Response of Gas-Purged Manometers to Oscillations in Water Level

GEOLOGICAL SURVEY WATER-SUPPLY PAPER 1869-E

Prepared in cooperation with the California Department of Water Resources

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By J. R. BECK *and* C. R. GOODWIN

RIVER HYDRAULICS

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CONTENTS

ILLUSTRATIONS

TABLES

RIVER HYDRAULICS

RESPONSE OF GAS-PURGED MANOMETERS TO OSCILLATIONS IN WATER LEVEL

J. R. Beck and C. R. Goodwin

ABSTRACT

This report describes tests conducted to evaluate the performance of bubble-gage servocontrolled manometers when operating in a pool or stream whose water level is oscillating at amplitudes and frequencies that are likely to be found at medium or high stages in streams or in larger bodies of water subject to wind action. Two types of units were tested—an Exactel Servomanometer[1] made by the Exactel Instrument Co. and a U.S. Geological Survey servocontrolled-manometer. Piezometer systems used in each case were those used in normal field applications.

These tests indicate that gas-purged manometers produce an accurate record of water stage under static conditions, but they record a value less than the average water-surface elevation when surface waves of significant amplitudes and frequencies are present. The test program imposed wave amplitudes of 0.1 and 0.2 foot and frequencies ranging from 2 to 10 cycles per minute, a range of wave conditions that commonly occur at field installations. The magnitude of error is shown to be dependent upon the frequency and amplitude of wave action, the gas-purge rate, the depth of water over the orifice, the internal volume of the manometer system, and the overriding effects of the servomechanism used to read the basic gas-purged manometer unit.

The tendency to underregister can be reduced—but not entirely eliminated—by increasing the gas-purge rate. A mathematical analysis is given which is verified by the experimental work. This analysis shows how the magnitude of underregistration can be predicted if the controlling factors—wave frequency and amplitude and the physical-system characteristics—are known.

The value of these findings is that the presence or absence of registration errors can be predicted from a knowledge of the regime imposed on the bubble-gage system so that the effects of such errors can be compensated for in subsequent analysis.

[1] Trade name of Exactel Instrument Co.

INTRODUCTION

Since 1958 there has been an increasing use of bubble gages, the term commonly used for the gas-purged servocontrolled-manometer water-level sensing and recording system used by the U.S. Geological Survey. Significant advantages of bubble gages include simplicity of installation resulting in lower initial cost, relative ease of changing orifice location in unstable channels, and high-salvage value when relocating or discontinuing a gaging station. Further, the recorder and sensing unit can be located in a gage house some distance away from the orifice at sites free from the threat of inundation or bank erosion.

Performance of bubble gages has generally been very satisfactory, and, because of the advantages noted above, many are now in service. However, in some instances anomalies in recorded stage have been noted, particularly in situations where short-term wave actions affect the hydrostatic pressure at the orifice. The pulsations under consideration here are those caused by minor wave action or oscillatory changes in the water surface. Field observations of this phenomenon show that the frequency of pulsations typically ranges from 2 to 10 cpm (cycles per minute). Such surging makes the accurate reading of staff gages impractical, and gage observations made under such conditions are generally qualified as accurate within a specified tolerance—perhaps ± 0.1 foot.

The purpose of this study was to determine and quantify the effects of short-period oscillating water-surface levels on the stage recorded by gas-purged manometer systems. Two servocontrolled-manometer systems, an Exactel Instrument Co. Servomanometer and a U.S. Geological Survey bubble gage, were tested. System responses to both abrupt and cyclic changes in stage were tested using two representative lengths of tubing and normally used bubble rates. For testing the effects of an abrupt change in stage, step-stage changes of 0.2 and 0.4 foot were used; for testing the effects of cyclic change in stage, sinusoidal oscillations at frequencies of about 2, 5, and 10 cpm and amplitudes of 0.1 and 0.2 foot were used.

This study was made in cooperation with the California Department of Water Resources. All the tests were made by the U.S. Geological Survey in conjunction with the Exactel Instrument Co. of Mountain View, Calif., who provided the Servomanometer, shop facilities, and assistance.

The test procedures were established by Winchell Smith, Geological Survey hydrologist. The report was prepared under the general supervision of Walter Hofmann and R. Stanley Lord, successive

district chiefs of the Water Resources Division of the U.S. Geological Survey in California, and under the immediate supervision of L. E. Young, chief of the Menlo Park subdistrict office.

THE GAS-PURGED MANOMETER SYSTEM

GENERAL DESCRIPTION

The bubble gage, a gas-purged servocontrolled-manometer system (fig. 1), is used by the Geological Survey to record water-surface levels. At one end of the system is a length of pressurized tubing that has one end submerged in the water whose level is to be recorded. The head of water above the tubing orifice establishes a pressure which is reflected through the length of tubing into a U-tube manometer at the other end of the system. Bottled gas is forced into the manometer system at a constant rate causing bubbles to rise in the water from the submerged orifice. The input bubble rate required depends primarily on the expected rate of change of stage of the stream and the length of tubing between the orifice and the manometer unit.

Mercury in a reservoir at one end of the U-tube is caused to stabilize at levels corresponding to the head of water above the submerged orifice. The mercury levels thus represent various stages of water and can be recorded by continuous analog or digital-chart recorders. Mercury levels may be detected by various means. One method, used in the survey system, is a float-actuated switch arrangement. As the mercury changes position in the reservior, a contact is made to complete a circuit through an electronic control unit to a drive motor, which then drives the manometer to a balanced position. The manometer movement is translated to a recorder, thus indicating a change in water level. A delay circuit, used in the survey system, can be manually switched in or out of operation. When the delay circuit is in operation, the manometer drive does not respond to water-level changes for about 35 seconds after electrical contact of the float-actuated switch. If the float-actuated switch opens within this time interval, no water-level change is recorded, thus giving the effect of damping the water-surface oscillations. Manometer response is immediate with the delay circuit off.

The Exactel Servomanometer uses a differential transformer to detect mercury levels. The transformer consists of moving coils around the mercury tubes. A magnetic armature floating on the mercury surface moves within the coils; any change in armature position causes an unbalanced voltage that is fed to an amplifier. The amplifier actuates a motor that moves the coils back to the null position and also drives a counter and the water-stage recorder.

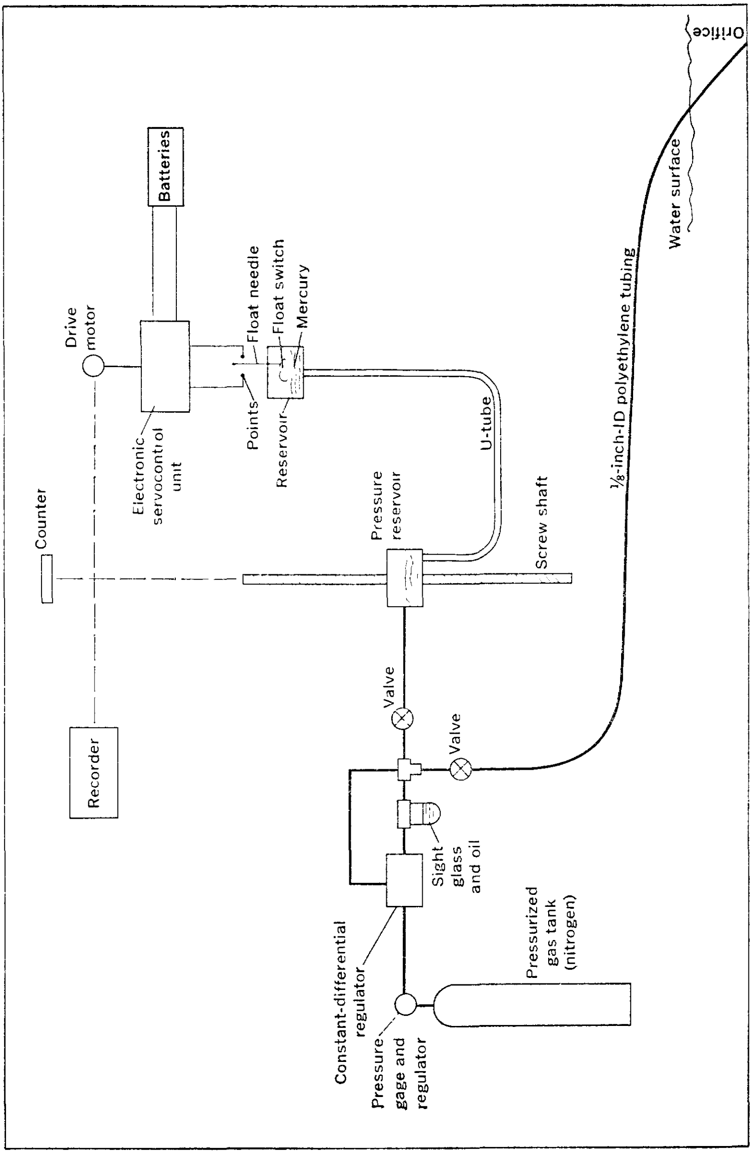

FIGURE 1.—Diagrammatic sketch of U.S. Geological Survey gas-purged servocontrolled-manometer.

MATHEMATICAL ANALYSIS

The bubble gage can be visualized as a simple gas-purged manometer which is read by an electromechanical servomechanism. Mathematical analysis of the gas-purged part of this system can be made, and an equation relating the internal pressure of the system to elapsed time can be developed. Factors controlling the internal pressure are the gas-bubble size and injection rate to the system, the temperature and density of the gas, the internal volume of the manometer and tubing, and the rate of external pressure change imposed. An instantaneous positive step change in stage is most easily analyzed. The falling-stage response of the system is almost instantaneous as far as the internal pressure changes are concerned. This is so because no mechanical system regulates the gas exhaust from the orifice, as is done with the gas supply. The gas, therefore, escapes very rapidly in response to decreasing external pressure at the orifice, and a practically constant equilibrium is maintained. For this analysis it has, therefore, been assumed that no internal pressure lag occurs in the manometer gas system during falling stages.

A differential equation, using the principle of mass conservation, is developed below for manometer operation. Rate of mass input of to the system (M_i) equals rate of mass output (M_o) plus rate of change in mass storage $\frac{dM}{dt}$ of the system; therefore

$$M_i=M_o+\frac{dM}{dt}. \tag{1}$$

For a rising stage, no bubbles exit from the orifice, and mass output is zero until bubbles appear at the orifice and the inside pressure stabilizes. Thus,

$$M_i=\frac{dM}{dt} \tag{2}$$

and

$$M_i=v_i r_i \rho_i, \tag{3}$$

where

v_i=volume of a gas bubble entering the system,
r_i=bubble rate into the system, and
ρ_i=mass density of input gas.

The rate of change in mass storage is equal to the product of the volume of the system and rate of density change of the gas with time; therefore

$$\frac{dM}{dt}=\frac{d\rho_i}{dt}V, \tag{4}$$

where V=total volume of system.

Substituting equations 3 and 4 into equation 2 gives

$$v_i r_i \rho_i = V \frac{d\rho_i}{dt}. \tag{5}$$

We now digress and examine the ideal gas law, $\rho = \frac{P}{gRT}$. If we assume constant temperature, we can express the term, $\frac{1}{gRT}$ by a constant of proportionality, k. Furthermore, P may be expressed as the sum of P_a and P_s, where: P=total absolute pressure inside the system, P_a=atmospheric pressure (33.9 ft of water), and P_s=inside pressure to balance outside static head of water. We can therefore rewrite the ideal gas law as follows:

$$\rho_i = k(P_a + P_s). \tag{6}$$

The total derivative of equation 6 with respect to time gives

$$\frac{d\rho_i}{dt} = k \frac{dP_s}{dt}. \tag{7}$$

Now, substituting equations 6 and 7 into equation 5 and rearranging terms

$$\frac{V}{v_i r_i}\left(\frac{dP_s}{dt}\right) - P_s = P_a. \tag{8}$$

Equation 8 is a first-order differential equation of the form

$$a_1 \frac{dy}{dt} + a_2 y = f(t) = b, \tag{9}$$

where the right-hand side of the equation for a step input is a constant and can be solved to give a relation between system pressure, in feet (P_s), and elapsed time, in minutes (t).

The general solution, y, of this equation is the sum of the complementary solution, y_c, and the particular solution, y_p, which can be solved by the methods for linear equations with constant coefficients and the inspection method of undetermined coefficients for nonhomogeneous equations (Rainville).[2]

thus

$$y = y_c + y_p, \tag{10}$$

[2] Rainville, E. D., 1964, Elementary differential equations: New York, Macmillan Co., p. 113–143.

where y_c is the solution of

$$a_1\frac{dy}{dt}+a_2y=0. \tag{11}$$

An exponential equation gives the complementary solution, thus

$$y_c=e^{mt}, \tag{12}$$

where

$$m=\frac{-a_2}{a_1}.$$

The particular solution for a step input is a constant, thus $y_p=A$, where

$$A=\frac{b}{a_2}. \tag{13}$$

The general solution of equation 8, with the constant of integration C, is now

$$y=Ce^{mt}+A, \tag{14}$$

where

$$y=P_s \text{ and } A=-P_a$$

for our tests; therefore, we get for a final equation

$$P_s=Ce^{mt}-P_a. \tag{15}$$

The constants C and m in equation 15 can be determined for known conditions. C can be found by substituting the initial stage at zero time in equation 15, and m can be found at various bubble rates from the following equation:

$$m=\frac{v_i r_i}{V}, \tag{16}$$

where v_i can be obtained from figure 2. Equation 15 can be plotted to indicate pressure in the system (in terms of feet of water) versus elapsed time for a rising stage-step input. Use of the equation is discussed on pages E20 and E22.

RELATION OF SUBSURFACE PRESSURE TO OSCILLATIONS OF WATER SURFACE

A bubble-gage installation senses pressure at the orifice location. This pressure may or may not oscillate with the same amplitude as oscillations in water-surface level, depending on depth to the orifice

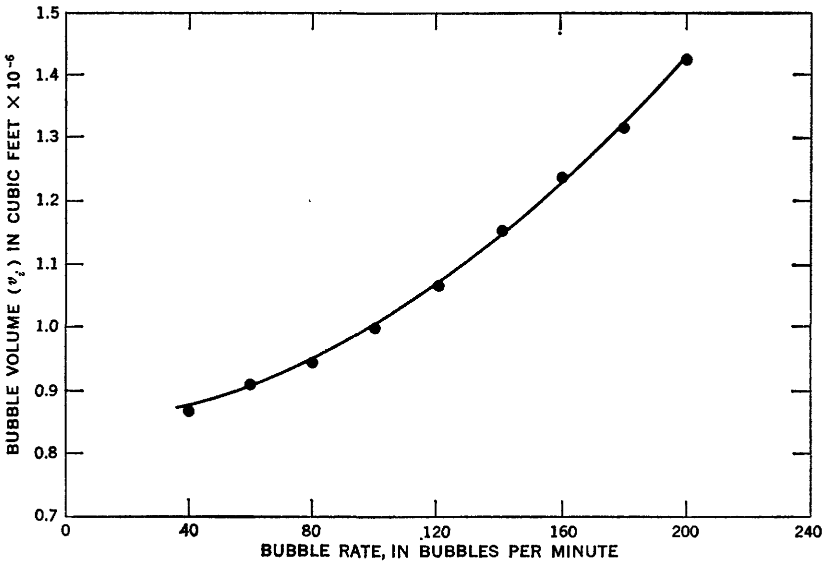

FIGURE 2.—Relation of bubble volume to bubble rate.

and the frequency of the surface oscillations. A study of surface wave motion and pressures indicated that surface-wave and subsurface-pressure profiles occur at virtually the same time (in phase) for both regular and slightly irregular wave oscillations. For highly irregular wave patterns, pressure and water depth would not be in phase for any appreciable length of time.

Folsom [3] discussed the subsurface pressure-response factor, K, considering two-dimensional irrotational theory of wave motion. K is defined as the ratio of pressure variation at a given point below the mean free surface to the equivalent pressure corresponding to a wave height. K is equal to a hyperbolic function which includes wave length or frequency, depth of water, and depth to the point of interest and is expressed as follows

$$K=\frac{\operatorname{Cosh}\left[\frac{2\pi D}{L}\left(1-\frac{Z}{D}\right)\right]}{\operatorname{Cosh}\frac{2\pi D}{L}}=\frac{\Delta P}{\Delta H},$$

where

$$\operatorname{Cosh}X=\frac{e^{x}+e^{-x}}{2}$$

[3] Folsom, R. G., 1947, Subsurface pressure due to oscillatory waves: Am. Geophys. Union Trans., v. 28, no. 5, p. 875–881.

and where

D=depth to bottom of water, in feet;
L=wave length, in feet;
Z=distance to point of interest from surface, in feet;
ΔP=pressure variation at a given point; and
ΔH=equivalent pressure corresponding to a wave height.

For our analysis, the point of interest is the orifice near the streambed. Thus, $Z=D$, which simplifies the numerator of the above equation to cosh $0=1$. The equation can be further reduced to

$$K=\frac{1}{\text{Cosh}\left(\frac{D^{1/2}}{T}C\right)}$$

by use of the following equalities:

$$T=\frac{1}{f}$$

$$C=\frac{2\pi}{\sqrt{g}}=1.11$$

$$L=T\sqrt{gD}$$

where

T=period of wave oscillation, in seconds;
f=frequency of wave oscillation, in cycles per second; and
$g=32.2$ ft per sec.^{2}

When pressure variation at depth is approximately 10 percent of the equivalent pressure corresponding to a wave height, we can consider the pressure variation to be negligible. Figure 3 is a plot of K values for various wave frequencies, in cycles per minute, as related to water depths. These curves indicate that pressure variations at the orifice will be nearly equal to surface wave heights for depths of water and the frequencies of wave motion that are likely to be encountered in field installations.

Figure 3 will be used in this report in the discussion of prediction of error in stage recorded by the bubble gage during cyclic oscillations in water level (p. E22). The tests described on the following pages are concerned entirely with response of the bubble gage to pressure changes at the gage orifice.

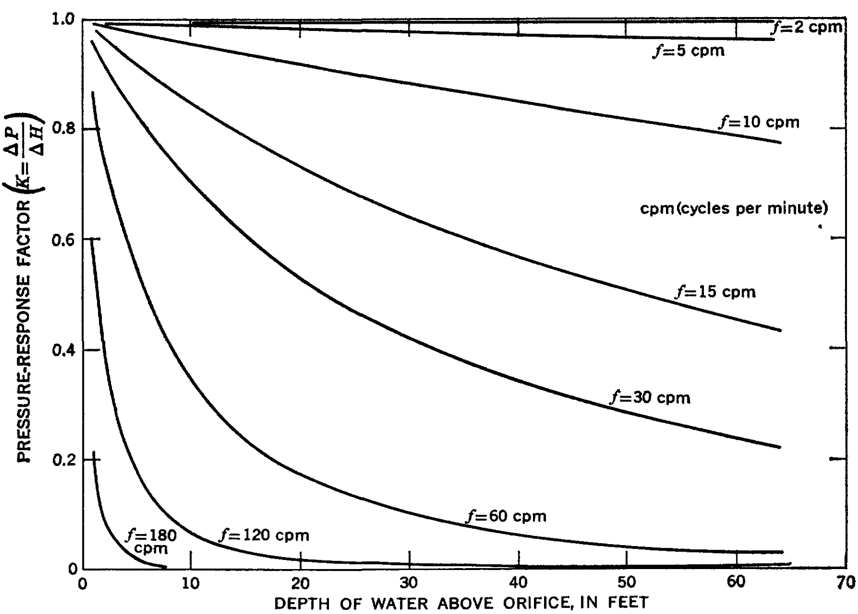

FIGURE 3.—Relation of pressure-response factor to depth of water for various wave frequencies.

TEST PROGRAM

Response tests for both cyclic and abrupt changes in stage were made for an Exactel Servomanometer and a survey servocontrolled-manometer. For testing the effect of cyclic change in stage, the manometer tubing orifice was placed in a 3-inch-diameter plastic cylinder about 2½ feet long that contained water, and change of stage was accomplished by cycling the orifice vertically at several rates by means of a motor and disk crank to generate a sine-wave input to the orifice. For testing the effect of abrupt change in stage, the crank motor was disconnected and movement was accomplished by manually rotating the crank from a minimum to a maximum head on the orifice.

The Exactel Servomanometer was tested at the Exactel plant. Two Esterline Angus 0–1 milliampere recorders were used, one to record the cycling rate of the orifice and the other to record the Servomanometer response. Both recorders operated from a potentiometer bridge between the recorder and the Servomanometer. A survey float-switch servocontrolled-manometer was available for testing at Menlo Park, Calif. Both cycling rate and response were recorded on a Leupold Stevens A–35 continuous chart recorder driven by a motor through a gear train.

Orifice tubing lengths and diameters and sight-glass bubble rates were chosen to represent typical field installations. Tubing lengths of 50 and 100 feet were used for the survey unit; only the longer tubing (100 ft) was used for the Exactel Servomanometer because this unit is usually used at installations where large ranges in stage are expected. Tubing ID (inside diameter) was ⅛ and 3/16 inch for the Exactel Servomanometer and ⅛ inch for the survey system. Bubble rates of 40, 80, and 160 per minute were used for both manometers.

MANOMETER RESPONSE TO CYCLIC CHANGE IN STAGE

The stage oscillations were in form of a sine wave and are referred to in this report as a sine-wave input. Cycling rates of about 2, 5, and 10 cpm and amplitudes of 0.1 and 0.2 foot were selected for these tests. These wave amplitudes and frequencies were used for the tests because they typify oscillations which often occur in field installations. Tests conducted with the Exactel Servomanometer were limited to amplitudes of 0.1 foot. Both 0.1- and 0.2-foot amplitudes were used for the survey servocontrolled-manometer tests. Testing of the survey unit was done first with the delay circuit off and repeated with the delay circuit on. Results of these tests are summarized in tables 1 and 2 and are plotted in figures 4–6; conclusions are discussed in the section on analysis.

TABLE 1.—*Error in stage indicated by Exactel Servomanometer and piezometer system with cyclic change in stage*

Bubble rate (bubbles per minute)	Error in recorded stage (foot of water)								
	2 cycles per minute			5 cycles per minute			10 cycles per minute		
	Minimum	Maximum	Mean	Minimum	Maximum	Mean	Minimum	Maximum	Mean
Tube length, 100 feet; inside diameter ⅛ inch; wave amplitude, 0.1 foot									
40	−0.082	−0.097	−0.089	−0.089	−0.097	−0.093	−0.092	−0.097	−0.095
80	−.059	−.095	−.077	−.077	−.094	−.085	−.082	−.089	−.085
160	−.003	−.089	−.046	−.059	−.091	−.075	−.066	−.081	−.073
Tube length, 100 feet; inside diameter 3/16 inch; wave amplitude, 0.1 foot									
40	−0.067	−0.094	−0.080	−0.075	−0.091	−0.083	−0.075	−0.095	−0.085
80	−.057	−.091	−.074	−.071	−.092	−.082	−.069	−.091	−.080
160	−.028	−.093	−.060	−.059	−.104	−.082	−.058	−.085	−.072

TABLE 2.—*Error in stage indicated by U.S. Geological Survey servocontrolled-manometer with cyclic change in stage*

Bubble rate (bubbles per minute)	Error in recorded stage (foot of water)											
	Delay circuit off									Delay circuit on		
	2 cycles per minute			5 cycles per minute			10 cycles per minute			2 cycles per minute	5 cycles per minute	10 cycles per minute
	Minimum	Maximum	Mean	Minimum	Maximum	Mean	Minimum	Maximum	Mean			
Tube length, 100 feet; inside diameter, ⅛ inch; wave amplitude, 0.1 foot												
40	−0.06	−0.10	−0.08	−0.08	−0.08	−0.08	−0.08	−0.08	−0.08	−0.05	−0.04	−0.06
80	−.02	−.06	−.04	−.06	−.06	−.06	−.07	−.07	−.07	0	−.04	−.05
160	+.04	−.03	0	−.01	−.03	−.02	−.03	−.03	−.03	0	0	0
Tube length, 100 feet; inside diameter, ⅛ inch; wave amplitude, 0.2 foot												
40	−0.15	−0.19	−0.17	−0.15	−0.17	−0.16	−0.16	−0.16	−0.16	−0.12	−0.15	−0.13
80	−.11	−.18	−.14	−.13	−.16	−.14	−.15	−.15	−.15	−.15	−.10	−.14
160	−.06	−.12	−.09	−.10	−.12	−.11	−.14	−.14	−.14	−.05	−.08	−.09
Tube length, 50 feet; inside diameter, ⅛ inch; wave amplitude, 0.1 foot												
40	−0.04	−0.10	−0.07	−0.05	−0.07	−0.06	−0.06	−0.07	−0.06	−0.07	−0.04	−0.05
80	0	−.07	−.04	−.05	−.05	−.05	−.05	−.05	−.05	0	−.04	−.04
160	+.03	−.04	0	0	−.03	−.02	−.03	−.03	−.03	0	0	−.05
Tube length, 50 feet; inside diameter, ⅛ inch; wave amplitude, 0.2 foot												
40	−0.12	−0.19	−0.16	−0.15	−0.16	−0.16	−0.15	−0.17	−0.16	−0.15	−0.13	−0.18
80	−.10	−.18	−.14	−.11	−.13	−.12	−.14	−.15	−.14	−.17	−.15	−.13
160	−.04	−.12	−.08	−.08	−.09	−.09	−.12	−.12	−.12	0	−.08	−.12

MANOMETER RESPONSE TO ABRUPT CHANGE IN STAGE

An abrupt change in stage (a step input) was applied to each system in order to determine maximum response rates. The magnitude of the step input for the Exactel servomanometer test was 0.2 foot, and for the survey servocontrolled-manometer tests, 0.2 and 0.4 foot. These amplitudes were adequate for comparison with computed response rates. Tests were made only for a rising stage because the internal pressure of the manometers responds with negligible lag to a falling stage. The absence of appreciable lag on falling stage was verified by observation of the rapid, but short-lived, rush of bubbles from the orifice when the stage was abruptly lowered followed by the almost immediate return to the normal exit bubble rate. During such a rapid pressure decrease, the recorded gage height is controlled primarily by the response of the servomechanism. The Exactel servosystem reacts almost instantaneously, thus providing a close replication of the actual internal-system pressure. The survey system is limited by the traverse rate of the drive motor and consequently records an abrupt pressure decrease as a relatively slow decrease equal to the motor traverse rate. For these tests the survey unit was tested with the 35-second delay circuit off rather than on, because the delay circuit inhibited manometer response to pressure changes in the system and precluded determination of its maximum response rate.

Results are given in tables 3 and 4 and are plotted in figures 7 and 8; they are discussed in the section on analysis.

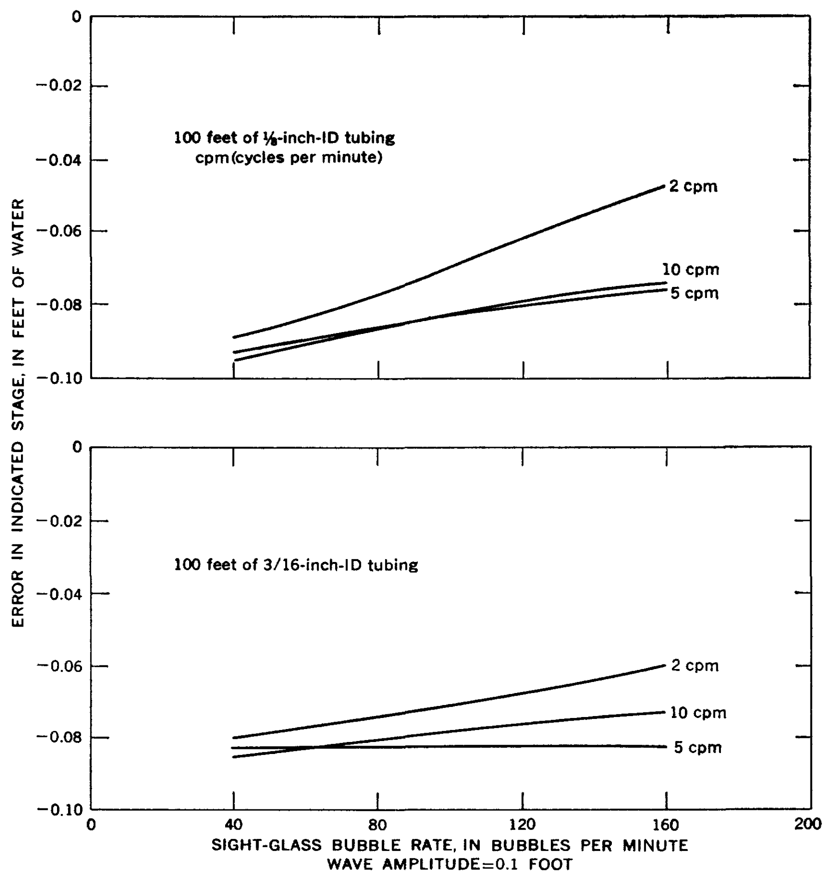

FIGURE 4.—Error in mean stage indicated by Exactel Servomanometer and piezometer for cyclic change in stage.

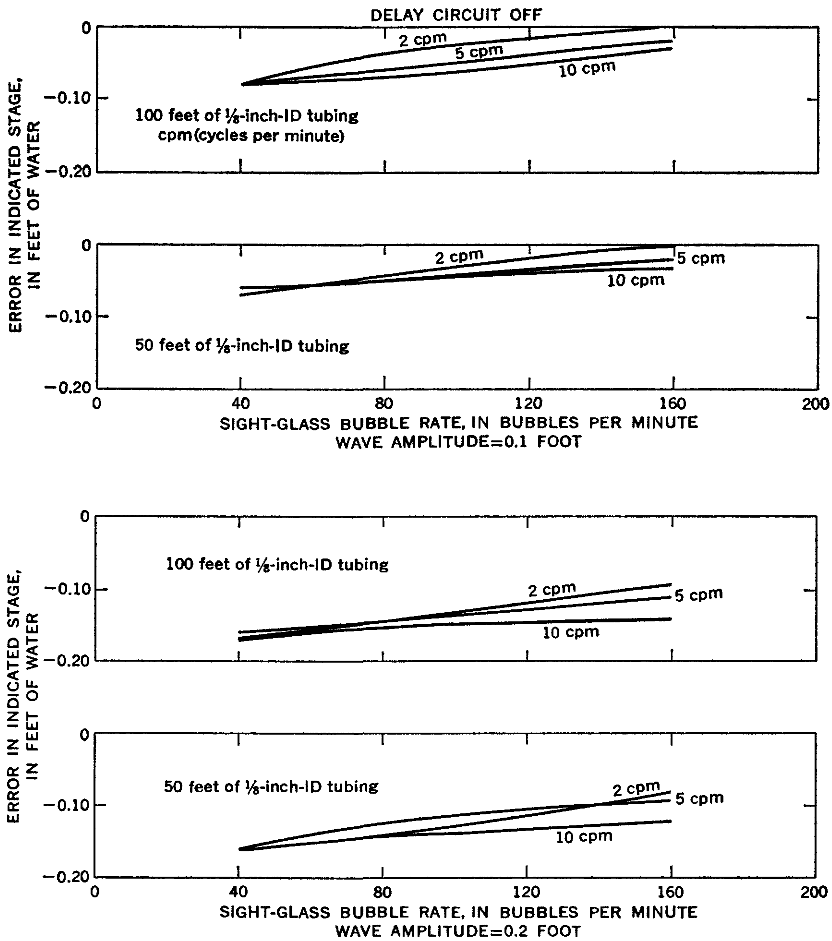

FIGURE 5.—Error in mean stage indicated by U.S. Geological Survey servo-controlled-manometer and piezometer system for cyclic change in stage, delay circuit off.

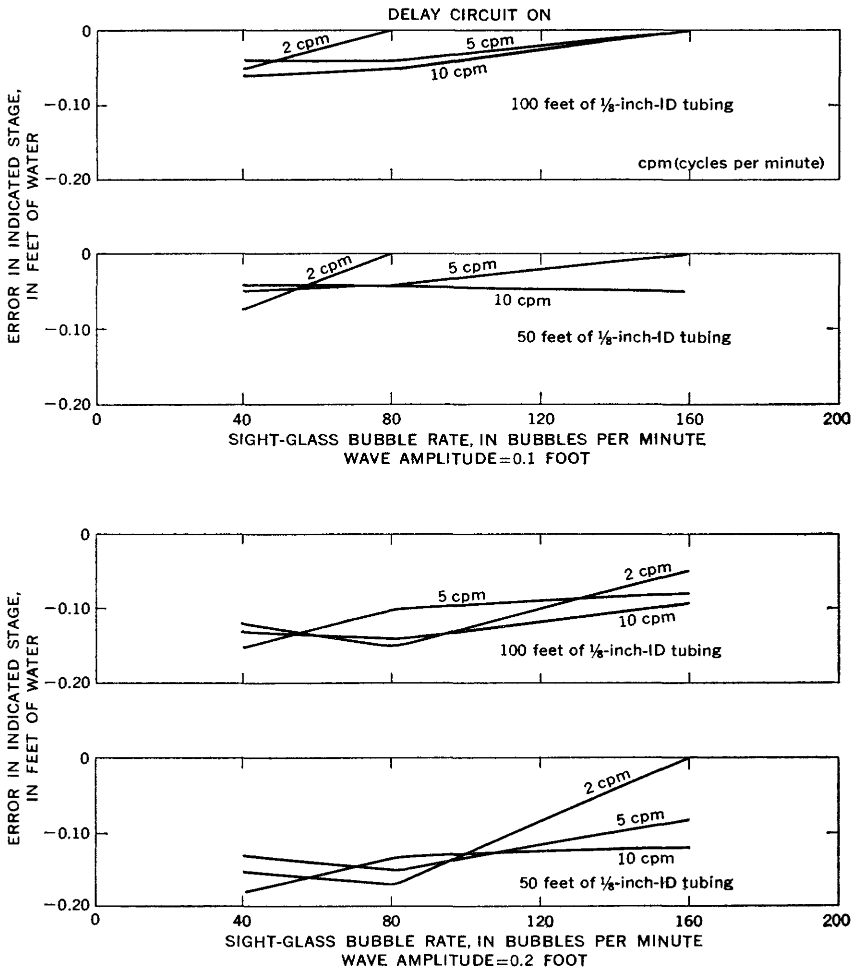

FIGURE 6.—Error in mean stage indicated by U.S. Geological Survey servo-controlled-manometer and piezometer system for cyclic change in stage, delay circuit on.

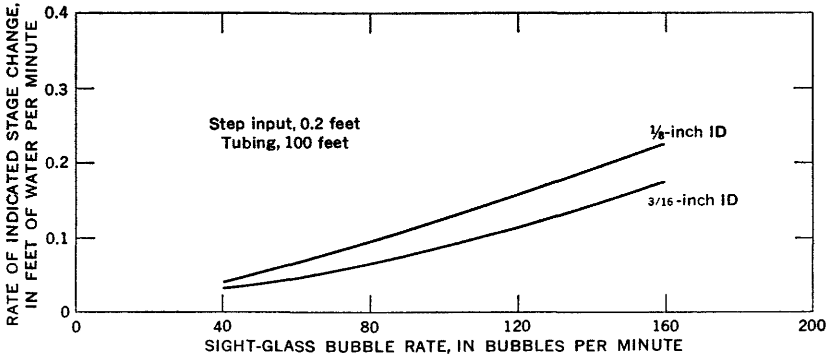

FIGURE 7.—Response rate of Exactel Servomanometer and piezometer system for abrupt rise in stage.

ANALYSIS OF TEST RESULTS

CYCLIC INPUT

Minimum, maximum, and mean stages were obtained from the recorded response traces at the various cycling and bubble rates. The orifice was stopped at its extreme and midpoint positions to obtain the static readings. The mean recorded stage for each test was compared to the static midpoint, and differences were plotted in figures 4–6. For an amplitude of 0.1 foot, the error in recorded stage ranged from about 0.09 foot of water at a bubble rate of 40 per minute to 0.05 foot of water at a rate of 160 per minute with the Exactel Servomanometer, and from about 0.08 to 0 foot of water with the Survey system. When the amplitude was doubled to 0.2 foot in the Survey system test, the range of error was approximately doubled—0.17–0.08 foot of water. Little difference was noted in the results between tests using 100 feet of tubing and those using 50 feet of tubing.

These errors were in all cases negative—that is, the recorded gage height was less than the true mean gage height when cyclic variations in stage were imposed on the system.

Cycling tests were run using the survey system with the delay circuit both on and off, as shown in the data of table 2. The orifice was set at midposition, and the manometer was stabilized at this elevation prior to each cycling run. When the orifice was cycled, the manometer float needle bounced back and forth between its contacts, and, with the delay circuit on, did not stay on each contact long

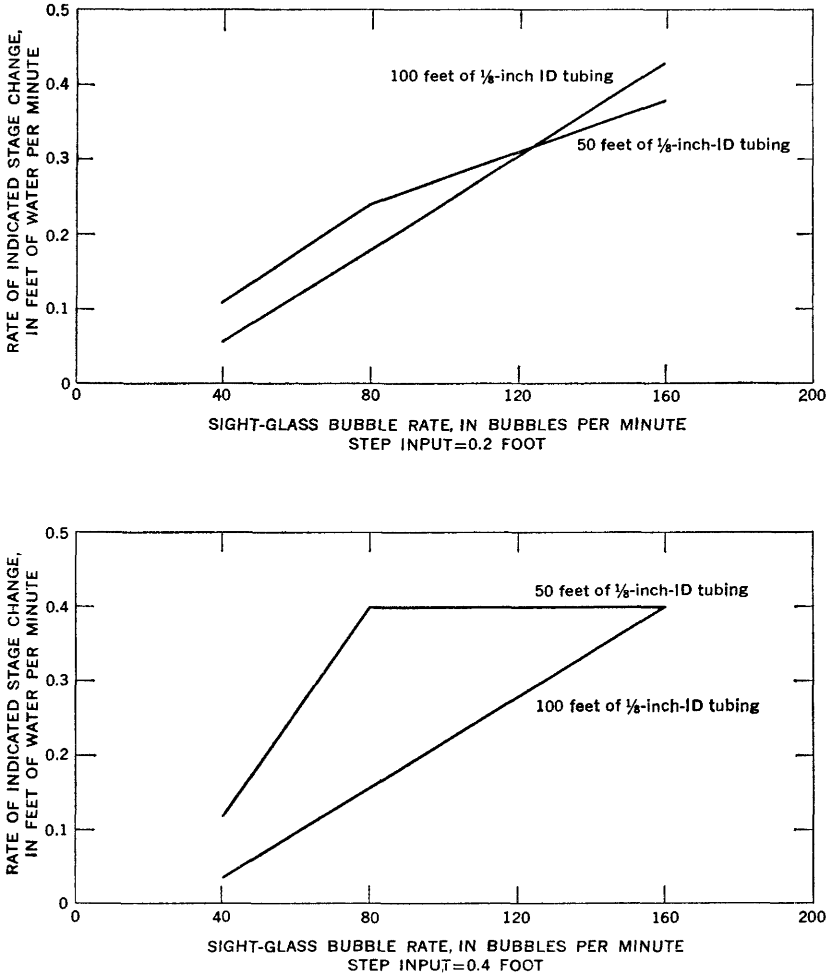

FIGURE 8.—Response rate of U.S. Geological Survey servocontrolled-manometer and piezometer system for abrupt rise in stage.

TABLE 3.—*Response rates of Exactel Servomanometer and piezometer system for abrupt rise in stage*

Bubble rate (bubbles per minute)	Rate of recovery (foot of water per minute)	Time to stabilize (minutes)	
		First bubble	Computed from rate of recovery
Tube length, 100 feet; inside diameter, 1/8 inch; 0.2-foot step input			
40	0.039	4.95	5.26
80	.093	1.99	2.15
160	.226	.77	.88
Tube length, 100 feet; inside diameter, 3/16 inch; 0.2-foot step input			
40	0.032	5.33	6.25
80	.064	2.18	3.13
160	.173	.92	1.16

TABLE 4.—*Response rates of U.S. Geological Survey servocontrolled-manometer and piezometer system for abrupt rise in stage (delay circuit off)*

Bubble rate (bubbles per minute)	Rate of recovery (foot of water per minute)	Time to stabilize (minutes)		
		First bubble	Including hunting	Computed from rate of recovery
Tube length, 100 feet; inside diameter, 1/8 inch; 0.2-foot step input				
40	0.056	2.8	2.8	3.6
80	.18	1.0	1.0	1.1
160	.43	.4	2.6	.5
Tube length, 100 feet; inside diameter, 1/8 inch; 0.4-foot step input				
40	0.037	7.4	7.4	10.8
80	.16	2.0	3.1	2.5
160	.40	.7	1.8	1.0
Tube length, 50 feet; inside diameter, 1/8 inch; 0.2-foot step input				
40	0.11	1.5	1.5	1.8
80	.24	.8	1.3	.8
160	.38	.3	2.5	.5
Tube length, 50 feet; inside diameter, 1/8 inch; 0.4-foot step input				
40	0.12	2.7	3.3	3.3
80	.40	1.0	1.0	1.0
160	.40	.6	3.2	1.0

enough to overcome the delay time (about 35 seconds). After about 5 minutes of cycling time, the recorded trace came to reasonable stability and trace-position values were obtained. Bouncing of the float needle depends upon point-gap spacing. If the points are close together, there is greater oscillation of the recorder trace; if they are farther apart, there is less oscillation of the trace. Total gap between points for test purposes was one-sixteenth inch. Results obtained for the survey system with the delay unit on seem to be anomalous in that the recording error, on some of the runs, tended to increase when the bubble rate was raised from 40–80 bubbles per minute and decrease, as would be expected, when the gas feed was increased to 160 bubbles per minute. The reason for this is not understood, but the significant facts are that the net recording error is of the same sign and general magnitude as that encountered with the delay circuit off and that this error can be decreased by an increase in gas-purge rate.

ABRUPT INPUT

The rate of response to an abrupt rise in stage was obtained by measuring the slope of the response trace resulting from an increasing-stage step input to the orifice. This was done for various bubble rates. Response rates are plotted in figures 7 and 8. Rates for the Exactel Servomanometer increase slowly with increased bubble rate because of the large volume of the total system; those for the survey system seem to vary, depending on whether the trace is increasing continuously or in intermittent steps and at low bubble rates, and are significantly greater for the shorter length of tubing. Response rates for the Exactel unit are more nearly equivalent to the pressure changes in the gas system and are not limited by the servo-operation because the servomanometer reacts very quickly. Rates for the survey system are controlled by a combination of pressure changes and the motor traverse rate. The maximum rate of rise of the servo-controlled-manometer trace was about 0.25 fpm (feet per minute) using 6-volt batteries and about 0.40 fpm using a 7½-volt power supply and was dependent primarily on the voltage applied and motor. With the fast bubble rate (160 bubbles per minute), the drive motor ran continuously, indicating that the motor could not keep up with the changing pressure in the system. In addition, a period of hunting was sometimes observed during stabilization. The Exactel Servomanometer was observed to follow accurately the pressure trend, and little or no hunting resulted.

The time required for the Exactel unit to stabilize was controlled by the purge rate and the time required for the relatively large internal volume of the manometer system (manometer tubes and piezometer tubing) to pressurize to the new stage condition. Because the survey system had less volume, it did not require as much time to stabilize pressure. However, because of the slower drive motor, the time necessary for the switch to make contact (which is dependent on point-gap setting), and the hunting period required, the overall stabilization time was increased. Operation of the survey system does not usually follow reproducible patterns; thus, table 4 shows internal-pressure stabilization times determined by timing the first appearance of a bubble at the orifice, overall time including hunting periods, and time computed from the rate of recovery.

THEORETICAL RESPONSE

The slope of the derived step-input response equation (eq 15), which is plotted in figure 9 for the Exactel Servomanometer, establishes a maximum rate of pressure recovery in the Servomanometer system for rising stages. The curve in figure 9 approximates a straight line for the times shown and is used as such in the analysis. Any rate of rise in stage which equals or exceeds this maximum rate of pressure recovery will, therefore, produce the maximum response rate. For the sine-wave input used in the experiment, the rate of rise during the rising-stage section of the waves was in all cases greater than that required to produce the maximum response. During falling stages, the observed response was so rapid that it was considered to be immediate for the purpose of this analysis. Minor errors due to fluid friction in the gas-purged tube and dynamic effect of the servomechanism system itself were not included in this simplified analysis.

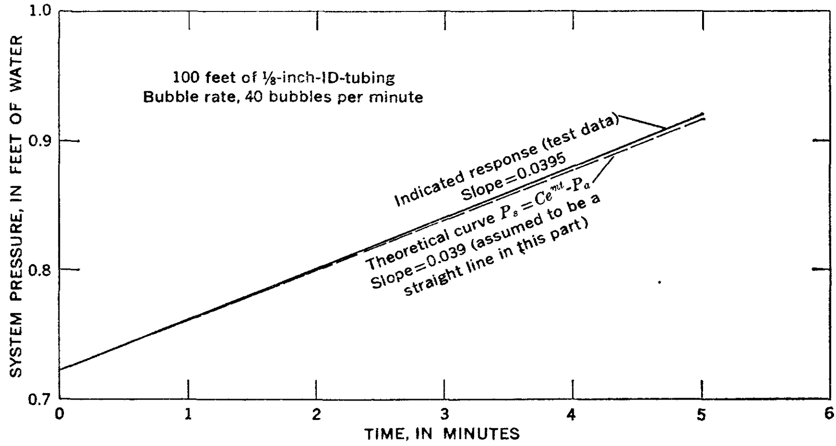

FIGURE 9.—Comparison of theoretical and recorded response of the Exactel Servomanometer and piezometer system to an abrupt rise in stage.

A theoretical response curve to a sine-wave input can be obtained by plotting the sine-wave input at the orifice using an amplitude equal to the lower excursion from static midpoint and then superimposing on the plot the theoretical maximum rising-stage response from figure 9. This has been done in figure 10 for a typical run. From the plot in figure 10, the theoretical average stage can be obtained. That stage can be compared with both the mean of the surface oscillations and the recorded mean stage. The comparison is made in table 5.

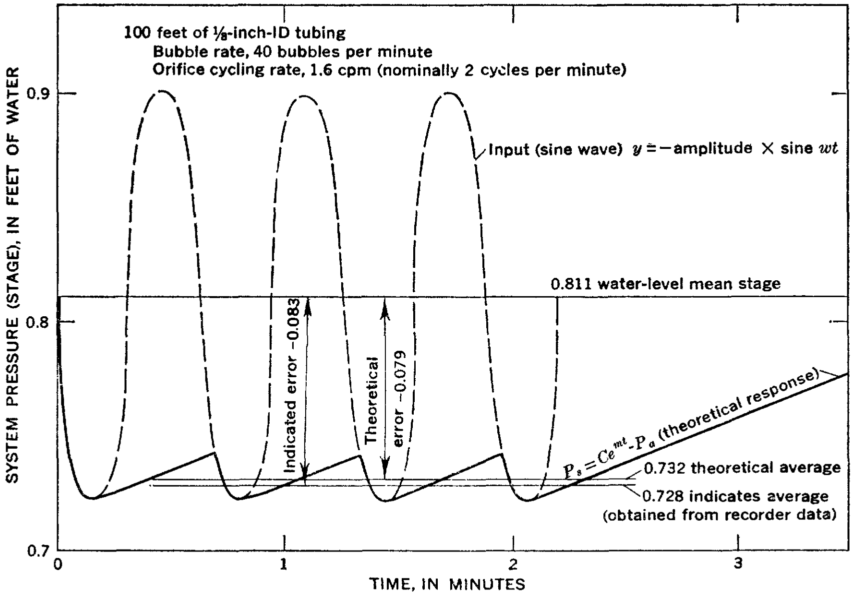

FIGURE 10.—Comparison of the average theoretical and recorded responses of the Exactel Servomanometer and piezometer system to a cyclic change in stage.

TABLE 5.—*Comparison of theoretical and recorded stage with mean surface oscillation for the Exactel Servomanometer and piezometer system*

Bubble rate (bubbles per minute)	2 cycles per minute			5 cycles per minute			10 cycles per minute		
	Mean stage (foot of water)			Mean stage (foot of water)			Mean stage (foot of water)		
	Surface	Theoretical	Recorded	Surface	Theoretical	Recorded	Surface	Theoretical	Recorded
Tube length, 100 feet; inside diameter, ⅛ inch; wave amplitude, 0.1-foot									
40	0.811	0.732	0.728	0.794	0.725	0.725	0.799	0.723	0.723
80	.807	.745	.741	.793	.731	.733	.800	.734	.733
160	.813	.775	.772	.797	.746	.743	.804	.746	.745

Response to a step input applied to the survey system orifice is limited by the relatively slow motor speed of the servosystem. The recorder response rate, therefore, does not always reflect the pressure in the gas system, but it may be indicative only of motor speed. Thus, a response trace such as that shown by the Exactel Servomanometer and piezometer system in figure 9 cannot be obtained with the survey servomanometer. A step-stage increase of 0.4 foot combined with a bubble rate of 40 bubbles per minute was the only condition under which the stage trace for the survey manometer closely followed the pressure in the system. The rate of recovery for this run was 0.037 fpm, and the average recorded stage for a sine-wave input of 0.2 amplitude was −0.08 foot below water-level mean stage. These values compare favorably with the slope and error for the Exactel Servomanometer when it is used with the gas-purged system as shown in figures 9 and 10.

PREDICTION OF ERROR IN STAGE RECORDED BY A BUBBLE GAGE DURING CYCLIC OSCILLATIONS IN WATER LEVEL

Error in recorded stage can be predicted by use of figure 3 and equation 15, if the depth of water over the orifice and the amplitude and frequency of the cyclic oscillations at the water surface are known. Figure 9 shows that the theoretical response computed from equation 15 closely matches the actual response for an Exactel system.

To predict error in response to water-surface oscillations of known amplitude and frequency, enter figure 3 with the depth of the orifice below water surface (abscissa) and the known frequency of the oscillation and obtain the ratio of orifice to water-surface pressures, k. From this ratio and the known amplitude at the surface, compute the effective amplitude at the orifice. The frequency of pressure oscillation at the orifice and at the surface will be approximately the same for all but highly irregular wave patterns. A plot can be made similar to figure 10 showing orifice pressure-head oscillations and the theoretical response computed from equation 15. The predicted error in stage is the difference between the mean of the surface oscillations and the mean of the theoretical response graph.

This method holds strictly true for the Exactel unit, but it must be modified slightly when applying it to the Survey unit. If the computed rate of pressure rise or the indicated rate of fall exceeds the motor traverse rate, the motor traverse rate must be used instead.

SUMMARY AND CONCLUSIONS

The stage indicated by a bubble gage is a reading equivalent to the internal pressure in the gas-purged part of the system. Under static conditions this internal pressure is an accurate analog of the water stage, but when oscillatory pressure variations occur at the system orifice in response to short-period wave action, the average internal pressure of the system will not be equal to the average pressure head at the orifice. This is so because such systems adjust rapidly to decreasing pressures (falling stage periods) and adjust slowly—at a rate controlled primarily by the purge rate—to increasing pressures (rising stage periods). The net effect is that bubble gages do not record the mean of the surge, but they underregister by a variable amount when subject to wave action.

The dominant factors controlling the magnitude of registration errors at a given installation are the size and frequencies of the waves and the purge rate of gas flowing into the system. Other factors, which may vary with time or from system to system, are the depth of water over the orifice, the internal gas-filled volume of the manometer itself, and the length and size of tubing used to connect with the orifice.

Under adverse conditions, where the purge rate is less than 80 bubbles per minute, where the depth of water over the orifice is less than 50 feet (as is usual with the survey unit), and where wave frequency is greater than 5 cpm, the gage height recorded may approach the trough of the waves. The magnitude of underregistration may be reduced by increasing the purge rate, but the laboratory tests herein reported still show significant errors at a purge rate of 160 bubbles per mniute.

The significance of these registration errors must be assessed in relation to the characteristics of the body of water involved and the purpose for which the stage is being recorded. State records on lakes and reservoirs are generally used to document the exact elevation of the water surface. Registration errors in the records may consequently have considerable importance. For lake gages, the errors due to wave action would probably occur at random times coincident with periods of high wind. The magnitude of errors would be less at installations where wave frequency was high, and the gage orifice was placed in deep water. In rivers the magnitude of wave action—sometimes referred to as surge—is a frequently repetitive phenomenon closely associated with the stage and discharge. At low stages, streams are typically calm with little or no wave action, whereas at high stages considerable turbulence develops and a consistent characteristic pattern of surge is apparent. If the primary purpose of the stage record on such a stream is for use in the computation of discharge records, then registration errors will probably be compensated for in the development of the stage-discharge relation and they will be of no significance whatsoever.

The significant findings of this study are that gas-purged servo-controlled-manometer systems (bubble gages) do tend to indicate a stage value less than the mean of the surge when surface waves are present. In some places these errors are negligible, or they may be compensated for in the manner in which the record is used. This report suggests a technique whereby the magnitude of errors may be predicted; but, unless the wave characteristics are actually known, minor errors must be expected. A knowledge of this system characteristic should improve the quality of interpretation of stage records obtained by use of a bubble gage and permit corrective steps to be taken in places where real problems exist.

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