G. Allan Brown

# Two-Well Tracer Test in Fractured Crystalline Rock

GEOLOGICAL SURVEY WATER-SUPPLY PAPER 1544-I

*Work done in cooperation with the U.S. Atomic Energy Commission*

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*By* D. S. WEBSTER, J. F. PROCTOR, *and* I. W. MARINE

GENERAL GROUND-WATER TECHNIQUES

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# SYMBOLS

| | |
|---|---|
| $C$ | Concentration of tracer from the discharge well |
| $C*$ | Concentration of tracer during pulse injection |
| $C_0$ | Concentration of tracer in step addition |
| $C_r$ | Reference concentration for pulse addition of tracer |
| $D$ | Axial dispersion coefficient; units are length squared divided by time |
| $l$ | Characteristic dispersion length, units are length (also termed "medium dispersion constant" by Orcutt and others, 1957, p. 793; and "mixing length" by Hennico and others, 1963, p. 10) |
| DRB | Deep rock boring, a cored well in crystalline rock |
| $\epsilon$ | Porosity |
| gpm | Gallons per minute |
| gpd/per sq ft | Gallons per day per square foot, a unit of permeability |
| $h$ | Head difference between the discharge and recharge wells |
| $K$ | Coefficient of permeability |
| $L$ | Straight line distance between the injection and discharge wells, or length of column in an experiment |
| $m$ | Thickness of aquifer |
| $\mu C/l$ | Microcuries per liter |
| $N$ | Number of characteristic dispersion lengths in a column length or flow path |
| $N_0$ | Number of dispersion lengths along the direct path between the two wells |
| $P$ | Peclet number, a dimensionless number that characterizes dispersion |
| $\psi$ | Angle at which streamline leaves injection well (or enters discharge well) measured from line connecting the two wells |
| $q_i$ | Pumping-injection rate while tracer was being injected |
| $q_0$ | Average pumping-injection rate during entire test |
| $r$ | Radius of the arc that defines a streamline between the injection and discharge wells |
| $r_w$ | Radius of the pumping well |
| $S$ | Length of flow path |
| $t$ | Time |
| $t_a$ | Transit time for nondispersed flow |
| $t_{a0}$ | Transit time for nondispersed flow along the direct path between the injection and the discharge wells. |
| $t_{a1}$ | Transit time for a specific streamline defined by angle $\psi_1$ |
| $t*$ | Duration of pulse injection of tracer |
| $\theta$ | Dimensionless time, the ratio of a selected time to the time required to fill completely a column with a displacing fluid |
| $U$ | Average interstitial velocity |
| $V$ | Volume |
| $V_f$ | Volume of fluid |

# GENERAL GROUND-WATER TECHNIQUES

## TWO-WELL TRACER TEST IN FRACTURED CRYSTALLINE ROCK

By D. S. WEBSTER,[1] J. F. PROCTOR,[2] and I. W. MARINE [3]

### ABSTRACT

A pulse injection of tritium (300 curies) was made to flow from an injection well to a discharge well through fractures in crystalline rock buried beneath about 1,000 feet of coastal plain sediments at the Savannah River Plant near Aiken, S.C. The wells were 1,765 feet apart, and the duration of the test was 2 years. The concentration of tritium arriving at the discharge well can be duplicated by calculations based on fluid dispersion in a homogeneous medium. In developing the theoretical curve, the only unknown variables were (1) the transit time for nondispersed flow along the line connecting the two wells and (2) the characteristic dispersion length in the medium (fractured crystalline rock). These two values were obtained by curve fitting. The results of the tracer test agree with computations based on the assumption that flow was through a homogeneous medium consisting of numerous intersecting fractures.

### INTRODUCTION

At the Savannah River Plant near Aiken, S.C., as at other locations where there are chemical separation plants for the processing of nuclear fuels, practically all the high-level radioactive wastes are stored in concrete and steel tanks buried just beneath the surface of the ground. This waste is of such activity and longevity that it cannot be dispersed into the environment, but it must be contained for periods of time extending at least into hundreds of years. One concept for the terminal containment of this waste is to store it in chambers excavated within crystalline basement rock, which is covered by about 1,000 feet of coastal plain sediments at the plant site.

As part of the safety evaluation of this concept, the hydrology of the buried crystalline rock has been intensively studied and was reported by Christl (1964), Siple (1964), Proctor and Marine (1965), and Marine (1966, 1967a, b). The present paper reports on the analysis of a 2-year tracer test in which a pulse injection of tritium was made to move from an injection well to a pumping well.

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The buried crystalline basement rock at the Savannah River Plant is predominantly chlorite-hornblende schist and hornblende gneiss with lesser amounts of quartzite. Water occurs under confined conditions in the small fractures that exist within the crystalline rock. The nature of the fractures in the buried crystalline rock was discussed by Marine (1966); from water injection and removal tests on packed-off sections of the rock, he concluded that there were two types of fractures. The first type consists of minute fractures that pervade the entire rock mass but transmit water extremely slowly. Rock that consists only of this type of fracture is termed "virtually impermeable" in this report. The other type of fracture is restricted to definite zones, and consists of larger openings that transmit water at a faster rate. Rock that includes this type of fracture is termed "hydraulically transmissive" in this report. Some zones of hydraulically transmissive rock are tapped by two or more wells.

The permeability of both types of rock was discussed by Marine (1967b). The apparent permeability of the hydraulically transmissive zones was estimated from pumping tests to average about 1 gallon per day per square foot, even though some fractured sections within these zones locally exceed this value.

The computation of hydraulic constants from pumping tests was necessarily based on the assumption that fractures were so uniformly distributed and interlaced that the idealized conditions of homogeneity and isotropy were reasonably well satisfied. However, it was realized that, if the hydraulically transmissive fracture zones did not consist of numerous interlacing fractures, the estimated hydraulic constants could depart significantly from values that would be applicable to actual water movement. A tracer test using tritium was designed to provide information on the nature of the flow through fractures in a known fracture zone.

## TRACER TEST

The previously inferred "inclined fracture zone" (Marine, 1966 p. D225) which is tapped by wells DRB 5 and DRB 6 was used for the test (fig. 1). A submersible pump was set in well DRB 6, the discharge well, at which power was available from an electric line. A 1-inch diameter plastic pipe was laid across the land surface to conduct the discharge water to well DRB 5, the injection well, where it was injected by a drop pipe that extended below the normal static water level. A semipermanent packer was set at a depth of 1,500 feet (1,212 ft below sea level) in DRB 5, so that all the water, as well as all the tritium tracer, would be injected into fractures that were known to be connected to the fractures near the bottom of DRB 6.

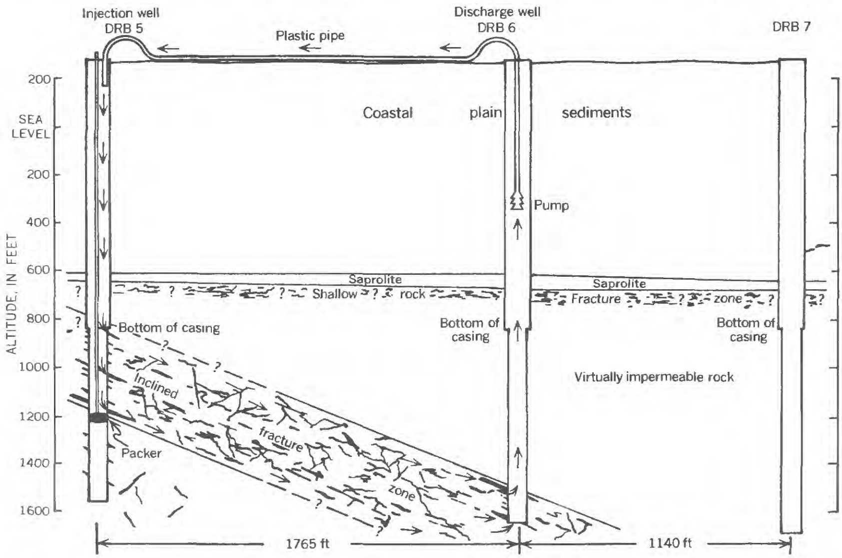


FIGURE 1.—Geologic section showing the wells and fracture zone used in the two-well tracer test. Arrows show the generalized direction of water flow.

On August 12, 1964, the discharge-injection system was started and pumped at about 10 gallons per minute for 6 days. It was assumed that hydraulic equilibrium was approximated, and a condition of near stabilization was achieved for the cones of depression and elevation in the area between the two wells.

On August 18–19, 1964, 297.44 curies of tritium was added to the system in 28¾ hours at a uniform rate, giving a tritium concentration of 4,750 microcuries per liter in the injected water. Tritium from a plastic jug was added through a plastic tube by a metering injection pump driven by electricity supplied from a portable generator, as shown in figure 2. Also shown, are a standby generator, the packer pipe extending about 2½ feet above the well casing, and the injection system itself with flow meter and pressure gage.

At DRB 6, the discharge well, there were flow control valves, seven sampling taps, a pressure gage, and a flow meter (fig. 3). The sampling taps were equipped with solenoid valves connected to electric timers. A sample of about 10 gallons was collected daily, and every week a part of each sample was transferred to a 1-liter polyethylene bottle for storage and analysis.

The pumping rate for the system and hydrographs for the discharge well (DRB 6), the injection zone (843–1,212 ft below sea level), and the zone below the semipermanent packer in DRB 5 are shown in figure 4. Ordinarily, the discharge and injection rates were the same;

however, if a leak occurred in the plastic pipe, the discharge rate exceeded the injection rate, and water was removed from the system. As shown in figure 4, when water was removed from the system, the water level declined in all three zones.

Initially, daily samples of the discharge water from DRB 6 were analysed for tritium concentration by the liquid scintillation method. Tritium was first detected 73 days after it was added to the injection well. From the 112th day to the end of the test on the 739th day,

FIGURE 2.—Tritium injection equipment at well DRB 5.

FIGURE 3.—Tritium sampling equipment at well DRB 6.

every fifth sample was analysed and corrected for radioactive decay. Figure 5 shows the tritium concentration at the discharge well plotted against time for the entire test.

From the overall average pumping rate at DRB 6 of 7.9 gpm or 43,000 liters per day, it was computed that the tritium recovered after adjustment for decay was about 108 curies; however, this includes some recirculated tritium (estimated to be 9.5 curies by methods presented below). With this estimate taken into account, about 33 percent of the original tritium injection was recovered.

## TRANSIT TIMES BETWEEN WELLS FOR NONDISPERSED FLOW

The flow paths from an injection to a discharge well, both of which have equal flow rates and fully penetrate a medium of great extent and of uniform thickness and flow properties, are arcs of circles that pass through the two wells and have their centers on the line perpendicular to, and bisecting, the line connecting the wells as shown in figure 6 (Muskat, 1937, p. 474; Jacob, 1950, p. 345; DeWiest, 1965, p. 249). Potential-flow theory permits calculation of the transit time for a packet of fluid moving from the injection to the discharge well along any streamline, in terms of the total flow, the porosity and thickness of the medium, and the distance between wells. Since the medium is assumed to be homogeneous and isotropic, the potential

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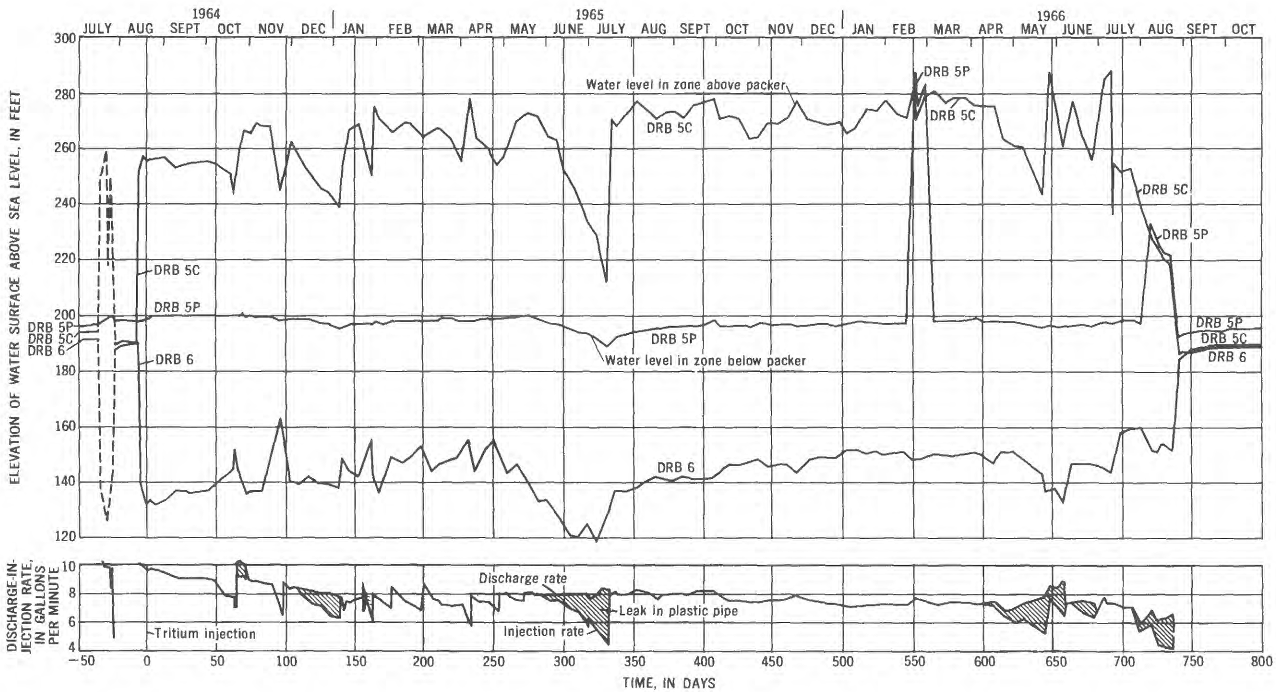



**FIGURE 4.**—Flow rates and water levels in the pumping and injection wells. DRB 5c is the water level in the injection zone which is above the packer set at depth of 1,500 feet. DRB 5p is the water level below this packer.

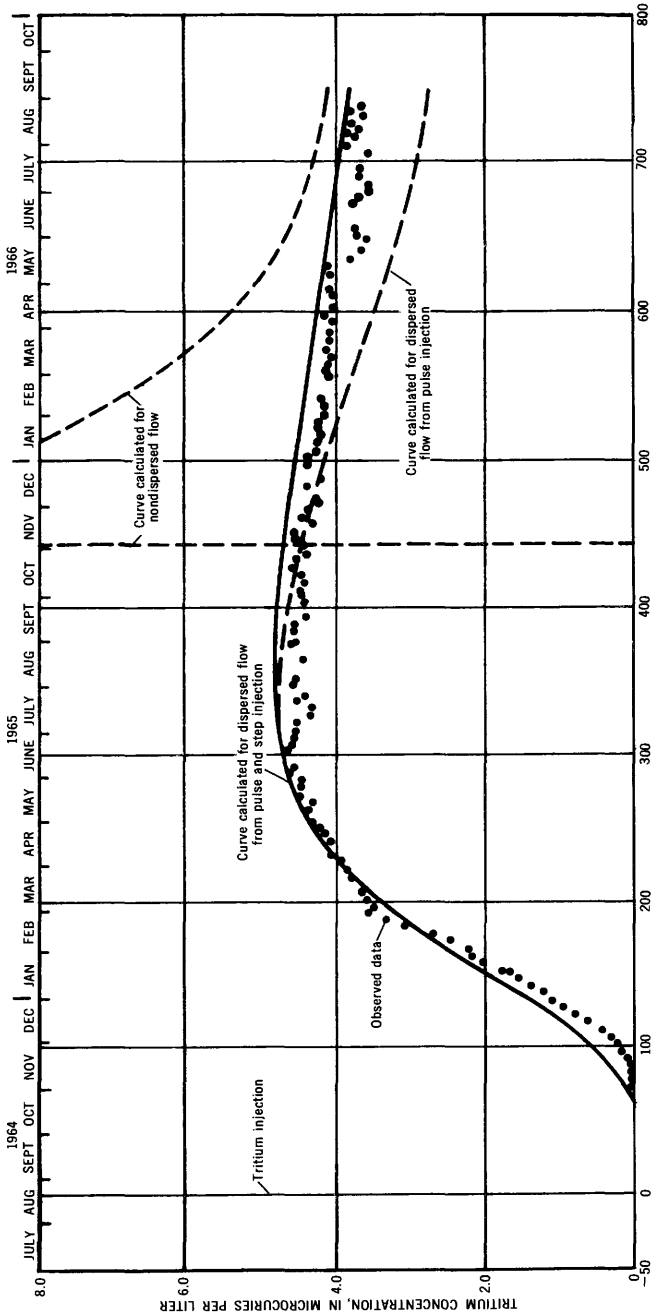
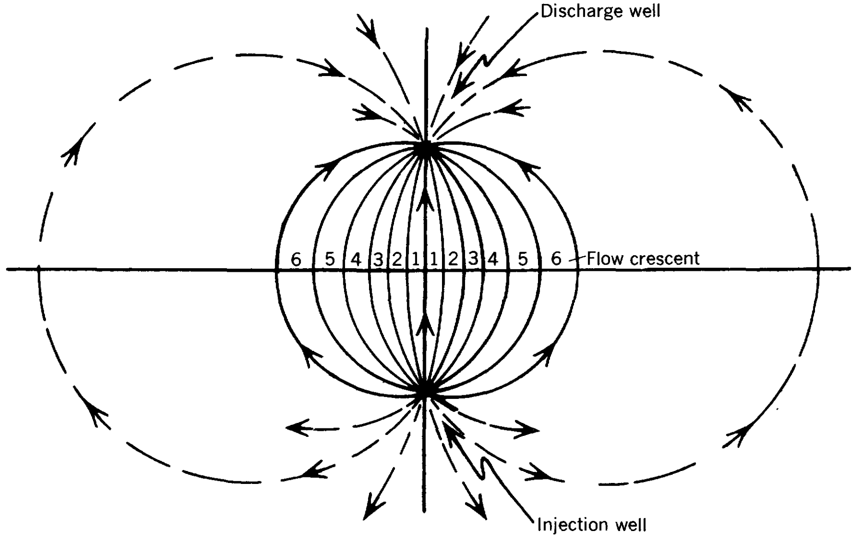



Figure 5.—Tritium concentration as observed at the discharge well and as predicted by three calculated curves.




$L$=distance between wells

$S$=length of flow path (arc)

$t_a$=time of transit

| Flow crescent | Range of $\psi$, in radians | $\frac{S}{L}$ (avg) | $\frac{t_a}{t_{a0}}$ (avg) |
|---|---|---|---|
| 1 | 0.0–0.3 | 1.005 | 1.010 |
| 2 | .3– .6 | 1.034 | 1.087 |
| 3 | .6– .9 | 1.10 | 1.265 |
| 4 | .9–1.2 | 1.21 | 1.60 |
| 5 | 1.2–1.5 | 1.39 | 2.20 |
| 6 | 1.5–1.8 | 1.64 | 3.45 |

FIGURE 6.—Streamlines between wells in a discharge-recharge system in a homogeneous and isotropic medium and the division of the flow field into crescents used to calculate the concentration of tracer arriving at the discharge well.

gradient at a given radius around, and near, a well is equal in all directions, and consequently, the outward (or inward) flow velocities at that radius will be equal. With this statement as a postulate, an analytical expression for time of transit along any streamline can be derived. The procedure involves obtaining an expression for the pore volume of a very narrow section cut through the medium from well to well along a streamline and dividing this volume by an expression for the rate of fluid inflow to the section. The result is the time required to fill the section with fluid, or alternatively, the time of transit along the streamline for nondispersed flow.

The construction for this derivation is given in figure 7. A streamline is identified by the angle, $\psi$, at which it leaves the injection well or enters the discharge well, the angle being measured relative to the straight line connecting the wells (ordinate). Since the inner streamline shown in figure 7 is the between-wells arc of the circle of radius $r$, a

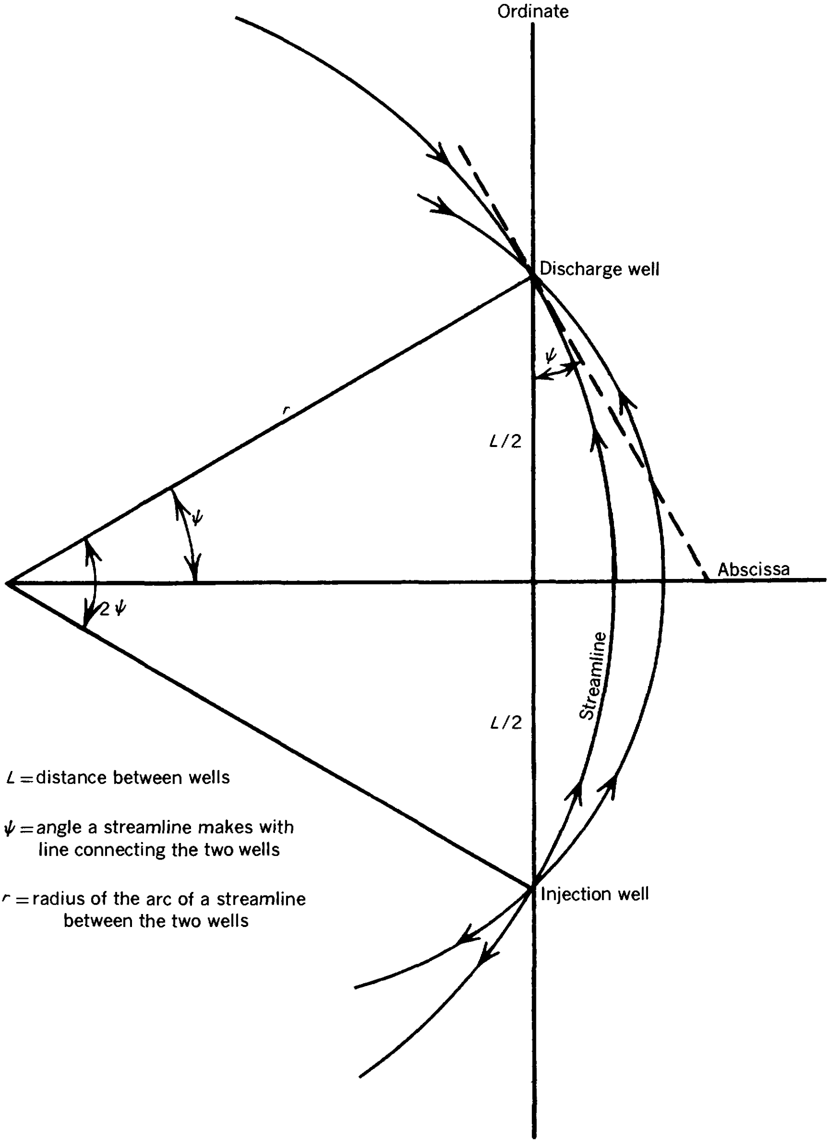



FIGURE 7.—Construction diagram for determining the relation between selected flow paths and the shortest flow path in a discharge-recharge system in a homogeneous and isotropic medium.

tangent to the circle (dashed line) passes through the well at angle $\psi$ and forms a triangle with the axes; this triangle is similar to that formed by the radius and the axes. Consequently, the radius is at an angle $\psi$ with the abscissa, and the included angle of the between-wells arc is $2\psi$.

The area of the circle-segment enclosed by the arc and the ordinate is a function of the radius and included angle:

$$A_{seg}=\frac{1}{2}r^2(2\psi-\sin 2\psi). \tag{1}$$

By inspection of figure 7

$$r=\frac{L/2}{\sin\psi}, \tag{2}$$

so that equation 1 becomes

$$A_{seg}=\frac{1}{2}\left(\frac{L}{2}\right)^2\left[\frac{2\psi-\sin 2\psi}{\sin^2\psi}\right]. \tag{3}$$

A circle-segment on the left side of the ordinate is a mirror image of that shown in figure 7 and is also specified by $\psi$. Consequently, the total area enclosed by the paired arcs specified by $\psi$ is twice the value given in equation 3. Therefore, the pore volume of the solid bounded by the paired arcs in a medium of uniform thickness $m$ and porosity $\epsilon$ is

$$V_{segs}=\epsilon m\left(\frac{L}{2}\right)^2\left[\frac{2\psi-\sin 2\psi}{\sin^2\psi}\right]. \tag{4}$$

The pore volume of the two slices of infinitesimal width along the paired streamlines through the medium is, then

$$dV_{segs}=\epsilon m\left(\frac{L}{2}\right)^2\frac{d}{d\psi}\left[\frac{2\psi-\sin 2\psi}{\sin^2\psi}\right]d\psi \tag{5}$$

The rate at which fluid enters this volume is obtained from the postulate that flow leaves the injection well equally at all angles. In the construction used, $\pi$ radians (180°) encompasses all possible angles of exit, hence represents the total flow; therefore, the fraction of flow which leaves through the sector between zero and $\psi$ radians on both sides of the ordinate is $\frac{\psi}{\pi}$.

For a pumping rate of $q_0$ and a pumping period of $t$, the volume of fluid $V_f$, injected into the sector is $q_0 t\frac{\psi}{\pi}$. Consequently, the volume of injection into the two infinitesimal slices defined by equation 5 is

$$dV_f=\frac{q_0 t}{\pi}d\psi. \tag{6}$$

If at time $t_a$ the pore volume to be filled ($dV_{segs}$) and the fluid volume injected ($dV_f$) are equal, then $t_a$ is the time of transit along the streamline defined by $\psi$. Equate the expressions on the right side of equations 5 and 6, and solve for $t_a$:

$$t_a=\frac{1}{4}\,\epsilon m\,\frac{\pi L^2}{q_0}\frac{d}{d\psi}\left[\frac{2\psi-\sin 2\psi}{\sin^2\psi}\right]. \tag{7}$$

The derivative of the bracketed expression is $4\left[\frac{1-\psi\cot\psi}{\sin^2\psi}\right]$.

Therefore, equation 7 becomes

$$t_a=\epsilon m\frac{\pi L^2}{q_0}\left[\frac{1-\psi\cot\psi}{\sin^2\psi}\right]. \tag{8}$$

This expression becomes indeterminate for $\psi=0$, the straight line between wells that gives the least time of transit, but by l'Hospital's rule the limiting value can be determined from its derivative, which is $\frac{1}{1+2\cos^2\psi}$. Since $\frac{1}{1+2\cos^2\psi}\to\frac{1}{3}$ as $\psi\to 0$, then $\frac{1-\psi\cot\psi}{\sin^2\psi}\to\frac{1}{3}$ as $\psi\to 0$. Therefore, when the value of $\psi$ in equation 8 is allowed to go to zero, the expression becomes

$$t_{a0}=\frac{1}{3}\,\epsilon m\,\frac{\pi L^2}{q_0}, \tag{9}$$

the transit time for the straight path between wells. This is the same relationship presented by Muskat (1937, p. 475) for the shortest path.

The transit time for the general streamline in equation 8 can be expressed as a ratio to the shortest transit time in equation 9:

$$\frac{t_a}{t_{a0}}=3\left[\frac{1-\psi\cot\psi}{\sin^2\psi}\right]. \tag{10}$$

Equation 10 was used to construct the log-log plot of $\left(\frac{t_a}{t_{a0}}-1\right)$ versus $\psi$ in figure 8. The log-log plot magnifies the region of interest in this study, that is, values of $t_a/t_{a0}$ less than 2, and will be used in subsequent calculations.

## TRITIUM CONCENTRATION AT DISCHARGE WELL FOR NONDISPERSED FLOW

The information of the preceding section on time of transit along the various streamlines can be used to calculate the concentration of tracer in the water being removed at any time from the discharge well, in the absence of dispersion effects. Traced water enters all

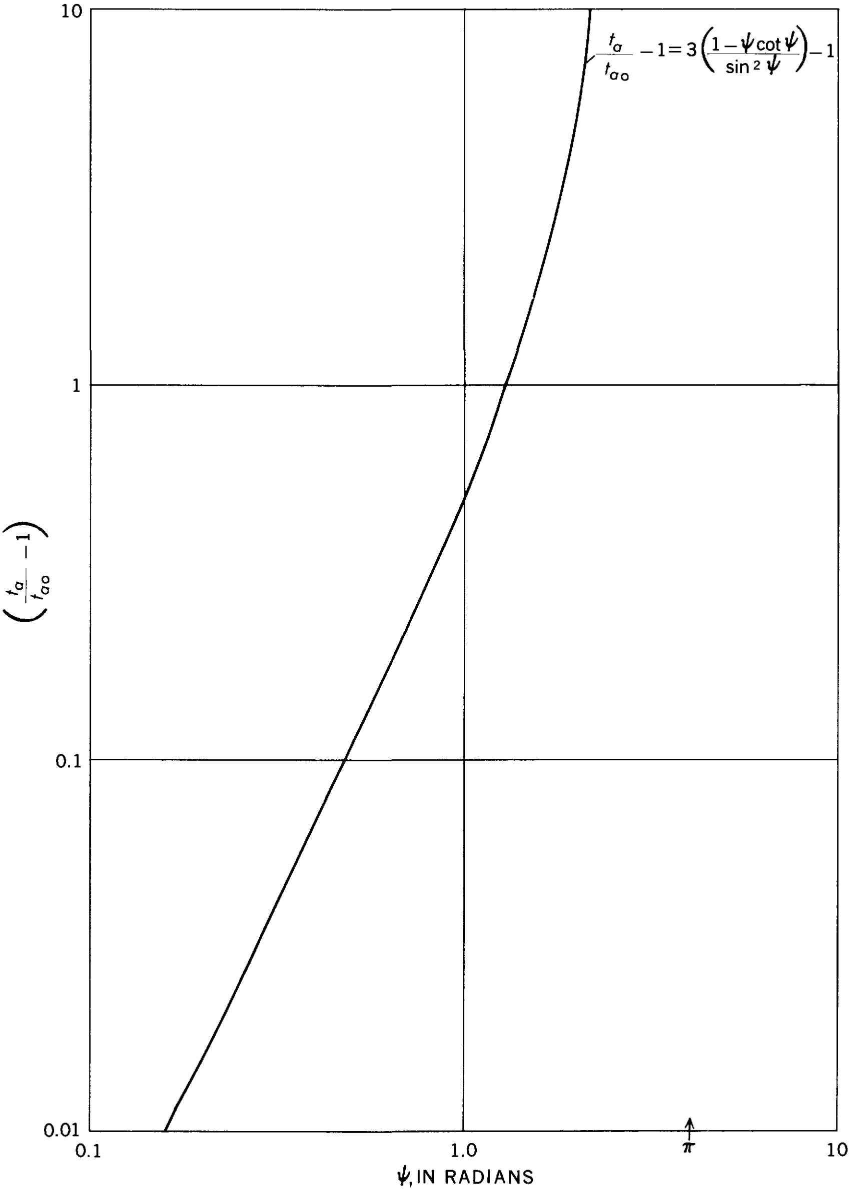



FIGURE 8.—Relative transit times along streamlines plotted on log-log scales.

streamlines from the injection well at time zero, and continues to enter for a time $t_*$ (1.2 days=28¾ hr). The traced water is assumed to move as a plug along each of a given pair of streamlines, the front eventually reaching the discharge well at time $t_a$; because of the symmetry of flow and the assumed absence of dispersion, a period $t_*$

is also required for the plug to pass into the discharge well. At the same time that the tail end of this plug is entering the discharge well ($t_{a1}+t_*$), the front of a plug having a transit time of ($t_{a1}+t_*$) is just starting to enter. If the streamlines with these two times of transit are designated $\psi_1$ and $\psi_2$, respectively, then the fraction of flow (consisting of the original traced water) that they include at any moment is $\frac{\psi_2-\psi_1}{\pi}$, and the original concentration of the tracer is reduced correspondingly by mixing with the main flow out of the well. Thus,

$$C=\frac{\psi_2-\psi_1}{\pi}C_* \qquad (11)$$

where $C$ is the concentration of tracer from the discharge well, and $C_*$ is the concentration of tracer initially put into the injection well.

Later analysis indicated that it takes about 440 days for the transit time, $t_{a0}$, of tracer if no dispersion occurred. So in the period between 440 days and 440+1.2 days, the concentration leaving the discharge well rises from zero to a maximum. The value of $t_a/t_{a0}$ for the end of this period is $\frac{441.2}{440}=1.00273$, and from a more detailed plot similar to figure 8, the corresponding $\psi$ is 0.0826 radians. The resulting concentration is given by equation 11 as $\frac{0.0826-0}{\pi}\times 4{,}750$ $\mu C/l$, or 125 $\mu C/l$; this is more than 25 times greater than the highest concentration observed. Twenty days later (that is, in the period from 460 to 461.2 days) the bounding values of $\psi$ are 0.337 and 0.347 radians, and the concentration had fallen to about 15 $\mu C/l$. It is obvious that this kind of profile bears no resemblance to the observed profile, as illustrated by the curve labeled "curve calculated for nondispersed flow" in figure 5.

## MODEL FOR DISPERSED FLOW

The most likely explanation for the shape of the observed curve in figure 5 is dispersion of the tracer by water movement through the inclined fracture zone. Dispersion occurs when flow paths for individual particles are not identical, and some elements of tracer move ahead of the rest. The result is a profile of tracer concentration that decreases in the direction of flow. When the mixing of tracer with water already in a pore or fracture is rapid and when the flushing time for all openings is nearly equal, the concentraction gradient is steep. When the mixing is slow or the rapidity of flushing varies widely, however, a "finger" of tracer can move much farther before attaining a dilution equivalent to that in other openings, and the concentration gradient is more gentle.

The whole process of "fingering" and mixing is highly complex, and can only be approximated by models that are mathematically tractable. A frequently used model is one obtained by assuming that mechanical dispersion is analogous to molecular diffusion and can be described with identical equations. Complete solutions to the diffusion model equations were obtained by Brenner (1962), and numerical results were computed. The results were specifically for "bounded" diffusion (Brenner 1962, p. 230), a situation which occurs in a column with entrance and exit holes so small relative to the column cross section that tracer is transferred in (and out) only by bulk flow—that is, transfer by dispersion at these points is negligible. This criterion is met by two wells in a discharge-recharge system, such as the one described for this test.

Other workers have also presented solutions for the general diffusion model equation (for example: Ogata and Banks, 1961; Shamir and Harleman, 1967); however, the work of Brenner (1962) was used here because the solution was for "bounded" diffusion (that is, a finite column length) and because of the tabular presentation of the results.

Brenner computed results for the "step" addition of a second fluid to a packed column. In step addition, flow of the second fluid is started abruptly and continued until displacement of the first-traced fluid from the column is acceptably complete. Miscibility and similar viscosities of the fluids are assumed. Brenner tabulated $C/C_0$, the ratio of tracer concentration at the column exit to tracer concentration in the original liquid in the column, for various Peclet numbers and at various times for each. Brenner (1962, p. 230) used the parameter $UL/4D$ which he referred to as the "Peclet number," where $U$ is the average interstitial velocity, $L$ is the length of bed, and $D$ is the axial dispersion coefficient. The extraneous factor of 4 was introduced in the denominator to simplify some of his equations. For the present purposes it is advantageous to remove the extraneous factor of 4; thus, $N=4P$ where $N$ is the dimensionless column length or the number of dispersion lengths in the column (Hennico and others, 1963, p. 10) and is the Peclet number as usually defined. The axial dispersion coefficient, $D$, is equal to the product of the characteristic dispersion length in the medium, $l$, and the interstitial velocity, $U$ (Orcutt and others, 1957, p. 793). Thus,

$$N=4P=\frac{4UL}{4D}=\frac{4UL}{4Ul}=\frac{L}{l}.$$

The units of $l$ are those of length, and $N$ is dimensionless.

Time is used dimensionlessly by Brenner as $\theta$, the ratio of the volume of second fluid that has entered the column by a given time, $t$, to the pore volume of the column. For a constant flow rate, $\theta$ is the ratio of the given time to the time required to fill completely the column. $C/C_0$ and $\theta$ are almost always measured quantities in an experiment; and $N$ is the sought quantity. Because of the complexity of the relationship, $N$ cannot be solved for explicitly, but is obtained by curve fitting.

Brenner tabulated values of the ratio of tracer concentration in the effluent to tracer concentration in the first fluid. The primary interest in the test reported in this paper is in the complementary situation—one in which the tracer is in the displacing liquid so that $C/C_0$ is the ratio of effluent concentration to concentration in the displacing liquid. Such values are obtained simply by subtracting Brenner's values from 1.0, and typical curves of $C/C_0$ versus $\theta$ for fixed values of $N$ are plotted in figure 9. It is apparent that as displacement continues, the effluent concentration approaches the concentration in the displacing liquid—that is, most of the first liquid is displaced. Plots similar to those of figure 9 were used to calculate the amount of tracer contributed to the discharge well after several hundred days by the recycling of traced water through the fracture zone.

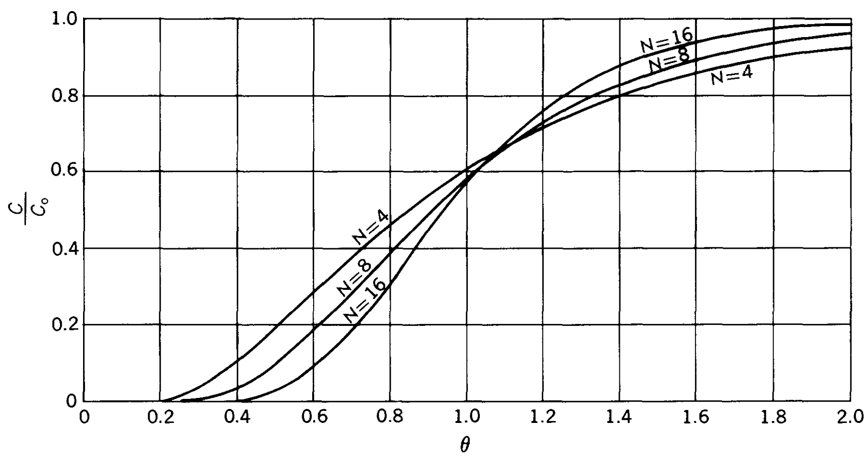



FIGURE 9.—Curves of the ratio of exit concentration to the initial concentration of tracer ($C/C_0$) versus the ratio ($\theta$) of selected times ($t$) to the transit time for nondispersed flow ($t_a$) for selected numbers of dispersion lengths ($N$) plotted for step addition of tracer from data given by Brenner (1962).

The initial passage of traced water through the formation in this experiment is of a different nature than that described. Addition of the tracer fluid was not continued indefinitely, once started; but consisted of a pulse of high concentration (4,750 $\mu C/l$) injected for only 1.2 days as the front of the "second fluid." As displacement progresses, the pulse is spread by dispersion.

To allow calculation of effluent concentrations for this case, a reference concentration is needed, similar to the concentration $C_0$ for step addition of tracer. The reference concentration, $C_r$, is that which would exist if the pulse of tracer were distributed uniformly throughout the column. As explained by Levenspiel (1962, p. 249), $C/C_r$ (for pulse addition) equals the derivative of $C/C_0$ (for step addition) with respect to $\theta$. Accordingly, values of $C/C_r$ for a given number of dispersion lengths, $N$, were calculated from pairs of entries in Brenner's table by dividing $\Delta(C/C_0)$ by $\Delta\theta$. Tables giving values for the differentiated equations (Brenner 1962, eq 20, 33) were presented by G. A. Latinen and F. D. Stockton in 1959, but were never published. However, use of these values gives the same results as the use of finite differences from Brenner's tables. Typical results are shown in figure 10 on rectangular coordinates. Similar plots were used for the calculations described in the following section.

## PROCEDURE FOR CALCULATING THE TRITIUM CONCENTRATION AT THE DISCHARGE WELL

The flow field was divided into crescents 0.3 radians "wide," as illustrated in figure 6. The crescents were considered as parallel-connected columns of different lengths, but with cross sections chosen

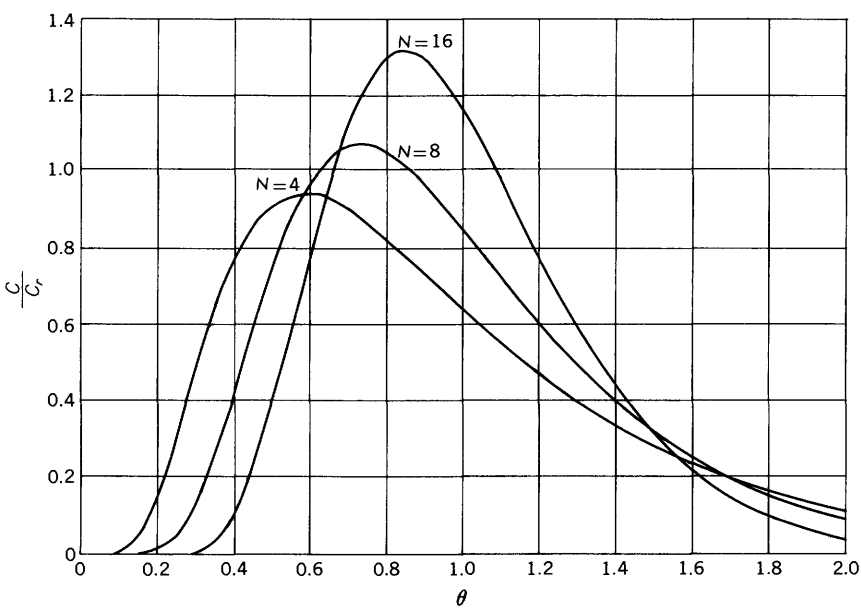



FIGURE 10.—Curves of the ratio of exit concentration of tracer to the reference concentration ($C/C_r$) to the ratio ($\theta$) of selected times ($t$) to the transit time for nondispersed flow ($t_a$) for selected numbers of dispersion lengths ($N$) plotted for a pulse addition of tracer from values obtained from those given by Brenner (1962).

(that is, equal $\Delta\psi$'s), so that flow is partioned equally among the columns. Only the 6 crescents that were short enough to become involved during the 2-year test, according to this analysis, are numbered. The time of transit for nondispersed flow, $t_a$, between wells is listed in figure 6 for each crescent, expressed as a ratio to the time $t_{a0}$ for nondispersed movement directly between wells; these values of $t_a/t_{a0}$ were read from figure 8 for the central value of $\psi$ in each crescent. The average path length, $S$, of each crescent is also listed, and was obtained as follows:

Inspection of figure 7 shows that the length of a streamline or arc ($S$) is simply the product of the radius, $r$, of the arc and the included angle $2\psi$. Since equation 2 states that $r=\frac{L/2}{\sin\psi}$, then

$$\frac{S}{L}=\frac{\psi}{\sin\psi}. \quad (12)$$

If the time of transit, or filling time, for a crescent column is $t_a$, then from equation 6 the pore volume of the crescent is

$$\Delta V_f=\frac{t_a q_0 \Delta\psi}{\pi}. \quad (13)$$

The tracer solution of concentration $C_*$ is injected as feed for a time $t_*$ (1.2 days) at a rate $q_i$. The fraction that enters each crescent is $\frac{\Delta\psi}{\pi}$; therefore the quantity of tracer in each is $C_* t_* q_i \frac{\Delta\psi}{\pi}$; the reference concentration, $C_r$, is this quantity divided by the crescent pore volume from equation 13, or

$$C_r=C_*\frac{t_* q_i}{t_a q_0}. \quad (14)$$

Ideally, $q_i$ would equal $q_0$, and the ratio would be unity. However, in the test reported here they were not equal (fig. 4), and it was essential to recognize this fact in the analysis.

Equation 14 gives the proper reference concentration for calculating the tracer concentration in effluent from any one crescent. But the tracer-test effluent concentrations were measured only after mixing together in the pipe of the discharge well the effluents from all crescents. Therefore, the concentration, $C$, from one crescent must be reduced (diluted) by $\frac{\Delta\psi}{\pi}$. It is convenient to include this multiplier with the reference concentration. With $C_*=4750\ \mu C/l$, $t_*=1.2$ days,

$q_t=9.63$ gpm, $q_0=7.9$ gpm, and $\Delta\psi=0.3$ radians,

$$\frac{\Delta\psi}{\pi}C_r=\frac{663}{t_a}\mu C/l\cdot \tag{15}$$

A likely value for $t_{a0}$ must be estimated from the plot of observed effluent concentrations versus time (fig. 5). For example, 400 days can be taken for this time of transit for nondispersed flow along the direct line between wells and adjusted in the direction indicated by curve fitting. Also, a value should be selected for $N_0$, the number of dispersion lengths along the direct line. To illustrate the method of calculation, $N_0$ will be taken as 8. For a given crescent, then,

$$t_a=t_{a0}\left(\frac{t_a}{t_{a0}}\right), \text{ with} \left(\frac{t_a}{t_{a0}}\right) \text{ from figure 8,}$$

$$\frac{\Delta\psi}{\pi}C_r=\frac{663}{t_a}\mu C/l \text{ (eq 15),}$$

$$N=N_0\left(\frac{S}{L}\right), \text{ with} \left(\frac{S}{L}\right) \text{ from equation 12.}$$

For a specific time, $t$ days,

$$\theta=\frac{t}{t_a},$$

$(C/C_r)$ is obtained as a function of $N$ and $\theta$ from a plot like figure 10,

$$C=\frac{\Delta\psi}{\pi}C_r\left(\frac{C}{C_r}\right)\cdot$$

For convenience, the calculations can be arranged as in table 1.

TABLE 1.—*Example of calculations for computing tritium concentration in discharge well from pulse injection of tracer*

| $t_{a0}=400$ days, $N_0=8$ | | | | $t=100$ days | | | $t=200$ days | | | $t=350$ days | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Crescent No. | $t_a$, days | $\frac{\Delta\psi}{\pi}C_r$ $\mu C/l$ | $N$ | $\theta$ | $(C/C_r)$ | $C$, $\mu C/l$ | $\theta$ | $(C/C_r)$ | $C$, $\mu C/l$ | $\theta$ | $(C/C_r)$ | $C$, $\mu C/l$ |
| 1 | 404 | 1.64 | 8.0 | 0.25 | 0.053 | 0.087 | 0.50 | 0.76 | 1.25 | 0.87 | 0.99 | 1.62 |
| 2 | 435 | 1.52 | 8.3 | .23 | .030 | .046 | .46 | .62 | .94 | .80 | 1.06 | 1.61 |
| 3 | 506 | 1.31 | 8.8 | .20 | .009 | .012 | .40 | .38 | .50 | .69 | 1.07 | 1.40 |
| 4 | 640 | 1.03 | 9.7 | .15 | .001 | .001 | .31 | .06 | .06 | .55 | .83 | .85 |
| 5 | 880 | .75 | 11.1 | .11 | | | .23 | .007 | .01 | .40 | .26 | .19 |
| 6 | 1,380 | .44 | 13.1 | .07 | | | .15 | | | .25 | .006 | .00 |
| Effluent concentration | | | | | | .146 | | | 2.76 | | | 5.67 |

Comparison of these points with the data in figure 5 shows that agreement is good at 100 days, but that the calculated value at 200 days is below the observed concentration, whereas the calculated value at 350 days is above. Other trial values for $t_{a0}$ and $N_0$ may be selected, and the calculated concentrations compared with the observed concentrations at various times. The best fit resulted for $N_0=4.0$ and $t_{a0}=440$ days; this fit is shown as the dashed line labeled "curve calculated for dispersed flow from a pulse injection" in figure 5.

This calculated curve drops below the observed curve after about 400 days. The difference is the result of ignoring tracer that has been through the system once and is appearing a second time because of recycling of water from the discharge well back to the injection well. This second appearance follows the mathematics of step addition of tracer, figure 9. For ease of explanation, this step addition will be considered to consist of the introduction of a steady flow of 4.5 $\mu C/l$ water into the injection well at 200 days. (Because of the gradual, instead of abrupt, introduction of recycled tracer into the injection well, the calculations were actually carried out as three step additions: 1.75 $\mu C/l$ at 150 days, 1.75 at 200 days, and 1.00 at 300 days. These three steps add together to give a total effect that is practically identical with that calculated for a single step addition of 4.5 $\mu C/l$ at 200 days.)

The concentration in the step addition will be diluted to $C_0=4.5 \frac{\Delta\psi}{\pi}=0.43$ $\mu C/l$ for purposes of calculation, as was done with the reference concentration in pulse addition. Values for $t_a$, $N$, and $\theta$ are computed as before, but with $t_{a0}=440$ days and $N_0=4.0$; $(C/C_0)$ comes from plots similar to those in figure 9. An example of these calculations is given in table 2.

This concentration of 0.72 $\mu C/l$ from recycling tritium is added to the previously calculated concentration from the pulse to give the

TABLE 2.—*Example of calculations for computing tritium concentration in discharge well from step injection of tracer*

| $t_{a0}=440$ days, $N_0=4.0$ | | | | $t=400$ days | | |
|---|---|---|---|---|---|---|
| Crescent No. | $t_a$, days | $\frac{\Delta\psi}{\pi}C_0$, $\mu C/l$ | $N$ | $\theta$ | $(C/C_0)$ | $\mu C/l$ |
| 1 | 444 | 0.43 | 4.0 | 0.90 | 0.54 | 0.23 |
| 2 | 478 | .43 | 4.2 | .84 | .49 | .21 |
| 3 | 556 | .43 | 4.4 | .72 | .38 | .16 |
| 4 | 704 | .43 | 4.9 | .57 | .22 | .09 |
| 5 | 968 | .43 | 5.6 | .41 | .07 | .03 |
| 6 | 1,520 | .43 | 6.6 | .26 | | |
| Concentration | | | | | | 0.72 |

total concentration at 400+200=600 days since start of the pulse test. That is, the 200 days of delay before starting the step addition must be added to the 400 days of flow to put the two tracer additions on the same time scale.

The solid line labeled "curve calculated for dispersed flow from pulse and step injection" in figure 5 shows the results of combining the two types of tracer addition. Agreement with the data is surprisingly good. The relatively high calculated concentrations at the start of the breakthrough curve, as well as other deviations, may be a result of an inexact model. For example, a portion of the first traced water passing through the system may be held in relatively dead pores, and reappears only slowly in the effluent at much later times. Lateral dispersion, which has been neglected in this analysis, may account for the early observed concentrations of tritium being lower than the calculated values. Tritium may be lost laterally from the earliest arriving central flow path, an action that is apparent only in the early stages before transfer back from adjacent crescents occurs. Finally, crescents are probably not identical, as assumed in the model.

## CONCLUSIONS

The purpose of the two-well tracer test using tritium was to determine hydraulic constants from the physical movement of fluid through fractures with which the hydraulic constants developed from induced head changes within the crystalline rock could be compared. Figure 5 shows the concentration of tritium arriving at the discharge well. This observed curve can be closely duplicated from dispersion theory and the assumption that the fracture zone behaves as a homogenous and isotropic medium with respect to fluid transmission and dispersion.

In fitting the theoretical tritium arrival curve to the observed curve, only two unknown variables occur. The first of these variables is the transit time for nondispersed flow along the direct path between the wells, $t_{a0}$. Variations as small as 3 percent in the value of $t_{a0}$ cause a noticeably poorer fit for the theoretical curve.

The second unknown variable, the characteristic dispersion length (the number of dispersion lengths between the two wells, $N$, was determined), is a convenient mathematical quantity with which to describe dispersion, but it is difficult to assign physical significance to this quantity. Mathematically, it is equivalent to the mean free path in the diffusion equations. The analysis showed that there were 4.0 dispersion lengths along the line between the two wells, 1,765 feet, making a dispersion length about 440 feet. Variations of 10 percent in $N_0$ cause a noticeably poorer fit for the theoretical curve.

The results of the tracer test agree with computations based on the assumption that flow was through a homogeneous medium consisting of numerous intersecting fractures. Any particular well in this area, however, is apt to penetrate individual fractures within the fracture zone that show large differences in permeability as concluded by Marine (1967b, p. B211). However, the larger, more permeable fractures that may be penetrated are probably connected with each other by less permeable fractures as shown schematically in figure 1. The tracer test suggests that large planar fractures do not form direct hydraulic connections between the wells used in the test and that the fractures in the "inclined fracture zone" in this area form an interlacing network.

From the hydraulic data obtained during the tracer test (that is, the calculation is not based on the physical movement of tracer but on the head relationships), the average permeability of the aquifer may be calculated.

$$K \approx \frac{q_0}{\pi \Delta h m} \ln \frac{L}{r_w} \text{ (DeWeist, 1965 p. 249)},$$

where

$K$=coefficient of permeability,
$q_0$=discharge-recharge rate (7.9 gpm),
$\Delta h$=difference in head between the discharge and recharge wells (120 ft),
$m$=thickness of the aquifer (250 ft),
$L$=distance between the two wells (1,765 ft), and
$r_w$=radius of the pumping well (0.23 ft).

The average coefficient of permeability so calculated is 1.1 gpd per sq ft. From pumping tests Marine (1967b, p. B210) estimated the average coefficient of permeability of fracture zones in the area to be 0.77 gpd per sq ft ($\approx$1 gpd per sq ft).

From the values obtained from the tracer test ($t_{a0}$=440 days), a value for the porosity due to fractures may be calculated from a rearrangement of equation 9: $\epsilon = \frac{3q_0 t_{a0}}{\pi L^2 m}$.

The value calculated is 0.08 percent.

The average velocity for nondispersed flow along the direct line between the wells was 1,765 feet÷440 days=4.0 feet per day.

If it is assumed that Darcy's law is valid for fluid movement in this medium, an average value for the coefficient of permeability can be calculated from the velocity (4.0 ft per day), the porosity (0.08 percent), and the average gradient between the wells (360 ft per mile). This value is 0.35 gpd per sq ft and is derived entirely from the analysis of the physical movement of fluid through the medium.

This value may be compared to the value of permeability of about 1 gpd per sq ft estimated from head changes in the medium resulting from the removal and injection of water.

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