

Techniques and Equipment Required for Precise Stream Gaging in Tide-Affected Fresh-Water Reaches of the Sacramento River, California

GEOLOGICAL SURVEY WATER-SUPPLY PAPER 1869-G

*Prepared in cooperation with the
California Department of Water
Resources*



Techniques and Equipment Required for Precise Stream Gaging in Tide-Affected Fresh-Water Reaches of the Sacramento River, California

By WINCHELL SMITH

R I V E R H Y D R A U L I C S

GEOLOGICAL SURVEY WATER-SUPPLY PAPER 1869-G

*Prepared in cooperation with the
California Department of Water
Resources*



UNITED STATES DEPARTMENT OF THE INTERIOR

ROGERS C. B. MORTON, *Secretary*

GEOLOGICAL SURVEY

W. A. Radlinski, *Acting Director*

Library of Congress catalog-card No. 75-611941

For sale by the Superintendent of Documents, U.S. Government Printing Office
Washington, D.C. 20402 - Price 75 cents (paper cover)

CONTENTS

	Page
Symbols.....	v
Abstract.....	G1
Introduction.....	2
Purpose and scope.....	2
Acknowledgments.....	3
The gaging problem.....	3
Current-meter measurements.....	5
Specialized stream-gaging procedures.....	7
Prych-Hubbel-Glenn system.....	7
Kallio system.....	9
Smoot system.....	9
System required for Chipps Island.....	10
Definition of the mean velocity in a vertical.....	11
Analysis of velocity-distribution data.....	11
Average velocity ratios for total observation period.....	13
Variation of velocity ratios with tide phase.....	15
Comparison of logarithmic- and power-law distribution functions.....	17
Conclusions from velocity-distribution study.....	19
Chipps Island measuring system.....	21
Basic theory of the measuring system.....	23
Resolution of velocity vector relations.....	23
Computation of discharge in each subsection.....	26
Computation of total instantaneous discharge.....	26
Development of the prototype equipment.....	27
Operational experience.....	29
Field test work in the Bolinas channel.....	29
Field test work in the Chipps Island channel.....	29
Measurement results.....	31
Probable accuracy of Chipps Island measurements.....	33
Errors in computation of subsection discharges.....	33
Evaluation of S_{r_t}	33
Evaluation of S_{r_s}	34
Evaluation of S_{r_α}	34
Evaluation of S_{r_p} and S_{r_b}	36
Evaluation of S_{r_α}	37
Evaluation of $S_{r_{vb}}$	37
Evaluation of S_{r_q}	37
Errors in computation of instantaneous total discharge.....	38
Evaluation of S_{R_q}	38
Evaluation of S_{R_N}	39
Evaluation of $S_{R_{IP}}$	40
Evaluation of $S_{R_{CM}}$	42
Evaluation of S_{R_Q}	42
Recommended stream-gaging system at Chipps Island.....	43
Selected references.....	45

ILLUSTRATIONS

	Page
PLATE	
1. Plot of time-related variations in velocity-distribution ratios.....	In pocket
FIGURE	
1. Map showing Chipps Island area.....	G2
2. Map and cross sections showing hydrographic detail at Chipps Island channel.....	4
3. Cross-section and horizontal-velocity profile near cross section 4 of figure 2 at 1215 hours, March 27, 1968....	5
4. Typical hydrographs of discharge and stage in the Sacramento River at Chipps Island.....	6
5. Sketch showing procedure used in the moving-boat method of streamflow measurement.....	8
6. Sample of computer analysis of velocity-distribution data..	14
7-11. Plots showing:	
7. Variation of ratio $V28/VM$ with rate of change of velocity at stations 1 and 2.....	16
8. Variation of ratio VL''/VM with rate of change of velocity at stations 1 and 2.....	17
9. Variation of ratio $V28/VM$ with rate of change of velocity at stations 1, 2, and 3.....	18
10. Variation of ratio VL''/VM with rate of change of velocity at stations 1, 2, and 3.....	19
11. Relation between mean velocity and the error index for distribution curves fitted to four observation points by the logarithmic-distribution law.....	20
12. Sketch of stream showing markers and course of boat during measurement.....	22
13. Diagram of velocity vector relations.....	24
14. Photograph showing data recording panel.....	28
15. Plot showing discharge computed from March 28, 1968, measurements.....	31
16. Plot showing discharge computed from August 27, 1968, measurements.....	32
17. Simplified vector diagram.....	35
18. Plot showing error introduced by parabolic interpolation procedures, assuming tidal cycle conforms to a sine function.....	41

TABLES

	Page
TABLE	
1. Average coefficients for total observation period.....	G15
2. Error indexes for various velocity-distribution curves....	18
3. Statistics of principle error sources in subsection discharges..	34

		Page
TABLE	4. Variation of error ratio r_α with water velocity and boat velocity.....	G36
	5. Statistics of principle error sources in computed instantaneous total discharge.....	38

SYMBOLS

a	Coefficient in the power-law velocity distribution function.
b	Exponent in the power-law velocity distribution function.
N	Number of subsections included in a measurement of total discharge.
Q	Total measured discharge.
Q_{CM}	Total measured discharge including error related to current-meter calibration.
Q_{IP}	Total measured discharge including error related to interpolation procedures.
Q_N	Total measured discharge including error related to number of subsections.
Q_q	Total measured discharge including errors in measured subsection discharge.
q	Subsection discharge.
q_b	Measured subsection discharge including error related to boat velocity.
q_d	Measured subsection discharge including error related to depth.
q_p	Measured subsection discharge including error related to boat position.
q_s	Measured subsection discharge including error related to vertical-velocity curve.
q_t	Measured subsection discharge including error due to velocity pulsation.
q_{vb}	Measured subsection discharge including error due to vertical movement of the boat.
q_α	Measured subsection discharge including error related to vector angle.
R_{CM}	Error ratio in total measured discharge due to current-meter calibration.
R_{IP}	Error ratio in total measured discharge due to interpolation procedures.
R_N	Error ratio in total measured discharge due to number of subsections used.
R_Q	Error ratio in computed instantaneous total discharge.
R_q	Error ratio in total measured discharge due to error in subsection discharge
R_{qa}	Absolute value of R_q .
r_b	Subsection error ratio related to boat velocity.
r_d	Subsection error ratio related to depth.
r_p	Subsection error ratio related to boat position.
r_q	Partial error ratio of subsection discharge.
r_{qa}	Absolute value of r_q .
r_s	Subsection error ratio related to vertical-velocity curve.
r_t	Subsection error ratio due to velocity pulsation.
r_{vb}	Subsection error ratio due to vertical movement of the boat.
r_α	Subsection error ratio related to vector angle.
S	Standard deviation of a parameter; subscripts indicate particular parameter.

T	Period of a tidal cycle in hours.
T_i	Time of observation.
V_2	Velocity at a point 0.2 of the depth below water surface.
V_6	Velocity at a point 0.6 of the depth below water surface.
V_8	Velocity at a point 0.8 of the depth below water surface.
V_{28}	Mean velocity computed as the average of velocities observed at points 0.2 and 0.8 of the depth below water surface.
VB	Boat velocity along the cross section.
VL	Mean velocity computed by least-squares fit of velocity observations at 9 points in the vertical to the logarithmic-distribution function.
VL'	Mean velocity computed by least-squares fit of velocity observations at points 0.2, 0.4, 0.6, and 0.9 of the depth below water surface to the logarithmic-distribution function.
VL''	Mean velocity computed by least-squares fit of velocity observations at points 0.2, 0.4, 0.6, and 0.8 of the depth below water surface to the logarithmic-distribution function.
VM	Mean velocity computed from the weighted arithmetic average of velocity observations at 9 points in the vertical.
VM_2	Average speed in the subsection recorded by the current meter placed at 0.2 of the depth below water surface.
VM_8	Average speed in the subsection recorded by the current meter placed at 0.8 of the depth below water surface.
VN_2	Average velocity component normal to the cross section at 0.2 of the depth below water surface.
VN_8	Average velocity component normal to the cross section at 0.8 of the depth below water surface.
VP	Mean velocity computed by least-squares fit of velocity observations at 9 points in the vertical to the power-law function.
VP'	Mean velocity computed by least-squares fit of velocity observations at points 0.2, 0.4, 0.6, and 0.9 of the depth below water surface to the power-law function.
VP''	Mean velocity computed by least-squares fit of velocity observations at points 0.2, 0.4, 0.6, and 0.8 of the depth below water surface to the power-law function.
V_f	Friction-velocity, a factor in the logarithmic velocity-distribution function
\bar{v}	Mean velocity in a subsection.
v_y	Velocity at a distance y above the riverbed.
Y_o	Constant in the logarithmic velocity-distribution function.
y	Distance above the riverbed.
α	Direction of the VM_2 velocity vector.

RIVER HYDRAULICS

TECHNIQUES AND EQUIPMENT REQUIRED FOR PRECISE STREAM GAGING IN TIDE-AFFECTED FRESH-WATER REACHES OF THE SACRAMENTO RIVER, CALIFORNIA

By WINCHELL SMITH

ABSTRACT

Current-meter measurements of high accuracy will be required for calibration of an acoustic flow-metering system proposed for installation in the Sacramento River at Chipps Island in California. This report presents an analysis of the problem of making continuous accurate current-meter measurements in this channel where the flow regime is changing constantly in response to tidal action. Gaging-system requirements are delineated, and a brief description is given of the several applicable techniques that have been developed by others. None of these techniques provides the accuracies required for the flowmeter calibration. A new system is described—one which has been assembled and tested in prototype and which will provide the matrix of data needed for accurate continuous current-meter measurements.

Analysis of a large quantity of data on the velocity distribution in the channel of the Sacramento River at Chipps Island shows that adequate definition of the velocity can be made during the dominant flow periods—that is, at times other than slack-water periods—by use of current meters suspended at elevations 0.2 and 0.8 of the depth below the water surface. However, additional velocity surveys will be necessary to determine whether or not small systematic corrections need be applied during periods of rapidly changing flow.

In the proposed system all gaged parameters, including velocities, depths, position in the stream, and related times, are monitored continuously as a boat moves across the river on the selected cross section. Data are recorded photographically and transferred later onto punchcards for computer processing. Computer programs have been written to permit computation of instantaneous discharges at any selected time interval throughout the period of the current-meter measurement program. It is anticipated that current-meter traverses will be made at intervals of about one-half hour over periods of several days. Capability of performance for protracted periods was, consequently, one of the important elements in system design.

Analysis of error sources in the proposed system indicates that errors in individual computed discharges can be kept smaller than 1.5 percent if the expected precision in all measured parameters is maintained.

INTRODUCTION

PURPOSE AND SCOPE

Drainage from the Great Central Valley of California is carried to the ocean by the Sacramento River in the channel opposite Chipps Island near Pittsburg, Calif. (fig. 1). A precise measurement of the flow at this point is desired for the economical operation of the California Aqueduct system and for the protection of the valuable agricultural lands in the delta region of the Sacramento and San Joaquin Rivers. Previous studies (Smith, 1969) indicated that the riverflow could be monitored with the desired precision if a suitable acoustic flowmeter could be procured and if stream-gaging techniques were developed for accurate calibration of such a meter.

This report describes the stream-gaging problem at the Chipps Island site, presents results of field surveys made to determine the

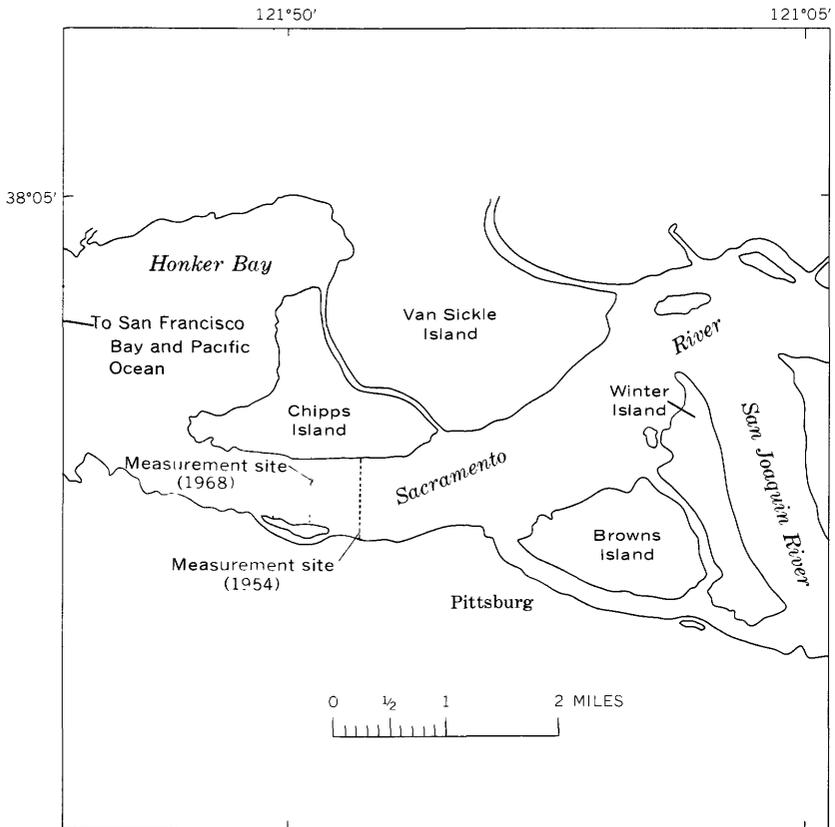


FIGURE 1.—Chipps Island area.

type of measuring equipment needed, and describes the prototype of equipment assembled and tested in the channel.

The report was prepared by the U.S. Geological Survey, Water Resources Division in cooperation with the California Department of Water Resources. The work was done during 1968 under the general supervision of R. Stanley Lord, district chief in charge of water-resources investigations in California.

ACKNOWLEDGMENTS

Assistance furnished by many of the technical personnel from the California and Oregon districts of the Geological Survey, Water Resources Division, has made this study possible, and this help is gratefully acknowledged. Particular recognition is owed to Carl Goodwin, who was an invaluable aid throughout the project, helping in assembly and operation of the prototype equipment and in the data analysis. The complex computer programs developed for the analysis of velocity data and for the computation of the tidal flow measurements were formulated and written by Mr. Goodwin.

THE GAGING PROBLEM

The Sacramento River at Chipps Island is about 3,000 feet wide and ranges in depth from 20 feet near the north shore to 60 feet in the ship channel which follows the south shore. Figure 2 shows the general configuration of the channel and several cross-section profiles in the reach. Figure 3 shows the cross section and velocities observed during a discharge measurement made March 27, 1968. Salt-water intrusion is generally moderate in this reach, with complete mixing in the vertical column. Separation of flow into salt- and fresh-water wedges does not occur.

Discharge is controlled primarily by the cyclic tidal action, with reversals in direction occurring within each of the twice-daily tidal cycles as shown in figure 4. Gross flows generally exceed 300,000 cfs (cubic feet per second) during both the ebbside and the floodside periods. The average daily fresh-water outflow, superimposed on this basic oscillatory tidal flow, may be as low as 2,000 cfs. A measure of this fresh-water outflow, or the net flow to the ocean, is the discharge of interest. It can only be computed as the difference between the measured floodside and ebbside discharges.

A net discharge computed in this manner will be meaningful only when averaged over several complete tidal cycles, and then only if the error characteristics of the measuring system are small and truly random. Thus, all facets of the measuring technique must be investigated to insure the elimination of systematic errors, particularly those

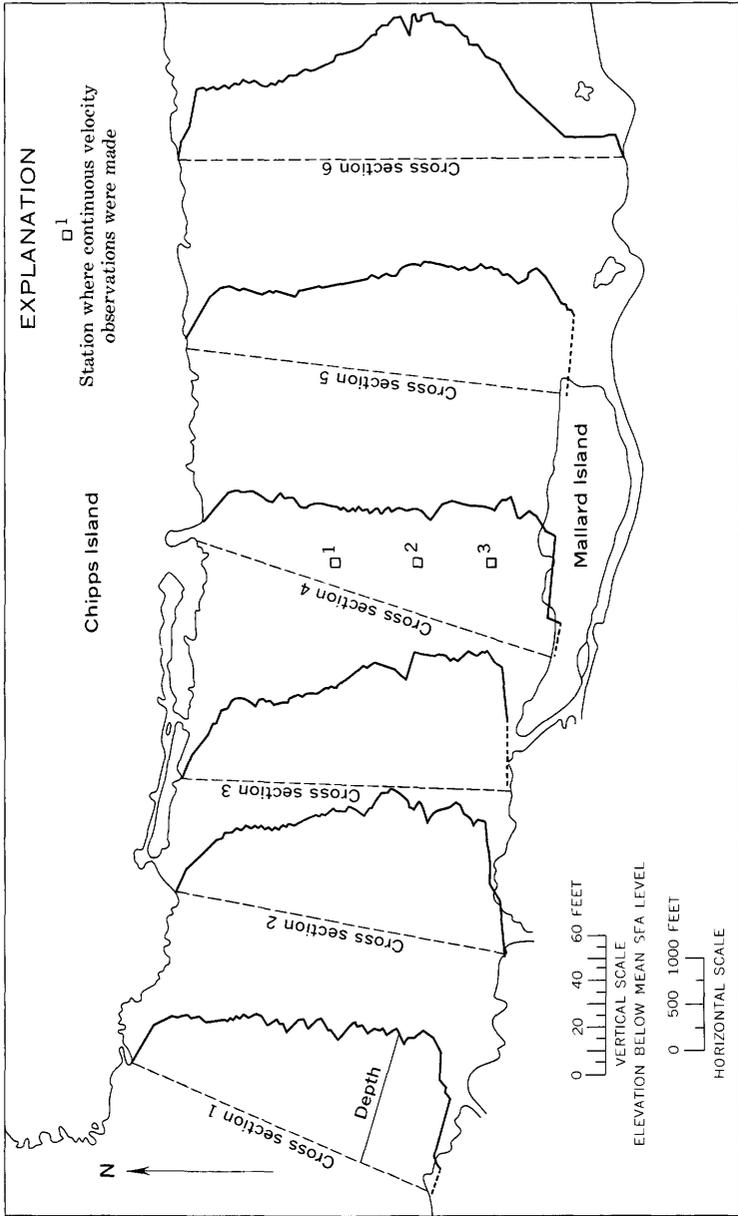


FIGURE 2.—Hydrographic detail at Chipps Island channel.

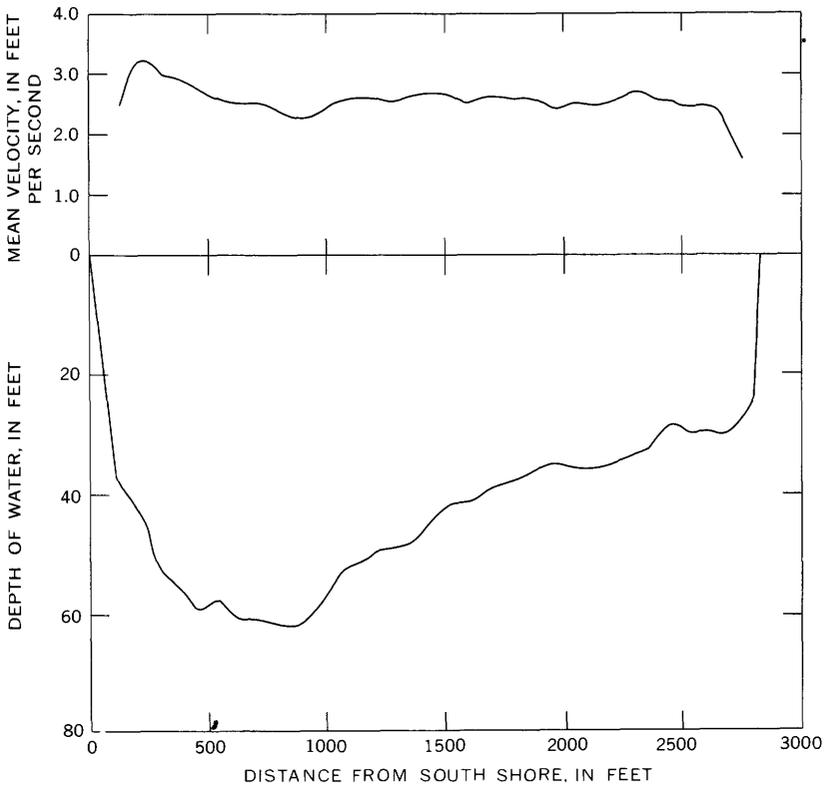


FIGURE 3.—Cross-section and horizontal-velocity profile near cross section 4 of figure 2 at 1215 hours, March 27, 1968.

related to the direction of flow. In addition, the system must be designed for operation on a continuous basis, providing definition of the flow over periods of several days. Problems of maintenance, of operator fatigue, and of safe operation in a busy ship channel will assume special significance in this stream-gaging program.

CURRENT-METER MEASUREMENTS

The conventional technique for making a current-meter measurement assumes a steady-state flow regime which will permit accurate computations of flow from observations spread out in time over a period of an hour or more. In making a conventional discharge measurement, the cross section of the stream is first divided into about 25 subsections. The mean velocity in a vertical line in the center of each subsection then is obtained, after which the discharge in each subsection is computed by multiplying each mean velocity by the area

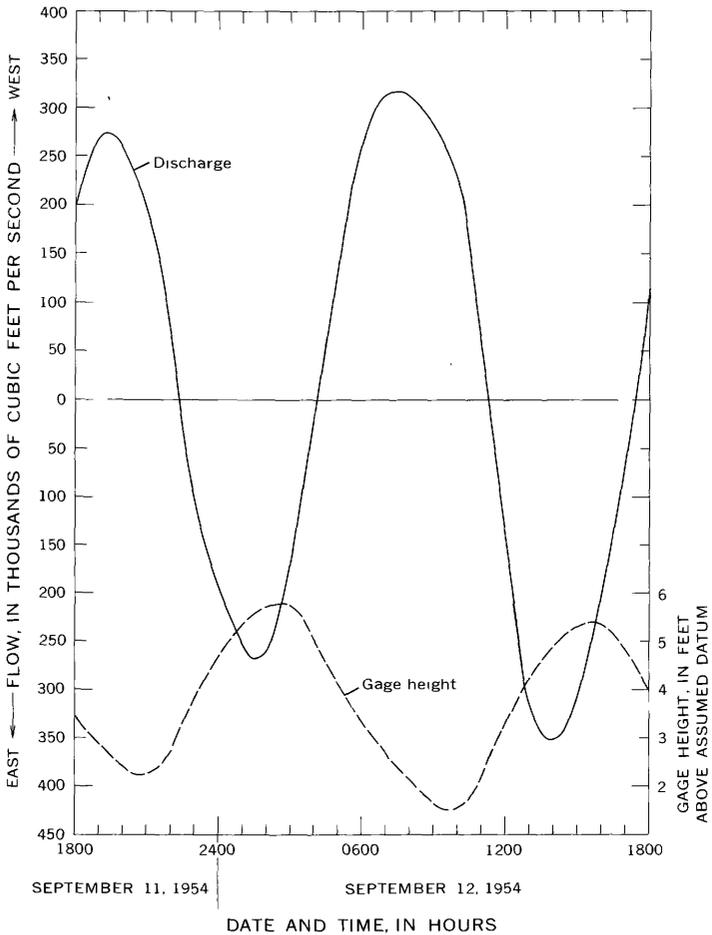


FIGURE 4.—Typical hydrographs of discharge and stage in the Sacramento River at Chipps Island.

of its respective subsection. The total discharge of the stream is then obtained by summing these incremental discharges. The measurements of width and depth required to obtain the area of a subsection are straightforward. The mean velocity in each vertical can be determined by any one of several methods; it is generally assumed equal to that observed at a point 0.6 of the depth below the water surface or equal to the mean of observations taken at points 0.2 and 0.8 of the depth below the water surface.

Discharge measurements made by these conventional techniques require an hour or more for execution and, in consequence, are poorly suited for gaging streams affected by tidal action where rapid changes

in discharge occur. For these streams a technique is required which will accurately sample the velocity-depth spectrum at frequent intervals and at many points in the channel cross section to provide a matrix of data from which discharges can be computed at time intervals short enough to properly define the discharge hydrograph. The decision as to the frequency of definition required rests, to some extent, on a subjective analysis of the cyclic flow pattern, which does not conform to a rigid mathematical model. However, definition about twice each hour would permit intermediate interpolation within a precision of less than 1 percent, and as this is within the expected accuracy of individual current-meter measurements, no greater frequency is necessary.

SPECIALIZED STREAM-GAGING PROCEDURES

One of the first attempts to accurately measure the flow in the lower Sacramento River was made by the California Department of Water Resources in 1954 (Ingerson, 1955). This measurement program, which used the moving-boat method illustrated in figure 5, was a classic application of direct measuring techniques. Buoys were placed in the channel to mark each of the subsections to be measured, and transits were placed on range lines to mark the ends of the short reach over which the boat traveled. Velocities were observed at positions 0.2 and 0.8 of the depth below water surface as the boat traversed upstream or downstream at right angles to the two ranges. The velocity of the boat (only enough to maintain steerageway) was determined by stopwatch measurement of the transit time of the boat between the established range lines. Water velocity was computed as the difference between the recorded current-meter velocities and the computed boat velocity. The primary criticisms of this system are that it takes too much time and will not provide sufficient data to accurately define the flow regime during periods of rapid change.

The problem of gaging large streams subject to tidal action has been given increased study since 1954, and several new systems have been developed. Most notable are the system introduced by Prych, Hubbel, and Glenn (1967) in the lower estuary of the Columbia River, the system developed by N. A. Kallio and others (written commun., 1968) for use in the upper Columbia River, and the moving-boat procedure devised by Smoot and Novak (1969). A brief description of each of these follows.

PRYCH-HUBBEL-GLENN SYSTEM

The system devised by Prych, Hubbel, and Glenn (1967) for use in the lower estuary of the Columbia River was designed specifically to permit measurement of flow in an estuary where fresh and salt water

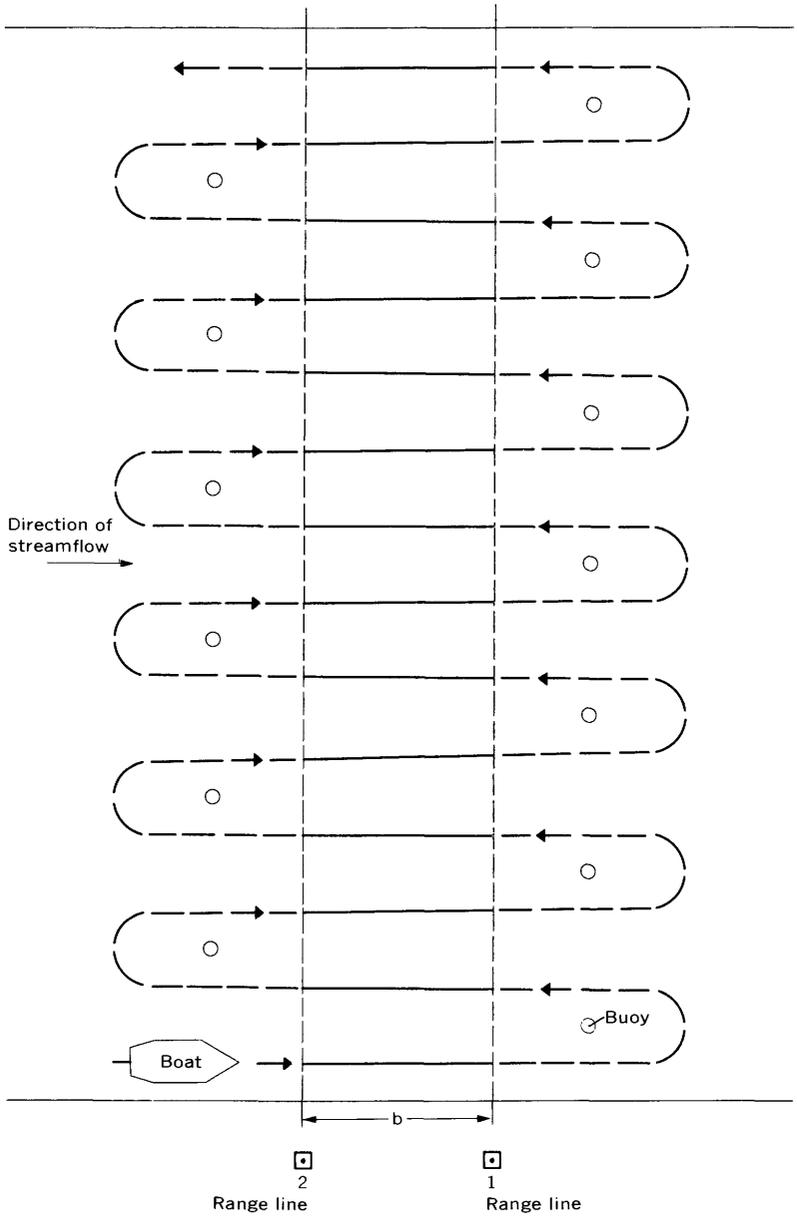


FIGURE 5.—Sketch showing procedure used in the moving-boat method of streamflow measurement.

mix. The unique feature of the system is that it can be used to define the entire vertical-velocity profile (magnitude and direction) regardless of its character. A sextant is used to determine boat position in the stream, a doppler navigation system is used to measure the rate of travel of the boat over the bottom during a velocity-observation period, a sonic sounder is employed for depth measurements, and a velocity-azimuth-depth assembly employing an Ott Cosine rotor is used to provide a continuous readout of velocity and direction as the sensing assembly is raised from the bottom to the water surface. Data from this system are recorded on magnetic tape for machine processing. Observations can be taken in about 15 verticals every hour. The system is well adapted to the problem for which it was designed, but is too slow to permit acquisition of the quantity of data required for the precise measurements that are needed at Chipps Island.

KALLIO SYSTEM

The Kallio system, designed and developed by N. A. Kallio and others (written commun., 1968) of the Portland district, Water Resources Division, Geological Survey, is a straightforward attempt to use routine stream-gaging methods from an unanchored boat. A specially built boat is used which has controls that will permit the operator to maintain position over one spot in the stream for the period necessary to obtain soundings and the usual 0.2- and 0.8-depth velocity observations. This boat is equipped with an open well through which the current meters and sounding weights are lowered. An integrated three-drum winch is employed to control and position the two current meters and the sounding weight. An electronic distance-measuring device is used to locate the boat on a line defined by range poles placed on the bank. Observations can be made in about 20 verticals in an hour.

This system works well and produces excellent results where a steady-state flow regime exists. However, it is not designed to meet the transient-flow problem and, because of the time element involved, is not well adapted to the tidal-cycle measurement problem.

SMOOT SYSTEM

The moving-boat procedure developed by Smoot and Novak (1969) is a technique for rapid measurement of large streams. During the traverse of a boat across a stream, a sonic sounder is used to record the geometry of the cross section, and a continuously operating current meter is used to sense the combined stream and boat velocity. A free-swinging vertical vane defines the direction of relative movement of the water, and an angle indicator, attached to the vane assembly,

is used to measure the angle between the velocity vector and the course of the boat. Data from these instruments provide the information necessary for computation of the discharge. Overall accuracies obtainable with this system are reported to be within 5 percent of those obtained by conventional methods.

Accuracy in the Sacramento River gaging investigation must be much higher than that obtainable even by conventional means. Hence, application of the Smoot system is contraindicated in spite of its attractive features of speed and simplicity.

SYSTEM REQUIRED FOR CHIPPS ISLAND

The foregoing discussion of the specialized systems than have been developed points up certain deficiencies in each that must be rectified for proper solution of the Chipps Island stream-gaging problem. A moving-boat procedure would seem to be the only procedure that will permit collection of data at the rate that is required, and the studies that follow relate to the development of a system of this type.

To achieve the desired accuracy levels, the system must define all significant variables, and unverified assumptions must be reduced to a minimum. Continuous measurements for periods of several days may be desired. Thus, the boat employed and the hardware assembled must be capable of continuous operation with a minimum of maintenance and operator fatigue. Onboard verification of the performance of all equipment items must be feasible in order to preclude the possibility of lost effort. This requires a redundancy of complex equipment items to insure proper performance, or the utmost simplicity in design so that onboard repairs can be made when needed.

The initial system concept was to continuously measure the velocity at four levels in the vertical as a boat traversed the river along a range line defined by a laser beam. Distance along the range—that is, distance from the shore—was to be monitored continuously with an electronic distance-measuring device, and provision was to be made for documenting the magnitude of drift upstream or downstream from the range during the measurement traverse. Depth of water would be recorded with a sonic sounder, and the bearing of the velocity vector at the upper current meter would be recorded by a remote-reading compass. Insofar as possible, data would be collected in digital form with a minimum amount of data transformation.

Two significant assumptions underlie this proposed system: First, it is assumed that an unbiased estimate of the mean velocity can be computed from current-meter observations at the selected points in the vertical and second, it is assumed that the streamlines of flow will have the same orientation throughout the vertical column. There is no stratification of fresh and salt water in the large Chipps Island

channel, and validity of this second assumption is considered axiomatic during periods other than those near slack water. However, verification of the first assumption was deemed necessary because of the significance that would attach to small undefined systematic errors related to the tidal phase. The study that follows was made to determine the manner in which the velocity distribution in the vertical column varied during the tidal cycle.

DEFINITION OF THE MEAN VELOCITY IN A VERTICAL

The average of velocities observed at points 0.2 and 0.8 of the depth below the water surface is generally considered a close approximation of the mean in the vertical. There is a question, however, whether the relation is exactly the same for flows in each direction in a tide-affected channel. In consequence, analysis of the actual velocity profiles in the Chippis Island channel was needed to determine how best to define the velocity in the cross section.

To obtain the necessary data, a test program was devised to monitor the velocity distribution during one or more complete tidal cycles at each of the three locations shown in figure 2. A boat was anchored in position with current meters suspended on separate suspension lines at each tenth of the depth from 0.1 to 0.9 of the depth below the water surface. A continuous count of the revolutions of each meter was recorded on a bank of electromechanical counters. The accumulated counts and the elapsed time, to the nearest tenth of a second, were recorded photographically each time the meter at the 0.1 depth made 50 revolutions. By this technique, a continuous record of velocity variations at nine points in the vertical was obtained. Data from the photographic record were transferred to punchcards for machine processing. Records of velocity variations were recorded at location 1 between 1057 and 1940 hours, March 14, 1968. At location 2, observations were made between 1010 and 2254 hours, March 15, 1968, and at location 3, observations covered the period from 1923 hours, March 19, to 1628 hours, March 20, 1968. Continuity of records was broken with each turn of the tide when it was necessary to raise the meters and turn the boat about.

This field investigation resulted in definition of the velocity distribution at intervals of about 30 seconds for the total period of observation at each of the three verticals included in the study.

ANALYSIS OF VELOCITY-DISTRIBUTION DATA

The primary objective of the study of the velocity variations in the vertical was to determine whether an unbiased estimate of the mean velocity in a vertical section could be obtained from observations taken at two points in the vertical or whether more observation

points would be required. Analysis of the data was, therefore, directed toward comparison of mean velocities computed by various techniques. The weighted-arithmetic mean, computed from all nine velocity observations, was chosen as the standard for comparison to minimize any systematic bias which might have been introduced by the assumption that the velocity distribution followed some fixed mathematical model. The basic computation was set up to do the following:

1. Compute mean velocities for each set of data by the eight methods described below.
2. Compute the ratios of the computed mean velocities to the weighted-arithmetic mean.
3. Compute an error index for the curves fitted to each data set, by using both the power-law velocity-distribution function and the logarithmic velocity-distribution function.

The eight methods used in computing mean velocities included the conventional value (V_{28}) computed as the mean of velocities observed at points 0.2 and 0.8 of the depth below the water surface; the weighted-arithmetic mean (VM), in which each observed velocity was assumed representative over half the distance to the point velocity above and below it, the velocity at the surface being assumed equal to that observed at 0.1 of the depth below the surface and the velocity at the bottom being assumed equal to one-half that observed at 0.9 of the depth below the surface; and computations based first on the assumption that the velocity distribution followed the power-law formula and then on the assumption that the distribution followed the logarithmic formula. Three computations were made by using each of the two distribution formulas.

If the velocity distribution is assumed to follow the power-law distribution formula, $v_y = ay^b$, where v_y is the velocity at a distance y above the bed, the coefficient a and exponent b can be computed by a least-squares fit of the observed velocities. A mean velocity can then be computed by integration of the relation over the depth involved

Mean velocities computed from the power-law distribution formula have been designated VP , VP' , and VP'' . VP was computed by using all nine observations in the vertical; VP' and VP'' were computed by using four points—those at 0.2, 0.4, 0.6, and 0.9 and those at 0.2, 0.4, 0.6, and 0.8 of the depth below the water surface.

A similar set of computations, producing VL , VL' , and VL'' , was made by assuming the velocity distribution followed the generally

accepted Prandtl-von Karman universal-velocity-distribution law,

$$v_y = 2.5 V_f \ln \frac{y}{Y_o}$$

Evaluation of the friction velocity V_f and the constant Y_o was made by a least-squares fit of the data, and the mean velocity was computed by integration of the equation over the observed depth.

Figure 6 shows the format of the computer output and illustrates the variety of statistics generated.

Computations were made for each set of velocity data in an effort to detect any time-related changes in the velocity distribution. Output from the computer included the individual values of the mean velocity, described above, and the ratio of each to VM , the weighted-arithmetic mean. To provide a measure of which function, logarithmic or power law, provided the best fit to the data, indexes of the standard errors of estimate were computed for curves fitted to each function. An index was used because meaningful standard errors of estimate could not be computed for those curves that were fitted to only four points. The error index, expressed as a percentage of the mean velocity, was computed from the standard deviation between velocities observed at the nine points in the vertical and velocities computed from curves fitted to these same points, first with all nine points and then with four points only. Thus, the error indexes show a comparison of the curve fit achieved by four points as compared with nine points and also a comparison of the relative fit to the functions chosen.

AVERAGE VELOCITY RATIOS FOR TOTAL OBSERVATION PERIOD

Averages of the ratios of the mean velocities, computed by various formulas, to the weighted-arithmetic mean velocity are given in table 1. Scatter of these coefficients, relative to the mean, is expressed in the standard deviations shown. Figures computed from nine observations in the vertical show less scatter, as would be expected, than those computed from four points or less. However, on the basis of the standard deviation, no significant difference is discernable between values derived from the power-law equation and those derived from the logarithmic function, nor is there any real gain apparent by using four points in each vertical as against two points. These data indicate that mean velocities computed by the conventional 0.2 and 0.8 method in this channel should be corrected by a coefficient of 0.984 (1/1.016) but that little is to be gained by increasing the number of observation points from two to four.

TABLE 1.—Average coefficients for total observation period

Coefficient	Average	Standard deviation
VP/VM -----	1. 013	0. 0075
VL/VM -----	1. 007	. 0080
VP''/VM -----	1. 010	. 0220
VL''/VM -----	. 998	. 0233
VP'/VM -----	1. 006	. 0167
VL'/VM -----	1. 000	. 0170
$V28/VM$ -----	1. 016	. 0233
$V2/VM$ -----	1. 144	. 0455
$V6/VM$ -----	1. 011	. 0396

Average ratios of velocities observed at the 0.2 and the 0.6 depth below the water surface to the weighted-arithmetic mean also are shown in table 1. Scatter in these coefficients, expressed in the standard deviation, is about twice that for the computations based on two or four points, a difference indicating a distinct drop in the accuracy of definition when the number of observations is reduced to a single point in the vertical.

Ratios shown for mean velocities computed from single-point observations or by the 0.2 and 0.8 method are almost identical with those described by Hulsing, Smith, and Cobb (1966, p. C7). This similarity is of interest because the figures shown in the reference were based on observations of steady-state flow in natural streams, while the data shown here relate to velocity distributions observed in a channel having two-directional flow. Thus the dynamic effects, relating to the rapid changes in discharge rate, possibly have, on the average, little effect on the velocity-distribution relations.

VARIATION OF VELOCITY RATIOS WITH TIDE PHASE

A second important objective in this study was to determine whether variations in vertical distribution were systematically related to the direction of flow or to the dynamic changes in the flow regime. Examination of the distribution of the mean-velocity ratios by the test of runs (Dixon and Massey, 1957, p. 287) showed the distribution to be nonrandom at the 5-percent level of significance. Thus there might be some correlation between the velocity distribution and tide phase. The plots of the mean-velocity ratios versus time, shown on plate 1, were accordingly generated to see if time-related variations might be apparent. The lower line in each of these figures shows VM , the weighted-arithmetic mean velocity, plotted against time. The other five lines are the respective time-related plots of the ratios of VP , VL , VL'' , VP'' , and $V28$ to VM .

The plot for station 1 (pl. 1) shows a rising trend in the ratio V_{28}/VM during the period of deceleration (1400–1445 hr) when velocity decreased with the approach of slack water. During that same 45-minute period, however, the trend of ratios VP''/VM and VL''/VM was negative.

Plate 1 shows that variations in velocity distribution, as indicated by changes in the mean-velocity ratios, seem to vary with the tidal cycle during periods when the flow regime is changing. This variation is most apparent in the plot for station 2. The trend of all the ratios is negative during the accelerating period of the floodtide from 1120 to 1220 hours and also during the accelerating period of the ebbtide from 1650 to about 1750 hours. The trends shown by examination of the transient periods defined in the plots for stations 1 and 2 seemed fairly consistent and are the basis for development of figures 7–10, which show the ratios plotted against the rate of change of velocity. Figures 7 and 8 show, on the basis of data from observations at stations 1 and 2, definite correlation between the magnitude of the ratios and the rate of acceleration. These trends were not duplicated at station 3, and when this last block of data is included, as in figures 9 and 10, the trends are obscured, except, perhaps, for the V_{28}/VM ratio (fig. 9). That ratio continues to have a positive trend during periods of increasing velocity only. Attempts to explain the differences between results from stations 1 and 2 and those at station 3 on the basis of direction of flow or magnitude of velocity were fruitless, and it can only be concluded that additional observations at several more stations—perhaps over several tidal cycles at each station—will be

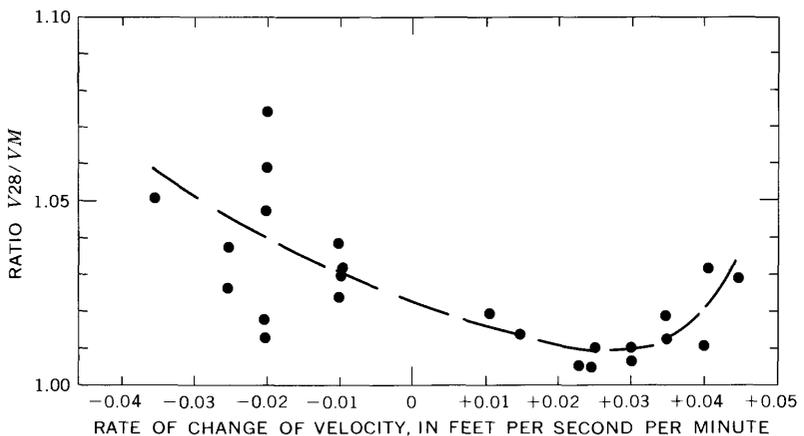


FIGURE 7.—Variation of ratio V_{28}/VM with rate of change of velocity at stations 1 and 2.

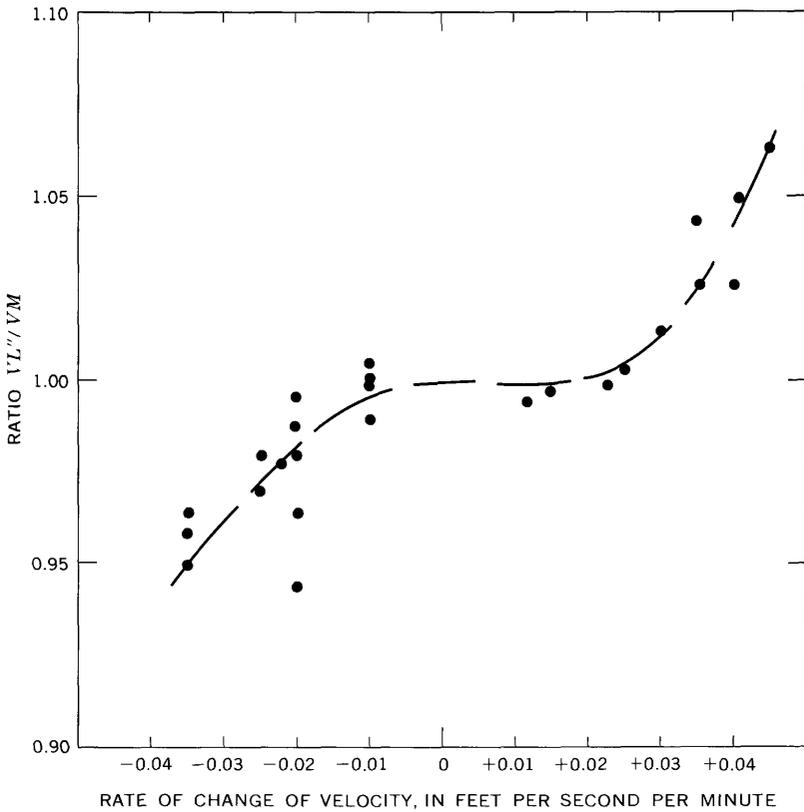


FIGURE 8.—Variation of ratio VL''/VM with rate of change of velocity at stations 1 and 2.

needed to demonstrate whether the distribution varies in a random fashion or whether trends, related to the phase of the tide, actually exist.

COMPARISON OF LOGARITHMIC- AND POWER-LAW DISTRIBUTION FUNCTIONS

Table 2 gives the error indexes for velocity distributions fitted to all nine data points and to four selected data points by the logarithmic and power-law functions. The error indexes, defined on page 13, range from 1.313 to 1.697 percent, showing a good fit to either function. The range, however, is too small to definitively indicate which function provides the best fit to the actual velocity distribution in this channel. It also is of interest to note that definition is improved only a small amount, 0.35 percent or less, by increasing the number of observations from four to nine.

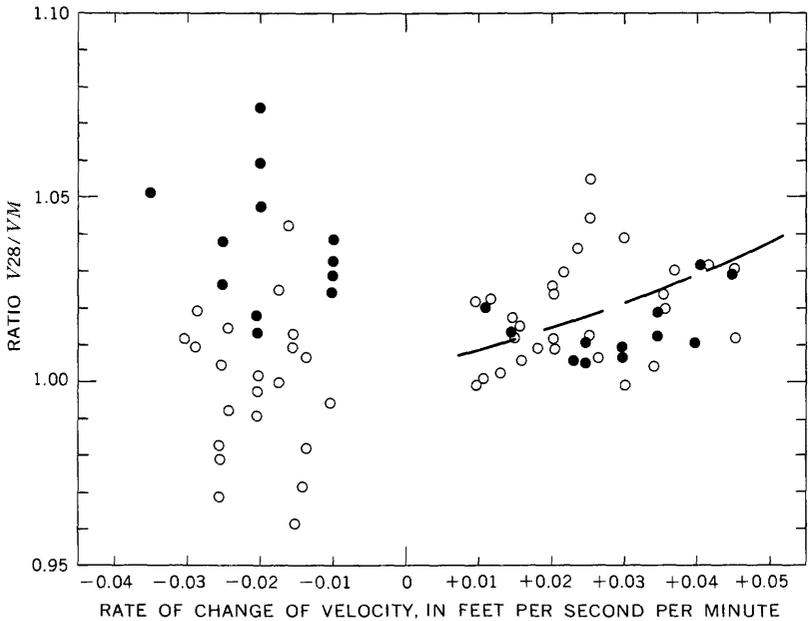


FIGURE 9.—Variation of ratio V_{28}/VM with rate of change of velocity at stations 1, 2, and 3. Solid circles are from observations at stations 1 and 2; open circles are from observations at station 3.

TABLE 2.—Error indexes for various velocity-distribution curves

Data used to define curves	Error index, in percent	
	Logarithmic distribution	Power-law distribution
Nine observations at points 0.1–0.9 of depth.....	1.351	1.313
Four observations at points 0.2, 0.4, 0.6, and 0.8 of depth..	1.697	1.543
Four observations at points 0.2, 0.4, 0.6, and 0.9 of depth..	1.552	1.529

Figure 11, in which the error index for the logarithmic-distribution formula is plotted against the instantaneous mean velocity, illustrates the manner in which the fit of the data improves as the velocity in the channel increases. During low-velocity periods the flow is rapidly increasing or decreasing, and the velocity distribution is not stable. In consequence, fit to either of the functions employed is not good, as evidenced by the scatter shown in figure 11. For velocities below 0.5 fps (feet per second) the error index may exceed 10 percent. For well-established flow regimes at velocities above 2 fps it is seldom greater than 1.5 percent. Data plotted in figure 11 are those for logarithmic curves fitted to velocities observed at points 0.2, 0.4, 0.6,

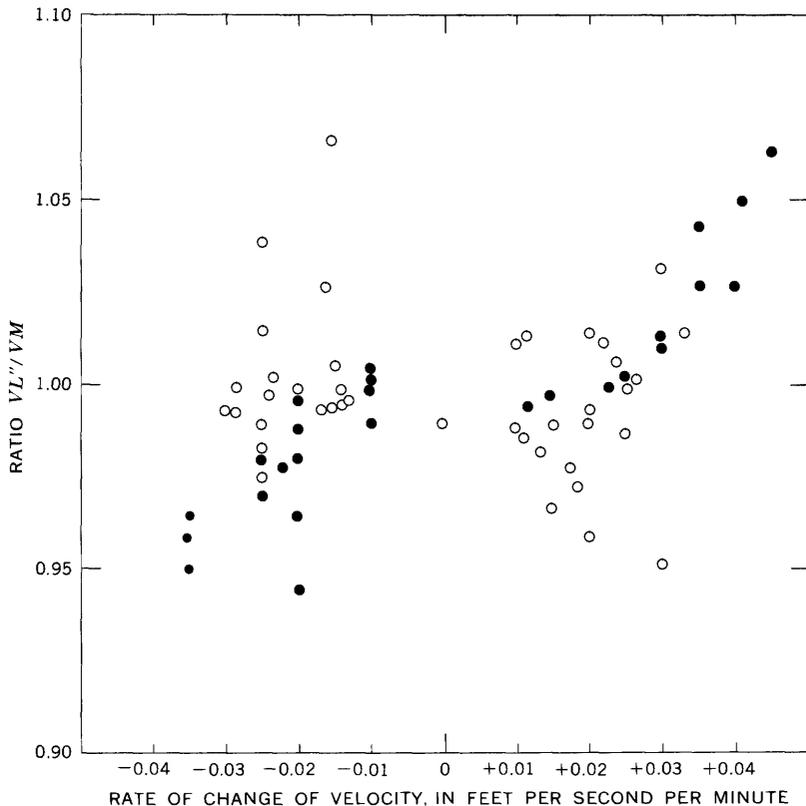


FIGURE 10.—Variation of ratio VL''/VM with rate of change of velocity at stations 1, 2, and 3. Solid circles are from observations at stations 1 and 2 open circles are from observations at station 3.

and 0.8 of the depth below water surface. Similar plots, indistinguishable from this one, could be made for curves fitted to the power-law equation or for curves fitted to all nine points of observation.

CONCLUSIONS FROM VELOCITY-DISTRIBUTION STUDY

The following conclusions can be made from this study of velocity data:

1. Velocity profiles in this tidal channel conform equally well to the logarithmic-distribution law or to the power law.
2. Definition of the mean velocity in the vertical can be achieved almost as well from two observations as from four observations. Very little is gained by placing meters at positions 0.2, 0.4, 0.6, and 0.8, or 0.2, 0.4, 0.6, and 0.9 of the depth, as compared with observations with meters at the 0.2 and 0.8 depth only.

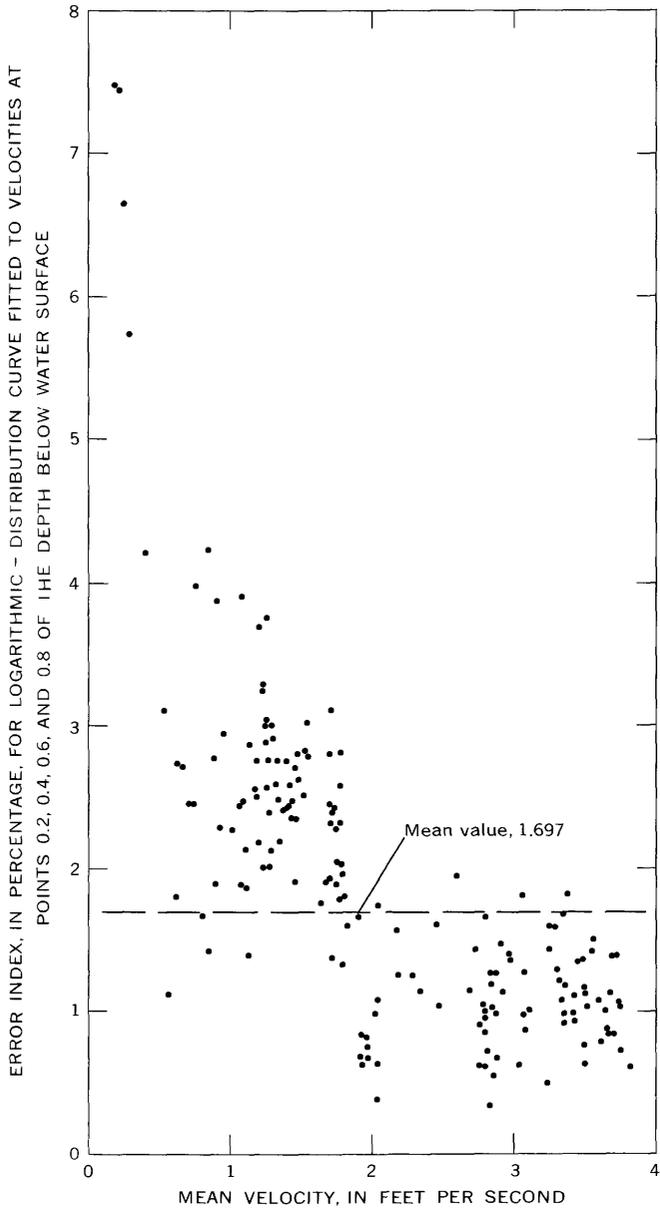


FIGURE 11.—Relation between mean velocity and the error index for distribution curves fitted to four observation points by the logarithmic-distribution law.

3. Precision of definition will decrease during periods of dynamic change, particularly at times near the slack-water periods when velocities are low.
4. Errors in mean velocities computed as the average of velocities observed at points 0.2 and 0.8 of the depth below the water surface or by application of the logarithmic- or power-law distribution formulas to observations at four points in the vertical are generally small, but may be nonrandom in character. The data obtained in this study at three stations in the cross section do not define systematic, tide-phase related trends nor do they clearly demonstrate random-error characteristics. Thus additional velocity-distribution data at several verticals (perhaps as many as 12) should be collected and analyzed as a preliminary part of the precision stream-gaging program required for calibration of an acoustic flowmeter at the Chipps Island site.

CHIPPS ISLAND MEASURING SYSTEM

The measuring system developed for use at Chipps Island follows the design concept already outlined. The measurement is made as a continuous process as a boat moves across the river on a course normal to the dominant streamlines of flow (fig. 12). The boat operator maintains his course on the range by visual reference to marker beacons placed at the far end of the range; an electronic measuring device is used to continuously monitor the distance the boat has moved along the selected range. A sonic sounder is used to record the bottom profile, which provides the depth record and the information needed to allow the equipment operator to keep the current meters at the proper positions in the vertical. The bearing of the velocity vector (the vector sum of water velocity and boat velocity) at the 0.2-depth position is indicated by a remote-reading compass. Readout from each of the instruments is registered on a continuous basis—either automatically or by manual input—on a bank of electromagnetic counters so that all data items may be recorded photographically at selected intervals. The complete data package includes the following:

Elapsed time, in tenths of seconds.

Distance from the edge of the water, in feet.

Depth of water, in feet.

Depth at which meters are set.

Accumulated meter revolutions at 0.2 depth below the water surface.

Accumulated meter revolutions at 0.4 depth below the water surface.

Accumulated meter revolutions at 0.6 depth below the water surface.

Accumulated meter revolutions at 0.8 depth below the water surface.

Observation number.

Time of day.

Azimuth of velocity vector at 0.2 depth below the water surface.

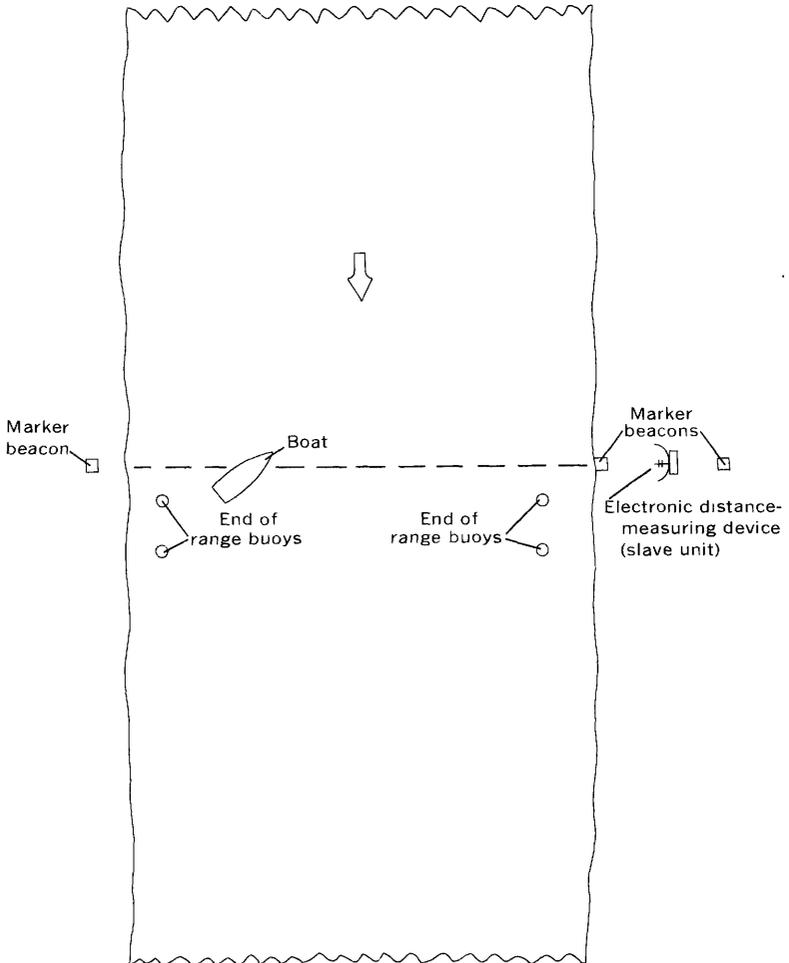


FIGURE 12.—Sketch of stream showing markers and course of boat during measurements.

Acquisition of velocity data at four points in the vertical was considered desirable in the original system concept. The prototype equipment was accordingly built to permit this, and four meters were used in the first series of measurements made at Chipps Island in March 1968. This same multiple-drum unit and the photographic recording system were used, with additional equipment, to obtain the data for the velocity-distribution studies discussed earlier. Analysis of this velocity-distribution data showed that only a small increase in accuracy resulted from increasing the number of meters used from 2 to 4; so the system was subsequently simplified by the elimination of the meters at the 0.4 and 0.6 depths.

BASIC THEORY OF THE MEASURING SYSTEM

Data recorded in the manner described above permit the average velocity in each subsection to be computed from the difference in total meter count at the subsection boundaries and the time required for the boat to move across the subsection. Computation procedures thus logically follow the mean-section method, wherein the subsection areas are computed as the product of incremental width and the mean of depths observed at each boundary, the velocities observed are the mean in the subsection, as noted above, and the orientation of the flow lines is computed as the average of resultant velocities computed at each boundary.

RESOLUTION OF VELOCITY VECTOR RELATIONS

The velocity observed by the current meter is the vector sum of the actual water velocity and the velocity of the boat, over the river bottom. This value must be resolved into its components to determine the water velocity normal to the selected cross section. In the system devised for Chipps Island the orientation of this vector is defined at the 0.2 depth only, and flow lines for velocities observed at other positions in the vertical are assumed to be parallel to those at the 0.2 depth.

Mathematical treatment of the vector relation is as follows. Referring to figure 13 and considering velocities of the 0.2 and 0.8 depth positions only, initial computations from the observed data yield:

VB = boat velocity along the cross section,

VM_2 = average speed in the subsection recorded by the meter at 0.2 of the depth,

VM_8 = average speed in the subsection recorded by the meter at 0.8 of the depth, and

α = direction of the VM_2 vector. This is the average of directions observed at the ends of the subsection.

From these values each of the velocity components normal to the cross section can be computed.

Since the speed and direction of the $VM2$ vector are known, VB can be vectorially subtracted to determine the true direction of all subsection streamlines, $\phi-C$. In the vector diagram (fig. 13), all true velocities must originate at ϕ and be translated by a constant amount, VB , to make up the resultant velocities recorded by the current meters. Line $A-B$, constructed parallel to $\phi-C$ and offset an amount VB , represents the locus of points terminating any subsection velocity

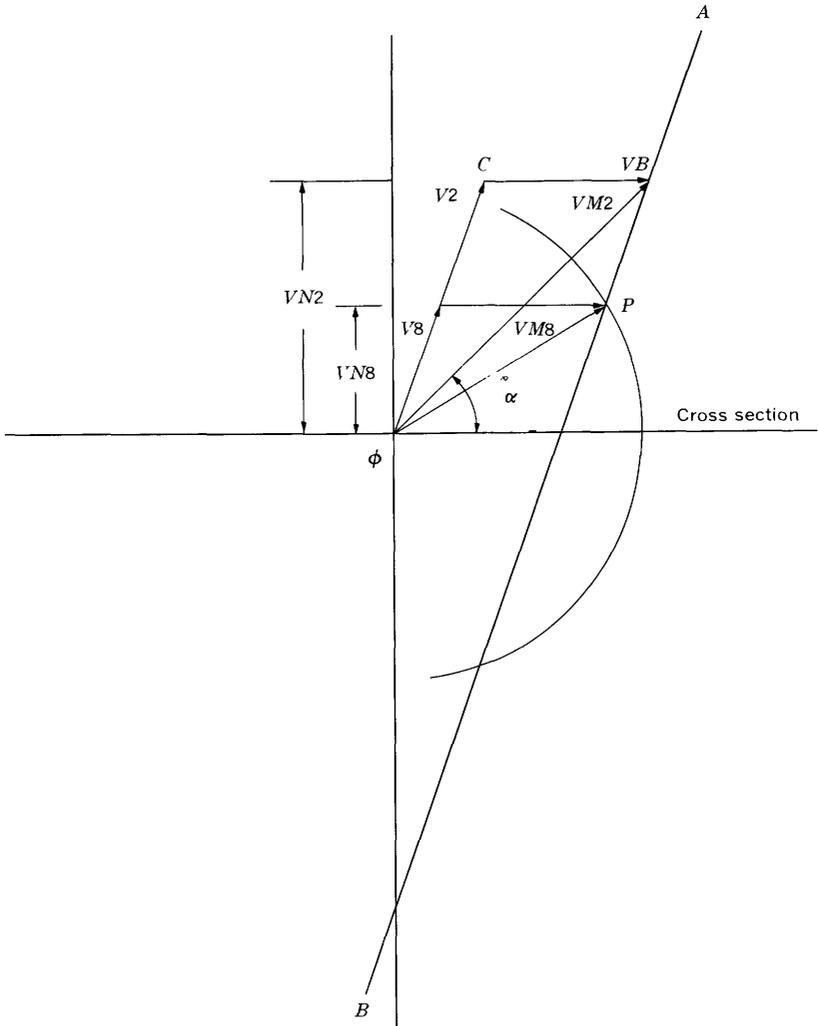


FIGURE 13—Diagram of velocity vector relations. See text for explanation.

vector. Thus, for example, the point P , where a circle of radius equal to $VM8$ intersects the line $A-B$, must be the unique end point of the velocity recorded by the meter at 0.8 of the depth. With $VM2$ and $VM8$ now located in space, their components normal to the cross section, $VN2$ and $VN8$, can be computed.

Referring again to figure 13, the value of $VN8$, or the y -coordinate of point P , is found by simultaneous solution of equations for line $A-B$ and the circle of radius equal to $VM8$. Line $A-B$ is of the form

$$y = mx + b; \tag{1}$$

the circle is of the form

$$x^2 + y^2 = a^2, \tag{2}$$

where

- m is the slope of the line,
- b is the y axis intercept of the line, and
- a is the radius of the circle.

Rewriting equation 1,

$$x = (y - b)/m; \tag{3}$$

substituting in equation 2,

$$\left(\frac{y-b}{m}\right)^2 + y^2 = a^2;$$

solving for y ,

$$y = \frac{b \pm \sqrt{b^2 - (m^2 + 1)(b^2 - m^2 a^2)}}{m^2 + 1}, \tag{4}$$

where

- a is equal to $VM8$ and
- b and m can be found from the geometry of the 0.2 meter velocity.

$$m = (VM2 \sin \alpha) / (VM2 \cos \alpha - VB) \tag{5}$$

$$b = \frac{(VB) (VM2 \sin \alpha)}{VM2 \cos \alpha - VB} \tag{6}$$

y is the value of $VN8$, that is sought
and

$$VN2 = VM2 \sin \alpha$$

The symbols a , b , m , x , and y used in the above derivation conform in usage to common algebraic notation. They are not shown in the list of symbols and should not be confused with earlier usage in the power-law and logarithmic-distribution formulas.

Computations for meters set at the 0.4 and 0.6 depth are made in the same manner. The computer program, based on this derivation, contains routines which convert any configuration of flow and boat movement into a first-quadrant problem. It also determines which of the two possible solutions of the quadratic equation is correct.

COMPUTATION OF DISCHARGE IN EACH SUBSECTION

Computation of \bar{v} , the mean velocity in the subsection, is made from the computed point velocities. If four meters are used in the measuring system, \bar{v} is computed from the least-squares fit of the data to the power-law equation. If two meters are used, at the 0.2- and 0.8-depth positions, \bar{v} is computed as the simple mean. Computer programs based on each of these formulas have been written. It must be emphasized that \bar{v} , as computed in this measurement system, is the average velocity across the subsection bounded by the verticals where data were recorded. This is in contrast to the conventional technique wherein meters are held stationary in a given vertical and \bar{v} is the average velocity in that vertical. The subsection discharge, q , expressed in cubic feet per second per foot of width, is computed as the product of \bar{v} and the average of depths recorded at the end points of the subsection. This discharge rate is assumed applicable at the midpoint of the subsection and at the midtime of observation.

COMPUTATION OF TOTAL INSTANTANEOUS DISCHARGE

The summation procedures used in computation of a conventional discharge measurement cannot be used for a precise measurement of tidal-cycle flow. Discharge in such rivers is changing rapidly during the major part of each tidal cycle, and hence, the sum of incremental discharges, defined sequentially in time, may not be truly representative of the total discharge during the period of measurement. The data recorded do, however, define a matrix of discharge rates that are fixed in space along the selected range and in time. The spatial location of these observations may vary between measurement runs, and the time interval between successive observations in a given part of the cross section will also vary.

This matrix of data can be manipulated to produce the desired output—computations of river discharge at specified time intervals within the total period during which measurement runs were made—by two successive interpolation procedures and a subsequent summation process. First, the incremental discharges are used to define a second matrix on fixed points along the cross section. For example, observations made at intervals of about 20 feet can be used to define, by linear interpolation, incremental discharge rates at fixed points with a spacing of 25 feet. Both the incremental discharge and the time of occurrence can be calculated and referenced to this fixed 25-foot spacing. Data from each successive observation run are treated in the same manner to complete the matrix of data.

The data matrix, produced from the first interpolation, provides discharge rates at fixed points in the cross section, but each defined discharge rate occurs at a different time.

A second interpolation, this one in the time frame, must be made to obtain corresponding time-referenced incremental discharges that can be summed up to compute the total discharge. For this second interpolation step, the times when total discharges are to be computed are first selected. These might be at 15-minute intervals throughout the several hours during which measurements were made. Then, incremental discharge rates, at these selected times, are computed for each of the fixed points in the cross section by a moving parabolic-interpolation procedure using four discharge rates, those computed from the two sets of data procured just before and the two sets of data procured just after the time for which the discharge is being computed.

The final step in the computation is the summation of the time-referenced incremental discharges at each of the fixed points in the cross section to produce the instantaneous total discharge in the river.

DEVELOPMENT OF THE PROTOTYPE EQUIPMENT

The first step in the development of the equipment was the assembly of hardware in prototype. Configuration of this prototype was dictated, in large part, by the space limitations imposed by the 21-foot outboard-powered boat which was available for use as a test platform.

The central unit is a compact multiple-drum power-operated reel assembly from which four separate sounding lines can be controlled. Four standard stream-gaging reels are chain connected to a common counter shaft in proportionate ratios so that for each turn of the counter shaft one reel releases 0.2 foot of cable, the second releases 0.4 foot of cable, the third releases 0.6 foot of cable, and the fourth reel releases 0.8 foot of cable. With this assembly, current meters can be simultaneously lowered or raised to positions 0.2, 0.4, 0.6, and 0.8 of the depth below water surface. Control is by means of a single lever. The meters are lowered, by the release of a brake, when the lever handle is pushed forward, and they are raised by power from a direct-current drive motor when the handle is pulled back. Release of the lever handle automatically applies the brake and stops motion in either direction. One revolution of the counter shaft is equivalent to 1 foot of depth. The dial reading on the register located on the upper right of the readout panel represents the depth of water, to tenths of a foot, for which the four meters are set at the selected positions in the vertical.

Proportionate position of the four current meters can thus be maintained by simply raising or lowering the meters until the depth displayed on the register is equal to the depth recorded by the sonic sounder. In this prototype unit the depth of water recorded on the sonic sounder is transferred manually to the readout panel so that all data can be recorded photographically. Accumulated distance,

measured by the operator of the electronic distance-measuring equipment, is transferred automatically to the readout panel.

Figure 14 is a photograph of the data recording panel used. The counters in the top row show the elapsed time, distance from shore, the depth of water, and the depth for which the meters are set. The second row of counters records the accumulated turns of the meters placed at 0.2, 0.4, 0.6, and 0.8 of the depth. The left-hand counter on the bottom row (registering 00759 in this picture) shows the observation number; the other counter on this row is a spare. Time of day is shown by a pocket watch, and the magnetic bearing of the velocity vector is indicated by the circular dial in the lower right.

Sequencing of the recording camera is controlled by a preset, subtracting impulse counter which can be connected in parallel with either the distance counter or one of the current-meter counters. Thus, the recording interval can be proportioned either to the distance moved

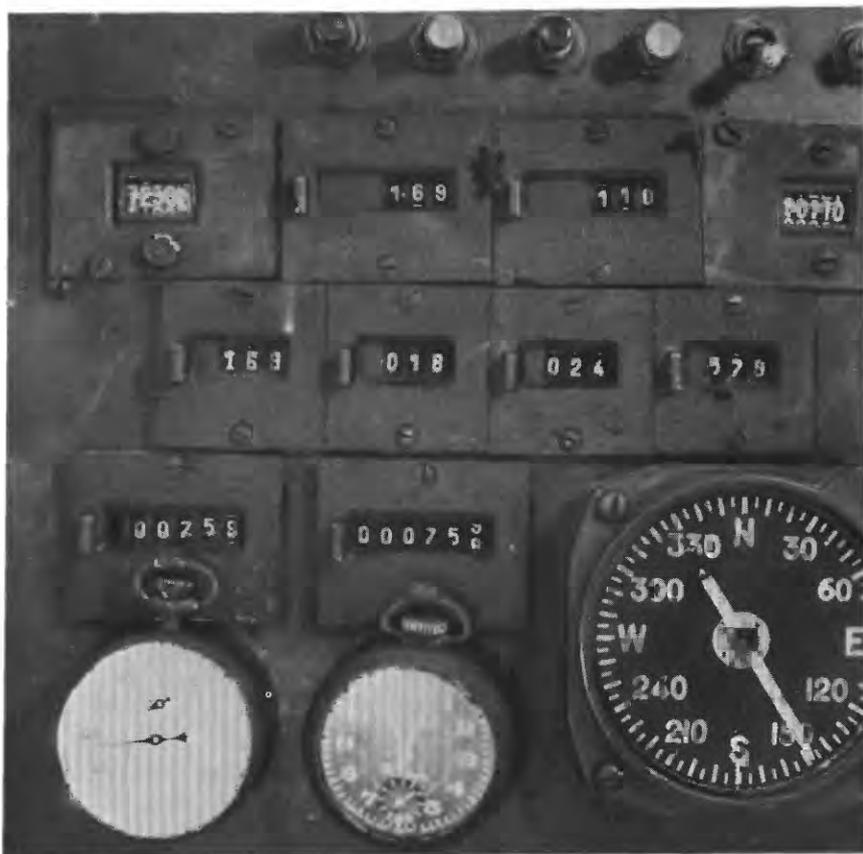


FIGURE 14.—Data recording panel.

along the range or to the velocity in the section. The latter method provides a variable distance between stations and results in a random sampling distribution on sequent runs across the stream.

OPERATIONAL EXPERIENCE

FIELD TEST WORK IN THE BOLINAS CHANNEL

The prototype equipment was first used to make a series of tidal measurements in the entrance channel to Bolinas Lagoon at Bolinas, Calif. This channel is about 300 feet wide—too narrow to permit accurate positioning of the boat under power—so a $\frac{5}{16}$ -inch plow-steel cable, marked every 20 feet, was used to hold the boat in position and provide a measure of its distance from the water's edge. This procedure eliminated the need for the electronic distance measurement and range markers and simplified the operational problems of the measurement so that primary attention could be placed on test of the meter-handling equipment and the recording techniques. Measurements were made on a continuous basis as the boat was moved back and forth along the cable by means of an electrically powered traversing mechanism.

Work at Bolinas met the requirements of a project study on the lagoon environment and also provided opportunity for field test of the prototype hardware. Several equipment modifications resulted from this preliminary trial, and data were procured for use in development of the computer programs needed for efficient handling of tidal-cycle flow computations.

FIELD TEST WORK IN THE CHIPPS ISLAND CHANNEL

An electronic distance-measuring device was added to the system after the test work in the Bolinas channel, and in March 1968 a series of trial runs was made in the Chipps Island channel. These trials showed that under favorable circumstances measurements could be made with a crew of three—a boat operator, an equipment operator, and a distance observer—but that a fourth man was needed in a rotating relief capacity for continuous operation.

The initial step in execution of the measurement program was the setting of channel markers, as shown in figure 12. These included the beacons defining the selected range line and the reference markers, about 50 feet offshore on either side, marking the ends of the traverse. At Chipps Island it was possible to use abandoned pilings near each bank for end-point reference, but at most sites small anchored buoys would be required. Illumination of these markers was provided to permit operation after sundown.

Experience in the 3,000-foot-wide channel at Chipps Island showed that a skilled operator guided by radio communication from a transitman at the end of the range could pilot the boat across the channel

with random deviations from the range of less than 5 feet. When the operator was dependent upon his own observation of distant range-markers—tall poles or electric lights—departures from the range were as much as 50 feet.

The laser beam, used so successfully by construction companies for positioning anchored barges, proved almost useless in this application. The beams from the lasers that have been designed for survey work spread to a width of about 6 inches at a distance of 3,000 feet and cannot be seen unless received head on. Such a beam provides no warning to a boat pilot as he is approaching the desired position, for by the time he can see the light he has crossed over the line. In consequence, he cannot gage the magnitude of departure from the range and make the precise changes in course that are needed to stay close to it.

A broader laser, flanked on either side by collimated colored beams, could be used; however, laser generators are expensive, and there is no point in using this type of a light source unless the narrow, precise beam is of value. Experiments made using a simple collimated light beam which spreads out about 10 feet in 3,000 have led to the conclusion that this type of a beacon would be satisfactory. With such a marking device a trained pilot would be able to control the course of the boat along the desired range with considerable precision. Drift would be slow and compensating, and failure to document the magnitude and direction of departures would probably introduce no significant error into the measurements.

A second series of measurement runs was made August 27, 1968. On this date two crews were assembled, and the plan was to continue the measurement for a 24-hour period. Limitations imposed by the small boat were much more apparent August 27 than March 28. A strong west wind producing waves about 4 feet high increased the pilot's problem tremendously and also affected the performance of the metering equipment. In spite of the operational problems, however, measurement runs were made about twice each hour from 1030 to 2400.

These measurement programs demonstrated the practicality of the system concept for use in tide-affected channels, such as the Sacramento River, where there are no abrupt changes in bottom profile. The winch operator had no difficulty in maintaining the current meters at the proper depth, and operational competence was rapidly achieved.

Application of the procedure in channels with large changes in bottom profile might require modification of the techniques. For example, the traverse rate could be slowed as the boat approached

regions of rapid change in bottom elevation. These points in a channel can be anticipated after the first cross section is run.

MEASUREMENT RESULTS

Data from the measurement runs made March 28 and August 27, 1968, at Chipps Island were used in the final development of the complex computer programs needed for definition of tidal-cycle flow. These programs, which follow the basic theories outlined earlier, were written in Fortran IV for use on the IBM 360/65. Detailed documentation is on file in the district office of the Water Resources Division, U.S. Geological Survey, Menlo Park, Calif. Figures 15 and 16 show results of computations for the two measurement periods. Discharges for each measurement run, computed by conventional summation procedures, are plotted at mean time of measurement for comparison with the instantaneous computations of discharge resulting from the more involved interpolation routines. These figures demonstrate that the interpolation routines are probably correct because they yield discharges that agree closely with the more direct, but less accurate, summation procedures. As is common with most complex computer programs, there is no inexpensive method for

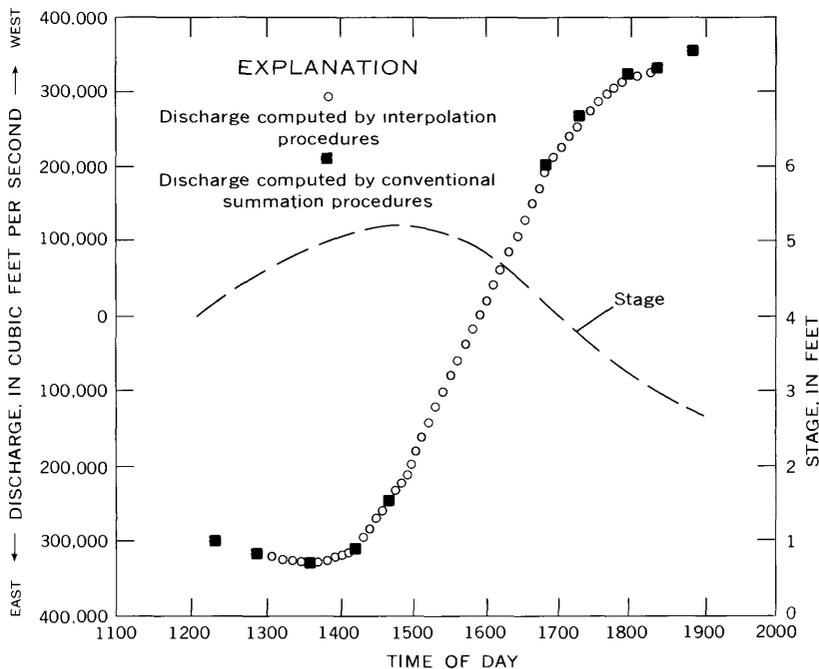


FIGURE 15.—Discharge computed from March 28, 1968, measurements.

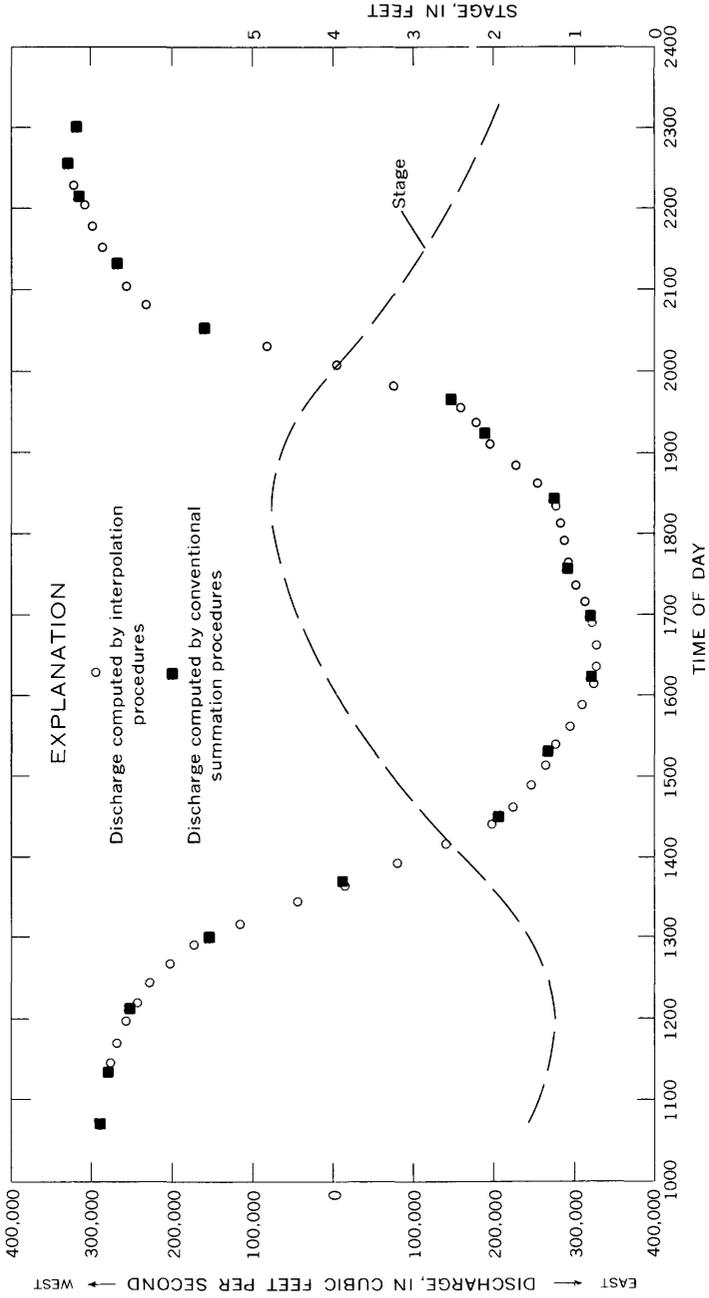


FIGURE 16.—Discharge computed from August 27, 1968, measurements.

proving the absolute accuracy of computations, and the laborious work of step checking the total program has been postponed until actual field use of the measuring system is to be made.

PROBABLE ACCURACY OF CHIPPS ISLAND MEASUREMENTS

Carter and Anderson (1963, p. 105-115) showed that the accuracy of current-meter measurements could be assessed by evaluation of the combined effects of instrument errors, errors due to velocity pulsations, errors due to variation in the velocity distribution in the vertical, and errors related to the number of subsections taken in a measurement. This same technique was extended by Smith (1969) to apply to a moving-boat measurement procedure by inclusion of errors related to positioning of the boat, errors in depth observations, errors due to variations in the angle of streamflow, and errors due to vertical motion of the boat. However, the moving-boat method outlined in the present report differs significantly from the procedure analyzed by Smith (1969); so reevaluation of the error sources must be made.

The error in the total measured discharge is related to the accuracy of the subsection discharges, the number of subsections included in the measurement, and the adequacy of the interpolation routines employed in the computation procedure. Rigorous evaluation of all error sources would be extremely complex as well as useless because of the many assumptions which would have to be made; however, a measure of the magnitude of probable error can be accomplished by consideration of the major error sources in the computed subsection discharge rates and the effect of these errors on the final discharges computed by the interpolation procedures employed.

ERRORS IN COMPUTATION OF SUBSECTION DISCHARGES

Each partial error ratio in table 3 is the percentage error in subsection discharges attributed to the specific source. The standard deviation S_r of a ratio r , is a measure of the distribution of the particular error ratio. The standard deviation of the computed subsection discharge, S_{rq} , can be obtained as follows:

$$S_{rq}^2 = S_{rt}^2 + S_{rs}^2 + \dots + S_{rvb}^2 \tag{7}$$

Values for the probable standard deviation of each of the error sources given in table 3 were estimated as described below.

EVALUATION OF S_{rt}

Carter and Anderson (1963) supplied data on the significance of velocity fluctuations at a point, and they tabulated values of S_{rt} as a function of the period of observation for periods from 240 to 15

TABLE 3.—*Statistics of principle error sources in subsection discharges*

Source of error	Partial error ratio	Standard deviation of ratio	Probable value of standard deviation, in percentage of true q
Velocity pulsation.....	$r_t = \frac{q - q_t}{q}(100)$	S_{r_t}	7.2
Variation of vertical velocity relation...	$r_s = \frac{q - q_s}{q}(100)$	S_{r_s}	2.3
Vector angle.....	$r_a = \frac{q - q_a}{q}(100)$	S_{r_a}	4.5
Boat position.....	$r_p = \frac{q - q_p}{q}(100)$	S_{r_p}	.2
Boat velocity.....	$r_b = \frac{q - q_b}{q}(100)$	S_{r_b}	.2
Subsection depth.....	$r_d = \frac{q - q_d}{q}(100)$	S_{r_d}	1.0
Vertical boat velocity.....	$r_{vb} = \frac{q - q_{vb}}{q}(100)$	$S_{r_{vb}}$	0

seconds. Extrapolation of these data downwards to the observation period of about 6 seconds, used in this moving-boat procedure, yields a value of 7.2 percent for S_{r_t} .

EVALUATION OF S_{r_s}

The magnitude of variations in the distribution of velocity in the vertical is also discussed by Carter and Anderson (1963). However, the figures they report relate to variations which might be expected between different streams. It is anticipated that detailed studies of the velocity distribution, similar to those reported in table 1 of the present report, would be made before an extensive measurement program was initiated in a tidal channel. Hence, vertical-velocity relations would be defined, and variations would probably be small and random. The value of 0.0233 (2.3 percent) given in table 1 for the standard deviation of the ratio $V28/VM$ was derived from observations at three verticals only. This value, 2.3 percent, is probably a conservative estimate of the magnitude of S_{r_s} .

EVALUATION OF S_{r_a}

Errors in the evaluation of the vector angle are a very significant error source. Dependence upon the uncompensated magnetic compass unit used in the prototype would probably limit the precision of this reading to $\pm 3^\circ$. This tolerance is too great and implies that better equipment must be used to gain the necessary precision. It is believed that the instrumentation scheme outlined briefly in the section

“Recommended stream-gaging system at Chipps Island” would permit evaluation of this parameter within a precision of $\pm 1^\circ$, and a probable error of this magnitude is accordingly used in this error analysis.

The magnitude of velocity error resulting from a 1° error in the vector angle is related to the speed of the water and the rate at which the boat moves across the section. The general vector equations derived previously, equations 4, 5, and 6, are too involved to permit assessment of this error, but a simplified case, wherein the velocities are specified normal to the cross section, as in figure 17, can be used for analysis.

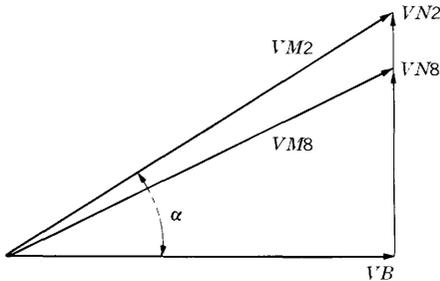


FIGURE 17.—Simplified vector diagram.
See text for explanation.

In figure 17, $VM2$ and α are the average magnitude and vector angle of the velocity measured in the subsection at 0.2 of the depth below water surface. $VM8$ is the average speed of the water recorded 0.8 of the depth below water surface, and VB is the velocity of the boat along the cross section. The direction of \bar{v} , the mean velocity in the subsection, is normal to the cross section. It is further assumed that the water velocity at 0.8 of the depth is equal to 0.8 of that at 0.2 of the depth. The following relations hold:

$$\begin{aligned}
 VN2 &= VM2 \sin \alpha \\
 VN8 &= (0.8)(VN2) \\
 \bar{v} &= (VN2 + VN8)/2 \\
 &= (VN2 + 0.8VN2)/2 = 0.9VM2 \sin \alpha
 \end{aligned}
 \tag{8}$$

If an error of 1° is made in the recording of α , the computed value of \bar{v} will be

$$\bar{v}' = 0.9VM2 (\sin (\alpha + 1)),
 \tag{9}$$

and the percentage error, r_α , introduced will be

$$r_\alpha = \frac{\bar{v} - \bar{v}'}{\bar{v}} (100), \quad (10)$$

or

$$r_\alpha = \frac{0.9VM2 \sin \alpha - 0.9VM2 \sin (\alpha \pm 1^\circ)}{0.9VM2 \sin \alpha},$$

which can be reduced to

$$r_\alpha = \frac{\sin \alpha - \sin \alpha \cos 1^\circ \pm \cos \alpha \sin 1^\circ}{\sin \alpha} (100),$$

and as $\cos 1^\circ \cong 1.000$ further simplified to

$$r_\alpha = \pm (\cot \alpha \sin 1^\circ) 100. \quad (11)$$

Experience has shown that most boats cannot be accurately controlled at speeds below about 3 fps. Water velocities in the Chipps Island channel range from 0 fps to a maximum of about 3 fps. The average during each tidal cycle is about 1.5 fps. Table 4 represents the evaluation of equation 11 for boat speeds from 3 to 6 fps and water velocities from 0.5 to 3.0 fps and shows the errors in subsection discharges that would result from errors of $\pm 1^\circ$ in measurement of the vector angle.

The error ratio r_α increases with increase in boat speed and decrease in water velocity. During the major part of each tidal cycle water velocities are greater than 1 fps, and boat speeds can be held near 3 fps. For these conditions a value of 4.5 percent seems a reasonable estimate for S_{r_α} , the standard deviation of r_α .

EVALUATION OF S_{r_p} AND S_{r_b}

The measurements of boat position and elapsed time will be the most precise elements in the data package. Distance measurements will be made by electronic equipment, and errors can be expected to be small and truly compensating. In addition, all measurements will be made over a known fixed distance so that an immediate check can

TABLE 4.—Variation of error ratio r_α with water velocity and boat velocity

Water velocity (fps)	Error ratio, r_α , in percent, at boat velocity indicated		
	3 fps	4 fps	6 fps
0.5	10.5	14.0	20.9
1.0	5.2	7.0	10.5
1.5	3.5	4.7	7.0
2.0	2.6	3.5	5.2
3.0	1.7	2.3	3.5

be made on any indicated misregistration. Similarly, the measurement of time can be made with very high precision. Hence, it is not unreasonable to assume that errors for S_{r_p} and S_{r_b} would be no greater than ± 0.2 percent.

EVALUATION OF S_{r_d}

Depth measurements will be made with a good-quality sonic sounder for which the manufacturers claim an accuracy of ± 0.25 feet for depths up to 100 feet. Experience has shown that an accuracy of ± 0.5 feet is a more reasonable expectancy. The average depth in the Chipps Island channel is about 35 feet; thus an error of ± 0.5 feet in measurement would result in an error range of ± 1.4 percent in individual recorded depths. Distribution of this error can be expected to be normal, and hence, the standard deviation of the error should be about two-thirds of the range. The value of 1.0 percent assigned to S_{r_d} seems a conservative estimate of the standard deviation of this error ratio.

The treatment given r_d in this analysis differs from that applied by Smith (1969) wherein discharge computations were made using a standard relation of area versus stage; the difference is minor, hinging only on the choice of computation methods employed. Either analysis could be applied to the projected measurement system. The procedure employed here, wherein each depth measurement is treated as an independent item, makes direct use of all recorded data and is simpler from the computation standpoint.

EVALUATION OF $S_{r_{vb}}$

Current meters, such as the Price meter or the Ott meter, tend to yield erroneous readouts when subject to vertical motion. Work done by Kallio (1966) showed that the error expressed as $S_{r_{vb}}$ can be virtually eliminated if an Ott meter is used and if the vertical motion of the meter is reduced below 0.6 fps. These conditions can be met by using a boat of several tons displacement and by avoiding operations during extreme weather conditions. $S_{r_{vb}}$ has accordingly been assumed equal to zero in this analysis.

EVALUATION OF S_{r_q}

The computation of S_{r_q} , the standard deviation of the computed subsection discharge, from equation 7 and from the error ratios given in table 3 yields the following:

$$S_{r_q}^2 = 7.2^2 + 2.3^2 + 4.5^2 + 0.2^2 + 0.2^2 + 1.0^2 + 0^2$$

$$S_{r_q} = \sqrt{77.56} = 8.8 \text{ percent}$$

This is the expected standard deviation in the measurement of individual subsection discharges.

ERRORS IN COMPUTATION OF INSTANTANEOUS TOTAL DISCHARGE

Error sources in the computed instantaneous total discharge include errors in the measurement of the subsection discharge, errors related to the number of subsections used, errors introduced by the interpolation routines employed, and errors in current-meter calibration. These error sources, expressed as error ratios, are summarized in table 5.

TABLE 5.—*Statistics of principle error sources in computed instantaneous total discharge*

Source of error	Partial error ratio	Standard deviation of ratio	Probable value of standard deviation, in percentage of true Q
Subsection discharge.....	$R_q = \frac{Q - Q_q}{Q} (100)$	$S_{R_q} = \frac{S_{r_q}}{\sqrt{N}}$	0.8
Number of subsections.....	$R_N = \frac{Q - Q_N}{Q} (100)$	$S_{R_N} = f(N)$.2
Interpolation procedure.....	$R_{IP} = \frac{Q - Q_{IP}}{Q} (100)$	$S_{R_{IP}}$.1
Current-meter calibration.....	$R_{CM} = \frac{Q - Q_{CM}}{Q} (100)$	$S_{R_{CM}}$	1.0

EVALUATION OF S_{R_q}

Instantaneous total discharges are computed by summation of incremental discharges which are interpolated from the time and spatially referenced subsection discharges. The absolute value of the standard deviation of the error ratio R_q attributed to errors in measurement of the subsection discharges can be computed from the equation

$$S_{R_{qa}}^2 = \sum_1^N S_{r_{qa}}^2 \quad (12)$$

This equation is valid if the absolute values of S_{r_q} , denoted as $S_{r_{qa}}$, are used. The following manipulations can be made to evaluate S_{R_q} , the percentage value of $S_{R_{qa}}$, from the percentage value computed for S_{r_q} .

In the large Chipps Island channel the discharge per foot of width is fairly uniform; thus, for the purposes of this evaluation, it can be

assumed that total discharge is equal to the sum of a large number of subsections of equal discharge. Thus,

$$Q = \sum_{N=1}^N q_N,$$

and by the above assumption,

$$q_N = \bar{q} = \frac{Q}{N}.$$

S_{r_q} is the percentage value of the standard deviation of \bar{q} . The absolute value of S_{r_q} , denoted as $S_{r_{qa}}$, equals $(S_{r_q}) \left(\frac{Q}{N}\right) 10^{-2}$, and

$$S_{R_{qa}}^2 = \sum_1^N S_{r_{qa}}^2. \tag{12}$$

By substitution,

$$S_{R_{qa}}^2 = \sum_1^N \left[S_{r_q} \left(\frac{Q}{N}\right) (10^{-2}) \right]^2 = \left(S_{r_q}^2 \frac{Q^2}{N} \right) 10^{-4} \tag{13}$$

or

$$S_{r_{qa}} = \left(S_{r_q} \frac{Q}{\sqrt{N}} \right) 10^{-2}. \tag{13a}$$

The percentage value of the standard deviation of the error ratio R_q can be expressed as

$$S_{R_q} = \frac{S_{R_{qa}}(100)}{Q}. \tag{14}$$

Substituting for $S_{R_{qa}}$ from equation 13a and simplifying yields

$$S_{R_q} = \frac{S_{r_q}}{\sqrt{N}}. \tag{15}$$

The procedure adopted for the Chipps Island measurement program result in definition of q in about 120 subsections; thus $N=120$. Substitution of this value of N and the value of 8.8 percent previously derived for S_{r_q} into equation 15 yields a value of 0.8 percent for S_{R_q} .

EVALUATION OF S_{R_N}

Conventional current-meter measurements generally include no more than 30 subsections, and errors are introduced because definition of the horizontal distribution of flow may not be adequately represented by a sample of this size. Carter and Anderson (1963) assumed that these errors would be reduced to zero for measurements containing

as many as 104 sections. No rational estimate of the residual error resulting from this density of definition can be made, but the logic followed in the cited reference suggests that this error source will be very small if more than 100 subsections are included in the measurement. However, the procedures used will require estimation of the flow in the subsections on each side of the channel next to the bank. These subsections carry less than 1 percent of the total discharge, but errors of as much as ± 20 percent, which could introduce an aggregate error in the total measurement of ± 0.2 percent, would not be unreasonable to expect. It follows that evaluation of the standard deviation of the error ratio related to number of subsections used, S_{RN} , should not be less than this, and a value of 0.2 percent has been arbitrarily assigned.

EVALUATION OF S_{RIP}

The interpolation procedures employed will introduce errors dependent primarily on how well the flow regime can be represented by the mathematical model that is assumed. Two interpolations are made in the computation process. The first is the linear interpolation made between the observed subsection discharges, which are made at random points across the channel, so that a fixed spatial matrix of time-referenced subsection discharges can be defined. A second interpolation is then made, in the time frame, by using parabolas fitted to two successive groups of three incremental discharges, to define a matrix of subsection discharges conforming to the selected spatial and time-interval grid. Total instantaneous discharges are computed from this final matrix of computed subsection discharges.

Errors introduced by the first interpolation process can be expected to be negligible because the horizontal distribution of flow is quite uniform and a large number of observations is made in each measurement traverse. However, the magnitude of errors resulting from the second interpolation will be of significance if the time interval between successive measurement runs is large.

The tidal cycles experienced in the Chippis Island channel correspond roughly to a sine function of slightly varying total period. If the period of the cycle were accurately known, interpolations based on integration of the sine function would be preferred. However, there is some variation in the period of the cycle, and for convenience, the parabolic interpolation procedures were adopted. There is no rigorous method for demonstrating the error introduced by these interpolation procedures, because accurate hydrographs of the total flow are not available; however, an approximation of the errors can be computed by application of the selected procedures to points abstracted from a simple sine function. Computations of this type, showing the percent-

age departure of points computed by the parabolic interpolation procedures from the true values computed from the sine curve, are summarized in figure 18. For these computations the tidal cycle was assumed to be exactly 12 hours in length, and interpolations were made at the midpoint between observations at 2-, 1-, $\frac{1}{2}$ -, and $\frac{1}{4}$ -hour

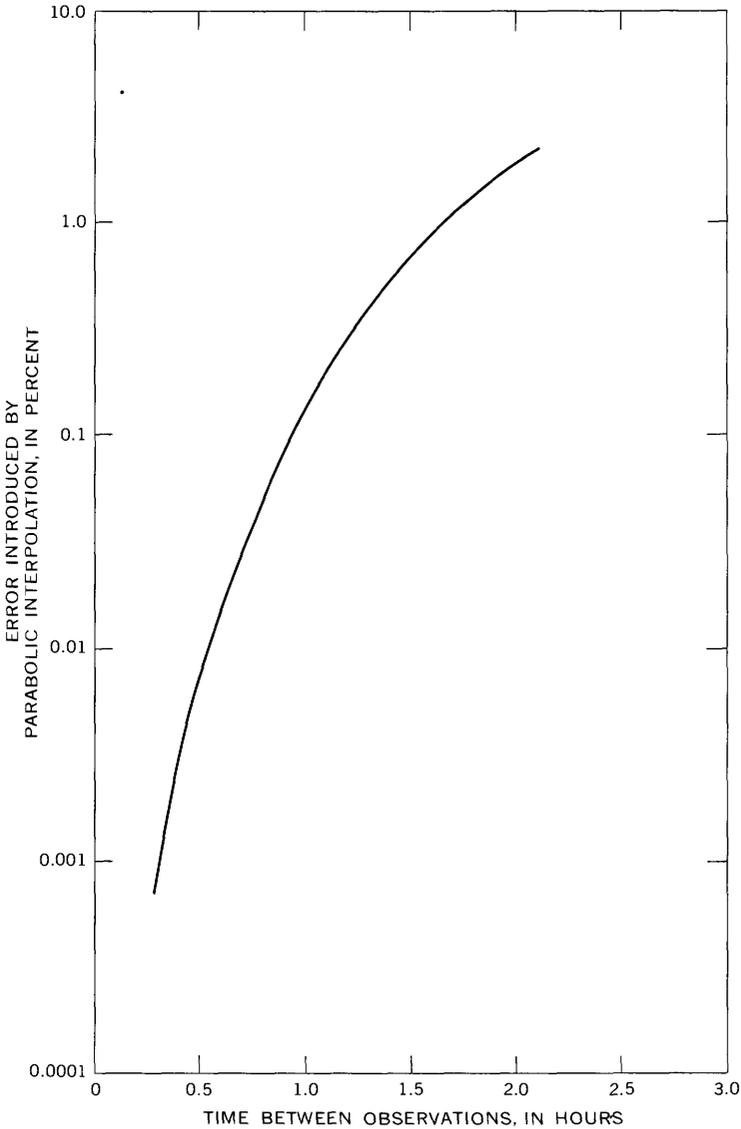


FIGURE 18.—Error introduced by parabolic interpolation procedures, assuming tidal cycle conforms to a sine function.

intervals. Discharge was assumed to follow the equation

$$Q = a \sin \frac{2\pi}{T} T_i,$$

where Q is instantaneous total discharge. T is cycle period equal to 12 hours, and T_i is time of observation.

The computations showed that the selected parabolic interpolation procedures gave a close fit during periods of positive or negative flow but that some error was introduced at points of inflection when the interpolation was carried through $Q=0$. This is not a serious deficiency, however, as alternate procedures, such as linear interpolation, can be inserted in the base program to provide a better fit during the transition period. The percentage departures plotted in figure 18 do not include values for the interpolations spanning the zero flow period.

With this minor exception, figure 18 shows that the adopted interpolation procedures would introduce errors of about 0.01 percent if the tidal cycle conformed to the assumed sine function and sequent measurement traverses were made at $\frac{1}{2}$ -hour intervals. The tidal cycle does not, however, conform exactly to a sine function, and it seems reasonable to assume that possible errors would be larger. The standard deviation of the error ratio related to interpolation procedure, $S_{R_{IP}}$, has accordingly been set at 0.1 percent, which should be a conservative estimate for this error source.

EVALUATION OF $S_{R_{CM}}$

Analysis of available data as to the stability and accuracy of current-meter calibration leads to the conclusion that the error ratio R_{CM} can be assumed to have a mean of zero and a standard deviation, $S_{R_{CM}}$, of 1 percent if several meters are used in the measurement program. If a single meter were used for all measurements, R_{CM} would not be random, but would appear as a small systematic error applicable equally to both ebb- and flood-flow measurements. For this analysis it is assumed that several current meters will be used in the measurement program and that R_{CM} is a random variable with a standard deviation of 1.0 percent.

EVALUATION OF S_{R_Q}

Computation of S_{R_Q} , the standard deviation of the error ratio in computed instantaneous total discharge, can be made by a truncated version of equation 7 and the error ratios given in table 5, as follows:

$$S_{RQ} = (S_{Rq}^2 + S_{RN}^2 + S_{RIP}^2 + S_{RCM}^2)^{1/2}$$

$$= ((0.8)^2 + (0.2)^2 + (0.1)^2 + (1.0)^2)^{1/2}$$

$$S_{RQ} = 1.3 \text{ percent}$$

This value can be considered as an approximation of the accuracy that might be expected for discharges computed from measurements made by the adopted moving-boat technique.

RECOMMENDED STREAM-GAGING SYSTEM AT CHIPPS ISLAND

The success achieved with the prototype equipment and the experience gained in the field trials with that unit lead to the general conclusions that a system of this type can be used for definition of flow in the lower reaches of the Sacramento River. The principal problem is the selection of the proper equipment and the assembly of the various components into a dependable and operable unit.

The environmental stresses placed on the operation coupled with the requirements of high accuracy and continuous, round-the-clock observations dictate the use of a vessel of significant size for stream gaging in the lower Sacramento River. A small twin-screw tugboat 40-50 feet long with a beam of 12-14 feet is the type of vessel needed. A boat of this size would be stable in the normal weather experienced and would be large enough to accommodate the equipment and personnel needed for continuous operation. Economics of the problem prohibit the purchase of such a boat for the single purpose of making periodic river measurements. It follows then that the equipment package should be designed for economical and convenient mounting on boats which might be rented.

Most small tugs have limited cabin facilities, providing only the space needed for normal operations. Thus a self-contained unit should be devised to provide the shelter needed for the stream-gaging equipment and personnel. The controls and readout units from all the hardware could be housed in a well-engineered enclosure about 5 by 8 by 6 feet.

Such a unit could be carried as a deckload on a craft of the size under consideration. Extendable booms, mounted on this same structure, would be used to carry sounding lines over the side. With this approach, fastenings and special designs, relating to installation on a particular vessel, would be reduced to a minimum, but the operating efficiency would not be sacrificed.

Definition of the direction of the velocity vector was achieved in the prototype by use of a magnetic compass device mounted in the weight suspended with the meter at the 0.2 depth. Sensitivity of this device was about $\pm 3^\circ$, and its use would be impractical if a steel vessel were employed as a measuring platform. There is no method by which such an unmounted compass could be compensated, because errors introduced by the proximity of the steel vessel would vary as the compass unit is moved up or down in the water relative to the vessel. These compensation problems could be solved if the velocity vector were computed from the azimuth of a third meter mounted on a free-swinging vane at a depth of 3 or 4 feet below water surface, as is done in the Smoot system. There are several ways in which a vane of this type could be instrumented to record the correct bearing of the velocity vector. Because the vane's position would be fixed relative to the magnetic influences on the vessel, an accurately compensated compass could be mounted with it. Alternately, the azimuth of the vane relative to the boat heading could be measured by a shaft-position indicator and recorded along with the boat heading as defined by a properly compensated remote-reading compass. Hardware of this type could be expected to provide readout accuracies within $\pm 1^\circ$.

Adequate definition of the velocity, its direction and variation in the vertical, could be obtained by use of system of a three meters, one at a fixed depth about 4 feet below water surface defining both the speed and direction of the velocity vector and two others maintained at positions 0.2 and 0.8 of the depth below water surface. These two meters, the vertical positions of which would be constantly varied in response to changes in water depth, would record speed only. Accuracy of the system will be better if Ott current meters are used in preference to Price current meters; in the preceding analysis of error it was assumed that Ott meters would be used. Work by Kallio (1966) shows that the Ott meter is less sensitive to registration errors resulting from vertical motion, which is caused by waves in the channel or by the controlled changes in meter position made during the measurement traverse.

The sonic sounder used on the prototype, a Model 723A Raytheon Fathometer, would be quite satisfactory for general use. However, units providing a digital readout of depth would be desirable.

Distance measurements can be satisfactorily made with the modified MRA-1 Tellurometer unit used in the prototype. This is the most economical unit found, but there are newer and more reliable systems which could be used if the economics of the program will permit. Use of highly specialized hardware, providing direct digital readout, may be too costly, even on a rental basis.

The decision as to how data should be recorded will hinge primarily upon how often current-meter measurements will be made. If the measuring system is to be used only three or four times each year, as is envisaged for the Chipps Island application, then trade offs between equipment costs and operating costs show that the photographic-recording technique, used in the prototype, will be most economical. The drawback to the photographic-recording system is that the data must be manually transferred to punched cards before computations can be made. This step is an added cost and an added source for error. In consequence, a shift to magnetic-tape recording will be desired if frequent use of the measuring system is required.

In either case it is believed that principal requisites of the recording system should be that faulty operation be easily detected, that provision be made for field repairs, and that data transforms—the conversion of data from mechanical analogs to electrical analogs and then to digital form—should be held to a minimum.

In summary then, if a measuring system were to be assembled at this date (1969) for use at Chipps Island it would be an integrated unit that could be carried as a deckload on a small tug or commercial fishing boat. All equipment would be housed in a fully enclosed shelter with work stations for two operators, one handling the current-meter and recording hardware and the other operating the distance-measuring equipment—probably an MRA-1 Tellurometer unit. A Raytheon, model 723A Fathometer would be used for depth measurement. Three Ott current meters, one mounted on a vane at a fixed depth and two carried on conventional sounding lines, would be used to define velocity. All data would be recorded photographically. Electrically powered beacons and range markers would be installed at the site for guidance of the boat pilot. Operating procedures and data-reduction routines would follow the general outline discussed previously.

The overall systems concept described here would produce, at minimum cost, discharge measurements of the accuracy required for calibration of an acoustic-velocity-meter installation at the Chipps Island site.

SELECTED REFERENCES

- Carter, R. W., and Anderson, I. E., 1963, Accuracy of current meter measurements: *Am. Soc. Civil Engineers Proc., Hydraulics Div. Jour.*, July 1963, p. 105-115.
- Dixon, W. J., and Massey, F. J., 1957, *Introduction to statistical analysis* [2d ed.]: New York, McGraw-Hill Book Co., p. 287-289.
- Hulsing, Harry, Smith, Winchell, and Cobb, E. D., 1966, Velocity-head coefficients in open channels: *U.S. Geol. Survey Water-Supply Paper 1869-C*, 45 p.

- Ingerson, I. M., 1955, Lunar-cycle measurement of estuarine flows: Am. Soc. Civil Engineers Proc. Paper 836, Nov. 1955, 18 p.
- Kallio, N. A., 1966, Effect of vertical motion on current meters: U.S. Geol. Survey Water-Supply Paper 1869-B, 20 p.
- Prych, E. A., Hubbell, D. W., and Glenn, J. L., 1967, New estuarine measurement equipment and techniques: Am. Soc. Civil Engineers Proc., Waterways and Harbors Div. Jour., v. 93, no. WW2, Proc. Paper 5219, May 1967, p. 41-58.
- Smith, Winchell, 1969, Feasibility study of the use of the acoustic velocity meter for measurement of net outflow from the Sacramento-San Joaquin Delta in California: U.S. Geol. Survey Water-Supply Paper 1877, 54 p.
- Smith, Winchell, and Wires, H. O., 1967, The acoustic velocity meter—a report on system development and testing: U.S. Geol. Survey open-file report., 43 p.
- Smoot, G. F., and Carter R. W. 1968 Are individual current-meter ratings necessary?: Am. Soc. Civil Engineers Proc. Hydraulics Div. Jour. March 1968 p. 105-115.
- Smoot G. F., and Novak, C. E., 1969, Measurement of discharge by the moving-boat method: U.S. Geol. Survey Techniques Water Resources Inv., book 3, chap. All, 24 p.