Discharge Ratings for Streams at Submerged Section Controls

GEOLOGICAL SURVEY WATER-SUPPLY PAPER 1779-L

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By Wm. S. EISENLOHR, Jr.

CONTRIBUTIONS TO THE HYDROLOGY OF THE UNITED STATES

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Theory and application of rating curves for section controls

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By WM. S. EISENLOHR, JR.

ABSTRACT

Many stream-gaging stations have section controls (which include riffles) that are subject to variable submergence. The rating of such stations requires two gages, one above and another below the control. Section controls are forms of weirs and must be analyzed as such. Weir formulas are not used to rate section controls because it is easier to measure the discharge and one or two other variables, and thus to calibrate the control in place, than it is to measure all the variables in a weir formula. Graphical multiple correlation provides a straightforward method of deriving a rating for any section cotrol. The functional relation is given by the equation $\log Q=f_2(\log H)+f_3\left(\log \frac{F_m}{F_r}\right)$, which may be expanded to include terms for other variables if the need arises. The shapes of the various relation curves are discussed in relation to theoretical considerations. Application of the method is given in detail, step by step, including an illustrative example.

INTRODUCTION

In considering the hydraulics of natural streams, a frequently made assumption is that the principles of open-channel flow can be applied. Sometimes the results are not as accurate as those desired, and sometimes the principles need so be modified slightly, but usually there is a broad group of natural streams that can be treated as open channels; most channels in alluvium can be classed as such. On the other hand, most streams in their headwaters, and many streams far down in their course, flow through a series of pools and riffles. Although pool and riffle flow may occur in only a small part of the total length of all streams, riffles play a dominant role in stream gaging.

Stream gaging is the process of obtaining a continuous record of the discharge of a stream. The discharge, or rate of flow, is a quantity that cannot be observed directly. A discharge measurement is usually computed from observations of depth, width, and velocity. The stage of a stream can be measured directly with more or less precision, and by using proper instrumentation a continuous record can be obtained. The practice of stream gaging is based on the principle that a relation exists between stage and discharge and that a continuous record of discharge can be computed from a continuous record of observed stages by means of such a relation.

The permanence of the stage-discharge relation is a major factor in determining the location of a stream-gaging station. The feature of a stream channel that fixes the stage-discharge relation is called a control. This feature may be a constricted cross section of the stream or a length of stream channel. A riffle in a stream having a pool above is thus a control. Inasmuch as riffles are frequently caused by rock outcrops, are usually fairly stable, and generally make good controls, they are sought as sites for gaging stations. Riffles are classed as section controls along with weirs, dams, and other constricted sections of stream channel that control the stage-discharge relation.

A characteristic of section controls is that they produce a convex water-surface profile at the control. A basic premise of this paper is that a convex water-surface profile identifies a section control. Concave water surfaces are characteristic of a backwater curve and represent an entirely different regimen of flow, which will not be discussed here.

The stage-discharge relation may be simple or complex. At most gaging stations the relation is simple, involving no other factors than stage and discharge. At the very beginning of stream gaging by the Geological Survey, however, it was recognized that other factors might affect this relation, for, as stated by Powell (1891), "it is assumed that for a given height of water the discharge varies within certain limits, depending upon circumstances such as amount of silt carried, condition of channel above and below, and other modifying features, and that rating tables [1] cannot be brought to the refinement of discriminating between all these conditions, but must represent an approximation at their mean." The purpose of this paper is to present a method of "discriminating between all these conditions" so that rating tables can "be brought to the refinement" of present-day requirements.

Early gaging stations were not continued in operation unless a simple stage-discharge relation could be used to obtain the discharge with reasonable accuracy. Gradually, however, the need for discharge records at points where other variables influenced the rating stimulated efforts to determine the effect of those other variables.

[1] Early stage-discharge curves were drawn freehand, seemingly because the ship curves used today were not then available to stream gagers. Therefore the rating table, by the care with which it was computed, was regarded as a refinement of the rating curve and thus the basic instrument. The rating table now is more often regarded as a close approximation, for use in computations, of a carefully drawn rating curve.

Methods were evolved for rating gaging stations under more and more complex situations. Although most of these methods were largely empirical, they served the immediate need. The present paper develops from theoretical considerations a rating procedure that is not much different from the empirical procedures. The differences, however, based on theoretical considerations, eliminate the pitfalls of the empirical procedures and make the method useful for general application.

The first part of the paper describes the physical quantities and relationships involved and discusses the principles used to correlate the observed data and develop the relation curves. The office engineer will probably be the one most interested in this part of the paper. The subsequent sections giving the mechanics of the process in step by step detail, and an illustrative example, were prepared with the subprofessional in mind.

The objective of a rating procedure is to find ways of adjusting the measured discharge for the effects of other variables in order to reduce the scatter of the plotted points in a stage-discharge diagram. This objective is based on the assumption that the original scatter is caused by some other hydraulic variable. Sometimes the scatter is small enough that use of a mean curve will yield a discharge record of acceptable accuracy. Such use is good practice, provided the other variables producing the scatter have been identified and the limits of use of the mean curve have been established. For example, the scatter may be due to filling and scouring, or to growth of vegetation, in a shallow approach channel to the control. A complete rating analysis will show how much of these effects can be tolerated if a mean rating curve is used. Of course, if measuring conditions are poor and the scatter is the result of inaccurate discharge measurements, no method of analysis will reduce the scatter.

This paper is a by-product of work done by the author during the period 1943–51, under the general direction of R. W. Davenport, chief, first of the Division of Water Utilization and then of the Technical Coordination Branch. During that time the author was engaged in the study of backwater problems relating to references before the International Joint Commission, United States and Canada. The ratings for Kootenai River near Copeland, Idaho, Kootenay River near Grohman, British Columbia, and Columbia River at international boundary, Washington, were developed as part of that work. The methods of anlyses used in these and other ratings were the subject of a series of lectures given for several years to engineers of the Surface Water Branch while on detail to the Washington office. The present paper draws heavily on the author's notes for those lectures.

DEFINITIONS AND NOTATION

Throughout this paper great emphasis is laid on definitions of terms. This emphasis is needed owing to the distinction that must be made between terms almost alike and to the different usage by others of the same terms. The usage in this paper was chosen on the basis of logic and simplicity commensurate with established practice and the needs of the problem. For example, the expression **discharge rating** or simply **rating** is used as the all-inclusive term to describe the one or more relations (defined by relation curves) used to determine the discharge from the measured parameters of flow. A **relation curve** defines the relation between two of the variables used in a discharge rating. A complete rating may require the use of several relation curves, but where only one relation curve is needed (the one between stage and discharge) is is called the **rating curve** in accordance with long-established usage.

The **control** is the section or reach of channel downstream from the gage that determines the shape of the relation curves in the rating. This paper will be concerned only with **section controls.** All controls can be classed also as **stable controls** or **shifting controls** according to their permanence with respect to time.

Often a given control is not effective throughout the range in stage that is experienced at a gaging station. In such circumstances, separate ratings need to be developed for each control. The final rating for the gaging station will then be a composite of the ratings for the different controls for the ranges through which they are effective. Generally there is a transition region between controls in which the rating procedures for both controls will yield about the same results with equal and acceptable accuracy.

Backwater in the hydraulic-engineering sense is defined as water backed up or retarded in its course as compared with its normal or natural condition of flow. With respect to a given stream channel, backwater is considered to be present at any point whenever the stage is greater than the minimum stage required to pass the same discharge, steady or variable as the case may be (subcritical depths excluded). The amount of such increase in stage is usually considered to be the amount of backwater.

The term **normal** is used to denote the condition that ordinarily occurs; it is a function of the frequency of occurrence and not a parameter of flow.

Fall is the drop (difference in elevation) of the water surface between two points on a stream—usually permanent gages. Where it is a factor in the rating, several classes of fall are defined as follows:

F = fall in general.

F_m = measured fall.

F_f=**free fall**—the minimum fall between two gages for which the stage discharge relation is unaffected by (free of) backwater. It is the upper limit of F_r and is the maximum fall that can affect the discharge; if the fall is greater than F_f, it has no effect on the discharge.

F_r=**rating fall**—the fall for which the stage-discharge relation is determined, the fall that occurs when the rating discharge occurs, and a fall that bears a fixed ratio to the free fall throughout the range of rating.

F_n=**normal fall**—the fall that ordinarily or normally exists and is dependent on the frequency of occurrence rather than channel hydraulics. A rating can be developed for normal-fall conditions but a normal-fall curve cannot be used to develop a rating.

Additional symbols used frequently in the paper are:

Q =discharge in general.

Q_m=measured discharge.

Q_r=**rating discharge**—the discharge given by the stage-discharge relation curve.

Q_f=**free-fall discharge**—the discharge when the water falls free over the control; that is, the water is not retarded or backed up by water below the control; it is the upper limit of Q at any stage. The rating discharge will be equal to or less than (by a fixed ratio) the free-fall discharge.

Q_{adj}=adjusted discharge—Q_m adjusted for the effect of all known variables.

J=discharge ratio—Q_m/Q_r or Q_m/Q_f.

J_F=discharge ratio explained by fall ratio.

J_x=discharge ratio explained by any variable x.

H=gage height—elevation of water surface above effective zero flow.

t=time.

T=temperature.

Other symbols used have the common meanings in hydraulics and are explained where used.

The symbol f is used throughout the paper indiscriminately to mean "function of." There seems no reason to differentiate between functions when there is no particular interest in the function other than that it exists. Thus in

$$y=f(x),\ y=f\left(\frac{1}{x}\right),\ \text{and}\ x=f(y)$$

the function f, if evaluated would be different in the three equations, but as used here it signifies only that there is a functional relationship between the two variables; when the nature of the function is considered, subscripts are used.

The evaluation of a function, furthermore, need not be in the form of an equation. It may be a graph, such as a rating curve; it may be a table of corresponding values, such as a rating table; or it may be simply a statement such as one of Newton's statements of a law of motion.

The above discussion suggests another point to be observed in considering the development given later. Although written in mathematical form, the equations are physical equations—that is, they state in mathematical shorthand the results of observational data. Physical equations may or may not be subject to mathematical manipulation; they are used to describe physical relationships rather that to solve problems. An important part of each physical equation is the text that accompanies it and describes how the quantities in it are to be measured. An example is the Seddon (1900) equation for the rate of travel of a specific discharge, U. In the nomenclature of this paper, Seddon's equation 2 becomes $UB=dQ/dH$, in which B is the width of the water surface. Seddon points out that dQ/dH "is given by the discharge curves." This statement can be misleading because it is not the steady-flow (or constant-stage) discharge rating curve that yields such information, but the rating curve for the conditions of flow then taking place. The equation is based on the stipulation that dQ/dH "is the rate of change of discharge with *its* change of stage" (italics supplied). The rating curve for this condition may be quite different from the steady-flow rating curve, and confusing the two could lead to serious error.

ANALYSIS OF PHYSICAL RELATIONSHIPS

Almost all stream-gaging stations are established to measure the flow of natural streams, which is usually a complex phenomenon. Flow-measuring devices suitable for use with pipes or for use in hydraulic laboratories have little practical value in stream gaging owing, in part at least, to the great range in flows to be measured. To cite the author's favorite—and a moderate—example, the maximum flow of the Potomac River at Washington, D.C., is more than a thousand times the minimum flow. Therefore in the following presentation, although rigorous development will be used as far as possible, practical considerations will require some deviations.

In order to arrive at a rating procedure that is quite general in its application, the physical quantities and dimensions that affect the flow will be chosen as a starting point. A section control has all the hydraulic characteristics of a weir. The quantities that affect the discharge, Q, over a weir are:

H=head—the height of the upstream water surface above the lowest point of the weir crest,
h_t=height of tailwater above lowest point of weir crest.
y=effective height of weir,
e=thickness of weir crest,
θ=angle of upstream face of weir,
B=breadth of channel at the weir,
k=height of roughness on weir,
g=acceleration of gravity,
ρ=density,
μ=viscosity,
σ=surface tension,
V=mean velocity,
C=weir coefficient.

A common procedure at this point would be to analyze these quantities according to the theory of dimensions. The result would be the usual general equation for flow over a weir:

$$Q=C\sqrt{g}BH^{3/2}, \tag{1}$$

in which

$$C=f\left[\frac{h_t}{H}, \frac{B}{H}, \frac{y}{H}, \frac{e}{H}, \theta, \frac{k}{H}, \frac{VH\rho}{\mu}, \frac{\rho HV^2}{\sigma}\right].$$

Such an equation is applicable to all types of weirs in all types of channels. A gaging-station rating, however, is developed for a particular channel containing a particular weir (riffle) that will be calibrated in place. In such a situation many of the variables in the above equation become constants.

For example, y, e, θ, and k are all dimensions of the channel and weir that are assumed to remain constant regardless of the discharge. There is the possibility that scour and fill of the approach channel will have the effect of varying the effective height of weir, y. Growth of vegetation may have the same effect; it could also change the effective breadth, B, and (especially if it is algae) it could change the effective thickness, e, and height of roughness, k. All these effects are the result of changes with time rather than the result of action of hydraulic variables. They can thus be analyzed in relation to time, t, whenever conditions warrant.

For natural streams we can assume that ρ, μ, and σ are constant, although it is possible that under some circumstances they will not be constant. Great variability in sediment load might cause enough variation in density (ρ) to affect the discharge. On very small streams, surface tension (σ) at a sharp-crested weir might affect the discharge significantly if the range in temperature were very large. The viscosity (μ) might also have some effect. Although it is improbable that these factors will affect a gaging station rating, the possibility should not be overlooked.

The breadth, B, can be expressed as a function of H. The velocity, V, can also be expressed as a function of H, subject to changes in channel geometry with time, t. Then, taking into consideration the assumptions made in the two preceding paragraphs, equation 1 becomes

$$Q=f\,(H, h_t) \tag{2}$$

if we neglect variations with time. Where time must be considered as a variable, the equation would be

$$Q=f\,(H, h_t, t) \tag{2A}$$

The quantity h_t, height of tailwater above lowest point of weir crest, can be measured directly on a regular weir, but for a riffle in a natural channel it is very difficult to determine. Experiment has shown that the discharge is a function of the ratio of tailwater to headwater [2]; that is

$$Q=f\left(\frac{h_t}{H}\right).$$

The ratio h_t/H is known as the "submergence", from the fact that when the tailwater is higher than the lowest point of the weir crest, the weir is considered to be submerged. In the past, submergence has been used to measure the effect on the discharge of submerging a weir. Herschel (1885) was probably the first to express the effect as a proportion of the free-fall discharge, although he actually used head ratio rather than discharge ratio.[3] Later investigators (see King, 1954) have used the discharge ratio directly. A slightly different scale for measuring the same effect can be obtained from data observed more easily in the field. It is easy to convert from one scale to the other, as has been done to show the Herschel (1885) submergence curve for a sharp crested weir (fig. 4). Figure 1 shows that $h_t=H-F_m$. Let F_1 be the fall when the tailwater is at the "lowest point of the weir crest." Then $F_1=H$ and

$$Q=f\left(\frac{h_t}{H}\right)=f\left(\frac{H-F_m}{H}\right)=f\left(1-\frac{F_m}{H}\right)=f\left(1-\frac{F_m}{F_1}\right).$$

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[2] Ratings of artificial controls made at the National Hydraulic Laboratory, Washington, D.C., 1934–35.

[3] The discharge ratio is easily obtained from the head ratio, for $(Q_m=CLH^{3/2})/(Q_f=CLd^{3/2})$ gives $\frac{Q_m}{Q_f}=\left(\frac{H}{d}\right)^{3/2}$ if Herschel's symbols for the heads are used.

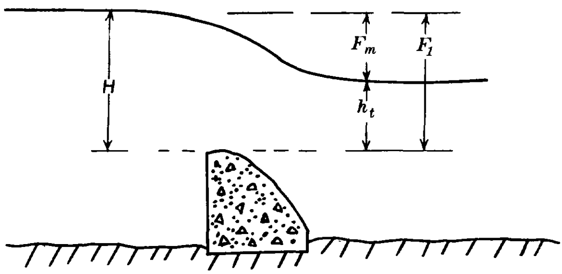

FIGURE 1.—Relation between height of tailwater and fall.

It has been shown by experiment (see footnote 2, p. L8) that F_f can be taken as a constant percentage of F_1; thus $F_f=a_1F_1$, in which a_1 is a constant. Then $Q=f\left(1-\frac{a_1}{F_f}F_m\right)$.

By definition $F_r=a_2F_f$, in which a_2 is a constant. Therefore

$$Q=f\left(1-a_1a_2\frac{F_m}{F_r}\right).$$

Absorbing constants into the function, we have

$$Q=f\left(\frac{F_m}{F_f}\right) \text{ and } Q=f\left(\frac{F_m}{F_r}\right).$$

Equation 2 can now be written

$$Q=f\left(H, \frac{F_m}{F_f}\right) \text{ and } Q=f\left(H, \frac{F_m}{F_r}\right). \tag{3}$$

The evaluation of these functions is discussed in succeeding sections.

The rating for a gaging station having a section control represents the flow over the control. If the gage for measuring the head on the control is very far upstream, there may be difficulties. For example, during rapidly rising stages the slope of the water surface between the gage and the control may be much steeper than usual. In such circumstances the effect would be to increase the velocity of approach beyond that on which the rating is based. This effect will be offset to some extent by the amount of flow going into storage between the gage and control. In this analysis it will be assumed that the gage is close enough to the control that the above effects can be ignored

Another effect associated with variable discharge, which might cause confusion, is the effect of variable submergence of the control. This effect can occur in a stream where there is a substantial pool above the control and another pool below the control that submerges

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the control at all times. On rising stages the upper pool (above the control) will fill faster than the lower pool because the upper pool is storing so much water the inflow to the lower pool is substantially less than the inflow to the upper pool. The effect is to reduce the submergence of the control and thus permit a greater flow over the control for a given gage height than under constant-flow conditions. On falling stages, the lower pool cannot drain as rapidly as the upper pool because the inflow to it includes the water being drained from storage in the upper pool. The effect is a greater submergence of the control, which, in turn allows less than constant-flow discharge. Although these effects can be correlated with rate of change of stage, they are measured directly and more accurately in the fall ratio, F_m/F_f or F_m/F_r.

There may be other occasions when the standard method of rating appears to give unsatisfactory results. In each case, adequate search should be made to make sure that the true cause of unsatisfactory results is found in some real and logical physical relationship.

CORRELATING THE OBSERVATIONAL DATA

A simple stage-discharge relation curve usually is well and easily defined. Such well-defined relations are common in the physical sciences where there is a functional relationship between two variables and where the errors in the observed data are quite small. When a third variable is added to the relationship, the problem is no longer simple. A three-dimensional diagram can always be constructed to show functional relations among three variables, but such a diagram requires a great many observations. However, there are procedures in the field of statistical analysis that can be used to simplify the work. There are many kinds of statistical analyses, but most of them deal with relationships that are never very well defined. For such relationships, a whole system of statistical measures has been developed to test their quality. Furthermore, such relationships usually are assumed to be straight-line functions.

In a discharge rating, the relation curves usually are curvilinear and often are very well defined. In fact they usually are so well defined that the investigator does not hesitate to fit curves of complex mathematical form. This fitting is done graphically and the relation curve is reduced to tabular form for use. The equations of the relation curves and the statistical measures such as standard deviation, probable error, correlation coefficients, and tests of significance have little value. The foregoing explanation shows that the statistical analyses used in the development of ratings belong to a rather specialized group—that of graphical multiple correlation. Few statistics texts deal with this subject, but Ezekiel (1941) gives a comprehensive discussion. Most of the following explanation is based on that discussion.

The best time to consider the physical factors that affect the analysis is at the very beginning. As pointed out by Ezekiel (1941, p. 222), "The conditions to be imposed on the shape of each curve, in view of the logical nature of the relations, are first thought through and stated." This procedure will be followed in connection with each of the relation curves as it is developed in a succeeding section.

In the physical sciences, every relationship can be explained by logical reasoning. The results of such reasoning then impose certain conditions on the analysis in addition to the statistical procedures for, no matter how good the statistical analysis, it will not yield the reasons why a given relationship exists. The investigator must therefore, be sure that all the relation curves can be explained by good logic; for example, the analysis should not be permitted to show a break in curvature of a relation curve where there is no apparent reason for such a break. Before accepting such a break in curvature, the analysis should be reexamined to see that there are no errors in the data and that all, and the right, variables affecting the rating have been considered. (See example of variable submergence, p. L9.

The general equation for the relations of any dependent variable, X_1, to two or more independent variables, X_2, X_3, etc., can be written, according to Ezekiel (1941, p. 221), as

$$X_1=a+f_2(X_2)+f_3(X_3)+\cdots \tag{4}$$

in which a is a constant. This equation is applicable only so long as each independent variable has a functional relationship with X_1 independent of the other functions, that is, f_2 must not be affected by f_3. Occasionally in gaging station ratings a joint relationship may appear. It can usually be removed by a method that is described on page L20. A procedure for evaluating the several relations of equation 4 is described below.

In the following descriptions it is assumed that the independent variables have been so chosen that any intercorrelations are at a low level. Because the relations basically are logarithmic, as explained on page L13, logarithmic plotting is used in figure 2 to illustrate the process. For the simpler arithmetic plotting, the reader is referred to the descriptions given by Ezekiel (1941). Arithmetic scales that are the equivalents of the logarithmic scales will be used for greater convenience in the "Procedures" section, page L20. In the general explanation following immediately, only the dimensions a, b, and c, etc. on figure 2 are considered. Stream-gaging units are taken up in the succeeding explanation showing the application of the method.

The first step is to plot, on diagram 1, the values of X_1 against the corresponding values of X_2. (See fig. 2 for example of diagram 1. "Diagram" as used in this discussion is not limited to the ones shown in fig. 2 and pl. 1.) Beside each point make a temporary notation of the value of X_3. Next, a first approximation curve for $f_2(X_2)$ is drawn among the points to represent a constant value of X_3. The departure of each point from this curve (*a*, fig. 2) is measured in the X_1 direction. These departures are then plotted as departures from the base line (see p. L14) in diagram 2 against the appropriate values of X_3. If there is a third independent variable X_4, the points just plotted would be labeled with their proper values of X_4, a first approximation curve would be drawn for a constant value of X_4, the departures (*b*, fig. 2) would be measured and plotted as departures from the base line in a new diagram against the next independent variable, and so on until a first approximation curve is obtained for each independent variable, all other independent variables being held constant.

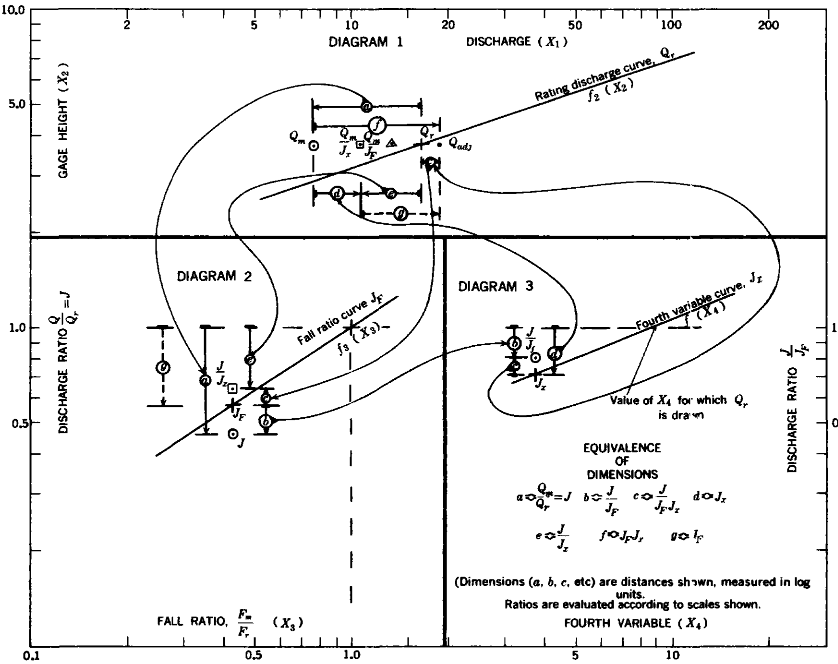

Figure 2.—Schematic plotting of relation diagrams to logarithmic scales.

In the diagram for the last independent variable, the first approximation curve is drawn as a mean of all the plotted points. The departures of the points from this curve (*c*, fig. 2) are the residuals. The plotted points in the first diagram are then adjusted (Q_{adj}, fig. 2) to make their departures from the first approximation curve equal to the residuals. This step adjusts the points for the effects of all the other variables as defined by the approximation curves.

A second approximation curve for $f_2(X_2)$ is next drawn as a mean of the adjusted points (Q_{adj}, fig. 2). The departures of these adjusted points from the second approximation curve are the new residuals. Because the residuals must be the same in all diagrams for any observation, the new residuals in diagram 1 can be transferred to diagram 2 and a second approximation curve can be drawn as a mean through these points (small squares, diagram 2, fig. 2). This process is carried through all diagrams and back to diagram 1 again and repeated until the residuals are as small as can be made.

As stated above, adjusting the plotted points to make their departures equal to their residuals adjusted the points for the effects of all the other variables as defined by the first approximation curves. This fact is the basis for a second method of adjusting the plotted points. It is sometimes easier in the end to compute the adjustment for a given observation from the approximation curves for the other variables. The departure of an observed point from $f_2(X_2)$ in diagram 1 (*a*, fig. 2) is the sum of the effects of all other variables plus the residual. Then to obtain the adjusted plotting position for any other diagram, such as diagram 2, the departures of the points in diagram 1 (*a*, fig. 2) are reduced by the curve values from diagram 3 (*d*, fig. 2), and so forth, before plotting in diagram 2 (small squares, fig. 2). These adjusted departures (*e*, diagram 1, fig. 2), measured from the base line (diagram 2), will give the same plotting position as the residuals measured from the curve (*c*, diagram 2, fig. 2).

For the description immediately following, stream gaging units now will be used and it will be assumed that there are only two independent variables. Then in the second diagram just described, the first approximation curve will be drawn as a mean through the points and the departures of the points from this curve are the residuals.

The equation (1) of the weir obtained by dimensional analysis shows that the discharge is a function of a product relation of the independent variables rather than the additive relation of equation 4. A product relation can be put in the additive form, however, simply by taking logarithms. Equation 3 in the form of equation 4 is then

$$\log Q = f_2(\log H) + f_3\left(\log \frac{F_m}{F_r}\right) \tag{5}$$

In measuring the departures (*a*) on diagram 1, the operation performed is $\log Q_m - \log Q_r$ which equals $\log\left(\frac{Q_m}{Q_r}\right)$ or $\log J$. The analyses may be carried out graphically by plotting the points on logarithmic paper (fig. 2) and scaling the departures in log units on diagram 1 for plotting in diagram 2. However, it is usually more convenient to use arithmetic scales for plotting and to compute the J ratios, as will be done later. For the present, the logarithmic plotting will be used in order to explain the fundamental relations that would not be apparent with arithmetic scales. In accordance with long established stream-gaging procedure, the discharge is plotted as abscissa and the gage height as ordinate. Diagram 1 then consists of a plot of measured discharge (Q_m), with the Q-scale horizontal and the H-scale vertical. The departures of the plotted points from the first approximation curve are computed as $\frac{Q_m}{Q_r}=J$

In diagram 2, the departures J from diagram 1, the dependent variable, are plotted as ordinates against the fall ratios, $\frac{F_m}{F_r}$, the independent variable, as abscissas according to the usual convention. The departures of points from the curve in diagram 1 are vector measurements. For additive relations, as represented by equation 4, the base line would be the zero of the ordinate scale and the departures would be plus or minus with respect to that base line. There is no difference when using the logarithms of product relations, provided the ordinate scale is marked in log units. Thus when Q_m equals Q_r, the departure is zero, the ratio $\frac{Q_m}{Q_r}=1.0$, and the log is 0, which gives a zero departure; furthermore, if Q_m is less than Q_r, the log of the ratio $\frac{Q_m}{Q_r}$ is negative. That is, if $\frac{Q_m}{Q_r}=0.7$, $\log \frac{Q_m}{Q_r}=-0.1549$,[4] which indicates the direction the departure is to be measured from the base line (0) in a diagram 2 (not shown) that has an ordinate scale marked in log units. This explanation is given only to show the similarity of procedures for additive and product relations. It is doubtful that ordinate scales marked in log units will ever be used—ordinary logarithmic paper is marked with antilogs of the true scale and the ratio $\frac{Q_m}{Q_r}$ can be plotted directly to this scale or to an arithmetic scale as used on plate 1. It should be kept in mind, however, that even though the ordinate scale in diagram 2 (fig. 2) is all positive numbers, the effect is that the departures are plotted plus and minus from the base line as explained above.

The departures of the plotted points from the J_F curve in diagram 2 are the residuals because there are no other independent variables

[4] This number is usually written 9.8451-10, but for use here the subtraction must be performed, because we are interested in the log as a vector quantity and not as something to be looked up in a log table.

being considered. The departures are computed as the ratio J/J_F, which is the equivalent of log $J-\log J_F$ in which J_F is that part of the departure of a point from the curve of diagram 1 that is explained by the fall ratio F_m/F_r.

The residuals of diagram 2 can be transferred to diagram 1 in two ways: by scaling the residual and plotting as a departure from the Q_r curve or by computing the position of Q_{adj} as was suggested on page L14. The second method is preferable for gaging-station ratings. The computations are made according to the equation

$$Q_{adj}=\frac{Q_m}{J_F}, \tag{6}$$

for, from figure 2, $\log Q_{adj}=\log Q_m-d-g$, in which d is zero because there is no diagram 3, and g in log units is $\log J_F$.

Although it was assumed for the preceding description that there were only two independent variables affecting the discharge, the possibility of a third variable should not be overlooked. Changes with time, t, might be taking place, some special complexity might be producing submergence that needs separate analysis, or one of the factors in equation 1 might be a variable instead of a constant as assumed. If such circumstances should exist, a diagram 3 will be required. Here the departures from the J_F curve ($\log J-\log J_F$ or J/J_F) are plotted as ordinates with the scale for the third independent variable, X_4, as the abscissa. The curve for J_x, the part of the discharge ratio explained by the new variable X_4, is drawn as a mean through the plotted points. The residual is now

$$\log r=\log J-\log J_F-\log J_x,$$

and the adjusted discharge is then

$$Q_{adj}=\frac{Q_m}{J_F J_x}. \tag{6A}$$

The plotted points in diagram 2 are adjusted for the effects of the variable X_4 according to J/J_x, and the points in diagram 3 are adjusted for the effects of fall ratio according to J/J_F.

The procedures for correlation of the observational data having been worked out, the forms of the relation curves will now be considered.

FORMS OF RELATION CURVES

STAGE DISCHARGE

The simple stage-discharge relation curve (rating curve) for section controls is well known to stream gagers and is described in all textbooks on stream gaging and in many textbooks on hydraulics. A characteristic of the simple rating curve is its smoothness. There are no reversals, kinks, or flat spots. Departures from a smooth curve indicate that more than one control is effective, and thus the stage-discharge relation is complex.

A common example of a complex stage-discharge relation (see fig. 3) is the situation in which a section control that is effective at low water is submerged at high water because the channel somewhere downstream from the gaging station does not have the capacity that it has at the station. Another example is the high-water channel that is much wider than the low-water channel and has different hydraulic characteristics. In both of these situations there is a pronounced break in curvature of the stage-discharge relation at the transition between the two controlling conditions.

The fact that a rating curve is a composite of two or more simple stage-discharge relation curves, one for the range of effectiveness of each control, presents no special problem so long as no other variable is involved. When another variable is involved, such as a varying amount of submergence at a given stage, then the stage discharge relation for each control must be recognized and analyzed separately. Every control in a natural stream has a unique stage discharge relation and a unique response to submergence. This uniqueness of response is illustrated in figure 4, in which the submergence characteristics of three artificial controls and a sharp-crested weir are shown to be widely different. Repeating the statement above, each control must be recognized and analyzed separately.

Variable submergence of a control implies variable backwater in the stream below the control. This backwater may be caused by another stream, either one that the stream enters as a tributary or one of its own tributaries. It could be caused also by tides or a regulated reservoir. In any of these cases a stage-discharge relation

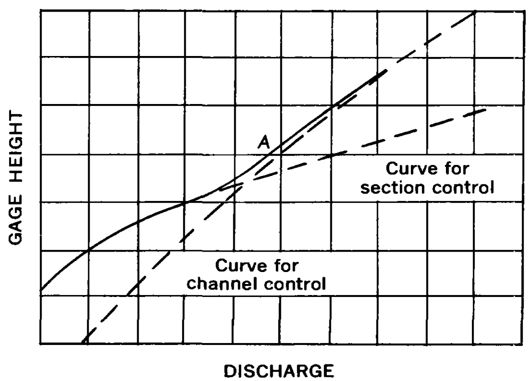

FIGURE 3.—A complex stage-discharge relation.

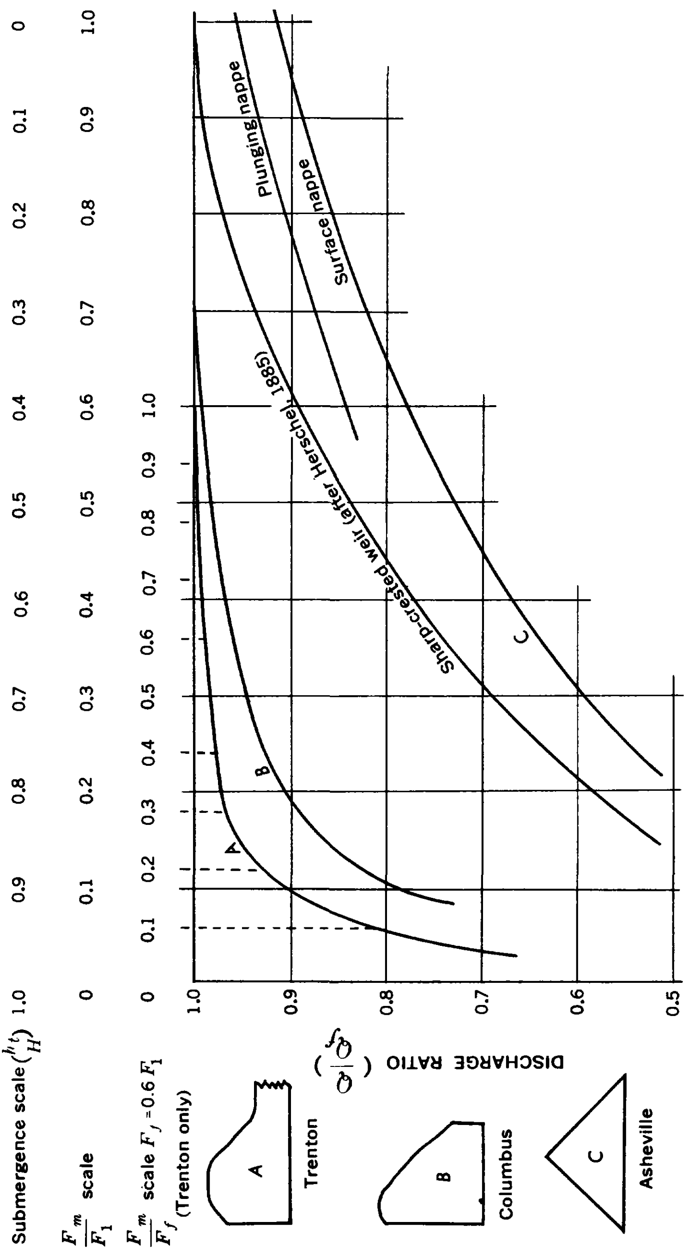

FIGURE 4.—Relation between fall ratio and submergence, and effect of type of control on fall-ratio curve.

curve will be insufficient to define the rating for the stream below the control. The necessity for defining this rating may never occur for, if the stage on the low-water control never gets above the middle of the transition region (*A*, figure 3), then the rating of that control is all that is necessary. If it is necessary to rate the stream below the control, the type of analysis will depend on the type of control involved, section or channel. A fairly broad range in stage will generally be found in the transition region (*A*, fig. 3), where the discharge can be determined from the ratings from both controls with about the same accuracy. Sometimes a three-dimensional diagram will provide the best rating for the transition region.

FREE FALL AND RATING FALL

In equation 3 it is shown that the discharge is a function of the ratio of measured fall (F_m) to either the free fall (F_f) or the rating fall (F_r). The laboratory study of artificial controls (see footnote 2, p. L8) and the analyses of many gaging-station ratings show that the free fall bears a constant ratio to the head (H). Therefore, if the measured falls are plotted as the abscissa against the head as ordinate (diagram 1A, pl. 1), the free-fall curve can be drawn as a straight line from the origin through the points at an angle with the vertical axis whose tangent is generally less than 1.00.

The origin here is the stage of effective zero flow—the stage at which the discharge would become zero if the control being rated were effective to zero flow. The stage of effective zero flow and the stage of actual zero flow, or lowest point on the crest, are usually the same for artificial controls without center notches. Natural section controls usually have the lowest point on their crest in some narrow channel, such as a gap between boulders or a break in ledge rock that forms the control. These channels are separate, very low flow controls and have their own rating, although they often are never defined because the flow never gets so low that the flow through them forms a significant part of the total flow.

The stage-discharge relation curve for a section control will usually plot as a straight line on logarithmic paper if the ordinate scale is the head (H). If, as usual, stage is used rather than head, the relation curve will not be linear. However, by substracting (assuming stage datum is below zero flow) a constant stage from all the points, they usually can be adjusted to show a linear relationship. The constant stage is determined by trial and error. When the correct value is obtained it is usually the stage of effective zero flow, and can be used as the origin for the F_f and F_r curves.

The free-fall line is drawn from the origin through the plotted F_m points for the smallest value for which the corresponding Q_m plots in the limiting position. If the data are sufficient, this line itself will provide a good estimate of the effective stage of zero flow.

When it is necessary to define the rating-discharge (Q_r) curve, it is more important to know the effective stage of zero flow. The rating fall, by definition, is a fixed percentage of the free fall. Therefore the rating-fall curve is a straight line through the origin making a smaller angle with the vertical axis than the free-fall line would make. The best position is a mean of the plotted points, for this position keeps the amount of adjustment small. For ease of computation, the rating-fall line is drawn near the mean of the plotted points but with a slope of an even tenth of a foot per foot or other slope for which it is equally easy to compute the value of F_r for any stage. The Q_r curve is then drawn through the plotted values of Q_m to correspond with the way the F_r curve is drawn through the plotted values of F_m.

Sometimes it is difficult to estimate the stage of effective zero flow. If the estimate is very far off, the plotted points for the fall-ratio curve will separate according to stage. The F_r curve should then be redrawn to reduce that separation to a minimum.

FALL RATIO

The purpose of the fall-ratio curve is to determine the values of J_F—the portion of the discharge ratio that is explained by the fall ratio. The shape of the curve is the same whether F_f or F_r is used as the base; the only difference is in the scales. Values of J, the ratio of measured discharge to the curve value (Q_f or Q_r), are plotted as ordinates with the fall ratios as abscissas. The ordinate scale is labeled simply Q/Q_f or Q/Q_r, or J.

The J_F curve for F_m/F_f will be considered first. The F_f curve was drawn so that no increase in discharge would occur at any stage with a further increase in fall. Therefore the J_F curve must become tangent to a horizontal line at the point $Q/Q_f=1.0$, $F_m/F_f=1.0$. At the other end, the curve must meet the vertical axis at the point (0, 0), for the discharge must be zero when the fall is zero. It is here that the points will indicate a family of curves if the stage of effective zero flow is in error or if there are errors in the measurement of fall as a result of errors in datum.

When the J_F curve is based on the ratio F_m/F_r, it must pass through the point $Q/Q_r=1.0$, $F_m/F_r=1.0$ as a consequence of the definitions of the quantities; however, it will still become tangent to the horizontal line at $Q/Q_r=Q_f/Q_r$, wherever that may occur. Because it is possible that a control will always be submerged, the J_F curve may never be defined as far as that point of tangency.

As explained under "Correlating the Observational Data" (p. L11) each functional relationship must be independent of all other functional relationships. The fall-ratio curve should be tested for this independence. Divide the measurements into several groups according to stage and on the fall-ratio diagram mark them distinctively, as with separate colors. There should be no tendency for each group to define a separate curve. If there is, the observations of fall should be examined to make sure there is no datum error; that is, the readings on both gages must be referred to exactly the same datum. If rating fall is used as a base, the stage of effective zero flow may need to be revised. Correction of these errors should eliminate any tendency for the points to show a joint relationship with stage.

OTHER

As explained under "Analysis of Physical Relationships," (p.L7,L15) it is possible for other variables to affect the discharge. The shape of relation curve to be used will depend entirely on the way in which the variable affects the discharge. For changes that take place with time, the departures are plotted against an abscissa scale of calendar time. This procedure, in effect, is the old Stout method for correcting for shifting controls (Hoyt and Grover, 1920, fig. 27).

PROCEDURES

This section describes the step-by-step procedures for obtaining a rating. The principles involved have been discussed under "Correlating the Observational Data," so this section will deal mostly with the mechanics of the process. Logarithmic cross-section paper can be used, but the author believes arithmetic scales are generally easier to work with, and for some uses are definitely superior, as for drawing a curve through the point of effective zero flow. A sheet of ordinary cross-section paper of suitable size having been selected, the steps in deriving a rating are given below. The diagrams referred to are illustrated in figure 2 and plate 1.

1. Plot, as diagram 1, the measurements for drawing the usual stage-discharge curve with gage heights as ordinates and discharges as abscissas. Make a temporary note of the observed fall (F_m) beside each plotted point.
2. Plot, as diagram 1A, to the right of diagram 1, using the same ordinate scale and the scale for fall as the abscissa, the fall (F_m) for each measurement. Plot also the point of effective zero flow, as observed or as determined by step 5. (Not shown in plate 1 because of space limitations; it was done on a separate sheet to get the correct slope of the F_f line.)
3. Draw the F_f or F_r curve in diagram 1A. To decide which curve to draw, examine the plotted points in diagram 1. If there seems to be a limiting position on the right, so that

at any stage for all falls greater than a certain minimum the plotted points do not exceed a certain discharge, draw the F_f curve as a straight line from the point of effective zero flow to average the minimum falls just described. If no such minimum fall can be discerned, draw the F_r curve from the point of effective zero flow as a straight line that averages all the plotted points, approximately. Inasmuch as the slope of the F_r curve is not critical, computations will be simplified if the slope is made at an even tenth of a foot of fall per foot of stage.

4. Draw the Q_f or Q_r curve in diagram 1 so that it has the same relation to the plotted points there that the F_f or F_r curve has to the corresponding points in diagram 1A—or as close to that relation as possible. That is, the plotted points for each measurement should be on the same side of the curve in both diagrams.
5. Test for accuracy of the point of effective zero flow. Plot the Q_f or Q_r curve on logarithmic paper. (See fig. 6.) If the point of effective zero flow has been chosen correctly, the curve will be a straight line. If it is not, add or subtract a constant gage-height adjustment that will make the curve a straight line. Apply this adjustment to the point of effective zero flow (step 2) and repeat steps 3 and 4.
6. Compute Q_m/Q_f or Q_m/Q_r (J) from diagram 1. Compute F_m/F_f or F_m/F_r from diagram 1A. (See table 1.) Plot as diagram 2 the values of F_m/F_f or F_m/F_r as abscissas against the values of J as ordinates. Draw the J_F curve as a mean of the plotted points. All J_F curves must pass through the points (0, 0) and (1.0, 1.0). In addition, the J_F curve based on the Q_m/Q_f ratio must become tangent to $J=1.0$ at the point (1.0, 1.0). The point at which a J_F curve based on the ratio Q_m/Q_r will become tangent to some larger value of J depends on the position of the F_r curve.

6A. If there is a third independent variable, X_4, that must be considered, plot as diagram 3 the values of X_4 as abscissas against the ratios J/J_F (computed from diagram 2) as ordinates. Draw the curve for J_x as a mean of the plotted points and through such other points as the physical nature of X_4 may dictate. For example, inasmuch as the Q_f or Q_r curve should be drawn for a constant value of X_4, the J_x curve in diagram 3 must intersect the $J/J_F=1.0$ line at that constant value of X_4.

7. Compute Q_{adj} (Q_m/J_F, see table 1) and plot the values in diagram 1. (If step 6A is used, $Q_{adj}=Q_m/J_FJ_x$.)

8. Redraw the Q_f or Q_r curve to give the best fit to the plotted Q_{adj} points.
9. Recompute the values of J and replot diagram 2. Redraw the J_F curve if necessary to give the best fit. (If X_4 is used, the points must be adjusted for this variable; that is, instead of plotting values of J, plot values of J/J_x.)

9A. Recompute the values of J/J_F from the new J_F curve and replot diagram 3. Draw a new J_x curve if necessary to give the best fit.

10. Test diagram 2 for a joint relationship with gage height if there is any appreciable scatter in the plotted points. A simple way to do this is to divide the range of gage heights into several zones and assign a different color to each zone. Then color the plotted points according to their gage height zone. A separation of colors indicates a joint relationship with gage height. This usually can be eliminated by adjustment of the point of effective zero flow.
11. Repeat steps 7–10 until satisfied with the rating.

These procedures are designed to evaluate all variables that may affect the rating. It can be expected, however, that for most ratings variables other than stage and fall will have little significant effect. It is always best to analyze a rating completely to identify all variables, and thus to determine their maximum effect on a given rating. Then if they are to be disregarded, the effect of such action can be evaluated. For example, the stage-discharge curves can be drawn for the value of the disregarded variables that will keep the error at a minimum.

ILLUSTRATIVE EXAMPLE

KOOTENAY RIVER AT GROHMAN, BRITISH COLUMBIA

Kootenay River flows generally south and west from its source in British Columbia to Bonners Ferry, Idaho, where it turns northward and flows into Kootenay Lake, British Columbia. From Kootenay Lake the river flows westward to Columbia River. The West Kootenay Power & Light Co. has a large concrete power dam, which has movable gates, at Corra Linn, British Columbia, about 8 miles downstream from the mouth of Kootenay Lake at Grohman Narrows. The channel at Grohman was enlarged by the power company to permit a greater outflow from the lake at a given stage, but Grohman Narrows is still the section that controls the outflow from the lake when enough gates at Corra Linn Dam are opened. (See fig. 5.) If the gates at Corra Linn Dam are closed to create a high head on the power plant, backwater extends upstream through Kootenay Lake to about Bonners Ferry, Idaho. This backwater adversely affects the agricultural lands along Kootenai River in Idaho. The resulting international problem is supervised by the International Joint Commission—United States and Canada—through its International Kootenay Lake Board of Control. This Board has specified rules for operation of the gates at Corra Linn Dam in order to limit the adverse effects of backwater in Idaho.

FIGURE 5.—Grohman Narrows showing old measuring cableway and car in foreground. Spoil bank from channel enlargement, on right. May 1942.

The flow of Kootenay River below the lake was gaged at Glade, downstream from Corra Linn Dam. This gaging station was discontinued in June 1944 when it was drowned out by backwater from a power dam downstream at Brilliant. The only method remaining for determining the flow was by computation of flow through the gates and trubines at Corra Linn Dam. These computations were made by the West Kootenay Power & Light Co. and regularly submitted to the International Kootenay Lake Board of Control as evidence that their operations were in compliance with the orders of the International Joint Commission. The records have not been published pending verification of ratings for the gates and turbines or acceptance of an alternate procedure.

To allay the fears of Idaho farmers, the gates on Corra Linn Dam usually were opened extra wide during the spring flood in order to make sure that no backwater would exist in Idaho as a result of Corra Linn Dam. This meant a large reduction in head on the turbines at Corra Linn.

During the period 1941–47 the author regularly computed the amount of backwater present in Kootenai River in Idaho as a result of operations at Corra Linn Dam. During the same period he also developed the rating procedures described in this report, and these principles were used to determine a rating for Kootenay River below the lake. The rating so determined, through the establishment of the free-fall relation, provided the International Kootenay Lake Board of Control with the technical data needed to establish an operating program for the gates on Corra Linn Dam that permitted the use of much higher heads on the power plant during the spring flood without danger of creating backwater in Idaho. The resulting increase in power generated helped to reduce a source of irritation between the power company and the farmers.

The rating for Kootenay River at Grohman, British Columbia, was first developed in 1946 on the basis of 47 current meter measurements made from 1943 to 1946, and was described in an unpublished memorandum (Nov. 21, 1946) by the author. Several years later in making a formal report to the International Kootenay Lake Board of Control, Waananen and Patterson [5] made only a few refinements in the 1946 rating on the basis of 33 additional discharge measurements. Later measurements indicate that a change in the stage-discharge relation took place during the flood of 1948, and a revised rating, based on the same procedures, was developed for use subsequent to that flood. The 1946 memorandum was "drawn upon freely for descriptions in this report" (Waananen and Patterson, p. 5). Rating computations taken from the Waananen and Patterson report are presented here only to illustrate the application of multiple correlation procedures. The several relation curves are shown in the diagrams in plate 1, and tables 1–4 are those usually computed for a rating of this type.

[5] Waananen, A. O., and Patterson, T. M., (joint report), 1951, Kootenay River discharge below Kootenay Lake—Development of the discharge curve for Kootenay River at Grohman, British Columbia, under present conditions: U.S. Geological Survey, Washington, D.C., and Canada Department of Resources and Development, Ottawa, Ontario, unpublished report to the International Kootenay Lake Board of Control.

TABLE 1.—*Discharge measurements for Kootenay River at Grohman, British Columbia*

Measurement		Gage height		Fall (F_m)	Free fall (F_f, table 3)	Fall ratio $\left(\frac{F_m}{F_f}\right)$	Discharge			Fall ratio part of discharge ratio, J_F (table 4)	Adjusted discharge $\left(Q_{adj}=\frac{Q_m}{J_F}\right)$
No.	Date	Nelson	Corra Linn				Measured (Q_m)	Free fall (Q_f, table 2)	Ratio $J=\frac{Q_m}{Q_f}$		
	1943										
1	Feb. 25	39.64	38.96	0.68	5.53	0.123	10,300	22,600	0.456	0.457	22,500
2	26	39.56	38.87	.69	5.49	.126	9,940	22,200	.448	.464	21,400
3	Mar. 1	39.43	38.73	.70	5.42	.129	9,470	21,700	.436	.470	20,100
4	Apr. 26	46.37	35.90	10.47	8.76	1.195	59,600	60,600	.984	1.000	59,600
5	May 4	45.04	35.98	9.06	8.12	1.116	51,900	51,900	1.000	1.000	51,900
6	June 16	48.13	35.96	12.17	9.60	1.268	73,200	73,400	.998	1.000	73,200
7	29	49.17	39.28	9.89	10.11	.978	79,800	81,800	.976	.999	79,900
8	July 8	49.93	40.06	9.87	10.46	.944	86,000	88,500	.972	.995	86,400
9	Aug. 28	45.24	44.95	.26	8.22	.032	9,830	53,200	.185	.179	55,000
10	Sept. 9	46.10	45.85	.25	8.63	.029	9,950	58,800	.169	.166	60,000
11	Oct. 26	46.99	46.79	.20	9.06	.022	9,950	64,900	.153	.134	74,200
12	Dec. 8	45.51	45.27	.24	8.34	.029	9,660	55,000	.176	.166	58,200
	1944										
13	Jan. 15	42.20	41.76	.44	6.76	.065	10,100	35,200	.287	.300	33,700
14	18	41.98	41.49	.49	6.65	.074	10,700	34,000	.315	.328	32,600
15	Feb. 14	40.20	39.79	.41	5.80	.071	8,240	25,100	.328	.319	25,800
16	23	39.54	39.06	.48	5.48	.088	8,080	22,200	.364	.370	21,800
17	Apr. 25	37.16	34.62	2.54	4.34	.585	12,100	13,800	.877	.870	13,900
18	May 15	40.91	37.97	2.94	6.14	.479	23,000	28,400	.810	.814	28,300
19	June 7	45.88	41.32	4.56	8.52	.535	48,700	57,400	.848	.844	57,700
20	24	44.44	37.19	7.25	7.83	.926	48,000	48,100	.998	.991	48,400
21	July 7	43.52	42.13	1.39	7.39	.188	24,200	42,600	.568	.568	42,600
22	Aug. 9	43.29	42.63	.66	7.28	.091	15,200	41,200	.369	.379	40,100
23	28	43.25	42.80	.45	7.26	.062	12,100	41,000	.295	.290	41,700
24	Oct. 29	45.22	45.00	.22	8.21	.027	7,490	53,000	.141	.157	47,700
25	Nov. 18	45.09	44.83	.26	8.14	.032	7,470	52,200	.143	.179	41,700
26	Dec. 15	44.58	44.31	.27	7.90	.034	7,160	49,000	.146	.187	38,300
27	20	44.24	43.97	.27	7.74	.035	7,510	46,900	.160	.191	39,300
	1945										
28	Jan. 8	43.30	43.03	.27	7.28	.037	7,920	41,300	.192	.199	39,800
29	Feb. 1	42.02	41.71	.31	6.67	.046	8,550	34,200	.250	.233	36,700
30	26	40.42	39.99	.43	5.90	.073	8,840	26,100	.339	.325	27,200
31	Apr. 24	37.59	36.51	1.08	4.55	.237	9,330	15,100	.618	.627	14,900
32	May 20	44.65	38.02	6.63	7.94	.835	47,200	49,400	.956	.972	48,600
33	24	44.74	38.01	6.73	7.98	.844	47,000	49,900	.942	.974	48,[illegible]00
34	July 9	46.33	38.56	7.77	8.73	.890	60,200	60,300	.998	.984	61,200
35	24	44.48	42.00	2.48	7.85	.316	33,600	48,400	.694	.700	48,000
36	Aug. 29	43.30	42.82	.48	7.28	.066	12,800	41,300	.310	.303	42,200
37	Oct. 6	45.25	45.00	.25	8.22	.030	8,080	53,200	.152	.171	47,300
38	Nov. 26	45.16	44.93	.23	8.18	.028	8,790	52,700	.167	.162	54,200
	1946										
39	Feb. 9	43.34	43.02	.32	7.30	.044	8,830	41,500	.213	.226	39,100
40	Mar. 5	41.67	40.79	.88	6.51	.135	15,100	32,300	.467	.482	31,300
41	Apr. 8	38.68	34.97	3.71	5.07	.732	18,300	18,800	.973	.935	19,600
42	May 2	43.36	34.98	8.38	7.31	1.146	42,100	41,700	1.010	1.000	42,100
43	7	45.45	35.01	10.44	8.32	1.255	55,300	54,600	1.013	1.000	55,300
44	June 3	52.49	34.97	17.52	11.70	1.497	113,000	113,600	.995	1.000	113,000
45	7	52.61	35.02	17.59	11.75	1.495	115,400	114,900	1.004	1.000	115,400
46	8	52.58	34.99	17.59	11.74	1.497	116,200	114,600	1.013	1.000	116,200
47	18	50.99	36.06	14.93	10.98	1.360	99,500	98,400	1.011	1.000	99,500
48	July 5	48.74	38.98	9.76	9.90	.986	77,400	78,300	.989	1.000	77,400
49	Aug. 19	43.24	42.19	1.05	7.26	.145	20,800	40,900	.509	.500	41,600
50	Sept. 1	43.73	43.46	.27	7.49	.036	9,590	43,900	.218	.195	49,200
51	20	45.03	44.66	.37	8.11	.046	12,300	51,800	.237	.233	52,800
52	Oct. 9	45.07	44.76	.31	8.14	.038	9,750	52,100	.187	.203	48,000
53	Nov. 8	45.14	44.90	.24	8.17	.029	7,700	52,500	.147	.166	46,400
54	Dec. 7	45.14	44.90	.24	8.17	.029	9,690	52,500	.185	.166	58,400
	1947										
55	Feb. 7	43.58	43.30	.28	7.42	.038	9,540	43,000	.222	.203	47,000
56	18	42.70	41.99	.71	7.00	.101	15,700	38,000	.413	.406	38,700
57	Mar. 19	40.05	38.59	1.46	5.72	.255	15,700	24,400	.644	.645	24,400
58	Apr. 12	39.36	34.89	4.47	5.39	.829	21,000	21,400	.981	.970	21,600
59	16	39.45	34.98	4.47	5.44	.822	21,900	21,800	1.004	.968	22,600
60	May 17	51.13	34.88	16.25	11.04	1.472	99,200	99,800	.994	1.000	99,200
61	21	50.20	34.96	15.24	10.60	1.438	92,400	90,900	1.017	1.000	92,400

TABLE 1.—*Discharge measurements for Kootenay River at Grohman, British Columbia*—Continued

Measurement		Gage height		Fall (F_m)	Free fall (F_f, table 3)	Fall ratio $\left(\frac{F_m}{F_f}\right)$	Discharge			Fall ratio part of discharge ratio, J_F (table 4)	Adjusted discharge $\left(Q_{adj}=\frac{Q_m}{J_F}\right)$
No.	Date	Nelson	Corra Linn				Measured (Q_m)	Free fall (Q_f, table 2)	Ratio $J=\frac{Q_m}{Q_f}$		
	1947										
62	June 3	50.14	35.43	14.71	10.57	1.392	91,800	90,400	1.016	1.000	91,800
63	July 22	44.74	37.15	7.59	7.98	.951	48,100	49,900	.964	.996	48,300
64	Aug. 5	43.17	41.53	1.64	7.23	.227	25,300	40,600	.623	.616	41,100
65	Oct. 8	44.84	44.02	.82	8.02	.102	21,500	50,500	.426	.408	52,700
66	24	44.68	38.30	6.38	7.95	.803	48,000	49,600	.968	.962	49,900
67	Dec. 11	45.06	44.79	.27	8.13	.033	9,620	52,000	.185	.183	52,600
	1948										
68	Feb. 11	43.02	42.72	.30	7.15	.042	9,530	39,700	.240	.218	43,700
69	Apr. 6	39.00	37.90	1.10	5.22	.211	12,000	20,000	.600	.597	20,100
70	May 11	43.48	35.50	7.98	7.37	1.083	42,500	42,400	1.002	1.000	42,500
71	June 2	54.94	34.14	20.80	12.87	1.616	140,000	140,000	1.000	1.000	140,000
72	3	55.41	34.54	20.87	13.09	1.594	146,000	145,000	1.007	1.000	146,000
73	4	55.79	34.92	20.87	13.28	1.572	150,000	150,000	1.000	1.000	150,000
74	5	56.10	35.16	20.94	13.43	1.560	152,000	153,000	.993	1.000	152,000
75	6	56.30	35.32	20.98	13.52	1.552	155,000	155,000	1.000	1.000	155,000
76	7	56.53	35.36	21.17	13.63	1.552	158,000	158,000	1.000	1.000	158,000
77	8	56.64	35.58	21.06	13.69	1.538	161,000	159,000	1.012	1.000	161,000
78	9	56.64	35.70	20.94	13.69	1.530	161,000	159,000	1.012	1.000	161,000
79	10	56.95	36.16	20.79	13.83	1.502	163,000	163,000	1.000	1.000	163,000
80	11	56.88	35.99	20.89	13.80	1.513	163,000	162,000	1.006	1.000	163,000

The base gage for this rating is at Nelson, British Columbia (Nelson, table 1), on the West Arm of Kootenay Lake about 2 miles above the control at Grohman. The auxiliary gage is on the forebay of the power plant at Corra Linn (Corra Linn, table 1), 10 miles downstream. Inasmuch as 1,700.00 feet added to the readings on both gages will convert the readings to height above mean sea level, the difference in gage readings gives directly the fall in water surface between the two gages (F_m, table 1). The possibility that the gage reading at Corra Linn may be affected by the proportion of flow passing through the turbines may explain some of the large departures of Q_{adj} from the Q_F curve for very small values of the fall (F_m).

Inasmuch as current-meter measurements of discharge (Q_m, table 1) are made from a cableway at Grohman (fig. 5), the measured flow is the true outflow from Kootenay Lake. There are a few small tributaries between the cableway and Corra Linn Dam but, because this inflow is all below the control, the only effect is to add a little more submergence to that caused by the dam. The effect of this increased submergence is included in the gage reading at Corra Linn.

By use of these data, the points were plotted in diagrams 1 and 1A of plate 1, Nelson gage heights against Q_m in diagram 1, and F_m against Nelson gage heights in diagram 1A. Curves of F_f and

Q_f were drawn as explained in steps 3 and 4 of "Procedures." Figure 6 shows the test for point of effective zero flow as explained in step 5—if zero flow is assumed at gage height 28.00 feet, Q_f is a straight line; using a gage height of 26.00 feet makes Q_f concave upward, and a gage height of 30.00 feet makes Q_f concave downward. Therefore 28.00 feet was taken as the best point of effective zero flow and the F_f line was drawn through that point. The Q_f curve was then adjusted to pass through the plotted discharge measurements the same way the F_f line passed through the plotted falls (step 4).

To compute Q_m/Q_f and F_m/F_f for step 6, the values of Q_f and F_f for each measurement were picked directly from the curves and entered in the appropriate column of table 1. The ratios were then computed, entered in the proper column, and plotted in diagram 2. The J_F curve was then drawn (step 6). The values of J_F for each measurement were picked directly from the curve and listed in table 1. Q_{adj} was computed next and plotted in diagram 1. As this rating was developed, it became apparent that for very small values of F the discharge could not be adjusted to the Q_f curve without appreciable error. A fall of 0.4 foot was picked as the limit below which a measurement would be given little weight in the final rating. Therefore, Q_{adj} points based on falls of 0.4 foot or more were shown as small solid circles; for falls of less than 0.4 foot, the points were shown as small open circles. When it is considered that these falls occur in a distance of 10 miles and that there is some question that the gages are at the best locations for measuring the fall (they were not established for that purpose), it would seem that the normal errors of observation would not allow much greater accuracy. Furthermore, for falls of less than 0.4 foot, the total fall in the reach may not be a good measure of the fall over the control.

The Q_f curve was next redrawn to give the best fit to the solid circles. The rating was completed by following steps 9–11 of "Procedures." Tables for the three relation curves (tables 2–4) were then prepared for use in making computations of daily discharge. Table 5 shows the computation of daily discharge for the month of April 1947. The gage heights were picked from the water-stage recorder charts; the difference in gage heights between the two gages is the fall. The rest of table 5 is self explanatory.

Of the 80 discharge measurements available for this rating, 59 were made when the fall was 0.4 foot or more. The maximum residual (p. L14) for any of the 23 measurements made under free-fall conditions was 1.7 percent. Of the remaining 36 measurements, after adjustment to free-fall discharge (Q_m/J_F), 30 have residuals of less than 4 percent, and only 1 has a residual of more than 5 percent.

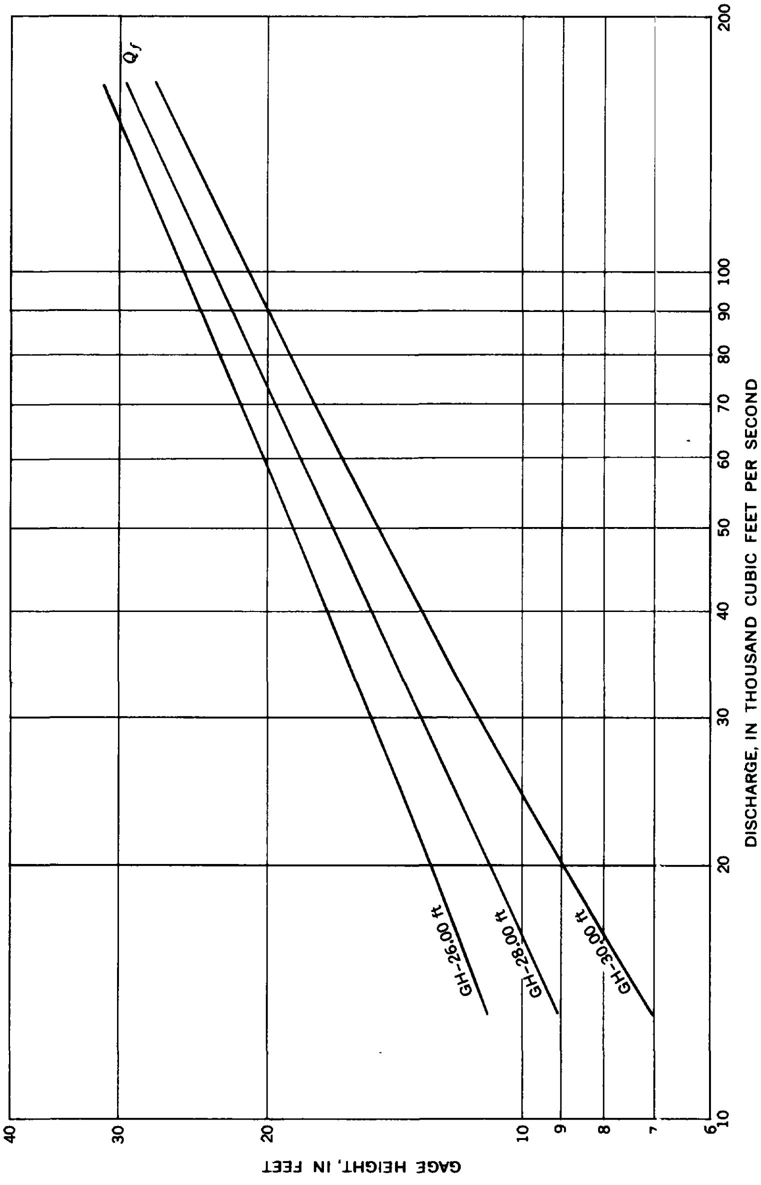

FIGURE 6.—Logarithmic plot to determine point of zero flow for Kootenay River at Grohman, British Columbia.

TABLE 2.—*Free fall discharge (Q_f) relation table for Kootenay River at Grohman British Columbia*

[Rating of August 17, 1951, which is applicable for the period 1943 to June 11, 1948]

Gage height (feet)	Discharge (cfs)	Gage height (feet)	Discharge (cfs)	Gage height (feet)	Discharge (cfs)
37.00	13,300	43.80	44,300	50.40	92,800
.10	13,600	.90	44,900	.50	93,800
.20	13,900	44.00	45,500	.60	94,700
.30	14,200	.10	46,100	.70	95,700
.40	14,500	.20	46,700	.80	96,600
.50	14,800	.30	47,300	.90	97,600
.60	15,100	.40	47,900	51.00	98,500
.70	15,400	.50	48,500	.10	99,500
.80	15,800	.60	49,100	.20	100,500
.90	16,100	.70	49,700	.30	101,500
38.00	16,500	.80	50,300	.40	102,500
.10	16,800	.90	51,000	.50	103,500
.20	17,200	45.00	51,600	.60	104,500
.30	17,500	.10	52,300	.70	105,500
.40	17,900	.20	52,900	.80	106,500
.50	18,200	.30	53,600	.90	107,500
.60	18,600	.40	54,200	52.00	108,500
.70	18,900	.50	54,900	.10	109,500
.80	19,300	.60	55,500	.20	110,600
.90	19,600	.70	56,200	.30	111,600
39.00	20,000	.80	56,800	.40	112,700
.10	20,400	.90	57,500	.50	113,700
.20	20,800	46.00	58,100	.60	114,800
.30	21,200	.10	58,800	.70	115,800
.40	21,600	.20	59,400	.80	116,900
.50	22,000	.30	60,100	.90	117,900
.60	22,400	.40	60,800	53.00	119,000
.70	22,800	.50	61,500	.10	120,000
.80	23,300	.60	62,200	.20	121,100
.90	23,700	.70	62,900	.30	122,100
40.00	24,200	.80	63,600	.40	123,200
.10	24,600	.90	64,300	.50	124,200
.20	25,100	47.00	65,000	.60	125,300
.30	25,500	.10	65,700	.70	126,300
.40	26,000	.20	66,400	.80	127,400
.50	26,400	.30	67,100	.90	128,400
.60	26,900	.40	67,900	54.00	129,500
.70	27,300	.50	68,600	.10	130,600
.80	27,800	.60	69,400	.20	131,700
.90	28,300	.70	70,100	.30	132,800
41.00	28,800	.80	70,900	.40	133,900
.10	29,300	.90	71,600	.50	135,000
.20	29,800	48.00	72,400	.60	136,100
.30	30,300	.10	73,200	.70	137,200
.40	30,800	.20	74,000	.80	138,300
.50	31,400	.30	74,800	.90	139,400
.60	31,900	.40	75,600	55.00	140,500
.70	32,500	.50	76,400	.10	141,600
.80	33,000	.60	77,200	.20	142,800
.90	33,600	.70	78,000	.30	143,900
.00	34,100	.80	78,800	.40	145,100
.10	34,700	.90	79,600	.50	146,200
.20	35,200	49.00	80,400	.60	147,400
.30	35,800	.10	81,200	.70	148,500
.40	36,300	.20	82,100	.80	149,700
.50	36,900	.30	82,900	.90	150,800
.60	37,400	.40	83,800	56.00	152,000
42.70	38,000	.50	84,600	.10	153,100
.80	38,500	.60	85,500	.20	154,300
.90	39,100	.70	86,400	.30	155,400
43.00	39,600	.80	87,300	.40	156,600
.10	40,200	.90	88,200	.50	157,700
.20	40,700	50.00	89,100	.60	158,900
.30	41,300	.10	90,000	.70	160,000
.40	41,900	.20	90,900	.80	161,200
.50	42,500	.30	91,900	.90	162,300
.60	43,100			57.00	163,500
.70	43,700				

TABLE 3.—*Free fall (F_f) relation table for Kootenay River at Grohman, British Columbia, dated August 17, 1951*

[Table is applicable for the period 1943 to June 11, 1948]

Gage height (feet)	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
36	3.78	3.83	3.88	3.92	3.97	4.02	4.07	4.12	4.16	4.21
37	4.26	4.31	4.36	4.40	4.45	4.50	4.55	4.60	4.64	4.69
38	4.74	4.79	4.84	4.88	4.93	4.98	5.03	5.08	5.12	5.17
39	5.22	5.27	5.32	5.36	5.41	5.46	5.51	5.56	5.60	5.65
40	5.70	5.75	5.80	5.84	5.89	5.94	5.99	6.04	6.08	6.13
41	6.18	6.23	6.28	6.32	6.37	6.42	6.47	6.52	6.56	6.61
42	6.66	6.71	6.76	6.80	6.85	6.90	6.95	7.00	7.04	7.09
43	7.14	7.19	7.24	7.28	7.33	7.38	7.43	7.48	7.52	7.57
44	7.62	7.67	7.72	7.76	7.81	7.86	7.91	7.96	8.00	8.05
45	8.10	8.15	8.20	8.24	8.29	8.34	8.39	8.44	8.48	8.53
46	8.58	8.63	8.68	8.72	8.77	8.82	8.87	8.92	8.96	9.01
47	9.06	9.11	9.16	9.20	9.25	9.30	9.35	9.40	9.44	9.49
48	9.54	9.59	9.64	9.68	9.73	9.78	9.83	9.88	9.92	9.97
49	10.02	10.07	10.12	10.16	10.21	10.26	10.31	10.36	10.40	10.45
50	10.50	10.55	10.60	10.64	10.69	10.74	10.79	10.84	10.88	10.93
51	10.98	11.03	11.08	11.12	11.17	11.22	11.27	11.32	11.36	11.41
52	11.46	11.51	11.56	11.60	11.65	11.70	11.75	11.80	11.84	11.89
53	11.94	11.99	12.04	12.08	12.13	12.18	12.23	12.28	12.32	12.37
54	12.42	12.47	12.52	12.56	12.61	12.66	12.71	12.76	12.80	12.85
55	12.90	12.95	13.00	13.04	13.09	13.14	13.19	13.24	13.28	13.33
56	13.38	13.43	13.48	13.52	13.57	13.62	13.67	13.72	13.76	13.81
57	13.86									

TABLE 4.—*Discharge-ratio (J_F) relation table for Kootenay River at Grohman, British Columbia, for given values of fall ratio (F/F_f)*

F/F_f	J_F	F/F_f	J_F	F/F_f	J_F
0.000	0	0.340	0.719	0.680	0.914
.010	.070	.350	.727	.690	.918
.020	.125	.360	.735	.700	.922
.030	.171	.370	.743	.710	.926
.040	.211	.380	.750	.720	.930
.050	.248	.390	.757	.730	.934
.060	.283	.400	.764	.740	.938
.070	.316	.410	.771	.750	.942
.080	.347	.420	.778	.760	.946
.090	.376	.430	.785	.770	.950
.100	.403	.440	.791	.780	.954
.110	.428	.450	.797	.790	.958
.120	.451	.460	.803	.800	.961
.130	.472	.470	.809	.810	.964
.140	.491	.480	.815	.820	.967
.150	.509	.490	.821	.830	.970
.160	.526	.500	.827	.840	.973
.170	.542	.510	.832	.850	.976
.180	.557	.520	.837	.860	.978
.190	.571	.530	.842	.870	.980
.200	.584	.540	.847	.880	.982
.210	.596	.550	.852	.890	.984
.220	.608	.560	.857	.900	.986
.230	.619	.570	.862	.910	.988
.240	.630	.580	.867	.920	.990
.250	.640	.590	.872	.930	.992
.260	.650	.600	.877	.940	.994
.270	.660	.610	.882	.950	.996
.280	.669	.620	.887	.960	.997
.290	.678	.630	.892	.970	.998
.300	.687	.640	.897	.980	.999
.310	.695	.650	.902	.990	1.000
.320	.703	.660	.906	1.000	1.000
.330	.711	.670	.910		

TABLE 5.—*Computation of daily mean discharge for Kootenay River at Grohman British Columbia, April 1947*

Day	Gage height		Fall	Free fall	Fall ratio	Discharge ratio	Free fall discharge	Daily discharge
	Nelson	Corra Linn	(F)	(F_f; table 3)	(F/F_f)	(J_F; table 4)	(Q_f; table 2)	($Q=J_F Q_f$)
1	39.30	34.96	4.34	5.36	0.810	0.964	21,200	20,400
2	39.35	34.98	4.37	5.38	.812	.965	21,400	20,700
3	39.41	34.94	4.47	5.41	.826	.969	21,600	20,900
4	39.45	34.97	4.48	5.43	.825	.968	21,800	21,100
5	39.50	34.96	4.54	5.46	.832	.971	22,000	21,400
6	39.52	34.99	4.53	5.47	.828	.969	22,100	21,400
7	39.51	34.99	4.52	5.46	.827	.969	22,000	21,300
8	39.48	34.94	4.54	5.45	.833	.971	21,900	21,300
9	39.43	34.95	4.48	5.42	.827	.969	21,700	21,000
10	39.36	34.96	4.40	5.39	.816	.966	21,400	20,700
11	39.37	34.93	4.44	5.40	.822	.968	21,500	20,800
12	39.35	34.94	4.41	5.38	.820	.967	21,400	20,700
13	39.33	34.94	4.39	5.37	.818	.966	21,300	20,600
14	39.30	35.02	4.28	5.36	.799	.961	21,200	20,400
15	39.36	35.00	4.36	5.39	.809	.964	21,400	20,600
16	39.46	35.00	4.46	5.44	.820	.967	21,800	21,100
17	39.62	35.02	4.60	5.52	.833	.971	22,500	21,900
18	39.82	35.04	4.78	5.61	.852	.976	23,400	22,800
19	40.04	34.98	5.06	5.72	.885	.983	24,400	24,000
20	40.35	34.95	5.40	5.86	.922	.990	25,800	25,500
21	40.70	35.02	5.68	6.04	.940	.994	27,300	27,100
22	41.01	34.97	6.04	6.18	.977	.999	28,800	28,800
23	41.25	34.97	6.28	6.30	.997	1.000	30,000	30,000
24	41.44	35.00	6.44	6.39	>1.0	1.000	31,000	31,000
25	41.59	34.97	6.62	6.46	>1.0	1.000	31,800	31,800
26	41.72	35.03	6.69	6.53	>1.0	1.000	32,600	32,600
27	41.90	34.97	6.93	6.61	>1.0	1.000	33,600	33,600
28	42.19	34.99	7.20	6.76	>1.0	1.000	35,200	35,200
29	42.61	34.98	7.63	6.95	>1.0	1.000	37,500	37,500
30	43.16	35.02	8.14	7.22	>1.0	1.000	40,500	40,500

CONCLUSIONS

The rating for Kootenay River at Grohman, British Columbia, is satisfactory except under conditions of extreme submergence. It is possible that the rating could be improved, even for these conditions, by use of improved instrumentation, such as a shorter reach and careful location of the gages. The big advantage of this free-fall rating is that the stage-discharge relation can be used directly for as much as several months in some years.

A rating based on rating falls can be obtained for Kootenay River at Grohman, British Columbia, by simple computations from the rating just described. Suppose, for example, it is desired to prepare a rating based on $F_r=0.5F_f$. Draw that F_r line in diagram 1A. From table 4 when $F_r/F_f=0.5$, $Q_r/Q_f=0.827$. Then draw the rating-fall discharge curve in diagram 1 so that $Q_r=0.827\ Q_f$. The scales in diagram 2 are then changed by dividing the scale of F/F_f by 0.5 to get the F/F_r scale and by dividing the scale of Q/Q_f by 0.827 to get the Q/Q_r scale; the curve remains the same. By use of these new curves and scales, the measurements will adjust to the Q_r curve with exactly the same results that were obtained in adjusting them to the Q_f curve.

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