headward in the form of a series of knickpoints. This upstream development is in general agreement with the geomorphic literature concerning stream initiation and the development of drainage systems.
Since detachment by rill flow indicates that the tractive force of concentrated runoff has exceeded the soil's critical shear stress, the controlling properties of soil resistance must be understood in order to simulate rill detachment. Prior to 1930, the erosion of cohesive soils was evaluated in terms of properties such as classification by particle size and critical erosion velocities of concentrated runoff (Partheniades, 1971). Lane (1955) developed critical velocity and tractive-force values for soils that were differentiated by density and soil classification.

Field studies in the 1950's and 1960's attempted to correlate various soil parameters with erosion thresholds. The effect of soil plasticity and particle size on critical erosive values was investigated by the Bureau of Reclamation (1953) with inconclusive results. In the laboratory, Flaxman (1963) correlated unconfined compressive strength with tractive power and derived a relation with a great deal of scatter. Smerdon and Beasley (1959) and Dunn (1959) attempted to define critical shear stress in terms of the plasticity index, percentage of silt-clay, and some statistical parameters of the sediment. Although the resulting relations showed a general increase in the critical tractive force with increasing vane shear strength or plasticity, the values varied by a factor of at least ten. Part of this problem can be attributed to the selection of subjective "states of failure" (Partheniades, 1971). Also, Arulanandam and others (1980) state that "the fact that cohesive soil erosion is essentially a surface phenomenon explains why bulk engineering properties of soils such as vane strength, unconfined compressive strength, and dry unit weight have not proved useful as erosion predictors."

Through the use of various soil properties that account somewhat for the behavior of cohesive materials and by expressing erodibility in a reproducible form, Grissinger (1966) produced some significant results. Erosion rates expressed in grains per minute were influenced as follows:

1. Inversely related to the degree of orientation of clay minerals.
2. Inversely related to the percentage of clay (except for calcium-montmorillonite).
3. Inversely related to antecedent moisture to a minimum, then directly related.
4. Directly related to water temperature.
5. No relation with the plasticity index, attributed to non-homogeneous sample structure.

The influence of clay orientation indicates the importance of the nature of the inter-particle bond and the type of clay mineral. In addition, the effects of antecedent moisture and water temperature imply that soil resistance varies with hydrologic conditions. The significance of moisture related parameters further infers that desiccation may play a dominant role.

The profound influence of the character of the pore fluid in causing dispersion of clays was presented by Sherard and others (1972) in a summary of research conducted on piping in dams in Australia. It was found that the presence of dissolved sodium in the pore water decreases the soil's resistance to erosion by enlarging the diffuse double layer, thereby reducing the net
attraction of discrete particles. However, the concentration of dissolved salts was shown to be inversely related to dispersion by piping. This was also found by Sargunam and others (1973) even though the sodium adsorption ratio (SAR; a ratio between the milliequivalents of sodium ions to the square root of milliequivalents of calcium and magnesium) was increasing with the concentrations of dissolved solids. This is also a double layer effect. Effectively the volume of a given diffuse double layer varies inversely with the soluble salt concentration.

A similar study was undertaken by Ariathurai and Arulanandam (1978) working with remolded, saturated soils. Their results in terms of the roles of SAR, temperature, and pore fluid concentration are in general agreement with Grissinger (1966) and Sherard and others (1972). Cation exchange capacity as a measure of particle attraction and potential adsorption was also investigated and found to affect a soil's resistance to erosion. Particle attraction was further described in terms of the number of interparticle bonds and was shown to increase erosion resistance (Kelly and Gularte, 1981).

A new laboratory analysis called the "pinhole dispersion test" was developed to qualitatively depict dispersion in terms of the inherent characteristics of the material (Sherard and others, 1976). The relative degree of dispersion occurring after water of a known head flows through a 1.0 millimeter (mm) hole punched in a cylindrical sample is classified. Although the technique was not developed for the purpose of deriving quantitative relations between flow velocity and erosion rates, a maximum noneroding pinhole velocity with measured erosion rates was successfully correlated by Grissinger and others (1981). This study further substantiates the results of previous studies emphasizing the role of sample morphology, hydrologic conditions, and percent clay.

Grissinger (1982) identifies the following primary soil characteristics that can be used to determine the resistance of cohesive soils to detachment by flow:

(1) mean particle size,
(2) percentage of clay,
(3) percentage of organic matter,
(4) clay mineralogy,
(5) bulk density,
(6) pH,
(7) SAR,
(8) calcium-sodium ratio, and
(9) concentration of exchangeable cations in the pore fluid.

Additional hydrologic factors influencing erodibility are as follows:

(1) water temperature,
(2) antecedent soil moisture conditions,
(3) rate of wetting, and
(4) pore water pressure.

This discussion has described significant parts of the massive research effort that has been, and is, taking place as an attempt to better understand the detachment of particles by flowing water. One of the most important
findings of all of this work is that the relative resistance of cohesive materials to flow detachment is both temporally and spatially variable. The heterogeneity of soil mechanical and physiochemical properties, within even small samples, in conjunction with potentially diverse hydrologic conditions makes parameter selection for simulation purposes difficult.

Rill Transport

The amount of soil transported in rills is governed by a balance between the availability of detached particles and the transport capacity of the concentrated flow. As discussed earlier, these detached particles may emanate from either rill or interrill areas and their availability is a function of the properties of the material and the upland slope, rainfall characteristics, and the hydraulics of flow.

Rill transport capacity, conceptualized in terms of the quantity and size of material in motion, is controlled by runoff velocity and turbulence (Vanoni, 1975). These in turn are directly related to slope steepness and runoff rates (Meyer and others, 1976). Flow turbulence is further enhanced by raindrop impact although this effect is reduced with increasing flow depth. Other variables which affect transport capacity include the hydraulic radius, slope of the energy grade-line, roughness, and particle size (Foot and Meyer, 1975). The latter factor must be viewed with caution in light of the physiochemical properties of cohesive materials and the fact that most detached material is composed of aggregates, not discrete particles. Although cohesive soil material is more difficult to detach than noncohesive material, cohesive soil is more easily transported due to its smaller mass.

Soil loss predictions from upland areas generally are overestimations. This occurs primarily because some of the eroded material does not reach a stream channel but is deposited somewhere within the upland area. This deposition takes place because of a reduction in transport capacity owing to a decrease in slope, an increase in roughness due to the density of vegetation, or the ponding of water. Thus the sediment yield at the basin outlet is usually less than the amount of gross erosion from all sources and as such, this difference defines the concept of the sediment-delivery ratio. Obviously, finer-grained material will be selectively transported further downslope indicating the importance of the size distribution of the eroded material in terms of predicting sediment yields.

PRMS APPROACH TO UPLAND EROSION

The Precipitation-Runoff Modeling System (PRMS) is a modular design watershed modeling system that has been developed to evaluate the impacts of various combinations of precipitation, climate, and land use on surface-water runoff, sediment yields, and general basin hydrology. The system is a deterministic physical-process modeling system. PRMS is designed to function as either a lumped- or distributed-parameter type model and will simulate both mean daily flows and stormflow hydrographs (Leavesley and others, 1983).
Upland soil erosion is simulated by PRMS during the storm simulation mode when the system is operating as a distributed parameter model. Surface runoff for storms is computed using the kinematic wave approximation to overland flow. Overland flow computations are performed on overland flow planes whose characteristics have been defined by the user. In PRMS, all overland-flow planes must discharge to a channel segment; cascading flow planes are not permitted (Leavesley and others, 1983).

The kinematic wave equations for overland flow on a plane are, the continuity equation (1) and an approximation of the momentum equation (2):

\[ \frac{\partial h}{\partial t} + \frac{\partial q}{\partial x} = R \]  

where \( h \) is depth of flow, 
\( q \) is runoff rate per unit width of flow plane, 
\( R \) is rainfall excess rate, 
\( t \) is time, and 
\( x \) is distance down the plane; and

\[ q = \alpha h^m \]  

where \( \alpha \) is a parameter including slope and roughness and 
\( m \) is an exponent reflecting the flow type (laminar or turbulent) and the roughness-velocity relation (Manning or Chezy relation).

The numerical technique developed by Leclerc and Schaake (1973) and described by Dawdy, Schaake, and Alley (1978) is used to approximate \( q(x,t) \) at discrete locations in the \( x-t \) plane. A rectangular grid of points spaced at intervals of time, \( t \), and distance, \( x \), is used. Values of \( t \) and \( x \) may vary from segment to segment as required to maintain computational stability, and to produce desired resolution in computed results (Leavesley and others, 1983).

The conservation of mass equation for sediment is used to describe sediment detachment and transport. The form of the equation used in the PRMS is one that was presented by Hjelmfelt, Piest, and Saxon (1975).

\[ \frac{\partial (c \cdot h)}{\partial t} + \frac{\partial (c \cdot q)}{\partial x} = E_f + E_r \]  

where \( c \) is sediment concentration, 
\( E_r \) is rainfall detachment rate, and 
\( E_f \) is flow detachment rate.

The rainfall detachment rate \( E_r \) is computed by an equation proposed by Meyer and Wischmeier (1969) as:

\[ E_r = K_r (I)^2 \]
and later modified by Smith (1976) to:

\[ E_r = K_r (I)^2 (e^{-Hh^2}) \]

(5)

where \( K_r \) is a parameter reflecting the erodibility of a soil, 
I is rainfall intensity, and 
\( H \) is a parameter reflecting the dampening effect of surface-water depth on raindrop impact.

The flow detachment rate \( E_f \) is computed using a relation proposed by Foster and Meyer (1972a):

\[ E_f = K_f (T_C - T_r) \]

(6)

then modified by Hjelmfelt, Piest, and Saxon (1975) to:

\[ E_f = K_f (\beta h^n - c_q) \]

(7)

where \( T_C \) is sediment transport capacity = \( \beta h^n \); 
\( K_f \) is a parameter that controls the rate of detachment when \( \beta h^n - c_q > 0 \); 
\( T_r \) is current sediment transport rate = \( c_q \); 
\( \beta \) is a parameter including, \( Y \) the weight density of runoff, \( S_e \) slope of the energy gradeline which is assumed equal to the slope of the overland flow plane (\( S \)), and a coefficient \( K_t \) that depends on particle size and density: \( \beta = K_t (Y S)^n \); and 
n is an exponent that is usually assigned a value of 1.5.

For a given time step, \( E_r \) is added to \( T_r \), and the sum is compared to \( T_C \). If the sum is greater than \( T_C \), then \( T_r \) is set equal to \( T_C \) and no detachment occurs. If the sum is less than \( T_C \), then the flow detachment is computed, and \( E_f \) is added to the sum to compute a new \( T_r \) (Leavesley and others, 1983). A summary of PRMS parameters and their physical significance is given in table 1.

### Table 1.—Summary of PRMS parameters

<table>
<thead>
<tr>
<th>PRMS parameter</th>
<th>Physical significance</th>
<th>PRMS parameter</th>
<th>Physical significance</th>
</tr>
</thead>
<tbody>
<tr>
<td>( K_r ) (coefficient)</td>
<td>Erodibility (rainfall)</td>
<td>( K_t ) (coefficient)</td>
<td>Effects of particle size and density on transport capacity.</td>
</tr>
<tr>
<td>( H ) (exponent)</td>
<td>Dampening raindrop impact by surface water.</td>
<td>( K_f ) (coefficient)</td>
<td>Rate of detachment when excess ( T_C ) exists.</td>
</tr>
<tr>
<td>( n ) (exponent)</td>
<td>Reflects nonlinearity of ( T_C ) versus ( I ) relation.</td>
<td>( \beta ) (coefficient)</td>
<td>( \beta = K_t (Y S)^n )</td>
</tr>
</tbody>
</table>
In order to fully understand the physical significance of the parameters used in PRMS, it is necessary to have some idea of how the erosion equations were developed.

On the basis of experimental data, Meyer and Wischmeier (1969) demonstrated that rainfall detachment was proportional to the square of the rainfall intensity (equation 4). Subsequently, Meyer (1981) has determined that equation 4 fits simulated erosion data well for soils with low clay content, however, as clay content increases, the exponent decreases. Figure 4 modified from Meyer (1981) shows the exponent decreasing from approximately 2.0 for a 10-percent clay soil to 1.6 for a 50-percent clay soil. Meyer (1981) found that the exponent (b) has the following relation to percentage clay:

\[ b = 2.1 - 0.01 \% \text{ clay} \]  

(8)

Figure 4.-- Effect of clay percentage on exponent b for different soils and cropping conditions (modified from Meyer, 1981).
There are several reasons why the exponent value decreases with increasing clay content. Wischmeier and Mannering (1969) found that both coarse-textured soils with high infiltration rates and fine-textured soils with high clay content were less erodible than medium-textured silty soils. Obviously, the coarse-textured soils do not promote surface runoff and the individual particles are not easily moved by rain splash. In areas containing high clay soils, surface runoff may occur quite easily but the increased cohesion resists detachment by rainfall and runoff. The medium-textured soils not only are more readily detached, but also are susceptible to surface sealing which promotes surface runoff. The decreasing exponent in figure 4 reflects the decrease in erodibility as the soil progresses from predominantly silt to predominantly clay.

Meyer and Harmon (1981) also found that in comparing different cover conditions on the same soil or comparing different soils with the same texture and cover, it is advantageous to use a constant exponent for equation 4. By doing this, the ratios of coefficients (K_r in eq. 4) quantify the relative interrill erodibility of different soils or the relative interrill erodibility of different cover conditions on the same soil (fig. 5) (Meyer and Harmon, 1981).

At present, the exponent and coefficient in equation 4 can only be determined by rainfall simulator studies as described by Meyer and Harmon (1979), Meyer (1981), and Meyer and Harmon (1981). This method of determining interrill erosion coefficients does not present a severe handicap because interrill erosion is a relatively uniform process, that does not appear to be greatly affected by the steepness or location of land slope (Meyer, 1979). Therefore, a determination of parameters for a given soil and surface condition should hold for the entire area where the specific soil and surface condition occur.

Rainfall simulators have had problems simulating the physical characteristics of rain such as intensity, drop size, and fall

![Figure 5.— Effect of rain intensity on interrill erosion at several stages of crop growth (modified from Meyer, 1981).](image-url)
velocity impact. Also, simulators have historically been quite bulky and time consuming, making their field application very difficult. Meyer and Harmon (1979), however, describe a portable rainfall simulator that applies simulated rainfall at a wide range of intensities with drop sizes and impact velocities near those of natural rainfall. Meyer and Harmon (1981) have demonstrated that the simulator can be used in pasture and forest as well as furrowed fields. Although these results are encouraging, rainfall simulators are expensive to use and are limited to areas accessible by vehicles.

Agricultural applications of soil erosion models are primarily concerned with erosion on furrowed fields. For this application, interrill erosion occurs on row sideslopes that are generally short and steep and have little potential for ponding water or developing a significant depth of interrill flow. To make erosion models generally applicable, equation 4 must be modified to account for the dampening effects of ponded water or significant interrill flow depth. Equation 5, which is currently incorporated into the PRMS sediment component, contains a surface-water depth term proposed by Smith (1976). Smith states that this term is purely conceptual and unverified, and gives no indication of how the parameter $H$ should be assessed.

Research by Mutchler and Larson (1971) and by Mutchler and Young (1975) shows that the erosive potential of rainsplash varies with the ratio of surface-water depth ($d$) to water drop diameter ($D$). As an indicator of the erosion potential, Mutchler and Young (1975) show total splash ($S_p$) as a fraction of waterdrop weight ($W_D$) versus $d/D$ (fig. 6). The equation for this relation is given by Mutchler and Larson (1971) as

$$\frac{S_p}{W_D} = 1 + 2.02e^{-2.56(d/D)} - 3.02e^{-16(d/D)}$$

The salient features of this relation are that for $d/D$ between 0.14 and 0.20, the function reaches its maximum, and at $d/D$ greater than 2.2, the function equals one. Thus rainsplash is presumed to be most erosive at $d$ equal to 0.17 $D$ and has no detachment effect at $d$ greater than or equal to 3$D$.

Based on the assumption that rainfall simulator results will adequately describe the rainfall detachment rate for surface-water depths up to $d/D$ equal to 0.17, the following modification is proposed to account for ratios greater than 0.17.

$$E_r = \frac{K_r}{[2.02e^{-2.56(d/D)} - 3.02e^{-16(d/D)}]}$$

Using the following relation from Laws and Parsons (1943), the median drop diameter ($D_{50}$) can be determined for a given rainfall intensity:

$$D_{50} = 2.2310^{0.182}$$

The median drop diameter determined using equation 11 can be substituted for $D$ in equation 10 and $d$ can be found from the rainfall excess overland flow equations.
Figure 6.—Total slash $S_D$ expressed as a fraction of waterdrop weight $W$ for various ratios of water depth $d$ to waterdrop diameter $D$ (modified from Mutchler and Young, 1975).

Equations 10 and 11 are proposed as more physically based alternatives to equation 5, but their adequacy in predicting the dampening effects of surface-water depth on rainfall detachment has not been determined. The work of Mutchler and Larson (1971) was conducted on a glass plate covered with water of various thickness. The effects of soil surfaces, suspended-sediment concentration, and surface roughness on their relation are not known. Whether or not $D_{50}$ is an adequate representation of raindrop size for the purpose of erosion prediction is also not very well documented.

Detachment by flowing water (rill erosion) is computed by the PRMS with equation 7. Foster and Meyer (1972a) arrived at the original form of this relation (eq. 6) by proposing the following relation between detachment by runoff and sediment load carried by runoff.

$$\frac{E_f}{D_c} + \frac{C_s}{T_c} = 1$$

(12)

where $D_c$ is the flow detachment capacity and $T_c$ is the flow transport capacity.

Foster and Meyer (1972a) arrived at this formulation through the following intuitive reasoning.
Meyer and Monke (1965) and Willis (1971) noted that the rate of erosion (detachment) at the head of a noncohesive bed where flow was introduced depended on the amount of sediment in the added flow. Furthermore, the sediment load at the end of the bed used by Meyer and Monke was independent of the sediment load of the inflow. This indicated that a channel sufficiently long will erode enough sediment to meet its transport capacity. Erosion rate decreased with distance downslope, which indicated a decrease in detachment rate as sediment load increased.

The similar process of stream channel degradation below dams has been analyzed by using a gradually varied flow analysis (Tinney, 1962; Komura and Simons, 1967; and Hales and others, 1970). The effect of sediment load on degradation was neglected in these analyses.

Deposition is very frequently observed at the toe of upland slopes. The flow at the toe of these slopes is very slow and has very little transport capacity. Maximum deposition occurs at the toe of the slope and decreases with distance from the toe. The decrease in deposition rate with distance from the toe is partially due to an interaction between the sediment load and the detachment (or deposition) rate.

On the basis of these observations, the rate of detachment or deposition by flow was concluded to be a function of the difference between the actual sediment load and the capacity of flow to transport sediment. This agrees with Einstein's (1968) assumption that the rate of deposition is directly proportional to the difference between the actual sediment concentration in the flow and the equilibrium concentration for those flow conditions.

Foster and Meyer (1972a) then assumed that the detachment capacity is related to the transport capacity.

\[ D_c = K_f T_c \]  \hspace{1cm} (13)

Substitution of 13 in 12 yields:

\[ E_f = K_f (T_c - c_q) \]  \hspace{1cm} (14)

which is identical to equation 6 with \( c_q = T_r \).

In order to find an expression for \( T_c \), Foster and Meyer (1972b), Neibling and Foster (1980), and Alonso and others (1981) reviewed several published sediment transport equations. All of these investigations agreed that the Yalin (1963) bedload equation not only provided the best fit with observed data but also that the assumptions made in deriving the equation were the most compatible with shallow overland flow. Foster and Meyer (1972b) state that the soil detached by concentrated rill flow consists of soil aggregates that have larger diameters but lower densities than the soil's primary particles. They have also observed that these aggregates tend to move along the bottom of small flow channels by saltation and rolling similar to bedload movement in alluvial channels.
Foster and Meyer (1972a) suggest that if the critical shear stress is zero, the Yalin equation reduces to:

$$T_c = K_t \tau^{3/2}$$  \hspace{1cm} (15)

where $K_t$ is a coefficient that depends on particle size and density. This assumption of zero critical shear stress essentially eliminates the need to test various transport equations because almost all tractive force or DuBoys type equations reduce to the form:

$$T_c = a \tau^b$$  \hspace{1cm} (16)

when $\tau_{cr} = 0$. As Shen (1971) points out, the general form of the bedload equation is

$$q_B = \alpha (\tau)^b$$ \hspace{1cm} (17)

or

$$q_B = \alpha (\tau - \tau_c)^b$$ \hspace{1cm} (18)

with the advantage of 18 being that it satisfies the boundary condition of $q_B = 0$ when $\tau - \tau_c = 0$.

In the PRMS, the following more general form of equation 15 is used.

$$T_c = K_t \tau^n$$ \hspace{1cm} (19)

where $n$ is an exponent. By using data from Partheniades (1965), Foster and others (1977) found that $n$ varies from 1 to 2 and they suggest using the average value of 1.5. This value also agrees with the findings of Meyer, Foster, and Nikolov (1975) for rill erosion.

The relation for shear stress at the bed is:

$$\tau = \gamma R S_e$$ \hspace{1cm} (20)

where $\gamma$ is unit weight of water,

$S_e$ is slope of the energy gradeline, and

$R$ is hydraulic radius.

For wide shallow flow, the hydraulic radius ($R$) is assumed to be equal to the depth ($h$); and for overland flow, the slope of the energy gradeline is assumed to be equal to the slope of the overland flow plane ($S$). Equation 20 then becomes

$$\tau = \gamma h S$$

and equation 19 can be written

$$T_c = K_t (\gamma S)^n (h)^n$$ \hspace{1cm} (21)

where the depth ($h$) will be supplied by the solution of the kinematic wave equations. Equation 21 is the expression that PRMS uses for $T_c$ (equation 7) with $\alpha = K_t (\gamma S)^n$. 

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The coefficient $K_t$ is often called the characteristic sediment coefficient and is usually determined empirically through laboratory flume experiments. For noncohesive material, $K_t$ can be shown to be a function of the specific gravity of the material ($S_s$) and some characteristic grain size $D_n$. In general, the transport capacity varies inversely with $S_s$ and $D_n$.

Observation of erosion on natural cohesive soils shows that the soil tends to be detached and transported in the form of aggregates having larger diameters but lower specific gravities than the primary particles (Foster and Meyer, 1972b). Foster and Meyer (1972b) state that aggregate diameters commonly range from 0.002 to 2.0 mm and Foster and others (1979) state that aggregate specific gravities range from 1.6 to 2.7.

Very few values of $K_t$ have been reported in the literature reviewed for this report; therefore, it is difficult to recommend an appropriate range of values for model use. The reduction of transport equations such as the Yalin (1963) equation to the form of equation 15 has been stated by Foster and Meyer (1972a) without proof or derivation, and therefore, the form of the relation between $K_t$ and $(S_s, D_n)$ is not obvious. Rohlf and Meadows (1980) use the Meyer-Peter equation as follows:

$$T_c = \frac{8}{\sqrt{\rho \gamma (S_s - 1.0)}} \left( \tau_0 - \tau_{cr} \right)^{1.5}$$

If $\tau_{cr} = 0$ then 22 becomes

$$T_c = \frac{8}{\sqrt{\rho \gamma (S_s - 1.0)}} (Y_hS)^{1.5}$$

substituting eq. 21 for $T_c$ with $n = 1.5$ yields,

$$K_t = \frac{8}{\sqrt{\rho (S_s - 1.0) \gamma}}$$

where $\rho$ is density of water and $\gamma$ is specific weight of water.

This expression for $K_t$ may serve as a preliminary guide until more detailed information about the characteristic sediment coefficients that accompany the Yalin and other commonly used transport equations becomes available.

Theoretically, the coefficient $K_f$ in equation 7 is related to the soil’s ability to resist detachment when excess transport capacity exists ($T_c - \tau_0 > 0$). However, in the PRMS, $K_f$ must also account for sediment delivery processes that are not simulated. The PRMS method of computation utilizing a single overland-flow plane from basin divided to channel boundary makes no provision for sediment deposition, and therefore sediment is delivered to the channel segment at the erosion rate determined for the plane. In cases where deposition may be occurring due to lower slopes or increased vegetation near the
channel, $K_f$ will have to be adjusted to lower the erosion rate for the plane. Therefore, $K_f$ can be considered as a function of both the delivery ratio for the plane and the soil's resistance to detachment.

A functional relation between $K_f$ and physically measurable soil parameters has not been found, and very little information is available in the literature on values of $K_f$ and its physical significance. Most studies of cohesive soil erosion have focused on methods for defining a value for the critical shear stress ($T_c$) and have neglected relations between detachment capacity and soil parameters.

Several authors including Partheniades (1965), Ariathurai and Arulanandan (1978), and Arulanandan and others (1980) have shown that erosion rate varies linearly with excess shear stress. They use the following equation to express their results

$$
\varepsilon = M(\tau - T_c)
$$

(24)

where $\varepsilon$ is erosion rate and $M$ is an erodibility constant equal to the slope of the erosion rate versus excess shear stress relation.

Equation 24 is a linear equation of the same form as equation 6. The difference between the two being that equation 6 assumes the erosion rate is a function of available transport capacity, and equation 24 assumes the transport capacity to be infinite and thus the erosion rate is a function of excess shear stress. Typically, equation 24 is used in estuarine environments where the eroded material consists of easily transported primary particles and flocs. Equation 6 reflects the tendency of soil material to detach in aggregates whose movement is a function of the transport capacity of the flow.

Although the literature contains little information on $M$ or $K_f$, studies of the constant $M$ are more prevalent. If $M$ is considered as an upper boundary for $K_f$ and the behavior of $M$ is assumed to reflect the behavior of $K_f$, then studies of $M$ can be used to gain some insight into the behavior of $K_f$.

Ariathurai and Arulanandan (1978) and Arulanandan and others (1980) have shown that $M$ decreases as critical shear stress increases indicating that once erosion starts, soils with higher critical shear stress erode slower than those with lower critical shear stress. Arulanandan and others (1980) found that for undisturbed soils with distilled water as the eroding fluid, and $T_c$ between 3 and 20 dynes per centimeter squared, $M$ can be calculated from:

$$
M = \frac{223}{1 + 0.13T_c}
$$

(25)

where $M$ is the slope of the soil erosion versus shear stress line for $\tau > T_c$. Soil erosion is in grams per centimeter squared minutes ($\text{gm/cm}^2\cdot\text{min}$), shear stress is in dynes/cm$^2$ and $M$ is in grams per dyne minute ($\text{gm/dyne-min}$) $10^{-4}$.

For $T_c$ greater than 20, $M$ is constant and for $T_c$ less than 3, $M$ becomes vertically asymptotic as shown in figure 7. Estimates of $M$ taken from this figure are likely to be high because $M$ should decrease as the salt content of the eroding fluid increases.
Figure 7.-- Rate of change of erosion rate $s$ versus critical shear stress $\tau_c$ for undisturbed soils tested in flume using distilled water as eroding fluid (modified from Arulanandan and others, 1980).

Determining $\tau_c$ for cohesive soils has received considerable attention as evidenced by the number of references listed in Kamphuis and Hall (1983). The results of these investigations vary over a wide range due to variations in experimental techniques and the definition of initiation of erosion. Because of this wide variation, only two methods of determining $\tau_c$ will be considered here.

Foster and Meyer (1975) suggest that for agricultural soils, the Smerdon and Beasley (1959) equation provides an adequate estimate of $\tau_c$. The Smerdon and Beasley (1959) equation is:

$$\tau_c = \frac{0.213}{d_r^{0.63}}$$

(26)

where $d_r$ is dispersion ratio.
Arulanandan and others (1980) use a predictive chart developed by Alizadeh (1974) and revised by Heinzen (1976). The chart shown as figure 8 requires that the soil SAR, dielectric dispersion (\( \Delta \varepsilon \)) and dissolved solids of soil pore water be known. The dielectric dispersion is defined as the decrease in the apparent dielectric constant when the frequency of an alternating current passing through the soil sample is increased from 5 to 50 megacycles per second (Arulanandan and others, 1980). The dielectric constant is a measure of the ability of the sample to store electrical potential energy on charged clay surfaces under the influence of an electric field (Arulanandan and others, 1980). According to Arulanandan and others (1980), the magnitude of the dielectric dispersion can be used as a quantitative method to characterize natural soils in terms of clay type and amount.

Figure 8 was developed for remolded soils being eroded by distilled water and, therefore, will yield estimates of \( T_c \) that are less than the actual values for undisturbed soils being eroded by water with dissolved solids. Initial estimates of \( M \) values selected from figure 7 or equation 25 will be greater than actual values and may have to be adjusted.

A quantitative determination of the coefficient \( K_f \) cannot be made at this time. However, if the assumption is made that values of \( K_f \) in equations 6 are similar to values of \( M \) in equation 24, then the methods described above can be used to at least provide initial estimates of \( K_f \) values. These initial values will have to be adjusted to account for possible overestimation and sediment delivery effects. These adjustments must still depend upon the subjective judgment of the model user.

A summary of PRMS parameters, their physical significance and methods of estimation is shown in table 2.

<table>
<thead>
<tr>
<th>PRMS parameter</th>
<th>Physical significance</th>
<th>Method of estimation</th>
<th>Reference</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>( H ) (exponent)</td>
<td>Dampening raindrop impact by surface water.</td>
<td>None</td>
<td>Smith, 1976</td>
<td>A different equation could be used (see equation 10).</td>
</tr>
<tr>
<td>( n ) (exponent)</td>
<td>Reflects nonlinearity of ( T_c ) versus ( T ) relation (eq. 21).</td>
<td>Ranges from 1 to 2, Usually set = 1.5</td>
<td>Foster and others, 1977</td>
<td></td>
</tr>
<tr>
<td>( \delta ) (coefficient)</td>
<td>See ( K_t )</td>
<td>( \delta = K_t(YS)^n )</td>
<td>Hjelmfelt, Piest and Saxton, 1975.</td>
<td></td>
</tr>
<tr>
<td>( K_f ) (coefficient)</td>
<td>Rate of detachment when excess ( T_c ) exists.</td>
<td>Empirical relations from literature.</td>
<td>Ariathurai and Arulanandan, 1978; Arulanandan and others, 1980.</td>
<td></td>
</tr>
</tbody>
</table>

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Figure 8.-- Critical shear stress $\tau_c$ versus SAR for different soil salt concentrations and different dielectric dispersion $\Delta \epsilon_0$ values (modified from Arulanandan and others, 1980).
SUMMATION AND RECOMMENDATIONS FOR MODEL IMPROVEMENTS AND FURTHER RESEARCH

The apparent trend in soil erosion modeling has been to pursue esoteric mathematical models such as those that simulate erosion by particle-size classes. This refinement of detail, which may produce additional parameters, can hardly be justified in light of the uncertainty and lack of refinement in current parameter estimation techniques. One of the objectives of this study is to recommend model improvements and suggest further research efforts that specifically address the physical basis of the erosion equation parameters.

Two areas of the PRMS upland erosion component could be changed to improve the physical basis of their associated parameters. The first concerns the component of the rainfall detachment equation (5) that accounts for the dampening effect of surface-water depth on rainfall impact. This revision has been discussed in the previous section and is presented as an example of how this dampening could be accommodated. Before proceeding with this change, however, it might be worthwhile to investigate how sensitive and important this component actually is.

The second recommended improvement is to allow for cascading planes. This modification would reduce the need to account for both erosion and sediment delivery on a single plane extending from a drainage divide to a channel segment. In addition to improving the physical basis of the associated parameters, it would also allow the model to identify areas of net erosion or deposition. This information would be valuable in identifying contributing areas and in assessing protective measures such as vegetated buffer strips.

The results of this investigation have shown that much more information is needed on the subject of parameter estimation. Optimum parameter values are rarely reported in the literature even though many modeling efforts have been completed using these or similar equations. It would be very beneficial if optimum parameter values could be obtained and compiled. At a minimum it would provide model users with a listing of parameter values from which initial estimates could be made and ultimately could be used to explore relations between the parameter values and physical conditions in the modeled area.

This study did not explore the mathematics of reducing bedload formulas to the form \( T_c = K_t T^{3/2} \). Shen (1971) states that this is a common form, several authors including Foster and Meyer (1972a) state that the Yalin equation reduces to this form, and Nordin and Beverage (1964) state that the Bagnold (1956) equation also reduces to this form. It would be very interesting to obtain the final form of the coefficients that accompany these reductions. Most of those coefficients are expressed in terms of the specific weight of the sediment and some characteristic particle size. By making some assumptions about the size and specific gravity of detached aggregates, initial values for the parameter \( K_t \) could be made.

Basic research into the factors controlling the rate of erosion of cohesive soils by flowing water is definitely needed. Studies designed to find some critical or incipient erosion condition yield little information
about the rate of erosion once the critical condition has been exceeded. Erosion rate studies would have to be conducted in the laboratory or under highly controlled field conditions. A relatively new laboratory test called the pinhole-dispersion test (Grissinger and others, 1981) seems to hold some promise for determining cohesive soil erosion rates.

Shirley and Lane (1978) and Lane and Shirley (1982) present a method for estimating initial parameter values for measured data from erosion plots. Their method is only valid for small plots that can be simulated as a single plane. Although their rainfall detachment equation is different from the PRMS equation, their flow detachment equations are the same. It may be possible to substitute the PRMS equation into their derivation and develop an estimation technique for PRMS.

Finally it should be emphasized that both the PRMS approach to soil erosion and most of the information in this report have been taken from the agricultural literature. The potential application of the PRMS covers a wide range of land disturbing activities. The extent to which the methods and techniques developed for agricultural research apply for different land uses is unknown. Meyer and Harmon (1981) have demonstrated the applicability of the portable rainfall simulator to land-use conditions different from those of row crops. However, all of these land uses occurred within a single small basin in an agricultural area. The methods developed for agricultural research and the assumptions made about the agricultural erosion process need to be tested under various combinations of land disturbance, physiographic, geologic, and climatic conditions.
SELECTED REFERENCES


Heinzen, R. T., 1976, Erodibility criteria for soils: Davis, California, University of California at Davis, unpublished thesis.


Lane, E. W., 1955, Design of stable channels: Transactions of the American Society of Civil Engineers, v. 120, paper no. 2776, p. 1234-1279


Nordin, C. F., Jr., and Beverage, J. P., 1964, Discussion on An expression for bed-load transportation: Journal of the Hydraulics Division of the American Society of Civil Engineers, v. 90, no. HY1, p. 303-313.

Partheniades, E., 1965, Erosion and deposition of cohesive soils: Journal of the Hydraulics Division of the American Society of Civil Engineers, v. 91, no. HY1, p. 105-139.


