Constant-Head Pumping Test of a Multiaquifer Well to Determine Characteristics of Individual Aquifers

GEOLOGICAL SURVEY WATER-SUPPLY PAPER 1536-G

Prepared in cooperation with the Pennsylvania Geological Survey, Department of Internal Affairs, Commonwealth of Pennsylvania

Constant-Head Pumping Test of a Multiaquifer Well to Determine Characteristics of Individual Aquifers

By GORDON D. BENNETT *and* EUGENE P. PATTEN, JR.

GROUND-WATER HYDRAULICS

GEOLOGICAL SURVEY WATER-SUPPLY PAPER 1536-G

Prepared in cooperation with the Pennsylvania Geological Survey, Department of Internal Affairs, Commonwealth of Pennsylvania

UNITED STATES GOVERNMENT PRINTING OFFICE, WASHINGTON : 1962

UNITED STATES DEPARTMENT OF THE INTERIOR

STEWART L. UDALL, *Secretary*

GEOLOGICAL SURVEY

Thomas B. Nolan, *Director*

Reprinted
1964

For sale by the Superintendent of Documents, U.S. Government Printing Office
Washington, D.C., 20402 - Price 15 cents (paper cover)

CONTENTS

ILLUSTRATIONS

TABLE

GROUND-WATER HYDRAULICS

CONSTANT-HEAD PUMPING TEST OF A MULTIAQUIFER WELL TO DETERMINE CHARACTERISTICS OF INDIVIDUAL AQUIFERS

By Gordon D. Bennett and Eugene P. Patten, Jr.

ABSTRACT

This report describes the theory and field procedures for determining the transmissibility and storage coefficients and the original hydrostatic head of each aquifer penetrated by a multiaquifer well. The procedure involves pumping the well in such a manner that the drawdown of water level is constant while the discharges of the different aquifers are measured by means of borehole flowmeters. The theory is developed by analogy to the heat-flow problem solved by Smith. The internal discharge between aquifers after the well is completed is analyzed as the first step. Pumping at constant drawdown constitutes the second step. Transmissibility and storage coefficients are determined by a method described by Jacob and Lohman, after the original internal discharge to or from the aquifer has been compensated for in the calculations. The original hydrostatic head of each aquifer is then determined by resubstituting the transmissibility and storage coefficients into the first step of the analysis.

The method was tested on a well in Chester County, Pa., but the results were not entirely satisfactory, owing to the lack of sufficiently accurate methods of flow measurement and, probably, to the effects of entrance losses in the well. The determinations of the transmissibility coefficient and static head can be accepted as having order-of-magnitude significance, but the determinations of the storage coefficient, which is highly sensitive to experimental error, must be rejected. It is felt that better results may be achieved in the future, as more reliable devices for metering the flow become available and as more is learned concerning the nature of entrance losses. If accurate data can be obtained, recently developed techniques of digital or analog computation may permit determination of the response of each aquifer in the well to any form of pumping.

INTRODUCTION

It is often desirable to know the coefficients of transmissibility (*T*) and storage (*S*) and the original hydrostatic head of each aquifer penetrated by a multiaquifer well. A conventional pumping-test (aquifer-test) analysis fails to give this information; the most that can be expected from a conventional test is the determination of composite values of transmissibility, storage, and hydrostatic head for all aquifers tapped by the well. Such results clearly have limited significance. This paper describes an aquifer-test technique designed to determine these constants for each aquifer penetrated by a well. Both the testing procedure and the method of analysis are more complex than those used in a conventional test, and the results are not yet satisfactory; however, refinements may be made in the testing equipment that will improve the accuracy of the test results and thus make the test a very useful tool in ground-water hydrology.

The research described in this paper was carried out by the U.S. Geological Survey under a cooperative program with the Pennsylvania Topographic and Geologic Survey. It constitutes one phase of an investigation into the application of borehole geophysics to hydrology.

REVIEW OF EARLIER WORK

The equation of ground-water flow, as given by Jacob (1950), for two-dimensional radial flow to a well at the origin is

$$\frac{\partial^2 h}{\partial r^2}+\frac{1}{r}\frac{\partial h}{\partial r}=\frac{S}{T}\frac{\partial h}{\partial t}, \tag{1}$$

where h represents head, or elevation of the piezometric surface above datum; S is the storage coefficient of the aquifer; T is the transmissibility of the aquifer; r is the radial coordinate; and t is time.

An analogy exists between the theory of ground-water flow and the theory of heat conduction in solids. Recognition of this analogy was of great assistance in formulating the mathematical treatment of ground water flow. The mathematical analysis developed for a given problem in heat flow has often been applied directly to the parallel problem in ground-water flow. The thermal analog of equation 1 may be obtained by substituting the inverse of thermal diffusivity for S/T, and temperature for h. The resulting equation deals with the problem of two-dimensional flow of heat in a slab of conducting material, with radial symmetry about the origin. The analogy may be further clarified by noting that transmissibility is mathematically equivalent to the product of thermal conductivity and slab thickness and that the storage coefficient is mathematically equivalent to the product of heat capacity per unit volume and slab thickness.

Smith (1937) presents a solution to the problem of heat flow in an infinite solid bounded internally by a very long cylinder, when the temperature at the cylinder wall is a step function of time. More precisely, Smith's solution deals with the problem in which (letting u represent the temperature at the cylinder wall)

$$\begin{aligned}
&u=u_0,\ t<t_0\\
&u=u_1,\ t_0<t<t_1\\
&u=u_2,\ t_1<t<t_2\\
&u=u_3,\ t_2<t<t_3 \text{ and so forth.}
\end{aligned}$$

It is assumed that prior to time t_0 the temperature u_0 prevails throughout the solid.

Smith's results may be stated in the notation of ground-water theory to obtain a quantitative analysis of the problem in which the water level in a well varies as some step function of the time. Let $h_0, h_1, h_2, \ldots h_n$ represent the succession of water levels which make up the step function, so that

$$h_w=h_0,\ t<t_0$$
$$h_w=h_1,\ t_0<t<t_1$$
$$h_w=h_2,\ t_1<t<t_2 \text{ and so forth;}$$

and let it be assumed that the well penetrates, at the origin of coordinates, an infinite artesian aquifer throughout which the head h_0 prevails prior to time t_0. It can be shown from Smith's analysis that during the stepwise variation, at a time t, such that $t_{(n-1)}<t<t_n$, the radial derivative of head at the cylindrical contact between borehole and aquifer is given by

$$\left(\frac{\partial h}{\partial r}\right)_{r_w,t}=(h_0-h_1)\cdot\frac{1}{r_w}\cdot G\left\{\frac{T}{Sr_w^2}(t-t_0)\right\}$$
$$+(h_1-h_2)\cdot\frac{1}{r_w}\cdot G\left\{\frac{T}{Sr_w^2}(t-t_1)\right\}$$
$$+\ldots\ldots(h_{(n-1)}-h_n)\cdot\frac{1}{r_w}\cdot G\left\{\frac{T}{Sr_w^2}(t-t_{(n-1)})\right\}, \quad (2)$$

where the function G is defined by

$$G(\beta)=\frac{4\beta}{\pi}\int_{X=0}^{X=\infty}Xe^{-\beta X^2}\left\{\frac{\pi}{2}+\arctan\frac{K_0(X)}{J_0(X)}\right\}dX, \quad (3)$$

in which

$J_0(X)$ = Bessel function of zero order of the first kind;

$K_0(X)$ = Bessel function of zero order of the second kind.

The volumetric rate of flow, $Q(t)$, from the aquifer into the well at any time t is related to the derivative at that time by the equation

$$Q(t)=2\pi r_w T\left(\frac{\partial h}{\partial r}\right)_{r_w,t} \quad (4)$$

Substitution of the expression from equation 2 for the derivative into equation 4 gives the following expression for $Q(t)$:

$$Q(t)=2\pi T(h_0-h_1)\cdot G\left\{\frac{T}{Sr_w^2}(t-t_0)\right\}+2\pi T(h_1-h_2)\cdot G\left\{\frac{T}{Sr_w^2}(t-t_1)\right\}+\ .\ .\ .\ .\ .\ +2\pi T(h_{(n-1)}-h_n)\cdot G\left\{\frac{T}{Sr_w^2}(t-t_n)\right\}\cdot \quad (5)$$

Equation 5 provides a means of computing the discharge from an aquifer to a well, or vice versa, at any instant, as a result of any pattern of changes in head that can be approximated as a series of steps, provided that the head h_0 prevails throughout the aquifer at time t_0.

Jacob and Lohman (1952) first applied Smith's solution to a hydrologic problem. They used the first step in his analysis to obtain the discharge of a flowing well suddenly opened to atmospheric pressure, after being closed long enough to permit the aquifer head to reach a uniform level. Thus t_0 was taken as the instant of opening of the well, and was set equal to zero. This led to the relation

$$Q(t)=2\pi T(h_0-h_w)\cdot G\left\{\frac{Tt}{Sr_w^2}\right\} \quad (6)$$

for the time-varying well discharge.

TABLE 1.—*Values of* G (β) *for values of* β *from* 10^{-4} *to* 10^{12}

[From Jacob and Lohman, 1952, p. 561]

No.	10^{-4}	10^{-3}	10^{-2}	10^{-1}	1	10	10^2	10^3
1	56.9	18.34	6.13	2.249	0.985	0.534	0.346	0.251
2	40.4	13.11	4.47	1.716	.803	.461	.311	.232
3	33.1	10.79	3.74	1.477	.719	.427	.294	.222
4	28.7	9.41	3.30	1.333	.667	.405	.283	.215
5	25.7	8.47	3.00	1.234	.630	.389	.274	.210
6	23.5	7.77	2.78	1.160	.602	.377	.268	.206
7	21.8	7.23	2.60	1.103	.580	.367	.263	.203
8	20.4	6.79	2.46	1.057	.562	.359	.258	.200
9	19.3	6.43	2.35	1.018	.547	.352	.254	.198
10	18.3	6.13	2.25	.985	.534	.346	.251	.196

No.	10^4	10^5	10^6	10^7	10^8	10^9	10^{10}	10^{11}
1	0.1964	0.1608	0.1360	0.1177	0.1037	0.0927	0.0838	0.0764
2	.1841	.1524	.1299	.1131	.1002	.0899	.0814	.0744
3	.1777	.1479	.1266	.1106	.0982	.0883	.0801	.0733
4	.1733	.1449	.1244	.1089	.0968	.0872	.0792	.0726
5	.1701	.1426	.1227	.1076	.0958	.0864	.0785	.0720
6	.1675	.1408	.1213	.1066	.0950	.0857	.0779	.0716
7	.1654	.1393	.1202	.1057	.0943	.0851	.0774	.0712
8	.1636	.1380	.1192	.1049	.0937	.0846	.0770	.0709
9	.1621	.1369	.1184	.1043	.0932	.0842	.0767	.0706
10	.1608	.1360	.1177	.1037	.0927	.0838	.0764	.0704

Jacob and Lohman present a table of values of the G function for a wide range of values of its argument. This table was obtained by numerical methods of approximation, as the G function cannot be integrated directly. Although used by Jacob and Lohman in a single-step analysis, the values of the G function given by them have general application, and are used in the present paper in an analysis of more than one step. The values of $G(\beta)$ for values of β ranging from 10^{-4} to 10^{12} given by Jacob and Lohman are reproduced in table 1.

CONDITIONS IN A MULTIAQUIFER WELL

BEHAVIOR OF THE WELL DURING CONSTRUCTION

During construction of a multiaquifer well several adjustments in water level normally occur. Consider, for example, a three-aquifer well (fig. 50), in which the undisturbed head of the upper aquifer is h_U, that of the middle aquifer is h_M, and that of the lower aquifer is h_L. Let it be assumed that $h_U > h_L > h_M$. When the well has penetrated only the upper aquifer, the water level stands at h_U. As the drill enters the middle aquifer, the water level begins to fall and flow commences between the aquifers. A cone of depression develops in the piezometric surface of the upper aquifer and a cone of elevation develops on that of the middle aquifer. The water level in the well normally stabilizes at some value between h_U and h_M within a relatively short time after the middle aquifer has been fully penetrated.

A further adjustment in water level occurs as the lowermost aquifer is entered. If the balance level between the heads of the upper and middle aquifers lies above the head of the lower aquifer, a drop in water level will occur, and flow into the lower aquifer will commence. If the balance between the upper and middle aquifers lies below the static head of the lower aquifer, a rise in water level will occur as flow from this aquifer enters the well. The latter situation is shown in figure 51. In either situation, however, the water level normally stabilizes at some new position within a relatively short time after the lower aquifer has been fully penetrated.

BEHAVIOR OF COMPLETED WELL UNDER CONDITION OF NO WITHDRAWAL

After the well is completed, the internal flow between aquifers generally continues indefinitely at a rate that diminishes slowly with time. If the well is assumed, for simplicity, to be free from periodic changes in head due to recharge to or discharge from one or more of the aquifers elsewhere, then significant changes in head are not possible after the early part of the well's existence. For the period of undisturbed internal flow between aquifers, the time derivative

644518 O—62——2

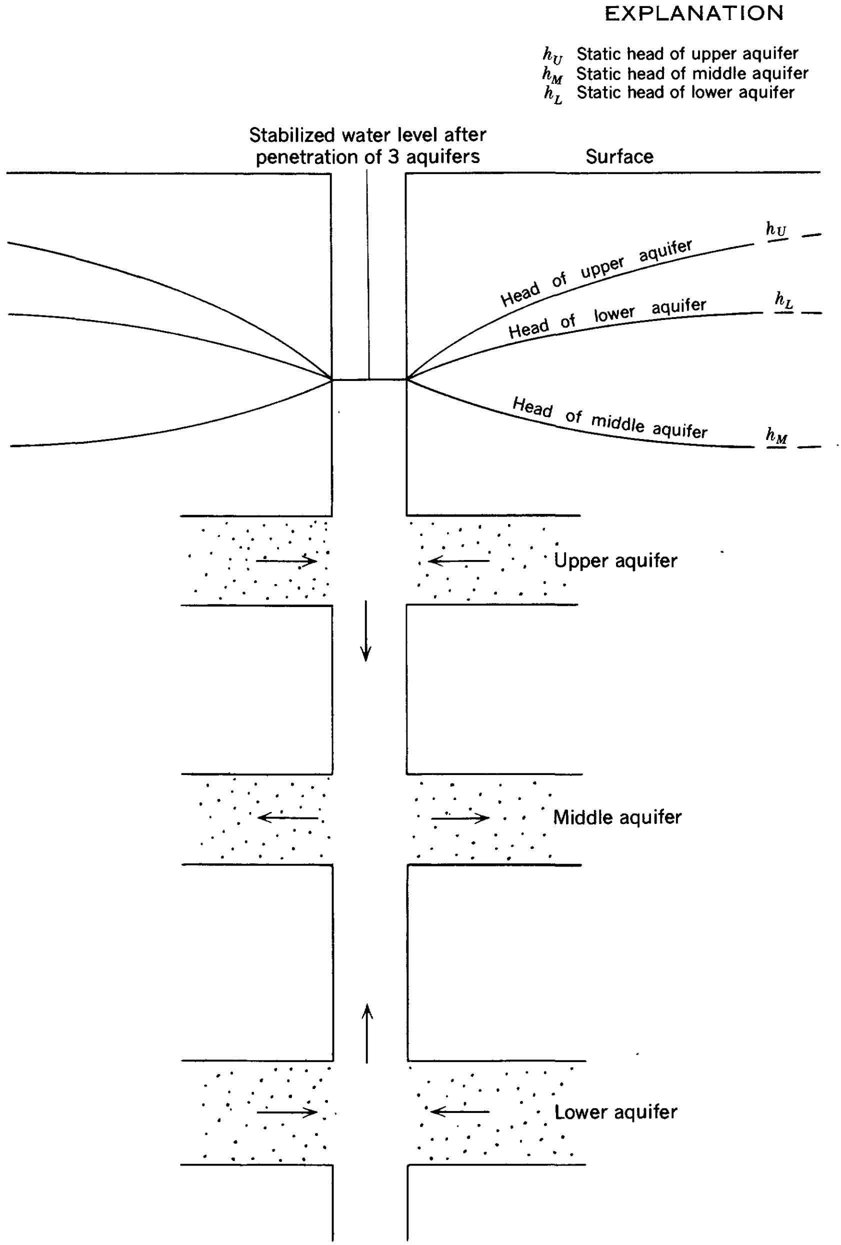

FIGURE 50.—Diagram showing conditions in a typical three-aquifer well shortly after completion.

$\partial h/\partial t$ must approach zero as time increases, and the head must always remain between the limits h_U and h_M. Thus the assumption that, for the condition of no withdrawal, the head stabilizes at a constant level a short time after the well is completed is a close approximation to the actual conditions in almost all wells. This condition can be checked easily by making periodic water-level measurements after the well is completed.

BEHAVIOR OF THE WELL WHEN PUMPED

Pumping the well produces a new set of conditions which may be summarized as follows: An equal change in head is applied to each aquifer over a given period of time. The flows entering or leaving each aquifer are changed from their values prior to pumping in such a way that the algebraic sum of these changes in discharge is essentially equal, at any instant of time, to the discharge of the well at that instant. In adjusting to pumping conditions, aquifers that were discharging to the well prior to pumping will increase their rate of discharge, while aquifers that were "thieving" from the well will decrease their rate of thieving and may even reverse flow completely and begin discharging.

The conditions outlined above for the discharge of a multiaquifer well are generally true regardless of the method of well discharge—that is, whether the well is discharged at a constant rate, at a constant drawdown, or in some other manner. For a further discussion of the internal hydraulics of a multiaquifer well during discharge, the reader is referred to Bennett and Patten (1959).

APPLICATION OF STEP-DRAWDOWN ANALYSIS TO THE PROBLEM

PERIOD FROM COMPLETION OF WELL TO BEGINNING OF TEST

Analysis of the multiaquifer problem by the methods proposed here is possible only insofar as the changes in head involved can be approximated by step functions. The analysis would become complex, moreover, if it were not possible to represent the interval from completion of the well to pumping as a single step. Therefore, the applicability of the step-function approximation must be considered.

Referring to the three-aquifer well shown in figure 50, let t_0 denote the time at which the middle aquifer is penetrated during drilling of the well. Let t_a denote a time after penetration of the middle aquifer but prior to penetration of the lower aquifer. Similarly, let t_b represent the time at which the lower aquifer is penetrated by the drill, and let t_c denote a time after penetration of the lowermost aquifer but prior to pumping of the well. Finally, let t_1 represent the time when the pumping test is begun, and let it be assumed that no pumping of the well occurs prior to t_1. The various times, as defined here.

are indicated on the abscissas of the graphs in figures 52–55. In order to facilitate the discussion of the fluctuations in head in each aquifer during drilling, the point (that is, the map location) at which the well penetrates each aquifer will be referred to as the origin of coordinates (or simply the origin) in that aquifer.

The changes in head in the upper aquifer at the origin during drilling are identical with the changes in the water level in the well. They are summarized in figure 51*A*. At time t_0 the head begins to decline as penetration of the middle aquifer begins; by the time t_a it has reached a nearly stable position. At time t_b, when the lowermost aquifer is penetrated, the head begins to rise, as it is assumed in this well that the static head of the lower aquifer is greater than the equilibrium head between the upper and middle aquifers. By the time t_c, the head has reached a new stable position. Figure 51*B* shows a stepwise approximation to the changes in head shown in figure 51*A*, and figure 51*C* shows a single-step change from the original head of the upper aquifer to the pretest equilibrium level of the completed well. It is assumed in this analysis that the time interval t_0–t_1 is very much greater than the interval t_0–t_c, as it would be if the tests were not begun until several weeks after completion of the well.

The fluctuations in head shown in figure 51*A* and *B* presume a discharge rate from the upper aquifer that at time t_1 has reached a value that can be calculated by an application of equation 5, as follows:

$$Q_U(t_1)=2\pi T_U(h_U-h')\cdot G\left\{\frac{T_U}{S_U r_w^2}\,(t_1-t_0)\right\}+2\pi T_U(h'-h_1)\cdot G\left\{\frac{T_U}{S_U r_w^2}\,(t_1-t_b)\right\}. \qquad (7)$$

The subscript *U* is used to indicate the properties of the upper aquifer.

The time interval from t_0 to t_1 is long relative to the time interval from t_0 to t_b, and the quantity (t_1-t_0) must therefore be relatively close in value to the quantity (t_1-t_b). Examination of table 1, moreover, indicates that at large values of β the change in $G(\beta)$ for a given change in β is small. Thus, as the quantities (t_1-t_0) and (t_1-t_b) are both large, and the difference between them is small, the quantities $G\left\{\frac{T_U(t_1-t_0)}{S_U r_w^2}\right\}$ and $G\left\{\frac{T_U(t_1-t_b)}{S_U r_w^2}\right\}$ can be considered to be essentially equal. Equation (7) can therefore be rewritten as

$$Q_U(t_1)=\left\{2\pi T_U(h_U-h')+2\pi T_U(h'-h_1)\right\}\cdot G\left\{\frac{T_U(t_1-t_0)}{S_U r_w^2}\right\}, \qquad (8)$$

which simplifies to

$$Q_U(t_1)=2\pi T_U(h_U-h_1)\cdot G\left\{\frac{T_U(t_1-t_0)}{S_U r_w^2}\right\}. \qquad (9)$$

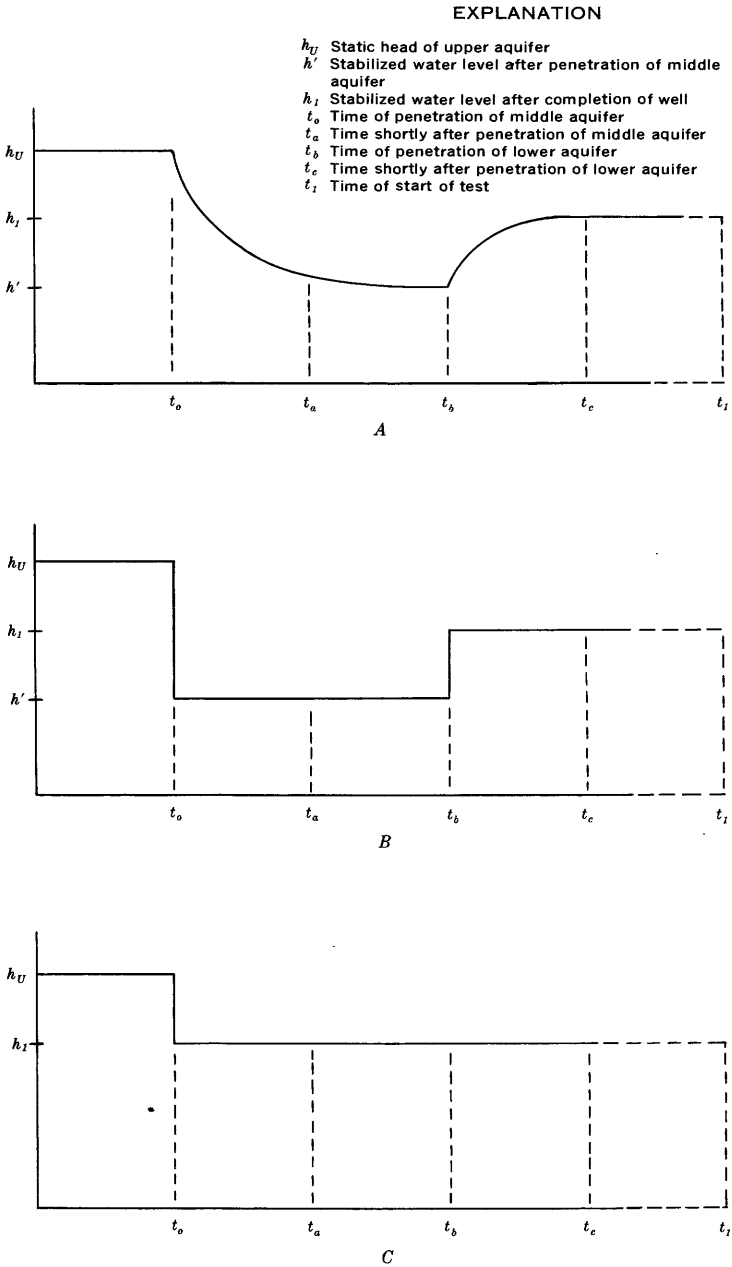

FIGURE 51.—Graphs showing fluctuations of artesian head in the upper aquifer of the three-aquifer well of figure 50, as the two lower aquifers are penetrated: (*A*) water-level fluctuations, (*B*) stepwise approximation, (*C*) single-step change from original head to equilibrium.

Equation 9 is exactly the expression that would be obtained from an application of equation 5 to the single-step drawdown illustrated in figure 51*C*. Thus it is established that the drawdown pattern of the upper aquifer, as illustrated in figure 51*A*, can be closely approximated by the single-step drawdown illustrated in figure 51*C*, provided only that the interval from completion of the well to time of measurement is much greater than the time spent in drilling the well. In other words, minor changes in head occurring early in an interval of time may be neglected, if they are followed by a long period of stability.

The changes in head in the middle aquifer at the origin of coordinates (at the point of well penetration) are shown in figure 52*A*. It is assumed that penetration of the middle aquifer occurs instantaneously at time t_0, and that prior to t_0 the middle aquifer is isolated hydraulically from the borehole—that is, that there is no gradually increasing leakage to the well through the material separating the middle aquifer from the bottom of the borehole as the well is drilled toward this aquifer. Under these assumptions, the head h_m will prevail at the origin, as well as throughout the rest of the middle aquifer, prior to t_0. At the moment of penetration, however, the head in the well, resulting from the upper aquifer, will be instantaneously applied to the middle aquifer at the origin; this water level is shown as h_u. Thus the graph of figure 52*A* shows an instantaneous rise from h_m to h_u occurring at t_0. Flow from the well into the middle aquifer begins immediately, however, and the water level begins to fall toward its new equilibrium level, h'. By the time t_a the water level has essentially stabilized at this new level.

Once the well has penetrated the middle aquifer, the head of this aquifer at the origin is identical with the water level in the well; thus, the fluctuations in head after time t_0 in figure 52*A* are simply the variations in water level in the well. At time t_b the lower aquifer is penetrated, and flow begins from this aquifer into the well. The resulting rise in water level is shown in figure 52*A* as an increase in the head of the middle aquifer at the origin. By time t_c this head has stabilized again to a new equilibrium at h_1.

In discussing the fluctuations in head of the upper aquifer, it was demonstrated that minor variations in head occurring early in an interval can be neglected if they are followed by a long period of stability. Application of this principle to the fluctuations in head shown in figure 52*A* leads to the single-step approximation of figure 52*B*, as the interval t_1-t_0 is assumed to be long relative to the interval t_c-t_0.

Figure 53*A* shows the variations in head of the lowermost aquifer at the origin during drilling. Again it is assumed that the aquifer is hydraulically isolated from the well until the time of its penetration,

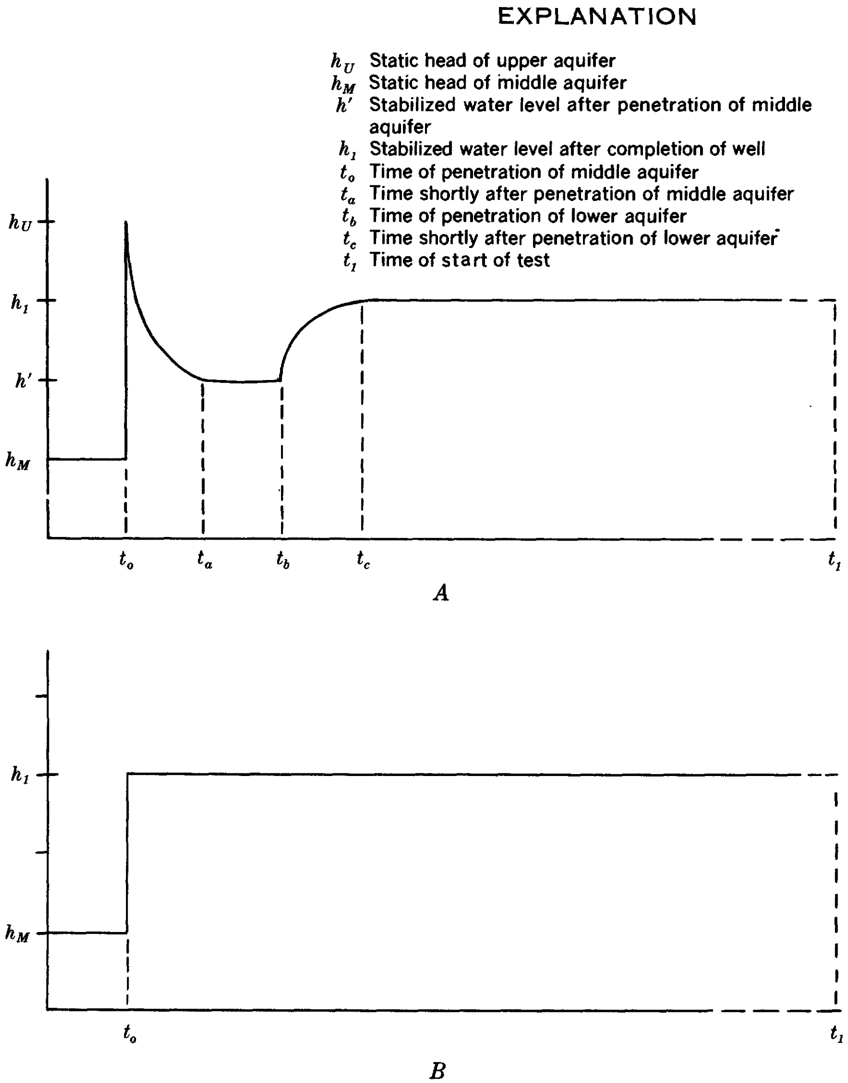

FIGURE 52.—Graphs showing fluctuations in artesian head in the middle aquifer of the three-aquifer well of figure 50 during penetration of middle and lower aquifers: (*A*) change in head during completion of well and (*B*) stepwise approximation of these changes.

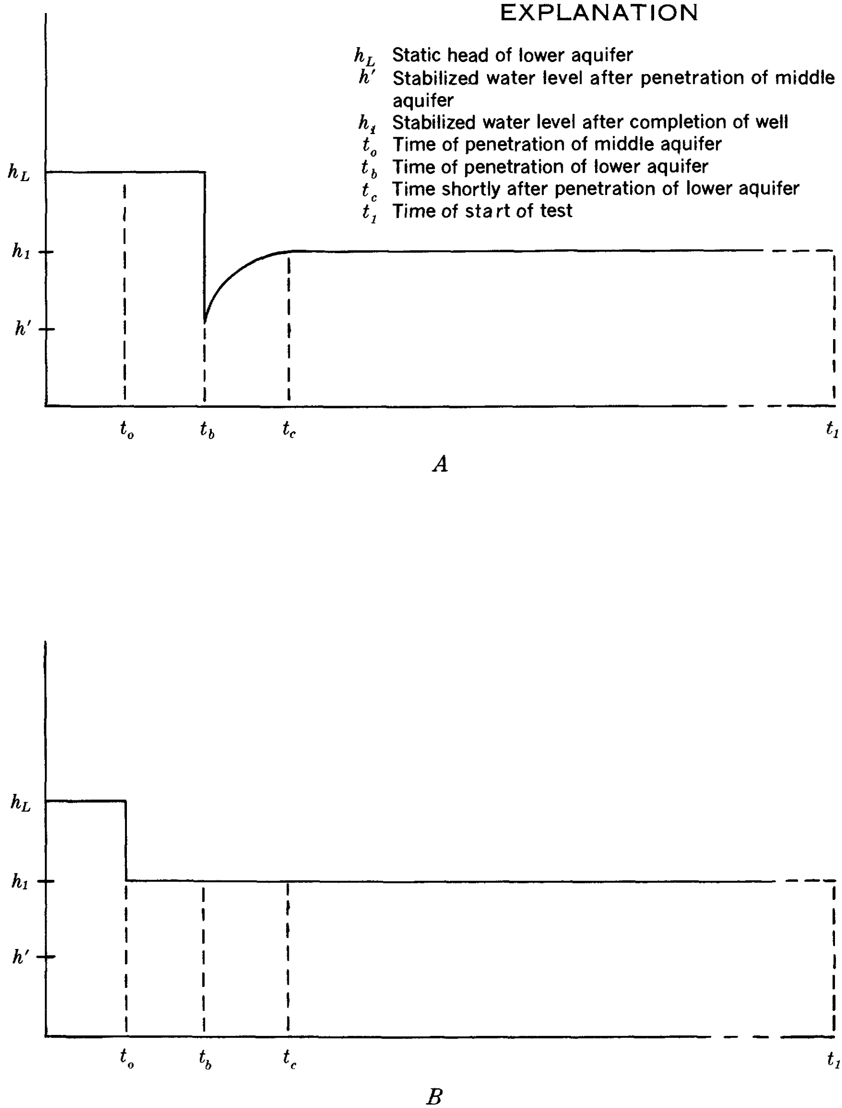

FIGURE 53.—Graphs showing fluctuations in artesian head in the lower aquifer of the three-aquifer well of figure 50: (*A*) Changes in head during completion and (*B*) stepwise approximations of these changes.

t_b. Until this time, therefore, the head at the origin in the lower aquifer is h_L, the static head of this aquifer. At the moment of penetration, the aquifer is instantaneously exposed to the head h'—that is, to the water column in the well. This is indicated in figure 53*A* by the sharp drop in head at t_b. The rise in head after t_b reflects the rise in water level in the well, due to the flow that begins from the lower aquifer at the instant of penetration. Again, since t_c-t_0 is assumed to be short relative to t_1-t_0, the single-step approximation of figure 53*B* may be used to represent the fluctuations shown in figure 53*A*.

The assumptions of instantaneous penetration and complete hydraulic separation until the moment of penetration do not represent conditions normally found in practice. However, the effects of gradual penetration and interconnection usually make the actual changes in head in the aquifers even more similar to the stepwise approximation of figures 52*B* and 53*B* than the graphs of figures 52*A* and 53*A* would indicate.

TEST PUMPING OF WELL AT CONSTANT DRAWDOWN

The change in head in each aquifer at the well, following completion of the well, can be approximated, as indicated, as a single-step change. A second-step change of head can be imposed on each aquifer at the well by beginning a constant-drawdown pumping test at time t_1. Figure 54*A* shows the drawdown created by such a test; figure 54*B* is a composite graph showing the relation of this drawdown to the changes of head in the various aquifers during drilling; and figure 54*C* shows the corresponding composite stepwise approximation. Neglecting entrance and well-bore losses, the same change of head is applied to each aquifer. Recording the discharges of the various aquifers in response to this change of head should, therefore, permit computation of the coefficients of transmissibility and storage. The discharge from the upper aquifer at a time t during this constant drawdown test can be calculated by a two-step application of equation 5, that is

$$Q_U(t)=2\pi T_U(h_U-h_1)\cdot G\,\frac{T_U(t-t_0)}{S_U r_w^2}+2\pi T_U(h_1-h_2)\cdot G\,\frac{T_U(t-t_1)}{S_U r_w^2}. \quad (10)$$

SUBTRACTION OF ORIGINAL DISCHARGE

If the time of measurement, t, is chosen a few hours after the beginning of the pumping test, whereas the time since completion of the well is measured in days or weeks, the difference between the intervals $(t-t_0)$ and (t_1-t_0) will be small relative to the total lengths

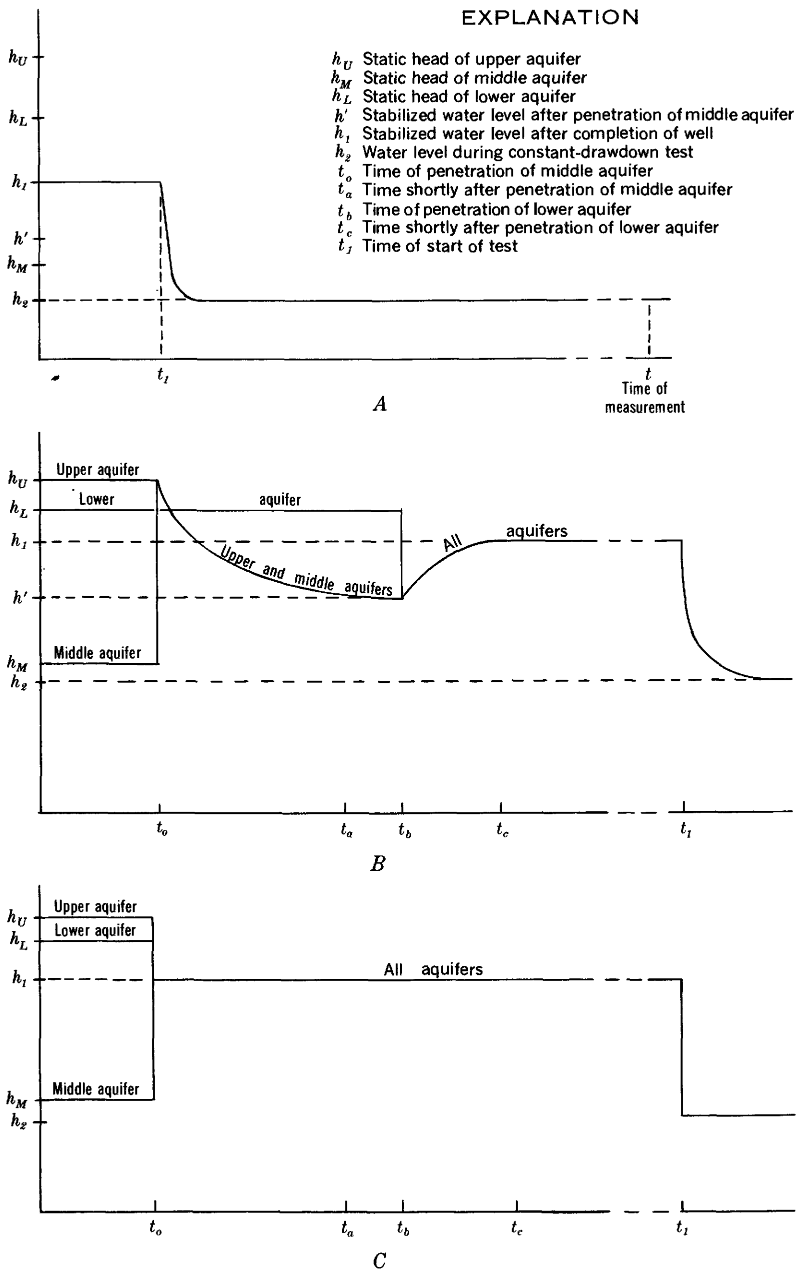

FIGURE 54.—Graphs showing fluctuations in artesian head in the three-aquifer well of figure 50: (*A*) drawdown pattern in a well during constant-head pumping test, (*B*) composite graph showing changes in head in all aquifers during completion of the well and constant-drawdown test, (*C*) final stepwise approximation to these changes.

of these intervals. The difference between the terms $G\left\{\frac{T_U(t-t_0)}{S_U r_w^2}\right\}$ and $G\left\{\frac{T_U(t_1-t_0)}{S_U r_w^2}\right\}$ will, accordingly, be negligible, and equation 10 can be modified by substitution to

$$Q_U(t)=2\pi T_U(h_U-h_1)\cdot G\left\{\frac{T_U(t_1-t_0)}{S_U r_w^2}\right\} +2\pi T_U(h_1-h_2)\cdot G\left\{\frac{T_U(t-t_1)}{S_U r_w^2}\right\}. \quad (11)$$

From equation 9, however, it can be seen that the first term on the right in equation 11 is equal to $Q_U(t_1)$, the internal discharge from the upper aquifer immediately prior to the beginning of the test. Substituting and transposing then gives

$$Q_U(t)-Q_U(t_1)=\Delta Q_U=2\pi T_U(h_1-h_2)\cdot G\left\{\frac{T_U(t-t_1)}{S_U r_w^2}\right\}. \quad (12)$$

The discharge from the upper aquifer at time t consists of two parts—one due to the first-step change in head occurring at completion of the well and the other due to the second-step change created by the pumping test.

It can be assumed that at the time of measurement the component of discharge due to the first step will be virtually equal to the value at the beginning of the test. This is true because the time difference $(t-t_1)$ is small relative to (t_1-t_0) and because the discharge due to the first-step change in head has been in progress for a considerable length of time and is, therefore, changing at a slow rate. Thus subtraction of the discharge due to the first step isolates the component which is due to the second step, and this component can then be equated to the expression corresponding to the second step for purposes of test analysis.

It should be noted that measurements must be taken at several times during the test, and the assumption that the discharge due to the first step has not changed appreciably since the start of the test must be true for the entire test interval. If this component of discharge is changing significantly with time at the beginning of the test, its trend may be established by taking successive measurements prior to pumping. Extrapolation will then give the value of this component corresponding to any later time, and subtraction will isolate the component corresponding to the second step of drawdown.

METHOD OF ANALYSIS OF TEST DATA

The test data will consist of records of aquifer discharge as a function of time for each aquifer penetrated by the well. Jacob and Lohman (1952) present two graphical methods of solution that may be applied to these records to compute the transmissibility and storage coefficients for the various aquifers.

The first method consists of plotting discharge (or, in this example, the net change in discharge obtained by subtracting the original internal discharge) against time on logarithmic paper. This plot is then overlain upon a plot of $G(\beta)$ versus (β) on logarithmic paper of the same scale. The curves are adjusted until the best possible fit or match is obtained while keeping the coordinate axes of the two graphs parallel. With the graphs aligned in this manner, a constant difference between the values of $[G(\beta)]$ and (ΔQ) is indicated by the separation of the horizontal axes of the curves. Similarly a constant difference between the values of β and t is indicated by the separation of the vertical axes. The significance of these constant differences can be seen by using logarithms in the equations

$$\Delta Q=2\pi T(h_1-h_2)\cdot G(\beta), \tag{13}$$

and

$$\beta=\frac{T(t-t_1)}{Sr_w^2}. \tag{14}$$

That is,

$$\log \Delta Q=\log [2\pi T(h_1-h_2)]+\log G(\beta), \tag{15}$$

and

$$\log (\beta)=\log \left(\frac{T}{Sr_w^2}\right)+\log (t-t_1). \tag{16}$$

The separation of the horizontal axes of the superposed logarithmic plots is thus proportional to $\log 2\pi T(h_1-h_2)$, while the separation of the vertical axes is proportional to $\log (T/Sr_w{}^2)$. When the curves are properly alined, therefore, the coordinates of any pair of overlying points may be used in equations 13 and 14 to compute T and S for the aquifer.

The second method is based upon the fact that when $(t-t_1)$ is large, the function $G\left\{\frac{T}{Sr_w^2}(t-t_1)\right\}$ is about equal to the function

$$\frac{2}{2.3\log\left[\frac{2.25T(t-t_1)}{Sr_w^2}\right]}.$$

This approximate equivalence is demonstrated by Jacob and Lohman (1952) by comparison of the solution for constant drawdown with the solution corresponding to constant rate of discharge. Substitution of the expression indicated above for the G function in the equation for net discharge change gives:

$$\Delta Q=\frac{4\pi T(h_1-h_2)}{2.3\log\left[\frac{2.25T(t-t_1)}{Sr_w^2}\right]}. \tag{17}$$

Equation 17 can be rearranged to read

$$\frac{1}{\Delta Q}=\frac{2.3}{4\pi T(h_1-h_2)}\cdot\log\frac{2.25T}{Sr_w^2}+\frac{2.3}{4\pi T(h_1-h_2)}\log(t-t_1). \tag{18}$$

Differentiating equation 18 gives

$$\frac{\partial\left(\frac{1}{\Delta Q}\right)}{\partial\log(t-t_1)}=\frac{2.3}{4\pi T(h_1-h_2)}. \tag{19}$$

The term on the right side of equation 19 is a constant. Thus if values of $\frac{1}{\Delta Q}$ are plotted against values of $(t-t_1)$ on semilogarithmic paper, the slope of the plot should become constant once the abscissa, $(t-t_1)$, enters the range of values for which equation 17 applies.

The second graphical solution is obtained by the following procedure. Values of $\frac{1}{\Delta Q}$ for the aquifer concerned are plotted against values of $(t-t')$ on semilogarithmic paper. When these values begin to define a straight line, the test has entered the region in which equation 17 applies. Measurement of the slope of this line then leads to a determination of the coefficient of transmissibility, because this slope may be substituted for

$$\frac{\partial\left(\frac{1}{\Delta Q}\right)}{\partial \log (t-t_1)}$$

in equation 19. Once T is known, it can be substituted, together with the coordinates of any point on the semilogarithmic plot, into equation 17 for a determination of the storage coefficient (S).

When the transmissibility and storage coefficients of an aquifer have been determined, the original hydrostatic head of the aquifer may be calculated by an application of equation 9. The discharge value employed is the original internal discharge from the aquifer, as measured prior to pumping. The water level in the well at the start of pumping (h_1), the time interval from completion of the well to the start of the test (t_1-t_0), and the appropriate values of T, S, and r_w are also inserted to obtain a solution for the original head (h_U in the notation of equation 9). This is a substitution of the newly determined values of T and S into the first step of the pattern of changes in head (sketched in figures 51–53) in order to find the height of the first step. The asymptotic expression

$$\frac{2}{2.3 \log \left[\frac{2.25T(t_1-t_0)}{Sr_w^2}\right]}$$

may be used in place of the G function in equation 9, if this seems to be more convenient.

FIELD PROCEDURES AND QUALITY OF RESULTS

In order to carry out a test of the sort under discussion, the following field procedures are necessary: (1) lithologic or geophysical logging of the well, to locate the possible aquifers, (2) measurement of the water level in the well prior to pumping, (3) measurement of the internal discharge to or from each aquifer prior to pumping, (4) pumping of the well at a measured constant drawdown, and (5) measurement of the internal discharge to or from each aquifer, as a function of time, during the constant-drawdown test.

The method was tested by the authors on a well in Chester County, Pennsylvania. Electric logging of the well indicated two major sandstone aquifers separated by a considerable thickness of shale. The internal discharge between the aquifers before pumping was measured by releasing a slug of brine in the region between the aquifers and tracing its movement with a fluid-resistivity logging device. The velocity of the brine slug multiplied by the borehole area (as determined from caliper logs) provided the measurement of internal discharge. The discharge was from the lower aquifer to the upper aquifer and was about 8 gpm.

The pump used in the test was a small centrifugal model powered by a gasoline engine. A 3-inch globe valve in the discharge line provided a means for controlling the rate of pumping. A float-operated water-level recorder was placed on the well, and a constant drawdown was obtained by controlling the pump manually while observing the recorder. A drawdown of 8.08 feet was obtained during the first three minutes of the test. This drawdown was not allowed to vary more than 0.15 foot during the remainder of the 6-hour test.

Measurements of discharge during the test were made frequently, both at the surface and, by use of a Au borehole flowmeter, in the interval between the two aquifers. The Au-meter measurements provided a record of the discharge of the lower aquifer, and subtraction of this discharge from the discharge of the well provided a record of the discharge of the upper aquifer. Algebraic subtraction of the original internal discharge provided a record of ΔQ as a function of time.

The data were analyzed by using the semilogarithmic-plot method of Jacob and Lohman (1952) to obtain values of S and T, and by using an equation of the form of equation 9 (but substituting the asymptotic expression for the G function) to calculate h. The results were poor and indicate that the method is unsatisfactory at present for accurate determination of the aquifer constants.

The results indicate that determinations of transmissibility and static head, using the field procedures employed by the authors, may be accepted as having order-of-magnitude significance. The determination of storage coefficient, which is highly sensitive to experimental error, cannot be accepted at all. The poor results are due to insufficient accuracy of measurement and to deficiencies in the theory. These topics are discussed in the following section.

GENERAL EVALUATION OF THE TEST METHOD

The method presented in this report leads to estimates of the transmissibilities and static heads of the various aquifers penetrated by a well; determination of storage coefficients may become feasible as the method is refined. Also it may sometimes be possible to recognize hydrologic boundaries in a given aquifer from the test results; a recharge boundary should be indicated by a levelling off in the aquifer's discharge, and an impermeable boundary should be indicated by an abnormal decrease in its discharge.

At present a number of difficulties detract from the utility of the method. The doubtful accuracy of the flow measurements is a serious problem because, even when the conditions assumed in the theory are fully met, reliable results cannot be achieved without an accurate means of measuring flow in the borehole and at the surface. Basically, however, this is a problem of developing better instruments, and it is to be expected that future development along these lines may minimize such sources of error.

The method is also subject to the limitations inherent in conventional methods for analyzing pumping tests—that is, the theories on which the solutions are based assume that each aquifer is infinite, homogeneous, isotropic, and free from discharge or recharge boundaries of any sort. Also, the head of each aquifer is assumed to be uniform prior to well completion. Because all these conditions must be fulfilled for several aquifers, rather than for only one, the chances of finding ideal test conditions are correspondingly lessened for a multiaquifer well.

The theory neglects any interflow between aquifers, other than through the well itself. Thus if the aquifers are not separated by perfect aquicludes and if certain aquifers are recharged vertically by loss from other aquifers, the test results may be misleading. In the same way, the theory does not consider flow that originates in the confining beds themselves and that enters the aquifers by vertical percolation, except insofar as this effect is considered to contribute to the storage coefficient of the aquifer concerned.

The greatest weakness in the test theory, however, is probably connected with well-entrance losses. Entrance losses were not considered. A test will, therefore, give good results only when these losses are negligible. In testing a well penetrating a single aquifer it is possible to isolate entrance losses by pumping at a number of different constant rates of discharge (Rorabaugh, 1953). In testing a multiaquifer well, however, there are two complicating factors. First, the entrance losses may differ from one aquifer to another, as is illustrated in figure 55, which shows the differences between the water level in the well and the piezometric surfaces of the various acquifers immediately outside the well. Second, the entrance losses of each aquifer, must vary with time in a constant-drawdown test, as they are proportional to some power of the aquifer's discharge. The complications that this may introduce into the analysis are obvious, as the constant total drawdown represents the sum of the formation loss and entrance loss, that is:

$$s_w = s_f + s_e, \tag{20}$$

where s_w is the drawdown in the well, s_f is the drawdown of the piezometric surface in the aquifer outside the well, and s_e is the entrance loss of the aquifer. Thus, if the entrance loss of an aquifer

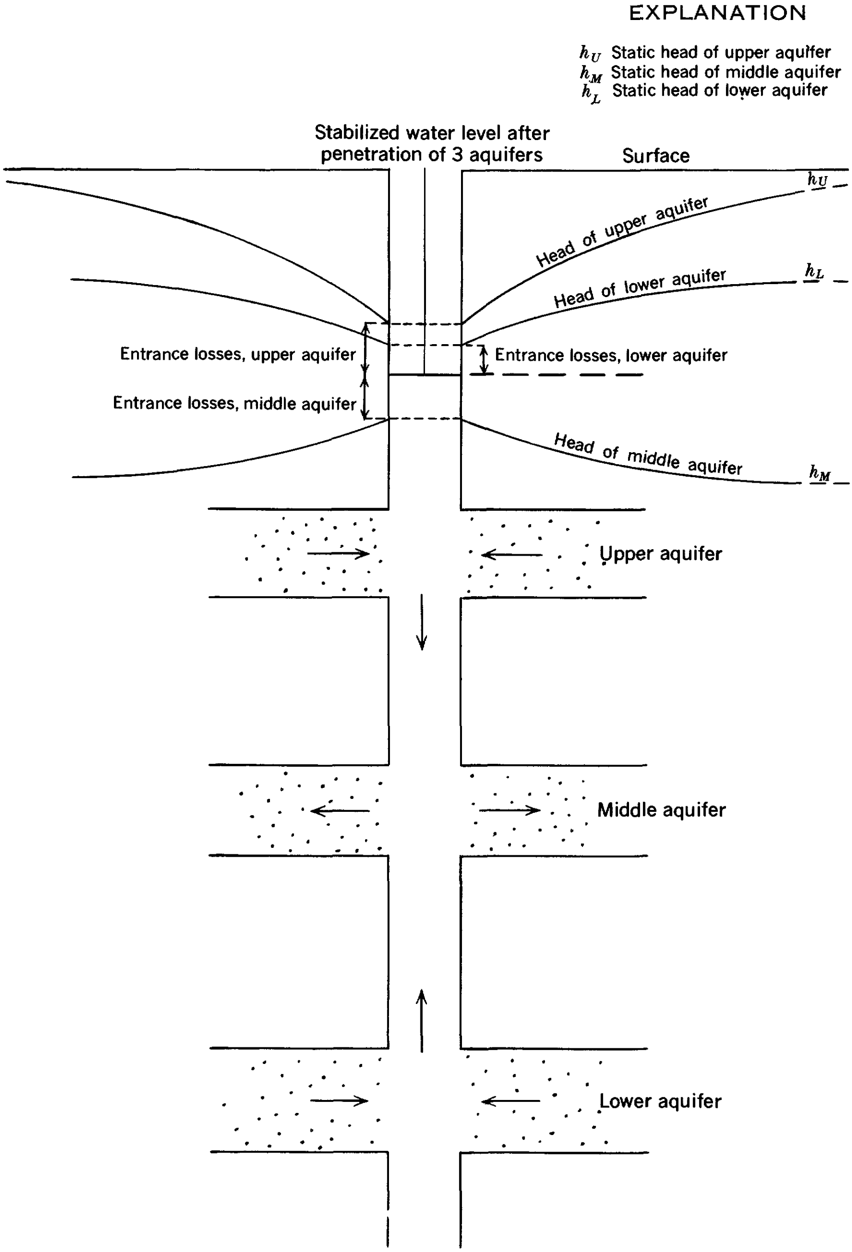

FIGURE 55.—Diagram showing entrance losses in a multiaquifer well.

(s_e) varies, the piezometric surface of the aquifer immediately outside the well must also vary in such a way that the total drawdown in the well remains constant. In such a situation, the boundary condition of constant drawdown at the borehole wall is no longer satisfied in the aquifer, and the test results are erroneous in proportion to the variation in the piezometric surface.

If the range of variation in an aquifer's discharge during most of the test is small compared to the absolute value of its discharge at the end of the test, the problem of entrance losses varying with time will probably be negligible. The problem of determining the magnitude of the entrance losses for each individual aquifer may be resolved as further research is carried on into the nature of entrance losses. For the present, however, the only way to deal with the entrance-loss problem is to use a drawdown step that will produce relatively moderate discharges during most of the test, thus minimizing the size of the losses. For any well, the results of the test should be questioned whenever significant entrance losses are suspected.

The most serious shortcomings of the test method are insufficient accuracy of measurement and incomplete control of certain variables. These shortcomings, at present at least, tend to reduce the test results to order-of-magnitude significance. In many tests, however, even a general idea of the constants of the various aquifers is of considerable value. If the method can be refined to the point of yielding reasonable results, it should be possible to obtain useful information concerning multiple aquifers penetrated by a single well. Using the results in an equation of the form of equation 5, the performance of each aquifer in the well can be estimated for any drawdown pattern that can be approximated by a series of steps. Modern methods of digital computation have made analyses of this sort practical. The data also might be used to construct an electric analog of the well and the surrounding aquifers; this in turn could be used to determine the response of the well to virtually any set of hydrologic conditions.

OTHER POSSIBLE METHODS OF OBTAINING DATA ON INDIVIDUAL AQUIFERS

If accurate data on the transmissibility and storage coefficients of the various aquifers are needed, it is probably best to run a constant-discharge test on each aquifer in turn. At present this involves isolating each aquifer tested from the other aquifers in the well. This might be accomplished by using packers in the tested well or by drilling new wells, each of which is open to only one of the aquifers.

It is possible that advances in instruments may lead to better methods of performing future tests on multiaquifer wells. For example, a system might be developed in which the well pump is governed by a servomechanism that is controlled by flowmeters placed above and below the particular aquifer being tested. The flowmeters would not only measure the discharge from this aquifer but would maintain it at a constant rate—decreasing the pump output to counteract increases in flow, and increasing the pump output to counteract decreases in flow. The drawdown pattern observed in the well would then correspond to the constant discharge measured from the aquifer in question, and the aquifer constants could be calculated by the conventional Theis analysis, provided that any internal discharge at the beginning of the test was compensated for in the calculations.

REFERENCES

Bennett, G. D., and Patten, E. P., Jr., 1959, Borehole geophysical methods for analyzing specific capacity of multiaquifer wells: U.S. Geol. Survey open-file report, 54 p., 8 figs.

Jacob, C. E., 1950, Flow of ground water, *in* Rouse, Hunter, Engineering hydraulics: New York, John Wiley and Sons, p. 321–386.

Jacob, C. E., and Lohman, S. W., 1952, Nonsteady flow to a well of constant drawdown in an extensive aquifer: Am. Geophys. Union Trans., v. 33, no. 4, p. 559–569.

Rorabaugh, M. I., 1953, Graphical and theoretical analysis of step-drawdown test of artesian well: Am. Soc. Civil Engineers Proc., v. 79, Separate 362, 23 p.

Smith, L. P., 1937, Heat flow in an infinite solid bounded internally by a cylinder: Jour. Appl. Physics, v. 8, p. 441–448.

○