Interpolating Water-Table Altitudes in West-Central Kansas Using Kriging Techniques

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Prepared in cooperation with the Western Kansas Groundwater Management District No. 1

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By L. E. Dunlap *and* J. M. Spinazola

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FIGURES

Conversion Factors

Inch-pound units of measurement in this report may be converted to International System (SI) of metric units using the following conversion factors:

Multiply inch-pound units	By	To obtain SI units
foot	0.3048	meter
mile	1.609	kilometer
square mile	2.509	square kilometer
gallon per minute	0.06309	liter per second

Note: Datum used in this report is National Geodetic Vertical Datum of 1929, formerly called mean sea level.

Glossary of Terms

Aquifer - A formation, group of formations, or part of a formation that contains sufficient saturated permeable material to yield significant quantities of water to wells and springs.

Autocorrelation - The mutual relation among members of a series of observations ordered in time or space.

Regionalized variable - A variable in space or time showing a certain structure.

Saturated thickness - The thickness of material in an aquifer in which all the voids are filled with water under hydrostatic pressure.

Semi-variogram - Graph representing the variability of the regionalized data versus the distance between data points.

Water table - That surface in an unconfined ground-water body at which the water pressure is atmospheric.

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By L. E. Dunlap and J. M. Spinazola

Abstract

Kriging is a regionalization technique that incorporates the autocorrelation between known data values in its estimation of values at unmeasured sites. This technique is reproducible, accepts irregularly spaced data, uses only measured values closely surrounding points at which values are estimated, is an exact interpolator at measured data points, and calculates an error of estimate.

Kriging was used to interpolate water-table altitudes for 1978 and an average 1978–80 for the Ogallala aquifer in the Western Kansas Groundwater Management District No. 1. An altitude and an error of estimate were provided at the center of each one-mile section of the district. The data were related to altitudes of the bedrock and the 1950 water table. A digital-contouring procedure was used to construct maps of the water table for 1978 and for the average 1978–80, the errors of estimate, the saturated thickness of the aquifer for 1978 and for the average 1978–80, and the percentage changes in saturated thickness since 1950. Maps made by kriging compared favorably with maps contoured manually by the point-intersection method.

INTRODUCTION

The continued availability of ground water for irrigation in west-central Kansas is important to the economy of the area. Ground-water management is becoming increasingly important as the resource decreases. Western Kansas Groundwater Management District No. 1 was formed to manage and conserve the ground-water resources of that area in west-central Kansas, as shown in figure 1. The management district uses information from maps showing saturated thickness and percentage change in saturated thickness of the Ogallala aquifer in making management decisions.

A study was made, in cooperation with the Western Kansas Groundwater Management District No. 1, to investigate mathematical interpolation techniques that would increase the objectivity and reproducibility of the required hydrologic maps. Of the various techniques analyzed, it was determined that the kriging technique would be the most useful for this study. This report describes (1) the reasons for the selection of kriging, (2) the application of kriging in the study, (3) the comparison of maps contoured from kriged estimates with manual-contoured maps, and (4) the use of kriging results to construct maps showing saturated thickness and percentage change in saturated thickness.

Although all of the area within the Western Kansas Groundwater Management District No. 1 was analyzed using the techniques and time periods presented in this report, comparisons for only part of the area (fig. 1) are shown to illustrate these techniques.

The authors wish to express gratitude to J. A. Skrivan and M. R. Karlinger (U.S. Geological Survey) for their help in applying their kriging program in west-central Kansas.

GEOHYDROLOGY OF STUDY AREA

The land surface in the Western Kansas Groundwater Management District No. 1 is a flat upland that is part of the High Plains of western Kansas. Drainage is mostly by ephemeral streams.

The Ogallala Formation of Miocene age is the principal source of ground water in west-central Kansas. The Ogallala consists of interbedded clay, silt, sand, gravel, and caliche with calcium-carbonate-cemented sand (mortar beds). In west-central Kansas, these deposits range in thickness from a few feet to more than 300 feet. The individual beds of silt, clay, sand, gravel, or caliche can be correlated with confidence only short distances from test holes (Kume and others, 1979, p. 12–23). The saturated part of this formation yields as much as 1,500 gallons per minute to wells in this area.

The Niobrara Chalk of Late Cretaceous age directly underlies the Ogallala Formation. The upper part of the Niobrara Chalk, the Smoky Hill Member, consists of yellow to orange-yellow chalk and light- to dark-gray beds of chalky shale that locally weather to ochre yellow. The Niobrara Chalk does not yield an adequate quantity of water for irrigation supplies and is referred to as "bedrock" in this report.

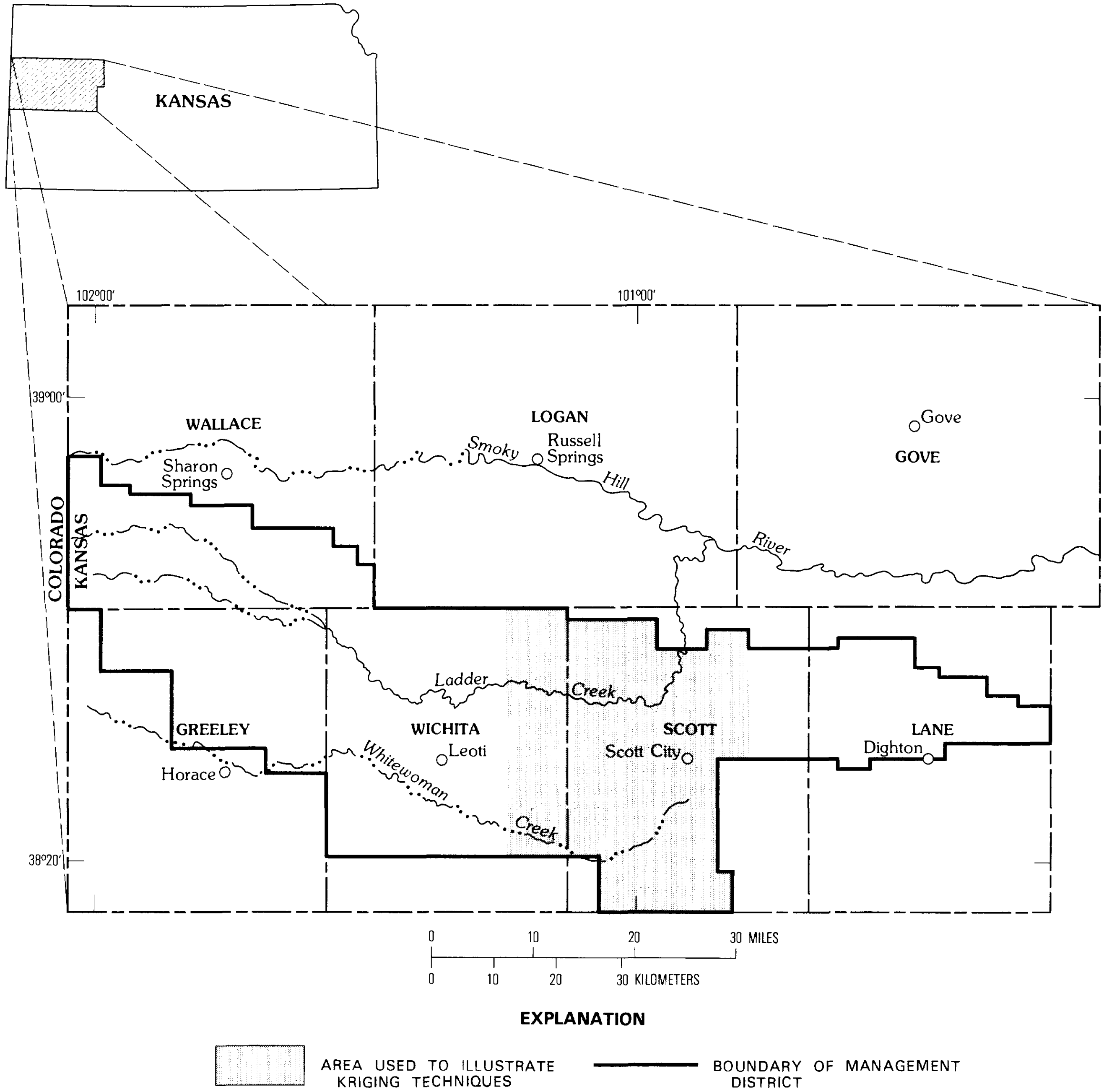

Figure 1. Location of Western Kansas Groundwater Management District No. 1 and area used to illustrate kriging techniques.

MATHEMATICAL INTERPOLATION TECHNIQUES

The study began with a literature search for mathematical interpolation techniques that would estimate the water-table altitude at unmeasured sites based on data from irregularly spaced observation wells. The study required a technique that was capable of estimating the water-table altitude at the center of every 1-square-mile section within the Management District (1,859 points). Classical interpolation, weighted interpolation, and the least-squares approach are among the techniques that were investigated. These techniques are described briefly here.

The surface or point to be interpolated with classical techniques usually is estimated by polynomial equations, such as in the Lagrange interpolation. These techniques are exact interpolators at data points; that is, data points are estimated at exactly their measured values. Classical interpolations yield very steep and unrealistic gradients and may yield inaccurate results when high-order polynomials are needed to estimate the surface

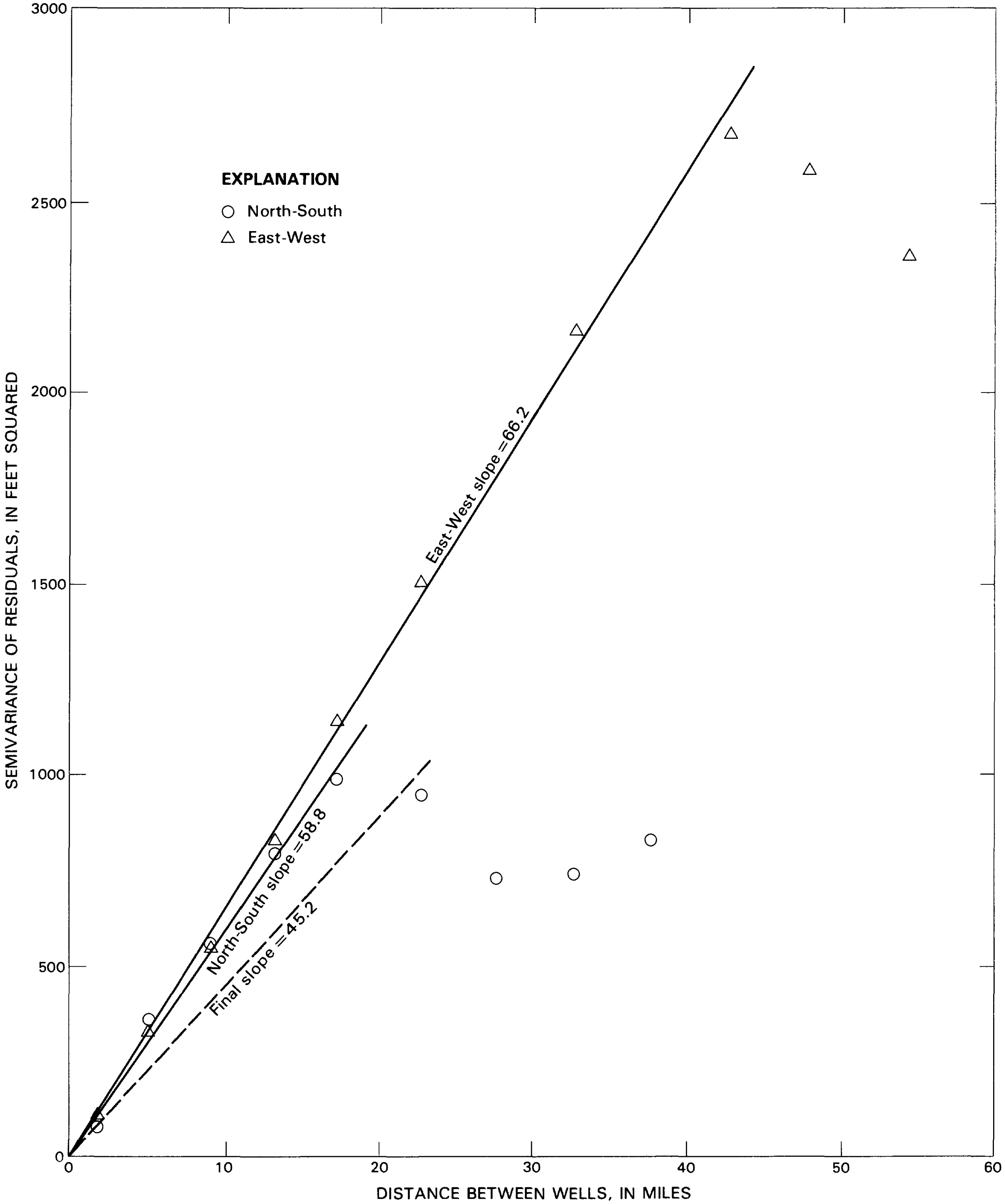

Figure 2. North-south, east-west semi-variograms and the final theoretical semi-variogram for 1978 water-table altitudes.

(Gambolati and Volpi, 1979b). The major disadvantage of these techniques is that data points (observation wells) must be regularly spaced, whereas observation wells in the Management District are irregularly spaced.

In weighted-interpolation techniques, the equations depend exclusively on the distance between the estimated point and the surrounding data points, not the direction of the data points. The major disadvantage of the technique in this study is that closely grouped data points having similar values are individually weighted with the

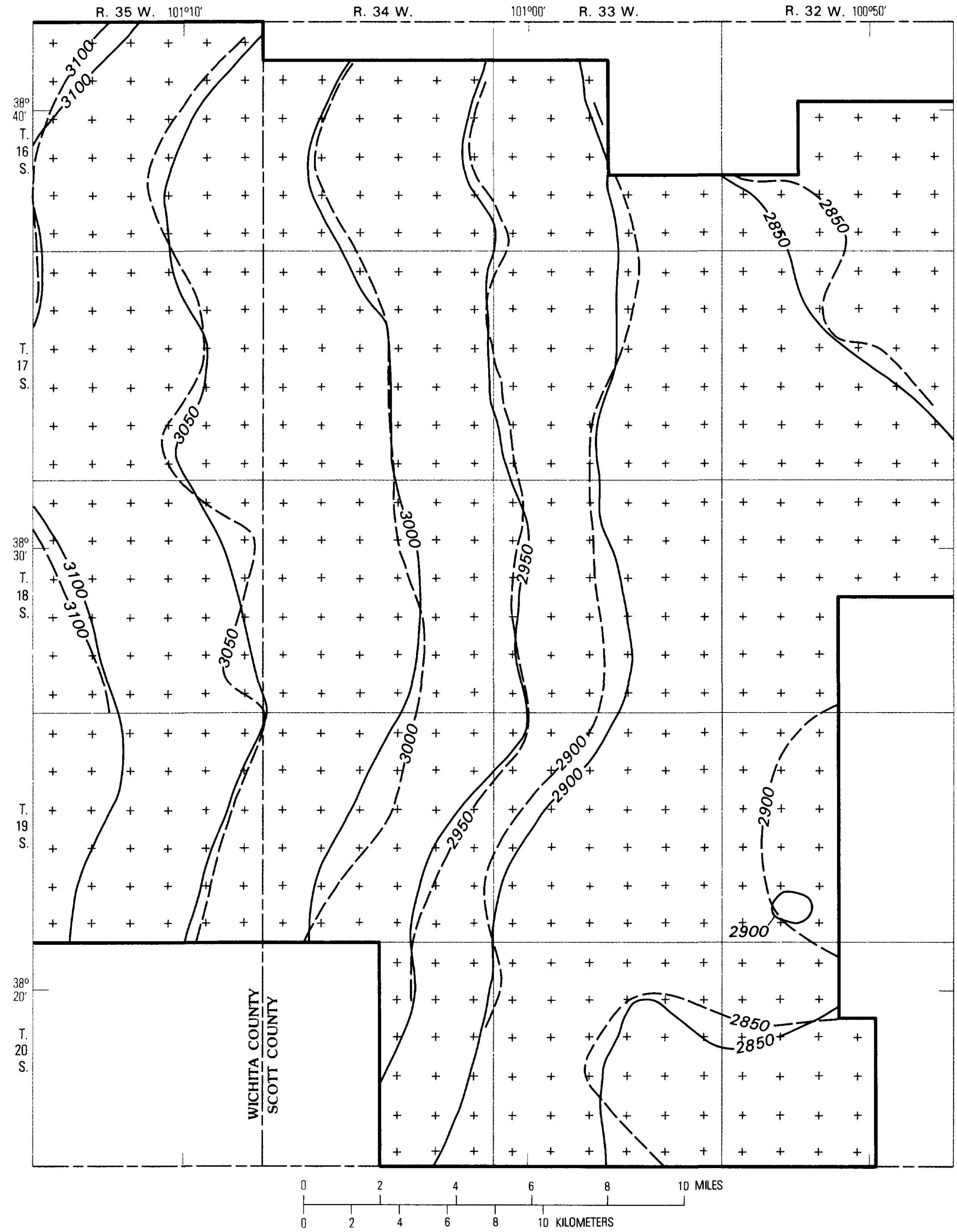

Figure 3. Water-table contours, 1978, constructed by center-of-section and manual-contouring techniques.

EXPLANATION

——*3000*—— WATER-TABLE CONTOUR—Constructed by center-of-section technique. Contour interval 50 feet. National Geodetic Vertical Datum of 1929

– –*3000*– – WATER-TABLE CONTOUR—Constructed by manual-contouring technique. Contour interval 50 feet. National Geodetic Vertical Datum of 1929

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same importance as other data points surrounding the estimated point. This redundancy of information leads to an "overweighting" of areas having the greatest number of data points (Gambolati and Volpi, 1979b).

The least-squares approach calculates one mathematical surface for a set of data points, such as in the trend-surface analysis. The technique is not an exact interpolator, which means that it will estimate data points at some value other than those measured. Trend-surface analysis commonly fits a medium- to high-order polynomial to the entire set of data points. Therefore, a value at an estimated point is interpolated from the entire set of data points instead of only its closest neighbors (Olea, 1975).

KRIGING

Kriging is an interpolation technique for autocorrelated, regionalized data. The technique is based on unbiased and minimum-variance conditions for estimation and involves a linear system of equations to calculate the weights of the data.

The kriging technique is an exact interpolator at points where data are given. Universal kriging uses a low-order polynomial in its equation and uses the implicit information from data points surrounding the interpolated point (Olea, 1975).

Kriging is the only known technique in which individual interpolation errors can be calculated. Therefore, this technique has the advantage of giving an actual measure of the reliability of the estimated points. A disadvantage is that any kriging problem has only one mathematically optimal solution, whereas several "engineering" solutions may have equally satisfactory results. The analyst uses kriging from an engineering perspective and chooses one of several optional applications of the technique to arrive at a satisfactory solution (Skrivan and Karlinger, 1980). Gambolati and Volpi (1979b) stated that the kriged estimates are not necessarily more reliable than those from other methods, but the ability to provide an estimation error is appealing.

Kriging was selected for mathematical interpolations in the study area for the following reasons:

1. It has the ability to estimate the reliability of the map data at unmeasured sites.
2. It is an exact interpolator at measured data points.
3. It uses the information from data points closely surrounding the point to be estimated by incorporating the autocorrelation structure of the data.
4. It provides a reproducible method of estimating the water-table altitude at unmeasured sites.

The general theory of kriging was developed by Matheron (1971), but the technique is named for D. R. Krige, who first applied the underlying theory to gold mining in South Africa. Recently, kriging has been applied to hydrologic data, such as precipitation (Karlinger and Skrivan, 1980), ground-water levels (Gambolati and Volpi, 1979a), and transmissivity (Delhomme, 1979).

A computer program containing the kriging technique (Skrivan and Karlinger, 1980) was used to apply kriging to water-table data in west-central Kansas. The program can calculate a theoretical semi-variogram (see section, "Semi-Variogram"), validate the theoretical semi-variogram, and estimate values at unmeasured sites.

The program by Skrivan and Karlinger (1980) uses the entire set of data points in its interpolation at unmeasured sites. Because of the large amount of data to be analyzed in the study area, use of all of the data would require unnecessary complications. Thus, the kriging computer program was modified for this study to use an arbitrarily specified number of neighborhood (or most closely situated) points to make the interpolations. The inclusion of only neighborhood points is the simplest and most desirable use of the technique in this study. A test of validity by comparing results from both the modified program and the original program showed that the kriged results differed only at the study-area boundaries.

Semi-Variogram

The most important item in the application of kriging is the semi-variogram. The semi-variogram is a graph of the variability of the difference of the regionalized data versus the distance between data points. In this study, the semi-variogram is a plot of the squared differences in water-table altitudes at pairs of observation wells that are separated by a certain distance. The equation for the semi-variogram is:

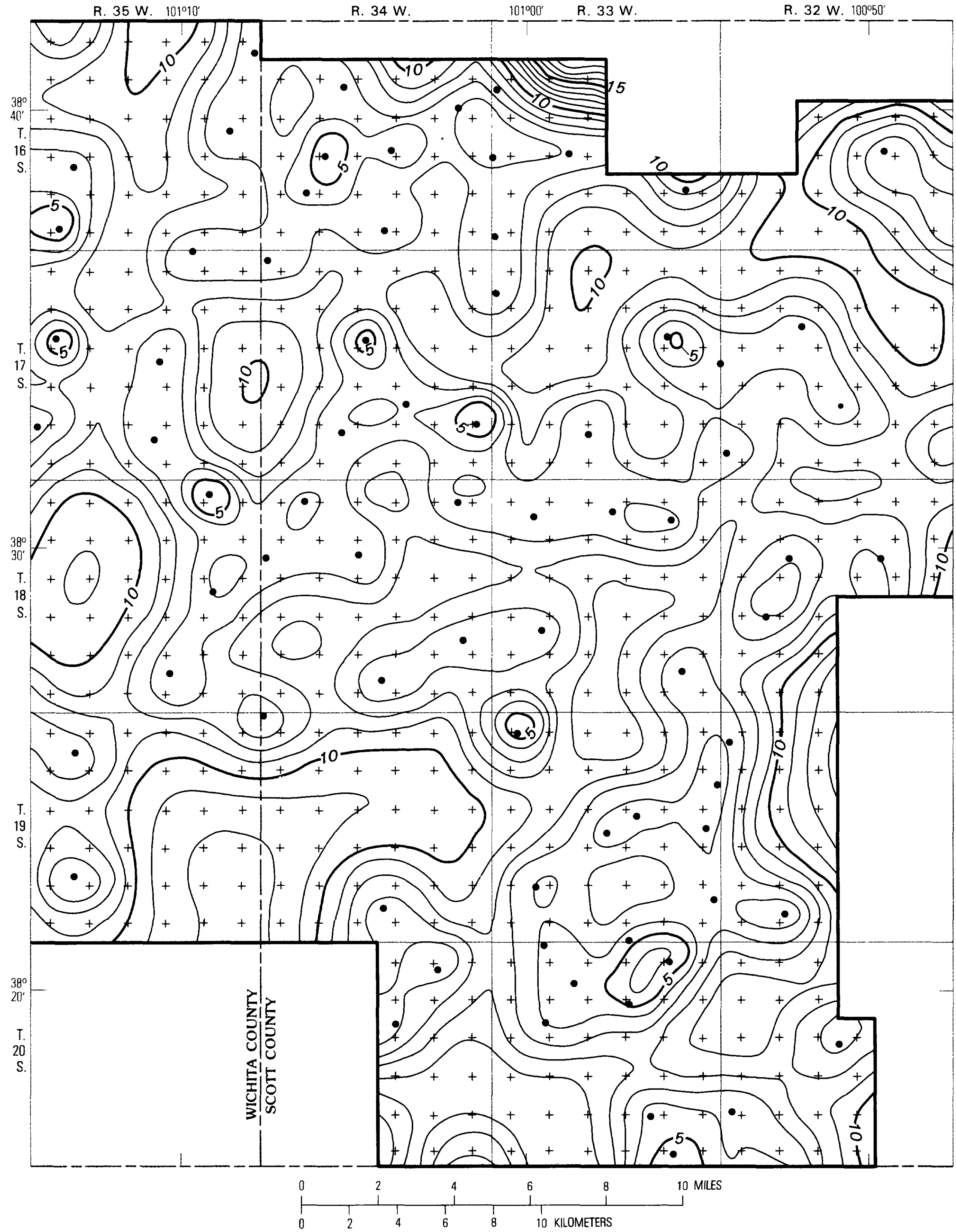

Figure 4. Error of estimate of 1978 water-table altitudes.

EXPLANATION

—5— LINE OF EQUAL ERROR OF ESTIMATE OF WATER-TABLE ALTITUDE—Interval 1 foot

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• WATER-TABLE DATA POINT

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$$\partial(h) = \tfrac{1}{2} \operatorname{Var}[Z(x+h) - Z(x)] \quad (1)$$

where

$\partial(h)$	=	semi-variogram;
Var	=	variance of data points;
$Z(x)$	=	regionalized variable at point x (such as a water-table altitude);
h	=	distance between data points; and
$Z(x+h)$	=	another regionalized variable a distance (h) from $Z(x)$.

The semi-variogram is approximated initially from the given set of measured values of the regionalized variable. This first approximation is called the *empirical semi-variogram*. The empirical semi-variogram needs to be "fit" as closely as possible to one of several *theoretical semi-variograms* and then tested before interpolation at unmeasured sites can begin. The most commonly used theoretical semi-variograms have the following shapes: linear, spherical, exponential, Gaussian, and DeWijsian. The reader is referred to Skrivan and Karlinger (1980) for a further explanation of semi-variograms.

The final equation of the semi-variogram is chosen by a validation process. This process is a modified split-sample technique where individual known data-point values are suppressed and then estimated using the remaining points. The errors in estimating the data points are then averaged, and a calculated and a theoretical variance are determined and averaged. The equation of the semi-variogram is adjusted until the kriged average error is approximately zero, the mean square error is made as small as practical, and the average ratio of theoretical to calculated variance (reduced mean square error) is near unity (Skrivan and Karlinger, 1980).

Isotropic Conditions and Simple Kriging

The regionalized variables are called *isotropic* when the semi-variogram is a function only of *distance* between data points. In other words, the semi-variogram does not change appreciably when comparing distance intervals in a north-south direction or in an east-west direction. In the case where the expected value of the regionalized variables is equal in all directions, kriging or simple kriging (a subset of universal kriging) may be used.

Anisotropic Conditions and Universal Kriging

Anisotropic conditions exist when the semi-variogram is a function of both the *distance* between data points and the *direction* of the distance intervals. If the expected value of the regionalized variables is not constant in all directions but changes in a gradual manner in one or more directions, the variables have a *drift*. This drift commonly is characterized by a low-order polynomial.

Universal kriging is used when drift is present. This technique uses the differences between the data values and the drift to determine the semi-variogram. Because a semi-variogram is needed initially to calculate the drift, the solution to universal kriging is an iterative process that uses the semi-variogram to calculate the drift and the residuals and then recalculates a semi-variogram from the residuals.

Error of Estimate

Kriging gives a variance estimate at each interpolated point. These variance estimates are called *kriging errors*, or *errors of estimate*. They define confidence intervals about the interpolated points if the errors are assumed to be normally distributed (Karlinger and Skrivan, 1980). The statistic actually calculated is the standard deviation, or square root of the variance. The kriging errors generally are greatest in areas of sparse data. A contour map of these errors shows areas of greatest uncertainty of the estimates and indicates where additional data points may be needed.

For a further and more detailed analysis of the theory of kriging, the reader is referred to Skrivan and Karlinger (1980), Karlinger and Skrivan (1980), or Olea (1975).

COMPARISON OF MANUAL CONTOURING TO THE CENTER-OF-SECTION TECHNIQUE

Prior to this investigation, the *point-intersection* technique was used to construct maps of the water table, saturated thickness of the aquifer, and percentage change in saturated thickness for the Western Kansas Groundwater Management District No. 1. The first step was to manually contour maps of the bedrock and water-table

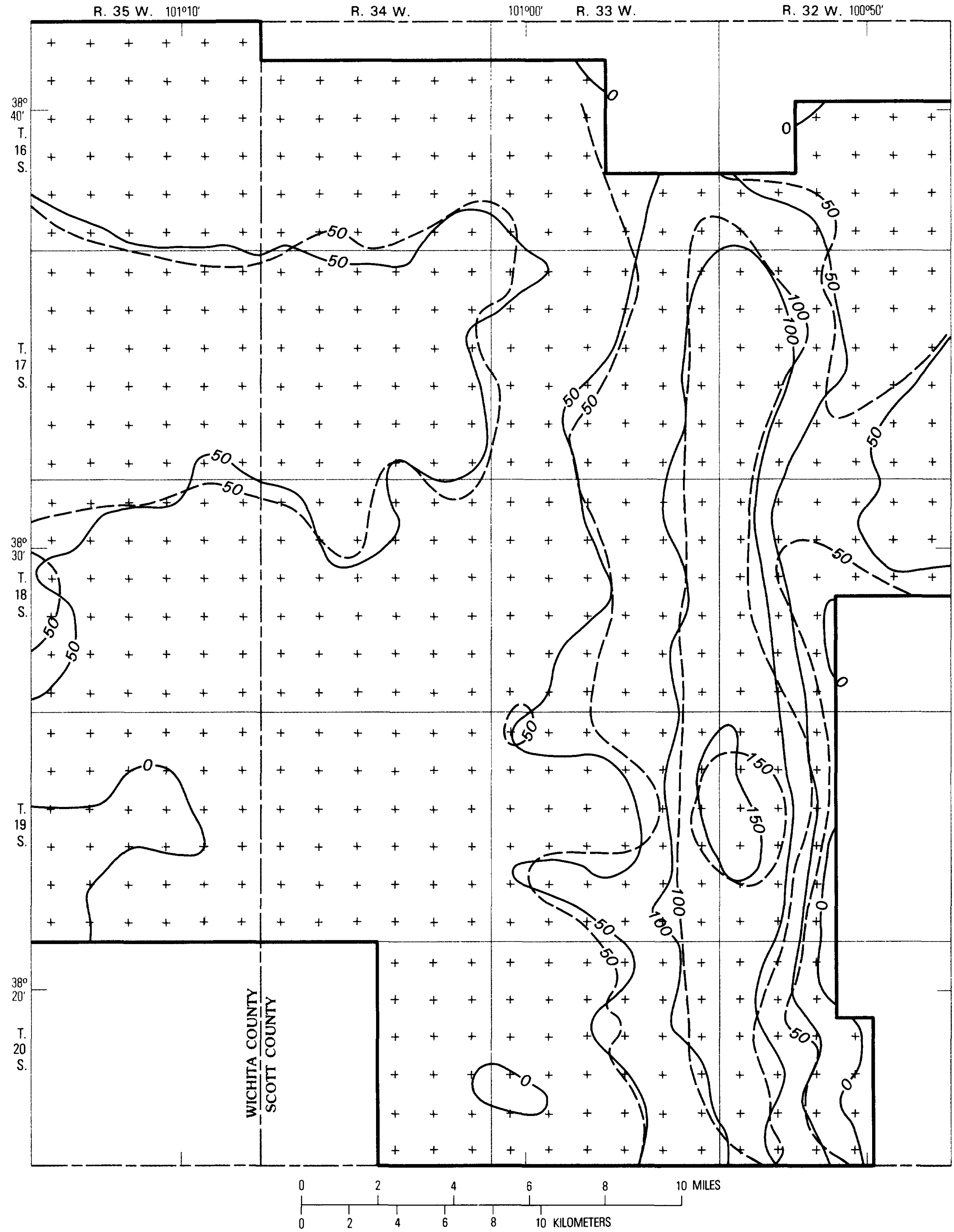

Figure 5. Saturated thickness in the aquifer, 1978, constructed by center-of-section and point-intersection techniques.

EXPLANATION

—50— LINE OF EQUAL SATURATED THICKNESS—Constructed by center-of-section technique. Interval 50 feet

– –50– – LINE OF EQUAL SATURATED THICKNESS—Constructed by point-intersection technique. Interval 50 feet

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surfaces using measured values from test holes and observation wells. A map showing saturated thickness of the aquifer was constructed by overlaying the bedrock-contour map with the water-table contour map. Data points for contouring the saturated thickness consisted of the measured saturated thickness at observation wells plus values indicated at the point where bedrock contours intersect water-table contours. These latter values commonly are called point-intersection values.

Maps of percentage change in saturated thickness of the aquifer were based on the change since 1950, a year of assumed steady-state conditions prior to major irrigation-well development. The maps of percentage change in saturated thickness by the point-intersection method were constructed by overlaying a map of the saturated thickness for 1950 on a map of water-level change since 1950. The data points used for contouring were the percentage change at observation wells plus the percentage change values indicated at the points of intersection.

As a result of this investigation, maps for the Management District were constructed using the *center-of-section* technique. In this technique, estimates of the water-table altitude at the center of each 1-square-mile section in the Western Kansas Groundwater Management District No. 1 were calculated by the kriging program. Estimated altitudes of the bedrock surface and the 1950 water-table surface at each section center were used, in conjunction with the water-table altitude data, to calculate saturated thickness of the aquifer and percentage change in saturated thickness at the center of each section. A total of 1,859 data points in the Kansas Groundwater Management District No. 1 were used to construct maps of the saturated thickness and percentage change in saturated thickness.

The bedrock and 1950 water-table altitudes were not kriged but were estimated visually at the center of the sections from the same bedrock and 1950 water-table maps used in the point-intersection technique. An empirical semi-variogram derived from bedrock data indicated that interpolation errors would be large because of the very irregular character of the erosion surface. Including these errors along with the errors of the water-table altitude gave unacceptable results when constructing saturated-thickness maps. The 1950 water-table altitudes were not kriged because the previously defined surface has been used as a standard in constructing previous percentage changes in saturated thickness since 1950. Therefore, values were not calculated for the errors of estimate associated with bedrock and 1950 water-table altitudes. The following sections describe the center-of-section technique in detail.

Altitudes of 1978 Water Table

Water-table altitudes for 1978 were entered into the kriging program to calculate the empirical semi-variogram. A substantial difference between empirical semi-variograms in the north-south and east-west directions indicates that anisotropic conditions exist. Because the water table in west-central Kansas has a general gradient from west to east, a drift is present in the data, and universal kriging should be used.

Using the iteration process, the semi-variograms in the north-south and east-west directions were constructed from residuals. After removing the effect of the drift, the semi-variograms are very similar (fig. 2) and are approximated best in the form of a linear equation:

$$\partial(h) = wh + c, \qquad (2)$$

where

$\partial(h)$ = semi-variogram;
w = slope of the line;
h = distance interval; and
c = intercept of the line.

The equations for the semi-variograms are:

north-south, $\partial(h) = 58.8(h) + 0;$ (3)

east-west, $\partial(h) = 66.2(h) + 0.$ (4)

The validation process resulted in the final slope for the theoretical semi-variogram of

$$\partial(h) = 45.2(h) + 0, \qquad (5)$$

and the result is shown in figure 2. The final kriged average error in the validation process is 0.0108 foot; the mean square error is 9.2186 feet; and the reduced mean square error is 1.0000 (dimensionless).

The kriging program was used to interpolate the

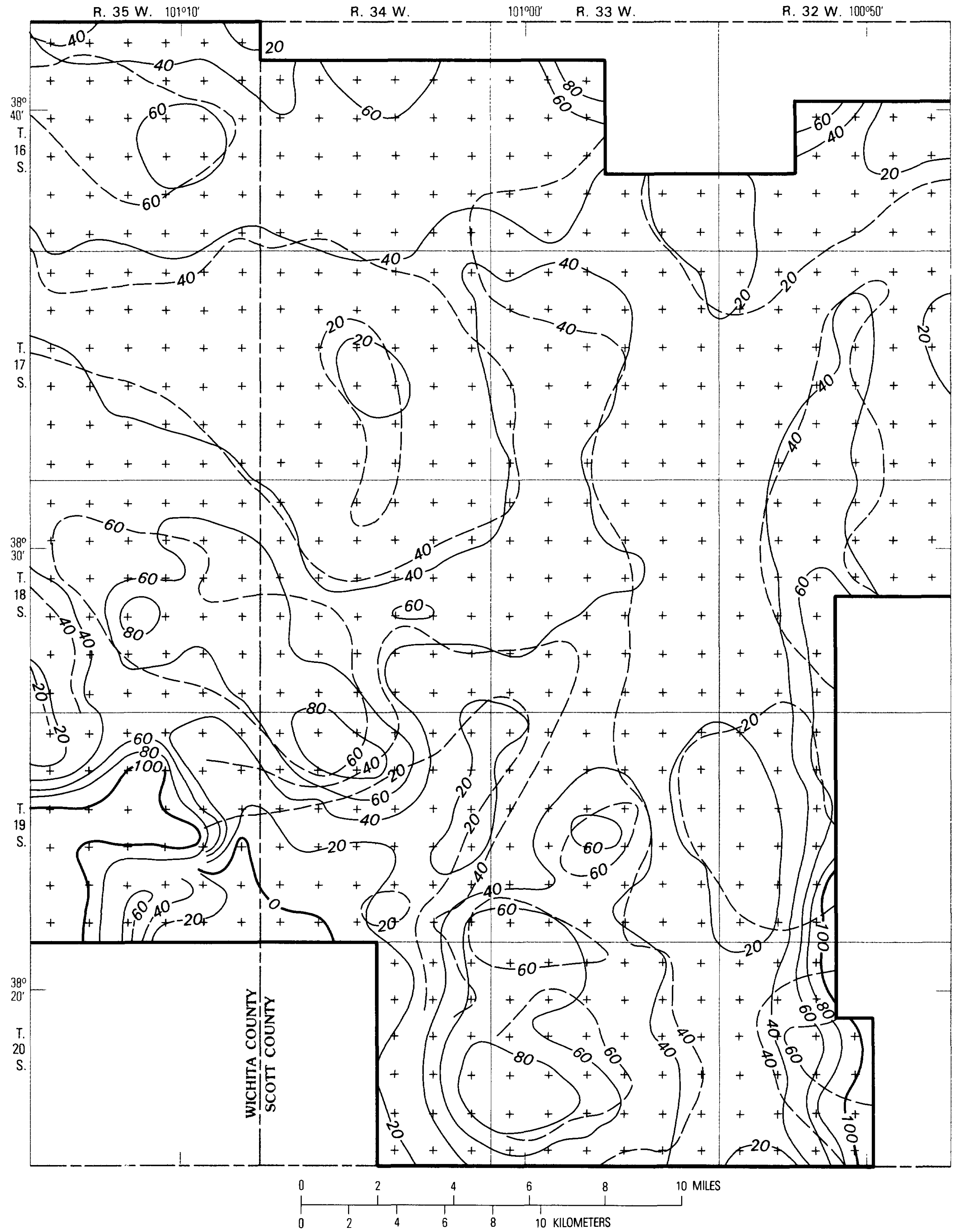

Figure 6. Percentage change in saturated thickness of the aquifer, 1950–78, constructed by center-of-section and point-intersection techniques.

EXPLANATION

—100— —20—	LINE OF EQUAL PERCENTAGE CHANGE IN SATURATED THICKNESS—Constructed by center-of-section technique. Positive value indicates a decrease in saturated thickness. Interval 20 percent
– –20– –	LINE OF EQUAL PERCENTAGE CHANGE IN SATURATED THICKNESS—Constructed by point-intersection technique. Positive value indicates a decrease in saturated thickness. Interval 20 percent
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1978 water-table altitude at the center of every 1-mile section in the Western Kansas Groundwater Management District No. 1. These estimates were used as the 1978 water-table altitudes in the center-of-section technique.

The kriged estimates of water-table altitudes along with observation-well altitudes were computer-contoured by GPCP (General Purpose Contouring Program).[1] The grid used was sufficient to eliminate most effects of GPCP gridding. Close inspection of the computer-contoured map showed that most contours accurately described the data; however, a contour was occasionally on the wrong side of a data point. Therefore, *all* computer-contoured maps were edited and revised where necessary to conform to the data points.

A part of the 1978 water-table contour map constructed by the center-of-section technique is compared to a corresponding part of the 1978 map constructed by manual contouring, as shown in figure 3. Contours constructed from kriged estimates will not match the exact values at the observation wells unless the well is at the center of the section. Therefore, measured values from observation wells also were used as data points in the water-table maps so that contours would not neglect these measured point values. Inspection of a 1978 water-table contour map constructed using only the kriged estimates shows only minor differences in the contours near a few measured values.

The contours on the maps of water table, saturated thickness of the aquifer, and percentage change in saturated thickness, constructed using the point-intersection technique, extend to what was previously stated to be the boundary of the main aquifer (Pabst, 1979b). The center-of-section technique extends contours to the Management District's boundary or to areas of zero saturated thickness (see section, "Saturated Thickness of the Aquifer, 1978").

[1]The use of the brand name in this report is for identification purposes only and does not imply endorsement by the U.S. Geological Survey.

The errors of estimate of the 1978 water-table altitudes for each interpolated point were computer-contoured, as shown in figure 4. These estimates are at the 67-percent significance level and imply that a normally distributed population was assumed. The errors are greatest around the boundary of the Management District where observation wells are lacking.

The observation wells plotted in figure 4 are not the total number of wells in the network but are only those that were measured during 1978. It is almost impossible to measure the water level in every observation well in the network in a given year due to measuring problems, such as the observation well pumping when visited or the inability to get a measuring tape down the well.

Saturated Thickness of the Aquifer, 1978

The map of saturated thickness in the aquifer during 1978, based on the center-of-section technique, was constructed by computer-contouring the differences between the 1978 kriged water-table altitude estimates and bedrock-altitude estimates at the center of the sections. The equation used to calculate saturated thickness is:

$$S_S = W_S - B_S, \tag{6}$$

where

S_S = saturated thickness of the aquifer at the center of the section;
W_S = kriged water-table altitude at the center of the section; and
B_S = bedrock altitude at the center of the section.

A part of the 1978 saturated-thickness map constructed by both the center-of-section technique and the point-intersection technique is shown in figure 5. The lines of equal saturated thickness constructed by the two techniques are very similar; however, the center-of-section technique shows the greatest detail because of the dense and regularly spaced pattern of data points. The computer-generated maps showing saturated thickness of the aquifer and percentage change in saturated thickness were contoured using the estimates at the center of each section.

Percentage Change in Saturated Thickness of the Aquifer, 1950–78

The map showing percentage change in saturated thickness of the aquifer from 1950 to 1978, using the center-of-section technique, was constructed by

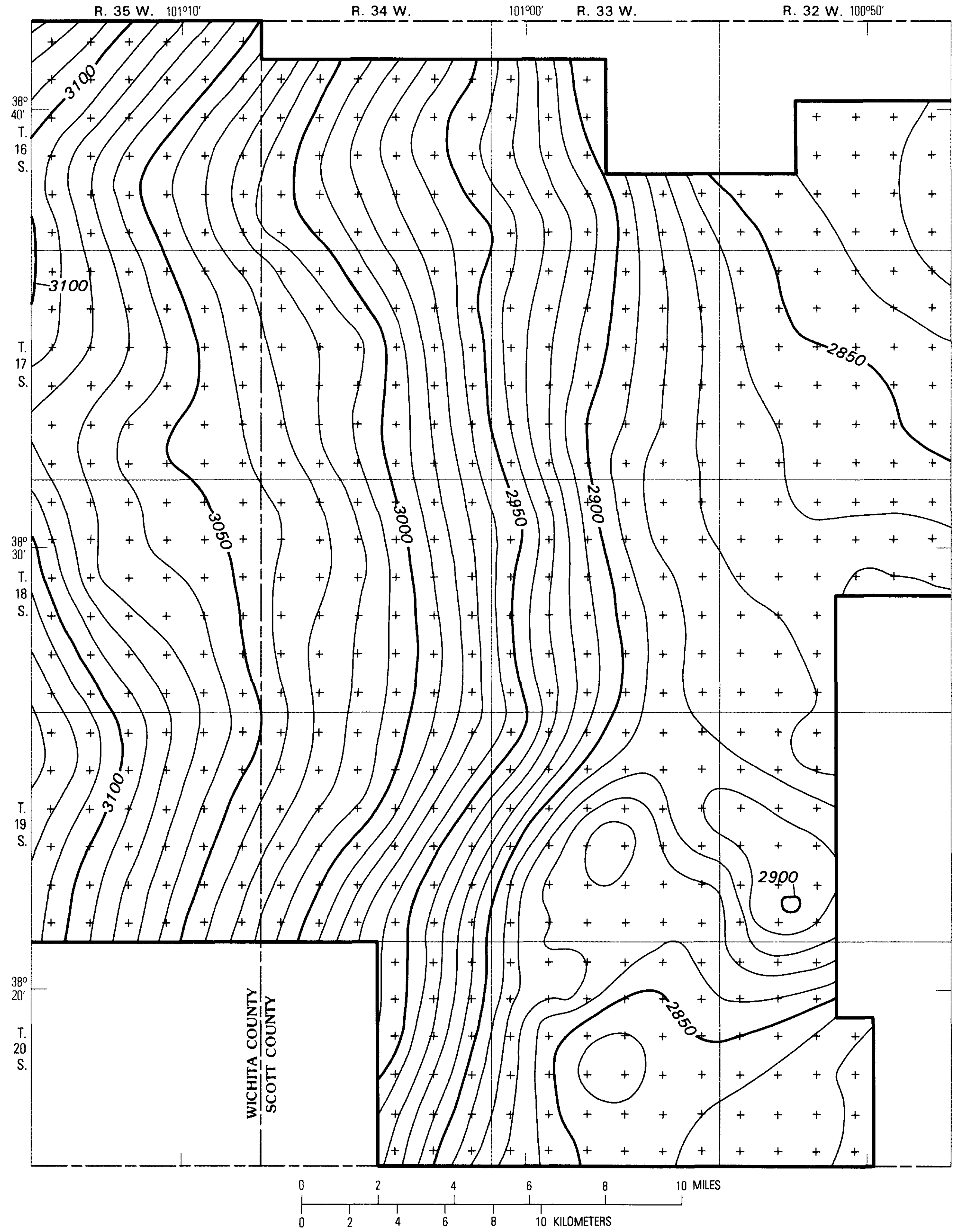

Figure 7. Average 1978–80 water table based on center-of-section technique.

EXPLANATION

—2900— AVERAGE 1978-80 WATER-TABLE CONTOUR—Contour interval 10 feet. National Geodetic Vertical Datum of 1929

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computer-contouring the results from the following equation:

$$P_s = 100 - \left(\frac{W_s - B_s}{T_s - B_s} \times 100\right) , \qquad (7)$$

where

P_s = percentage change in saturated thickness of the aquifer, 1950–78, at the center of the section;
W_s = kriged 1978 water-table altitude at the center of the section;
T_s = 1950 water-table altitude at the center of the section; and
B_s = bedrock altitude at the center of the section.

The lines of equal percentage change in saturated thickness constructed by both the center-of-section technique and the point-intersection technique (Pabst, 1979b) are compared in figure 6. Like the saturated-thickness maps, both techniques provide similar configurations in the lines of equal change, but the center-of-section technique results in more detail.

The percentage change in saturated thickness was considered to be 100 percent in areas where the saturated thickness is zero. An area with a 100 percent change may indicate that: (1) the area has been dewatered, or (2) zero saturated thickness existed in the area since data collection began during 1950.

In some areas of the map, the percentage change is negative, meaning the water-table altitude has risen since 1950. These changes need to be viewed with caution because they occur in areas of sparse data where the error-of-estimate map indicates that large errors are possible (fig. 4).

Areas with saturated thicknesses of a few tens of feet or more commonly have large variations in percentage change within short distances. This is evident around the boundaries of the Management District. When saturated thickness is small, the ratio in equation (7) can change rapidly when a small change in water levels occurs. Another rapid change results when saturated thickness decreases to zero, causing a change of 100 percent.

RESULTS OF 1978–80 HYDROLOGIC DATA

Personnel of the Western Kansas Groundwater Management District No. 1 suggested that the maps showing saturated thickness of the aquifer and percentage change in saturated thickness should be based on a 3-year moving-average altitude of the water table. This suggestion was made because of the problems that occur when water-table altitudes in various observation wells do not recover to the static water level during January. The observation wells are measured in January because most of the irrigation wells are idle during the winter. Water-table altitudes in a few wells may be lower in 1 year, due to recent pumping or pumping by nearby wells, and then seem to rise the next year when recent or nearby pumping was not a factor.

The Management District does not recommend approval of a new irrigation well in an area where the change in saturated thickness of the aquifer since 1950 is greater than 50 percent. In some areas, however, the change since 1950 may be greater than 50 percent during 1 year (closed to new well development) and less than 50 percent during the next year (open to development) because the water level in an observation well rose compared to the water level in the previous year. Therefore, the January water-table altitudes from 1978 through 1980 were averaged at sites where two or three measurements were available. If only one measurement was available for the 3-year period, it was considered as the average water-table altitude for that site and weighted equally with other sites. Average water-table altitudes were used to construct the 1978–80 maps.

Average 1978–80 Water-Table Altitudes

The 1978–80 water-table altitudes were estimated with the kriging program using the same procedure as with the 1978 water-table altitudes. As expected, 1978–80 water-table altitudes also showed anisotropic conditions. The final equation of the theoretical semi-variogram after the iteration and the validation process is:

$$(h) = 38.2(h) + 0. \qquad (8)$$

More data points were used for the 1978–80 semi-variogram than for the 1978 semi-variogram. There were more data points for 1978–80 because every well in the observation-well network was measured at least once during the 3 years.

The 1978–80 water-table map was computer-contoured using kriged water-table estimates at the center

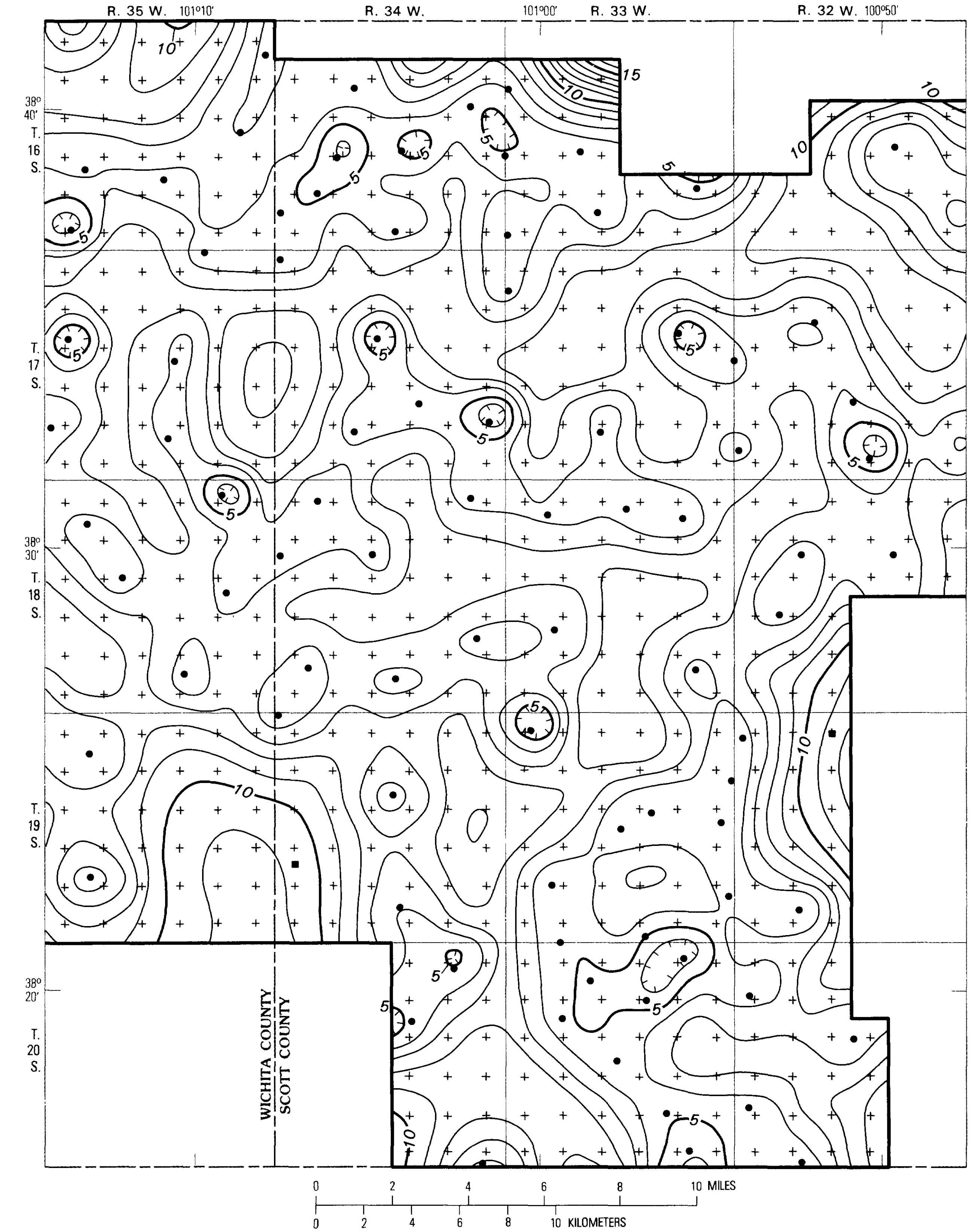

Figure 8. Error of estimate of average 1978–80 water-table altitudes.

EXPLANATION

—5— LINE OF EQUAL ERROR OF ESTIMATE OF WATER-TABLE ALTITUDE—Interval 1 foot

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• WATER-TABLE DATA POINT

▪ FUTURE WATER-TABLE DATA POINT

CENTER OF SECTION

of the sections and the average of the measured values (fig. 7). The errors of estimate of average 1978–80 water-table altitudes for each interpolated point were computer-contoured, as shown in figure 8. The errors of estimate from each interpolated point are at the 67-percent significance level.

Two new observation wells will be added to the network for the January 1981 measurements, as shown in figure 8. These two wells were chosen to reduce the large errors of estimate and to strengthen the reliability of the water-table estimates in their respective areas.

Average 1978–80 Saturated Thickness in the Aquifer

A map showing the average 1978–80 saturated thickness of the aquifer was constructed by the center-of-section technique in the same manner as the 1978 saturated-thickness map. A part of the map of average 1978–80 saturated thickness is shown in figure 9.

Percentage Change in Saturated Thickness of the Aquifer, 1950 to Average 1978–80

The map of the percentage change in saturated thickness of the aquifer from 1950 to average 1978–80 was constructed by the center-of-section technique in the same manner as the map of the percentage change in saturated thickness from 1950 to 1978. A part of the map of the percentage change in saturated thickness from 1950 to average 1978–80 is shown in figure 10.

SUMMARY AND CONCLUSIONS

Kriging is a statistical interpolation technique for autocorrelated, regionalized data. Other interpolation techniques include classical interpolation, weighted interpolation, and least-squares approach. Kriging is the most useful technique in west-central Kansas for the following reasons:

1. It has the ability to estimate the reliability of the map at unmeasured sites.
2. It is an exact interpolator at measured data points.
3. It uses measured values closely surrounding the point it estimates by incorporating the autocorrelative structure of the data.

The semi-variogram is a graph of the variability of the differences among data as a function of distance. Kriging is used to approximate first an empirical semi-variogram and then to fit one of several theoretical semi-variograms to the data. The 1978 water-table altitudes were fitted to a linear theoretical semi-variogram with a slope of 45.2. The average 1978–80 water-table altitudes had a slope of 38.2.

The water-table altitudes in west-central Kansas proved to be anisotropic; that is, the semi-variogram was a function of the direction, as well as the distance between data points, because water-table altitudes change more rapidly in a west-to-east direction (downgradient) than in a south-to-north direction.

Kriging gives an estimate of variance or estimate of error at each interpolated point. This provides a means of assessing the reliability of the map and designates areas where there is a great uncertainty of the estimates because of sparse data points. After examination of the error map, two additional observation wells are to be added to the network in order to decrease the error of future estimates in these areas.

The center-of-section technique was used to construct maps of saturated thickness of the aquifer and percentage change in saturated thickness. This technique uses the kriged estimates of the water-table altitude at the center of every section in the Western Kansas Groundwater Management District No. 1. By visually estimating bedrock and 1950 water-table altitudes from manually contoured maps, saturated thickness of the aquifer and percentage change in saturated thickness are easily calculated at the center of the sections. The data are then put into map form by a computer-contouring program.

A comparison of maps constructed using this technique to the same data previously contoured manually by the point-intersection technique shows that kriging is a comparable method for constructing maps of the water table, saturated thickness of the aquifer, and percentage change in saturated thickness. Both techniques gave a similar configuration of the contours, but the maps by the center-of-section technique had more detail.

The center-of-section technique was used to construct maps of the water table, saturated thickness, and percentage change in saturated thickness based on the

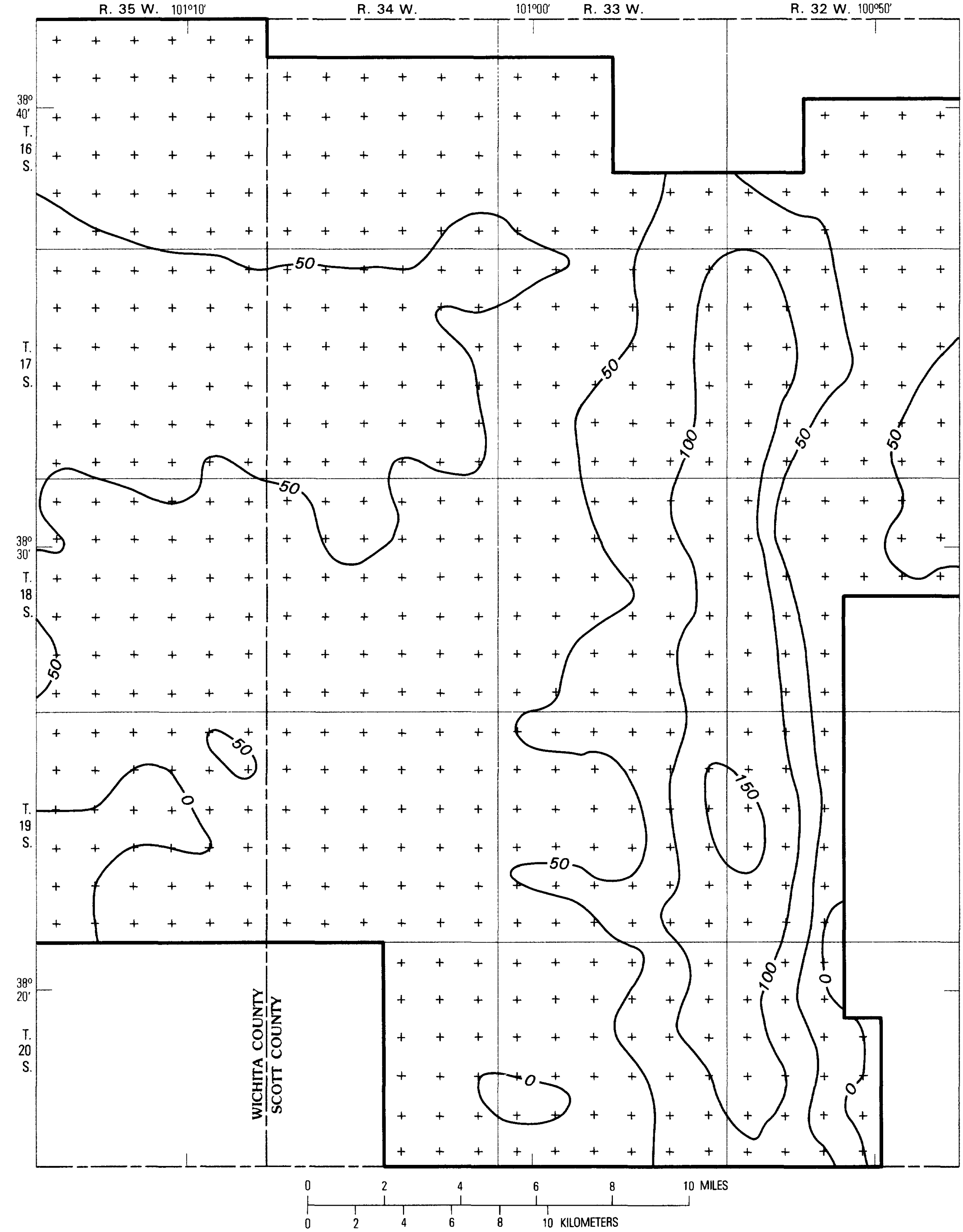

Figure 9. Average 1978–80 saturated thickness of the aquifer based on center-of-section technique.

EXPLANATION

—— 50 —— LINE OF EQUAL SATURATED THICKNESS—Interval 50 feet

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average 1978–80 water-table altitudes for the Western Kansas Groundwater Management District No. 1. Three-year average water-table altitudes resulted in an increased number of data points and an increased reliability in the water-table contour map.

The center-of-section technique has advantages over the manually contoured, point-intersection technique for the following reasons:

1. Maps are produced in a reproducible manner, and the variability and inconsistencies found in manual contouring are reduced.
2. More than six times the number of data values, each with an associated error of estimate, are used for contouring while the time spent in using the extra data is comparable or less, and the extra data provide a more detailed map.
3. It is very easy to update the maps when new data need to be added.

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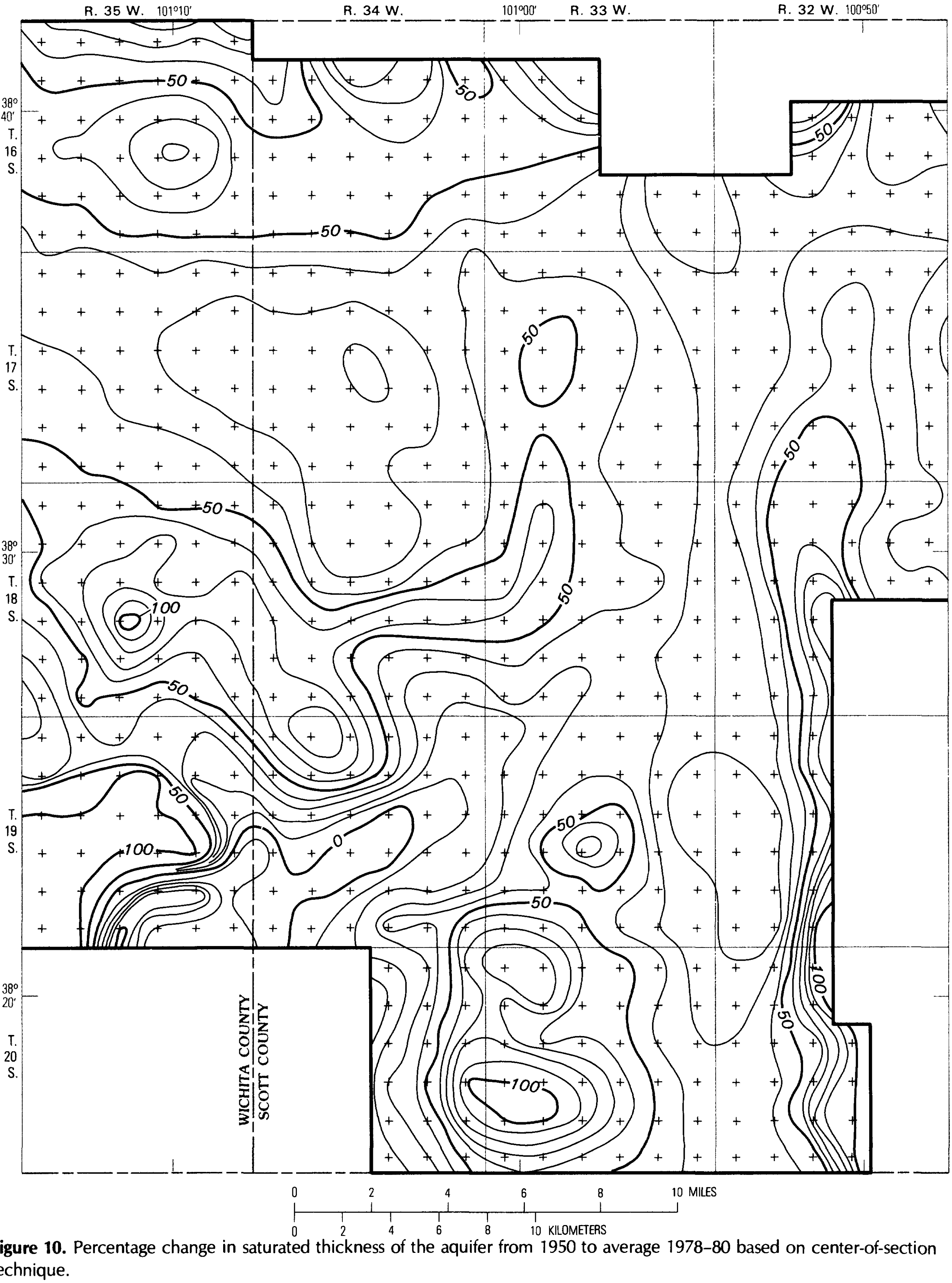

Figure 10. Percentage change in saturated thickness of the aquifer from 1950 to average 1978–80 based on center-of-section technique.

EXPLANATION

—50— LINE OF EQUAL PERCENTAGE CHANGE IN SATURATED THICKNESS—Interval 10 feet

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