On Determining Pollutant Distribution in Tidal Estuaries

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On Determining Pollutant Distribution in Tidal Estuaries

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HYDROLOGY OF TIDAL STREAMS

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ON DETERMINING POLLUTANT DISTRIBUTION IN TIDAL ESTUARIES

By E. E. Pyatt

ABSTRACT

Pollution of the Nation's estuaries is a complex problem that is increasingly attracting the attention of scientists and engineers. Because estuaries are used for diverse purposes such as commercial fishing, shellfish harvesting, navigation, recreation, and water supply, as well as for waste disposal, receiving-water quality criteria must be established to provide a balance between economical treatment of the wastes before disposal and acceptable degradation of the receiving body of water. Hence, the causal relationship between discharged wastes and estuarine response must be established.

Pollutants in estuaries are of three general classes—organic, inorganic, and radioactive. This paper emphasizes the seriousness of organic pollutants in our Nation's tidal bodies of water. Because the study of mixing processes in an estuary is an exceedingly complicated undertaking involving at least 28 independent variables, investigators must make some compromise between the desirability of a rigorous mathematical approach and the practical necessity for achieving workable engineering solutions and evaluations.

Approaches that various investigators have taken to determine the assimilative capacity of tidal estuaries may be listed under four general classes: theoretical, quasi-theoretical, experimental (hydraulic models), and statistical. Of these approaches, the author considers the statistical approach as the most useful, at least for the forseeable future. In the meantime, attempts to derive a mathematical, or theoretical, solution should not be abandoned. Future attempts to improve theoretical approaches should perhaps be directed first toward achieving a better understanding of the coefficient of eddy diffusivity, K.

Although the approach used by the author in his analysis of the St. Johns estuary probably cannot be used in other specific situations, it is presented for consideration. Any approach that yields useful results should be published.

INTRODUCTION

Because the necessity for free access in navigation has historically been recognized, the major centers of population on the eastern seaboard were located on or near tidal bodies of water. To a lesser extent, the same is true for the western part of the United States—for example, Seattle, Portland, San Francisco, Los Angeles, and San

Diego. Population complexes necessarily produce wastes, which are discharged into nearby estuaries.

PURPOSE AND SCOPE

Pollution of the Nation's estuaries is a reality and is a problem that is increasingly attracting the attention of scientists and engineers. Pollution is a reflection of demographic and cultural factors, as well as of the physical environment—for example, suspended silt. Man's activities contribute to tidal bodies of water three broad classes of wastes—organic, inorganic, and radioactive. This paper considers only dissolved organic pollutants, which exert oxygen demand on the receiving body of water.

The most important economic use of tidal estuaries is for navigation. Estuaries are also used for other purposes such as waste disposal, recreation, as sources of industrial and municipal water supplies (provided the salinity characteristics are appropriate), and for commercial fishing and shellfish harvesting. One task of a river-basin planner is to study and to decide upon the proper division of the sometimes conflicting legitimate uses. In general, the more a body of water is utilized for waste disposal, the less suitable it becomes for recreation. Although a tremendous amount of money has been allocated for the treatment of wastes before estuarine disposal, still more is needed for research on how to achieve the optimum balance between technical, economic, and social factors.

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Data for the numerical example contained herein were generously supplied by the Office of the City Engineer, Jacksonville, Fla. Throughout the study, liaison has been maintained with the author by Mr. E. T. Owens, Design Engineer for the city of Jacksonville Engineering Office. For the provision of these data and the supplying of copies of graphs and other illustrations pertinent to the St. Johns River Estuary at Jacksonville, the author expresses his appreciation to Mr. Owens and his associates.

WATER QUALITY SAMPLING PROGRAMS

Analytical approaches for studying the assimilation capacity of fresh-water streams are reasonably well established, but tidal estuaries pose problems that are more complex than those of fresh-water streams because tidal estuaries have more independent variables in their assimilation process. From a hydrodynamic point of view, development of equations that would enable one to calculate the velocities and accelerations of fluid particles at any point in the three-dimensional coordinate system within a tidal estuary is an enormous undertaking. Someone has estimated that the estuarine milieu has at least 28 independent variables.

How to design and execute a proper water quality sampling program for such a complex hydrodynamic situation is a problem. In programs of various regulatory agencies on tidal estuaries in the United States "shotgun" sampling procedures are generally used. Lacking an appropriate analytical methodology for interpreting the data, the regulatory bodies have proceeded systematically to collect any data that could possibly be significant in any future analysis. Although these regulatory agencies have undoubtedly been aware of the necessity for economy in their collection procedures, they have also been aware of the possibility that by "economizing" they might omit potentially essential data.

A typical tidal estuary sampling program might include the following determinations: (a) tidal stage and phase, (b) coliform MPN (most probable number), (c) dissolved-oxygen content, DO, (d) 5-day biochemical oxygen demand, BOD, (e) conductivity (chloride content), (f) turbidity, (g) pH, (h) air temperature, (i) water temperature, (j) weather data, and (k) fresh-water discharge. Although this paper is scarcely the place to discuss the implications of each of these determinations (see American Public Health Association and others, 1960), brief reference to them may be helpful. The coliform MPN is a statistical computation based upon the results of the inoculation of lactose tubes with serial dilutions of samples of water from the estuary. Because the coliform group is assumed to be an indicator of potential contamination by pathogenic organisms, the MPN is of considerable significance to public-health regulatory bodies. Although they are aware of health implications, water-pollution control agencies tend to place more importance upon the DO of an estuary. The saturation concentration of DO in water is a function of temperature and conductivity of the water (that is, chloride content), and, therefore, temperature and chloride content must be determined. Determinations of tidal stage and phase are made in an attempt to understand the motion of the "salinity front" of the estuary or to enable a comparison of test results made at different times of the tidal cycle. Unless the pH determination is generally very much above or below 7.0, the sampling agency tends to ignore this determination completely in its appraisal of the data. Although turbidity may be important in assessing the potential of a receiving body of water to serve as a source for industrial or municipal water and to an understanding of the metabolism of phytoplankton, it is considerably less important than either DO or coliform MPN. BOD is a measure of the oxygen demanding properties within a receiving body of water, and its determination, in conjunction with the DO, may provide a basis for establishing some relationship between cause and effect. Usually, however, the causal relationship has given way to a statistical study of the effect; that is, a study of DO profiles. Although an understanding of the relationship between cause and effect is preferable to the understanding of effect only, an understanding of the effect is clearly a step forward. This effect can perhaps be related to the total quantity and strength of sewage imposed upon the stream; these factors are usually known with a fair degree of reliability. Some simultaneous determinations of BOD and DO, on the other hand, seem to have no relationship to each other nor to the quantity and quality of the load imposed upon the receiving body of water.

Four basic techniques are used for collecting data in tidal estuaries: (1) a series of fixed stations along the estuary for the duration of a tidal cycle, (2) "same slack" technique, (3) continuous recorders, and (4) random sampling.

Before deciding upon which of the four sampling techniques will be used in a specific situation, one should know something about the mixing characteristics of an estuary. Some estuaries, such as the Mississippi River exhibit a well-defined "salt wedge," whereas others, such as the Delaware River at Philadelphia, Pa., exhibit no salt wedge but, rather, a gradual decrease in salinity as one proceeds upstream. Naturally, intermediate characteristics also occur. Classification of estuaries is difficult, and perhaps meaningless, because their salinity characteristics do not fit into neat subdivisions. In addition to the longitudinal salinity characteristic, estuaries may exhibit relatively homogeneous salinity in a particular cross section, or they may exhibit pronounced salinity gradients. Lateral salinity gradients are common because of the Coriolis effect or because of the geomorphology of the channel. Thus, examples of estuaries that are homogeneous in cross section are difficult to locate; the Delaware River seems to approach this ideal, and it has frequently served as a mathematical model assumed to have neither a lateral nor vertical salinity gradient. Where a particular estuary approaches the ideal condition of homogeneous mixing in a cross section, stations need to be spaced in only the longitudinal, or axial, direction of the estuary. Each station in the longitudinal can be onshore because such a sample is representative of all points in the cross section. Where these near-ideal conditions do not prevail, several stations must be established within a cross section—perhaps one in the center and one on either bank. In addition, if vertical salinity gradients exist at the section, each of the three lateral stations must be sampled vertically. Such sampling procedure is not only expensive but also dangerous for crews must be anchored in midstream in small crafts. Also, obstruction to the passage of navigation may result. Although navigational interests can be coordinated to permit midstream sampling, frequent use of this procedure is time consuming for officials and annoying to commercial vessels. This the choice of the sampling procedure will be dictated by knowledge of the mixing characteristics of the estuary and by the goals of the sampling program.

In technique 1, samples are usually taken every hour on the hour for 12 hours at each station. Data obtained through such a procedure provide a quick, reliable, and thorough way of determining relative concentrations of a particular sampling parameter along the axis (and possibly also in the lateral and vertical) of the estuary. The technique, however, has the disadvantage of requiring numerous personnel, considerable coordination between laboratory and field personnel, and a lot of money.

Technique 2 was developed by Kaplovsky (1957) for application on the Delaware River Estuary. Instead of using several fixed points, as in technique 1, Kaplovsky utilized a high-speed craft capable of moving as rapidly as the tide in the estuary. Thus, he was able to sample each point along the estuary at approximately the same tidal phase. Although this technique, may be advantageous on some occasions, it is almost impossible to correlate these data with data from technique 1. Perhaps this difficulty is more than offset by the fact that one crew can sample all points and thus greatly reduce the cost of the program.

Recently, equipment capable of continuous sampling of rivers and estuaries has been developed. Although continuous sampling equipment is still subject to operational difficulties, such factors as DO, salinity, and temperature can be determined and recorded continuously. Such data can even be transmitted electronically to a central office for processing and analysis. Although continuous recording of pollution parameters at a point in a stream or estuary is desirable, the nature of the equipment requires that it be anchored firmly on a pier or bridge or piling. Therefore, installation of continuous-recording equipment at any desired point in an estuary would be difficult. If the estuary is homogeneous in cross section, or nearly so, installation of sampling equipment on a bridge may be possible. The Philadelphia, Pa., district office of the U.S. Geological Survey is using continuous-recording sampling equipment on the Delaware River Estuary. ORSANCO (Ohio River Valley Water

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Sanitation Commission) and the Public Health Service also are using this type of equipment in their water quality network investigations.

Random sampling refers to randomness in the time sequence rather than in the sampling points. Some hydrologists argue that random sampling is statistically superior to the predetermined collection pattern of a scheme such as the "same slack" technique. Although this may be true, the technique has attracted little attention in estuarine sampling programs.

POSSIBLE APPROACHES TO THE PROBLEM

Sanitary engineers are concerned chiefly with the assimilation and dispersion of the dissolved organic phase of estuarine pollutants. Despite the nonconservative (subject to decay independently of the properties of the receiving body of water) nature of such wastes, a hydrodynamic analog exists with the three dimensional distribution of salinity, in time and space. Thus, dissolved organic waste in an estuary should have a dispersion pattern characterized by: (a) the hydrodynamic environment in relation to salinity concentrations and (b) its own decay curve, or BOD. One possible approach to the estuarine pollution problem, whether general or specific, is to attempt a solution of the classical diffusion equation (equation 8, p. F10) for a conservative waste, or tracer, such as salinity or stable dye. Then, superimpose the self-decay parameter that distinguishes organic-waste loading from a conservative tracer. To date, attempts to solve the diffusion equation for estuarine application have not been successful. Almost no progress has been made on the superposition aspects of the overall problem. The author has chosen to designate all approaches that rest either completely or partially upon the diffusion equation as theoretical approaches. In this context, the nontheoretical approaches should be self-explanatory by definition. Although there are various degrees of overlapping between the subsequent terms used in this report, the terms should be convenient to use.

THEORETICAL (DIFFUSION EQUATION) APPROACH

Suppose that a variable, such as salinity (s), is a function of the spatial coordinates x, y, and z (longitudinal, lateral, and vertical, respectively) and of time, t Then,

$$s=f(x, y, z, t),$$

$$ds=\frac{\partial s}{\partial x}\,dx+\frac{\partial s}{\partial y}\,dy+\frac{\partial s}{\partial z}\,dz+\frac{\partial s}{\partial t}\,dt \text{ (by definition)},$$

and,

$$\frac{ds}{dt}=\frac{\partial s}{\partial x}\frac{dx}{dt}+\frac{\partial s}{\partial y}\frac{dy}{dt}+\frac{\partial s}{\partial z}\frac{dz}{dt}+\frac{\partial s}{\partial t}.$$

If we let

$$u=\frac{dx}{dt}=\text{velocity in } x \text{ direction},$$

$$v=\frac{dy}{dt},$$

and

$$w=\frac{dz}{dt},$$

then

$$\frac{ds}{dt}=u\frac{\partial s}{\partial x}+v\frac{\partial s}{\partial y}+w\frac{\partial s}{\partial z}+\frac{\partial s}{\partial t}. \tag{1}$$

Equation 1 states that the total derivative of s with time equals the effect of advection (motion under the influence of velocity components u, v, and w) plus the effect of local change, $\frac{\partial s}{\partial t}$. Physical meaning attaches to the local change if one transposes to a Lagrangian frame of reference wherein the coordinate system x, y, z itself is in motion with components u, v, w. Equation 1 then would become independent of x, y, z if s were always measured at the origin. In that particular measurement

$$\frac{ds}{dt}=\frac{\partial s}{\partial t} \tag{2}$$

because t would be the only variable. Diffusion is the physical mechanism which accounts for changes in s with time. Note that diffusion is not confined to molecular diffusion but also includes eddy diffusion, which is much greater than molecular diffusion. For all practical purposes, therefore, "diffusion" means eddy diffusion. Thus, returning to the Eulerian system: diffusion (eddy) = advection + local change, or

$$\frac{ds}{dt}=u\frac{\partial s}{\partial x}+v\frac{\partial s}{\partial y}+w\frac{\partial s}{\partial z}+\frac{\partial s}{\partial t}. \tag{3}$$

It is now germane to formulate eddy diffusion mathematically, much of this work having been done by G. I. Taylor and L. Prandtl between 1915 and 1920. On atmospheric diffusion Taylor (1959, p. 101–111) stated:

The rate at which this wave of cooling penetrated into the atmosphere could be accounted for by a transfer process of the same nature as molecular conductivity but much more vigorous. This idea was not new; it had been used by many meteorologists. The only possibly new element that I introduced (Taylor, 1915) into it was the use of a definite "mixture length." Formally this is a perfectly definite conception. The rate of heat transfer and the mean temperature gradient could both be measured and their ratio determines a coefficient of diffusivity for heat. This has the dimensions of length x velocity. The transfer is in a vertical direction so that the relevant velocity is the vertical component, and this could be measured and the time-mean of its absolute value or the root-mean-square value determined. Thus the length which is defined as (diffusivity/vertical turbulent velocity) is a measurable length.

Some years later [1920(?)] * * * Prandtl independently introduced the idea of a mixture length and his name is usually associated with it. Though, as I mentioned, the idea that a mixture length can be defined and measured was brought out in my paper of 1915 I did not attempt to associate the magnitude of the turbulent velocity with this length. This was the new feature of Prandtl's theory which assumed that the turbulent velocities at any point depend only on the mean rate of shear and the mixture length and were in fact proportional to the product of the two. This additional assumption was needed in order to make a mixture-length theory into a tool which could be used in interpreting hydrodynamic experiments, but even then a further assumption was needed, namely that the mass of fluid or "austausch" which carries the transferable property is unaffected by fluctuating pressure gradients or by molecular diffusibility during the period in which it preserves its identity. It is clear that if the transferable property with which one is concerned is momentum the fluctuating pressure gradients can alter the momentum of the transporting mass without necessarily causing it to mix with its surroundings. However, Prandtl made the assumption that fluctuating pressure gradients have no effect on the average transfer of momentum and the theory so produced is his well-known Momentum Transport theory. In my 1915 paper I did not take that bold step, but I realized that if the whole field of flow, the mean as well as the turbulent velocities, were limited to two dimensions, vorticity would be a transferable property and would be unaffected by fluctuating or steady pressure gradients, if the effects of viscosity were neglected, and so give rise to a theory of vorticity transfer.

Though mixture-length theories are not in general capable of giving a realistic description of the relationship between mass and momentum transport, they do lead to the expectation that transferable properties other than momentum will be transported in exactly the same way * * *. The chief difficulty in a mixture-length theory is to form a physical picture of the process by which a body of fluid carrying some transferable property discharges its load into the surrounding fluid. Taken literally the original mixture-length theories would envisage the unrealistic conception that a mass of fluid retains its load unchanged till at a certain instant it meets with a sudden catastrophe and disintegrates into fine threads or drops which then discharge their load by molecular processes * * *.

The model which I have just described was Eulerian in the sense that no attempt was made to follow particles of the diffusing matter or property, and it could even be regarded as describing a steady state. To devise a more realistic description of turbulent diffusion I had to base it on the Lagrangian conception in which attention is fixed on a particle of the diffusing matter which preserves its identity as it moves in the turbulent field of flow. I therefore tried to describe the diffusive properties of turbulence by defining the correlation between the velocity of a particle at one instant and that of the same particle at a later time (Taylor, 1921). I found in fact that the diffusion from a fixed point source can be described when only this Lagrangian time-correlation function and the mean square velocity is known. I found that if this correlation vanishes after an interval of time, the diffusion for much longer periods proceeds as though it were due to a constant virtual coefficient of eddy diffusivity. Since, as I have already mentioned, a mixture length can be formally defined without reference to the physical process of mixture, the time-correlation function makes it possible to think of mixture length roughly proportional to the average length of path which a particle traverses before its velocity becomes uncorrelated with that which it had at the beginning of the path. This method of describing diffusion makes it possible to define a mixture length without considering any physical processes, but it involves the assumption that the particles of the diffusing substance are indistinguishable so far as their motion is concerned from those of the transporting fluid * * *

It seems * * * that no physical meaning can be attached to the use of equations in which the coefficient of diffusion varies with the time of diffusion, even though the formulae produced by their use do represent adequately the concentrations in particular cases. This point was well understood by L. F. Richardson who [1926] * * * initiated the modern approach to the subject * * *. He discussed diffusion in one dimension and layed down a statistical method for describing the number of particles, n, which are situated in unit length at distance 1 from each particle. He then showed that the principal agents in separating two particles are eddies of length comparable in size with l and he summed up the situation by showing that n must obey an equation of the form

$$\frac{\partial n}{\partial i}=\frac{\partial}{\partial l}\left[K(l)\,\frac{\partial n}{\partial l}\right] \tag{4}$$

K is therefore of the nature of a diffusion coefficient which depends on the separation of particles and does not suffer from the logical difficulty of attaching a meaning to a diffusibility which is a function of the time of diffusion. To find out how K varies with l Richardson took a large number of recorded values of K. These had been determined by interpreting diffusion measurements over a limited range of l as being due to a constant diffusivity in that range. He found that

$$K=0.2l^{4/3} \tag{5}$$

and had the idea that the index (4/3) was determined by something connected with the way energy was handed down from larger to smaller and smaller eddies. He perceived that this is a process which, because of its universality, must be subject to some simple universal rule. It is perhaps rather surprising that he did not take the step which Kolmogoroff (1941) and Obukhov took fifteen years later, namely to express his equation non-dimensionally using only the two physical quantities which could be relevant to a universal rule regulating the handing down of energy, namely e the rate of energy dissipation and v the dynamical viscosity * * *. Richardson's equation (4) can be expressed non-dimensionally using Kolmogoroff's transformation:

$$\frac{\partial \sigma}{t'}=0.2e^{-1/3}\,\frac{\partial}{\partial l'}\left[l'^{4/3}\,\frac{\partial \sigma}{\partial l'}\right] \tag{6}$$

one cannot go further without knowing a value to take for e. Brunt made an estimate $e=5$ cm^2 sec^{-3} as a general average, which would make (6) become:

$$\frac{\partial \sigma}{\partial t}=0.12\,\frac{\partial}{\partial l'}\left[l'^{4/3}\,\frac{\partial \sigma}{\partial l'}\right] \tag{7}$$

All that can be said of (7) is that the factor 0.12 is of order 1 which is what one would intuitively expect from such a general non-dimensional equation. It would be interesting if one could make a more precise determination of this number * * *.

Since it is theoretically possible, using the Navier-Stokes equations, to trace the changes in a field of flow which follow any given initial distribution of velocity, it might be thought possible, though no doubt beyond our power to find average values at subsequent times for a set of initial conditions which themselves are only known through a number of averages. If it were possible to do this the dispersive power of turbulence would be one of the averages so calculated. This line of approach, however, fails at a very early stage. Kármán and Howarth (1938) showed that the rate of change of a two-point Eulerian velocity correlation can only be found if the values of certain three-point correlations are known, and the rate of change of a three-point correlation is only known when the values of some four-point correlations are known. Thus though the essential parameter e of the Richardson-Kolmogoroff universal law of diffusivity is known only when the second differential coefficients near the origin of the two-point correlations in the Eulerian description are known, theory gives no indication that there is any direct connection between the Eulerian two-particle velocity correlation at a given time and the Lagrangian correlation between the velocities of a particle at different times.

Though it is difficult to connect experimental measurements of e made under the conditions assumed in Kolmogoroff's theory, and so to determine the constant in the universal diffusion law, there is one case in which a connection between the Eulerian and the Lagrangian methods of describing turbulence has been traced, namely that of dispersion in a turbulent flow through a straight pipe.

The left-hand side (eddy diffusion) of equation 3 can therefore be stated as a function of the eddy diffusivity, $K(x, y, z)$, thus

$$\frac{\partial}{\partial x}\left[K(x)\frac{\partial s}{\partial x}\right]+\frac{\partial}{\partial y}\left[K(y)\frac{\partial s}{\partial y}\right]+\frac{\partial}{\partial z}\left[K(z)\frac{\partial s}{\partial z}\right]=u\frac{\partial s}{\partial x}+v\frac{\partial s}{\partial y}+w\frac{\partial s}{\partial z}+\frac{\partial s}{\partial t}. \quad (8)$$

K is not assumed to vary with time because s is conservative but in general, does vary with the spatial coordinates x, y, z. Before equation 8 could be solved,

$$K=f(x, y, z)$$

would have to be known or assumed. K is sometimes assumed to be independently constant in the x direction, y direction and z direction, or

$$\frac{\partial K(x)}{\partial x}=\frac{\partial K(y)}{\partial y}=\frac{\partial K(z)}{\partial z}=0. \quad (9)$$

however if

$$K_x\neq K_y\neq K_z,$$

then

$$K_x \frac{\partial^2 s}{\partial x^2}+K_y \frac{\partial^2 s}{\partial y^2}+K_z \frac{\partial^2 s}{\partial z^2}=u \frac{\partial s}{\partial x}+v \frac{\partial s}{\partial y}+w \frac{\partial s}{\partial z}+\frac{\partial s}{\partial t}. \quad (10)$$

Now that the nature of the theoretical basis for pollution computations (equations 8 and 10) has been established, the work of several estuarine pollution investigators will be examined to appraise the validity of their simplifying assumptions.

Kent (1958, 1960) in a doctoral study on the Delaware River Estuary defined a "sectionally homogeneous" estuary as one in which at any longitudinal (axial) station there is no variation in the contaminant (for example, salinity) across the transverse section. According to Pritchard (1955), such an estuary is known as type D, in which tidal mixing is especially intense. Although no estuary is apt to fit such a classification perfectly, the Delaware River Estuary seemed to fit sufficiently well that the y and z components in equation 8 could be dropped. Thus for a sectionally homogeneous estuary,

$$\frac{\partial}{\partial x}\left[K(x) \frac{\partial \bar{s}}{\partial x}\right]=u \frac{\partial \bar{s}}{\partial x}+\frac{\partial \bar{s}}{\partial t}, \quad (11)$$

which became the basis for Kent's determination of the theoretical distribution of mean contaminant, $\bar{s}$, in the x direction. As Kent (1958, 1960) admitted, "the applicability of equation 11 to problems in estuarine diffusion depends on how well the unknown turbulence mechanism is incorporated into the formulation of the coefficient of eddy diffusion." He then illustrated the exact solution of equation 11 for three idealized situations. Kent also said: "Since it is almost certain that any natural estuary will be nonlinear in either u or A (cross-sectional area), or in both, it is unlikely that a useful general solution to equation 11 will emerge. Lacking an elementary analytic solution of equation 11 alternative methods of solution must be considered." Using finite difference approximations of the derivatives, Kent rewrote equation 11 as

$$S_{x,\,t+\Delta t}=S_{x,\,t}\left[1-\frac{1}{A_x}\frac{\Delta t}{\Delta x^2}\left\{(AK)_{x+\frac{\Delta x}{2}}+(AK)_{x-\frac{\Delta x}{2}}\right\}\right]$$
$$+S_{x+\Delta x,\,t}\left[-\frac{u\Delta t}{2\Delta x}+\frac{(AK)_{x+\frac{\Delta x}{2}}}{A_x}\frac{\Delta t}{\Delta x^2}\right]$$
$$+S_{x-\Delta x,\,t}\left[\frac{u\Delta t}{2\Delta x}+\frac{(AK)_{x+\frac{\Delta x}{2}}}{A_x}\frac{\Delta t}{\Delta x^2}\right]. \quad (12)$$

where:

A_x=cross sectional area at X,

Δx=longitudinal distance between grid points (sampling stations.

Δt =time interval, and

K =eddy diffusivity in x direction for the contaminant S.

Kent did not determine eddy diffusivity directly; rather, he used salinity distribution to calculate eddy diffusivity for salt and, then, used that determination to calculate eddy diffusivity for a contaminant (methylene blue). The pertinent equations are:

$$K_S=\frac{RS_x 2\Delta x}{A_x(S_{x+\Delta x}-S_{x-\Delta x})} \qquad (13)$$

and

$$K_q=\frac{K_s L_q}{L_s}, \qquad (14)$$

where

K_s=salt eddy diffusivity,

R =fresh water discharge,

S =salt concentration,

K_q=methylene-blue eddy diffusivity,

L_q =measurable longitudinal extent of methylene blue, and

L_s =measurable longitudinal extent of salt.

Knowing K_S and, hence K_q, and replacing S with q and K with K_q, Kent used a digital computer to solve equation 12 for $q=f(x, t)$ for the given boundary conditions. These results were then compared with experimental data from the hydraulic model of the U.S. Army Corps of Engineers at Vicksburg, Miss. The fit of the data was good.

Huiswaard, Banks, and Bell (1959) have used the general techniques of Kent to study the longitudinal dispersion of a continuous dye release.

Not all investigators have been so careful to specify assumptions as were Kent and Huiswaard. After Pritchard (1952) criticized Stommel's earlier work on the basis of inadequate specification of boundary conditions, Stommel (1953) revised his methodology and was most precise in defining the limitations of the technique. He said that his method is valid "only in vertically unstratified estuaries in which mixing is due to tides." The method depends upon equating the net seaward flux $F(x)$ of pollutant across any section to the sum of the fluxes due to advection and to turbulent mixing. No suppositions have been formulated about the turbulent mixing process, and it is proposed that the turbulent eddy diffusivity $A(x)$ be calculated from a knowledge of the concentration of any other property in the estuary such as the fresh water distribution $f(x)$. Stommel (1953) said that "in some estuaries where ground water, precipitation, or evaporation is important, it might be almost impossible to use the concentration of fresh water as a means of computing $A(x)$."

On the basis of $F(x)$, $A(x)$, and $f(x)$ as defined above, the fundamental equation for flux is:

$$F(x)=Qc-SA=\frac{dc}{dx}, \tag{15}$$

where

Q=discharge,
c=concentration of pollutant,
S=cross sectional area, and
x=longitudinal distance.

Equation 15 is virtually the same as equation 11 used by Kent. But, unlike Kent, Stommel proceeds to note that $c(x)$ may be either conservative or nonconservative in equation 15. Because nearly all pollution indices are nonconservative (for example BOD and radioactivity), let

$$c=c_0e^{-t/\tau}, \tag{16}$$

where

c_0=initial concentration at zero time,
t=time, and
τ=decay constant.

Then, for a nonconservative waste, "the net flux, $F(x)$ cannot be the same for all values of x * * * but must diminish in the following manner (Stommel, 1953):

$$\frac{d}{dx}F(x)=\frac{-S}{\tau}c, \tag{17}$$

From equations (15) and (17)

$$\frac{d}{dx}\left[Qc-SA\frac{dc}{dx}\right]+\frac{S}{\tau}c=0 \tag{18}$$

and

$$\frac{d}{dx}\left[Qc-SA\frac{dc}{dx}\right]+\frac{S}{\tau}c=\psi, \tag{19}$$

where ψ is the total rate of pollution at the outfall. In a given estuary, Q, S, and τ together with salinity data are used to calculate A and then $c(x)$ can be determined by using equations 18 and 19.

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Stommel arbitrarily segmented the estuary into equal longitudinal steps of length a. Numbering begins upstream at section 1 (S_1) and extends downstream. At each segment, or cross section, the eddy diffusivity is computed from known salinity data. The pertinent equation derives from equation 15 substituting the concentration of fresh water f for c.

$$A=\frac{Q(f-1)}{S\,df/dx} \tag{20}$$

because $F(x)=Q$. Note that f is dimensionless and would be, for example, 1.0 for fresh water. If equation 20 is written in finite difference form,

$$A_n=\frac{2aQ\ (1-f_n)}{S_n(f_{n-1}-f_{n+1})}, \tag{21}$$

which is similar to equation 13. Stommel, however, did not distinguish between K_s and K_q in equation 14; so, no further adjustment is required in A_n (equation 21).

Eddy diffusivity, A_n, is tabulated for each section, S_n, under the given flow condition, Q, and known salinity distribution. Equations 18 and 19 are written in finite difference form to give, respectively:

$$P_nc_{n-1}+Q_nc_n+R_nc_{n+1}=0 \tag{22}$$

$$P_nc_{n-1}+Q_nc_n+R_nc_{n+1}=\frac{\psi}{2a}, \tag{23}$$

where

$$P_n=-\frac{1}{2a}\left[Q-\frac{A_{n+1}S_{n+1}-A_{n-1}S_{n-1}}{2a}\right]-\frac{A_nS_n}{a^2},$$

$$Q_n=\frac{2A_nS_n}{a^2}+\frac{S_n}{\tau},$$

$$R_n=-P_n-\frac{2A_nS_n}{a^2}.$$

Equations 22 and 23 form the basis for calculating the distribution of pollutant $c(x)$ through the use of relaxation analysis. Stommel (1953) provided a "numerical example" that consisted of hypothetical values. Thus, his "example" served neither to validate nor to invalidate his theoretical treatment.

In 1959 the author attempted to apply Stommel's technique to the St. Johns River at Jacksonville, Fla., even though certain of Stommel's criteria were lacking. For example, the St. Johns River is not sectionally homogeneous; ground water, precipitation, and evaporation are important factors; and experimental data were collected from non-equidistant stations. As expected, the technique failed. Of course, this failure neither proves nor disproves the validity of Stommel's equations; on the other hand, if the attempt had succeeded, it would have been the first known successful application of Stommel's technique to a real situation.

O'Connor (1960) in his theoretical treatment has followed the form of equation 8 but has modified it for direct application to DO. Again, assuming a sectionally homogeneous estuary and steady-state conditions, O'Connor formulated:

$$O = e\frac{d^2c}{dx^2} - u\frac{dc}{dx} + K_2[c_s - c] - K_dL, \tag{24}$$

where

e = turbulent transport coefficient,
c = oxygen concentration,
c_s = saturation value of DO at the ambient temperature (in water),
L = ultimate BOD,
u = forward velocity of river flow,
K_d = coefficient of deoxygenation, and
K_2 = reaeration coefficient.

It was then necessary to express L as a function of x; from a consideration of material balance, O'Connor found:

$$L = L_O \text{exponential } j_1x, \tag{25}$$

in which

$$j_1 = \frac{u}{2e}\left[1 - \sqrt{1 + \frac{4K_de}{u^2}}\right]. \tag{26}$$

Substituting equation 25 into equation 24 and integrating, the resulting second-order linear differential equation is obtained:

$$c = c_O \text{exponential } j_2x + c_s(1 - \text{exponential } j_2x) - FL_0(\text{exponential } j_1x - \text{exponential } j_2x) \tag{27}$$

where

$$j_2 = \frac{u}{2e}\left[1 - \sqrt{1 + \frac{4K_2e}{u^2}}\right]. \tag{28}$$

Thus, with a knowledge of u, e, K_2 and K_d, it is possible to calculate $c = f(x)$ for steady-state (mean over a tidal cycle) conditions. It is not possible to find $c = f(x, t)$ as by using Kent's treatment. Working to calculate e from a knowledge of the salinity distribution, O'Connor used equation 13. More will be said of K_2 and K_d under "Evaluation of possible approaches."

The three preceding theoretical approaches of Kent, Stommel, and O'Connor represent present knowledge in utilizing the classical diffusion equation for a determination of the distribution of pollution in tidal estuaries.

QUASI-THEORETICAL APPROACH

Perhaps Ketchum (1951a, b) may be credited with focusing interest upon the pollution of tidal estuaries and providing the first reasonably logical technique. According to Redfield (1952, p. 130) sanitary engineers have long employed the "tidal prism" calculation, which assumes that the "volume of water which enters and leaves a basin with the rise and fall of the tide is available to dilute the increment of pollution added to it during each tidal cycle. Ketchum points out that this assumption could only lead to correct estimates if the water in the basin were completely mixed and if none of the water which escaped during the ebb returned on the following flood tide. Since these conditions are not met generally, the tidal prism concept gives an exaggerated estimate of the rate of flushing." Ketchum modified the classical tidal-prism approach by dividing the estuary into a number of segments, the lengths of which were determined by the mean excursion on a rising tide. In essence, then, the tidal-prism approach was applied to segments of an estuary, rather than to the estuary as a whole. Such a procedure considers variations in gross mixing phenomena along the estuary and allows steady-state computations of escaping volumes of fresh water and mean age of fresh water in any given segment. In salinity studies of the Raritan River, the procedure yielded good results. However, even though Ketchum's method might yield a reliable estimate of "flushing time" as a function of the x coordinate along an estuary, the serious problem of correlating the flushing number with probable DO for a particular contaminant loading remains.

Arons and Stommel (1951), stimulated by the work of Ketchum, devised an impressive mathematical solution for an idealized estuary. The model estuary was assumed of constant width and depth and of constant salinity. By equating the seaward advective flux of salt to the landward eddy diffused salt (necessary statement of salt balance for steady state), Arons and Stommel (in Redfield, 1952, p. 130) arrived at an equation relating the longitudinal salinity distribution to "a constant F, designated the flushing number. The flushing number is a function of the fresh water discharge, the tidal amplitude, angular frequency, the depth of the channel and its volume." The work of Arons and Stommel, according to Redfield (1952, p. 130),

> is useful in showing how the salinity will change if any one of the factors on which the flushing number depends is varied. The curves correspond well enough with

the observed salinity in selected narrow estuaries to suggest that the flushing number may serve usefully in classifying estuaries of the narrow type. Unfortunately, the values of the proportionality factor (of which F is a function) obtained from the data on real estuaries vary by an order of magnitude in different cases, which means that the equation fails in evaluating the intensity of mixing from given parameters of the system. The restrictions imposed by the assumption of uniform dimensions prohibits the application of this treatment to the many basis of irregular morphology found in nature.

Gameson and Barrett (1958) have reported on pollution computations for the Thames, carried out in cooperation with Preddy (1954). The analysis entails mixing, or flushing, concepts not unlike those of Ketchum. If P_1 is defined as the proportion of original water at point 0 which becomes dispersed downstream after a tidal cycle, and P_2 is defined as the proportion dispersed upstream, then $1-P_1-P_2$ represents the return to point 0. It is desired to calculate P_1 and P_2 for various values of x (or points 0) along the Thames. Field studies indicated that the dispersion of P_1 and P_2 about point 0 could be mathematically represented by an asymmetric distribution having a spread equal to 9 miles downstream and 9 miles upstream after two tidal cycles. Consideration of equilibrium requirements that there be no net transfer of either salt or water across any boundary due to mixing alone led to:

$$\frac{1}{9}\left[\int_0^9 \bar{S}Y(9-x)dx+\int_0^{-9} \bar{S}X(9+x)dx\right]=\overline{QS}+(m-z)/n \qquad (29)$$

and

$$\int_0^9 Y(9-x)dx+\int_0^{-9} X(9+x)dx=0, \qquad (30)$$

where

z=total amount of salt discharged to the estuary above the point considered,
m=increase in total salt content to the estuary above the point,
$\overline{QS}$=average product of flow and salinity,
$\bar{S}$=mean salinity,
A=cross-sectional area,
$X=AP_1$,
$Y=AP_2$, and
n=number of double tidal cycles.

Solution of Equations 29 and 30 by iterative methods for X and Y will yield P_1 and P_2, the "mixing constants" of the system. It is then possible to calculate the distribution of ultimate oxygen demand under equilibrium conditions and, hence, the oxygen deficiency (c_s-c) in equilibrium with this distribution. Gameson and Barrett (1958) reported fairly good agreement between predicted and observed oxygen profiles.

The quasi-theoretical methods of Ketchum (1951a, b), Arons and Stommel (1951), and Gameson and Barrett (1958) share in common the concepts of an "exchange ratio," "mixing ratio," and "flushing number." Whereas Ketchum, and then Arons and Stommel, restricted their treatment to salinity distributions, Gameson and Barrett, prompted by practical requirements, related mixing ratios to oxygen profiles. This is a significant development.

EXPERIMENTAL (HYDRAULIC MODELS) APPROACH

Studies have been conducted with hydraulic models at the U.S. Army Corps of Engineers Waterways Experiment Station in Vicksburg, Miss. Most of the models were constructed and operated by the corps to study proposed flood-control facilities and (or) structures and channel changes for navigation. Because of their availability, the models have been used for purposes other than those originally intended; for example, the dispersion of pollutants. Because the Delaware River is perennial and has nearly ideal mixing characteristics, it has been studied as a model thoroughly especially for its pollution-dispersion capabilities (Kent, 1958; Pritchard, 1954; and Kaplovsky and others, 1961). In all studies, a conservative pollutant such as methylene blue, released either continuously at a point source or instantaneously as a "slug," has been used. Although model studies have shed considerable light on the regimen of these conservative pollutants, using them to infer the nonconservative regimen is risky. Despite certain potential operational difficulties, one wonders why a nonconservative pollutant such as iodine-131 or synthetic sewage has not been used.

In a study of a conservative pollutant, Kaplovsky and others (1961) used duPont methylene blue ZX in nine continuous tests run for a period of 180 cycles. Tests 1 to 5 corresponded to a fresh-water flow of 3,400 cfs (cubic feet per second), at Trenton, N.J., whereas tests 6, 7, 8, and 9 were for flows of 4,500, 5,500, 7,000 and 8,000 cfs, respectively. Three other continuous tests were run using station 0+00 as the pollution injection point, and then five more tests were run for instantaneous discharges. Each test used only one (not necessarily the same each time) injection point. Samples were withdrawn and analyzed for methylene blue by photometric techniques. In analyzing the data, several important interrelationships were found to occur between fresh water flow, rate of buildup of methylene blue concentration at a particular station, and equilibrium (steady-state) concentration "plateaus." Rate coefficients were derived and relationships established. All these relationships are useful in understanding more completely the prototype in its capacity to assimilate conservative pollutants. The nonconservative or oxygen-demanding contaminants, however, are more important than the conservative contaminants in controlling pollution in estuaries. Although Kaplovsky (1961) recognized the need to understand nonconservative pollutant behavior, he was also dubious about the application of his findings to the nonconservative milieu. He suggested that by "combining the transit time findings with the stabilization rates derived from the prototype studies and employing a geometric progression buildup, coupled with pollution loadings equivalent to the oxygen supplied to the river through reaeration alone, river BOD values similar to those observed during prototype investigations resulted."

STATISTICAL APPROACH

By a statistical analysis (multivariate analysis), or approach, is meant a study of sampling data for correlations between and among easily determined parameters. No attempt is made nor is it possible to define and understand the fundamental relationships in the mixing process. For example, in a specific estuary, a strong relationship may exist between the n-day antecedent fresh-water inflow and the DO profile. If so, a least-squares analysis of the data will yield an equation relating flow to DO. Thus, it becomes feasible to predict the DO profile that will exist for any assumed antecedent discharge. The immediate technical problem of assessing pollution assimilation capacity can be solved, but one would know nothing of the physical details through deriving the equation regardless of the number of independent variables used in the statistical analysis. (Only one variable was used in the above example.) There are entirely too many potential independent parameters (at least 28) operative in an estuary and too many statistical techniques to allow a complete summary here. Probably, each estuary should be appraised before the most appropriate approach can be chosen. (See Blackman and Tukey, 1959, p. 190; Diachishin 1957; Rainwater and Avrett, 1961; and Thomann and others, 1959.)

As an example of one statistical procedure, consider the work of Diachishin, (1957). His interest was in the distribution of coliform bacteria as measured by the MPN, in New York Harbor. It was assumed that the MPN variation was cyclical and, specifically, that it was sinusoidal over a tidal cycle of 12.38 hours. Then,

$$Y=K_1+K_2 \sin \frac{t-t_0}{p}, \tag{31}$$

where

$Y = \log_{10} MPN$,

K_1, K_2 and t_0 are constants, and

p = tidal period = 12.38 hr.

Expanding $\sin \frac{t-t_0}{p}$,

$$Y = K_1 + K_2 \sin \frac{t}{p} \cos \frac{t_0}{p} - \cos \frac{t}{p} \sin \frac{t_0}{p}; \tag{32}$$

hence

$$Y = K_1 + K_3 \sin \frac{t}{p} + K_4 \cos \frac{t}{p}, \tag{33}$$

if,

$$K_3 = K_2 \cos \frac{t_0}{p}$$

$$K_4 = -K_2 \sin \frac{t_0}{p}.$$

The constants of equation 33—K_1, K_3, K_4—are readily found by the standard statistical method of deriving least squares normal equations:

$$K_1 = \frac{\Sigma Y}{N} \tag{34}$$

$$K_3 = \frac{\Sigma Y \sin t/p}{\Sigma \sin^2 t/p} \tag{35}$$

$$K_4 = \frac{\Sigma Y \cos t/p}{\Sigma \cos^2 t/p}. \tag{36}$$

If one has collected sampling data (Y) for specific times (t), equations 34, 35, and 36 may be solved for K_1, K_3, and K_4. These, in turn, are substituted in equation 33 to yield a prediction equation for Y as a function of the continuous variable t. The reliability of equation 33 may be, and should be, assessed by any of several available statistical tests. Diachishin (1957) employed the X^2 test at a 5 percent-significance level.

Presumably as an extension of earlier work, Diachishin (1957) and Thomann and others (1959) discussed the three-cycle analysis of variables in estuaries. The techniques employed did not differ from Diachishin's (1957) previous work; the significant difference was in the number of variables considered. Oxygen concentrations were assumed to vary sinusoidally with three independent cyclic changes—tidal, diurnal, and seasonal—rather than one. The necessary multivariate analysis, although more laborious, presented no new concepts, but it provided a better correspondence between predicted and observed data.

F. H. Rainwater and J. R. Avrett (written commun. 1961) have considered the proper design of a sampling program (not specifically for estuaries), the frequency of sampling, and the reliability of computed results. If a statistical approach is chosen beforehand for the treatment of collected data from an estuary, it is most important to execute the program so that only a minimum of errors will occur.

EVALUATION OF POSSIBLE APPROACHES

Before the various approaches mentioned in the section "Possible approaches to the problem" are evaluated, several critiques of these approaches will be discussed. Pearson (1958), in his discussion of a paper by O'Connor, stated that recently various investigators concerned with the assimilation capacity of receiving bodies of water have employed the Fickian diffusion model to describe mathematically the dispersion of pollutants. He gave the pertinent equation as:

$$\frac{\partial c}{\partial t}=\frac{\partial\left(k\frac{\partial c}{\partial x}\right)}{\partial x}, \tag{37}$$

where

c = concentration of waste or tracer,
t = time,
x = space coordinate, and
k = coefficient of eddy diffusivity.

Equation 37, which Pearson has designated the Fickian diffusion equation, is a simplification of equation 8, which the author has called the "classical diffusion equation." Equation 37 deals with the one-dimensional situation in that only the x coordinate is under consideration. Pearson further stated that "the coefficient of eddy diffusion is really a measure of horizontal turbulence or dispersion affected by fluctuating velocity components and eddy systems. It is a direct measure of the rate of dispersion of a tracer and as such should be proportional to the rate of surface renewal. Moreover, the coefficient of eddy diffusivity is frequently determined by measurement of the rate of surface dispersion of a dye spot or stream." Thomas (1958), on the other hand, termed K the coefficient of longitudinal dispersion. Although both Pearson and Thomas were specifically concerned with the oxygen relationships in fresh water streams rather than in estuaries, their concept of K would be no different for estuaries.

To illustrate that there is neither common agreement in terminology nor in the conceptual notion of K, the following discussion between H. A. Thomas and Pearson (1958) is presented verbatim:

E. A. PEARSON. I would like to ask Professor Thomas if he has any special reason why he chose to call the K in his expressions, or the formulations of Taylor, the coefficient of longitudinal dispersion. Isn't this the same K we have described as the coefficient of eddy diffusion?

H. A. THOMAS, JR. No!

E. A. PEARSON. How does it differ?

H. A. THOMAS, JR. There are cases in which the geometry is very simple where they are identical. In other cases, common in inland waterways, they are not because of irregularities in the boundary of the flow such as dams. In the streams we have investigated, the flow patterns are complex and the anisotrophy or turbulence is marked because of the irregular outline of the solid boundaries of the flow. As a result, the mixing process is very complicated. Fortunately, however, the practical assessment of the overall degree of mixing is simple since it is incorporated in the longitudinal mixing coefficient, K, which is proportional to the standard deviation of the temporal distribution of the tracer cloud.

Now the eddy diffusion coefficient is a different thing entirely. As defined in elementary texts, it is simply the ratio of the shear stress at a point in the fluid divided by the mean velocity gradient at this point in a direction normal to that of the velocity vector. As such it is merely a local parameter. In rivers we have found that it varies markedly from point to point in the flow. The eddy diffusion coefficient only provides a measure of mixing intensity at a point in the flow, but it does not give a measure of the overall degree of mixing in the entire stream. The longitudinal mixing coefficient does this since it represents the effective average of the mixing at all points.

E. A. PEARSON. I think maybe there is a little bit of longitudinal dispersion in our analysis. Oceanographers have been measuring this type of dispersion probably longer than anyone and they use K and call it eddy diffusivity. It includes all the factors that are included in your expression. The oceanographic literature includes expressions of the type that you credit to Taylor for the solution of the general diffusion equation that I put on the board. These pre-date Taylor's equations by many years. In the general oceanographic field they have been concerned with the dispersion phenomena, laterally as well as vertically. They chose to use the term eddy diffusivity.

H. A. THOMAS, JR.: In oceans, K and the eddy diffusion coefficient may for certain purposes be taken as the same thing; in rivers they are not the same. In oceans it, the mixing process, is fairly simple; in rivers it is much more complicated. It is true that if you knew the spatial and temporal distribution of the local diffusion coefficients throughout the river, one could compute the overall effect by an appropriate method of averaging these. In effect this is precisely what is done by tracer test and the calculation of the coefficient K. The eddy diffusion coefficient is a local parameter; the longitudinal mixing coefficient indicates the overall effect.

The foregoing discussion between Pearson and Thomas illustrates at least one divergence of opinion among the experts concerning the concept of K. Other references might be cited to illustrate this point.

Ogura (1959, p. 151) discussed the influence of finite sampling period on mean concentrations. He derived the following equation:

$$\overline{R}_T(\tau)=\int_0^\infty F_\infty(n)\left[1-\frac{\sin^2\frac{nT}{2}}{\left(\frac{nT}{2}\right)^2}\right]\cos(n\tau)dn \tag{38}$$

where

$\overline{R}_T(\tau)$=mean correlation function,
t=independent variable,
$F_\infty(n)$=power spectrum, and
T=time interval.

In Ogura's (1959, p. 151) words: "Eddies whose frequency are lower than 2π divided by T are excluded from the observation during the interval T and only eddies with frequencies higher than 2π divided by T can contribute to the turbulent power." Although in equation 38 Ogura does not deal directly with the coefficient of eddy diffusivity, K, his formulation and comments show that the techniques utilized for empirically evaluating the coefficient of eddy diffusivity in any specific estuary are related to what can be determined.

O'Connor's paper has been discussed by Thomann, McPherson, and Hull (1960). In his examination of the hydraulic aspects of the technique proposed by O'Connor, McPherson objected to the uncertainties in the determination of the "forward advective component." Because of the difficulty of assessing the fresh-water inflow to a tidal estuary with any degree of accuracy, McPherson stated: "It is the writer's firm opinion that oxygen balance analysis will not reach a degree of precision and adequacy urgently needed until the true contribution, large or small, of fresh water inflow can be appraised." Hull, on the other hand, did not criticize the procedures suggested by O'Connor, but rather objected to the mathematical model; that is, Hull objected to the assumption that oxygen is supplied by reaeration and by physical processes of advection and mixing. Specifically, Hull suggested that the evidence to date, not only on the Delaware River Estuary but on many other estuaries as well, strongly supports his thesis that photosynthesis is one of the more important factors that O'Connor neglected. Later unpublished studies by Hull on the Delaware River Estuary have confirmed his contention that photosynthesis is, indeed, highly important, and should not be omitted from any analysis of oxygen dispersion in an estuary. In a rebuttal to Hull's objection, O'Connor (1961) responded that his mathematical formulation omitted consideration of photosynthesis not because of the assumption that it is always unimportant but rather that photosynthesis is too complex a phenomenon to be currently susceptible to mathematical quantification.

In a discussion of Kent's (1958) paper, McPherson (1960) commented on the danger of attempting to apply results obtained with the use of a conservative dye in a hydraulic model to the prototype problem of a nonconservative waste. McPherson said:

One of the objections to the use of continuous dye release results from the model for evaluating the effects of nonconservative wastes arises from the fact that convective mixing is involved in an important way in the oxygen balance while the diffusion of oxygen across the water-air interface is usually regarded as predominantly a molecular type. While the eddy diffusivity, or turbulent diffusion, might be conceived as the product of the intensity and scale of the turbulent mixing process, a realistic allocation of the contributions from molecular and turbulent diffusion would necessarily have to be appraised using discrete measurements made within both model and prototype systems. It is commonly assumed that an analogy exists between molecular and turbulent diffusion. The author's results imply that such an analogy does exist in the instantaneous dispersion of a conservative contaminant in the model. Any literal attempt to stretch this limited analogy to the dispersion of a non-conservative contaminant in the prototype must be considered highly suspect.

McPherson then summarized his discussion and critique of Kent's analysis thus: "It may be fairly stated that the several attempts to establish reasonably exact relationships between wastes loadings, temperature, dissolved oxygen and inflow have fallen far short of the mark."

Several generalizations may be made about the theoretical approaches for determining eddy diffusivity. Kent, O'Connor, and Stommel all utilized a mathematical model in which a one-dimensional aspect is assumed in the dispersion of a pollutant or in the calculation of a DO profile. Admittedly, the problem is sufficiently complicated without assuming two-dimensional or three-dimensional aspects. Yet, in reality, very few, if any, estuaries have conditions approaching such an idealized mathematical model.

All theoretical approaches, as well as quasi-theoretical approaches, have employed the concept of steady state. Steady state, of course, does not infer that there is no variation during a particular tidal cycle. Rather, the inference is that tidal stage, stage-current phase relationship, and fresh-water flow remain constant for various points in the time sequence, wherein prevail similar tidal cycle conditions. Although such an inference is no doubt much more realistic than the assumption of one dimensionality, the estuarine environment is dynamic whether it be between or within tidal cycles.

Statistical approaches are not usually subject to criticism if the investigator is reasonably competent. However, a serious shortcoming of the statistical approach is that correlations of sampling data for a particular time may become invalid for subsequent years because of the change in the loads imposed upon the system. Therefore, the sampling procedure must be a continuing and costly one. Also, because of its very nature, the statistical approach is limited in its applicability; that is, a given statistical approach can be applied to a given estuary to yield a variety of particular solutions, but this procedure and its results cannot be applied to another estuary.

Because estuaries are highly variable in their individual characteristics, anyone who attempts to suggest a specific approach, statistical or otherwise, for application on other estuaries would be presumptuous, to say the least. Furthermore, it becomes apparent that forbidding objections can be made about each of the approaches. However, the seriousness of the pollution problem in the Nation's rivers and tidal bodies is so great that the problem cannot be abandoned because of its complexity. Following are a few remarks about the apparent promise of each approach.

Of the various theoretical approaches, the one proposed by Kent (1958, 1960) seems most sound. Although O'Connor's (1960) paper represents an important piece of research, the author concurs with Hull (Thomann, McPherson, and Hull, 1960) that O'Connor has not considered a sufficient number of factors to enable him to obtain an accurate picture of the oxygen balance of an estuary. Kent's approach, therefore, seems to be on firmer ground, even though the successful solution of his equations would still require that the investigator relate the contaminant load to the resultant oxygen distribution in the estuary. The author emphasizes the desirability of more research on the nature of eddy diffusivity, K, and methods for its proper evaluation. Attention should then be directed toward establishing a relationship between calculated contaminant loading and resultant oxygen distribution. Perhaps the latter relationship would be susceptible to statistical treatment. Studies for the eddy diffusivity, K, might be directed in one of two possible approaches: (1) a deductive approach based upon analytical work or (2) an inductive approach which would involve experimental data obtained from either the prototype or from a hydraulic model.

Of the various quasi-theoretical treatments, the author believes that Gameson's (Gameson and Barrett, 1958) is the most promising. The combination of a statistical treatment and a logical mathematical model has resulted in significant successes in computing pollution distributions in the Thames River Estuary. Such an approach should be more widely employed in the United States.

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The author also believes, as does McPherson (1960), that the use of hydraulic models employing an instantaneous dye can only result in relationships that are pertinent to the distribution of conservative wastes in the prototype. As previously suggested, future model studies at the U.S. Army Corps of Engineers Station in Vicksburg, Miss., should, if possible, utilize nonconservative wastes as tracers.

Agencies or individuals faced with the task of immediately determining pollution assimilation capacities for administrative purposes would probably do well to use statistical approaches; others engaged in research should attempt to extend the theoretical approaches to the problem. Although statistical methods would seem to offer only limited applicability, such an objection should not concern most administrators of water pollution programs, as the general utility of an approach is usually only of academic interest. Because of the multiplicity of parameters involved in the study of estuaries, any statistical treatment that yields reasonably accurate results is preferable to a rigorous mathematical approach in which so many of the independent variables are neglected that inaccuracies result.

NUMERICAL EXAMPLE: ST. JOHNS RIVER ESTUARY AT JACKSONVILLE, FLA.

The author studied estuarine data provided by the city of Jacksonville, Fla. A synopsis of that study (Pyatt, 1959) is presented to illustrate the statistical approach for determining assimilative capacity in tidal bodies of water. Again, the reader is cautioned against assuming that the specific procedure herein reported for the St. Johns River Estuary is necessarily that which should be used on other estuaries.

DESCRIPTION

Florida derives its name from the Spanish expedition of 1513 led by Ponce de Leon in search of the legendary fountain of youth. Whether Florida was so named because of the landing there coinciding with "Pascua Florida" (Feast of Flowers) or the proliferation of flowers along the coast is unknown (Federal Security Agency, 1951). Settlement of the area was slow and colonial conflict and Indian uprisings were common. Difficulties with the Indians persisted even after Florida became part of the United States.

The small index map on plate 1 shows the relationship of the St. Johns River basin to Florida and to the Southern States. Probably the most striking feature shown on this map is the occurrence of a a tremendous number of lakes and marshy areas. These water areas account for 12 percent of the watershed. Of course, this condition is not unusual for Florida, but it is not prevalent elsewhere in the United States. The river has its headwaters about 50 miles north of Lake

Okeechobee in the great grassy plain and then flows north 250 miles to Jacksonville before cutting sharply eastward for 26 miles to the coast.

Agreement on the size of the watershed is difficult because of the numerous sinks, fissures, springs and subsurface rivers, some of which are saline. Saline ground-water discharges probably result from the aquifers. As might be supposed, this situation complicates hydrologic study, at least in the upper reaches of the river.

Normally, the St. Johns River is tidal to a point above Palatka (pl. 1), 80 miles above the mouth. This condition has resulted from extremely flat riverbed gradient. At times of spring tide, the river has been reported to be tidal for about 283 miles (Federal Security Agency, 1951).

Climate, besides being influenced by the usual factors, is also influenced by the Gulf of Mexico and the Atlantic Ocean. Florida is an area of significant lateral convergence of onshore winds and, hence, is subject to more or less spontaneous rainfall. Around the basin, average annual rainfall ranges from 48 to 54 inches with the summer months receiving more than their proportionate share. Mean annual temperatures range from 69° F at Jacksonville to 72° F in the southern part of the area with a range of about 28° F in the expected extremes.

The basic population of the St. Johns River basin in 1940 was 418,482 and in 1950 was 580,710 (Federal Security Agency, 1951). The 1950 population of Jacksonville was 200,000 and is currently (1959) about 225,000. Jacksonville is the largest city in the St. Johns River basin and together with Orlando (51,826) and Gainesville (26,577) (Federal Security Agency, 1951) accounts for nearly all the urban influence. There are a few basin communities in the 5,000–10,000 population range. Because the most important economic activity in Florida is tourist trade, of which the St. Johns River basin receives its share, population can be expected to fluctuate within any given year. Other economic activities within the basin include the raising of citrus crops, canning (especially frozen concentrates), beef production, and industries using forest materials.

Of the 86 public water supplies in the St. Johns River basin, 84 utilize ground water (Federal Security Agency, 1951). Subsurface water is also used rather extensively for irrigation. As might be supposed, there are localized problems of overdraft of ground-water resources, as well as some pollution problems resulting from the fairly common subsurface sewage disposal in those areas not sewered.

The St. Johns River is dredged to a depth of 34 feet (mlw) between Jacksonville and the Atlantic Ocean, and therefore that city is qualified as an important port. In 1948 water-borne commerce at Jacksonville was 2,411,000 tons (Federal Security Agency, 1951).

A channel 13 feet deep is maintained between Jacksonville and Palatka 12 feet from Palatka to Sanford, and 5 feet from Sanford to Lake Harney. (See pl. 1.) Shell fishing formerly was important in the St. Johns River, but today the river is used primarily for navigation. The upstream lakelike reaches prevalent around Jacksonville are used for recreational purposes such as boating but not swimming.

SAMPLING PROGRAM OF 1954–55

The city of Jacksonville discharges its sewage through some 70 outfalls into the St. Johns River. In general, the sewage is discharged in a raw state; only a small fraction of the total load on the river is subjected to primary sewage treatment. As indicated in plate 1, the estuary at Jacksonville is constricted and has a high degree of total curvature; turbulence, therefore, seems to cause almost immediate mixing of the major releases from the 70 outfalls and the river water. In 1954 and 1955, the city of Jacksonville, in cooperation with a number of other official agencies, undertook the collection of large quantities of data to assess the impact of the organic wastes from the city of Jacksonville on the receiving estuary.

For the studies of 1954–55, eight permanent sampling stations were selected (pl. 2). Each station was occupied continuously for 12 hours. Such a 12-hour sampling period was designated a "phase investigation." Five phase investigations were made and are referred to as phase 1, phase 2, phase 3, phase 4, and phase 5. They were conducted at different times of the year, and so the effect of such variables as tidal stage, weather, location of salinity front, and fluvial discharge could be considered. Dates for the studies weie as follows:

Phase	*Date*
1	Oct. 25, 1954
2	Feb. 14, 1955
3	May 23, 1955
4	Aug. 5, 1955
5	Nov. 16, 1955

The studies were exceptionally thoroughly planned and involved the participation of many individuals and agencies. Including salaries and overhead, as well as direct expenditures, the five phase investigations cost about $125,000 (E. T. Owens, oral commun., 1959.) Comparatively few estuaries, notably the Delaware and East Rivers, have been subject to such careful sanitary-engineering scrutiny.

The eight sampling stations (pl. 2) extended from Mayport (sta. 1) to Orange Point (sta. 8), a distance of 34.57 statute miles. Main Street Bridge (sta. 5) was considered the primary reference. Because each station was located approximately at midchannel, the cooperation of the navigational interests was required. A knowledge of the vertical distribution of pollution indices was needed; accordingly, three depths were sampled at each station: depth A, at 2 feet below the surface; depth B, at one-third of total depth below the surface; and depth C, at 2 feet above channel bed. Station 5 (Main Street Bridge) was an exception to this arrangement: at station 5, depth A was 2 feet below the surface, depth B was at 38 feet (about middepth), and depth C was at 73 feet (several feet above the bottom).

For all stations, except No. 5, the following factors were determined:

Depth	*Factors determined*
A	Chloride content, turbidity, pH.
B	Water temperature, coliform MPN, dissolved oxygen, 5-day BOD, chloride content, turbidity, pH.
C	Water temperature, chloride content, pH.

In addition, tidal phase, current direction, and air tenperature were recorded. Station 5, considered to be of primary importance, was sampled at depths A, B, and C for all the factors listed for depth B.

In each phase study each of the eight stations was occupied from 0700 to 1900 hours. At each station, a sample was taken at depth A, B, and C every hour on the hour during the 12-hour investigation. Samples were refrigerated or fixed and were collected by speed boat a number of times during the 12-hour period and were then taken to the laboratory for analysis. Concurrently, the U.S. Geological Survey ran velocity tranverses at Main Street Bridge so that net discharge would be known more accurately than the multiple-correlation technique afforded. The multiple-correlation technique is a correlation among three stage recorders.

ANALYSIS OF SOME INDEPENDENT PARAMETERS

HYDROLOGY

Data obtained since February 10, 1954, when the U.S. Geological Survey installed a system of gages for continuous recording of daily discharges at Jacksonville, show that the flow at Jacksonville over a 24-hour period is often negative, or in the upstream direction. This phenomenon may be explained by the flat riverbed gradient and the broad, shallow estuarine configuration; water goes into storage upstream at times of high tidal stage (not necessarily high tide) when the hydraulic gradient is reversed. The stored water, of course, is subsequently released. For highly variable discharge, often in the negative direction, the pollution distribution may have almost nothing to do with the corresponding instantaneous flow. Antecedent discharge is usually important in explaining pollutant distribution, and it is especially so at Jacksonville.

The U.S. Geological Survey flow data at Jacksonville from February 10, 1954, to October 14, 1957, were analyzed. Such a period of record is extremely short and hence of questionable statistical reliability. However, the 44-month record, with its more than 1,200 daily discharges provides a sounder basis for estimating the discharge of the St. Johns River at Jacksonville than was previously possible by studying the few velocity traverses made at the time of sampling. Table 1 gives the results of a statistical analysis.

TABLE 1.—*Discharge at Jacksonville, February 10, 1954, to October 14, 1957*

(1) Month	(2) Mean daily discharge (cfs)	(3) Standard deviation (cfs)	(4) Probability of a negative discharge (percent)
January	7, 226	11, 826	26. 5
February	4, 615	11, 415	34. 0
March	3, 161	13, 161	40. 0
April	867	12, 467	47. 0
May	1, 838	10, 638	43. 0
June	2, 467	11, 167	42. 0
July	3, 774	9, 874	35. 0
August	3, 548	11, 650	38. 0
September	2, 933	10, 800	39. 0
October	5, 476	11, 076	31. 0
November	9, 244	10, 244	18. 0
December	8, 150	13, 350	27. 0

The mean of column 2 (the mean monthly flow) is 4,441 cfs (cubic feet per second). However, because water years 1954 and 1955 were slightly deficient in rainfall and water year 1956 was considerably deficient, the long-term mean at Jacksonville most certainly is greater than 4,441 cfs. To determine the long-term mean, a correlation procedure was employed.

Attempts were made to correlate the discharge at Jacksonville with ground-water levels, upstream gage records, and rainfall records. It was decided that the best correlation could be made with the data of U.S. Geological Survey gage on the St. Johns River near DeLand (pl. 1). Records for the DeLand gage are available (as of 1959) from January 1934 to September 1956 (U.S. Geological Survey, 1958). The drainage area at DeLand is 2,960 square miles, and the 22-year mean discharge is 3,083 cfs. Maximum daily discharge was recorded on October 15, 1953, as 17,100 cfs whereas the maximum reverse (negative) flow was 1,630 cfs on June 9, 1955. The mean daily discharge was used in pairing the flow at DeLand with the flow at Jacksonville by months. These paired values were correlated by least squares to give:

$$y=0.22x+45 \tag{39}$$

where

y=Jacksonville flow,

x=DeLand flow.

Although the 22-year mean at DeLand is 3,083 cfs, the synthesized 22-year mean at Jacksonville is 6,895 cfs in accordance with equation 39. Mean discharge at Jacksonville has been rounded off to 7,000 cfs. Determination of the correlation coefficient for equation 39 yielded +0.61. Although such a correlation coefficient leaves something to be desired, the 7,000-cfs mean discharge probably is entirely consistent with the short-term rainfall value of 4,441 cfs. Even if 7,000 cfs is not exact, subsequent sections show that for the St. Johns River a fairly large standard error can be tolerated for purposes of appraising pollution assimilative capacity.

It was necessary to establish an index antecedent discharge for each of the five phase investigations. As pointed out, the discharge on the date of a phase had no utility, for it failed in every way to correlate with observed longitudinal chlorinity distributions. Various schemes—such as the 5-day, 10-day, and 15-day antecedent discharge—were employed to establish the index antecedent discharge. Also, weighting factors of diminishing intensity were applied backwards from the date of each phase. All schemes were studied in context with the longitudinal chlorinity distribution, and the best index proved to be the 30-day mean discharge preceding the date of each investigation. In succeeding pages, the 30-day antecedent discharge is termed "the discharge" corresponding to each phase. The discharges, as taken from U.S. Geological Survey records, were as follows:

Phase	*Discharge (cfs)*
1	7, 509
2	12, 985
3	2, 528
4	3, 629
5	4, 675

CHLORINITY DISTRIBUTION

SOURCE SALINITY

Because most pollution computations employ distributions of salinity (or chlorinity) in an estuary as a measure of eddy diffusivity, it is important to have a knowledge of the salinity source or the salinity of the sea water that enters into the exchange dynamics. A simple measurement of salinity at the mouth of an estuary is not sufficient, for the fresh-water discharge influences the measurement. The best procedure seemingly is to determine salinity 10 miles or so offshore from the mouth of the estuary. Such a distance, in any specific estuary, is necessarily a function of the mean fresh-water discharge.

During each season in 1953, the Fish and Wildlife Service of the U.S. Department of Interior made a cruise in the waters off Florida and Georgia. Among other data reported obtained during the cruises was the salinity at several stations at various depths (Anderson and Gehringer, 1956a, b and 1957a, b). The stations that seemed most representative of source water for the St. Johns River were designated 24, 25, and 26 (Anderson and Gehringer, 1956a). Mean salinity of source water at these stations for each of the four cruises is given on table 2.

TABLE 2.—*Mean salinity, in parts per thousand, of source water*

(1) Month	(2) Salinity	(3) Chlorinity
February	36. 15	20. 00
May	36. 27	20. 08
July	35. 95	19. 89
October	35. 73	19. 78

Column 3 was calculated from the relationship established by an international commission in 1902 and cited by Sverdrup, Johnson, and Fleming (1942, p. 51):

Salinity in parts per thousand

$$= (1.805) \text{ (chlorinity in parts per thousand) } + 0.03 \qquad (40)$$

MIXING

The eight sampling stations employed for the phase investigations (pl. 2) were approximately at midchannel. Samples were collected in the vertical at each station but not laterally; thus, although little is known of the lateral salinity gradient, it seems reasonable to suppose that one does exist. The data that were collected, however, provide a basis for plotting the vertical and longitudinal distribution of chlorinity. This has been done in two ways: first, for each phase at each station the chlorinity for each of the three depths has been plotted against time for an entire tidal cycle (see figs. 1–8 for plots of phase 1; see Pyatt, 1959 for plots of other four phases) and, second, for each phase at a particular sampling time, chlorinity has been plotted for all stations and all depths (see figs. 9–13 in this report for phase 1; for the other four phases, see Pyatt, 1959). Figures 9–13 were obtained by interpolation of plotted grid points.

Analysis of the plotted chlorinity data, although helpful in understanding the behavior of the St. Johns River in the Jacksonville area, cannot be expected to yield a definitive explanation for mixing phenomena. Nevertheless, some generalizations are suggested.

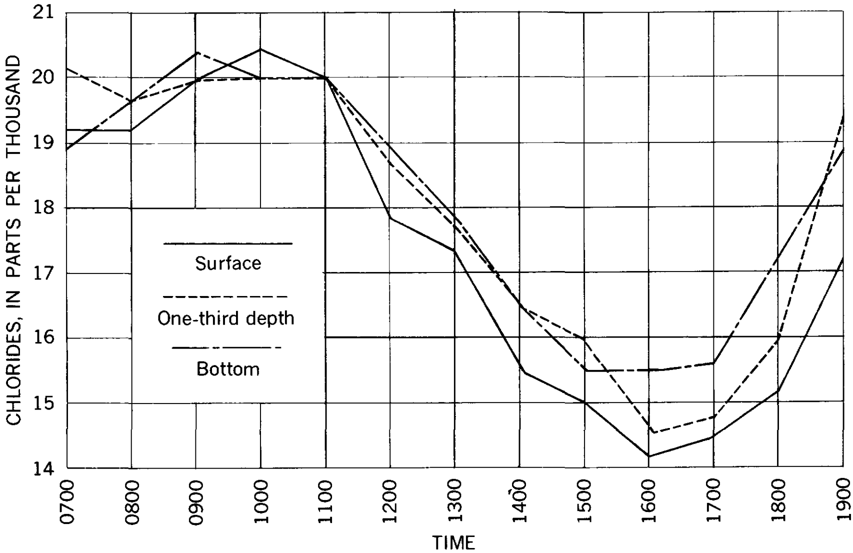

FIGURE 1.—Chlorinity-time-depth curves, phase 1 at station 1, Oct. 25, 1954.

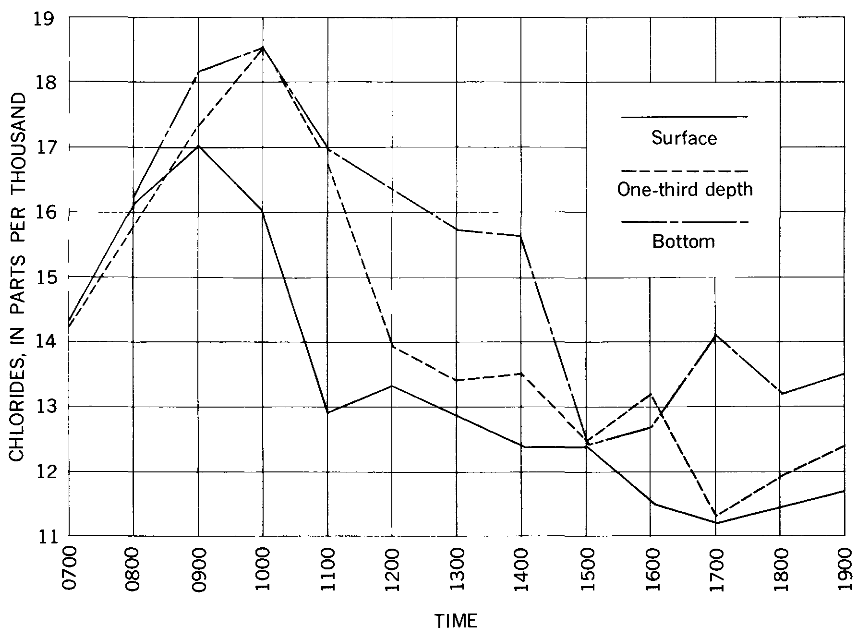

FIGURE 2.—Chlorinity-time-depth curves, phase 1 at station 2, Oct. 25, 1954.

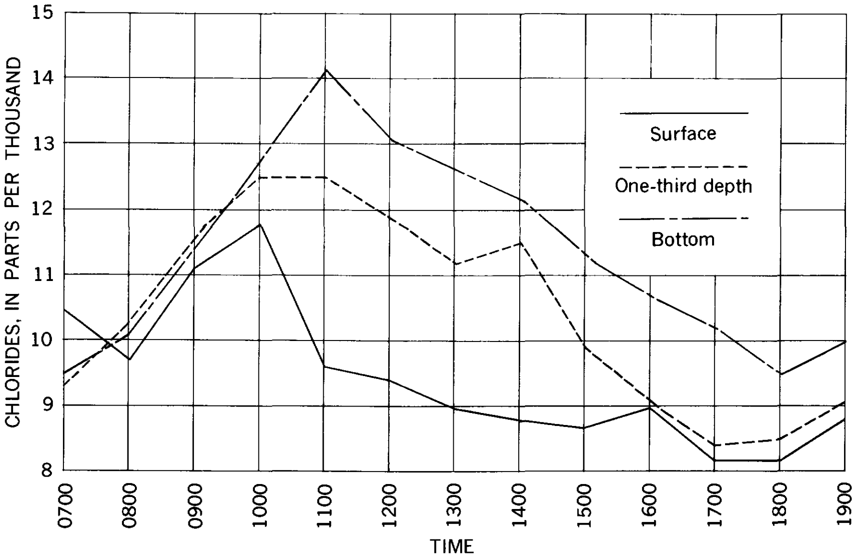

FIGURE 3.—Chlorinity-time-depth curves, phase 1 at station 3, Oct. 25, 1954.

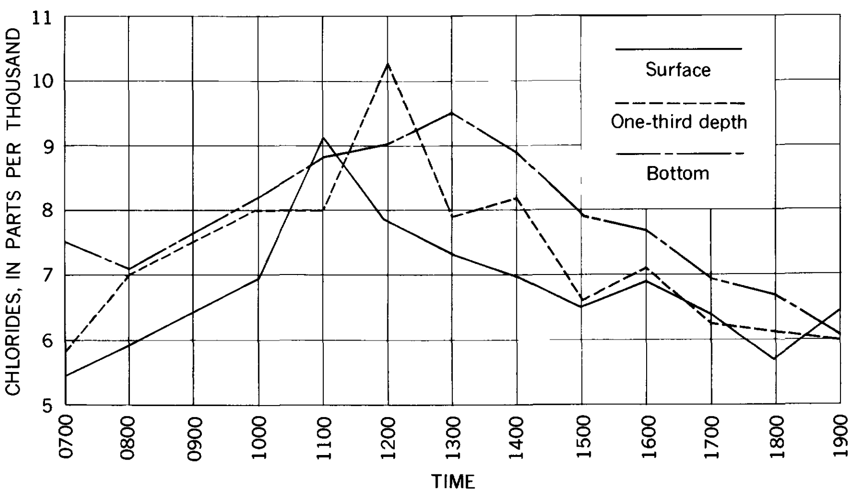

FIGURE 4.—Chlorinity-time-depth curves, phase 1 at station 4, Oct. 25, 1954.

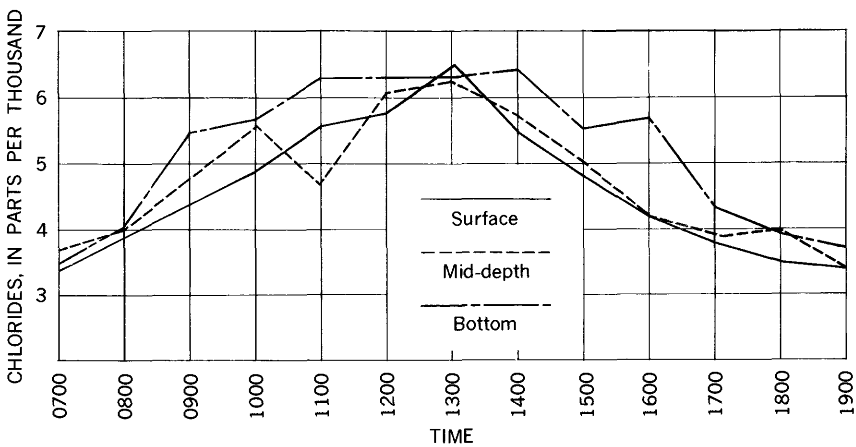

FIGURE 5.—Chlorinity-time-depth curves, phase 1 at station 5, Oct. 25, 1954.

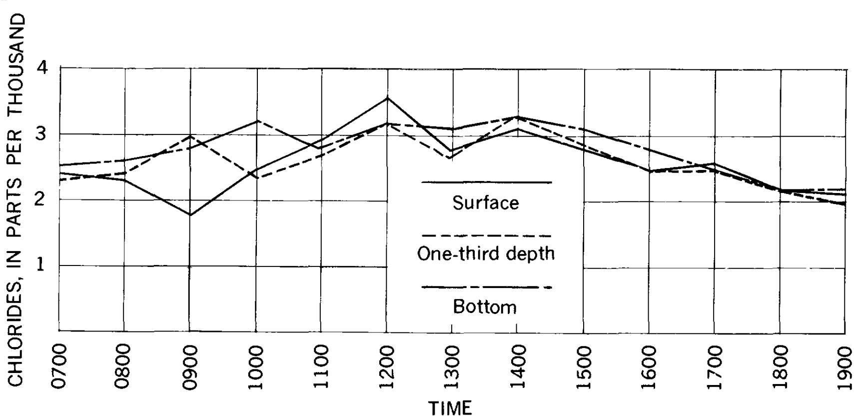

FIGURE 6.—Chlorinity-time-depth curves, phase 1 at station 6, Oct. 25, 1954.

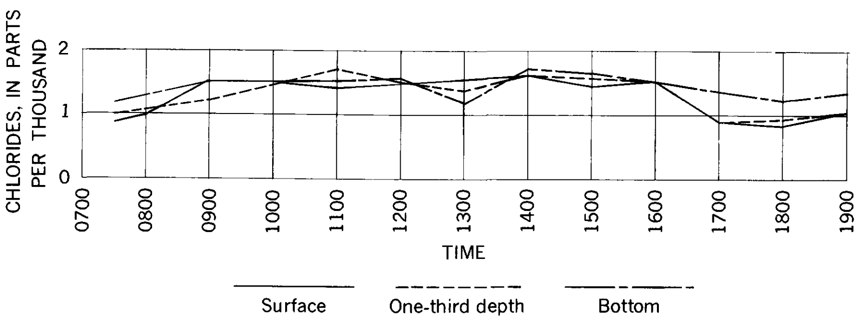

FIGURE 7.—Chlorinity-time-depth curves, phase 1 at station 7, Oct. 25, 1954.

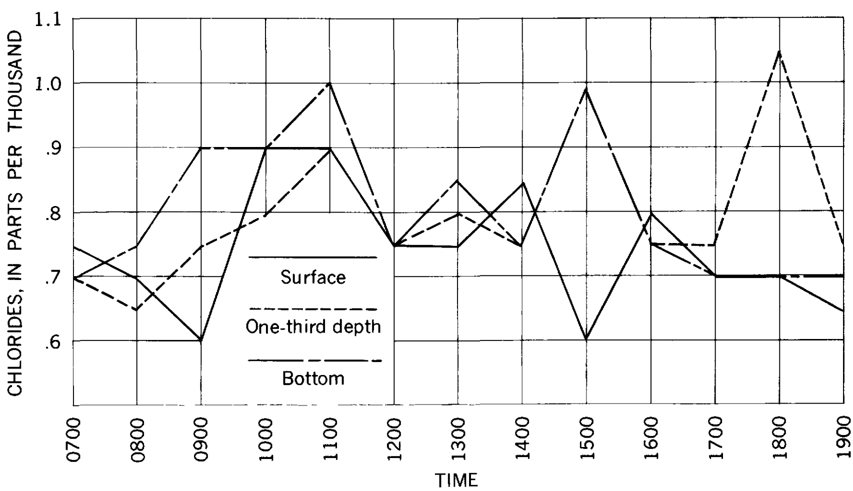

FIGURE 8.—Chlorinity-time-depth curves, phase 1 at station 8, Oct. 25, 1954.

At a particular station, chlorinity distribution at the three depths tends to be homogeneous during the latter part of flood, but stratification becomes apparent at high slack. This phenomenon is especially apparent in the plots for stations 1, 2, and 3 but occurs also at the upstream stations. Mixing is good during late flood stages but at high slack bottom waters underrun the quiescent surface waters. On ebb, underrunning occurs at low slack. O'Brien (1952) in his study of density currents in the Columbia River noted a similar pattern.

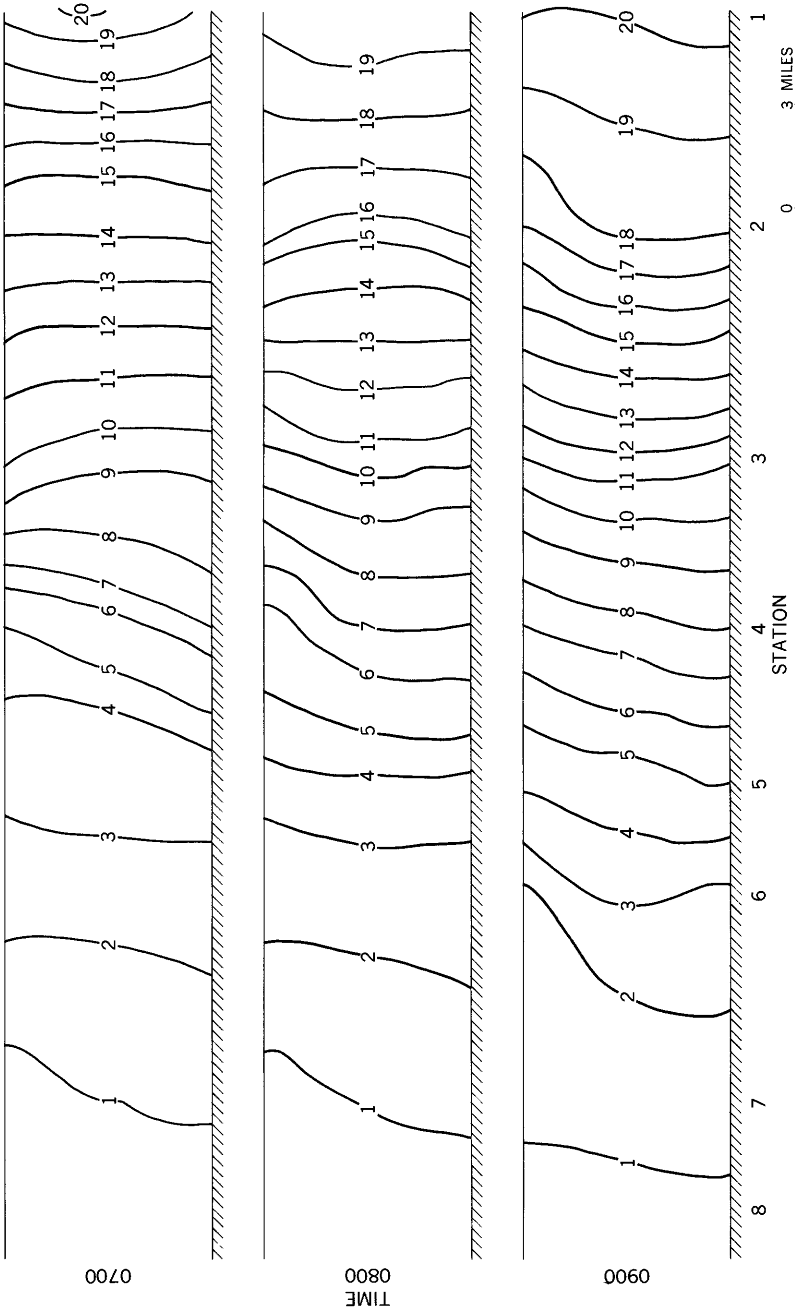

FIGURE 9.—Longitudinal isochlors, phase 1 at stations 1–8, 0700–0900 hours, Oct. 25, 1954. Isochlor intervals for figures 9–13 are expressed in parts per thousand.

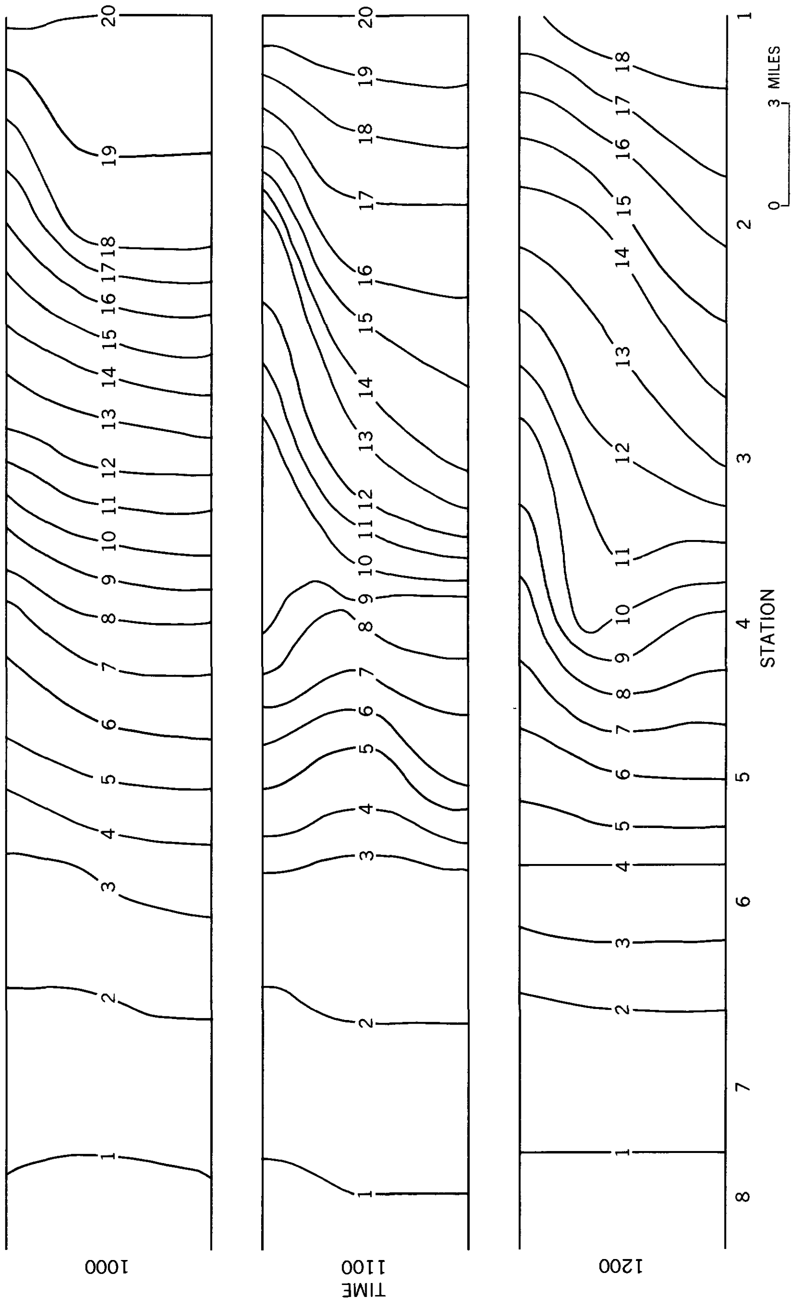

FIGURE 10.—Longitudinal isochlors, phase 1 at stations 1–8, 1000–1200 hours, Oct. 25, 1954.

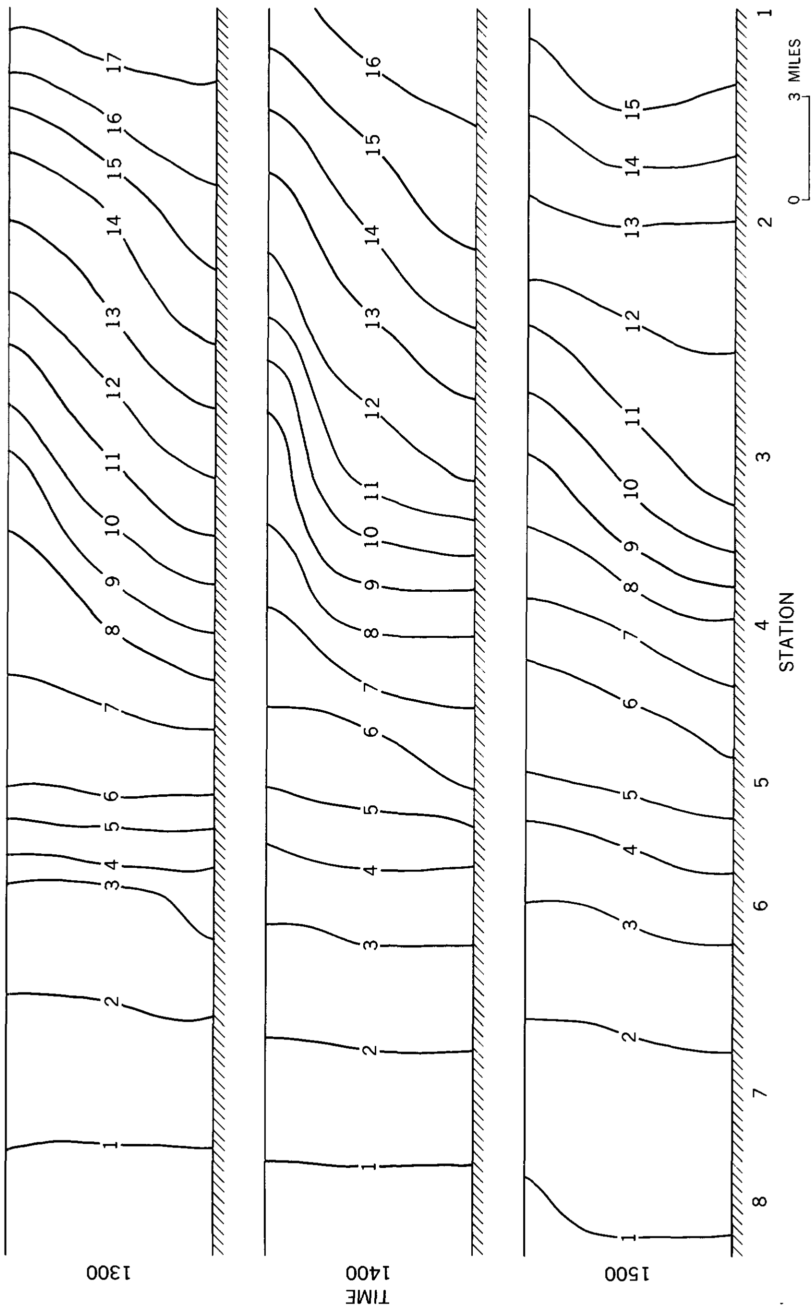

FIGURE 11.—Longitudinal isochlors, phase 1 at stations 1–8, 1300–1500 hours, Oct. 25, 1954.

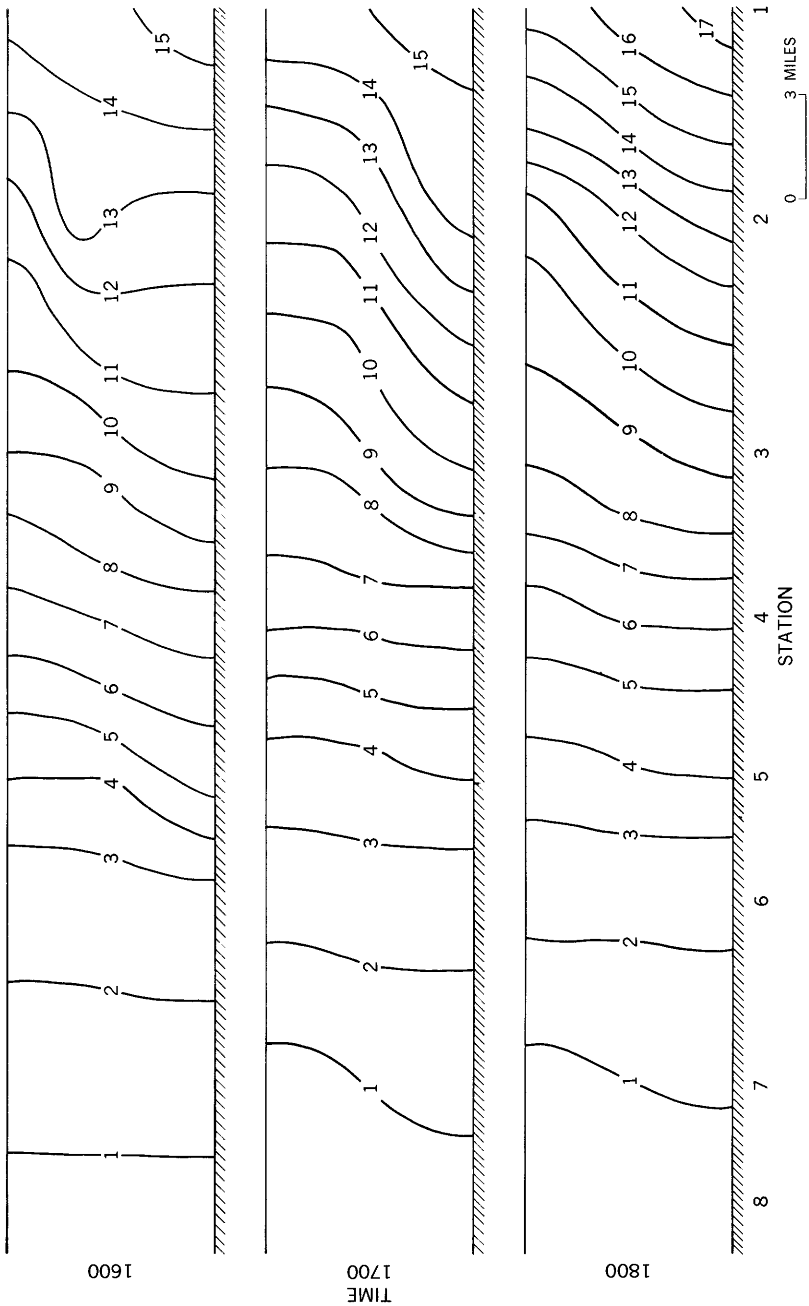

Figure 12.—Longitudinal isochlors, phase 1 at stations 1–8, 1600–1800 hours, Oct. 25, 1954.

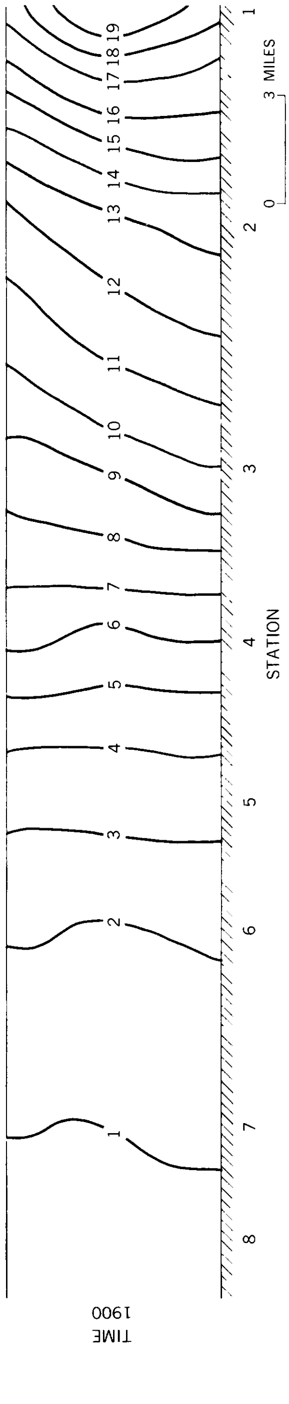

FIGURE 13.—Longitudinal isochlors, phase 1 at stations 1–8, 1900 hours, Oct. 25, 1954.

The St. Johns River does not exhibit a well-defined salinity front. There is no salt wedge such as exists in the Mississippi River near New Orleans, but there is a definite longitudinal translation and vertical distortion of the chlorinity distribution in response to varying fresh-water discharge. The five phase studies indicate that about 4,000 cfs is the critical discharge. For example when the mean fresh-water flow remains at less than 4,000 cfs for 10 to 15 days and the chloride concentration at Main Street Bridge (station 5) reaches 6 parts per thousand the chlorinity distribution translates upstream until vertical mixing becomes fairly good at all stations. On the other hand, discharges of more than 4,000 cfs for a similar period of time will translate the chlorinity distribution downstream, and vertical stratification is then appreciable. Thus, phases 3 and 4 show good vertical mixing whereas phases 2 and 5 show stratification. Phase 1 falls between these extremes. According to Pritchard's (1952) scheme of estuarine typing, the St. Johns River seems to fluctuate between type A and type B.

The degree of vertical homogeneity for all phases was greatest at station 1 and decreased progressively at stations 5, 2, 4, and 3. This characteristic can be inversely correlated with cross-sectional area; the smaller the cross-sectional area, the greater the degree of homogeneity.

The chlorinity determinations made in the phase studies lead to conclusions that are substantiated by the work of the U.S. Army Corps of Engineers (1947). To the author's knowledge, no other systematic chlorinity data at Jacksonville are available for study.

CHLORINITY-DISCHARGE CORRELATION

Stommel and Farmer (1952) suggested that upstream constrictions may exert a "control" action and that an estuary may be "over mixed" as a consequence. They present equations showing that if a straight-line plot of discharge in relation to $\Delta\bar{s}/s$ obtains on log-log paper, an estuary may be termed "over mixed." $\Delta\bar{s}$ was defined as the mean difference in the top and bottom salinity, and s was defined as the bottom salinity. Discharge in relation to $\Delta\bar{s}/s$ was plotted for the St. Johns River. By least-squares analysis, equations were derived relating antecedent discharge q to $\Delta\bar{s}/s$. At station 1, for example, the correlation coefficient was computed as 0.913. This high degree of correlation indicates that Stommel and Farmer were probably correct in contending that a control section exists in the St. Johns River; the narrow constriction at Main Street Bridge is possibly that control.

METEOROLOGY AND TIDAL PHENOMENA

EFFECT OF WIND UPON TIDAL STAGE

The data collected by various agencies during the phase investigations included continuous stage-recorder strips from the U.S. Naval Air Station above Jacksonville. Stage is a function of several factors such as the stage existing at the estuary mouth, the tidal phase relationship between the mouth and the upstream point, and the fresh-water discharge. In addition, fluctuations in barometric pressure and winds modify the stage. These variables were separated so that their relative importance could be assessed.

The predicted mean-tide elevation at Mayport (Station 1) was taken from published data of the U.S. Coast and Geodetic Survey (issued annually) for each day of the year. Mean sea level was subtracted from those data so that the results would reflect the predicted mean daily departures from mean stage at Mayport. The daily values were then multiplied by the ratio of mean tidal range at the Naval Air Station to mean tidal range at Mayport so that the results would reflect the predicted mean daily astronomical departures from the mean gage at the Naval Air Station.

The next step consisted of studying the continuous stage-recorder strips at the Naval Air Station over a 1-year period. The data for 1953 were used because they were readily available. By averaging the two high-tide readings with the two low-tide readings, the actual mean stage that existed at the station for each day of 1953 was calculated.

The third step was to analyze the Weather Bureau Local Climatological Data (LCD) sheets for the Imeson Airport near Jacksonville for 1953. Actual daily departures, expressed in feet of water, of mean barometric pressure from mean annual barometric pressure were computed from the four daily readings reported on the LCD sheets.

No convenient way of separating the runoff parameter was found. Therefore, the barometric-pressure effect and the astronomical effect (with due regard to algebraic signs) were subtracted from the actual daily mean stage for the Naval Air Station. Mean gage value was then subtracted. The resultant daily values, reflecting stage due to winds, were termed "wind setup," but wind setup as used here includes the minor effect of fresh-water discharge.

When the processed data were studied, the following conclusions were reached:

1. The predicted departure of daily mean tide from annual mean tide at the naval air station ranges from −0.07 to +0.10 feet.
2. Fluctuations in barometric pressure produce mean stage departures at the Naval Air Station from −0.43 to +0.50 feet.
3. Wind setup ranges from −0.92 to +3.20 feet.

4. Winds at the Imeson Airport average 8.4 mph; significant wind setup usually occurs at sustained velocities of more than 7.0 mph.
5. Winds from south to west account for most precipitation, but there are exceptions, especially during the summer months when winds from northeast to southeast may be associated with precipitation.
6. Winds from 0° to 180° (north to south in a clockwise direction) cause an increase in stage at the Naval Air Station, whereas winds from 180° to 360° cause a decrease. This statement, however, must be somewhat qualified; for example, there is a limit to the amount of water that can set up by northeast winds. Northeast winds of more than 10 mph may cause a large increase in stage at the station, but continued northeast winds of gradually diminishing intensity will allow the stage to fall. Thus, stage fluctuation depends upon where the stage is initially and upon the intensity, duration, and direction of wind. Stage fluctuations caused by runoff from storms may obscure the wind-stage relationship. For example, the wind pattern at the Naval Air Station may be such that, although one would expect a decrease in stage, an increase would actually be observed on the hydrograph because of the amount of runoff received.

Thus, wind setup has an extremely important effect on the tidal stage of the St. Johns River. Other factors in order of importance are barometric pressure and tidal variations. The effect that winds have upon the mixing pattern of the St. Johns River cannot be deduced from these data, but if the St. Johns River is typical of most estuaries, however, winds that cause setup upstream (here, winds from 0° to 180°) imply surface current directed upstream and a concurrent return flow along the stream bottom of about one-tenth magnitude. A study of climatological-stage data for the five phase investigations provided evidence supporting this observation.

TIDES AND CURRENTS

Knowledge of the currents and the tide-current relationship in the St. Johns River results almost entirely from the work of Haight (1938) of the U.S. Coast and Geodetic Survey in 1933 and 1934, and the Hydraulic Model Investigations of the U.S. Army Corps of Engineers (1947) in 1945 and 1946.

If one looks in the direction of the prevailing flow, the higher velocities occur on the right and the current turns first on the left. This evidence suggests that the Coriolis acceleration term is significant in the dynamics of the St. Johns River and indicates that a strong lateral salinity gradient occurs frequently.

Using a current pole and a Price current meter, Haight (1938) and his party obtained instantaneous longitudinal profiles of the surface currents. At some stations, lateral and vertical distribution were also established. The party noted that the currents were considerably more erratic than the variations in the tidal range. As expected, velocities were small where the river is broad and larger in constricted sections. From Mayport (sta. 1) to Jacksonville (sta. 5), the current ranged from 1 to 3 knots and was generally less than 1 knot above Jacksonville. Haight noted that the currents in the St. Johns River are at times modified considerably by winds and freshets. For example, northerly and northeasterly winds increase the velocity and duration of the flood stream and decrease the ebb whereas southerly and southwesterly winds have a reverse effect.

Figures 14 and 15 have been reproduced after Haight (1938), who concluded:

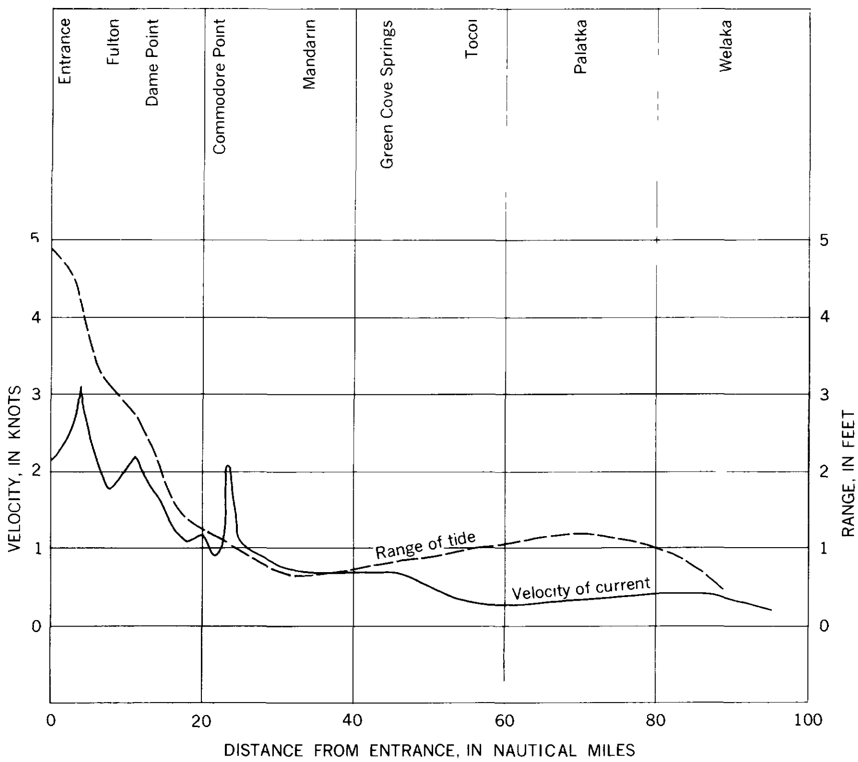

FIGURE 14.—Range of tide and velocity of current, St. Johns River, Fla. (After Haight, 1938.)

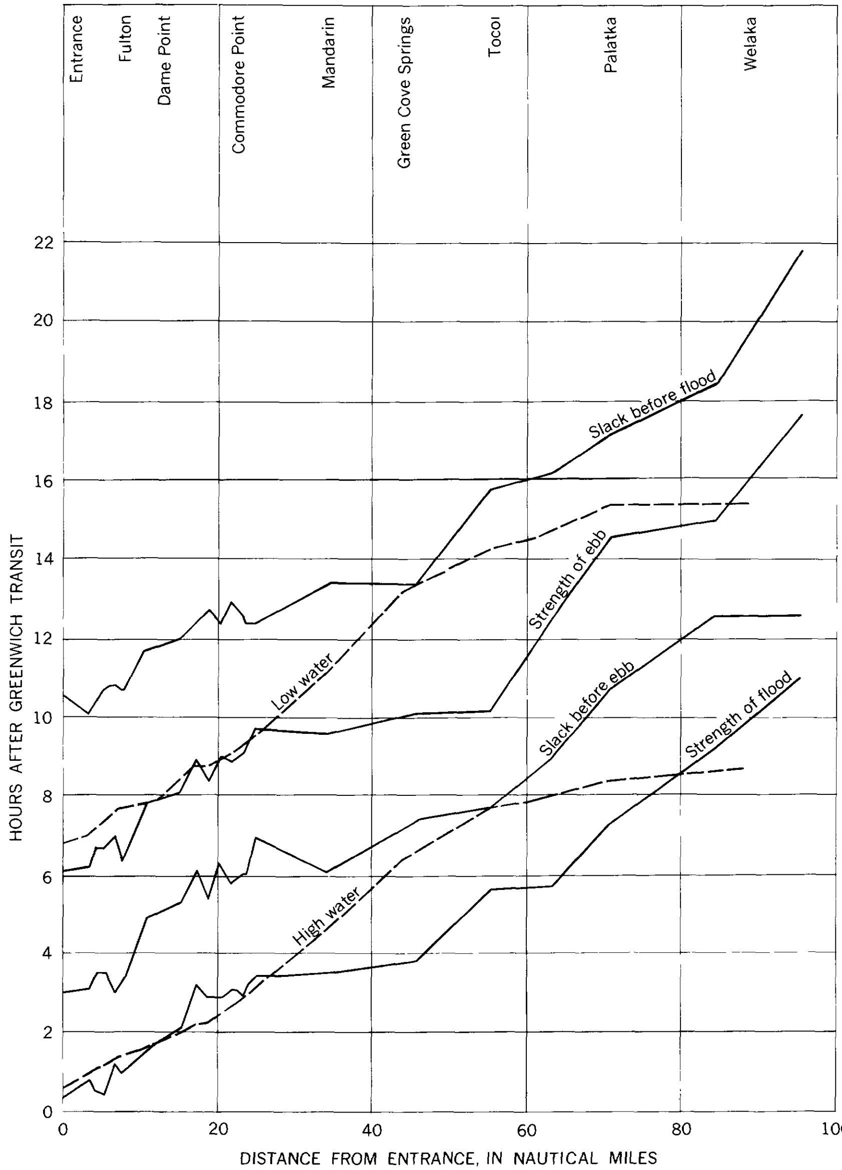

FIGURE 15.—Tide and current intervals, St. Johns River, Fla. (After Haight, 1938.)

The curves show that the time relation of current to local tide varies from place to place along the river. In the lower portion of the river the strengths of flood and ebb occur near the times of high and low water respectively. Above Jacksonville the current becomes rapidly earlier with respect to the local tide and fifty miles from the sea the strengths of flood and ebb precede the high and low waters by about three hours, the slack waters occurring near the times of the highs and lows. Advancing up the river the current occurs later and later with respect to the tide and at a distance of 85 miles from the sea the strengths again come at about the times of high and low tide, which is the same relationship that exists at Jacksonville.

The time relations shown by the curves of fig. 17 give further evidence of the existence of a stationary tide wave in combination with a progressive wave, for the time relation of current to tide in the lower part of the river and also between Palatka and Welaka are approximately those of a progressive wave movement—whereas the time relation in a region about midway between these two localities is that of a stationary wave.

ORGANIC LOADING AND POINTS OF LOAD APPLICATION

A rather thorough study of population records of Jacksonville and the surrounding area, together with extrapolation into the future, serves as a primary basis for the computation of organic loading to the estuary. Although the per-capita quantity and quality characteristics of sewage may vary somewhat about the United States, the most critical factor in any assessment of organic-waste loading is population. Using data on file in the Office the City Engineer in Jacksonville, Fla., the author estimated certain quantity and quality characteristics of the city sewage at 10-year intervals for the period 1960–2010 (table 3).

TABLE 3.—*Quantity and quality of Jacksonville, Fla., sewage*

(1) Year	(2) Flow (million gallons per day)	(3) Biochemical oxygen demand (lb per day)	(4) Suspended solids (lb per day)
1960	24.8	56,250	63,000
1970	32.8	71,250	79,800
1980	43.8	87,500	98,000
1990	55.6	105,000	117,600
2000	68.0	121,250	136,000
2010	77.5	133,750	150,000

Jacksonville employs about 70 outfalls to discharge its sewage. These outfalls range from 8 to 54 inches in diameter, although the majority range from 10 to 20 inches. Most of the outfalls discharge sewage several hundred feet offshore, but some extend to 1,000 feet. If one used the present multiple-outfall system at Jacksonville as a basis to compute pollution, he would be faced with a nearly hopeless task of specifying conditions of pollution distribution from each outfall. However, one might consider the effect at each outfall separately and then superimpose the resultant pollution distributions. Because of the uncertainty of the amount of discharge and the nature of the sewage in each outfall, lack of vertical mixing throughout the estuary, and the highly irregular morphology of the river, one should calculate the center of gravity of the discharge weighted outfalls and consider the total discharge to be a point source of pollution. Although, in general, such a procedure may not yield an equivalent theoretical pollution distribution, such an assumption seems warranted, under the circumstances prevailing in the St. Johns River Estuary. Calculations indicate that station 5 (Main Street Bridge) may be assumed to be the equivalent point source of organic-waste loading.

STATISTICAL APPROACH

RELATIONSHIP BETWEEN DISCHARGE AND MEAN PERCENTAGE SATURATION

Because the best theoretical approaches to the problem of ascertaining the pollution distribution in the St. Johns River were found inapplicable, it was necessary to resort to statistical studies.

Figure 16 is a plot of longitudinal profiles showing percentage saturation of dissolved oxygen (mean over a complete tidal cycle) in relation to station location for each of the five phases. The

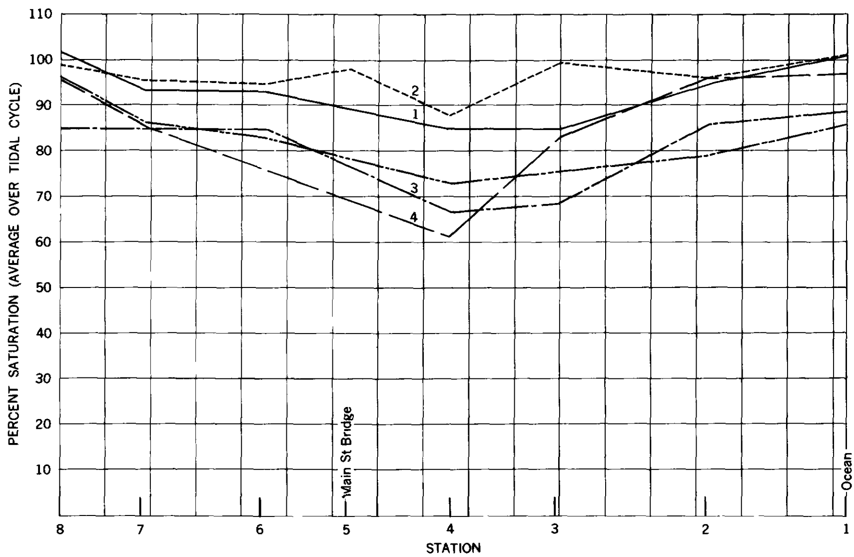

FIGURE 16.—Dissolved-oxygen profiles, phases 1-5.

literature refers to these profiles as oxygen sag curves because rivers generally show a gradual decrease in DO downstream from the point of discharge; recovery occurs subsequently as the forces of self-purification overcome the BOD being exerted. Thus, there is a "critical deficit," or an extreme minimum, to the curve. The curves shown on figure 16 are somewhat more irregular than the idealized curves to be found in textbooks; they exhibit the expected characteristics. For all five phases, the critical deficit occurs at station 4, which is 4.55 miles downstream from the effective point of organic waste loading. Because the critical deficit at station 4 varies from phase to phase, there is the possibility of correlating it with some independent parameter. Antecedent discharge, Q, is the most obvious parameter because of the presumed constant pollution load in all five phases.

Correlation of 30-day antecedent discharge with the mean critical deficit at station 4 (expressed as percentage saturation) for the five phases led to:

$$\text{Log } Q = 2.237 + 0.0198 \text{ (percentage saturation)} \qquad (41)$$

The correlation coefficient, r, was $+0.953$. An excellent basis for appraising river quality is thereby provided. Before proceeding with an evaluation, however, it is useful to find an equation for station 5, because that station is more accessible for sampling and is considered the primary station. Correlation of the mean percentage saturation at station 5 with antecedent discharge gave:

$$\text{Log } Q = 1.92 + 0.0218 \text{ (percentage saturation)} \qquad (42)$$
$$r = +0.93$$

The station 5 correlation is also high.

Data upon which equations 41 and 42 were calculated can be employed to establish the correlation between the mean percentage saturation at stations 4 and 5. The resulting equation is

$$\text{Percentage saturation at station 5}$$
$$= 9.8 + 0.972 \text{ (percentage saturation at station 4)} \qquad (43)$$
$$r = +0.97.$$

There is a 94.1-percent probability (r^2) that equation 43 did not result from chance.

Equations 41, 42 and 43 cannot yet be applied indiscriminately because the reader will note that the correlations refer to mean percentage saturation over a tidal cycle. Therefore, the extreme minimum DO at stations 4 and 5 was something less than shown in figure 16. Appraisal of water quality, of course, depends upon extreme minimum DO as well as upon mean critical deficit. An investigation of fluctuations about the mean percentage saturations was indicated. Hourly values of DO, expressed as percentage saturation at stations 4 and 5 were treated statistically for all phases. Standard deviations were determined and are tabulated in table 4. By definition, there were 15.9 percent of the hourly dissolved oxygen values that fell below the mean percentage saturation minus one standard deviation.

TABLE 4.—*Standard deviations of dissolved-oxygen data*

Phase	Standard deviation (percent saturation) at—	
	Station 4	Station 5
1	5. 6	6. 3
2	2. 6	4. 7
3	3. 9	11. 3
4	12. 3	7. 8
5	1. 9	2. 5

INTERPRETATION OF RESULTS

Equation 41 may be used with temperature, chlorinity, and discharge data to determine the probable minimum mean oxygen content, in milligrams per liter, in the river. This value must be compared with the minimum acceptable value of DO that is consistent with the established priority uses. Determination of the minimum acceptable value is not always straightforward, even though there is considerable published information relating to such topics as fish tolerance and aesthetic nuisance. Recommended minimum DO values fall generally in the range of 3.0 to 5.0 mg per l (milligrams per liter), 4.0 mg per l being recommended most frequently.

Saturation values of DO decrease as temperature and chlorinity increase. The smaller the antecedent discharge, the smaller the percentage saturation at station 4. (See equation 41.) Thus, the minimum probable mean DO at station 4 will occur under conditions of high temperature and low discharge. Low discharge contributes in two ways: first, it means high chlorinity at station 4 and therefore a lower saturation value of DO; secondly, it means a smaller percentage saturation in accordance with equation 41. The author believes that the quality of the St. Johns River may best be determined by comparing the mean DO at station 4, under the combined adverse conditions of high temperature and low discharge with a minimum acceptable mean DO value of 4.0 mg per l. This allows some hourly values of DO to fall below 4.0 mg per l in accordance with the standard deviation in table 4.

If the assumed adverse conditions of high temperature and low flow prevail, it is unreasonable to select the minimum daily flow of record. In fact, equation 41 utilizes the mean 30-day antecedent flow and not a daily flow. The value needed is probably the minimum mean 30-day flow that occurs once every 20 years. That value cannot be determined from the available period of record of slightly more than 3 years, and it cannot be synthesized with any degree of reliability. After reviewing the short-term mean monthly flow records for July, August, and September (period of high temperature), 2,500 cfs was arbitrarily selected as the adverse discharge. The discharge during phase 3 was approximately the same as the adverse discharge; the study during phase 3 provided the following chlorinity data: Station 4, 10.09 parts per thousand; station 5, 7.24 parts per thousand. During study phase 4, which was conducted in August, high temperatures prevailed: station 4, 30.0° C; station 5, 30.5° C. From tables for saturation DO values, one finds that saturation DO at station 4 was 6.9 mg per l and at station 5; 7.0 mg per l. For 2,500 cfs, equations 41 and 42 give percentage saturation at station 4 as 58.7 percent and at station 5, 67.8 percent. Thus, the minimum probable mean DO at station 4 was 4.0 mg per l and at station 5, 4.7 mg per l.

Under 1955 pollution loads, then, when flow and temperature conditions were adverse, the mean DO over a complete tidal cycle at station 4 may have been 4.0 mg per l. That value is the lowest mean value that can be expected to occur in the river because station 4 is the critical point on the oxygen-sag curve.

It is interesting to formulate an idea of river quality under conditions of mean flow and mean annual water temperature. As previously seen the mean flow may be taken as 7,000 cfs. Mean annual water temperature at station 4 is 22.4 °C and at station 5, 22.7 °C. Because selected discharge corresponds closely to the conditions of phase 1, chlorinity data may be taken from that investigation. Repeating the preceding steps, one finds that under mean annual flow and temperature, the mean DO values over a tidal cycle are: station 4, 6.6 mg per l; station 5, 7.4 mg per l. If 4.0 mg per l is accepted as the permissible minimum for mean DO over a tidal cycle, as suggested by the author, then under the loading conditions of 1954–55 the DO of the river was generally very satisfactory. Occasionally, when flow and temperature conditions were adverse, the DO at station 4 was nearly equal to the minimum acceptable mean DO.

As a check on the previous correlation technique for evaluating water quality, the somewhat different approach of Klegerman and Niles (1954) was also tried. In their study, which was primarily concerned with finding the assimilative capacity of the Potomac River at Washington, D.C., they took 366 longitudinal DO profiles and grouped the data by discharge and temperature. The extreme minimum DO was found for each profile in each group, and the corresponding saturation value of DO was calculated. These investigators considered 4.0 mg per l as the minimum acceptable value of DO, and for each extreme minimum they calculated the assimilative capacity of the river.

$$\text{Assimilative capacity} = \text{present BOD (lb per day)} \; \frac{\text{saturated DO} - \text{observed DO.}}{\text{saturated DO} - 4.0 \text{ mg per l}}$$

These data, when plotted, gave a scatter of points from which a reasonable overall value of assimilative capacity was selected.

The approach of Klegerman and Niles was applied also to the St. Johns River to find the ratio by which the present BOD should be multiplied to give the assimilative capacity. (Because the present BOD is not known with accuracy, no useful purpose seemed to be served by reporting the assimilative capacity.) Table 5 (col. 6) shows the pertinent ratio. The ratio is significant because it shows that as long as it is more than 1.0, the river has not reached the limit of assimilative capacity. Ratios less than 1.0 indicate the need for treatment of the sewage and also enable one to calculate for the required degree of treatment. Column 6 substantiates the previous findings that occasionally the river was in a marginal, or even submarginal, condition.

TABLE 5.—*Evaluation of relative loading of the St. Johns River, Fla.*

(1) Phase	(2) Extreme minimum DO at any station at any time (mg per l)	(3) Saturation DO (mg per l)	(4) Column 3—column 2	(5) Column 3—4.0 mg per l	(6) Ratio of column 5 to column 4
1	6.1	8.0	1.9	4.0	2.1
2	8.6	10.04	1.44	6.04	4.2
3	4.5	7.37	2.87	3.37	1.17
4	3.6	7.0	3.4	3.00	0.88
5	6.1	8.12	2.02	4.12	2.04

SUBSEQUENT APPLICATION OF THE DERIVED EQUATIONS

The city of Jacksonville installed a permanent sampling station in a shed on one of the piers beneath Main Street Bridge (station 5). It was hoped that after the five phase studies had been analyzed, it would be possible to sample only at station 5 and yet know the river quality at other stations. Several test runs of less duration than a complete tidal cycle were made at station 5. Operational difficulties with automatic sampling equipment were resolved. On June 16 and 17, 1958, complete tidal cycle data were collected.

Observed mean percentage saturation of DO at station 5 was 68.3 for the first cycle and 60.1 for the second. The 30-day antecedent discharge, as taken from U.S. Geological Survey records, and equation 42 were used to calculate corresponding mean percentage saturations of 77.7 and 76.8. Thus, the observed values were 9.4 and 16.7 percent lower than those predicted by equation 42. Although the loading undoubtedly increased between 1955 and 1958 and thus, the functional relationship in equation 42 was altered slightly, such an explanation would seem to fall short of accounting for variations of this magnitude.

A review of meteorological data indicated that the test was preceded by rain and by winds out of the west of higher than average intensity. These conditions account for the low chloride contents at station 5, which are lower than those normally associated with antecedent discharge. Hence, the saturation DO was higher than normal, and the observed mean percentage saturation was lower than predicted.

Phases 1 through 5 were largely associated with sunshine and consequent photosynthesis. Equations 41, 42, and 43 reflect that fact. However, the followup test was preceded by heavy cloud cover, and cycle 2 of the test was conducted at night. Photosynthesis is probably important to the oxygen balance of the St. Johns River, and, so, in cycle 1, a variation of 9.4 percent in the percentage saturation of DO could primarily be due to antecedent cloud cover, whereas a drop from 9.4 to 16.7 percent below the predicted DO for cycle 2 could be due to the fact that no photosynthesis takes place at night.

It would be most desirable to express the relationship between cloud cover, algal respiration, and oxygen content in a stream, river, or estuary in quantitative terms. To date, no one has found a satisfactory general approach to the problem, but empirical approaches have been applied with success to specific situations. An empirical approach for the St. Johns River, is not now feasible because of the nature of the data. Equations 41, 42, and 43 can be applied, but one must realize that under cloudy conditions or at night the values of mean percentage saturation will be too high. Although cloud cover prevails fairly frequently, the appraisal of water quality in "Interpretation of Results" does not represent the most adverse combination of circumstances. Statistically speaking, the methods described remain an adequate and at present the most appropriate index of water quality.

SUMMARY

This paper has sought to emphasize the very serious problem of organic-waste pollution of our Nation's tidal bodies. The study of mixing processes in an estuary is an exceedingly complicated undertaking involving at least 28 independent variables. Thus, investigators are obliged to make some compromise between the desirability of a rigorous mathematical approach and the practical necessity for achieving workable engineering solutions and evaluations.

Because of the complexity of the problem and the lack of an appropriate approach that clearly defines the necessary sampling variables, one is chary of criticizing the "shotgun" procedures used by official agencies and regulatory bodies for their sampling programs of estuaries. Nevertheless, a great deal of money is being spent for sampling programs on both tidal and fresh-water bodies, and it certainly behooves those concerned to make maximum use of the data that are collected.

Approaches that various investigators have taken to determine the assimilative capacity of tidal estuaries may be broadly classified under four general categories: (1) theoretical, (2) quasi-theoretical, (3) experimental (hydraulic models), and (4) statistical. Of these various approaches, the author regards the statistical approach as being most useful, at least in the foreseeable future. Certainly attempts to reach a mathematical, or theoretical, solution to the problem should not be abandoned in the meanwhile. Future attempts to improve theoretical approaches should perhaps be directed toward first achieving a better understanding of the coefficient of eddy diffusivity, *K*.

Although the procedure probably cannot be followed in other specific situations, the approach used by the author in his analysis of the St. Johns River Estuary is presented for consideration. Strictly speaking, no estuary approaches the mathematical ideal of type D, as defined by Pritchard (1955), but the St. Johns River probably departs from it more than any other. Thus, a discussion of any approach that yields useful results should be published.

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