A METHOD OF CORRECTING RIVER DISCHARGE FOR A CHANGING STAGE.

By Benjamin E. Jones.

When a river is rising fast it has a greater velocity and a greater discharge than it has at the same height when its stage is constant. Likewise, when it is falling fast it has a lesser velocity and a lesser discharge. For this reason the relation of gage height to discharge which applies under conditions of constant stage will not apply during times of changing stage. This is especially true if the slope of the river is small. For, as has been known for some time,[1] the increased discharge at a given gage height during a rising stage is due to the increase in slope, and the opposite is true for a falling stage. Therefore the smaller the natural slope of the stream the greater is the proportional change in slope for any given rate of change in stage.

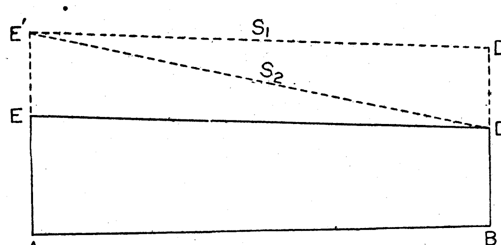

Figure 19.—Theoretical slope diagram.

The current-meter measurement gives the actual mean discharge of the stream during the time of the measurement, even during a change of stage, provided the change is uniform over that period. Changes that are not uniform may be allowed for by reading the gage frequently and correcting the results. But although the mean discharge during the time of measurement is thus obtained, it does not correspond to the discharge for the mean gage height for the period.

This may be shown in figure 19, where E–D represents the slope of the stream at a constant stage and at a gage height E. E′–D′ represents the slope under similar conditions at a gage height E′ and may be the same as or different from E–D. Suppose there is a rise at the gage at station A from E to E′, and this rise has just reached B, but the river has not started to rise at B. Then there is a gage height at station A of E′ and a slope of E′–D, whereas under

[1] The idea that the rate of change of stage divided by the velocity would give the increase in slope due to this rate of change of stage was suggested several years ago by F. F. Henshaw, and later, independently, by R. R. Randell, and possibly by other members of the United States Geological Survey.

normal conditions for a gage height of E′ there should be a slope of E′–D′.

Chezy's formula for the flow of water in open channels is $V=C\sqrt{RS}$. Let the slope E′–D′ equal S_1 at a velocity V_1 and at a gage height E′. Let V_2 be the velocity with a slope E′–D, equal to S_2, at the same gage height. The hydraulic radius R will be the same in both cases, and as C depends on the channel conditions it also will be the same in both cases. There may possibly be a slight change in C, owing to the small increase in velocity, but it is so small that it is negligible, as the results will show. Then at gage height E′ under normal conditions $V_1=C\sqrt{RS_1}$, and for gage height E′ during changing stage $V_2=C\sqrt{RS_2}$. Combining the two, we have[1]

$$\frac{V_1}{V_2}=\frac{\sqrt{S_1}}{\sqrt{S_2}}$$

Assume at first that the normal slope at gage heights E and E′ is the same. Let E–E′ represent the rise in stage in one second of time, and A–B the distance the water travels in one second of time. Then $\frac{E-E'}{A-B}$ equals the increase in slope and $\frac{E-E'}{A-B}+S_1=S_2$.

As E–E′ represents the change of stage per second—that is, the rate of change of stage—and A–B represents the distance the water travels in one second, or the velocity, the expression may be written $S_2=S_1+\frac{\text{rate of change of stage}}{\text{velocity}}$. Substituting this value of S_2 in the formula gives

$$\frac{V_1}{V_2}=\frac{\sqrt{S_1}}{\sqrt{S_1+\frac{\text{rate of change of stage}}{\text{velocity}}}}$$

If the normal slope at E′ differs from that at E, the same results can be obtained more easily by the use of the calculus. In figure 20, let h represent the initial stage, Δh the increment in stage, V the initial velocity, and ΔV the increment in velocity corresponding to the increase in stage Δh in the time Δt. Then the increase in slope ΔS is seen to be

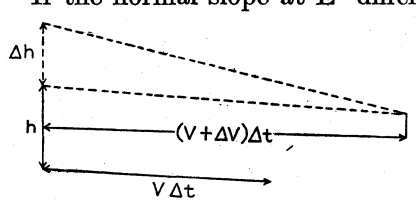

FIGURE 20.—Theoretical slope diagram.

$$\Delta S=\frac{\Delta h}{(V+\Delta V)\Delta t}=\frac{\frac{\Delta h}{\Delta t}}{V+\Delta V}$$

[1] See Hall, M. R., Hall, W. E., and Pierce, C. H., A method of determining the daily discharge of rivers of variable slope: U. S. Geol. Survey Water-Supply Paper 345, pp. 53–65, 1914.

Passing to limits, we have

$$dS=\frac{\frac{dh}{dt}}{V+dV}$$

that is, the increase in slope equals the rate of change of stage divided by the velocity, the same as before.

As at the given stage of E′ the area is constant, the formula may be written

$$\frac{Q_1}{Q_2}=\frac{\sqrt{S_1}}{\sqrt{S_1+\frac{\text{rate of change of stage}}{\text{velocity}}}}$$

Now, the question arises, what velocity should be used? Let A and B, in figure 21, represent two points a short distance apart in a channel. The water at A is traveling at the different rates X_1, X_2, etc., depending on the depth, as indicated by the curve. The surface water is arriving at B at the velocity X_1. Now, suppose more water is turned in at A, raising the stage and the head. Both the mean and surface velocities are increased to carry away this excess. Then that part of the increased flow which travels at the rate of the surface velocity—that is, approximately the upper third of the stream—will arrive at B at the rate of the surface velocity. As fast as this increased flow reaches B, it increases the head by raising the stage and increases R by increasing the cross section. Thus the mean velocity at B is increased, not at the rate of the slow-moving mean velocity at A, but more nearly at the rate of the surface velocity. For this reason it is assumed that the flood travels very nearly at the rate of the surface velocity.

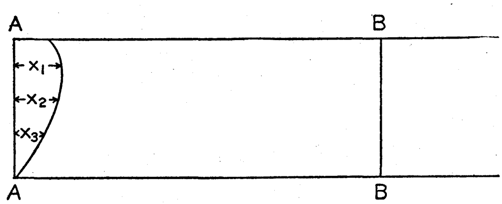

FIGURE 21.—Theoretical vertical velocity curve.

The relation of surface velocity to mean velocity will be the same in this case as under conditions of constant stage, for the mean velocity used is the actual mean velocity measured during the rise. This velocity is automatically governed by the slope, which in turn is governed by the velocity of the flood. If the rise or fall is fairly uniform, the change in slope is fairly uniform, and the relation of mean velocity to surface velocity under such conditions should be exactly the same as if the slope were governed by some other cause. Therefore, to obtain the velocity of the flood—that is, the surface velocity—the mean velocity of the measurement should be divided by 0.9 for large streams and by 0.85 for smaller ones. The formula then finally becomes

$$\frac{Q_1}{Q_2}=\frac{\sqrt{S_1}}{\sqrt{S_1+\frac{\text{rate of change of stage}}{\text{surface velocity.}}}}$$

If K stands for the change of stage per second, V for the mean velocity of the measurement made during the changing stage, and N for the coefficient for obtaining mean velocity from surface velocity, the formula may be written

$$\frac{Q_1}{Q_2}=\frac{\sqrt{S_1}}{\sqrt{S_1+\frac{K}{\frac{V}{N}}}}=\frac{\sqrt{S_1}}{\sqrt{S_1+\frac{NK}{V}}}$$

The main difficulty to be met in the field observations is in obtaining the slope accurately. Although the slope at a station usually varies with the gage height, even under conditions of constant stage, still at any given gage height under normal conditions the slope is practically permanent. This is especially true of medium and high stages, at which rapid changes of stage are most likely to occur. By making careful observations at different gage heights, being sure to allow for changing stage, and plotting these observations the engineer can obtain a fairly accurate slope curve for each station.

Any error in S_1 enters into both numerator and denominator. Suppose S_1 were really 0.04 and S_2 were 0.05, but that S_1 were determined as 0.05, which would make S_2 0.06. Here is an error of 25 per cent. But the factor $\frac{Q_1}{Q_2}$ is 0.895 in the first case and 0.913 in the second, an error of about 17 per cent in the correction and of only 2 per cent in the total discharge.

It is obviously necessary that the change of stage and mean velocity must be observed at the same section. If the permanent gage is not at the measuring section, it will be necessary to install a temporary gage to use during the measurement; or, if the cross section at the gage is available, the mean and surface velocity at the gage can be obtained by dividing the discharge from the measurement by the area at the gage.

During the summer of 1914 some experiments were made to test the method, and tables and curves have been prepared giving the results of these tests.

The first experiments were made on Agency ditch, on the Fort Belknap Indian Reservation, near Harlem, Mont. This canal has a capacity of 100 second-feet, but only about half of this flow could be obtained. For a gage, a 2 by 4 inch post graduated to tenths was driven into the bed of the stream near the left bank. Measurements were all made by wading about 2 feet above the gage. It was difficult to read the gage accurately, but the error is probably not over a couple of hundredths in any one reading. The channel below the gage is straight for several hundred feet, with a few weeds in the bottom. In obtaining the slope, the level rod was set firmly on the bottom, the surface of the water read, and this reading subtracted from the rod reading. This operation was repeated several times as a check. Readings were taken 100 feet up and down stream from the gage, and the mean was taken as the slope. This was only a rough method, but it gave fairly good results. Four measurements were

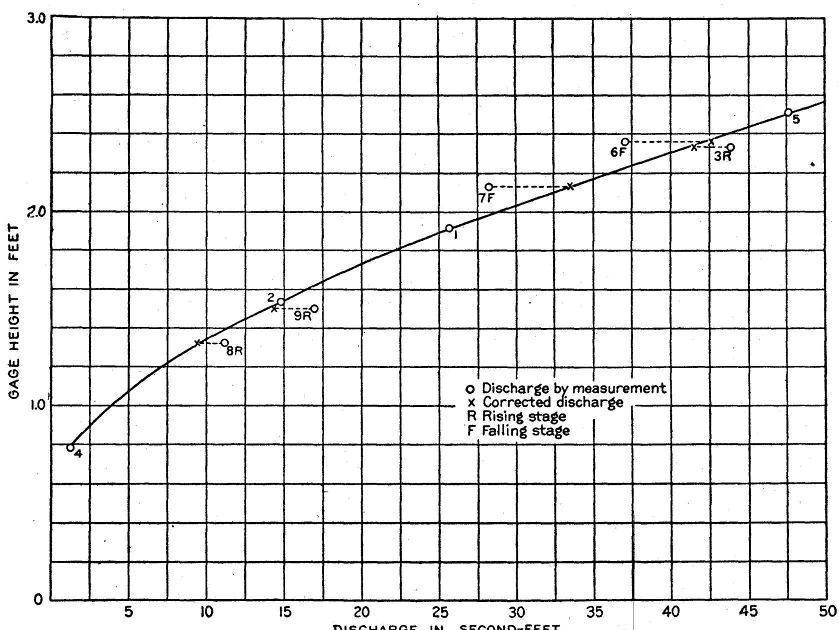

FIGURE 22.—Rating curve for Agency ditch near Harlem, Mont.

made at a constant stage, and five during a changing stage. A curve was drawn through the former and assumed to be correct. Figure 22 shows the measurements plotted to the measured and corrected discharge. Table 1 shows the results of the measurements and the error in those made during a changing stage, compared with the curve for a constant stage. It also shows the percentage of error after the measurements were corrected.

From August 1 to 6, 1914, 13 measurements were made on Little Missouri River near Alzada, Mont. During this time the river rose from a stage of 3.2 feet to 11.3 feet and then fell to 2.5 feet. All but two of the measurements were made during a changing stage of considerable amount. This station is located about 150 feet above a riffle, and the water at the gage is considerably deeper than on the riffle. The slope and velocity at the gage are therefore very small at low stages, and as the stream rises both increase rapidly at first and then become nearly constant. The slope was determined in the manner described above at five different gage heights, allowance being made for changing stage. A slope curve was drawn to average these five points as shown, and the slope taken from it. The results are not large enough to show up well on a small scale, but Table 2 and figure 23 together give a very good idea of the results. Particular attention is called to measurements 14 and 16, also to 9 and 20.

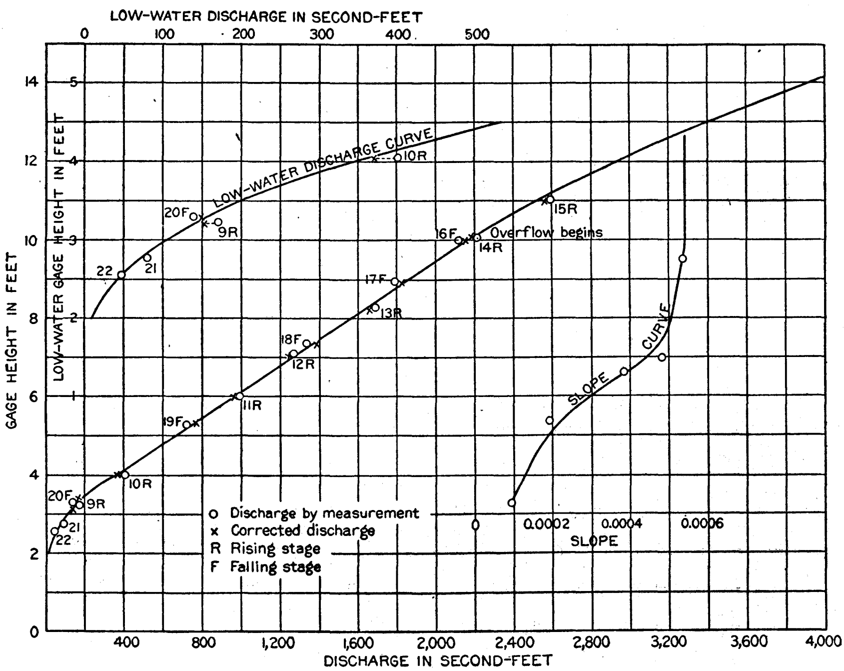

FIGURE 23.—Rating curve for Little Missouri River near Alzada, Mont.

These measurements were made at nearly the same gage height, 9 and 14 being on a rising stage and 16 and 20 on a falling stage. The rate of change for 14 and 16 is much greater than for 9 and 20, but on account of the increased slope and high velocity the percentage of error in 14 and 16 is small. On the other hand, although for 9 and 20 the change of stage is slight, on account of the small slope and low velocity the percentage of error is high. The method takes care of these two extreme cases fairly well, and, although the corrected discharges for 9 and 20 do not agree exactly, they are nearly as good as the average measurement with a constant stage. The error is probably due to errors in obtaining the slope, which is very slight and difficult to obtain accurately, for the correction in both cases is too small.

Table 3 and figure 24, showing the application of the method to the measurements on Ohio River at Wheeling, W. Va., are given, not as a proof of the method, but to show the possibilities. The slope at this station is not available, but the Army engineers give the mean slope of the river bed below Wheeling at 0.6 foot to the mile, and it has been assumed that at high stages the surface slope would be practically that of the bed. The rating curve with which the results were compared was drawn as a mean between the measurements made on rising and falling stages. The results show that the curve as drawn is as good as could be obtained by this method, but it is seldom that measurements so well distributed on both rising and falling stages are available.

In addition to this application of the method for correcting discharge measurements made during a changing stage it can be used to determine the actual daily discharge during rapidly fluctuating stages. This determination is especially important on streams where flood control is under consideration, for to study the effect of storage on the peak of a flood the actual daily discharge must be known and also the discharge at all stages, whether rising, constant, or falling. The assumption that the errors due to a rising stage are balanced by those due to the corresponding falling stage may lead to large errors in the estimates of daily discharge. Furthermore, the method will be equally useful in showing for any gaging station what rates of change of stage may be disregarded without introducing an appreciable error in the result.

In order to compute the actual discharge from gage heights observed during a changing stage it is necessary to have the cross section at the gage and a slope curve in addition to the discharge curve and mean-velocity curve for constant stage. Then the discharge may be assumed to be equal to that at the same gage height at a constant stage, and the approximate mean velocity, surface velocity, and corrected discharge may be obtained. The corrected discharge may then be used in obtaining a new surface velocity and a second correction obtained. This can be repeated until the error is negligible, but once will usually be sufficient.

As an example, take measurement No. 5 on Ohio River at Wheeling. (See Table 3.) The mean gage height is 28.2 feet, for which the rating table gives a discharge of 205,000 second-feet at constant stage. Dividing this by the area in square feet (38,890), to get the mean velocity and by 0.9 to obtain the surface velocity, and then applying the formula with the observed rate of change of +0.68 foot per hour, we get a first approximation of 232,000 second-feet.

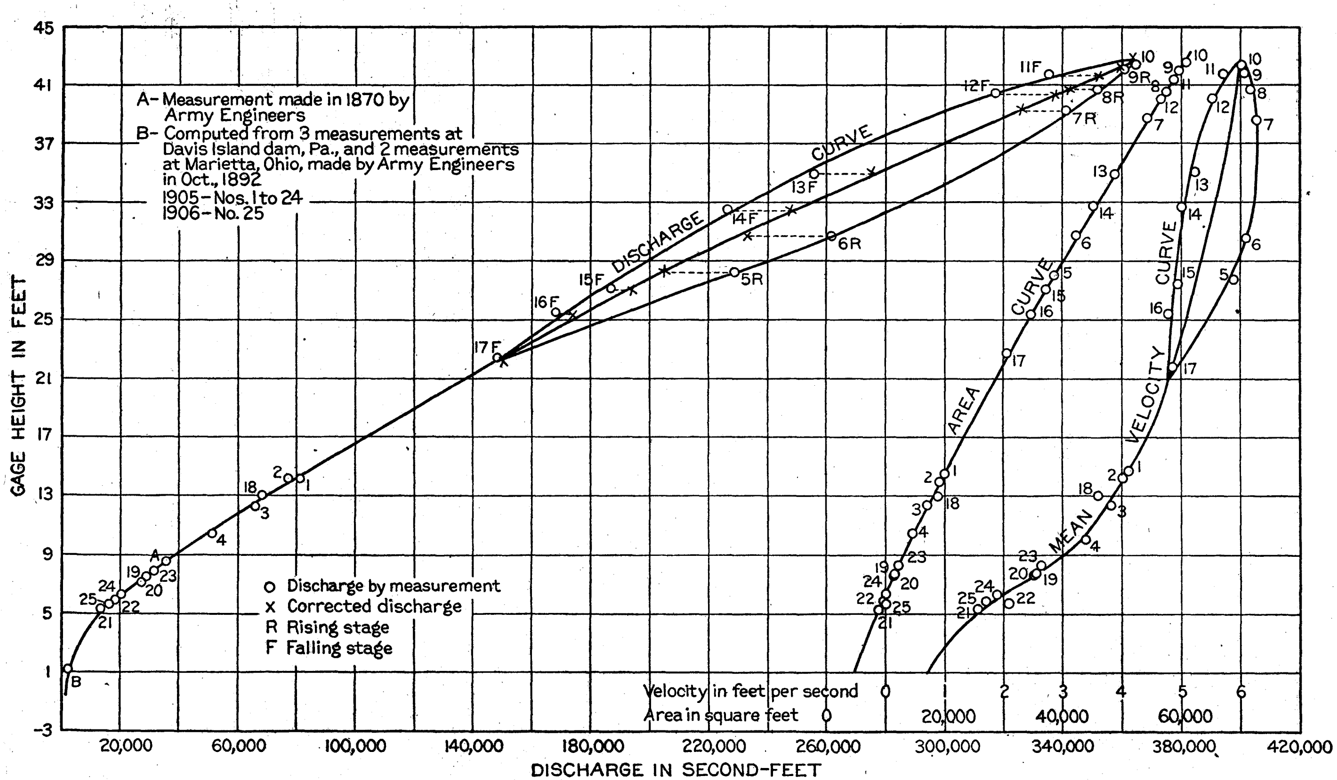

Figure 24.—Discharge, area, and mean-velocity curves for Ohio River at Wheeling, W. Va., with measurements at changing stage plotted to both measured and corrected discharge. From Hoyt and Grover's "River discharge."

$$\frac{205{,}000}{38{,}890}=5.27=\text{mean velocity at constant stage}=V.$$

$$Q_1=205{,}000$$
$$S_1=0.0001135$$
$$K=+\frac{0.68}{3{,}600}=+0.000189$$
$$N=0.90$$

By tho formula

$$\frac{Q_1}{Q_2}=\frac{\sqrt{S_1}}{\sqrt{S_1+\frac{NK}{V}}}$$

$$\frac{205{,}000}{Q_2}=\frac{\sqrt{0.0001135}}{\sqrt{0.0001135+\frac{0.000189\times 0.9}{5.27}}}$$

$$Q_2=232{,}000$$

Dividing again by 38,890 to obtain a second approximate mean velocity and proceeding as before gives a second approximation of 229,000 second-feet as the discharge at this gage height and rate of change of stage.

$$\frac{232{,}000}{38{,}890}=5.97=V$$
$$Q_1=205{,}000$$
$$S_1=0.0001135$$
$$K=0.000189$$

Substituting in the formula, we have

$$\frac{205{,}000}{Q_2}=\frac{\sqrt{0.0001135}}{\sqrt{0.0001135+\frac{0.000189\times 0.9}{5.97}}}$$

$$Q_2=229{,}000$$

The current-meter measurement showed a discharge of 229,200 second-feet.

If many discharges are to be corrected, a table can be prepared giving corrections for different rates of change at different stages.

In the tables all the data are given. In Table 1 in the column headed "Discharge by rating curve" is given the discharge by the constant-stage rating curve for the mean gage height of the measurement. The rating curve is based on measurements 1, 2, 4, and 5, all made under conditions of constant stage. This discharge from the rating curve is compared with the actual discharge as shown by the current-meter measurement, and the percentage of error in the latter is shown in the next column. Then the measured discharge after being corrected for changing stage is given, together with its percentage of error, when compared with the discharge by the rating curve. The slope given is the one used in the computations. In the last column is given the coefficient used for reducing surface velocity to mean velocity.

Table 2 gives the same items as Table 1, except that the rating curve from which the discharges at constant stage were obtained was drawn as a mean between measurements made on rising and falling stages. In Table 3 the discharges at constant stage were obtained directly from a rating table prepared several years ago for this station. They are given in the column headed "Discharge by rating table."

Figure 22 shows the measurements on Agency ditch, with the discharge as measured and also after being corrected for changing stage. The curve is drawn through the measurements at constant stage and is the one referred to in Table 1.

Figure 23 is similar to figure 22 except that it has a slope curve and a low-water discharge curve plotted to a large scale. All the measurements at the higher gage heights were made under conditions of changing stage, and the curve is drawn so as to average them as nearly as possible.

Figure 24, taken from Hoyt and Grover's "River discharge," is similar to the others, with the addition of the area and mean-velocity curves. At the upper end the outer curve or loop shows the actual discharge during the rising and falling stages for that particular flood. Measurements 5 to 17 were all made in a period of seven days. The inner line, a continuation of the lower curve, shows the discharge at constant stage.

As all the data shown were gathered under ordinary field conditions, the results should give a good idea of the reliability of the method in actual use, and the tables and curves offer fairly good proof of its accuracy. If sufficient care is taken in obtaining a good slope curve, as good or better results should be obtained by any hydrographer with no other equipment than he ordinarily carries into the field. Taken as a whole, the results shown are nearly as close as the average measurement at constant stage. Care must be exercised in applying the method to stations affected by backwater from dams or similar obstructions. If the the cross section at the gage is very large compared with the cross section on the dam or riffle, because of this backwater, the method outlined should not be applied at the gage without considering and allowing for conditions below the gage.

All the computations given have been made on a slide rule.

Acknowledgments are due to Mr. C. R. Hauke, assistant engineer for the United States Indian Office, for the use of Agency ditch in making the experiments and for furnishing a man to assist in raising and lowering the head gates; also to Messrs. R. R. Randell, W. A. Lamb, and A. H. Horton for assistance, suggestions, and criticisms, and to Mr. C. H. Pierce for reading and criticising the finished paper.

TABLE 1.—*Measurements made during changing stage on Agency ditch near Harlem, Mont., July, 1914.*

No.	Date.	Gage height.			Time.		Gage-height change, rate per hour.	Area.	Mean velocity.	Discharge by measurement.	Discharge by rating curve.	Per cent error of actual measurement.	Corrected discharge.	Per cent error of corrected measurement.	Slope.	Coefficient for surface velocity.
		Begin.	End.	Mean.	Begin.	End.										
		Feet.	*Feet.*	*Feet.*			*Feet.*	*Sq. ft.*	*Ft. per sec.*	*Sec.-ft.*	*Sec.-ft.*		*Sec.-ft.*			
1	8	[a] 1.91	1.91	1.91			0.00			25.6	25.6	0.0				
2	8	1.53	1.53	1.53	1.45 p. m.	2.05 p. m.	.00	13.20	1.12	14.75	14.8	.0				
3	8	2.30	2.37	2.335	5.12 p. m.	5.25 p. m.	+.32	26.85	1.635	43.89	41.3	+ 6.3	41.6	+0.7	0.0004	0.85
4	9	.78	.78	.78	9.00 a. m.	9.20 a. m.	.00	2.00	.60	1.21	1.2	.0				
5	9	2.53	2.50	2.51	7.35 p. m.	8.00 p. m.	−.07	30.25	1.575	47.65	47.6	.0				
6	9	2.47	2.25	2.36	8.06 p. m.	8.29 p. m.	−.57	27.00	1.37	37.08	41.7	−11.1	42.6	+2.2	.0004	.85
7	9	2.24	2.02	2.13	8.31 p. m.	8.53 p. m.	−.60	23.10	1.22	28.23	33.3	−15.2	33.5	+ .6	.0004	.85
8	10	1.23	1.42	1.325	9.45 a. m.	10.01 a. m.	+.71	10.64	1.047	11.12	9.6	+15.8	9.4	−2.1	.00048	.85
9	10	1.43	1.56	1.495	10.02 a. m.	10.11 a. m.	+.87	13.64	1.24	16.96	14.0	+21.1	14.3	+2.1	.0004	.85

[a] Discharge obtained from measurement one-half mile upstream.

NOTE.—Surface slope measurements: Gage height 0.78, slope 0.0006; gage height 1.53, slope 0.0004.

TABLE 2.—*Measurements made during changing stage on Little Missouri River near Alzada, Mont., August, 1914.*

No.	Date.	Gage height.			Time.		Gage-height change, rate per hour.	Area.	Mean velocity.	Discharge by measurement.	Discharge by rating curve.	Per cent error of actual measurement.	Corrected discharge.	Per cent error of corrected measurement.	Slope.	Coefficient for surface velocity.
		Begin.	End.	Mean.	Begin.	End.										
		Feet.	*Feet.*	*Feet.*			*Feet.*	*Sq. ft.*	*Ft. per sec.*	*Sec.-ft.*	*Sec.-ft.*		*Sec.-ft.*			
9	1	3.17	3.28	3.23	2.40 p. m.	3.28 p. m.	+0.137	130	1.32	172	145	+18.6	153	+5.3	0.000095	0.85
10	1	3.94	4.16	4.05	5.50 p. m.	6.46 p. m.	+ .236	166	2.42	402	366	+ 9.8	371	+1.4	.000134	.85
11	2	5.89	6.11	6.00	4.50 a. m.	6.10 a. m.	+ .165	273	3.62	988	980	+ 0.8	970	−1.0	.000300	.90
12	2	6.94	7.19	7.06	11.37 a. m.	1.10 p. m.	+ .161	336	3.78	1,270	1,290	− 1.6	1,260	−2.3	.000450	.90
13	2	8.11	8.47	8.29	6.38 p. m.	8.15 p. m.	+ .222	416	4.06	1,690	1,660	+ 1.8	1,670	+ .6	.000510	.90
14	3	9.89	10.21	10.05	4.17 a. m.	6.20 a. m.	+ .156	538	4.09	2,200	2,180	+ 0.9	2,180	.0	.000540	.90
15	3	10.89	11.12	11.00	10.52 a. m.	1.00 p. m.	+ .108	609	4.25	2,590	[a] 2,520	[a] + 2.8	[a] 2,570	[a] +2.0	.000540	.90
16	4	10.11	9.87	9.99	5.10 a. m.	6.50 a. m.	− .144	531	3.99	2,120	2,160	− 1.9	2,140	− .9	.000540	.90
17	4	9.22	8.64	8.93	10.03 a. m.	11.40 a. m.	− .359	458	3.91	1,790	1,840	− 2.7	1,830	− .5	.000525	.90
18	4	7.74	6.97	7.36	1.30 p. m.	2.57 p. m.	− .53	352	3.81	1,340	1,380	− 2.9	1,390	+ .7	.000471	.90
19	4	5.44	5.14	5.29	6.30 p. m.	7.30 p. m.	− .30	230	3.17	729	770	− 5.3	770	.0	.000218	.90
20	5	3.32	3.27	3.29	5.54 a. m.	6.51 a. m.	− .0526	131	1.08	141	155	− 9.0	150	−3.2	.000098	.85
21	5	2.78	2.78	2.78	4.00 p. m.	4.35 p. m.	.0	109	.75	82	78	+ 5.1				
22	6	2.57	2.56	2.56	4.40 a. m.	5.15 a. m.	− .017	100	.49	49	51	− 3.9				

[a] Error due to using mean of soundings for measurements 12 to 14.

TABLE 3.—*Measurements made during changing stage on Ohio River near Wheeling, W. Va., March, 1905.*

No.	Date.	Mean gage height.	Gage-height change, rate per hour.	Area.	Mean velocity.	Discharge by measurement.	Discharge by rating table.[a]	Per cent error of actual measurement.	Corrected discharge.	Per cent error of corrected measurement.	Slope.	Coefficient for surface velocity.
		Feet.	*Feet.*	*Sq. ft.*	*Ft. per sec.*	*Sec.-feet.*	*Sec.-feet.*		*Sec.-feet.*			
5	20.........	28.2	+0.68	38,890	5.89	229,200	205,000	+11.8	205,000	0.0	0.0001135	0.90
6	20.........	30.8	+ .60	42,750	6.13	261,900	232,000	+12.9	237,000	+2.2	.0001135	.90
7	21.........	38.9	+ .37	54,780	6.23	341,100	320,000	+ 6.6	321,000	+ .3	.0001135	.90
8	21.........	40.7	+ .20	57,360	6.18	354,400	341,000	+ 3.9	342,000	+ .3	.0001135	.90
9	22.........	42.05	+ .05	59,580	6.07	361,600	357,000	+ 1.3	358,000	+ .3	.0001135	.90
10	22.........	42.5	+ .05	60,510	6.05	365,700	362,000	+ 1.0	362,000	.0	.0001135	.90
11	23.........	41.6	− .20	58,830	5.73	336,900	351,000	− 4.0	351,000	.0	.0001135	.90
12	23.........	40.3	− .27	56,790	5.60	318,100	336,000	− 5.3	336,000	.0	.0001135	.90
13	24.........	35.2	− .35	49,250	5.20	255,800	279,000	− 8.3	277,000	− .7	.0001135	.90
14	24.........	32.7	− .40	45,550	4.99	227,300	252,000	− 9.8	250,000	− .8	.0001135	.90
15	25.........	27.2	− .23	37,560	4.95	186,100	195,000	− 4.6	197,000	+1.0	.0001135	.90
16	25.........	25.5	− .14	35,050	4.80	168,100	179,000	− 6.1	174,000	−2.8	.0001135	.90
17	27.........	22.44	− .05	30,830	4.83	149,100	150,000	− .6	151,000	+ .7	.0001135	.90

[a] From U. S. Geol. Survey Water-Supply Paper 169, p. 23, 1906.

NOTE.—List of measurements taken from Hoyt and Grover's "River discharge," p. 98.

NOTE.—The foregoing paper was read at a conference of the engineers connected with the water-resources branch of the Geological Survey held in Washington December 12 to 19, 1914, and was discussed as follows:

Mr. Johnson said that on a stream where the gage is a short distance above rapids and where the water has a high velocity a change in stage does not affect the plotting of the measurement. He cited localities in the St. Lawrence River basin to illustrate the point.

Mr. Lamb said that at the localities mentioned by Mr. Johnson the slope and velocity were necessarily high because the gages were above rapids, and that a given change in stage would consequently make a small percentage of change in the slope and would therefore make the coefficient derived from the formula approach unity; on the other hand, if the slope were small and the velocity low, the same rate of change in stage would make a greater change in slope and also a greater change in the ratio of the normal slope to the slope during a changing stage, which would result in a lower coefficient for rising stages and a higher coefficient for falling stages derived from the formula. He said also that the principles involved in the formula are applicable for all slopes, velocities, and rates of change in stage, but that the practical application is confined largely to streams of medium and low velocities.

Mr. Lamb said that where the rise or fall of a stream continued over one or more days an appreciable error would be introduced in the daily discharges if corrections for the changing stages were not applied.

In this connection the following extract from an article by Sir William Willcocks, published in the Engineering Record of July 4, 1914, page 19, commenting on the report of the Pittsburgh Flood Commission, was read:

"CAPACITY OF RESERVOIRS.

"Still, in an important city like Pittsburgh, in addition to whatever you do, something in the way of reservoirs ought to be done if it can be done. In a recent book which you have written on this reservoir question I see that the quantity of water you consider necessary to impound in these reservoirs seems to be in excess of what you need. In all the calculations it has been assumed that when the river rises its discharge increases up to its maximum gage. As a matter of fact, it does just the opposite. When it is rising fast it has a great velocity and a great discharge, but when it comes to within 7 or 8 feet of the top the velocity has begun to decrease and with it the discharge. On the Tigris, which jumps up and down very much like this, when the river gage is 15 feet rising, the discharge is 180,000 second-feet. When it has risen to 20 feet and reached its maximum for that rise, its discharge is 120,000, and when it has come down on the other side to 15 feet the discharge is 90,000 second-feet. As in all these estimates you have allowed for an increasing discharge and not reduced by half for the falling gage, a much smaller quantity of water than you have assumed would suffice to shelter you from these hours of high flood which produce all the worry."

It was stated that, while the conclusions reached by Sir William Willcocks in this particular case might be questioned, they emphasize the fact that a changing stage does affect the discharge, and that the errors introduced may be much greater than the errors introduced in computing daily discharge by applying the mean gage height to the rating table rather than by taking the average of the hourly discharges, also much greater than the error due to diurnal fluctuation.