Effect of Reservoir Storage on Peak Flow

GEOLOGICAL SURVEY WATER-SUPPLY PAPER 1580-C

U.S. DEPARTMENT OF THE INTERIOR
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By WILLIAM D. MITCHELL

FLOOD HYDROLOGY

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ABSTRACT

For observation of small-basin flood peaks, numerous crest-stage gages now are operated at culverts in roadway embankments. To the extent that they obstruct the natural flood plains of the streams, these embankments serve to create detention reservoirs, and thus to reduce the magnitude of observed peak flows. Hence, it is desirable to obtain a factor, I/O, by which the observed outflow peaks may be adjusted to corresponding inflow peaks. The problem is made more difficult by the fact that, at most of these observation sites, only peak stages and discharges are observed, and complete hydrographs are not available.

It is postulated that the inflow hydrographs may be described in terms of Q, the instantaneous discharge; A, the size of drainage area; P_e, the amount of rainfall excess; H, the time from beginning of rainfall excess; D, the duration of rainfall excess; and T and k, characteristic times for the drainage area, and indicative of the time lag between rainfall and runoff. These factors are combined into the dimensionless ratios (QT/AP_e), (H/T), (k/T), and (D/T), leading to families of inflow hydrographs in which the first ratio is the ordinate, the second is the abscissa, and the third and fourth are distinguishing parameters.

Sixteen dimensionless inflow hydrographs have been routed through reservoir storage to obtain 139 corresponding outflow hydrographs. In most of the routings it has been assumed that the storage-outflow relation is linear; that is, that storage is some constant, K, times the outflow. The existence of nonlinear storage is recognized, and exploratory nonlinear routings are described, but analyses and conclusions are confined to the problems of linear storage.

Comparisons between inflow hydrographs and outflow hydrographs indicate that, at least for linear storage,

$$I/O=f(k/T,\ D/T,\ K/T)$$

in which I and O are, respectively, the magnitudes of the inflow and the outflow peaks, and T, k, D, and K are as defined above. Diagrams are presented to show the functional relation indicated by the foregoing equation.

INTRODUCTION

Comprehensive programs of observations of small-basin flood peaks have been inaugurated in many States. Practical considerations have dictated that many observations be made at culverts situated in embankments which tend to pond flood runoff in the reach upstream from the culvert. Observations at such points should be classified as outflow peaks, and if these peaks are materially affected by embankment storage, they should be adjusted to corresponding inflow peaks before being used as a basis for any regional study of small-basin floods.

The problem of adjustment is complicated by the fact that, at most observation sites, complete hydrographs are not available; only the peak stage and discharge being observed. Thus, the usual methods of flood routing cannot be used to determine inflow from outflow. Furthermore, reservoir routing in the upstream direction—from outflow to inflow—is difficult, time consuming, and generally unsatisfactory for determination of inflow peaks. The problem, then, simply stated, is this: How can an outflow peak be transformed to the corresponding inflow peak without the use of upstream routing?

The effect of embankment storage in reducing the peak discharge is illustrated by table 1, which has been prepared from observations on a small stream draining 0.9 square mile near Paxton, Ill. The ratio of inflow peak to outflow peak is shown for the existing 8-foot culvert and for hypothetical culverts with similar flow characteristics and assumed spans of 4 and 16 feet.

TABLE 1.—*Comparison of peak inflow and peak outflow discharges as related to size of culvert for a small stream near Paxton, Ill.*

Storm	Inflow discharge, in cfs	Size of culvert, in feet			Size of culvert, in feet		
		4	8	16	4	8	16
		Outflow discharge, in cfs			Ratio—Inflow/Outflow		
May 28, 1956, a.m.	118	88.6	112	118	1.33	1.05	1.00
May 27, 1956, a.m.	146	93.8	136	146	1.56	1.07	1.00
1 in. excess in 15 min [1]	165	97.0	------	------	1.70	------	------
May 27, 1956, p.m.	228	105	178	226	2.17	1.28	1.01
2 in. excess in 30 min [1]	325	--------	220	------	------	1.48	------
4 in. excess in 2 hr [1]	568	--------	------	464	------	------	1.22

[1] Computed from unit hydrographs derived from observed data.

The relations between outflow peaks and inflow peaks depend, primarily, on two classes of hydrologic factors, namely, those which describe the inflow hydrograph, and those which describe the storage-outflow relation. In this report, the characteristics of the inflow graph have been considered in detail, and on a base sufficiently broad as to include most, if not all, of the types of inflow that may be expected from culvert-size drainage areas. With respect to storage-outflow relations, the coverage is broad, but is not complete. Storage-outflow relations may be considered to fall into two major subclasses, namely, linear and nonlinear. Linear storage, which is applicable to many reservoirs, has been treated in detail; but owing to the vastly more tedious nature of the necessary computations, nonlinear storage has been touched upon only lightly.

The investigation includes a general appraisal of basic relations as derived from theoretical considerations and as obtained from application of field observations. Solution of the problem requires use of hydrologic parameters, such as amount and duration of precipitation excess and time of basin lag or runoff concentration which in many cases will be unavailable or at least not readily available. This will tend to limit application for the originally intended purpose. As progress is made on developing basin investigations which study in greater detail the flood hydrograph as influenced by natural factors and by the works of man, the results of this investigation will be put to use.

INFLOW-OUTFLOW-STORAGE RELATIONS

The basic relation between inflow, outflow, and storage is expressed by the equation of continuity, which for an incompressible fluid such as water, may be written:

$$I=O+\Delta S/\Delta t \qquad (1)$$

that is, the inflow, I, is equal to the outflow, O, plus the change of storage, $\Delta S/\Delta t$. If the time increment, $\Delta t=t_2-t_1$, be made sufficiently small, the values of I and O may be considered to be the average of their respective values at times t_1 and t_2, or $(I_1+I_2)/2$, and $(O_1+O_2)/2$. Further, ΔS may be expressed as the difference in storage volume at times t_2 and t_1, or $\Delta S=S_2-S_1$. Equation 1 may then be expressed as

$$O_2=(2S_1/\Delta t)-O_1-(2S_2/\Delta t)+(I_1+I_2). \qquad (2)$$

Thus, the value of O_2, the outflow at the end of the increment of time, Δt, may be computed from the following information:

1. An inflow hydrograph, from which to determine values of I_1 and I_2 for successive times at intervals of Δt,

2. A storage curve, or storage relation, from which to determine values of S_1 and S_2 for successive times at intervals of Δt, and

3. The value of O_1, the outflow at the beginning of the increment of time, Δt.

To obtain a value of O_1, it is only necessary that the computations be started at a time for which the outflow is known. At the beginning of a flood period, the outflow generally is equal to the inflow. Unless the storage values are extremely large, a large error in the assumed outflow at the beginning of the flood will have little, if any, effect upon the computed value of outflow peak discharge. For this reason, further discussion of O_1 is unnecessary. However, the outflow-storage relation and the inflow hydrograph must be given close attention.

It will be noted that equation 2 has been set up to compute an outflow hydrograph from a given inflow hydrograph. This, at first glance, appears to be incompatible with the requirements of the basic problem, namely, that an inflow peak discharge be determined from an observed outflow peak discharge. It should be noted, however, that even if equation 2 were solved for I_2, rather than for O_2, it still would be impossible to compute the inflow hydrograph, owing to the fact that observed data are limited to stage and discharge of the outflow peak; the complete outflow hydrograph normally is not observed. Obviously, to use equation 2, either an inflow hydrograph or an outflow hydrograph must be assumed. Of these two alternatives, the assumption of an inflow hydrograph is simpler and more satisfactory, inasmuch as it is unaffected by the embankment storage, and hence is more closely related to the characteristics of the drainage area.

OUTFLOW-STORAGE CONSIDERATIONS

From a topographic survey of the channel and flood plain upstream from the embankment, a stage-storage curve may be derived. The assumption is made that the reservoir surface is a horizontal plane, and that for any stage the storage capacity is the volume between this plane and the ground surface. This assumption neglects the possible existence of wedge storage above the assumed horizontal plane due to slope of the water surface in the reservoir. It seems likely, however, that the principal effect of wedge storage would be to alter to some slight extent the position, or possibly the slope, of the stage-storage curve, and would not otherwise affect either the methods or the results of this study.

For convenience in use with equation 2, storage should be related to outflow, rather than to stage. This is readily accomplished by developing a relation between stage and outflow. For various values of stage, a point is read from both the stage-storage curve and the stage-discharge curve, and the value of outflow discharge is plotted as abscissa against the value of storage as ordinate, thereby obtaining points to define the outflow-storage relation.

A generalized expression for the shape of the outflow-storage curve may be stated as

$$S=K\,O^x \tag{3}$$

in which the values of K and x may be variable, but for many sites they appear to be nearly constant. The magnitude of K is, of course, a function of the relation between the capacity of the reservoir and the capacity of the outlet. For a large reservoir served by a small outlet, the value of K will be large. For a small reservoir served by a large outlet, the value of K will be small, and will be zero when the culvert becomes so large as to have no effect upon the passage of the flood wave. The value of x is a function of the relative slopes of the stage-discharge and the stage-storage curves. The minimum value of x is about 0.67, and would occur for outflow at critical depth from a reservoir with vertical side walls. The maximum possible value of x is indeterminate. In general, the value of x will increase as the side slopes of the reservoir become flatter. A common assumption in the solution of many reservoir routing problems is that

$$S=KO \tag{4}$$

and that the storage is said to be linear storage.

Nonlinear storage is a condition that occurs with sufficient frequency to warrant careful consideration, but the required computations are extremely tedious and time consuming. The present study is directed primarily to the consideration of linear storage, and nonlinear storage is touched upon only briefly.

ROUTING COMPUTATIONS

In equation 4, S has the dimension of cubic feet, and O has the dimension of cubic feet per second. It follows, therefore, that K must have the dimension of time. In the following section dealing with inflow hydrographs, it will be found that all time factors pertaining to the hydrographs are expressed as a ratio to T, a characteristic time for the drainage area, and indicative of the time lag between rainfall and runoff. Routing computations may be greatly simplified by also expressing K as a ratio to T. Thus, if $K/T=r$, then $S=rTO$.

In the generalized routing expression, equation 2, Δt also has the dimension of time. Here, again, routing computations will be facilitated by expressing Δt as a ratio to T. Let $\Delta t/T=p$. Then

$$2S/\Delta t=2rTO/pT=(2r/p)O.$$

Equation 2 then becomes

$$O_2=(2r/p)O_1-O_1-(2r/p)O_2+(I_1+I_2)$$

or

$$O_2(2r/p+1)=O_1(2r/p-1)+(I_1+I_2). \tag{5}$$

For any specific problem with linear storage, r will have a fixed specific value, such as $r=K/T=1.00$, and p will have a fixed specific value, such as $p=\Delta t/T=0.02$; hence, $2r/p$ will have a specific value, such as $2 \times 1.00/0.02 = 100$, and equation 5 will assume the specific form

$$O_2 = (99O_1 + I_1 + I_2)/101. \tag{5a}$$

It should be noted that this simple form of the routing equation is appropriate only when storage is a linear function of outflow, and that the constants in the equation will vary with the values of K and Δt. For example, if K becomes $0.5T (r=0.5)$ while Δt remains $0.02T (p=0.02)$, then

$$O_2 = (49O_1 + I_1 + I_2)/51. \tag{5b}$$

Values of K used in this report cover a considerable range, but the value of Δt has been kept the same for all computations. As stated on page C–3, Δt should be small enough so that the average values of I and O during the interval will be virtually the same as the average of their respective values at the beginning and end of the interval. For this purpose alone a time increment of $0.1T$ might be sufficiently small. However, in order that there may be no doubt as to the accuracy of the computations, and also to provide better information concerning time relationships in the outflow hydrographs, the value of Δt has been taken as $0.02T$ for all routing computations.

INFLOW HYDROGRAPH

The ratio between inflow discharge and outflow discharge, I/O, at the peak may be a function of volume of inflow, but more generally it is a function of the shape of the inflow hydrograph. For a flood of given volume passing through storage of a given volume and controlled by a given stage-outflow relation, the value of I/O may be only slightly greater than unity if the inflow hydrograph has a sufficiently flat peak; but I/O may be much greater if the peak of the inflow graph is sufficiently sharp. This is illustrated by figure 1. Each of the four hydrographs has a runoff volume of 1 inch. The I–1 inflow graph, when modified by a particular storage and outflow, becomes the O–1 outflow graph; the I–2 inflow, when modified by the same storage and outflow, becomes the O–2 outflow. In the first example the ratio between inflow and outflow is 1.86; in the second, 1.59.

If the results of this study are to cover the range of conditions normally met in the field, it will be necessary to include not only a wide range of storage and outflow conditions, but also an appropriate range of magnitudes and shapes of the inflow hydrograph. The possible variations in both inflow graphs and in storage-outflow relations seems, at first glance, to be so great as to preclude study of any significant portion of them. But further investigation indicates that, although the total number of possible combinations may be almost

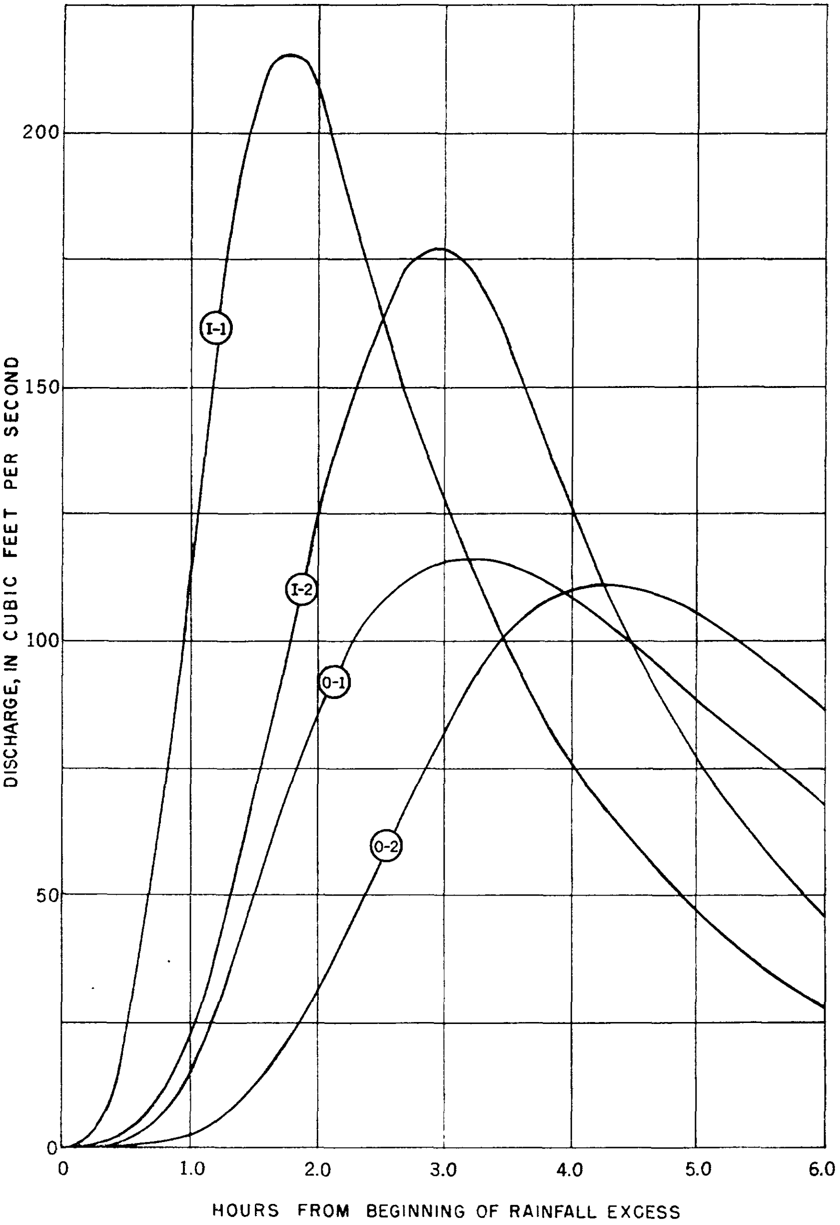

FIGURE 1.—Typical inflow and outflow hydrographs.

infinite, the results obtained from the study of a few may be arranged in such a way that interpolations may be made for a large part of the others. This simplification is brought about by the use of dimensional analysis.

INITIAL PARAMETERS AND RATIOS

It was postulated, initially, that the graph of surface runoff from any drainage area might be described by the relation

$$Q=f(A,P_e,H,D,T)$$

in which Q is the instantaneous discharge, in cfs (cubic feet per second)

A is the size of drainage area, in square miles

P_e is the amount of rainfall excess, in inches

H is the time from beginning of rainfall excess, in hours

D is the duration of rainfall excess, in hours

and T is a characteristic time for the drainage area, indicative of the time lag between rainfall and runoff, in hours.

Dimensional analysis suggests that these characteristics be combined into three dimensionless ratios, QT/AP_e, H/T, and D/T, indicating a family of inflow hydrographs for which the first ratio is the ordinate; the second, the abscissa; and the third, the family parameter.

SHAPE OF THE INFLOW HYDROGRAPH

Dimensional analysis provides no clue as to the shape of the inflow hydrographs, but hydrologic considerations suggest that they might

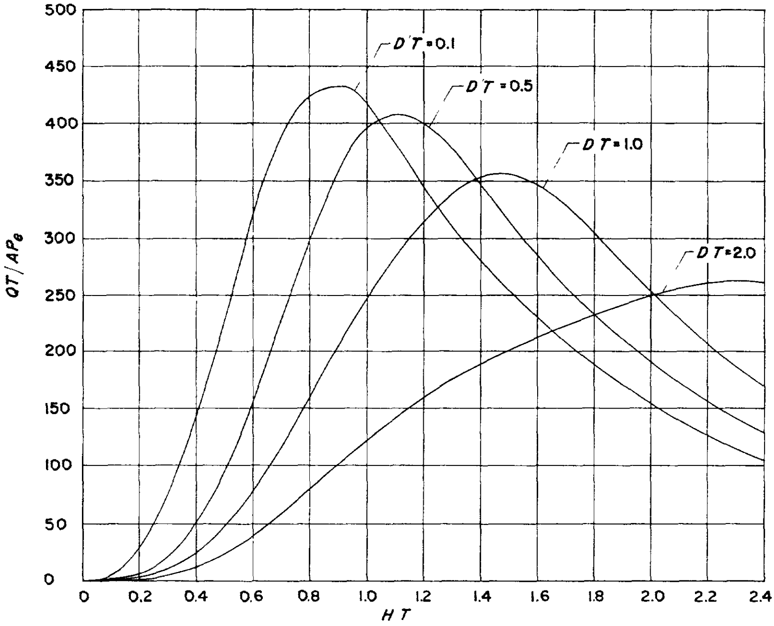

FIGURE 2.—Typical dimensionless hydrographs.

appear as shown in figure 2. It should be noted that, although only four curves are shown here, the figure represents an endless number of graphs. The dependent variables, Q and H, will be determined when specific values are substituted for the independent variables, A, P_e, D, and T; and changing any one or more of these independent variables will produce a different hydrograph. These substitutions, however, need not be made until later, after routing studies have been completed. Thus, by making a study of a few graphs and arranging the results so that they are applicable to a wide range of values of the independent variables, information will become available for a very large number of inflow hydrographs.

The graphs of figure 2 are mathematical curves and are derived from the further postulation that the appropriate shape of the graph may be obtained by routing an isosceles hydrograph [1] through appropriate linear storage.

Consider the isosceles hydrograph shown as figure 3. This triangle is completely described by the values of T, A, and P_e. Since AP_e represents the volume of runoff from a given storm, it also must represent the area of the triangle. Dividing this area by the length of the base, T, gives the average ordinate, which, of course, will be half of the maximum ordinate. Letting the maximum ordinate be represented by h,

$$h=2AP_e/T$$

If A is expressed in square miles, P_e in inches, and T in hours, this relation becomes:

$$h=2\times(5{,}280)^2\ \text{ft}^2/\text{sq mi}\times 1/12\ \text{ft/in}/3{,}600\ \text{sec/hr}$$
$$h=1{,}290.667\ \text{ft}^3/\text{sec}\times\text{hr/sq mi}\times A\ \text{sq mi}\times P_e\ \text{in}/T\ \text{hr}$$
$$h=(1{,}290.667\ AP_e/T)\ \text{ft}^3/\text{sec}.$$

Thus, the ordinates of the isosceles hydrograph increase uniformly from zero at $H/T=O$ to 1,290.667 AP_e/T at $H/T=0.5$, then decrease uniformly to zero at $H/T=1.0$. Or, to state the matter more appropriately for computing the dimensionless ordinates for the isosceles hydrograph, the maximum value of QT/AP_e, at $H/T=0.5$, is 1,290.667, and decreases uniformly to zero at $H/T=O$ and at $H/T=1.0$. A complete tabulation of the ordinates, at intervals of $H/T=0.02$, is included in table 2.

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[1] A thesis entitled "Synthetic Unit Hydrographs Based on Triangular Inflow," by James C. I. Dooge, submitted to the University of Iowa in 1956, contained information helpful to the author.

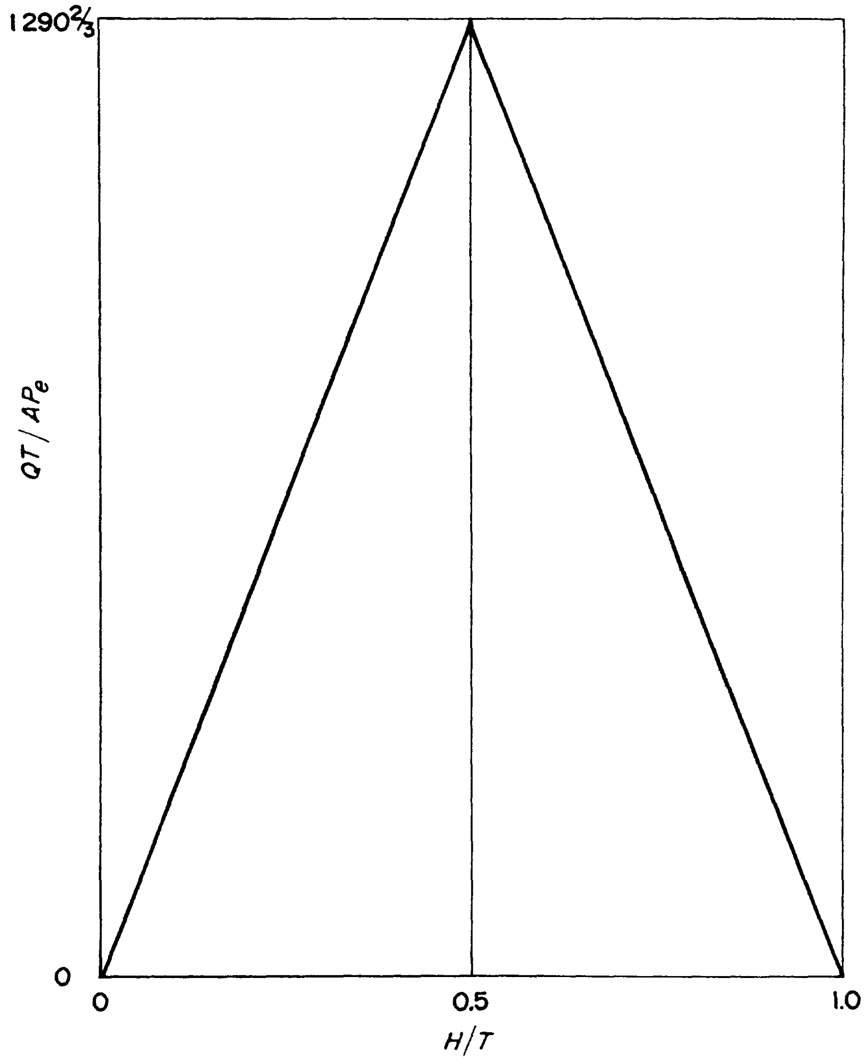

FIGURE 3.—The isosceles dimensionless hydrograph.

The isosceles hydrograph is assumed to represent an inflow hydrograph for a storm of instantaneous duration with no channel storage. But each actual storm has a finite duration, D, or finite value of the ratio D/T. It is necessary to transform the graph representative of the instantaneous storm to a graph having appropriate values of D/T, such as 0.5, 1.0, and 1.5.

TABLE 2.—*Section of computations for typical inflow hydrographs, using* S=*1.0* TO

[Inst hgh, instantaneous hydrograph; hgh, hydrograph]

$\frac{Hr}{T}$ (1)	$\frac{QT}{AP_e}$ (2)	$\frac{QT}{AP_{e1}}-\frac{QT}{AP_{e2}}$ (3)	Inst hgh (101,99) (4)	0.02T-hgh (5)	Summation 0.02T-hgh (6)	Summation lag 0.1T (7)	0.1T-hgh (8)	Summation lag 0.5T (9)	0.5T-hgh (10)	Summation lag 1.0T (11)	1.0T-hgh (12)
0.50	1,290.667	2,529.707	274.965	264.966	2,385.062	1,250.496	226.913	0	95.402	0	47.701
.52	1,239.040	2,529.707	294.567	284.766	2,669.828	1,440.674	245.831	.256	106.783	0	53.397
.54	1,187.413	2,426.453	312.758	303.662	2,973.490	1,648.554	264.987	1.528	118.878	0	59.470
.56	1,135.787	2,323.200	329.567	321.162	3,294.652	1,874.808	283.969	4.820	131.593	0	65.893
.58	1,084.160	2,219.947	345.021	337.294	3,631.946	2,120.096	302.370	11.113	144.843	0	72.639
.60	1,032.533	2,116.693	359.146	352.084	3,984.030	2,385.062	319.794	21.371	158.506	0	79.681
.62	980.907	2,013.440	371.969	365.558	4,349.588	2,669.828	335.952	36.537	172.522	0	86.992
.64	929.280	1,910.187	383.516	377.742	4,727.330	2,973.490	350.768	57.537	186.792	0	94.547
.66	877.653	1,806.933	393.812	388.664	5,115.994	3,294.652	364.268	85.277	201.229	0	102.320
.68	826.027	1,703.680	402.882	398.347	5,514.341	3,631.946	376.479	120.645	215.748	0	110.287
.70	774.400	1,600.427	410.750	406.816	5,921.157	3,984.030	387.425	164.514	230.266	0	118.423
.72	722.773	1,497.173	417.440	414.095	6,335.252	4,349.588	397.133	217.738	244.700	0	126.705
.74	671.147	1,393.920	422.975	420.208	6,755.460	4,727.330	405.626	281.153	258.972	0	135.109
.76	619.520	1,290.667	427.378	425.176	7,180.636	5,115.994	412.928	355.580	273.002	0	143.613
.78	567.893	1,187.413	430.672	429.025	7,609.661	5,514.341	419.064	441.823	286.714	0	152.193
.80	516.267	1,084.160	432.878	431.775	8,041.436	5,921.157	424.056	540.671	300.031	0	160.829
.82	464.640	980.907	434.018	433.448	8,474.884	6,335.252	427.926	652.896	312.880	0	169.498
.84	413.013	877.653	434.113	434.066	8,908.950	6,755.460	430.698	779.256	325.188	0	178.179
.86	361.387	774.400	433.184	433.648	9,342.598	7,180.636	432.392	920.494	336.884	0	186.852
.88	309.760	671.147	431.251	432.218	9,774.816	7,609.661	433.031	1,077.336	347.899	0	195.496
.90	258.133	567.893	428.334	429.792	10,204.608	8,041.436	432.634	1,250.496	358.164	0	204.092
.92	206.507	464.640	424.453	426.394	10,631.002	8,474.884	431.224	1,440.674	367.613	0	212.620
.94	154.880	361.387	419.626	422.040	11,053.042	8,908.950	428.818	1,648.554	376.180	0	221.061
.96	103.253	258.133	413.872	416.749	11,469.791	9,342.598	425.439	1,874.808	383.799	0	229.396
.98	51.627	154.880	407.210	410.541	11,880.332	9,774.816	421.103	2,120.096	390.409	0	237.607
1.00	0	51.627	399.658	403.434	12,283.766	10,204.608	415.832	2,385.062	395.948	0	245.675
1.02	0	0	391.744	395.701	12,679.467	10,631.002	409.693	2,669.828	400.386	.256	253.584
1.04	0	0	383.987	387.866	13,067.333	11,053.042	402.858	2,973.490	403.754	1.528	261.316
1.06	0	0	376.383	380.185	13,447.518	11,469.791	395.545	3,294.652	406.115	4.820	268.854
1.08	0	0	368.930	372.657	13,820.175	11,880.332	387.969	3,631.946	407.529	11.113	276.181
1.10	0	0	361.624	365.277	14,185.452	12,283.766	380.337	3,984.030	408.057	21.371	283.282

Transformations normally are effected by means of the summation curve, as follows: Prepare a summation curve for any available hydrograph, whose D/T value is x. Prepare a second summation curve identical to the first, but lagged by a time interval equal to the desired storm duration, $D/T=y$. Subtract the ordinates of the second curve from those of the first curve, and divide the remainders by the value of the ratio y/x. Unfortunately, this method cannot be applied directly in the case where the only available inflow graph is one for instantaneous rainfall, since here the ratio y/x becomes infinity. However, negligible error will result in computing the inflow hydrograph for a finite time of short duration such as $\Delta t=0.02T$, merely by averaging the successive ordinates, at intervals of Δt, of the instantaneous inflow graph. After the graph for $D/T=\Delta t$ has been obtained in this way, it is used as the basic hydrograph for further D/T transformations as described above.

The graphs of figure 2 were derived from the isosceles hydrograph by the methods just described, using as the appropriate linear storage the value $S=1.0TO$. A part of the pertinent computations is included here as table 2. Columns 1 and 2 are a listing of the coordinates of the isosceles hydrograph at intervals of $\Delta t=0.02T$. Since $r=1.0$, $(S=1.0TO)$, and $\Delta t=0.02T$ it follows then, $2r/p=100$, and the routing equation assumes the specific form of equation 5a. Any given line in column 3 is the sum of column 2 on that line and column 2 on the preceding line, thus providing the (I_1+I_2) terms of the routing relation, equation 5a. Column 4 is the instantaneous hydrograph and is obtained by taking a value from column 4, multiplying it by 99, then adding the value of column 3 of the following line, and dividing the sum by 101. (*See* equation 5a.) The quotient is entered as the value of the instantaneous hydrograph on the second line. The $0.02T$-hydrograph, column 5, is obtained by averaging the value of instantaneous hydrograph on any line with the instantaneous value from the preceding line. The continuous summation of the $0.02T$-hydrograph is given in column 6. In column 7 the continuous summation, $0.02T$-hydrograph, has been lagged $0.10T$. Column 8 is the $0.1T$-hydrograph, obtained by taking the difference between the two columns immediately preceding, and dividing by the ratio of the D/T's involved—in this case, $0.1/0.02=5$.

Similar procedure was followed for columns 9 through 12, for the $0.5T$ and $1.0T$-hydrographs. That is, for column 9 the summation for $0.02T$-hydrograph has been lagged $0.5T$ and the $0.5T$-hydrograph (column 10) was obtained by taking the difference between columns 6 and 9, then dividing by 25, $(0.50/0.02=25)$.

AN ADDITIONAL PARAMETER

There is, of course, no compelling argument for expecting that all inflow hydrographs would conform to the family of curves presented as figure 2. Hydrologic considerations indicated this as a likely possibility, but there are others. Hence, an important part of this study has been the determination, from observed hydrographs, of the variations from this assumption.

It was found that some hydrographs could, indeed, be reduced to this form, but that variations did occur. To indicate the nature and range of the variations, let attention be concentrated on one of the curves of figure 2, namely, that designated $D/T=0.1$. It is replotted as one of the graphs of figure 4. This new figure is another family of dimensionless graphs, but for all of these there is a common value of D/T, namely 0.1. The curve from figure 2 has been drawn heavier than the others, and is now identified by the ratio $k/T=1.0$. Present experience indicates that this curve may vary in shape throughout the range shown in figure 4, with values of k/T from about 0.3 to about 1.2. In other words, it is necessary to add another parameter to those previously considered in order to provide a generalized description of inflow hydrographs. This new factor is assumed to describe the effect of miscellaneous storage of the drainage basin, that is, water storage in ponds, channels, and minor depressions upstream from the principal reservoir. This element of storage, reflected in the inflow hydrograph, is distinguished from the reservoir storage, reflected only in the outflow hydrograph, by use of the symbol k, rather than K.

RANGE OF SELECTED INFLOW HYDROGRAPHS

In the earlier phases of this study attention was concentrated upon the family of inflow hydrographs for which k/T is equal to 1.0. Curves for eight different values of D/T, as shown in table 3, were computed, analyzed, and routed through various degrees of reservoir storage. Later, when it was determined that k/T must be regarded as a variable, two additional families of curves were added, each with a more limited number of D/T values, but still sufficient to allow for appropriate correlations. These additional graphs also are included in table 3.

TABLE 3.—*Parameters of selected inflow hydrographs*

k/T	D/T							
1.0	(1)	0.1	0.3	0.5	0.7	1.0	1.5	2.0
.7	------	.1	--------	.5	--------	1.0	1.5	------
.3	------	.1	--------	.5	--------	1.0	1.5	------

[1] Instantaneous.

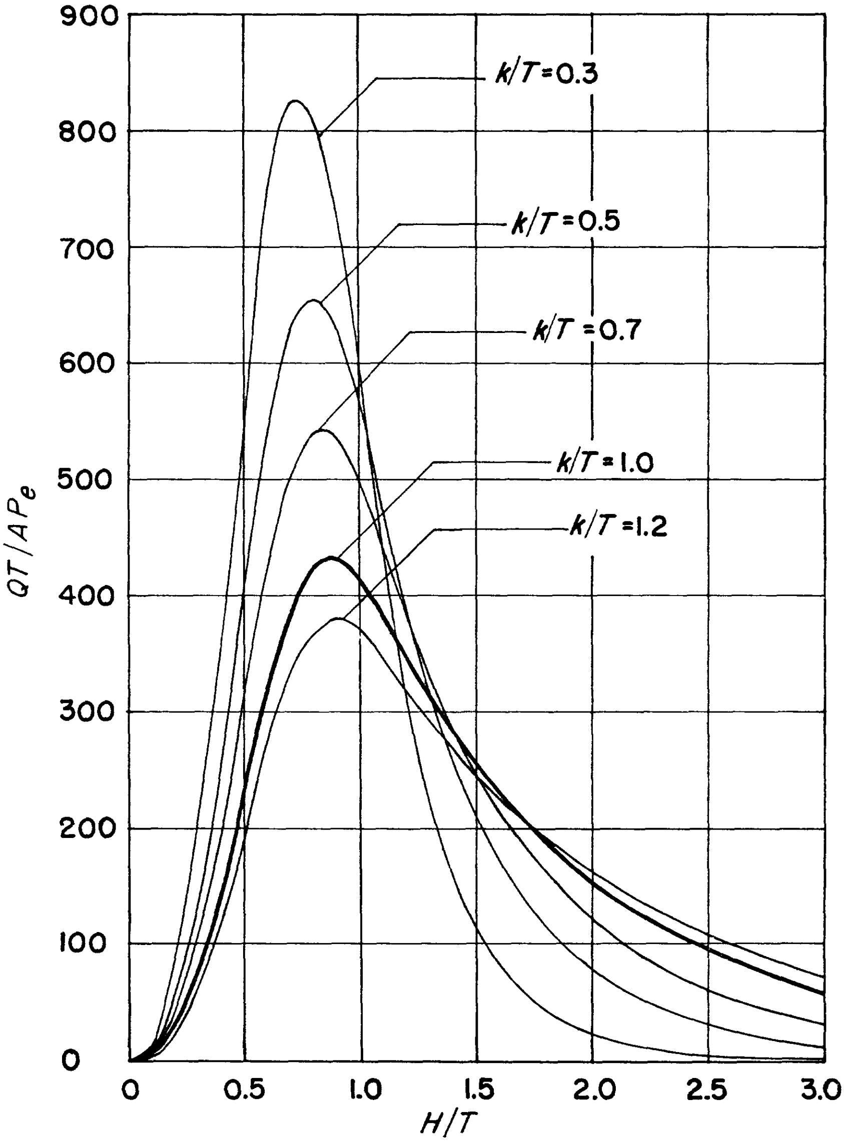

FIGURE 4.—Possible variation in inflow graphs for constant value (0.1) of D/T.

One of the principal items commonly selected for comparison between inflow and outflow hydrographs is the value of the peak discharges. For linear storage, the ratio between peak inflow and peak outflow, I/O, is dependent upon the relative, but not the absolute, ordinates of the inflow hydrograph. Hence, it is sufficient to assume that, for all inflow graphs, the value of AP_e/T is unity. For nonlinear storage the relation of I/O also is a function of the absolute magnitude of the inflow ordinates, and other values, in addition to 1.0, will need to be used for AP_e/T, thus greatly increasing the number of inflow hydrographs needed for such a study. However, the development of these data are beyond the scope of the present study.

SUMMARY OF ROUTINGS

Using routing relations of the type shown by equations 5a and 5b, each of the inflow hydrographs listed in table 3 has been routed through each of eight different reservoirs, namely, those for which the value of K/T (or r) is 0.1, 0.2, 0.3, 0.5, 0.7, 1.0, 1.5, and 2.0. In addition, a few exploratory routings through nonlinear storage also have been made. Thus, the total of 16 inflow hydrographs has been expanded to a total of 139 outflow hydrographs.

LINEAR STORAGE

The pertinent characteristics of these hydrographs, both inflow and outflow, for linear storage have been summarized in tables 4–6. Each table pertains to a single family of graphs, identified in the table heading by the value of k/T. Within the table, each block of data pertains to a particular member of the family, identified in the first column by the value of D/T. Within each block, the first line pertains to the inflow hydrograph, identified by the fact that $K/T=O$; the succeeding lines pertain to the several outflow hydrographs, arranged in order of increasing magnitude of reservoir storage (K/T). For each line, there is shown the time (t_p/T) and the magnitude (QT/AP_e) of the peak discharge, and the time (t_v/T) corresponding to the center of volume of the hydrograph. In the last column of each block is shown the ratio of inflow peak to outflow peaks (I/O).

TABLE 4.—*Results of routing inflow hydrographs through linear storage* (k/T=*1.0*)

D/T	K/T	t_p/T	QT/AP_e	t_s/T	I/O	D/T	K/T	t_p/T	QT/AP_e	t_s/T	I/O
(1)	0.0	0.84	434.1	1.214	--------	0.7	0.0	1.24	387.8	1.584	--------
(1)	.1	.94	421.1	1.319	1.031	.7	.1	1.34	379.5	1.688	1.022
(1)	.2	1.02	391.3	1.429	1.109	.7	.2	1.44	360.2	1.796	1.077
(1)	.3	1.10	360.1	1.537	1.205	.7	.3	1.52	338.2	1.902	1.147
(1)	.5	1.26	309.7	1.740	1.402	.7	.5	1.66	297.6	2.101	1.303
(1)	.7	1.38	272.6	1.929	1.592	.7	.7	1.78	264.9	2.286	1.464
(1)	1.0	1.54	232.5	2.187	1.867	.7	1.0	1.94	227.9	2.545	1.702
(1)	1.5	1.76	188.6	2.562	2.302	.7	1.5	2.14	186.0	2.943	2.085
(1)	2.0	1.92	159.7	2.962	2.718	.7	2.0	2.30	158.1	3.318	2.453
0.1	0.0	0.88	433.0	1.264	--------	1.0	0.0	1.48	355.6	1.754	--------
.1	.1	.98	420.0	1.369	1.031	1.0	.1	1.56	350.1	1.857	1.016
.1	.2	1.08	390.4	1.480	1.109	1.0	.2	1.66	336.6	1.962	1.056
.1	.3	1.16	359.6	1.588	1.204	1.0	.3	1.74	319.8	2.066	1.112
.1	.5	1.30	309.4	1.790	1.399	1.0	.5	1.88	286.2	2.262	1.242
.1	.7	1.44	272.4	1.977	1.590	1.0	.7	1.98	257.4	2.446	1.382
.1	1.0	1.60	232.4	2.237	1.863	1.0	1.0	2.12	223.3	2.703	1.592
.1	1.5	1.80	188.5	2.636	2.297	1.0	1.5	2.23	183.5	3.100	1.938
.1	2.0	1.96	159.6	3.012	2.713	1.0	2.0	2.48	156.4	3.474	2.274
0.3	0.0	0.98	424.4	1.368	--------	1.5	0.0	1.88	305.3	2.046	--------
.3	.1	1.10	411.7	1.473	1.031	1.5	.1	1.98	302.5	2.147	1.009
.3	.2	1.18	384.4	1.582	1.104	1.5	.2	2.04	295.3	2.248	1.034
.3	.3	1.28	355.6	1.690	1.193	1.5	.3	2.12	285.5	2.348	1.069
.3	.5	1.42	307.4	1.892	1.381	1.5	.5	2.24	262.9	2.539	1.161
.3	.7	1.54	271.1	2.079	1.565	1.5	.7	2.34	240.9	2.719	1.267
.3	1.0	1.70	231.7	2.338	1.832	1.5	1.0	2.48	212.8	2.972	1.435
.3	1.5	1.90	188.1	2.738	2.256	1.5	1.5	2.66	177.6	3.366	1.719
.3	2.0	2.08	159.4	3.113	2.662	1.5	2.0	2.80	152.6	3.739	2.001
0.5	0.0	1.10	408.1	1.474	--------	2.0	0.0	2.30	262.8	2.340	--------
.5	.1	1.22	397.4	1.579	1.027	2.0	.1	2.40	261.2	2.440	1.006
.5	.2	1.32	373.8	1.688	1.092	2.0	.2	2.46	257.3	2.538	1.021
.5	.3	1.40	348.3	1.795	1.172	2.0	.3	2.52	251.8	2.635	1.044
.5	.5	1.54	303.4	1.996	1.345	2.0	.5	2.64	237.5	2.821	1.107
.5	.7	1.66	268.6	2.182	1.519	2.0	.7	2.72	221.9	2.998	1.184
.5	1.0	1.82	230.1	2.441	1.774	2.0	1.0	2.86	199.8	3.248	1.315
.5	1.5	2.02	187.3	2.840	2.179	2.0	1.5	3.02	170.0	3.639	1.546
.5	2.0	2.18	158.8	3.215	2.570	2.0	2.0	3.16	147.6	4.000	1.780

[1] Instantaneous.

TABLE 5.—*Results of routing inflow hydrographs through linear storage* (k/T=*0.7*)

D/T	K/T	t_p/T	QT/AP_e	t_s/T	I/O	D/T	K/T	t_p/T	QT/AP_e	t_s/T	I/O
0.1	0.0	0.84	543.8	1.066	--------	1.0	0.0	1.42	425.0	1.559	--------
.1	.1	.94	525.1	1.171	1.036	1.0	.1	1.50	418.0	1.661	1.017
.1	.2	1.02	484.6	1.277	1.122	1.0	.2	1.60	400.6	1.762	1.061
.1	.3	1.10	442.8	1.378	1.228	1.0	.3	1.66	379.0	1.858	1.121
.1	.5	1.22	374.4	1.565	1.452	1.0	.5	1.78	336.3	2.040	1.264
.1	.7	1.32	324.9	1.737	1.674	1.0	.7	1.88	299.9	2.208	1.417
.1	1.0	1.44	272.4	1.977	1.996	1.0	1.0	1.98	257.4	2.446	1.651
.1	1.5	1.60	216.4	2.351	2.513	1.0	1.5	2.12	208.3	2.817	2.040
.1	2.0	1.72	180.6	2.708	3.011	1.0	2.0	2.24	175.5	3.172	2.422
0.5	0.0	1.06	507.3	1.278	--------	1.5	0.0	1.80	350.2	1.846	--------
.5	.1	1.16	491.6	1.382	1.032	1.5	.1	1.90	346.9	1.946	1.010
.5	.2	1.26	458.6	1.486	1.106	1.5	.2	1.98	338.9	2.044	1.033
.5	.3	1.32	423.9	1.586	1.197	1.5	.3	2.04	327.5	2.139	1.069
.5	.5	1.44	364.1	1.771	1.393	1.5	.5	2.14	300.7	2.317	1.165
.5	.7	1.54	318.4	1.942	1.593	1.5	.7	2.24	274.5	2.484	1.276
.5	1.0	1.66	268.6	2.182	1.889	1.5	1.0	2.34	240.9	2.719	1.454
.5	1.5	1.82	214.4	2.555	2.366	1.5	1.5	2.48	199.0	3.088	1.760
.5	2.0	1.94	179.3	2.911	2.829	1.5	2.0	2.60	169.5	3.440	2.066

TABLE 6.—*Results of routing inflow hydrographs through linear storage* (k/T=*0.3*)

D/T	K/T	t_p/T	QT/AP_e	t_v/T	I/O	D/T	K/T	t_p/T	QT/AP_e	t_v/T	I/O
0.1	0.0	0.72	826.6	0.802	--------	1.0	0.0	1.26	560.5	1.274	--------
.1	.1	.84	785.3	.902	1.053	1.0	.1	1.36	550.1	1.373	1.019
.1	.2	.92	709.3	.994	1.165	1.0	.2	1.44	524.3	1.467	1.069
.1	.3	.96	637.3	1.078	1.297	1.0	.3	1.50	492.5	1.554	1.138
.1	.5	1.04	523.7	1.233	1.578	1.0	.5	1.60	430.6	1.713	1.302
.1	.7	1.10	442.8	1.378	1.867	1.0	.7	1.66	379.0	1.858	1.479
.1	1.0	1.16	359.6	1.588	2.299	1.0	1.0	1.74	319.8	2.066	1.753
.1	1.5	1.24	274.8	1.929	3.008	1.0	1.5	1.82	253.2	2.404	2.214
.1	2.0	1.30	223.0	2.269	3.707	1.0	2.0	1.90	209.6	2.738	2.674
0.5	0.0	0.96	743.3	1.008	--------	1.5	0.0	1.62	415.8	1.538	--------
.5	.1	1.06	712.7	1.108	1.043	1.5	.1	1.72	413.1	1.637	1.007
.5	.2	1.12	653.1	1.201	1.238	1.5	.2	1.78	405.1	1.733	1.026
.5	.3	1.18	593.3	1.286	1.253	1.5	.3	1.86	392.2	1.824	1.060
.5	.5	1.26	495.4	1.441	1.500	1.5	.5	1.96	359.9	1.989	1.155
.5	.7	1.32	423.9	1.586	1.753	1.5	.7	2.04	327.5	2.139	1.270
.5	1.0	1.40	348.3	1.795	2.134	1.5	1.0	2.12	285.5	2.348	1.456
.5	1.5	1.48	268.9	2.135	2.764	1.5	1.5	2.22	233.2	2.681	1.783
.5	2.0	1.54	219.4	2.474	3.388	1.5	2.0	2.28	196.5	3.013	2.116

NONLINEAR STORAGE

Exploratory investigations of routings through nonlinear storage have resulted in outflow hydrographs for which the pertinent characteristics are as shown in table 7.

TABLE 7.—*Results of routing inflow hydrographs through nonlinear storage*

k/T	D/T	AP_e/T	Inflow			Outflow-storage relation	Outflow		
			t_p/T	QT/AP_e	t_v/T		t_p/T	QT/AP_e	t_v/T
1.0	(1)	1.0	0.84	434.1	1.214	$S=0.01\ O^2$	1.88	166.8	2.624
1.0	(1)	2.0	.84	868.2	1.214	$S=0.01\ O^2$	2.12	257.9	3.250
1.0	1.0	1.0	1.48	355.6	1.754	$S=0.01\ O^2$	2.44	162.7	3.095
1.0	1.0	2.0	1.48	711.3	1.754	$S=0.01\ O^2$	2.68	253.9	3.712
2.0	.5	2.0	1.20	496.0	2.139	$S=0.01\ O^2$	2.98	214.0	4.106
1.0	(1)	1.0	0.84	434.1	1.214	$S=10\ O^{0.5}$	1.20	324.3	1.848
1.0	(1)	2.0	.84	868.2	1.214	$S=10\ O^{0.5}$	1.08	742.0	1.648
1.0	1.0	1.0	1.48	355.6	1.754	$S=10\ O^{0.5}$	1.82	300.4	2.385
1.0	1.0	2.0	1.48	711.3	1.754	$S=10\ O^{0.5}$	1.70	656.8	2.186
1.0	(1)	1.0	0.84	434.1	1.214	$S=7.2\times10^{-8}\ O^4$	1.58	223.5	1.941
1.0	(1)	2.0	.84	868.2	1.214	$S=7.2\times10^{-8}\ O^4$	2.00	296.6	2.704
1.0	1.0	1.0	1.48	355.6	1.754	$S=7.2\times10^{-8}\ O^4$	2.16	215.9	2.387
1.0	1.0	2.0	1.48	711.3	1.754	$S=7.2\times10^{-8}\ O^4$	2.56	291.4	3.124

[1] Instantaneous.

ANALYSIS

THREE-RATIO DIAGRAM FOR SINGLE VALUE OF *k*

The type of information given by tables 4–6 becomes more useful when presented in the form of diagrams, such as figure 5. In this diagram, the dimensionless ratio, I/O, has been plotted as ordinate against another dimensionless ratio, D/T, as abscissa. In any one of the tables, the last line of each block of data pertains to a common value of a third dimensionlesss factor, K/T, and thus these serve to define each of the several curves of the figure.

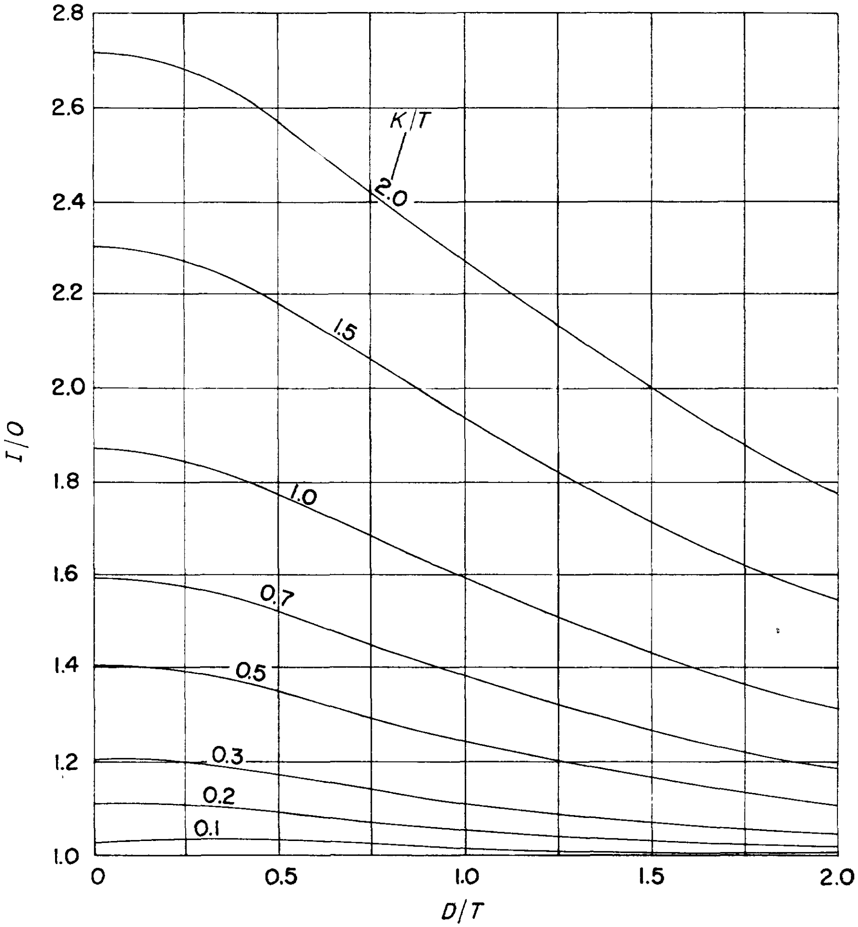

FIGURE 5.—*I/O* as a function of *D/T* and *K/T* (*k/T*=1.0).

Figure 5 has been prepared for the family of inflow hydrographs, $k/T=1.0$, and from this figure the correction factor, I/O, for adjusting an observed outflow peak to the corresponding inflow peak, may be determined, provided: (1) the storage is linear, (2) the inflow graph has a k/T ratio of 1.0, and (3) values are known for D, K, and T. D may be estimated from rainfall records; and K, from the slope of the outflow-storage curve. T may be determined by a formula involving the physiographic characteristics of the basin and is approximately equivalent to time of concentration.

To illustrate the use of this figure, suppose an outflow peak of 100 cfs is observed from an area for which T is 2 hours, D is 0.9 hour, and K is 1.4 hours. Then, $D/T=0.45$, and $K/T=0.70$; and interpolating from figure 5, $I/O=1.54$. Hence, the corresponding inflow peak is 154 cfs.

THE t_r/t_o RATIO

One of the disadvantages of diagrams such as figure 5 is that a separate diagram is required for each value of k, and even a series of such diagrams will provide solutions only for discrete, rather than continuous variations of k. The key to a more comprehensive solution for determination of I/O lies in the fact that there is a relation between the reduction in peak discharge and the time of travel through the reservoir. The accuracy obtained by this relationship is slightly less than that given by diagrams such as figure 5, but still is well within acceptable limits.

Let t_i denote the time to center of volume of inflow; t_r, the time of travel through the reservoir; and t_o the time to center of volume of outflow. By generally accepted hydrologic principles, it would be expected that

$$t_i/T=0.50(T/T)+0.50(D/T+k/T) \tag{6}$$
$$t_r/T=K/T \tag{7}$$
$$t_o/T=t_i/T+t_r/T. \tag{8}$$

Analysis of the data of tables 4–6 indicates, however, that, for the graphs considered here, the relations expressed by equations 6 and 7 are only approximate, and that more accurate expressions are:

$$t_i/T=0.554(T/T)+0.54(D/T)^{1.05}+0.66(k/T) \tag{6a}$$

and

$$t_r/T=a(K/T)^{0.86}-0.03(k/T) \tag{7a}$$

in which a is a function of (k/T), and for which representative values are shown in table 8.

TABLE 8.—*Representative values of coefficient* a

k/T	a
0.3	0.81
0.7	.91
1.0	.98

Fortunately, for this study—the determination of I/O ratios—the effects of variations in k/T upon the values for t_i/T and t_o/T appear not to be significant. In fact, better correlations with I/O values result from the use of the less precise, but greatly simplified, equations:

$$t_i/T=(T/T)+0.7(D/T) \tag{9}$$

$$t_r/T=0.9(K/T) \tag{10}$$

and

$$t_o/T=(T/T)+0.7(D/T)+0.9(K/T) \tag{11}$$

whence

$$t_r/t_o=0.9(K/T)/[1.00+0.7(D/T)+0.9(K/T)]. \tag{12}$$

For the 128 outflow hydrographs using linear storage the values of t_r/t_o, as computed by equation 12 are shown in column 3 of tables 9–11.

TABLE 9.—*Computations of* I/O, *and ratios to true values, hydrographs routed through linear storage* (k/T=*1.0*)

D/T	K/T	t_r/t_o	$(I/O)_e$	Ratio	D/T	K/T	t_r/t_o	$(I/O)_e$	Ratio
(1)	0.1	0.083	1.032	1.001	0.7	0.1	0.057	1.014	0.992
(1)	.2	.153	1.104	.995	.7	.2	.108	1.054	.979
(1)	.3	.213	1.203	.998	.7	.3	.153	1.104	.962
(1)	.5	.310	1.432	1.021	.7	.5	.232	1.243	.954
(1)	.7	.386	1.671	1.050	.7	.7	.297	1.396	.954
(1)	1.0	.474	2.013	1.078	.7	1.0	.376	1.634	.960
(1)	1.5	.574	2.481	1.078	.7	1.5	.475	2.017	.967
(1)	2.0	.643	2.859	1.052	.7	2.0	.531	2.269	.925
0.1	0.1	0.078	1.027	0.996	1.0	0.1	0.050	1.009	0.993
.1	.2	.144	1.095	.987	1.0	.2	.096	1.040	.985
.1	.3	.201	1.180	.980	1.0	.3	.137	1.086	.977
.1	.5	.296	1.396	.998	1.0	.5	.209	1.198	.965
.1	.7	.370	1.616	1.016	1.0	.7	.270	1.328	.961
.1	1.0	.457	1.940	1.041	1.0	1.0	.346	1.540	.967
.1	1.5	.558	2.400	1.045	1.0	1.5	.443	1.882	.971
.1	2.0	.627	2.768	1.020	1.0	2.0	.514	2.188	.962
0.3	0.1	0.069	1.022	0.991	1.5	0.1	0.042	1.009	1.000
.3	.2	.129	1.076	.975	1.5	.2	.081	1.032	.998
.3	.3	.182	1.148	.962	1.5	.3	.116	1.058	.990
.3	.5	.271	1.328	.962	1.5	.5	.180	1.144	.985
.3	.7	.342	1.526	.975	1.5	.7	.235	1.248	.985
.3	1.0	.426	1.814	.990	1.5	1.0	.305	1.418	.988
.3	1.5	.527	2.251	.998	1.5	1.5	.397	1.711	.995
.3	2.0	.598	2.611	.981	1.5	2.0	.468	1.986	.945
0.5	0.1	0.062	1.018	0.991	2.0	0.1	0.036	1.004	0.998
.5	.2	.118	1.063	.973	2.0	.2	.070	1.022	1.001
.5	.3	.167	1.126	.961	2.0	.3	.101	1.045	1.001
.5	.5	.250	1.279	.951	2.0	.5	.158	1.112	1.005
.5	.7	.318	1.454	.957	2.0	.7	.208	1.194	1.008
.5	1.0	.400	1.720	.970	2.0	1.0	.273	1.338	1.017
.5	1.5	.500	2.125	.975	2.0	1.5	.360	1.585	1.025
.5	2.0	.571	2.467	.960	2.0	2.0	.428	1.824	1.025

[1] Instantaneous.

TABLE 10.—*Computations of* I/O, *and ratios to true values, hydrographs routed through linear storage* (k/T=*0.7*)

D/T	K/T	t_r/t_o	$(I/O)_e$	Ratio	D/T	K/T	t_r/t_o	$(I/O)_e$	Ratio
0.1	0.1	0.078	1.032	0.996	1.0	0.1	0.050	1.010	0.993
.1	.2	.144	1.110	.989	1.0	.2	.096	1.047	.987
.1	.3	.201	1.210	.985	1.0	.3	.137	1.100	.981
.1	.5	.296	1.462	1.007	1.0	.5	.209	1.231	.974
.1	.7	.370	1.719	1.027	1.0	.7	.270	1.383	.976
.1	1.0	.457	2.097	1.051	1.0	1.0	.346	1.630	.987
.1	1.5	.558	2.633	1.048	1.0	1.5	.443	2.029	.995
.1	2.0	.627	3.063	1.017	1.0	2.0	.514	2.386	.985
0.5	0.1	0.062	1.021	0.989	1.5	0.1	0.042	1.010	1.000
.5	.2	.118	1.074	.971	1.5	.2	.081	1.037	1.004
.5	.3	.167	1.147	.958	1.5	.3	.116	1.068	.999
.5	.5	.250	1.326	.952	1.5	.5	.180	1.168	1.002
.5	.7	.318	1.530	.960	1.5	.7	.235	1.289	1.010
.5	1.0	.400	1.840	.974	1.5	1.0	.305	1.488	1.023
.5	1.5	.500	2.312	.977	1.5	1.5	.397	1.830	1.040
.5	2.0	.571	2.712	.959	1.5	2.0	.468	2.150	1.041

TABLE 11.—*Computations of* I/O, *and ratios to true values, hydrographs routed through linear storage* (k/T=*0.3*)

D/T	K/T	t_r/t_o	$(I/O)_e$	Ratio	D/T	K/T	t_r/t_o	$(I/O)_e$	Ratio
0.1	0.1	0.078	1.038	0.986	1.0	0.1	0.050	1.012	0.993
.1	.2	.144	1.131	.971	1.0	.2	.096	1.056	.988
.1	.3	.201	1.250	.964	1.0	.3	.137	1.119	.983
.1	.5	.296	1.550	.982	1.0	.5	.209	1.276	.980
.1	.7	.370	1.856	.994	1.0	.7	.270	1.456	.984
.1	1.0	.457	2.306	1.003	1.0	1.0	.346	1.750	.998
.1	1.5	.558	2.944	.979	1.0	1.5	.443	2.225	1.005
.1	2.0	.627	3.456	.932	1.0	2.0	.514	2.650	.991
0.5	0.1	0.062	1.025	0.983	1.5	0.1	0.042	1.012	1.005
.5	.2	.118	1.088	.956	1.5	.2	.081	1.044	1.018
.5	.3	.167	1.175	.938	1.5	.3	.116	1.081	1.020
.5	.5	.250	1.388	.925	1.5	.5	.180	1.200	1.039
.5	.7	.318	1.631	.930	1.5	.7	.235	1.344	1.058
.5	1.0	.400	2.000	.937	1.5	1.0	.305	1.581	1.086
.5	1.5	.500	2.562	.927	1.5	1.5	.397	1.988	1.115
.5	2.0	.571	3.038	.897	1.5	2.0	.468	2.369	1.120

I/O AS A FUNCTION OF t_r/t_o, VARIABLE k

In figure 6 the ratio t_r/t_o has been shown as abscissa, with values of I/O as ordinates. It should be noted that, to compute the abscissa, there is need for the same information that was required for use of figure 5; that is, the abscissa of figure 6 is a function of T, D, and K. But now, to read the best value of I/O from figure 6, it is necessary also to have an estimate of k.

It is worthy of note, however, that an approximate value generally within about 15 percent of the true one of I/O may be obtained without regard to k. For Illinois streams, values of k/T seem to range from about 0.3 to about 1.2, with the most common value being about 0.7. In the absence of information as to the actual value of k/T, it is suggested that it may be assumed to be 0.7, and that the approximate value of I/O read directly from the middle curve of figure 6.

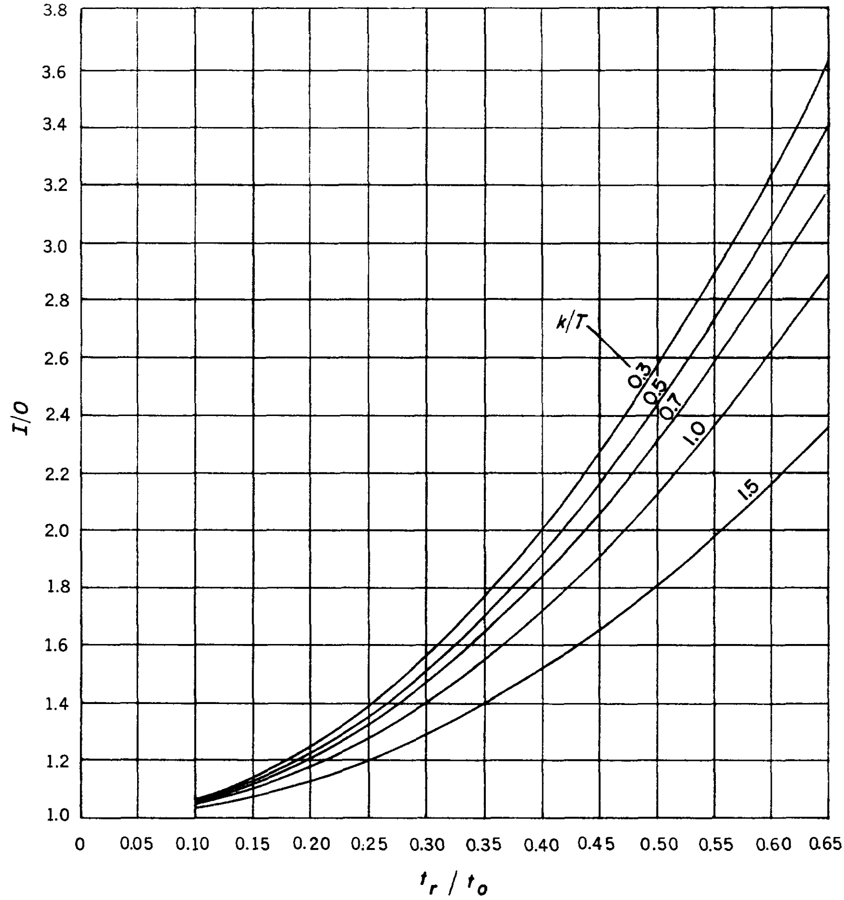

FIGURE 6.—I/O as a function of t_r, t_o, and k/T.

Even with knowledge of the actual values of k/T, there is some scatter of the data with respect to figure 6. Correlations between the variables t_r/t_o (as computed from equation 12), I/O and k/T yield the relation:

$$I/O=1+[7.0-2.5(k/T)]\ [t_r/t_o]\ 2, \qquad (13)$$

which is the expression used for computing the curves of figure 6. This expression also has been used to compute the values of $(I/O)_e$, shown in column 4 of tables 9–11. In the last column of these tables are the results obtained by dividing the values in column 4 by the true values of I/O as given in tables 4–6. Thus, the values in this last column are a direct indication of the variations between the true values of I/O and those read from figure 6. It will be noted that the maximum departure is 12 percent, but only rarely is the departure more than 5 percent, and approximately two-thirds of the values are correct within 3 percent.

DETERMINATION OF T AND k

It has been shown that the relation between inflow peaks and outflow peaks may be determined without use of either an inflow or an outflow hydrograph, but only when certain parameters of the inflow graph and the outflow-storage relation can be determined. The needed factors are D, K, T, and, for the best values of I/O, k. Values of D may be estimated from rainfall records, and those for K, from the slope of the storage-outflow relation curve. But what about the values for T and k?

COMPUTATION FROM PHYSIOGRAPHIC CHARACTERISTICS

The values of T and k are functions of the physiographic characteristics of the drainage basin. If the appropriate physiographic characteristics are adequately measured and described for a sufficient number of instances in which T and k are known, it should be possible to establish relationships by which values of T and k may be predicted for other areas. Work toward this end is now in progress as a part of another project, but as yet no results are available.

Until further information becomes available, it is suggested that T may be estimated by one of the formulas now available for computing time of concentration, and that it be assumed that $k=0.7T$.

MEASUREMENT FROM OBSERVED HYDROGRAPHS

To determine the relationships mentioned above, between T and k on the one hand and the physiographic characteristics on the other, there must be available for a sufficient number of areas not only the description of the appropriate physiographic characteristics, but also the actual values of T and k as measured from observed hydrographs. To this end, hydrographs now are being obtained at appropriate sites in Illinois, and techniques have been developed which seem to be appropriate for obtaining T and k from these hydrographs.

To determine T from an observed hydrograph, use is recommended of the relation:

$$T=P-0.9D, \tag{14}$$

in which P is the time to the point of inflection on the recession side of the hydrograph, measured from the beginning of rainfall excess, and D is as described heretofore, namely, the duration of rainfall excess.

To determine k from an observed hydrograph, the procedure is as follows: After base flow has been deducted from the hydrograph, and starting at a point near to, but slightly beyond, P, read four to six ordinates of the hydrograph at successive, equal, time intervals, Δt. Divide each of these discharge values by the one preceding, thus obtaining three to five values of the recession coefficient, r. Determine the average value of r, and substitute in the formula:

$$k=(1+r)/[(1-r)(2/\Delta t)]. \qquad (15)$$

Equations 14 and 15 have been tested with respect to the 16 inflow graphs used for this report (table 3) with results as shown in table 12.

TABLE 12.—*Computed values of* T *and* k

k/T	D/T	P	T	k
1.0	(1)	1.02	1.02	1.00 T
1.0	0.1	1.08	.99	1.00 T
1.0	.3	1.26	.99	1.00 T
1.0	.5	1.44	.99	1.00 T
1.0	.7	1.62	.99	1.00 T
1.0	1.0	1.90	1.00	1.00 T
1.0	1.5	2.38	1.03	1.00 T
1.0	2.0	2.86	1.06	1.00 T
.7	.1	1.08	.99	.70 T
.7	.5	1.40	.95	.70 T
.7	1.0	1.84	.94	.70 T
.7	1.5	2.32	.97	.70 T
.3	.1	1.04	.95	.30 T
.3	.5	1.28	.83	.30 T
.3	1.0	1.70	.80	.30 T
.3	1.5	2.18	.83	.30 T

[1] Instantaneous.

For all instances in the above table, the true value of T, is, of course, 1.00; thus, the computed values, as shown in the table and as derived by application of equation 14 are not precisely correct, and better values might be obtained by using a more complicated expression, such as $T=P-aD^x$, in which a and x were functions of k/T. However, it is anticipated that the accuracy with which P and D may be determined for field data will not be sufficient to warrant this additional refinement.

SUMMARY

Sixteen dimensionless inflow hydrographs have been routed through various degrees of reservoir storage to provide 139 outflow hydrographs, of which 11 are for nonlinear storage.

Analysis of the remaining 128 outflow hydrographs, all for conditions of linear storage, indicate that peak inflow may be determined from peak outflow, without benefit of a complete hydrograph or the need for routing computations, provided estimates can be made for specified parameters, namely, K, the slope of the storage-outflow relation, D, the duration of rainfall excess, and T, a characteristic time for the drainage area, and indicative of the time lag between rainfall and runoff. A better determination of the peak inflow may be obtained if an estimate is available, also, for k, a second characteristic of the drainage basin, indicative of the miscellaneous storage capacity of the basin, upstream from the principal reservoir. Values of D may be estimated from rainfall records, and K may be estimated from the storage-outflow curve. Work is continuing, as a part of another project, from which it is hoped to derive methods of estimating T, and also k, from the physiographic characteristics of the drainage area. Until these become available, it is suggested that T may be computed by one of the methods now available for computing time of concentration, and that k may be assumed to be approximately $0.7T$.

It is shown that, with proper values for the four needed parameters, maximum error in determination of I/O, the ratio of inflow peak discharge to outflow peak discharge, is about 12 percent, and that, in about two cases out of three, the error should not exceed 3 percent. Within these limits of accuracy, and subject to the accuracy with which values may be determined for the four parameters, outflow peaks may be adjusted to the corresponding inflow peaks.

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