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ACCURACY OF STREAM MEASUREMENTS.—MURPHY

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CHARLES D. WALCOTT, DIRECTOR

ACCURACY

OF

STREAM MEASUREMENTS

BY

EDWARD CHARLES MURPHY

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DEPARTMENT OF THE INTERIOR,
UNITED STATES GEOLOGICAL SURVEY,
Washington, D. C., December 30, 1901.

SIR: I have the honor to transmit herewith a manuscript prepared by Prof. E. C. Murphy, of Cornell University, Ithaca, N. Y., giving the results of investigations made by him into the accuracy of stream measurements as ordinarily conducted by the hydrographers of this Survey.

Professor Murphy was for some time resident hydrographer for eastern Kansas, and carried on in the field measurements of several rivers, being thus by actual practice thoroughly familiar with the operations of the hydrographic branch of the Geological Survey. Upon his removal to Ithaca Professor Murphy began a series of experiments, using the facilities offered by the hydraulic laboratory of Cornell University.

While the methods of river measurement at present in use are believed to be sufficiently accurate for the purposes of ascertaining the water resources of the United States, yet it is important to know their probable accuracy, and particularly the accuracy of the instruments which are commonly employed. Through such knowledge and experience gained from time to time it will be possible to improve upon the methods, in the interest of greater exactness or wider diffusion of the work.

I therefore request that this manuscript be published in the series of Water-Supply and Irrigation Papers.

Very respectfully,

F. H. NEWELL,
Hydrographer in Charge.

Hon. CHARLES D. WALCOTT,
Director United States Geological Survey.

ACCURACY OF STREAM MEASUREMENTS.

By EDWARD C. MURPHY.

FACTORS CONTROLLING ACCURACY OF STREAM MEASUREMENTS.

The accuracy of a stream measurement depends largely upon the accuracy with which the cross-sectional area and the velocity are measured. There is no special difficulty in measuring the first factor, but the second factor—the velocity—is very difficult to measure, chiefly for the reason that it is constantly changing. It not only varies from the surface to the bottom and from one bank of the stream to the other, so that it is necessary to measure it at many points, but it is constantly changing at every point, even when the cross-sectional area and the discharge (and consequently the mean velocity) remain constant. Several experimenters have observed the phenomenon of "pulsation of moving water," and a few have tried to measure it, but as yet little is known of the magnitude and frequency of the pulsations or of the laws governing them. J. B. Francis says:[a] "It is observed that there is a continual change in all parts of these channels, although there may be no sensible change in the volume of water flowing and consequently in the mean velocity." Captain Cunningham says:[b] "One of the most important conclusions of modern experiments is that the motion of water, even when tranquil to the eye, is extremely unsteady, so that there is no definite velocity at any point, but the velocity varies everywhere, largely from instant to instant. * * * It is analogous to the unsteady motion of wind, which is exemplified by the swaying of wind vanes and by the fluttering of pinions." D. F. Henry says:[c] "All water in motion has an intermittent velocity, increasing and decreasing according to some undiscovered law," and Henry found this true of streams of all sizes, from small mill races to the great St. Lawrence River.

A knowledge of these phenomena is evidently of vital importance in making and computing stream measurements. If only a few observations of velocity are made, these may all, or nearly all, be

[a] Trans. Am. Soc. Civ. Eng., Vol. VII, p. 111.
[b] Recent Hydraulic Experiments: Proc. Inst. Civ. Eng., Vol. LXXI, p. 7.
[c] Jour. Franklin Inst., Vol. LXII, p. 323.

made at a time of maximum impulse, and thus the measured mean velocity be too large; or it is possible that most of the observations may be made at a time of minimum impulse, and thus the mean velocity be too small. They also have an important bearing on the kind of instrument best suited to measure the velocity, as some—the float rod, for example—give the velocity of a single impulse, while others, as the current meter, show the average velocity due to all the impulses during the observation.

The motion of water in an open channel is not, however, simply a succession of impulses. On the contrary, it is exceedingly complex, very different from the uniform flow in parallel straight lines that is assumed in deriving the common hydraulic formulæ. When closely observed the water of the most undisturbed streams is seen to "boil" and swirl and to be very unsteady, some particles moving up, others down, others across, but all as a rule having a general motion downstream.

The accuracy of a discharge measurement also depends much upon the physical features of the stream at the discharge section or point of measurement. When possible this section should be on a straight reach and far enough from a bend to be out of its influence, the bed should be permanent and not stony, and the slope and wetted perimeter such that at high and low stages of the stream the velocity in all parts of the section will be easily measurable. The banks should be sufficiently high not to be overflowed at flood stage, and the section should be free from the influence of milldams and bridge piers. In addition, economy requires that the section selected be easily accessible from a railway station and that there be a person living near who will read the elevation of the water surface at stated times. Seldom if ever are all of these conditions even approximately satisfied. Scour of bed in some cases and silting in other cases give much trouble; too great a velocity at high stages and too low a velocity at low stages to measure with accuracy are common difficulties; and milldams often give trouble, on account of storage and irregular discharge through their wheels; so that the hydrographer must make the most of the best location he can find, always having in mind the desirable and the undesirable qualities.

Rapid fluctuations of the water surface or river height during measurement and the condition of the velocity-measuring instrument are other factors which affect the accuracy of a stream measurement.

A great many discharge measurements of natural and artificial channels have been made with various kinds of instruments and in various ways. The earlier ones were made with crude instruments, and in some cases the surface velocity only was observed, the mean velocity being computed from a formula which we now know is not correct. Very little appears to have been done in the way of determining the degree of accuracy of the measurements. Even when the experimenter has used two or more instruments to measure velocity, he does not appear to have made simultaneous measurements with different instruments, or employed different methods with the same instruments, in order to test the accuracy of the results.

These matters are all discussed in detail on the following pages.

METHODS OF MEASURING VELOCITY.

The methods that have been used for measuring velocity may be divided into two general classes—the direct and indirect. The direct methods include all ways of ascertaining the velocity of water from bodies floating in it, such as surface floats, float rods, double floats, etc. The indirect methods include measurements with the current meter, pressure plates, the thermometer, etc., by which the velocity of the water is inferred from its impact, its pressure, or its temperature. Measurements by the direct methods are simple, requiring only the measurement of the time that it takes a body in the water to move over a certain distance, while in the indirect methods the constant of the instant must first be found from experiment. The latter methods are, however, theoretically better than the former.

By the direct methods the mean velocity of a comparatively few particles of water is found for the time required for the float to move between the sections of observation—that is, the float is acted upon by the particles about it during the time it is passing over the measured distance. It is carried along by a single impulse of the water, and is not affected by the succeeding impulses, which may be greater or less than the impulse which moves it. Instruments of the second class, on the other hand—as, for instance, the meter—measure instantaneously the velocity of all the particles that strike it during the observation. If there were no pulsations of water, and if the particles following one another at any given point moved with uniform velocity, then the direct methods might, in the absence of wind, give good results; but since all moving water, even the most undisturbed, has these pulsations, velocity measurements by the indirect methods are decidedly more accurate than those by the direct methods. One measurement with a current meter is worth several measurements with floats.

There is another reason why indirect measurements are preferable to direct measurements. Since in the latter the mean velocity is found over a certain distance, usually from 50 to 200 feet, it is necessary to know the mean area over that distance, and this requires the measurement of many cross sections between the upper and the lower section, while by the indirect methods the measurement of only one area is required.

DIRECT METHODS.

SURFACE FLOATS.

Surface floats are small, light bodies, such as wood or wax, which float on the surface of the water and can readily be seen from the shore. Measurement by this means is rapid, but the velocities obtained may be greatly in error, due to the action of wind on the float. At best it only indicates the velocity for a short time of a comparatively few particles of water, and the mean velocity must be found from the relation between surface velocity and mean velocity, which is not well established. The time consumed by the floats in passing over a measured distance—usually from 50 to 200 feet—is observed, and the distance divided by the observed time gives the surface velocity at that point for that time.

This method was used by Dubuat in 1779 in gaging the Canal du Jard, in France, and in 1782 in gaging the Haine River, Belgium; by Trechsel, in 1825, to gage the Outer Aar, near Thun; by Wampfler, in 1867, to gage the Simme Canal, Switzerland; by Harlacher, in 1881, to gage the Elbe, in Bohemia (at high water only); by Ellet, in 1858, in gaging the Ohio; and by others.

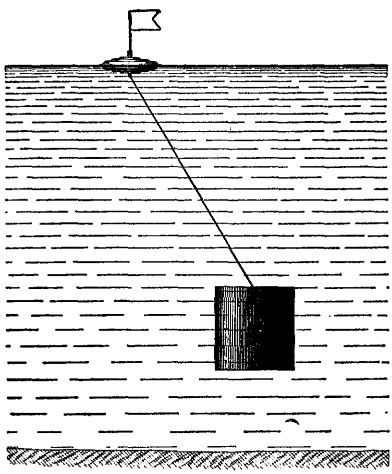

FIG. 1.—Double float used by Ellis in Connecticut River survey in 1874.

DOUBLE FLOATS.

The double float consists of a light surface float and a subsurface float somewhat heavier than water connected to it by a cord or a small rope. The office of the upper float is to support the lower float and indicate its position. The connecting cord can be lengthened at will and the lower float be placed at any desired depth. Fig. 1 shows the double float used by T. G. Ellis in the Connecticut River survey in 1874. This subsurface float was a hollow annulus of tin $8\frac{1}{2}$ inches high, $8\frac{1}{2}$ inches outside diameter, and $7\frac{1}{2}$ inches inside diameter. Two brass wires were soldered across the bottom at right angles to each other, to which was attached 28 ounces of lead as a sinker. Two other wires were soldered at right angles to each other at the center, to which the connecting cord was attached. The surface float was an ellipsoid of tin 6 inches in diameter and $1\frac{1}{2}$ inches thick, with a cork in the top holding a small flag and an eye in the bottom for the connecting cord. The connecting cord had a diameter of 0.036 inch.

The method of measuring stream flow by double floats as used by

Humphreys and Abbot in gaging the Mississippi River in 1851 and 1858 is substantially as follows:[a] A suitable place was selected on the river, a base line 200 feet long was measured off on one bank parallel to the axis of the current, and a section at each end of this base was marked out at right angles to it. The time consumed by each float in passing between these sections and the position of each float when it passed them were noted by four men, two at each end of the base, each party having a stop watch and a theodolite. At a signal from the engineer the floats were placed in the river from a boat about 100 feet above the upper section and were picked up by a man in a boat some distance below the lower section. At the instant a float passed the upper section a signal was given, the watches were started, and the angular position of the float was read with both theodolites. The instant the float passed the lower section a signal was again given, the watches were stopped, and the angular position of the float was again read with both theodolites. These readings gave the distance of the float from the base line when it passed the sections and two independent measurements of the time consumed by it in passing between the sections. Soundings were taken at the end sections and at one or more intermediate sections, from which the area of the mean section was computed.

In some cases where this method has been used the base was 300 feet long and the time of run of the floats has been recorded on a chronograph.

This method has its advantages and its disadvantages. In very deep rivers, such as the Mississippi, or in streams carrying weeds and grass, it is almost the only available method. Humphreys and Abbot, in their report on the Mississippi River, say:[b] "Saxton's current meter was tried but found to be unsuited to measurements in a river of such great depth and violence of current. Only double floats were found to give reliable results."

In regard to the disadvantages, D. F. Henry, who had charge of the field work of the gaging of the outlets of the Great Lakes and who has used this and other methods for deep rivers, says:[c] "All the objections to the surface float apply with greater force to the double float, and additional ones peculiar to itself." It is impossible to determine the exact position or depth of the lower float. Its position is determined from that of the surface float, but it varies with the direction and velocity of the wind and the length of the cord connecting the floats. The depth of the float is determined from the length of the connecting cord, but on account of the upward "boiling" motion of the water, and also the pressure of the water on a long connecting cord, the depth may be much less than the length of the cord. The upper float may drag the lower one or be dragged by it. At best

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[a] Report on the Mississippi River, by Humphreys and Abbot, p. 224.

[b] Op. cit., p. 225.

[c] Jour. Franklin Inst., Vol. LXII, p. 167.

it can only give the velocity of a few particles for a short space of time.

This method of gaging was used in the Mississippi River and tributaries from 1857 to 1881. Since that time the current meter has been largely used. In 1869 it was used in conjunction with the current meter for gaging the outlets of the Great Lakes. It was also used by T. G. Ellis in 1874 in his survey of the Connecticut River, in conjunction with the current meter, and by Gordon in 1873 to gage the Irawadi River. For comparisons of accuracy of this method with others, see pages 47 to 51.

FLOAT RODS.

Professor Cabeo[a] was the first to use the float rod for measuring velocity. This was in 1646. The rod is of wood or tin, from 1 to 2 inches in diameter, weighted at the lower end so as to float vertically. Its lower end should nearly touch the bottom and its upper end project a few inches above the surface of the water, so as to be visible. The method of using float rods as followed by the writer in 1900 in the New York State canal survey is as follows: Two surveyor's 100-foot chains were stretched across the canal from 10 to 30 feet apart and at right angles to the axis of the canal. Fifteen feet above the upper chain a rope was stretched across the canal, and to this a boat was attached by pulleys, so that a man in the boat could easily move himself back and forth by pulling on the rope. Another rope was stretched across the canal from 6 to 8 feet below the lower chain and a boat operated from it in a similar way. The floats were put in by the man in the upper boat, two at a time and about 5 feet apart, and were observed by him until they passed the upper chain, when he called to the recorder the position of each. As they passed under the lower chain the man in the lower boat called out their position, and when they reached his boat he took them out of the water and brought them to the shore. An observer with two stop watches noted the time of passage of each pair of floats between the chains. A short run was used on account of the low velocity of the water.

The advantages and disadvantages of the method may be summed up briefly as follows:

Advantages.—For artificial channels of moderate and uniform depth and with floating grass and weeds this is probably the best method to use. The advantages of float rods as given by Captain Cunningham, who has used them to a large extent in his gagings of the Ganges Canal, are as follows:[b] (1) They interfere less with the natural motion of the water; (2) they measure velocity direct; (3) they can be used in a stream of any size; (4) they are not affected by silt and weeds; (5) they measure forward velocity; (6) they can be made by a common workman; and (7) they are cheap.

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[a] See Report of New York Barge Canal, 1901, p. 869.

[b] Proc. Inst. Civil Eng., Vol. LXXI, p. 1.

Disadvantages.—The rods are affected to some extent by wind; they do not give mean velocity, but velocity of impulses; they can not be used in deep streams or in streams of rough or irregular bed, and they are expensive to operate.

Float rods have been used by Cunningham, on the Ganges Canal, in 1880; by Francis, in 1852, on a canal at Lowell, Mass.; by Buffon, on the Tiber River, in 1821; by Krayenhoff, on the Rhine, in 1812; and by the Mississippi River Commission, on the Upper Mississippi, in 1881.

FLOAT FRAME.

Hirn used, for obtaining velocity, a light frame of wood which nearly filled the cross section of the channel, but there are practical difficulties in the way of the use of this instrument.

INDIRECT METHODS.

The following instruments and methods have been used at one time or another for measuring velocity indirectly. Several of them are now of historical interest only.

FLOAT WHEEL.

The float wheel was used by Borda and Dubuat for measuring surface velocities. That used by the latter was 2 feet in diameter, and recorded the revolutions on dials.

PRESSURE PLATE.

In 1779 Gaunthy invented the pressure plate, which consists of a disk of metal opposed to the pressure of water, the velocity being computed from the weight necessary to keep it vertical. The tachometer used by Brünings to gage the Rhine and the one used by Racourt in his gagings of the Neva were made on this principle. Captain Boileau also used a pressure plate in his tachometer.

BOX WITH HOLE IN SIDE.

A box with a hole in the side was used by Grandi in 1730 to measure velocity. It was lowered gradually from the surface to the bottom and raised again at the same rate, the velocity being inferred from the amount of water in the box.

HYDROMETRIC TUBE.

This instrument was used by Captain Boileau in 1850. It consists of a glass tube suspended in a frame, having a full-sized opening at one end and a small opening at the other end. The tube is filled with water and a bubble of air, and is placed in the water with the small end upstream. The large end is then opened and the time required for the bubble to traverse the tube is noted. From this time and the areas of the ends of the tube the velocity is computed.

HYDROMETRIC PENDULUM.

This instrument was used by Castelli in 1628. It consists of a ball suspended from the center of a graduated arc. The velocity is computed from the weight of the ball and the angle of the string when the water impinges on it.

THERMOMETER.

Leslie used a thermometer for measuring velocity. The principle of this method is that the temperature of water in motion is greater than the temperature when at rest.

PITOT TUBE.

This instrument was invented by Pitot in 1730. Fig. 2 shows it as improved and used by Darcy and Bazin.[a] It consists essentially of two tubes, one drawn to a fine point and pointing upstream, the other straight and with an opening at the lower end the size of the tube. The air in both tubes is partly exhausted, so that the water columns will rise to a convenient height. The velocity is found from the difference in the readings of the water columns. Like floats, this is an impulse-measuring instrument, but in using it both maximum and minimum impulses are observed and a mean is found. After the instrument is in place and the stop cocks R and R' are opened one of the observers applies his mouth to the tube O, removes the air until the surface of the water in the tubes stands at a convenient height to read, and then closes the upper cock (R). He observes the columns until they reach a maximum height, when he closes the cock R' and reads the surface of both columns; then he opens the cock R and observes the surface of both until the columns reach a minimum position, when he closes cock R' and reads both surfaces again. He now has a maximum and a minimum reading of each column, from which he makes the following deduction: If a' is the maximum reading of column A, and b' is the maximum reading of column B, and a'' is the minimum reading of column A, and b'' is the minimum reading of column B, then $a'-b'=d'$, the difference for the maximum impulse, and $a''-b''=d''$, the difference for the minimum impulse, and $\frac{d'+d''}{2}$ is the mean difference of the column readings for the two impulses. Two or three sets of these readings are taken at each point. Then if $\Sigma\frac{d}{2}$ is the mean of the values of these column differences ($\Sigma d = d'+d''+ \ldots$); the velocity at that point is given by the formula $V=c\sqrt{2g\Sigma\frac{d}{2}}$, c being the coefficient found by rating and g the accel-

[a] Recherches hydrauliques, entreprises par M. H. Darcy continuées par M. H. Bazin. Première partie de Recherches expérimentales sur l'écoulement de l'eau dans les canaux découverts: Extrait des Mémoires présentés par divers savants à l'académie des sciences de Institut Impé rial de France, Paris, 1865, Vol. XIX, Pl. XVII, fig. 5.

eration of gravity. The velocity can very quickly be found by one accustomed to using the instrument. The accuracy of V depends on that of c and on $\sqrt{\Sigma d}$, the greater the number of d's, or sets of observations, the more nearly accurate will be the value of V.

CURRENT METER.

The current meter had its beginning in the float wheel used by Borda and Dubuat to obtain surface velocities. In 1790 Waltman modified this wheel so that it could be used below the surface. His meter had helicoidal blades and an endless screw on its axis, which by gearing into a train of wheels caused a record of the number of revolutions of the wheel to be made on dials. It was moved up and down on a rod the lower end of which was driven firmly into the river bed. The recording apparatus was thrown in or out of action by a pull on a string. It was necessary to lift the meter to the surface to read the revolutions, which was a great drawback to its use, as was also the fact that dirt in the water retarded the train of wheels. Lepont sought to remove these difficulties by bringing the recording apparatus to the surface, away from the dirty water and where it could be read without lifting the meter. He did this by introducing a

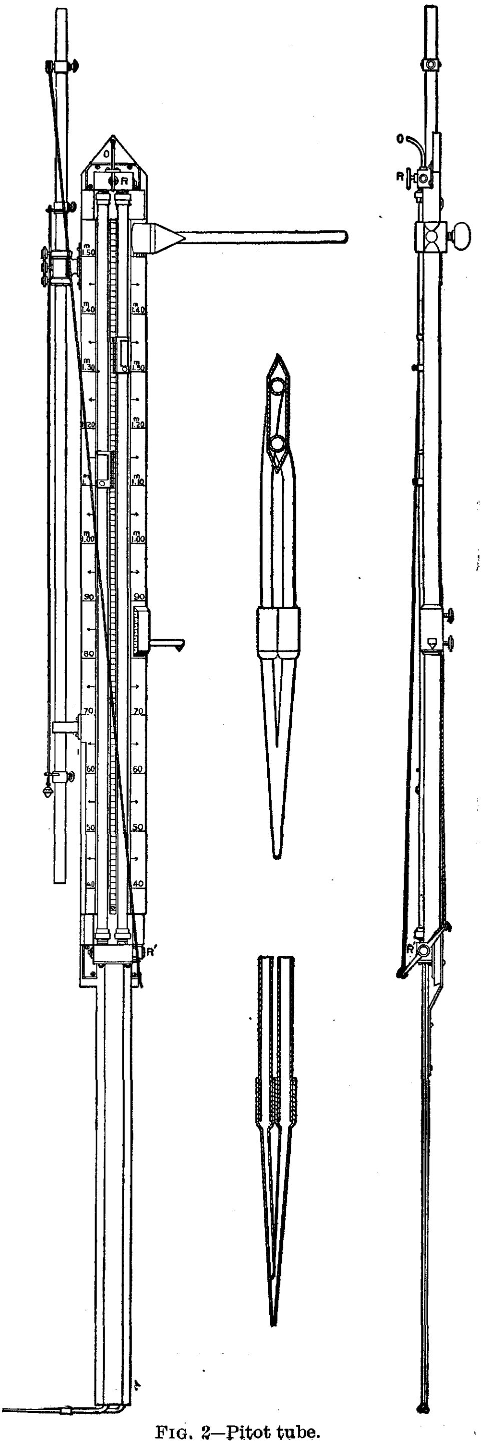


FIG. 2—Pitot tube.

vertical shaft or rod with beveled gearing, but this increased the friction so that it could only be used in shallow water. Baumgarten, Saxton, and others following modified its form somewhat, but did not materially improve it. Brewster made one in which the axis of the meter was a long screw and the revolutions were given by the distance the wheel traveled along this screw. The friction was thus

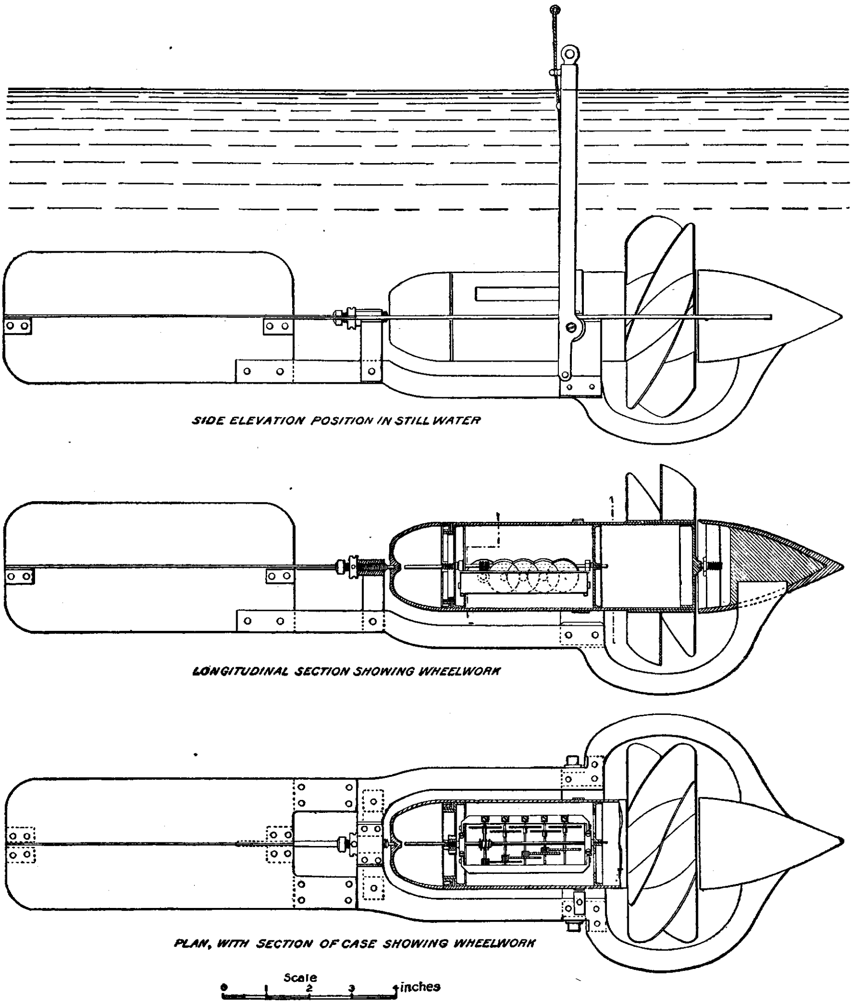
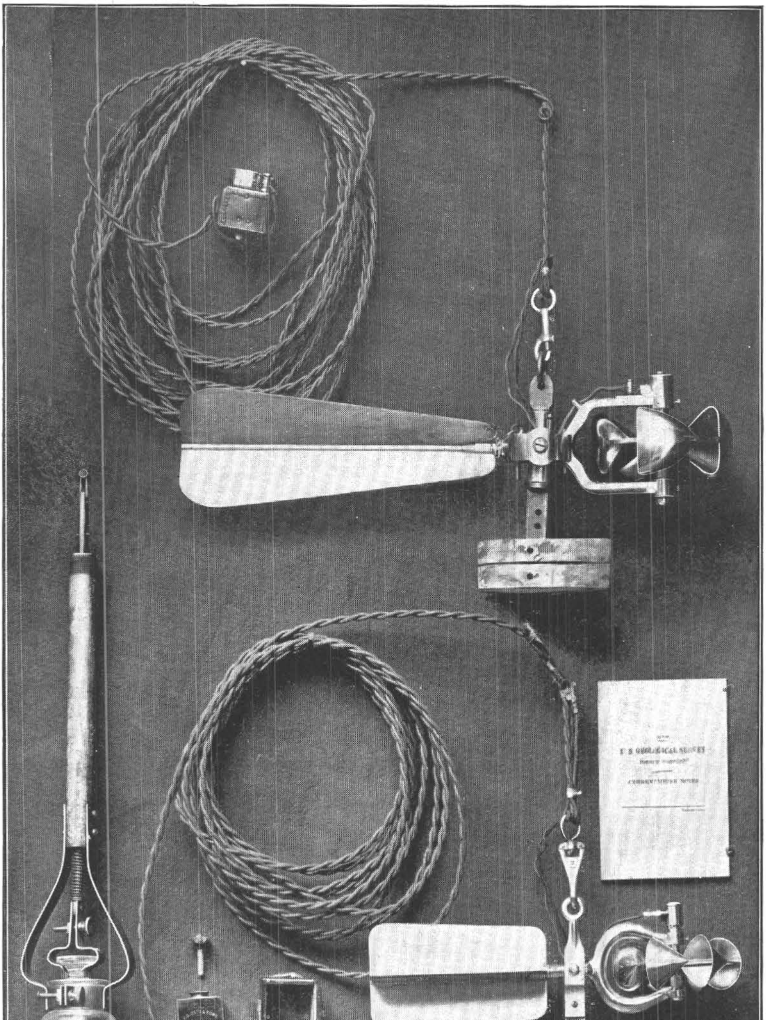


FIG. 3.—Moore current meter.

lessened, but the meter had to be stopped before the wheel traveled the whole length of the screw. It was a long step in advance that was taken by D. F. Henry when he applied an electric recording device to the meter, for by it the difficulties that Lepont and others sought to avoid are very successfully overcome, the friction of the train of recording wheels being entirely done away with. If a recording device



PRICE ELECTRIC CURRENT METERS.

is used it is worked by a spring and not by the meter, and, furthermore, it is not necessary to lift the meter to the surface to read the number of revolutions, which are recorded electrically, or may be

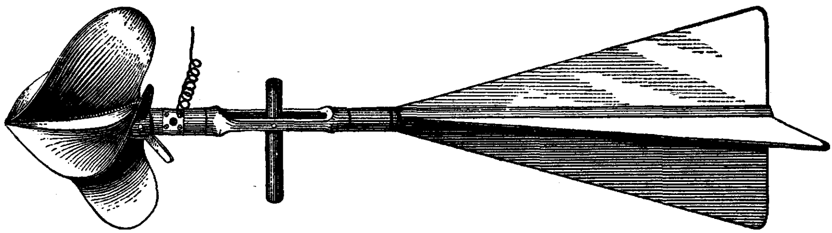


FIG. 4.—Haskell current meter.

counted from the indications of a buzzer. The electric meter as used by Henry is shown in fig. 10, page 34; that used by Moore in fig. 3.

Meters in use at the present day may be divided into two classes: (1) Those in which the revolving part turns about a horizontal axis, and (2) those in which it revolves about a vertical axis. The former class is illustrated by the Haskell and Fteley meters, shown in figs. 4 and 5, respectively, and the latter class by the Price meter, shown in Pl. I. Meters of the latter class have some advantages over those of the former class: First, friction is usually less, since it nearly all comes on one point, and this point can be protected from the action of grit in the water and from jars; and, second, for a given high velocity the wheel will not revolve as rapidly as the wheel of a meter of the first class under the same conditions, and at the same time the wheel will start in a less velocity than will the wheel of a meter of the former class, so that both low and high velocities can be measured more accurately with meters of the second class than with those of the first class. For example, by comparing the rating table of Haskell meter No. 3 on page 81 with that of small Price meter No. 363 on page 42, we find that the former requires a velocity of about 0.20 foot per second to start it, and that it makes 6.5 revolutions per second in water having

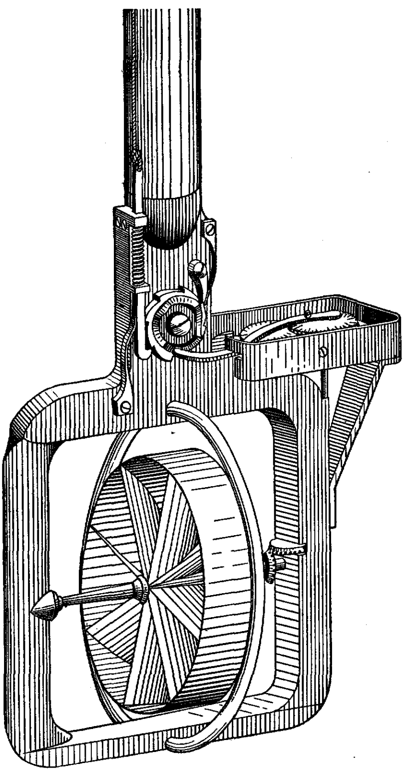


FIG. 5.—Fteley current meter.

a velocity of 7 feet per second, while the latter requires a velocity of about 0.06 foot per second to start it and makes only 3.02 revolutions in water having a velocity of 7 feet per second. A rate of 3 revolutions per second can easily be recorded or counted, but a rate of 6.5 revolutions per second can not be counted or easily recorded.

The disadvantages of the current meter are, briefly, as follows: (1) It can not be used where there is much floating grass or weeds, as on canals where the weeds are cut and allowed to float in the water, float rods being much better in such cases; (2) it requires rating before use and frequently thereafter; and (3) it must be used with care. These last two conditions, however, are true for all instruments used for measuring velocity indirectly.

The meter without some electrical device for indicating to the observer the revolutions in a given time, and which must be lifted out of the water to read the revolutions, is a thing of the past. Too much time is lost in making the readings. The acoustic meter is not a success. The clicks are not sufficiently loud to be heard when the water is deep and the place noisy.

The advantages of the meter are as follows: (1) It gives integrated or mean velocity (any kind of a float gives the velocity of a comparatively few particles at some stage of a pulsation of the water, not mean velocity); and (2) it can be used on streams of all sizes.

For large streams the choice of instruments is between the current meter and the double float, and the meter is preferable except where a large amount of weeds and grass is carried along in the water. For artificial channels the choice is between the meter and the float rod, and the meter has the preference except where there is a considerable amount of floating weeds, and possibly for very slow velocities also, but the latter is still in doubt. For velocities of 0.25 foot per second the meter will give the mean velocity as accurately and more rapidly than the rods, and at less cost, for at least four persons are necessary with the rod method, whereas one person, or at most two persons, can make the gaging with the meter, and the expense of moving the equipment from place to place is much greater for the rods than for the meter.

SLOPE FORMULA.

The velocity of a stream can be computed from measurements of the slope of the surface, the dimensions of the cross section, and a knowledge of the roughness of the bottom and sides. The formula which is now almost universally used for this purpose was devised by Kutter in 1869[a] and is as follows:

$$V=\left[\frac{41.6+\frac{1.811}{n}+\frac{.00281}{s}}{1+\left(41.6+\frac{.00281}{s}\right)\frac{n}{\sqrt{Rs}}}\right]\sqrt{Rs}$$

[a] Flow of Water in Rivers and Other Channels, by E. Ganguillet and W. R. Kutter.

In this formula V=velocity, in feet per second; s=surface slope=$\frac{h}{l}$, h being the fall of the surface in the distance l; R=hydraulic radius=$\frac{F}{w}$, F being the cross-sectional area and w the wetted perimeter; and n a ratio whose value varies from 0.009 for a well-planed wooden channel to 0.035 for a channel overgrown with weeds or covered with stones.

This formula is said to apply to streams of all sizes, from creeks and sewers to large rivers like the Mississippi, its constants being determined from Bazin's gagings of small channels, from Humphreys and Abbot's gagings of the Mississippi River, from Cunningham's gagings of the Ganges Canal, and from gagings by many others.

The disadvantage of the method is that it is very difficult to measure accurately the surface slope (s) of large rivers. T. G. Ellis says[a] that on the Connecticut River the slope of parts 100 to 400 feet long was very carefully measured and the discharge computed, using the best modern slope formulæ. The discharge found from these formulæ differed by 50 to 250 per cent from that found by gaging. He thinks the slope is so uncertain an element that slope formulæ are of little value. Captain Cunningham, after an examination of five hundred slope measurements of the Ganges Canal, says:[b] "It [the slope of surface measurement] is so delicate a matter that the results are of doubtful use." He found that the slope was very different at different parts of a reach from 1 to 2 miles long, and that the slope at opposite sides differed 50 per cent. Ellet says,[c] in regard to the slope of the Mississippi River: "It not unfrequently happens that while the mass of the water which its channel bears is sweeping to the *south* at a speed of four or five miles per hour, the water near the shore is running *to the north* at a speed of one or two miles per hour. It is no unusual thing to find a swift current and a corresponding fall on one shore toward the south, and on the opposite shore a visible current and an appreciable slope toward the north."

Only in very rare cases is it advisable to use this method.

MEASUREMENT OF SURFACE FLUCTUATIONS.

Any change in the elevation of the surface of a stream affects the discharge in two ways: The cross-sectional area is changed and as a rule the velocity also is changed.[d] It is necessary, therefore, in accurate work to measure the surface fluctuations with great care. Wave action, due to wind or other causes, should be eliminated.

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[a] Trans. Am. Soc. Civ. Eng., Vol. XI, p. 23.

[b] Proc. Inst. Civ. Eng., Vol. LXXI, p. 11.

[c] Report on the Mississippi River, by Humphreys and Abbot, p. 218.

[d] On some streams there are places where the mean velocity remains nearly constant for several feet variation in stage of river. This is true of the Verdigris River at Liberty, Kans. See Nineteenth Ann. Rept. U. S. Geol. Survey, Pt. IV, p. 374.

There are two classes of instruments in use at the present time for indicating surface elevations: (1) Self-registering gages, which give a continuous record of the changes in surface elevation; and (2) a fixed scale from which the elevation of the surface is read at any time, or a moving scale with a fixed index from which the elevation of the surface is read.

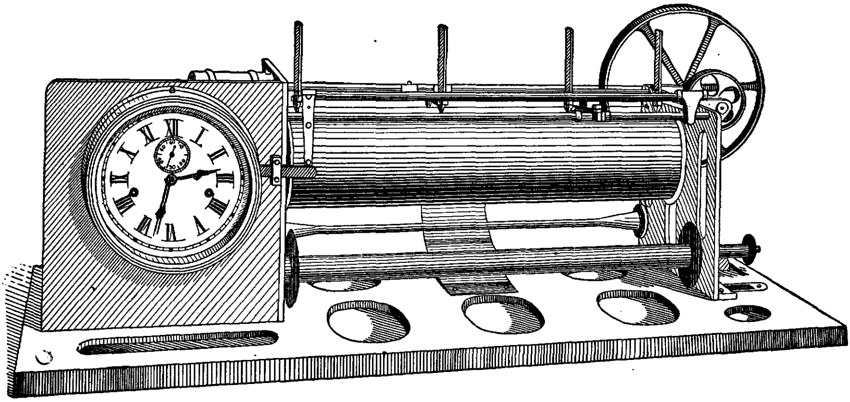

FIG. 6.—Self-registering water gage used by United States Lake Survey.

The United States Lake Survey self-registering water gage, shown in fig. 6, is a good illustration of instruments of the first class. The center pen is connected to a float on the surface of the lake, and its

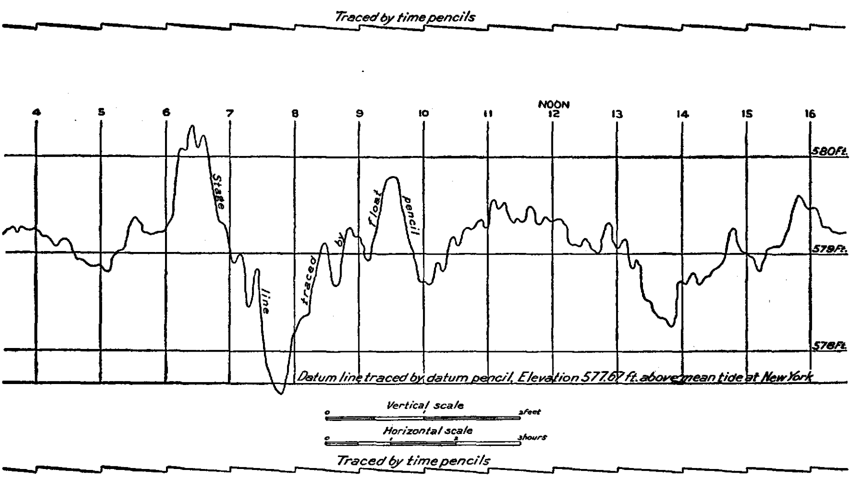

FIG. 7.—Record made by self-registering water gage used by United States Lake Survey.

motion is proportional to the change in elevation of the float. This motion is traced by the pen on a sheet of paper on the drum, which is moved at a uniform rate by clockwork. The two outer pens record the time. Fig. 7 is a reproduction of the record made by this instrument on May 17, 1899. The elevation of the surface at any instant is given by the corresponding ordinate of this curve. This figure illustrates the need of an accurate record of the change in stage, for it shows a change of surface elevation of 2.5 feet in one and five-tenths hours.

The hook gage, shown in fig. 8, is a good illustration of a gage of the second class. The frame is fastened firmly to a post in the water, and the elevation of the zero reading is determined. By turning the milled nut the hook is raised until its point comes to the surface of the water. This can be done very accurately in still water, by means of the little spot of light on the point of the hook. The scale is then read, and from it the surface elevation is found.

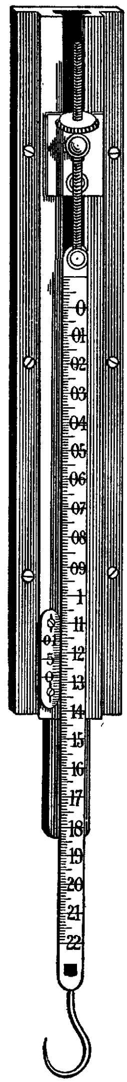

FIG. 8.—Hook gage for measuring surface fluctuations.

The water gage used by the United States Board of Engineers on Deep Waterways in 1897–98, on the Niagara River at Buffalo, N. Y., was of the box-with-float type. The box was of wood, 7 inches high, 7 inches wide, and 7 feet long, with a closed bottom and a removable cover. The box was fastened firmly in the water, with half its length below the surface. Water was admitted to it through one or more of three ¼-inch holes, allowing the float to change its elevation as the river stage changed and at the same time eliminating the effect of waves. The float was a 2-quart bottle with a 7-foot staff, marked to feet, tenths, and hundredths, wedged into it. This staff extended up through the cover, and was marked so as to give the rise of the water surface above a certain plane as read at the cover of the box.

VERTICAL VELOCITY CURVES.

The relation between velocity and depth in a vertical section of a stream parallel to the thread of the current, or the vertical velocity curve, is very desirable. From such a curve the ratio of velocity at any depth to mean velocity can easily be found and the discharge be computed from observations at one depth only in verticals.

Much effort has been expended in investigating this relation and the change in it for change in river stage. As might be expected, the results of experiments in this direction do not agree. Each investigator adopts a new form of curve to fit his observations. In 1791 Waltman made some observations on the Rhine, from which he concluded that the vertical velocity curve is a reversed parabola with its vertex below the bed of the river. In 1820 De Fontaine found the greatest velocity of the Rhine to be at the surface. He states that two inclined right lines intersecting about mid depth satisfy his vertical-curve observations. In 1824–1826 Racourt made observations on the Neva River, from which he concluded that this curve is an ellipse whose minor axis is a little below the surface. Funk adopted a logarithmic curve for his observations on the Weser in 1820. In 1844 Boileau found this curve for a small canal to be a parabola with its axis near the surface. Darcy and Bazin found it to be a reversed parabola for their experiment canal, the perimeter changing with the character of the bed. For the Mississippi River, Humphreys and Abbot found it to be a parabola whose axis is three-tenths depth below the surface. Henry found it to be an ellipse for the St. Clair River, while Baumgarten says that no simple curve will fit the observations in a vertical. From the results of these investigations and others that might be mentioned it appears that the shape of the vertical velocity curve depends much on local conditions, as roughness of bed, slope, ratio of depth to width, wind, obstructions, etc. Two facts must be borne in mind, however: (1) All the vertical velocity-curve observations obtained with a single meter will not when plotted fall on any one line, but the results will be scattered more or less, and it will be possible to pass a number of curves among them, one satisfying the observations about as well as another; and (2) there is some part of each of the curves mentioned which if placed by the side of some part of another curve will coincide with it closely, so that it is quite possible for any or all of these curves to fit more or less closely a set of imperfect vertical velocity-curve observations.

There are two methods of obtaining vertical velocity curves with current meters: (1) The single-meter method, and (2) the multiple-meter method. In the former a meter is held at as many points in a vertical as desired—at each tenth of the depth, for example—for a time sufficiently long to eliminate the effect of pulsations. This requires one or more hours to a vertical, during which time a permanent change may occur in the mean velocity. In the latter method, however, this difficulty is overcome, for enough meters are used simultaneously to give the whole curve in one operation.

In the vertical velocity-curve work on the St. Clair River, under the direction of E. E. Haskell, United States assistant engineer, eleven meters were used at a time. These were fastened between two cables, which were attached to a 200-pound sinker at the lower end and to a drum at the upper end, and operated from a catamaran. The lower meter was 1.75 feet above the bottom; the other ten were placed at each tenth of the depth. Each meter was connected with an electric register, and by means of a switch all of the circuits could be opened in one operation. The observer, with a stop watch in one hand and the switch handle in the other, started the watch and at a given instant closed all of the circuits through the meters and registers. At the end of the observation he stopped the watch and opened the circuits at the same instant. One observation of six hundred seconds was made with the meters arranged as described, and from one to ten others of one hundred seconds each with the meters arranged in other ways in each vertical.

There should be some criterion for the rejection of doubtful observations and the grouping of vertical velocity curves. The following table gives the position of the vertex of sixty-nine vertical velocity curves obtained on the Mississippi River by Humphreys and Abbott, and illustrates this need:

Table showing position of vertex of vertical velocity curves obtained on Mississippi River.

Depth.	First series.		Second series.		Total.	
	Number of verticals.	Number of floats.	Number of verticals.	Number of floats.	Number of verticals.	Number of floats.
Surface	12	63	8	17	20	80
One-tenth	11	46	1	2	12	48
Two-tenths	5	33			5	33
Three-tenths	3	20	1	3	5	23
Four-tenths			4	9	4	9
Five-tenths	2	22	1	3	3	25
Six-tenths	5	30	1	3	6	33
Seven-tenths			6	18	6	18
Eight-tenths			6	15	6	15
Nine-tenths			2	5	2	5
Bottom	1	8			1	8
Sum	39	222	30	75	69	297

The vertex of 34 per cent of the curves in the foregoing table is at or below mid depth of the river. Using all of them the vertex is three-tenths depth below the surface. This combination of curves so dissimilar is, to say the least, very questionable. Those in which the vertex is at or below seven-tenths depth should be rejected, or at least not combined with the others, for they are due either to errors or to abnormal conditions, and should not be used in deriving a general law.

Vertical velocity curves obtained in different parts of the discharge section for different depths are frequently combined by combining the velocities at each tenth of the depth. This assumes that the curves are similar in all parts of the discharge section, which is not true except in rare cases.

There is very little available data on the change in the vertical velocity curve with change of river stage. In the data on this point that have come to the writer's observation the change in stage is small and the inference to be drawn not conclusive. The effect of roughness of bed decreases, while that of slope and hydraulic radius increases with increase of stage. The effect of these on the shape of the curve can only be determined by careful measurements in which there is a wide range of stage.

PULSATION OF MOVING WATER.

We have already called attention to the fact that one of the reasons it is difficult to measure accurately the velocity of water is that the velocity at any given point in a stream is constantly changing. This is true of both natural and artificial channels of all sizes. Captain Cunningham, who observed this pulsation of moving water in his experiments on the Ganges Canal, of India, states that he considers fifty repetitions with float rods necessary in order to get a good average value of the velocity in one vertical of a canal.[a] J. B. Francis has made numerous measurements of the velocity with float rods in a rectangular wooden flume at Lowell, Mass., every precaution being taken to have the usual causes of irregularity of motion suppressed. The time of run was obtained by the use of an electric telegraph and a chronograph, and the depth was very carefully measured with a hook gage, and yet he found that the velocity shown by these floats varied from 8.57 per cent above the mean to 11.4 per cent below it. He says: "Similar and probably greater variations occur in different parts of the depth from the same cause."[b] Adam Baum has made some observations on the variation of velocity in the Rhine near the Bridge of Constance.[c] He fixed a current meter in the river and noted the time of each 100 revolutions of the meter wheel for a period of two hours. The results show a continual change in the velocity for a constant depth of water. He concludes that for an accurate measurement of velocity it is necessary to extend the observations over a period of one hour at each point.

Professor Unwin has made some observations with a current meter on the variation in velocity in the river Thames.[d] He noted the time of each 100 revolutions, and upon plotting the time and the revolutions found that they gave a very irregular curve. The mean of each 500 revolutions, however, when plotted with the corresponding time gave a rather regular curve. The following are some of the results of Professor Unwin's observations:

[a] Proc. Inst. Civ. Eng., Vol. LXXI, p. 8.
[b] Trans. Am. Soc. Civ. Eng., Vol. VII, p. 117.
[c] Proc. Inst. Civ. Eng., Vol. LXXI, pp. 456-459.
[d] Op. cit., pp. 348, 349.

Variation in velocity at three points of a vertical of Thames River as found by Unwin.

At 0.5 meter depth; velocity found from—		At 3 meters depth; velocity found from 100 revolutions.	At 6 meters depth; velocity found from 100 revolutions.	Variation $\left(\frac{Vm-V}{Vm}\right)$.[a]		
100 revolutions.	500 revolutions.			At 0.5 meter depth.	At 3 meters depth.	At 6 meters depth.
Met. per sec.	*Met. per sec.*	*Met. per sec.*	*Met. per sec.*	*Per cent.*	*Per cent.*	*Per cent.*
1.909	1.877	1.861	1.111	−1.8	+6.6	− 3.6
1.942	1.846	2.030	1.069	−3.6	−1.9	+ 0.3
1.987	1.846	1.942	0.977	−6.0	+2.6	+ 8.9
1.942	1.846	2.030	1.180	−3.6	−1.9	−10.1
1.861	1.832	2.030	1.044	+0.7	−1.9	+ 2.6
1.861	1.832	2.060	0.937	+0.7	−3.4	+12.6
1.887	1.823	1.909	1.021	−0.7	+4.2	+ 4.8
1.861	1.787	1.909	0.998	+0.7	+4.2	+ 7.0
1.861	1.809	1.861	0.977	+0.7	+6.7	+ 8.9
1.861	1.787	2.030	1.094	+0.7	−1.9	− 2.1
1.942	1.861	2.125	0.900	−3.6	−6.7	+16.1
1.760	1.823	2.030	1.094	+6.1	−1.9	− 2.1
1.831	....	2.086	1.021	+2.4	−4.7	+ 4.7
1.719	....	1.994	1.180	+8.3	0.0	−10.1
1.942	....	2.086	1.079	−3.6	−4.7	− 0.7
1.831	....	1.909	1.472	+2.4	+4.2	−37.4
1.875		1.993	1.072			

[a] V=velocity, Vm=mean velocity.

It is seen that during the period required for the meter to make 1,600 revolutions the greatest variations from the mean velocity at 0.5 meter depth are +8.3 per cent and −6 per cent and at 6 meters depth +16.1 per cent and −37.4 per cent. The greatest difference in velocity, as shown by two consecutive 100-revolution periods, is 12 per cent at 0.5 meter depth and 36.7 per cent at 6 meters depth. A velocity measurement of 100 revolutions 0.5 meter below the surface may be in error 8 per cent and one 6 meters below may be in error 37 per cent.

D. F. Henry[a] has observed this fluctuation in velocity at a given point in large and in small streams. He says: "The lesser fluctuations have a duration of 30 to 60 seconds and the larger ones from 5 to 10 minutes. They do not seem to be synchronous with the surface fluctuations, and are smaller at the surface than at the bottom."

Harlacher[b] found the velocity near the surface of the Rhine to vary 20 per cent in a few seconds, and near the bottom he found it to vary 50 per cent in the same period.

Marr's simultaneous observations of velocity in the Mississippi River at Burlington, Iowa,[c] with five current meters are described on page 48. The following table gives the velocity, as shown by two of his current meters, one near the surface and the other near the bottom, for consecutive periods of one minute each. The depth in

[a] Jour. Franklin Inst., Vol. LXII, p. 323.
[b] Trans. Am. Soc. Civ. Eng., Vol. XII, p. 311.
[c] Report on Current Meter Observations, Burlington, Iowa, by Maj. A. McKenzie, 1884.

the first section is 11 feet, in the second section 27.6 feet, and the meter is 4 feet above the bottom:

Variation in velocity per minute in Mississippi River at Burlington, Iowa.

Simultaneous observations in section No. 1.				Simultaneous observations in section No. 2.			
Velocity 1 foot below surface (V).	Variation $\left(\frac{Vm-V}{Vm}\right)$.	Velocity 9.1 feet below surface (Vm).	Variation $\left(\frac{Vm-V}{Vm}\right)$.	Velocity 3.6 feet below surface (V).	Variation $\left(\frac{Vm-V}{Vm}\right)$.	Velocity 23.6 feet below surface (Vm).	Variation $\left(\frac{Vm-V}{Vm}\right)$.
Ft. per sec.	*Per cent.*	*Ft. per sec.*	*Per cent.*	*Ft. per sec.*	*Per cent.*	*Ft. per sec.*	*Per cent.*
2.242	+2.0	1.724	0.0	2.286	+4.8	1.966	+ 2.4
2.267	+0.9	1.675	+ 2.9	2.353	+1.3	1.817	+ 9.8
2.252	+1.1	1.761	− 2.2	2.437	−2.3	1.985	+ 1.5
2.236	+2.2	1.744	− 1.2	2.300	+3.5	2.008	+ 0.3
2.267	+0.9	1.797	− 4.3	2.404	−0.9	2.065	− 2.5
2.299	−0.5	1.766	− 2.4	2.378	+0.2	2.119	− 5.2
2.289	0.0	1.772	− 2.8	2.414	−1.3	1.926	+ 4.4
2.274	+0.6	1.819	− 5.5	2.392	−0.4	2.008	+ 0.3
2.211	+3.3	1.712	+ 0.7	2.313	+3.0	1.983	+ 1.5
2.224	+2.8	1.644	+ 4.7	2.296	+3.8	2.081	− 3.3
2.280	+0.3	1.698	+ 1.5	2.383	0.0	2.104	− 4.5
2.239	+2.1	1.915	−11.1	2.397	−0.6	2.276	−13.3
2.280	+0.3	1.670	+ 3.1	2.420	−1.6	2.092	− 3.9
2.289	0.0	1.775	− 3.1	2.361	+0.9	2.008	+ 0.3
2.321	−1.5	1.582	+ 8.3	2.278	+4.4	2.047	− 1.6
2.420	−6.0	1.664	+ 3.5	2.445	−2.6	1.848	+ 8.3
2.445	−6.9	1.741	− 1.0	2.459	−3.2	2.024	− 1.0
2.308	−0.9	1.717	+ 0.5	2.428	−1.9	2.070	− 2.8
2.333	−2.0	1.800	− 4.4	2.456	−3.1	1.796	+10.8
2.264	+1.0	1.695	+ 1.7	2.460	−3.2	2.055	− 2.4
2.327	−1.3	1.812	− 5.1	----------	----------	----------	----------
2.311	−1.0	1.689	+ 2.0	----------	----------	----------	----------
2.305	−0.8	1.792	− 4.0	----------	----------	----------	----------
2.311	−1.0	1.627	+ 5.7	----------	----------	----------	----------
2.277	+0.4	1.554	+ 9.9	----------	----------	----------	----------
2.274	+0.6	1.678	+ 2.7	----------	----------	----------	----------
2.292	−0.2	1.731	− 0.4	----------	----------	----------	----------
2.271	+0.7	1.704	+ 1.1	----------	----------	----------	----------
2.321	−1.5	1.698	+ 1.5	----------	----------	----------	----------
2.261	+1.1	1.723	0.0	----------	----------	----------	----------
2.217	+3.6	1.757	− 1.9	----------	----------	----------	----------
2.280	+0.3	1.743	− 1.1	----------	----------	----------	----------
2.287	----------	1.724	----------	2.383	----------	2.014	----------

It is seen that in the first section the greatest departures of velocity from the mean are +3.6 per cent and −6.9 per cent at 1 foot below the surface, and +9.9 per cent and −11.1 per cent at 9.1 feet below the surface, or 2 feet above the bottom. The greatest difference in velocity obtained from two consecutive 1-minute periods is 6 per cent at 1 foot below the surface and 12.5 per cent at 9.1 feet below the surface, or 2 feet above the bottom. In the second section the greatest departures of velocities from the mean are +4.8 per cent and −3.2 per cent at 3.6 feet below the surface, and +10.8 per cent and −13.3 per cent at 23.6 feet below the surface, or 4 feet above the bottom. The greatest difference in velocity obtained from two consecutive 1-minute periods is 7 per cent at 3.6 feet below the surface and 13.6 per cent at 4 feet above the bottom. In the first section a velocity measurement near the surface may be in error 7 per cent and a measurement 2 feet above the bottom may be in error 11 per cent. In the second section a velocity measurement 3.6 feet below the surface may be in error 4.8 per cent and a measurement 4 feet above the bottom in error 13.3 per cent.

In 1899, w. . measuring the discharge of the St. Clair River, Mr. L. C. Sabin, assistant engineer, made some experiments to determine the pulsations or fluctuations in the velocity of the river at the discharge station. Four meters were used, placed 50 feet apart and at the same depth. In the first series of observations the meters were in a line across the river and at right angles to the current; in the second series they were in a line with the axis of the current. Simultaneous readings of the four meters were taken every fifteen seconds for several periods of ten minutes each. The results were plotted, using time as abscissæ and velocity as ordinates, and a curve drawn for each meter, showing the fluctuations in velocity in each fifteen seconds of time at four points 50 feet apart. These curves have two sets of waves—small ones of fifteen to sixty seconds amplitude and larger ones of three to six or more minutes amplitude. The range of velocity as found from the large waves is in some cases 35 per cent of the mean velocity shown by the meter for ten minutes. The curves for two adjacent meters are at times nearly parallel, but they soon diverge and cross each other occasionally. The curves for the meters in a line with the axis of the river resemble one another more closely and are more nearly parallel than those of the meters in a line across the river. These experiments indicate that the pulsations are very limited in extent in a direction at right angles to the current, but that they can be traced for some distance in the direction of the thread of the current. The whole depth of the river is affected by them, and their effect decreases from the bottom toward the surface.

The practical lesson to be drawn from the experiments by Sabin and others is that velocity observations of short duration are of little value unless they are numerous and well distributed over the discharge area. In order to eliminate the effect of the pulsations, each observation should extend over a period of from six to ten minutes. Long single observations and rapid fluctuations of water surface elevation require that discharge be found from single observations in comparatively few verticals, and that the ratio of velocity at this depth to mean velocity be known from vertical velocity curves. This emphasizes the need of a thorough investigation of the relation between velocity and depth in a vertical longitudinal plane and the change in this relation with change of river stage.

METHODS OF MEASURING AND COMPUTING STREAM FLOW.

WEIRS.

The discharge of a stream can be obtained by causing it to flow over a weir or dam the coefficient of which is known, by the substitution of observed data in a weir formula. Three sharp-crested or standard weir formulæ are in use, viz, Francis's, Fteley and Stearns's, and Bazin's. They differ much in form, and each is derived from experiments on small weirs with low heads. A brief discussion of each formula is here given because of its bearing on the accuracy of discharge measurements. Bazin has determined the values of the coefficient of discharge over weirs of many other shapes of crest, and the Cornell University experiments of the Board of Engineers on Deep Waterways have extended the values of the coefficients of a few of these to the higher heads. Time and space, however, will not permit more than the mention of them here.[a]

The weir is the most accurate method of measuring discharge yet used for small streams if the coefficient of the weir be accurately known. The head on the weir and the velocity of approach are the only variables, and these can easily and accurately be measured. The disadvantages are (1) the cost of constructing a weir, or the difficulty of finding one in use whose shape closely resembles one whose coefficient is known, and (2) the limited range of values of the coefficient of a weir of any shape and the limited number of shapes that have been investigated.

FRANCIS'S EXPERIMENTS AND FORMULA.

In 1852 J. B. Francis made numerous very careful measurements of the volume of water passing over small weirs and the corresponding heads at Lowell, Mass.[b] His measuring tank was a canal lock which had a capacity of 12,138 cubic feet for a depth of 9.5 feet; the greatest length of crest was 10 feet; the head on the weir varied from 5 to 19 inches, and was measured with two hook gages 6 feet upstream from the crest; the width of the channel was about 14 feet. His formula, derived from these experiments, is as follows:

$$Q=\tfrac{2}{3}\times 0.622\, h_2(b-\tfrac{1}{10}\, nh_2)\sqrt{2\, gh_2}=3.33(b-\tfrac{1}{10}\, nh_2)h_2^{\frac{3}{2}}$$

Q=discharge in cubic feet per second; h=head on weir, or difference in elevation of weir crest and water surface; n is a constant whose value is 0 when both ends of the weir are flush with the sides of the channel, 1 when one side only is flush, and 2 when neither side is flush; g=acceleration of gravity. (See fig. 9.) The head on the weir should be not less than 0.5 foot nor more than 2 feet, the depth of water in the canal should be at least three times the head on the weir, and the air should have free entrance under the sheet. The velocity of approach of the water to the weir must be

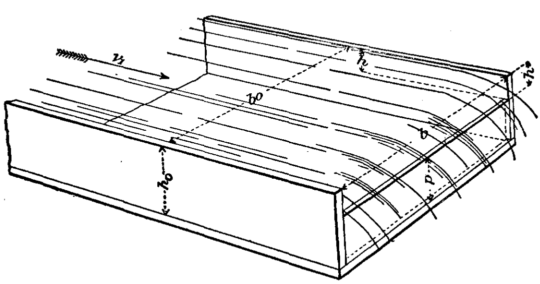

FIG. 9.—Sharp-crested weir.

[a] A discussion of these experiments, which were conducted by George W. Rafter, will be found in Trans. Am. Soc. Civ. Eng., Vol. XLIV, pp. 220-398.

[b] Lowell Hydraulic Experiments, by J. B. Francis, p. 133.

measured and the head corrected for it. The formula for this purpose is $h_2=[(h+h_1)^{\frac{3}{2}}-h_1^{\frac{3}{2}}]^{\frac{2}{3}}$, h being the measured head, h_1 the head due to the velocity of approach $=\frac{V_1^2}{2g}$ and $V_1=\frac{Q}{b_0h_0}$. (See fig. 9.)

Hamilton Smith, jr., says of this formula:[a] "It stands in the first rank in reliability."

Francis made eighty-eight experiments with two weirs of 10 and 8 feet length, in addition to those to derive his formula, and found that the greatest percentage difference between his measured discharge and the discharge computed by his formula was 0.9 per cent for the mean of two experiments and 0.68 per cent for the mean of eight experiments; for the other eleven means the difference is less than 0.4 per cent.

FTELEY AND STEARNS'S EXPERIMENTS AND FORMULA.

In 1877 and 1879 Fteley and Stearns[b] measured the discharge over weirs of 5 and 19 feet crest length and the corresponding head on weir, from which they derived the following formula for a standard weir with end contractions suppressed:

$$Q=3.31b\left[h+1.5\frac{V_1^2}{2g}\right]^{\frac{3}{2}}-0.007b.$$

Q=discharge in cubic feet per second, b=length of crest, h=head on weir, V_1=velocity of approach in feet per second. (See fig. 9.) Their measuring tank was a section of the Sudbury conduit having a capacity of 300,000 cubic feet for an increase in depth of 3 feet. The head on the weir was measured with a hook gage 6 feet upstream from the crest. The term $1.5\frac{V_1^2}{2g}$ corrects for velocity of approach. The head on the weir must be not less than 0.06 foot, and the air must have free access under the sheet. This formula is not used as generally as is that of Francis or Bazin. It was used by Fteley and Stearns to compute the discharge of the weir used in their current-meter and weir comparisons. (See p. 57.)

BAZIN'S FORMULA.

In 1886 and 1887 Bazin[c] made numerous measurements of the volume passing over weirs having lengths of from 0.5 meter to 2 meters and the corresponding heads. From these he derived the following formula for a standard weir with end contractions suppressed:

$$Q=\tfrac{2}{3}\mu\left[1+0.55\left(\frac{h}{p+h}\right)^2\right]bh\sqrt{2gh}$$

$$\text{and } \mu=0.6075+\frac{0.0148}{h\ (\text{in feet})}.$$

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[a] Hydraulics, by Hamilton Smith, p. 93.
[b] Trans. Am. Soc. Civ. Eng., Vol. XII, p. 1.
[c] Annales des ponts et chaussées, 1888, p. 416.

This formula is true for any system of units (feet or meters). Q=discharge, b=length of crest, h=head on weir, p=distance of crest above bottom. (See fig. 9.) This formula needs no correction for velocity of approach. The measuring tank was a section of a rectangular channel 200 meters long, 2 meters wide, and 1.2 meters deep. The head on the weir varied from 0.05 meter to 0.6 meter. The air had free access under the sheet. Each of these formulæ will give the discharge to within about 1 per cent if the conditions existing when the observations on which the formula is based are exactly duplicated. Discharges computed from any two of these formulæ may differ 3 per cent.

CURRENT METERS.

If the stream be small, meter measurements are made from a bridge, if one can be found in a suitable location. Bridge piers, however,

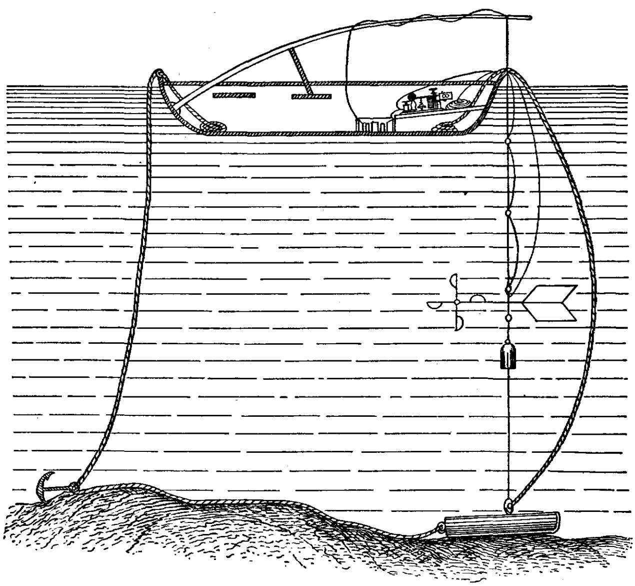

FIG. 10.—Method of using current meter on large river.

disturb the natural flow of a stream, as well as distort the vertical and transverse velocity curves and render them of only local application. Discharge measurements made where there are piers in the channel are, as a rule, less accurate than measurements made where there are no piers to obstruct the flow.

In case there is not a bridge at the desired place, the observer wades, if the water be shallow, taking observations of depth and velocity at as many points in the cross section as desired. If the water be deep and swift a cable is stretched across the stream at a convenient height above the water, and a car or box is suspended from it. From this car, which is moved back and forth across the stream by the observer in it, the depth and velocity of the water are measured. If the current be not swift a surveyor's chain or a cable can be stretched across the stream temporarily and the depth and velocity be measured from a rowboat at points marked on the chain or cable.

In a large river like the Mississippi, where a steamboat can conven-

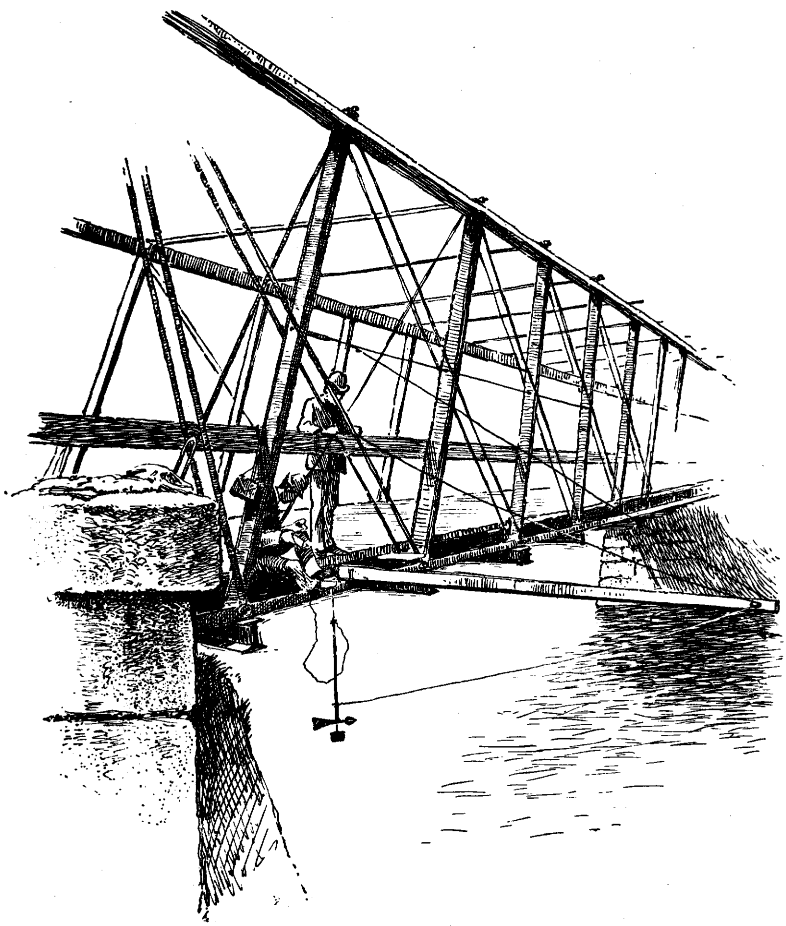

FIG. 11.—Device for holding meter in place.

iently anchor, a method that has been used satisfactorily is shown in fig. 10.[a] The boat is anchored in the proper place, and a lead weight of 50 pounds connected to the anchor is lowered over the stern. A copper wire is connected to this weight and to the spring pole in the boat, and the meter is moved up and down on this wire by a rope. The revolutions of the meter are recorded electrically in the boat, the cop-

[a] Jour. Franklin Inst., Vol. LXII, p. 171.

per wire being connected to one pole of the battery and an insulated wire connected to the meter and the other pole.

On large rivers where boats can not anchor the measurements are made from a boat towed by a steamboat.[a] The meter is kept in position by two lines of range poles at right angles to each other. The pilot on the steamboat steers it so as to keep it in line with one range, while the engineer, with his hand on the steam valve and his eye on the other range, controls the speed of the boat so that it will remain in a fixed position.

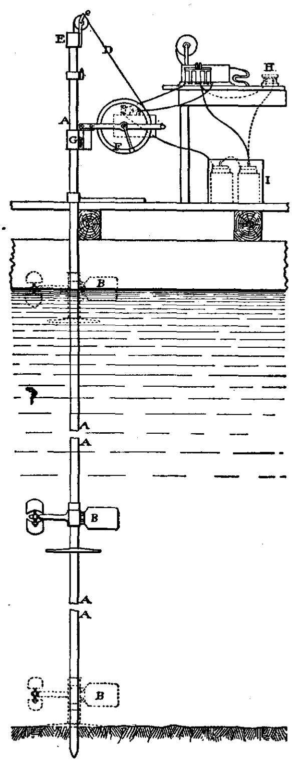

FIG. 12.—Harlacher's apparatus for moving a meter with uniform velocity.

In a swift current the meter and its sinker will be carried downstream a distance depending on the weight of the sinker and the depth and velocity of the water. There are two methods of treating this difficulty. One is to apply a special device like that shown in fig. 11 to keep the meter in place;[b] the other is to compute from the weight of the sinker and the depth and approximate velocity of the water the additional length of cable to use in order that the meter may sink to the required depth.[c] The first is the better for streams of ordinary size. In the device shown in fig. 11 the stay line is attached to the meter and is passed over a pulley at the end of the stay pole, being held by an assistant. By means of this stay line and the vertical cord the meter can be kept nearly in position. Instead of a stay pole a wire cable is sometimes used to keep the stay line in place.

POINT AND INTEGRATION METHODS.

There are two methods of using a meter in a cross section, as follows: (1) The point method, in which the meter is held at certain points in the cross section for a stated period of time, usually fifty seconds, and the number of revolutions of the wheel of the meter during that time are observed, either by counting the clicks of a buzzer or reading a register; and (2) the integration method, in which, instead of holding the meter at certain points, it is moved with a uniform speed through the section, the time and number of revolutions being observed as before. The point method is the more accurate if sufficient observations are taken in the section. The velocity found by the integration method is the resultant of

[a] Jour. West. Soc. Eng., Vol. III, No. 3.
[b] Twentieth Ann. Rept. U. S. Geol. Survey, Pt. IV, p. 20.
[c] Jour. West. Soc. Eng., Vol. IV, No. 6.

the velocity of the water and that of the meter as it is carried by the observer, and must always be greater than the true velocity. The error increases with the speed of the meter, and also increases as the velocity of the water decreases. When the point method is used the meter may be held at several points in certain verticals, in which case we will call it the ordinary method; or it may be held

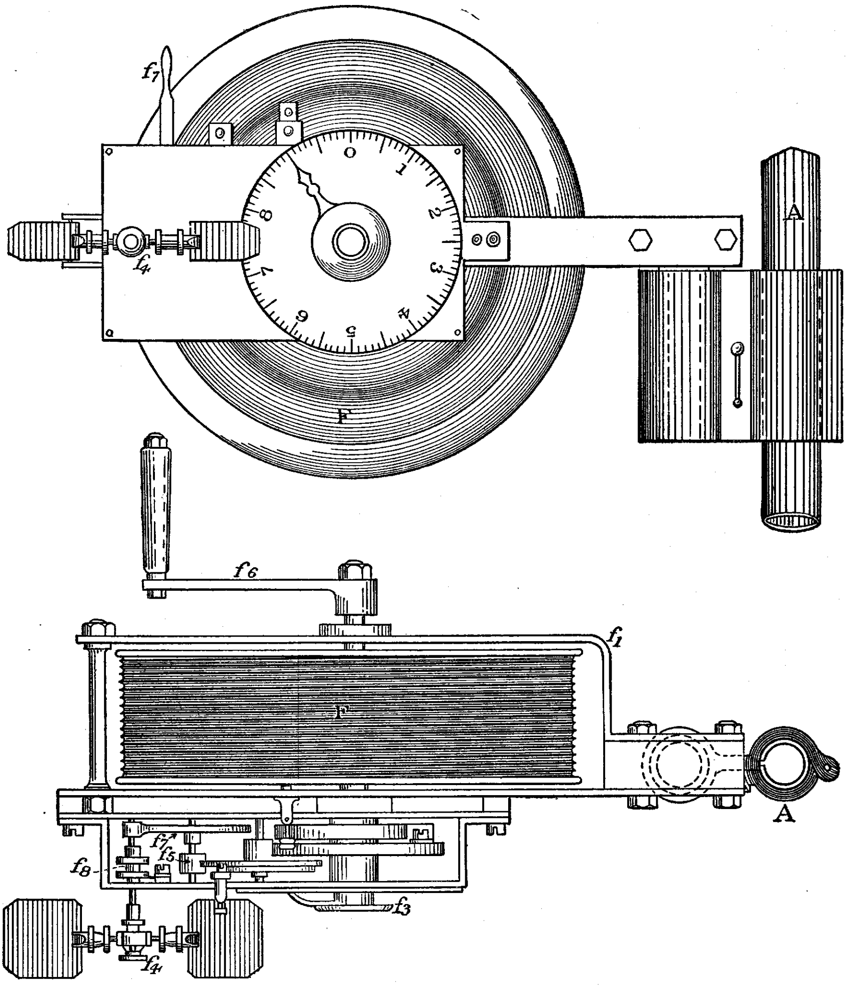

FIG. 13.—Details of Harlacher's apparatus for moving a meter with uniform velocity.

at the surface, at mid depth, and at the bottom in certain verticals, and the discharge be found from these (see Moore's method, p. 45); or it may be held at the surface and the bottom only in several verticals, and the mean velocity be found from these; or it may be held at three-tenths, at six-tenths, or at mid depth in certain verticals, and the mean velocity be found by applying a factor. The three-tenths-depth point[a] is used because it is said to be the point of maximum velocity in a vertical, and a small error in the position of this point will not affect the result much. The mid-depth point[b] is employed because the factor which is used to obtain the mean velocity is more nearly constant for it than for any other point on a vertical curve. The six-tenths-depth point[c] is used because it is believed to be the depth of the thread of mean velocity.

In the integration method the meter may be moved down several times in certain verticals only, as used by Harlacher,[d] or across the stream at a given depth, as used by Price on the Mississippi River at Carrollton, La.,[e] or diagonally across from one side to the other at the same time that it is moved from the surface to the bottom several times, as used by the writer and others. Harlacher's apparatus for moving a meter with a uniform velocity is shown in figs. 12 and 13. The meter *B* is moved up or down on the iron tube *AA* by the rope *D* attached to the drum *F*. *E* is a bracket holding the pulley *e*. *G* is an arm supporting the drum *F*, and f_3 (fig. 13) is a dial on which is registered the depth. The fan f_4 (fig. 13) and the gearing f_5 regulate the rate of motion of the meter; f_6 is a crank lever for raising the meter, and f_8 a ratchet wheel for arresting the motion. *I* (fig. 12) is an electric battery, and *H* (fig. 12) is the registering apparatus.

When accurate results are desired the meter should be held with a rod, if the depth be not too great, in which case it is not free to turn sidewise, but keeps its axis parallel to the axis of the stream and can be placed close to the bottom. Ordinarily it is held with an insulated wire, and is free to take the direction of the water at the point where it is held.

COEFFICIENT WORK.

Coefficient work in river gaging consists of obtaining the ratio of the velocity at any point in the depth of the stream (called the index point), as, for example, the three-tenths-depth point, to the mean velocity in the whole depth. When this ratio or coefficient is known for a vertical, the mean velocity in that vertical can be found by making an observation at the index point and applying the coefficient.

Let C_1, C_2, C_3, etc., be the coefficients for the component parts, F_1, F_2, F_3, etc., of the discharge area F; V_1, V_2, V_3, etc., the observed velocities at the index points of these areas; and V the mean velocity for the whole discharge area. The discharge $Q = FV = C_1F_1V_1 + C_2F_2V_2 + C_3F_3V_3 + \ldots$ (1).

A value of each of these coefficients can be found for each foot variation of river stage. The discharge of the stream is then computed by equation (1) from observed values of V_1, V_2, V_3, etc., at the

[a] Jour. West. Soc. Eng., Vol. III, No. 3.
[b] Report on Current Meter Observations, Burlington, Iowa, by Maj. A. McKenzie, 1884.
[c] Proc. Inst. Civ. Eng., Vol. LXXI, p. 60.
[d] Ibid., Vol. LXVII, p. 358.
[e] Jour. West. Soc. Eng., Vol. III, No. 3.

index points, using the set of values of the coefficients which correspond with the observed river stage.

Coefficient work can be done with one current meter, but it can be done more rapidly with two or more meters. When two meters are used, one is placed at the index point and the other is held for several minutes at as many points in the vertical as desired, and simultaneous readings of the two meters are obtained at each point. The two meters are occasionally held at the same depth and relative readings of the meters are obtained. A chart can be prepared from the ratings of the meters in still water which will give the velocity ratio from the simultaneous readings of the two meters. For example: The index meter at three-tenths depth indicates 2.34 revolutions per second. The other meter, at eight-tenths depth, indicates 2.05 revolutions per second. The chart shows the velocity ratio for these meter speeds to be 0.89. From the velocity ratio at each tenth of the depth the ratio of the mean velocity in the vertical to the index velocity, or C, is found.

This coefficient method of gaging is better and more accurate for large streams than the slower one in which velocity observations are made at several points in each vertical, on account of the rapid changes of surface elevation of the stream.

RATING OF METERS AND CONSTRUCTION OF RATING TABLES.

The accuracy of a discharge measurement depends largely on the accuracy of rating the meter used. Errors of observation are as likely to be too large as too small, and are compensating. Errors in a rating table always have the same sign and are cumulative, hence they should be eliminated or reduced to a minimum.

There are two kinds of rating of a current meter, absolute and relative. From the former the absolute velocity at a single point is found; from the latter the ratio or coefficient of velocity at any two points is found. The former is usually employed in still water, the latter in running water.

Four methods have been used for rating meters: (1) By the use of surface floats; (2) by moving the meter through still water with a known velocity; (3) by observing velocity at many points in a cross section and comparing this with the known mean velocity; and (4) by attaching the instrument to a long arm and revolving it about a vertical axis in a body of still water. The second method is the one now universally used. The meter is suspended from a car or a boat and is moved with a uniform velocity through still water at a depth of 2 or more feet below the surface. The length of a run varies from 100 to 300 feet, with a starting run sufficient to allow the meter wheel to reach the proper speed before entering the run. It is moved across and back over the run to eliminate the effect of velocity in either direction. The rating equipment should be such as to give time of run to the tenth of a second and speed of meter to the tenth of a revolution (a chronograph recording time, revolutions, and the instants of beginning and ending the run should meet this requirement).

The range of velocities employed in rating should be those for which the meter is to be used. If it is to be employed mainly to measure low velocities the speeds in rating should be low. The rating table should not be extended beyond the limits of velocity used in the rating.

The relation between distance passed over by the meter and the corresponding revolutions of the meter wheel is often assumed to be a straight line. It is always a curved line, and must be so on account of friction of bearings and inertia of moving wheel. A velocity of from 0.1 to 0.5 foot per second is necessary to start the wheel. As the velocity increases the effect of friction and inertia becomes proportionately less and the curve approaches a straight line. The relation between velocity and revolutions of wheel per second is of the second degree and concave to the axis of velocity. For velocities of 2 or more feet per second this curve differs little from a straight line.

The results of the rating are usually plotted on squared paper, using revolutions per second as ordinates and velocity as abscissæ, and all observations that fall much from a well-defined line are rejected. If this line be nearly straight between the limits of the observed revolutions its equation is assumed to be of the form $y=a+bx$, y being velocity, x revolutions per second, and a and b constants. If the results indicate a curved line its equation is assumed to be of the form $y=a+bx+cx^2$. There are two methods of preparing the rating table from the observations. The first method is rapid. The most probable curve is drawn among the plotted observations, and the velocities to tenths of a revolution are read from the curve. In the second method the equation of the most probable relation is derived by the method of "least squares" and then the table is prepared from the equation. The writer has found it better to plot values of x and the ratio y to x (see fig. 29, p. 89) instead of x and y. The reason is that the ratio y to x changes rapidly as x increases and can be read more accurately from this curve than from a nearly straight line. It can be shown mathematically that these curves are approximately branches of hyperbolas.

The second method, which is to derive the equation of the most probable relation between velocity and revolutions by the method of "least squares," first rejecting the observations that fall much outside of a well-defined curve, is the most accurate, but it requires the most time. If the meter is to measure low velocities to within 1 per cent, the second method must be used. It gives n observation equations of the form $y-a+bx=v$ if the relation be linear, or $y-a-bx-cx^2=v$ if it be a parabolic curve. v is a small quantity called a residual, or a residual error. Its value would be zero if the observations were perfect. According to the principle of least squares, the best values of the constants are those that make the sum of the squares of the residuals a minimum.

Squaring each of these n equations and adding, we have

$$\Sigma v^2 = v_1^2 + v_2^2 + v_3^2 + \ldots = [y' - a - bx' - cx'^2]^2 + [y'' - a - bx'' - cx''^2]^2 + [y''' - a - bx''' + cx'''^2]^2 + \ldots$$

Taking the first derivative of this equation with respect to a, b, and c and placing these equal to 0, we have

$$d\frac{(\Sigma v^2)}{da} = -2\,[y' - a - bx' - cx'^2] - 2\,[y'' - a - bx'' - cx''^2] \ldots = 0$$

$$d\frac{\Sigma v^2}{db} = -2x'\,[y' - a - bx' - cx'^2] - 2x''\,[y'' - a - bx'' - cx''^2] \ldots = 0$$

$$d\frac{\Sigma v^2}{dc} = -2x'^2\,[y' - a - bx' - cx'^2] - 2x''^2\,[y'' - a - bx'' - cx''^2] \ldots = 0$$

From these normal equations the values of a, b, and c can be found.

For the linear function these equations become two in number and have the form

$$a\Sigma x + b\Sigma x^2 = \Sigma xy$$
$$na + b\Sigma x = \Sigma y$$

and a and b are found from these equations.

This relation between revolutions per second and velocity, whatever it may be, is not constant. Below are given the ratings of a Price meter, obtained by J. C. Bailey, United States assistant engineer.[a]

Results of four ratings of a Price meter used in 1892 on Niagara River.

Date.	Number of observations.	Velocity.	Meter coefficients.		Remarks.
			a	b	
		Ft. per sec.			
April 28	20	2.5 to 7.3	4.239	+0	Still water; base 150 feet long.
April 29	34	2.1 to 6.7	4.256	+0.061	Do.
May 4	49	1.7 to 7.7	4.046	+0.207	Do.
May 17	43	2.0 to 8.0	4.130	+0.067	Still water; base 300 feet long.
			4.168	+0.0848	

Values computed from two ratings of current meter on April 29 and May 4, 1892.

[April 29, $V_1 = 4.256\,R + 0.061$; May 4, $V_2 = 4.046\,R + 0.207$.]

R	V_1	V_2	$V_2 - V_1$	$\frac{V_2 - V_1}{V_2}$
0.05	0.274	0.409	0.135	0.314
0.10	0.487	0.612	0.125	0.204
0.15	0.699	0.814	0.115	0.141
0.20	0.912	1.016	0.104	0.102
0.25	1.125	1.219	0.094	0.077
0.30	1.338	1.421	0.083	0.058
0.35	1.551	1.623	0.072	0.044
0.40	1.763	1.825	0.062	0.034
0.45	1.967	2.028	0.052	0.026
0.50	2.189	2.230	0.041	0.018

[a] Engineering News, Vol. XXIX, 1893.

The foregoing table shows that if on May 4 velocity measurements had been made from 0.05 to 0.50 revolution per second and the rating table of April 29 had been used the results would be in error from 1.8 per cent to 31.4 per cent. It is true that neither set of values of a and b was determined for low velocities. This, however, emphasizes the fact that values of a and b found for high velocities may be greatly in error for the low velocities if the linear relation is assumed. These four ratings of a meter in less than one month show considerable variation in the values of a and b, and indicate that when a meter is in constant use its rating should be examined once a week if accurate work is required.

A rating table that gives velocity to only two places of decimals is not sufficiently accurate for low velocities. To illustrate this we will take the case of the rating table of small Price meter No. 363, rated at Chevy Chase, Md., June 26, 1900. The velocity corresponding with 0.05 revolution per second is given as 0.17 foot per second. This may mean any value from 0.166 foot per second to 0.174 foot per second. This is 2.04 per cent of 0.17. Hence if the computed revolutions were either 0.166 or 0.174 the error introduced by using this rating table would be 2.4 per cent. The following table shows the error for other velocities given in this rating table. If the velocity be 0.25 foot per second it may be in error nearly 2 per cent if found from this table.

Table showing greatest percentage error in rating table of small Price meter No. 363, which gives velocity to only two places of decimals.

Revolutions per second.	Velocity.	Error $\frac{0.004}{V}$.	Revolutions per second.	Velocity.	Error $\frac{0.004}{V}$.
	Ft. per sec.	*Per cent.*		*Ft. per sec.*	*Per cent.*
0	0.06	6.7	0.25	0.63	0.6
0.05	0.17	2.4	0.30	0.75	0.5
0.10	0.29	1.4	0.35	0.86	0.5
0.15	0.40	1.0	0.40	0.98	0.4
0.20	0.52	0.8	0.45	1.09	0.4

Darcy and Bazin employed the first three methods to rate the Pitot tube which they used in their hydraulic investigations on canals.[a] Fig. 2, page 19, shows this instrument as they used it. A discussion of the results of the ratings is given here on account of the light they throw on accuracy tests.

[a] Recherches hydrauliques, by Darcy and Bazin, 1865, pp. 63-70.

Results of Pitot tube ratings by Darcy and Bazin.

First method.[a]			Second method.[b]			Third method.[c]					
No. of experiment.	Value of coefficient.	Number of experiments.	Velocity of boat.	Value of coefficient.	Number of experiments.	Designation of experiment. Series.	Designation of experiment. No.	Value of coefficient.	Mean Velocity.	Channel. Shape.	Channel. Width.
			Meters.						*Meters.*		*Meters.*
2	0.987	8	0.609	1.040	2	51	1	1.021	0.508	(d)	0.8
3	1.024	8	0.692	1.053	1	51	2	1.010	0.768	(d)	0.8
4	0.981	8	0.785	1.032	3	51	3	1.018	1.028	(d)	0.8
5	1.013	8	0.938	1.033	1	52	1	1.029	0.376	(d)	0.48
6	1.006	8	0.980	1.040	3	52	2	1.000	0.542	(d)	0.48
7	0.988	8	1.120	1.015	1	52	3	1.006	0.694	(d)	0.48
8	1.012	8	1.231	1.028	1	58	4	1.005	1.429	(d)	1.994
9	1.008	4	1.333	1.032	1	59	4	0.969	2.318	(d)	1.994
10	1.008	4	1.385	1.048	1	60	2	0.968	2.571	(d)	1.994
11	1.009	4	1.470	1.029	1	61	4	0.966	1.267	(e)	1.994
12	1.007	4	1.500	1.040	1	62	4	0.965	1.979	(e)	1.994
13	1.015	4	1.611	1.033	2	63	3	0.988	2.199	(e)	1.994
14	1.039	4	1.661	1.027	1	64	1	0.996	0.856	(e)	1.994
15	0.994	4	1.775	1.042	1	64	2	0.988	0.948	(e)	1.994
16	1.007	4	1.819	1.031	3	65	4	0.978	1.511	(e)	1.994
17	1.023	4	1.863	1.027	3	66	1	1.015	1.464	(e)	1.994
			1.930	1.039	1	66	2	0.984	1.675	(e)	1.994
			1.976	1.025	4	68	4	0.976	1.497	(f)	1.984
			2.034	1.037	1	71	6	0.968	1.810	(g)	1.40
						72	5	1.017	1.679	(g)	1.40
						73	4	0.967	1.612	(g)	1.40
						74	4	1.014	1.229	(g)	1.40
						84	2	1.014	1.033	(h)	1.990
						84	3	0.984	1.246	(h)	1.990
						85	1	0.998	1.011	(h)	1.990
						85	2	0.997	1.218	(h)	1.990
						85	3	0.990	1.473	(h)	1.990
						88	1	0.995	0.894	(h)	1.990
						88	2	0.986	0.921	(h)	1.990
						88	3	0.980	0.955	(h)	1.990
						88	4	0.993	1.010	(h)	1.990
	1.006			1.034				0.993			

[a] By surface floats.
[b] In still water.
[c] In moving water.
[d] Rectangular covered wooden channel.
[e] Rectangular open wooden channel covered with cloth.
[f] Trapezoidal open wooden channel covered with cloth.
[g] Semicircular.
[h] Rectangular.

In the first method the floats were run over a distance of 40 meters and the tube was held with its point 0.02 meter below the surface of the water. C is found from $v=C\sqrt{2g\frac{\Sigma d}{2}}$, v being known from the floats. In the second method the tube was fastened in front of a boat and was drawn through still water at different observed velocities. In the third method the instrument was held at many points in a cross section of an experiment canal 2 meters wide, the mean velocity in which was known, and the mean velocity as shown by the tube was computed.

It is seen that the three values of C found by these three methods differ somewhat, although each is the mean of many observations. The value found by the second method (the one now commonly used) is the largest of the three and is rejected on account of the action of the boat on the instrument. The pull on the boat as it was drawn through the water tipped the point of the tube down slightly, making its indications less than they should be and making C too large. By the first method the mean of experiments Nos. 2 to 8 is 1.0016 and the mean of experiments Nos. 9 to 17 is 1.0122. The difference between these is nearly 1 per cent of the former. The greatest variations from the mean of all the observations by the first method are +1.8 per cent and −2.5 per cent. The value of C found by the third method is 0.993, obtained from thirty-one measurements of discharge of the experiment canal. The tube was held at many points in the cross section. The variations from this mean are +3.6 per cent and −2.8 per cent. Darcy uses the mean of the values found by the first and third methods, which makes $C=1$. The extreme variations from this mean are +3.9 per cent and −3.5 per cent. The velocity of the water during these ratings varied from 0.5 to 2.5 meters per second, or from 1.64 feet to 8.20 feet per second.

GENERAL FORMULÆ FOR COMPUTING DISCHARGE.

Whatever method is used in computing discharge, the depth must be known at a sufficient number of points in the cross section to determine the cross-sectional area with the required degree of accuracy. For a small stream it should be known at each foot of width, for large streams at each 5 or 10 feet of width.

The cross-sectional area (F) is found from the following formula:

$$F=\frac{b}{-}\left[d_1+4\,(d_2+d_4+d_6+d_{n-1})+2(d_3+d_5+d_7+dn)+dn+1\right]$$

d_1, d_2, d_3, etc., being depths and b the distance between the points where the depths are measured.

The general formula for the discharge per second is

$$Q=F_1\,V_1+F_2\,V_2+F_3\,V_3+F_n\,V_n. \qquad (1)$$

F_1, F_2, F_3, etc., are the component parts of the cross section, and V_1, V_2, V_3, etc., the mean velocity in each of these parts. If the width of these component parts is b and the mean depth in each is d_1, d_2, d_3, this formula can be written

$$Q=b\,(V_1d_1+V_2d_2+V_3d_3+\text{——}). \qquad (2)$$

If $d_1=d_2=d_3$—, that is, if the channel is rectangular in cross section, then formula (2) becomes

$$Q=bd\,(V_1+V_2+V_3+\text{——}). \qquad (3)$$

The component velocities are found (1) from a single observation at some depth by applying a coefficient, as a mid-depth velocity observation multiplied by 0.95, or a six-tenths-depth observation multiplied by unity, and (2) from a vertical velocity curve. The latter, as already explained, is found from numerous observations at points in the depth, and the mean velocity in this vertical is then the area inclosed between the vertical curve and a vertical line representing no velocity divided by the depth. Harlacher's method of integrating [a] in a vertical gives each component velocity very readily and quickly. Any component velocity is $\frac{RC}{T}$, R being the observed revolutions, T the observed time in seconds, and C the number, which changes from revolutions to velocity.

In measuring the discharge of a large river the component velocities are always found from an observation at some given depth and the use of a coefficient. As the depth or stage of a river is almost constantly changing, it is necessary to make the velocity observation in a short time—one or two hours at most. The value of the coefficient to use to reduce observed velocity at any depth to mean velocity in that vertical should be found from measurement if possible.

The discharge measurements of the Niagara River made in 1897 and 1898 by the United States Board of Engineers on Deep Waterways [b] from the international bridge 3 miles below the head of the river, were computed from velocity observations at three-tenths depth, using coefficients whose values were found from vertical velocity curves. The meter stations were about 80 feet apart and were at the middle of each half span of the short spans and at the middle of each one-third span of the long spans. The value of the coefficient for reducing observed velocity to mean velocity ranged from 0.61 to 1.17, being influenced by the piers of the bridge.

T. V. Moore in his discharge measurements of the Thames [c] assumed the vertical velocity curve to be a parabola, and computed its area from the formula $A=\frac{d}{6}[V_s+4V_m+V_b]$, d being the depth, V_s the surface velocity, V_m the velocity at mid depth, and V_b the bottom velocity. The discharge per second flowing between the end sections he computed from the formula $V=\frac{h}{3}[\Sigma A_1+4\Sigma A_2+2\Sigma A_3]$, h being the distance between the consecutive sections, ΣA_1 the sum of the two end sections, ΣA_2 the sum of the odd sections, and ΣA_3 the areas of the even sections. The discharge is found by adding to this the small volume flowing between each end section and the shore.

T. G. Ellis, in computing the discharge of the Connecticut River in 1874, multiplied the velocity at mid depth in each component area by 0.95 to find the mean velocity in that area.[d] In computing the discharge of the Mississippi River in 1882 the velocity at mid depth in each component area was multiplied by 0.91 for the observations taken when the river was covered with ice, and by 0.96 when free from ice, to reduce the mean velocity in a vertical.[e] From 1885 to 1887 the velocity at six-tenths depth was observed in the component areas, and this was taken as the mean velocity in each area.

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[a] Proc. Inst. Civ. Eng., Vol. LXVII, p. 358.
[b] Jour. West. Soc. Eng., Vol. IV, No. 6.
[c] Proc. Inst. Civ. Eng., Vol. XLV, p. 220.
[d] Trans. Am. Soc. Civ. Eng., Vol. IV, p. 303.
[e] Jour. West. Soc. Eng., Vol. III, No. 3.

Harlacher's method of computing flood discharge where the surface velocity only could be measured is illustrated in fig. 14.[a] The surface velocity was measured at a sufficient number of points (1, 2, 3, etc.) in the cross section *ACB* to determine the surface velocity curve *ADB*. The line *ACB* shows the depth at each point and is plotted from soundings taken immediately before or after the flood.

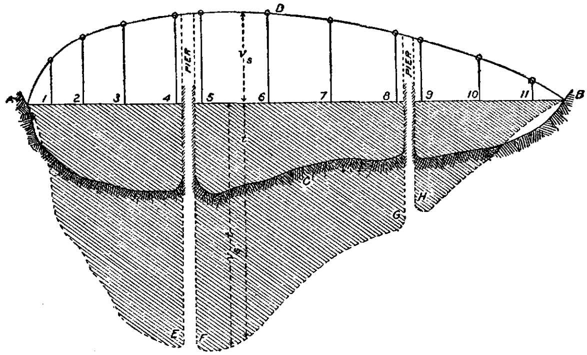


FIG. 14.—Diagram illustrating Harlacher's method of computing flood discharge.

The openings between *EF* and *GH* are bridge piers, the measurements being made from a bridge. The product of the surface velocity v_s and the corresponding depth t is found for a sufficient number of points, depending on the irregularity of the bottom and the curve *AEFGHB* drawn. The discharge per second is the area between the line *AB* and the curve *AEFGHB*, less the piers, multiplied by 0.85.

COMPARISONS OF RESULTS WITH DIFFERENT INSTRUMENTS AND METHODS.

SURFACE FLOATS AND PITOT TUBE.

On page 43 we have given the results of three ratings of a Pitot tube by Darcy and Bazin.[b] In the first method the surface velocity was found from surface floats and a Pitot tube, as already described. Ninety-two experiments were made. The results are combined, according to velocity, into seven groups of eight experiments each and nine groups of four experiments each. The value of the coefficient as found from the first seven groups differs from that found from the mean of the last groups by about 1 per cent. The extreme variations of these individual means from the mean of all the experiments are — 1.8 per cent and + 2.5 per cent. The individual values of the coefficient are not given. Their extreme variation from the mean of all must be at least twice that of the individual means from the general mean.

[a] Proc. Inst. Civ. Eng., Vol. XCI, p. 399.

[b] Recherches hydrauliques, by Darcy and Bazin, 1865, pp. 63-70.

PITOT TUBE AND RECTANGULAR ORIFICES.

On page 43 are given the results of a third rating of a Pitot tube by Darcy and Bazin (third method). This is also a comparison of the discharge of a canal 2 meters wide, as found from rectangular orifices and as found from numerous velocity measurements in a cross section of the canal with the same Pitot tube. The discharge of the rectangular orifices admitting water to the canal was determined by experiment for different heads on them. The number of velocity measurements in each experiment or discharge measurement varied with the depth of the water in the canal, being from twenty-seven in the smallest channel, which was 0.30 meter by 0.48 meter, to ninety-one in the canal 2 meters wide. The points of measurement were nearest together where the velocity changed most rapidly. The results of thirty-one discharge measurements are given, the mean velocity varying from 0.38 meter to 2.57 meters per second. The extreme variations from the mean of all the values of the coefficient are + 3.6 per cent and − 2.8 per cent.

DOUBLE FLOATS AND CURRENT METER.

ELLIS'S EXPERIMENTS.

In 1874 T. G. Ellis made numerous velocity measurements of the Connecticut River at Thompsonville, Conn., with current meters and with double floats like that shown in fig 1.[a] Vertical velocity curves were obtained in planes parallel with the axis of the river and 100 feet apart, with the meters and with the floats. Each of these curves was plotted on cross-section paper and divided into tenths of the depth. All of the curves in which the mean velocity was less than 1.86 feet were combined by combining all the observations at each tenth of depth; all in which the mean velocity was more than 1.86 feet were combined in the same way. From the resultant vertical velocity curves the results in the following table are taken:

Comparison of velocity by double floats and current meters from measurements by T. G. Ellis on Connecticut River in 1874.

Observations.	Mean velocity, in feet per second, at various depths.											Mean.	Variation.
	Surface.	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	Bottom.		
													P. ct.
Meter below av. velocity 1.86	1.15	1.19	1.22	1.22	1.19	1.14	1.11	1.03	0.94	0.79	0.46	1.063	25.8
Float below av. velocity 1.86	1.50	1.52	1.52	1.53	1.49	1.46	1.35	1.29	1.20	0.97	0.58	1.337	
Meter above av. velocity 1.86	2.96	3.08	3.12	3.06	2.99	2.91	2.78	2.62	2.42	2.16	1.47	2.735	5.7
Float above av. velocity 1.86	3.26	3.24	3.17	3.17	3 11	3.01	2.94	2.80	2.60	2.35	1.78	2.891	
All meter observations	2.05	2.12	2.15	2.13	2.08	2.01	1.93	1.81	1.67	1.47	0.96	1.887	12.0
All float observations	2.38	2.38	2.35	2.35	2.30	2.23	2.15	2.04	1.90	1.60	1.18	2.114	

[a] Rept. Chief Eng. U. S. A., 1878, Appendix B.

It is seen from the foregoing table that the velocity as found with the meter is from 6 to 26 per cent less than that found with the floats, and that the difference between the meter and float velocities increases as the velocity decreases.

Ellis gives also the following values of the ratio of mean velocity in a vertical (Vm) to mid-depth velocity in that vertical ($V_{\frac{1}{2}} D$) and the ratio of distance below the surface of mean velocity (m) to total depth (D).

Mean ratios from vertical velocity curves obtained by Ellis on Connecticut River in 1874.

Observations.	$Vm \div V_{\frac{1}{2}}D$	$m \div D$
Meter at low velocities	0.933	0.656
Floats at low velocities	0.918	0.622
Mean of low velocities	0.930	0.652
Meter at high velocities	0.939	0.628
Floats at high velocities	0.961	0.637
Mean of high velocities	0.944	0.629
Mean of meter measurements	0.937	0.638
Mean of float measurements	0.947	0.632
Mean of all measurements	0.940	0.636

From the foregoing table it is seen that $\frac{Vm}{V_{\frac{1}{2}}D}$ varies from about 0.92 to 0.96 and $\frac{m}{D}$ from 0.62 to 0.66—about the same range of values.

MARR'S EXPERIMENTS.

In October, 1879, G. A. Marr made simultaneous velocity measurements of the Mississippi River at Burlington, Iowa, with double floats like those shown in fig. 1 and with five current meters.[a] The river where measured was about 2,000 feet wide, and the path of the floats was near the center, where the depth was about 16.4 feet. Ten floats were run in succession in a period of about twenty-five minutes over a distance of 200 feet, and the time of entering and leaving each quarter of the run was recorded on a chronograph of eight pens. The five current meters were fastened to a cable held vertically 16 feet below the end of the run. One meter was at mid depth (the depth of the lower float), and the other four were arranged two above and two below it, so as to divide the depth into equal parts. One pen of the chronograph recorded the time of passage of each float, two recorded the time as shown by a chronometer, and the other five recorded the revolutions of the five meters. From a table giving the velocity of each float over each quarter of the run and the velocity shown by each of these meters during the time of passage over these distances the following table has been prepared:

[a] Report on Current Meter Observations, Burlington, Iowa, by Maj. A. McKenzie, 1884.

Comparison of velocity by double floats and current meters from measurements by G. A. Marr on Mississippi River at Burlington, Iowa, October, 1879.

Number of float.	Mean velocity found from 200-foot run.		Difference ($Vm-Vf$).	Mean velocity found from 50-foot run.		Difference ($Vm-Vf$).
	Float (Vf).	Meter (Vm).		Float (Vf).	Meter (Vm).	
	Ft. per sec.	*Ft. per sec.*	*Ft. per sec.*	*Ft. per sec.*	*Ft. per sec.*	*Ft. per sec.*
1......	2.543	2.604	+0.061	2.660	2.597	−0.063
2......	2.741	2.546	−0.195	2.976	2.657	−0.319
3......	2.634	2.520	−0.114	2.674	2.630	−0.044
4......	2.446	2.435	−0.011	2.456	2.481	+0.025
5......	2.850	2.468	−0.382	2.847	2.590	−0.257
6......	2.672	2.700	+0.028	2.778	2.649	−0.129
7......	2.514	2.545	+0.031	2.674	2.484	−0.190
8......	2.832	2.457	−0.375	2.857	2.749	−0.108
9......	2.476	2.625	+0.149	2.569	2.821	+0.252
10......	2.829	2.758	−0.071	3.012	2.823	−0.189
	2.654	2.566	....	2.750	2.648	....

In the foregoing observations the velocity of a float was not the same over each 50-foot length of the run. Each velocity given in the second column is a mean for the whole run of 200 feet, and the meter velocity in the third column is for the time of the full run. The velocity in the fifth column is for the fourth section of the run, and the corresponding number in that column is the meter velocity for the next consecutive and equal interval of time. The chief thing to be noticed is that the float velocity, whether computed from the whole run of 200 feet or from the shorter run of 50 feet, is greater than the meter velocity by about 3.5 per cent. We may note also that we have here forty mid-depth velocities in nearly the same vertical taken in about twenty-five minutes. For the ten found from the floats on the long run the variations of velocity from the mean are from −7.4 per cent to +7.8 per cent, and for the short run of 50 feet the variations are from −9.5 per cent to +10.7 per cent. For the meter long run the variations are from −7.5 per cent to +12.1 per cent, and for the short run from −6.5 per cent to +6.3 per cent.

Individual measurements of velocity by either of these methods differ much from the mean, but the means by the two methods differ little from each other. The excess of mid-depth velocity by double floats over that by meter may be due to the upper float dragging the lower one.

HENRY'S EXPERIMENTS.

D. F. Henry also gives a comparison of velocity measured with double floats like that shown in fig. 1 and current meter, made on the St. Clair River in 1869.[a] The results of the comparison are given in the following table:

[a] Jour. Franklin Inst., Vol. LXII, p. 322.

Comparison of velocity by double floats and current meter from measurements by D. F. Henry on St. Clair River in 1869.

Number of observations.	Direction and velocity of wind.	Depth of observation below surface.	Velocity of current.		
			Floats.	Meter.	Difference.
	Miles per hour.	*Feet.*	*Ft. per sec.*	*Ft. per sec.*	*Ft. per sec.*
50	3.26 up	1	3.619	3.655	−0.036
62	1.92 up	5	3.759	3.783	−0.024
56	1.27 up	10	3.703	3.674	+0.029
50	0.48 up	15	3.590	3.516	+0.074
54	0.29 down	20	3.598	3.405	+0.193
31	0.53 up	25	3.637	3.441	+0.196
37	0.80 up	30	3.546	3.279	+0.267
29	2.18 down	35	3.556	3.166	+0.390
12	4.85 down	40	3.636	3.142	+0.494
7	0.74 up	45	3.542	2.985	+0.557

It is seen that the float velocity is less than the meter velocity to a depth of about 5 feet. Below that depth the float velocity is greater than the meter velocity, the difference increasing with the depth. The discharge as found with the floats is 10 per cent greater than that found with the meter. While the action of the upper float on the lower float may not account for all the difference between the float and meter velocities, it will account for some of it. The velocity as shown by the vertical velocity curve increases from the surface to a depth of from 5 to 10 feet and then decreases to the bottom. The surface float will therefore move more slowly than the lower float for depths of from 0 to 5 or 10 feet, and hence the lower float will be retarded by the upper float to that depth. For depths greater than 10 feet the upper float will move faster than the lower float and will drag it, and the velocity shown will be too large.

Henry gives also a comparison of velocity as found with double floats and current meter in a small canal at Ogdensburg, N. Y.[a] The floats were run over a distance of 200 feet at 3 feet below the surface, the time being recorded on a chronograph. The current meter was held at the same depth at the middle of the run. The results are as follows:

Comparison of velocity by double floats and current meter from measurements made by D. F. Henry in small canal at Ogdensburg, N. Y., 3 feet below surface.

Number of observations.	Velocity.		
	Floats.	Meter.	Difference.
	Ft. per sec.	*Ft. per sec.*	*Ft. per sec.*
24	1.992	1.980	−0.012
6	1.876	1.916	+0.040
6	1.476	1.434	−0.042

[a] Ibid., p. 259.

These results indicate that although the mean of a few float observations may differ much from the corresponding meter velocity, the means for thirty or forty observations differ little. The meter gives a little smaller velocity than the floats.

Henry also gives a comparison of velocity found with floats and a propeller meter in the St. Clair River. The lower float was 1 foot below the surface, and the meter was held at the same depth. The mean velocity given by fifty floats was 3.619 feet per second; that by meter was 3.655 feet per second, or 0.036 foot per second greater than by floats. There was a light wind blowing upstream, and that may have retarded the floats somewhat.

GORDON'S EXPERIMENTS.

In 1873 Gordon made comparisons of the discharge of the Irawadi River at Saiktha, Burma, as measured with double floats and with current meter. From these he concluded that the double-float discharge was about 10 per cent too large. He therefore reduced all discharges found with double floats by that amount.

FLOAT RODS AND CURRENT METER.

NEW YORK STATE CANAL SURVEY EXPERIMENTS.

During the progress of the New York State canal survey of 1900 comparative discharge measurements were made on four occasions for the purpose of comparing the discharge of the canal as found by one rod-measuring party with that found by another party using similar rods and methods, and of comparing the discharge found with rods with that found simultaneously with a current meter. The results of the comparisons are summarized in the following table:

Comparison of results of experiments of New York State canal survey during the summer of 1900.

Place.	Date.	$V'r$	$V''r$	Vm	$Q'r$	$Q''r$	Qm	$Q'r-Q''r \div Q'r$	$Q'r-Qm \div Q'r$	$Q''r-Qm \div Q''r$
	1900.	*Ft. per sec.*	*Ft. per sec.*	*Ft. per sec.*	*Cu. ft. per sec.*	*Cu. ft. per sec.*	*Cu. ft. per sec.*	*Per ct.*	*Per ct.*	*Per ct.*
Lockport	Sept. 14		1.440	1.531		805.70	799.85			+0.73
Rochester	Sept. 20		0.704	0.736		235.12	251.57			−7.00
Do	Sept. 21	0.893	0.884		297.82	302.04		−1.41		
Do	do		0.889	0.929		301.35	310.83			−3.14
Do	do		0.899	0.939		307.08	317.54			−3.46
Boonville	Sept. 25	1.073	0.813		227.22	227.51		−0.13		
Do	do		0.850	1.079		238.81	228.66			+4.25
Do	do		0.836	1.111		233.54	234.70			−0.49
Glens Falls	Oct. 8	0.821	0.690		164.73	172.77	161.50	−4.88	+1.94	+6.52
Do	do	0.756	0.640		149.66	158.57	155.23	−5.92	−3.72	+2.11

$V'r$=mean velocity found by party No. 1, with rods.
$V''r$=mean velocity found by party No. 2, with rods.
Vm=mean velocity found with meter, using six-tenths-depth method.
$Q'r$=discharge found by party No. 1, with rods.
$Q''r$=discharge found by party No. 2, with rods.
Qm=discharge found with meter, using six-tenths-depth method.

The Lockport experiments were made on the Erie Canal about 1 mile above the locks at Lockport, in a rock cut with rough, nearly vertical sides. The meter measurements were made 700 feet above the place where the discharge was measured with rods. The observations extended over a period of thirty-four minutes, during which time 60 rods were passed over a run of 21 feet, with an immersion of 80 to 90 per cent of the depth, and 13 meter observations of 50 seconds each were made at six-tenths depth below the surface. There was no measurable fluctuation of surface level during the test.

The Rochester experiments were made on the Rochester Aqueduct, which has smooth and nearly vertical sides. The width was 43 feet and the depth 7.5 feet. The flow was undisturbed for a half mile above the section of measurement, but there was a sharp curve less than 100 feet below the point where the lower rod-measuring party was stationed. In the first comparison of rods with meter, September 20, the period of observation was forty-two minutes; 64 rods were passed over a 20-foot run with 92 per cent depth of immersion; 19 meter observations of fifty seconds each were made by the six-tenths-depth method; the surface level fluctuated 0.05 foot during the observations, and rain interfered somewhat with the work. In the first rod comparison, September 21, the observations extended over a period of sixty-five minutes; 79 rods were passed by party No. 1, with a depth of immersion of from 90 to 97 per cent, and 100 rods by party No. 2, with a depth of immersion of 92 per cent. In the first rod-and-meter comparison of September 21 the period of observation was forty-two minutes; 92 rods were passed by party No. 2, with 90 per cent depth of immersion, and 36 meter observations of fifty seconds each were taken by the six-tenths-depth method. In the second rod-and-meter comparison the period of observation was forty minutes; 80 rods were passed, with 92 per cent depth of immersion, and 36 meter observations of fifty seconds each were made at six-tenths depth.

The Boonville comparisons were made on the Black River Canal at Boonville, where the top width of the canal is 43 feet, with gradual sloping sides, and a maximum depth at center of 7.7 feet. There was an unobstructed flow for at least one-fourth mile above and a half mile below the place selected. The parties were stationed about 200 feet apart. There was practically no water entering or leaving the canal between these points. In the rod comparison, which lasted one hundred and forty-four minutes, with some interruptions, 185 rods were passed over a 20-foot run by party No. 1, and 143 rods by party No. 2. In the first meter comparison, which lasted fifty-eight minutes, 55 rods were passed and 36 meter observations were made at six-tenths depth. In the second meter comparison, which lasted fifty-three minutes, 77 rods were passed and 42 meter observations of fifty seconds each were made by the six-tenths-depth method.

The Glens Falls comparisons were made on the Glens Falls feeder, near Glens Falls. At the place selected for the tests the feeder is 32 feet wide and 6 feet deep, with nearly vertical sides. There is a bend in the feeder about 100 feet above the upper place of measurement, and another about 1,000 feet below the lower place of measurement. The meter measurements were made from a bridge about 100 feet from each of the points where the rod measurements were made. The first comparison lasted seventy-six minutes, during which time party No. 1 passed 56 rods, party No. 2 passed 90 rods, and 28 meter observations of fifty seconds each were made. In the second comparison, which lasted fifty-eight minutes, party No. 1 passed 63 rods, party No. 2 ran 108 rods, and 28 meter observations of fifty seconds each were made.

The method of making these comparisons is the same as that used in the canal discharge measurements, which is described on page 16.

The Rochester and Glens Falls comparisons are not so satisfactory as those at Lockport and Boonville. In the former the measurements were made too near a curve. The large variation in the time of run of the individual rods indicates this fact, and surface-level fluctuations and passing boats interfered somewhat with the work. There is a possibility of leakage from the feeder between the points of measurement at Glens Falls. No leakage was visible on the surface, but there is a large amount of leakage not far distant from the place of measurement. The measurements there indicate a gradual loss from the upper to the lower point of measurement.

The results of the rod comparisons agree more closely than those of meter with rods. It must not be concluded from the Rochester comparisons that because the rod discharges agree closely and the meter discharge differs from the rod discharge 7 per cent that this error is all due to the meter. The two sets of velocity measurements obtained with the meter during this comparison agree closely, whereas the time of run of the individual rods differs considerably. At least half of this difference is due to errors in the rod discharge measurements.

MURPHY'S EXPERIMENTS AT CORNELL UNIVERSITY.

In connection with the writer's experiments at Cornell University to determine the relative accuracy of measurements by the weir and by the current meter, the results of which are discussed on pages 58 to 95, surface velocity experiments were made with float rods and with the current meter, the results of which are given on pages 89 to 92.

FLOAT RODS AND WEIR.

FRANCIS'S EXPERIMENTS.

In 1856 J. B. Francis compared the discharge of a canal at Lowell, Mass., as found with float rods with that shown by a standard weir.[a] This wooden canal or flume was 27.75 feet wide where the first 63 experiments were made and 14 feet wide where the remaining 52

[a] Lowell Hydraulic Experiments, by J. B. Francis, p. 170.

experiments were made. The length of run was 70 feet, with a starting run of 28 feet. The time of run was obtained to a tenth of a second and the depth to 0.0001 foot. Every precaution was taken to obtain accurate results. The floats were tin tubes 2 inches in diameter, loaded at the lower end so as to float upright. They were put in the water at each foot of width of the canal, and the time of run was recorded on a chronograph. The observed velocities were plotted on cross-section paper and a mean curve was drawn among the points showing change in velocity from one side of the canal to the other. The mean velocity in the cross section was found from this curve by dividing the velocity area by the width. The variation of the observed velocities of individual rods from the velocity curve was from 8.57 per cent above to 11.4 per cent below this curve in experiment No. 1. The rods used were not all of the proper depth of immersion, and Francis derived the following formula for computing the correct discharge: $Q'''=Q''[1-0.116(\sqrt{D}-0.1)]$, in which Q'' is the measured discharge and D the depth of water in the flume less the depth of the immersed part of the rod and divided by the depth of water in the flume. After applying this formula he found that the variation of the rod discharge from the weir discharge for the 115 experiments is as follows: For 9 it is less than 0.1 per cent, for 79 it is less than 1 per cent, for 23 it is from 1 per cent to 1.9 per cent, for 2 it is from 2 per cent to 2.9 per cent, and for 1 it is 3.69 per cent. The mean velocity in these experiments varied from 0.5 foot to 5 feet per second.

CORNELL UNIVERSITY EXPERIMENTS.

At the close of the field work of the New York State canal survey of 1900 a series of experiments was made at the hydraulic laboratory of Cornell University for the purpose of comparing discharge as found with rods and current meter on the canals with that shown by the Cornell standard weir. These experiments were planned by the consulting engineer of the New York State canal survey, Mr. E. Kuichling, and were carried out, under the direction of Prof. G. S. Williams, in charge of the hydraulic laboratory, by the writer and by Mr. W. P. Boright, C. E., assisted by members of the class in hydraulics.

The surface level of the sections of the canals fluctuates rapidly on account of lockage of boats and irregular feed of water to lower sections. It was therefore necessary to make discharge measurements rapidly and to make short runs and starting runs. The depth of water in the different sections varied from 5 to 10 feet at the center, so that the immersed portion of the rods used varied from 75 per cent to 95 per cent of the depth of the water. The mean velocity varied from nearly 2 feet per second west of Lockport to 0 on the summit levels. In the laboratory experiments the conditions were duplicated as closely as possible and the work was performed in the same way as on the canals, so that the results of the comparisons give a fair idea of the accuracy of the discharge measurements of the State canals and feeders.

Pl. II shows the laboratory canal and the observers employed in making the tests. A description of the laboratory and of the method of obtaining standard weir discharge will be found on pages 59 to 64. The gates at the head of the canal were opened until the head on the weir was such as to give the desired discharge; then the slits in the bulkhead and the height of the gates at the lower end of the canal were adjusted so as to give the desired depth of water in the channel. As soon as the flow became steady three parties began measuring the discharge of the canal, one with rods and two with current meters. Two series of rods were used, one of 75 per cent depth of immersion, the other of 90 per cent depth of immersion, the former lettered, the latter numbered, so that they could easily be recognized as they passed under each wire. The lengths of the run and of the starting run were the same as in the canal work, being from 25 to 7 feet for the former and from 10 to 8 feet for the latter, depending on the velocity. The number of rods used in an experiment varied from fifty to one hundred and fifty. Meter party No. 1 measured the discharge by the six-tenths-depth and the integration methods, and then assisted the rod-measuring party, four stop watches being used and the number of rods run in a given time nearly doubled. During each experiment the head on the weir, the depth of water in the canal, and the slope of the surface were read every thirty seconds and were checked occasionally by readings of the hook gage. The time of run of each rod was plotted on squared paper, using distance from side of canal as abscissæ and time of run, in seconds, as ordinates, and a mean time curve was drawn among them. The length of run divided by the mean ordinate of this curve gave the mean velocity, which multiplied by the cross-sectional area gave the discharge. The results of this rod-and-weir discharge comparison are given in the following table:

Comparison of float rod and weir velocity and discharge measurements at Cornell hydraulic laboratory.

Date.	Discharge over standard weir per second.	Rod gagings in laboratory canal.						Error of discharge referred to standard weir.		Mean error of discharge referred to standard weir.	
		Velocity from weir discharge per second.	Depth of water in canal.	Immersion, 75 per cent.		Immersion, 90 per cent.					
				Velocity per second.	Discharge per second.	Velocity per second.	Discharge per second.	75 per cent immersion.	90 per cent immersion.	75 per cent immersion.	90 per cent immersion.
1900.	*Cu. ft.*	*Feet.*	*Feet.*	*Feet.*	*Cu. ft.*	*Feet.*	*Cu. ft.*	*Per. ct.*	*Per. ct.*	*P. ct.*	*P. ct.*
October 24	214.588	1.428	9.356	1.422	213.674	....	....	+0.43	....	1.32	0.75
Do	140.514	0.945	9.253	0.962	142.966	0.949	141.034	−1.74	−0.37		
Do	71.670	0.484	9.222	0.490	72.621	0.476	70.546	−1.33	+1.57		
Do	38.308	0.269	8.789	0.276	38.990	0.272	38.425	−1.78	−0.31		
October 26	178.553	1.473	7.543	1.510	183.185	1.525	184.832	−2.59	−3.52	3.96	2.09
Do	125.298	1.033	7.549	1.080	131.000	1.058	128.332	−4.55	−2.42		
October 27	61.272	0.510	7.471	0.530	63.658	0.520	62.457	−3.89	−1.93		
Do	30.089	0.254	7.376	0.266	31.543	0.255	30.239	−4.83	−0.50		
Do	196.387	1.983	6.163	2.067	204.695	2.044	202.407	−4.23	−3.06	4.22	2.95
Do	132.408	1.308	6.299	1.363	137.956	1.341	135.730	−4.19	−2.51		
Do	91.056	0.910	6.228	0.953	95.372	0.944	94.471	−4.74	−3.64		
Do	50.309	0.535	5.847	0.555	52.176	0.549	51.612	−3.71	−2.59		
October 31	198.258	1.475	8.365	1.537	206.554	1.507	202.523	−4.18	−2.15	4.68	2.78
Do	125.299	0.930	8.393	0.958	129.174	....	....	−3.09	....		
October 29	65.287	0.486	8.355	0.512	68.766	0.501	67.288	−5.33	−3.07		
November 1	31.140	0.235	8.255	0.249	33.043	0.242	32.114	−6.11	−3.13		
										3.54	2.14

It is seen that the discharge obtained with the rods is larger for all depths, velocities, and percentage depths of immersion than the corresponding weir discharge, except in two cases. The greatest variation from the weir discharge is 6.11 per cent for the 75 per cent depth of immersion and 3.64 per cent for the 90 per cent depth of immersion. This variation from the weir discharge, or error, increases as the depth, velocity, and depth of immersion decrease. The mean error for all depths and velocities is 3.54 per cent for the 75 per cent depth of immersion and 2.14 per cent for the 90 per cent depth of immersion. The greatest variations from these means are +1.14 per cent and −2.22 per cent for the 75 per cent depth of immersion and +0.81 per cent and −1.39 per cent for the 90 per cent depth of immersion, a range of 3.4 per cent in the former and 2.2 per cent in the latter. From the last two columns is seen the extent to which the error increases as the depth decreases.

It is to be expected that the rod discharge will be greater than that of the weir, especially for the 75 per cent depth of immersion, for as the rods do not extend to the bottom they are not affected by the bottom layers of slowly moving water, and hence move faster than the mean velocity. By applying Francis's correction formula (p. 54) to these results others will be obtained agreeing more closely with those shown by the weir. It will be shown further on (p. 93) that the weir discharges used in these experiments and computed from Bazin's formula are probably too small by from 0.5 per cent to 2.5 per cent, the former for a head on weir of 1 foot, the latter for a head of 2.5 feet.

When these two sets of corrections are applied it will be found that the rod discharge is correct to within about 2 per cent. This degree of accuracy is only attained, however, by many observations, equivalent to six or twelve gagings, with the rods started 2 feet apart from side to side of canal.

CURRENT METER AND WEIR.

FTELEY AND STEARNS'S EXPERIMENTS.

Fteley and Stearns have made some comparisons of the discharge of the Sudbury conduit as obtained by weir measurement and by current meter.[a] The weir was of the sharp-crested type, 19 feet long, and was located at the head of the conduit. The flow to it was controlled by rectangular gates, which were changed slightly from time to time so as to keep the head on the weir constant. The current-meter measurements were made from a manhole in the conduit 6,000 feet below the weir. The meter used was a Fteley like that shown in fig. 5. Two methods of operating the meter were employed, viz, the point and the integration. When the point method was used the meter was held for thirty seconds at several points in the cross-sectional area, depending on the depth. For the greatest depth of 4.5 feet the number of points of observation was 167. The rate of moving the meter while integrating was from 0.1 foot to 1 foot per second. The results for the 8-vane meter are as follows:

Comparison of discharge measured with 8-vane Fteley meter and over weir.

No. of experiment.	Number of comparisons included in experiment.	Rate of integration.	Depth of center of meter from surface.	Ratio of speed of meter to speed of water.	Mean velocity of water.	Variation of meter discharge from weir discharge.
			Feet.		*Ft. per sec.*	*Per cent.*
1	1	0	1.5	0	1.83	+1.2
2	2	0	2.0	0	2.13	0.0
3	1	0	3.0	0	2.56	+0.1
4	1	0	4.0	0	2.83	+0.1
5	1	0	4.5	0	2.93	—0.4
6	1	0	3.0	0	1.71	—0.2
7	3	0.1	1.5	0.055	1.83	+0.8
8	3	0.1	2.0	0.047	2.13	+0.1
9	8	0.1	2.5	0.042	2.37	—0.6
10	3	0.1	3.0	0.039	2.56	—0.2
11	6	0.1	3.5	0.037	2.71	—0.1
12	4	0.1	4.0	0.035	2.83	—0.5
13	4	0.1	4.5	0.034	2.93	—0.8
14	4	0.1	2.5	0.042	2.37	+0.2
15	16	0.5	2.5	0.211	2.37	—3.6
16	15	0.5	3.5	0.185	2.71	—1.8
17	20	1.0	3.5	0.369	2.71	—6.5
18	3	0.1	3.0	0.058	1.71	—1.0
19	4	0.5	3.0	0.292	1.71	—4.5
20	7	1.0	3.0	0.585	1.71	—9.4

The results of experiments Nos. 2 to 6 show a remarkably close agreement between measurements by this meter and those over the

[a] Trans. Am. Soc. Civ. Eng., Vol. XII, p. 301.

weir. The length of an observation, thirty seconds, is too short to give a good average indication of the velocity. The discharge varied somewhat during the experiment. The depth of water in the conduit being small, any error in the depth must have introduced considerable error in the result. We would not expect, therefore, so close an agreement, which appears all the more remarkable when it is seen from the following table that the discharge shown by two weirs may differ by 3 per cent:

Values of m in formula $Q = mlh\sqrt{2gh}$, as found by Bazin and by Fteley and Stearns.[a]

Head on weir (h).	Values of m.		$m_1 - m_2$	$\frac{m_1 - m_2}{m_1}$
	Fteley and Stearns (m_2).	Bazin (m_1).		
Meters.	*Meters.*	*Meters.*	*Meters.*	*Per cent.*
0.1428	0.4149	0.4290	0.0141	3.29
0.1969	0.4166	0.4263	0.0097	2.28
0.2497	0.4191	0.4259	0.0068	1.60
0.3008	0.4218	0.4266	0.0048	1.12
0.3530	0.4247	0.4276	0.0029	0.68
0.3957	0.4272	0.4285	0.0013	0.33
0.4434	0.4296	0.4295	0.0001	0.02
0.4888	0.4323	0.4310	−0.0013	−0.33

It is seen that for heads on the weir of from 0.5 foot to 1 foot the variation in m is from 3 to 1 per cent.

MURPHY'S EXPERIMENTS.

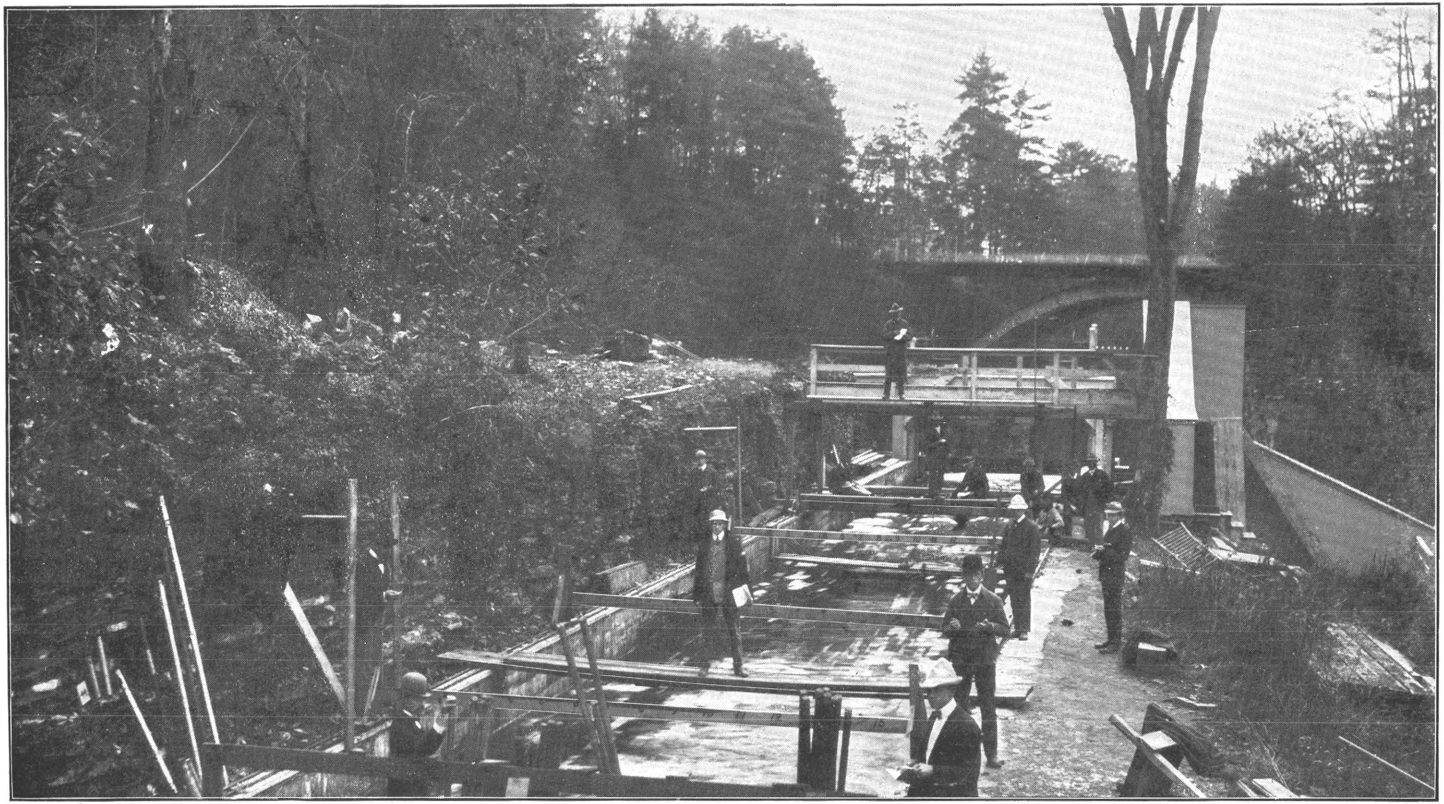
The results of numerous weir, current-meter, and other experiments by the writer at the Cornell hydraulic laboratory are discussed in detail on the following pages; also the application of the results to ordinary river gaging and the general conclusions reached from these recent studies and investigations.

EXPERIMENTS AT CORNELL UNIVERSITY HYDRAULIC LABORATORY.

INTRODUCTORY.

This investigation was begun by the writer in May, 1900, while a fellow in civil engineering at Cornell University. It was interrupted from July 1 to December 4 of that year, on account of the New York State canal survey, in which the writer participated, and again from December 9 to April 1, on account of freezing weather, and was completed in May, 1901. Its object was to compare the discharge of the Cornell University experiment canal as obtained with a sharp-crested weir with that obtained with current meters operated by different

[a] Annales des ponts et chaussées, mémoires et documents, 1888, p. 416.



HYDRAULIC LABORATORY CANAL OF CORNELL UNIVERSITY, SHOWING CORPS ENGAGED IN MAKING DISCHARGE EXPERIMENTS.

methods, and thus determine the accuracy of velocity and discharge obtained by current-meter methods. The scarcity of meters available for the work, the lack of ready facilities for frequent testing and rating of the meters used, and the scarcity of water during the first months of the investigation render the results somewhat less valuable than they otherwise would be.

The writer takes this occasion to acknowledge his indebtedness to Mr. F. H. Newell, chief hydrographer of the United States Geological Survey, for his hearty cooperation; to Prof. E. A. Fuertes, director and dean of the College of Civil Engineering, Cornell University, for the use of the hydraulic laboratory and a chronograph, computing machine, current meter, and electric register, and for kindly interest in the work; to Prof. Gardener S. Williams, in charge of the hydraulic laboratory, for many valuable suggestions and for the use of a chart for converting head on the Cornell University standard weir into cubic meters; also to Mr. C. E. Torrance, scholar in civil engineering, to members of the senior class in civil engineering, and to Mr. C. D. Cass, assistant mechanician of the college, for their cheerful assistance.

DESCRIPTION OF CORNELL HYDRAULIC LABORATORY.

The Cornell hydraulic laboratory consists essentially, as shown in Pls. II, III, and IV and fig. 15, of a canal 16 feet wide, 10 feet deep, and 415 feet long, with concrete bottom and sides and with a bed slope of 1 foot in 500. There are six rectangular gates at the upper end for controlling water entering the canal and four gates near the lower end, two on each side, for controlling the depth of water in the canal. Fifty-nine feet below the entrance gates is a bulkhead about 10.5 feet high built of 12-inch by 12-inch timbers. On the top of this, or 11.1 feet above the bottom of the canal, is fastened a steel angle iron 3.5 inches by 5 inches by 16 feet long, with edges five-sixteenths inch thick, which forms the standard sharp-crested weir. The lower end of the canal is closed with 12-inch by 12-inch timbers, which can be moved up or down if desired and the water be allowed to pass out of the canal at the end instead of under the side gates. In the upper chamber are two sets of baffles for checking the velocity of the water after it passes under the entrance gates. One of these is 7.4 feet from the gates and is made of 4-inch by 12-inch timbers placed horizontally, with the wide face toward the current and spaced from 8 to 12 inches apart. On the upper side of these timbers are 1-inch by 6-inch strips, fastened diagonally and spaced 12 inches from center to center. The second baffle is about 6.5 feet below the first one and is similar in construction, except that the 1-inch by 6-inch strips are placed vertically and are spaced 10 inches from center to center. Twenty-eight feet below the weir is a baffle made of 1-inch by 6-inch strips placed 3 inches from center to center, with the narrow side toward the current. To still further quiet the waters after passing the weir, when the depth is small and the velocity large, a second baffle was constructed of 1-inch by 6-inch pieces placed vertically, spaced 2 inches from center to center and fastened on the upper side of the baffle just described.

Air is admitted under the falling sheet of water through 12-inch by 10-inch passages in the concrete, one on each side of the weir. The head on the weir and the depth of water in the canal are measured by piezometers and a hook gage. As rough checks on these readings gage boards and a dial gage were occasionally read. The piezometer for giving the head on the standard weir is formed of three 1-inch pipes (*P*, *P*, *P*, fig. 15) perforated for 3 feet of their length with ¼-inch holes 6 inches apart around the pipes. These are placed 27 feet upstream from the weir, with their axes parallel with the axis of the

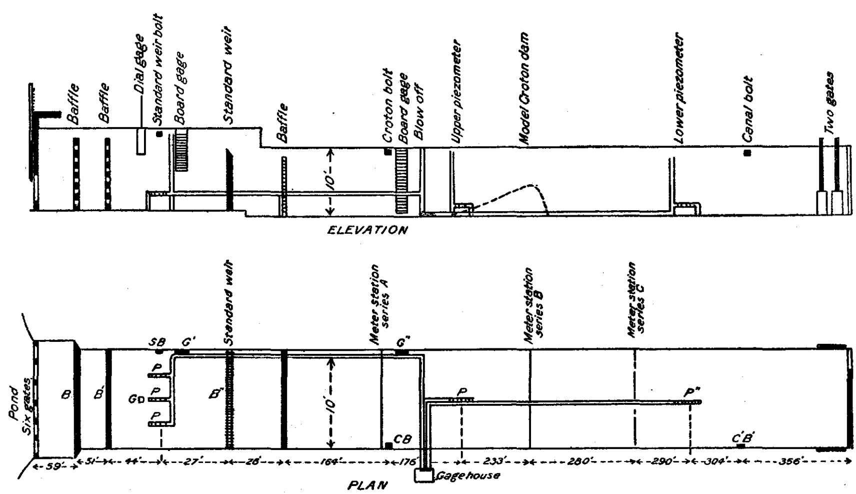
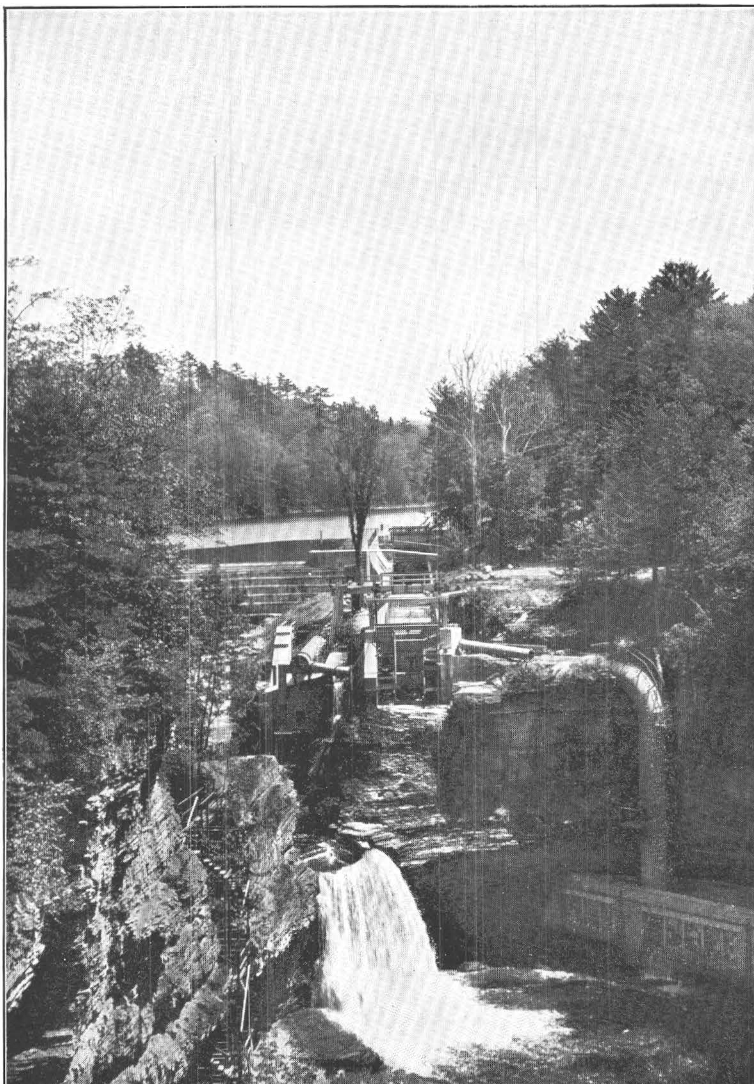


FIG. 15.—Plan and elevation of Cornell University experiment canal.

canal, 4 feet from the bottom, and each connected to a pipe which extends down the south side of the canal to a point opposite the gage house. From this point it is carried across the canal along the bottom and up into the gage house and is connected to a glass tube by a rubber hose. The tube is fastened to a scale marked to 2 mm. spaces. The canal piezometers are similar to those of the weir, but have only one entrance pipe, which is in the center of the canal. The surface elevation of the water was read with the hook gage at the three points in fig. 15 marked "Standard weir bolt," "Croton bolt," and "Canal bolt," the elevation of each of which is known. The dial gage and the board gage *G'* show approximately the head on the standard weir in feet and hundredths and enable the man operating the gates to keep a nearly constant head. A board gage (*G''*) opposite the "Croton bolt" (*CB*) shows approximately the depth of water in the canal.



FLUME, PENSTOCK, AND WHEEL HOUSE OF CORNELL HYDRAULIC LABORATORY CANAL.

SCOPE OF EXPERIMENTS.

The experiments naturally divide themselves into five groups, which we have called series A, B, C, D, and E. The experiments of series A were made at a section 158 feet below the standard weir and 75 feet upstream from a model of the Croton dam the height of which was 6 feet and the upstream face of which had a gradual slope and extended to within 29 feet of the meter station. This model dam formed a pool above it at the meter station and gave a zero velocity for 6 feet depth of water. The range of depths and velocities was therefore small. The water surface during the experiments was smooth, without boiling or swirls, with most of the foam made by the water falling over the weir moving down in two parallel lines 3 or 4 feet from each side. Three current meters were used in this series of experiments, viz, large Price meter No. 88, small Price meter No. 351, and the Cornell University Fteley meter. The latter was always held with a rod, the small Price meter was held with a cable, and the large Price meter with a cable in some of the experiments and with a rod in other experiments. About 3¼ inches of mud had accumulated on the bottom of the canal above the model dam. After the first two experiments the mud was removed for a space of 4 or 5 feet above and below the meter station. For all other experiments the bottom was clean and smooth.

The experiments of series B were made 234 feet below the standard weir and after the model dam had been removed. They are with high velocities and shallow depths, the scarcity of water at the time restricting the depth somewhat. In all these experiments except Nos. 6 and 8 the tail gates were open, so that there was no backwater. The surface of the water was quite rough, being a succession of waves, without swirls or boiling. (See Pl. IV.) The same three meters were used as in series A. The Fteley and the large Price were held with rods, the small Price was held with a rod in some of the experiments and with a cable in other experiments.

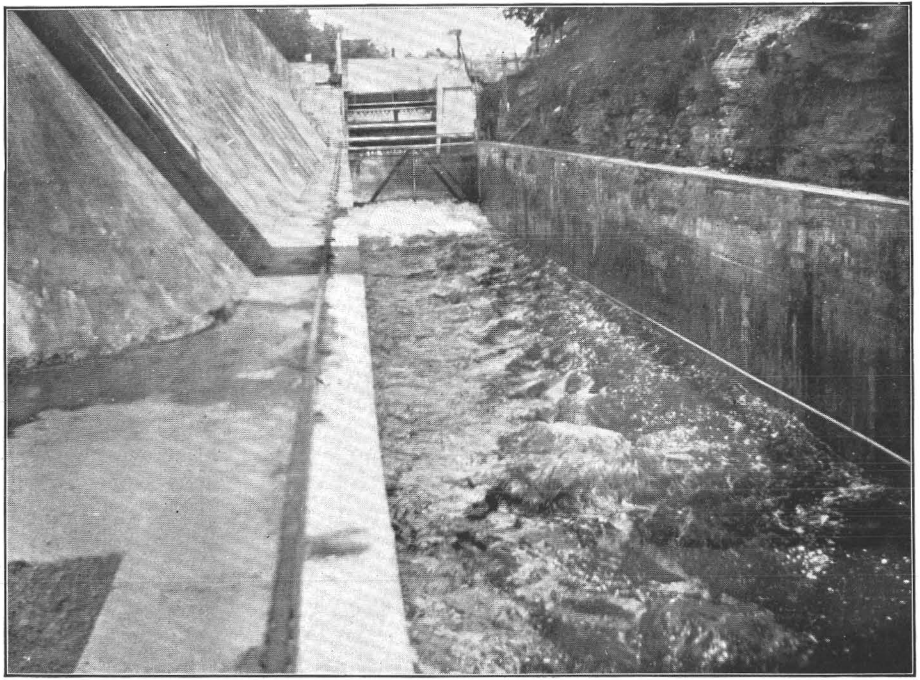
The experiments of series C were made 280 feet below the standard weir, as part of the tests of accuracy of the discharge measurements of the New York State canals and feeders made during the summer of 1900. They cover a range of depths from 6 to 9.5 feet and velocities from 0.23 foot to 2 feet per second. In most of the experiments of this series nearly all of the water was passed out of the canal through horizontal slits in the bulkhead. The water surface was comparatively smooth, being a succession of long waves, without boiling or swirls. Two meters were used—small Price No. 363, operated by the six-tenths-depth and integration methods, and small Price No. 351, operated by the ordinary method. The former was held with a cable, the latter with a rod.

The experiments of series D were made 220 feet below the standard weir, with Haskell meter No. 3 and small Price meter No. 363, each held with a cable. The range of depth was from 5 to 9.5 feet, and the velocities from 0.25 foot to 2 feet. The water surface was like that in series C—long waves, without boiling or swirls. Three methods of obtaining velocity were used—the ordinary, the six-tenths, and the integration. This series is not complete, the work being stopped by the freezing of the canal. The piezometers could not be used on account of the freezing of the pipes. The hook gage only was used to measure the head on the weir and the depth of water in the canal.

The experiments of series E were made with two small Price meters and a Haskell meter, held with cables at station 220 feet below the standard weir. The depth in this series varied from 4.6 feet to 9.3 feet, and the mean velocity from 1.5 feet to 3 feet per second. It was designed to extend the results of the previous year to the higher velocities. The discharge measurements were made in pairs, as in series D, so as to compare the results of two meters with each other as well as the results of each meter with the corresponding weir discharge. The head on the weir and the elevation of water surface were obtained from readings of the portable hook gage. The water surface was quite rough for these higher velocities, being a succession of waves, without swirls. To quiet it and enable the surface elevation to be more accurately measured, in experiments 9 to 22 two planks, 2 inches by 12 inches by 15 feet long, were placed across the canal about 3 feet apart and 154 feet from the weir, fastened at the ends so as to float on the surface and break the force of the waves. In experiments 23 to 50 three of these planks were used. The planks served their purpose very well, but distorted somewhat the vertical velocity curves.

METHOD OF EXPERIMENTATION.

The first operation in each experiment was to adjust the entrance gates to admit such a quantity of water that with the desired depth of water at the meter station the velocity would be about the desired rate. The tail gates were then opened or closed somewhat, as the case required, until the water reached the desired depth and remained steady. The standard weir gage was then read every thirty seconds and the canal gages (one or both) were read every five minutes during the time the meter observations were being taken. When the hook gage was used it was read six times in succession, as quickly as possible, at each of the three points in fig. 15 marked "Standard bolt," "Croton bolt," and "Canal bolt," already described. When the ordinary method of operating the meter was used the meter was held at from two to six points, depending on the depth, in each of from five to eight verticals, and the number of the revolutions of the meter wheel in from one to two periods of from thirty to sixty seconds each was obtained. Usually it was two periods of fifty seconds each. In series C, ordinary method, one period of sixty seconds was used and the revolutions were read on a recorder. In all the other experiments except those



CORNELL HYDRAULIC LABORATORY CANAL, SHOWING DISTURBED WATER SURFACE DURING EXPERIMENTS IN SERIES B.

with the Fteley meter the revolutions were obtained from indications of an electric buzzer. When the meter was operated by the six-tenths-depth method it was held at about six-tenths depth below the surface in eight verticals, viz, 1, 3, 5, 7, 9, 11, 13, and 15 feet from the south side of the canal, and the revolutions were obtained for two periods of fifty seconds each. The meter was then held at the same depth in the same verticals in the inverse order, i. e., 15, 13, etc., 1 foot from the south side of the canal, and the revolutions were again counted for two periods of fifty seconds each. When the integration method was used in series C the meter was moved in the following way: It was started at 1 foot from the bottom (center of meter 0.5 foot above bottom) and 1 foot from the side of the canal, and was passed slowly to the surface at 4.5 feet from the side, then to the bottom at 8 feet from the side, then to the surface at 11.5 feet from the side, and then to the bottom at 15 feet from the side. The time at the beginning and end was noted and the total number of revolutions were counted. The meter was then carried back over the same path at about the same speed, and the time of passage and the number of revolutions were obtained as before. The mean time and the mean number of revolutions of these two passages across the canal are used as a single experiment. When this method was used in series A the meter was carried only once across the canal, and it was started and stopped at the bottom (center of meter 0.5 foot above bottom) and close to each side, and reached the surface 4 feet and 12 feet from the south side of the canal. When this method was used in series E the meter was moved slowly from the surface to the bottom (center of meter from 2.5 inches below surface to 5.5 inches above bottom) and back to the surface in each of the eight verticals in which the meter was held when the ordinary point method was used. The time of passage of the meter down and back again in each vertical was noted and the revolutions of the meter wheel were counted.

REDUCTION OF EXPERIMENTS.

The standard weir gage readings were averaged for the time of each discharge measurement, and the reading for zero head subtracted, giving the head on the weir in double centimeters. By means of a chart for converting head on weir into cubic meters per second and a computing machine the discharge in cubic feet per second was easily obtained. The depth of water in the canal was found by adding to the mean canal gage reading in feet the difference in elevation of the canal bottom and the zero of the scale. The discharge when the ordinary method was used was computed from the vertical velocity curve for that experiment (see figs. 23 to 28). The observations were usually taken in the eight verticals at 1, 3, 5, 7, 9, 11, 13, and 15 feet from the south side of the canal, and at the same distance above the bottom. When observations in any vertical were missing, as in verticals 7 and 11 in series C, they were supplied by interpolation. The mean number of revolutions at the depth of each observation for these eight verticals was then found, converted into velocity by the rating table, plotted (as in figs. 23 to 28), and a mean curve drawn. The mean abscissa of this curve is the mean velocity in the cross section; the discharge is the product of this velocity and the cross-sectional area.

When the six-tenths-depth method was used a horizontal curve of velocity was drawn in a similar manner and the mean velocity was found from it. The mean velocity by the integration method was obtained by dividing the revolutions by the corresponding time and converting the quotient into velocity.

DISCUSSION OF RESULTS.

COMPARISONS OF DISCHARGE MEASUREMENTS.

The following tables give the results of the discharge comparisons of series A to D. The standard weir discharge (third column) is the mean for the time over which the meter observations (ordinary method) extended. It is assumed to be the same for the shorter period of measuring the discharge by the six-tenths-depth and the integration methods. This is not strictly true for the larger discharges, but it is near enough for practical purposes. The true velocity is obtained by dividing the weir discharge by the cross-sectional area at the meter station. The other columns are self-explanatory.

Comparisons of discharge over Cornell University standard weir with measurements by current meters.

SERIES A.

Date.	No. of experiment.	Standard weir discharge (Q).	Meter observations.								Variation.	
			Depth of water in canal at meter section.	True velocity (V) at meter section.	Kind of meter.	Number of observations.	Ordinary method.		Integration method.			
							V_0	Q_1	V_1	Q_1	$\frac{Q-Q_0}{Q}$	$\frac{Q-Q_1}{Q}$
1900.		Cu. ft. per sec.	Feet.	Feet p. sec.	(a)		Feet p. sec.	Cu. ft. per sec.	Feet p. sec.	Cu. ft. per sec.	Per ct.	Per ct.
May 5	1	147.991	7.996	1.166	L. P. 88.	76	1.097	138.478	......	......	+ 6.43	......
Do	2	65.024	7.167	0.573	do	29	0.447	50.714	......	......	+22.01	......
May 7	3	78.481	7.258	0.683	Fteley	31	0.677	77.780	......	......	+ 0.90	......
May 8	4	162.684	8.104	1.268	S. P. 351.	38	1.267	162.464	1.252	160.62	+ 0.14	+1.26
Do	5	100.379	7.542	0.841	do	20	0.790	94.318	......	......	+ 6.03	......
May 9	6	193.624	8.369	1.462	do	35	1.435	190.109	......	......	+ 1.81	......
Do	7	131.178	7.845	1.056	do	18	1.036	128.657	1.031	128.04	+ 1.92	+2.32
Do	8	131.037	7.848	1.056	L. P. 88.	18	0.982	122.000	1.006	124.98	+ 6.90	+4.63
Do	9	131.072	7.845	1.055	Fteley	18	1.148	142.570	......	......	− 8.77	......
May 15	10	148.556	7.984	1.175	do	40	1.286	162.540	......	......	− 9.43	......
Do	11	119.947	7.729	0.980	S. P. 351.	32	0.939	114.887	0.978	119.66	+ 4.22	+0.26
May 16	12	74.137	7.281	0.643	L. P. 88.	36	0.533	61.432	......	......	+17.14	......
Do	13	93.457	7.484	0.788	do	32	0.744	88.110	......	......	+ 5.72	......
Do	14	74.137	7.280	0.643	Fteley	29	0.642	73.985	......	......	+ 0.20	......
Do	15	93.386	7.484	0.788	do	32	0.834	98.804	......	......	− 5.80	......

[a] L. P.=large Price; S. P.=small Price. The figures given are the meter numbers.

Comparisons of discharge over Cornell University standard weir with measurements by current meters—Continued.

SERIES B.

Date.	No. of experiment.	Standard weir discharge (*Q*).	Meter observations.						Variation. $\frac{Q-Q_0}{Q}$
			Depth of water in canal at meter section.	True velocity (*V*) at meter section.	Kind of meter.	Number of observations.	Ordinary method.		
							V_0	Q_0	
		Cu. ft. per sec.	*Feet per sec.*	*Feet per sec.*	([a])		*Feet per sec.*	*Cu. ft. per sec.*	*Per ct.*
1900.									
May 19	1	50.366	1.115	2.823	S.P.351	16	2.790	49.774	+ 1.11
Do	2	50.260	1.115	2.823	Fteley	16	3.599	64.206	−27.75
Do	3	50.154	1.110	2.824	L.P. 88	16	3.103	53.790	− 7.26
May 21	4	65.625	1.299	3.157	Fteley	24	3.344	69.502	− 5.91
Do	5	65.625	1.299	3.157	L.P. 88	24	3.085	64.118	+ 2.30
Do	6	66.896	2.000	2.091	Fteley	24	2.500	80.000	−18.09
Do	7	21.616	0.786	1.719	L.P. 88	16	1.538	19.341	+10.54
Do	8	66.731	2.029	2.056	....do....	24	1.900	61.682	+ 9.07
Do	9	33.060	0.741	2.766	Fteley	16	3.273	32.295	+ 2.30
May 22	10	21.369	0.550	2.428	....do....	16	2.945	25.916	−21.28
Do	11	94.410	1.885	3.130	....do....	24	3.789	114.250	−21.01
Do	12	94.410	1.885	3.130	L.P. 88	24	3.102	93.556	+ 0.90
Do	13	57.854	1.198	3.018	Fteley	24	3.720	71.305	−23.27
Do	14	57.854	1.198	3.018	L.P. 88	24	2.947	56.589	+ 2.18
Do	15	43.761	0.953	2.870	S.P.351	16	2.698	43.055	+ 1.62
Do	16	43.761	0.953	2.870	Fteley	16	3.474	52.944	−20.98
Do	17	29.598	0.693	2.671	S.P.351	16	2.425	26.876	+ 9.19
Do	18	29.598	0.693	2.671	Fteley	16	3.137	34.768	−17.47
Do	19	18.932	0.464	2.553	S.P.351	8	2.290	16.983	−10.30
Do	20	18.932	0.464	2.553	Fteley	8	2.932	21.744	−14.84

[a] L. P. = large Price; S. P. = small Price. The figures given are the meter numbers.

Comparison of discharge over Cornell University standard weir with measurements by current meters.

SERIES C

Date.	Number of experiment.	Standard weir discharge (Q).	Meter observations.							Variation.		
			Depth of water in canal at meter section.	True velocity (V) at meter section.	Small Price No. 351, ordinary method.		Small Price No. 363.			Small Price No. 351, $\left(\frac{Q-Q_0}{Q}\right)$.	Small Price No. 363.	
					V_0	Q_0	Ordinary method. V_6	Ordinary method. Q_6	Integration method, (Q_1).		$\frac{Q-Q_6}{Q}$	$\frac{Q-Q_1}{Q}$
1900.		*Cu. ft. per sec.*	*Feet.*	*Ft. per sec.*	*Ft. per sec.*	*Cu. ft. per sec.*	*Ft. per sec.*	*Cu. ft. per sec.*	*Cu. ft. per sec.*	*Per cent.*	*Per cent.*	*Per cent.*
October 24	1	216.131	9.460	1.409	1.446	219.836	1.465	222.980	220.393	− 1.77	−4.11	− 0.63
Do	2	140.515	9.374	0.932	0.969	146.060	0.985	148.528	143.099	− 4.13	−6.13	− 5.37
Do	3	71.681	9.338	0.477	0.493	73.050	0.487	73.167	71.514	− 2.38	−2.08	− 1.59
Do	4	38.254	8.910	0.265	0.259	37.420	0.278	40.113	33.330	+ 2.03	−6.00	−16.16
October 26	5	178.553	7.673	1.444	1.449	179.130	1.489	184.075	174.679	− 0.33	−3.09	− 5.17
Do	6	125.298	7.640	1.018	0.999	122.974	1.047	128.880	120.789	+ 1.85	−2.78	− 5.12
October 27	7	61.272	7.582	0.502	----------	----------	0.522	63.771	61.327		−4.88	− 4.08
Do	8	31.176	7.557	0.255	0.215	26.180	0.258	31.488	28.070	---------- +16.01	−1.04	−22.10
Do	9	50.310	6.005	0.515	0.494	48.208	0.540	52.696	46.451	+ 4.17	−5.92	− 7.65
Do	10	196.387	6.278	1.940	1.890	192.799	1.989	201.367	188.232	+ 1.83	−1.46	− 4.86
Do	11	132.414	6.399	1.283	1.251	129.078	1.318	135.991	123.816	+ 2.52	−3.18	− 4.27
Do	12	91.056	6.319	0.893	0.847	86.318	0.920	93.758	83.872	+ 5.20	−5.08	− 2.96
October 29	13	65.319	8.476	0.478	0.464	63.325	0.492	67.146	60.459	+ 3.07	−2.35	− 2.80
October 31	14	198.258	8.470	1.454	1.460	199.115	1.514	206.480	187.795	− 0.44	−3.71	− 6.76
Do	15	125.301	8.508	0.915	0.890	121.921	0.949	130.001	114.522	+ 2.69	−3.75	− 9.54
November 1	16	31.223	8.455	0.229	0.201	27.364	0.240	32.674	25.186	+12.37	−2.47	−39.30

Comparison of discharge over Cornell University standard weir with measurements by current meters—Continued.

SERIES D.

Date.	Number of experiment.	Standard weir discharge (Q).	Meter observations.							Variation.		
			Depth of water in canal at meter section.	True velocity (V) at meter section.	Kind of meter.[a]	Ordinary method.		Six-tenths-depth method (Q_6).	Integration method (Q_1).	$\frac{Q-Q_0}{Q}$	$\frac{Q-Q_6}{Q}$	$\frac{Q-Q_1}{Q}$
						V_0	Q_0					
1900.		*Cu. ft. per sec.*	*Feet.*	*Ft. per sec.*		*Ft. per sec.*	*Cu. ft. per sec.*	*Cu. ft. per sec.*	*Cu. ft. per sec.*	*Per cent.*	*Per cent.*	*Per cent.*
December 6	1	233.262	9.505	1.528	S. P. 363	1.519	231.976	....	235.082	+0.55	....	−0.77
Do	2	233.262	9.505	1.528	Hask. 3	1.484	226.380	....	239.968	+2.95	....	−2.88
December 7	3	229.731	9.317	1.535	do	....	....	239.120	235.828	....	− 4.09	−2.66
Do	4	117.673	[b]9.536	0.768	do	0.832	[b]127.421	[b]126.498	....	−8.28	− 7.50	....
Do	5	117.673	[b]9.536	0.768	S. P. 363	0.794	[b]121.601	[b]119.301	....	−3.34	− 1.40	....
Do	6	95.176	[b]9.524	0.622	Hask. 3	0.676	[b]103.401	[b]105.846	....	−8.63	−11.21	....
Do	7	95.176	[b]9.524	....	S. P. 363	0.563	[b]99.883	[b]97.892	....	−4.93	− 2.84	....
December 8	8	221.749	7.245	1.905	Hask. 3	1.864	216.932	218.680	228.340	+2.13	+ 1.38	−3.08
Do	9	221.749	7.245	1.905	S. P. 363	1.867	217.281	222.521	....	+2.01	− 0.35	....
Do	10	191.165	7.217	1.649	Hask. 3	1.675	194.183	200.209	202.647	−1.58	− 4.73	−6.00
Do	11	191.165	7.217	1.649	S. P. 363	1.651	191.400	192.793	....	+0.20	− 0.84	....
Do	12	110.186	7.183	0.955	Hask. 3	1.022	117.919	118.802	119.609	−7.01	− 7.84	−8.55
Do	13	110.186	....	0.955	S. P. 363	0.955	110.188	110.858	....	0.00	− 0.68	....
Do	14	56.717	7.113	0.496	{Hask. 3 S. P. 363	....	....	53.244 58.270	....	....	− 6.14 − 2.73	....
Do	15	57.071	5.154	0.689	Hask. 3	0.730	60.455	62.277	62.277	−5.94	− 9.10	−9.10
Do	16	57.071	5.154	0.689	S. P. 363	0.672	55.651	57.308	....	+2.49	− 0.42	....

[a] S. P.=small Price; Hask.=Haskell. The figures given are the meter numbers.

[b] Hook gage broken; depth not accurately measured.

It is seen from the foregoing tables that the meter discharge differs from the weir discharge all the way from 0 to 40 per cent. One might say, without careful examination, that the current meter must be a very unreliable velocity-measuring instrument; but when we examine these results carefully and plot them according to velocity we find the case very different. Take first the results by the ordinary method.

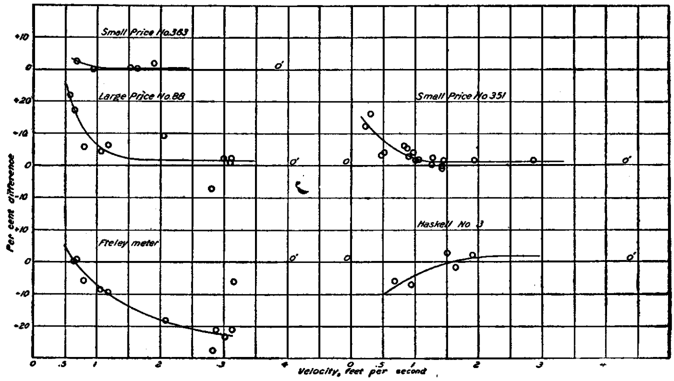


FIG. 16.—Diagram showing difference between meter discharge by ordinary method and weir discharge, from measurements in Cornell University experiment canal. Line 0–0′ represents weir discharge.

These are plotted for each meter in fig. 16, using mean velocity as abscissæ and percentage difference as ordinates. It is seen that nearly all of these points fall on well-defined lines. The few exceptions are those in which the depth is small and the errors of depth and position of meter have a large effect on the measured discharge. The discharge obtained with the Fteley meter differs more from the corre-

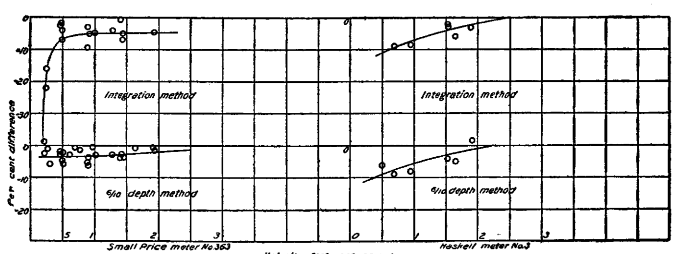


FIG. 17.—Diagram showing difference between meter discharge by six-tenths-depth and integration methods and weir discharge, from measurements in Cornell University experiment canal. Line 0–0′ represents weir discharge.

sponding weir discharge than that obtained with the other meters. This is because it had never been rated. The constant we have used in reducing the observations with this meter—viz, 1 revolution per second at 1 foot velocity = 1.072—was obtained from observations by students. It is not, however, much in error for this velocity, but is largely in error for other velocities. The rating obtained from these weir experiments is given in the table on page 81. For the other three meters the percentage error is small for velocities above 1 foot per second. For the Price meters the error is positive and increases rapidly as the velocity decreases. For the Haskell meter the error is negative and increases, negatively, rapidly as the velocity decreases. The error in rating, if we may call it such, is not so large as it appears from these diagrams, for a 10-per cent error in a small quantity is only a 1-per cent error in a quantity ten times as large. The rating tables as derived from these curves and as found by moving the meter through still water are given in the table on page 81. Experiments with depths of 0.75 foot or less (Nos. 9 and 17 to 20 of series B) are not plotted in the diagram (fig. 16), as the probable error is large; nor are experiments 1 to 4 of series C, because the upper and lower parts of the vertical curves are not well defined; nor experiments 4 to 7 of series D, because there appears to be an error in the hook-gage work of those experiments.

In fig. 17 are plotted the percentage differences between weir discharge and discharge obtained by the six-tenths-depth and the integration methods. It is seen that the meter discharge by these methods is greater than the weir discharge, being 4 or 5 per cent greater for velocities above 1 foot per second. In the integration method, for small velocities the difference increases very rapidly as the velocity decreases. It is seen from figs. 16 and 17 that while the ordinary method gives discharge 1 or 2 per cent less than the weir for velocities above 1 foot per second, the six-tenths-depth and the integration methods give discharge 4 or 5 per cent greater than the weir.

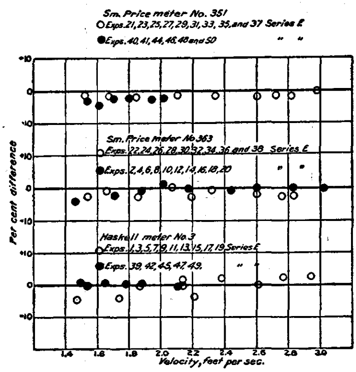


FIG. 18.—Diagram showing difference between meter discharge with different meters and weir discharge in experiments of series E in Cornell University experiment canal. Zero line indicates weir discharge.

The results of the discharge measurements of series E are given in the following table and are plotted in fig. 18, which shows three sets of double comparisons. Experiments 1 to 20 give a comparison of simultaneous measurements with small Price meter No. 363 and Haskell meter No. 3, and also with the discharge obtained at the same time with the weir. Experiments 21 to 38 give similar comparisons for the two small Price meters and the weir, and experiments 39 to 50 give similar comparisons for Haskell and Price meters and the weir.

Comparison of discharge over Cornell University standard weir with measurements by current meters.

SERIES E.

No. of experiment.	Date.	Depth of water in canal at meter section.	True velocity at meter section.	Standard weir discharge (Q).	Meter discharge (Q_W).	Variation $\left(\frac{Q-Q_W}{Q_W}\right)$.	Kind of meter.[a]	Difference in meter discharges.
	1901.	*Feet.*	*Ft. per sec.*	*Cu. ft. per sec.*	*Cu. ft. per sec.*	*Per cent.*		*Per cent.*
1	April 1	9.075	1.472	204.41	214.08	−4.73	Hask. 3	0.86
2	do	9.075	1.456	204.41	212.33	−3.87	S. P. 363	
3	April 2	6.079	2.208	208.88	216.28	−3.54	Hask. 3	0.99
4	do	6.079	2.187	208.88	214.22	−2.55	S. P. 363	
5	do	7.453	1.744	200.81	208.85	−4.00	Hask. 3	2.03
6	do	7.453	1.710	200.81	204.77	−1.97	S. P. 363	
7	do	5.742	2.137	197.64	197.02	−0.31	Hask. 3	1.58
8	do	5.742	2.171	197.64	200.15	+1.27	S. P. 363	
9	April 5	7.409	1.869	222.67	222.68	0.00	Hask. 3	0.75
10	do	7.409	1.883	222.67	224.35	−0.75	S. P. 363	
11	do	6.295	2.137	220.33	216.00	+1.96	Hask. 3	1.82
12	do	6.295	2.177	220.33	220.03	+0.14	S. P. 363	
13	do	5.628	2.377	219.38	214.82	+2.08	Hask. 3	2.51
14	do	5.628	2.438	219.38	220.33	−0.43	S. P. 363	
15	do	5.294	2.611	221.96	222.02	−0.03	Hask. 3	0.34
16	do	5.294	2.602	221.96	221.27	+0.31	S. P. 363	
17	do	4.892	2.764	222.81	217.32	+2.46	Hask. 3	2.22
18	do	4.892	2.827	222.81	222.27	+0.24	S. P. 363	
19	do	4.617	2.936	224.75	218.32	+2.86	Hask. 3	2.85
20	do	4.617	3.019	224.75	224.50	+0.01	S. P. 363	
								1.60
21	April 6	4.808	2.985	230.36	230.66	−0.13	S. P. 351	0.71
22	do	4.808	3.006	230.36	232.30	−0.84	S. P. 363	
23	April 10	8 744	1.670	230.75	234.68	−1.70	S. P. 351	0.71
24	do	8.744	1.657	230.75	232.85	−0.99	S. P. 363	
25	do	7.874	1.846	229.28	233.61	−1.89	S. P. 351	0.88
26	do	7.874	1.862	229.28	235.64	−2.77	S. P. 363	
27	do	6.889	2.108	229.91	233.25	−1.45	S. P. 351	1.68
28	do	6.889	2.073	229.91	229.38	+0.23	S. P. 363	
29	April 11	6.129	2.340	227.16	230.25	−1.36	S. P. 351	0.86
30	do	6.129	2.320	227.16	228.28	−0.50	S. P. 363	
31	do	5.483	2.596	225.01	228.59	−1.60	S. P. 351	0.31
32	do	5.483	2.604	225.01	229.30	−1.91	S. P. 363	
33	do	5.185	2.725	224.06	226.98	−1.30	S. P. 351	1.23
34	do	5.185	2.758	224.06	229.73	−2.53	S. P. 363	
35	do	5.003	2.816	222.60	226.37	−1.69	S. P. 351	0.65
36	do	5.003	2.834	222.60	227.82	−2.34	S. P. 363	
37	do	9.256	1.522	223.40	226.38	−1.33	S. P. 351	1.05
38	do	9.256	1.538	223.40	228.71	−2.38	S. P. 363	
								0.90
39	April 12	9.344	1.486	224.63	223.11	+0.68	Hask. 3	3.68
40	do	9.344	1.541	224.63	231.37	−3.00	S. P. 351	
41	do	8.934	1.613	221.39	231.58	−4.60	S. P. 351	4.67
42	do	8.934	1.543	221.39	221.53	+0.07	Hask. 3	
43	do	8.297	1.646	221.39	219.50	+0.81	do	3.21
44	do	8.297	1.700	221.39	226.70	−2.40	S. P. 351	
45	do	7.806	1.775	221.49	222.69	+0.54	Hask. 3	2.61
46	do	7.806	1.803	221.49	226.19	−2.07	S. P. 351	
47	do	7.266	1.879	221.21	219.34	+0.85	Hask. 3	3.38
48	do	7.266	1.943	221.21	226.81	−2.53	S. P. 351	
49	do	6.439	2.132	221.03	220.43	−0.27	Hask. 3	1.80
50	do	6.439	2.182	221.03	225.61	−2.07	S. P. 351	
								3.22

[a] S. P. = small Price; Hask. = Haskell. The figures given are the meter numbers.

The principal results of the foregoing comparisons have been condensed in the following table:

Table showing percentage variation between weir discharge and meter discharge, and between meter discharge by different instruments of experiments in series E.

Experiments.	Kind of meter.	Difference between standard weir discharge and meter discharge.				Difference in discharge of simultaneous meter measurements.			
		Max.	Min.	Mean.[a]	Range.	Max.	Min.	Mean.	Range.
		Per ct.	*Per ct.*	*Per ct.*	*Per cent.*	*Per ct.*	*Per ct.*	*Per ct.*	*Per ct.*
1 to 20	Haskell No. 3	−4.73	0.00	−0.33	−4.73 to +2.86				
Do.	S. Price No. 363	−3.87	+0.01	−0.76	−3.87 to +1.27	2.85	0.34	[a]0.29 [b]1.60	2.51
21 to 38	S. Price No. 351	−1.89	−0.13	−1.38	−1.89 to −0.13				
Do.	S. Price No. 363	−2.77	+0.23	−1.56	−2.77 to +0.23	1.68	0.31	[a]0.18 [b]0.90	1.37
39 to 50	S. Price No. 351	−4.60	−2.07	−2.78	−4.60 to −2.07				
Do.	Haskell No. 3	+0.85	−0.27	+0.40	+0.85 to −0.27	4.67	1.80	[a]3.22 [b]3.22	2.87

[a] Regarding signs. [b] Disregarding signs.

It is seen from this that in none of these fifty experiments does a meter discharge differ from a corresponding weir discharge 5 per cent, and in no case do two simultaneous meter discharges differ from each other 5 per cent. In experiments 1 to 20 the mean variation from the weir discharge is −0.33 per cent for the Haskell meter and −0.76 per cent for the Price meter, while the mean difference between corresponding meter discharges is 0.29 per cent. In experiments 21 to 38 the mean variation from the weir discharge is 1.38 per cent for Price meter No. 351 and −1.56 per cent for Price meter No. 363, while the mean difference between corresponding meter discharges is 0.18 per cent. In experiments 39 to 50 the mean variation from the weir discharge is 2.78 per cent for the Price meter and 0.40 per cent for the Haskell meter, while the mean difference between corresponding meter discharges is 3.22 per cent.

The range of variation of meter discharge from weir discharge is greater than that of corresponding meter discharges. This is to be expected, however, since there are a number of possible errors in the former which are not in the latter. Errors in measuring head, depth, calibration of weir, velocity, and rating of meter affect the former, while only the two latter errors affect corresponding meter discharges.

It is also seen that the mean of the difference between simultaneous discharge measurements Nos. 21 to 38 is only 0.18 per cent, and the extreme range in these eighteen consecutive measurements is only 1.37 per cent. How much of this error is due to errors in the rating table and how much to errors in observing velocity we were unable to determine, as good facilities for rating were not at hand. We are, however, justified by these experiments in the statement that the discharge of this canal can, with a small Price meter, be measured to within less than 1 per cent when the mean velocity in the canal is from 1.5 feet to 3 feet per second.

VELOCITY CURVES.

Curves of equal velocity in the cross-sectional area of observation are shown in figs. 19, 20, 21, and 22. Four experiments are selected (one from each series) from the sixty-five experiments of series A to D by the ordinary method, to show the variation of velocity in the sections. They are self-explanatory. Fig. 20 illustrates a case of small depth, high velocity, and no back-water effect. The maximum velocity is at the surface, nearly midway between the banks. In figs. 19, 21, and 22 the maximum velocity is below the surface and not midway between the banks. It is less at the center than 3 or 4 feet from either side. This is true of all the experiments of series A. We tried admitting the water to the upper chamber of the canal in various ways, by opening some of the gates and closing or partly closing others, but the distribution of velocity appeared to be nearly independent of the openings through which the water was admitted to the canal.

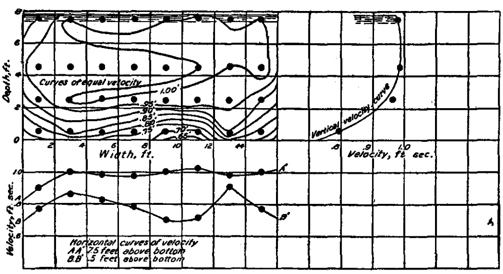


FIG. 19.—Curves of equal velocity in experiment No. 11, series A.

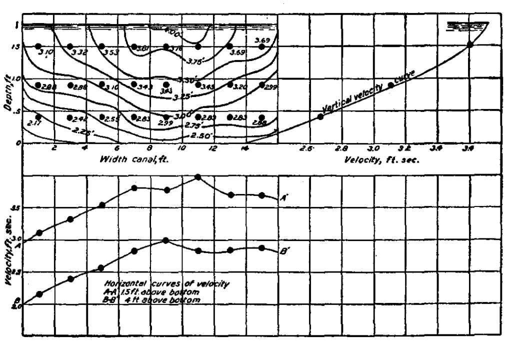


FIG. 20.—Curves of equal velocity in experiment No. 12, series B, with large Price meter No. 88.

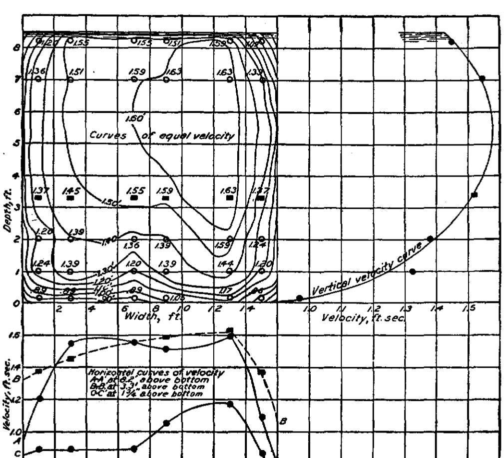


FIG. 21.—Curves of equal velocity in experiment No. 14, series C. The dots are for small Price meter No. 351; heavy black squares for small Price meter No. 363.

Horizontal velocity curves are shown in these same figures (19 to 22), the velocity curve at six-tenths depth below the surface being shown, and also the curves for surface and bottom. In these figures the velocity near the north side of the canal is somewhat less than that near the south side. This is true of all the experiments of series C and D. It is due to the north side of the canal being much rougher than the south side, the concrete of the north side having been cut away preparatory to its repair just previous to making the experiments of series C.

Vertical velocity curves for these four series of experiments (A to D) are shown in figs. 23 to 28. Each of these curves is the mean of observations in the eight verticals 1, 3, 5, 7, 9, 11, 13, and 15 feet from the south side of the canal. For series C and D, figs. 26, 27, and 28, the observations obtained with small Price meter No. 363 at six-tenths depth are shown by the little squares. As a rule they fall outside the curve, meter No. 363 indicating a somewhat greater velocity than meter No. 351. This fact is also shown by the corresponding curves of figs. 16 and 17. The two sets of velocity curves (full and dotted) of fig. 28 were found from simultaneous velocity measurements. Two parties measured velocity in the same section at the same time. One party, with the Haskell meter, started near the south

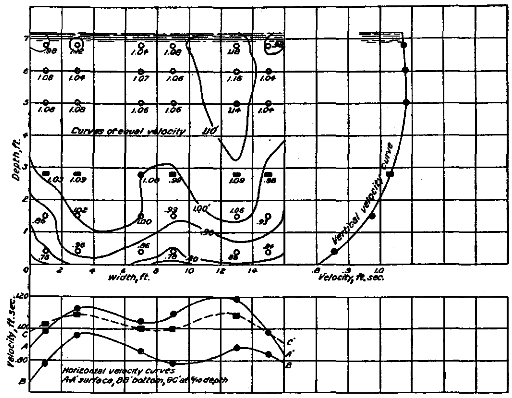


FIG. 22.—Curves of equal velocity in experiment No. 20, series D. The dots are for Haskell meter No. 3; heavy black squares for small Price meter No. 363.

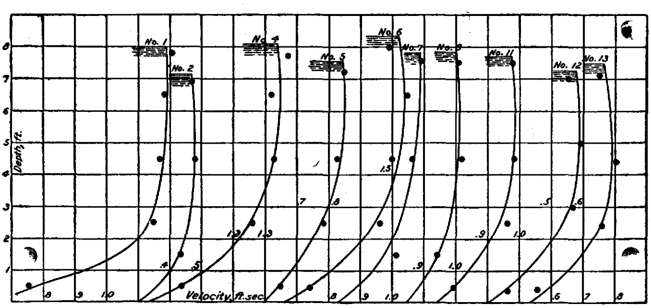


FIG. 23.—Vertical velocity curves obtained with current meters in experiments in series A. Nos. 1, 2, 8, 12, and 13 are for large Price meter No. 88; Nos. 4, 5, 6, 7, and 11 are for small Price meter No. 351.

side of the canal and worked toward the north side; the other party, with the Price meter, started at the north side and worked toward the south side. It is seen from these curves (fig. 28) that the bottom velocities agree more closely than those at the surface, and that the Haskell meter gives a much larger surface velocity than the Price meter. We should expect a little difference, since the revolving head of the Haskell meter is larger than the wheel of the Price meter, and its center being farther below the surface it must revolve faster. But the difference in velocity is much greater than this little difference in depth of axis would indicate. (See p. 91.)

The shape of these curves evidently depends on the depth and velocity. For depths not exceeding 1 foot it is a nearly straight line,

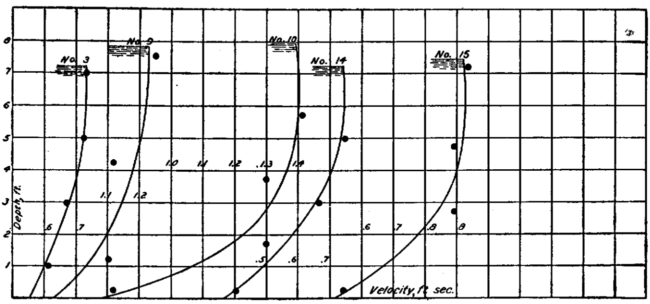


Fig. 24.—Vertical velocity curves obtained with Fteley current meter in experiments in series A.

with the maximum velocity at the surface and the mean velocity at mid depth. For depths of 8 or 9 feet and small velocities the curve is very flat, with the maximum velocity at two-tenths to three-tenths depth and the mean velocity at sixty-five-hundredths to seven-tenths depth below the surface. The radius of curvature decreases as the velocity increases. In all of these curves the least velocity is at the bottom. Nos. 6 and 8, series B, fig. 25, show the effect on the vertical

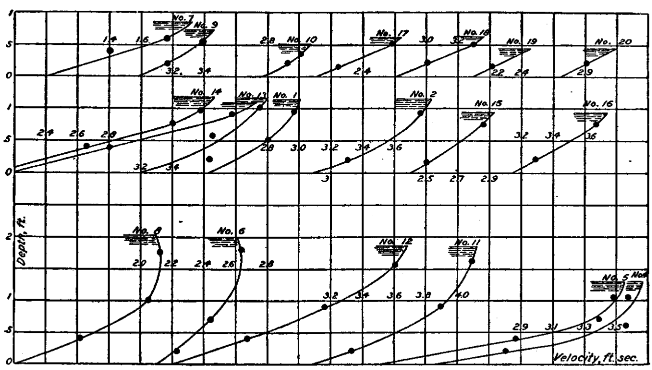


Fig. 25.—Vertical velocity curves obtained with current meters in experiments in series B. Nos. 1, 15, 17, and 19 are for small Price meter No. 351; Nos. 3, 5, 7, 8, 12, and 14 are for large Price meter No. 88; Nos. 2, 4, 6, 9, 10, 11, 13, 16, 18, and 20 are for Fteley meter.

velocity curve of checking the discharge at the lower end of the canal. For all the other curves of fig. 25 there is free discharge. The effect of checking the discharge at the lower end is to force the thread of maximum velocity from the surface to a depth of two-tenths or more. The table on page 77 gives the position of the threads of maximum and mean velocity taken from the curves shown in figs. 23 to 28. It is difficult to locate the threads with accuracy, since very little change in the shape of the curve would make a great change in the position of the threads. It is easily seen that the shape may be changed somewhat and yet fit the observations nearly as well as the one drawn and give a discharge differing from it but little. For velocities of a foot or more per second, however, the threads can be located with a fair

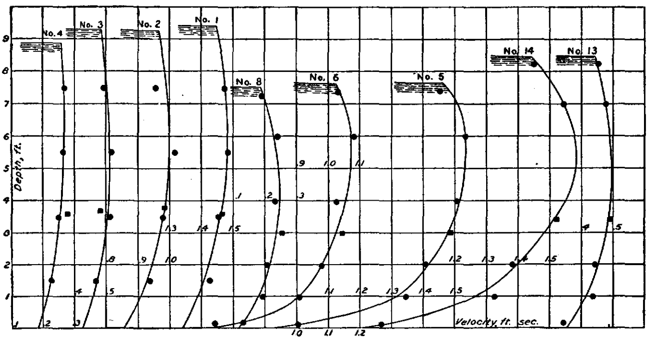


FIG. 26.—Vertical velocity curves obtained with current meters in experiments in series C. Heavy round dots are for small Price meter No. 351; the square blocks are for small Price meter No. 363.

degree of accuracy. The position of the thread of maximum velocity as found from series A is eighteen-hundredths depth below the surface; for series B it is one-tenth depth; for series C, thirty-eight-hundredths depth; and for series D, twenty-four-hundredths depth. For the mean of C and D, in which the conditions are nearly the same, it is thirty-one-hundredths depth. The position of the thread of mean

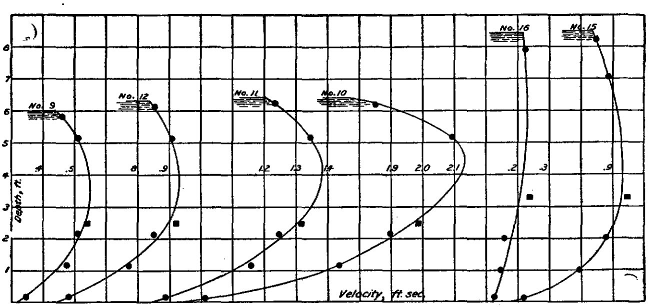


FIG. 27.—Vertical velocity curves obtained with current meters in experiments in series C. Heavy round dots are for small Price meter No. 351; the square blocks are for small Price meter No. 363.

velocity as shown by this table is sixty-six-hundredths depth for series A, fifty-four-hundredths depth for series B, and sixty-six-hundredths depth for series C and D. It is seen that the threads of mean and maximum velocity are nearer the surface in series A than in series C or D, although the depths do not differ much. This is probably due to the influence of the Croton model dam, the long upstream slope of which has had the effect of forcing the thread of maximum velocity up to thirteen-hundredths depth. As the ratio of width to depth

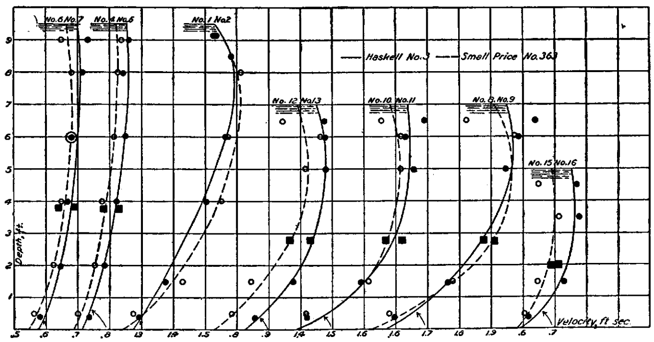


FIG. 28.—Vertical velocity curves obtained with Haskell meter No. 3 and small Price meter No. 363 in experiments in series D. Open circles are for small Price meter No. 363, point method; heavy black dots are for Haskell meter No. 3, point method; heavy black squares are for small Price meter No. 363, six-tenths-depth method.

changed from 16 to 2 the thread of mean velocity moved from fifty-four-hundredths depth to sixty-six-hundredths depth below the surface.

Table showing position of threads of maximum and mean velocity in Cornell University experiment canal as found from current-meter discharge measurements made in May, November, and December, 1900.

No. of experiment.	Series A.				Series B.				Series C.				Series D.			
	Velocity (V).	Depth (D).	Ratio $\left(\frac{dm}{D}\right)$.	Ratio $\left(\frac{d'}{D}\right)$.	Velocity (V_o).	Depth (D).	Ratio $\left(\frac{dm}{D}\right)$.	Ratio $\left(\frac{d'}{D}\right)$.	Velocity (V_o).	Depth (D).	Ratio $\left(\frac{dm}{D}\right)$.	Ratio $\left(\frac{d'}{D}\right)$.	Velocity (V_o).	Depth (D).	Ratio $\left(\frac{dm}{D}\right)$.	Ratio $\left(\frac{d'}{D}\right)$.
	Feet per second.	*Feet.*			*Feet per second.*	*Feet.*			*Feet per second.*	*Feet.*			*Feet per second.*	*Feet.*		
1	1.17	8.00	0.81	0.35	2.79	1.12	1.00	0.40	1.45	9.46	0.65	0.31	1.52	9.51	0.75	0.29
2	0.57	7.17	0.77	0.29	3.60	1.12	1.00	0.49	0.97	9.37	0.61	0.32	1.48	9.51	0.80	0.47
3	0.68	7.26	0.92	0.28	3.10	1.11	1.00	0.49	0.49	9.34	0.52	0.27	....	9.32	....	....
4	1.27	8.10	0.89	0.29	3.34	1.30	1.00	0.42	0.26	8.92	0.62	0.28	0.83	9.54	0.79	0.42
5	0.84	7.54	0.80	0.49	3.09	1.30	1.00	0.38	1.45	7.67	0.70	0.33	0.79	9.54	0.84	0.42
6	1.46	8.37	0.75	0.30	2.50	2.00	0.85	0.44	1.00	7.64	0.72	0.33	0.68	9.52	0.77	0.39
7	1.06	7.85	0.84	0.45	1.54	0.79	1.00	0.45	....	7.58	....	....	0.65	9.52	0.80	0.34
8	1.06	7.85	0.75	0.31	1.90	2.03	0.85	0.37	0.22	7.56	0.60	0.28	1.86	7.25	0.82	0.36
9	1.06	7.85	0.95	0.43	3.27	0.75	1.00	0.47	0.49	6.01	0.60	0.28	1.87	7.25	0.69	0.32
10	1.18	7.98	0.79	0.37	2.95	0.55	1.00	0.50	1.89	6.28	0.71	0.32	1.68	7.22	0.77	0.34
11	0.98	7.73	0.71	0.36	3.79	1.89	1.00	0.40	1.25	6.40	0.66	0.32	1.65	7.22	0.72	0.33
12	0.64	7.28	0.75	0.30	3.10	1.89	1.00	0.43	0.85	6.32	0.60	0.31	1.02	7.18	0.72	0.33
13	0.79	7.48	0.65	0.21	3.72	1.20	1.00	0.46	0.46	8.48	0.56	0.27	0.96	7.18	0.70	0.36
14	0.64	7.28	0.86	0.39	2.95	1.20	1.00	0.49	1.46	8.47	0.65	0.31	....	7.11	....	....
15	0.79	7.48	0.84	0.33	2.70	0.95	1.00	0.48	0.89	8.51	0.53	0.27	0.73	5.15	0.70	0.33
16	....	....	....	....	3.47	0.95	1.00	0.48	....	....	0.623	0.30	0.67	5.15	0.70	0.44
17	....	....	....	....	2.43	0.69	1.00	0.50	....	....	0.755	0.37	....	....	....	....
18	....	....	....	....	3.14	0.69	1.00	0.50	....	....	....	....	....	....	....	....
19	....	....	....	....	2.29	0.46	1.00	0.50	....	....	....	....	....	....	....	....
20	....	....	....	....	2.93	0.46	1.00	0.50	....	....	....	....	....	....	....	....
	....	....	0.815	0.343	....	....	0.985	0.457	....	....	0.69	0.34	....	....	0.755	0.37

V =velocity found from the weir.

V_o=velocity found with meter by ordinary method.

dm=distance above bottom to thread of maximum velocity.

d' =distance above bottom to thread of mean velocity.

The first of the following tables gives the mean, maximum, and bottom velocities in the experiments of series A, C, and D, also the ratio of bottom velocity to mean velocity and of mean velocity to mid-depth velocity. It is seen that the ratio of mean velocity to mid-depth velocity is much more nearly constant, and hence is better to use than that of mean velocity to bottom velocity. The second table gives the ratio of mean velocity to three-tenths-depth velocity, and of mean velocity to six-tenths-depth velocity in series A, C, and D. The ratio at three-tenths depth is seen to be more nearly constant than that at six-tenths depth.

Velocities at various depths and velocity ratios in Cornell University experiment canal as found from standard weir and current-meter discharge measurements.

No. of experiment.	Depth (*D*) in feet.	Series A. Velocities in feet per sec.*				Series A. Ratios.		Depth (*D*) in feet.	Series C. Velocities in feet per sec.*				Series C. Ratios.		Depth (*D*) in feet.	Series D. Velocities in feet per sec.*				Series D. Ratios.	
		V	V_m	V_b(†)	$V_{\frac{1}{2}D}$	$\frac{V_b}{V}$	$\frac{V}{V_{\frac{1}{2}D}}$		V	V_m	V_b(†)	$V_{\frac{1}{2}D}$	$\frac{V_b}{V}$	$\frac{V}{V_{\frac{1}{2}D}}$		V	V_m	V_b(†)	$V_{\frac{1}{2}D}$	$\frac{V_b}{V}$	$V \div V_{\frac{1}{2}D}$
1	8.00	1.17	1.36	0.80	1.24	0.69	0.90	9.46	1.41	1.45	1.05	1.44	0.74	0.98	9.51	1.53	1.62	1.29	1.57	0.84	0.97
2	7.17	0.57	0.59	0.50	0.56	0.88	0.98	9.37	0.93	0.96	0.83	0.95	0.89	0.98	9.51	1.53	1.64	1.33	1.56	0.87	0.98
3	7.26	0.68	0.76	0.60	0.70	0.88	0.97	9.34	0.48	0.50	0.43	0.50	0.89	0.96	9.32	1.54	......	......	......	......	......
4	8.10	1.27	1.37	1.01	1.33	0.80	0.96	8.92	0.27	0.29	0.22	0.29	0.82	0.94	9.54	0.77	......	......	......	......	......
5	7.54	0.84	0.87	0.64	0.85	0.76	0.99	7.67	1.44	1.54	1.12	1.52	0.78	0.95	9.54	0.77	......	......	......	......	......
6	8.37	1.46	1.55	1.22	1.53	0.82	0.96	7.64	1.02	1.09	0.78	1.06	0.76	0.96	9.52	0.62	......	......	......	......	......
7	7.85	1.06	1.10	0.92	1.08	0.87	0.98	7.58	0.59	......	......	......	......	......	9.52	0.62	......	......	......	......	......
8	7.85	1.06	1.09	0.92	1.07	0.87	0.99	7.56	0.26	0.29	0.15	0.28	0.58	0.93	7.25	1.91	2.01	1.65	1.96	0.86	0.97
9	7.85	1.06	1.10	0.92	1.07	0.87	0.99	6.01	0.52	0.58	0.37	0.57	0.71	0.91	7.25	1.91	2.01	1.61	1.98	0.84	0.96
10	7.98	1.18	1.24	0.83	1.20	0.70	0.98	6.28	1.94	2.17	1.42	2.07	0.73	0.94	7.22	1.65	1.77	1.42	1.69	0.85	0.98
11	7.73	0.98	1.01	0.79	1.00	0.81	0.98	6.40	1.28	1.41	0.94	1.37	0.73	0.94	7.22	1.65	1.72	1.44	1.70	0.87	0.97
12	7.28	0.64	0.71	0.49	0.70	0.77	0.92	6.32	0.89	0.98	0.63	0.96	0.71	0.93	7.18	0.96	1.01	0.80	0.99	0.83	0.97
13	7.48	0.97	0.89	0.65	0.88	0.83	0.90	8.48	0.48	0.54	0.40	0.53	0.79	0.91	7.18	0.96	1.03	0.76	1.00	0.80	0.96
14	7.28	0.64	0.73	0.42	0.68	0.66	0.94	8.47	1.45	1.60	1.04	1.58	0.72	0.92	7.11	......	......	......	......	......	......
15	7.48	0.79	0.87	0.56	0.85	0.71	0.93	8.51	0.92	0.95	0.67	0.95	0.73	0.97	5.15	0.69	0.71	0.59	0.68	0.86	0.99
16	......	......	......	......	......	......	......	8.46	0.23	0.26	0.15	......	0.65	......	5.15	0.69	0.72	0.62	0.68	0.89	0.99
						0.795	0.958						0.747	0.944						0.851	0.974

* Correct velocities found from standard weir.

† Velocity 2½ inches above the bottom.

NOTE.—V=mean velocity; V_m=maximum velocity; V_b=bottom velocity; $V_{\frac{1}{2}D}$=mid-depth velocity.

Velocity ratios in Cornell University experiment canal as found with current meters in May, November, and December, 1900.

No. of experiment.	Series A.				Series C.				Series D.			
	Velocity (v_0).	Depth (D).	Ratio $\left(\frac{v_0}{v_3}\right)$.	Ratio $\left(\frac{v_0}{v_6}\right)$.	Velocity (v_0).	Depth (D).	Ratio $\left(\frac{v_0}{v_3}\right)$.	Ratio $\left(\frac{v_0}{v_6}\right)$.	Velocity (v_0).	Depth (D).	Ratio $\left(\frac{v_0}{v_3}\right)$.	Ratio $\left(\frac{v_0}{v_6}\right)$.
	Ft. per sec.	*Feet.*			*Ft. per sec.*	*Feet.*			*Ft. per sec.*	*Feet.*		
1	1.10	8.00	1.082	1.050	1.45	9.46	1.022	1.009	1.52	9.51	1.059	1.012
2	0.45	7.17	1.062	1.036	0.97	9.37	1.029	1.017	1.48	9.51	1.071	1.002
3	0.68	7.26	1.066	0.997	0.49	9.34	1.037	1.022	-------	9.32	-------	-------
4	1.27	8.10	1.060	1.014	0.26	8.92	1.031	0.981	0.83	9.54	1.031	0.988
5	0.79	7.54	1.073	1.010	1.45	7.67	1.057	1.021	0.79	9.54	1.039	1.000
6	1.44	8.37	1.070	1.033	1.00	7.64	1.073	1.025	0.68	9.52	1.034	1.000
7	1.04	7.85	1.040	1.003	0.50	7.58	-------	-------	0.65	9.52	1.046	1.010
8	0.98	7.85	1.034	1.013	0.22	7.56	1.091	1.082	1.86	7.25	1.058	1.011
9	1.15	7.86	1.055	1.000	0.49	6.01	1.091	1.069	1.87	7.25	1.056	1.027
10	1.29	7.98	1.083	1.025	1.89	6.28	1.127	1.042	1.68	7.22	1.038	1.010
11	0.94	7.73	1.050	1.021	1.25	6.40	1.094	1.045	1.65	7.22	1.039	1.018
12	0.53	7.28	1.103	1.043	0.85	6.32	1.090	1.056	1.02	7.18	1.060	1.023
13	0.74	7.48	1.070	1.046	0.46	8.48	1.056	1.043	0.96	7.18	1.070	1.017
14	0.64	7.28	1.146	1.001	1.46	8.47	1.081	1.036	-------	7.11	-------	-------
15	0.83	7.48	1.155	1.030	0.89	8.51	1.025	1.042	0.73	5.15	1.041	1.011
16	-------	-------	-------	-------	0.20	8.46	1.139	1.030	0.67	5.15	1.045	1.015
			1.077	1.021			1.069	1.035			1.048	1.010

v_0 = velocity obtained by ordinary method.
v_3 and v_6 = velocity at three-tenths and at six-tenths depth below the surface.

MEAN VELOCITY BY INTEGRATION IN VERTICALS.

The mean velocity in the discharge section was obtained by the method of integration in the eight verticals 1, 3, 5, etc., immediately after making experiments 1, 3, 9, and 37 of series E. In the following table the velocity thus obtained is compared with that obtained by the ordinary point method in the experiment just preceding:

Comparison of mean velocity obtained by the ordinary point method and by integration in verticals in Cornell University experiment canal.

Date.	Depth.	Kind of meter.	Velocity.		$\frac{V_p - V_1}{V_p}$
			Point method (V_p).	Integration method (V_1).	
1901.	*Feet.*		*Ft. per sec.*	*Ft. per sec.*	*Per cent.*
April 1	9.08	Haskell No. 3	1.468	1.460	+0.6
April 2	6.08	do	2.208	2.225	−0.8
April 5	7.41	do	1.883	1.921	−2.0
April 11	9.26	S. Price No. 351	1.522	1.568	−3.0

It will be seen that the agreement is close, the greatest difference being 3 per cent.

RATING METERS IN STILL AND IN MOVING WATER.

The results of meter and standard-weir comparisons are given in the tables on pages 64 to 67. With a few exceptions, already mentioned, these are shown graphically in figs. 16, 17, and 18, pages 68 and 69. The straight lines 0–0′ represent in each case the weir discharge and the curved lines drawn through the plotted points represent the percentage variation of the meter discharge from the corresponding weir discharge. The meter discharge is computed from a rating table prepared from ratings in still water. If we assume that the weir discharge is correct, then these curves show the percentage difference between a rating in still water and a rating in moving water, and furnish the data for preparing a rating table for moving water from the one for still water. We have only to apply the correction to any velocity for any given number of revolutions per second, shown on the proper diagram, to obtain the velocity in moving water. This has been done, and the rating tables of the meters in still and in moving water are given in the following table. The velocity in moving water for any given number of revolutions per second is greater for all speeds of the Price meters than the velocity in still water. For the Haskell meter (a propeller instrument) the velocity is less in moving water than in still water for low velocities, and probably about the same in moving water as in still water for the higher velocities.[a] The discharges might now be computed, using the rating tables for moving water, and they would agree closely with the weir discharges. New vertical velocity curves have been found for series A, and the mean velocity ascertained from them. The shape of the new curve differs little from the old one, as the range of velocities for any curve is small. The new velocities obtained from the corrected curves are given in the second column of the table.

Rating tables of current meters in still water and in moving water in Cornell University experiment canal.

Revolutions per second.	Small Price No. 351.[b]		Large Price No. 88.[b]		Haskell No. 3.[c]		Fteley.[b]		Revolutions per second.	Haskell No. 3.[c]		Fteley.[b]	
	V	V_w	V	V_w	V	V_w	V	V_w		V	V_w	V	V_w
									1.10	1.30	1.28	1.18	1.07
									1.20	1.40	1.39	1.29	1.15
0.10	0.284	0.318							1.30	1.50	1.49	1.39	1.23
0.15	0.397	0.437	0.54	0.660					1.40	1.60	1.60	1.50	1.30
0.20	0.510	0.552	0.70	0.804					1.50	1.71	1.70	1.61	1.38
0.25	0.625	0.664	0.86	0.950					1.60	1.81	1.81	1.72	1.45
0.30	0.741	0.776	1.03	1.097					1.70	1.92	1.92	1.82	1.53
0.35	0.857	0.887	1.19	1.244					1.80	2.02	2.03	1.93	1.60
0.40	0.972	0.998	1.35	1.392					1.90	2.13	2.14	2.04	1.67
0.45	1.088	1.109	1.52	1.550					2.00	2.23	2.24	2.14	1.74
0.50	1.204	1.220	1.68	1.709	0.70	0.647	0.536	0.552					
0.55	1.319	1.332	1.84	1.874	0.75	0.698	0.589	0.598					
0.60	1.435	1.447	2.01	2.042	0.80	0.751	0.643	0.644					
0.65	1.550	1.562	2.17	2.207	0.85	0.803	0.697	0.690					
0.70	1.660	1.677	2.34	2.373	0.90	0.855	0.750	0.735					
0.75	1.782	1.796	2.50	2.538	0.95	0.907	0.804	0.779					
0.80	1.897	1.915	2.66	2.702	1.00	0.960	0.858	0.822					
0.85	2.013	2.033	2.83	2.868	1.05	1.014	0.911	0.864					
0.90	2.129	2.150	2.99	3.034	1.10	1.068	0.965	0.905					
0.95	2.244	2.266	3.15	3.199	1.15	1.121	1.018	0.946					
1.00	2.36	2.38	3.32	3.366	1.20	1.174	1.072	0.987					

[a] Recent experiments with a Fteley meter do not verify this statement.
[b] Meter held with rigid rod.
[c] Meter held with cable.

NOTE.—V=velocity, in feet per second, from still-water rating; V_w=velocity, in feet per second, from moving-water rating.

A reason for the percentage increase in the variation of the meter discharge from the weir discharge as the velocity decreases is to be found in the rating table from which the velocities are computed. The experiment of November 1 shows that the small Price meter will measure a velocity of 0.23 foot per second, as the mean velocity found from the weir is 0.229 foot per second and the meter wheel revolved with regularity at all points where it was held. The observations in the rating do not extend to this velocity, however, and do not warrant the preparation of a rating table giving velocities of less than 0.5 foot per second.

In the three ratings of small Price meter No. 351 on May 9, 1901, shown in fig. 29, page 89, there are only four observations for velocities less than 0.5 foot per second and none below 0.45 foot per second. The extension of the table to velocities less than 0.5 foot per second is probably made on the assumption that this part of the rating curve is a straight line. That this is very far from the truth can be seen from an inspection of the three curves of fig. 29. Another reason is that discharge shown by the Cornell University standard weir is probably from 1 to 3 per cent too large for low heads on the weir. Recent experiments seem to indicate this fact. These two causes, however, account for only a comparatively small part of the difference between current-meter and weir discharge at low velocities which the experiments of series A and C, and others made since the writing of the body of this paper, indicate. The failure of the meter to indicate as high a velocity when held in a very slow-moving current, as when the meter is dragged through still water in the direction of the current, is probably due to the stream lines becoming more and more oblique to the axis of the canal as the velocity decreases, and consequently their impact on the meter being less than when they moved more nearly in the direction of the axis of the canal.

ERROR IN USING AVERAGE OF BOTTOM AND SURFACE VELOCITIES FOR MEAN VELOCITY.

The following table gives the mean velocity and the average of the velocities 0.5 foot above the bottom and 0.5 foot below the surface for the experiments of series A, C, and D; also the error, in per cent, made by using the average of surface and bottom velocities instead of the mean velocity. It is seen that in all these experiments except two the mean velocity is greater than the half sum of the surface and bottom velocities, the difference varying from − 2.2 per cent to + 30.6 per cent.

Table showing relation between mean velocity and average of surface and bottom velocities as found from Cornell University standard weir and from current meters in Cornell experiment canal.

No. of experiment.	Series A.			Series C.			Series D.		
	V	V'	$\frac{V-V'}{V}$	V	V'	$\frac{V-V'}{V}$	V	V'	$\frac{V-V'}{V}$
	Feet per second.	*Feet per second.*	*Per cent.*	*Feet per second.*	*Feet per second.*	*Per cent.*	*Feet per second.*	*Feet per second.*	*Per cent.*
1	1.164	0.987	+15.3	1.409	1.412	− 0.2	1.528	1.436	+6.2
2	0.573	0.416	+27.5	0.932	0.954	− 0.2	1.528	1.436	+6.2
3				0.477	0.468	+ 0.2			
4	1.268	1.190	+ 6.1	0.265	0.231	+12.8			
5	0.841	0.749	+10.9	1.444	1.364	+ 5.6			
6	1.462	1.384	+ 5.4	1.018	0.940	+ 7.7			
7	1.056	0.965	+ 8.6	0.502					
8	1.056	0.946	+10.4	0.255	0.177	+30.6	1.905	1.807	+5.2
9				0.515	0.435	+15.5	1.905	1.779	+6.6
10				1.914	1.730	+ 9.6	1.649	1.613	+2.2
11	0.980	0.887	+ 9.5	1.283	1.129	+12.0	1.649	1.590	+3.6
12	0.643	0.476	+25.9	0.893	0.759	+15.0	0.955	0.973	−1.9
13	0.788	0.687	+12.7	0.478	0.422	+11.8	0.955	0.897	+6.1
14				1.454	1.311	+ 9.8			
15				0.915	0.784	+14.3	0.689	0.704	−2.2
16				0.229	0.179	+21.8	0.689	0.659	+4.3

V = mean velocity found from standard weir.
V' = mean of velocities 0.5 foot above bottom and 0.5 foot below surface, as found by meters, uncorrected.

From the foregoing table, and from the table on page 79, in which is given the ratio of bottom velocity to mean velocity, it appears that bottom velocity is not a desirable quantity to use in computing discharge, for it varies between too wide limits. In a stream with a stony or gravelly bottom the limits must be considerably greater than in the Cornell canal with its smooth bottom. Observations at mid depth or at six-tenths depth give better results.

COMPARISON OF RESULTS BY THREE METHODS OF OPERATING METER.

Three methods of operating meters have been used in these experiments, viz, the ordinary method, the six-tenths-depth, and the integration. A detailed account of the manner of operating the meter by each of these methods has been given on pages 62 to 63. The time required to make a discharge measurement was from thirty to sixty minutes with the ordinary method, from twenty to thirty minutes with the six-tenths-depth, and from five to ten minutes with the integration.

The results by the ordinary method in series A to D (fig. 16, p. 68) give for each meter a well-defined line which agrees with the weir line (0–0′) within about 2 per cent for velocities greater than 1.5 feet per second, the meter discharge being less than the corresponding weir discharge.

The agreement with the weir discharge is closer in series E than in series A to D, being −0.03 per cent for the Haskell meter, −1.13 per cent for Price meter No. 363, and −1.88 per cent for Price meter No. 351. The results by the six-tenths-depth method do not give as well defined a line (fig. 17) for each meter as do those by the ordinary method, the meter discharge being from 1 to 6 per cent greater than the corresponding weir discharge, and the mean being 3.5 per cent for small Price meter No. 363. If the observations with the latter meter had been taken at sixty-four-hundredths depth instead of at six-tenths depth the line representing the mean would coincide with the weir line 0–0′, and the greatest percentage variations from the mean would be −2.5 per cent and +2.5 per cent.

The results by the integration method do not give a well-defined line. The meter discharge is from 1 per cent to 9 per cent greater than the weir discharge for velocities greater than 0.5 foot per second in the experiments of series C and D, and from 0 to 5 per cent less than the weir discharge for the experiments of series A. The following table gives the true velocity as found from the weir and the velocity found by the integration method for different speeds of the meter, also the ratio of difference between these velocities and the true velocity, expressed in per cent:

Table showing effect of variation of speed of meter in integration method.

No. of experiment.	Date.	Depth.	True velocity by weir (V) per second.	Speed of meter per second.	Velocity of water by meter (V_1) per second.	$\frac{V-V_1}{V}$	Kind of meter.
	1900.	*Feet.*	*Feet.*	*Feet.*	*Feet.*	*Per cent.*	([a])
1	May 8	8.10	1.268	0.101	1.268	0.00	S. P. 351.
2	do	8.10	1.268	0.152	1.235	+ 2.60	Do.
3	do	8.10	1.268	0.270	1.200	+ 5.35	Do.
4	do	7.54	0.841	0.079	0.834	+ 0.09	Fteley.
5	do	7.54	0.841	0.055	0.842	− 0.01	Do.
6	May 9	7.85	1.056	0.056	1.028	+ 2.65	S. P. 351.
7	do	7.85	1.056	0.043	1.008	+ 4.54	L. P. 88.
8	May 15	7.73	0.980	0.032	0.978	+ 0.02	S. P. 351.
9	October 24	9.46	1.409	0.118	1.429	− 1.42	S. P. 363.
10	do	9.37	0.932	0.142	0.982	− 5.36	Do.
11	do	9.34	0.477	0.164	0.485	− 1.68	Do.
12	do	8.92	0.265	0.107	0.308	−16.23	Do.
13	October 26	7.67	1.444	0.137	1.519	− 5.20	Do.
14	do	7.64	1.[illegible]	0.135	1.079	− 5.99	Do.
15	October 27	7.58	0.5[illegible]	0.143	0.522	− 3.99	Do.
16	do	7.56	0.255	0.128	0.312	−22.35	Do.
17	do	6.01	0.515	0.142	0.555	− 7.77	Do.
18	do	6.28	1.940	0.180	2.030	− 4.64	Do.
19	do	6.40	1.283	0.154	1.338	− 4.29	Do.
20	do	6.32	0.893	0.188	0.920	− 3.02	Do.
21	October 29	8.48	0.478	0.151	0.492	− 2.93	Do.
22	October 31	8.47	1.454	0.293	1.552	− 6.74	Do.
23	do	8.51	0.915	0.258	1,002	− 9.51	Do.
24	November 1	8.46	0.229	0.185	0.319	−39.30	Do.
25	December 6	9.51	1.528	0.253	1.540	− 0.78	Do.
26	do	9.51	1.528	0.284	1.572	− 2.87	Hask. 3.
27	December 7	9.32	1.535	0.243	1.576	− 2.66	Do.
28	December 8	7.25	1.905	0.252	1.962	− 2.99	Do.
29	do	7.22	1.649	0.295	1.748	− 6.00	Do.
30	do	7.18	0.955	0.254	1.039	− 8.79	Do.
31	do	5.15	0.689	0.179	0.752	− 9.14	Do.

[a] S. P. = small Price; L. P. = large Price; Hask. = Haskell. Figures given are the numbers of the meters.

Table showing effect of speed of meter on observed velocity as found by the integration method in experiments of table on page 84.

Experiments Nos. 9 to 25, omitting Nos. 12, 16, and 24.						Experiments Nos. 1 to 8.			
Speed of meter 0.10 to 0.14.	$\frac{V-V_1}{V}$	Speed of meter 0.14 to 0.18.	$\frac{V-V_1}{V}$	Speed of meter 0.18 to 0.30.	$\frac{V-V_1}{V}$	Speed of meter 0.05 to 0.15.	$\frac{V-V_1}{V}$	Speed of meter 0.15 to 0.30.	$\frac{V-V_1}{V}$
Feet per second.	*Per cent.*	*Feet per second.*	*Per cent.*	*Feet per second.*	*Per cent.*	*Feet per second.*	*Per cent.*	*Feet per second.*	*Per cent.*
0.118	−1.42	0.142	−5.36	0.180	−4.64	0.056	+2.65	0.152	+2.60
0.137	−5.20	0.164	−1.68	0.188	−3.02	0.043	+4.54	0.270	+5.35
0.135	−5.99	0.143	−3.99	0.293	−6.74	0.032	+0.02	----------	----------
0.130	−4.20	0.142	−7.77	0.258	−9.51	0.101	+0.00	----------	----------
----------	----------	0.154	−4.29	0.253	−0.78	0.079	+0.09	----------	----------
----------	----------	0.151	−2.93	0.234	−4.94	0.055	−0.01	----------	----------
----------	----------	0.159	−6.33	----------	----------	0.061	+1.22	0.211	+3.97

This ratio is positive for all the experiments of series A and is negative for those of C and D. This difference of sign is due to two causes: (1) The meter was moved faster in the experiments of C and D than in those of series A; and (2) it was moved through only 14 feet of the width in series C and D and through nearly 15.5 feet of the width in series A, and the additional 1.5 feet of width had a small velocity. It is seen from the above table that for low velocities the error in velocity increases as the speed of the meter increases. The table on page 84 shows this effect for velocities from 0.5 foot to 2 feet per second.

COMPARISON OF RESULTS OBTAINED IN 1900 WITH THOSE OBTAINED IN 1901.

Examining the plotted results in figs. 16 (page 68) and 18 (page 69), it is seen that the meter discharge, compared with the corresponding weir discharge, is somewhat larger for the work of 1901 than for that of 1900 for velocities greater than 1.4 feet per second. This can not be due to change in rating of the meters, for an increase in the friction in a meter would reduce the observed velocity and make the meter discharge too small. There was considerably more seepage into the canal when the experiments of series E (1901) were made than at the time those of A, B, C, and D (1900) were made. There was also quite a little surface water entering the canal between the weir and the meter section in the experiments of series E. It is hardly possible, however, that this inflow between the weir and the meter section could amount to 2 per cent of the canal discharge. It must be borne in mind in this connection that the discharges obtained in 1900 for velocities greater than 1.4 feet were in shallow water, and therefore are not so reliable as those obtained in series E.

EFFECT OF METHOD OF HOLDING METER IN WATER.

In some of the experiments the meter was suspended from a cable; in others it was held in the water with a rigid rod. In the latter cases its axis was free to move about a horizontal axis; in the former cases it was free to move about a vertical as well as a horizontal axis. When used with a cable a sinker is necessary, which, being attached to the under side of the meter, keeps the axis of the revolving wheel farther from the bottom than is desirable in some cases. It is also difficult to keep the meter in any desired place when held with a cable, and it usually changes its position considerably in a swift current. It was in order to measure the velocity close to the bottom and keep the meter at a desired place during the observation that in many of the experiments the meter was held with a rod. In ordinary river gaging, however, the Price and Haskell meters are held with cables and the meters are rated in still water suspended from a car and cable. The question then arose: "Is the rating table for a meter used suspended from a cable applicable to the same meter held with a rigid rod?"

On December 30, 1900, Mr. E. G. Paul made two ratings of small Price meter No. 351 in still water, one with the meter held with a rigid rod and the other with the meter held with a cable and a 5-pound lead sinker. In the following tables (pp. 87 and 88) are given the observations and computations of these ratings. The second column ("time in seconds") gives the average time required for the car to which the meter was attached to move over a run of 100 feet, and again over the same course in the opposite direction; the third column gives the number of revolutions observed in this average time; the seventh column gives for each of these cases the difference between the observed velocity and the velocity computed from the old rating table of April 23, 1900, for the same meter held with a cable. The tenth column of the first table (p. 87) gives the difference between the observed velocities and those computed from the new rating table, which is based on meter rated in still water and held by a rigid rod. The portion of this new rating table which was used in reducing the Cornell experiments is given on page 81. By comparing the corresponding quantities in the seventh and tenth columns of this table it will be seen how much more closely the values in the new rating table fit the observations than do those of the old one. The tenth column shows also how much the relation between the revolutions per second of this meter and the velocity as shown by these observations differs from the "most probable" straight line, $y=0.047+2.313x$, y being velocity in feet per second and x revolutions of meter wheel per second. From the seventh column of the table on page 87 it is seen that the observed velocities are larger than the computed velocities, indicating more friction in the meter on December 30, 1900, than on April 23, 1900.

The "most probable" linear relation between x and y for this case of the meter held with a cable was found to be $y=0.067+2.321x$. The differences in the tenth column show that this line does not fit the observations. It crosses the curve given by the observations twice as

y increases from 1 to 8 feet per second. The rating table was prepared by using this line as a basis, applying corrections as indicated by the corresponding difference of the tenth column, and also that the rating table will difference smoothly. The numbers in column 12 show how closely the observed velocities agree with those computed from the new rating table with the meter held with a cable.

It should be noted that the smallest observed velocity is about 0.5 foot per second in one case and 0.35 foot in the other case.

Observations and computations of a rating of small Price meter No. 351 held by rigid rod, made at Chevy Chase, Md., December 30, 1900.

1.	2 and 3.		4.	5.	6.	7.	8.	9.	10.	11.
No. of observation.	Length of run, 100 feet. Time.	Length of run, 100 feet. Revolutions of meter.	Revolutions per second (x).	Observed velocity (y).	Computed velocity (y^1).[a]	Difference ($y-y^1$).	x^2	xy	Residual (v).	v^2
	Seconds.			*Feet per second.*	*Feet per second.*	*Ft. per second.*				
1	9.25	43.00	4.648	10.811	10.815	−.004	21.604	50.245	+.059	.003481
2	10.5	42.50	[b]4.048	[b]9.524	9.405	+.119				
3	16.0	43.00	2.688	6.250	6.231	+.019	7.225	16.800	−.014	.000196
4	22.5	43.00	1.911	4.444	4.414	+.030	3.652	8.493	−.022	.000484
5	35.0	42.25	1.207	2.857	2.797	+.060	1.457	3.448	+.018	.000324
6	45.0	42.00	0.933	2.222	2.183	+.039	0.870	2.073	+.017	.000289
7	53.0	42.00	0.794	1.887	1.867	+.020	0.630	1.498	+.003	.000009
8	54.5	42.00	0.774	1.835	1.823	+.012	0.599	1.420	−.003	.000009
9	68.0	41.75	0.614	1.471	1.461	+.010	0.377	0.903	+.004	.000016
10	85.5	41.75	0.488	1.170	1.174	−.004	0.238	0.571	−.006	.000036
11	106.5	41.00	0.385	0.939	0.944	−.005	0.148	0.362	+.001	.000001
12	113.0	41.00	0.363	0.885	0.891	−.006	0.132	0.321	−.002	.000004
13	154.0	40.50	0.263	0.649	0.669	−.020	0.069	0.171	−.006	.000036
14	197.0	39.50	0.201	0.508	0.522	−.014	0.040	0.102	−.004	.000016
15	220.0	39.00	0.177	0.455	0.569	−.014	0.031	0.081	−.001	.000001
16	293.0	36.90	0.126	0.341	0.351	−.010	0.016	0.043	−.003	.000009
			15.572	36.724			37.088	86.531		.004911

[a] Computed from rating table prepared from rating of April 23, 1900.
[b] Omitted in computing Σx and Σy.

NOTE.—Observation equation, $y-a-bx=v$; normal equations, $a\Sigma x+b\Sigma x^2=\Sigma xy$, $na+b\Sigma x=\Sigma y$. For these observations these become $15.572\,a+37.088\,b=86.531$, $15\,a+15.572\,b=36.724$. The solution of these gives $b=2.313$ and $a=.047$. The mean error of a single observation is $\sqrt{\frac{\Sigma v^2}{n-1}}=.019$. The probable error of result is $6{,}745\sqrt{\frac{\Sigma v^2}{n(n-1)}}=.003$.

Observations and computations of a rating of small Price meter No. 351 held by cable, made at Chevy Chase, Md., December 30, 1900.

1.	2 and 3.		4.	5.	6.	7.	8.	9.	10.	11.	12.
No. of observation.	Length of run, 100 feet.		Revolutions per second (x).	Observed velocity (y).	Computed velocity (y').[a]	Difference (y-y').	x^2	xy	Residual (v).	Computed velocity (y'').[b]	Difference (y-y'').
	Time.	Revolutions of meter.									
	Seconds.			*Ft. per second.*	*Ft. per second.*	*Ft. per second.*				*Ft. per second.*	
1	10.5	42.00	4.000	9.524	9.300	+.224	-------	-------	-------	-------	-------
2	12.5	42.50	3.400	8.000	7.900	+.100	11.560	27.200	+.042	7.98	+.02
3	16.5	42.50	2.576	6.061	5.967	+.094	6.636	15.613	+.015	6.04	+.021
4	21.5	42.75	1.990	4.651	4.588	+.063	3.960	9.256	−.034	4.68	−.029
5	26.0	42.50	1.635	3.846	3.767	+.079	2.673	6.288	−.015	3.837	+.009
6	29.0	42.50	1.465	3.448	3.383	+.065	2.146	5.051	−.020	3.443	+.005
7	31.0	42.25	1.363	3.226	3.151	+.075	1.858	4.397	−.004	3.290	+.017
8	41.0	42.50	1.037	2.439	2.411	+.028	1.075	2.529	−.034	2.451	−.012
9	50.0	42.50	0.850	2.000	1.990	+.010	0.723	1.700	−.039	2.024	−.024
10	61.0	42.00	0.688	1.639	1.624	+.015	0.473	1.128	−.024	1.655	−.016
11	70.0	42.00	0.600	1.429	1.430	−.001	0.360	0.857	.030	1.454	−.025
12	80.5	41.25	0.512	1.242	1.226	+.016	0.262	0.636	−.013	1.256	−.014
13	90.5	40.00	0.442	1.105	1.072	+.033	0.195	0.488	+.012	1.098	+.007
14	105.5	39.00	0.370	0.949	0.908	+.041	0.137	0.351	+.024	0.938	+.011
15	125.0	38.00	0.304	0.800	0.759	+.041	0.092	0.243	+.028	0.790	+.010
16	140.0	37.00	0.264	0.714	0.671	+.043	0.070	0.189	+.035	0.697	+.017
17	171.0	36.00	0.215	0.585	0.556	+.029	0.046	0.126	+.019	0.585	−.000
18	184.0	36.00	0.196	0.543	0.507	+.036	0.038	0.106	+.022	0.541	+.002
19	211.0	35.75	0.169	0.474	0.452	+.022	0.029	0.080	+.015	0.479	−.005
			18.076	43.151			32.334	76.239			

[a] Computed from rating of April 23, 1900.

[b] Computed from rating of December 30, 1900.

NOTE.—Most probable straight line given by these observations, $y=.067+2.321x$.

The following table shows the percentage difference of velocity for each 0.05 revolution for these two ways of holding the meter. It is seen that a given number of revolutions per second of this meter indicates a higher velocity when it is held with a cable than when it is held with a rigid rod, and that this percentage difference decreases as the velocity increases. In other words, in a given velocity the meter will revolve faster when held with a rigid rod than when held with a cable. It must be remembered that the values given in this table for velocities less than about 0.5 foot per second are only approximate, since the observations do not extend below 0.35 foot per second. We may say that if the rating table for this meter held with a cable be used to reduce observations made with the meter held with a rigid rod, the results will be in error from 1.3 per cent to 8.5 per cent for velocities from 1.5 feet to 0.5 foot per second.

Table showing percentage difference in velocity as shown by small Price meter No. 351 when rated in still water on a rigid rod and on a cable.

Speed of meter.	V_r	V_c	$\frac{V_r - V_c}{V_r}$	Speed of meter.	V_r	V_c	$\frac{V_r - V_c}{V_r}$
R. per sec.	*F. per sec.*	*F. per sec.*	*Per cent.*	*R. per sec.*	*F. per sec.*	*F. per sec.*	*Per cent.*
0.15	0.397	0.433	9.1	0.60	1.435	1.455	1.4
0.20	0.510	0.550	7.8	0.65	1.550	1.568	1.2
0.25	0.625	0.666	6.6	0.70	1.666	1.682	1.0
0.30	0.741	0.781	5.4	0.75	·1.782	1.796	0.8
0.35	0.857	0.894	4.3	0.80	1.897	1.910	0.7
0.40	0.972	1.006	3.5	0.85	2.013	2.024	0.5
0.45	1.088	1.118	2.8	0.90	2.129	2.138	0.4
0.50	1.204	1.230	2.2	0.95	2.244	2.252	0.4
0.55	1.319	1.342	1.7	1.00	2.36	2.37	0.3

V_r = velocity with meter rated on a rigid rod.
V_c = velocity with meter rated on a cable.

Small Price meter No. 351 was again rated three times on May 9, 1901. It was held (1) with a cable, using an 18-pound sinker; (2) with a rod, the meter being free to tip; and (3) with a rod and the meter not free to tip. Space will not permit the insertion of the observed data or the computations, but the results are plotted in fig. 29, using R= revolutions per second as abscissæ and $K=\frac{V}{R}$=ratio of velocity to revolutions as ordinates. It is seen that these rating observations give a separate curve for each of these cases. The value of K for any value of R for one curve is very different from that for the other curves for velocities less than about 3 feet per second. For example, for R=0.3, K=2.755 for case 1 (curve AE), 2.585 for case 2 (curve BE), and 2.485 for case 3 (curve DF). The coefficient is largest when the meter has the most freedom of motion and least when it has no freedom of motion. As the velocity for a given number of revolutions varies directly with K, the velocity decreases as the freedom to tip about a horizontal axis and to swing around a vertical axis decreases. Or, stated in the other way, in a current of a given velocity the revolutions increase as the freedom of motion decreases. Each of these cases therefore requires a separate rating table.

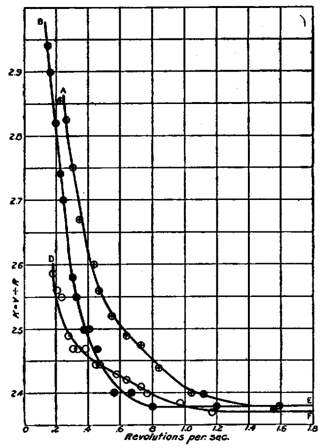


FIG. 29.—Curves showing relation between revolutions of meter and ratio between velocity of water and revolutions of meter from three ratings of small Price meter No. 351 at Chevy Chase, Md., May 9, 1901. Crossed circles are for observations with meter held with cable, an 18-pound sinker being used. Heavy black dots are for observations with meter held with rod and free to tip. Open circles are for observations with meter held with rod and not free to tip.

SURFACE VELOCITY EXPERIMENTS WITH FLOAT RODS AND WITH CURRENT METERS.

It was noticed during the weir and current-meter comparisons that the shape of the vertical velocity curve near the surface obtained with a small Price meter was quite different under certain conditions from that obtained with the Haskell meter under the same conditions. This peculiarity can be seen in the simultaneously obtained curves of fig. 30. The Haskell meter indicates a higher surface velocity than the Price meter. By surface velocity obtained with a meter is meant that obtained when the revolving part of the meter is entirely under water all of the time and yet as close to the surface as possible. For the three meters used in these surface velocity observations the distance of the center below the surface was as follows: Small Price meter No. 351, from 2.5 to 3 inches; Fteley No. 107, from 3 to 3.5 inches; Haskell No. 3, from 4 to 4.5 inches.

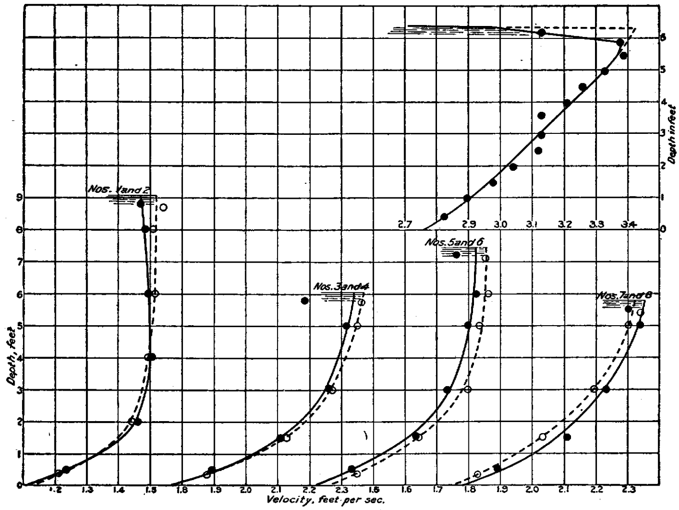


FIG. 30.—Vertical velocity curves from simultaneous experiments of series E with small Price meter No. 363 and with Haskell meter No. 3. Large black dots are for Price meter, open circles for Haskell meter.

In order to investigate this matter further the surface velocity of a part of the canal was measured simultaneously with floats and with two current meters. The floats used were 6-inch cubes of wood loaded with lead so as to have an immersion of $5\frac{1}{4}$ inches. Surface floats of wood 6 inches by 6 inches by 1 inch thick were also used. These were started near the center of the canal 18 feet above the upper wire and timed over a run of 60 feet. Any that passed the lower wire within 2 feet of either side were not used in computing velocity. The meters were held 10 feet below the lower wire, the Haskell at 5 and 7 feet from the south side of the canal, the Price at 9 and 11 feet from the south side, and the revolutions were counted for periods of forty seconds. The results of the experiments are given in the following table:

Comparison of surface velocity measured with float rods and with current meters.

Small Price meter No. 351.				Haskell meter No. 3.			
Number of float observations.	Velocity with floats.	Number of 40-second periods.	Velocity with meter.	Number of float observations.	Velocity with floats.	Number of 40-second periods.	Velocity with meter.
	Feet per second.		*Feet per second.*		*Feet per second.*		*Feet per second.*
11	3.122	10	2.794	13	3.083	13	3.171
12	3.076	12	2.806	12	3.144	12	3.187
13	3.030	13	2.818	10	3.135	10	3.165
36	3.076	35	2.806	35	3.121	35	3.174

During the foregoing observations the depth of water in canal was 6.04 feet, its mean velocity 2.8 feet, and the discharge 275.3 cubic feet per second. The float velocity that is compared with the Price meter velocity is obtained from the floats passing the lower wire from 2 to 8 feet from the south side of canal; that which is compared with the Haskell meter velocity is found from the floats passing the lower wire from 8 to 14 feet from the south side of canal. It is seen that in each experiment the float velocity is greater than that of the Price meter and less than that of the Haskell meter. The mean float velocity for the three experiments is 8.8 per cent greater than the velocity with the Price meter and 1.7 per cent less than that with the Haskell meter.

It was thought that possibly this failure of the small Price meter to give correct surface velocity indications was due to wave motion of the meter. Another experiment was therefore made with the floats and with the meter held with a rod and not free to tip. Fteley electric meter No. 107 was also used in place of the Haskell meter. The floats were started at many points in the width of the canal and were timed over a run of 50 feet. Those that came within 2 feet of either bank were not used in computing the velocity. The meters were held 24 feet below the lower wire at points from 0 to 7 feet from the center, and the revolutions were counted for 50-second periods.

The mean surface velocity shown by the small Price meter from thirty-two 50-second periods is 1.967 feet, and the corresponding float velocity obtained from ten floats passing from 2 to 8 feet from the south side of canal is 2.146 feet. The corresponding surface velocity shown by the Fteley meter is 2.074 feet, and by the floats from 8 to 14 feet from the south side of canal, 2.137 feet. The float velocity is 8.3 per cent greater than the Price meter velocity and 2.95 per cent greater than the Fteley meter velocity. The difference between the float and the Price meter velocities is about the same as in the previous experiments.

During this experiment the Price meter was held 3.5 feet from the south side of canal, with its center $2\frac{3}{4}$ inches below the surface for seven 50-second periods, and again with its center 4.5 inches below the surface for about the same length of time. The mean velocity at $2\frac{3}{4}$ inches depth was 1.884 feet, and at 4.5 inches depth it was 2.035 feet. The float velocity at this point was 2.137 feet. The indicated velocity increased 7.4 per cent as the meter was lowered $1\frac{3}{4}$ inches. The float work in this experiment is not very satisfactory.

On May 20 vertical velocity-curve data were taken with small Price meter No. 351 and with Haskell meter No. 3 for different mean velocities, with a view to determining the magnitude of this error at different velocities and depths below the surface. The Haskell meter was held with a cable 1 foot south of the center of the canal and 244 feet from the weir; the Price meter was held with a cable 1 foot north of the center of canal. The following data were obtained:

Velocities near the surface of Cornell University experiment canal as found with small Price meter No. 351 and with Haskell meter No. 3.

Number of run.	Small Price meter No. 351.[a] Depth below surface, in inches.				Haskell meter No. 3.[b] Depth below surface, in inches.		
	2.5	4.5	6.5	8.5	4.5	6.5	8.5.
Second	0.482	0.490	0.482	....	0	0	0
Third	0.556	0.595	0.575	....	0.592	0.615	0.602
Fourth	0.936	0.958	0.958	0.942	1.015	1.032	1.010
Fifth	1.297	1.281	1.320	1.299	1.348	1.325	....
Sixth	1.480	1.457	1.389	1.434	1.450	1.392	1.412
Seventh	1.611	1.556	1.533	....	1.552	1.572	....

[a] Meter held with cable 1 foot north of canal center and 244 feet from weir.
[b] Meter held with cable 1 foot south of canal center and 244 feet from weir.

Each run lasted about half an hour, during which time the mean velocity remained constant. These results indicate that for velocities less than 1.5 feet the difference between the velocities indicated by these meters is small from the surface to a depth of 8.5 inches.

In the upper right-hand corner of fig. 30 is a vertical velocity curve prepared from thirty-five 50-second observations taken in forty minutes 7 feet from the south side of the canal, at a station 220 feet from the weir, with small Price meter No. 351. It shows a decrease of 8.5 per cent in the velocity $2\frac{3}{4}$ inches below the surface.

From these float and current-meter surface velocity experiments we conclude that a small Price meter will not measure velocity correctly when its center is within 0.5 foot of the surface, if the velocity be greater than 1.5 feet per second, and that this error increases from 0 at about 0.5 foot below the surface to 8 or 9 per cent at 2.5 inches below the surface.

APPLICATION OF RESULTS TO ORDINARY RIVER GAGING.

It has been shown that the discharge of the Cornell University canal can be measured with a small Price current meter by the ordinary point method with an error of not more than 1 per cent under favorable conditions. This degree of accuracy, however, can seldom be attained in ordinary river gaging, as the conditions are not so favorable for accurate work. The bed of the Cornell canal is hard, smooth, and regular in shape; the bed of a river is frequently of soft material, into which the meter will settle, or is gravelly, stony, and irregular in shape. There are no shallow parts in the canal, the depth and velocity of which it is difficult to measure with accuracy, but frequently there are one or more of these in a river discharge section. There is a good measurable velocity in all parts of the canal for all depths, while in a river there are frequently parts in which the velocity is too small for accurate measurement. The change in velocity from one point to another in a discharge section is frequently greater in a river than in the canal, on account of obstructions. Fluctuations of the surface elevation are also larger in a river than in the canal. On these accounts it is to be expected that ordinary river discharge measurements may be several per cent less accurate than those of this canal.

In river gaging work, however, it is not alone accurate discharge measurements that are required, but it is the accurate measurement of the volume passing the gaging station each day and each month. The accuracy of daily and monthly flow of a stream depends on the accuracy of the discharge curve, or the relation between the total discharge and the river stage, and on the accuracy of measurement of the fluctuation of river stage. The latter is quite as important as the former, and improvement in it should keep pace with improvements in the former.

ACCURACY OF RESULTS.

The discharge of the Cornell University standard weir is computed from Bazin's formula. It is not possible, however, to measure the head on the weir in exactly the same way that Bazin measured the head on his weir, so that this formula is not strictly applicable to this weir. From experiments made at the Cornell hydraulic laboratory, however, Prof. G. S. Williams believes that the actual discharge of the Cornell weir will not differ from that computed by Bazin's formula by more than 1 per cent for heads up to 1.5 feet, nor more than 3 per cent for heads up to 2.5 feet.

The results given in the tables have been obtained with five current meters operated in three ways by several observers. In several of the experiments simultaneous measurements were made with two meters, so that we have several checks on the work. We have not carried any of the computations beyond the third decimal place, believing that the length of a meter observation, the method of obtaining fractional parts of a revolution of the meter wheel, the uncertainty in the exact location of the vertical velocity curve, and the pulsations in the water do not warrant even this degree of accuracy. In closely studying the results some little inaccuracies will be found, as all the results are not obtained in exactly the same way; but they are believed to be substantially correct.

In comparing the results of these experiments with results obtained by other experimenters it must be remembered that we have used smaller velocities in many of our experiments than were used by them. In Fteley and Stearns's comparisons (p. 57) the smallest mean velocity is 1.7 feet, in Henry's (p. 50) the smallest is 3 feet, in Bazin's (p. 43) the smallest is 1.25 feet, in Francis's (p. 53) it is 0.5 foot, and in Marr's (p. 48) 2.4 feet. For these and higher velocities our meter results by the ordinary method agree closely with those given by the weir. We are unable to find any comparisons by other experimenters for velocities less than 1 foot per second, except a few by Francis with rods and those at Cornell with rods (p. 51). The large disagreement between meter and weir is for the low velocities, a field into which apparently few experimenters have entered.

CONCLUSIONS.

The conclusions to be drawn from these experiments may be briefly summarized as follows:

(1) Discharge measured with current meter by the ordinary (point) method agrees with that given by the Cornell standard weir within 2 per cent for velocities above 1.5 feet per second. (See tables on pages 64 to 67 and 70.)

(2) For velocities less than 1.5 feet per second the discharge found with the Price meters is less than the corresponding weir discharge, and the difference increases rapidly as the velocity decreases. (See tables on pages 64 to 67.)

(3) For velocities less than about 1.5 feet per second the discharge found with the Haskell meter is greater than that of the weir, and the difference increases as the velocity decreases. This difference is 6 per cent for a velocity of 0.75 foot per second. (See fig. 16.)

(4) The discharge of the Cornell canal can be measured with a small Price meter, ordinary point method, with an error of not more than 1 per cent under favorable conditions, a velocity observation lasting fifty seconds being taken in each 2.3 square feet of discharge area.

(5) When the most accurate results are desired the meter should be held with a rod and not given freedom to tip.

(6) Velocities of 1.5 feet per second and upward obtained with a small Price meter when its center is closer to the surface than 0.5 foot are too small by from 0 to 9 per cent. This error, however, decreases from maximum at the surface to 0 at about 0.5 foot depth.

(7) The small Price meter will measure velocities of 1 foot per second and less more accurately than either the large Price meter or the Haskell meter. It appears from these experiments that the smallest velocity that these meters will measure with a fair degree of accuracy is 0.5 foot for the large Price meter and the Haskell meter and 0.22 foot for the small Price meter. (See fig. 16.)

(8) The small Price meter should be frequently rated, and it should be used with much care if accurate results are required.

(9) The six-tenths-depth method gives discharge from 2 to 6 per cent in excess of that by the weir, depending on the ratio of width to depth. (See fig. 17.)

(10) The integration method as a rule gives results in excess of those given by the weir, the difference increasing with the speed of the meter and decreasing as the velocity increases. (See fig. 17 and tables on pages 84 and 85.) By using special care in moving the meter at a slow speed and a uniform rate it may be possible to obtain better results by this method than those shown by these experiments. A device like Harlacher's (page 36) for giving the meter a uniform motion will increase the accuracy somewhat. We do not believe, however, that the little saving of time of this method over that by the mid-depth or the six-tenths-depth will warrant the use of any such device. The method is useful only as a rough check on one of the other methods.

(11) The thread of maximum velocity is at the surface for depths less that 2 feet and unobstructed flow at the lower end of the canal. For depths of 5 feet or more and discharge checked at the lower end of the canal this thread is from two-tenths to four-tenths depth below the surface, the mean for thirty-one experiments being thirty-one-hundredths depth. (See table on page 77.)

(12) The position of the thread of mean velocity varies from five-tenths depth for small depths to seventy-three-hundredths depth for the larger depths. For the thirty-one experiments by the ordinary method of series C and D it is sixty-four-hundredths depth below the surface. (See table on page 77.)

(13) The surface velocity is always greater than the bottom velocity with center of meter 0.25 foot above bottom. (See figs. 23 to 28.)

(14) The ratio of bottom velocity to mean velocity varies from 0.6 to 0.9, being 0.8 for the experiments of series A, 0.75 for series C, and 0.85 for series D. (See table on page 79.)

(15) The ratio of mean velocity to mid-depth velocity varies from 0.90 to 0.99, the mean of forty experiments being 0.95. (See table on page 79.)

(16) The average of the velocities 0.5 foot above the bottom and 0.5 foot below the surface is from −2.2 per cent to +30 per cent less than the mean velocity shown by the weir. (See table on page 83.)

(17) A small Price meter will revolve faster in moving water of a given velocity when held with a rigid rod than when held with a cable. (See table on page 89.) Hence the same rating table will not answer for both.

(18) The bottom velocity varies between so wide limits that it is not a desirable quantity to use in computing discharge; the mid-depth or the six-tenths-depth is better.

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