Considerations Involved in Evaluating Mathematical Modeling of Urban Hydrologic Systems

By DAVID R. DAWDY

HYDROLOGIC EFFECTS OF URBAN GROWTH

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CONSIDERATIONS INVOLVED IN EVALUATING MATHEMATICAL MODELING OF URBAN HYDROLOGIC SYSTEMS

By David R. Dawdy

ABSTRACT

Techniques are available for assessing the performance of mathematical models of catchment behavior. These system analysis techniques include optimization methods and use of goodness of fit criteria, error analysis, and sensitivity analysis. Further research, however, in model assessment is needed, particularly in the application of mathematical modeling to the urban hydrologic environment.

INTRODUCTION

Quantitative hydrology is modeling. The development of hydrology is the development of mathematical statements about hydrologic processes. Thus flood routing is a mathematical model which describes how a flood wave is translated and attenuated as it travels down a channel. Whenever a past event is reconstructed or a future event is predicted, a model is involved. Interest in the hydrology of the urban environment leads to interest in modeling as soon as knowledge is sufficient so that quantitative answers are desired to hydrologic questions.

The field of mathematical modeling can be divided by at least two different methods of classification. First is the stochastic-deterministic classification. A stochastic model relates input to output statistically. This can be by the relation of an input as an independent variable to the output as a dependent variable, as in Carter (1961), who related change in flood peaks as a result of urbanization to change in lag time. It can also be by a statistical simulation of a synthetic streamflow trace. Fiering (1967), for instance, computes the statistics of a given recorded trace, then uses those statistics to generate "equally likely" traces. A deterministic model relates input to output in such a manner that, once the input is known, the output is wholly predictable. The most often used deterministic models in hydrology are those based on the laws of hydraulics, which use equations of continuity and motion.

Examples are Liggett's (1959) study of open channel flow and Grace and Eagleson's (1965) study of similitude for modeling runoff from small, impervious areas.

A second division of hydrologic modeling is the analytic-synthetic classification. An analytic model usually describes a restricted area of hydrology in which the laws governing the process are fairly well known and accepted. Given these laws, a means of solution is developed, either in closed form or by finite difference approximations. A synthetic model specifies a conceptual relating function, and system parameters are identified through the use of input and output data. The conceptual model may vary between modelers, if the physical laws operating are not well defined or generally agreed upon. Thus, an analytic model usually describes a narrow or restricted subsystem in hydrology so that the problem is made manageable, whereas a synthetic model can be made as complex and cover as broad an area of hydrology as desired. Grace and Eagleson's study is an example of the analysis approach, and Crawford and Linsley's (1966) rainfall-runoff model of the synthesis approach.

During the process of model building, the model builder is continually faced with a decision concerning simplicity versus completeness in the model. The simpler the model is, the easier it is to understand and use, and probably the cheaper it is to use. However, no important element of the system which bears on the process to be modeled should be omitted from the model. For example, the use of hydraulic equations is familiar to most hydrologists. Because the general form of the equations is difficult to solve, simplifications are made. Each simplification requires a statement of an assumption, such as that steady, uniform flow occurs in a trapezoidal channel. Once these assumptions are made the problem is reduced to a manageable size and can be solved, but the solution does not apply if the assumptions are violated.

The same problem faces the synthetic model builder as faces the analytic model builder, but the decisions are not so easy nor the assumptions so obvious. First the process to be modeled must be broken down into subsystems, each subsystem must be judged as to relative importance, each must be modeled, and all must be tied together in a master program. However, generally accepted concepts such as steady, uniform flow or gradually varied flow do not exist. Rather, the question facing a rainfall-runoff model builder might be the relative importance of interception storage as opposed to detention storage, and how the two can be differentiated if the two are to be modeled separately. Equations for both must be derived from empirical studies and data for neither will be available for most areas of use. Both interception and detention may exist on a basin, yet neither may be necessary in a model which aims at a given level of output accuracy. A desire for completeness in a model tends to lead toward inclusion of all components which intuitively are known to exist. However, the desire for completeness may lead to the inclusion of many parameters which are merely curve fitting factors rather than physical parameters describing the process they supposedly are modeling. The optimal rule to follow would be that of Occam's Razor, which states that if a simple model will suffice, none more complex is necessary. This fails in practice unless "suffice" is better defined.

An important problem which faces both the model builder and the model user is that concerning the transferability of results. This is particularly true in urban hydrology, for little data exist for urban watersheds. Practically no data exist for the modeling of the quality of urban runoff. Therefore, any model developed must be applicable to ungaged areas. Transferability implies that model parameters must be derivable from physical measures of the drainage basin. For instance, Carter's study requires measures of length, slope, drainage area, and degree of urbanization. With these, an estimate of the mean annual flood can be computed. Fiering's streamflow synthesis model requires a mean flow, a variance, and a first order serial correlation between adjacent flows. From these a synthetic trace can be generated. Urbanization studies require that the change in parameters as a result of man's influence must be estimated. Carter estimated man's influence by using the percentage of impervious area as a parameter. If parameters have true physical significance, such methods may be very effective.

The physical significance of the parameters in a typical rainfall-runoff model and their changes as a result of urbanization are discussed by James (1965) in his study of the hydrology of Sacramento, Calif. The lack of direct physical significance of the statistical parameters in a streamflow synthesis program is a possible shortcoming. Means, variances, and serial correlations must be related to a mappable measure. In addition, the statistical parameters used to establish any relation must be defined on the basis of a measured period of record. Adequate records to define a range for the statistical parameters for the urban environment are inadequate or nonexistent. For this reason, stochastic models are not considered feasible at this time for modeling urban basins, and therefore are not discussed further. However, many of the points which are discussed are applicable both to stochastic and deterministic models.

CRITERIA OF GOODNESS OF FIT AND THE FITTING PROCESS

Any model must be easily usable and must give satisfactory results. While a model builder is concerned with how a model is derived, a user is more concerned with what is derived and how well it can predict results for his particular problem. "Satisfactory results," however, have no meaning by themselves. There must be some criteria for judging goodness of fit.

A best linear predictor for linear systems can be derived by the least squares criterion if residual errors are independent, normally distributed, and homoscedastic. Even when these conditions are not met, some least squares fitting is often used. Yet, even a least squares criterion requires that deviations from something must be computed in order to have a squared measure to minimize. Eagleson and others (1966) explained the rationale for the use of a least squares analysis when they stated that the basic problem of linear black box analysis is to solve for a meaningful response function when the input and output are related by a system that is not truly linear. Eagleson obtained an optimal realizable unit hydrograph by using linear programing to solve a Weiner-Hopf formulation for the rainfall-runoff system. A solution of the Weiner-Hopf equations gives the least-squares fit or, in Eagleson's language, minimizes the integral square error. The Weiner-Hopf equation, in Eagleson's analysis, minimized the squared differences between the simulated and observed total streamflow traces. This was an operational decision made in order to obtain a hopefully meaningful response function. Any other squared error term could have been used, would have given a different Weiner-Hopf solution, and would have been equally optimal.

Dawdy and Thompson (1967) indicated that the criterion set for optimization influences the resulting set of optimal parameters for a given model. They indicated that the modeling process can be considered as analogous to a linear programing problem. Eagleson transformed the problem to an explicit formulation of a linear programing problem. As Eagleson showed, the goodness of fit criterion in modeling is an objective function, and the model itself, expressed by the Weiner-Hopf convolution equations, is a set of constraints.

Sensitivity analysis in linear programing studies the changes in the optimal solution as any set of coefficients are varied, including those of the objective function. Dawdy and Thompson indicated that in their study, three different sets of optimal parameters existed, each corresponding to a different objective function. If all three of their objective functions were combined into one weighted function, their three solutions would be end-member solutions with weights of

(1,0,0), (0,1,0), and (0,0,1), where the numbers in each set of parentheses represent the relative weights given to each of the three objective functions. Each weighting produces a different set of optimal parameters. Research into the variation between sets of optimal parameters produced by the use of different objective function weightings would give insight into the modeling process and into the interpretation of a given model. Each optimal set of parameters is optimal only in terms of its objective function. Most workers have observed this seeming paradox in simpler cases of an optimal solution not being a unique solution, and it is here merely extended to the general field of simulation. Eagleson summarizes the problem when he states, "When each storm is analyzed independently, the several unit hydrographs obtained are thus likely to have widely varying geometrical properties, and the method of averaging them * * * is not clear." A measure of the effect of averaging of parameters on peak estimation is shown in figure 1. A rainfall-runoff model was fitted to four different years of record for Arroyo Seco near Pasadena, Calif. Each year was fitted separately, and the simulated peaks are shown. In addition, the optimum parameters for each year were averaged, and these average parameters were used to simulate the same peaks. The scatter increases, particularly for the lower peaks.

There is a direct interaction between the objective function and the fitted parameters. Therefore, the choice of the objective function itself should be optimal in some sense to the model user. There is no objective method for choosing the objective function, however, so that the choice of the objective function is a very subjective decision. Research in this area is vitally needed.

An example of the effect of the choice of the objective function on the fitting process is shown in figure 2. For the same rainfall-runoff model, parameters were fitted to a given control period on the basis of two different objective functions. The first objective function minimized the sum of squared deviations of simulated discharges from observed discharges for peaks, the second for days. The purpose of the fitting was to develop a model for estimating peak discharges. The goodness of fit was tested by estimating peak discharges for a test period on the basis of the two sets of averaged parameters. As can be seen, the scatter is about the same for the two sets of simulated peaks and is on the same order as the scatter during the control period, as shown in figure 1 for the average parameters. However, the parameters based on fitting to daily values result in unbiased estimates, whereas fitting to peak values resulted in what seems to be a biased estimate. This effect in terms of a frequency diagram is shown in figure 3. Part

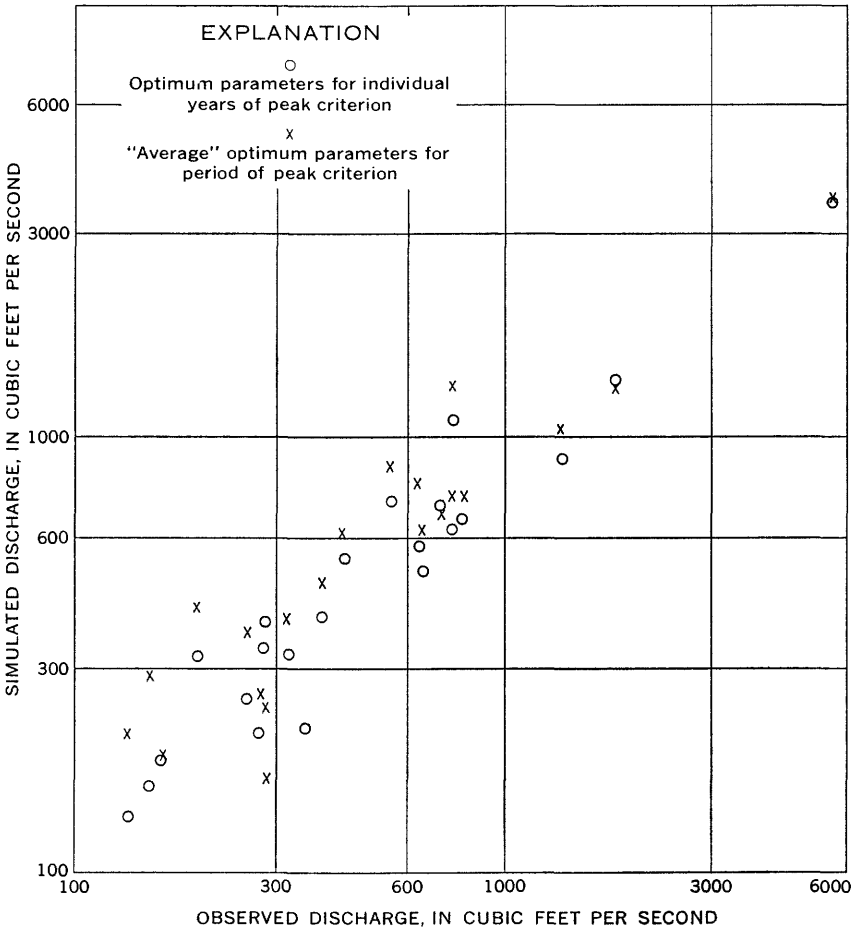

FIGURE 1.—Comparison of simulation results using optimum parameters for each year with results using averaged parameters.

of the hydrologic model became a curve-fitting function and therefore did not retain its supposed physical significance when peaks alone were used for fitting. Although parameters were "reasonable," the water balance was grossly erroneous. This created errors of prediction in the test period.

Sequential fitting methods can use different criteria for fitting different subsets of parameters in a model. A first stage could hold routing constants at some first estimate and fit the parameters for water balance. Then the water balance parameters can be held fixed while

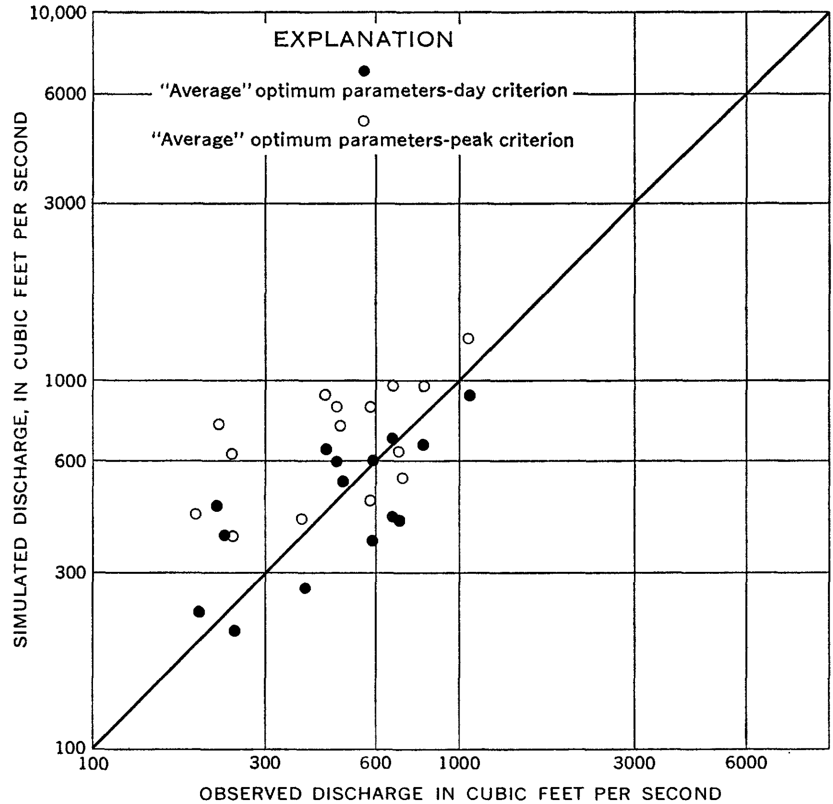

FIGURE 2.—Comparison of prediction using two sets of optimal parameters. Fitting for one is to peak flows, for the other, to daily flows.

routing parameters are fitted to peaks. However, there is much interaction among parameters, and some parameters may influence equally both water balance and peak flows. Research in fitting methods can go hand-in-hand with research in the effects of choice of objective function on the optimal fitted solution.

Interactions cannot be eliminated, for they exist in the physical system. Their importance in the curve fitting process can be minimized, however, by determining as many parameters outside the model as possible. For instance, the Stanford Watershed Model has 20 parameters. Two are based on meteorological data, four on hydrograph separation, five are computed from physical measures, three are estimated from empirical tables, and six are fitted. The six fitted parameters all are involved in the loss function, which includes infiltration, drainage, and evaporation. Chawford and Linsley (1966) discuss the

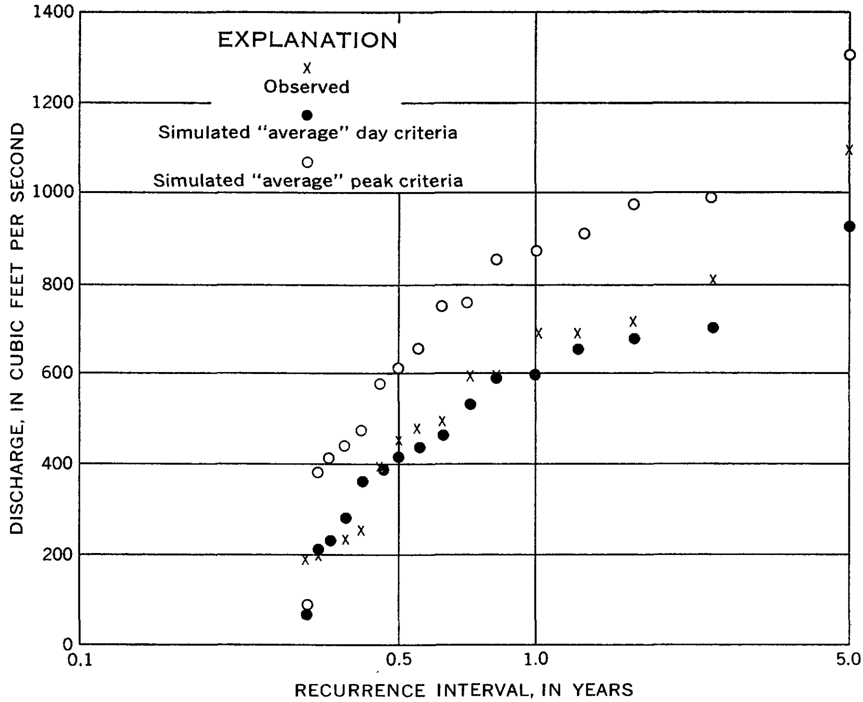

FIGURE 3.—Resulting flood frequency curves for scatter diagrams of figure 2.

interactions among the fitted parameters and suggest a combination of fitting both sequentially and to selected data to define the parameters as independently as possible.

The determination of parameters a priori eliminates to a degree the use of a simulation model as a black box device, and adds a degree of grayness. It masks a part of the interaction of parameters because some are held constant. However, certain "physical" parameters may be indices rather than measures. Resistance coefficients and slope when applied to basins as "average values" are examples. If fitted parameters are obtained in order to correlate with measured indices, the interactions plus errors in data and in the model may result in physically meaningless values for some or all parameters.

To summarize, fruitful areas of research exist in the development of fitting methods, the choice of criteria for judging goodness of fit, and the effect of choice of criteria upon fitted parameters.

EFFECTS OF ERRORS OF DATA ON MODELING

Errors in data are reflected in errors in the fitted parameters in a simulation model. If perfect input data are routed through a perfect model, the output produced would agree perfectly with an error-free output record. If errors are introduced into the input or output record or both, the output will not be exactly reproduced even by a perfect model. If a fitting process is used, the parameters will deviate from their true values in order to minimize the deviations between the simulated and recorded traces as specified in the objective function. The "optimal" set of parameters will now be in error, and the fitted values of the objective function will be less than its "true" value.

This process is analogous to statistical least squares analysis. The fitted parameters deviate from their population values because of random errors in the data. The standard error of estimate is a measure of error of reproduction of the fitted data. The standard error of prediction, however, is somewhat greater than the standard error of estimate, for it includes both the measure of lack of fit of the data used to fit the model and the measure of error in the fitted parameters. These relationships are shown in table 1.

TABLE 1. *Qualitative comparison of errors involved in hydrologic modeling with analogous errors in standard statistical analysis*

Source of error	Size of error	Statistical analog
Data	a	Measurement and sampling error.
Comparison of measured to simulated during period used for fitting.	a−b	Standard error of estimate.
Comparison of measured to simulated during period not used for fitting.	a+c	Standard error of prediction.

If the assumptions of regression theory hold (a linear model with normally distributed and homoscedastic errors of the dependent variable), the error of prediction can be computed from the standard error, the deviations of the independent variables from their mean, and the error in the coefficients for the independent variables. The assumptions seldom hold, however, so that statisticians often resort to split sample testing. A similar situation holds for hydrologic simulation, except that there is no theory by which to compute the error of prediction. In order to present a measure of utility of a model to the potential user, the data used to test a model should not include any data used to develop the model or its parameters.

Nonlinearity of the hydrologic process precludes any theoretical description of the mechanism by which errors in data are transferred to model parameters and then combined with input errors in the test period to produce errors in the simulated streamflow trace. An empirical study for the response for a particular model can be made as in table 2. A recorded rainfall trace was assumed error free and routed through an optimized set of parameters, which were assumed correct values, to obtain a "true" streamflow trace. Then a random error with mean zero and standard deviation of 10 percent was applied to all rainfall values. These "erroneous" rainfall values were then routed through the true model, and the resulting standard error of the simulated streamflow trace computed. An optimization run then was made which adjusted the parameters to minimize the standard error. The "optimized" set of parameters is shown, along with the resulting standard error. The "true" rainfall trace was then routed through the optimized parameters, and the standard error computed. Assuming independence of the two souces of error, one in the input data and the other in the model parameters, the error of prediction should be approximately equal to the square root of the sum of the squares of the two separate estimates. Similar results are shown for random errors in the input with 20 percent standard error, and for random errors

TABLE 2. *The effect of errors in data on the fitting process*

[The model parameters have the following meaning: SWF is suction at the wetted front, KSAT is saturated soil permeability, KSW is surface storage coefficient, EVC is evaporation pan coefficient, SMSN is a nominal soil-moisture storage, RGF is a multiplicative factor controlling the range of effective SWF, RR is a runoff ratio, and DRN is the soil drainage rate]

Parameter	Parameter value				
	True	Optimized errors (percent)			
		Rainfall		Runoff	
		10	20	5	10
SWF (in.)	3.6	3.6	3.8	3.7	3.7
KSAT (in. per hr)	.063	.063	.06	.063	.061
KSW (hr.)	1.0	1.04	1.06	.98	.98
EVC	.56	.57	.58	.559	.56
SMSN (in.)	4.0	4.02	3.98	4.04	4.04
RGF	12.0	11.9	11.94	12.12	12.21
RR	.8	.796	.8	.796	.796
DRN (in. per hr)	.020	.018	.017	.020	.019
U, Pd[1] (given)		[2].405(13)	1.45(26)	.063(4.9)	.247(9.4)
pd (fitted)		.261(10.5)	1.06(19.8)	.046(4.0)	.191(8.3)
pD		.105(6.5)	.41(13)	----------	----------
U, Test[3] predicted		.530(14)	2.40(30)	----------	----------

[1] P=True parameters, p=optimized parameters, D=correct data, d=erroneous data, U=value of objective function.
[2] Nota bene. First value is sum of squares of difference of natural logarithms of the 18 peaks plus half the squares of the 18 storm volumes. The second value converts the first value to an equivalent "percent standard error" by SE=antilog $\sqrt{U/27}$, and averaging plus and minus percentages.
[3] Average of nine separate test runs.

in the output of 5 and 10 percent. A further test is then made for each case by applying a new set of random errors to the rainfall and routing this new set of erroneous data through the "optimized" parameters and computing the resulting errors of prediction. These errors also are shown in table 1. Such an analysis gives insight into the fitting process in simulation models, shows how errors in data are transmitted to the model, and gives an empirical measure of the effect of data errors on the accuracy of prediction in simulation.

Data errors have different results depending upon the type of data considered. All transfer part of their errors into the model parameters, but each differently. Each therefore must be considered separately.

INPUT ERRORS—RAINFALL

The major source of error results from the rainfall input data. Rainfall errors have several sources. There is sampling error as evidenced by variability between measured catches for different designs of rain gages, and changes in catch when minor changes are made in the configuration of a given rain gage. There are changes with time as buildings are built or trees grow nearby, changing wind patterns and thus changing catch. There are changes which occur when the gage is physically moved, and most long-term records have come from gages which had been moved at some time. There is spatial variability of rainfall over the basin which a rain gage does not measure because it measures point rainfall. Finally, any rainfall record must be considered representative of the basin to be simulated, and there may be adjustments made before it is considered representative.

Lumped parameter models of basin response take a rainfall record as representative of a basin or subbasin and assume uniform rainfall in space. As distributed parameter components or subsystems are included in a given model, input data needs are increased. For example, an areally distributed rainfall input becomes necessary in order to be compatible with the model, and therefore to realize the benefits of the detailed description of the model. The problems of rainfall network density for both lumped and distributed parameter modeling have been discussed by Amorocho, Brandstetter, and Morgan (1968) and Eagleson (1967). Both agree that one or two gages are sufficient to estimate mean convective storm precipitation for drainage areas of size similar to those found in urban areas. In addition, Amorocho and Brandstetter (1966) have studied the problem of rainfall input for a distributed system in space. Their conclusions state, "The investigation of questions regarding catchment damping is also essential in order to determine the range of departures from the real input field which these systems can accept without significant change in the output."

The study of the relation of rainfall network density to accuracy still is a fruitful area for research. But errors in data by themselves are not of as much importance as the effect of those errors on the decision-making process. The next logical step, as Amorocho and Brandstetter imply, is to study the effect of rainfall errors on simulated runoff records. In particular, the use of distributed parameter models requires a study of the effect upon simulated streamflow traces of an assumption of uniformity of rainfall in space as opposed to the use of a spatially varied rainfall.

INPUT ERRORS—EVAPOTRANSPIRATION

"Evapotranspiration (ET)" in the main is a depletion from soil moisture. An "antecedent precipitation index (API)" is a simplified model of the persistence of the effect of past rainfall on future events. ET is used in simulation models to construct a better index, or perhaps a "time variable API." ET determines soil moisture depletion, and therefore affects the rate of losses of rainfall to soil moisture through infiltration. If any concept of a limiting value of minimum infiltration is included in a simulation model, major storms of long duration and high intensity approach that limiting value, and the effect of initial soil-moisture conditions is decreased. Thus streamflow traces for periods of major storms, years of high precipitation, and regions of high precipitation should generally be fitted more accurately than traces where initial soil moisture conditions are critical.

Measured values of soil moisture are seldom available for testing depletions from soil moisture through the process of drainage and ET. If models of ET loss are to be constructed and then tested by means of rainfall-runoff data, the ET model can best be tested in regions where soil moisture conditions are critical but variable, or else only selected periods of record should be used for model construction. Such periods should be selected so as to cover the range of effects of soil moisture on storm runoff and peak flows. With such a test, the problems of errors in ET can be studied in the manner suggested by Amorocho and Brandstetter (1966) for the study of rainfall errors by measuring the impact of errors on simulated outputs.

True measures of ET seldom exist. Rather, pan evaporation data is collected, pan coefficients are determined, which supposedly adjust pan data to a potential evapotranspiration (PET), and then the calculated PET is used as input. Alternatively some PET can be calculated by an empirical (that is, Blaney-Criddle) or semi-empirical

(Thornthwaite or Penman) equation. Penman (1963) presents various theories of "availability of water for evapotranspiration." These different theories represent different modulating functions which convert PET to ET. Errors of output result from errors in PET, errors in the conceptual modulating function, and errors of the parameters in the modulating function. Studies of the relative importance of these components of error would be worthwhile. In addition, the impact of various levels of urbanization on the simulation results will give some measure of the degree of attention to these data necessary for hydrologic simulation in the urban environment.

OUTPUT ERRORS—STREAMFLOW

A discussion of errors of recorded output data centers around errors in stream gaging. The output of interest will depend upon the objective function, whether that objective function is stated explicitly or is an implicit "let's see how it looks." However, generally the only output data available is recorded streamflow. Streamflow data are much more accurate than rainfall data because they measure an integrated runoff from the total basin. Most streamflow records are rated as "good" by the U.S. Geological Survey (1966), which can be interpreted as meaning that daily mean discharges have a standard error of 5 percent (95 percent are within 10 percent). Peak discharges are somewhat more inaccurate. Peaks fairly well defined by discharge measurements would have a standard error of about 5 percent. By "fairly well defined" is meant that the peak flow is no more than twice the highest current meter measurement. When not so defined, peak flows may be computed by means such as slope-area measurements or other indirect methods, and the standard error then may be about 10 percent.

Errors in the output are transferred to the parameters through the fitting process just as are the errors in input. As indicated by the results of table 1, however, output errors are not as critical as input errors. Errors in input are magnified because a residual of excess precipitation is used in the routing, and any absolute error in input becomes an absolute error in the residual before routing. The generally minor amount of surface storage in urban basins does not greatly attenuate input errors. The objective function usually is stated in terms of some measure of streamflow, so that proportional errors in measured streamflow are transferred as proportional errors in output.

Errors in input and output impute errors in different parts of the model. Random, unbiased errors in input usually are compensated for by adjustments in the parameters of the loss function, which includes infiltration, drainage, interception, and detention. Similar errors in output usually are compensated for in the routing function. Biased errors in either may act differently. For rainfall, the bias may be an error in the adjustment of the recorded record to "average basin precipitation." For streamflow the bias may vary with discharge, with little error for the smallest peak and a maximum error for the largest peak, if the general slope of the stage-discharge relation used is computed in error.

INTERPRETATION OF ERRORS

The impact of errors upon the simulation process depends in part upon whether the error is a random error of a quantity which is measured or whether the error is in an index which is used as an approximation to something which cannot be measured. Data and parameters cover the spectrum from streamflow discharge (truly measurable) through point rainfall (measurable but used as an index to basin rainfall) to PET (not directly measurable and used as an index to relate water demand and water availability) to basin slope (some index of a varying quantity (Benson, 1959) the effect of which can only be estimated in itself). Even grosser indices might be "basin roughness" or "transbasin ground-water seepage" measures.

Errors of indices are errors of approximation in the model. Even a "best" value of an index may lead to serious errors in simulation when the data are outside the range for which the particular approximation applies. Therefore, errors in indices generate both random errors and errors of approximation. It is errors in indices which usually cause outliers, for gross errors usually are the results of poor approximations in a case where the approximation is the controlling factor in simulation of a given event.

Errors of approximation become masked in the fitting process. The subsystems within the hydrologic cycle are highly interrelated, with many interactions among parameters taking place even in the simplest models. Therefore the handling of outliers introduces an important area of subjectivity in any modeling process. If methods of systems identification and systems specification can be developed to separate the modeling process into a series of relatively independent problems, a major step forward will result. Sequential fitting methods, as mentioned earlier, are a first step in this direction, but much more could be done to advance the model development phase if the independent factors could somehow be treated as is done in statistical hydrology through eigen vector analysis and synthesis.

SENSITIVITY ANALYSIS

Sensitivity analysis studies the effect on the optimal solution of changes in the input-output coefficients and in the objective function. According to Dantzig (1963),

> In many applications, the information thus obtained [through sensitivity analysis] is as valuable as the specification of the optimum solution itself.
>
> Sensitivity analysis is important for several reasons:
>
> (a) Stability of the optimum solution under changes of parameters may be critical. For example, using the old optimum solution point, a slight variation of a parameter in one direction may result in a large unfavorable difference in the objective function relative to the new minimum, while a large variation in the parameter in another direction may result in only a small difference. * * * it may be desirable to move away from the optimum solution in order to achieve a solution less likely to require essential modification.
>
> (b) Values of the input-output coefficients, objective function coefficients, and/or constraint constants may be to some extent *controllable*, and in this case we want to know the effects which would result from changing these values.
>
> (c) Even though the input-output and objective function coefficients and constraint constants are *not* controllable, the estimates for their values may be only approximate, making it important to know for what ranges of these values the solution is still optimum. If it turns out that the optimum solution is extremely sensitive to their values, it may become necessary to obtain better estimates.

Sensitivity analyses in hydrologic simulation are not as straightforward as in linear programing. The same principles apply, however. Discussions of sensitivity of results to parameter variability are given by Crawford and Linsley (1966) and Dawdy and O'Donnell (1965). In addition, sensitivity to changes in the objective function (Dawdy and Thompson, 1967) were mentioned earlier. A conclusion drawn by Dawdy and O'Donnell was that "Any further development of automatic parameter optimization techniques must use some criterion of response sensitivity (or its equivalent) in selecting what can be considered adequately optimized parameters. Indeed, the minimization of differences from recorded data cannot be the sole criterion in interpreting the fit of any model."

Although all the parameters discussed by Dawdy and O'Donnell (1965) apparently were equally sensitive to either positive or negative changes in parameter values, this does not necessarily occur in all cases. A case in point is shown in figure 4. A parameter representing the rate of drainage of moisture from the soil through percolation to the ground-water table was estimated for a simulation run. The sensitivity of the objective function to changes in rate of drainage shows that the rate is critical until a certain value is reached. Beyond that

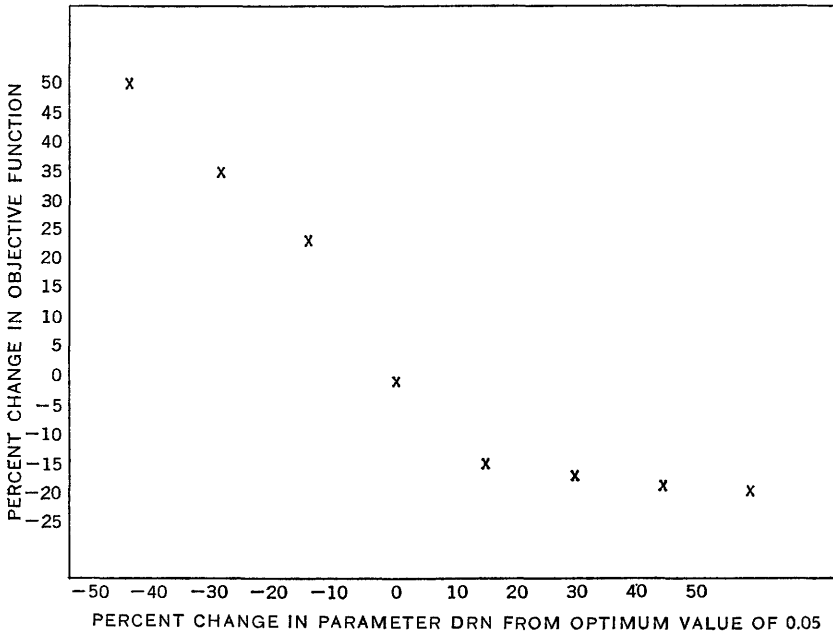

FIGURE 4.—Changes in goodness of fit as soil moisture drainage rate is varied, illustrating nonsymmetry of response.

critical drainage rate, the objective function is quite insensitive to increases in the drainage rate. Therefore, for this model and data it is far better to overestimate than to underestimate drainage.

Sensitivity plots for both optimized and computed parameters give insight into the modeling process and the interpretation of the physical meaning of the parameters in a given model. Sensitivity plots cannot be plotted unless the objective function is stated in objective terms. Yet, as Dawdy and Thompson (1967) indicated, the formulation of the objective function itself influences the results. At the present time, the use and interpretation of sensitivity analyses is quite subjective. A valid field of research which is to date almost untouched is the methodology of developing and using sensitivity analyses for comparing different models and for comparing results for a given model under differing conditions, as touched upon by Crawford and Linsley (1966) and Dawdy and O'Donnell (1965).

OPTIMIZATION VERSUS SUBOPTIMIZATION

Up to this point, all discussion has centered upon hydrology as a system. Optimization has been in terms of errors in simulation of hydrologic data. In fact, hydrology is a subsystem, and the discussion has concerned suboptimization. Hydrology is one input into a decision process concerning resources development. In optimization of the decision process, accuracy of hydrologic simulation has both costs and benefits with reference to the ultimate development scheme. Costs are incurred in gathering data of a certain type at a certain level of accuracy for a given length of time. Benefits result from better decisions which result from better data. Discussion of this broader systems study of the marginal cost and marginal worth of hydrologic data is beyond the scope of the present study. Nevertheless the broader problem must be considered. It also presents many areas where research is needed.

CONCLUSIONS

Hydrologic simulation today is half science and half art. It is an art to the extent that subjective decisions enter into the modeling process and its assessment. Research is needed to move the field closer toward the area of science by developing and using the necessary systems-analysis techniques. Measures of error are necessary in order to judge between models, whether on the part of the model builder or the model user. In addition, measures of error must usually be stated in some form of an objective function in order to us the tools of systems analysis.

Fruitful areas of research are:

1. The construction of meaningful objective functions and the interplay between objective function and simulation results. The use of objective functions for the disparate aims of fitting and predicting within a single model and of comparing between models should be considered.
2. The development of tools for sensitivity analyses and of methods for using sensitivity analyses for developing and using simulation models.
3. The development of mathematical tools for fitting the independent parts of hydrology. This would be in the sense of eigen vector analysis, in which the independent factors are not necessarily the original variables nor subsets of the original variables, but some combination of the original variables.

4. The determination of guidelines for error analysis in hydrologic simulations. Hydrologic models are nonlinear, so that standard linear theory does not apply. Errors in data are transferred into fitted parameters. These errors in fitted parameters plus those in computed parameters are transferred into predictions. Little is understood of this mechanism.
5. The comparative effects of input errors and output errors on simulation results so that some concept of network density can be applied to urban hydrologic data in terms of the end results of hydrologic prediction through simulation.

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