Depth-Discharge Relations of Alluvial Streams—Discontinuous Rating Curves

GEOLOGICAL SURVEY WATER-SUPPLY PAPER 1498-C

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By DAVID R. DAWDY

STUDIES OF FLOW IN ALLUVIAL CHANNELS

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DEFINITIONS

Median diameter of bed material. That size for which 50 percent of bed material is finer. Size is determined either by sieve or visual accumulation-tube analysis.

Sand-channel stream. A stream which has an unlimited source of sand of a given size available to the channel. The topmost bed layer consists statistically of grains of the same size.

Standard deviation of bed material. Obtained from formula $\frac{1}{2}[(D_{84}/D_{50})+D_{50}/D_{16})]$ where D_{16}, D_{50}, and D_{84} denote size of bed material of which 16 percent, 50 percent, and 84 percent, respectively, are progressively finer in a given sample or combination of samples.

SYMBOLS

A area of cross section of stream

C Chezy roughness parameter; $C=V/\sqrt{RS}$

F Froude number; $\mathsf{F}=V/\sqrt{gR}$

g gravity constant

k intercept constant in relation $V=k\,R^{1/2}$ at each station

n Manning roughness parameter; $n=1.5\,R^{2/3}\,S^{1/2}/V$

R hydraulic radius, approximated by $R=A/\left(W+2\frac{A}{W}\right)$

S stream-channel slope, in feet per foot, generally abstracted from topographic maps

V mean velocity of the stream

W top width for individual measurement

σ standard deviation of bed material

STUDIES OF FLOW IN ALLUVIAL CHANNELS

DEPTH-DISCHARGE RELATIONS OF ALLUVIAL STREAMS—DISCONTINUOUS RATING CURVES

By David R. Dawdy

ABSTRACT

A discontinuity occurs in the depth-discharge relation of many alluvial streams. For the higher part of the relation, after the discontinuity, Froude number and Chezy C remain constant. For gaged sites, Froude number may be used for the extension of the relation. For ungaged sites, Chezy C may be estimated on the basis of bed-material properties.

THE PROBLEM OF RATING ALLUVIAL STREAMS

The development of methods for determining the discharge of alluvial streams has been a problem since the beginning of systematic stream gaging in this country. The usual approach is to relate discharge to stage and thus to determine discharge from a record of stage. Inspection of the stage-discharge relation shows that the relation is poor for many streams. On some streams, and particularly on many alluvial streams, an abrupt discontinuity occurs.

The Geological Survey began laboratory studies of the mechanics of flow in alluvial streams at Colorado State University in 1956. These studies are conducted by D. B. Simons and E. V. Richardson. One of the primary contributions of the laboratory study has been the definition of the various regimes of configuration of the sand-bed channel. These are described below.

Regime of flow	*Descriptions*
Plane bed	For flow prior to movement.
Ripples	Small, uniform sand waves, with little sediment movement.
Dunes	Much larger, more irregular sand waves, with a great deal of turbulence.

Regime of flow	*Descriptions*
Plane-bed transition	Dunes are smoothed out. Both the water surface and the bed are plane, with little turbulence.
Standing waves	Both the water surface and the bed are characterized by standing waves, often termed "sand waves."
Antidunes	The "sand waves" move upstream, until at some critical point they break, then reform.

Idealized diagrams of these various regimes of flow are shown in figure 1.

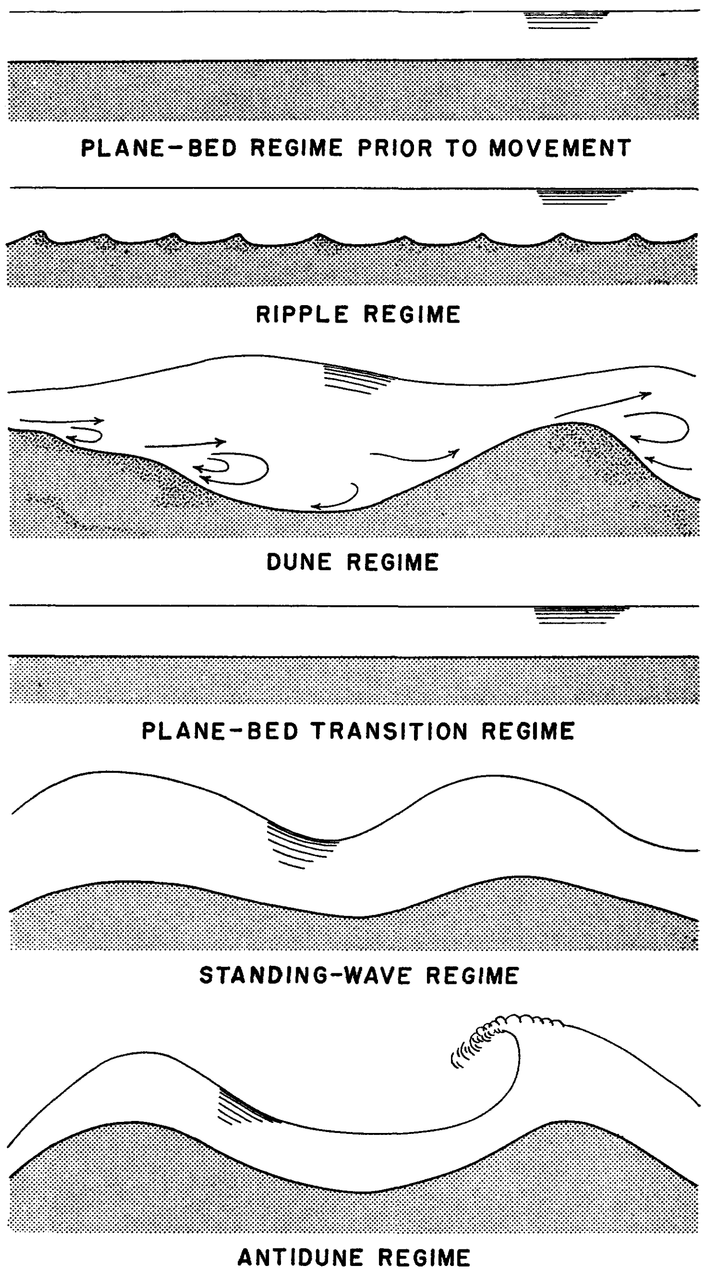

FIGURE 1.—Idealized diagram of bed and surface configuration of alluvial streams with various regimes of flow.

For the laboratory data there is a distinct discontinuity in the relation of $V \propto \sqrt{RS}$, where V is the mean velocity, R is the hydraulic radius, and S is the channel slope which is assumed to equal the energy slope. The first three regimes, blend into each other in such a relation; the last three also blend together. The discontinuity occurs between the dune and plane-bed transition regimes. This is shown graphically by the flume data for a 0.45-mm sand in figure 2.

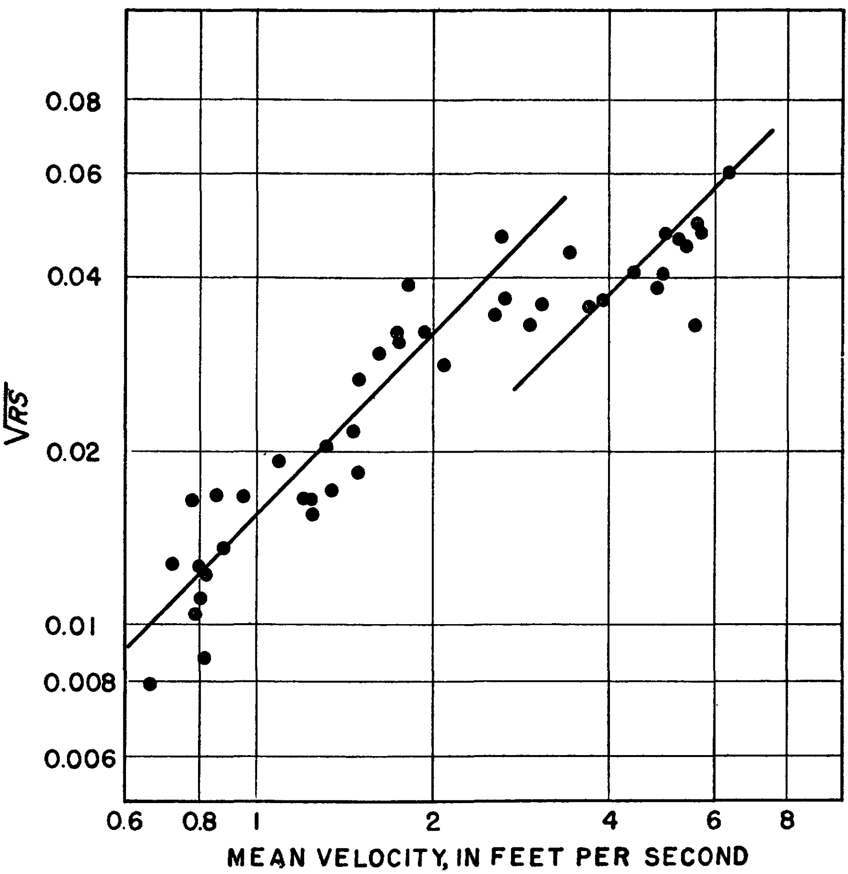

FIGURE 2.—Variation of velocity with $\sqrt{RS}$, 0.45 mm sand in laboratory.

The average Chezy C for this sand for the lower regimes is about 65; for the upper regimes it is about 110.

AN APPROACH TO THE SOLUTION OF THE PROBLEM

The various regimes of flow and bed configuration observed in the flume have been seen in sand-channel streams by most stream gagers, but their significance to depth-discharge relations has been realized only recently (Colby, 1960). The method of determining discharges by a stage-discharge relation often obscures any underlying relation. This is because in alluvial streams neither the bottom nor the sides of the channel are fixed. Figure 3 shows as a typical example the stage-discharge plot for Huerfano River near Undercliffe, Colo., for 1941 and 1942.

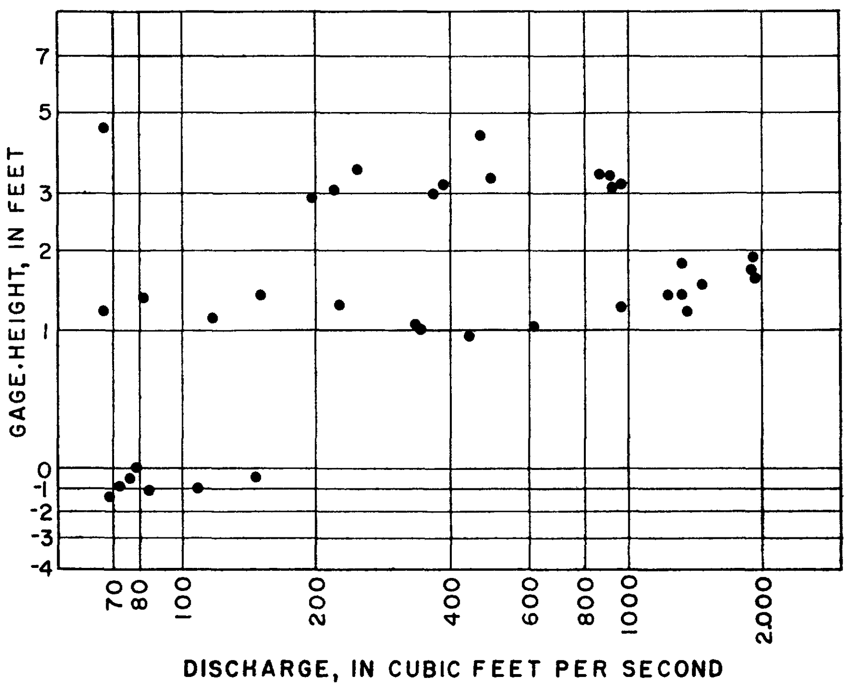

FIGURE 3.—Stage-discharge relation for Huerfano River near Undercliffe, Colo.

The underlying relation may be revealed by a change of variables. The effect of variation in bottom elevation is eliminated by replacing stage by mean depth or hydraulic radius. The effect of variation in width is eliminated by using mean velocity. Figure 4 shows the same measurements for Huerfano River, as were plotted in figure 3, replotted on the basis of velocity and hydraulic radius. Figures 5–9 show a similar plot for five other streams. Each graph shows a similar pattern with a trend for the lower and a trend for the higher measurements, and an amorphous range of discontinuity. On each graph the trend for the upper measurement is represented by a curve of relation. Each of these curves can be expressed as

$$V=kR^{\frac{1}{2}}$$

where k represents a constant in the relation. According to this relation, the Froude number, F, remains constant, because the other term, g, in the formula

$$\mathsf{F}=k/g^{\frac{1}{2}}$$

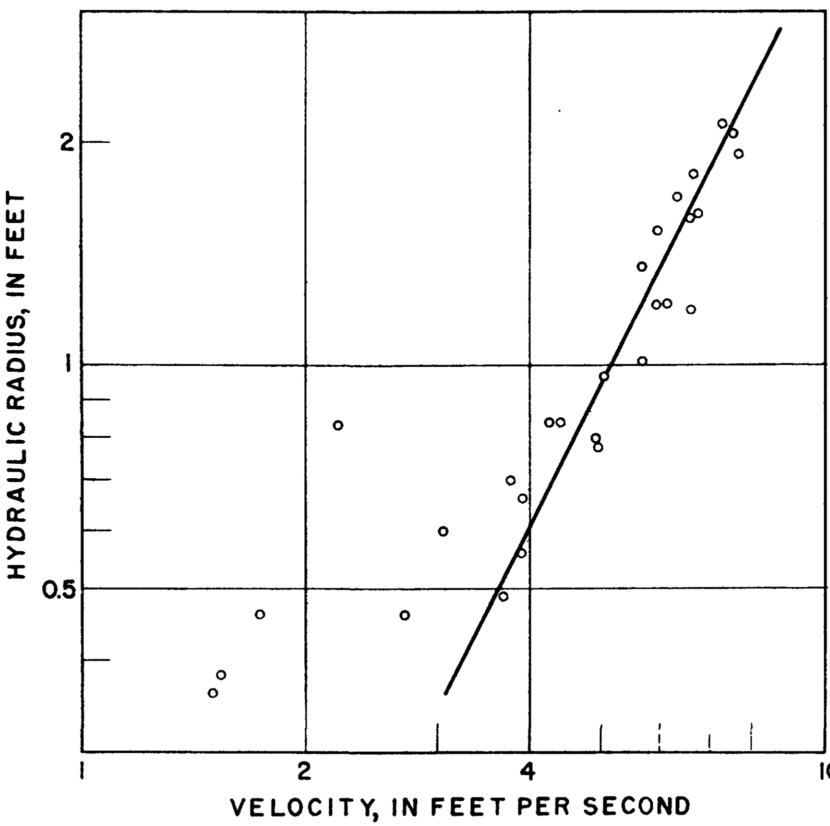

FIGURE 4.—Relation of velocity to hydraulic radius for Huerfano River near Undercliffe, Colo.

is also a constant. If the energy slope remains constant at a given stream site, each curve also represents a constant value of the Chezy roughness coefficient, C, for

$$C=k/S^{\frac{1}{2}}$$

contains only constant terms.

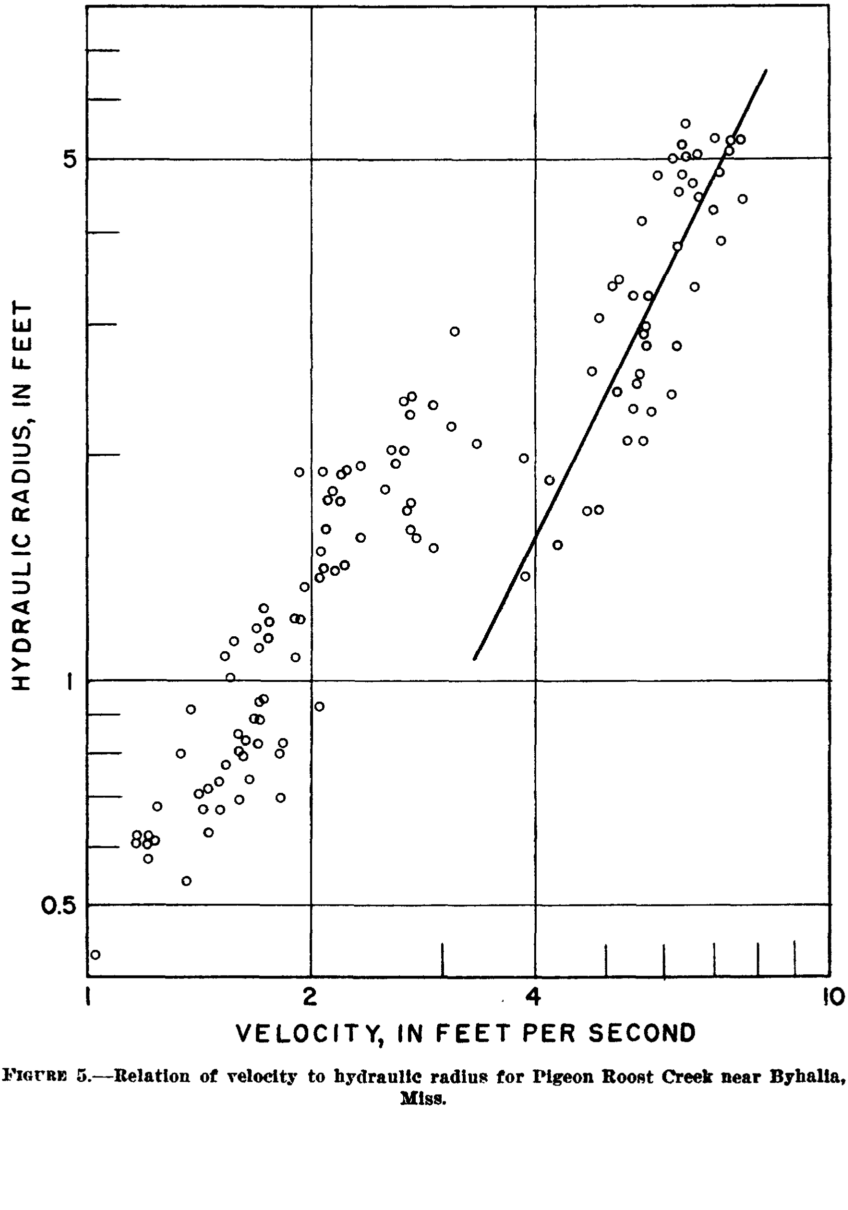

FIGURE 5.—Relation of velocity to hydraulic radius for Pigeon Roost Creek near Byhalia, Miss.

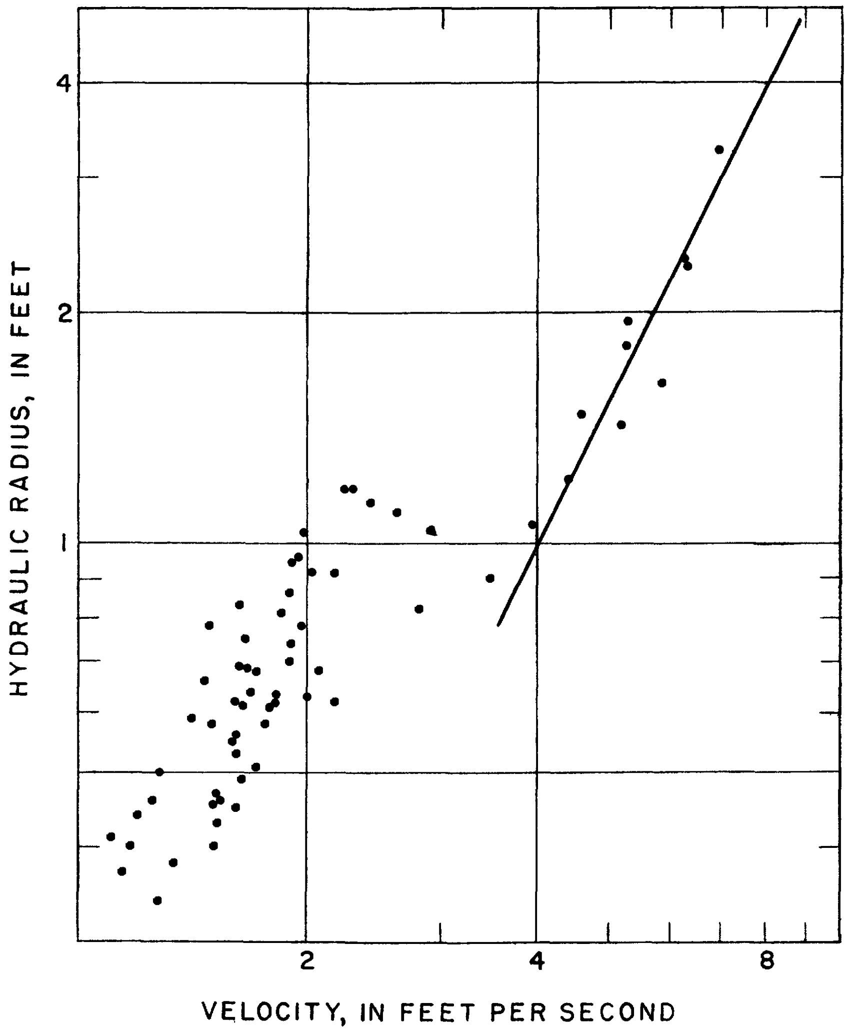

FIGURE 6.—Relation of velocity to hydraulic radius for Republican River at Stratton, Nebr.

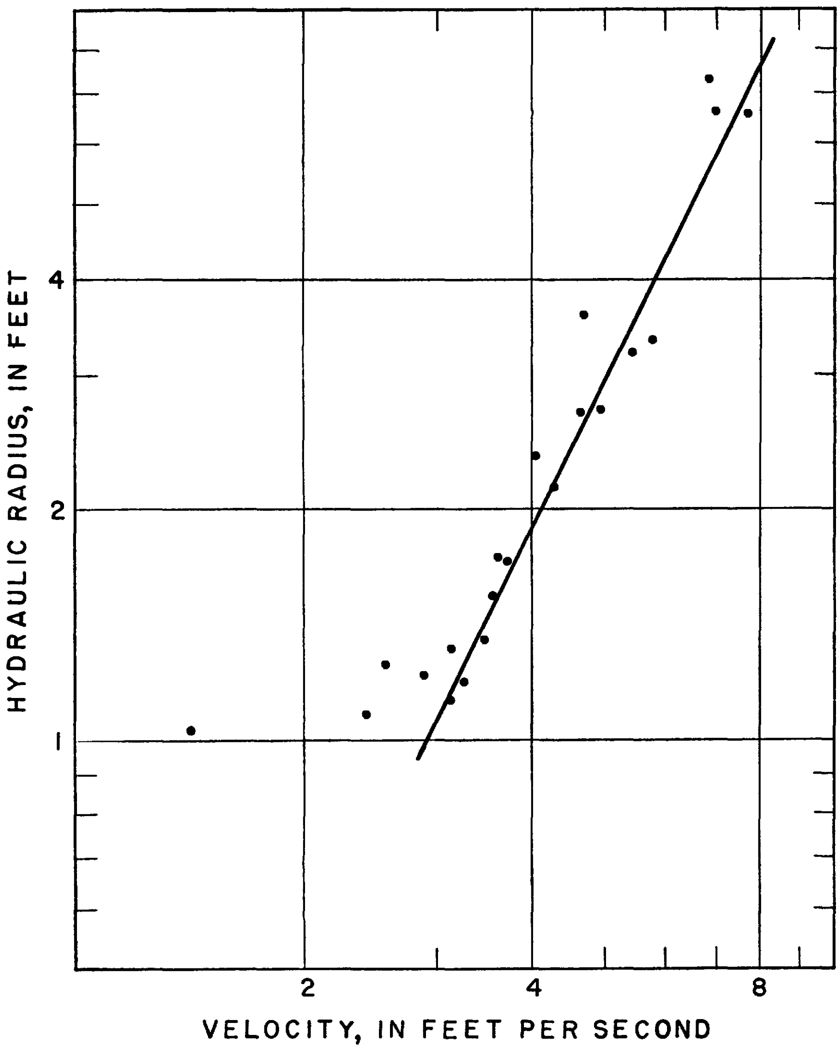

FIGURE 7.—Relation of velocity to hydraulic radius for Cheyenne River near Spencer, Wyo.

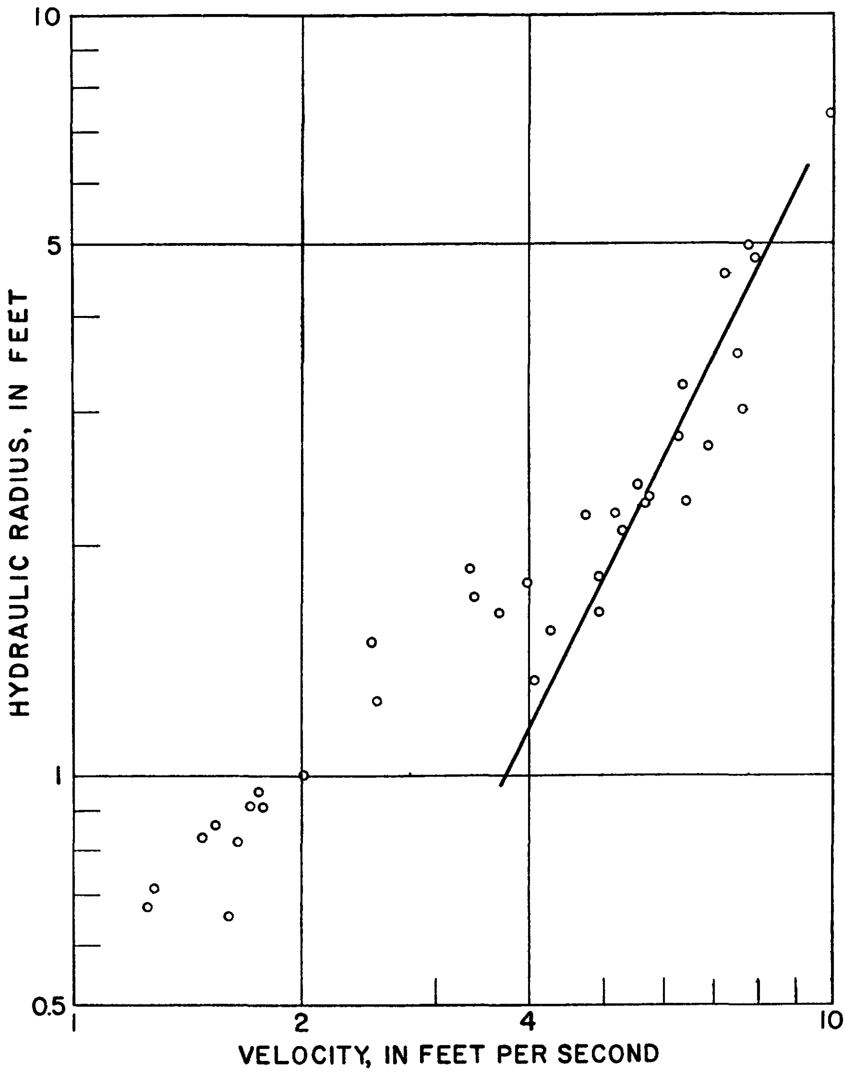

FIGURE 8.—Relation of velocity to hydraulic radius for Middle Loup River at St. Paul, Nebr.

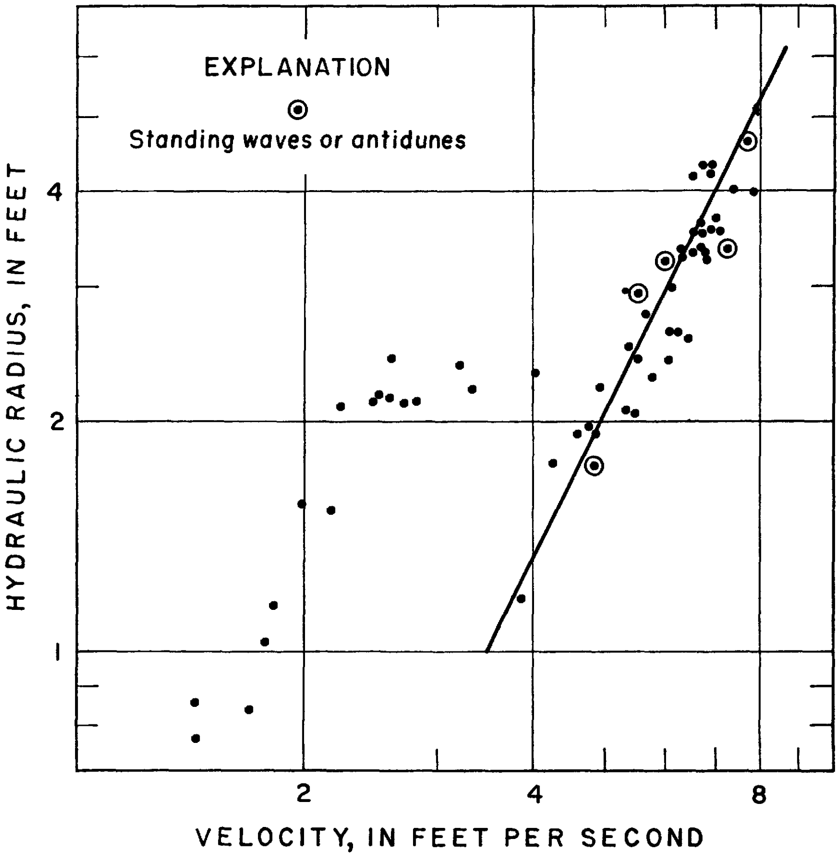

FIGURE 9.—Relation of velocity to hydraulic radius for Rio Grande near Bernalillo, N. Mex.

Chezy *C* is not constant for all stations, however. The parameters most likely to explain the variation between stations would be the channel slope and the characteristics of the bed material. This is assuming all sands are of about the same specific weight, and that temperature variations are averaged out in the relation at each station. If the slope remains virtually constant at a station, as one normally would expect in alluvial channels, then the Chezy *C* is constant. The hypothesis easily is made that the magnitude of this constant Chezy *C* should be related to properties of the bed material.

RELATION OF ROUGHNESS TO SIZE OF BED MATERIAL

To test the hypothesis that a general relation for the upper curve existed which could be related to slope and bed material, a study was undertaken and records were obtained and studied of the relation be-

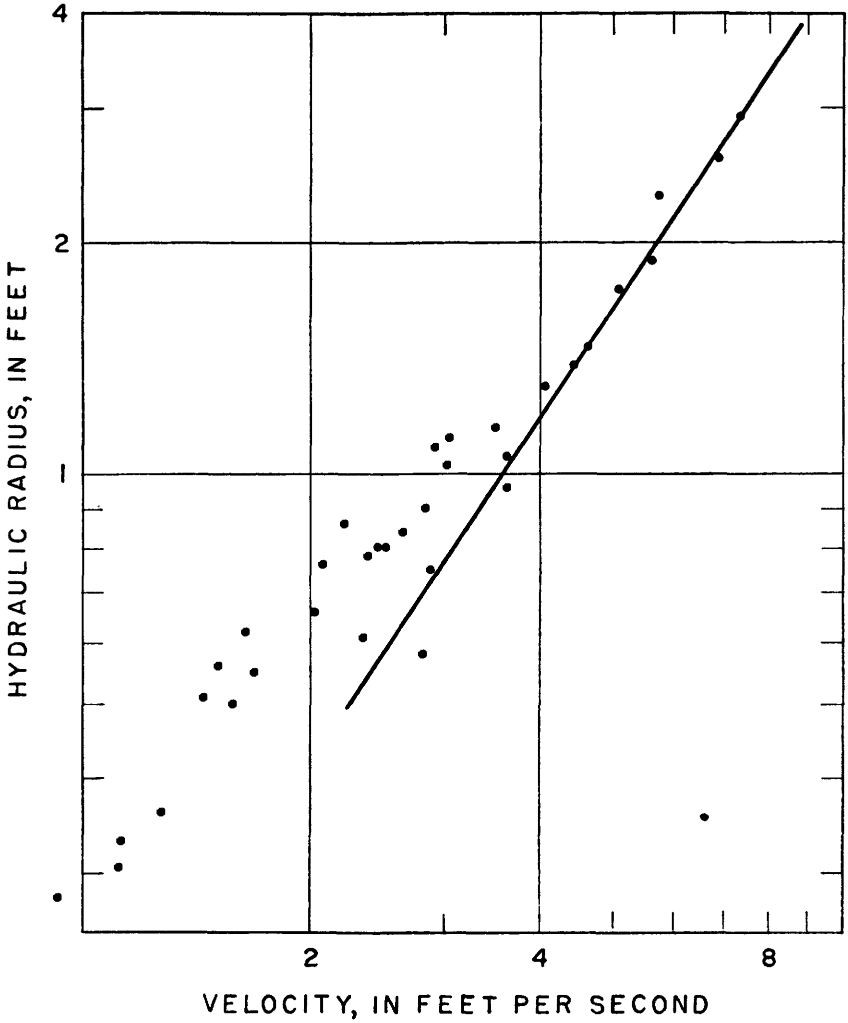

FIGURE 10.—Relation of velocity to hydraulic radius for South Fork Powder River near Kaycee, Wyo.

tween hydraulic radius and velocity for stream-gaging stations having sand channels.

For many stations no discontinuity in the rating was found. The slope and depth of some streams apparently were too small to create sufficient shear to wash out the dunes. The gaging station sometimes was established at a constriction or above a diversion dam or outcrop of bedrock. At places, attempts had been made to stabilize the ratings by dumping rock in the stream or by building low-control structures. Quite often, for convenience in servicing, gaging stations were established on bridges. All of these conditions tend to complicate the hydraulics, inducing variable acceleration and backwater, and no break in the rating could be defined. In addition, many streams have high flows of such short duration that measurements were difficult to obtain, and enough were not available to define the upper curve. For other streams, notably those draining the Sand Hills of Nebraska, the flow is so uniform that a sufficient range in discharge was not experienced to define any relation.

The stations showing a discontinuity in rating were chosen for this study. Generally they were stations in relatively typical reaches with no abrupt constrictions. Measurements of high water were made either from a cableway or from a single-span bridge which did not obstruct the flow. The streams had slopes of 0.0004 feet per foot or greater, and had a wide range in discharge.

The relation $R \propto V$ was determined by plotting the data on logarithmic scales and fitting by eye a straight line to the data. For most of the ratings studied, the slope of one-half (abscissa to ordinate) represented the line of best fit For some ratings, however, the range in velocity in the upper regime was not sufficient to determine the slope, and for them a slope of one-half was fitted arbitrarily. For one station, South Fork Powder River near Kaycee, Wyo., shown on figure 10, the measurements define a slope of two-thirds rather than one-half, indicating a constant Manning's n rather than a constant Chezy C. The scatter about the line fitted for each station includes error in measured discharge and the effect of variations in slope, temperature, or sediment properties which may alter the relation. The curve represents a constant Froude number or Chezy C. The scatter about the curve partly is natural scatter introduced by measuring error, and partly is due to minor variations in the relation. Thus, for all but one of the sites studied, rating extensions could be made by using a constant Froude number for higher flows. For ungaged sites, a constant Chezy C could be used and acceptable results be obtained in a great majority of cases.

In order to determine the Chezy *C* at each station, the slope of the stream was determined, generally from topographic maps. It was assumed that the energy slope could be approximated by the average bed slope through a long reach. The median diameter and the standard deviation of the bed material were abstracted, wherever possible, from published reports. In many cases, however, no information was available, and single samples of bed material were obtained in the field. Some of the samples were taken during periods of no flow, so that both median diameter and standard deviation are subject to considerable error. These data are shown in table 1.

Despite the possibility of error in both variables, a plot of the Chezy *C* against median diameter shows a trend in the expected direction. The larger the bed material the greater the resistance to flow (fig. 11). A refinement in the measurement of both slope and bed material probably would reduce the scatter somewhat. The standard error about the line of relation is +25 percent and −20 percent.

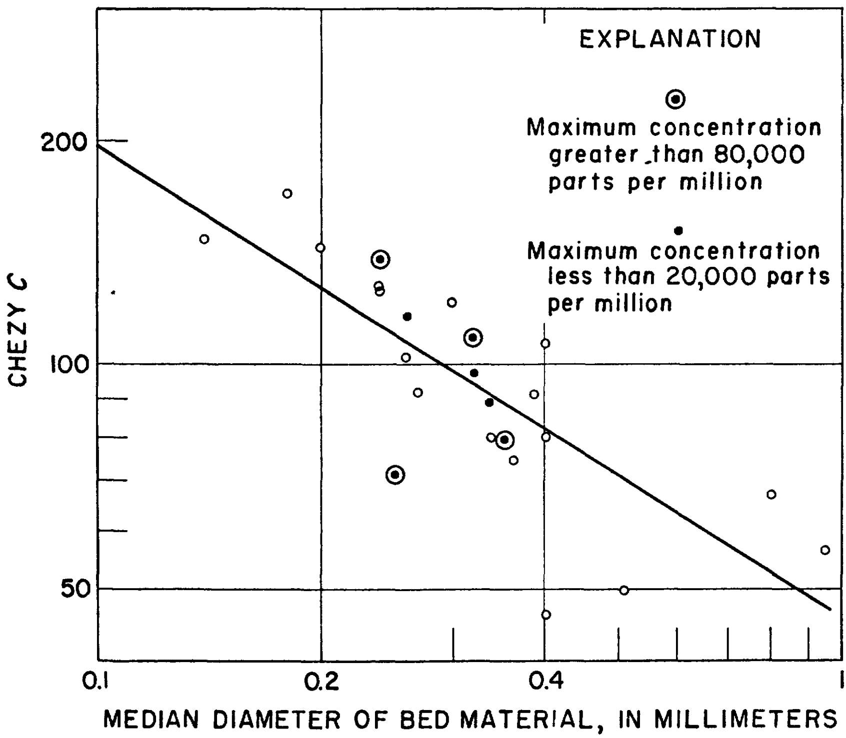

FIGURE 11.—Relation of Chezy *C* to median diameter of bed material.

TABLE 1.—*Variables measured*

Station	Intercept	Slope (feet per foot)	Chezy *C*	Bed material: Median diameter (mm)	Bed material: Standard deviation
Bighorn River at Manderson, Wyo	2.3	0.000947	75	-------	-------
Bighorn River near Manderson, Wyo	2.6	-------	84	0.34	2.3
Browns Wash near Green River, Utah	4.6	.0025	92	.27	2.1
Canadian River at Logan, N. Mex	4.1	.00057	172	.18	1.43
Cherry Creek near Franktown, Colo	4.9	.0074	57	.95	2.75
Cherry Creek near Melvin, Colo	5.7	.0051	80	.40	1.96
Cheyenne River near Spencer, Wyo	2.85	.0018	67	.8	5
Cuffawa Creek at Chulahoma, Miss	4.1	.002	92	.39	1.45
Elkhorn River at Waterloo, Nebr	2.5	.0004	125	.24	1.68
Galisteo Creek at Domingo, N. Mex	5.6	.0055	75	.40	2.35
Huerfano River near Undercliffe, Colo	5.2	.0113	49	-------	-------
Lance Creek at Spencer, Wyo	3.3	.00073	122	.3	-------
Middle Loup River at St. Paul, Nebr	3.7	.001	117	.26	1.53
Muddy Creek near Shoshoni, Wyo	4.7	.0044	71	.25	3.75
Pigeon Roost Creek near Byhalia, Miss	3.2	.0009	107	.40	1.4
Powder River, South Fork, near Kaycee, Wyo	3.6	.0016	[1]0.0165	.5	9.4
Republican River at Stratton, Nebr	4.0	.0017	97	.32	1.62
Rio Grande at Bernalillo, N. Mex	3.5	.00095	114	.30	1.60
Rio Grand near Belen, N. Mex	3.6	.00082	126	.24	1.50
Rio Grande at San Antonio, N. Mex	3.5	.00055	149	.20	1.51
Rio Grande at San Marcial, N. Mex	3.0	.00041	148	.14	1.46
Rio Puerco near Bernardo, N. Mex	4.35	.001	138	.24	2.42
San Francisco Riverside Drain near Bernardo, N. Mex	2.75	.00075	101	.26	1.42
San Juan River at Shiprock, N. Mex	3.5	.0019	80	.35	1.58
Walkers Bottom Creek, near Holly Springs, Miss	5.1	.0125	46	.40	1.49
White River, South Fork, below White River, S. Dak	3.25	.0015	90	.34	1.54
Willie Wilkins Creek near Holly Springs, Miss	3.8	.0058	50	.51	1.49

[1] Manning's *n*.

OTHER VARIABLES CONSIDERED

The use of the standard deviation of the bed material as a third variable indicated a trend that one might predict, but the results were not significant. The standard deviation was negatively correlated with the Chezy *C*, but it reduced the variance by about 3 percent only, being significant at about the 40-percent level. The standard deviation, however, probably is the least well defined of all the variables measured. The relation may be determined better by refinement of the measurements of the variables.

It has been suggested (Brooks, 1958; Vanoni and Nomicos, 1959) that suspended sediment is one of the independent variables which determines the velocity of the stream. This was not found to be true once the discontinuity occurred, although it may influence the point of discontinuity. At each of the stations used in this study, there is a range of suspended loads in the regimes of flow beyond the discontinuity, yet Chezy *C* remains virtually constant at each station. There also is a wide variation in the rates of sediment transport among the various stations. On figure 11, 3 stations for which the maximum measured suspended-sediment concentration is less than 20,000 ppm are compared with 4 stations for which the maximum is greater than 80,000 ppm. The differences in the value of Chezy *C* are not explained by the gross differences in sediment transport.

CONCLUSIONS

Sand-channel streams with wide ranges in discharge often have a discontinuous rating. The upper part of the rating represents plane-bed, standing-wave, and antidune regimes. For this upper part, the Froude number and the Chezy *C* are constant at a site, and the variation of *C* from site to site appears to be related primarily to grain size. The relation of roughness to bed-material size should prove valuable in extending rating curves above the highest measured discharge and in determining peak discharge of ungaged streams from floodmarks.

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