Analysis of Aquifer Tests in the Punjab Region of West Pakistan

GEOLOGICAL SURVEY WATER-SUPPLY PAPER 1608-G

Prepared in cooperation with the West Pakistan Water and Power Development Authority under the auspices of the U.S. Agency for International Development

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By GORDON D. BENNETT, ATA-UR-REHMAN, IJAZ AHMED SHEIKH, *and* SABIR ALI

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UNITED STATES GOVERNMENT PRINTING OFFICE, WASHINGTON : 1967

UNITED STATES DEPARTMENT OF THE INTERIOR

STEWART L. UDALL, *Secretary*

GEOLOGICAL SURVEY

William T. Pecora, *Director*

For sale by the Superintendent of Documents, U.S. Government Printing Office
Washington, D.C. 20402 - Price 55 cents (paper cover)

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ANALYSIS OF AQUIFER TESTS IN THE PUNJAB REGION OF WEST PAKISTAN

By GORDON D. BENNETT, ATA-UR-REHMAN, IJAZ AHMED SHEIKH, and SABIR ALI

ABSTRACT

The results of 141 pumping tests in the Punjab Plain of West Pakistan are reported. Methods of test analysis are described in detail, and an outline of the theory underlying these methods is given. The lateral permeability of the screened interval is given for all tests; the specific yield of the material at water-table depth is given for 106 tests; and the vertical permeability of the material between the water table and the top of the screen is given for 14 tests. The lateral permeabilities are predominantly in the range 0.001 to 0.006 cfs per sq ft; the average value is 0.0032 cfs per sq ft. Specific yields generally range from 0.02 to 0.26; the average value is 0.14. All vertical permeability results fall in the range 10^{-5} to 10^{-3} cfs per sq ft.

INTRODUCTION

PURPOSE AND SCOPE OF REPORT

Between 1954 and 1963, aquifer tests were completed at 164 sites in the Punjab Plain of West Pakistan by the Ground Water Development Organization and its successor, the Water and Soils Investigation Division (WASID). (For the sake of brevity, the organization will hereafter be referred to as WASID, whether the reference is to before or after the reorganization and change in name.) This report is based upon the analyses of 141 of these aquifer tests. Determination of lateral permeability proved impossible in the remaining 23 tests, either because data on the length of screen were lacking or because of generally poor test results owing to various difficulties encountered in the field. These 23 tests were accordingly excluded from the study.

It has been the policy of WASID to release the initial results of pumping-test analyses as soon as possible, as preliminary information subject to revision. No test analysis was concluded at the time of the release of the preliminary data, and for most, revised or more extensive results were achieved in later analysis. The results given in this report supersede those quoted in all previous WASID releases.

Figure 1 shows the area covered in the pumping-test program reported in this paper. The area consists of Rechna, Chaj, Bāri and Thal Doābs. The report does not cover pumping tests performed by WASID in adjacent areas, such as Bahāwalpur, Bahāwalnagar and Dera Īsmāīl Khān. Of the 141 tests upon which this report is based, 47 were in Rechna Doāb, 36 in Chaj, 35 in Thal, and 21 in Bāri.

All the test wells were screened in the unconsolidated alluvial deposits of the Punjab. These deposits, consisting mainly of interbedded and lenticular sands, silts, and clays, extend to depths of one

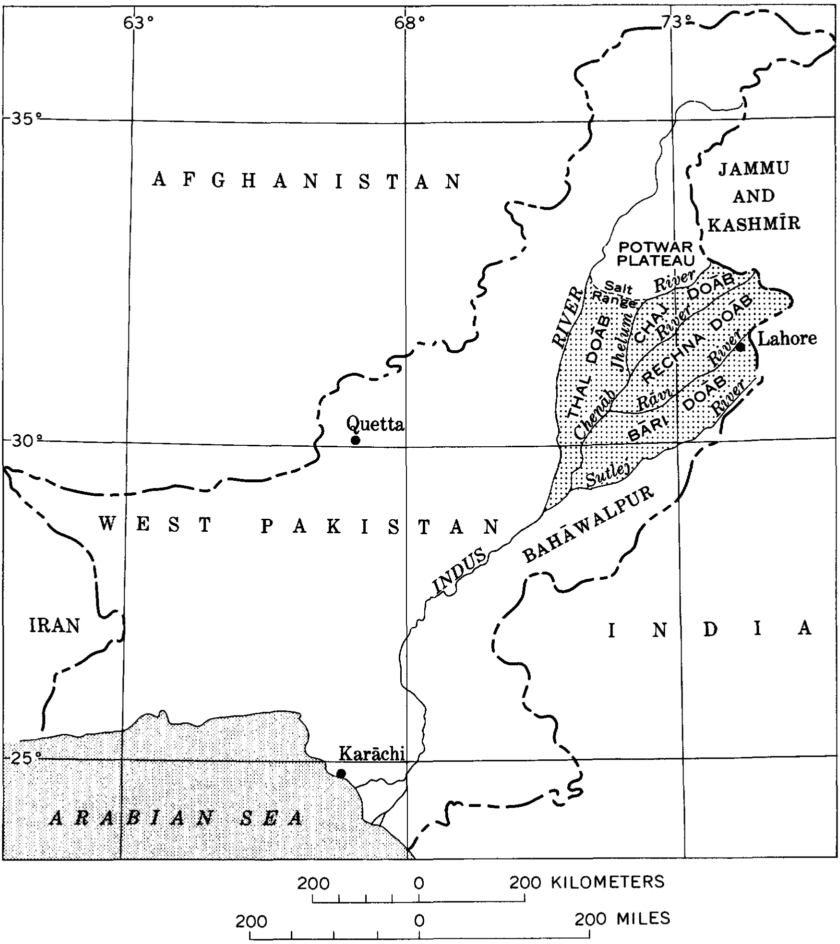

FIGURE 1.—West Pakistan showing the Punjab Region (stipple pattern).

thousand to several thousand feet over most of the area. They constitute a very extensive heterogeneous unconfined aquifer. None of the test wells exceeded 400 feet in depth; the test results thus reflect the characteristics of the upper few hundred feet of the aquifer.

The tests were analyzed by methods which the authors considered the most realistic at their disposal. These methods are based upon certain physical assumptions, and errors due to the deviation of field conditions from these assumptions are present in many of the results. For example, it was assumed that flow occurred in a horizontal-radial pattern, and was confined to a specific depth interval, at short radial distances from the test well. The values obtained for the lateral permeability are undoubtedly high in many instances where these conditions were not fulfilled. Similarly, it was assumed that virtually all the water discharged by a well was derived from unconfined storage, and that the rate of drawdown at the water table was equal to that at greater depths in the aquifer. Deviations from these conditions probably caused errors in the values obtained for the specific yield in many tests. The methods of analysis and the assumptions underlying these methods are outlined in some detail in the text, so that the reader may evaluate both the merits and the shortcomings of the analysis.

ACKNOWLEDGMENTS

Many persons have contributed to the success of the aquifer test program, either through actual participation or through constructive suggestions and criticisms. Particular credit is due to Messrs. A. H. Arif and M. A. Moghul, of WASID, for their part in both fieldwork and test analysis; to Messrs. M. A. Sultan, A. S. Saffi, M. Hassan Mian, and M. Asghar Khan, of WASID, for their part in the field program; to Messrs. Ansar Hussain, M. Sarwar, and K. F. Sheikh, of WASID, for their assistance in test analysis; and to Mr. Z. U. Kidwai and the staff of the Geohydrology Circle, WASID, for lithologic logging at the test sites.

Special credit is due to Mr. R. L. Cushman, of the U.S. Geological Survey, under whose guidance much of the field program and the initial test analyses were carried out. Special credit is also due to Mr. M. J. Mundorff, also of the Survey, for his assistance in developing the method used for specific-yield determination and for his suggestions regarding many other facets of analysis. The authors are also indebted to Messrs. G. A. LaRocque and D. W. Greenman, of the U.S. Geological Survey, who planned the test program and assisted in many phases of its execution.

The initial phases of the program were carried out under the overall supervision of Mr. Sayyid Hamid, then Superintending Engineer,

Ground Water Development Organization; and under the direct supervision of Mr. S. M. Minhas, and his successor Mr. M. Shemeem Ahmed, of the Ground Water Development Organization. Later phases of the program were carried out under the overall supervision of Mr. S. M. Said, Chief Engineer, WASID; and under the direct supervision of Mr. M. A. Lateef, Superintending Engineer, General Hydrology Circle, WASID.

DESCRIPTION OF THE TEST PROGRAM

TEST PROGRAM IN RECHNA, CHAJ, AND THAL DOĀBS

The aquifer test program in Rechna, Chaj, and Thal Doābs extended from 1954 to 1961. At 78 sites in these 3 doābs, the test well—that is, the pumped well—was installed by WASID; at the remainder, the test was made on an existing well. At all sites the observation wells were installed by WASID.

The test wells installed by WASID were all of the design shown in figure 2. About 80 feet of blank pipe was installed as pump housing; below this, if the formation was favorable, screen was installed. If clay or other low-permeability material was present just below the housing pipe, additional blank pipe was installed through this material, and screen was placed opposite permeable deposits lower in the section. The length of screen usually ranged from 100 to 200 feet depending upon the geology. Many of the existing wells were of this same general design, although the length of blank pipe was sometimes less. In a few places the existing wells were relatively shallow, with little or no blank pipe below the water table.

In most of the tests, four to eight observation wells were installed. These wells were generally provided with a short screen near the center of the depth interval of the test-well screen; they were located at distances ranging from 100 to 2,500 feet from the test well. A common arrangement included observation wells at 100, 200, 400, 600, 800, and 2,500 feet from the test well.

The discharge of the test well was maintained at a constant rate during the test; at most wells this rate was between 1 and 4 cfs. The majority of the tests were of 4–6 days' duration, and measurements of discharge, drawdown in the pumped well, and drawdown in each observation well were made throughout each test.

ORIGINAL TEST ANALYSES

The tests performed in Rechna, Chaj, and Thal Doābs were analyzed initially using the methods developed by Theis (1935) and Cooper and Jacob (1946). In order to make the preliminary conclusions available to the planning groups as quickly as possible, the results of the initial

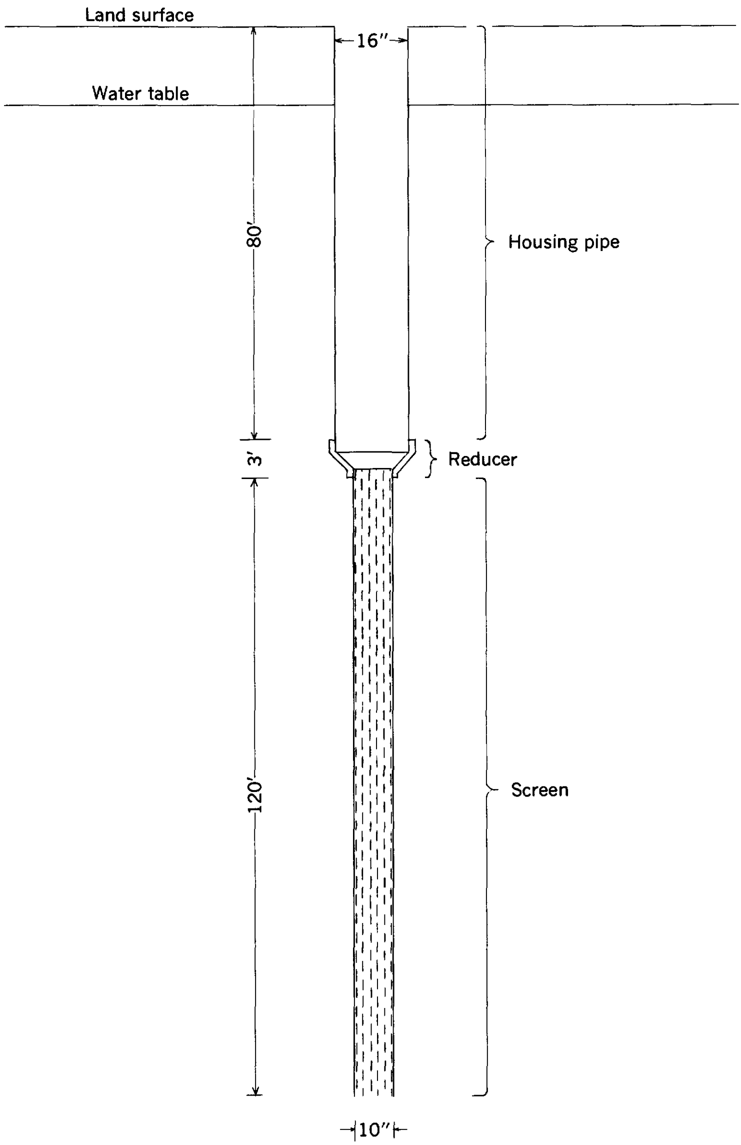

FIGURE 2.—Typical discharging well installed by WASID for aquifer testing.

analyses for Rechna and Chaj Doābs were presented in a preliminary report (Arif and Rehman, 1960), and the results of the initial analyses for Thal Doāb were circulated within WASID in an informal release. It was recognized, however, that the initial analyses were inadequate in certain respects. The value of transmissibility obtained by time-drawdown analysis differed in many tests from that obtained by distance-drawdown analysis, and frequently seemed questionably high. In addition, values obtained for the storage coefficient in the initial analyses were generally very low, suggesting artesian conditions, whereas geologic evidence indicated that the aquifer was for the most part unconfined. Finally, it was recognized that the drawdowns recorded in the observation wells actually represented head changes occurring at the depths at which these wells were screened—that is, at depths as much as 200 feet below the water table. The question as to what effect pumping from the lower strata would have on the water table was largely unanswered by the tests.

PROGRAM OF SPECIAL TESTS IN BĀRI DOĀB

In view of the inconsistencies and inadequacies in the original analyses, several methods of reanalyzing the tests were attempted, and as testing was commenced in Bāri Doāb, certain modifications in the test procedure were introduced.

Shallow observation wells, extending only a short depth below the water table, were installed in addition to the deep observation wells at 19 sites in Bāri Doāb. Each shallow well was paired to one of the deep observation wells at the site; that is, the shallow well was installed 10 feet from one of the deep wells and at the same radial distance from the test well as the deep well. The shallow wells provided data on the behavior of the water table during pumping, whereas the deep observation wells provided data on pressure changes in the depth interval of the test-well screen. The total number of observation wells varied from one site to another. At 4 sites extensive arrays consisting of 8–10 shallow-deep pairs were utilized. At the remaining sites, two or three deep wells and one or two shallow wells were used.

These special tests in Bāri Doāb established certain facts regarding the response of the aquifer to discharge by a well of the design employed in the test program. On the basis of this information, it was possible to select or devise methods by which the earlier tests in Rechna, Chaj, and Thal Doābs could be reanalyzed. Before the results of the special tests are discussed in detail, however, it will be useful to review certain aspects of the theory of flow to a well.

THEORY OF FLOW TO A WELL

GENERAL THEORY OF UNCONFINED FLOW TO WELLS

The differential equation governing flow to a well in an unconfined aquifer may be written

$$P_r\left[\frac{\partial^2 h}{\partial r^2}+\frac{1}{r}\frac{\partial h}{\partial r}\right]+P_z\frac{\partial^2 h}{\partial z^2}=S'\frac{\partial h}{\partial t}, \qquad (1)$$

where P_r is the permeability of the aquifer to flow in a horizontal plane; P_z is the permeability to flow along a vertical; S' is the unit storage coefficient, defined as the quantity of water released from storage in a unit volume of the aquifer per unit decline in head; h is the hydraulic head measured from an arbitrary datum; r and z are the cylindrical coordinates; and t is time. The angular coordinate, θ, does not appear in the equation because flow to a well is generally assumed to exhibit radial symmetry in any horizontal plane. Equation 1 is based upon the assumption that the aquifer exhibits simple two-dimensional anisotropy, in which the principal axes of permeability coincide with the r and z axes. In obtaining equation 1 the aquifer is divided into ring-shaped elements coaxial with the well. The inflow and outflow for a given element are expressed in terms of permeabilities and the directional derivatives of head; their difference is equated to the rate of accumulation of fluid in the element, expressed in terms of storage coefficient and the time derivative of head.

Equation 1 applies at all points in the interior of the aquifer within the region affected by the discharging well; additional relations must be specified which define conditions along the various boundaries of the system. The free water surface constitutes the upper boundary of the aquifer; the boundary condition applying at all points on this free surface has been given by Boulton (1954) as

$$S_y \frac{\partial h}{\partial t} = P_r \left[\frac{\partial h}{\partial r}\right]^2 + P_z \left[\left(\frac{\partial h}{\partial z}\right)^2 - \frac{\partial h}{\partial z}\right], \qquad (2)$$

where S_y represents the specific yield of the aquifer material, defined as the quantity of water that can be drained by gravity from a unit volume of the material.

The boundary condition of equation 2 is obtained by considering the motion of an individual particle of fluid in the free surface. In ground-water flow the hydraulic head, h, is taken as the sum $\frac{p}{\rho g}+z$, where p is the pressure, ρ the fluid density, and g the gravitational constant. The pressure at various points on the free surface is in effect constant, as it is simply the atmospheric pressure. The term $(h-z)$ is thus constant for particles in the free surface, and the total derivative of $(h-z)$ with respect to time for a particle moving in the free surface is zero. This total time derivative is given by

$$\frac{d(h-z)}{dt} = \frac{\partial (h-z)}{\partial t} + \frac{V_r}{S_y}\frac{\partial}{\partial r}(h-z) + \frac{V_z}{S_y}\frac{\partial}{\partial z}(h-z) = 0, \qquad (3)$$

where V_r is the apparent velocity and $\frac{V_r}{S_y}$ the actual particle velocity, in the radial direction; and V_z is the apparent velocity and $\frac{V_z}{S_y}$ the actual particle velocity, in the vertical direction. The boundary condition of equation 2 is obtained by simplifying equation 3 after expressing the apparent velocities in terms of permeabilities and the directional derivatives of head. The development rests on the assumptions that a particle once in the free surface remains in it throughout the problem, and that the specific yield, S_y, gives the fraction of a given cross-sectional area in the aquifer which is actually available for flow.

Throughout the development of theory in this section the quantities P_r, P_z, S' and S_y are considered constants. It will be obvious as the development is continued that this approximation is acceptable for the particular purposes of this paper. In the general case, however, these quantities would have to be treated as functions of position; and in applying the boundary condition given in equation 2, variation of P_r, P_z, and S_y with time would have to be considered, because of the variation in the position of the water table with time.

A comparison of definitions will indicate that specific yield is actually the unit storage coefficient of the aquifer at points occupied by the free surface. It should not be supposed, however, that when the water table reaches a point r_1, z_1 at which the unit storage coefficient has exhibited a value M, the specific yield will also exhibit a value M at that point. The specific yield of equation 2 describes the release of water from storage by the process of drainage or unwatering; in this process the free surface moves downward through the aquifer material, so that parts of the aquifer which were fully saturated initially are left containing only that water which is held by capillary retention. The unit storage coefficient of equation 1 describes the release of water from storage within a segment of the aquifer in which the free surface is not present. The physical processes involved are very different from those involved in "dewatering," and depend upon such factors as the compressibility of the water and of the aquifer. Prior to the time at which the free surface reaches a given point in the aquifer, the release of water from storage in the vicinity of the point is described by the unit storage coefficient; when the free surface reaches the point, the release of water from storage is described by the specific yield.

A further boundary condition which may be applied to most problems of flow into wells may be expressed as

$$\int_{z_1}^{z_2} \left(\frac{\partial h}{\partial r}\right)_w dz = \frac{-Q}{2\pi r_w P_r}, \tag{4}$$

where Q is the discharge of the well at a given instant; $\left(\frac{\partial h}{\partial r}\right)_w$ represents the radial head gradient along the well face at the given instant, expressed as a function of z; z_1 is the elevation of the bottom of the well screen; z_2 is the elevation of the top of the well screen; and r_w is the radius of the well. Equation 4 assumes that there is no vertical flow entering at the bottom of the well, and that the permeability of the material immediately surrounding the screen is the same as that of the formation as a whole. In a gravel-packed well the boundary condition would hold at the radius of the gravel pack rather than at the radius of the well, assuming that there was negligible vertical flow into the gravel pack at its top and bottom. In many cases the term "$\left(\frac{\partial h}{\partial r}\right)_w$" is apparently constant along the length of the screen. In these cases the condition expressed in equation 4 becomes simply

$$\left(\frac{\partial h}{\partial r}\right)_w=\frac{-Q}{2\pi r_w L P_r}, \tag{5}$$

where L is the length of the screen. The condition expressed in equations 4 and 5 is obtained by applying Darcy's Law to the flow through the cylindrical face of the well.

A given well-flow problem will be characterized by several other boundary conditions, in addition to those of equations 2 and 4 or 5. These additional conditions normally differ from one situation to another, depending upon such factors as the discharge pattern of the well, the conditions of recharge and discharge throughout the aquifer, the degree of penetration of the aquifer by the well screen, and so on.

The head changes resulting from a given pattern of well discharge are described by an appropriate solution to the differential equation 1—that is, by an expression $h=f(r, z, t)$ which satisfies equation 1, the boundary conditions of equations 2 and 4, and whatever other boundary conditions apply in the particular problem under study. Thus, in studying any problem of well flow, we are actually seeking particular solutions to equation 1. The possible solutions to equation 1 are of course infinite in number, and may take on a variety of forms depending upon the boundary conditions in effect. In many cases it is difficult or impossible to obtain the required particular solution by rigorous analytical methods, and approximate methods or empirical techniques must be utilized.

Certain simplifications can sometimes be introduced in equations 1 and 2 which make the analysis of well-discharge problems somewhat less difficult. In many problems, for example, the unit storage coefficient is extremely small, and the quantity of water released from storage at points below the water table is negligible. In these cases the left side of equation 1 can be set equal to zero, and the differential equation of flow becomes

$$P_r\left[\frac{\partial^2 h}{\partial r^2}+\frac{1}{r}\frac{\partial h}{\partial r}\right]+P_z\frac{\partial^2 h}{\partial z^2}=0. \tag{6}$$

In many problems, moreover, the quantities $\left(\frac{\partial h}{\partial r}\right)^2$ and $\left(\frac{\partial h}{\partial z}\right)^2$ are very small in comparison to $\frac{\partial h}{\partial z}$. In such cases these terms may be dropped from equation 2, leaving as the boundary condition

$$-P_z\frac{\partial h}{\partial z}=S_y\frac{\partial h}{\partial t}. \tag{7}$$

In applying equation 7 the assumption is made that movement at the free water surface is nearly vertical; the equation relates two expressions for the rate of downward movement at the surface.

THE THEIS EQUATION

C. V. Theis (1935) developed an expression for the drawdown in a homogeneous and isotropic artesian aquifer of infinite lateral extent, due to the discharge of a well at a constant rate. The Theis equation, which is the basis for the conventional methods of pumping-test analysis, is actually a solution to the differential equation

$$\frac{\partial^2 h}{\partial r^2}+\frac{1}{r}\frac{\partial h}{\partial r}=\frac{S}{T}\frac{\partial h}{\partial t}, \tag{8}$$

where S is the storage coefficient of the aquifer, or the quantity of water released from storage per unit decline in head, in a prism of the aquifer having unit base area and extending through the full thickness of the aquifer; and T is the transmissibility of the aquifer, or its lateral permeability multiplied by its thickness. In a homogeneous artesian aquifer, the storage coefficient would theoretically equal the unit storage coefficient, S', multiplied by the thickness of the aquifer, which is assumed constant. Equation 8 can thus be obtained from the more general equation 1, simply by multiplying each side of (1) by the aquifer thickness, and setting $\frac{\partial^2 h}{\partial z^2}$ equal to zero, since the flow is assumed to be entirely horizontal in the confined case.

The Theis equation may be written

$$h=h_0-\frac{Q}{4\pi T}\int_{\frac{r^2S}{4Tt}}^{\infty}\frac{e^{-\Phi}}{\Phi}\,d\Phi, \tag{9}$$

where h_0 is the head in the aquifer prior to pumping; Q is the discharge of the well; and h is the head at a distance r from the well and a time t after the start of pumping.

Although it was developed for confined systems, equation 9 may in some cases provide a reasonable approximation for the head in an unconfined aquifer during the discharge of a well at a constant rate. Certain restrictive conditions must apply in order for the approximation to be valid. The drawdown at any point in the aquifer must be small relative to the total saturated thickness of the aquifer so that the effective aquifer thickness may be considered constant. Vertical head gradients must be virtually negligible; this implies, if the water is derived largely from water-table storage, that downward movement from the water table and reorientation of the flowlines into a radial pattern must involve relatively small head losses. Finally, the various assumptions which govern the application of the Theis equation to artesian systems must be satisfied also for the water-table system, that is, the aquifer must be homogeneous, isotropic, of great lateral extent, and so on.

Where the Theis equation is found to be an acceptable approximation for a water-table system, the storage coefficient to be applied in equations 8 and 9 is in effect the specific yield of the aquifer material. A prism through the aquifer, bounded at the top by the water table, will contribute water from storage by the two processes of unwatering at the surface and release from "artesian" storage below the surface. Normally, the quantity released from storage below the surface will be very small, so that unwatering at the surface will account for virtually all the water released from storage in the prism. Thus, the storage coefficient as defined above will be virtually equal to the specific yield of the material, S_y.

SEMICONFINED FLOW WITH LEAKAGE PROPORTIONAL TO DRAWDOWN

Another well system which has received considerable attention in the literature is that in which radial flow to a well in a semiconfined aquifer is supported by vertical leakage proportional to the drawdown. This "leaky aquifer" problem was considered by Jacob (1946); although much additional work has been done on the problem since that time, the methods proposed in Jacob's original paper seemed best suited to the analysis of pumping-test data in the Punjab.

The theory given here will therefore be confined to a review of Jacob's original work.

The problem is diagrammed in figure 3. The aquifer tapped by the well is overlain by semipervious clay which, in turn, is overlain by a water-table aquifer of high permeability and specific yield. Both the water-table aquifer and the lower aquifer exhibit a head h_0 prior to pumping, and it is assumed that pumping from the lower aquifer produces negligible drawdowns in the water-table zone. Thus, if h represents the head at some point in the lower aquifer during the course of pumping, a head difference given by h_0-h will be present across the confining bed above that point. h_0-h is of course simply the drawdown, s, in the lower aquifer during the course of pumping. It follows that if P' is the vertical permeability of the confining bed and m' is its thickness, the vertical flow per unit area across the semipervious clay is given by Darcy's Law as

$$\frac{Q}{A}=P'\frac{s}{m'}. \tag{10}$$

If it is assumed that the flowlines are refracted 90° immediately upon entering the lower sand, so that flow within the sand is purely radial, the differential equation of flow in the lower sand can be written

$$\frac{\partial^2 s}{\partial r^2}+\frac{1}{r}\frac{\partial s}{\partial r}-\frac{P'}{m'T}s=\frac{S}{T}\frac{\partial s}{\partial t}, \tag{11}$$

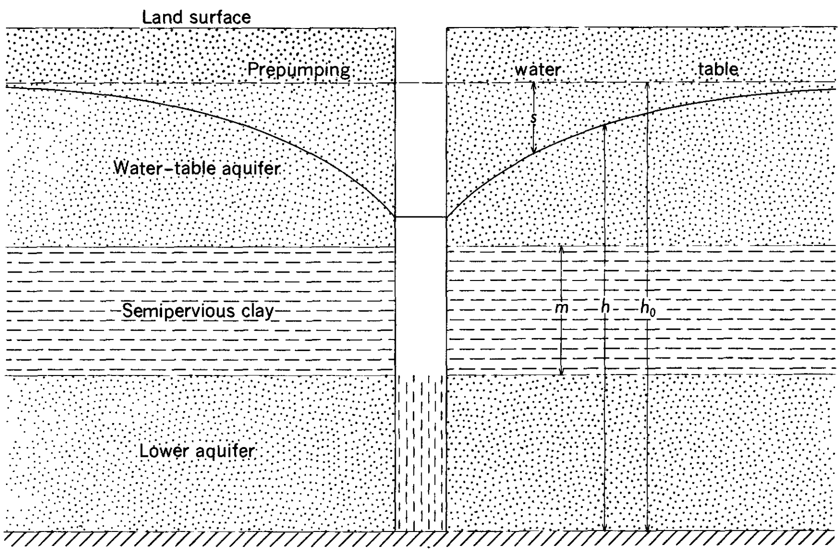

FIGURE 3.—Schematic diagram of drawdown in a semiconfined aquifer.

where s is the drawdown in the lower sand, and S is the storage coefficient and T the transmissibility of the lower sand.

In obtaining equation 11 the lower aquifer is divided into cylindrical elements coaxial with the well. Radial inflow and outflow for a given element are expressed in terms of T and the radial derivative of head, whereas vertical inflow is expressed by an application of equation 9; the net inflow minus outflow for the element is then equated to the rate of accumulation of fluid in the element, expressed in terms of storage coefficient and rate of drawdown, to obtain the equation.

If the drawdowns in the sand have reached equilibrium $\left(\text{that is, if the term } \frac{\partial s}{\partial t} \text{ is zero or at least is very small}\right)$, the equation of flow is simply

$$\frac{\partial^2 s}{\partial r^2}+\frac{1}{r}\frac{\partial s}{\partial r}-\frac{P'}{m'T}s=0. \qquad (12)$$

Jacob presents a solution to equation 12 for the condition in which drawdown is zero at an infinite distance from the well, during discharge of the well at a constant rate Q. The well is considered a line sink, and the boundary conditions in explicit terms are

$$\lim_{r\to 0} r\frac{\partial s}{\partial r}=-\frac{Q}{2\pi T}, \qquad (13)$$

which is obtained by applying Darcy's Law to the flow into the well bore and

$$\lim_{r\to\infty} s=0. \qquad (14)$$

The solution to equation 12 for these conditions is

$$s=\frac{Q}{2\pi T}K_0\left[r\sqrt{\frac{P'}{Tm'}}\right], \qquad (15)$$

where K_0 indicates the modified Bessel function of the second kind of zero order. As a solution to equation 12, equation 15 gives the distribution of drawdown with radial distance from the well at equilibrium for the assumed boundary conditions. A graphical method of pumping-test analysis, based upon equation 15, is outlined in the reference. Some details of this method are given on page G34.

Jacob presents in addition a solution to equation 11—that is, a nonequilibrium solution—giving drawdown as a function of radial distance and time, for the case in which drawdown is zero at some finite radius r_e, during discharge of the well at a constant rate Q. Graphs of drawdown as a function of time, constructed using assumed values of T, S, P', m', and r in this solution, are presented in the reference and compared to the standard Theis pattern of drawdown. The illustration is reproduced as figure 4. It can be seen that the transient effects die out at large values of time for each assumed value of r; that is, at large values of time, the solution approaches a solution to equation 12 for similar boundary conditions. The fraction of the well discharge derived from storage in the lower sand thus becomes negligible at large times. In each case the drawdown pattern at very small values of time closely follows the Theis curve; as time increases, the drawdown in the semiconfined system begins to deviate from the Theis curve, and at large values of time it approaches the steady-state condition.

Although Jacob's analysis applies strictly only for problems of the sort shown in figure 3, it undoubtedly also provides an approximate description for many problems which should actually be analyzed using equation 1 and the boundary conditions of equations 2 and 4. For example, if the water table actually did draw down, in the situation shown in figure 3, appropriate solutions to equations 11 or 12 might still be good approximations for the drawdown in the lower sand,

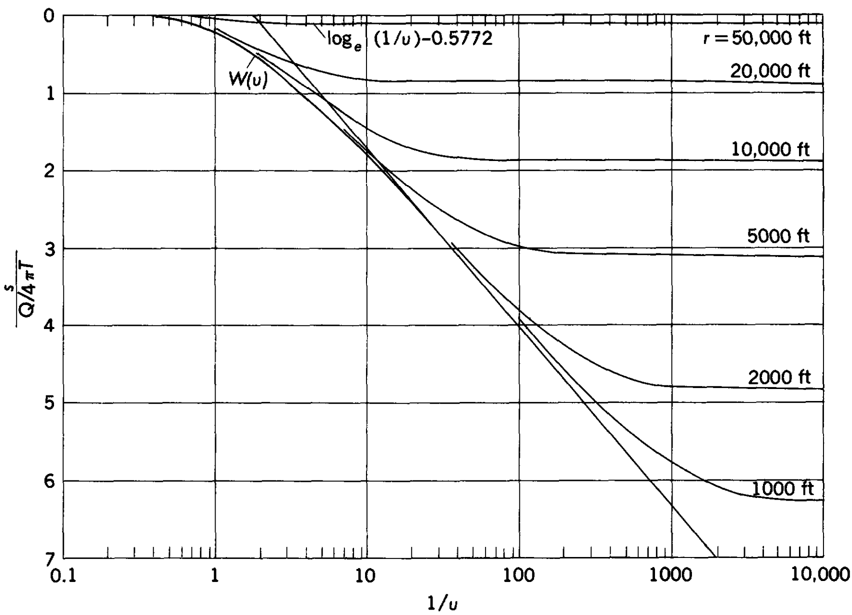

FIGURE 4.—Time-drawdown curves for "leaky" artesian aquifer with well at center discharging at rate Q. Drawdown is 0 at r_e=100,000 ft. T=20,000 sq ft per day; $\sqrt{\frac{Tm'}{P'}}$=20,000 ft $\left(u=\frac{r^2S}{4Tt}\right)$; after Jacob (1946).

provided that the drawdown in the water table was everywhere a relatively small fraction of that just below in the lower sand.

RESULTS OF THE SPECIAL TESTING IN BĀRI DOĀB

The series of special tests demonstrated that, except in special cases, flow to a well of the design employed in the tests is not described by either the Theis equation or equation 15. The flow is described by a solution to equation 1 or, after a short period of pumping, by a solution to equation 6. The boundary conditions include those stated in equations 2 and 4 and whatever other conditions apply for the particular test. The special tests confirmed that the term $\frac{\partial h}{\partial z}$ is usually much greater than $\left(\frac{\partial h}{\partial z}\right)^2$ or $\left(\frac{\partial h}{\partial r}\right)^2$ at points in the water table in the vicinity of the discharging well. Thus, the boundary condition of equation 2 reduces to that of equation 7.

The required solution will vary from one test site to the next, depending upon the boundary conditions which apply, in addition to equations 4 and 7, and upon the values of the parameters P_r, P_z, and S_y in the vicinity of the well. Theoretically, to analyze the data of pumping tests or predict the effects of well discharge, solutions to 1 or 6 satisfying the necessary boundary conditions would have to be obtained. In practice, exact solutions are very difficult to obtain. The series of special tests, however, established empirically that the flow systems in question usually exhibit certain common characteristics. Utilizing this information, approximate methods of pumping-tests analysis were devised, which do not require a full solution of the flow problem for every site. In the following paragraphs, the results of the special tests, and the common characteristics of the flow patterns that can be inferred therefrom, are discussed in some detail. The methods of test analysis based upon these characteristics are outlined on pages G24–G35.

TIME-DRAWDOWN CHARACTERISTICS

Significant drawdown of the water table was recorded at each test site at which shallow observation wells were installed, confirming the theory that the aquifer in effect is an unconfined system. Figure 5 shows the time-drawdown record of a shallow-deep observation-well pair located 300 feet from the test well at Renāla Khurd. The shallow well was screened from 15–20 feet below land surface, and the deep well was screened from 150–165 feet below land surface. The water table at this site prior to pumping was 10 feet below land surface, and the static levels of the deep observation wells were at about the same elevation. The test-well screen extended from 100–220 feet below

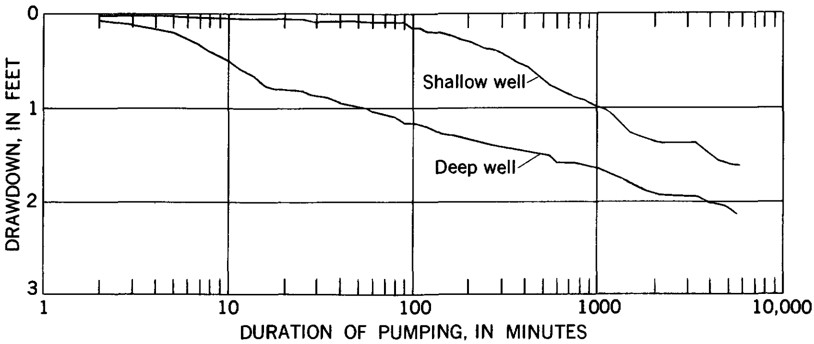

FIGURE 5.—Time-drawdown curves for shallow and deep observation wells located 300 feet from the test well at Renāla Khurd.

land surface. The drawdown of the deep observation well during the first 15 minutes of pumping resembled the standard Theis pattern, suggesting that in the initial moments of pumping much of the discharge was derived from "artesian" storage in the depth interval of the tubewell[1] screen. The drawdown curve then deviated from the Theis pattern in a logarithmic trend roughly resembling the deviation from the Theis pattern occurring in the curves of figure 4. Instead of approaching equilibrium as in figure 4, however, the drawdown continued in the same logarithmic trend to the end of the test. The drawdown of the water table remained negligible for about 90 minutes of pumping. It then increased measurably, and ultimately assumed a logarithmic trend slightly steeper than that of the deep well. During the interval from 1,500–5,000 minutes the rate of drawdown in the shallow well, in feet per minute, can be represented approximately by the function $0.28/t$ where t is the time of pumping in minutes. The rate of drawdown in the deep well during the same interval can be represented approximately by the function $0.25/t$. Thus, the rate of water-table drawdown was approximately 1.12 times the rate of drawdown at depth during this interval, and the head difference between the water table and the deeper zone decreased very slowly as pumping continued.

The fact that the water-table drawdown was negligible during the first 90 minutes of pumping suggests that, in the particular test under consideration, the assumptions underlying equation 11 were approximately satisfied during this interval. The drawdown pattern of the deep well tends to support this theory; the initial 15 minutes of pumping corresponds to the early portions of the curves of figure 4, when the discharge of the well is supported largely by the release of water from artesian storage. The period from 15–90 minutes corresponds to the period of transition in figure 4, when both artesian

[1] "Tubewell," as used in Pakistan, denotes a drilled well; it is used interchangeably with "test well" in this report.

storage and vertical flow proportional to the drawdown are effective. The balance of the test is characterized by appreciable water-table drawdown; vertical flow components between the water table and the deeper zone are no longer proportional to the drawdown at depth, and the assumptions underlying equation 11 are no longer satisfied. The flow pattern during this latter portion of the test involves unwatering at the free surface, downward flow toward the depth interval of the screen, and radial flow toward the well.

Once the rate of drawdown at the water table becomes appreciable throughout the area of influence of the well, the discharge of the well is almost entirely accounted for by unwatering, assuming that recharge is not a factor. To demonstrate this, consider a vertical prism extending through an aquifer from the water table to the impervious base of the aquifer. The cross-sectional area of the prism is A, and we assume that the rate of drawdown is the same throughout the length of the prism. The rate at which water is released from water-table storage in the uppermost part of the prism is given by

$$q=S_y A \frac{\partial h}{\partial t}, \tag{16}$$

where S_y is the specific yield of the aquifer material. The rate at which water is released from "artesian" storage throughout the rest of the prism is given by

$$q'=SA \frac{\partial h}{\partial t}, \tag{17}$$

where S is the "artesian" storage coefficient of the aquifer, or the unit storage coefficient, S', multiplied by the height of the prism. For most unconfined aquifers the specific yield is several hundred or several thousand times as great as the "artesian" storage coefficient. Accordingly, as the rates of drawdown at the water table and at depth are assumed equal, the quantity of water derived from water-table storage within the prism will greatly exceed that derived from "artesian" storage. In the test represented by figure 5, the rate of drawdown at the water table actually becomes slightly greater than that lower in the aquifer. Once this condition is realized in a given segment of the aquifer, the contribution of that part of the aquifer to the well discharge is provided largely by water-table storage, if recharge is negligible.

It should be noted that the behavior illustrated in figure 5 could be expected, on theoretical grounds, in certain well systems. For example, consider a well tapping a sand overlain by a semipervious clay, as shown in figure 6. The water table occurs in the clay; it is assumed that as the well is pumped, water is removed from storage

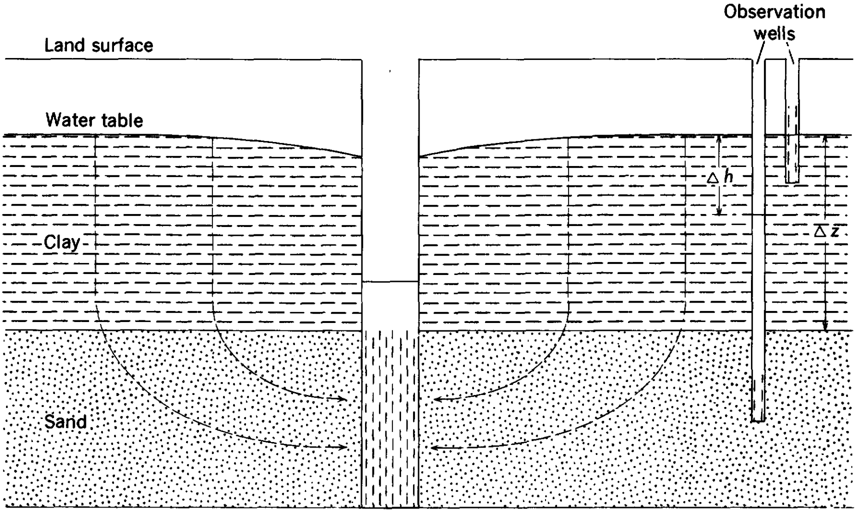

FIGURE 6.—Well system in which vertical flow occurs from the water table to an underlying sand.

at the water table, moves vertically downward through the clay and is reoriented into a radial pattern in the sand. In this system the vertical flow crossing a small horizontal area at any point in the clay is equal to the rate of release of water from storage at the water table directly above that point. (It is assumed that the release of water from storage below the water table is negligible.) The term $\frac{\partial h}{\partial z}$, then, does not vary along a vertical within the clay, at a given time; thus we could write

$$S_y \frac{\partial h}{\partial t} = -P_z \frac{\partial h}{\partial z}, \tag{18}$$

where $\frac{\partial h}{\partial z}$, instead of representing the gradient just at the water table as in equation 7, now represents the gradient at any point in the clay along a given vertical, and is equal to the ratio $\frac{\Delta h}{\Delta z}$, where Δh is the difference in head between an observation well screened in the sand and a shallow well just penetrating to the water table, and Δz is the vertical distance from the water table to the top of the sand in the vicinity of this observation-well pair. If, as is normally true after a sufficient time of pumping, the rate of drawdown of the water table, $\frac{\partial s}{\partial t}$, decreases slowly as pumping is continued, the magnitude of the gradient, $\frac{\partial h}{\partial z}$, must also decrease with time. The relation between these rates of decrease, for the particular flow system we have assumed, may be obtained by making the substitution

$$\frac{\partial s}{\partial t} = -\frac{\partial h}{\partial t} \tag{19}$$

in equation 18 and differentiating the result with respect to time. The rate of drawdown of the water table must in any case slightly exceed that in the sand, so that the head difference, Δh, decreases at a rate sufficient to overcome the continual decrease in the interval Δz, bringing about a net reduction in the gradient.

These conditions, moreover, are not necessarily restricted to geologic systems of the sort illustrated in figure 6. Any unconfined system in which water released from storage at the water table moves almost vertically downward to the zone of the test-well screen, and is there reoriented into a radial pattern, should exhibit the characteristics outlined for the system shown in figure 6. It is very possible, moreover, that numerous other flow systems will exhibit the same or closely similar characteristics. Thus, the behavior of the shallow-deep observation-well pairs during the special tests in Bāri Doāb is in agreement with conditions that could be expected on theoretical grounds.

The similarity of the earliest part of the drawdown curve of the deep well to the Theis curve was observed in almost every instance in which shallow-deep pairs were installed. The final condition, in which the rate of drawdown in the shallow well became equal to, or slightly greater than, that in the corresponding deep well was observed in most pairs located 300 feet or more from the test well. At distances less than 300 feet from the test well, the rate of water-table drawdown was often considerably greater than that at depth; in a few cases, at short distances from the test well, the rate of drawdown of the water table eventually became as much as twice that at depth. For the test analysis, however, the significant fact is that at distances of 300 feet or more the rates of drawdown of corresponding deep and shallow wells eventually became approximately the same. By assuming a similar behavior in the tests in Rechna, Chaj, and Thal Doābs, a method for determining specific yield from the data of those tests where there were no shallow observation wells, was devised. This method is described under "methods of test analysis."

The average rates of drawdown of shallow and deep observation wells are compared in table 1. The table includes all the shallow-deep pairs in the series of tests that were more than 200 feet from the pumping well. The average rate of drawdown of each well during the latter part of the test is shown as an approximate function of the time. The ratio of the rate of drawdown in each shallow well to that in the corresponding deep well is also given.

In most of the special tests, the early portion of the drawdown curves of the deep wells bore some resemblance to the early portions of the curves in figure 4. The resemblance was generally strongest where the drawdown of the water table was a small fraction of that at depth. The assumptions underlying equation 11 were therefore probably approximated at least to some extent, during the initial moments of pumping in many of the tests. The equilibrium condition exhibited in the latter portions of the curves of figure 4, was actually attained in only one test; however, the final logarithmic rate of drawdown in four other tests was extremely low, so that equilibrium was at least approximated. The assumptions underlying equation 15 were thus probably approximated in these instances.

TABLE 1.—*Comparison of rates of drawdown in shallow and deep observation wells*

Test	Site	Radial distance of observation-well pair from test well (feet)	Rate of drawdown in shallow well [1] (ft/min)	Rate of drawdown in deep well [1] (ft/min)	Ratio of rate of drawdown in shallow well to rate of drawdown in deep well
B–5	Renāla Khurd	300	0. 28/*t*	0. 25/*t*	1. 12
		500	. 26/*t*	. 26/*t*	1. 00
		700	. 19/*t*	. 19/*t*	1. 00
		2, 500	. 09/*t*	. 09/*t*	1. 00
7	Chak 27 near Ghamber Rl. St.	300	. 08/*t*	. 07/*t*	1. 14
8	Pākpattan	300	. 45/*t*	. 34/*t*	1. 32
		500	. 28/*t*	. 26/*t*	1. 08
		1, 000	. 15/*t*	. 15/*t*	1. 00
		2, 000	. 02/*t*	. 02/*t*	1. 00
9	Near Harrapa	300	. 22/*t*	. 22/*t*	1. 00
		500	. 34/*t*	. 32/*t*	1. 06
		1, 000	. 30/*t*	. 28/*t*	1. 07
		2, 000	. 11/*t*	. 11/*t*	1. 00
10	Ārifwāla	300	. 19/*t*	. 15/*t*	1. 27
		500	. 21/*t*	. 17/*t*	1. 24
		1, 000	. 13/*t*	. 11/*t*	1. 18
		2, 000	. 22/*t*	. 28/*t*	. 79
12	Luddan	500	. 04/*t*	. 06/*t*	. 67
16	MS 621/4 Lahore-Multān Road	300	. 28/*t*	. 24/*t*	1. 17

[1] Expressed as an approximate function of the time; *t* is the time of pumping in minutes. The indicated drawdown rate applies during the latter portion of each test. Where approximate equilibrium was eventually attained, the indicated rate applies after the initial period of rapid drawdown at depth, but prior to the period of approximate equilibrium.

DISTANCE-DRAWDOWN CHARACTERISTICS

The series of special tests served also to confirm a result suggested by the earlier tests regarding one characteristic of the flow system.

Figure 7 shows an imaginary cylindrical surface in an aquifer, coaxial with the screen of a well and having a height equal to the length of the screen, L. If the radius of this surface is r, its area will be $2\pi rL$. Application of Darcy's Law to the flow through this surface gives

$$\frac{Q_r}{2\pi rL}=-P_r\frac{\partial h}{\partial r}, \qquad (20)$$

where Q_r is the discharge crossing the surface, P_r is the lateral permeability, and $\frac{\partial h}{\partial r}$ is the radial head gradient at the surface. From the ordinary rules of differentiation,

$$\frac{\partial h}{\partial r}=\frac{\partial (\log r)}{\partial r}\frac{\partial h}{\partial (\log r)}=\frac{0.43}{r}\frac{\partial h}{\partial (\log r)}. \qquad (21)$$

Substituting (21) in (20) we obtain

$$\frac{\partial h}{\partial (\log r)}=\frac{-2.3Q_r}{2\pi LP_r}, \qquad (22)$$

or, in terms of drawdown

$$\frac{2.3Q_r}{2\pi LP_r}=\frac{\partial s}{\partial (\log r)}. \qquad (23)$$

If the discharge crossing successive cylindrical surfaces is increasing appreciably toward the screen, owing to contributions from storage, inflow from above, or any other cause, the term $\frac{\partial s}{\partial (\log r)}$ will increase with diminishing radius. If, on the other hand, increases in the term Q_r are negligible between the radius r and the screen—that is, if for

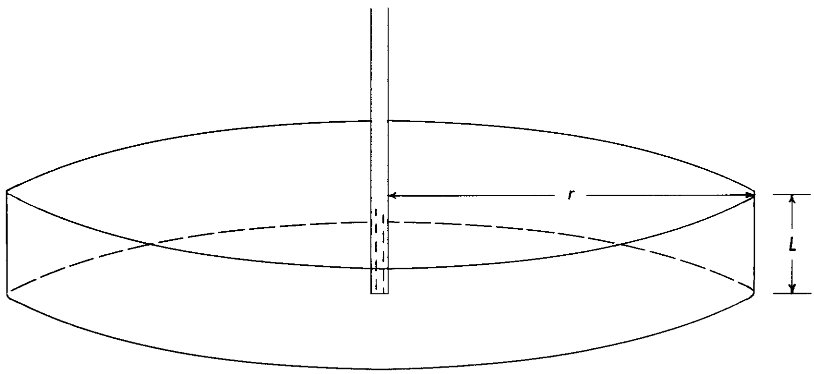

FIGURE 7.—Cylindrical flow surface in an aquifer.

the most part the full discharge of the well, Q_w, is crossing the surface at r—the term $\frac{2.3Q_r}{2\pi LP_r}$ will have virtually the constant value $\frac{2.3Q_w}{2\pi LP_r}$ over the interval from r to the screen. If values of drawdown are plotted versus radial distance on semilog paper, therefore, the plot should be a straight line with a slope $\frac{2.3Q_w}{2\pi LP_r}$. Conversely, if drawdown data for the interval between a given radius and the screen plot along a straight line on semilog paper, it normally can be concluded that within this radius almost the full discharge of the well is moving radially toward the screen in the depth interval penetrated by the screen.

In the various tests performed in Rechna, Chaj, and Thal Doābs, draw down data for observation wells between 100 and 400 feet from the tubewell were generally observed to plot on a straight line on semilog paper by the time several hours of pumping had elapsed. Observation wells closer than 100 feet from the tubewell were generally not used in these tests. In certain of the special tests run in Bāri Doāb, arrays of observation wells from within a few feet of the test well to 2,000 feet from the test well were used. In these tests, drawdown data for observation wells at all radii to 400 feet were observed to fall on an essentially linear semilog plot after several hours of pumping. This behavior would seem to confirm the indications of the earlier tests, that after several hours of pumping, flow in the depth interval of the screen within 400 feet of the well was nearly constant, horizontal and radial, and equal to the discharge of the well. This characteristic of the flow pattern is significant in that it permits an approximate determination of lateral permeability from the pumping-test data.

Both the special tests and the earlier series of tests provide an indication of the area of influence of a well of the design used in the tests. The radius of influence of the well after 3 or 4 days of pumping, as inferred from direct observation or from the extrapolation of observation well data, was in most cases at least 2,000 feet. In some it extended to 5,000 feet. This indicates, among other things, that in reclamation schemes in which one tubewell is installed per square mile, interference effects can be expected relatively early in the course of pumping.

Figure 8 shows the distribution of head in a vertical plane in the vicinity of a discharging well of the design used in the tests. The figure shows the head pattern after pumping of the test well at Pākpattan for 5,000 minutes. The dashed lines represent lines of equal head; the datum is taken as 20 feet below the static water level. As control was limited, consisting only of observations along the water

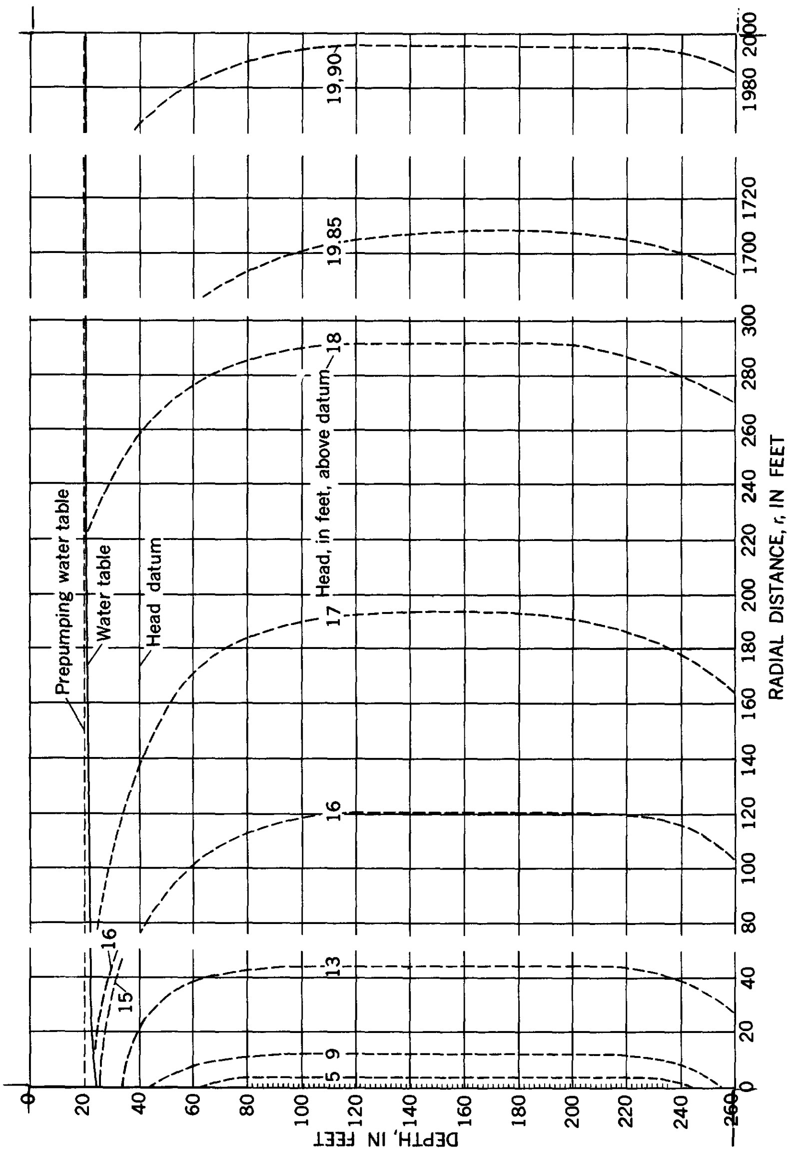

FIGURE 8.—Distribution of head in a vertical plane around test well at Pākpattan.

table and in the depth interval from 165–180 feet, the equipotential pattern involves a considerable degree of interpretation. Nevertheless, there can be relatively little doubt regarding the general features of the head distribution or regarding the general characteristics of the flow pattern that can be inferred therefrom. The unwatered segment of the aquifer differs somewhat in shape, although probably not in volume, from that which would result if an equal quantity of water were discharged by a shallow well screened through the water table. In the latter case one would expect greater drawdowns in the immediate vicinity of the discharging well and a somewhat smaller radius of influence—in other words, a generally steeper and more localized water table "cone" than is shown in figure 8.

In summary, the special tests indicated that the sediments of the Punjab Plain constitute a virtually unconfined anisotropic aquifer; that discharge from a tubewell of the design employed in the tests is sustained largely by unwatering or by recharge, and causes water-table drawdown over a considerable area; and that reanalysis of the earlier pumping tests by revised techniques was necessary.

METHODS OF TEST ANALYSIS

In the light of the information provided by the special tests in Bāri Doāb, the pumping tests in Rechna, Chaj, and Thal Doābs, as well as those in Bāri, were analyzed by revised methods. These methods are described briefly in this section.

LATERAL-PERMEABILITY DETERMINATION

Semilog plots of drawdown versus distance from the pumping well were made from the data of each pumping test, for various times of pumping. The data from observation wells within 400 feet of the pumping well was observed to fall on a straight line in almost all cases after a sufficient duration of pumping. Lateral permeability was determined by inserting the slope of this line, $\frac{\partial s}{\partial (\log r)}$, in the equation

$$P_r = \frac{2.3Q_w}{2\pi L \frac{\partial s}{\partial (\log r)}}, \qquad (24)$$

which is obtained by putting Q_w in place of Q_r in equation 23 and solving the resulting equation for P_r. This method is identical to the Thiem method or the distance-drawdown method of Cooper and Jacob, except that the length of screen, L, is used in place of the thickness of aquifer.

The method depends upon the assumptions that (1) a horizontal radial flow, equal in magnitude to Q_w, occurs within the interval L at radii between the well screen and 400 feet; and that (2) this flow is uniformly distributed throughout the interval L. These assumptions were made somewhat arbitrarily, in the absence of more definite information on the actual distribution of flow. The linearity of the semilog plots, especially those for the tests in which observation wells were installed at short distances from the pumping well, suggests that the first assumption was in effect satisfied in most of the tests. However, analog-model studies since performed in WASID indicate that as much as 25 percent of the flow may occur outside the interval L at a distance of 200 feet from the well, in an aquifer exhibiting the sedimentary characteristics of the Punjab alluvium. A situation of this sort would of course lead to a calculated permeability considerably higher than the true value. It seems probable, therefore, that in many tests the values obtained for the permeability are too high. The permeability results for Bāri Doāb, where observation wells were installed at short distances from the test well, were generally lower than those for the Punjab as a whole. This could reflect, at least partially, errors in some of the earlier tests due to flow outside the limits of the screened interval. Efforts are continuing to obtain more accurate permeability figures, through additional testing and through refinements in the analyses of the existing tests. For the present, however, the figures calculated according to the above assumptions will be given. As a conservative approach, these figures should be taken as upper limits for the permeability.

The assumption that the flow was uniformly distributed throughout the depth interval of the screen was probably not satisfied in many cases. Flowmeter surveys were run in a limited number of the test wells and indicated that in most cases there was some nonuniformity in the pattern of inflow through the screen. In many instances there were short sections, presumably opposite clays, in which little or no flow entered the screen. The lateral-permeability figure thus represents the average lateral permeability of the entire screened interval.

In the construction of each test well by WASID, and presumably also in the construction of the various existing wells that were tested, an effort was made to exclude thick deposits of clay from the screened interval. In the installation of a well by WASID, for example, where the section between the base of the housing pipe and a depth of 130 feet was observed to be predominantly clay, whereas the section between 130 feet and 300 feet was observed to be predominantly sand, blank pipe was installed from the base of the housing pipe to 130 feet. In most cases, therefore, the screened interval consisted predominantly of sand, silt, and thin beds or lenses of clay. The lateral-permeabilities determined in the pumping tests are thus averages for the sediments at the site exclusive of the thick deposits of clay. As such, they are undoubtedly somewhat higher than the average lateral permeability of the section as a whole.

SPECIFIC-YIELD DETERMINATION

One result derived from the special tests conducted in Bāri Doāb was the observation that, after a certain time of pumping, the rate of drawdown of the water table at a given point became almost equal to the rate of drawdown at a point directly below, in the depth interval of the tubewell screen, over most of the area of influence of the well. A method of determining specific yield from the data of the tests in Rechna, Chaj, and Bāri Doābs was developed on the assumption that the same behavior occurred at these test sites. As the range of geologic conditions encountered in Bāri Doāb was representative of those encountered throughout the Punjab, and the design of the test wells was the same in all doābs, the assumption would appear to be justified.

If the volume of water, v, drained by gravity from a porous material, is plotted as a function of the volume of unwatered material, V, the slope of this graph, $\frac{\Delta v}{\Delta V}$, will by definition be equal to the specific yield of the material. For a pumping test, this graph would be made by plotting the volume of water discharged by the well against the volume of the unwatered "cone" of aquifer around the well, for various times during the test. This was in effect the procedure employed in the calculation of specific yields for the pumping-test sites in the Punjab, except that a volume defined by the drawdowns in the deep observation wells was used in place of the true unwatered volume of sediment. The justification for this procedure is based upon the observation that the rates of drawdown in corresponding shallow and deep wells were virtually equal after a sufficient time of pumping in the tests in Bāri Doāb. Details of this justification as well as of the method of calculation are presented in this section.

For a well discharging from an extensive unconfined aquifer, the volume of unwatered material can usually be determined by performing the integration

$$V = 2\pi \int_0^\infty r \cdot s(r)dr, \tag{25}$$

where r is the radial distance from the well, and $s(r)$ is the drawdown of the water table, expressed as a function of r. In equation 25 the unwatered volume around the well is divided into coaxial cylindrical shells of height s and base area $2\pi r dr$; the volumes of these shells are summed in the integration to obtain the total unwatered volume.

The change in the unwatered volume, ΔV, occurring over a given interval of pumping is given by

$$\Delta V = 2\pi \int_0^\infty r[s_2(r) - s_1(r)]dr, \tag{26}$$

where $s_2(r)$ is the drawdown of the water table as a function of r, at a time t_2 marking the end of the interval of pumping; and $s_1(r)$ is that at a time t_1 marking the beginning of the interval. The specific yield is accordingly given by

$$\frac{\Delta v}{\Delta V} = \frac{v_2 - v_1}{2\pi \int_0^\infty r[s_2(r) - s_1(r)]dr}, \tag{27}$$

where v_1 is the total quantity of water discharged by the well from the beginning of pumping to time t_1, and v_2 is the quantity discharged from the beginning of pumping to time t_2.

Data on drawdown of the water table were generally lacking in the tests in Rechna, Chaj and Thal Doābs; the drawdowns recorded in these tests represent head changes that occurred in the depth interval of the test-well screen. Thus, the individual functions $s_2(r)$ and $s_1(r)$ could not be determined for any of these tests. However, if $f_2(r)$ represents the drawdown in the deep horizon, expressed as a function of r, at the time t_2, and $f_1(r)$ represents this drawdown at time t_1, then the assumption that the rates of drawdown are the same along a vertical at any given r, for all values of t between t_1 and t_2, implies that

$$s_2(r) - s_1(r) = f_2(r) - f_1(r). \tag{28}$$

Substituting (28) in (26), we have for the change in unwatered volume

$$\Delta V = 2\pi \int_0^\infty r\,[f_2(r) - f_1(r)]\,dr. \tag{29}$$

Thus, provided the time t_1 is such that the rates of drawdown have become virtually equal, changes in drawdown measured in the deep observation wells may be used in place of changes in water-table drawdown to calculate changes in the unwatered volume. The assumption that the rates of drawdown are equal also implies that most of the withdrawal from storage occurs as unwatering. These changes in unwatered volume may therefore be used, together with changes in total quantity pumped, to determine the specific yield of the material at water-table depth in the vicinity of the test well. We thus have

$$\frac{\Delta v}{\Delta V}=\frac{v_2-v_1}{2\pi \int_0^\infty r[f_2(r)-f_1(r)]dr} \tag{30}$$

for the specific yield.

In practice, the volume between the original water level of the deep observation wells and the piezometric surface defined by their drawdowns during pumping were calculated for several different times of pumping. For a given time of pumping t_i, for which the drawdowns at depth are defined by the function $f_i(r)$, this volume will be

$$V=2\pi \int_0^\infty rf_i(r)dr. \tag{31}$$

The total volume of water discharged by the test well prior to the time in question was plotted opposite the volume calculated by equation 31 for that time. The points representing later times of pumping, for which the rates of drawdown at depth and at the water table could be assumed equal, were generally observed to fall on a straight line. The slope of this line is given by equation 30 and is thus equal to the specific yield of the material at water-table depth.

The calculation indicated in equation 31 was facilitated by a technique suggested by Mr. M. J. Mundorff, of the U.S. Geological Survey. Semilog plots of drawdown versus distance were constructed for several different times of pumping. As noted in the preceding section, the drawdowns within a few hundred feet of the test well normally fell on a straight line on the semilog plot. At larger radial distances the drawdown plot began to deviate from this straight line. The radial flow toward the well was generally maintained by progressive additions from storage along the path of flow; thus, the radial discharge, Q_r, of equation 23 diminished with increasing distance from the pumping well, and the slope, $\frac{\partial s}{\partial (\log r)}$, also diminished. The drawdowns actually deviated from the straight-line segment along a curve of gradually decreasing slope. However, it was found that by drawing a second straight-line segment, as shown in figure 9, or in some cases two additional segments, a plot could be obtained which gave a close approximation to the drawdown at all radii. Figure 9 is based on the data for 1,080 minutes pumping in the test at Mankera in Thal Doāb. The drawdown pattern is typical of that recorded throughout the test series at comparable times of

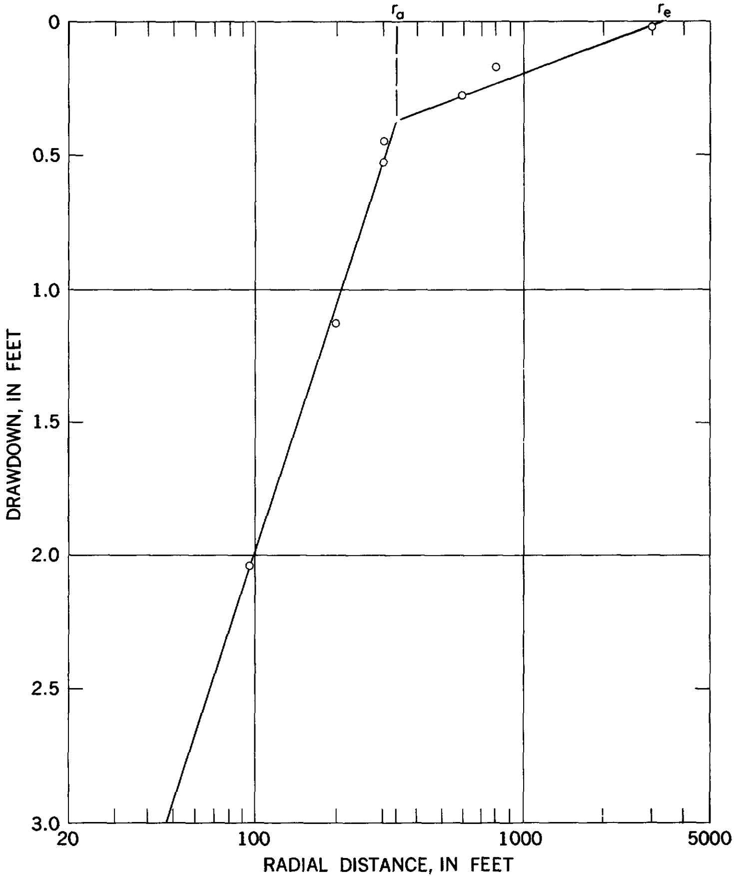

FIGURE 9.—Drawdown versus distance for aquifer test at Mankera.

pumping. Up to a certain radius, which is denoted r_a, the drawdown is given by the expression

$$f(r) = C_1 - K_1 \log r, \tag{32}$$

where C_1 is the intercept of the first line segment on the $r=1$ axis, and K_1 is the slope of this segment in feet per log cycle. The drawdown between r_a and r_e is given approximately by the expression

$$f(r) = C_2 - K_2 \log r, \tag{33}$$

where C_2 is the intercept of the second segment on the $r=1$ axis, and K_2 is its slope. r_e is taken as the effective radius of influence of the well at the time in question. Thus, we may evaluate the integral of equation 31 by dividing the region of integration into three segments, 0 to r_a, r_a to r_e, and r_e to ∞. Throughout the first segment, neglecting the small interval 0 to r_w, the drawdown will be given by equation 32; throughout the second segment it will be given by equation 33; throughout the third segment it will be zero. Equation 31 therefore becomes simply

$$V=2\pi\int_0^{r_a} r(C_1-K_1\log r)dr+2\pi\int_{r_a}^{r_e} r(C_2-K_2\log r)dr. \qquad (34)$$

The integrals in equation 34 are easily evaluated. The terms $(C_1-K_1\log r_a)$ and $(C_2-K_2\log r_a)$ are each equal to the drawdown, s_a, at the radius r_a. The term $(C_2-K_2\log r_e)$ is equal to zero, the drawdown at r_e. Utilizing these relations the integrals may be evaluated as

$$2\pi\int_0^{r_a} r(C_1-K_1\log r)dr=\pi r_a^2\left[s_a+\frac{K_1}{4.6}\right] \qquad (35)$$

and

$$2\pi\int_{r_a}^{r_e} r(C_2-K_2\log r)dr=\frac{\pi K_2}{4.6}(r_e^2-r_a^2)-\pi r_a^2 s_a. \qquad (36)$$

Substituting (35) and (36) in (34), the expression for the volume becomes

$$V=\frac{\pi}{4.6}[K_2r_e^2+(K_1-K_2)r_a^2]. \qquad (37)$$

Following a similar procedure, it can be shown that if n straight-line segments are taken on the semilog plot to represent the drawdown pattern, the volume will be given by

$$V=\frac{\pi}{4.6}[(K_1-K_2)r_1^2+(K_2-K_3)r_2^2+\ldots+(K_{n-1}-K_n)r_{n-1}^2+K_nr_n^2], \qquad (38)$$

where K_1 is the slope, in feet per log cycle, of the first segment of the semilog plot, extending from r_w to r_1; K_2 is the slope of the second segment, extending from r_1 to r_2; and so on. The nth segment, having a slope K_n, extends from r_{n-1} to r_n, where r_n is here the outer limit of the cone of depression.

In computing the specific yield, therefore, semilog plots of drawdown versus distance were first constructed for a number of times of pumping. Reasonably good representations of the drawdown pattern could be obtained using two or three line segments in most instances. The volume calculation indicated in equation 38 was then made for each semilog plot and a graph of pumpage versus V was constructed. In about 75 percent of the tests the points representing later times of pumping fell nearly on a straight line, the slope of which gave reasonable specific capacity figures. Points representing very early times of pumping frequently departed from this line, indicating that at these times the assumption that most of the pumpage was derived from water-table storage was not justified. It should be evident that specific-yield figures determined by this method refer to the material at water-table depth in the vicinity of the test well.

In 35 of the 141 tests the method failed to give reasonable results; virtually all these failures could be attributed to one of two causes: an insufficient array of observation wells or the appearance of excessive recharge. In some tests the observation-well array did not extend beyond a few hundred feet from the pumped well; thus, the second or third segments on the semilog plot could not be determined, or in other words, the cone of depression could not be outlined in sufficient detail to permit application of the method. This shortcoming was particularly evident in Bāri Doāb, where the radial extent of the observation-well array was sacrificed in many tests to provide for the installation of shallow piezometers. The appearance of excessive recharge was indicated on the plot of pumpage versus *V* by a progressive increase in slope. It was of course also indicated by decreases in the slopes of the time drawdown plots of individual wells. The effect of recharge was to make the specific-capacity figure too high, since an appreciable fraction of the pumpage was derived in these instances from sources other than unwatering.

Even for those sites at which recharge was not excessive and the array of observation wells was adequate, the specific-yield figures determined by the method outlined above should be treated as approximations, owing to certain weaknesses inherent in the method. In most instances the rate of drawdown of the water table was probably slightly greater than that of the deep observation wells, as indicated by the results given in table 1. The average ratio of the rate of water-table drawdown to the rate of drawdown in the deep wells for the data in table 1 is 1.06. Thus, calculations of specific yield based upon the assumption that these rates of drawdown are equal may tend to be slightly high. A further source of error lay in the determination of r_e, which was taken as the intercept, on the zero drawdown axis, of the outermost segment of the semilog plot. It is likely that the actual radius along which drawdowns became completely negligible differed in many cases from this intercept. Additional errors were probably introduced in many of the results by the effects of partial or limited recharge. Finally, the method of analysis gave no consideration to the effects of delayed release of water from storage. In developing the method it was assumed that all water retained in the sediments by capillary forces following the fall of the free surface remained permanently in retention—that is, that there was no gradual drainage of this retained water to the water table. If such a process of gradual drainage were occurring during the period used for specific-yield calculation—say, between 1000 and 4000 minutes of pumping—the results would not reflect the ultimate or long-range specific yield of the material.

Because of these weaknesses in the method, each individual specific-yield figure should be taken as a rough approximation. The entire set of figures, however, should give a reasonable idea of the range of specific yields in the Punjab, and the relative frequency with which different specific-yield figures can be expected to occur.

VERTICAL-PERMEABILITY DETERMINATION

If it is assumed that the discharge of the test well is derived entirely from water-table storage or from surface recharge, the discharge of the well will be given by

$$Q_w=P_z\int_0^{r_e}\left(\frac{\partial h}{\partial z}\right)_r 2\pi r dr, \qquad (39)$$

where Q_w is the well discharge; P_z is the vertical permeability of the material between the water table and the depth interval of the test-well screen; and $\left(\frac{\partial h}{\partial z}\right)_r$ is the vertical derivative of head at some specified depth between the water table and the top of the test-well screen, expressed as a function of radial distance from the well. Equation 39 is obtained by applying Darcy's Law to the flow across an imaginary horizontal surface located at some depth between the top of the screen and the water table. The surface is divided into concentric elements, each having an area $2\pi r dr$. The magnitude of the vertical component of flow across each element of area is given by Darcy's Law as $P_z\left(\frac{\partial h}{\partial z}\right)_r 2\pi r dr$; the integration of equation 39 sums these increments of flow for the entire radius of influence of the well, to obtain the total flow crossing the horizontal surface. The discharge of the well must equal this total vertical flow by virtue of the assumption that the entire well discharge is supplied from water-table storage or from surface recharge.

In utilizing equation 39 it is convenient to replace the integration by a summation, in which the concentric elements of area are given by $\pi(r_{0i}^2-r_{ai}^2)$ where r_{0i} is the outer radius of the ith element and r_{ai} is the inner radius of this element. The term $\left(\frac{\partial h}{\partial z}\right)_r$ is approximated by the term $\left(\frac{\Delta h}{\Delta z}\right)_i$, where Δh represents the head difference and Δz the vertical distance between the water table and the depth interval of the screen, within the ith element of area. When these substitutions are made and the resulting equation is solved for P_z, we have

$$P_z=\frac{Q_w}{\pi \sum_{i=1}^{n} \left(\frac{\Delta h}{\Delta z}\right)_i (r_{0i}^2-r_{ai}^2)}, \tag{40}$$

where the summation is carried over n elements of area, covering the full radius of influence of the well.

Equation 40 can be applied to determine vertical permeability only where an extensive array of both shallow and deep observation wells, sufficient to cover the entire radius of influence of the test well, is available. Arrays of this extent were installed at four of the sites in Bāri Doāb, and calculations of vertical permeability were made using equation 40 for each of these sites. It should be evident that the vertical permeability so calculated is an average value for all the material between the water table and the top of the screened interval.

Other computations of vertical permeability were made using Jacob's graphical method for semiconfined flow with leakage proportional to drawdown. This method is based upon equation 15, which describes semiconfined flow under steady-state conditions; the assumptions underlying the method are those underlying equation 15. The series of special tests demonstrated that in many instances the assumptions underlying equation 11—the differential equation for semiconfined flow under nonequilibrium conditions—were approximately satisfied during a short period early in the test. In a few of the special tests, moreover, the more restrictive assumptions underlying the steady-state solution, equation 15, were apparently approximated; and it seems reasonable to assume that these assumptions were also approximated in a few of the earlier tests. The assumptions should be roughly satisfied at those sites where the total drawdown of the water table is a small fraction of that at depth, whereas the rates of drawdown ultimately attained both at depth and at the water table are low, approximating equilibrium.

Ten tests in Rechna and Chaj Doābs were accordingly selected as suitable for analysis by Jacob's method. The selection was based largely upon close similarity of the time drawdown curves to the patterns shown in figure 4. This criterion was used in preference to lithologic data, as it was recognized that a variety of lithologic combinations might produce the required hydraulic conditions, and, conversely, that these hydraulic conditions might be lacking even where lithology seemed favorable.

In applying the method of test analysis based upon equation 15, a plot of drawdown versus radial distance from the pumped well is prepared on logarithmic coordinates. This plot is overlain on a graph of the Bessel Function, $K_0(x)$, versus its argument, x, on logarithmic coordinates at the same scale. The curves are adjusted until they exactly overlie one another at all points. The relation between the curves can be demonstrated by rewriting equation 14 in the form

$$s=\frac{Q}{2\pi T}K_0(x), \tag{41}$$

where the symbol x is used to represent the term $r\sqrt{\frac{P'}{Tm'}}$, so that we may write

$$r=x\sqrt{\frac{Tm'}{P'}}. \tag{42}$$

Values of s are related to the corresponding values of $K_0(x)$ by the constant factor $\frac{Q}{2\pi T}$, whereas values of r are related to the corresponding values of x by the constant factor $\sqrt{\frac{Tm'}{P'}}$. Thus, when the two graphs are properly overlaid, corresponding horizontal axes are separated by a constant distance proportional to log $\frac{Q}{2\pi T}$ whereas corresponding vertical axes are separated by a constant distance proportional to log $\sqrt{\frac{Tm'}{P'}}$. With the curves thus alined, a convenient point (r_1, s_1) is noted on the distance drawdown graph, and the point directly underlying it, $[x_1, K_0(x_1)]$, is marked on the Bessel Function graph. The values s_1 and $K_0(x_1)$ are substituted in equation 41 to determine T, and the values r_1 and x_1 are inserted, together with this value of T, in equation 42 to determine P'.

In applying this method to the selected tests, m' was taken as the interval between the water table and the top of the test-well screen. The vertical permeability thus calculated should be taken as the average vertical permeability of the material between the water table and the top of the screen; and because the assumptions underlying the method of analysis were probably only approximated, the results should be treated as having only order of magnitude significance.

Values of transmissibility for the screened interval were also determined in the graphical procedure and were compared with values of the product P_rL, determined using equation 24. If the semilog plot used to determine $\frac{\partial s}{\partial(\log r)}$ is based upon data for relatively small values of r, and if the assumptions underlying Jacob's method are satisfied, the two values should be reasonably in agreement, since the leakage to the screened interval within a short radial distance of the well will normally be a small fraction of the total flow.

It should be noted that whereas the method based on equation 15 may be suitable for the analysis of certain short-term pumping tests, subject to the conditions outlined in this section, the various relations for semiconfined flow should not be employed to predict the effects of long-range pumping in the Punjab. If pumping is continued for long periods of time, the drawdown of the water table will eventually become appreciable, and the vertical leakage to the screened interval will no longer be proportional to the drawdown in that interval. Since this is the fundamental assumption underlying all the relations for semiconfined flow, it follows that these relations will give misleading figures for the drawdown at depth. In analyzing the effects of long-term pumpage, moreover, it is often drawdown of the water table that is of primary interest. The relations for semiconfined flow are inherently incapable of giving information on water-table drawdown, since they are based upon the assumption that the water table is undisturbed by the pumping.

PRESENTATION AND DISCUSSION OF RESULTS

Table 2 summarizes the results of the lateral permeability and specific-yield determinations for the various tests performed in the Punjab. In addition to the lateral permeability and specific yield, the table lists various details of the test-well construction, the water temperature and static water level at the test site, the duration of the test, the discharge during the test, and the maximum drawdown observed in the test well itself (that is, the drawdown just prior to the end of the test). Plate 1 shows the locations of the various test sites and the lateral-permeability figures determined in each test. Figures 10–12 show semilog plots of drawdown versus distance for several times of pumping in representative tests. These plots were utilized both for the lateral-permeability calculation, which was based upon the innermost straight line segment in each case, and for the volume calculation indicated in equation 38. Figures 13–15 show plots of total pumpage versus the corresponding volume, V, obtained by using equation 38, for three representative tests.

Table 3 summarizes the results of the vertical-permeability calculations. In addition to the vertical-permeability figures, the table lists the values of transmissibility obtained for the screened interval by Jacob's Bessel function method, values of the product P_rL as determined from equation 24, and a brief description of the lithology

TABLE 2.—*Lateral permeabilities, specific yields, and general test information*

Abbreviations: P_r, average lateral permeability of screened interval; S_y, specific yield of material at water-table depth expressed as a dimensionless fraction; L, length of test-well screen; D, depth of test well; DTW, depth to water at test site; γ, temperature of pumped water; Q_w, discharge of test well; Drn, duration of test; Max s_w, maximum drawdown observed in pumping well (just prior to end of test).

Test	Site	P_r cfs per ft ^{2}	S_y decimal fraction	L (ft)	D (ft)	DTW (ft)	γ (°F)	Q_w (cfs)	Drn (hr)	Max s_w (ft)
R-1	Chuharkana Rest House	0.0020		146	226	2.8		2.05	144	15.00
2	do	.0025		120	158	2.8		1.00	144	15.00
6	Chīchoki Mallīan Drain	.0031	0.01	144	288	2.5		2.50	144	22.00
8	U.C. Br.	.0016	.22	197	234	4		1.82	144	13.00
9	Jarānwala	.0024	.09	119	203	7		2.00	144	20.00
10	Miānwāli Br.	.0046	.18	132	202	4		1.25	144	13.00
11	Jhang Branch	.0009		140	243	12	78	1.25	144	14.00
16	Jalālpur	.0048	.12	132	205	6	83	2.00	98	
17	Jahānshāh Drain	.0044	.11	100	202	6	78	1.34	144	16.88
18	Mangoki Drain	.0011	.14	150	301	1.5		2.50	144	18.42
19	Nokhar	.0038	.09	130	150	3	78	.83	72	10.00
20	Qila Dīdār Singh	.0040	.15	100	150	12		1.50	96	10.00
21	Jarānwala Area	.0031	.08	160	345	10.8	76	2.50	144	23.20
22	Lahore-Sargodha Road Mile 40	.0040	.06	144	300	8	91	2.50	156	10.40
24	Mānānwāla Disty	.0050		120	299	9	84	2.54	144	11.73
25	Lakarmandi R.H.	.0060	.09	140	300	10	86	2.75	144	19.87
27	Hitar Wali R.H.	.0055	.09	116	291	16	86	2.54	144	15.64
28	Tail Rody Disty	.0052	.06	139	300	8	88	2.54	144	10.63
29	Māngtānwāla R.H.	.0038	.09	130	276	8	74	2.50	144	17.05
31	Lundiānwāla R. H.	.0041		133	296	5	81	2.54	144	13.06
32	Jhumra	.0039	.08	140	234	8.1	84	2.50	72	10.68
33	Chiniot R. H.	.0070		99	210	13		3.00	120	11.83
34	Aminpur R. H.	.0042	.14	135	300	18	72	2.50	144	16.13
35	Murādwāla R. H.	.0101	.30	129	300	16.5	81	2.54	144	25.17
36	Pacca Anna	.0027	.18	146	300	20	81	2.58	144	19.20
37	Gojra	.0047	.29	143	300	20	84	2.75	144	12.70
38	Tāndliānwāla	.0041	.21	145	282	25	76	3.06	144	15.78
39	Chak No. 210	.0085	.25	120	300	19	90	3.50	144	17.53
40	Buchrinwāla	.0078	.18	146	300	26.7	83	3.00	144	14.58
41	Kamālia	.0029	.20	140	300	27	83	3.00	48	14.00
42	Bhangu Disty	.0051	.17	155	300	18	84	3.50	48	14.26
43	Gujrānwāla	.0052	.10	140	300	12.4	77	3.00	52	13.38
44	Daska	.0020	.02	140	236	12		3.00	48	20.00
45	Karial	.0025		146	246	5	78	3.00	63	
46	Nandipur	.0028	.02	140	300	6	77	3.50	93	15.80
47	Kālāshah Kāku	.0032	.07	140	300	10		3.00	98	12.40
48	Mehta Suja R. H.	.0026	.03	140	289	7	82	3.00	80	19.04
49	Miānwāli	.0019	.03	110	232	13	78	3.00	92	16.06
50	Qila Sattār Shāh	.0041	.13	140	303	14.3	84	3.50	98	18.43
51	Dhoka Mandi	.0018	.10	140	290	12	81	3.50	96	34.60
52	Buchiāna	.0038		140	305	21	85	3.00	116	14.73
53	Sialwala Rest House	.0027	.28	140	283	14	84	3.50	120	22.58
54	Lyallpur	.0028	.06	151	296	10	82	3.50	120	16.40
55	Toba Tek Singh	.0028	.24	150	296	28	83	3.50	96	14.49
56	Sultānpur	.0025	.33	150	300	10	82	3.00	96	15.29
57	Kamrana R. H.	.0036	.11	120	275	10	81	3.00	110	15.45
58	R.D. 64 Vanike Disty	.0027	.12	140	260	9	85	3.50	98	25.95
59	R.D. 55 LCC Pandori	.0012	.11	140	280	10	77	3.00	98	26.18
60	R.D. 19800 Arūri Disty	.0030	.06	140	300	16	84	3.00	98	23.00
C-3	R.D. 5400 Sulki Br.	.0029	.08	140	303	8		1.37	144	
4	R.D. Ghazni Disty	.0023	.39	141	243	4		1.42	144	16.50
5	Lower Jhelum Canal	.0024	.08	140	187	7		1.41	144	16.70
6	R.D. 41000, Southern Br.	.0029		140	210	5	77	1.45	142	16.20
7	R.D. 2700 Lower Jhelum Canal	.0036	.08	140	214	9		1.47	144	17.20
9	R.D. 10000 Khādir Br.	.0040		130	214	12		1.72	144	
10	Alipur Noon	.0036		88	201	3		1.55	144	24.20
11	Melowāl Village	.0040		100	165	15		1.54	144	18.00
14	Kot Moman Village	.0036	.05	110	142	9.1	82	1.05	144	8.00
16	Rakh Bohiwal	.0033		137	160	7		1.27	144	13.20
17	Bhairowāl	.0045	.22	100	150	8	75	1.7	144	17.30
20	Kot Khān	.0019	.21	150	170	9	81	1.50	144	23.80
21	Chiliānwāla	.0018	.13	110	284	5	83	2.10	192	14.20
22	Dharema	.0031	.18	120	206	7		3.50	72	17.30
23	Wazīr Kot	.0022	.20	179	283	14	82	3.30	96	16.76
24	Kot Moman	.0021	.18	140	216	5	83	3.50	96	16.39
25	Lālūwāli	.0014	.25	180	295	6	84	3.55	96	18.24
26	Mitha Lak	.0028	.14	180	306	9	86	3.06	96	30.15
27	Ghauswāla	.0032	.17	113	208	9	80	3.50	96	19.75
28	Choranwala	.0015	.04	130	290	3	82	3.50	144	37.45

TABLE 2.—*Lateral permeabilities, specific yields, and general test information*—Cont.

Test	Site	P_r cfs per ft^2	S_y decimal fraction	L (ft)	D (ft)	DTW (ft)	γ (°F)	Q_w (cfs)	Drn (hr)	Max s_w (ft)
C-29	Chak Saidu Phālia	0.0033	0.10	134	220	7	76	4.00	96	13.64
30	Sāhiwāl	.0021	.12	181	276	6	80	3.00	96	16.20
31	Tail Sobi	.0029	.36	137	215	13		3.50	144	19.90
32	Jhaniān Shāh	.0030	.02	148	236	4	84	3.00	96	14.20
33	Machar Khadi R.D. 19–20	.0024	.14	170	272	6	82	3.40	96	17.72
34	Manghat	.0032	.10	136	218	15	81	3.25	96	12.39
35	Shāh Jiwana	.0016		231	296	9	81	3.05	144	12.15
36	Tail Nūrewāla Disty	.0018	.12	180	296	7	80	3.50	140	19.72
37	Tāalib Wāla	.0046	.29	160	236	10	82	2.75	96	11.74
38	Hassan Khān	.0025		200	296	9	82	3.50	144	12.64
39	Kishangarh	.0029	.16	159	260	10		3.30	100	14.70
40	R. D. 170 Shāhpur Br	.0019	.20	108	184	5		3.50	140	30.10
41	Takht Hazara	.0038	.17	140	216	13		3.06	96	14.33
42	Bhalwāl	.0025	.08	140	260	4		3.60	96	17.98
43	Chak 27 Faqiriān	.0024		115	240	5		3.37	144	26.98
44	Gujrāt	.0021	.08	135	292	7		3.00	96	18.94
T-3	Liaqatabad Textile Mill	.0074	.06	140	345	10	79	1.27	96	10.29
4	Rakh Kasor	.0043	.13	140	264	40	88	1.76	96	9.20
5	Rakh Daggranwali	.0033	.17	120	232	58.8	83	1.97	120	15.20
6	Sarai Krishna	.0053		136	248	24	83	1.91	120	8.80
7	Kalūr Kot	.0022	.31	152	262	30		1.82	120	10.35
8	R. D. 380 Main Line (Thal)	.0065	.26	98	250	26.9		1.62	120	15.92
9	Rakh Mankera	.0031	.09	162	300	43	83	1.65	97	10.40
10	Leiah Sugar Cane Farm	.0032	.14	152	250	10	82	1.24	96	6.89
11	Tail Indus Sugar	.0020		152	230	7	83	1.10	120	2.01
12	Khundian Reclamation Farm	.0036	.05	160	304	55	83	2.04	111	8.69
14	Tibba Mushaqil	.0020	.25	152	232	14		2.70	120	15.53
15	Rakh Dingh	.0025	.09	152	282	44	79.7	1.50	120	11.50
16	Rakh Tuhra Shamali	.0023	.04	128	246	53		1.67	120	12.00
17	Rakh Jamāli	.0017	.08	168	288	56	85	1.40	119	11.00
18	Rakh Jendan Wāla	.0035	.09	136	272	55.7	86	2.23	120	12.14
20	Harnoli	.0026	.10	140	300	12.7		2.92	62	26.54
21	Bundial	.0019		150	300	7.5		3.04	144	22.26
23	Hamoka	.0030	.26	150	300	9.1		2.41	96	16.53
25	Dullewāla	.0036	.09	150	240	44.3	80	3.30	144	13.02
27	Athārān Hazāri	.0016	.07	150	250	14.4	84	2.81	96	21.24
29	Mankera	.0026	.07	150	250	37.7	84	2.35	96	14.15
30	Fatehpur	.0024		150	302	18.6	85	2.70	96	14.75
31	Garh Mahārāja	.0012	.16	140	302	10.8	85	3.00	96	23.89
32	Kapūri	.0019	.01	150	260	11.9		3.00	96	19.82
33	Chaubāra	.0074	.03	140	298	23.9	84	2.90	96	23.68
34	Dād Minor	.0036	.17	146	300	9.1	81	3.15	96	19.23
35	Bhagul Distributary	.0029	.42	150	300	18.1		3.10	96	15.80
36	Jaman Shāh	.0062		156	300	11.3		2.90	144	16.39
37	Munda Crossing	.0022	.11	158	280	19	84	3.75	44	20.74
38	Rangpur	.0072	.03	140	300	9.5	81	3.27	144	20.95
39	Muhammadwāla	.0017	.04	140	301	9.5	84	3.00	96	22.90
40	Dāira Dīn Panāh	.0053		140	280	5.9		3.05	96	13.89
43	M.R.A. I	.0016	.12	150	300	6.3		2.50	116	34.75
45	M.R.A. III	.0020	.09	125	310	8.4	80	3.00	96	18.25
46	M.R.A. IV	.0025	.02	150	302	8.4	84	3.00	96	23.70
B-1	Kot Lakhpat	.0010		120	287	9.4	88	2.00	456	19.13
2	Lajhenwali Village	.0006		100	205	13.7	82	2.00	144	35.92
3	Bhai Pheru	.0013		120	225	34.3	80	2.50	92	22.26
4	Veeram (Kasūr)	.0013		120	225	6.7	78	2.50	96	19.50
5	Renāla Khurd	.0015	.06	120	225	10.1	78	1.75	96	15.81
6	B.S. Link	.0025		120	205	11.1	82	2.00	96	27.70
7	Chak 27 Near Ghamber Rl. St	.0097		125	260	21.2	82	3.00	96	28.90
8	Pākpattan L.R	.0012	.24	152	235	19.7	74	3.00	384	20.50
9	Near Harrapa	.0014	.04	120	315	35	76	2.73	240	46.10
10	Ārifwāla	.0011	.31	135	228	21.9		2.50	244	22.14
11	Iqbālnagar	.0024		120	205	27.0	78	2.50	144	10.45
12	Luddan	.0077		120	355	17.3	82	2.50	87	25.60
13	Near Sheranwāla R.H	.0096		135	355	33.6	76	2.00	96	21.50
14	Porowala Jungle	.0025		120	250	30.4	79	3.00	144	11.30
15	Toba Sultanpur	.0018		140	215	23.0	82	2.44	187	12.08
16	MS 621/4 Lahore-Multān Road	.0015		120	228	23.4	80	2.75	144	17.10
17	Munirabad	.0019		120	230	16.0	82	2.48	144	12.55
18	Korar Pacca	.0018		120	205	27.3	84	2.00	96	24.45
19	Village Khai'a Wāla	.0011		135	220	14.1	80	3.81	68	22.55
20	Near Chak Pathanwāla	.0013		135	215	25.4	82	2.50	144	13.60
21	Shujāābad	.0012		140	220	10.0	82	2.73	72	18.00

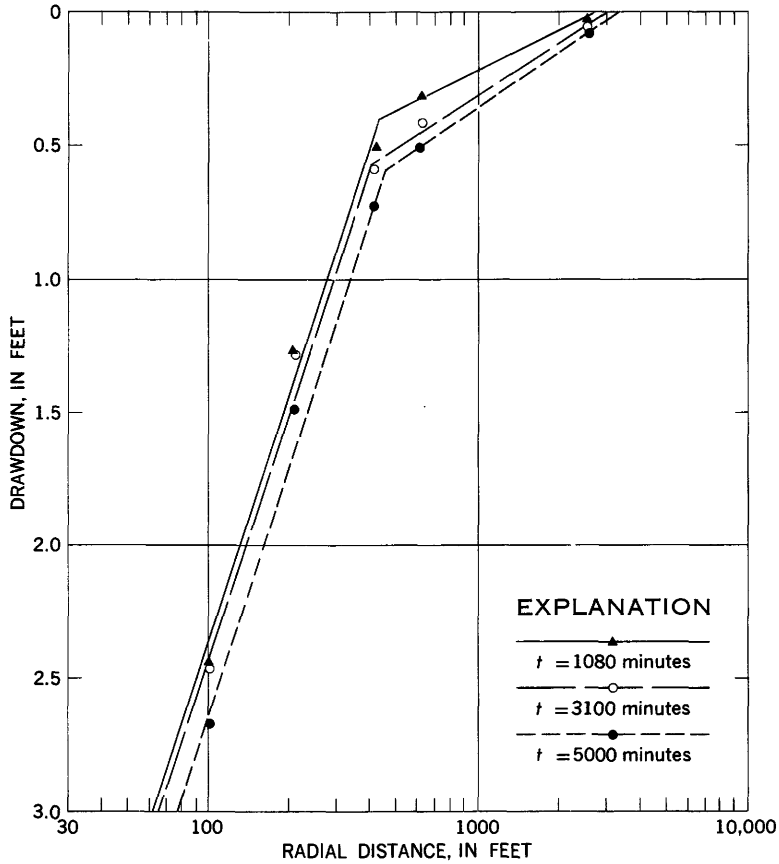

FIGURE 10.—Distance-drawdown plots for various times of pumping, aquifer test at Tibba Mushaqil.

between the water table and the top of the screen at each site. The drawdown curves of the deep wells at test site 10 in Bāri Doāb were observed to be very similar to the curves of figure 4, and the water-table drawdown at this site was observed to be very small. For this site, therefore, vertical permeability was calculated both by equation 40 and by Jacob's graphical procedure; both results are given in the table.

Some brief comments on the results listed in tables 2 and 3 are presented in the following sections.

LATERAL-PERMEABILITY RESULTS

As noted in the discussion of their method of calculation, the lateral-permeability figures should be taken as the average for the screened interval of the test well, which usually consisted predominantly of

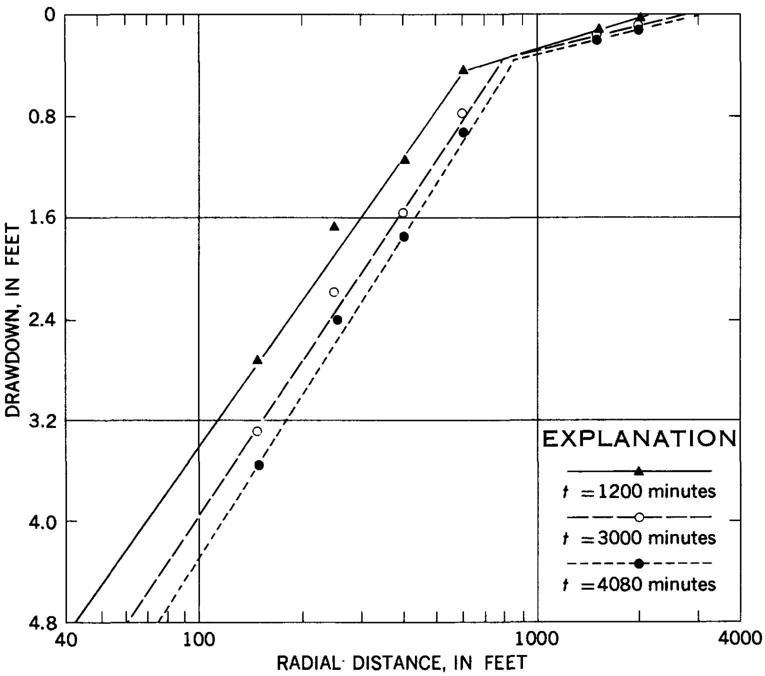

FIGURE 11.—Distance-drawdown plots for various times of pumping, aquifer test at Dharema.

TABLE 3.—*Vertical-permeability results*

Test	Site	Vertical permeability determined in Jacob's Bessel function analysis cfs per ft [2]	Vertical permeability determined by equation 40 cfs per ft [2]	Transmissibility determined in Jacob's Bessel function analysis cfs per ft	P_rL from equation 24 cfs per ft	Lithology of interval between water table and top of screen [1]
R-32	Jhumra	0.00002		0.45	0.55	2 ft clay; 24 ft fine sand; 16 ft medium sand.
35	Murādwāla R.H.	.00006		1.17	1.30	154 ft sand with many clay streaks.
46	Nandipur	.00005		.38	.39	17 ft clay; 137 ft sand.
55	Toba Tek Singh	.00004		.40	.42	43 ft fine sand and silt.
57	Kamrana R.H.	.00004		.53	.43	10 ft clay; 60 ft sand.
58	R.D. 64 Vanike Disty.	.00003		.41	.38	5 ft clay; 68 ft fine sand.
C-9	R.D. 10000 Khādir Br.	.00008		.48	.52	2 ft clay; 16 ft fine sand; 24 ft coarse sand.
20	Kot Khān	.00001		.23	.29	5 ft clay; 6 ft fine sand.
29	Chāk Saidu Phalia	.00012		.47	.44	70 ft fine sand and silt.
40	R.D. 170 Shāhpur Br.	.00003		.20	.21	3 ft clay; 68 ft sand.
B-5	Renāla Khurd		0.00004			2 ft clay; 93 ft sand.
8	Pākpattan		.00015			2 ft clay; 61 ft sand.
9	Near Harrapa		.00042			14 ft clay; 46 ft sand.
10	Ārifwāla	.00007	.00008	.15	.15	11 ft clay; 55 ft sand.

[1] The figures in this column indicate only the total thicknesses of various types of material in the interval between the water table and the top of the screen; they do not constitute logs of the lithologic sequence in this interval.

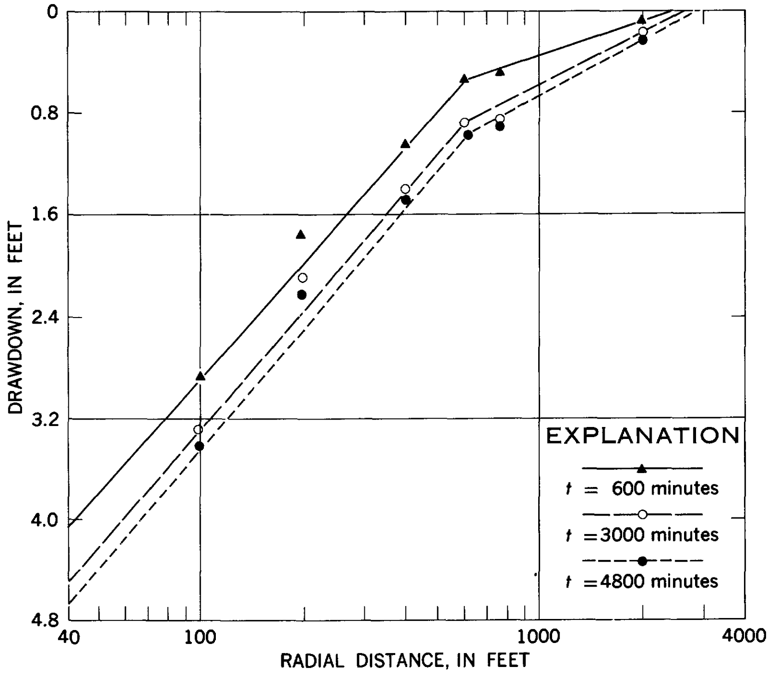

FIGURE 12.—Distance-drawdown plots for various times of pumping, aquifer test at Wazīr Kot.

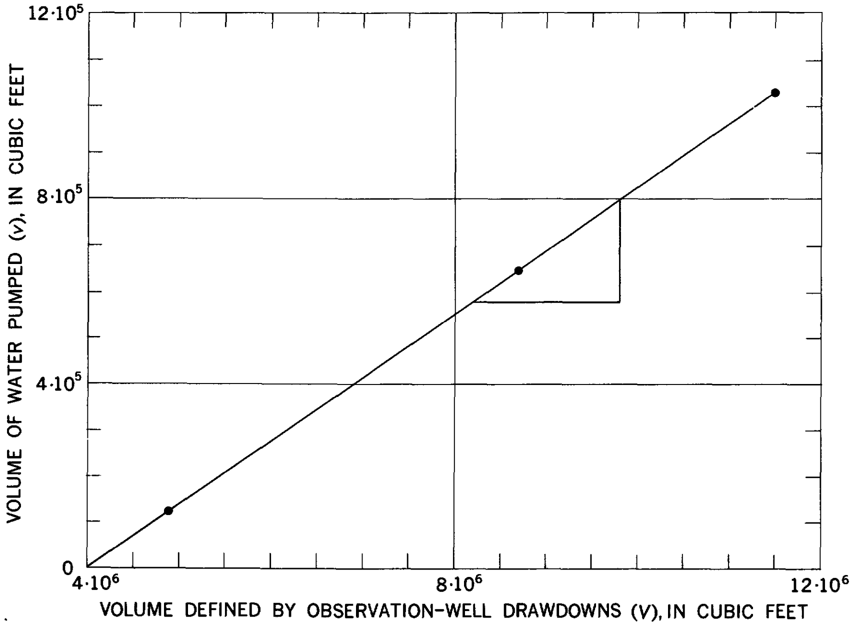

FIGURE 13.—Plot of pumpage versus volume defined by drawdowns, aquifer test at Mitha Lak.

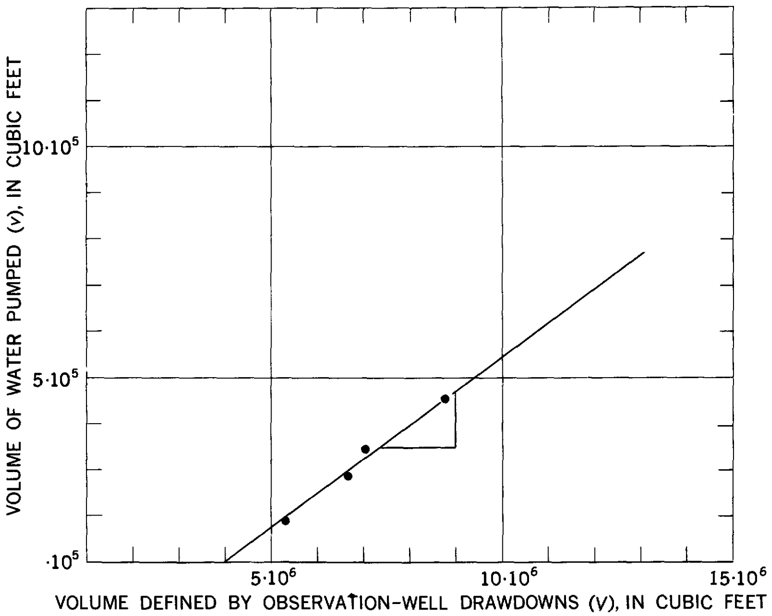

FIGURE 14.—Plot of pumpage versus volume defined by drawdowns, aquifer test at Rakh Jamāli.

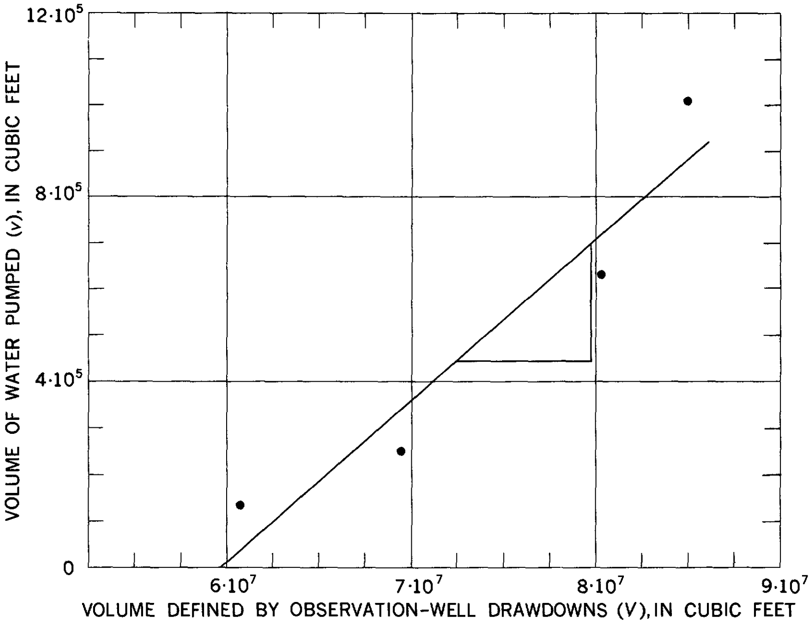

FIGURE 15.—Plot of pumpage versus volume defined by drawdowns, aquifer test at Choranwāla.

sands, silts, and thin beds or lenses of clay. In some instances the figure may be high because of failure of the test conditions to satisfy the assumptions underlying equation 24. As indicated previously, a conservative approach might be to consider the permeability figure quoted in the report as an upper limit in each test.

An attempt was made to correlate the permeability figures with water temperature, inasmuch as fluid viscosity is dependent upon temperature, and the permeability figure obtained in a test is dependent upon viscosity. No meaningful correlation could be obtained, indicating that for the relatively narrow range of ground-water temperatures encountered in the Punjab, the effects of viscosity variation are negligible in comparison with the effects of lithologic variation.

The average lateral permeability figure for the 141 tests was found to be 0.0032 cfs per sq ft. About 71 percent of the results fall in the range 0.0012 to 0.0039 cfs per sq ft. Figure 16 illustrates the distribution of lateral-permeability results for the 141 tests; the percentage of the tests yielding a permeability figure within a given range is plotted versus the range itself on this graph.

For the 49 tests in Rechna Doāb, the average lateral permeability was found to be 0.0038 cfs per sq ft; about 69 percent of the results fall in the range 0.0025 to 0.0053 cfs per sq ft. The distribution of permeability results for these 49 tests is illustrated in figure 17. For the 36 tests in Chaj Doāb, the average lateral permeability was 0.0028 cfs per sq ft; about 69 percent of the results fall in the range 0.0018 to 0.0034 cfs per sq ft. The distribution of results for Chaj is illustrated in figure 18. The average lateral permeability for the 35 tests in Thal Doāb was 0.0033 cfs per sq ft; about 68 percent of the results fall in the range 0.0017 to 0.0037 cfs per sq ft. The incidence of permeability values greater than 0.50 is 20 percent, which is slightly higher than that for any other doab. The permeability distribution for Thal is illustrated in figure 19. The average lateral permeability for the 21 tests in Bāri Doāb was 0.0026 cfs per sq ft; about 67 percent of the results fall in the range 0.0010 to 0.0020 cfs per sq ft. The permeability distribution is shown in figure 20. As noted above, the lower results in Bāri may be due in part to the fact that observation wells closer to the pumped well were used in this doab; if this is so, the results for Bāri may reflect the true characteristics of the Punjab alluvium more closely than do the results for the other doabs.

Geologic sampling has indicated that the proportion of coarse material in the section is slightly higher in Thal Doāb than in the other doabs, presumably because the alluvium was deposited by the larger rivers of the Indus system, the Jhelum and Indus Rivers. Geologic sampling has also suggested a higher proportion of fine

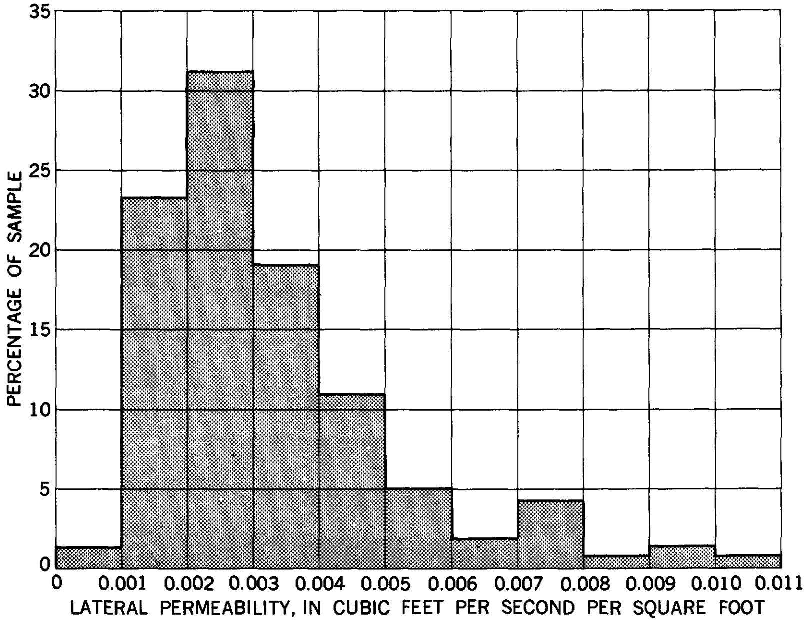

FIGURE 16.—Distribution of lateral permeabilities of screened intervals, Punjab Region.

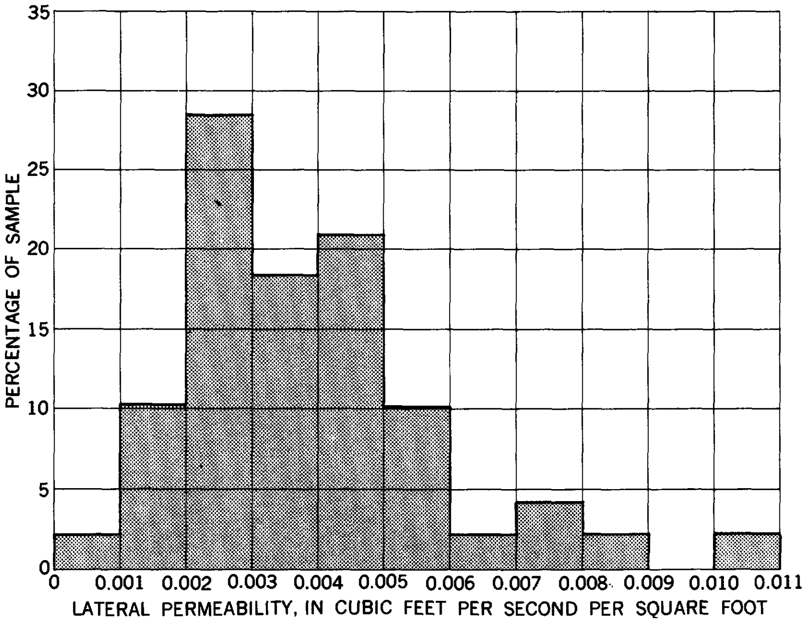

FIGURE 17.—Distribution of lateral permeabilities of screened intervals for 49 tests in Rechna Doāb.

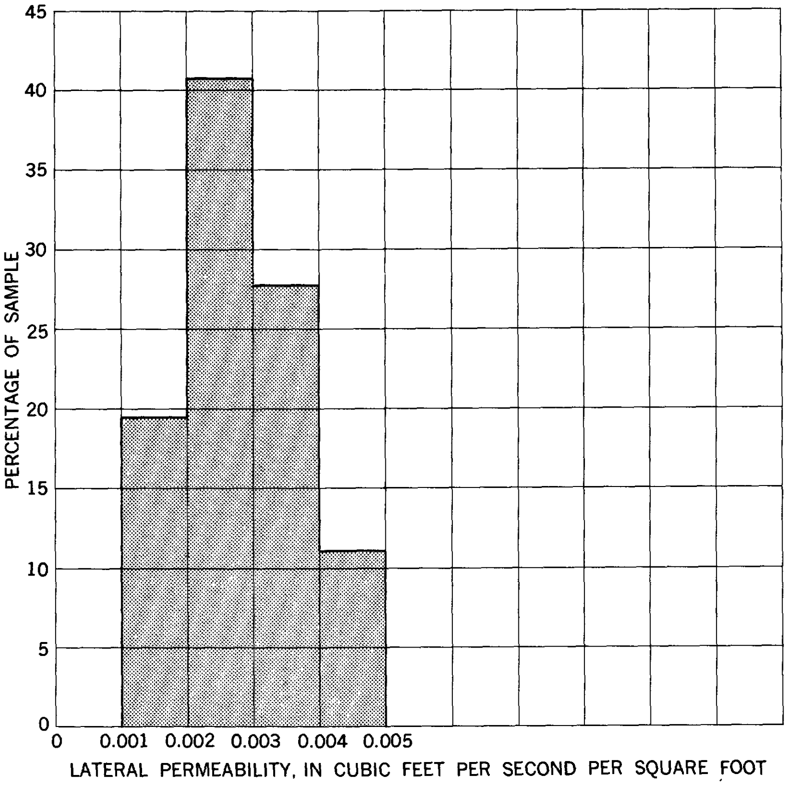

FIGURE 18.—Distribution of lateral permeabilities of screened intervals for 36 tests in Chaj Doāb.

sediment in Chaj than in the other doabs. The pumping-test results seem to be in partial agreement with these conclusions, although they do not wholly confirm them. In this connection it should be borne in mind that geologic sampling indicates the fraction of sand or silt found in each section; whereas pumping tests on wells screened selectively in sand serve rather to compare the permeabilities of the sands at various localities. The two should agree only insofar as the sections exhibiting the highest percentage of sand also include the sands of highest permeability.

Correlation of lateral permeability with lithology of the screened interval was not attempted in this study, owing to the limitations of time and the serious difficulties inherent in such an analysis. As clays of appreciable thickness were deliberately avoided in selecting the screened intervals, and as gravels are rarely found in the alluvium, the range of lithologic variation represented by the screened intervals extended for the most part only from silts to coarse sands.

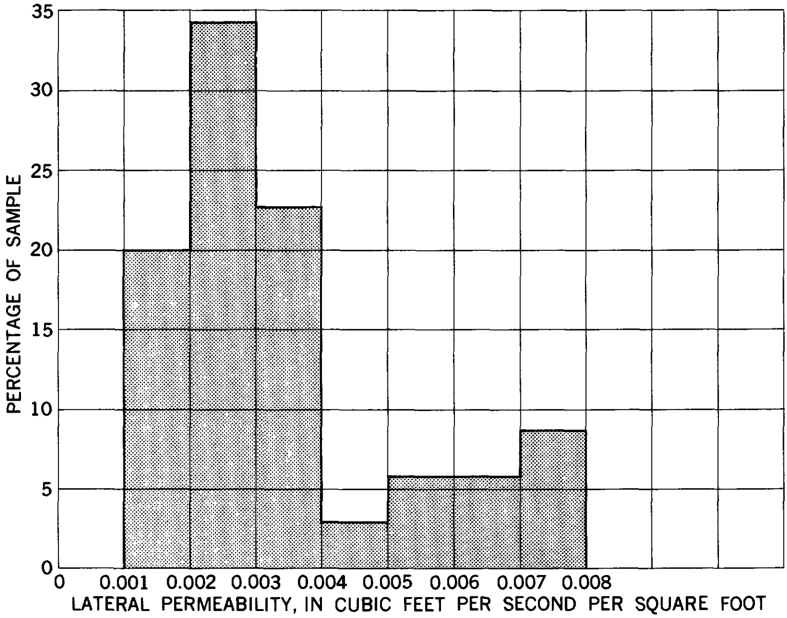

FIGURE 19.—Distribution of lateral permeabilities of screened intervals for 35 tests in Thal Doāb.

Meaningful permeability correlation would of course require accurate and consistent identification of narrow lithologic subdivisions within this range. It is doubtful whether the lithologic data for the test wells possess this requisite refinement, in view of the fact that it was collected by many different geologists, through field logging of the wells during rotary drilling, frequently under adverse field conditions. Each screened interval, moreover, generally contained numerous layers of differing lithologic character and thickness. It would be difficult to develop an adequate quantitative means of relating the various combinations of lithology and thickness to the resultant average lateral permeability figures. However, although these difficulties have precluded a lithology-permeability correlation in the amount of time available for the present study, it is nevertheless hoped that such a correlation can be attempted as a separate study in the future.

In many problems an average permeability for the entire section, including thick deposits of clay, may be required, rather than the figures given above for the screened intervals alone. It is not possible at present to discuss ranges for this overall permeability with any degree of precision. However, an approximate idea of the magnitude of this overall permeability can be derived from geologic considerations. Lithologic evidence suggests that about 20 percent of the sedimentary material consists of clays in thick deposits—that

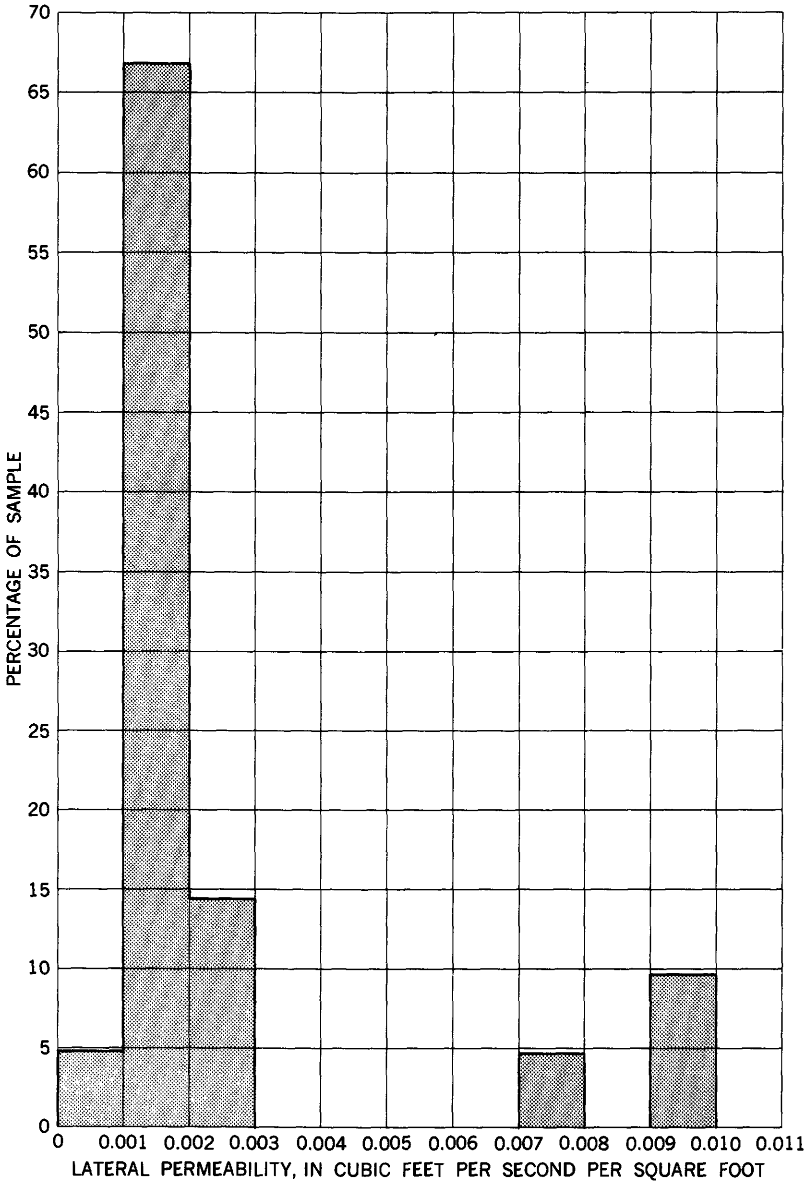

FIGURE 20.—Distribution of lateral permeabilities of screened intervals for 21 tests in Bāri Doāb.

is, in deposits of sufficient thickness to merit exclusion from the screened portion of a well. This percentage varies of course from one locality to another; it appears to be generally high in Chaj Doāb and low in Thal Doāb. The figure quoted is believed to be a representative average for the Punjab as a whole. On this basis, taking the permeability of these clay deposits as virtually equal to zero, the lateral permeability of a representative section of the Punjab alluvium, including these clays, could normally be expected to fall between 0.0005 and 0.0050 cfs per sq ft; an average figure would be about 0.0025 cfs per sq ft.

In using any of the above lateral-permeability results, it should be kept in mind that the screened intervals were all confined to the upper 400 feet of sediment. The lateral permeabilities at greater depths may tend to be generally lower, owing to compression and compaction.

SPECIFIC-YIELD RESULTS

As already noted, the method of specific-yield calculation proved inapplicable in 35 of the 141 tests. In about two-thirds of the remaining 106 tests, the plots of pumpage versus V gave a fairly well defined straight line, similar to those shown in figures 13 and 14; in the remaining third the plot was less clearly defined, as in figure 15. The plot of figure 15 suggests that limited, though gradually increasing, recharge is affecting the cone of depression; this condition was observed in many instances. In such cases an effort was made to select a slope based upon relatively early data—that is, data taken before recharge became dominant, but after the effects of "artesian" storage had disappeared.

Figure 21 illustrates the distribution of specific-yield results; the percentage of the 106 tests giving a specific-yield figure within a certain range is plotted against the range itself in this figure. Seventy percent of the specific-yield determinations lie in the range 0.03 to 0.19, whereas nearly 90 percent lie in the range 0.02 to 0.26. The average figure for the entire 106 tests was 0.14, whereas the most frequently observed value was 0.09.

The specific-yield results apply to the material at water-table depth in the vicinity of the test site, and should be taken as rough approximations because of the weaknesses in the method of analysis. It should be recognized that, if the water table in a given locality declines or rises into a strata of different type, the specific yield for that locality will change.

In the description of the method of specific-yield calculation, four sources of error were indicated: recharge, difference between the drawdown rates at depth and at the water table, errors in the determination of the radius of influence of the well, and delayed drainage of initially retained water. The first two of these would tend to produce specific-yield results larger than the true values. Thus, it would seem that positive errors are slightly more probable than

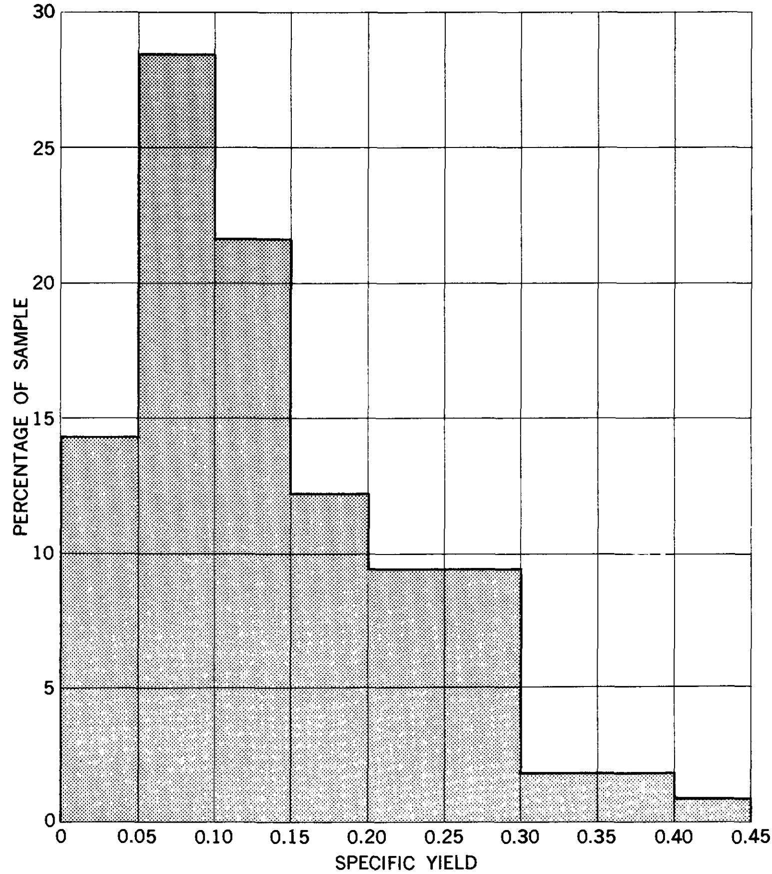

FIGURE 21.—Distribution of specific-yield results for all tests.

negative errors; in particular, it seems likely that the highest results—those above 0.30, for example—contain appreciable positive errors.

Because of their approximate nature, any correlation of the specific-yield results with lithologic or hydrologic factors is of questionable value. Two such correlations were nevertheless attempted. In the first of these the data were separated according to the material present at water-table depth. In 67 of the tests a reasonably definite classification of sand or clay could be assigned to the material at water-table depth. In the remaining 39 tests the lithologic data for this depth were inconclusive, because of rapid vertical change in lithology within the interval of drawdown, or because of the general difficulties noted previously in obtaining lithologic data for the test wells. These 39 tests were therefore excluded from the specific yield-lithology correlation.

At 34 sites the material at water-table depth was reported as sand. The distribution of specific-yield results for these tests is shown in figure 22. About 70 percent of the determinations fall in the range 0.04 to 0.19. The average specific yield for these tests was 0.16. The distribution is relatively broad—the most frequently occurring results are 0.09, 0.10, 0.17 and 0.18, all of which occur three times. At 33 sites the material at water-table depth was clay. The distribution of specific-yield results for these sites is shown in figure 23. About 70 percent of the determinations fall in the range 0.02 to 0.13, whereas the average for this group is 0.11. It should be noted, however, that in six tests of the clay group, the specific-yield determination falls in the range 0.20 to 0.30. It is relatively certain, on the basis of general experience with aquifer materials, that either the specific-yield figures or the lithologic data are in error for these tests. If these tests were excluded, the average for the clay group would be 0.08, which is also the most frequently occurring result for the clay group.

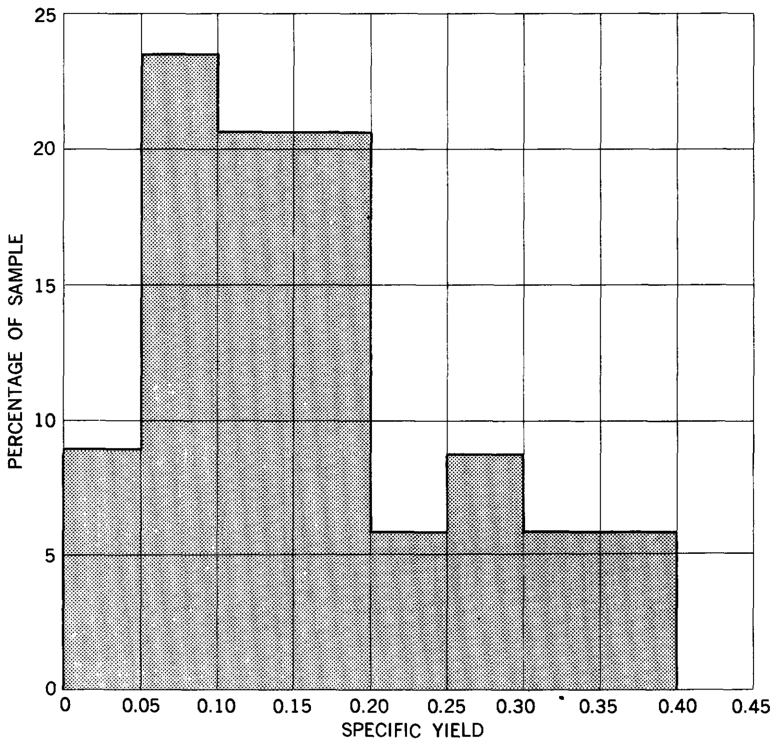

FIGURE 22.—Distribution of specific-yield results for 34 tests in aquifers where the water table is in sand.

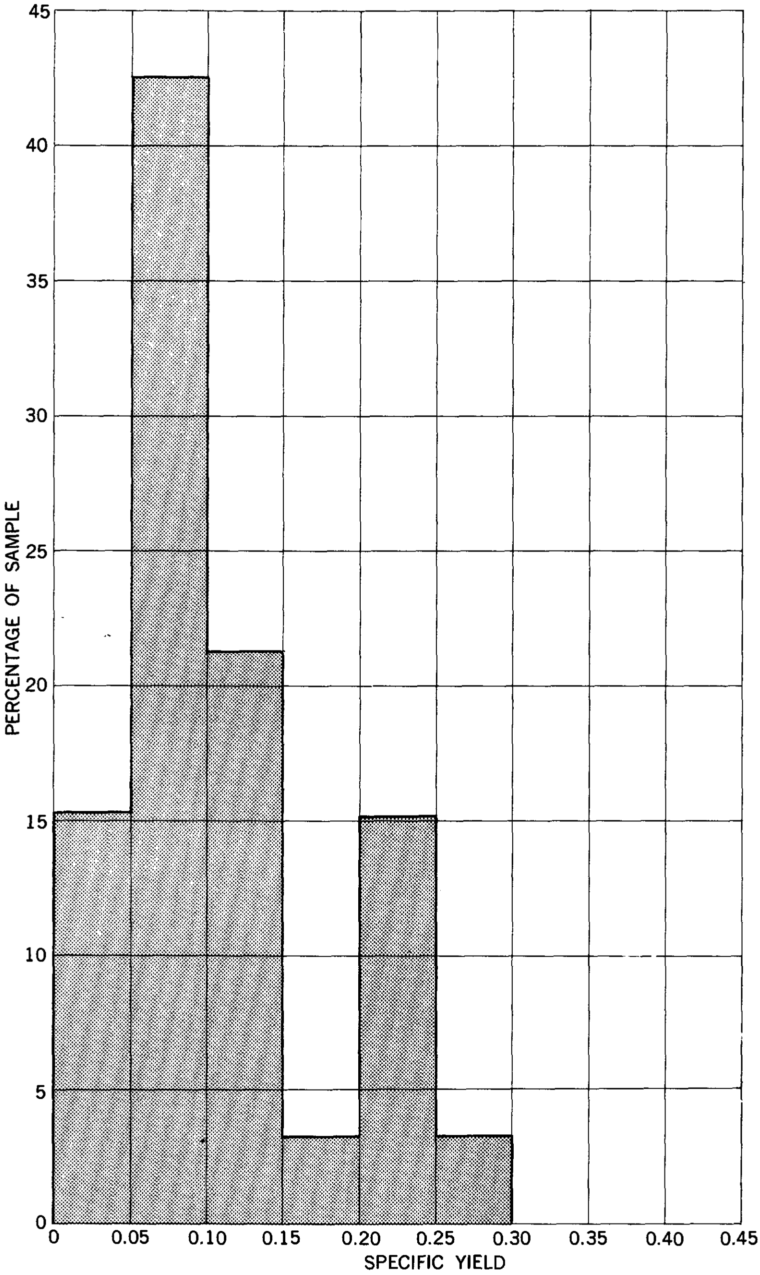

FIGURE 23.—Distribution of specific-yield results for 33 tests in aquifers where the water table is in clay.

In spite of the fact that both the lithologic and specific-yield data are approximate, it seems reasonable to conclude that a specific yield between 0.04 and 0.19 can be expected in about 70 percent of the localities at which the water table lies in sand; whereas a specific yield between 0.02 and 0.13 can be expected in at least 70 percent of the sites at which the water table lies in clay.

The second correlation was made to determine whether the depth of the water table below land surface had any measurable effect upon the specific-yield value. If it is assumed that a capillary fringe a few feet in thickness overlies the water table at all points, there is reason to expect that the specific yields determined by pumping tests will be low in areas of shallow water table, where the capillary fringe extends to the surface. Additions to storage in these instances would involve excessive rise of the water table through material already highly saturated; whereas withdrawals from storage would involve excessive fall of the water table, leaving a highly saturated unwatered volume. A study was accordingly made to determine whether specific-yield figures in the Punjab showed any tendency toward lower values in areas of shallow water table. The results of this study were negative; no clear tendency toward lower values at shallow depths to water could be detected. It should be noted, however, that only those tests having known lithology at water-table depth were considered in the study; the various tests showing sand at the water table were compared in one group to ascertain the effect of depth to water, and the tests showing clay at the water table were compared in another group. The number of tests in each group may not have been sufficient to provide a meaningful result. Another possible explanation for the negative results is that the width of the highly saturated part of the capillary fringe may have been measured in inches, rather than feet, at many of the sites, whereas the minimum depth to water recorded was 1 foot. Finally, it may be that the entire effect of depth to water is negligible compared to the effects of slight lithologic variation.

VERTICAL-PERMEABILITY RESULTS

Each vertical-permeability figure in table 3 should, as already noted, be interpreted as an average for the entire section between the water table and the top of the test-well screen. In the instance in which vertical permeability was computed by both methods, the results agreed to within 13 percent of the higher value. The transmissibility values determined in Jacob's curve-matching procedure, moreover, are all reasonably close to the values of P_rL determined by equation

23. These results suggest that the vertical-permeability figures can be accepted as having at least order of magnitude significance.

The tests selected for analysis by Jacob's method were those in which the drawdown pattern particularly suggested the applicability of this method. It might seem that tests selected on this basis would tend to represent sites where a section of relatively low permeability separates the water table from the tubewell screen. The lithologic data for the selected sites, however, give little support to this hypothesis. Ultimately, the factors that determine whether or not Jacob's method can be applied are the ratio of drawdown at the water table to drawdown at depth, and the degree to which the entire system approximates equilibrium. In some cases a high specific yield or a source of recharge at the surface might be more effective than a confining bed of low permeability in producing low drawdown of the water table and approximate equilibrium. In others a thick section of moderately permeable material might be as effective as a thinner section of low permeability material in producing a large difference in drawdown between the water table and the screened interval. In other words, it is the interrelation of all the lithologic and hydrologic factors at a given site which determines whether the hydraulic conditions underlying Jacob's method will be approximated. The presence of a low permeability bed would contribute to the appearance of these conditions, but might not be sufficient in itself to produce them. At the same time, other factors might combine to produce the requisite conditions at sites where no low permeability interval were present. Thus, although the vertical permeabilities at these 10 selected sites may be somewhat lower than the average for the Punjab, they are probably not of a lower order of magnitude. This conclusion is supported by the fact that the four vertical permeabilities calculated by equation 40 are of virtually the same order of magnitude as those calculated by Jacob's method.

The results indicate that the vertical permeability of a section of unconsolidated alluvium in the Punjab can be expected to fall in the range 10^{-5} to 10^{-3} cfs per sq ft. The sampled sections were all confined to the uppermost 200 feet of the aquifer; it is possible that due to compaction and other processes, the vertical permeability at greater depths may be generally lower. It is important to realize that in no case do the test results provide the true anisotropy of the alluvial material at a point. The lateral permeability is an average for the screened interval, whereas the vertical permeability is an average for the overlying material. However, an idea of the overall anisotropy of a sizable volume of the sediments can be gained from the entire set of test results. Taking 0.0025 as an average for the lateral permeability of the section, including clays, and 10^{-4} as an approximate average for the vertical permeability, an average anisotropy ratio, P_r/P_z, of 25 to 1 is obtained. This figure refers, of course, to the ratio of the overall lateral permeability of a section to its overall vertical permeability and not to the true anisotropy ratio at any particular point.

For the tests listed in table 3, the ratio of the lateral permeability of the screened interval to the vertical permeability of the overlying section varies between 3.3:1 and 195:1; the average of these ratios for the 14 tests is 76:1. Although ratios of this type do not constitute a measure of anisotropy, they should prove useful in the prediction of well performance.

UTILIZATION OF THE TEST RESULTS

A detailed discussion of the various ways in which permeability and specific-yield figures can be used is somewhat beyond the scope of this paper, as it is intended primarily as a report of data on aquifer constants. Studies dealing with utilization of the data in predicting the effects of pumpage, as well as in other problems, will be undertaken by WASID in the future. The discussion given in this section will be confined to a few preliminary remarks.

Generally speaking, the data will be used in analysis of the ground-water system in the Punjab and in evaluation of the effects of various engineering works upon that system. Such analysis and evaluation are accomplished through solutions to the differential equations of ground-water flow. In some instances, known solutions, such as the Theis equation, are applied to the problems encountered, whereas in others, entirely new solutions must be obtained. Thus, the utilization of permeability and specific-yield data is in effect a process of finding solutions to the differential equations of flow.

Probably the most versatile means of obtaining such solutions at present is the electric analog model, in which the flow and storage of water in an aquifer are simulated by the flow and storage of charge in a resistance-capacitance network. Using the permeability and specific-yield data, analog models of this type can readily be constructed; solutions to a wide variety of hydrologic problems may then be obtained by performing the corresponding model experiments. Numerical methods of solving differential equations offer another relatively versatile means of analyzing hydrologic problems, and the permeability and specific-yield figures should permit application of these methods in obtaining solutions to the equations of flow for a variety of boundary conditions.

Formal analytical solutions to the differential equations of flow are available for only a small fraction of the problems normally encountered, and even when available, they usually apply only as approximations to the actual field conditions. In spite of these limitations, however, formal solutions constitute an important means of attacking hydrologic problems. Probably the best known of these solutions is the Theis equation; the question must therefore arise as to whether the Theis equation, and the various relations built up from it, can be used to predict the effects of pumping in the Punjab. For wells of the type constructed by WASID in the test series, the answer to this question is obvious. The drawdown curves of these wells seldom followed the Theis pattern, and the Theis time-drawdown and distance-drawdown methods of analysis generally gave inconsistent results. The conclusion is that the Theis analysis seldom applies to this type of well and that its use in drawdown prediction would lead to erroneous results. The reason evidently lies in the fact that the well screen is separated from the water table by a considerable thickness of sediment, generally of low average vertical permeability. The assumption that vertical head gradients are negligible and the flow nearly horizontal is not, therefore, justified. However, if the regional water table is eventually lowered to the depth of the well screens in the reclamation projects of the Punjab, further drawdowns may possibly be estimated by Theis methods; and even at present, Theis methods may prove suitable for estimating the effects of discharge from shallow wells, where no clay intervenes between the water table and the screen.

As these remarks on the applicability of the Theis equation suggest, formal solutions must always be evaluated for conformity to conditions existing in the field before they are applied. If the various assumptions underlying a solution are found to approximate field conditions, the solution can be utilized; otherwise it will give misleading results. Many formal solutions, among them the Theis equation, are phrased in terms of transmissibility and storage coefficient; in adapting these solutions to conditions in the Punjab it will usually be found that storage coefficient should be taken as the specific yield of the sediment, whereas transmissibility should be taken as the lateral permeability of the sediment multiplied by the effective thickness of flow. Since in most problems the lateral flow can be assumed to occur largely through the sands or silts rather than through the clays, a good effective transmissibility for a section can probably be obtained by multiplying a permeability figure (representative of those obtained for the screened intervals in the test series) by the thickness of sand and silt present in the section.

Analog-model techniques, numerical methods, and formal mathematical solution constitute the principal means of solving the flow equations. In special cases, however, various processes of approximation may be combined to obtain rough solutions. For example, it may sometimes be possible to divide the region of flow into two or more subregions, in each of which a differential equation somewhat simpler than the original may be applied. Solutions describing the flow system in each subregion may then be obtained and fitted together to obtain an approximate description of the entire flow system. Graphical methods, such as trial-and-error flownet construction, constitute another useful method of arriving at approximate solutions. It is difficult to generalize on the application of methods of this sort. They depend as much upon a grasp of the physical characteristics of the particular flow problem in question as upon mathematical techniques. When they can be utilized, however, they may prove more convenient and fruitful than the more rigorous approaches.

In summary, any quantitative appraisal of ground-water resources involves, in one way or another, solutions to the differential equations of flow. These solutions, in turn, depend upon the aquifer properties—in particular the permeability and specific yield. The data presented in this paper, if used in solutions that actually represent hydrologic conditions in the Punjab, should permit reasonable quantitative appraisal of the ground-water resources of the area.

SUMMARY AND CONCLUSIONS

The alluvial material of the Punjab constitutes an extensive heterogeneous and anisotropic unconfined aquifer. The lateral permeability of the sediments, exclusive of thick deposits of clay, in the upper 300 feet of the section normally ranges from 0.001 to 0.006 cfs per sq ft. The average value is 0.0032 cfs per sq ft. These figures may, however, be somewhat higher than the actual values, owing to certain weaknesses in the method of calculation.

Discharge from tubewells as much as 300 feet in depth results in drawdown of the water table and is in fact almost wholly sustained by dewatering or surface recharge, even where the top of the well screen is 150 feet below the water table. The drawdown of the water table is always less than the drawdown at depth in the aquifer, and in some cases, during 4 or 5 days of pumping, is only a small fraction of that at depth; the rate of water-table drawdown, however, usually becomes approximately equal to that at depth after a short time of pumping. The specific yield of the alluvial material normally varies between 0.02 and 0.26. The average is 0.14.

The few calculations of vertical permeability that were made indicate that the average vertical permeability of the alluvial material is much less than the average lateral permeability. The vertical permeability of the material between the water table and the top of the test-well screen was observed to range between 10^{-5} and 10^{-3} cfs per sq ft.

The results of the test program should facilitate quantitative management of the aquifer in conjunction with the overall development of the water resources of West Pakistan. The current program of WASID involves still further analyses of the completed tests. In these analyses, attempts will be made to determine the entrance losses of the test wells; to obtain further vertical-permeability data; to estimate the effects of recharge in short-term pumping; to achieve improved correlations between lithology and aquifer constants; and to develop improved methods of predicting the effects of pumpage. Any significant results that are derived from these analyses will be reported in subsequent WASID releases.

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