Simplified Methods for Computing Total Sediment Discharge with the Modified Einstein Procedure

GEOLOGICAL SURVEY WATER-SUPPLY PAPER 1593

Prepared as part of a program of the Department of the Interior for development of the Missouri River basin

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By B. R. COLBY *and* D. W. HUBBELL

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UNITED STATES GOVERNMENT PRINTING OFFICE, WASHINGTON : 1961

UNITED STATES DEPARTMENT OF THE INTERIOR

STEWART L. UDALL, *Secretary*

GEOLOGICAL SURVEY

William T. Pecora, *Director*

First printing 1961
Second printing 1967

**For sale by the Superintendent of Documents, U.S. Government Printing Office
Washington, D.C. 20402 - Price $1.75 (paper cover)**

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ILLUSTRATIONS

[All plates are in pocket]

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SYMBOLS

$'$	Single prime mark, when associated with an integral, indicates that the limits of integration are the bottom of the sampled zone and the water surface.
$''$	Double prime mark, when associated with an integral, indicates that the limits of integration are the top of the bed layer and the water surface.
A'	Fraction of the depth not sampled.
A''	Thickness of the bed layer of a given size divided by the depth of a stream.
Conc.	Mean concentration of measured suspended sediment.
d	Average depth of the cross section.
d_n	Depth of the unsampled zone.
d_s	Depth at the sampled verticals.
D	Geometric mean size of a size range.
D_{35}	Particle size at which 35 percent by weight of the bed material is finer.
g	Gravity constant.
i_b	Fraction by weight of bed material in a size range.
i_BQ_B	Sediment discharge through the bed layer of particles of a size class, in tons per day.
i_{sM}	Fraction by weight of measured suspended sediment in a size range.
$\mathrm{I}_{sM}\mathrm{Q}_{sM}$	Measured suspended-sediment discharge in a size range.
I_1	A mathematical abbreviation that contains J_1.
I_2	A mathematical abbreviation that contains J_2.
J_1	An integral.
J_2	An integral.
k_s	Roughness diameter, that particle size of bed material for which 65 percent by weight is finer.
P	Equals $2.303 \log_{10} (30.2\ xd/k_s)$.
Q_s'	Discharge of suspended sediment of a size range through the sampled zone.
Q_{sM}	Measured suspended-sediment discharge.
Q_w	Total water discharge.
R	Ratio of measured suspended-sediment discharge through the sampled zone to total suspended-sediment discharge.
R_o	Ratio of flow through the sampled zone to total flow.
$(RS)_m$	Product of the hydraulic radius and the energy gradient as computed by solving equation (1) by using a known mean velocity.
S_s	Specific gravity of the solid sediment particles.
Temp.	Water temperature.
$\overline{u}$	Average velocity for the cross section.
u_*	Shear velocity.
V_s	Fall velocity of sediment particles of a given size.
w	Width of the stream.
x	A dimensionless parameter determined from plate 1.

z	Theoretical exponent for vertical distribution of suspended sediment of a given size.
z_2	Type of z computed indirectly by the trial-and-error solution of equation (5).
δ	Thickness of laminar sublayer.
ν	Kinematic viscosity.
Φ_*	Intensity of bedload transport from Einstein.
$\Phi_*/2$	Intensity of bedload transport from the modified procedure.
Ψ_m	Shear intensity on the sediment particles from the modified procedure.
Ψ_*	Shear intensity on the sediment particles from Einstein.

SIMPLIFIED METHODS FOR COMPUTING TOTAL SEDIMENT DISCHARGE WITH THE MODIFIED EINSTEIN PROCEDURE

By B. R. Colby and D. W. Hubbell

ABSTRACT

A procedure was presented in 1950 by H. A. Einstein for computing the total discharge of sediment particles of sizes that are in appreciable quantities in the stream bed. This procedure was modified by the U.S. Geological Survey and adapted to computing the total sediment discharge of a stream on the basis of samples of bed sediment, depth-integrated samples of suspended sediment, streamflow measurements, and water temperature. This paper gives simplified methods for computing total sediment discharge by the modified Einstein procedure. Each of four nomographs appreciably simplifies a major step in the computations. Within the stated limitations, use of the nomographs introduces much less error than is present in either the basic data or the theories on which the computations of total sediment discharge are based. The results are nearly as accurate mathematically as those that could be obtained from the longer and more complex arithmetic and algebraic computations of the Einstein procedure.

INTRODUCTION

Equipment currently available for sampling suspended sediment does not collect water-sediment mixture throughout the entire depth of flow; consequently, the concentration and particle-size distribution in only part of the flow can be determined from suspended-sediment samples. The unsampled flow near the stream bed, or flow in the unsampled zone, normally contains higher concentrations and coarser particle-size distributions than the sampled flow, or flow in the sampled zone. Thus, the concentration of suspended-sediment samples is usually lower than the suspended-sediment concentration for the entire depth, and the particle sizes of the samples are usually smaller than the particle sizes for the entire depth.

The sediment discharge computed from the concentration of depth-integrated sediment samples and water discharge is called measured suspended-sediment discharge, and the difference between the total sediment discharge and the measured suspended-sediment discharge is called the unmeasured sediment discharge. Ratios of unmeasured sediment discharge to measured suspended-sediment discharge are highly variable from stream to stream, from cross section to cross section in any one stream, and from time to time at a given cross section. They vary with depths, velocities, sediment concentrations, particle sizes of the suspended and bed sediments, and other factors. Ratios may be considerably greater than 1.0. Also, the unmeasured sediment discharge is composed mostly of sand or coarser sediments, and knowledge of the rate of discharge of sediment of these larger sizes is often more helpful in design and other problems than knowledge of the rate of discharge of the fine sediment.

H. A. Einstein in 1950 presented a procedure for computing the total discharge of sediment of sizes found in appreciable quantities in the stream bed. The procedure, which is based on both theoretical considerations and experimental findings, requires data from a long reach of the stream channel. Colby and Hembree in 1955 (p. 66–98) presented the modified Einstein procedure, which computes the total discharge of all particle sizes and requires data from only a single cross section. In the modified procedure, relationships are based as far as possible on such measurable quantities as the concentration and particle-size distribution of suspended sediment and mean velocity. Computations are made for several ranges of particle sizes and involve many variables; therefore, the computations are undesirably tedious and complex. This paper presents and explains simpler methods, mainly graphical, for computing the total sediment discharge and the approximate size distribution of the total sediment discharge. The graphs are in pound-foot-second units, and sediment loads are computed in tons per day.

SIMPLIFIED METHODS OF COMPUTATION

The simplified methods described in this paper can be applied without an extensive knowledge of the developments leading to the modified Einstein procedure and the assumptions involved in the procedure; therefore, such information is mentioned only briefly if at all. However, some of the essential steps and fundamental equations of the procedure are given so that the simplified computations can be understood and so that graphs can be prepared, if necessary, for different units of measure, other size ranges, or unusual cross sections of streams. Einstein (1950) has stated the theories on which the method is based, and Colby and Hembree (1955, p. 66–98) have explained the reasoning, assumptions, and experimental basis from which the modified procedure was developed. Accuracy and consistency of total sediment discharges that have been computed with the modified Einstein procedure have been discussed by Schroeder and Hembree (1956), Hubbell and Matejka (1959), and Colby and Hembree (1955, p. 106–113).

An example of the simplified computation of total sediment discharge with the modified Einstein procedure is given. (See p. 12–17.) Throughout the example, use is made of drafting dividers and the logarithmic scales of the graphs. As with a slide rule, logarithmic scales can be used for multiplication and division by adding and subtracting distances that represent the logarithms of numbers. The dividers are convenient for measuring and retaining the sum or differences of distances.

COMPUTATION OF $\sqrt{(RS)_m}$ AND P

The mean velocity at a cross section can be expressed by the equation

$$\bar{u}=5.75\sqrt{g(RS)_m}\log_{10}(12.27\,xd/k_s) \qquad (1)$$

in which

- $\bar{u}$ is the average velocity for the cross section
- g is the gravity constant
- $(RS)_m$ is the product of the hydraulic radius and the energy gradient as computed by solving equation (1) by using a known mean velocity
- d is the average depth of the cross section
- x is the dimensionless parameter to be determined from plate 1
- k_s is the roughness diameter, that particle size of bed material for which 65 percent by weight is finer

If all quantities are expressed in pound-foot-second units

$$\sqrt{(RS)_m}=\bar{u}/[32.6\log_{10}(12.27\,xd/k_s)] \qquad (2)$$

The first step in computing total sediment discharge by the modified Einstein procedure is the solution of equation (2) for $\sqrt{(RS)_m}$. The trial-and-error solution of equation (2) can be made graphically from plate 1.

Equation (2) can be represented on a semilogarithmic graph of $\bar{u}$ against xd/k_s by a straight line that passes through the point ($\bar{u}=0$, $xd/k_s=1/12.27$); this fact is readily seen when the equation is written as

$$\bar{u}=32.6\sqrt{(RS)_m}\log_{10}(12.27\,xd/k_s)$$

or

$$=32.6\sqrt{(RS)_m}\log_{10}12.27+32.6\sqrt{(RS)_m}\log_{10}(xd/k_s)$$

in which the slope of the line equals $32.6\sqrt{(RS)_m}$. Because the slope of the line can be read directly from the velocity scale along a vertical line one logarithmic cycle to the right of $xd/k_s=1/12.27=0.0815$, $\sqrt{(RS)_m}$ can be represented by a vertical scale. [$\sqrt{(RS)_m}$=slope/32.6] The $\sqrt{(RS)_m}$ scale can be placed in a convenient relationship to the velocity scale. On plate 1, for example, $\sqrt{(RS)_m}$ equals 0.008 was placed on the same ordinate as $\bar{u}$ equals 1.0. The lateral position of the $\sqrt{(RS)_m}$ scale was determined by solving for xd/k_s in one of the forms of equation (2). For $\bar{u}=1.0$ and $\sqrt{(RS)_m}=0.008$,

$$\log_{10}(xd/k_s)=\frac{1.0}{(32.6)(0.008)}-\log_{10}12.27$$

$$=2.745$$

and

$$xd/k_s=556$$

Hence, the $\sqrt{(RS)_m}$ scale was placed along the vertical at which xd/k_s equals 556. For mean velocities greater than 6.0 feet per second, the numbers on the $\sqrt{(RS)_m}$ and velocity scales can be multiplied by some convenient coefficient. For example, if the velocity scale is changed to read from 0 to 12.0 instead of from 0 to 6.0, each value on the $\sqrt{(RS)_m}$ must be doubled.

Equation (2) cannot be solved for $\sqrt{(RS)_m}$ until x is known at least approximately; and x is a function not only of k_s and the kinematic viscosity, ν, but also of $\sqrt{(RS)_m}$. Thus, equation (2) can be solved only by a trial-and-error process. The left-hand graph of plate 1 is used in the trial process. This graph is merely a transposure of the x versus k_s/δ graph presented by Einstein (1950) by the addition of constants to the abscissa. By definition,

$$\frac{k_s}{\delta}=\frac{k_s}{\dfrac{11.6\nu}{\sqrt{g(RS)_m}}}$$

$$=\frac{k_s\sqrt{(RS)_m}}{2.05\nu} \tag{3}$$

where

δ is the thickness of the laminar sublayer

By multiplying both sides of the equation by 1,000/100,000 and transposing 2.05,

$$0.0205\frac{k_s}{\delta}=\frac{1{,}000k_s}{100{,}000\nu}\sqrt{(RS)_m}$$

Abscissa values of the Einstein graph have been multiplied by 0.0205 to give the abscissa $1{,}000k_s\sqrt{(RS)_m}/100{,}000\nu$. The narrow graph in the middle of plate 1 gives the relationship between kinematic viscosity and water temperature.

Equation (2) can be solved from plate 1 by trial and error as follows:

1. From the known quantities d and k_s, the ratio d/k_s is computed.
2. A numerical value is assumed for x, and the point whose coordinates are the known mean velocity and the approximate numerical value of xd/k_s is roughly located.

3. The intersection with the $\sqrt{(RS)_m}$ scale of a straight line between this point and the circled point (velocity=0 and xd/k_s=0.0815) on the left-hand graph determines an approximate $\sqrt{(RS)_m}$. This procedure can be done accurately enough by eye without actually multiplying by x or using a straight edge between the two points.
4. On the narrow middle graph of plate 1, a pair of dividers is used to span horizontally along the line for the known water temperature from the line that represents the kinematic viscosity times 10^5 to the vertical line that represents 1,000 k_s. The divider span is a measure of the logarithm of the ratio of 1,000 k_s to the kinematic viscosity times 10^5. If the ratio is greater than 1.0, the quantity $1{,}000k_s\sqrt{(RS)_m}/\nu\times10^5$ is determined by stepping off the divider span horizontally to the right on the left-hand graph from the vertical line that represents the approximate numerical value of $\sqrt{(RS)_m}$. If the ratio is less than 1.0, the span is measured to the left. A vertical line through the point on which the leg of the dividers falls intersects the smooth curve of the left-hand graph at the horizontal line that represents the approximate numerical value of x.
5. If this numerical value of x is appreciably different from the original estimate of x, the general procedures of steps 2, 3, and 4 are repeated. This time, however, dividers are used to add the logarithms of d/k_s and of the second estimate of x, and a straight edge is used to read the numerical value of $\sqrt{(RS)_m}$. The whole procedure of steps 2 through 5 should be repeated until the estimated x and the computed x are about the same. Generally, agreement is satisfactory on the second trial.

By definition,

$$P=2.303\ \log_{10}(30.2\ xd/k_s) \tag{4}$$

This equation, which can be represented by a straight line on a semilogarithmic graph of P against xd/k_s, is plotted in plate 1. The position of the line was fixed by values of P that were computed from assumed values of xd/k_s. When xd/k_s equals 1,000, the logarithm to the base 10 of 30.2 xd/k_s is 4.48, and P equals 2.303 times 4.48, or 10.32. When xd/k_s is 100, P equals 10.32−2.303, or 8.017; and when xd/k_s equals 10,000, P equals 12.623. Thus the position of the scale for P is readily fixed, and P can be determined directly from xd/k_s.

COMPUTATION OF SEDIMENT DISCHARGED AS BEDLOAD

Einstein (1950) showed a relationship between the intensity of bedload transport, Φ_*, and the shear intensity on the sediment particles, Ψ_*. This relationship was somewhat modified by Colby and Hembree

(1955, p. 83–89). According to the modified relationship, the intensity of bedload transport is $\Phi_*/2$, if Φ_* is determined from the relationship between Φ_* and Ψ_* (Einstein, 1950, figs. 10 and 11) by substituting Ψ_m for Ψ_*. Ψ_m is the function that correlates the shear intensity with the intensity of bedload transport in the modified procedure and is defined as equal to $(S_s-1)D_{35}/(RS)_m$ or to $0.4(S_s-1)D/(RS)_m$, whichever is larger. By definition, D is the geometric mean size of a bed-material size range, D_{35} is the particle size at which 35 percent by weight of the bed material is finer, and S_s is the specific gravity of the solid sediment particles. Equating and simplifying the two equations for Ψ_m give D equal to 2.5 D_{35}. Thus, whenever D is greater than 2.5 D_{35}, Ψ_m from the equation with D is larger than Ψ_m from the equation with D_{35}. The converse is true when D is less than 2.5 D_{35}.

In the modified procedure, the rate of bedload discharge, in tons per day, of particles of a given size range, i_BQ_B, is computed from the equation

$$i_BQ_B = i_b w(43.2)(1{,}200)D^{3/2}\Phi_*/2$$

in which

i_b is the fraction by weight of the bed material in the size range
w is the width of the stream, in feet
D is the geometric mean size (the square root of the product of the upper and lower sizes of the range), in feet
$\Phi_*/2$ is the intensity of bedload transport

Plate 2 was prepared for determining i_BQ_B from $\sqrt{(RS)_m}$ according to the preceding relationships. It has been drawn for the size ranges of 0.062 to 0.125 mm, 0.125 to 0.25 mm, 0.25 to 0.50 mm, etc., of which the geometric mean sizes are 0.00029 ft, 0.00058 ft, 0.00116 ft, etc., respectively. Of course, similar nomographs can be drawn for whatever ranges of particle sizes are desired.

The upper part of the main graph of plate 2 was drawn for the particle-size ranges for which D is less than 2.5 D_{35}. The curves of this graph are for individual geometric mean sizes and were established by solving for values of $(43.2)(1{,}200)D^{3/2}\Phi_*/2$ from various values of $\sqrt{(RS)_m}$ and a D_{35} equal to 0.001; the parameter 1.65 $D_{35}/(RS)_m$ was used to determine Ψ_m, which in turn was used to determine $\Phi_*/2$ from the Φ_*-Ψ_* relationship. Consequently, whenever D_{35} is not 0.001, these curves cannot be used to determine $(43.2)(1{,}200)D^{3/2}\Phi_*/2$ unless an adjustment is made. This adjustment is evident from the following equation:

$$\Psi_m = 1.65\frac{D_{35}}{(RS)_m}$$

$$= \frac{1.65(0.001)}{(RS)_m\left(\dfrac{0.001}{D_{35}}\right)}$$

Thus, correct values of Ψ_m, and therefore of $(43.2)(1{,}200)D^{3/2}\Phi_*/2$, can be determined for any given $\sqrt{(RS)_m}$ and D_{35} from $1.65(0.001)/(RS)_m$ by multiplying $\sqrt{(RS)_m}$ by the adjustment, $\sqrt{0.001/D_{35}}$.

The lower set of curves in the main graph was drawn for particle-size ranges for which D is greater than 2.5 D_{35}. For these size ranges, Ψ_m is determined from $(0.4)(1.65)D/(RS)_m$. Consequently, the curves were defined for the separate size ranges by computing values of $(43.2)(1{,}200)D^{3/2}\Phi_*/2$ for various values of $\sqrt{(RS)_m}$.

The first step in computing bedload discharge from plate 2 is to note which geometric mean sizes are larger and which are smaller than 2.5 D_{35}. On the graph in the lower right-hand corner of plate 2, geometric mean sizes (shown vertically) to the left of any D_{35} are smaller than 2.5 D_{35} and those to the right are larger. Also, the curve on this graph gives the adjustment $\sqrt{0.001/D_{35}}$ from D_{35}. For the small sizes, the adjustment is applied as a coefficient to $\sqrt{(RS)_m}$, and the upper main graph is entered to determine the rates of bedload discharge per foot of width and for 100 percent of the bed material in each size range $[(43.2)(1{,}200)D^{3/2}\Phi_*/2]$. (If $\sqrt{0.001/D_{35}}$ is greater than 1.0, a vertical divider span from $\sqrt{0.001/D_{35}}$ to 1.0 is graphically added to $\sqrt{(RS)_m}$. If less than 1.0, the span is graphically subtracted from $\sqrt{(RS)_m}$.) The points of intersection between the curves and the horizontal line that represents the product of $\sqrt{0.001/D_{35}}$ and $\sqrt{(RS)_m}$ give these rates. The bedload discharge for each size range is determined by multiplying the various values of $(43.2)(1{,}200)D^{3/2}\Phi_*/2$ by the appropriate i_b and width. The multiplication can be done with a pair of dividers by obtaining a span from the i_b and w scales in the main graph of plate 2 and applying the span to the right or left of the points of intersection between the curves and the $\sqrt{0.001/D_{35}}\sqrt{(RS)_m}$ line. If the product of w and i_b is greater than 1.0, the span is measured to the right of the point of intersection to obtain the bedload discharge; if the product is less than 1.0, the span is measured to the left. The rates of bedload discharge for the geometric mean sizes that are larger than 2.5 D_{35} are obtained from the lower part of the main graph by a similar procedure except that the $\sqrt{(RS)_m}$ requires no adjustment for D_{35}.

If the specific gravity of the bed sediment varies appreciably from 2.65, two types of adjustment are required in the application of the nomograph of plate 2. One adjustment is to the vertical scale of the nomograph, and the other is to the horizontal scale. The small diagram near the bottom of plate 2 gives these adjustments for different specific gravities of the bed material. The adjustments are relatively easy to apply and are based on the assumption that particle sizes of the bed material were determined by actual sizes rather than by fall velocities. No adjustments are given for specific gravity on the other nomographs because, if the specific gravity of the suspended sediment is much different from 2.65, adjustments would be more questionable and would depend mainly on the method that was used in the size analysis of the suspended sediment.

COMPUTATION OF z_2

One of the principal changes from the Einstein procedure was the replacement of z as defined in z equals $V_s/(0.4\ u_*)$ by z_2, the z that can be computed indirectly by trial-and-error solution of the equation

$$\frac{Q_s'}{i_B Q_B}=(PJ_1'+J_2')\ \frac{0.216\ (A'')^{z-1}}{(1-A'')^z} \tag{5}$$

in which

- z is the theoretical exponent for the vertical distribution of suspended sediment of a given size
- V_s is the fall velocity of the sediment particles of the given size
- u_* is the shear velocity
- Q_s' is the discharge of suspended sediment of the given size range through the sampled zone
- A'' is the thickness of the bed layer of the given size divided by the depth of the stream, or A'' equals $2D/d$
- J_1 and J_2 are mathematical abbreviations for integrals defined by Einstein (1950, p. 19, 24), and the single prime marks mean that the limits of integration are the bottom of the sampled zone and the water surface

Equation (5) is solved for z_2 for a size range that has appreciable quantities of both suspended-sediment discharge and bedload discharge. The geometric mean size for this size range is called the reference size. For other size ranges, z_2 is computed from the z_2 for the reference size; the procedure is explained on page 10.

Graphs for the trial-and-error solution for z_2 from equation (5) are shown on plate 3. The main graph of this plate is based on the assumption that because A'' is on the order of 0.001 ft for stream depths of 1 ft and is even less for greater depths, $(1-A'')^z$ is close to 1.00. Thus, equation (5) can be rewritten as

$$\frac{Q_s'}{i_B Q_B}=(PJ_1'+J_2')\ 0.216\ (A'')^{z-1}$$

from which $\log_{10}(Q_s'/i_B Q_B)$ can be determined from the sum of $\log_{10}(PJ_1'+J_2')$ and $\log_{10}[(0.216)(A'')^{z-1}]$. ($J_1'$ and J_2' are functions of A', the fraction of depth not sampled, and z.) Substitution in the latter quantity gives $\log_{10}[0.216\ (2D/d)^{z-1}]$. This quantity can be expressed graphically as a straight line with z as the ordinate and

$\log_{10}[0.216\ (2D/d)^{z-1}]$ as the abscissa. At $z=1.0$, the abscissa, whatever the ratio of D to d, will equal $\log_{10} 0.216$. At $z=0$, the abscissa will equal $\log_{10}(0.216\ d/2D)$. If the graph is based on D equal to 0.00058 ft, as in plate 3, the abscissa at $z=0$ will equal $\log_{10}(186.2d)$. A line for any depth can be drawn on this graph by connecting the points (abscissa$=\log_{10} 0.216$, $z=1.0$) and (abscissa$=\log_{10} 186.2+\log_{10}d$, $z=0$). If $\log_{10}(PJ_1'+J_2')$ is added horizontally from the intersection of the appropriate depth line and the z that solves equation (5), the sum will equal $\log_{10}(Q_s'/i_B)$. For many computations, $PJ_1'+J_2'$ is equal to about 8.0; thus, for convenience, the $\log_{10} 8.0$ has been added to the quantity $\log_{10}[0.216\ (2D/d)^{z-1}$. The addition shifts the d lines horizontally to the right by a distance that represents $\log_{10} 8.0$. (Such shifted lines are drawn on plate 3 for depths of 1.0, 10, and 100 ft.) Thus, if $PJ_1'+J_2'$ is not 8.0, the lines should be shifted a vector distance $\log_{10}(PJ_1'+J_2')-\log_{10} 8.0$ or the horizontal scale should be shifted a vector distance $\log_{10} 8.0-\log_{10}(PJ_1'+J_2')$. The upper right-hand graph of plate 3 gives $10J_1'+J_2'$ in terms of A' and z. If P does not equal 10, then $10J_1'+J_2'$ must be adjusted to obtain the correct value of $PJ_1'+J_2'$. The adjustment can be made approximately by adding the product of $10J_1'+J_2'$ and 11 percent of $(P-10)$ to $10J_1'+J_2'$. A simpler, though generally less precise, adjustment can be made by multiplying $10J_1'+J_2'$ by 0.1 P. This simpler method is ordinarily used in computations.

When plate 3 is used, the ratio Q_s'/i_BQ_B is computed first. The bedload discharge, i_BQ_B, is determined from plate 2. Q_s' is computed as the product of a conversion constant, the flow through the whole cross section, the fraction of the flow that is through the sampled zone, the measured suspended-sediment concentration (suspended-sediment concentration of the flow through the sampled zone), and the fraction of the measured suspended sediment that is in the given size range. The last two quantities are based on analyses of suspended-sediment samples, usually depth integrated. The flow through the whole cross section is determined by a streamflow measurement or from the stage–discharge relationship. The fraction of the total flow that passes through the sampled zone can be computed from the theoretical vertical distribution of velocity for different numerical values of P and for different fractions of depth not sampled. (See pl. 4.) The fraction of depth not sampled, A', is dependent on the configuration of the sampler and on the depth of the water.

After the ratio Q_s'/i_BQ_B is computed, it is used to establish a first approximation of z_2 from the main graph in plate 3. Then, $PJ_1'+J_2'$ is determined in the right-hand graph by multiplying (adding graphically) 0.1 P by $10J_1'+J_2'$, which is given in the graph by the intersection of the estimated z_2 and A' lines. The distance between

$PJ_1'+J_2'$ and 8.0 is measured and applied horizontally to the point on the appropriate d line where the addition or subtraction of the distance from the line gives Q_s'/i_BQ_B. This point defines a second approximation for z_2. An adjusted z_2 may then be estimated on the basis of the difference between the first and the second approximations of z_2 by noting on the right-hand graph which way a change in z will affect $PJ_1'+J_2'$. Plate 3 can be used for other reference sizes. For example, if the reference size is 0.00116 foot, the depth is divided by 2 before the main graph is used.

For each geometric mean size other than the reference size, z_2 can be determined on plate 5 from the z_2 for the reference size. Plate 5 was given by Colby and Hembree (1955, pl. 1) and shows for several geometric mean sizes the ratio of z_2 for that size to z_2 for a geometric mean size of 0.00058 ft. The ratios are based on the empirical relationship that z for a given shear velocity and vertical distribution of velocity varies with about the 0.7 power of the fall velocities as given by Rubey (1933, p. 332), rather than with the first power.

TOTAL DISCHARGE OF FINE SEDIMENT

The total sediment discharge of size ranges having a z_2 less than about 0.8 is the sum of the total suspended-sediment discharge of the range and, if significant, the bedload discharge of the range determined from plate 2. The total suspended-sediment discharge is computed from the measured suspended-sediment discharge and the theoretical distribution of velocity and suspended sediment in the vertical.

The ratio, R, of measured suspended-sediment discharge through the sampled zone to total suspended-sediment discharge is given by the equation

$$R=\frac{PJ_1'+J_2'}{PJ_1''+J_2''} \qquad (6)$$

The double prime marks indicate that the limits of integration for J_1 and J_2 are the top of the bed layer and the water surface, whereas the single prime marks mean that the limits are the bottom of the sampled zone and the water surface. R varies with P, A', A'', and z, but the effect of changes in P and A'' is relatively small for fine sediment. For z equal to 0, R is equal to R_o, the ratio of the flow through the sampled zone to the total flow. The total suspended-sediment discharge for a range of small sediment sizes can be computed from the product of the measured suspended-sediment discharge in the size range and R_o/R.

Both A'' and A' are related to the depth, d. A'' equals $2D/d$, and A' equals d_n/d or the ratio of the depth of the unsampled zone,

d_n, to the depth of the water. Hence, A'' equals $2DA'/d_n$. The geometric mean size of 0.00029 ft is likely to be the largest or next to the largest geometric mean size to be used for the fine sediment, and d_n is usually about 0.3 ft. Therefore, as a useful approximation, A'' equals 0.00058 $A'/0.3$ or roughly 0.002 A'. For appreciably smaller particle sizes, z is so small that even large changes in the relationship between A' and A'' make insignificant differences in R_o/R. Also, d_n can vary from 0.1 to 1.0 and not produce noticeable differences in R_o/R for given numerical values of z and A'.

Ratios of R_o/R for P equal to 10 and A'' equal to 0.002 A' were computed for several numerical values of A' up to 0.6 and values of z up to 0.8. Smooth curves were drawn through the computed points to define the main graph of plate 6. As long as either z or A' is low, changes in P affect R_o/R only a little. As z and A' increase, changes in P become appreciable. Hence, R_o/R was computed for P equal to 8 and P equal to 14 for many different numerical values of z and A'. The differences for P equal to 14 rather than to 10 are numerically about equal to the differences for P equal to 8 rather than to 10, but the differences are of opposite algebraic sign. Zones on the main graph indicate the general effect of changes in P. In zone 1 the effect of changing P from 10 to 14 ranges from about 0 to 2 percent, in zone 2 from about 2 to 4 percent, etc. The small diagram at the left of plate 6 indicates the adjustment to be applied in each zone for different numerical values of P.

TOTAL DISCHARGE OF COARSE SEDIMENT

When z_2 is larger than about 0.8, a different method of computation is likely to be more accurate than that on which plate 6 is based. According to either the Einstein procedure or the modified procedure, the discharge of the coarser sediment can be computed from $i_BQ_B(PI_1''+I_2''+1)$. I_1 and I_2 are integrals defined by Einstein (1950, p. 24), and the double prime marks indicate that the limits of integration are the top of the bed layer and the water surface. Of course, a graph of 10 $I_1''+I_2''+1$ can be prepared, and zones for the effect of changes in P can be outlined on the graph; but the effect of large changes in P would be difficult to read accurately when z becomes greater than about 0.8. Several other types of approximate graphs are also possible, but perhaps the original I_1 and I_2 graphs given by Einstein (1950, figs. 1 and 2) and reproduced as plates 7 and 8 are as satisfactory as any for the computation of the total discharge of the coarser sediment. Plate 8 could be prepared in terms of I_2 plus 1, or the numbers on the vertical scale could be reduced by 1 and the grid left unchanged. (I_2 is always negative.)

TOTAL SEDIMENT DISCHARGE OF ALL PARTICLE SIZES

The total sediment discharge through the cross section is obtained by adding the total sediment discharges for all the size ranges of the transported sediment. The particle-size distribution of the computed total sediment discharge can be determined from the sediment discharges in the different size ranges. The unmeasured sediment discharge, the difference between the measured suspended-sediment discharge and the total sediment discharge, and its size distribution can also be computed from the results of the computations of total sediment discharge.

SAMPLE COMPUTATION OF TOTAL SEDIMENT DISCHARGE

A sample computation is summarized in table 1 to illustrate the use of the simplified methods of computing total sediment discharge by the modified Einstein procedure. This computation has been made with the same basic data that were used by Colby and Hembree (1955, pl. 3). However, the computed total sediment discharge and some of the values within this sample computation differ slightly from those in Colby and Hembree (1955, pl. 3) because of the use of the nomographs and a rounded value for A'. The basic data are lettered vertically in table 1 to distinguish them from computations that are part of the procedure.

The first computation is for d/k_s, which is 0.98/0.00105 or 933. This number and the mean velocity of 2.08 fps determine point A on the right-hand graph of plate 1. Let x be assumed equal to about 1.2 (purposely a poor estimate to show that accuracy is not necessary). Point B is roughly located by eye about 20 percent horizontally to the right from point A. An imaginary straight line from point B to the index circle C on the baseline and near the left edge of the left-hand graph would pass close to 0.0155 on the $\sqrt{(RS)_m}$ scale. The kinematic viscosity times 10^5 at a water temperature of 64° F is given by point D on the middle graph of plate 1. A pair of dividers is set to span from point D to 1,000 k_s, which is 1.05 on the horizontal scale and is indicated by point E. On any convenient horizontal line of the left-hand graph of plate 1 note point F, which is determined by $\sqrt{(RS)_m}$ equal to 0.0155, and span horizontally the divider setting D–E from point F to determine point G. (If point E is to the left of point D, point G will be to the left of point F, and vice versa.) A vertical line through point G intersects the curve of the left-hand graph of plate 1 at point H, which indicates that x is about 1.54. This is much higher than the first guess of 1.2. As x increases on the right-hand graph, $\sqrt{(RS)_m}$ decreases. Also, a decrease in $\sqrt{(RS)_m}$ below 0.0155 will slightly decrease x on the left-hand graph, so use 1.53 for x. Return

Station *Niobrara River near Cody, Nebr.* Section *C-2* Date *June 19, 1952* Time *12:10 p.m.*

Q_w =	239	cfs	Temp. =	64	°F	$A' = d_n/d_s$ =	0.3/1.22 = *0.25*
w =	118	ft	k_s =	0.00105	ft	Percentage of flow in sampled zone =	*79*
$\overline{u}$ =	2.08	fps	D_{35} =	0.00075	ft	Reference size is	*0.00058* ft
d =	0.98	ft	d/k_s =	*933*		Q'_s for reference size =	*(163)(.39)(.79) = 50* tons/day
d_s =	1.22	ft	x =	*1.53*		$\frac{Q'_s}{i_B Q_B}$ for reference size =	*50/8.3 = 6.0*
Conc. =	262	ppm	$\sqrt{(RS)_m}$ =	*0.0150*	$ft^{\frac{1}{2}}$		
Q_{sM} =	163	tons/day	P =	*10.68*		z_2 for reference size =	*0.77*

Size (mm)	D (ft)	i_{sM}	$i_{sM} Q_{sM}$	i_b	$i_B Q_B$	z_2	A″	PI_1''	I_2''	$PI_1''+I_2''+1$	Total load (tons/day)
< 0.062	0.000036	0.28	*46*	---		*0.016*	*0.000074*				*46*
.062–.125	.00029	.24	*39*	a0.03	*0.24*	*.34*	*.00059*				*52*
.125–.250	.00058	.39	*64*	.38	*8.3*	*.77*	*.00118*	*30.0*	*8.30*	*22.70*	*188*
.250–.500	.00116	.09	*15*	.50	*31.0*	*1.34*	*.00237*	*5.40*	*2.12*	*4.28*	*133*
.500–1.00	.00232			.05	*5.1*	*1.99*	*.00474*	*2.27*	*.93*	*2.34*	*12*
1.00–2.00	.00464			.01	*.24*	*2.63*	*.00947*	*1.38*	*.52*	*1.86*	*0*
2.00–4.00	.00928			.01							
4.00–8.00	.01856										
Remarks:											*431*

Computed by ______ Date ______ Checked by ______ Date ______

a-From Colby and Hembree (1955, fig. 32).

TABLE 1.—*Sample computation of total sediment discharge*

to the right-hand graph, and set the dividers to span from 1,000 to 1,530 on the horizontal scale (any convenient cycle of the logarithmic scale can be used). Measure this span horizontally to the right from point A to establish point I. A straight edge passing through points I and C crosses the $\sqrt{(RS)_m}$ scale at 0.0150, point J. Thus, equation (2) has been solved by trial with x equals 1.53 and $\sqrt{(RS)_m}$ equals 0.0150. A line projected vertically upward from point I to point K on the P scale indicates that P is 10.68.

This procedure shows clearly that considerable changes in x have little effect on $\sqrt{(RS)_m}$. Hence, the steps leading to the approximate determination of x from the left-hand graph of plate 1 can be done hurriedly and inexactly.

Bedload discharges are computed from plate 2. D_{35} for the bed material is 0.00075 foot. A vertical line on the lower right-hand graph of plate 2 at this particle size intersects the 1.00 horizontal line and the sloping adjustment line at points A and B, respectively. The geometric mean size of 0.00116 is to the left of points A and B, and for this and smaller geometric mean sizes the upper part of the main graph of plate 2 is to be used. Point C on the left margin of the upper part of the main graph is determined by $\sqrt{(RS)_m}$, which is 0.0150. Measure the distance A–B vertically upward from point C to establish point D. A horizontal line through point D intersects the curves for geometric mean sizes of 0.00029, 0.00058, and 0.00116 foot at points E, F, and G, respectively. Set one leg of the dividers at a stream width of 118 feet, point H, and the other leg at an i_b of 0.03, point I. Measure the distance I–H horizontally to the right from point E to determine point J at which the bedload discharge is 0.24 ton per day. Points K and L at which the bedload discharge is 8.3 and 31 tons per day are similarly located by measuring the distances M–H and N–H horizontally to the right from points F and G, respectively.

The lower part of the main graph of plate 2 is for geometric mean sizes to the right of points A and B. On this part of the main graph, the line for $\sqrt{(RS)_m}$ equal to 0.0150 intersects curves for the geometric mean sizes of 0.00232 and 0.00464 ft at points O and P, respectively. Bedload discharges for larger mean particle sizes will be insignificant. Points Q and R indicate bedload discharges of 5.1 and 0.24 tons per day, respectively, and are horizontally to the right from points O and P by the respective distances S–H and T–H.

Appreciable amounts of bedload discharge were computed for three ranges of particle size. Reasonably large percentages of suspended sediment were measured for two (geometric mean sizes of 0.00058 and 0.00116 ft) of these size ranges. Either of these two size ranges or both could be used as representative size ranges for which z_2 is to be computed, but probably the better one is the size range of 0.125 to

0.25 mm (geometric mean size of 0.00058 ft) for which a considerably larger discharge of suspended sediment was measured. In general, 0.00058 ft is a good reference size for shallow streams that flow over sandy beds.

The depth of the unsampled zone, d_n, for the U.S. DH–48 sediment sampler is about 0.3 ft, and the average depth at the sampling verticals, d_s, is 1.22 feet. Hence, A' equals 0.3/1.22 or 0.25. According to plate 4, the percentage of the flow in the sampled zone is 79 percent for A' equal to 0.25 and P equal to 10.68. The measured suspended-sediment discharge was 163 tons per day of which 39 percent was in the size range from 0.125 to 0.25 mm. Therefore, Q_s' equals (163) (0.39)(0.79) or 50 tons per day, and Q_s'/i_BQ_B equals 50/8.3 or 6.0.

The z_2 for the reference size is computed with the graphs of plate 3. A vertical line through point A, at $Q_s'/i_BQ_B=6.0$ in the left-hand graph, intersects a line from the circled intersection of the depth lines to 0.98 ft on the depth scale at a z_2 of about 0.81 (point B). Point C in the upper right-hand graph is defined by $A'=0.25$, $z_2=0.81$, and point D is about 7 percent (0.1 of 10.68 is approximately 1.07) greater than the value of point C. At point C, $10\ J_1'+J_2'=5.7$, and, at point D, $10.68\ J_1'+J_2'=6.1$ approximately. One leg of the dividers is set on point D and the other leg on point E, which is at the intersection of the line for $A'=0.25$ and the line for $10\ J_1'+J_2'=8.00$. Measure the divider distance D–E horizontally to the right from point A on the baseline of the left-hand graph to point F. (If point D is to the left of point E, point F is to the right of point A, and vice versa.) A vertical line through point F intersects a line for a depth of 0.98 ft at point G, for which z_2 is 0.77. The difference between 0.77 and the first estimate of 0.81 is so small that no recheck on the right-hand graph is necessary.

The z_2's for the other geometric mean sizes are determined by multiplying the z_2 of 0.77 for the reference size by factors, called multipliers, that are given in plate 5. The multiplier for each size range is found at the intersection of the multiplier curve and the 64°F water-temperature line. The z_2's can also be determined graphically. The 0.77 and the temperature of 64°F establish point A on plate 5. One leg of the dividers is set on point A and the other on point B, which is at the intersection of the line for the temperature of 64°F and the line for the geometric mean size of 0.00058 ft. Points C, D, E, F, and G are established by measuring the divider distance B-A horizontally to the left from the lines for the different geometric mean sizes. These points indicate z_2's, on the horizontal scale, of 0.016, 0.34, 1.34, 1.99, and 2.63 for the geometric mean sizes of 0.000036, 0.00029. 0.00116, 0.00232, and 0.00464 ft, respectively.

The particle-size analysis of the sediment samples showed 28, 24, and 39 percent in the size ranges whose geometric mean sizes are 0.000036, 0.00029, and 0.00058 ft, respectively. The measured suspended-sediment discharge of 163 tons per day is multiplied by these percentages and divided by 100 to obtain the measured suspended-sediment discharges ($i_{sM}Q_{sM}$) of 46, 39, and 64 tons per day for the three size ranges. On plate 6 the vertical line for $A'=0.25$ crosses the estimated lines for z equal to 0.016, 0.34, and 0.77 at ratios of total suspended-sediment discharge to measured suspended-sediment discharge of 1.01, 1.34, and 2.75 as indicated by points *A*, *B*, and *C*, respectively. These ratios are for *P* equal to 10.0. The left-hand graph of adjustment for *P* shows that adjustments would be negligible for *P* equal to 10.68 for the 2 smaller ratios but would be about 2 percent for the ratio of 2.75. Mentally increase the 2.75 by 2 percent to 2.81. The multiplication can also be done readily by adding logarithms with the dividers. The measured suspended-sediment discharges of 46, 39, and 64 tons per day are multiplied by their respective ratios of 1.01, 1.34, and 2.81 to obtain the total suspended-sediment discharges of 46, 52, and 180 tons per day. To the 180 tons per day must be added the bedload discharge of 8 tons per day to compute the total sediment discharge for the size range.

An alternative computation of total suspended-sediment discharge from plate 6 may be preferable and will be illustrated by an example for the 0.125- to 0.25-mm size range. The dividers are set to span vertically from the baseline of the main graph to point *C*. The dividers are then placed on the left-hand graph and along the vertical line for zone 5 so that the upper leg is at point *E* on the horizontal line $P=10$. While the lower leg of the dividers is held stationary, the upper leg is moved upward from point E to point *F* ($P=10.68$) to give a total span that represents the ratio of total suspended-sediment discharge to measured suspended-sediment discharge. This total span is measured vertically upward from point *G* on a horizontal line $i_{sM}Q_{sM}=64$ to point *H* at which the total discharge of suspended sediment in the size range is read as 180 tons per day.

The last major computation is that of the discharge of sediment of the larger particle sizes. A'' is computed for individual geometric mean sizes by multiplying each size in feet by 2.00 and dividing by the mean depth, 0.98 ft. A'' is 0.00118 and z_2 is 0.77 for the geometric mean size of 0.00058 ft. The lines representing these quantities intersect at point *A* on plate 7. A divider span representing the logarithm on the vertical scale of 10.68 is measured upward from point *A* to point *B*, which indicates that 10.68 I_1'' is about 30.0. Points *C*, *D*, and *E* at which 10.68 I_1'' is about 5.4, 2.27, and 1.38 are similarly determined for the three next larger geometric mean sizes. Numerical values of I_2'' are read as 8.3, 2.12, 0.93, and 0.52 from plate 8 for the four geometric mean sizes of the coarser sediment particles. The total sediment discharge of particles of each of the four size ranges is computed by multiplying the bedload discharge, $i_B Q_B$, in tons per day, by the algebraic sum of 10.68 $I_1''+I_2''+1.00$. (I_2'' is always negative.) The computed total sediment discharges are 188, 133, and 12 tons per day for the size ranges that are represented by the geometric mean sizes of 0.00058, 0.00116, and 0.00232 ft, respectively. (See table 1.) The computed discharge for the size range of next larger particles is less than 0.5 ton per day.

The computed total sediment discharge of particles from 0.125 to 0.25 mm was 188 tons per day whether computed from plate 6 or from plates 7 and 8. This exact agreement is a coincidence. Ordinarily a difference of about 1 to 3 percent is expected in total sediment discharge for the reference size range if both types of computation are made. The only reason for making both types of computation is to check on the computation of z_2 for the reference size range and on the computed sediment discharge in the size range.

Total computed discharge of sediment of all size ranges is 431 tons per day as compared to a measured suspended-sediment discharge of 163 tons per day.

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