

Weaver

Storage and Flood Routing

Manual of Hydrology: Part 3. Flood-Flow Techniques

GEOLOGICAL SURVEY WATER-SUPPLY PAPER 1543-B



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By R. W. CARTER and R. G. GODFREY

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*Methods and practices of
the Geological Survey*



UNITED STATES DEPARTMENT OF THE INTERIOR

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MANUAL OF HYDROLOGY: PART 3, FLOOD-FLOW TECHNIQUES

STORAGE AND FLOOD ROUTING

By R. W. CARTER and R. G. GODFREY

ABSTRACT

The basic equations used in flood routing are developed from the law of continuity. In each method the assumptions are discussed to enable the user to select an appropriate technique.

In the stage-storage method the storage is related to the mean gage height in the reach under consideration. In the discharge-storage method the storage is determined from weighted values of inflow and outflow discharge. In the reservoir-storage method the storage is considered as a function of outflow discharge alone.

A detailed example is given for each method to illustrate that particular technique.

INTRODUCTION

The storage in the reaches of stream channels is used extensively as an index of the timing and shape of flood waves at successive points along a river. Among the principal users of the technique are the Corps of Engineers who route hypothetical floods through river systems to determine the effects from proposed flood-control projects, the U.S. Weather Bureau whose forecasts of river stages are based largely on flood routing, and the operators of hydroelectric power systems who schedule their operations according to the predicted progress of a flood wave. The storage index and the techniques of flood routing may also be used to advantage in computing and evaluating streamflow records.

The primary use of these methods in the Geological Survey is in testing and improving the overall consistency of records of discharge during major floods in a river basin. The number of direct observations of discharge during such flood periods is generally limited by the short duration of the flood and the inaccessibility of certain stream sites. Through the use of flood-routing techniques, all observations of discharge and other hydrologic events in a river basin may be combined and used to evaluate the discharge hydrograph at a single site. For

an example of a study of this type the reader is referred to Water-Supply Paper 1139 (U.S. Geological Survey, 1952).

Storage-routing techniques also are used frequently to determine the effect of artificial storage on peak discharges from small drainage basins. For economy of operation, gaging stations on many small streams are located at the outfall of small detention reservoirs impounded by dams or by highway crossings. The measured peak discharges are adjusted for the effects of artificial storage, thereby increasing their usefulness in areal studies of the frequency and magnitude of peak flow.

Another use of flood routing in hydrologic studies is in computing natural flood hydrographs for long-term gaging stations on streams where major storage reservoirs have been constructed. These computations are made to extend the time period for a single condition of basin development for use in discharge-frequency studies.

In this report, several different methods of storage routing are presented in detailed form. These methods are not new and, in general, were not originated in the Geological Survey. They are presented for the convenience of engineers engaged in hydrologic studies. A previous manual on this subject was prepared by W. B. Langbein and R. W. Carter (written communication, 1947). The methods and techniques described in this manual are the product of development in many engineering offices concerned with floods.

GENERAL RELATION BETWEEN UPSTREAM AND DOWNSTREAM HYDROGRAPHS

Flood waves are subject to two principal kinds of movements—uniformly progressive flow and reservoir action. A uniformly progressive flow designates downstream movement of a flood wave without a change in shape, which would occur only under ideal conditions in a prismatic channel in which the stage and discharge are uniquely defined at all places. Reservoir action refers to the modification of a flood wave by reservoir pondage. Flood-wave movement in natural-channel systems is probably intermediate between the two ideal conditions cited, one or the other predominating in a particular place. However, the actual behavior of the wave is sometimes obscured by the effects of local tributary inflow.

The attenuation effect of what has been called reservoir-type or pondage storage is illustrated by the performance of a detention or retarding basin—a reservoir impounded by a dam with an outlet at stream level. The outlet conduit is designed to permit the normal flows of the stream to pass unobstructed, but its capacity is exceeded by flood discharges. The impounded water creates a sensibly level pool and discharge through the conduit is related solely to the head

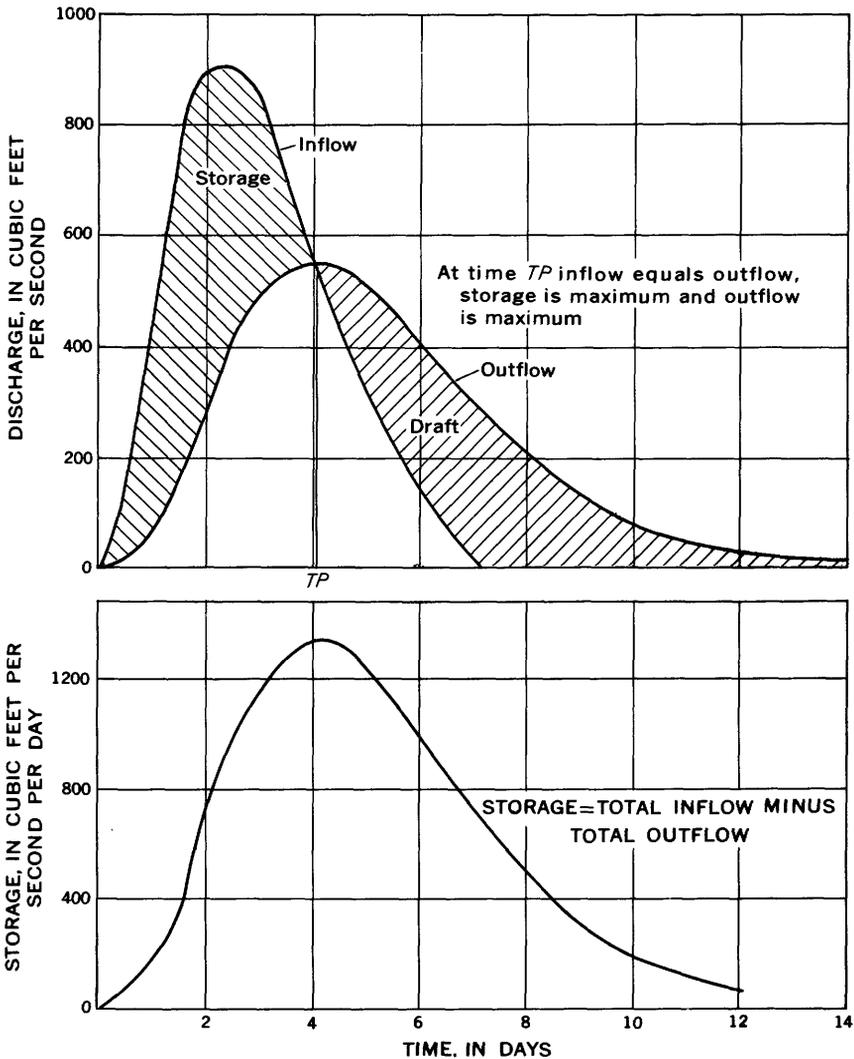


FIGURE 31.—Schematic representation of reservoir-type storage.

and hence to the storage. Storage in the reservoir will increase so long as inflow exceeds outflow, and is still increasing when peak inflow occurs. Figure 31 shows that storage and outflow reach a maximum (*TP*) when inflow equals outflow, but this equality cannot occur until after inflow has receded from its peak. Therefore, reservoir or pondage storage always attenuates peak discharge.

The storage generated in a reach of channel by a uniformly progressive wave, illustrated in figure 32, is completely different. The storage prism varies in shape and in volume. River stages and storage in a reach are conditioned by the inflow and the outflow rates. Hence,

in the course of a flood that is controlled only by channel action, the storage increases rapidly in relation to outflow. Conversely, the storage decreases rapidly when the flood recedes. Storage is increasing when peak inflow occurs, and it continues to increase until outflow rises to equal the receding inflow. At this time the flood crest occupies a central position in the reach. At the instant of the maximum storage, the net change in storage equals zero and inflow equals outflow; the upstream stages are decreasing, the downstream stages are increasing and volume of storage in the reach at that instant is constant. After this point is passed, outflow continues to increase,

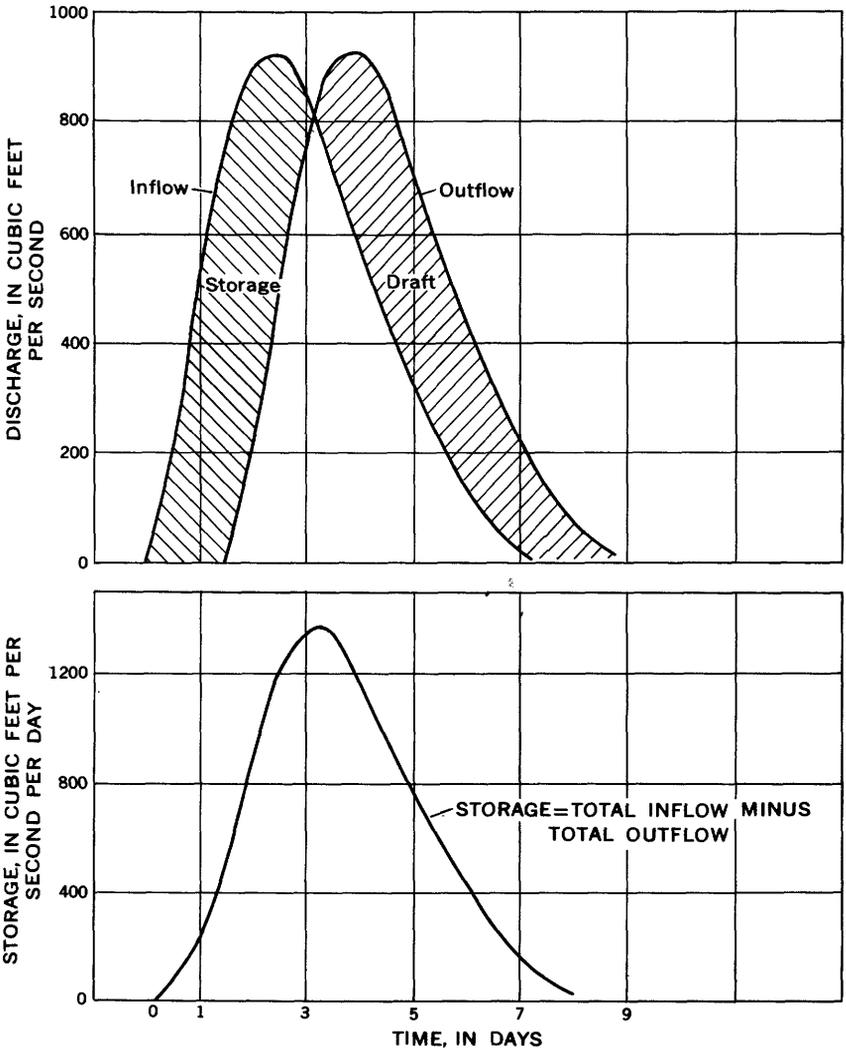


FIGURE 32.—Schematic representation of inflow, outflow, and storage in a uniformly progressive wave.

partly at the expense of storage in the reach, until the crest passes the lower end.

Figures 31 and 32 show the difference in the behavior of storage controlled by a fixed outlet dam and by channel action. The inflow hydrographs, total storage, and lag between centers of mass of inflow and outflow, are the same in both illustrations. In reservoir-type storage, as in figure 31, the peak outflow is much less than the peak inflow, whereas in the uniformly progressive wave, as in figure 32, there is no reduction in peak discharge.

River channel systems are commonly characterized by alternations of "pools" and "rapids" and of "narrows" and "intervals" which in the aggregate have much the same attenuating effect as reservoir storage. However, where the cross section and bottom slope are virtually uniform, little or no attenuation due to storage can be expected. The action of storage must not be confused with the attenuating effect of the desynchronized timing of tributary and mainstream flood peaks.

METHODS OF FLOOD ROUTING

BASIC CONCEPTS

Many different methods are used to route flood waves through river reaches. All these methods are based on the law of continuity—the volume of water that is discharged from a reach during any interval must equal the volume of inflow during the interval plus or minus any increment in stored water during the period. In equation form it becomes:

$$\bar{O} = \bar{I} - \frac{\Delta S}{\Delta t} \quad (1)$$

where \bar{O} = mean outflow during routing period Δt

\bar{I} = mean inflow during routing period Δt

ΔS = net change in storage during routing period Δt

Equation (1) is general. A modification frequently used is:

$$\frac{\Delta t(O_1 + O_2)}{2} = \frac{\Delta t(I_1 + I_2)}{2} - (S_2 - S_1) \quad (2)$$

where O , I , S and Δt are as before, and the subscripts identify the beginning and ending of routing period Δt . The assumption that mean discharge is equal to the simple arithmetic average of the flows at the end points of the interval can be justified if the period is equal to, or less than, the time of travel through the reach and no abrupt changes in flow occur during the routing period.

Two problems arise in using the storage equation in a particular reach. The first is to express the storage volume in terms of some practical index and the second is to solve equation (2) for O_2 . The equation can be readily solved by graphical or analytical methods. The exact nature of the solution depends on the method used to determine the storage in the reach.

Expressing the storage volume in terms of some practical index is much more difficult, but it is extremely important because the

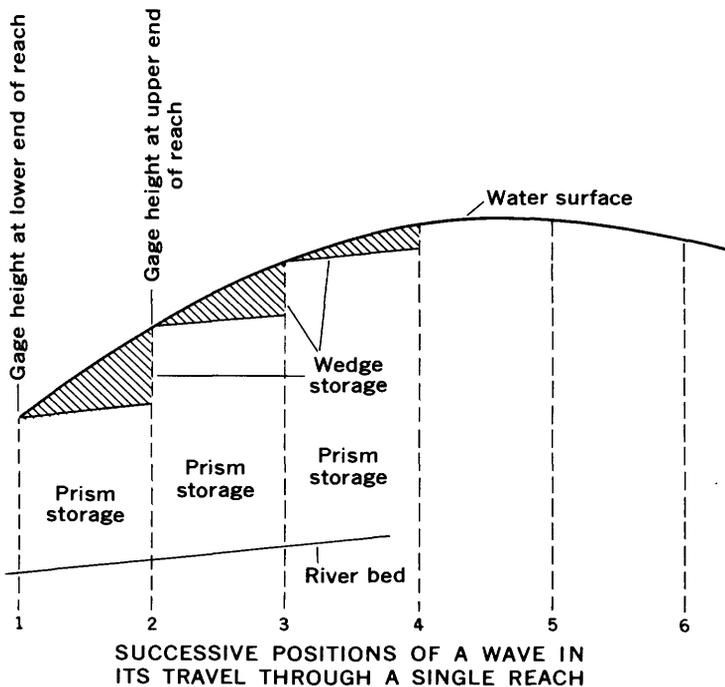


FIGURE 33.—Schematic representation of prism and wedge storage.

accuracy of flood routing depends on the closeness with which the storage volume can be approximated. Assume that the reach in figure 33 represents successive positions of a wave traveling through a single reach. One element of storage commonly called "prism storage" may be easily defined in terms of stage or discharge at the downstream station. The simpler routing methods attempt to use this approximation alone, neglecting a second important volume of "wedge storage" which is superimposed on the prism. Wedge storage is a secondary approximation, but it is much more difficult to define in terms of a simple index because of the variety of shapes of the wedges. However, a routing method that ignores wedge storage is subject to serious error except in most unusual conditions.

The storage in a reach may be determined by direct measurement from detailed maps or channel cross sections, as if computing reservoir or earthwork volumes. This approach is laborious and data are rarely adequate. Except where stage records are available during the flood to be routed, it would be necessary to compute several backwater curves defining the water-surface profile under various conditions of unsteady flow.

The method most often used is to compute the volume of storage by comparing the actual flood hydrographs. The volume of water in storage at any instant exclusive of intervening inflow between the stations is equal to the accumulated difference between inflow and outflow.

STAGE-STORAGE METHOD OF FLOOD ROUTING

The simplest method of flood routing defines the storage in terms of the mean gage height in the reach. Thus a gage-height record for both ends of the reach must be available if the flood is to be routed. The necessary stage-storage relation is generally defined on the basis of past flood-discharge records; although in certain reaches it may be defined from topographic data. Areal photographs of rivers in flood are also valuable in defining this relation. Because storage is directly related to stage, $\bar{A}\Delta h$ may be substituted in equation (1) for ΔS

$$\bar{O} = \bar{I} - \frac{\Delta S}{\Delta t} \quad (1)$$

$$\bar{O} = \bar{I} - \bar{A} \frac{\Delta h}{\Delta t} \quad (3)$$

where \bar{A} = average area of water surface in the reach during time Δt and Δh = average change in water surface elevation in the reach in time Δt .

In equation (3), the value of \bar{I} is obtained from the given inflow hydrograph and from the value $\frac{\Delta h}{\Delta t}$ which is obtained from the stage record. The water-surface area at a given stage is the slope of the stage-storage curve and may be easily computed from a stage-storage table, since it is the "first difference." Hence the slope or first difference is a function of the mean stage in the reach. As mentioned previously, areal photographs made during record floods can be used to construct or extend a stage-water surface relation. The outflow may be computed from the mean stage, the rate of change of stage, and the inflow.

As an example, the method is applied to the Cochrane-Gainesville reach on the Tombigbee River in Alabama. This reach, as shown

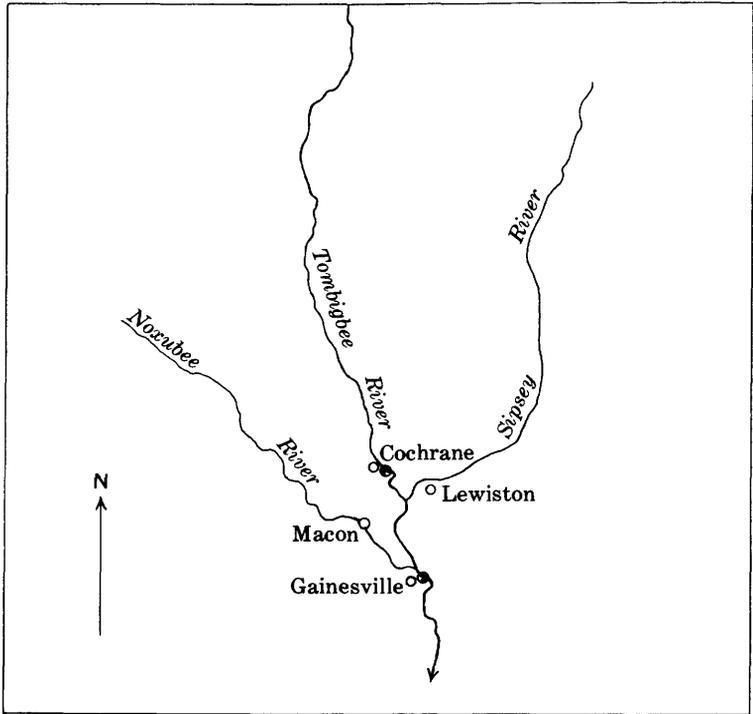


FIGURE 34.—Cochrane-Gainesville reach on Tombigbee River.

in figure 34, is about 25 miles long and the flow from the main tributaries entering the reach is measured. Records for the flood of July 1940 were used to define the stage-storage relation. The small amount of unmeasured inflow was estimated by increasing the flow of the measured tributaries by the drainage-area ratio. In table 1, columns 2, 3, and 6 show the several components of inflow, and column 7 shows the total inflow. Listed in columns 8 and 9 are the accumulated inflow and the accumulated outflow. The difference in accumulated inflow and accumulated outflow, which is the total storage in the reach at the end of the selected time interval, is shown in column 10. The storage is represented by water within the channel, on the river flood plain, in riverbank voids, and in contiguous swamps which are affected by river levels.

The mean stage shown in column 11 of table 1 is the mean of the midnight gage heights at each end of the reach. Intermediate stage records in the reach may be included in this average if available. The mean stage and storage in the reach at the time of the mean are plotted in figure 35.

TABLE 1.—Computation of storage for Cochrane-Garnesville reach, Tombigbee River

Date	Partial inflow (cfs) discharge				Cochrane	Total inflow discharge (cfs)	Cumulative discharge		Storage (1,000 cfs-days)	Mean gage height at 12 p.m. (feet)
	Noxabee	Sipsey	Noxabee plus Sipsey	Column 4 times 1.16			Inflow (1,000 cfs-days)	Outflow (1,000 cfs-days)		
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
June 28, 1940	294	243	537	623	1,360	1,980	2	2	0	4.2
29	380	274	654	760	1,680	2,700	6	8	0	6.4
30	1,480	208	1,798	2,081	2,880	3,670	12	5	3	7.9
1	1,420	206	1,716	1,990	4,700	4,700	18	16	2	8.4
July 2	1,080	432	1,462	1,696	4,680	6,380	23	22	8	9.0
3	2,050	1,100	3,150	3,654	14,500	18,200	43	34	9	19.9
4	6,400	1,590	7,990	9,293	29,000	38,300	81	61	20	27.1
5	6,900	1,570	8,490	9,850	31,000	41,200	122	90	26	30.1
6	6,980	1,550	8,530	9,895	36,100	46,000	168	134	34	32.8
7	7,180	1,760	8,940	10,370	40,400	50,800	219	176	43	33.1
8	7,900	2,026	9,920	11,500	42,000	53,500	273	221	52	36.3
9	11,100	2,030	13,130	15,200	41,700	56,900	330	269	38.1	38.1
10	22,400	2,410	24,810	28,800	40,400	69,200	399	325	74	39.6
11	20,400	2,810	23,210	26,900	36,800	63,700	462	386	76	40.2
12	19,500	3,630	23,130	26,800	36,100	62,900	525	447	78	40.5
13	18,500	4,780	23,280	27,000	36,700	63,700	589	508	81	40.7
14	14,700	5,760	20,460	23,700	38,700	62,400	652	567	85	41.1
15	11,500	6,460	17,960	20,800	40,500	61,300	713	625	88	41.4
16	10,800	7,020	17,820	20,700	40,300	61,000	774	684	90	41.5
17	9,840	6,690	16,530	19,200	39,300	58,500	832	742	90	41.2
18	8,620	5,280	13,900	16,100	36,700	52,800	885	799	86	40.8
19	6,930	5,040	11,970	12,800	33,700	46,500	932	854	78	40.0
20	6,100	4,230	10,330	12,000	30,900	42,900	974	907	67	39.0
21	6,160	3,500	9,660	11,200	27,200	38,400	1,013	957	56	37.6
22	6,540	3,550	10,090	11,700	23,300	35,000	1,048	1,004	44	35.4
23	6,420	3,040	9,460	11,000	20,000	31,000	1,079	1,046	33	32.3
24	5,030	2,230	7,260	8,420	12,700	21,100	1,100	1,080	20	27.0
25	3,760	1,850	5,610	6,510	7,670	14,200	1,114	1,104	10	19.6
26	3,936	1,810	5,746	3,185	3,790	6,980	1,121	1,118	3	13.2
27	846	1,670	2,516	2,920	3,610	6,530	1,128	1,127	1	9.0
28	410	1,500	1,910	2,220	3,360	5,580	1,133	1,132	1	7.4

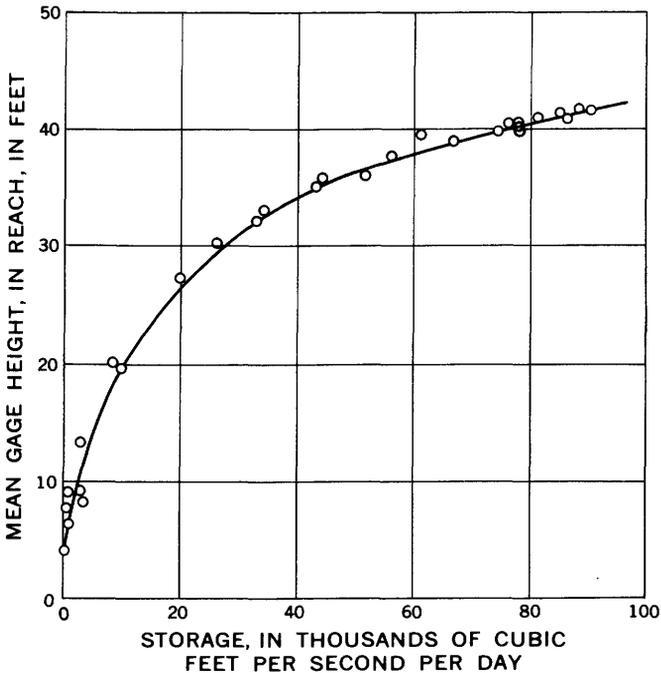


FIGURE 35.—Relation of mean gage height to storage in Cochrane-Gainesville reach.

The slope of the stage-storage curve, $\Delta s : \Delta h$, is shown plotted against the mean stage in figure 36.

The above operation should be plotted for several floods, and an average stage-storage relation adopted. Major differences for one or more floods indicate that backwater conditions may exist or that there are errors in ratings or stage data.

The treatment of the above reach is simplified somewhat, because the storage volume is treated as a function of the straight average of the stage at each end of the reach. In reaches where a large percentage of the storage volume is near one end of the reach, the stages generally have to be weighted. Records from gages on the tributaries should be given some weight in determining the mean stage in the reach. These weights may be assigned from a knowledge of the physical characteristics of the channel or may be determined by trial and error.

To test the stage-storage relation, the outflow hydrograph is computed from the inflow hydrograph and from the record of stage at both ends of the reach (table 2). Column 2 lists the daily mean gage height in the reach, which is obtained by averaging the observed mean daily stages at both ends of the reach. The change in storage per foot of gage height is listed in column 3. This change is equivalent to the

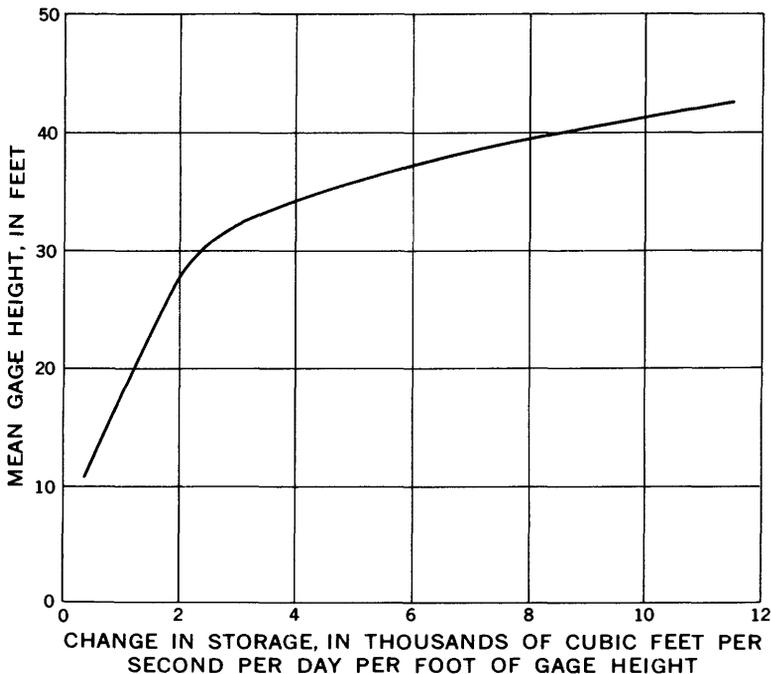


FIGURE 36.—Relation of change in storage per foot of gage height to mean gage height for Cochrane-Gainesville reach.

surface area (see equation 3) and is the abscissa of figure 36 associated with the ordinate which is the mean stage from column 2. Listed in column 4 is the change in stage in the reach during the time interval selected, which is 1 day in this example. The product of the change in storage per foot of stage (column 3) and the stage per day (column 4) is shown in column 5. The outflow which is listed in column 7 is equal to the inflow minus the change in storage. A comparison of the computed and the actual outflow is shown in figure 37.

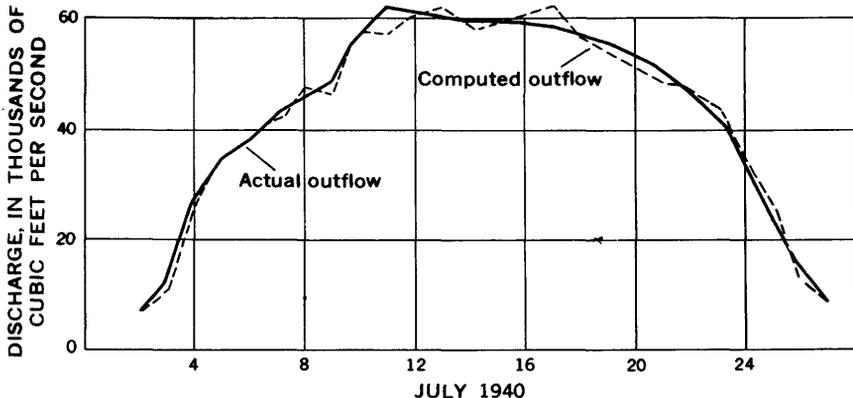


FIGURE 37.—Comparison of actual and computed hydrographs for Tombigbee River at Gainesville, Ala.

TABLE 2.—Example of routing for Cochrane-Gainesville reach

Date	Mean daily gage height (<i>h</i>) (feet)	$\frac{\Delta S}{\Delta t}$ (1,000 cfs-days)	$\frac{\Delta h}{\Delta t}$ (feet per day)	$\frac{\Delta S}{\Delta t}$ (cfs-days)	Total inflow (cfs)	Outflow (cfs)	
						Computed	Actual
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
July 1940							
2	8.3	0.2	+0.6	+120	6,380	6,260	6,160
3	14.1	.7	+10.9	+7,630	18,200	10,600	12,200
4	25.1	1.8	+7.2	+13,000	38,300	25,300	27,100
5	28.7	2.1	+3.0	+6,300	41,200	34,900	34,400
6	31.4	2.8	+2.7	+7,560	46,000	38,400	37,900
7	34.1	4.0	+2.3	+9,200	50,800	41,600	42,000
8	35.7	4.9	+1.2	+5,880	53,500	47,600	45,200
9	37.1	6.0	+1.8	+10,800	56,900	46,100	48,100
10	38.9	7.7	+1.5	+11,600	69,200	57,600	55,700
11	40.0	8.6	+ .80	+6,880	63,700	56,800	61,700
12	40.4	9.1	+ .30	+2,730	62,900	60,200	61,000
13	40.6	9.2	+ .20	+1,840	63,700	61,900	60,100
14	40.9	9.5	+ .40	+3,800	62,400	58,600	59,300
15	41.2	9.9	+ .30	+2,970	61,300	58,300	58,600
16	41.4	10.1	+ .10	+1,010	61,000	60,000	58,200
17	41.4	10.1	- .30	-3,030	58,500	61,500	58,100
18	41.0	9.6	- .40	-3,840	52,800	56,600	56,900
19	40.4	9.1	- .80	-7,280	46,500	53,800	55,100
20	39.6	8.2	-1.00	-8,200	42,900	51,100	52,900
21	38.3	7.1	-1.4	-9,940	38,400	48,300	50,200
22	36.6	5.6	-2.2	-12,300	35,000	47,300	46,900
23	34.0	3.9	-3.1	-12,100	31,000	43,100	41,800
24	29.8	2.4	-5.3	-12,700	21,100	33,800	34,300
25	23.3	1.6	-7.4	-11,800	14,200	26,000	23,800
26	16.3	.9	-6.4	-5,760	6,980	12,700	14,700
27	10.3	.4	-4.2	-1,680	6,530	8,210	8,280

DISCHARGE-STORAGE METHOD OF FLOOD ROUTING

Most methods of flood routing now in use define the storage volume in terms of the inflow and the outflow rather than the stage. The simplest, but least accurate, routing procedures define the reach storage in terms of the outflow only. If the storage in the reach is plotted against the outflow, an irregular loop is developed rather than a single-valued graph. The loop reflects the influence of wedge storage. A much more satisfactory definition of storage was introduced by McCarthy (written communication, 1938) who used both the inflow and the outflow rates, to express storage as a function of the weighted mean flow through the reach, as follows:

$$\text{Storage} = K[xI + (1-x)O] \quad (4)$$

I = inflow rate at a given time

O = outflow rate at a given time

K = slope of storage-weighted discharge relation and has the dimension of time

x = a dimensionless constant which weights the inflow and outflow.

This method, which is known as the Muskingum method, assumes that the water-surface profile is uniform and unbroken between the

upstream and downstream points on the reach, that the stage and discharge are uniquely defined at these two places, and that K and x are sensibly constant throughout the range in stage experienced by the flood wave.

The factor x (equation 4) is chosen so that the indicated storage volume is the same whether the stage is rising or falling. For spillway discharges from a reservoir, x may be shown to be zero, because the reservoir stage and hence the storage are uniquely defined by the outflow; hence, the rate of inflow has a negligible influence on the storage in the reservoir at any time. For uniformly progressive flow, x equals 0.50, and both the inflow and the outflow are equal in weight. In this wave no change in shape occurs and the peak discharge remains unaffected. Thus, the value of x will range from 0 to 0.50 with a value of 0.25 as average for river reaches. No way is known for determining the value of x from the hydraulic characteristics of a channel system in the absence of discharge records.

The factor K has the dimension of time and is the slope of the storage-weighted discharge relation, which in most flood problems approaches a straight line. Analysis of many flood waves indicates that the time required for the center of mass of the flood wave to pass from the upstream end of the reach to the downstream end is equal to the factor K . The time between peaks only approximates the factor K . Ordinarily, the value of K can be determined with much greater ease and certainty than that of x . Equations (2) and (4) may be combined into:

$$K = \frac{\Delta t \frac{I_2 + I_1}{2} - \frac{O_2 + O_1}{2}}{x(I_2 - I_1) + (1 - x)(O_2 - O_1)} \quad (5)$$

or

$$O_2 = -\frac{(Kx - 0.5\Delta t)}{(K - Kx + 0.5\Delta t)} I_2 + \frac{(Kx + 0.5\Delta t)}{(K - Kx + 0.5\Delta t)} I_1 + \frac{(K - Kx - 0.5\Delta t)}{(K - Kx + 0.5\Delta t)} O_1 \quad (6)$$

or simplified as

$$O_2 = C_0 I_2 + C_1 I_1 + C_2 O_1 \quad (7)$$

where x and K have the same meaning as in equation 4 and:

Δt = time unit of computation in fractions of a day

I_1, I_2 = total instantaneous inflow in cubic feet per second to a reach at the beginning of successive time units (Δt).

O_1, O_2 = instantaneous outflow in cubic feet per second at the beginning of successive time units (Δt)

NOTE: Instantaneous values at the beginning of successive time units are used to derive the values of x and K . Average values of discharge for the successive time intervals may be used in applying equation (7) to a set of data.

The numerator of equation (5) is the storage increment, which is equal to the inflow minus the outflow in cubic feet per second days, whereas the denominator is the corresponding weighted-flow increment in cubic feet per second. Equation (6) gives O_2 in terms of 3 "routing coefficients" and 3 known discharges: I_1 , I_2 , and O_1 . The routing coefficients may be computed from known values of x and K . Equation (7) reduces the routing procedure by the Muskingum method to tabular multiplication and addition.

SELECTION OF TIME INTERVAL

The time increment, Δt , between successive values of inflow or outflow should be greater than $2Kx$ to avoid negative values of C_0 . For some reaches, a value of Δt greater than $2Kx$ will be too long to define the hydrograph adequately. In such reaches, trial routings should be made, using different values of x through n subreaches of travel time, $K_s = \Delta t$, such that nK_s is equal to the value of K for the entire reach. The value of x which satisfactorily reproduces the outflow hydrograph would then be used in subsequent routings through the subreaches.

COMPUTATION OF STORAGE VOLUME

The storage volume is computed from equation (1) and from a record of the inflow and outflow hydrograph. Starting at a time when the inflow and outflow are about equal, that is, where both are equal to base flow, successive values of inflow and outflow are cumulated. If the inflow is from several sources such as main stem, tributary, or local, then the combination of these is used as the inflow. Local inflow from an ungaged area may be estimated from rainfall records or may be estimated on the basis of the flow from a drainage area of similar size in the region. A check should be made to determine whether the measured inflow and outflow volumes and the estimated unmeasured volume are in balance. Minor adjustments may be made to the estimated unmeasured inflow, but if major adjustments seem necessary, possible errors in the main stream discharge records should be investigated. This may be done by choosing additional reaches above and below the main reach, and computing the storage volumes for these reaches for the same floods. It should then be apparent whether either of the main stream records is in error, and if so, which one. The difference between the cumulated inflow and the cumulated outflow represents the volume of storage in the reach at that time. An example of the computation procedure is given in columns 2 to 6 of table 3.

TABLE 3.—*Computation of storage for Andalusia-Brooklyn reach, Conечuh River*

Date	Partial inflows (cfs)				Total inflow (cfs)	Cumulative inflow (cfs-days)	Outflow Brooklyn (cfs)	Cumulative outflow (cfs-days)	Storage (1,000 cfs-days)
	Andalusia		McKenzie	Local					
	(2)	(3)	(4)	(5)					
(1)					(6)	(7)	(8)	(9)	(10)
1944									
March 16.....	2,510	552	698	500	4,260	4,260	4,180	4,180	0.1
17.....	3,140	766	1,080	2,710	7,650	11,910	5,600	9,780	2.1
18.....	4,170	947	1,480	4,570	11,100	23,010	7,270	17,050	6.0
19.....	7,000	1,630	2,210	5,890	16,700	39,710	9,620	26,670	13.0
20.....	9,600	2,620	3,400	6,070	21,900	61,310	17,000	43,670	17.6
21.....	10,100	2,280	4,550	4,020	21,000	82,310	18,300	61,970	20.3
22.....	12,300	2,660	5,080	6,680	26,600	108,910	19,400	81,370	27.5
23.....	21,000	7,260	8,420	9,380	46,000	154,910	25,400	106,770	48.1
24.....	26,800	8,390	16,400	5,370	60,000	214,910	34,100	140,870	74.0
25.....	26,800	8,390	16,900	2,650	57,700	272,610	44,600	185,470	87.1
26.....	29,500	5,440	10,300	2,650	47,900	320,510	51,200	236,670	83.8
27.....	24,000	3,190	5,680	1,590	34,500	355,010	51,900	288,570	66.4
28.....	14,600	2,440	4,120	500	21,700	376,710	41,700	330,270	46.4
29.....	18,400	3,790	6,680	5,860	34,700	411,410	35,800	366,070	45.3
30.....	19,300	5,860	15,500	4,520	45,200	456,610	36,100	402,170	54.4
31.....	18,500	10,000	18,800	1,840	49,100	505,710	35,100	437,270	68.4
April 1.....	22,000	5,980	11,500	1,840	41,300	547,010	38,500	475,770	71.2
2.....	22,000	6,210	3,300	2,320	33,800	580,810	44,600	520,370	60.4
3.....	14,200	2,120	2,690	1,500	20,500	601,310	40,100	560,470	40.8
4.....	10,100	1,570	1,980	1,120	14,700	616,010	30,000	590,470	28.5
5.....	7,950	1,220	1,480	788	11,400	627,410	29,600	611,070	16.3
6.....	6,340	898	1,170	788	9,250	636,610	13,700	624,770	11.9
7.....	3,410	650	825	666	6,440	644,580	10,300	635,070	9.5
8.....	4,210	718	800	500	6,230	650,780	8,300	643,600	7.2
9.....	4,170	671	732	500	6,080	656,840	7,710	651,310	5.5

EVALUATION OF THE CONSTANT WEIGHTING THE INFLOW AND OUTFLOW

The value of x for a reach is determined by a trial and error procedure. A value of x is assumed and the weighted outflow-inflow discharge is computed. The weighted discharge is then plotted against the corresponding volume of storage in the reach. If the correct value of x has been chosen, the storage loop should close and the plot usually approximates a straight line through the medium and high discharge range. An example of the procedure is given in table 4 and in figure 39 and will be discussed subsequently.

The value of x ranges from 0 for reservoir-type storage to 0.5 for uniformly progressive flow. For natural stream channels, a common value of x is 0.25. In determining the value of x , great accuracy is not required because the method is relatively insensitive to the value of this coefficient.

EVALUATION OF THE SLOPE OF THE STORAGE-WEIGHTED DISCHARGE RELATION

The coefficient K has the dimension of time and is the slope of the weighted-discharge storage plot described in the preceding section. The value of K thus determined is the average K for the reach, but if tributaries enter the reach, probably not all the inflow will rise and fall simultaneously. In such streams the storage will be influenced by the variation in inflow and a more accurate evaluation of its effect can be made by separately routing each inflow from its point of entry into the reach to the lower end of the reach. To accomplish this, it is necessary to determine individual coefficients for each component of the flow. This may be approximated by the following equations:

$$K_m = \frac{(Q_c)}{(Q_c M)} K \quad (8)$$

and

$$K_c = M K_m \quad (9)$$

where K = average K for the reach in units of days

K_m = K per mile

K_c = individual K for separate routing of inflows

Q_c = total inflow at one point during a given flood

M = distance in miles from point of inflow to lower end of reach.

When determining the value of K for several reaches of the main stem of a river, a plot of K and the length of the reach is useful in determining the value of K for subreaches.

The time interval represented by K is equivalent to the time required for an elemental discharge wave to traverse the routing reach. In the absence of complete discharge records, the value of K for natural stream channels may be approximated by dividing the length of the reach by the mean velocity in the channel and a coefficient, C , that varies with the shape of the channel:

<i>Channel</i>	<i>Coefficient, C</i>
Wide rectangular.....	1. 67
Wide parabolic.....	1. 44
Triangular.....	1. 33

The value of K in natural channels is also about equal to the time interval between the peak flow at successive points along the channel.

EXAMPLE OF COMPUTATION PROCEDURE

The computation of routing coefficients for a reach on the Conecuh River in Alabama is presented as an example. This reach is composed of the Conecuh River from Andalusia to Brooklyn, Pigeon Creek from Thad to its mouth, and Sepulga River from McKenzie, to its mouth (see fig. 38). The inflow at Andalusia, Thad, and McKenzie, and the outflow at Brooklyn are gaged. All other inter-

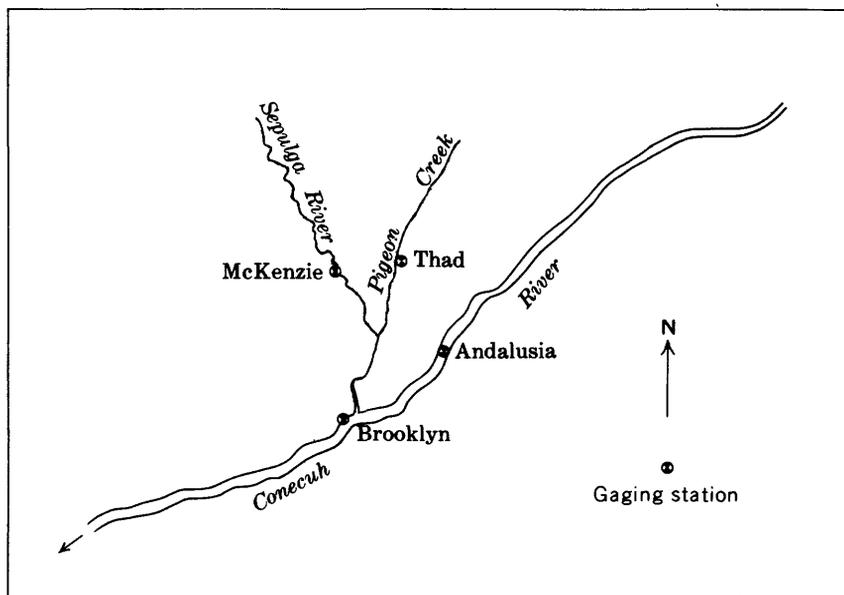


FIGURE 38.—Andalusia-Brooklyn reach on Conecuh River.

vening inflow is unengaged and is classed as local inflow. The drainage areas for the various sources are:

Point	Drainage area (square miles)
Andalusia.....	1,300
McKenzie.....	296
Thad.....	470
Local.....	334
Brooklyn.....	2,400

DETERMINATION OF STORAGE

Records for the flood of March–April 1944 were used in computing storage. Local inflow was estimated on the basis of rainfall and runoff per square mile of other streams in the region. Since the streams rise fairly slowly, the time unit used was the day. In table 3, the daily inflows at points in the reach are listed in columns 2, 3, 4 and 5. The total daily inflow is listed in column 6 and the daily outflow at Brooklyn is listed in column 8. The cumulative inflow and the cumulative outflow are listed in columns 7 and 9, respectively, and the difference in cumulative inflow and cumulative outflow, which is equal to the storage in the reach up to midnight of each indicated day, is listed in column 10.

DETERMINATION OF CONSTANTS

In table 4, the total instantaneous inflow at 12 p.m. is listed in column 2 and the outflow discharge and total storage in the reach at 12 p.m. are listed in columns 3 and 4. The weighted discharge for various assumed values of x was computed by using equation (10) and is listed in columns 5 to 7.

$$\text{weighted discharge} = xI + (1-x)O \quad (10)$$

The weighted discharge-storage plots are shown on figure 39. The storage loop is closed and the relation best approximates a

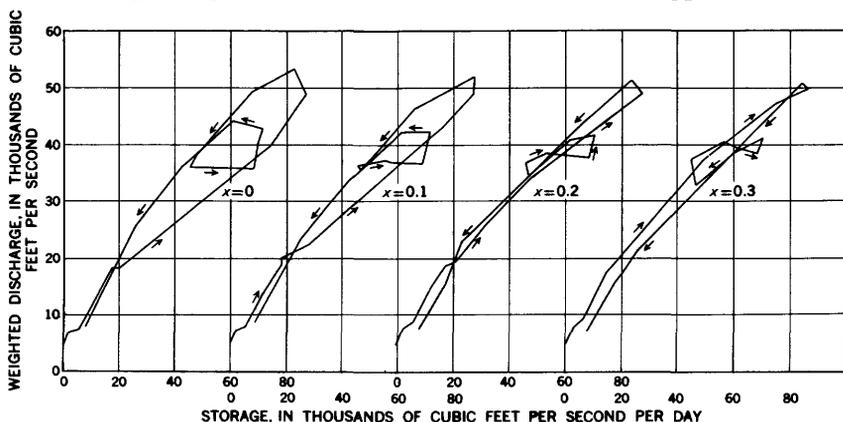


FIGURE 39.—Relation of storage to weighted discharge for different values of x .

straight line when $x=0.2$. The average slope of the weighted discharge-storage relation is equal to K . On the graph for $x=0.2$ the slope averages about 2.0 days, hence, for the reach as a whole, $x=0.2$ and $K=2.0$ days.

TABLE 4.—Computation of weighting factor x for Andalusia-Brooklyn reach, Conecuh River

Date (1)	Total inflow 12 p.m. (cfs) (2)	Total outflow 12 p.m. (cfs) (3)	Storage volume 12 p.m. (1,000 cfs- days) (4)	Weighted discharge		
				$x=0.1$ (5)	$x=0.2$ (6)	$x=0.3$ (7)
<i>1944</i>						
March 16	5,870	4,180	0.1	4,350	4,520	4,690
17	9,310	6,970	2.1	7,200	7,440	7,670
18	12,900	7,560	6.0	8,090	8,650	9,160
19	20,500	14,200	13.0	14,800	15,590	16,100
20	21,000	18,300	17.6	18,600	18,800	19,100
21	23,400	18,500	20.3	19,000	19,500	20,000
22	32,500	21,300	27.5	22,400	23,500	24,700
23	55,400	29,300	48.1	31,900	34,590	37,100
24	62,700	39,700	74.0	42,000	44,300	46,600
25	52,600	48,700	87.1	49,100	49,500	50,000
26	43,200	53,300	83.8	52,300	51,300	50,300
27	25,200	48,700	66.4	46,400	44,000	41,700
28	22,800	37,100	48.4	35,700	34,200	32,890
29	41,200	35,800	45.3	36,300	36,900	37,400
30	50,400	35,800	54.4	37,300	38,700	40,200
31	45,300	35,800	68.4	36,800	37,700	38,600
April 1	38,800	42,700	71.2	42,300	41,900	41,600
2	27,000	44,100	60.4	42,400	40,800	39,000
3	16,200	35,400	40.8	33,500	31,600	29,600
4	12,400	25,200	25.5	23,900	22,600	21,400
5	10,200	16,400	16.3	15,800	15,200	14,600
6	8,080	11,590	11.9	11,200	10,800	10,500
7	6,010	9,380	9.5	9,040	8,710	8,370
8	5,050	7,860	7.2	7,580	7,300	7,020
9						

The inflow, which enters the reach at four places, necessitates the determination of a separate K for each component inflow as is shown in the following table:

K for each component inflow

[Equations 8 and 9 used]

Point	Miles to Brooklyn M	Total discharge during flood Q (cfs-days)	$Q \times M$	K_c	x
Andalusia	20	350,000	7,000,000	1.9	0.2
Thad	25	83,380	2,084,500	2.4	.2
McKenzie	25	148,640	3,716,000	2.4	.2
Local	12	74,720	896,640	1.1	.2
Total		656,740	13,697,140		

$$K_m = \frac{656,740}{13,697,140} \times 2.0 = 0.0959 \text{ days per mile.}$$

$$K_c = M K_m$$

From the value of x and K for each component of the inflow, the flood-routing coefficients for each inflow may be computed from equation (6).

To test the routing coefficients, the outflow hydrograph is computed from the record of inflow discharge. These computations, along with the routing coefficients used, are shown in table 5. At the

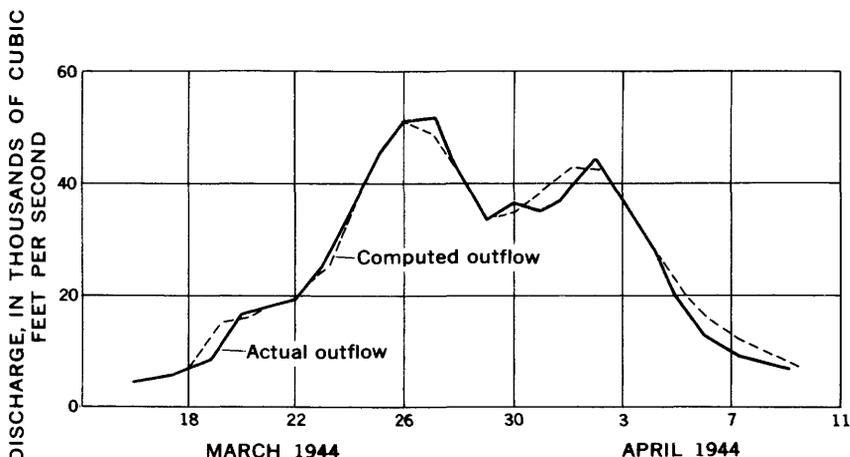


FIGURE 40.—Comparison of computed and actual hydrograph, Conecuh River near Brooklyn, Ala.

beginning of the rise the inflow and outflow are assumed to be equal. Then, since I_1 , I_2 and O_1 are known, O_2 may be computed from equation (7).

The total of the computed outflows is compared with the actual outflow on figure 40.

The procedure used in testing the coefficients may be used also in routing other floods through the reach in either the upstream or downstream direction.

TABLE 5.—Example of routing for Andalusia-Brooklyn reach, Conocuh River

$$[O_2 = C_2 I_2 + C_1 I_1 + C_3 O_1]$$

$[O_2 = 0.20 (4570) + 0.52 (2710) + 0.28 (942) = 2,590$ cfs—Local inflow to Brooklyn, March 18]

Date	Local inflow to Brooklyn		Thad to Brooklyn		McKenzie to Brooklyn		Andalusia to Brooklyn		Total routed outflow	Measured outflow
	$I = 0.20 \quad K = 1.1$ $C_2 = .28 \quad C_1 = .52$		$I = 0.2 \quad K = 2.4$ $C_2 = .01 \quad C_3 = .58$ $C_1 = .41$		$I = 0.2 \quad K = 2.4$ $C_2 = .01 \quad C_3 = .58$ $C_1 = .41$		$I = 0.2 \quad K = 1.9$ $C_2 = .06 \quad C_3 = .50$ $C_1 = .44$			
	Inflow	Outflow	Inflow	Outflow	Inflow	Outflow	Inflow	Outflow		
March 16	500	500	552	552	698	698	2,510	2,510	4,260	4,180
17	2,710	942	766	554	1,030	702	3,410	2,610	4,810	5,600
18	4,570	2,590	947	645	1,480	844	4,170	2,940	7,020	7,270
19	5,890	4,280	1,630	778	2,210	5,520	7,000	3,720	14,300	9,620
20	6,070	5,480	2,520	1,150	3,400	4,140	9,600	5,040	15,800	17,000
21	4,020	5,490	2,280	1,720	4,550	3,840	10,100	6,840	17,900	18,300
22	6,630	4,950	2,560	1,960	5,080	4,140	12,300	8,600	19,600	19,400
23	9,330	6,700	2,260	2,260	8,420	4,570	21,000	11,000	24,500	25,400
24	5,370	7,800	8,390	4,370	16,400	6,270	29,800	16,500	34,100	34,100
25	2,650	5,510	8,390	6,060	16,900	10,500	29,800	23,200	45,300	44,000
26	2,650	3,450	5,440	7,010	10,300	13,100	29,500	26,500	50,100	51,200
27	1,590	2,660	3,190	6,330	5,080	11,900	24,000	27,700	48,600	51,600
28	1,900	1,670	2,440	3,000	4,120	9,270	14,600	21,200	41,200	41,700
29	5,860	1,900	3,790	3,940	6,650	7,130	18,400	20,200	33,200	33,500
30	4,320	4,480	5,860	3,900	13,300	7,010	19,300	19,400	34,800	36,100
31	1,840	3,970	10,050	4,760	18,800	10,600	18,500	19,300	38,600	35,100
1	1,840	2,440	5,950	6,930	18,500	14,000	22,000	19,100	42,500	38,500
2	2,330	2,100	3,300	6,460	6,210	12,900	22,000	20,600	42,100	44,600
3	2,100	2,100	5,140	5,140	2,600	10,100	14,200	20,800	38,100	40,100
4	1,120	1,590	1,570	3,870	1,930	6,980	10,100	17,300	29,700	30,000
5	786	1,180	1,220	2,900	1,480	4,850	7,950	13,600	22,500	20,600
6	896	896	2,100	2,100	1,170	3,430	6,340	10,700	17,200	13,700
7	666	793	1,690	1,690	1,925	2,480	5,410	8,460	13,400	10,300
8	500	668	1,330	718	800	1,830	4,210	7,190	11,000	8,530
9	500	547	671	1,070	742	1,490	4,170	5,700	8,700	7,710

RESERVOIR-STORAGE METHOD

If the storage in a pond or reservoir can be related solely to the outflow discharge, as schematically shown in figure 31, then the general equation (2) can be rearranged to permit rapid calculation of the outflow hydrograph.

$$(I_1 + I_2) + \frac{2S_1}{\Delta t} - O_1 = \frac{2S_2}{\Delta t} + O_2 \quad (11)$$

The only data needed to use this method are the inflow hydrograph and the storage-outflow relation. The latter may be determined by combining the stage-outflow relation and the stage-storage relation. To use the method conveniently, values of $\frac{2S}{\Delta t} + O$ are computed and plotted against corresponding values of O . Selected values are shown in table 6 to illustrate the computation.

TABLE 6.—Selected values of $\frac{2S}{\Delta t} + O$ for $\Delta t = 15$ minutes

Stage (feet)	Storage (cubic feet)	Outflow discharge (cfs)	$\frac{2S}{\Delta t} + O$ (cfs)
10.29	530	0	1.18
10.3	535	.01	1.20
10.4	680	.32	1.83
10.6	1,050	2.55	4.88
10.8	1,550	7.50	10.9
11.0	2,240	14.4	19.4
11.2	3,210	21.6	28.7
11.4	4,580	28.9	39.1
11.6	6,430	36.3	50.6
11.8	8,800	44.3	63.9
12.0	11,900	53.0	79.4
12.2	16,800	62.0	99.3
12.4	23,400	71.5	123
12.6	31,600	81.5	152
12.8	41,800	92.0	185
13.0	53,600	103	222
13.2	67,900	114	265
13.4	86,200	126	318
13.6	111,000	138	385
13.8	143,000	150	468
14.0	190,000	163	585
14.2	256,000	177	746
14.4	352,000	191	973
14.6	493,000	205	1,300
14.8	658,000	219	1,680
15.0	952,000	233	2,350

A sample of the routing computations is given in table 7. The computations begin at a known value of outflow and inflow. The steps then are as follows:

1. Entries in columns 1 and 2 are known. The first entry of outflow in column 5 is given.
2. Column 3 contains successive sums of values of inflow in column 2 for the particular time period under consideration and for the previous period.
3. Table 6 is entered with the known value of O , and the initial value of $\frac{2S}{\Delta t} + O$ is selected for column 4 of table 7.
4. From the value in column 4 subtract twice the value in column 5 and enter the result in column 6.
5. Add the value in column 6 to the value in column 3 of the succeeding time period and enter the result in column 4 for the new time period under consideration.
6. The new outflow discharge in column 5 is again obtained from the relation of $\frac{2S}{\Delta t} + O$ with O in table 6.
7. The steps 4, 5, and 6 are then repeated until the entire outflow hydrograph is generated.
8. The stage corresponding to this outflow is then selected from table 6 and entered in column 7 of table 7.

TABLE 7.—Routing through a reservoir

Time	Inflow cfs	$I_1 + I_2$	$\frac{2S}{\Delta t} + O$	Outflow cfs	$\frac{2S}{\Delta t} - O$	Gage height (feet)
(1)	(2)	(3)	(4)	(5)	(6)	(7)
0:00.....	0	0	1.18	0	1.18	10.29
:15.....	1.44	1.44	2.62	.84	.94	10.47
:30.....	4.34	5.78	6.72	4.04	-1.36	10.67
:45.....	8.90	13.24	11.88	8.30	-4.72	10.82
1:00.....	18.7	27.6	22.9	17.2	-11.5	11.08
1:15.....	38.8	57.5	46.0	33.5	-21.0	11.52
1:30.....	106	144.8	123.8	71.6	-19.4	12.40
1:45.....	216	322	303	123	57	13.35
2:00.....	291	507	564	161	242	13.97
2:15.....	320	611	853	185	483	14.31
2:30.....	325	645	1,128	198	732	14.50
2:45.....	309	634	1,366	208	950	14.64
3:00.....	285	594	1,544	214	1,116	14.73
3:15.....	260	545	1,661	219	1,223	14.80
3:30.....	235	495	1,718	220	1,278	14.81
3:45.....	211	446	1,724	220	1,284	14.81
4:00.....	188	399	1,683	219	1,245	14.80
4:15.....	165	353	1,598	217	1,164	14.77
4:30.....	145	310	1,474	212	1,050	14.70
4:45.....	129	274	1,324	206	912	14.61
5:00.....	116	246	1,158	199	760	14.51
5:15.....	106	222	982	191	600.4	14.40
5:30.....	96.4	202.4	802.4	181	440.4	14.26
5:45.....	88.0	184.4	624.8	166	292.8	14.04
6:00.....	80.2	168.2	461.0	149	163.0	13.78
15:00.....	0.75	1.82	2.61	.83	.95	10.47
15:15.....	.47	1.22	2.17	.54	1.09	10.43
15:30.....	.22	.69	1.78	.30	1.18	10.39
15:45.....	0	.22	1.40	.11	1.18	10.33
16:00.....	0	0	1.18	0	-----	10.29
Total.....	4,504	-----	-----	4,504	-----	-----

The relation $\frac{2S}{\Delta t} + O$ versus O may be used in either a graphic or a tabular form. If a number of peaks are to be routed through the reservoir, the tabular form is more convenient because machine computation can be more readily utilized.

In table 7 the computations are shown only for the first 6 hours and the final hour. Figure 41 is a plot of the entire inflow and

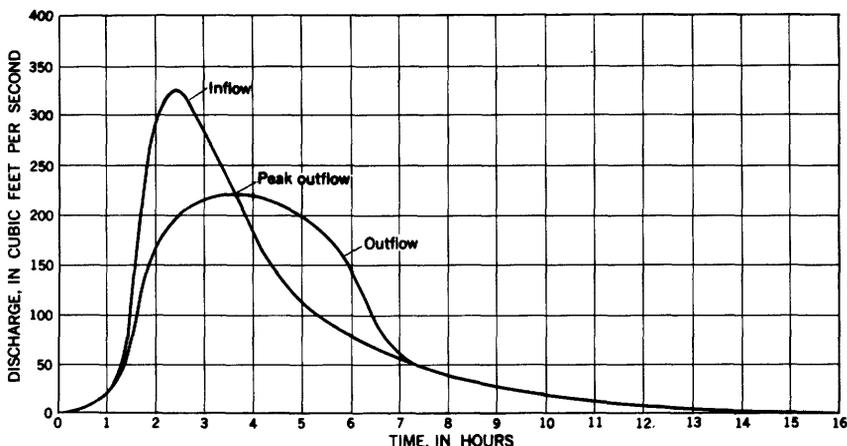


FIGURE 41.—Inflow and outflow hydrographs for reservoir.

outflow hydrographs. The sums of the ordinates of the two hydrographs, at the bottom of table 7, are equal.

The method can be used also to determine the inflow hydrograph if the outflow hydrograph is known.

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