Free-Surface Instability Correlations *and* Roughness-Concentration Effects on Flow Over Hydrodynamically Rough Surfaces

GEOLOGICAL SURVEY WATER-SUPPLY PAPER 1592-C, D

Prepared in cooperation with the Iowa Institute of Hydraulic Research

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LABORATORY STUDIES OF OPEN-CHANNEL FLOW

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SYMBOLS

f = Resistance coefficient: $8gRS/U^2$ based on hydraulic radius; $8gy_0^3S/q^2$ based on two-dimensional flow.

g = Acceleration of gravity.

k = Height of roughness element.

log = Logarithm to base 10.

q = Discharge per foot of width of channel.

t = Time.

u = Velocity at a point.

x = Distance in mean-flow direction.

y_0 = Depth of flow measured normal to channel floor.

y_0' = Wave-altered depth of flow measured normal to channel floor.

A = Cross-sectional area.

B = Channel width.

C_1, C_2 = Constants.

F_B = Boundary retarding force on an elemental section of fluid.

F_G = Gravitational force on an elemental section of fluid.

L = Roll-wave development distance.

$\mathbf{F}_s$ = Limiting value of the Froude number for stable flow. It is called the stable-flow limit.

$\mathbf{F}$ = Froude number, $U/\sqrt{(gA \cos \theta)/(dA/dy_0)}$

$\mathbf{R}$ = Reynolds number: $4RU/\nu$ based on hydraulic radius; $4y_0U/\nu$ based on two-dimensional flow.

R = Hydraulic radius; ratio of area to wetted perimeter.

S = Sin θ.

U = Average velocity.

U' = Wave-altered average velocity.

U_w = Velocity of wave or of region of increased surface agitation.

β = Velocity distribution factor, $\frac{\int u^2 dA}{U^2 A}$.

θ = Angle of inclination of channel.

λ = Roughness-concentration factor. It represents the ratio of the sum of the projected areas of the roughness elements normal to the mean direction of fluid movement, to the floor area.

ν = Kinematic viscosity of the fluid.

ρ = Mass density of fluid.

τ = Average boundary shear.

LABORATORY STUDIES OF OPEN-CHANNEL FLOW

FREE-SURFACE INSTABILITY CORRELATIONS

By H. J. Koloseus and Jacob Davidian

ABSTRACT

The correlation between free-surface stability criteria and the physical characteristics of stable and unstable flow is studied through a consideration of the few data found in the literature and of many new and previously unpublished data. Stability criteria for flow in smooth and rough rectangular channels, predicated upon the logarithmic law of resistance, are also presented. Many observations disclose a fair correlation between the flows classified as either stable or unstable and the associated absence or presence of roll waves. Results also indicate that the channel resistance is greater for unstable flow than for stable flow. When the flow is classified as unstable, the channel resistance is a function of the Froude number. Because of the increase in channel resistance and the possibility of roll waves, the usual resistance relations for open-channel flow can not be extrapolated indefinitely.

INTRODUCTION

Because of the presence of the free surface and of gravitational effects, steady uniform open-channel flow can become unstable and give way to unsteady nonuniform flow. Observations of the physical manifestation of this type of instability (roll waves) prompted analyses that have led to the formulation of numerous stability criteria. These set forth the conditions under which the flow is classified as either stable or unstable. Few empirical data are available in the literature, however, to either support or refute the criteria.

A consideration of free-surface instability is not of academic interest alone. The stability criterion is important because, if valid, limits for steady uniform flow in long open channels will be established. Fully developed roll waves not only reduce the carrying capacity of a canal appreciably, but they may also bring about serious structural damage through varying pressures and overtopping of walls.

When flow is classified as unstable, free-surface perturbations of infinitesimal size, having the characteristics of shallow-water waves, are supposed to become larger as they travel downstream and give rise to what are termed "roll waves"; this phenomenon can take place in ether turbulent flow or initially laminar flow. If the flow is stable, waves are supposed to be damped out. The term "roll wave" is usually associated with a breaking-front type of wave traveling downstream. In this report, however, "roll wave" is used to denote any wave of spontaneous origin, regardless of size or shape, that is attributable to no cause other than a superiority of the gravitational force over the boundary retarding force. As a consequence of this superiority, a roll wave is capable of traveling indefinitely. Even barely perceptible waves are included in this definition. Roll waves are depicted in figures 1–3. These photographs were taken in one of two flumes used in this study to make observations on roll waves and channel resistance in the unstable regime. The arrow in these and subsequent photographs indicates direction of flow. The photographs in figures 1 and 2 were taken from the downstream end of the flume by means of a still camera. Although the waves in figure 2 are readily discernible, they are not as obvious and well developed as those in figure 1. The photographs in figure 3 are single frames from movies taken at the indicated distances from the flume entrance; these illustrate the growth of roll waves, perhaps to a limited extent, as they move downstream. No distinction is drawn, as was done by Mayer (1961), between the occurrence of this phenomenon in laminar and turbulent flows.

Laminar flow in a wide channel has been classified as unstable when the Froude number is greater than 0.5; this is about one-third of that for a comparable state in turbulent flow. In discussing four regimes of open-channel flow, Robertson and Rouse (1941) mention the laminar supercritical one (Froude number, **F**, is greater than 1), and they point out that this type is uncommon because of its inherent instability and degeneration into roll waves. Although laminar flow is important in certain fields, in this report consideration will be devoted chiefly to turbulent flow, because it is the more common of the two for flows in rivers, streams, and channels.

A stability criterion would be considered valid if there were a one-to-one correspondence between it and the presence or absence of roll waves. As a consequence of the fluid acceleration and deceleration that accompanies roll waves, it is rational that the channel resistance would be greater for flow of this type than that for steady uniform flow. Therefore, increased channel resistance in the unstable regime, over that found in the stable regime, is also regarded as evidence of the validity of the stability criteria, even when roll waves are not particularly apparent.

This report is concerned with the correlation that exists between the observable physical characteristics of flow which is classified as stable or unstable and those dictated by stability criteria. To this end, many new and previously unpublished data relative to stable and unstable flow in both smooth and rough laboratory channels are presented. Because many comparisons of flow are made on the basis

FIGURE 1.—Well-developed roll waves. Boundary VIII; $q=0.101$ cfs per ft; $y_0=0.028$ foot; $S=0.050$; $\mathbf{F}=3.9$. Arrow indicates direction of flow.

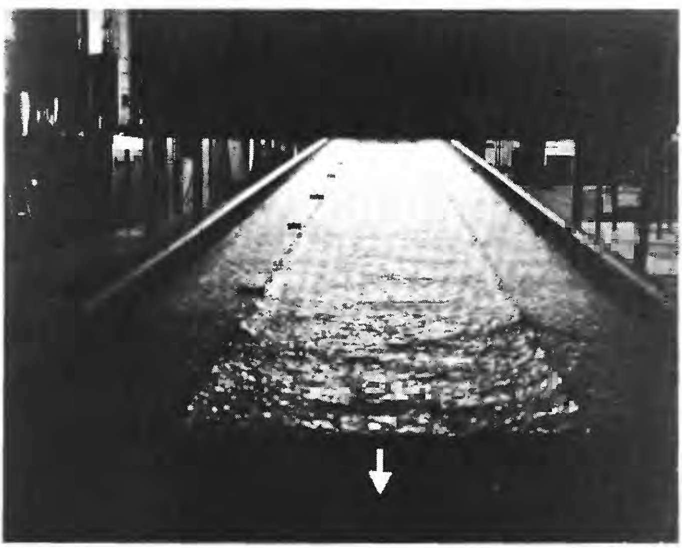
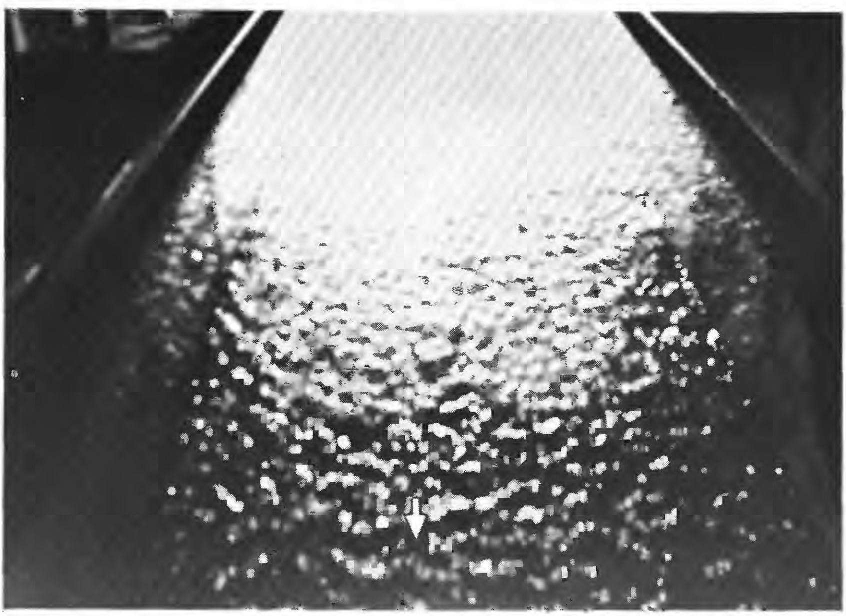


FIGURE 2.—Discernible roll waves. Boundary VIII; $q=0.41$ cfs per ft; $y_0=0.066$ foot; $S=0.050$; $\mathbf{F}=4.3$.

of the logarithmic laws of resistance and because stability criteria based on these laws have not been developed heretofore, stability criteria for flow in smooth and rough rectangular channels utilizing these forms of resistance equations are presented. These criteria are particularly relevant to the results contained herein because these

A



B

FIGURE 3.—Growth of roll waves with distance traveled along flume. Boundary V; $q=0.25$ cfs per ft; $y_0=0.076$ foot; $S=0.050$; $\mathbf{F}=2.1$. *A*, 15 feet from entrance; no

C

D

waves evident. *B*, 35 feet from entrance; questionable waves. *C*, 55 feet from entrance; faint waves. *D*, 75 feet from entrance; definite waves.

data conform to the logarithmic laws of resistance. Because the correlation between the flows that are classified as unstable and the actual presence of roll waves for these flows is not perfect, the small differences between the various criteria are overlooked. As a consequence, the question of which of the many criteria is best is not considered in this report.

ACKNOWLEDGMENT

This work was performed under a cooperative agreement between the U.S. Geological Survey and the Iowa Institute of Hydraulic Research, Dr. Hunter Rouse, director. It represents an extension of the doctoral dissertation of the senior author.

These findings would not have reached their present state of completeness without the encouragement and guidance of Dr. Hunter Rouse. Gratitude is also due Mr. Dale Harris and the craftsmen ot his shop for conforming to what at times may have seemed like unwarranted standards of workmanship. The contributions of the graduate students, Messrs. A. Schultz, H. Meyer, T. Carmody, and J. Nemec, who so patiently collected data, are greatly appreciated.

GENERAL FORM OF THE STABILITY CRITERION ACCORDING TO IWASA

Analysts of open-channel flow stability have in general used two approaches in the formulation of stability criteria: the study of the time growth of a free-surface perturbation (Jeffreys, 1925; Keulegan and Patterson, 1940; Vedernikov, 1946; Escoffier, 1950; Craya, 1952; Iwasa, 1954; Koloseus, 1958) and the study of a quasi-steady nonuniform flow regime (Thomas, 1940; Dressler, 1949). The former pertains to unsteady flow, whereas the latter is concerned with a quasi-steady regime. The results of all of these works are strikingly similar; differences between them arise from the varying assumptions made by each investigator with regard to the law of resistance, the velocity distribution, and the channel shape.

One form of the dynamic equation of motion for gradually varied unsteady flow that has been used in the development of stability criteria is

$$\frac{\partial U}{\partial t}+\beta\frac{U\partial U}{\partial x}+g\cos\theta\frac{\partial y_0}{\partial x}+(1-\beta)\frac{U}{A}\frac{\partial A}{\partial t}=gS-\frac{\tau}{\rho R} \qquad (1)$$

where

g=acceleration of gravity,
t=time,
u=velocity at a point,
x=distance in mean-flow direction,

y_0=depth of flow measured normal to the channel floor,
A=cross-sectional area,
R=hydraulic radius; ratio of area to wetted perimeter,
$S=\sin\theta$,
U=average velocity,
β=velocity distribution factor, $\frac{\int u^2 dA}{U^2 A}$
θ=angle of inclination of the channel,
ρ=mass density of the fluid,
τ=average boundary shear.

A lucid derivation of equation 1 along with that for the equation of continuity,

$$\frac{\partial A}{\partial t}+\frac{U\partial A}{\partial x}\frac{A\partial U}{\partial x}=0, \tag{2}$$

has been set forth by Keulegan and Patterson (1943). One of the basic assumptions underlying equation 1 is that the pressure distribution is hydrostatic. This same assumption is incorporated in the analyses of the propagation of shallow-water waves.

Iwasa (1954) obtained through equations 1 and 2 the relation

$$-\frac{A}{\left(\frac{\partial\tau}{\partial U}\right)}\left[\frac{\partial\tau}{\partial A}-\frac{\tau}{R}\left(\frac{dR}{dA}\right)\right]\begin{bmatrix}\text{Unstable}\\ \text{Stable}\end{bmatrix}\begin{matrix}>\\<\end{matrix}\left[\beta-1\pm\sqrt{\beta^2-\beta+\frac{S}{\left(\frac{U^2}{gR\cos\theta}\right)\left(\frac{dA}{dy_0}\right)}}\right]U, \tag{3}$$

as a general form of the stability criterion. The final form was obtained by assuming that equations 4 and 5 pertained:

$$\tau=\rho g R S, \tag{4}$$

and

$$U=C_1^{C_2}R^{C_3}S^{C_4}, \tag{5}$$

where

C_1=boundary-roughness factor,
C_2, C_3, C_4=constants for limited ranges of depth.

The utilization of the equations 4 and 5 in this manner is tantamount to assuming that these expressions are as applicable to gradually varied unsteady flow as they are to steady uniform flow. Iwasa's criterion, equation 3, reduces to that of Vedernikov (1946) for comparable conditions.

DERIVATION OF A STABILITY CRITERION FOR ROUGH RECTANGULAR CHANNELS

In the derivation of the stability criterion for flow in rough channels that is presented herein, Iwasa's (1954) general stability criterion, equation 3, is considered applicable, as well as the logarithmic law of resistance,

$$f=\frac{1}{\left[C_5 \log\left(\frac{4R}{k} C_6\right)\right]^2}, \tag{6}$$

where

f=resistance coefficient that is defined as

$$f=\frac{8\tau}{\rho U^2}, \tag{7}$$

or, in view of equation 4, as

$$f=\frac{8gRS}{U^2}, \tag{8}$$

and where

k=height of roughness,
C_5 and C_6=constants.

Through the adoption of equations 4 and 6, it is tacitly assumed, as was done by Iwasa (1954) and by others, that these relations are as applicable to this gradually varied unsteady state as they are to the steady uniform state. The utilization of equations 6 and 8 in the differentiation indicated by equation 3 results in

$$\frac{U}{\sqrt{\frac{gA\cos}{\frac{dA}{dy_0}}}}=\mathbf{F}\begin{bmatrix}\text{Unstable}\\ \text{Stable}\end{bmatrix}\gtrless\mathbf{F}_s=$$

$$\frac{1}{\left\{\left[\frac{A\left(\frac{dR}{dy_0}\right)}{R\left(\frac{dA}{dy_0}\right)}(0.4343C_5 f^{1/2}+0.5)-(\beta-1)\right]^2-\beta(\beta-1)\right\}^{1/2}}, \tag{9}$$

where

$\mathbf{F}$=Froude number, defined by the expression on the left side of equation 9,

$\mathbf{F}_s$=Limiting value of $\mathbf{F}$ for stable flow, henceforth to be called the stable-flow limit, and equal in magnitude to the expression on the right side of equation 9.

Equation 9 is the stability criterion for flow in rough channels. The flow is classified as stable if the Froude number is less than the stable-flow limit, and it is classified as unstable if the reverse is true. A convenient way of setting forth the relative magnitudes of $\mathbf{F}$ and $\mathbf{F}_s$ is by means of the ratio $\mathbf{F}/\mathbf{F}_s$, henceforth called the degree of instability. Although they are not exactly equal, the degree of instability and what Powell (1948) has called the Vedernikov number are identical in significance. If $\mathbf{F}/\mathbf{F}_s$ is less than 1, the flow is classified as stable; if it is greater than 1, the flow is unstable. It is rational that the inherent instability of the flow would increase as the magnitude of $\mathbf{F}/\mathbf{F}_s$ becomes progressively greater than 1.

If the velocity distribution is considered to be logarithmic, which, incidentally, is in keeping with the logarithmic law of resistance, then in a wide channel the equation given by Iwasa (1954) pertains:

$$\beta=1+0.781f. \tag{10}$$

The stability criterion, equation 9, depends upon C_5 which has, in general, been found to have a value of 2. If equation 10 applies, if $C_5=2$, and if the channel is rectangular, then equation 9 reduces to

$$\frac{U}{\sqrt{gy_0}\cos\theta}=\mathbf{F}\begin{bmatrix}\text{Unstable}\\ \text{Stable}\end{bmatrix}\begin{matrix}>\\<\end{matrix}=\mathbf{F}_s=$$

$$\frac{1}{\left[\left(\dfrac{0.8686f^{1/2}+0.5}{\dfrac{2}{\left(\dfrac{B}{y_0}\right)}+1}-0.781f\right)^2-0.781f(1+0.781f)\right]^2}, \tag{11}$$

where

B=width of the channel.

Equation 11 is the stability criterion for flow in rough rectangular channels. The right side of this equation is graphed in figure 4. Each line in figure 4 is the locus of the stable-flow limit for a particular value of the channel aspect ratio, B/y_0. A ready evaluation of the state of stability is forthcoming from figure 4 if the ordinate represents both $\mathbf{F}$ and $\mathbf{F}_s$ because a particular state of flow would be unstable if its Froude number and resistance coefficient were such that the point plotted above its locus of the stable-flow limit; the reverse would be true if it plotted below the locus. If in the derivation of the stability criterion the velocity distribution is considered to be uniform, β will have a value of 1. The graph of the stability criterion under this condition for an infinitely wide rough channel is that depicted by the dashed line in figure 4.

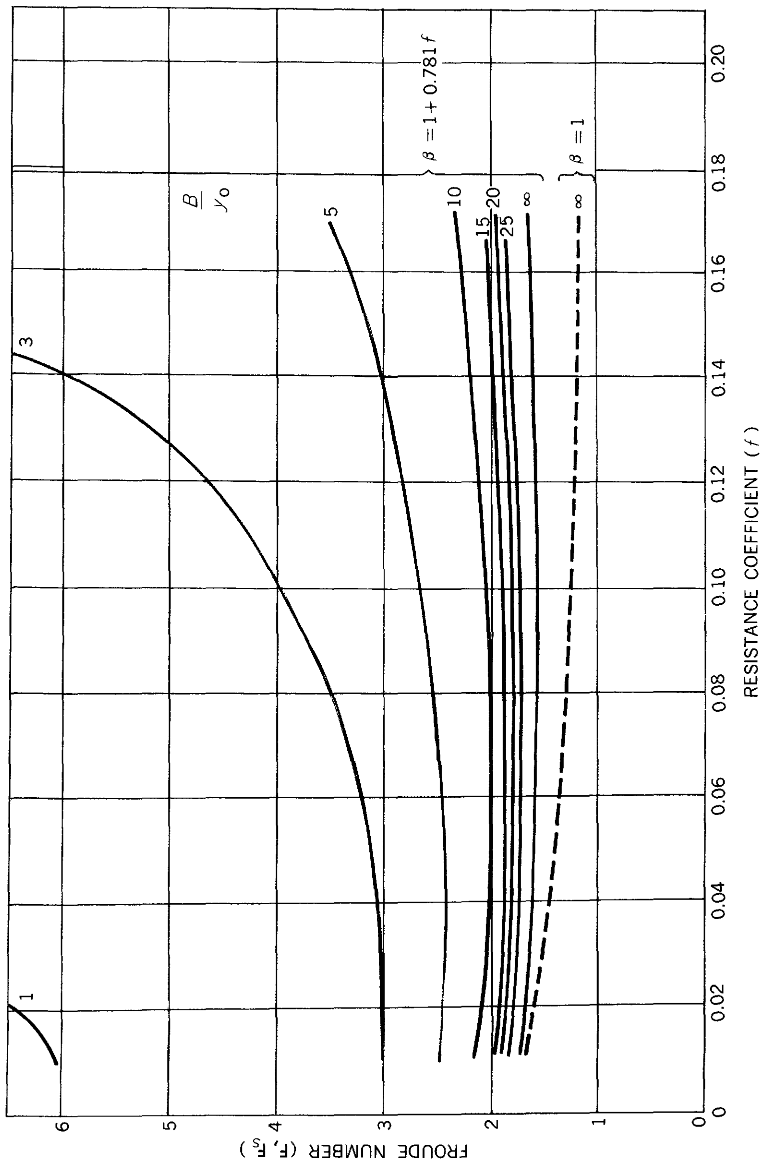


FIGURE 4.—Stability criterion for hydrodynamically rough rectangular channels. Relation between Froude number at limit of stable flow and resistance coefficient at various values of aspect ratio and for two velocity-distribution factors.

DERIVATION OF A STABILITY CRITERION FOR SMOOTH RECTANGULAR CHANNELS

For flow in smooth channels, equations 3 and 8 are again considered appropriate; in addition, the law of resistance for this type of boundary is taken as

$$f=\frac{1}{[C_5 \log (\mathbf{R}\sqrt{f}C_7)]^2}, \tag{12}$$

where

R=Reynolds number; $4RU/\nu$ based on the hydraulic radius, $4y_0U/\nu$ based on two-dimensional flow.

Through equations 3, 8, and 12, the stability criterion for flow in smooth channels is obtained:

$$\frac{U}{\sqrt{\frac{gA\cos\theta}{\frac{dA}{dy_0}}}}=\mathbf{F}\begin{bmatrix}\text{Unstable}\\ \text{Stable}\end{bmatrix}\gtrless \mathbf{F}_s=$$

$$\frac{1}{\left[\left\{\frac{A\left(\frac{dR}{dy_0}\right)}{R\left(\frac{dA}{dy_0}\right)}\left[\frac{3}{2}(0.4343)C_5f^{1/2}+0.5\right]-(\beta-1)\right\}^2\beta-\beta(\beta-1)\right]^{1/2}}. \tag{13}$$

As it does for the rough channel, the stability criterion depends upon C_5 which, in general, has a value of 2. If equation 10 applies, if $C_5=2$, and if the channel is rectangular, equation 13 reduces to

$$\frac{U}{\sqrt{gy_0\cos\theta}}=\mathbf{F}\begin{bmatrix}\text{Unstable}\\ \text{Stable}\end{bmatrix}\gtrless\mathbf{F}_s=$$

$$\frac{1}{\left[\left(\frac{1.303f^{1/2}+0.5}{\frac{2}{\left(\frac{B}{y_0}\right)}+1}-0.781f\right)^2-0.781f(1+0.781f)\right]^2}, \tag{14}$$

which is the stability criterion for flow in smooth rectangular channels. A graphical representation of equation 14 is shown in figure 5. The stability criterion for flow in an infinitely wide channel with a uniform velocity distribution is depicted in figure 5 by a dashed line.

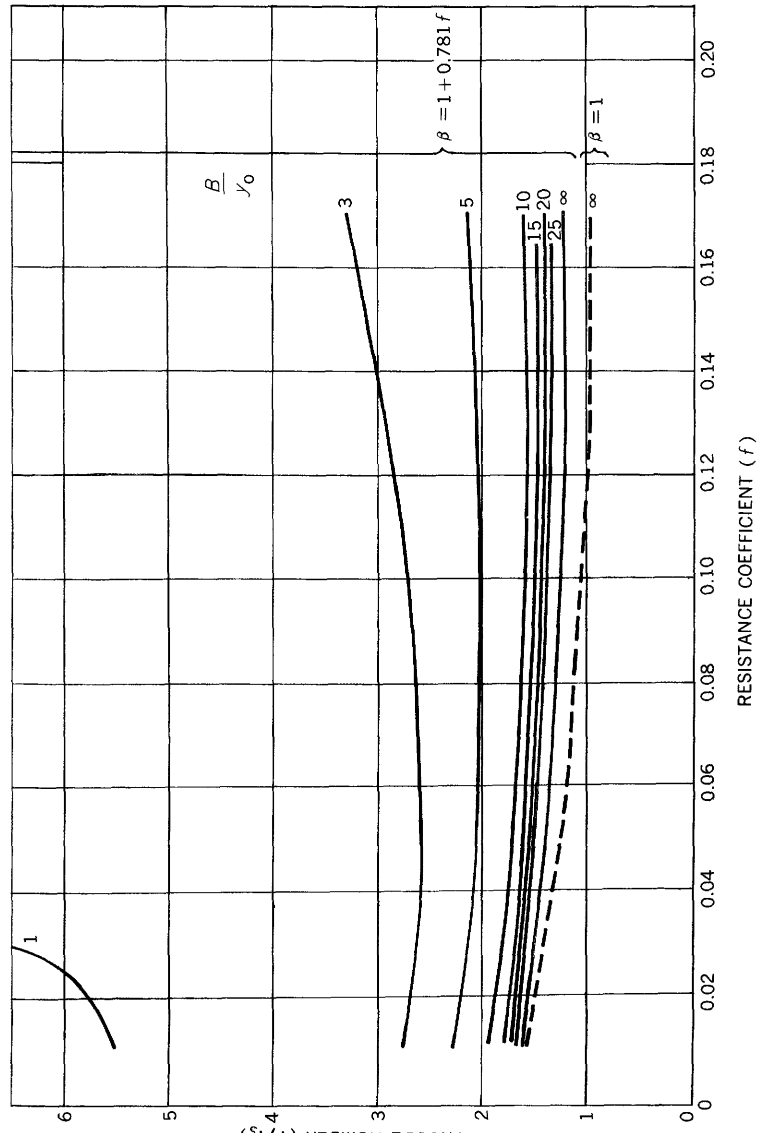


Figure 5.—Stability criterion for hydrodynamically smooth rectangular channels. Relation between Froude number at limit of stable flow and resistance coefficient at various values of aspect ratio and for two velocity-distribution factors.

PARAMETERS AFFECTING STABILITY CRITERIA AND INSTABILITY OF FLOW

It becomes apparent through a comparison of the numerous stability criteria that in all instances the state of stability is a function of the Froude number. Therefore, the Froude number not only has its usual significance so far as the characteristics and manner of flow it the subcritical, critical, and supercritical regimes are concerned, but it is, in addition, a significant parameter that is associated with the state of stability of the flow. These criteria also depend upon channel shape and other parameters associated with the approximations and assumptions incorporated in the derivations. In some instances (Jeffrey, 1925; Thomas, 1940; Keulegan and Patterson, 1940), the assumptions are such that the state of stability is a function of the Froude number alone.

Figures 4 and 5 show that the stable-flow limit increases as the aspect ratio for a rectangular channel decreases; this apparent stabilizing effect of decreasing aspect ratio is worthy of consideration.

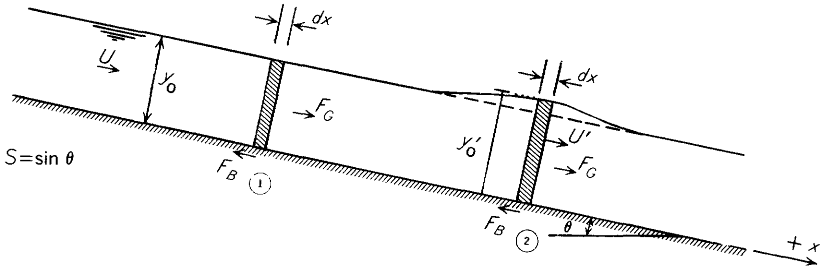
The notable dependency of the stable-flow limit on the velocity distribution is brought out in figures 4 and 5 for an infinitely wide channel through a comparison of the loci for $\beta=1$ and for $\beta=1+0.781f$. Iwasa (1954) not only called attention to the influence of these two factors but also pointed out the great effect that shapes other than rectangular have on the stable-flow limit. According to his work, the stable-flow limit for an infinitely wide channel is 1.7, whereas that for a 90° triangular channel is 5.9.

Roll waves are a consequence of the imbalance that exists between the boundary retarding force and the motivating gravitational force. An indication of the relative magnitude of these forces for the different states of stability can be obtained through a consideration of one form of the stability criterion which was given by Keulegan and Patterson (1940),

$$\rho g y'_0 S - \frac{f\rho (U')^2}{8} \begin{bmatrix} \text{Unstable} \\ \text{Stable} \end{bmatrix} \gtrless 0. \qquad (15)$$

The heretofore undefined symbols are delineated in figure 6. Equation 15 shows that flow is unstable when the gravitational force exerted on the elemental section of a wave, F_G in figure 6, exceeds the channel retarding or frictional force, F_B. Flow is stable when the reverse is true.

During the initial roll-wave formational stages, the waves are considered to be infinitesimal in size and to have the characteristics of shallow-water waves. Consequently, their celerity is equal to



Flow classification	Force relations section 1	section 2
Stable, $\frac{\mathbf{F}}{\mathbf{F}_s}<1$	$F_G=F_B$	$F_G<F_B$
Unstable, $\frac{\mathbf{F}}{\mathbf{F}_s}>1$	$F_G=F_B$	$F_G>F_B$

$\mathbf{F}$=Froude number.
$\mathbf{F}_s$=Limiting value of the Froude number for stable flow.
F_G=Gravitational force on an elemental section of fluid, $\rho g y_0 S dx$ and $\rho g y_0' S dx$.
F_B=Boundary retarding force on the elemental section of fluid, $\frac{f\rho U^2}{8}\,dx$ and $\frac{f\rho U'^2}{8}\,dx$.

FIGURE 6.—Free-body diagram showing force relations for shallow-water waves.

$\sqrt{gy_0}$. Doubling the amplitude of a small wave (that is, doubling $(y_0'-y_0)/y_0$, which for a small wave might be smaller than 0.01) will result in less than 1 percent change in celerity but will increase the mass of the wave above the undisturbed surface by more than 100 percent. The force imbalance for unstable flow indicated in figure 6 gives rise to an impulse that increases the momentum of the wave. It follows, then, that this increase in wave momentum is achieved chiefly through an increase in the mass of the wave above the undisturbed surface rather than a substantial change in celerity. Because it is a wave phenomenon, however, that is under discussion, the impulse must increase the wave celerity to some extent if it is to increase the momentum at all. As long as this imbalance exists, the size of the wave will increase. In the derivations of stability criteria based on the time growth of infinitesimal disturbances, it is implicit that the forces exerted on the fluid in the reaches between waves (for example, section 1 in fig. 6) are in equilibrium. Rouse (1938) said that the larger waves travel faster and in the end overtake and coalesce with the smaller waves, and thereby always increase in size and velocity; he also stated that roll waves would be separated by comparatively dry sections of channel if the channel were sufficiently long.

Dressler (1949) investigated the role of channel resistance in free-surface instability and pointed out that roll waves cannot form if the boundary resistance is zero. In a later paper on flow in a wide channel, he and Pohle (1953) indicated that the constants C_2 and C_3 in

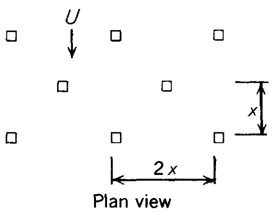
$$U=C_1 y_0^{C_2} S^{C_3} \qquad (16)$$

must be greater than zero if roll waves are to develop. This roll-wave resistance requirement is met for all situations for which the Manning and Chezy equations are applicable. Jeffrey (1934, p. 154), in discussing roll waves, said in effect that roll waves depend upon friction, whereas bores probably occur in spite of it. These comments indicate that boundary resistance is a necessary prerequisite for the formation of roll waves. Rouse (1938, p. 388), however, also said that they could be eliminated by making the channel rougher, which, in effect, would increase the channel resistance. This comment by Rouse is rational in light of the stability criteria. Greater channel roughness would bring about an increase in f which would, according to figure 4, have little effect on the stable-flow limit when B/y_0 is greater than 10. The increased resistance, however, for a channel of constant width, slope, and discharge would result in a greater depth that in turn would bring about a decrease in the magnitude of both the Froude number and the aspect ratio. Figure 4 indicates that a lessening of either or both of these parameters shifts data toward the region of stable flow. The effect of channel resistance on the formation of roll waves can be summarized in the words of Dressler (1949), who said "roll waves cannot occur either if the resistance is zero or if the resistance exceeds a certain critical value."

LABORATORY EQUIPMENT

All data pertinent to the study of the correlation between the stability criteria and the physical manifestations of instability in free-surface flow were collected in the laboratories of the Iowa Institute of Hydraulic Research, Iowa City, Iowa. Two rectangular variable-slope flumes were used. One flume was 2.0 feet wide and 30 feet long; it had plate-glass walls and a steel floor. The other was 2.5 feet wide and 85 feet long; it had plate-glass walls and floor. Flows over both smooth boundaries (boundary VIII, table 1, page C22) and roughened boundaries (boundaries I–VII, table 1) were studied.

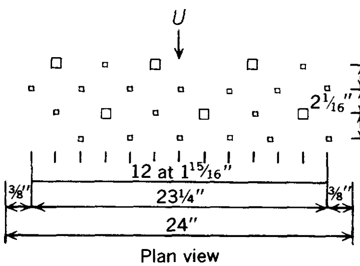
Brass cubes and a plastic louver having square openings, placed only on the floor, formed the rough surfaces as shown in figure 7. The louver type of roughness was meant to simulate a high concentration of cubes. Various concentrations of cubes, 3/16 inch on a side, were used for boundaries I, II, IV, V, and VI. These were placed in a diamond pattern with a face normal to the mean direction of fluid motion. The roughness concentration factor, λ, is defined as the ratio of the total upstream projected area of the protuberances to the total floor area of the flume. Boundary III was similar to the other cube-roughened boundaries, except that cubes of two different sizes were used, three 3/8-inch cubes to ten 3/16-inch cubes. Boundary VII was formed by filling the alternate 3/8-inch square openings of the plastic louver. This in essence formed a flat surface having square holes arranged in a checkerboard pattern. Boundary VIII was simply the smooth plate glass that formed the sides and floor of the 85-foot flume. Photographs of some of the boundaries and equipment are shown in figures 8–12.



CUBE-ROUGHENED BOUNDARIES

BOUNDARY	Cube (in.)	λ	x (in.)
I	3/16	1/512	3
II	3/16	1/128	1½
IV	3/16	1/32	¾
V	3/16	1/32	¾
VI	3/16	1/8	⅜

$k_l = 0.01563$ ft $\qquad \lambda_I = \frac{(3/16)^2}{3(6)} = \frac{1}{512}$



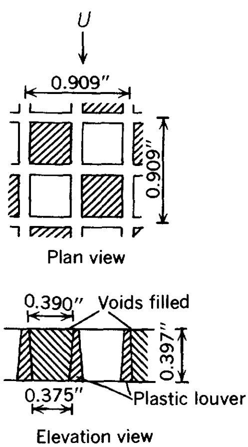
BOUNDARY III-MIXED CUBES

Ten 3/16-in. cubes to three ⅜-in. cubes

$$k_l = \frac{\lambda_a k_a + \lambda_b k_b}{\lambda_a + \lambda_b} = \frac{\frac{10}{24}\frac{(3/16)^2}{(33/8)}\cdot\frac{3}{16} + \frac{3}{24}\frac{(3/8)^2}{(33/8)}\cdot\frac{3}{8}}{\frac{10}{24}\frac{(3/16)^2}{(33/8)} + \frac{3}{24}\frac{(3/8)^2}{(33/8)}}\cdot\frac{1}{12}$$

$k_l = 0.02415$ ft

$$\lambda = \frac{10(3/16)^2 + 3(3/8)^2}{24(33/8)} = \frac{1}{128}$$



BOUNDARY VII LOUVER

$$\lambda = \frac{2(0.397)\left[0.909 - \frac{(0.390+0.375)}{2}\right]}{(0.909)^2} = 0.506$$

$k = 0.03308$ ft

FIGURE 7.—Schematic diagrams of rough boundaries.

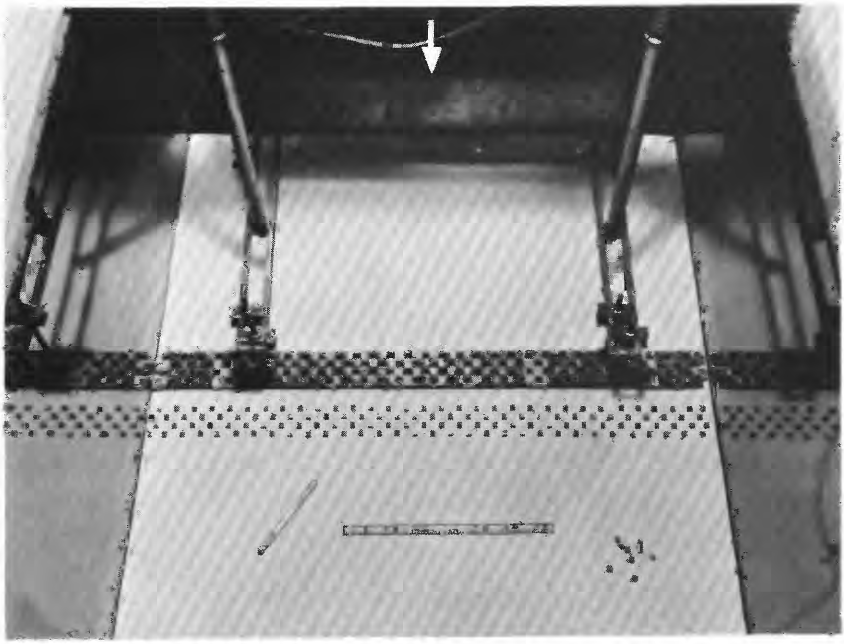


FIGURE 8.—Cube-placement jig for boundary VI; 2- by 30-foot flume; smallest scale division, 0.01 foot.

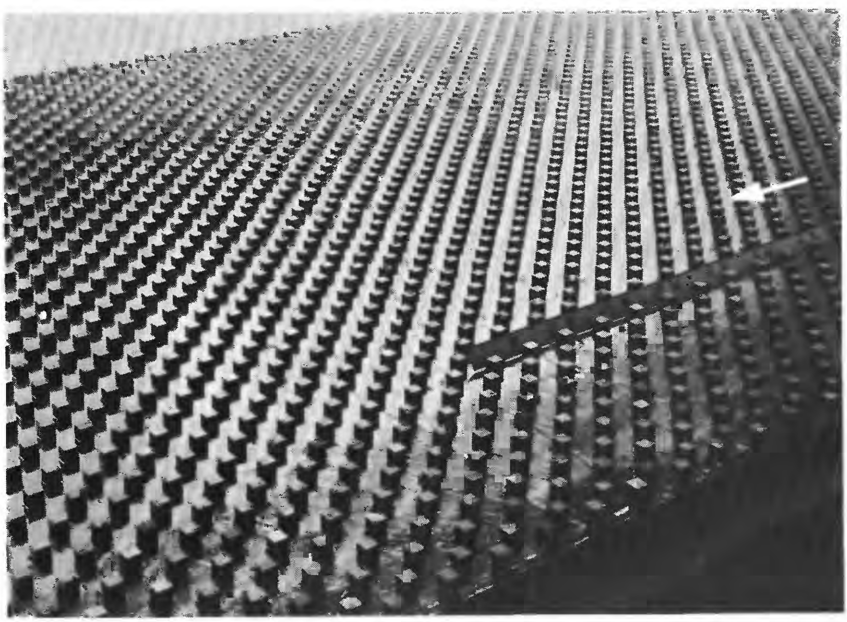


FIGURE 9.—Boundary VI; 2- by 30-foot flume; smallest scale division, 0.01 foot.

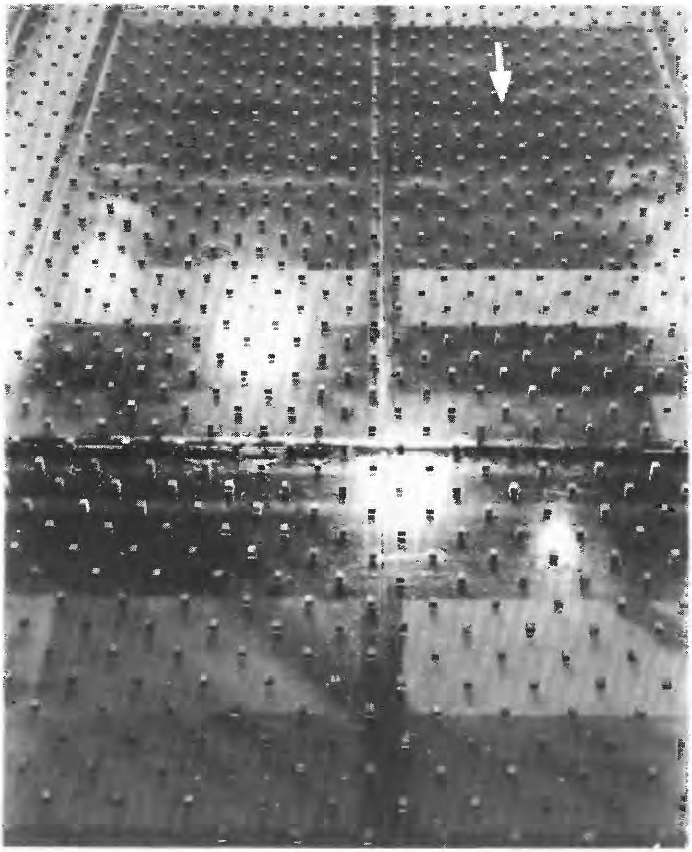


FIGURE 10.—Boundary V; 2.5.- by 85-foot flume.

In locating the datum from which the depth of flow over rough boundaries I–VI was measured, the criterion followed was the same as that adopted by Schlichting (1936). The depth was considered to be the distance from the free surface to the plane that would be formed if the volumes of the roughness elements were distributed evenly over the flume floor. For boundary VII, y_0 was taken as the distance from the free surface to the top of the louver. For boundary VIII, the depth was measured from the free surface to the smooth glass floor.

Because of the large number that would have been required and because of the interference effects from the cubes, it was not feasible to install a piezometer at each location where the depth was desired. Therefore, use was made of a special technique and instrument to determine the depths at the many positions. In order to overcome the uncertainties associated with the ordinary point gage when the water surface was rough, an electrical vibrating-needle gage was employed (fig. 13). This device was used in a null-sensing fashion and was attached to a movable scale. By means of this combination of instruments, a measure of the difference in depth between a location in the flume and a reference piezometer could be obtained. The depth at this location could then be determined through a consideration of the difference in depth and the depth as indicated by the reference piezometer. Except for boundary VII, the reference piezometers were installed in the floors of the flumes. In order to eliminate the possibility of the cubes affecting the piezometers, they were omitted from the area in the immediate vicinity of the floor opening. This area was approximately 6 inches on a side. The piezometer arrangement for boundary VII is shown in figure 11.

The summary of collected data in table 1 shows that the scopes of the data for the various boundaries are not the same. The reason is that the data presented herein were gathered as just one facet of another investigation; the collection commenced only after the other study had been in progress for some time.

When observations of roll waves were made and also when data relative to the resistance coefficient were collected, a particular discharge was established and maintained—q = constant cubic feet per

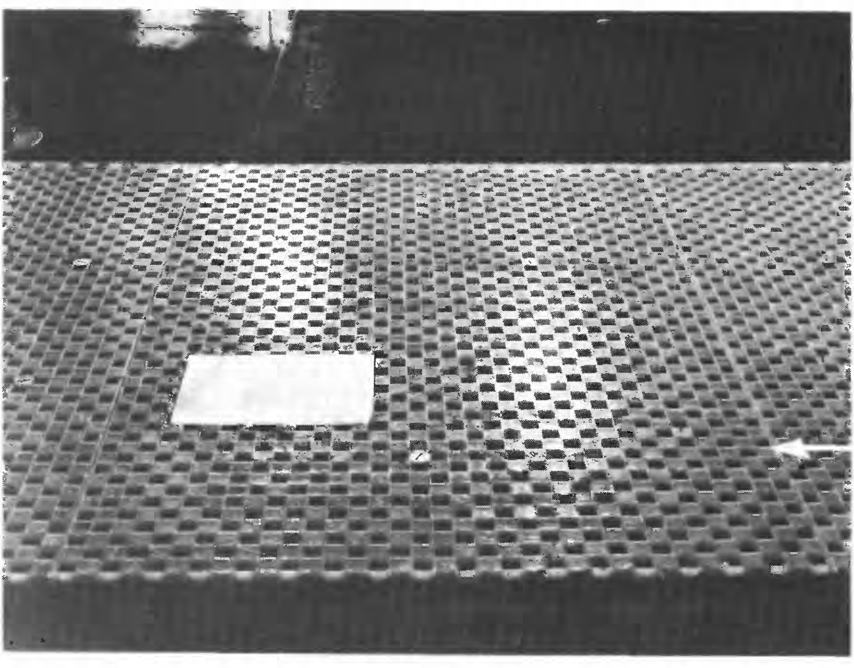


FIGURE 11.—Boundary VII; 2- by 30-foot flume. Rectangular plate contains reference piezometer for measurement of water depth.

FIGURE 12.—Boundary VIII; 2.5- by 85-foot flume.

second per foot—while the slope was varied. Owing to the fact that the roughness of the floor was materially greater than that of the glass walls when either the cubes or the grid was in place, flows in the roughened flumes were treated as being two dimensional. As a consequence of this assumption and the fact that the kinematic viscosity of the water varied little (temperature changes were small), the Reynolds number for the rough boundaries was practically constant and independent of the slope for a particular discharge, $\mathbf{R}=4y_0\mathrm{U}/\nu=4\ q/\nu$. In the analysis of the smooth-channel results, the length parameter in the Reynolds number was taken as the hydraulic radius; with a constant discharge, the hydraulic radius changed with variations in slope because the depth became less as the slope was increased. Even so, for a particular discharge in the smooth channel of relatively large aspect ratio, the Reynolds number did not vary greatly with slope because changes in RU and ν were small. For a constant discharge in

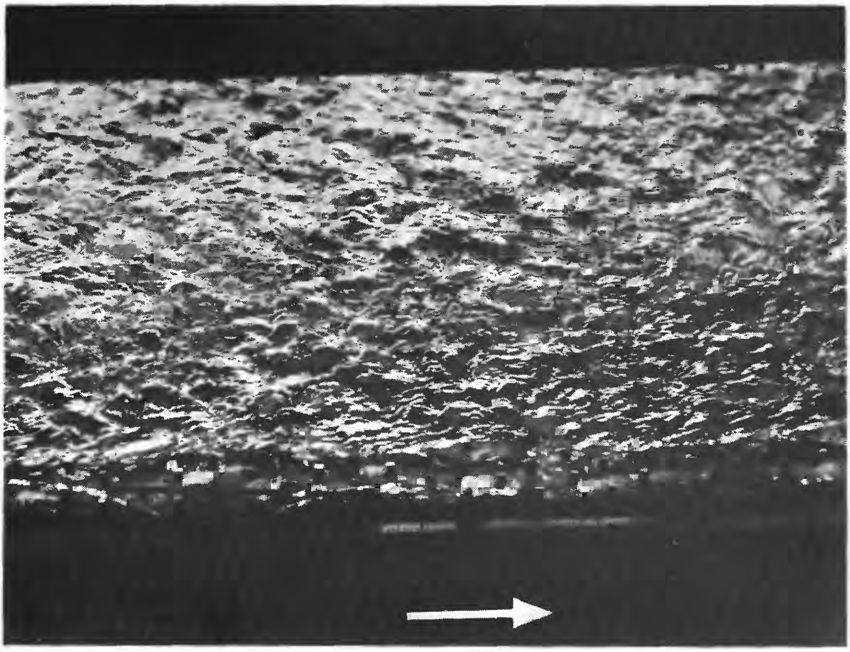


FIGURE 13.—Agitated free surface in roughened channel. Boundary IV; q=0.50 cfs per ft; y_0=0.11 foot S=0.067; $\mathbf{F}$=2.5; smallest scale division, 0.01 foot.

either the smooth or the rough flume, the Froude number naturally increased with slope because of the decreasing depth. When data relative to the presence of roll waves were collected, no great effort was made to determine y_0 accurately because the resulting precision would not be commensurate with the uncertainties associated with the presence and location of roll waves. Owing to this lack of precision, these roll-wave data, which are given in table 2 (page C58), were not used in defining the resistance relations. A great effort, however, was made to measure accurately the parameters used in the study of the resistance relations; these results are tabulated in table 3 (page C64).

The channel slope was determined from a reading of a counter geared to the slope-changing mechanism. The discharge in the 30-foot flume was measured by means of an orifice and that in the 85-foot flume by a weir; both devices were calibrated gravimetrically. A mercurial thermometer indicated the temperature. The value of $\mathbf{F}_s$ in the ratio $\mathbf{F}/\mathbf{F}_s$ for the rough boundary was based on a logarithmic velocity distribution, the resistance coefficient, and an infinite aspect ratio, and was determined through use of figure 4. For the smooth boundary, $\mathbf{F}_s$ was based on a logarithmic velocity distribution, the resistance coefficient, and the aspect ratio, and was determined from figure 5. The roll-wave development distance, L, refers to the dis-

TABLE 1.—*Catalog of comparable results*

	Boundary							
	I	II	III	IV	V	VI	VII	VIII
Roughness	3/16-inch cubes	3/16-inch cubes	3/16- and 3/8-inch cubes.	3/16-inch cubes	3/16-inch cubes	3/16-inch cubes	Plastic louver	None (smooth).
Concentration, λ	1/512	1/128	1/128	1/32	1/32	1/8	0.506	
Flume dimensions, feet	2×30	2×30	2×30	2×30	2.5×85	2×30	2×30	2.5×85.
	Results; Roll-wave data							
Correlation:								
Stability criterion			Fig. 14			Fig. 18	Fig. 21	Fig. 25.
q, L, and S			Fig. 15					Fig. 26.
$\mathbf{F}/\mathbf{F}_s$, L, and q			Fig. 16			Fig. 19	Fig. 22	Fig. 27.
$\mathbf{F}/\mathbf{F}_s$, and fL/y_0			Fig. 17			Fig. 20	Fig. 23	Fig. 28.
	Results; Resistance data							
Range of values:								
y_0 feet	0.059–0.25	0.074–0.36	0.072–0.36	0.068–0.38	0.075–0.42	0.096–0.38	0.061–0.33	
q cfs per ft	0.31–1.3	0.35–1.4	0.31–1.3	0.25–1.3	0.25–1.8	0.31–1.3	0.31–1.3	
f	0.022–.035	0.030–.056	0.034–.066	0.042–.087	0.040–.085	0.061–.16	0.029–.043	
$\mathbf{F}$	1.2–4.1	0.98–3.5	1.1–3.2	0.95–2.8	0.49–2.6	0.95–2.2	1.0–4.1	
$\mathbf{F}_s$	1.63–1.68	1.59–1.65	1.59–1.63	1.57–1.61	1.57–1.62	1.56–1.59	1.61–1.65	
$\mathbf{R}\times10^{-5}$	1.2–5.2	1.4–5.5	1.2–6.0	0.96–4.9	0.94–7.1	1.2–5.2	1.3–5.6	
Flow classification:								
Stable					Fig. 29			
Stable and unstable	Fig. 32	Fig. 32	Fig. 33	Fig. 32	Figs. 30 and 32	Fig. 32	Fig. 34	
Unstable					Fig. 31			
Composite—stable	Fig. 35	Fig. 35		Fig. 35	Fig. 35	Fig. 35		
Composite—unstable	Fig. 36	Fig. 36		Fig. 36	Fig. 36	Fig. 36		

tance from the flume entrance to that location where roll waves first became noticeable.

During the observation of roll waves, lighting and viewing points were found to be important. The waves could best be seen when the observer was at the downstream end of the flume viewing the water surface by means of reflected light that emanated from a diffused source located at the flume entrance (figs. 1 and 2). Another satisfactory method of visually detecting roll waves was by sighting diagonally upstream from the sides of the flume slightly above the plane of the free surface.

ROLL-WAVE OBSERVATIONS

When roll waves were at the initial formational stage, opinion varied amongst the observers regarding their presence or absence. Likewise, the distance from the entrance at which they first became discernible was subjective. When the results of two observers were compared, it was noted that one consistently detected the roll waves at lower values of the Froude number and at smaller distances from the flume entrance. These comments serve to indicate that the roll-wave data are subject to personal bias.

For a constant discharge, the water surface became rougher as the flume slope was increased; the water-surface roughness also increased with discharge when the slope was kept constant. In general, the water surface was less agitated for the smooth boundary than for the rough; this had its advantages and its disadvantages. The smoother water surface made it easier to detect waves, including those of dubious origin. When the water surface above the smooth and rough boundaries was very disturbed, it was difficult to make an unequivocal statement concerning the presence or absence of very small roll waves.

It became apparent after numerous observations in the 2.5- by 85-foot smooth flume that the source and classification of water-surface disturbances of almost imperceptible magnitude were subject to question and could have been due to one or more of the following causes:

1. Free-surface instability, the phenomenon under discussion.
2. Flume vibration from the laboratory recirculating pumps and vehicular traffic on the adjacent highway.
3. Surges, particularly at the higher discharges, due to high-velocity currents and waves in the head box.
4. Moving regions of fluid of excessive surface agitation, some of which traveled at a velocity equal to U; these regions were not considered to be roll waves. Others moved with a velocity, U_w, which satisfied the equation

$$U_w \geq U + \sqrt{gy_0}. \tag{17}$$

These regions were regarded as roll waves. Cornish's measurements (1934, p. 97) indicate that equation 17 pertained to the roll waves he observed in the Alps.

5. Headgate control. To expedite the establishment of normal flow when the Froude number was greater than 1, use was made of an inclined headgate. When this gate controlled the entering flow, waves were detected at lower Froude numbers, and they appeared to originate closer to the entrance than when the gate was not employed. It was also found during the course of experimenting with the gate that an unsteady nonuniform flow condition, similar to that for roll waves, was established near the entrance when the headgate was so set that the ratio of the Froude number of the entering flow to that downstream was significantly greater than 1, perhaps 5–10 times greater than 1. Equipment limitations prevented the attainment of high values of this ratio when the rates of flow were large, which may account for the observance of this phenomenon at only the smaller discharges.

When the waves were very small, it was impossible to differentiate between water-surface disturbances due to the natural instability of the flow and those due to the causes just enumerated. If the flume were of unlimited length and free of vibration, the uncertainty concerning the presence of roll waves and their origin would not exist, because only those surface perturbations that could travel indefinitely would be roll waves. Equation 15 and figure 6 indicate that only where there is unstable flow is the balance of forces such as to foster the formation and the perpetual movement of a wave in an inclined channel.

The preceding remarks of uncertainty pertain only to the barely perceptible waves, particularly in the smooth flume. More pronounced waves had to exist in the rough than in the smooth flume in order that the waves could be detected amidst the increased water-surface roughness. For this reason, there was less doubt regarding the presence or absence of roll waves in the rough flumes. Once perturbations grew to unmistakable waves, there was no question that they were roll waves because the only explanation for their development to such a state was the instability of the flow associated with the presence of the free surface.

Another phenomenon that gave rise to uncertainty regarding the presence of roll waves was seen in the smooth flume. In the course of increasing the flume slope with the discharge kept constant, the flow was noted to change from one where no waves existed to one where they were barely perceptible; the flow in the latter were classifiable as unstable according to the stability criterion in figure 5. As did previous slope changes, further increases in the slope resulted in a further increase above 1 in the degree of instability and in an increase in water-surface roughness. This change in $\mathbf{F}/\mathbf{F}_s$, it was thought, would be accompanied by a visible increase in the size of the waves. Within some unknown but limited range of $\mathbf{F}/\mathbf{F}_s$, however, the minute waves gave no visual evidence of growing proportionately larger with increasing slope; beyond this range, they grew as anticipated. Although the increase in water-surface roughness lent uncertainty to these observations, this seemingly limited state of roll-wave development probably existed. This phenomenon was not noted for the flows over the rough boundaries, but it would not be surprising if it had also occurred there to a lesser extent. The occurrence of roll waves in an arrested state of growth that is short of the breaking-front phase may be rationalized in the following manner. Equation 15 indicates that roll waves will exist when the gravitational force on an elemental wave exceeds the boundary-retarding force. The resistance coefficient, f, in equation 15 is that for steady uniform flow. It is reasonable to assume, however, that f will become greater under these slightly unsteady nonuniform conditions; that such is actually so will be shown in a subsequent section. With this increase in f it is plausible that equilibrium between the gravitational and retarding forces in equation 15 might be reestablished for this state of unsteady flow. As a consequence, roll waves in an arrested state of development might be anticipated instead of the continual growth of the waves to the breaking point. Because roll waves are usually thought of in terms of waves having breaking fronts, the actual manifestation of the effects of free-surface instability in the form of waves of limited development might be more prevalent in both the laboratory and in the field than heretofore suspected.

Two objects were successively placed on the floor of the 2.5- by 85-foot flume near the entrance in order to study their effect on roll waves. One was a piece of plastic material molded roughly in the form of a disc, ¼-inch thick and 3 inches in diameter, positioned in the center of the flume. The other was a ⅛-inch brass rod, 2.5 feet long, laid transversely to the flow. The shifting of the point where waves were first noted toward the flume entrance appeared to be the only effect these objects had on the flow.

CORRELATION BETWEEN THE STABILITY CRITERIA AND THE PRESENCE OF ROLL WAVES

Numerous observations of flow in the roughened 30- and 85-foot flumes and in the smooth 85-foot flume were made in the course of studying the correlation between the stability criteria, figures 4 and 5, and the presence of roll waves. Data pertinent to flows over rough boundaries when roll waves were evident are presented commencing with boundary III. These are followed by the data for the smooth boundary. A catalog of all of the figures pertaining to roll waves is to be found in table 1.

In order to establish an appreciation for the magnitudes of the variables measured and for the relations between certain dimensional quantities, some of the results are depicted in dimensional form. As might be anticipated, the maximum coalescing of the data is achieved when the data are expressed in dimensionless form.

The correlation between the stability criterion and the presence of roll waves for boundary III is depicted in figure 14. Because the degree of the correlation is dependent upon the smallest values of $\mathbf{F}/\mathbf{F}_s$ for which roll waves were seen, only that point for each discharge when roll waves were first noted is plotted in this figure. The extent of the collected data for which roll waves were present is indicated by the shaded area of the diagram.

If the flume had been sufficiently long and if the correlation between the presence of roll waves and the stability criterion had been perfect, the points for each discharge in figure 14 would have plotted on a

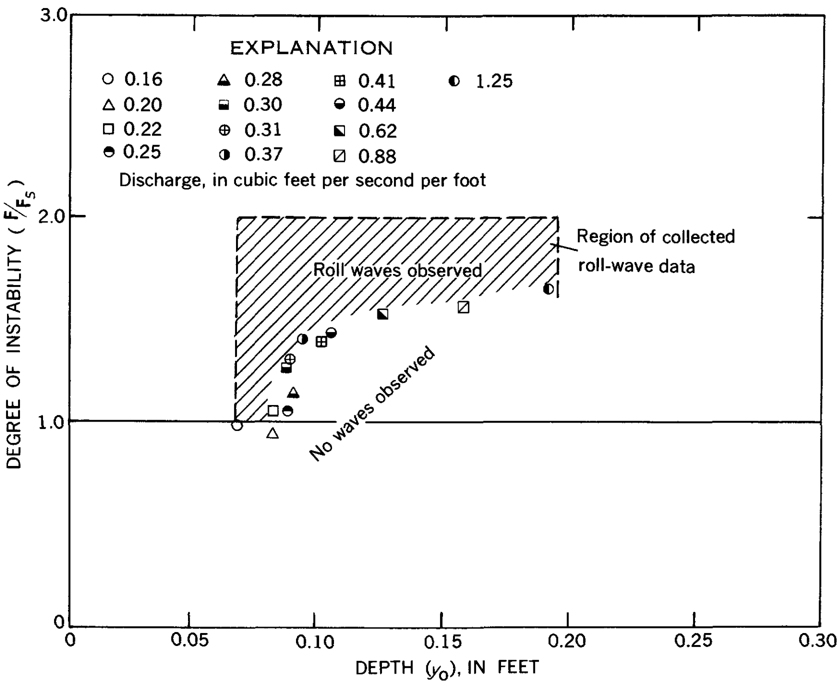


FIGURE 14.—Correlation between degree of instability, depth, and presence of roll waves—boundary III.

common horizontal line at a value of $\mathbf{F}/\mathbf{F}_s$ equal to 1. Such is true for discharges equal to and less than 0.25 cfs per ft. For discharges equal to and greater than 0.28 cfs per ft, there seems to be a relation between the depth, the degree of instability, and the presence of roll waves. It will be brought out in a subsequent section, however, that this correlation is attributable, at least in part, to a flume of insufficient length.

Two facts concerning the roll-wave development distance, L, are evident from figure 15: For a constant slope the required development distance increases with discharge, and for a constant discharge the development distance increases as the slope decreases. The effect of a change in $\mathbf{F}/\mathbf{F}_s$ on L is made apparent through figure 16; as might be expected, the development distance increases as the degree of instability decreases. The fairly systematic change in L with $\mathbf{F}/\mathbf{F}_s$ depicted in figure 16 can be used as a guide in determining the minimum length of flume required for the formation of barely perceptible roll waves when $\mathbf{F}/\mathbf{F}_s=1$. Figure 16 discloses, through an extrapolation of the curves for discharges of 0.25 cfs per ft and less, that the required development distance is equal to or less than the 30-foot length of flume in which the data were collected; with regard to these same data, it will be recalled that the correlation between the stability criterion and the presence of roll waves depicted in figure 14 at $\mathbf{F}/\mathbf{F}_s=1$ is surprisingly good. Through similar extrapolations of the lines for discharges of 0.28 cfs per ft and greater, a development distance in excess of the length of the flume seems to be required. The necessity for development lengths that are greater than the length of the flume for values of $\mathbf{F}/\mathbf{F}_s=1$ accounts, in part if not entirely, for the apparent correlation in figure 14 between the degree of instability, the depth, and the presence of barely perceptible roll waves for discharges equal to and greater than 0.28 cfs per ft.

The data of figure 16 coalesce to a fair degree as shown in figure 17 when the dimensionless ratio, fL/y_0, is introduced. For all practical purposes this parameter is identical with that of Montuori (1961), gSL/U^2. The coalescing effect of the term fL/y_0 indicates for a constant depth that the roll-wave development distance, L, decreases as the channel resistance, f, increases.

Data for boundaries VI and VII, comparable with that in figures 14, 16, and 17 for boundary III, are presented in figures 18–23.

A comparison of the results for boundary VI in figure 18 with the extrapolated lines in figure 19 again supports the contention that the 30-foot channel was not long enough for the formation of roll waves for all discharges when $\mathbf{F}/\mathbf{F}_s=1$; a similar statement based on figures 21 and 22 can be made for boundary VII. Aside from the following exceptions, the trends of the data for these two boundaries support the conclusions based on a consideration of the data for boundary III. The data for boundaries VI and VII contrast with those for boundary III in that the scatter of the data for the former is much greater than that for the latter. The data for these two boundaries differ significantly from those for boundary III in that roll waves were also found in the stable regimes, that is, waves were found when $\mathbf{F}/\mathbf{F}_s$ was less than 1. This difference shows that an anomaly exists between the data as interpreted herein and the stability criterion depicted in figure 4. This discrepancy between the actual data and the stability criterion could be due to either or both of the following causes:

1. The use of unwarranted assumptions and approximations in the derivation of the stability criterion. As an example, a measure of the effect of an improper assumption regarding the velocity distribution on the stability criterion can be gleaned from figures 4 and 5 for infinitely wide channels; the value of $\mathbf{F}_s$ is less for a uniform velocity distribution than for a logarithmic velocity distribution. It is conceivable that the velocity distribution tended to be more uniform than logarithmic especially at the shallower depths for these two roughnesses. Assumptions regarding velocity distribution can, however, at most, account for only part of the discrepancy between the stability criterion and the presence of roll waves depicted in figures 18 and 21.
2. Inapplicability of the stability criterion to rough surfaces for which the roughness concentration is large and for which $4\,y_0/k$ is small, perhaps 25 or less. The Froude number in figure 18 represents an average value; however, the local Froude number at the tops of the cubes would necessarily be higher than the average because of the decreased depth and the increased velocity. Therefore, it is possible, even though the flow was classified as stable on the basis of average values of the parameters, that there existed a sufficient number of locally unstable regions to give rise to the formation of perceptible roll waves.

Of the three sets of results presented so far, that for boundary VII exhibits the least amount of order.

The results for flow over the smooth boundary, boundary VIII, figures 24–27, are similar to those for boundary III; consequently, the rough-boundary conclusions also pertain to the smooth surface. A study of figures 24 and 26 indicates that the 85-foot flume may also have not been long enough for the formation of roll waves at all discharges for which $\mathbf{F}/\mathbf{F}_s=1$.

Supposedly, free-surface flow is unstable and roll waves can develop whenever the degree of instability, $\mathbf{F}/\mathbf{F}_s$, is greater than 1. It has

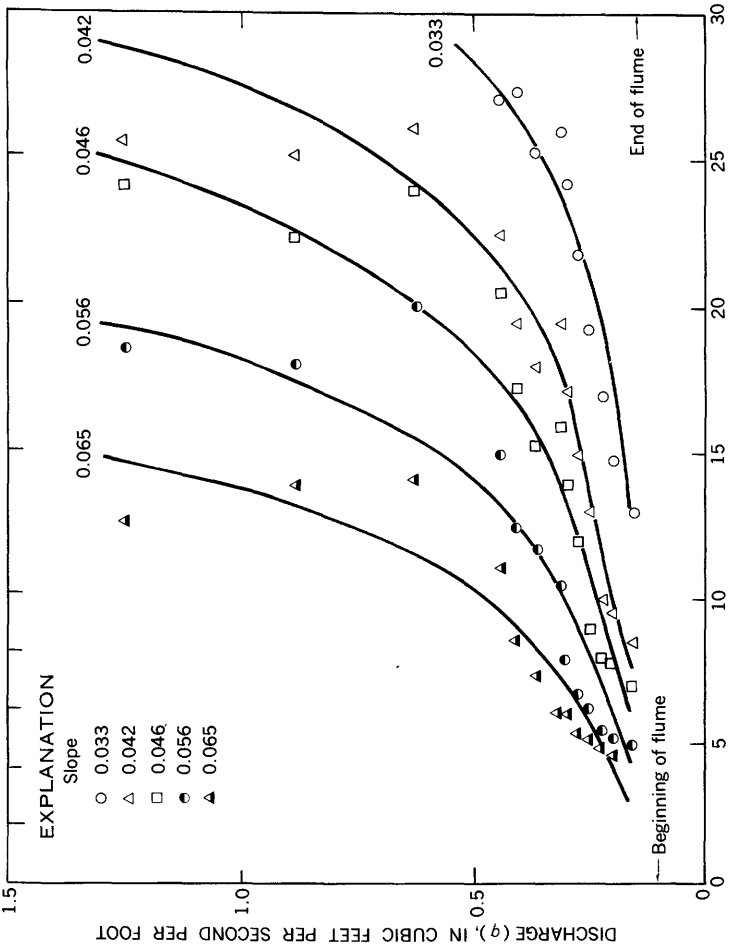


FIGURE 15.—Relation between discharge, roll-wave development distance, and slope—boundary III.

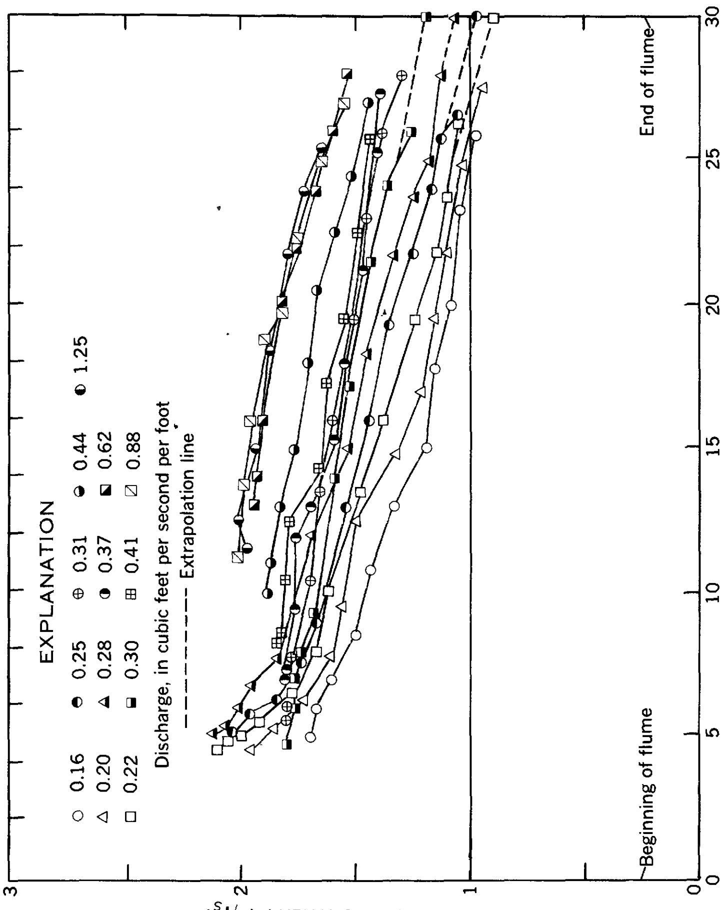


FIGURE 16.—Relation between degree of instability, roll-wave development distance, and discharge—boundary III.

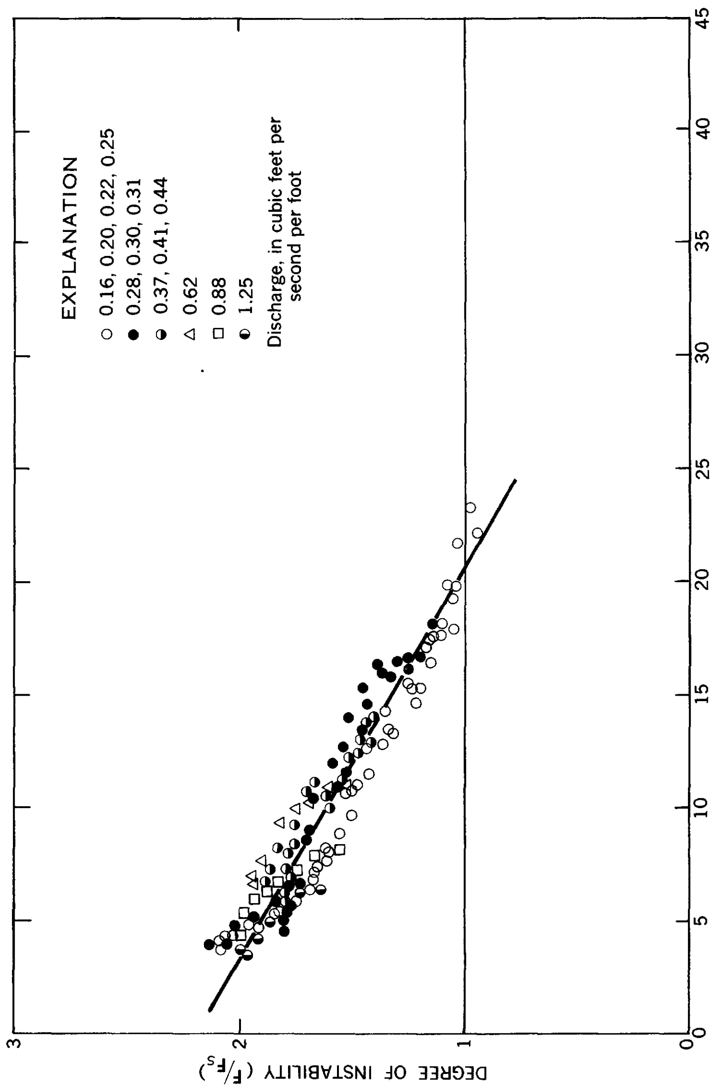


FIGURE 17.—Relation between degree of instability and ratio fL/y_0—boundary III.

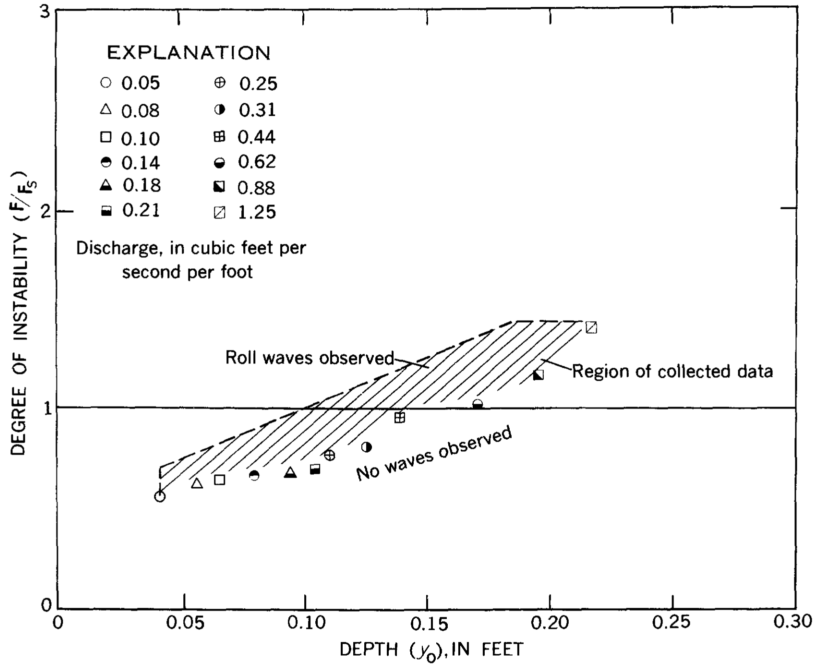


FIGURE 18.—Correlation between degree of instability, depth, and presence of roll waves—boundary VI.

been brought out, however, that the channel length, channel resistance, and the magnitude of $\mathbf{F}/\mathbf{F}_s$ have a bearing on whether or not roll waves are to be found in a particular channel. Montuori (1963), in his discussion of the work of Escoffier and Boyd (1962), arrived at analytically and set forth graphically a free-surface stability criterion that takes into account these factors. On the basis of field data, Montouri (1963) concluded that his criterion, as set forth in figure 28, was quite satisfactory. The agreement in figure 28 between Montouri's stability criterion and the laboratory data of this report is far from satisfactory. Definitely growing and vigorous roll waves of the type observed in the laboratory, as shown in figures 1–3, however, may be difficult to observe in a natural stream under different lighting and viewing conditions. The lack of agreement between Montuori's criterion and that proposed here, $\mathbf{F}/\mathbf{F}_s=1.0$ as is shown in figure 28, and the fact that all the laboratory data do not coalesce along a single line serve to point out that more research is required in this area.

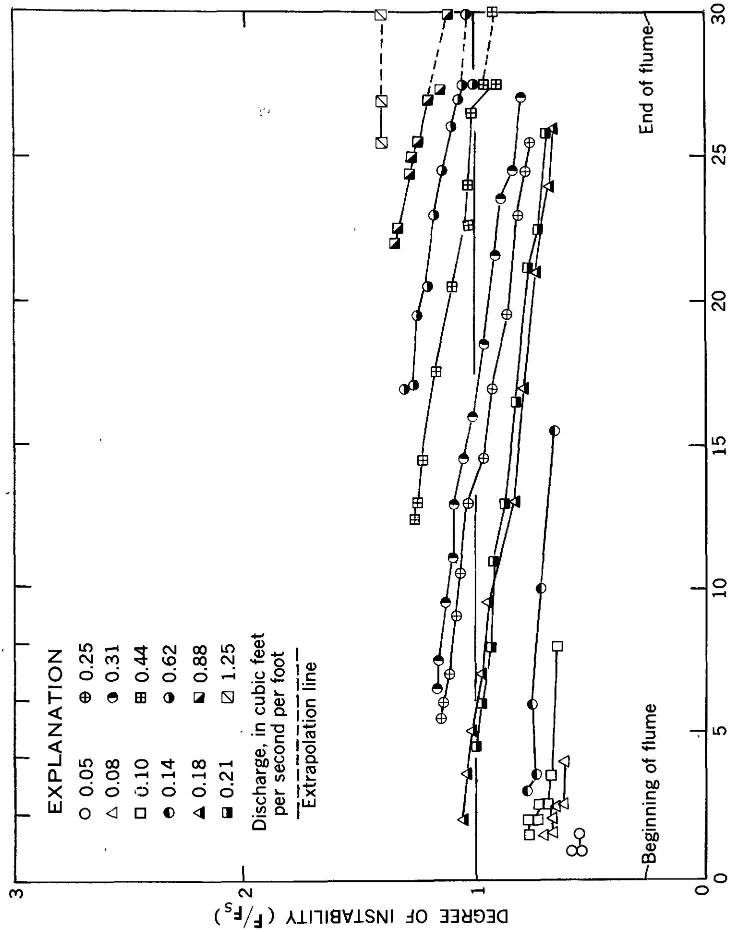


FIGURE 19.—Relation between degree of instability, roll-wave development distance, and discharge—boundary VI.

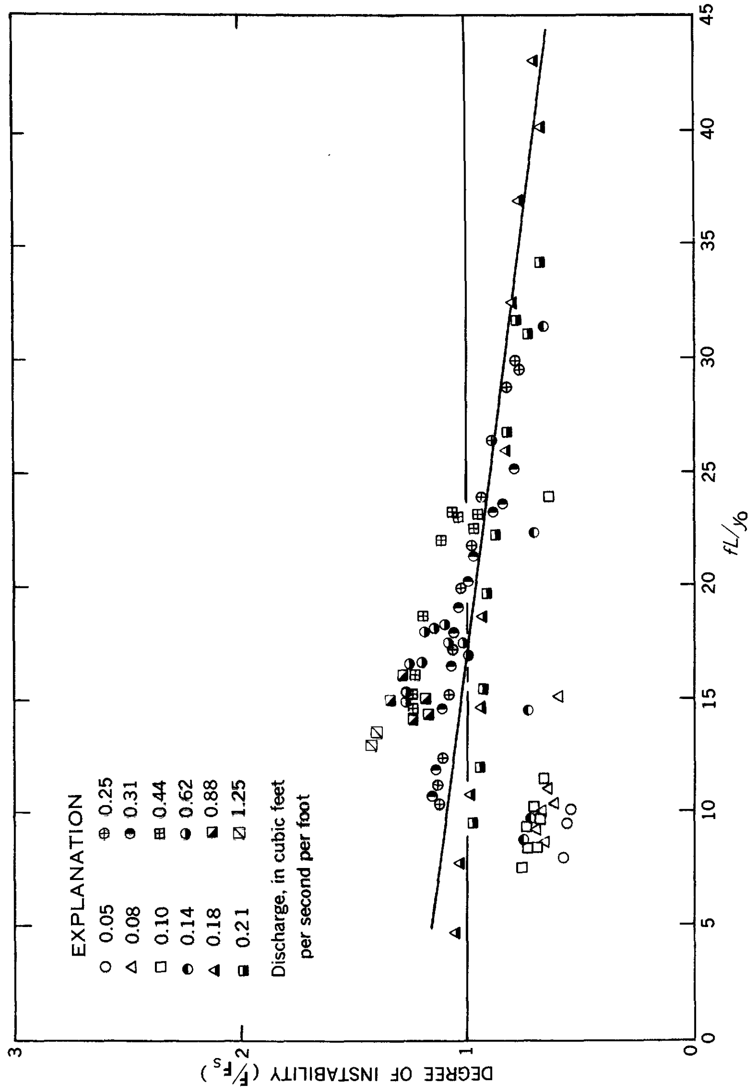


FIGURE 20.—Relation between degree of instability and ratio fL/y_0—boundary VI.

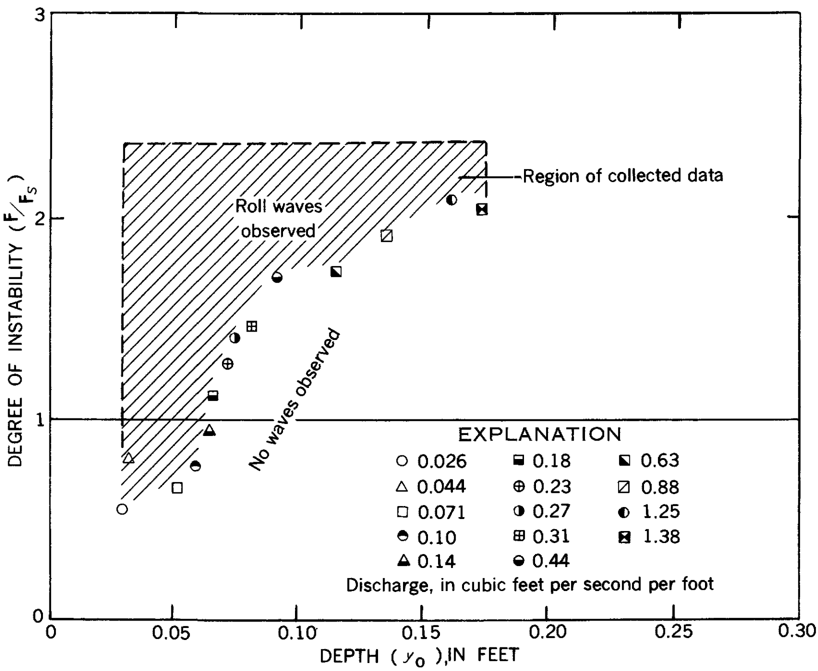


FIGURE 21.—Correlation between degree of instability, depth, and presence of roll waves—boundary VII.

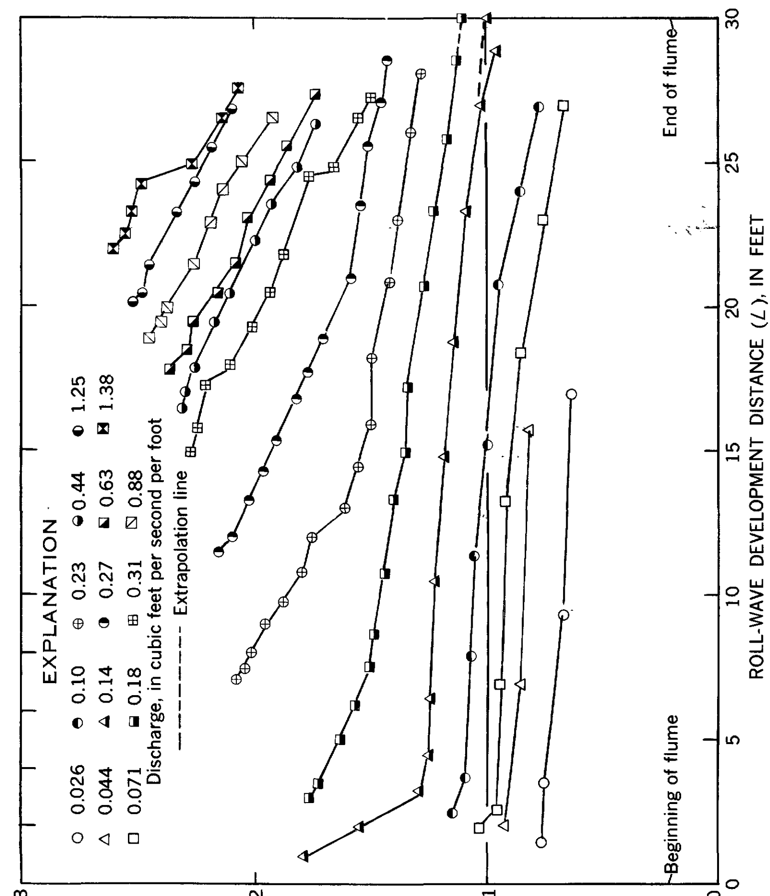


FIGURE 22.—Relation between degree of instability, roll-wave development distance, and discharge—boundary VII.

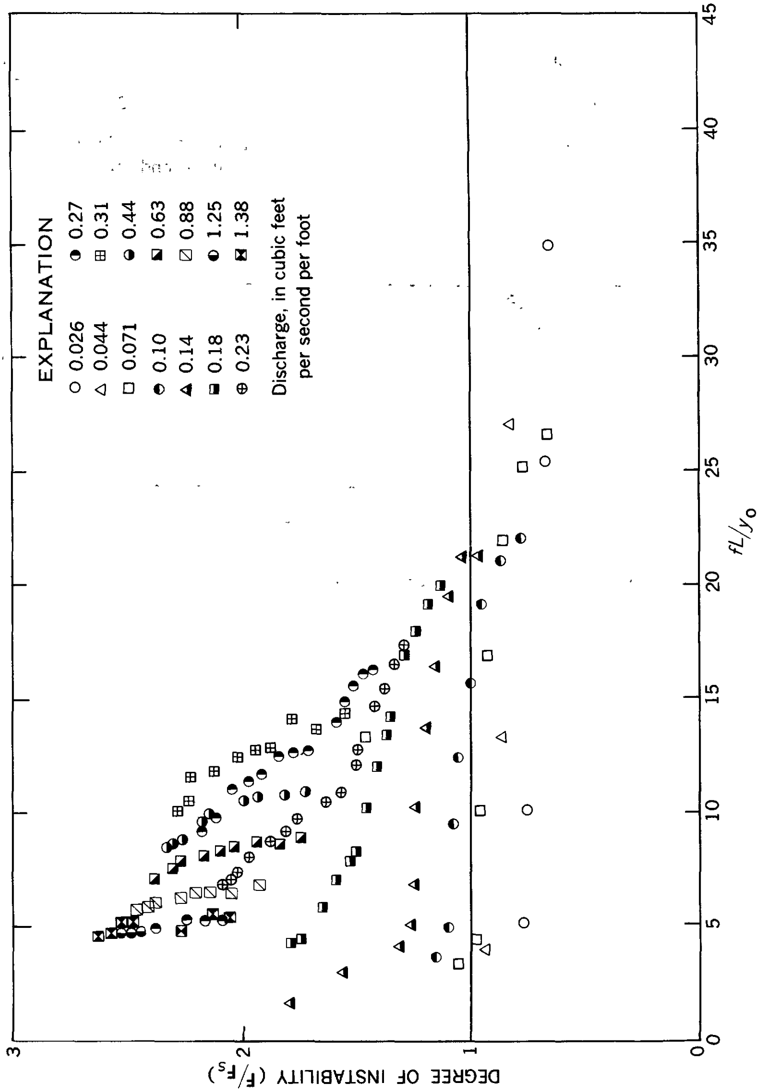


FIGURE 23.—Relation between degree of instability and ratio fL/y_0—boundary VII.

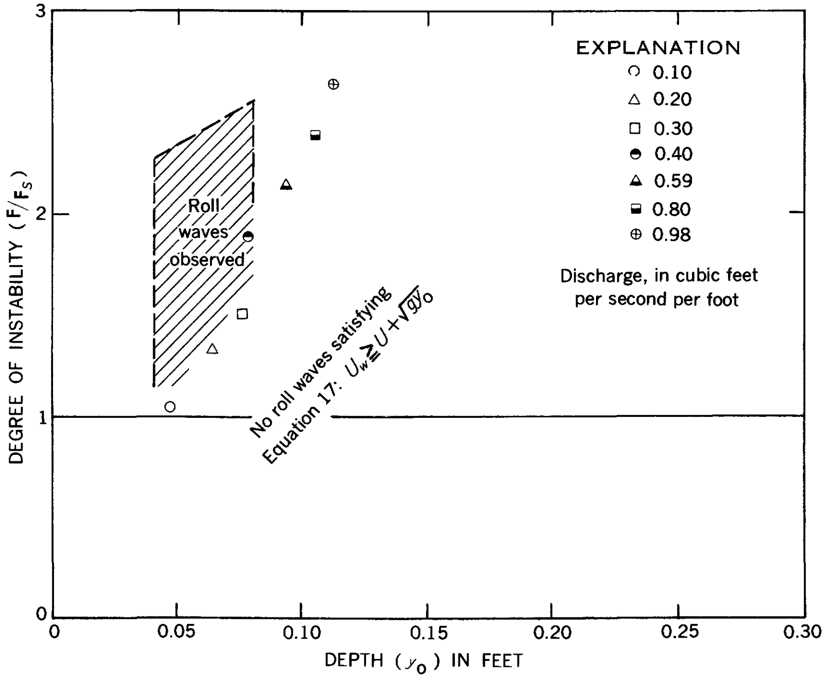


FIGURE 24.—Correlation between degree of instability and presence of roll waves—boundary VIII.

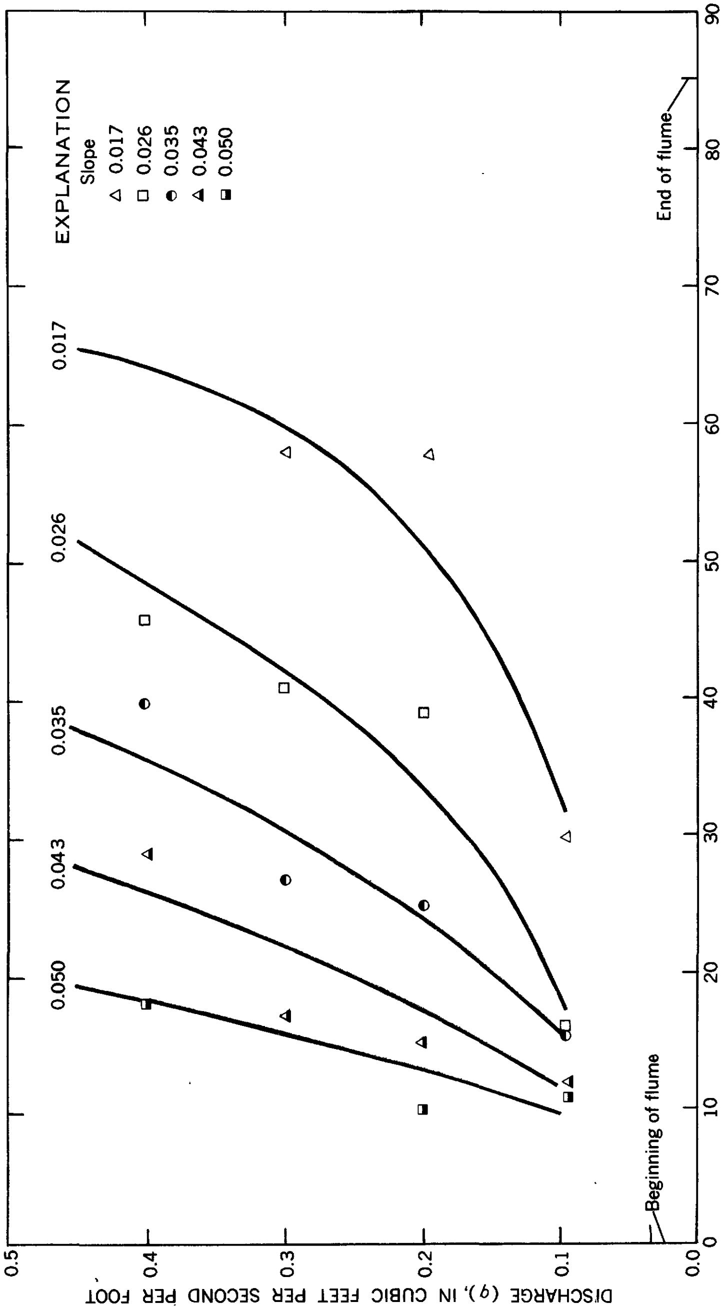


FIGURE 25.—Relation between discharge, roll-wave development distance, and slope—boundary VIII.

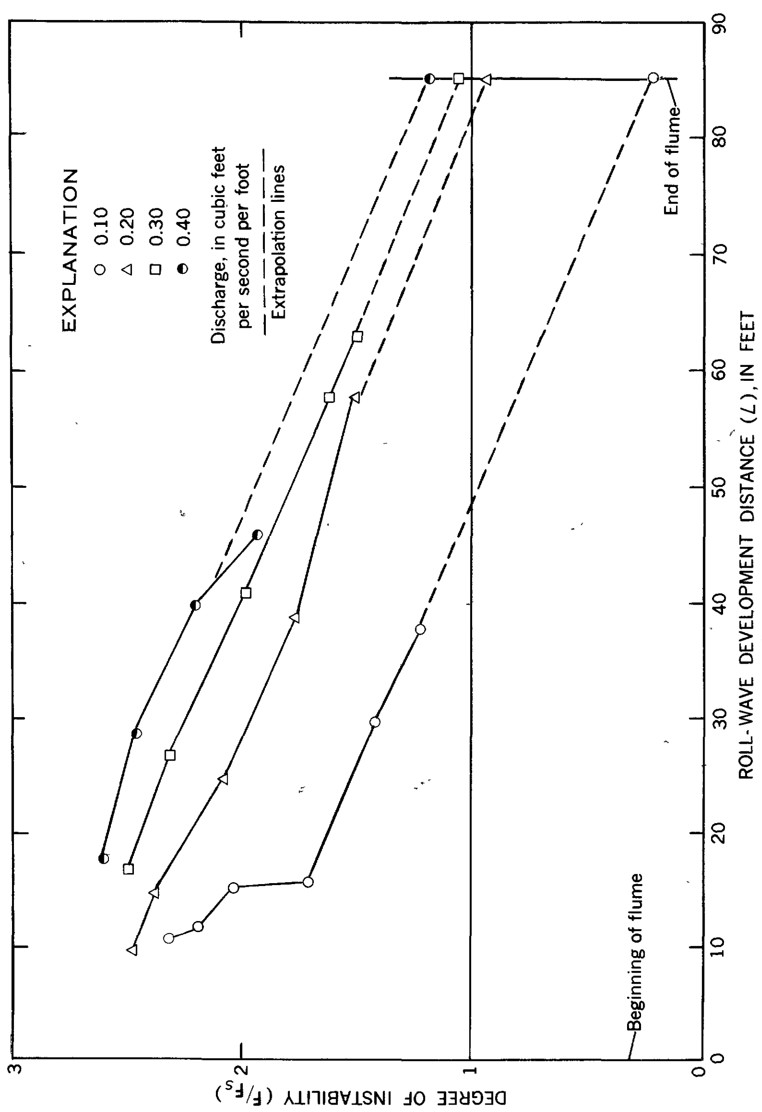


FIGURE 26.—Relation between degree of instability, roll-wave development distance, and discharge—boundary VIII.

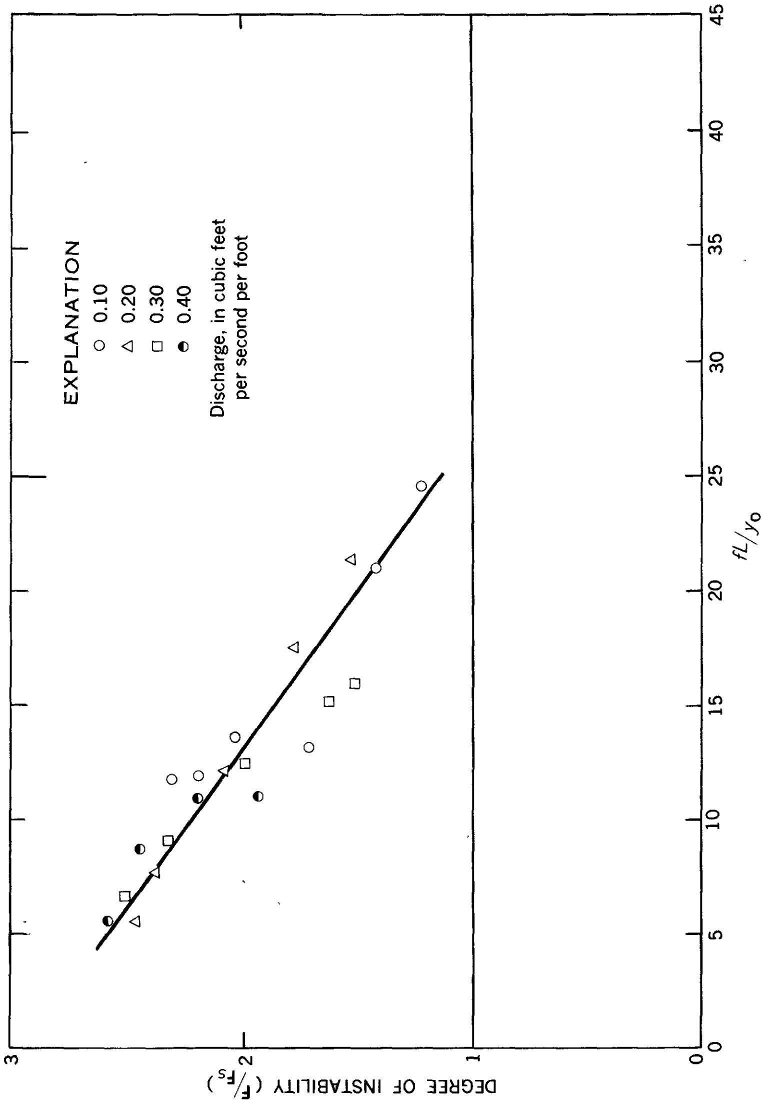


FIGURE 27.—Relation between degree of instability and ratio fL/y_0—boundary VIII.

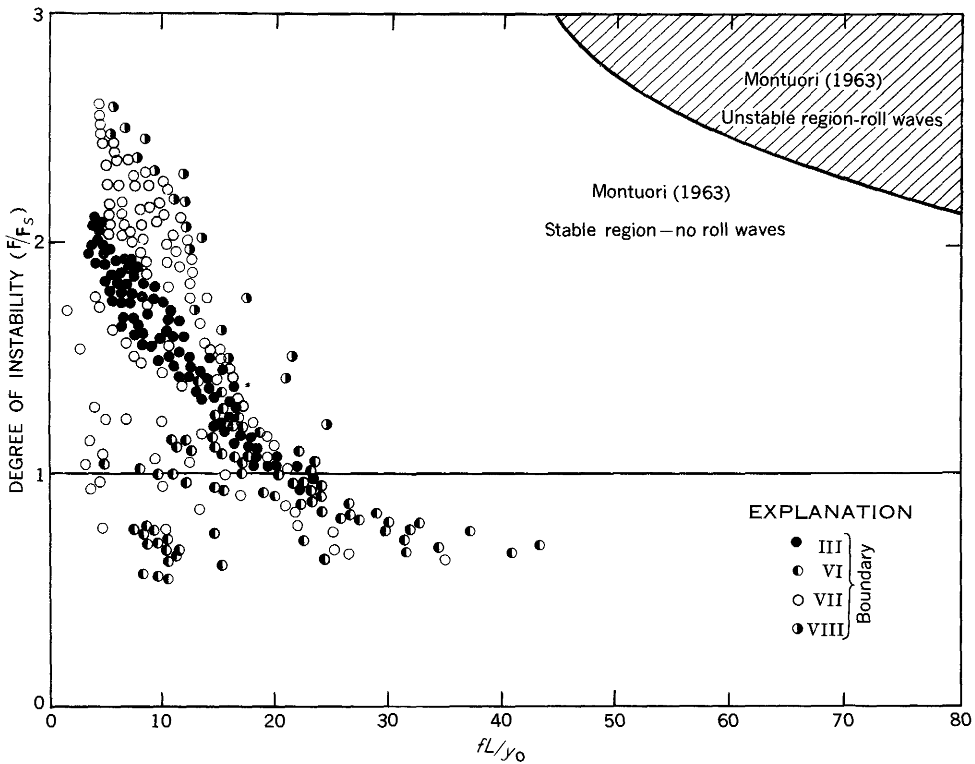


FIGURE 28.—Comparison between stability criterion of Montuori (1963) and roll-wave observations for boundaries III, VI, VII, and VIII, showing relation between degree of instability and ratio fL/y_0.

CORRELATION BETWEEN THE ROUGHENED-CHANNEL STABILITY CRITERION AND INCREASED CHANNEL RESISTANCE

Information relative to channel resistance for stable and unstable flow has been collected only in the roughened channels. The scope of the results for boundary V is more comprehensive than that for the others because it encompasses the subcritical regime in addition to the supercritical one. Because the results relative to boundary V are the most complete, they will be considered in detail and then the data for boundaries I–IV, VI, and VII will be discussed.

Only in the longer flume, the 85-foot one in which boundary V was studied, was it possible to establish uniform flow in the subcritical region with sufficient accuracy to warrant data collection; however, that was not completely satisfactory at the smaller values of the aspect ratio because the wall effects became noticeable. Because most of the data appeared to be unaffected by the walls and because it was desirable, in the interest of simplicity, to exclude wall effects, those subcritical data which correlated with the aspect ratio were omitted from the figures. The omitted results have been tabulated along with the others in table 3 (page C64) and are marked "Not plotted."

The Froude number, a parameter associated with gravitational effects, is an essential parameter of open-channel flow and resistance when the nonuniformity of such flow is due to changes in boundary alinement. Its role, however, as deduced from empirical data for flow in uniform channels has not been delineated as clearly.

Shallow-water waves represent a hydraulic phenomenon related to gravitational effects. Owing to the lack of a wave-perpetuating situation for stable flow (figure 6), it is reasonable that flow that is classified as stable in a uniform channel would be independent of the Froude number. The systematic arrangement of the data for boundary V in figure 29 indicates that the resistance coefficient is independent of the Froude number when the flow is stable, and, in addition, lends support to the growing accumulation of results that indicate that a single relation pertains to both the stable subcritical and the stable supercritical regimes (Jegorow, 1940; Powell, 1946; Homma, 1952). These data are represented by

$$\frac{1}{\sqrt{f}}=2\log\left[\frac{0.14(4y_0/k)}{\lambda^{0.9}}\right]. \qquad (18)$$

When the flow is classified as unstable, the growth of waves is due to the imbalance that exists between the gravitational force and the boundary retarding force (fig. 6). As a consequence the unaided growth of roll waves, except for gravitational effects, is feasible, and

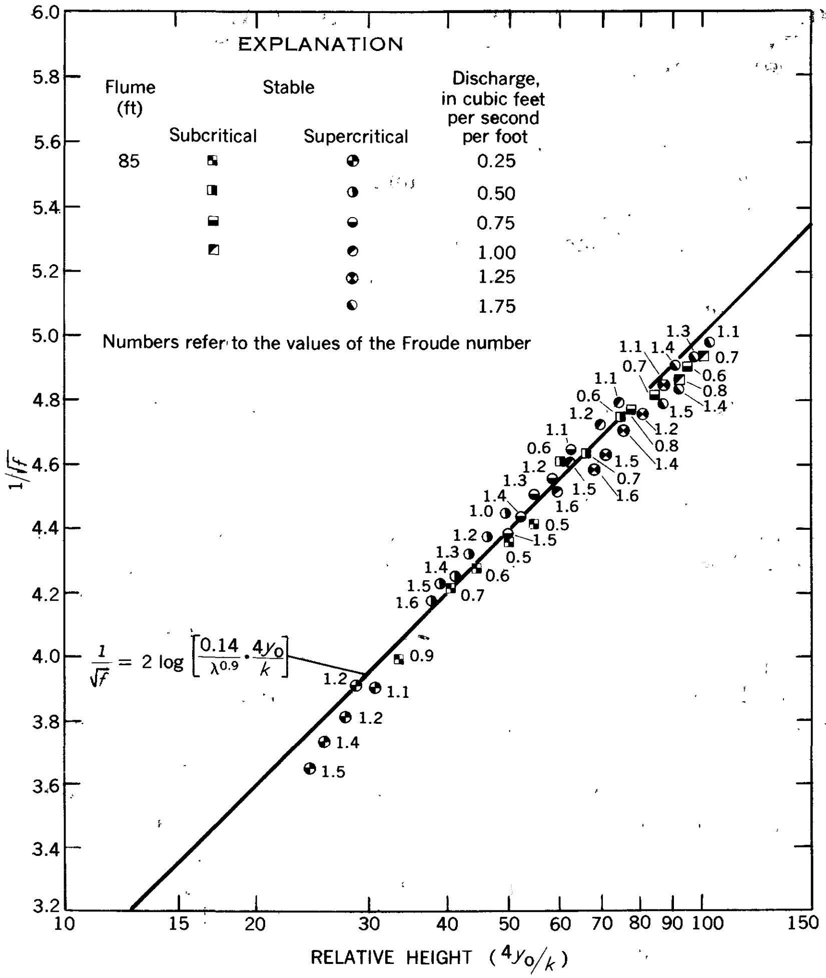


FIGURE 29.—Relation between resistance coefficient and relative height for stable flow—boundary V.

the flow might reasonably be a function of the Froude number. Figure 30 discloses that the points for the unstable data deviate systematically, according to the magnitude of the Froude number, from the line for stable flow. The departure of the data is such that the channel resistance for unstable flow is shown to be greater than that for stable flow as might be anticipated and as was assumed in a preceding section (p. C15). The roll waves associated with some of the unstable-flow data depicted in figure 30 were, in some instances, barely perceptible, and in others, were unmistakably present though not of the breaking-front type depicted by Cornish (1934, p. 95).

In figure 30 the increase in f due to instability ranges from 0 to 30 percent. The maximum change, 30 percent, corresponds to either a

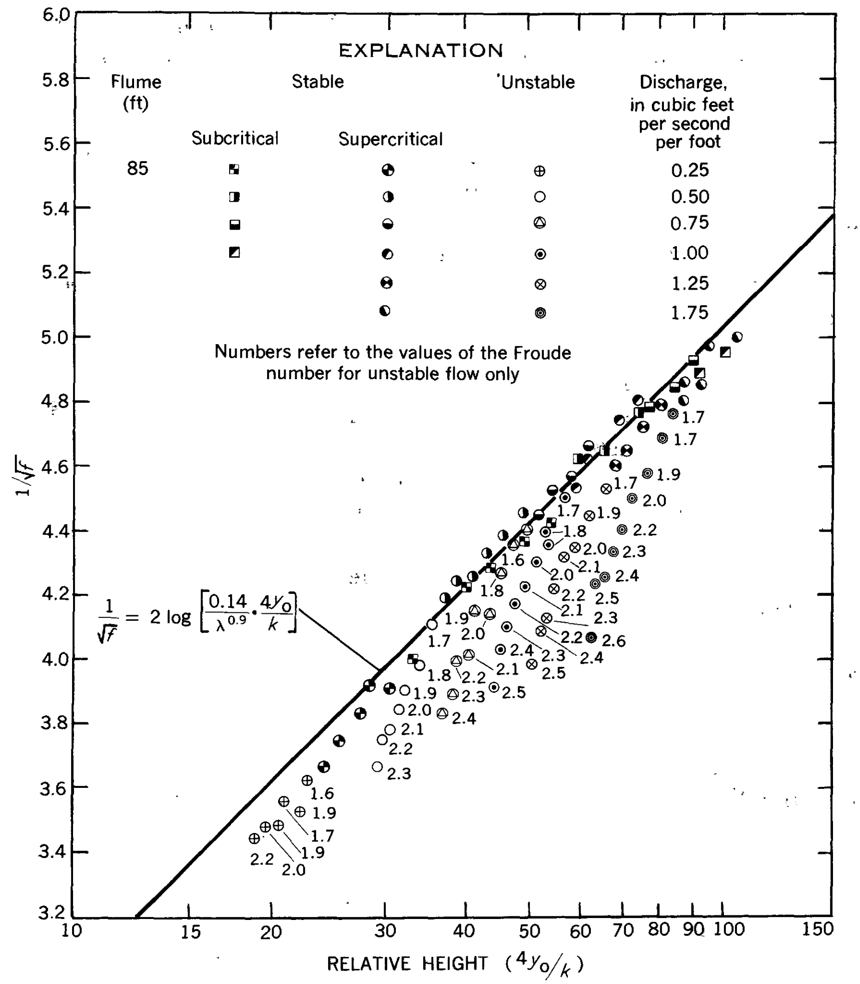


FIGURE 30.—Relation between resistance coefficient and relative height for stable and unstable flow—boundary V.

15 percent decrease in discharge or a 30 percent increase in slope from that for stable flow for the same relative height, $k/4y_0$. The unstable-flow data coalesce when the degree of instability is taken into account as in figure 31. This figure discloses that the channel resistance is a function of the Froude number when the flow is unstable. These data are represented by

$$\frac{1}{\sqrt{f}} = 2 \log \left[\frac{0.14 \left(\frac{4y_0}{k}\right)}{\lambda^{0.9} \left(\frac{\mathbf{F}}{\mathbf{F}_s}\right)^{2/3}} \right]. \qquad (19)$$

The expressions for stable and unstable flow, equations 18 and 19, differ by only one factor, $(\mathbf{F}/\mathbf{F}_s)^{2/3}$, which is a function of the degree of instability of the flow.

The data collected in both flumes for boundaries I–VI and VIII are plotted in figures 32–34. Because of the similarity of the trends of the data for boundary V and of those for boundaries I–IV and VI, most of the previous remarks relative to boundary V also pertain to these other rough surfaces. Owing to a lack of empirical data, those statements concerning subcritical flow are naturally excluded; it seems reasonable, however, that they too would apply to these surfaces as well. Although the trends of the data for boundary III (fig. 33) are similar to those for boundary V, the scatter of the plotted results is greater. With regard to the plastic-louver roughness, boundary VII, the channel resistance for unstable flow was in many instances less than for stable flow (fig. 34). The reason for the nonconformity of these results with those for cubes is unknown; it is well to recall at this point that the roll-wave results for boundary VII (figs. 21–23) were also not in accord with those for the other roughnesses.

When account is taken of roughness concentration, all of the stable-flow data for the boundaries formed by cubes of a constant size coalesce as shown in figure 35, and can be represented by equation 18. The unstable-flow data similarly coalesce about the line representing equation 19 in figure 36.

The use of $\mathbf{F}_s$ in equation 19 requires qualification. Plots of the unstable-flow data left little doubt that the Froude number was a significant consolidating parameter; however, a comparable statement cannot be made about $\mathbf{F}_s$. Because the flows reported upon herein are regarded as two dimensional, $\mathbf{F}_s$ is practically constant according to figure 4, and, as a consequence, could just as well have been omitted from this equation; in other words, the data of figures 14, 16–24, 26, 27, 31, and 36 would have plotted just as well if $\mathbf{F}_s$ had been omitted. Therefore, these data in no way show that $(\mathbf{F}/\mathbf{F}_s)^{2/3}$ is any more significant as a coalescing parameter than $\mathbf{F}^{2/3}$. Nonetheless, it does seem reasonable that the increase in channel resistance and the correlation of data relative to the presence of roll waves would be associated with some measure of the degree of instability rather than the magnitude of the Froude number alone. The ratio $\mathbf{F}/\mathbf{F}_s$ is considered to be more significant than $\mathbf{F}$ and consequently has been used throughout this report.

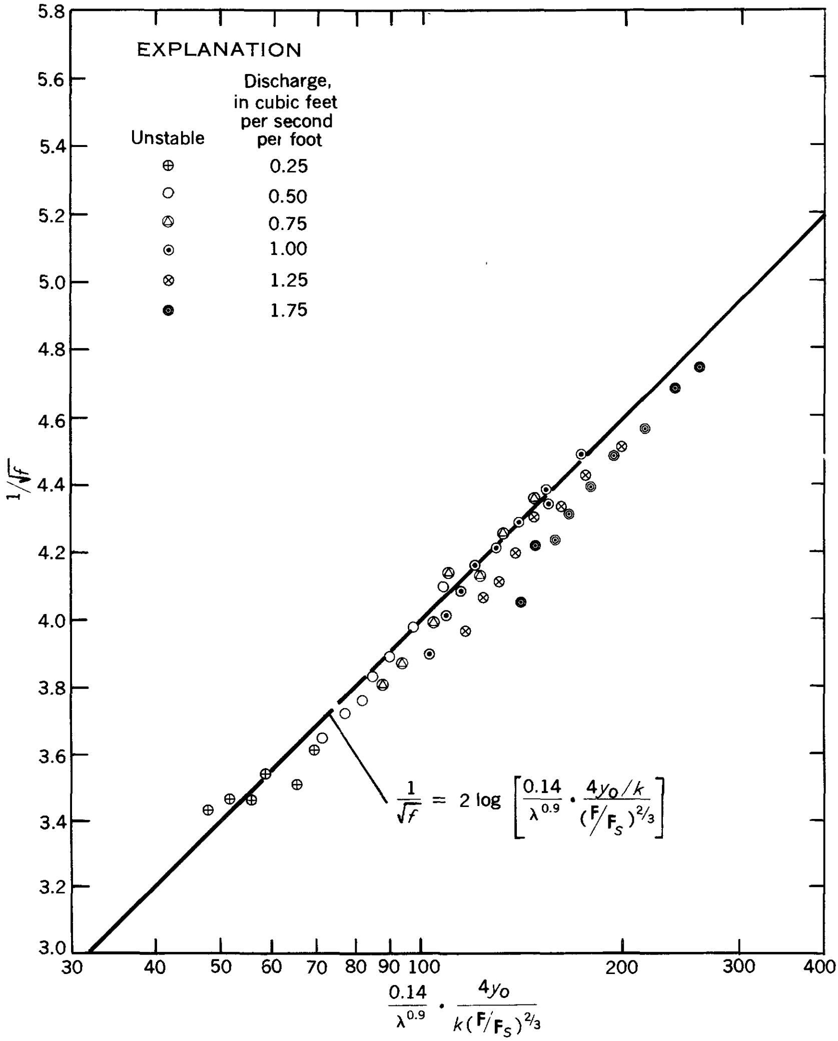


FIGURE 31.—Coalesced unstable-flow data—boundary V.

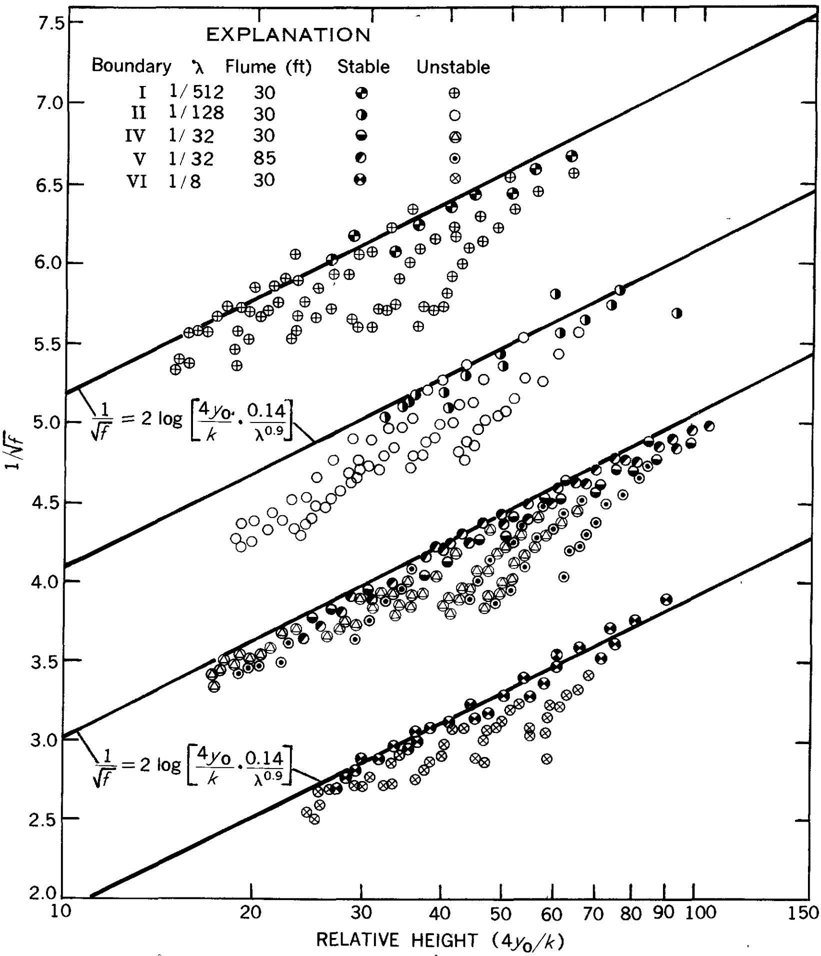


FIGURE 32.—Relation between resistance coefficient and relative height for stable and unstable flow—boundaries I, II, IV–VI.

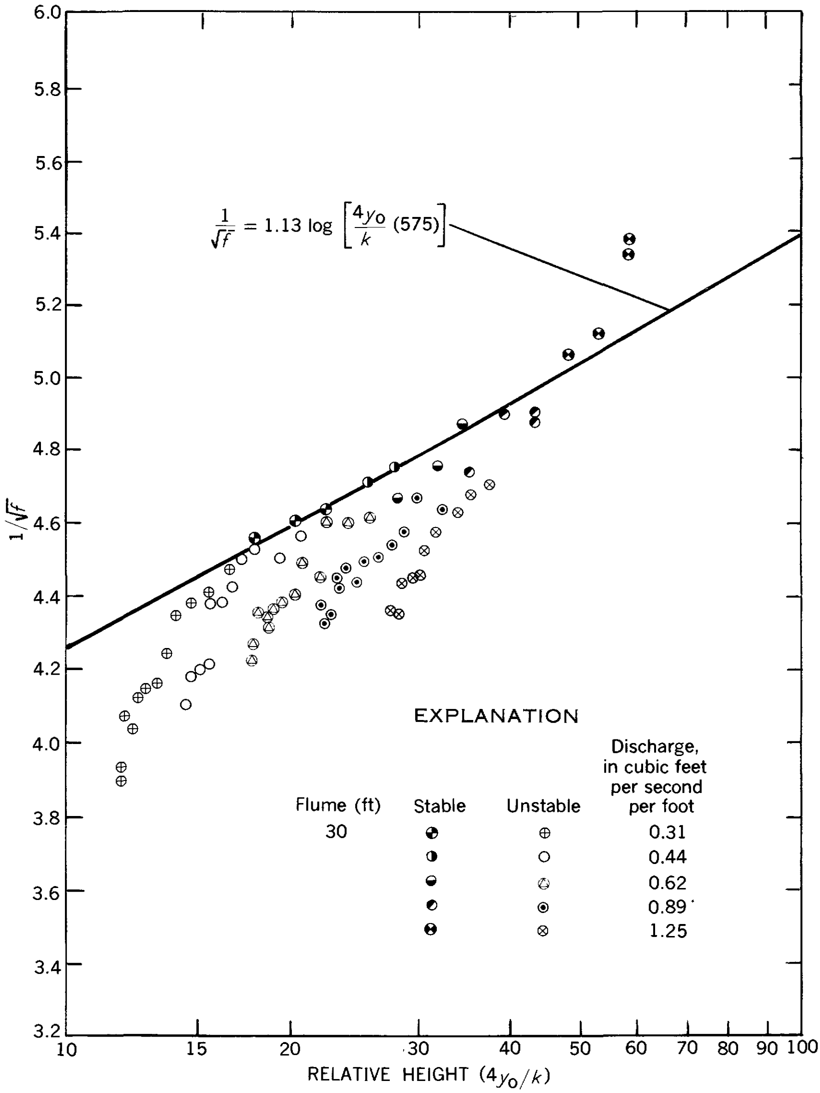


FIGURE 33.—Relation between resistance coefficient and relative height for stable and unstable flow—boundary III.

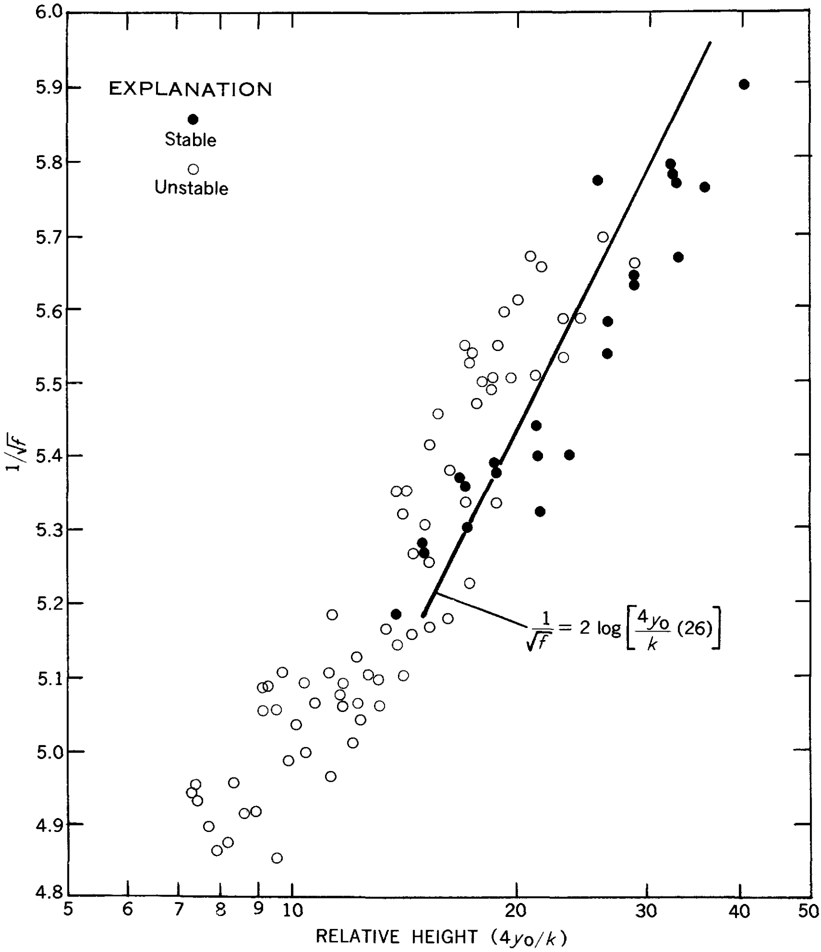


Figure 34.—Relation between resistance coefficient and relative height for stable and unstable flow—boundary VII.

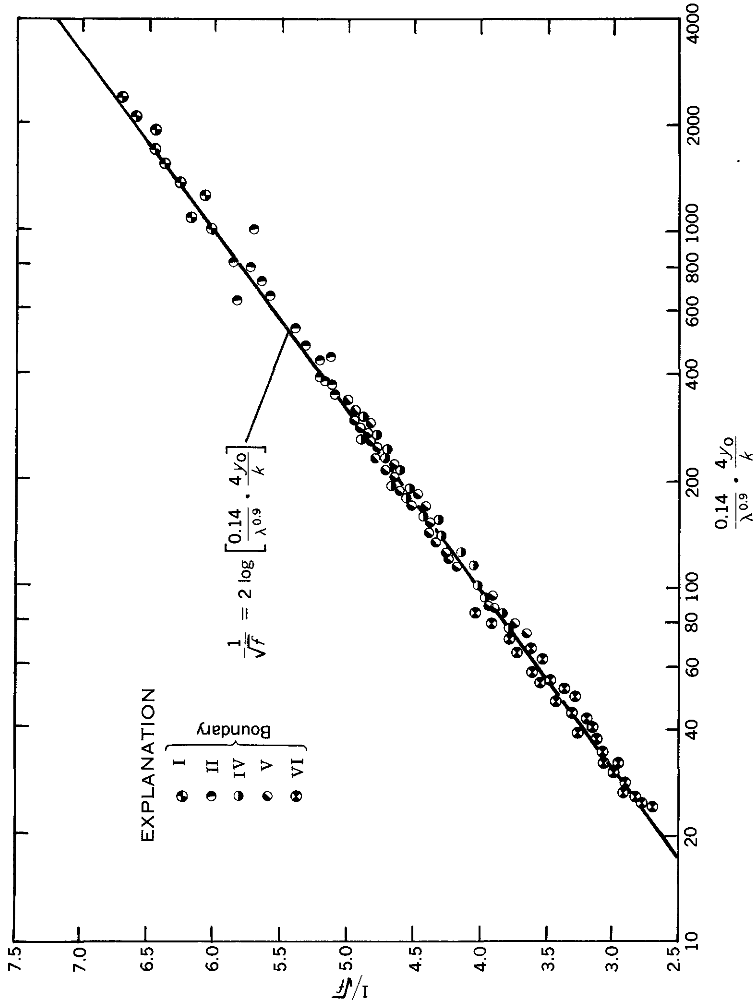


FIGURE 35.—Stable-flow data for uniform roughness.

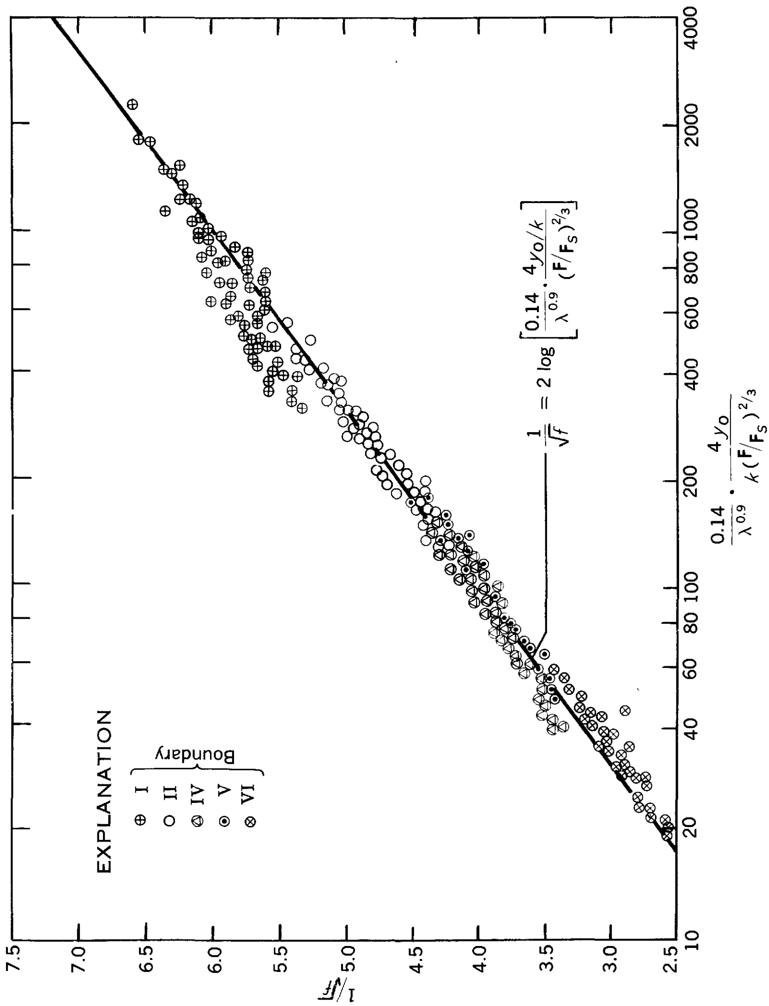


FIGURE 36.—Unstable-flow data for uniform roughness.

RELEVANT INVESTIGATIONS OF UNSTABLE FLOW

Besides the work of Montuori (1963), there are several other investigations dealing with the actual characteristics of flows that are classified as unstable. As indicated in the following paragraphs, some of these results substantiate and others conflict with those already presented.

Keulegan (1938), in analyzing Bazin's data (series 6–11) for flow in a 2-meter-wide channel, stated that at times the resistance varied in a manner similar to what might be expected for flow over a smooth wavy boundary, and that at other times, the resistance variation was similar to what might be anticipated for flow over a hydrodynamically rough surface. A stability classification of these data according to figure 4 indicated that the degree of instability of some of the data was just slightly greater than 1. The results from an effort to rationalize this dual effect of a boundary on channel resistance from the stability point of view were inconclusive.

Jegorow (1940) collected data for flow in a smooth channel and found that the resistance coefficient first increased with increasing Froude numbers and then decreased with still further increases in the Froude number. It was maximum for a Froude number of about 2. The reason for the subsequent decrease of the resistance coefficient with increasing Froude number is not apparent.

Powell (1948) found that the resistance for unstable flow in a smooth channel was somewhat greater than that for stable flow and that the resistance coefficient was a function of the degree of instability. These results for a smooth boundary support those reported upon herein for rough boundaries. Powell also mentioned that waves were found in the unstable regime but did not elaborate on them. Vedernikov (1946) regarded Powell's results as confirmation of a stability criterion that Powell subsequently called the Vedernikov number.

The slight increase in channel resistance for both smooth and rough boundaries reported upon by Homma (1952) for Froude numbers greater than 2 also tends to support the correlation between unstable flow and increased channel resistance.

Iwagaki (1952 and 1954), working with the mixing length, endeavored to take into account effects of the free surface and the instability on channel resistance for smooth and rough surfaces in open-channel flow. This approach indicated that the resistance coefficient is a function of the Froude number even when the flow is subcritical and stable; the scatter of the data that Iwagaki used to corroborate his analytical work is great. After a consideration of figures 29–36 of this report and of the works of other investigators, it was concluded by the authors that, at least for all practical purposes, channel resistance is independent of the Froude number when flow in a uniform channel is classified as stable and a function of the Froude number when it is unstable.

The work of Tracy and Lester (1961) in a smooth 80-foot channel was also scrutinized. Figure 24 indicates that roll waves may have been present for a few of their runs, although Tracy and Lester make no mention of having found them in the course of their work. Furthermore, according to their results, the channel resistance for unstable flow is the same as that for stable flow. Even though Tracy and Lester's results disagree with those presented in this report and other findings, they are understandable, particularly when the degree of instability and length of flume are not great. Where these conditions exist, the practically imperceptible roll waves could easily go unnoticed. Similarly, because the increase in channel resistance due to instability is not large, the change in the resistance coefficient could likewise go undetected amidst the experimental scatter unless the testing program were designed to ferret out this change.

The accounts by Cornish (1934) and by Holmes (1936) disclose that roll waves are also to be found in the field. Cornish (1934) reported upon roll waves that existed in channels in the Alps. It was found that the velocity of these agreed in general with equation 17, but the greatest depth of flow reported upon was but 3 inches. Holmes (1936) described roll waves that overtopped a channel 8 feet deep; this description and his photograph of waves of somewhat lesser size are evidence that such a phenomenon can and does occur at great depths. The discharge of the channel under these severe conditions was estimated to be $\frac{1}{7}$–$\frac{1}{8}$ of that for steady uniform flow. This example cited by Holmes points out the great extent to which roll waves can reduce the carrying capacity of a canal.

As far as is known, no data on unstable flow in the field have been collected that show an increase in the channel resistance over that for stable flow. In view of the laboratory results, it is questionable whether such an increase will ever be detected. Because of the rather small change in f, any increase in the resistance coefficient due to instability would hardly be differentiable from that brought about by variations in such other factors as channel roughness, channel shape, and channel size. From the designer's point of view, if the possibility of roll waves exists or if they are present, attention would be focused upon their prevention or elimination rather than on the small increase in channel resistance.

Because of the undesirable characteristics of roll waves, the potential instability of open-channel flow is an important consideration in the design of any free-surface conveyance. The existence of unstable flow as manifested by the presence of roll waves and increased channel resistance is supported fairly well by the previously described correlations. To aid the designers in avoiding this regime of flow, both Iwasa (1954) and Escoffier and Boyd (1962) have presented graphs that readily permit the stability classification of flows in channels of various shapes. Koloseus (1962) has cautioned against the use of the usual relations between the resistance coefficient and the relative height and the Reynolds number for unstable flow. Montuori (1961) has proposed stability criteria that are not as conservative as those of Iwasa (1954) and Escoffier and Boyd (1962); this difference arises from the fact that the roll-wave development distance has been taken into account. As an approximate guide in determining the state of stability of flow in a smooth or rough channel of large aspect ratio, turbulent flow can be considered to be unstable when the Froude number is greater than 1.6.

CONCLUSIONS

From the present study and other works it is concluded that:

1. The regimes of open-channel flow should include the stable and unstable, in addition to the laminar, turbulent, subcritical, critical, and supercritical.
2. All stability criteria for free-surface flow are based in part, if not entirely, on the Froude number. Channel shape is another important parameter.
3. Channel resistance in a uniform open channel is independent of the Froude number when the flow is stable and a function of it when the flow is unstable.
4. The correlation between flow, which is classified as unstable, and its associated physical manifestation—roll waves—is fair. This less-than-perfect correlation may be due to limitations of the equipment, peculiarities of the roughness, or shortcomings of the stability criteria.
5. Roll waves and the increased channel resistance associated with unstable flow decrease the maximum carrying capacity of a channel. The diminutive effect on maximum discharge brought about by the containment of the roll waves within the confines of a channel is much greater than that from increased channel resistance.
6. As a consequence of the increased channel resistance of unstable flow, roll waves in an arrested state of development may be possible.
7. The minimum distance required for the development of roll waves to a discernible state is a function of channel roughness, channel shape, depth of flow, and degree of instability.

8. For a given slope, the required roll-wave development distance increases with discharge.
9. For a particular discharge, the required roll-wave development distance decreases as the slope increases.
10. For equal depths and degrees of instability in identically shaped channels of different roughness, the required roll-wave development distance decreases as the channel roughness increases.
11. Because of the potential instability of open-channel flow, the usual relations between the resistance coefficient, relative height, and Reynolds number can not be extrapolated indefinitely.

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TABLE 2.—*Roll-wave data*

[Method of computation: for smooth boundary, $f=8gRS/U^2$, $\mathbf{F}=q/\sqrt{gy_0^3}$, and $\mathbf{R}=4RU/\nu$; for rough boundary, $f=8gy_0^3S/q^2$, $\mathbf{F}=q/\sqrt{gy_0^3}$, and $\mathbf{R}=4y_0U/\nu=4q/\nu$]

q (cfs per ft)	*S*	y_0 (feet)	**F**		$\mathbf{R}\times 10^{-5}$	$\mathbf{F}_s$	*L* (feet)
Boundary III, ten 3/16-in. to three 3/8-in. cubes, λ=1/128, 2- by 30-ft flume							
0.159	0.0186	0.0687	1.56	0.0616	0.616	1.59	25.8
.159	.0210	.0663	1.65	.0618	.618	1.59	23.3
.159	.0233	.0646	1.70	.0641	.615	1.58	20.0
.158	.0256	.0614	1.84	.0607	.615	1.59	17.8
.161	.0279	.0608	1.90	.0620	.626	1.58	15.0
.162	.0325	.0569	2.10	.0588	.629	1.59	13.0
.162	.0372	.0541	2.27	.0578	.628	1.59	10.8
.161	.0418	.0522	2.38	.0593	.624	1.59	8.5
.162	.0464	.0501	2.54	.0574	.628	1.59	7.0
.161	.0511	.0486	2.65	.0583	.624	1.59	6.0
.161	.0556	.0482	2.69	.0617	.627	1.59	5.0
.200	.0186	.0822	1.50	.0663	.773	1.58	27.5
.200	.0210	.0770	1.65	.0618	.770	1.59	24.8
.199	.0233	.0734	1.76	.0598	.767	1.59	21.8
.200	.0256	.0715	1.85	.0601	.773	1.59	19.5
.200	.0279	.0693	1.93	.0599	.771	1.59	17.0
.199	.0325	.0654	2.10	.0590	.769	1.59	14.8
.200	.0372	.0600	2.40	.0517	.771	1.60	12.5
.200	.0418	.0587	2.48	.0545	.773	1.59	9.5
.199	.0465	.0573	2.56	.0566	.771	1.59	7.7
.199	.0511	.0536	2.83	.0509	.771	1.60	6.2
.200	.0556	.0520	2.98	.0503	.775	1.60	5.2
.200	.0649	.0503	3.12	.0534	.772	1.59	4.5
.223	.0210	.0825	1.66	.0608	.855	1.59	26.3
.223	.0233	.0797	1.75	.0609	.855	1.59	23.8
.224	.0256	.0785	1.79	.0637	.857	1.58	21.8
.223	.0279	.0737	1.96	.0579	.855	1.59	19.5
.223	.0325	.0689	2.18	.0550	.856	1.59	16.0
.223	.0372	.0651	2.36	.0532	.855	1.59	13.5
.223	.0418	.0612	2.59	.0498	.854	1.60	10.0
.223	.0465	.0600	2.68	.0518	.857	1.60	8.0
.224	.0511	.0582	2.81	.0518	.858	1.60	6.5
.224	.0556	.0547	3.08	.0468	.859	1.61	5.5
.224	.0602	.0532	3.21	.0467	.858	1.61	5.0
.224	.0649	.0523	3.30	.0476	.860	1.60	4.7
.224	.0667	.0516	3.36	.0472	.859	1.61	4.5
.250	.0210	.0886	1.67	.0601	.954	1.59	26.5
.249	.0233	.0844	1.79	.0579	.952	1.59	25.8
.252	.0256	.0832	1.85	.0597	.963	1.59	24.0
.252	.0279	.0796	1.98	.0569	.964	1.59	21.8
.252	.0325	.0749	2.17	.0553	.964	1.59	19.3
.252	.0372	.0721	2.29	.0566	.962	1.59	16.0
.252	.0418	.0693	2.44	.0564	.963	1.59	13.0
.251	.0465	.0656	2.64	.0534	.961	1.59	9.0
.252	.0511	.0635	2.78	.0530	.964	1.60	7.5
.251	.0556	.0608	2.95	.0510	.960	1.60	6.2
.251	.0602	.0583	3.14	.0488	.959	1.60	5.7
.250	.0649	.0571	3.24	.0496	.958	1.60	5.0
.252	.0667	.0560	3.35	.0475	.963	1.60	4.5
.275	.0233	.0903	1.79	.0583	1.12	1.59	28.0
.274	.0256	.0873	1.88	.0582	1.12	1.59	25.0
.275	.0279	.0843	1.98	.0570	1.12	1.59	23.8
.275	.0325	.0809	2.10	.0588	1.12	1.59	21.8
.275	.0372	.0763	2.30	.0562	1.12	1.59	18.3
.275	.0418	.0735	2.43	.0566	1.12	1.59	15.0
.274	.0465	.0685	2.70	.0511	1.12	1.60	12.0
.275	.0511	.0645	2.96	.0467	1.12	1.61	7.7
.274	.0556	.0619	3.14	.0451	1.12	1.61	6.7
.274	.0602	.0609	3.22	.0466	1.12	1.61	6.0
.275	.0649	.0596	3.33	.0469	1.12	1.61	5.2
.275	.0667	.0588	3.40	.0463	1.12	1.61	5.0
.296	.0279	.0879	2.00	.0556	1.21	1.59	26.0
.299	.0325	.0839	2.17	.0553	1.22	1.59	24.2
.299	.0372	.0812	2.28	.0572	1.22	1.59	21.5
.299	.0418	.0778	2.43	.0567	1.22	1.59	17.2
.299	.0465	.0762	2.51	.0590	1.22	1.59	14.0
.299	.0511	.0734	2.65	.0580	1.22	1.59	9.2
.298	.0556	.0717	2.74	.0593	1.22	1.59	8.0
.298	.0602	.0709	2.78	.0622	1.22	1.58	7.0
.298	.0649	.0707	2.79	.0666	1.22	1.58	6.0
.298	.0667	.0700	2.84	.0662	1.22	1.58	4.7

TABLE 2.—*Roll-wave data*—Continued

q (cfs per ft)	S	y_0 (feet)	**F**	f	**R** × 10^{-5}	$\mathbf{F}_s$	L (feet)
Boundary III, ten 3/16-in. to three 3/8-in. cubes, λ=1/128, 2- by 30-ft flume—Continued							
0.314	0.0279	0.0893	2.07	0.0521	1.22	1.60	28.0
.313	.0325	.0852	2.22	.0530	1.22	1.59	26.0
.312	.0372	.0826	2.32	.0555	1.22	1.59	23.0
.313	.0418	.0807	2.41	.0577	1.23	1.59	19.5
.313	.0465	.0780	2.53	.0581	1.23	1.59	16.0
.313	.0511	.0758	2.64	.0586	1.22	1.59	13.5
.313	.0556	.0748	2.70	.0613	1.22	1.59	10.5
.313	.0602	.0726	2.82	.0605	1.23	1.59	7.7
.313	.0649	.0723	2.84	.0646	1.23	1.58	6.0
.313	.0667	.0721	2.85	.0658	1.23	1.58	5.5
.366	.0302	.0940	2.24	.0482	1.43	1.60	27.3
.365	.0325	.0934	2.26	.0511	1.43	1.60	25.3
.365	.0349	.0912	2.34	.0510	1.43	1.60	23.0
.365	.0372	.0906	2.36	.0533	1.43	1.60	21.0
.364	.0418	.0879	2.46	.0552	1.42	1.59	18.0
.367	.0465	.0864	2.55	.0572	1.44	1.59	15.3
.367	.0511	.0833	2.69	.0564	1.43	1.59	13.0
.367	.0556	.0812	2.79	.0570	1.43	1.59	11.8
.367	.0602	.0808	2.82	.0608	1.43	1.59	9.2
.367	.0649	.0802	2.85	.0639	1.44	1.58	7.2
.367	.0667	.0796	2.88	.0642	1.44	1.58	7.0
.410	.0325	.102	2.22	.0527	1.60	1.60	27.3
.410	.0349	.100	2.28	.0536	1.60	1.60	25.8
.410	.0372	.0982	2.35	.0538	1.61	1.59	22.5
.409	.0418	.0955	2.44	.0561	1.60	1.59	19.5
.411	.0465	.0924	2.58	.0559	1.61	1.59	17.3
.409	.0511	.0895	2.69	.0564	1.60	1.59	14.3
.411	.0556	.0869	2.83	.0557	1.61	1.59	12.5
.412	.0602	.0867	2.85	.0595	1.61	1.59	10.5
.412	.0649	.0864	2.86	.0636	1.61	1.58	8.5
.411	.0667	.0856	2.89	.0638	2.61	1.58	8.2
.443	.0325	.105	2.30	.0494	1.75	1.60	27.0
.443	.0372	.102	2.41	.0511	1.75	1.60	24.5
.443	.0418	.0981	2.54	.0519	1.73	1.60	22.5
.443	.0464	.0948	2.67	.0520	1.73	1.60	20.5
.442	.0511	.0942	2.70	.0562	1.73	1.59	18.0
.442	.0556	.0916	2.81	.0565	1.73	1.59	15.0
.442	.0602	.0895	2.91	.0569	1.74	1.59	13.0
.442	.0649	.0885	2.96	.0593	1.74	1.59	11.0
.442	.0667	.0877	3.00	.0593	1.74	1.59	10.0
.625	.0372	.126	2.45	.0495	2.65	1.60	28.0
.626	.0418	.123	2.55	.0514	2.65	1.60	26.0
.625	.0465	.119	2.68	.0518	2.64	1.60	24.0
.625	.0511	.116	2.79	.0524	2.60	1.60	22.0
.625	.0556	.113	2.90	.0529	2.60	1.60	20.0
.626	.0602	.110	3.03	.0526	2.60	1.60	16.0
.625	.0649	.108	3.08	.0547	2.57	1.59	14.0
.625	.0667	.108	3.09	.0558	2.58	1.59	13.0
.882	.0372	.157	2.50	.0477	3.49	1.60	27.0
.884	.0418	.152	2.64	.0480	3.50	1.60	25.0
.882	.0465	.145	2.80	.0473	3.45	1.60	22.3
.883	.0511	.142	2.92	.0479	3.47	1.60	19.8
.886	.0556	.139	3.01	.0492	2.48	1.60	18.0
.886	.0602	.137	3.09	.0505	3.50	1.60	16.0
.889	.0649	.135	3.17	.0518	3.53	1.60	13.8
.890	.0667	.134	3.20	.0520	3.54	1.60	11.3
1.25	.0418	.191	2.64	.0481	5.22	1.60	25.5
1.25	.0465	.185	2.76	.0487	5.25	1.60	24.0
1.25	.0511	.180	2.87	.0495	5.27	1.60	21.8
1.25	.0556	.176	2.99	.0499	5.28	1.60	18.5
1.25	.0602	.172	3.08	.0509	5.29	1.60	15.0
1.26	.0649	.168	3.21	.0505	5.32	1.60	12.5
1.25	.0667	.170	3.14	.0542	5.31	1.59	11.5

TABLE 2.—*Roll-wave data*—Continued

q (cfs per ft)	S	y_0 (feet)	**F**		**R** $\times 10^{-5}$	$\mathbf{F}_s$	L (feet)
Boundary VI, 3/16-in. cubes, λ=1/8, 2- by 30-ft flume							
0.0487	0.0372	0.0410	1.03	0.278	0.201	1.88	1.5
.0488	.0511	.0383	1.15	.310	.201	1.99	1.0
.0487	.0667	.0363	1.24	.346	.201	2.21	1.0
.0767	.0279	.0556	1.03	.210	.314	1.70	4.0
.0765	.0325	.0538	1.08	.223	.313	1.73	2.5
.0765	.0372	.0519	1.14	.229	.313	1.75	2.5
.0765	.0464	.0497	1.22	.251	.313	1.81	2.0
.0765	.0556	.0484	1.27	.278	.313	1.88	1.5
.0765	.0649	.0466	1.34	.289	.314	1.92	1.5
.101	.0279	.0653	1.06	.198	.408	1.67	8.0
.101	.0325	.0630	1.13	.205	.410	1.69	3.5
.101	.0372	.0610	1.18	.213	.410	1.71	2.5
.101	.0418	.0599	1.21	.228	.408	1.75	2.5
.100	.0464	.0584	1.25	.236	.408	1.77	2.5
.100	.0511	.0568	1.31	.237	.409	1.77	2.0
.101	.0602	.0552	1.37	.257	.408	1.82	2.0
.101	.0667	.0540	1.41	.267	.408	1.85	1.5
.136	.0233	.0792	1.08	.160	.565	1.63	15.5
.136	.0279	.0756	1.15	.169	.562	1.63	10.0
.136	.0325	.0730	1.22	.176	.564	1.63	6.0
.136	.0372	.0721	1.24	.195	.562	1.67	3.5
.136	.0418	.0701	1.29	.202	.562	1.68	3.0
.175	.0210	.0941	1.07	.147	.698	1.60	26.0
.175	.0256	.0913	1.12	.164	.699	1.63	24.0
.175	.0279	.0870	1.20	.154	.700	1.61	21.0
.176	.0325	.0840	1.27	.161	.703	1.62	17.0
.176	.0372	.0812	1.34	.165	.704	1.63	13.0
.177	.0418	.0759	1.49	.150	.711	1.61	9.5
.178	.0464	.0747	1.54	.157	.716	1.62	7.0
.178	.0511	.0721	1.62	.155	.718	1.62	5.0
.179	.0556	.0710	1.66	.161	.720	1.62	3.5
.179	.0602	.0697	1.72	.164	.723	1.63	2.0
.211	.0210	.104	1.10	.138	.874	1.60	26.0
.211	.0233	.102	1.15	.141	.876	1.60	22.5
.211	.0279	.0970	1.23	.147	.877	1.61	21.0
.212	.0325	.0933	1.31	.152	.878	1.61	16.5
.212	.0372	.0896	1.39	.153	.882	1.61	13.0
.212	.0418	.0864	1.47	.154	.882	1.62	11.0
.211	.0464	.0848	1.51	.163	.879	1.63	8.0
.211	.0511	.0821	1.58	.163	.878	1.63	6.0
.211	.0556	.0806	1.62	.169	.877	1.63	4.5
.251	.0233	.110	1.21	.128	.995	1.58	25.5
.251	.0256	.108	1.25	.131	.996	1.59	24.5
.250	.0279	.104	1.31	.131	.994	1.59	23.0
.250	.0325	.100	1.39	.136	.995	1.60	19.5
.251	.0372	.0960	1.48	.135	.995	1.60	17.0
.250	.0418	.0933	1.55	.140	.993	1.60	14.5
.250	.0464	.0892	1.65	.136	.993	1.60	13.0
.250	.0511	.0879	1.69	.143	.993	1.60	10.5
.250	.0556	.0858	1.75	.145	.993	1.61	9.0
.250	.0602	.0846	1.79	.150	.994	1.61	7.0
.250	.0649	.0834	1.83	.156	.992	1.62	6.0
.249	.0667	.0829	1.84	.157	.992	1.62	5.5
.314	.0233	.125	1.26	.118	1.25	1.57	27.0
.314	.0256	.121	1.32	.118	1.25	1.57	24.5
.313	.0279	.117	1.38	.117	1.24	1.57	23.5
.313	.0325	.114	1.44	.126	1.24	1.57	21.5
.313	.0372	.109	1.52	.128	1.24	1.57	18.5
.314	.0418	.107	1.58	.134	1.24	1.57	16.0
.313	.0465	.104	1.64	.138	1.24	1.58	14.5
.313	.0511	.101	1.71	.140	1.24	1.58	13.0
.313	.0556	.101	1.72	.151	1.24	1.59	11.0
.312	.0602	.0985	1.78	.152	1.24	1.59	9.5
.312	.0649	.0965	1.83	.154	1.24	1.59	7.5
.312	.0667	.0963	1.84	.158	1.24	1.59	6.5

TABLE 2.—*Roll-wave data*—Continued

q (cfs per ft)	S	y_0 (feet)	**F**		**R** × 10^{-5}	$\mathbf{F}_s$	L (feet)
Boundary VI, 3/16-in. cubes, λ=1/8, 2- by 30-ft flume—Continued							
0.443	0.0325	0.138	1.52	0.113	1.76	1.57	27.5
.442	.0349	.136	1.56	.115	1.76	1.57	27.5
.442	.0372	.133	1.60	.116	1.76	1.57	26.5
.441	.0418	.132	1.62	.127	1.75	1.57	24.0
.441	.0465	.130	1.66	.135	1.75	1.58	22.5
.442	.0511	.126	1.74	.136	1.76	1.58	20.5
.442	.0556	.121	1.86	.129	1.76	1.57	17.5
.442	.0602	.118	1.91	.132	1.76	1.57	14.5
.442	.0649	.117	1.95	.137	1.76	1.58	13.0
.442	.0667	.116	1.98	.136	1.76	1.58	12.5
.625	.0325	.170	1.58	.105	2.51	1.57	27.5
.625	.0349	.166	1.62	.106	2.51	1.57	27.5
.625	.0372	.163	1.68	.106	2.51	1.57	27.0
.626	.0418	.160	1.73	.112	2.51	1.57	26.0
.625	.0464	.156	1.79	.116	2.51	1.57	24.5
.625	.0511	.152	1.85	.119	2.51	1.57	23.0
.626	.0556	.149	1.92	.121	2.51	1.57	20.5
.625	.0602	.146	1.97	.125	2.51	1.57	19.5
.626	.0649	.144	2.01	.128	2.52	1.57	17.0
.625	.0667	.143	2.03	.129	2.51	1.57	17.0
.884	.0418	.195	1.82	.102	3.50	1.57	27.5
.882	.0465	.190	1.88	.106	3.49	1.57	27.0
.886	.0511	.185	1.96	.106	3.51	1.57	25.5
.885	.0556	.182	2.00	.111	3.51	1.57	25.0
.885	.0602	.183	1.99	.122	3.53	1.57	24.5
.886	.0649	.177	2.10	.117	3.54	1.57	22.5
.885	.0667	.175	2.12	.118	3.54	1.57	22.0
1.25	.0649	.215	2.21	.107	5.03	1.57	27.0
1.25	.0667	.215	2.21	.109	5.04	1.57	25.5
Boundary VII, louver, λ=0.506, 2- by 30-ft flume							
0.0247	0.00701	0.0263	1.02	0.0540	0.110	1.59	17.0
.0233	.00932	.0246	1.06	.0658	.104	1.58	9.5
.0233	.0116	.0229	1.19	.0663	.104	1.58	3.5
.0236	.0140	.0229	1.20	.0779	.105	1.58	1.5
.0438	.0116	.0329	1.30	.0556	.194	1.59	16.0
.0437	.0140	.0318	1.36	.0605	.194	1.58	7.0
.0442	.0163	.0304	1.47	.0604	.196	1.58	2.0
.0708	.00701	.0522	1.05	.0512	.313	1.59	27.0
.0704	.00932	.0475	1.20	.0519	.311	1.59	23.0
.0689	.0116	.0433	1.35	.0513	.305	1.59	18.5
.0696	.0140	.0415	1.45	.0530	.308	1.59	13.3
.0697	.0163	.0407	1.50	.0583	.308	1.59	7.0
.0697	.0186	.0403	1.52	.0646	.308	1.58	2.7
.0696	.0210	.0381	1.65	.0617	.308	1.58	2.0
.104	.00932	.0604	1.23	.0493	.457	1.60	27.0
.105	.0116	.0565	1.37	.0494	.462	1.60	24.0
.105	.0140	.0529	1.52	.0487	.462	1.60	20.8
.104	.0163	.0511	1.59	.0517	.461	1.59	15.5
.104	.0186	.0496	1.67	.0536	.462	1.59	11.5
.104	.0210	.0488	1.70	.0576	.462	1.59	8.0
.104	.0233	.0485	1.72	.0630	.461	1.58	3.7
.104	.0299	.0468	1.81	.0681	.460	1.58	2.5
.144	.0140	.0652	1.53	.0478	.636	1.60	29.0
.145	.0163	.0625	1.63	.0490	.637	1.60	27.0
.144	.0186	.0603	1.72	.0505	.636	1.59	23.3
.144	.0210	.0582	1.81	.0510	.636	1.59	18.8
.145	.0233	.0570	1.88	.0529	.638	1.59	14.8
.145	.0256	.0553	1.96	.0532	.639	1.59	10.5
.144	.0279	.0551	1.96	.0578	.636	1.59	6.5
.144	.0302	.0548	1.98	.0614	.637	1.58	4.5
.144	.0349	.0537	2.04	.0671	.635	1.58	3.2
.144	.0511	.0478	2.44	.0684	.636	1.58	2.0
.144	.0685	.0433	2.82	.0688	.636	1.58	1.0

TABLE 2.—*Roll-wave data*—Continued

q (cfs per ft)	S	y_0 (feet)	F	f	R × 10^{-5}	F_s	L (feet)
Boundary VII, louver, λ=0.506, 2- by 30-ft flume—Continued							
0.175	0.0186	0.0667	1.79	0.0465	0.782	1.60	28.5
.176	.0210	.0653	1.86	.0484	.787	1.60	25.8
.176	.0233	.0631	1.96	.0486	.786	1.60	23.3
.176	.0256	.0616	2.02	.0500	.784	1.60	20.8
.175	.0279	.0594	2.13	.0491	.784	1.60	17.3
.176	.0302	.0593	2.14	.0527	.787	1.59	15.0
.176	.0325	.0577	2.23	.0522	.786	1.59	13.3
.175	.0349	.0566	2.29	.0533	.783	1.59	10.8
.179	.0372	.0563	2.36	.0532	.803	1.59	8.7
.180	.0418	.0559	2.39	.0584	.804	1.59	7.5
.179	.0465	.0545	2.48	.0603	.803	1.58	6.2
.180	.0511	.0532	2.58	.0614	.806	1.58	5.0
.179	.0602	.0511	2.74	.0644	.804	1.58	3.5
.179	.0685	.0503	2.79	.0703	.802	1.58	3.0
.226	.0233	.0721	2.06	.0440	1.00	1.61	28.0
.227	.0256	.0703	2.14	.0446	1.01	1.61	26.0
.226	.0279	.0690	2.20	.0463	1.01	1.60	23.0
.226	0302	.0677	2.26	.0473	1.01	1.60	21.0
.226	.0325	.0651	2.40	.0454	1.00	1.60	18.3
.226	.0349	.0655	2.38	.0494	1.01	1.60	16.0
.226	.0372	.0635	2.49	.0480	1.01	1.60	14.5
.226	.0418	.0618	2.59	.0500	1.00	1.60	13.0
.226	.0465	.0587	2.80	.0475	1.01	1.60	12.0
.228	.0511	.0579	2.88	.0493	1.01	1.60	10.8
.227	.0556	.0565	2.98	.0502	1.01	1.60	9.7
.227	.0602	.0548	3.12	.0494	1.01	1.60	9.0
.228	.0649	.0536	3.23	.0497	1.01	1.60	8.0
.227	.0667	.0532	3.25	.0504	1.01	1.59	7.5
.228	.0685	.0526	3.32	.0496	1.01	1.60	7.2
.270	.0279	.0759	2.28	.0431	1.20	1.61	28.5
.269	.0302	.0745	2.33	.0444	1.20	1.61	27.0
.270	.0325	.0729	2.41	.0447	1.20	1.61	25.5
.270	.0349	.0718	2.47	.0457	1.20	1.60	23.5
.269	.0372	.0708	2.51	.0470	1.20	1.60	21.0
.271	.0418	.0676	2.72	.0452	1.21	1.60	19.0
.271	.0465	.0661	2.81	.0471	1.20	1.60	17.8
.271	.0511	.0646	2.91	.0483	1.20	1.60	16.8
.272	.0556	.0628	3.04	.0481	1.21	1.60	15.3
.271	.0602	.0614	3.14	.0490	1.20	1.60	14.3
.271	.0649	.0603	3.23	.0499	1.20	1.60	13.3
.270	.0667	.0586	3.35	.0474	1.20	1.60	12.0
.270	.0685	.0573	3.47	.0455	1.20	1.60	11.5
.313	.0279	.0818	2.36	.0402	1.34	1.62	27.0
.312	.0325	.0790	2.47	.0425	1.33	1.61	26.5
.312	.0372	.0749	2.68	.0415	1.33	1.61	24.8
.312	.0418	.0719	2.85	.0413	1.33	1.61	24.5
.313	.0465	.0694	3.02	.0407	1.34	1.61	21.8
.313	.0511	.0679	3.12	.0421	1.34	1.61	20.5
.312	.0556	.0660	3.24	.0423	1.35	1.61	19.3
.312	.0602	.0639	3.40	.0417	1.34	1.61	18.0
.311	.0649	.0619	3.56	.0411	1.34	1.61	17.3
.312	.0667	.0613	3.62	.0407	1.34	1.61	15.8
.312	.0685	.0608	3.66	.0409	1.34	1.61	15.0
.442	.0372	.0922	2.78	.0384	1.90	1.62	26.3
.442	.0418	.0891	2.93	.0390	1.90	1.62	24.8
.443	.0465	.0858	3.10	.0385	1.87	1.62	23.5
.442	.0511	.0837	3.22	.0394	1.88	1.62	22.3
.441	.0556	.0804	3.41	.0383	1.88	1.62	20.5
.443	.0602	.0791	3.51	.0392	1.88	1.62	19.5
.443	.0649	.0768	3.66	.0387	1.89	1.62	17.8
.442	.0667	.0761	3.71	.0388	1.88	1.62	17.0
.443	.0685	.0758	3.74	.0392	1.89	1.62	16.5

TABLE 2.—*Roll-wave data*—Continued

q (cfs per ft)	S	y_0 (feet)	**F**		$\mathbf{R} \times 10^{-5}$	$\mathbf{F}_s$	L (feet)
Boundary VII, louver, λ=0.506, 2- by 30-ft flume—Continued							
0.626	0.0372	0.116	2.81	0.0377	2.85	1.62	27.3
.625	.0418	.111	2.99	.0375	2.85	1.62	25.5
.624	.0465	.108	3.11	.0385	2.85	1.62	24.3
.626	.0511	.105	3.26	.0384	2.85	1.62	23.0
.626	.0556	.102	3.36	.0393	2.86	1.62	21.5
.625	.0602	.100	3.48	.0398	2.86	1.62	20.5
.624	.0649	.0969	3.65	.0390	2.86	1.62	19.5
.625	.0667	.0959	3.70	.0389	2.87	1.62	18.5
.625	.0685	.0938	3.84	.0373	2.87	1.62	17.8
.884	.0418	.136	3.11	.0346	3.80	1.63	26.5
.882	.0465	.130	3.32	.0336	3.80	1.63	25.0
.885	.0511	.127	3.46	.0341	3.82	1.63	24.0
.884	.0556	.125	3.54	.0355	3.83	1.62	23.0
.886	.0602	.122	3.66	.0360	3.84	1.62	21.5
.884	.0649	.118	3.85	.0349	3.84	1.62	20.0
.885	.0667	.117	3.89	.0353	3.85	1.62	19.5
.883	.0685	.116	3.96	.0349	3.85	1.62	19.0
1.25	.0465	.161	3.41	.0320	5.58	1.63	26.8
1.25	.0511	.157	3.55	.0325	5.60	1.63	25.5
1.25	.0556	.153	3.67	.0331	5.60	1.63	24.3
1.25	.0602	.149	3.82	.0331	5.57	1.63	23.3
1.25	.0649	.145	3.99	.0326	5.56	1.63	21.5
1.25	.0667	.144	4.04	.0327	5.57	1.63	20.5
1.25	.0685	.142	4.11	.0325	5.58	1.63	20.3
1.38	.0465	.174	3.36	.0329	6.13	1.63	27.5
1.37	.0511	.170	3.46	.0341	6.12	1.63	26.5
1.37	.0556	.163	3.67	.0331	6.11	1.63	25.0
1.37	.0602	.152	4.08	.0289	6.10	1.64	24.3
1.36	.0649	.150	4.13	.0304	6.06	1.64	23.3
1.36	.0667	.149	4.18	.0305	6.07	1.64	22.5
1.36	.0685	.147	4.26	.0302	6.08	1.64	22.0
Boundary VIII, smooth, 2.5- by 85-ft flume							
0.0948	0.0130	0.0423	1.92	0.0273	0.349	1.58	38
.0952	.0174	.0382	2.25	.0267	.351	1.58	30
.0952	.0260	.0338	2.70	.0278	.352	1.58	16
.0952	.0347	.0301	3.21	.0263	.354	1.58	15
.0948	.0434	.0286	3.45	.0284	.352	1.58	12
.0948	.0499	.0276	3.64	.0294	.353	1.58	11
.200	.0174	.0588	2.48	.0216	.746	1.64	58
.199	.0260	.0527	2.90	.0236	.747	1.64	39
.200	.0347	.0474	3.42	.0229	.755	1.64	25
.200	.0434	.0434	3.90	.0220	.757	1.64	15
.201	.0495	.0424	4.05	.0232	.762	1.64	10
.299	.0152	.0784	2.40	.0198	1.13	1.60	63
.300	.0174	.0747	2.59	.0195	1.14	1.60	58
.300	.0260	.0654	3.16	.0198	1.15	1.59	41
.300	.0347	.0594	3.65	.0198	1.15	1.58	27
.300	.0434	.0567	3.91	.0216	1.16	1.57	17
.401	.0260	.0783	3.21	.0188	1.52	1.66	46
.401	.0347	.0720	3.66	.0196	1.53	1.66	40
.401	.0434	.0672	4.05	.0200	1.53	1.65	29
.401	.0482	.0647	4.29	.0199	1.53	1.65	18
.591	.0317	.0940	3.61	.0170	2.34	1.68	------------
.797	.0408	.1057	4.09	.0165	3.14	1.71	------------
.980	.0495	.1131	4.54	.0160	3.84	1.72	------------

TABLE 3.—*Boundary-resistance data*

[Method of computation: for rough boundary, $f=8gy_0^3S/q^2$, $\mathbf{F}=q/\sqrt{gy_0^3}$, $\mathbf{R}=4y_0U/\nu=4q/\nu$]

q (cfs per ft)	S	y_0 (feet)	Temperature (° F)	$4y_0/k$	f	$\mathbf{F}$	$\mathbf{F}_s$	$\mathbf{R}\times 10^{-5}$
Boundary I, 3/16-in. cubes, λ=1/512, 2- by 30-ft flume								
0.3124	0.004695	0.1298	71.0	33.22	0.02710	1.177	1.65	1.199
.3122	.007009	.1122	71.0	28.72	.02617	1.463	1.66	1.198
.3122	.009322	.1038	71.2	26.57	.02755	1.645	1.65	1.200
.3123	.01398	.0905	71.2	23.16	.02736	2.021	1.65	1.201
.3122	.01864	.0839	71.2	21.47	.02909	2.263	1.65	1.200
.3128	.02326	.0781	71.2	19.99	.02917	2.525	1.64	1.203
.3130	.02792	.0746	70.5	19.09	.03048	2.706	1.64	1.194
.3126	.03254	.0708	71.2	18.12	.03044	2.924	1.64	1.202
.3127	.03717	.0682	70.5	17.45	.03106	3.093	1.64	1.193
.3126	.04184	.0663	71.3	16.97	.03215	3.226	1.64	1.204
.3126	.04645	.0640	70.5	16.38	.03210	3.402	1.64	1.193
.3122	.05107	.0620	71.3	15.87	.03216	3.563	1.64	1.203
.3122	.05564	.0616	70.7	15.77	.03437	3.598	1.63	1.193
.3109	.06024	.0598	71.0	15.30	.03433	3.746	1.63	1.193
.3118	.06488	.0589	70.8	15.07	.03513	3.843	1.63	1.193
.3130	.06670	.0585	70.9	14.97	.03511	3.897	1.63	1.199
.4418	.004695	.1582	71.0	40.49	.02453	1.237	1.67	1.696
.4422	.007009	.1405	71.4	35.96	.02561	1.479	1.66	1.707
.4430	.009322	.1281	71.1	32.79	.02572	1.702	1.66	1.703
.4414	.01398	.1139	71.8	29.15	.02731	2.023	1.65	1.710
.4424	.01864	.1050	71.1	26.88	.02839	2.291	1.65	1.701
.4419	.02326	.0984	71.8	25.19	.02923	2.522	1.64	1.712
.4412	.02792	.0920	71.1	23.55	.02877	2.786	1.65	1.696
.4407	.03254	.0872	72.0	22.32	.02862	3.015	1.65	1.713
.4420	.03717	.0850	71.1	21.76	.03010	3.142	1.64	1.700
.4412	.04184	.0821	72.0	21.01	.03064	3.304	1.64	1.715
.4422	.04645	.0798	71.1	20.42	.03110	3.456	1.64	1.700
.4417	.05107	.0771	72.2	19.73	.03090	3.635	1.64	1.722
.4418	.05564	.0765	71.1	19.58	.03287	3.679	1.64	1.699
.4427	.06024	.0740	72.4	18.94	.03209	3.875	1.64	1.731
.4402	.06488	.0729	71.2	18.66	.03341	3.940	1.63	1.694
.4420	.06670	.0734	72.6	18.79	.03478	3.916	1.63	1.731
.6243	.004695	.1978	75.1	50.63	.02401	1.250	1.67	2.525
.6240	.007009	.1730	73.4	44.28	.02401	1.528	1.67	2.471
.6250	.009322	.1610	74.1	41.21	.02565	1.704	1.66	2.500
.6253	.01398	.1392	71.7	35.63	.02484	2.121	1.66	2.423
.6251	.01864	.1298	74.6	33.22	.02687	2.355	1.66	2.515
.6243	.02326	.1207	71.7	30.89	.02703	2.623	1.65	2.419
.6237	.02792	.1141	74.9	29.21	.02746	2.851	1.65	2.520
.6253	.03254	.1098	71.7	28.10	.02838	3.028	1.65	2.423
.6258	.03717	.1043	74.2	26.70	.02774	3.369	1.65	2.505
.6245	.04184	.1033	71.8	26.44	.03046	3.314	1.64	2.422
.6258	.04645	.0988	74.2	25.29	.02947	3.550	1.65	2.505
.6264	.05107	.0976	74.6	24.98	.03117	3.620	1.64	2.520
.6256	.05564	.0937	74.3	23.98	.03013	3.843	1.64	2.507
.6234	.06024	.0919	74.2	23.52	.03099	3.943	1.64	2.496
.6251	.06488	.0909	74.3	23.27	.03212	4.019	1.64	2.505
.6250	.06670	.0905	74.3	23.17	.03260	4.045	1.64	2.502
.8820	.004695	.2432	73.0	62.25	.02236	1.295	1.68	3.475
.8836	.007009	.2149	73.3	55.01	.02295	1.562	1.61	3.495
.8834	.009322	.1964	73.5	50.27	.02331	1.788	1.67	3.498
.8834	.01398	.1760	73.5	45.05	.02515	2.108	1.66	3.498
.8831	.01864	.1620	73.7	41.47	.02617	2.386	1.66	3.511
.8836	.02326	.1508	73.2	38.60	.02631	2.658	1.66	3.492
.8826	.02792	.1427	73.3	36.53	.02683	2.885	1.66	3.492
.8836	.03254	.1369	73.5	35.04	.02755	3.073	1.65	3.502
.8840	.03717	.1326	73.6	33.94	.02857	3.226	1.65	3.507
.8880	.04184	.1304	73.8	33.38	.03031	3.323	1.64	3.537
.8844	.04645	.1258	73.9	32.20	.03046	3.492	1.64	3.530
.8834	.05107	.1219	74.1	31.40	.03053	3.657	1.64	3.533
.8836	.05564	.1202	74.8	30.77	.03188	3.736	1.64	3.562
.8846	.06024	.1171	75.1	29.97	.03184	3.889	1.64	3.581
.8840	.06488	.1140	75.2	29.18	.03168	4.046	1.64	3.582
.8846	.06670	.1127	75.5	28.85	.03143	4.119	1.64	3.599

TABLE 3.—*Boundary-resistance data*—Continued

q (cfs per ft)	S	y_0 (feet)	Temperature (° F)	$4y_0/k$	f	$\mathbf{F}$	$\mathbf{F}_s$	$\mathbf{R} \times 10^{-5}$
Boundary I, 3/16-in. cubes, λ=1/512, 2- by 30-ft flume—Continued								
1.247	0.009322	0.2464	76.8	63.07	0.02308	1.797	1.67	5.160
1.250	.01398	.2179	76.9	55.78	.02381	2.166	1.67	5.178
1.249	.01864	.2005	75.1	51.32	.02479	2.452	1.66	5.058
1.249	.02326	.1885	75.5	48.25	.02570	2.690	1.66	5.084
1.247	.02792	.1790	75.8	45.82	.02649	2.903	1.66	5.098
1.250	.03254	.1711	76.0	43.80	.02685	3.115	1.66	5.124
1.249	.03717	.1653	76.0	42.31	.02768	3.277	1.65	5.122
1.251	.04184	.1605	76.2	41.08	.02844	3.430	1.65	5.140
1.250	.04645	.1567	76.3	40.11	.02945	3.552	1.64	5.140
1.248	.05107	.1532	76.4	39.21	.03033	3.669	1.64	5.144
1.250	.05564	.1493	76.5	38.22	.03049	3.820	1.64	5.157
1.248	.06024	.1451	76.6	37.14	.03041	3.980	1.64	5.148
1.249	.06488	.1440	76.6	36.86	.03196	4.029	1.64	5.158
1.249	.06670	.1425	76.8	36.48	.03182	4.094	1.64	5.170
Boundary II, 3/16-in. cubes, λ=1/128, 2- by 30-ft flume								
0.3540	0.004611	0.1592	73.0	40.76	0.03825	0.9822	1.62	1.394
.3539	.006475	.1406	74.3	35.99	.03701	1.183	1.63	1.420
.3528	.006941	.1379	73.0	35.30	.03767	1.214	1.62	1.390
.3533	.007407	.1357	74.2	34.74	.03820	1.246	1.62	1.416
.3521	.009271	.1270	73.2	32.51	.03946	1.371	1.62	1.390
.3520	.01392	.1127	74.1	28.85	.04143	1.639	1.62	1.408
.3569	.01857	.1053	73.4	26.96	.04384	1.840	1.61	1.414
.3527	.02318	.0985	74.0	25.22	.04588	2.010	1.61	1.409
.3519	.02783	.0941	73.6	24.09	.04825	2.148	1.60	1.398
.3529	.03246	.0899	73.9	23.01	.04878	2.307	1.60	1.409
.3522	.03710	.0876	73.7	22.43	.05181	2.394	1.60	1.400
.3535	.04168	.0839	73.8	21.48	.05075	2.563	1.60	1.408
.3535	.04636	.0823	73.7	21.07	.05327	2.638	1.60	1.407
.3536	.05094	.0791	73.7	20.25	.05195	2.801	1.60	1.407
.3539	.05560	.0783	73.8	20.04	.05486	2.847	1.59	1.411
.3539	.06014	.0750	73.6	19.20	.05217	3.036	1.60	1.406
.3540	.06475	.0748	73.9	19.15	.05570	3.049	1.59	1.413
.3540	.06662	.0736	73.6	18.84	.05461	3.124	1.59	1.405
.4988	.004611	.1935	71.7	49.54	.03459	1.033	1.63	1.933
.5000	.006941	.1704	73.3	43.62	.03539	1.252	1.63	1.980
.4992	.009271	.1568	71.7	40.14	.03693	1.417	1.62	1.937
.4983	.01392	.1397	73.2	35.76	.03938	1.682	1.62	1.970
.5000	.01857	.1281	72.0	32.79	.04024	1.922	1.62	1.944
.5005	.02318	.1204	73.1	30.82	.04160	2.111	1.62	1.976
.4980	.02783	.1150	72.0	29.44	.04397	2.250	1.61	1.938
.5005	.03246	.1109	73.0	28.39	.04552	2.388	1.61	1.972
.4993	.03710	.1074	72.2	27.49	.04750	2.499	1.61	1.945
.5000	.04168	.1042	72.9	26.68	.04858	2.619	1.60	1.966
.5000	.04636	.1013	72.7	25.93	.04966	2.733	1.60	1.963
.4996	.05094	.0982	72.6	25.14	.04979	2.862	1.60	1.959
.5003	.05560	.0965	72.4	24.70	.05142	2.940	1.60	1.956
.4992	.06014	.0943	72.3	24.14	.05214	3.038	1.60	1.950
.5000	.06475	.0929	72.6	23.78	.05349	3.112	1.59	1.961
.4991	.06662	.0916	72.3	23.45	.05295	3.172	1.60	1.948
.7073	.004611	.2318	78.7	59.34	.02957	1.117	1.65	2.988
.7045	.009271	.1914	79.0	49.00	.03373	1.483	1.64	2.988
.7090	.01392	.1696	82.8	43.42	.03479	1.788	1.63	3.152
.7045	.01857	.1555	79.4	39.81	.03623	2.025	1.63	3.004
.7060	.02318	.1473	82.5	37.71	.03829	2.201	1.62	3.132
.7048	.02783	.1404	79.7	35.94	.03994	2.360	1.62	3.016
.7030	.03246	.1337	82.3	34.23	.04043	2.534	1.62	3.104
.7050	.03710	.1303	79.9	33.36	.04254	2.641	1.61	3.024
.7045	.04168	.1260	82.1	32.26	.04328	2.776	1.61	3.104
.7065	.04636	.1232	80.1	31.54	.04473	2.879	1.61	3.040
.7063	.05094	.1189	81.8	30.44	.04421	3.035	1.61	3.104
.7080	.05560	.1162	80.8	29.75	.04484	3.149	1.61	3.076
.7045	.06014	.1135	81.6	29.06	.04564	3.247	1.61	3.088
.7060	.06475	.1117	81.0	28.60	.04664	3.333	1.61	3.068
.7055	.06662	.1105	81.3	28.29	.04651	3.385	1.61	3.080

TABLE 3.—*Boundary-resistance data*—Continued

q (cfs per ft)	S	y_0 (feet)	Temperature (° F)	$4y_0/k$	f	F	F_s	$R \times 10^{-5}$
Boundary II, 3/16-in. cubes, λ=1/128, 2- by 30-ft flume—Continued								
1.000	0.004611	0.2930	71.7	75.01	0.02987	1.111	1.65	3.876
.9995	.006941	.2598	72.1	66.51	.03139	1.330	1.64	3.889
1.000	.009271	.2378	71.8	60.88	.03212	1.520	1.64	3.880
1.000	.01392	.2085	72.0	53.38	.03251	1.850	1.64	3.890
1.003	.01857	.1940	71.9	49.66	.03472	2.068	1.63	3.895
.9970	.02318	.1814	71.9	46.44	.03585	2.274	1.63	3.876
.9980	.02783	.1737	72.1	44.47	.03772	2.429	1.62	3.883
1.001	.03246	.1658	71.8	42.44	.03799	2.614	1.62	3.889
1.001	.03710	.1609	72.2	41.19	.03969	2.734	1.62	3.904
1.003	.04168	.1552	71.7	39.73	.03989	2.890	1.62	3.888
.9990	.04636	.1512	72.3	38.71	.04137	2.994	1.62	3.898
.9995	.05094	.1471	71.6	37.66	.04181	3.121	1.62	3.874
1.002	.05560	.1444	72.3	36.97	.04295	3.218	1.61	3.910
1.001	.06014	.1409	71.5	36.07	.04321	3.337	1.61	3.870
1.001	.06475	.1390	72.3	35.58	.04466	3.406	1.61	3.912
1.000	.06662	.1375	71.4	35.20	.04461	3.456	1.61	3.857
1.349	.004611	.3612	75.4	92.47	.03073	1.095	1.64	5.484
1.350	.009271	.2850	74.5	72.96	.03033	1.175	1.65	5.420
1.349	.01392	.2535	75.2	64.90	.03207	1.862	1.64	5.468
1.350	.01857	.2345	74.5	60.03	.03382	2.096	1.63	5.428
1.352	.02318	.2228	75.1	57.04	.03613	2.265	1.63	5.472
1.349	.02783	.2084	74.7	53.35	.03563	2.499	1.63	5.440
1.349	.03246	.2015	75.0	51.58	.03757	2.629	1.62	5.464
1.350	.03710	.1946	74.7	49.82	.03864	2.771	1.62	5.448
1.352	.04168	.1879	74.6	48.10	.03897	2.925	1.62	5.448
1.350	.04636	.1820	74.8	46.59	.03949	3.064	1.62	5.452
1.353	.05094	.1780	74.3	45.57	.04040	3.176	1.62	5.424
1.350	.05560	.1749	74.8	44.77	.04204	3.252	1.61	5.452
1.349	.06014	.1699	74.1	43.49	.04175	3.394	1.62	5.396
1.350	.06475	.1684	74.9	43.11	.04370	3.442	1.61	5.456
1.349	.06662	.1655	73.9	42.37	.04275	3.531	1.61	5.380
Boundary III, ten 3/16-in. to three 3/8-in. cubes, λ=1/128, 2- by 30-ft flume								
0.3122	0.007009	0.1358	72.6	22.49	0.04639	1.099	1.60	1.225
.3130	.009322	.1242	72.6	20.57	.04696	1.260	1.60	1.227
.3123	.01398	.1091	72.7	18.07	.04794	1.527	1.60	1.225
.3127	.01864	.1005	72.7	16.64	.04983	1.729	1.60	1.228
.3136	.02326	.0944	71.7	15.63	.05124	1.905	1.60	1.215
.3136	.02792	.0893	71.9	14.79	.05208	2.070	1.60	1.217
.3128	.03254	.0852	72.0	14.11	.05298	2.216	1.60	1.217
.3119	.03717	.0826	72.3	13.68	.05547	2.315	1.60	1.218
.3133	.04184	.0807	72.6	13.36	.05771	2.408	1.60	1.229
.3126	.04645	.0780	72.8	12.92	.05811	2.528	1.60	1.228
.3128	.05107	.0758	72.5	12.55	.05855	2.641	1.60	1.224
.3129	.05564	.0748	72.5	12.39	.06127	2.695	1.59	1.224
.3133	.06024	.0726	72.6	12.02	.06050	2.822	1.59	1.228
.3128	.06488	.0723	72.7	11.97	.06455	2.835	1.59	1.227
.3128	.06670	.0721	72.7	11.94	.06582	2.847	1.59	1.227
.4439	.007009	.1688	75.8	27.96	.04407	1.127	1.60	1.815
.4423	.009322	.1540	75.8	25.51	.04483	1.289	1.60	1.809
.4421	.01398	.1360	74.1	22.52	.04634	1.553	1.60	1.770
.4423	.01864	.1250	74.0	20.70	.04793	1.763	1.60	1.767
.4409	.02326	.1169	73.1	19.36	.04923	1.943	1.60	1.739
.4414	.02792	.1097	73.1	18.17	.04873	2.140	1.60	1.742
.4434	.03254	.1050	73.2	17.39	.04936	2.296	1.60	1.752
.4433	.03717	.1016	73.2	16.83	.05110	2.412	1.60	1.752
.4428	.04184	.0981	72.5	16.25	.05190	2.539	1.60	1.733
.4426	.04645	.0948	72.6	15.70	.05204	2.671	1.60	1.734
.4425	.05107	.0942	72.6	15.60	.05616	2.696	1.60	1.733
.4415	.05564	.0916	72.6	15.17	.05651	2.806	1.60	1.731
.4421	.06024	.0895	72.7	14.82	.05692	2.909	1.60	1.737
.4421	.06488	.0885	72.8	14.66	.05927	2.958	1.59	1.738
.4420	.06670	.0877	72.9	14.52	.05932	2.998	1.59	1.740

TABLE 3.—*Boundary-resistance data*—Continued

q (cfs per ft)	S	y_0 (feet)	Temperature (° F)	$4y_0/k$	f	F	F_s	$R \times 10^{-5}$
Boundary III, ten 3/16-in. to three 3/8-in. cubes, λ=1/128, 2- by 30-ft flume—Continued								
0.6256	0.007009	0.2087	88.3	34.57	0.04193	1.156	1.61	2.979
.6253	.009322	.1928	87.5	31.93	.04402	1.301	1.60	2.946
.6256	.01398	.1706	87.2	28.25	.04568	1.564	1.60	2.940
.6250	.01864	.1562	87.1	25.87	.04683	1.784	1.60	2.930
.6253	.02326	.1454	80.8	24.08	.04710	1.987	1.60	2.712
.6258	.02792	.1369	80.6	22.67	.04712	2.177	1.60	2.712
.6256	.03254	.1329	78.9	22.01	.05027	2.275	1.60	2.650
.6252	.03717	.1264	78.9	20.93	.04947	2.451	1.60	2.649
.6256	.04184	.1231	78.7	20.39	.05137	2.552	1.60	2.645
.6253	.04645	.1192	78.6	19.74	.05183	2.677	1.60	2.644
.6250	.05107	.1159	77.3	19.19	.05243	2.791	1.60	2.604
.6254	.05564	.1130	77.3	18.71	.05287	2.901	1.60	2.603
.6256	.05564	.1133	77.3	18.76	.05326	2.890	1.60	2.601
.6258	.06024	.1099	77.3	18.20	.05260	3.026	1.60	2.602
.6246	.06488	.1085	76.4	17.97	.05472	3.079	1.60	2.570
.6253	.06670	.1083	76.5	17.94	.05582	3.091	1.60	2.578
.8838	.004695	.2935	79.7	48.61	.03915	0.979	1.62	3.781
.8860	.007009	.2619	79.9	43.38	.04132	1.164	1.61	3.798
.8846	.007009	.2620	80.1	43.40	.04149	1.162	1.61	3.812
.8840	.009322	.2382	79.7	39.45	.04153	1.339	1.61	3.781
.8853	.01398	.2129	80.0	35.26	.04433	1.588	1.60	3.807
.8839	.01864	.1961	80.8	32.48	.04633	1.793	1.60	3.838
.8859	.02326	.1815	81.9	30.06	.04564	2.018	1.60	3.898
.8862	.02792	.1732	82.1	28.69	.04758	2.166	1.60	3.912
.8840	.03254	.1651	82.2	27.34	.04827	2.322	1.60	3.907
.8850	.03717	.1589	82.4	26.32	.04905	2.462	1.60	3.924
.8852	.04184	.1531	82.6	25.36	.04936	2.603	1.60	3.929
.8852	.04645	.1491	82.8	24.69	.05062	2.709	1.60	3.934
.8834	.05107	.1434	81.3	23.75	.04971	2.866	1.60	3.861
.8844	.05564	.1399	81.6	23.17	.05017	2.977	1.60	3.874
.8844	.05564	.1407	82.3	23.30	.05104	2.952	1.60	3.909
.8852	.06024	.1385	82.5	22.94	.05261	3.026	1.60	3.921
.8843	.06488	.1355	82.7	22.44	.05317	3.124	1.60	3.930
.8837	.06670	.1333	83.1	22.08	.05211	3.199	1.60	3.940
1.249	.004695	.3558	83.8	58.93	.03490	1.037	1.63	5.621
1.251	.004695	.3543	83.8	58.68	.03434	1.045	1.63	5.631
1.248	.007009	.3205	84.4	53.09	.03815	1.212	1.62	5.660
1.249	.009322	.2933	84.2	48.58	.03880	1.386	1.62	5.647
1.249	.01398	.2618	84.4	43.36	.04138	1.643	1.61	5.667
1.249	.01864	.2380	85.0	39.42	.04143	1.896	1.61	5.706
1.249	.02326	.2271	85.2	37.61	.04495	2.034	1.60	5.718
1.250	.02792	.2148	85.4	35.58	.04557	2.213	1.60	5.736
1.251	.03254	.2055	84.9	34.04	.04647	2.366	1.60	5.706
1.247	.03717	.1957	85.8	32.41	.04610	2.539	1.60	5.749
1.248	.04184	.1903	86.5	31.52	.04764	2.650	1.60	5.807
1.248	.04645	.1849	87.0	30.62	.04855	2.766	1.60	5.846
1.251	.05107	.1815	87.8	30.06	.05024	2.851	1.60	5.922
1.250	.05564	.1765	88.3	27.23	.05040	2.971	1.60	5.954
1.247	.06024	.1719	88.4	28.47	.05065	3.084	1.60	5.947
1.250	.06488	.1701	88.9	28.17	.05259	3.141	1.60	5.998
1.249	.06670	.1682	89.2	27.86	.05240	3.190	1.60	6.027
Boundary IV, 3/16-in. cubes, λ=1/32, 2- by 30-ft flume								
0.2489	0.006941	0.1291	71.8	33.05	0.06210	0.9456	1.59	0.9657
.2500	.009271	.1189	72.6	30.44	.06423	1.074	1.59	.9796
.2481	.01392	.1054	71.8	26.98	.06821	1.277	1.58	.9626
.2502	.01857	.0970	72.6	24.83	.06974	1.459	1.58	.9810
.2519	.02318	.0917	71.8	23.48	.07256	1.599	1.58	.9783
.2527	.02783	.0869	72.6	22.25	.07367	1.738	1.57	.9910
.2500	.03246	.0833	71.9	21.32	.07733	1.832	1.57	.9709
.2501	.03710	.0805	71.9	20.61	.07971	1.930	1.57	.9713
.2501	.04168	.0778	71.9	19.92	.08087	2.031	1.57	.9713
.2514	.04636	.0751	72.7	19.23	.08000	2.152	1.57	.9859
.2500	.05094	.0732	71.8	18.74	.08237	2.224	1.57	.9699
.2474	.05560	.0704	72.7	18.02	.08165	2.334	1.57	.9711
.2498	.06014	.0697	71.8	17.84	.08410	2.392	1.57	.9692
.2480	.06475	.0680	72.7	17.41	.08528	2.464	1.57	.9725
.2496	.06662	.0682	71.8	17.46	.08741	2.470	1.57	.9684

TABLE 3.—*Boundary-resistance data*—Continued

q (cfs per ft)	S	y_0 (feet)	Temperature (° F)	$4y_0/k$	f	**F**	$\mathbf{F}_s$	$\mathbf{R} \times 10^{-5}$
Boundary IV, 3/16-in. cubes, λ=1/32, 2- by 30-ft flume—Continued								
0.4998	0.006941	0.1958	66.7	50.12	0.05373	1.017	1.60	1.814
.5002	.009271	.1787	68.3	45.75	.05447	1.167	1.59	1.854
.5000	.01392	.1597	66.7	40.88	.05842	1.380	1.59	1.815
.4987	.01857	.1470	68.5	37.63	.06109	1.559	1.59	1.852
.5000	.02318	.1374	66.5	35.17	.06197	1.730	1.58	1.810
.5004	.02783	.1307	68.8	33.46	.06391	1.865	1.58	1.867
.5004	.03246	.1247	66.4	31.92	.06476	2.002	1.58	1.806
.5002	.03710	.1205	66.3	30.85	.06684	2.107	1.58	1.806
.5000	.04168	.1171	67.8	29.98	.06897	2.199	1.58	1.840
.5000	.04636	.1141	68.8	29.21	.07097	2.286	1.58	1.866
.5004	.05094	.1105	68.2	28.29	.07070	2.401	1.58	1.853
.4984	.05560	.1080	68.9	27.65	.07262	2.474	1.58	1.863
.4987	.06014	.1056	68.6	27.03	.07336	2.561	1.57	1.857
.4998	.06475	.1035	68.8	26.50	.07403	2.644	1.58	1.865
.5000	.06662	.1029	68.8	26.34	.07479	2.669	1.57	1.866
.7500	.006941	.2436	73.3	62.36	.04595	1.099	1.60	2.967
.7500	.009271	.2249	72.7	57.57	.04830	1.239	1.60	2.944
.7545	.01392	.2011	73.1	51.48	.05121	1.474	1.60	2.976
.7500	.01857	.1836	72.8	47.00	.05262	1.680	1.60	2.944
.7522	.02318	.1726	72.9	44.19	.05427	1.847	1.59	2.959
.7470	.02783	.1642	72.8	42.04	.05687	1.978	1.59	2.935
.7520	.03246	.1582	72.8	40.50	.05853	2.105	1.59	2.955
.7500	.03710	.1532	72.9	39.22	.06108	2.204	1.59	2.953
.7505	.04168	.1479	72.7	37.86	.06167	2.325	1.59	2.946
.7470	.04636	.1447	73.0	37.04	.06482	2.391	1.58	2.944
.7480	.05094	.1405	72.4	35.97	.06506	2.502	1.58	2.925
.7480	.05560	.1378	73.1	35.28	.06697	2.577	1.58	2.954
.7490	.06014	.1344	72.2	34.41	.06704	2.678	1.58	2.917
.7470	.06475	.1324	73.2	33.89	.06938	2.732	1.58	2.953
.7490	.06662	.1312	71.2	33.59	.06909	2.777	1.58	2.914
.9985	.004611	.3275	67.2	83.84	.04185	.9390	1.61	3.651
.9990	.006941	.2923	70.7	74.83	.04475	1.114	1.61	3.817
.9985	.009271	.2703	67.5	69.20	.04729	1.252	1.60	3.664
.9980	.01392	.2381	70.5	60.95	.04859	1.514	1.60	3.809
.9975	.01857	.2200	67.8	56.32	.05519	1.703	1.60	3.674
.9980	.02318	.2075	70.3	53.12	.05357	1.860	1.60	3.802
.9970	.02783	.1978	68.1	50.64	.05581	1.997	1.59	3.689
.9975	.03246	.1890	70.2	48.38	.05673	2.140	1.59	3.793
1.0035	.03710	.1848	68.3	47.31	.05989	2.226	1.59	3.724
.9990	.04168	.1775	69.9	45.44	.06016	2.354	1.59	3.784
1.001	.04636	.1744	68.4	44.65	.06323	2.422	1.58	3.714
.9985	.05094	.1690	69.2	43.26	.06352	2.532	1.58	3.747
.9985	.05560	.1661	68.5	42.52	.06583	2.599	1.58	3.715
.9980	.06014	.1617	68.9	41.40	.06577	2.704	1.58	3.731
.9975	.06475	.1598	68.8	40.91	.06840	2.751	1.58	3.722
.9970	.06662	.1572	68.8	40.24	.06708	2.818	1.58	3.724
1.249	.004611	.3799	70.7	97.25	.04174	.9403	1.61	4.776
1.249	.006941	.3367	73.0	86.20	.04375	1.127	1.61	4.922
1.249	.009271	.3089	71.1	79.08	.04512	1.282	1.60	4.804
1.254	.01392	.2742	73.0	70.20	.04700	1.538	1.60	4.732
1.249	.01857	.2543	71.3	65.10	.05043	1.716	1.60	4.813
1.251	.02318	.2392	72.9	61.24	.05222	1.884	1.60	4.920
1.250	.02783	.2279	71.6	58.34	.05432	2.025	1.59	4.840
1.249	.03246	.2186	72.8	55.96	.05598	2.154	1.59	4.908
1.249	.03710	.2108	71.8	53.96	.05738	2.275	1.59	4.846
1.251	.04168	.2048	72.6	52.43	.05894	2.377	1.59	4.906
1.250	.04636	.2006	72.0	51.35	.06170	2.452	1.59	4.859
1.250	.05094	.1951	72.5	49.95	.06237	2.556	1.59	4.892
1.250	.05560	.1915	72.1	49.02	.06437	2.628	1.58	4.864
1.246	.06014	.1868	72.3	47.82	.06505	2.719	1.58	4.862
1.250	.06475	.1846	72.2	47.26	.06715	2.777	1.58	4.873
1.249	.06662	.1832	72.1	46.90	.06764	2.806	1.58	4.865

TABLE 3.—*Boundary-resistance data*—Continued

q (cfs per ft)	S	y_0 (feet)	Temperature (° F)	$4y_0/k$	f	**F**	$\mathbf{F}_s$	$\mathbf{R} \times 10^{-5}$
				Boundary V, 3/16-in cubes, λ=1/32, 2.5- by 85-ft flume				
[1] 0.2514	0.0004340	0.3073	71.6	78.67	0.05132	0.2903	1.60	0.9726
[1] .2500	.0008681	.2444	72.6	62.57	.05220	.3646	1.60	.9805
.2508	.001302	.2129	73.2	54.50	.05145	.4867	1.60	.9904
.2500	.001736	.1946	73.3	49.81	.05271	.5133	1.60	.9903
.2503	.002604	.1723	73.4	44.11	.05475	.6168	1.60	.9925
.2497	.003472	.1577	73.4	40.37	.05622	.7028	1.59	.9893
.2509	.006944	.1303	70.4	33.36	.06284	.9401	1.59	.9569
.2497	.009115	.1204	70.6	30.82	.06573	1.053	1.59	.9540
.2506	.01128	.1122	70.2	28.72	.06532	1.175	1.59	.9540
.2494	.01302	.1085	70.7	27.78	.06879	1.230	1.58	.9552
.2502	.01736	.1002	70.1	25.65	.07187	1.390	1.58	.9495
.2496	.02170	.0943	71.0	24.14	.07523	1.519	1.58	.9591
.2505	.02604	.0895	69.9	22.91	.07668	1.648	1.58	.9486
.2506	.03038	.0867	71.4	22.20	.08118	1.730	1.57	.9668
.2500	.03472	.0823	70.7	21.07	.07976	1.866	1.57	.9561
.2499	.03906	.0802	69.6	20.53	.08310	1.939	1.57	.9431
.2502	.04340	.0776	70.5	19.87	.08342	2.040	1.57	.9552
.2503	.04991	.0745	70.2	19.07	.08482	2.170	1.57	.9528
[1] .4990	.0004340	.4660	72.2	119.30	.04542	.2765	1.60	1.946
[1] .4986	.0008681	.3676	70.5	94.11	.04468	.3942	1.60	1.903
[1] .4979	.001302	.3213	73.1	82.25	.04487	.4818	1.60	1.964
.4997	.001736	.2913	71.3	74.57	.04427	.5601	1.60	1.926
.4999	.002604	.2588	73.0	66.25	.04651	.6692	1.60	1.969
.4972	.003472	.2353	73.4	60.24	.04712	.7675	1.60	1.969
.5022	.006944	.1926	70.5	49.31	.05066	1.047	1.60	1.915
.5002	.008681	.1801	69.8	46.11	.05222	1.153	1.60	1.893
.5004	.01085	.1687	71.6	43.19	.05359	1.273	1.60	1.938
.5015	.01302	.1609	69.8	41.19	.05554	1.369	1.60	1.896
.5006	.01519	.1530	71.5	39.17	.05590	1.474	1.60	1.933
.5006	.01736	.1475	71.4	37.76	.05725	1.557	1.59	1.931
.5027	.02127	.1401	69.8	35.87	.05961	1.689	1.59	1.901
.4997	.02604	.1332	71.4	34.10	.06348	1.811	1.58	1.909
.4936	.03038	.1272	70.2	32.56	.06612	1.917	1.58	1.877
.5002	.03472	.1241	71.4	31.77	.06831	2.011	1.58	1.911
.4992	.03906	.1205	70.4	30.82	.07061	2.104	1.58	1.902
.4997	.04340	.1170	71.4	29.95	.07170	2.200	1.58	1.909
.5013	.04991	.1138	70.7	29.13	.07540	2.301	1.57	1.917
[1] .7507	.0004340	.5859	72.3	150.0	.03989	.2950	1.62	2.933
[1] .7491	.0008681	.4680	76.5	119.8	.04088	.4121	1.61	3.088
[1] .7483	.001302	.4109	75.4	105.1	.04155	.5006	1.61	3.044
.7503	.001736	.3741	75.8	95.77	.04151	.5778	1.61	3.069
.7471	.002604	.3296	74.0	84.38	.04303	.6957	1.61	2.986
.7519	.003472	.3029	75.8	77.54	.04396	.7947	1.61	3.072
.7519	.006944	.2446	72.4	62.62	.04631	1.095	1.60	2.940
.7475	.008681	.2292	72.6	58.68	.04819	1.200	1.60	2.931
.7475	.01085	.2144	72.4	54.89	.04930	1.327	1.60	2.923
.7467	.01302	.2037	72.6	52.15	.05078	1.432	1.60	2.929
.7487	.01519	.1953	72.4	50.00	.05201	1.528	1.60	2.927
.7483	.01736	.1877	72.6	48.05	.05282	1.621	1.60	2.931
.7523	.02170	.1775	72.5	45.44	.05524	1.772	1.59	2.944
.7515	.02604	.1704	72.6	43.62	.05876	1.883	1.59	2.944
.7503	.02995	.1621	72.5	41.50	.05838	2.026	1.59	2.937
.7495	.03472	.1578	72.6	40.40	.06255	2.107	1.58	2.937
.7507	.03906	.1524	72.5	39.01	.06318	2.223	1.58	2.939
.7483	.04340	.1495	72.6	38.27	.06671	2.281	1.58	2.932
.7487	.04991	.1443	72.6	36.94	.06891	2.407	1.58	2.933
[1] 1.000	.0004340	.7044	75.5	180.3	.03904	.2982	1.62	4.075
[1] .9996	.0008681	.5578	75.0	142.8	.03884	.4229	1.62	4.043
[1] 1.000	.001302	.4941	73.2	126.4	.04043	.5075	1.62	3.954
[1] .9996	.001736	.4471	74.0	114.4	.03998	.5893	1.62	3.999
1.001	.002604	.3944	74.7	100.9	.04102	.7125	1.61	4.039
1.000	.003472	.3618	73.9	92.62	.04229	.8103	1.61	3.999
.9992	.006076	.3035	71.6	77.70	.04382	1.053	1.61	3.869
1.003	.006944	.2904	72.9	74.34	.04353	1.130	1.61	3.946
1.001	.008681	.2720	71.8	69.63	.04485	1.244	1.60	3.886
1.002	.01085	.2560	73.0	65.54	.04666	1.364	1.60	3.951
1.002	.01302	.2415	71.9	61.80	.04705	1.488	1.60	3.895
1.003	.01519	.2328	73.1	59.60	.04906	1.574	1.60	3.961
1.001	.01736	.2233	72.1	57.16	.04963	1.673	1.60	3.901

[1] Not plotted.

TABLE 3.—*Boundary-resistance data*—Continued

q (cfs per ft)	S	y_0 (feet)	Temperature (° F)	$4y_0/k$	f	F	F_s	$R \times 10^{-5}$
Boundary V, 3/16-in. cubes, λ=1/32, 2.5 by 85-ft flume—Continued								
0.9956	0.02170	0.2111	73.2	54.04	0.05304	1.809	1.60	3.935
.9984	.02170	.2102	73.1	53.81	.05208	1.825	1.60	3.938
1.002	.02604	.2012	72.2	51.51	.05437	1.956	1.59	3.908
1.000	.03038	.1932	73.2	49.46	.05639	2.076	1.59	3.954
.9992	.03472	.1862	73.2	47.67	.05783	2.191	1.59	3.950
.9996	.03906	.1813	73.2	46.41	.06002	2.281	1.59	3.951
1.000	.04340	.1771	73.2	45.34	.06211	2.364	1.59	3.952
1.001	.04991	.1726	73.2	44.19	.06594	2.460	1.58	3.958
[1] 1.251	.0008681	.6577	75.4	168.3	.04066	.4132	1.62	5.085
[1] 1.255	.001302	.5723	76.4	146.5	.03987	.5110	1.62	5.173
[1] 1.253	.001736	.5212	75.1	133.4	.04028	.5871	1.62	5.081
[1] 1.252	.002604	.4559	75.5	116.7	.04054	.7167	1.62	5.095
[1] 1.250	.003472	.4170	75.3	106.7	.04147	.8182	1.61	5.078
[1] 1.246	.004340	.3879	72.3	99.30	.04200	.9091	1.61	4.874
1.254	.006510	.3419	70.6	87.53	.04261	1.105	1.61	4.918
1.248	.008681	.3130	72.4	80.13	.04402	1.256	1.61	4.880
1.254	.01085	.2941	70.7	75.29	.04518	1.386	1.60	4.924
1.249	.01302	.2787	72.4	71.35	.04650	1.497	1.60	4.886
1.254	.01519	.2673	70.8	68.43	.04749	1.599	1.60	4.929
1.248	.01736	.2576	72.4	65.95	.04900	1.683	1.60	4.884
1.254	.02170	.2424	70.9	62.05	.05066	1.851	1.60	4.931
1.247	.02604	.2311	72.4	59.16	.05318	1.979	1.60	4.879
1.256	.03038	.2219	71.0	56.81	.05396	2.117	1.60	4.822
1.247	.03472	.2145	71.2	54.91	.05669	2.213	1.59	4.804
1.247	.03906	.2092	71.3	53.56	.05918	2.297	1.59	4.808
1.247	.04340	.2035	71.3	52.10	.06051	2.395	1.59	4.809
1.248	.04991	.1976	71.2	50.59	.06364	2.506	1.58	4.807
[1] 1.744	.0008681	.8058	75.3	206.2	.03845	.4249	1.62	7.076
[1] 1.741	.001302	.7016	75.9	179.6	.03819	.5222	1.62	7.123
[1] 1.753	.001736	.6469	74.3	165.6	.03939	.5937	1.62	7.033
[1] 1.747	.002604	.5597	74.9	143.2	.03851	.7354	1.62	7.061
[1] 1.748	.003472	.5117	74.6	131.0	.03920	.8417	1.62	7.039
[1] 1.749	.004340	.4778	73.9	122.3	.03984	.9334	1.62	6.977
1.749	.006510	.4189	74.3	107.2	.04031	1.137	1.62	7.009
1.751	.008681	.3832	74.0	98.10	.04102	1.301	1.62	6.999
1.749	.01085	.3569	74.3	91.37	.04150	1.446	1.61	7.014
1.750	.01085	.3600	74.5	92.16	.04254	1.428	1.61	7.038
1.753	.01302	.3417	74.1	87.48	.04354	1.546	1.61	7.016
1.755	.01519	.3271	74.7	83.74	.04443	1.654	1.61	7.072
1.754	.01736	.3156	74.2	80.79	.04566	1.744	1.61	7.029
1.754	.02170	.2980	74.8	76.29	.04807	1.900	1.61	7.077
1.754	.02604	.2837	74.3	72.63	.04975	2.046	1.61	7.036
1.743	.03038	.2723	75.5	69.71	.05196	2.163	1.60	7.103
1.747	.03472	.2638	75.6	67.28	.05379	2.272	1.60	7.117
1.750	.03906	.2569	75.6	65.77	.05566	2.369	1.59	7.131
1.754	.04340	.2492	75.0	63.80	.05623	2.485	1.59	7.098
1.750	.04991	.2440	75.6	62.46	.06097	2.558	1.59	7.136
Boundary VI, 3/16-in. cubes, λ=1/8, 2.5- by 85-ft flume								
0.3128	0.01397	0.1424	71.4	36.45	0.1062	1.025	1.58	1.207
.3124	.01863	.1318	71.5	33.74	.1126	1.150	1.58	1.207
.3119	.02325	.1245	72.0	31.87	.1188	1.251	1.58	1.213
.3102	.02791	.1168	72.1	29.90	.1190	1.369	1.58	1.208
.3124	.03254	.1136	72.3	29.08	.1259	1.437	1.59	1.221
.3117	.03717	.1094	77.9	28.00	.1290	1.517	1.59	1.306
.3109	.04184	.1069	77.0	27.36	.1362	1.567	1.59	1.290
.3143	.04645	.1041	77.0	26.65	.1366	1.648	1.58	1.304
.3139	.05106	.1013	77.0	25.93	.1387	1.715	1.58	1.301
.3144	.05563	.1009	74.4	25.83	.1489	1.728	1.59	1.262
.3131	.06024	.0985	74.6	25.21	.1512	1.784	1.59	1.261
.3129	.06487	.0965	76.2	24.70	.1533	1.839	1.59	1.286
.3135	.06670	.0963	76.4	25.16	.1561	1.848	1.59	1.290
.4431	.01397	.1731	72.3	44.31	.09512	1.084	1.58	1.732
.4421	.01863	.1612	72.3	41.26	.1028	1.203	1.58	1.728
.4420	.02325	.1507	72.4	38.57	.1049	1.331	1.58	1.728
.4418	.02791	.1434	72.4	36.71	.1086	1.433	1.58	1.727
.4408	.03254	.1384	73.0	35.43	.1143	1.508	1.58	1.738
.4416	.03717	.1333	72.9	34.63	.1163	1.598	1.57	1.738
.4422	.04184	.1301	78.5	33.30	.1213	1.660	1.57	1.865
.4429	.04645	.1299	77.2	33.25	.1337	1.666	1.58	1.841

[1] Not plotted.

TABLE 3.—*Boundary-resistance data*—Continued

q (cfs per ft)	S	y_0 (feet)	Temperature (° F)	$4y_0/k$	f	**F**	$\mathbf{F}_s$	$\mathbf{R} \times 10^{-5}$
Boundary VI, 3/16-in. cubes, λ=1/8, 2.5- by 85-ft flume—Continued								
0.4446	0.05106	0.1263	77.4	32.33	0.1340	1.745	1.58	1.852
.4419	.05563	.1208	78.8	30.92	.1293	1.854	1.57	1.872
.4429	.06024	.1184	78.9	30.31	.1313	1.915	1.57	1.878
.4426	.06487	.1169	79.0	29.92	.1362	1.951	1.58	1.877
.4427	.06670	.1155	79.0	29.56	.1350	1.987	1.58	1.879
.6258	.009322	.2349	73.3	60.13	.07948	.9686	1.59	2.476
.6256	.01397	.2112	73.0	54.06	.08667	1.135	1.58	2.463
.6250	.01863	.1956	73.2	50.07	.09197	1.273	1.58	2.470
.6252	.02325	.1857	73.7	47.53	.09815	1.376	1.58	2.488
.6253	.02791	.1764	73.9	45.15	.1009	1.487	1.58	2.498
.6248	.03254	.1698	74.2	43.46	.1051	1.573	1.57	2.501
.6252	.03717	.1630	72.2	41.72	.1060	1.674	1.57	2.439
.6256	.04184	.1596	72.4	40.85	.1119	1.728	1.57	2.448
.6252	.04645	.1560	72.2	39.93	.1177	1.788	1.57	2.439
.6244	.05106	.1523	72.7	38.98	.1191	1.851	1.57	2.451
.6256	.05563	.1491	71.4	38.17	.1213	1.914	1.57	2.415
.6245	.06024	.1463	71.5	37.45	.1245	1.966	1.57	2.413
.6253	.06487	.1443	71.4	36.94	.1284	2.010	1.57	2.414
.6256	.06670	.1433	71.4	36.68	.1291	2.032	1.57	2.415
.8862	.009322	.2867	73.1	73.39	.07206	1.017	1.59	3.499
.8856	.01397	.2571	73.2	65.81	.07802	1.197	1.58	3.503
.8834	.01397	.2566	73.2	65.69	.07795	1.197	1.58	3.491
.8837	.01863	.2377	73.5	60.85	.08256	1.343	1.58	3.503
.8842	.02325	.2254	73.6	57.70	.08775	1.456	1.58	3.512
.8850	.02791	.2159	73.8	55.27	.09241	1.554	1.58	3.522
.8858	.03254	.2073	73.9	53.06	.09517	1.653	1.56	3.536
.8848	.03717	.1997	73.4	51.12	.09741	1.747	1.56	3.504
.8843	.04184	.1946	73.6	49.81	.1015	1.815	1.57	3.516
.8844	.04645	.1900	73.7	48.64	.1049	1.881	1.57	3.520
.8849	.05106	.1850	73.5	47.36	.1063	1.959	1.57	3.511
.8849	.05563	.1823	73.7	46.66	.1108	2.003	1.57	3.522
.8864	.06024	.1833	75.0	46.92	.1216	1.990	1.57	3.585
.8827	.06487	.1766	74.3	45.21	.1181	2.095	1.57	3.541
.8826	.06670	.1753	74.7	44.87	.1188	2.118	1.57	3.558
1.247	.007009	.3757	76.5	96.17	.06148	.9549	1.59	5.146
1.249	.009322	.3496	76.7	89.49	.06568	1.065	1.59	5.164
1.249	.01397	.3128	76.9	80.07	.07055	1.258	1.59	5.175
1.249	.01863	.2921	77.0	74.77	.07668	1.394	1.58	5.177
1.252	.01325	.2759	76.5	70.63	.08027	1.522	1.58	5.168
1.251	.02791	.2643	76.7	67.66	.08477	1.623	1.57	5.171
1.247	.03254	.2553	76.7	65.35	.08957	1.704	1.57	5.156
1.249	.03717	.2462	74.5	63.02	.09151	1.802	1.57	5.023
1.250	.04184	.2405	74.7	61.56	.09591	1.868	1.57	5.041
1.250	.04645	.2323	76.5	59.46	.09596	1.967	1.57	5.155
1.248	.05106	.2281	76.8	58.39	.1001	2.019	1.56	5.171
1.249	.05563	.2270	77.0	58.11	.1074	2.035	1.57	5.183
1.252	.06024	.2294	77.0	58.72	.1195	2.007	1.57	5.189
1.246	.06487	.2150	77.1	55.04	.1068	2.203	1.57	5.183
1.249	.06670	.2149	77.3	55.01	.1048	2.210	1.57	5.207
Boundary VII, Louver., λ=0.506, 2- by 30-ft flume								
0.3108	0.004695	0.1406	80.3	17.00	0.03482	1.039	1.63	1.342
.3109	.004695	.1405	80.3	16.98	.03468	1.041	1.63	1.343
.3122	.007009	.1248	80.2	15.08	.03600	1.248	1.63	1.347
.3119	.007009	.1246	80.2	15.05	.03587	1.250	1.63	1.345
.3120	.009322	.1147	80.2	13.86	.03722	1.416	1.62	1.346
.3126	.01398	.1010	80.2	12.21	.03801	1.715	1.62	1.349
.3121	.01864	.0937	79.5	11.33	.04058	1.916	1.62	1.335
.3122	.02326	.0867	79.6	10.48	.04011	2.153	1.62	1.337
.3126	.02792	.0818	79.6	9.879	.04021	2.357	1.62	1.339
.3116	.03254	.0790	79.6	9.542	.04250	2.475	1.61	1.334
.3115	.03717	.0749	79.6	9.056	.04153	2.676	1.61	1.334
.3115	.04184	.0719	79.6	8.692	.04134	2.845	1.61	1.334
.3132	.04645	.0694	79.6	8.382	.04070	3.021	1.62	1.342
.3126	.05106	.0679	79.7	8.201	.04207	3.116	1.61	1.340
.3120	.05564	.0660	80.1	7.975	.04231	3.243	1.61	1.346
.3117	.06024	.0639	80.2	7.722	.04167	3.400	1.61	1.344
.3114	.06488	.0619	80.2	7.479	.04111	3.564	1.61	1.344
.3115	.06670	.0613	80.2	7.403	.04071	3.620	1.62	1.345
.3115	.06852	.0608	80.3	7.349	.04091	3.660	1.61	1.345

TABLE 3.—*Boundary-resistance data*—Continued

q (cfs per ft)	S	y_0 (feet)	Temperature (° F)	$4y_0/k$	f	**F**	$\mathbf{F}_s$	$\mathbf{R} \times 10^{-5}$
Boundary VII, Louver λ=0.506, 2- by 30-ft flume								
0.4417	0.004695	0.1760	80.5	21.27	0.03382	1.054	1.64	1.912
.4425	.004695	.1772	80.3	21.41	.03434	1.046	1.63	1.912
.4417	.007009	.1552	80.0	18.75	.03456	1.274	1.63	1.901
.4416	.007009	.1549	80.6	18.72	.03440	1.276	1.63	1.916
.4418	.009322	.1425	79.9	17.22	.03561	1.447	1.63	1.899
.4424	.01398	.1253	79.9	15.14	.03619	1.758	1.63	1.901
.4423	.01864	.1161	79.8	14.03	.03843	1.969	1.62	1.898
.4424	.02326	.1085	79.8	13.11	.03908	2.182	1.62	1.899
.4419	.02792	.1020	79.7	12.32	.03904	2.392	1.62	1.897
.4425	.03254	.0966	79.7	11.67	.03858	2.597	1.62	1.898
.4419	.03717	.0922	79.7	11.14	.03838	2.783	1.62	1.896
.4423	.04184	.0892	79.7	10.77	.03901	2.929	1.62	1.898
.4427	.04645	.0858	78.6	10.37	.03854	3.105	1.62	1.874
.4420	.05106	.0837	78.8	10.11	.03902	3.219	1.62	1.875
.4410	.05564	.0804	79.1	9.72	.03834	3.407	1.62	1.878
.4427	.06024	.0791	79.0	9.56	.03916	3.508	1.62	1.884
.4429	.06488	.0768	79.1	9.29	.03865	3.664	1.62	1.886
.4417	.06670	.0761	79.1	9.19	.03878	3.709	1.62	1.882
.4426	.06852	.0758	79.2	9.16	.03918	3.740	1.62	1.887
.6256	.004695	.2194	79.6	26.52	.03266	1.072	1.64	2.680
.6259	.007009	.1952	81.5	23.59	.03427	1.279	1.63	2.777
.6256	.009322	.1792	83.1	21.65	.03529	1.454	1.63	2.794
.6246	.01398	.1562	83.2	18.87	.03514	1.784	1.63	2.727
.6254	.01864	.1440	83.4	17.40	.03666	2.016	1.62	2.804
.6250	.02326	.1345	83.8	16.25	.03732	2.233	1.62	2.815
.6250	.02792	.1267	84.1	15.31	.03745	2.442	1.62	2.825
.6266	.03254	.1208	84.3	14.59	.03761	2.631	1.62	2.841
.6256	.03717	.1156	84.7	13.96	.03774	2.806	1.62	2.849
.6246	.04184	.1107	84.8	13.38	.03752	2.986	1.62	2.848
.6245	.04645	.1079	84.8	13.04	.03852	3.106	1.62	2.845
.6259	.05106	.1046	84.8	12.64	.03842	3.261	1.62	2.852
.6259	.05564	.1024	84.9	12.38	.03934	3.363	1.62	2.856
.6248	.06024	.1000	85.1	12.09	.03981	3.479	1.62	2.858
.6240	.06488	.0969	85.3	11.70	.03900	3.648	1.62	2.860
.6248	.06670	.0959	85.4	11.59	.03887	3.705	1.62	2.868
.6253	.06852	.0938	85.5	11.34	.03726	3.835	1.62	2.872
.8883	.004695	.2697	85.3	32.60	.03008	1.117	1.65	4.070
.8882	.004695	.2691	85.1	32.52	.02986	1.121	1.65	4.060
.8879	.004695	.2695	85.2	32.56	.03002	1.118	1.65	4.064
.8867	.007009	.2394	85.3	28.94	.03152	1.334	1.64	4.063
.8879	.007009	.2395	85.2	28.94	.03146	1.335	1.64	4.064
.8867	.009322	.2191	85.3	26.48	.03214	1.523	1.64	4.063
.8830	.009322	.2137	79.3	25.82	.03005	1.575	1.65	3.768
.8839	.01398	.1910	79.9	23.08	.03210	1.866	1.64	3.800
.8887	.01864	.1758	80.3	21.24	.03301	2.125	1.64	3.840
.8874	.02326	.1632	80.5	19.72	.03304	2.373	1.64	3.841
.8869	.02792	.1538	80.5	18.58	.03324	2.592	1.64	3.840
.8860	.03254	.1463	80.7	17.68	.03343	2.790	1.64	3.847
.8859	.03717	.1423	80.9	17.20	.03516	2.908	1.63	3.853
.8835	.04184	.1358	79.9	16.41	.03459	3.111	1.63	3.800
.8822	.04645	.1298	80.0	15.69	.03362	3.324	1.63	3.798
.8854	.05106	.1267	80.3	15.31	.03414	3.459	1.64	3.825
.8843	.05564	.1247	80.4	15.07	.03555	3.538	1.63	3.826
.8855	.06024	.1221	80.6	14.76	.03602	3.657	1.62	3.841
.8841	.06488	.1178	80.7	14.23	.03492	3.855	1.63	3.840
.8847	.06670	.1172	80.8	14.16	.03533	3.886	1.63	3.847
.8829	.06852	.1156	81.1	13.96	.03494	3.960	1.63	3.850
1.249	.004695	.3336	80.8	40.32	.02879	1.142	1.65	5.432
1.248	.007009	.2965	81.3	35.82	.03021	1.362	1.65	5.458
1.252	.009322	.2730	81.6	33.00	.03121	1.546	1.64	5.491
1.249	.01398	.2385	82.1	28.82	.03130	1.890	1.64	5.519
1.248	.01864	.2156	81.9	26.06	.03089	2.197	1.64	5.500
1.246	.02326	.2025	82.0	24.47	.03206	2.409	1.64	5.497
1.252	.02792	.1925	82.2	23.26	.03270	2.613	1.64	5.536
1.249	.03254	.1800	82.3	21.75	.03133	2.882	1.64	5.531
1.247	.03717	.1717	82.6	20.75	.03115	3.090	1.64	5.543
1.249	.04184	.1665	82.9	20.12	.03187	3.240	1.64	5.565
1.249	.04645	.1611	83.0	19.46	.03204	3.405	1.64	5.576
1.250	.05106	.1568	83.4	18.95	.03248	3.546	1.64	5.602
1.248	.05564	.1531	83.5	18.50	.03305	3.670	1.63	5.600
1.250	.06024	.1494	82.8	18.06	.03310	3.815	1.63	5.569
1.248	.06488	.1448	82.9	17.50	.03262	3.988	1.64	5.562
1.247	.06670	.1436	83.0	17.35	.03273	4.037	1.64	5.567
1.246	.06852	.1420	83.2	17.16	.03250	4.106	1.64	5.579

Roughness-Concentration Effects on Flow Over Hydrodynamically Rough Surfaces

By H. J. KOLOSEUS *and* JACOB DAVIDIAN

LABORATORY STUDIES OF OPEN-CHANNEL FLOW

GEOLOGICAL SURVEY WATER-SUPPLY PAPER 1592-D

Prepared in cooperation with the Iowa Institute of Hydraulic Research

CONTENTS

ILLUSTRATIONS

TABLE

SYMBOLS

f = Resistance coefficient, $8gRS_0/U^2$ based on hydraulic radius; $8gy_0S_0/U^2$ based on two-dimensional flow.

g = Acceleration of gravity.

k_s = Equivalent Nikuradse sand-grain size.

k_1 = Height of roughness element.

k_2 = Length of roughness element in a horizontal direction transverse to the mean-flow direction.

k_3 = Length of roughness element in mean-flow direction.

log = Logarithm to base 10.

m = Coalescing factor.

r_0 = Radius of pipe.

u = Velocity at distance y from the boundary.

x = Spacing of rows of roughness elements measured in the direction of flow.

y_0 = Depth of flow measured perpendicular to the channel floor.

B_1 = Spacing of roughness elements in a row.

C = Constant.

C_D = Coefficient of drag for roughness element.

$\mathbf{R}$ = Reynolds number, $4RU/\nu$ based on the hydraulic radius; $4y_0U/\nu$ based on two-dimensional flow.

R = Hydraulic radius, ratio of area to the wetted perimeter.

S_0 = Energy gradient which is equal to sin θ in uniform flow.

U = Average velocity.

U_* = Shear velocity, $\sqrt{gRS_0}$.

θ = Angle of inclination of the channel.

λ = Roughness-concentration factor. It represents the ratio of the sum of projected areas of roughness elements normal to mean direction of fluid movement to the total floor area.

ν = Kinematic viscosity of the fluid.

LABORATORY STUDIES OF OPEN-CHANNEL FLOW

ROUGHNESS-CONCENTRATION EFFECTS ON FLOW OVER HYDRODYNAMICALLY ROUGH SURFACES

By H. J. Koloseus and Jacob Davidian

ABSTRACT

A general relation has been found between the roughness-element concentration and the resistance coefficient for flow over hydrodynamically rough surfaces. This relation pertains to many different forms of roughness in both open and closed conduits over wide ranges of concentration. The effect of changes in roughness concentration on the resistance coefficient is the same as that for equal numerical variations in roughness size.

INTRODUCTION

Through the work of Nikuradse (1933), the relation between the relative height ($k_1/4R$) of the maximum density of a sand-grain roughness and the resistance coefficient (f) became known. Numerous other studies have been directed toward correlating changes in the resistance coefficient with roughness concentration (λ). The results from these studies, however, were more of a specific than of a general nature, owing in part to the limited ranges of concentration tested. Concentration effects not readily apparent in earlier works were found in an investigation of a 64-fold variation in cube density by Koloseus and Davidian (1961). From these concentration effects a relation between the resistance coefficient and the roughness concentration can be formulated. It is applicable to many forms of roughness in both open and closed conduits for wide ranges of λ.

The work reported herein reflects the interest of the U.S. Geological Survey in a better understanding of flow over rough boundaries. This report discusses the mechanics of stable uniform flow. A companion report by Koloseus and Davidian (1966) discusses in detail the correlation between unstable flow and both roll waves and increased channel resistance.

ACKNOWLEDGMENT

This investigation was conducted under a cooperative agreement between the U.S. Geological Survey and the Iowa Institute of Hydraulic Research. It represents a continuation of the rough-boundary studies begun by Dr. Hunter Rouse, director of the institute. The use of the facilities of the institute for the gathering of data and the guidance and assistance of the staff, especially Dr. Rouse, are greatly appreciated.

NOTATION

The letter symbols used in this report are defined in the illustrations, in the text, and on page IV. Some of the symbols used are the same as those defined in the report on free-surface instability correlations (Koloseus and Davidian, 1966).

BOUNDARY RESISTANCE

The tendency to retard the movement of fluid past a boundary stems from the form drag on each roughness element and from the surface drag on the smooth part of the boundary between protuberances. The shear on the smooth sections of the boundary is increased beyond that which would exist if the boundary irregularities were not present. In this report the increase in smooth-boundary shear is considered to be a part of the total drag associated with the roughness elements; the increase is also assumed to be independent of the Reynolds number ($\mathbf{R}$).

Whether the resistance coefficient is a function of either or both the Reynolds number and the geometric characteristics of the roughness elements ($k_1/4R$, λ, shape, and pattern) is a relative matter. When the roughness concentration is so small that the shear on the smooth boundary is approximately equal to the total boundary resistance, then the resistance coefficient will be a function of only $\mathbf{R}$ for all practical purposes. If the boundary resistance is due chiefly to the drag on the roughness elements and if the zones of separation about the elements do not change with the Reynolds number, then the resistance coefficient (f) will be a function of only the geometry of the conduit and of the boundary protuberances and will be independent of the Reynolds number.

Between these two extremes, a transition region exists wherein f is a function of both $\mathbf{R}$ and the characteristics of the surface irregularities. To simplify the study of changes in f with roughness concentration (λ), only those results will be considered for which the resistance coefficient is a function of the relative geometry. The boundaries under these conditions are classified as hydrodynamically rough surfaces.

The boundary shear in flow over hydrodynamically rough surfaces is practically equal to the total drag on the roughness elements. This shear depends not only on the resistance of an individual particle but also on the number of particles. The drag on a protuberance is equal to the decrease in momentum flux of the fluid moving about the element. In "uniform flow" this momentum defect is distributed through the flow by turbulence and is decreased to the vanishing point through the reduction in piezometric head as the fluid moves downstream. If the roughness spacing is such that the momentum defect is practically eliminated before another element is reached, then one form of relation between the resistance coefficient and roughness concentration might be anticipated. If the roughness concentration, however, is such that each element is appreciably in the momentum-defect zone of the preceding one, then f might reasonably be related to λ in another manner. Under these latter conditions the magnitude of the velocity past and the flow pattern about each element varies with the concentration; in other words, each element affects the flow about each succeeding roughness element. Morris (1954) viewed the flow phenomenon in the vicinity of the protuberances in a somewhat similar manner, but he did not take into account the combined effect of both roughness size and spacing as is done in this work.

DISCUSSION OF PARAMETERS

The roughness-concentration parameter (λ) is defined as the ratio of the sum of the projected areas of the roughness elements normal to the mean direction of fluid movement, to the total floor area. The use of this area of the roughness is appropriate because both Wieghardt (1942) and Roberson (1961) have shown that the drag on bodies in nonuniform velocity fields is equal to the integral of the product of the local velocity head ($V^2/2g$), a constant drag coefficient (C_D), the specific weight of the fluid, and a differential area normal to the mean direction of fluid movement. Roberson has also shown that the drag coefficient for a sharp-edged body, a cube, in nonuniform velocity fields is approximately equal to the drag coefficient for a square plate in a uniform velocity field, that is, $C_D=1.2$. If the drag coefficient for a sharp-edged body is independent of the velocity distribution and if the drag on an element is equal to the indicated integral, then a correspondence between the drag coefficient of the elements forming a surface and the effective roughness of that surface might be expected. In view of the known dependence of C_D on body shape, the resistance coefficients for flow past boundaries that are roughened by different forms of roughness need not be equal, even though the roughness concentrations are the same. The symbols associated with the various geometric dimensions of the protuberances are set forth in figure 1.

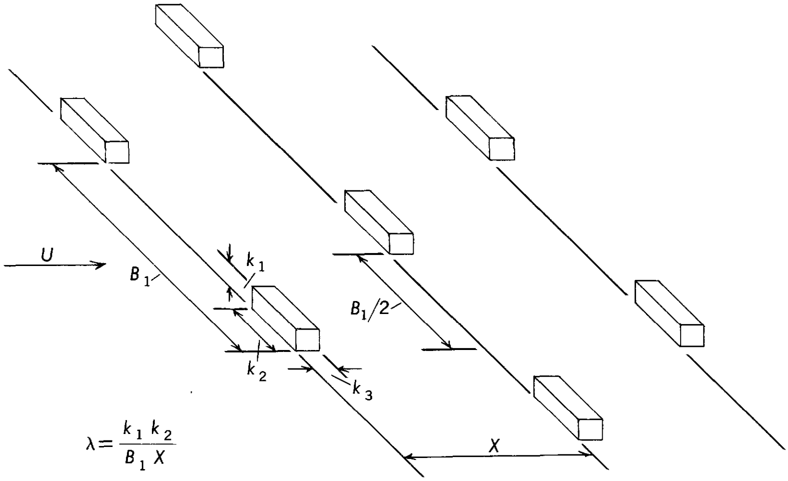


FIGURE 1.—Definition sketch of roughness parameters.

Nikuradse's (1933) tests on sand-grain roughened pipes showed that in the hydrodynamically rough range equation 1 pertained:

$$\frac{u}{U_*}=5.75 \log \left(\frac{y}{k_s} 30\right), \tag{1}$$

where

g=acceleration of gravity,
k_s=Nikuradse sand-grain size,
u=velocity at distance y from the boundary,
y=distance from the boundary,
U_*=shear velocity, $\sqrt{gRS_0}$,
R=hydraulic radius, ratio of area to the wetted perimeter,
S_0=energy gradient.

Nikuradse also found that the law of resistance based on the empirical data,

pipe:

$$\frac{1}{\sqrt{f}}=2 \log \left(\frac{4R}{k_s} 3.706\right), \tag{2}$$

where

f=resistance coefficient, $8gRS_0/U^2$,

was the same as that obtained through integration of equation 1 except for minor variations in coefficients.

If the law of resistance for flow over other rough-pipe surfaces is pipe:

$$\frac{1}{\sqrt{f}}=C_1 \log \left(\frac{4R}{k_1} C_2\right), \tag{3}$$

where

k_1=height of roughness elements,
C_1=constant,
C_2=constant whose value depends upon the geometric characteristics of the roughness,

then the equivalent Nikuradse sand-grain size can be obtained through the simultaneous solution of equations 2 and 3; the expression for it is

pipe:

$$k_s=k_1 \frac{3.706}{C_2^{(0.5C_1)}\left(\frac{4R}{k_1}\right)^{(0.5C_1-1)}}. \tag{4}$$

If $C_1=2$, as it frequently does, equation 3 becomes

pipe:

$$\frac{1}{\sqrt{f}}=2 \log \left(\frac{4R}{k_1} C_2\right), \tag{5}$$

and equation 4 simplifies to

pipe:

$$k_s=k_1 \frac{3.706}{C_2}. \tag{6}$$

The advantage of expressing all surface roughnesses in terms of the equivalent Nikuradse sand-grain size is that the surface roughnesses are then put in terms of a single type of roughness that is easily visualized and appreciated. It has the further advantage that the equivalent Nikuradse sand-grain size is independent of concentration considerations, because Nikuradse's roughness concentrations were constant and were fixed at the maximum value. Therefore the adoption of the equivalent Nikuradse sand-grain size as a roughness standard has merit. It is used as a basis of comparison in this report.

An empirical expression for flow in a wide channel in terms of the Nikuradse sand-grain size that is comparable to empirical equation 2 for pipes is not available. If it is assumed, however, that equation 1 applies to any rough surface, including a plane one, then the law of resistance for flow in a wide rectangular channel in terms of the Nikuradse sand-grain size is

wide channel:

$$\frac{1}{\sqrt{f}}=2 \log \left(\frac{4y_0}{k_s} 3.06\right), \tag{7}$$

where

f=resistance coefficient, $8gy_0S_0/U^2$,
y_0=depth.

Equation 7 is the wide-channel counterpart of equation 2.

As for the circular pipe, if the law of resistance for flow in a rough wide channel is

wide channel:

$$\frac{1}{\sqrt{f}}=C_3 \log\left(\frac{4y_0}{k_1} C_4\right), \tag{8}$$

where

C_3=constant,

C_4=constant whose value depends upon the geometric characteristics of the roughness,

then the equivalent Nikuradse sand-grain size would be given by

wide channel:

$$k_s=k_1\frac{3.06}{C_4^{(0.5C_3)}\left(\frac{4y_0}{k_1}\right)^{(0.5C_3-1)}}. \tag{9}$$

Equation 8 becomes

wide channel:

$$\frac{1}{\sqrt{f}}=2 \log\left(\frac{4y_0}{k_1} C_4\right), \tag{10}$$

and equation 9 becomes

$$k_s=k_1 \frac{3.06}{C_4}, \tag{11}$$

when $C_3=2$. If the resistance coefficient for flow in a sand-roughened pipe as proposed by Nikuradse is equal to that for a sand-roughened wide channel as proposed by Nikuradse, then according to equations 2 and 7 the following relation must exist between the size of the sand in the pipe and the size of the sand in the wide channel:

$$(k_s)_{\text{channel}}=0.825(k_s)_{\text{pipe}}. \tag{12}$$

Equation 12 can be used to convert from a pipe to a wide-channel sand-grain roughness.

The study of surface-roughness and roughness-concentration effects for different forms of irregularities is best accomplished when the results pertinent to these different shapes are expressed in terms of a common roughness; the Nikuradse sand-grain roughness serves this purpose very well. Through equations 4, 6, 9, 11, and 12 the equivalent Nikuradse pipe or wide-channel sand-grain size can be obtained for any surface for which equations 3 and 8 pertain. The roughness-height ratio (k_s/k_1), is even more meaningful than the equivalent Nikuradse sand-grain size, because geometrically similar irregularities of different sizes will have the same value; this ratio was also used by Johnson (1944) in his study of various bar concentrations.

DISCUSSION OF ROUGHNESS-CONCENTRATION DATA

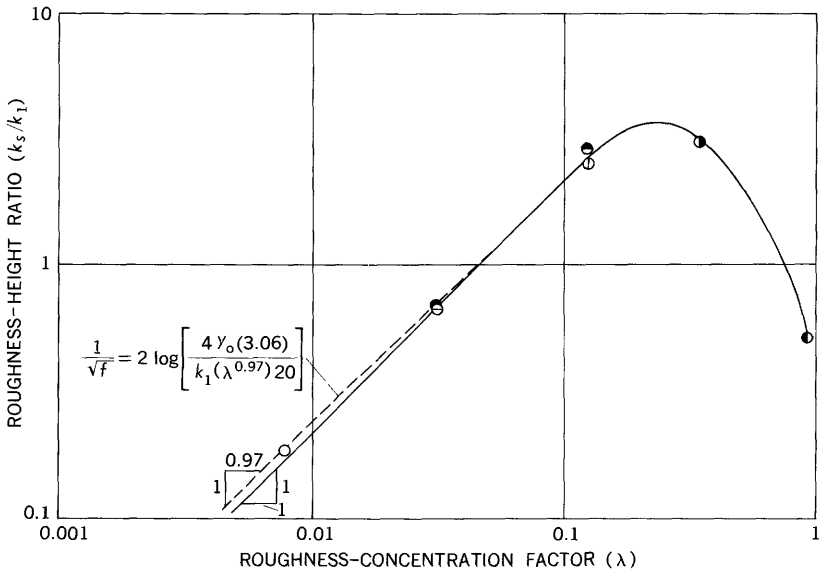
Roughness-concentration data are available from many sources. They pertain to two- and three-dimensional elements and to open- and closed-conduit flow. The concentration effects are brought out in this report through a comparison of the variation of the roughness-height ratio (k_s/k_1) with the roughness concentration (λ). It can readily be concluded from equations 2 and 7 that the effective surface roughness increases as k_s/k_1 increases for a constant value of k_1. For similarly shaped roughness elements, results in terms of k_s/k_1 for flow through roughened pipes can be grouped with those for open channels if k_s for the pipe flow is determined by equation 4 or 6 and that for the wide-channel flow is determined by equation 9 or 11. Not all the roughness-concentration results considered herein are amenable to mutual comparison because of the heterogeneous geometric conditions under which they were gathered; some channels were wide, others were narrow; some roughness elements were placed only on the floor, others were placed on both the walls and the floor. Nevertheless, the trends of the data are strikingly similar despite this lack of uniformity.

Schlichting (1936) gathered data, shown in figure 2, on spheres in a closed rectangular conduit for more than a 100-fold variation in concentration. The near superposition of the 0.21-cm and 0.41-cm sphere data proves the worth of the parameter k_s/k_1 and indicates that the anticipated dynamic similarity existed. The results for concentrations of 0.125 and less form a straight line with a slope of 0.97. The linearity of the trend of these discloses that f varies as an easily formulated and constant function of λ, which is

$$\frac{1}{\sqrt{f}}=2\log\left(\frac{4y_0}{k_1}\frac{(3.06)}{\lambda^{0.97}(20)}\right). \qquad (13)$$

The interpolation curve for concentrations greater than 0.125 represents a more complex relation. This change in function between λ and f beginning at $\lambda=0.125$ is interpreted as evidence of the expected interference effects between the roughness elements at the higher densities. Schlichting expressed the roughness of these sphere-covered boundaries in terms of the equivalent Nikuradse sand-grain size for pipes. Because the spheres covered only one wall of a rectangular conduit, it was assumed that the equivalent Nikuradse sand-grain size for a wide channel would be more appropriate. Accordingly, Schlichting's results were converted from pipe to wide channel by means of equation 12.

Data on various concentrations of 3/16-inch cubes and on a louver roughness that was used to simulate a high cube concentration were collected by Koloseus and Davidian (1961 and 1966). The work was done in the laboratories of the Iowa Institute of Hydraulic Research, Iowa City, Iowa. The geometry of these roughnesses is shown in figures 3–5. The open-channel results, which are tabulated in detail in Koloseus and Davidian (1966), are shown in figure 6. The trend is similar to that shown in figure 2 for tests by Schlichting of spheres in closed channels. For densities of 0.125 and less, log k_s/k_1 varies linearly with log λ, the slope of the line being 0.9. The two equations in figure 5 were used to determine values of k_s/k_1; these are nothing more than the appropriate forms of equation 11.



Designation used in cited reference	Symbol	λ	k_1 (cm)	k_1 (cm) Pipe [1]	k_1 (cm) Channel	k_s/k_1 channel	m
XII	⊙	0.00785	0.41	0.093	0.0767	0.187	0.220
III	●	.0314	.41	.344	.284	.693	.220
I	⦶	.126	.41	1.26	1.039	2.53	.220
II	◑	.349	.41	1.56	1.288	3.14	.220
V	◐	.907	.41	.257	.212	.517	.220
VI	⊖	.0314	.21	.172	.142	.676	.220
IV	◓	.126	.21	.759	.626	2.98	.220

[1] Data as presented in cited reference. Closed rectangular channel; one boundary roughened.

FIGURE 2.—Relation between roughness-height ratio and concentration for spherical roughness elements. Data from Schlichting (1936).

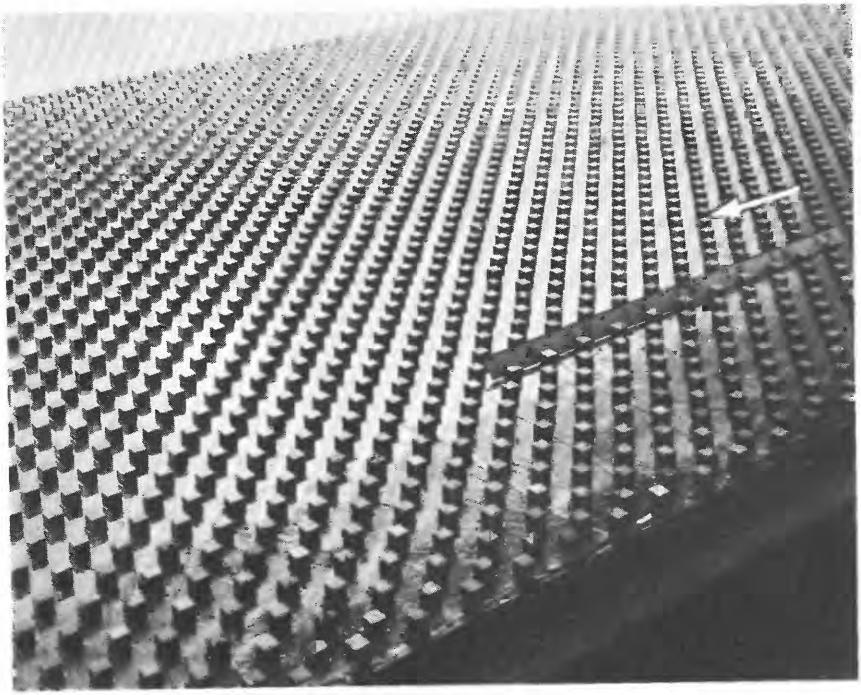


FIGURE 3.—Cube-roughened boundary; $k_1=k_2=k_3=3/16$ inch; $\lambda=1/8$; smallest scale division=0.01 foot. Arrow indicates direction of flow. From Koloseus and Davidian (1966).

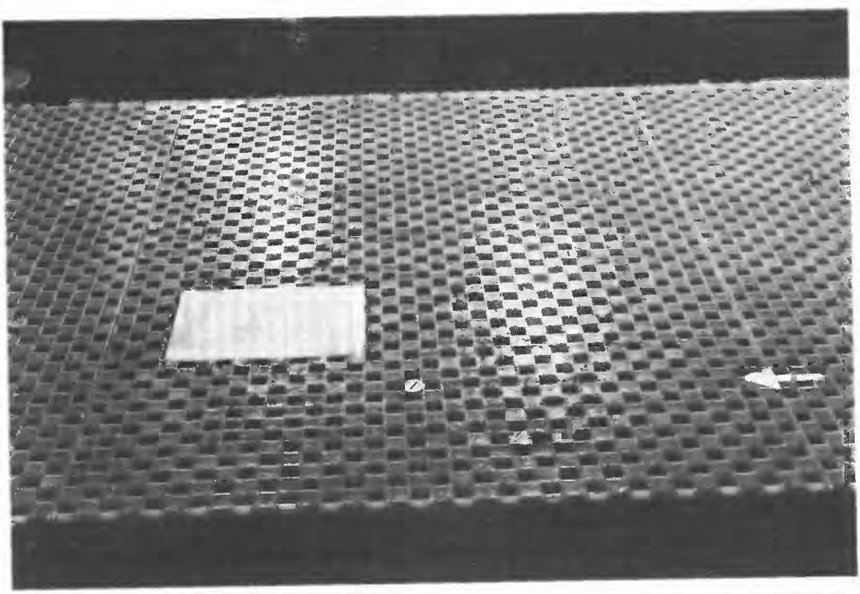
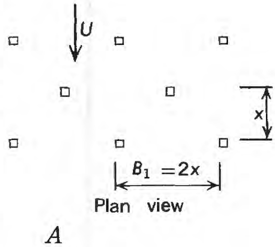


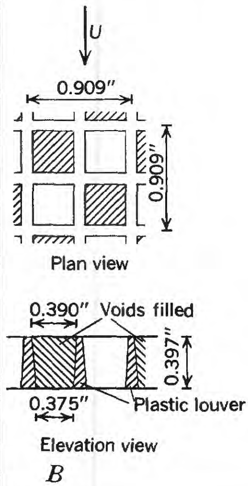
FIGURE 4.—Louver roughness. Rectangular plate contains piezometer for measurement of water depth. From Koloseus and Davidian (1966).



$k_1 = 0.01563$ ft

$$\frac{1}{\sqrt{f}} = 2 \log \left[\frac{4Y_o}{k_1} \frac{0.14}{\lambda^{0.9}} \right]$$ Koloseus and Davidian (1961)

$$\frac{k_s}{k_1} = \frac{3.06}{0.14} \lambda^{0.9}$$



$k_1 = 0.03308$ ft

$$\lambda = \frac{2(0.397\left[0.909 - \frac{(0.390+0.375)}{2}\right]}{(0.909)^2} = 0.506$$

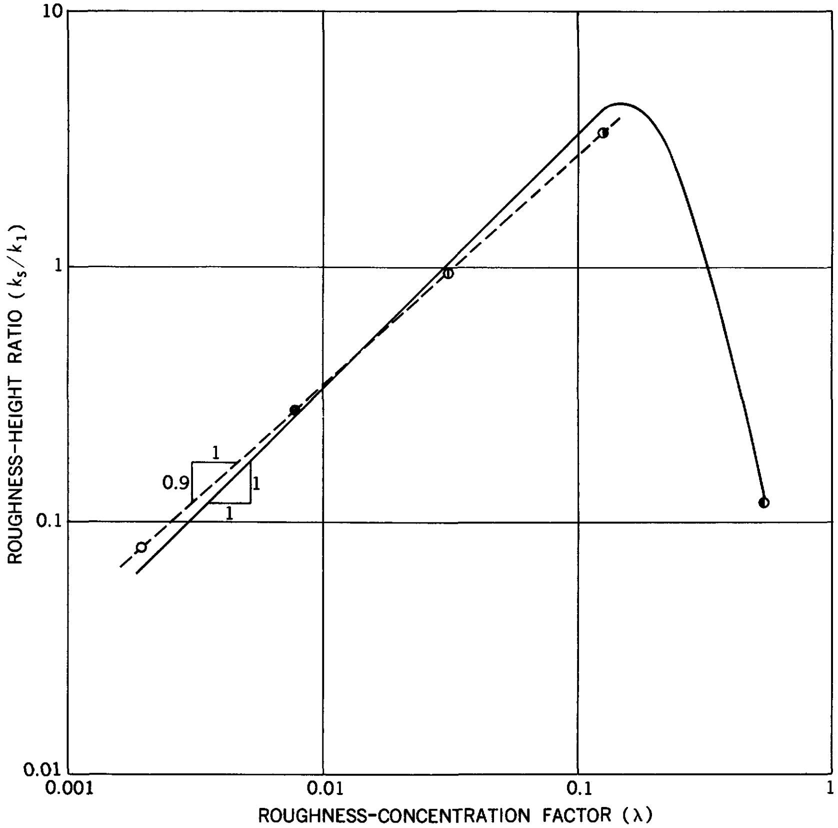
$$\frac{1}{\sqrt{f}} = 2 \log \left[\frac{4Y_o}{k_1} 26 \right]$$ Koloseus and Davidian (1966)

$$\frac{k_s}{k_1} = \frac{3.06}{26} = 0.118$$

Designation used in cited reference	Cube (inches)	λ	x (inches)
I	3/16	1/512	3
II	3/16	1/128	1½
IV	3/16	1/32	¾
VI	3/16	1/8	⅜

FIGURE 5.—Schematic drawing of *A*, cube-roughened boundaries, and *B*, louver-roughened boundaries.

A large range in concentration for long, square roughness elements ($k_3/k_1=1$) is obtained through the grouping of the open-channel data of Powell (1944 and 1946) and Rand (1952) and the circular-pipe data of Chu and Streeter (1949); this is shown in figure 7. For concentrations of 0.025 and less, a straight line has been shown at a slope of 1.0 for reference purposes. The correspondence depicted in figure 7 between Rand's results and those of Chu and Streeter for the very limited number of data discloses that the concentration effects are the same for flow in open and closed conduits. In evaluating k_s/k_1,



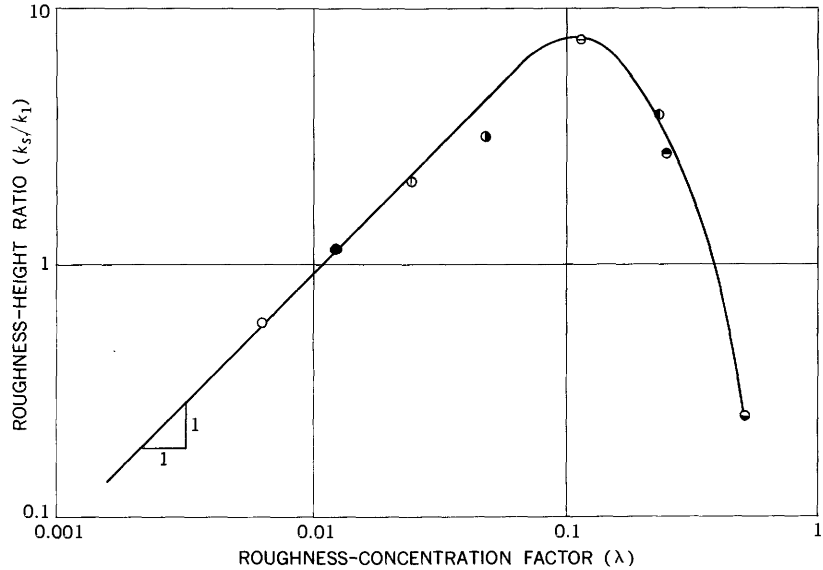
Designation used in cited reference	Symbol	λ	k_1 (feet)	Channel		m	Roughness element
				k_s[1] (feet)	k_s/k_1		
I	⊙	0.00195	0.01563	0.00124	0.0796	0.338	3/16-inch cubes.
II	●	.00781	.01563	.00434	.277	.338	Do.
IV	⦶	.03125	.01563	.0151	.965	.388	Do.
VI	◑	.125	.01563	.0526	3.36	.338	Do.
VII	◐	.368	.03308	.00184	.118	.338	Plastic louver.

[1] See fig. 5 for evaluation. Rectangular open channel; only floor roughened.

FIGURE 6.—Relation between roughness-height ratio and concentration for cubical and louver roughness elements. Data from Koloseus and Davidian (1966).

equation 9 was used for Rand's data and equation 6 for the results of Chu and Streeter, and values of k_s as given by Powell (1944) were converted from a pipe to wide-channel sizes by means of equation 12.

The findings of Johnson (1944) for long, rectangular bars ($k_3/k_1=4$) for the flow of water in an open channel are shown in figure 8, with a reference straight line at a slope of 1.0 for concentrations less than 0.05. Owing to the small interval between concentrations, Johnson's



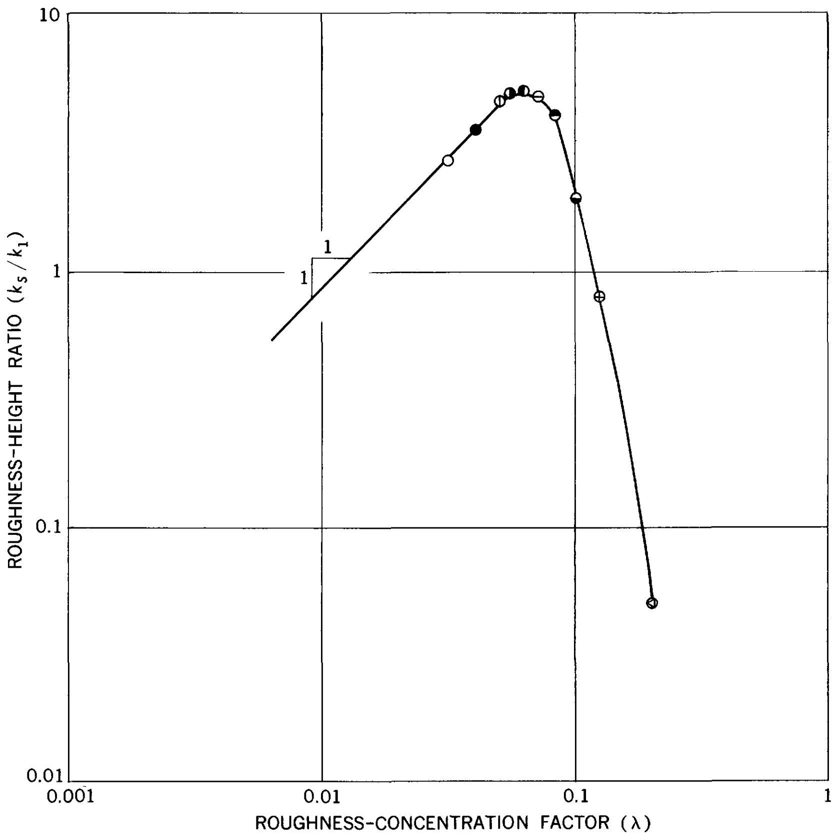
Designation used in cited references	Symbol	λ	k_1 (feet)	Pipe [1]		Channel		m
				k_s (feet)	k_s/k_1	k_s (feet)	k_s/k_1	
X, XI	⊙	0.00625	0.0104 .0208	----------	[2] 0.72	----------	0.594	[3] 0.92
IX, I	●	.0125	.0104 .0208	----------	[2] 1.40	----------	1.16	[3] .92
VIII, II	⦶	.0250	.0104 .0208	----------	[2] 2.60	----------	2.14	[3] .92
VII, III	◑	.0500	.0104 .0208	----------	[2] 3.89	----------	3.21	[3] .92
	◐	.25	.022	----------	----------	0.0603	2.74	[4] .92
III	⊖	.116	.00425	0.0338	7.94	----------	----------	[5] .92
II	◓	.231	.00425	.0168	3.94	----------	----------	[5] .92
I_m	◒	.505	.00425	.00106	.251	----------	----------	[5] .92

[1] $k_3/k_1=1$.
[2] Data as presented by Powell.
[3] Powell; bar, open channel; walls and floor roughened.
[4] Rand; bar, open channel; only floor roughened.
[5] Chu and Streeter; square helical thread, pipe.

FIGURE 7.—Relation between roughness-height ratio and concentration for long, square roughness elements. Data from Powell (1944), Rand (1952), and Chu and Streeter (1949).

work also sets forth the nature of the functional relation between log k_s/k_1 and log λ in the region of maximum boundary roughness; this relation was the basis for the form of the nonlinear parts of the interpolation curves in figures 2, 6, and 7. The results of Tripp (1936) for a similar form of roughness in a closed rectangular channel with air as the fluid medium extend Johnson's work to a high concentration. Johnson's and Tripp's values of k_s were converted from pipe to wide channel by means of equation 12.

In figures 9–11, the data of Schlichting (1936) for cones, spherical segments, and short angles in closed rectangular conduits and, in



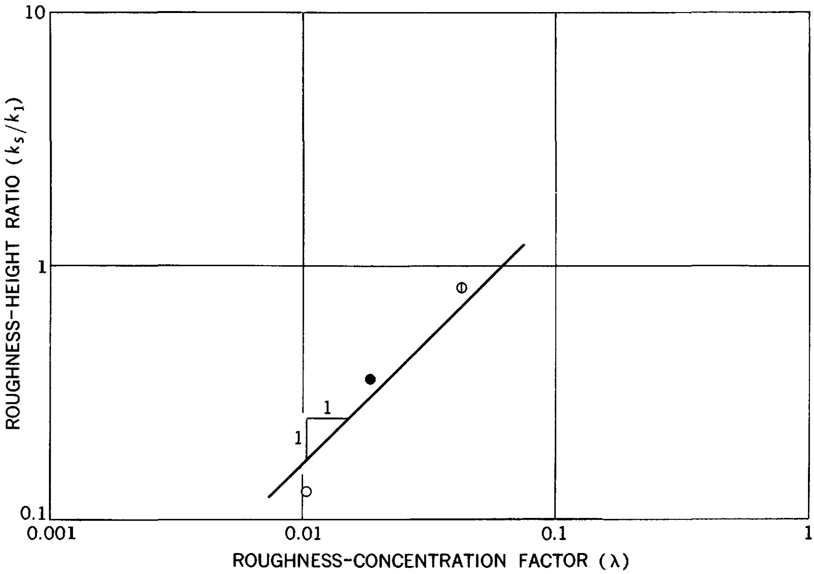
Designation used in cited reference	Symbol	λ	k_1 (feet)	k_s (feet)		k_s/k_1 channel	m
				Pipe [1]	Channel		
C	⊙	0.0315	0.0208	0.0698	0.0576	2.77	[2] 0.880
G	●	.0417	.0208	.0940	.0766	3.68	[2] .880
E	⦶	.0500	.0208	.119	.0978	4.70	[2] .880
I	◑	.0555	.0208	.127	.105	5.04	[2] .880
B	◐	.0625	.0208	.130	.107	5.14	[2] .880
H	⊖	.0714	.0208	.126	.104	4.97	[2] .880
F	◓	.0833	.0208	.105	.0866	4.17	[2] .880
D	◒	.100	.0208	.0500	.0413	1.98	[2] .880
A	⊕	.125	.0208	.0202	.0167	.801	[2] .880
	◬	.200	.0104	.000652	.000538	.0516	[3] .880

[1] Data as presented in cited references.
[2] Johnson; rectangular open channel, $k_3/k_1=4$; only floor roughened; fluid, water.
[3] Tripp; rectangular closed channel, $k_3/k_1=4$; fluid, air.

FIGURE 8.—Relation between roughness-height ratio and concentration for rectangular-bar roughness elements. Data from Johnson (1944), and Tripp (1936).

figure 12, the data of Sayre and Albertson (1961) for short angles in an open channel are shown with straight reference lines at slopes of 1.0 for concentrations less than about 0.05. Despite the paucity of data in these figures, the relation between log k_s/k_1 and log λ in this range of density could well be linear, with slopes from 1.0 to near 1.3. Equation 12 was used to convert Schlichting's values of k_s from pipe to wide channel, and equation 9 was used to evaluate k_s for the data of Sayre and Albertson.

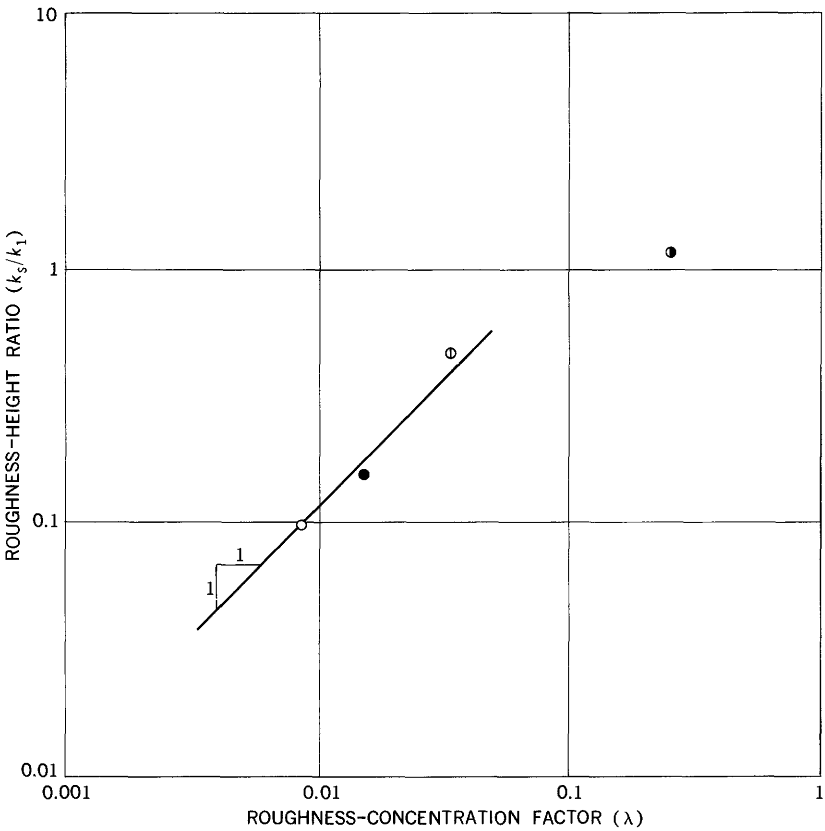
Figures 2 and 6–12 show that these data are well represented by a linear relation between log k_s/k_1 and log λ over a large range in roughness concentration. The slopes of these lines vary in an unsystematic fashion about a value of 1; this lack of order is due in all likelihood to the heterogeneous conditions under which the data were collected. Until more definitive data become available regarding these slopes, the slopes can be taken as 1 without compromising the results greatly.



Designation used in cited reference	Symbol	λ	k_1 (cm)	k_s (cm)		k_s/k_1 channel	m
				Pipe [1]	Channel		
XXIII	⊙	0.0106	0.375	0.059	0.0487	0.130	0.167
XXIV	●	.0189	.375	.164	.135	.360	.167
XXV	⊖	.0425	.375	.374	.308	.821	.167

[1] Data as presented in cited reference. Closed rectangular channel, one boundary roughened.

FIGURE 9.—Relation between roughness-height ratio and concentration for conical roughness elements. Data from Schlichting (1936).



Designation used in cited reference	Symbol	λ	k_1 (cm)	k_s (cm)		k_s/k_1 channel	m
				Pipe [1]	Channel		
XIII	○	0.0087	0.26	0.031	0.0256	0.0985	0.116
XIV	●	.0155	.26	.049	.0404	.155	.116
XV	⦶	.0348	.26	.149	.123	.473	.116
XIX	◑	.251	.26	.365	.301	1.16	.116

[1] Data as presented in cited reference. Closed rectangular channel, one boundary roughened.

FIGURE 10.—Relation between roughness-height ratio and concentration for spherical-segment roughness elements. Data from Schlichting (1936).

As a consequence, equations 5 and 10, modified to take roughness-concentration effects into account, are

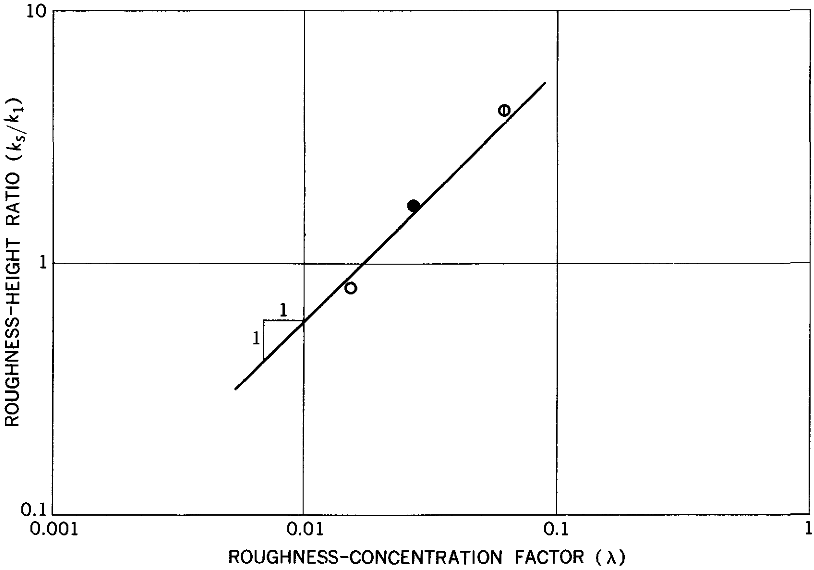
pipe:

$$\frac{1}{\sqrt{f}}=2\log\left(\frac{4R}{k_1\lambda}C_5\right), \qquad (14)$$

and

wide channel:

$$\frac{1}{\sqrt{f}}=2\log\left(\frac{4y_0}{k_1\lambda}C_6\right). \qquad (15)$$

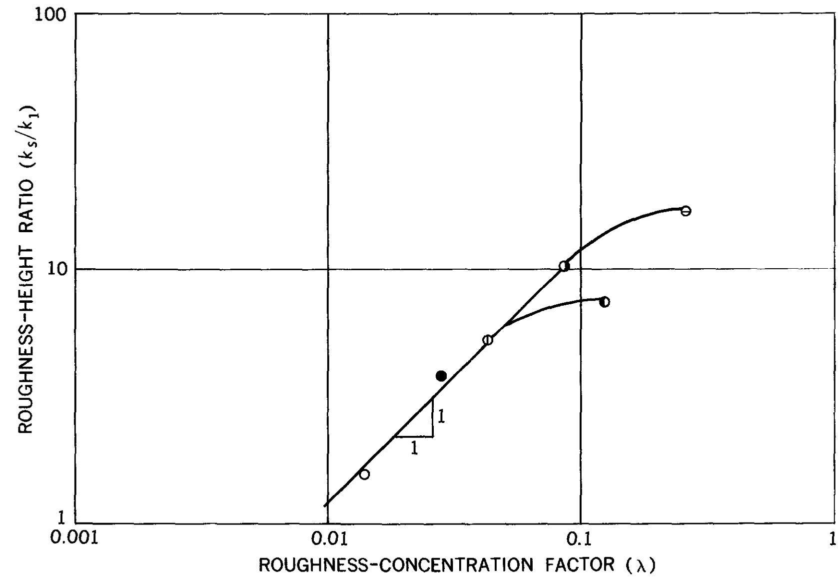


Designation used in cited reference	Symbol	λ	k_1 (cm)	k_s (cm)		k_s/k_1 channel	m
				Pipe [1]	Channel		
XVI	⊙	0.0151	0.30	0.291	0.240	0.800	0.580
XVIII	●	.0269	.30	.618	.509	.170	.580
XVII	⦶	.0605	.30	1.47	1.21	4.03	.580

[1] Data as presented in cited reference. Closed rectangular channel, $k_2/k_1=2.7$, one boundary roughened.

FIGURE 11.—Relation between roughness-height ratio and concentration for short, angle roughness elements. Data from Schlichting (1936).

The linearity between log k_s/k_1 and log λ at a nearly common slope of 1 for all these different forms of roughness indicates that the functional relation between f and λ is, for all practical purposes, independent of roughness shape. A consideration of figures 2 and 6–12 indicates that the constants C_5 and C_6, in equations 14 and 15, vary with the form of the roughness. It is rational that C_5 and C_6 would be functions of pattern as well; however, the extent to which they depend upon arrangement is unknown. Of the investigators cited, only Sayre and Albertson varied the roughness pattern. Their results indicate that C_6 is independent of the flat-plate pattern for $\lambda \leq 0.043$. Further work may disclose that equations 14 and 15 are only applicable to a 10-fold variation or less in Reynolds number; this range has been exceeded only in the works of Powell (1944, 1946), Rand (1952), and Chu and Streeter (1949).



Run numbers in cited reference	Symbol	λ	k_1 (feet)	Channel [1] k_s (feet)	k_s/k_1	m	x/k_1	B_1/k_1
24–27	○	0.0141	0.125	0.196	1.57	1.24	18	15.8
12–15	●	.0281	.125	.482	3.86	1.24	18	7.9
20–23	⦶	.0422	.125	.674	5.39	1.24	6	15.8
8–11	◑	.0844	.125	1.30	10.4	1.24	6	7.9
16–19	◐	.127	.125	.930	7.44	1.24	2	15.8
1–7	⊖	.253	.125	2.11	16.9	1.24	2	7.9

[1] Open rectangular channel, $k_2/k_1=4$, only floor roughened.

FIGURE 12.—Relation between roughness-height ratio and concentration for short, angle roughness elements. Data from Sayre and Albertson (1961).

Equations 14 and 15 indicate that doubling the size of the protuberance has the same effect on f as doubling the concentration. Owing to the lack of data for very small concentrations, the lower limiting concentration for which these equations are still applicable cannot be established. In all likelihood, this limit is a function of the roughness shape, the roughness pattern, and the Reynolds number. In subsequent paragraphs, consideration is given to the upper limiting value of λ for which equations 14 and 15 are applicable.

Because all the data in figures 2 and 6–12 are considered to have a common slope (taken to be 1), these data can be made to coalesce about a single line if log k_s/mk_1 is used as the ordinate, as shown in figure 13, instead of log k_s/k_1. The magnitude of m for each form of roughness is taken to be such that k_s/mk_1 is equal to 1 for a concentration of 0.01. The solid lines in the various figures have been used in determining m. Figure 13 adds nothing to the relations between log k_s/k_1 and log λ which have already been presented; however, it does permit a more ready comparison of the concentrations at which interference effects set in for the different forms of boundary roughness. The surface irregularities are considered to interfere markedly with the flow about succeeding elements when log k_s/k_1 no longer varies directly with log λ. That concentration at which this change takes place marks the upper limiting value of concentration for which equations 14 and 15 pertain. Figure 13 discloses that the interference effects associated with increasing concentrations occur first for the rectangular bar followed by those for the square bar, the cubes, and finally the spheres. It can also be noted from figure 13 that the concentrations at which the boundary resistance are maximum for each of these four forms of roughness are not the same. This difference indicates that there is no one concentration applicable to all forms of roughness at which the boundary resistance is greatest.

The two sets of results for long bars depicted in figure 13 indicate that interference effects are first noted for those having the larger value k_3/k_1. For example, the effects are first noted for the data of Johnson (1944) and Tripp (1936), in which $k_3/k_1=4$, and then for the data of Powell (1944), Rand (1952), and Chu and Streeter (1949), in which $k_3/k_1=1$. When consideration is given to the downstream displacement of the zone of separation as k_3/k_1 is increased, this result is rational.

Within those ranges of concentration for which equations 14 and 15 are applicable, the parameter m is an indicator of relative surface rugosity for like values of λ; m varies directly with surface rugosity. Therefore, the tabulated values of m in figure 13 indicate for the forms of roughness reported upon that the short angle of Sayre and Albertson (1961) is the roughest, whereas Schlichting's (1936) spherical segments offer the least resistance to flow.

That there exists the plausible correspondence between surface roughness and the coefficient of drag (C_D), suggested earlier in the text, is brought out in table 1 through a comparison of the values of m and C_D. Owing to a lack of values of drag coefficients for flow in uniform velocity fields about bodies identical with those in figures 6–8, recourse is made to coefficients for which the geometric variations are the same even though the shapes of the bodies are different. Values of C_D are from Rouse and Howe (1955). For similar geometric alteration as shown in table 1, the nature of the change in m, increase or decrease, is consistent with the change in C_D. However, the correlation between changes in m and changes in C_D is far from perfect because the percentage change in m does not vary directly with that in C_D. Nonetheless, since the trends are the same, the values of C_D can be used as qualitative indicators of surface rugosity.

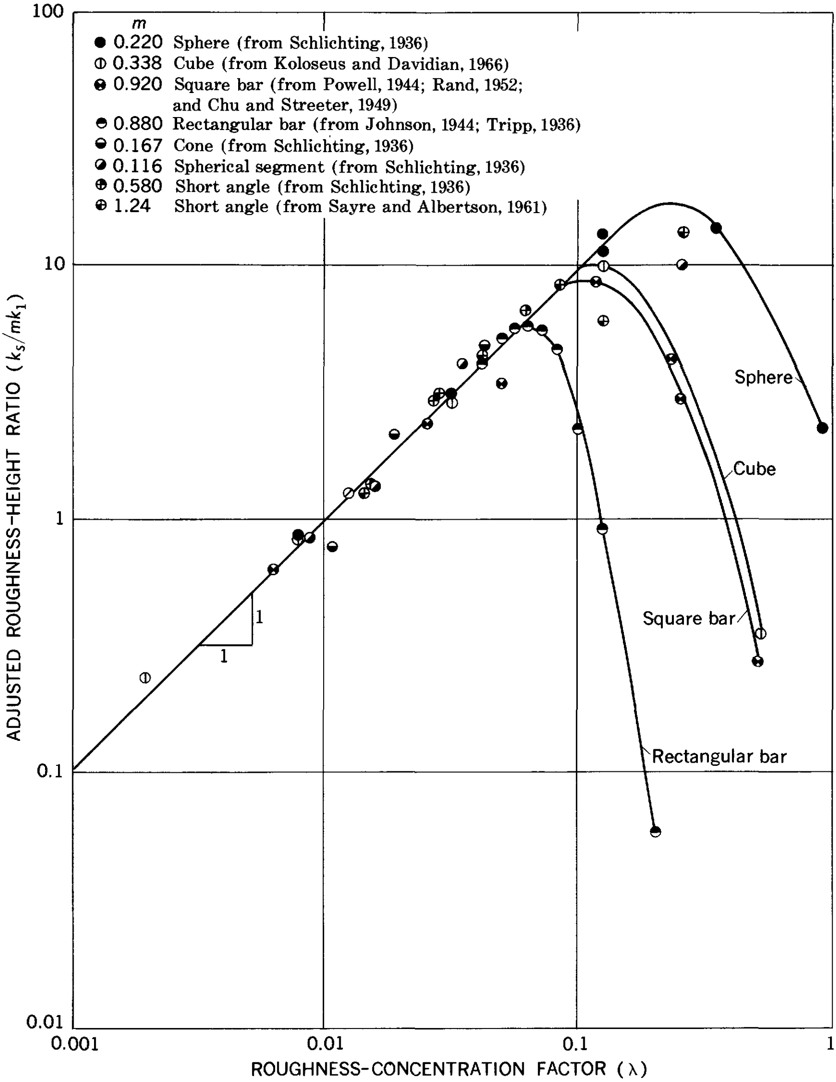


FIGURE 13.—Interference effects of roughness elements as concentration increases.

TABLE 1.—*Correspondence between changes in* m *and* C_D *for similar geometric variations*

	m	C_D	
Variation of k_3/k_1:			
Long bar			Cylinder—axis parallel to flow.
Johnson—$k_3/2k_1=2$	0.880	0.85	Length/diameter=2.
Powell, Chu and Streeter, Rand—$k_3/2k_1=0.5$.	.920	1.02	Length/diameter=0.5.
Variation of k_2/k_1:			
Square bar—$k_3/k_1=1$			Rectangular plate—$k_3/k_1 \doteq 0$.
Koloseus and Davidian—$k_2/2k_1=0.5$.	.338	1.17	$k_2/2k_1=0.5$.
Powell, Chu and Streeter, Rand—$k_2/2k_1=16$.	.920	1.42	$k_2/2k_1=16$.
Rectangular plate—$k_3/k_1 \doteq 0$			Rectangular plate—$k_3/k_1 \doteq 0$.
Schlichting—$k_2/2k_1=1.33$	.580	1.16	$k_2/2k_1=1.33$.
Sayre and Albertson—$k_2/2k_1=2$.	1.23	1.17	$k_2/2k_1=2$.
Axisymmetric bodies:			
Spheres			
Schlichting	.220	.200	Spheres.
Cubes			Rectangular plate—$k_3/k_1 \doteq 0$.
Koloseus and Davidian	.338	1.17	$k_2/2k_1=0.5$.

CONCLUSIONS

From a study of investigations of roughness concentrations for different forms of irregularities in both open and closed conduits, it is concluded that:

1. The ratio of the sum of the upstream projected areas to the total floor area is, within some range of density, a satisfactory measure of roughness concentration.
2. A simple relation between the resistance coefficient and roughness concentration, which is independent of the roughness shape (and, possibly, the pattern as well), pertains over some range of concentration. The same relation is applicable to both open- and closed-channel flow. The upper limit of this range of concentration varies with the roughness shape and, presumably, the pattern as well. The upper limit decreases as the length of the body in the direction of flow increases.
3. Above some concentration, the relation between the resistance coefficient and the concentration becomes more involved than that for lesser concentrations.
4. The effective roughness of a surface increases with like changes in the drag coefficient of the elements forming the surface.

5. There is no one concentration applicable to all forms of roughness at which the boundary resistance is greatest.
6. There is no one concentration applicable to all forms of roughness at which interference effects first become noticeable.

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