Methods for Determining the Proper Spacing of Wells in Artesian Aquifers

METHODS OF AQUIFER TESTS

GEOLOGICAL SURVEY WATER-SUPPLY PAPER 1545-B

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The Theis nonequilibrium formula is modified to obtain equations useful in determining proper well spacing in artesian aquifers

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METHODS FOR DETERMINING THE PROPER SPACING OF WELLS IN ARTESIAN AQUIFERS

By S. M. Lang

ABSTRACT

The Theis nonequilibrium formula can be modified to provide for convenient solution of innumerable problems of limited scope in ground-water hydraulics. An example of the type of modification of the formula that can be made is presented herein.

Well spacing is a problem that cannot be solved directly by the application of the nonequilibrium formula or any of the available modifications. The formula is altered to obtain equations useful in determining the proper well spacing in artesian aquifers for the following situations: (a) pumped wells in an areally extensive aquifer, (b) pumped wells in an aquifer bounded on one side by a perennial stream, the wells being arranged in a line either parallel or at an angle to the boundary, and (c) pumped wells in an aquifer bounded by an impervious barrier, the wells being arranged in a line either parallel or at an angle to the boundary.

INTRODUCTION

A great advance in ground-water hydrology was made with the development of the nonequilibrium formula (Theis, 1935). This formula provides a means of determining the hydraulic characteristics of an aquifer; these characteristics define the ability of the aquifer to store and transmit water. The formula has also been used extensively in evaluating well performance and in predicting changes in water levels in aquifers of large areal extent and relatively uniform permeability.

The nonequilibrium formula may be used to obtain answers to many quantitative problems having to do with the recharge, movement, and discharge of ground water. However, problems remain that cannot be solved by the direct application of the formula, although it is possible to modify the nonequilibrium formula to provide shortcuts for solving innumerable problems of limited scope. In the past, modifications have been developed that have simplified methods of applying the formula (Cooper and Jacob, 1946) and have extended its application to situations where the pumped aquifer is limited by geologic or hydrologic boundaries (Stallman, 1952). This paper presents a further example of the sort of modification of the nonequilibrium formula that can be made.

Well spacing remains as one of the problems that cannot be solved directly by the application of the nonequilibrium formula or of any available modifications. Theis (1957) presented a method for determining well spacing in a thick and areally extensive aquifer; however, the prime consideration in this method was economics. According to the method, the farther apart the wells are the less their mutual interference caused by pumping but the greater the cost of pipeline and electrical installations. By equating the added cost of pumping due to mutual interference against the capitalized cost of these installations, Theis was able to derive an equation for optimum well spacing. In many situations, however, economics may be of secondary importance, or other considerations may be involved.

One or more of the following factors may be of importance in determining the proper spacing of pumped wells: (a) physical limitations of the aquifer, such as depth, thickness, piezometric head, or areal extent; (b) depths of existing wells; (c) limitations on available pumping equipment, such as suction lift, setting of bowls, or horsepower of motors; or (d) place and purpose of use of the water.

Presented herein is a modification of the nonequilibrium formula by means of which it is possible to determine the proper spacing of pumped wells in an artesian water-bearing formation, taking into account the physical limitations of the pumped formation.

AREALLY EXTENSIVE AQUIFER

The nonequilibrium formula is

$$s=\frac{114.6Q}{T}W(u) \qquad (1)$$

where $u=\frac{1.87\,r^2S}{Tt}$ (2)

s=drawdown produced by the pumped well, in feet,
Q=discharge rate of the pumped well, in gallons per minute,
T=coefficient of transmissibility of the aquifer, in gallons per day per foot,

$$W(u)=-0.5772-\log_e u+u-\frac{u^2}{2.2!}+\frac{u^3}{3.3!}-\frac{u^4}{4.4!}+\cdots$$

($W(u)$ is known as the "well function of u"),
r=distance from the pumped well to the point of observation, in feet (where the pumped well is the point of observation, r is the effective radius of the well),
S=coefficient of storage of the aquifer, dimensionless,
t=time since pumping began, in days.

Most of the terms are self-explanatory. However, the coefficients of transmissibility and storage need further elaboration. The coefficient of transmissibility of an aquifer, T, is the rate of flow of water, under prevailing conditions including water temperature, in gallons per day, through a vertical strip of the aquifer 1 foot wide extending the full saturated height of the aquifer, under a hydraulic gradient of 100 percent. The coefficient of storage of an aquifer, S, is the volume of water it releases from or takes into storage per unit surface area of the aquifer per unit change in the component of head normal to that surface.

When two wells are pumped simultaneously, the total drawdown in each well is the sum of the effects due to its own pumping and to the interference of the other well. If their pumping rates are the same, then

$$s_p=s_1+s_2=\frac{114.6Q}{T}\left(W(u)_1+W(u)_2\right) \tag{3}$$

where s_p = total drawdown in the first pumped well (well 1), in feet,
s_1 = drawdown in the first pumped well (well 1) due to its own pumping, in feet,
s_2 = drawdown in the first pumped well produced by interference of the second pumped well (well 2), in feet,
$W(u)_1$ = value of $W(u)$ at well 1 resulting from its own pumping, and
$W(u)_2$ = value of $W(u)$ at well 1 resulting from the pumping of well 2.

When t becomes sufficiently large that u is equal to or less than 0.02, the value of $W(u)$ can be approximated by the first and second terms of the series expression, and equation 3 may be rewritten in the form

$$s_p=\frac{114.6Q}{T}\left(-2(0.5772)-\log_e u_1u_2\right) \tag{4}$$

$$\log_e u_1u_2=-\left(\frac{s_pT}{114.6Q}+1.1544\right). \tag{5}$$

Converting to logarithms to the base 10,

$$\log u_1u_2=-\left(\frac{s_pT}{264Q}+0.501\right). \tag{6}$$

By substituting the proper values for T and Q and the desired value for s_p in equation 6, the product of u_1u_2 can be computed directly. Then, from equation 2,

$$u_1u_2=(Kr_1r_2)^2 \tag{7}$$

where

$$K = \frac{1.87S}{Tt}$$

r_1 = the effective radius of well 1,

and r_2 = the distance from well 2 to well 1.

Therefore,

$$r_2 = \frac{\sqrt{u_1 u_2}}{K r_1}. \tag{8}$$

Within the limitation that $u \leq 0.02$, equation 8 can be used to determine the well spacing in an areally extensive aquifer. The following example illustrates the use of the method.

Example 1.—A water user requires about 500 gpm (gallons per minute) for 10 hours per day for 5 days per week. Ground-water investigations have identified an areally extensive shallow artesian aquifer (values of T and S are 50,000 gpd per foot and 1×10^{-4}, respectively). Test drilling has demonstrated that it is feasible to construct a 24-inch supply well capable of being pumped at a rate of 500 gpm. However, another nearby water user already has a well tapping this particular aquifer operating at the same rate and schedule as the proposed well. Inasmuch as two 500-gpm wells will be pumping simultaneously, it is desired to determine the minimum spacing of the wells at which the artesian head of the aquifer will not be lowered excessively. It is determined that the permissible total drawdown in each well is 30 feet. If it is assumed that the total effect of all well losses (losses in head due to entrance of water into the well and travel of water in the well) and the residual drawdown due to the intermittent operation of the wells is 2 feet, the drawdown caused by the daily operation of the wells can be only 28 feet. It is assumed also that the effective radius of the proposed well is equal to the actual radius of the well.

Thus, $s_p = 28$ feet, $T = 50{,}000$ gpd per foot, $S = 1 \times 10^{-4}$, $Q = 500$ gpm, $t = 0.417$ day, and $r_1 = 1$ foot.

From equation 6,

$$\log u_1 u_2 = -\left(\frac{s_p T}{264Q} + 0.501\right)$$

$$= -\left(\frac{28 \times 50{,}000}{264 \times 500} + 0.501\right)$$

$$\log u_1 u_2 = -11.107$$

$$= 8.893 - 20.$$

Therefore,

$$u_1u_2=7.82\times10^{-12}$$

$$K=\frac{1.87S}{Tt}=\frac{1.87\times1\times10^{-4}}{50,000\times0.417}=8.97\times10^{-9}.$$

From equation 8,

$$r_2=\frac{\sqrt{u_1u_2}}{Kr_1}$$

$$r_2=\frac{\sqrt{7.82\times10^{-12}}}{8.97\times10^{-9}\times1}$$

$$=312 \text{ feet.}$$

PUMPED WELLS NEAR A GEOLOGIC OR HYDROLOGIC BOUNDARY

Stallman (1952) modified the nonequilibrium formula to broaden the scope of its use to situations where the pumped formation is adjacent to another formation of vastly different transmissibility, and the boundary between the formations approximates a straight line. The most common examples of this situation might be (a) permeable material of a buried valley abutting the relatively impermeable walls of the valley, and (b) an aquifer that is hydraulically connected with a body of surface water. In either example, the solution is made by use of the image-well theory (Ferris, 1949). In the first example, the condition is imposed that no water flows across the boundary; in the second example, the condition is imposed that no drawdown occurs along the boundary. These conditions are simulated by assuming that the aquifer is infinite in extent and that an imaginary (image) well lies the same distance from the boundary as the pumping well but on the opposite side of the boundary. In the first example, the image well would operate at the same time and rate as the pumped well. In the second example, the image well would operate at the same time and rate but would be a recharging well instead of a discharging well. In this manner, the problem is converted from one involving boundary conditions to one of a multiple-well system in an infinite aquifer.

PUMPED WELLS NEAR AN IMPERMEABLE BARRIER

Each pumped well near a boundary has its own image well. Therefore, the total drawdown in each pumped well is the algebraic sum of the effects of all pumped and image wells.

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WELLS ARRANGED IN A LINE PARALLEL TO THE BOUNDARY

First, consider the case where two wells are arranged in a line parallel to the boundary (fig. 1*A*):

$$s_p = s_1 + s_2 + s_a + s_b = \frac{114.6\,Q}{T}\,(W(u)_1 + W(u)_2 + W(u)_a + W(u)_b). \quad (9)$$

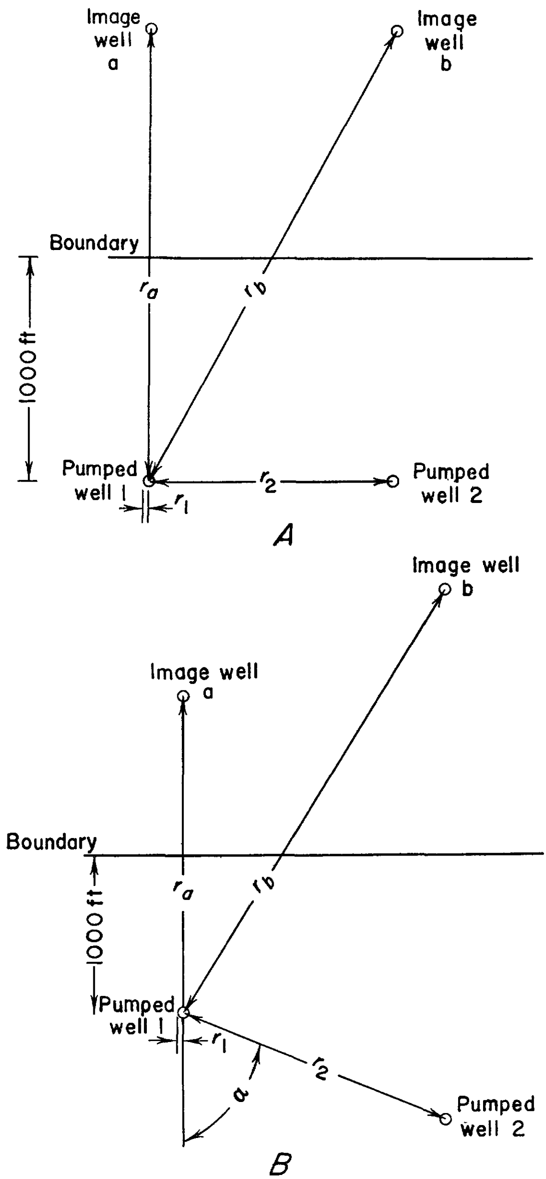

FIGURE 1.—Location of real and image wells near a straight-line boundary. *A*. Wells parallel to the boundary; *B*. wells at an angle to the boundary.

When u is less than 0.02, the value of $W(u)$ is given by $W(u)-0.5772$ $-\log_e u$ as discussed above.
Hence,

$$s_p=\frac{114.6\,Q}{T}\,(-0.5772-\log_e u_1-0.5772$$
$$-\log_e u_2-0.5772-\log_e u_a-0.5772-\log_e u_b) \quad (10)$$

$$s_p=\frac{114.6\,Q}{T}(-4\,(0.5772)-\log_e u_1u_2u_au_b) \quad (11)$$

$$\log_e u_1u_2u_au_b=-\left(\frac{s_pT}{114.6Q}+2.3088\right). \quad (12)$$

Converting to logarithms to the base 10,

$$\log u_1u_2u_au_b=-\left(\frac{s_pT}{264Q}+1\right). \quad (13)$$

The value of the second term on the right side of equation 13 is approximate; the actual value is 1.002.

By substituting the proper values of s_p, T, and Q into equation 13, the product $u_1u_2u_au_b$ may be computed directly. From equation 2

$$u_1u_2u_au_b=K^4r_1^2r_2^2r_a^2r_b^2,\text{ where }K=\frac{1.87S}{Tt}. \quad (14)$$

It follows that

$$r_2^2r_b^2=L,\text{ where }L=\frac{u_1u_2u_au_b}{K^4r_1^2r_a^2}. \quad (15)$$

However, note from figure 1A that

$$r_b^2=r_2^2+r_a^2$$

and from equation 15

$$r_2^2(r_2^2+r_a^2)=L. \quad (16)$$

Thus,

$$r_2^4+r_a^2r_2^2-L=0. \quad (17)$$

Again within the limitation that $u\leq 0.02$, equation 17 can be used to determine well spacing when the wells are to be arranged in a line parallel to an impermeable boundary. This equation is a quadratic of the classic form $ax^2+bx+c=0$, where in this case $x=r_2^2$, $a=1$,

$b=r_a^2$, and $c=-L$. The quadratic is solved by the formula $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$; the negative root of this formula has no significance to the solution. The following example demonstrates the use of these equations.

Example 2.—A new project is to have its own ground-water supply. It is estimated that it will require an average of 300 gpm to meet its expected demands. Although it is possible to obtain this amount from one well, it is decided to construct a second well in order to insure an adequate supply during periods of peak demand, as well as for a safeguard in case of well or pump failure. It is possible that the two wells will be operated simultaneously for periods as long as 5 days.

Ground-water investigations have identified a relatively shallow artesian aquifer ($T=30{,}000$ gpd per foot, $S=1\times10^{-4}$) that is cut off by an impermeable boundary. Test drilling has demonstrated the feasibility of constructing 12-inch wells, each capable of being pumped continuously at 300 gpm, arranged in a line about 1,000 feet from the boundary. It is desired to determine the well spacing if it is assumed that the well losses are negligible and that the permissible total drawdown is 40 feet in each well at the end of the 5 days.

The analysis evidently will require the use of two discharging image wells placed across the boundary opposite the supply wells and at a distance of 2,000 feet from the supply wells (fig. 1*A*).

Therefore, $s_p=40$ feet, $T=30{,}000$ gpd per foot, $S=1\times10^{-4}$, $Q=300$ gpm, $r_a=2{,}000$ feet, $r_1=0.5$ foot, $t=5$ days. From equation 13,

$$\log u_1u_2u_au_b=-\left(\frac{s_pT}{264Q}+1\right)$$

$$=-\left(\frac{40\times30{,}000}{264\times300}+1\right)$$

$$=-16.2$$

$$=3.8-20$$

$$u_1u_2u_au_b=6.3\times10^{-17}$$

$$K=\frac{1.87S}{Tt}=\frac{1.87\times1\times10^{-4}}{30{,}000\times5}=1.25\times10^{-9}$$

$$L=\frac{u_1u_2u_au_b}{K^4r_1^2r_a^2}=\frac{6.3\times10^{-17}}{2.41\times10^{-36}\times0.25\times4\times10^6}$$

$$=2.6\times10^{13}.$$

Solving equation 17 by the formula

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$x=r_2^2,\ a=1,\ b=r_a^2=4\times10^6,\ c=-L=-2.6\times10^{13}$$

$$r_2^4+4\times10^6r_2^2-2.6\times10^{13}=0$$

$$r_2^2=\frac{-4\times10^6\pm\sqrt{16\times10^{12}+4\times2.6\times10^{13}}}{2}$$

$$=\frac{-4\times10^6\pm11\times10^6}{2}=3.5\times10^6$$

$r_2=1{,}900$ feet.

WELLS ARRANGED IN A LINE AT AN ANGLE TO THE BOUNDARY

A more complex problem would result from arranging the wells in a line at an angle to the boundary (fig. 1*B*).

Restating equation 15,

$$r_2^2r_b^2=L,\ \text{where}\ L=\frac{u_1u_2u_au_b}{K^4r_1^2r_a^2}.$$

Inasmuch as the image-well distances from the boundary are the same as for the real wells, except that they are on opposite sides of it, r_b may be stated in terms of the known distance, r_a, the angle, α, and the unknown well spacing, r_2. Thus, applying the law of cosines, and recognizing that r_b is also equal to the distance from image well a to pumped well 2,

$$r_b^2=r_2^2+r_a^2-2r_ar_2\cos(180°-\alpha)$$

and

$$r_b^2=r_2^2+r_a^2+2r_ar_2\cos\alpha. \qquad (18)$$

Substituting equation 18 into equation 15,

$$r_2^2(r_2^2+r_a^2+2r_ar_2\cos\alpha)=L' \qquad (19)$$

and

$$r_2^4+2r_ar_2^3\cos\alpha+r_a^2r_2^2-L'=0. \qquad (20)$$

Equation 20 is a quartic equation that can be solved readily by graphics. By substituting various values of r_2 into the modified equation 20

$$y=f(r_2)=r_2^4+2r_a\cos\alpha r_2^3+r_a^2r_2^2-L' \qquad (21)$$

corresponding values for y are obtained. The values for r_2 and y then serve as coordinates for points plotted on rectangular coordinate paper. If the r_2 values are taken along the horizontal axis, a curve fitted to the plotted points will yield the proper value for r_2 where it crosses the horizontal axis. The following example illustrates the use of these equations.

Example 3.—Assume the same field situations as described in example 2 except that well 2 is located farther from the boundary and the line of wells makes an angle (α) of 60° with the impermeable boundary.

Therefore,

$$r_a=2{,}000 \text{ feet}, \; s_p=40 \text{ feet}, \; T=30{,}000 \text{ gpd per foot},$$

$$S=1\times10^{-4}, Q=300 \text{ gpm},$$

$$r_1=0.5 \text{ foot}, \; \alpha=60°, \; \cos\alpha=0.5, \; t=5 \text{ days}.$$

As previously determined, $L'=26\times10^{12}$. Equation 20 is then

$$r_2^4+2\times10^3r_2^3+4\times10^6r_2^2-26\times10^{12}=0.$$

Substituting values for r_2 into the above,

r_2	$y=r_2^4+2\times10^3r_2^3+4\times10^6r_2^2-26\times10^{12}$
1×10^3	$(1+2+4-26)10^{12}=-19\times10^{12}$
1.5×10^3	$(5.1+6.8+9.0-26)10^{12}=-5.1\times10^{12}$
1.75×10^3	$(9.4+10.7+12.2-26)10^{12}=6.3\times10^{12}$
2×10^3	$(16+16+16-26)10^{12}=22\times10^{12}$

The computations shown above indicate that the wells should be spaced at a distance between 1,500 and 1,750 feet. The values for r_2 and y are plotted (fig. 2) and the place where the curve fitted to the points crosses the horizontal axis represents the exact solution—in this case about 1,600 feet.

An inspection of equation 20 indicates that the sum of the terms containing r_2 must equal the value of the L' term. Thus, a simple means of estimating the magnitude of r_2 is to take a fourth of the power to which 10 is raised in the L' term. In the above example, 10 is raised to the 12th power in the L' term. A fourth of 12 is 3. Therefore, all assumed values of r_2 have a magnitude of 10^3, and all terms in equation 21 have a magnitude of 10^{12}.

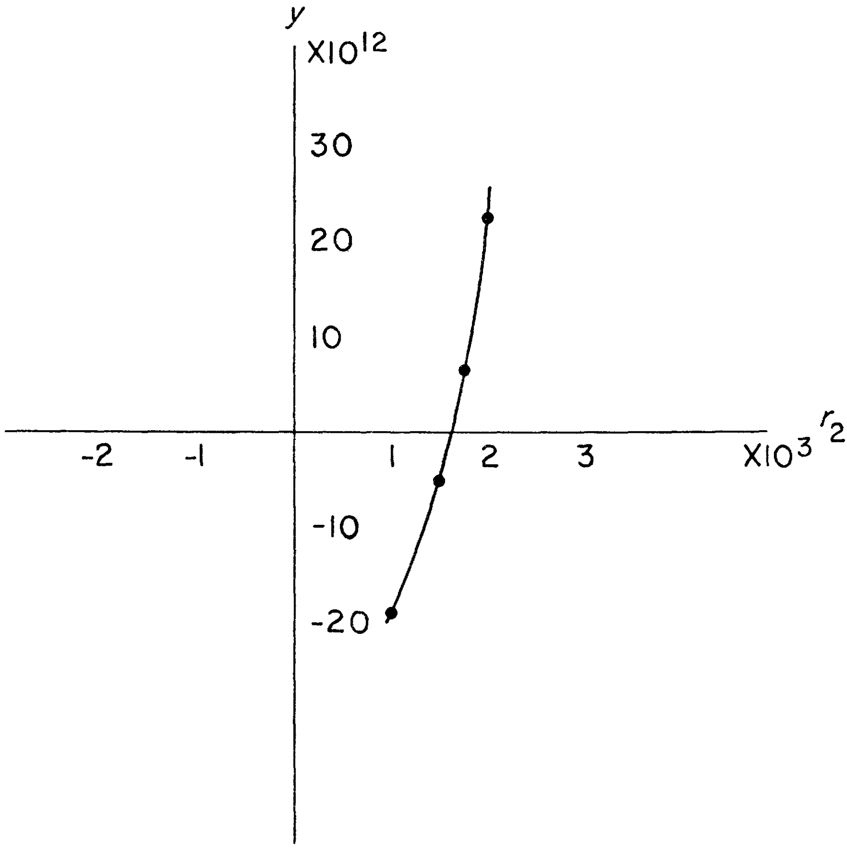

FIGURE 2.—Plot of the curve $y=f(r_2)=r_2^4+2\times10^3r_2^3+4\times10^6r_2^2-26\times10^{12}$. The curve indicates the solution of the problem where the pumped wells are arranged in a line at an angle to an impervious barrier.

PUMPED WELLS NEAR A RECHARGE BOUNDARY

The total drawdown in each of two pumped wells near a recharge boundary is the algebraic sum of the individual effects of the pumped and image wells

$$s_p=s_1+s_2-s_a-s_b=\frac{114.6Q}{T}\left(W(u)_1+W(u)_2-W(u)_a-W(u)_b\right). \qquad (22)$$

When u is less than 0.02, $W(u)$ is approximately

$$W(u)=-0.5772-\log_e u.$$

Therefore,

$$s_p=\frac{114.6Q}{T}\left(-0.5772-\log_e u_1-0.5772-\log_e u_2 +0.5772+\log_e u_a+0.5772+\log_e u_b\right) \qquad (23)$$

and

$$s_p=\frac{114.6Q}{T}\log_e\frac{u_a u_b}{u_1 u_2}. \qquad (24)$$

Converting to logarithms to the base 10,

$$s_p=\frac{264Q}{T}\log\frac{u_a u_b}{u_1 u_2}. \qquad (25)$$

If the wells operate simultaneously, the only variable in the u is r^2 and

$$s_p=\frac{264Q}{T}\log\frac{r_a^2 r_b^2}{r_1^2 r_2^2}. \qquad (26)$$

Rearranging,

$$\log\frac{r_a^2 r_b^2}{r_1^2 r_2^2}=\frac{s_p T}{264Q} \qquad (27)$$

and

$$\frac{r_a^2 r_b^2}{r_1^2 r_2^2}=M,\ \text{where } M=\text{antilog}\,\frac{s_p T}{264Q}. \qquad (28)$$

When the wells are placed parallel to the boundary (fig. 1*A*),

$$r_b^2=r_a^2+r_2^2. \qquad (29)$$

Hence,

$$\frac{r_a^2(r_a^2+r_2^2)}{r_1^2 r_2^2}=M \qquad (30)$$

$$\frac{r_2^2+r_a^2}{r_2^2}=\frac{r_1^2 M}{r_a^2} \qquad (31)$$

$$1+\frac{r_a^2}{r_2^2}=\frac{r_1^2 M}{r_a^2} \qquad (32)$$

$$\frac{r_a^2}{r_2^2}=\frac{r_1^2M}{r_a^2}-1 \tag{33}$$

$$r_2^2=\frac{r_a^4}{r_1^2M-r_a^2} \tag{34}$$

and

$$r_2=\frac{r_a^2}{\sqrt{r_1^2M-r_a^2}}. \tag{35}$$

The determination of the proper well spacing when the wells are placed at an angle to the boundary (fig. 1*B*) is made as follows: Repeating equation 28,

$$\frac{r_a^2r_b^2}{r_1^2r_2^2}=M,\text{ where } M=\text{antilog } \frac{s_pT}{264Q}$$

Applying the law of cosines, we repeat equation 18:

$$r_b^2=r_2^2+r_a^2+2r_ar_2 \cos \alpha$$

$$\frac{r_a^2(r_2^2+r_a^2+2r_ar_2 \cos \alpha)}{r_1^2r_2^2}=M \tag{36}$$

$$r_2^2+r_a^2+2r_ar_2 \cos \alpha=\frac{r_1^2r_2^2}{r_a^2} M \tag{37}$$

$$r_a^2+2r_ar_2 \cos \alpha=\frac{r_1^2r_2^2M}{r_a^2}-r_2^2 \tag{38}$$

$$r_a^2+2r_ar_2 \cos \alpha=r_2^2 \left(\frac{r_1^2}{r_a^2} M-1\right) \tag{39}$$

$$r_2^2 \left(\frac{r_1^2}{r_a^2} M-1\right)-2r_ar_2 \cos \alpha-r_a^2=0. \tag{40}$$

Equation 40 is a quadratic of the form $ax^2+bx+c=0$, where $a=\frac{r_1^2M}{r_a^2}-1$, $b=-2r_a \cos \alpha$, and $c=-r_a^2$. Thus, equations 35 and 40 permit the determination of proper well spacing near a recharge boundary when the line of wells is parallel to and at an angle to the boundary, respectively. The following example illustrates the use of these equations.

Example 4.—It is desired to install a water-supply system. Ground-water investigations on a flood plain have identified a shallow artesian aquifer ($T=50{,}000$ gpd per foot, $S=1\times10^{-4}$) capped by about 35 feet of fine-grained alluvium. Test drilling has located a site about 1,000 feet from the stream for a 12-inch well capable of producing 600 gpm continuously. A site for a second well is to be located, and it is desired to determine the proper well spacing if the second well is to be arranged in a line either parallel to the stream or at an angle to the stream such that $\alpha=30°$. Because of the small thickness of confining material, the total drawdown in each well should not exceed 30 feet. See figure 1 for an arrangement of wells. Evidently the analysis now requires the use of two recharging image wells placed across the recharge boundary.

Assuming that well losses are negligible, $T=50{,}000$ gpd per foot, $S=1\times10^{-4}$, $r=0.5$ foot, $r_a=2{,}000$ feet, $s_p=30$ feet, $Q=600$ gpm, $\alpha=30°$, $\cos\alpha=0.866$

$$\frac{s_pT}{264Q}=\frac{30\times50{,}000}{264\times600}=9.5 \quad \therefore \quad M=3.2\times10^9.$$

From equation 35, we can determine the proper spacing for the case where the wells are arranged in a line parallel to the stream:

$$r_2=\frac{r_a^2}{\sqrt{r_1^2M-r_a^2}}=\frac{4\times10^6}{\sqrt{0.25\times3.2\times10^9-4\times10^6}}=\frac{4\times10^6}{2.8\times10^4}$$

$r_2=140$ feet if the wells are arranged in a line parallel to the stream.

Equation 40 is solved by means of the quadratic formula

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

which gives the proper spacing for the case where the wells are arranged in a line at an angle to the stream.

$$x=r_2$$

$$a=\frac{r_1^2M}{r_a^2}-1=\frac{0.25\times3.2\times10^9}{4\times10^6}-1=199$$

$$b=-2r_a\cos\alpha=-2\times2\times10^3\times0.866=-3.5\times10^3$$

$$c=-r_a^2=-4\times10^6.$$

Therefore, $r_2=\dfrac{3.5\times10^3\pm\sqrt{12.2\times10^6+3.2\times10^9}}{398}$

$$=\frac{3.5\times10^3\pm56.7\times10^3}{398}$$

$r_2=150$ feet if the wells are arranged in a line at an angle of $\alpha=30°$ from the stream.

An inspection of equation 26 indicates that the total drawdown in a pumped well near a recharge boundary is dependent upon the rate of discharge, the transmissibility of the aquifer, and the well distances. Time is not involved in the equation; therefore, the drawdown or pumping level stabilizes and becomes a constant factor when u is less than 0.02. The time required for the drawdown to stabilize can be determined from the following analysis: The value of u varies with the square of the distance (r); if the time necessary for u to become as small as 0.02 is determined for the greatest distance in the well setup, u will be less than 0.02 for any other distance for that same time. In other words, the pumping level in the operating wells may be considered to have reached approximate stabilization at the time necessary for u at the maximum distance to equal 0.02.

When only one operating well is involved, the maximum distance is r_a, the distance between the pumped well and its image well. With two operating wells, the maximum distance is r_b, the distance between the first pumped well and the image well of the second pumped well (fig. 1). Thus, for the example given above, when the r_2 values have been determined the maximum distances may be computed from equation 18 or 29 and then substituted in the following form of equation 2:

$$t=\frac{1.87\, r_b^2 S}{Tu}. \tag{41}$$

If the wells are arranged in a line parallel to the boundary, from equation 29

$$r_b^2=r_a^2+r_2^2$$

$$r_b^2=(2{,}000)^2+(140)^2=4{,}020{,}000$$

$$r_b=2{,}000 \text{ feet.}$$

Thus,

$$t=\frac{1.87\times(2{,}000)^2\times1\times10^{-4}}{50{,}000\times0.02}=0.75 \text{ day.}$$

If the wells are arranged in a line at an angle to the boundary, from equation 18

$$r_b^2=r_2^2+r_a^2+2r_ar_2\cos\alpha$$

$$r_b^2=(150)^2+(2{,}000)^2+2\times2{,}000\times150\times0.866=4{,}542{,}000$$

$$r_b=2{,}130 \text{ feet.}$$

Thus, from equation 41

$$t=\frac{1.87\times(2{,}130)^2\times1\times10^{-4}}{50{,}000\times0.02}=0.85 \text{ day.}$$

It must be remembered that the total permissible drawdown in the pumped wells must be specified before the equations presented in this paper can be applied. This may be done on the basis of (a) past experience with other wells tapping a particular formation; (b) physical limitations of the aquifer, such as depth, thickness, or areal extent; and (c) limitations imposed by pumping equipment, such as pumping by suction, setting of bowls, or horsepower of motors. The maximum permissible drawdown having been estimated, the accompanying equations then provide a convenient means of determining the proper spacing of pumped wells.

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