# Application of a Parameter-Estimation Technique to Modeling the Regional Aquifer Underlying the Eastern Snake River Plain, Idaho

By S. P. GARABEDIAN

A contribution of the
Regional Aquifer-System
Analysis Program

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# CONTENTS



## PLATES

[Plates are in pocket]

## FIGURES



## TABLES

# Application of a Parameter-Estimation Technique to Modeling the Regional Aquifer Underlying the Eastern Snake River Plain, Idaho

*By* S. P. Garabedian


**Abstract**

A nonlinear, least-squares regression technique for the estimation of ground-water flow model parameters was applied to the regional aquifer underlying the eastern Snake River Plain, Idaho. The technique uses a computer program to simulate two-dimensional, steady-state ground-water flow. Hydrologic data for the 1980 water year were used to calculate recharge rates, boundary fluxes, and spring discharges. Ground-water use was estimated from irrigated land maps and crop consumptive-use figures. These estimates of ground-water withdrawal, recharge rates, and boundary flux, along with leakance, were used as known values in the model calibration of transmissivity. Leakance values were adjusted between regression solutions by comparing model-calculated to measured spring discharges. In other simulations, recharge and leakance also were calibrated as prior-information regression parameters, which limits the variation of these parameters using a normalized standard error of estimate.

Results from a best-fit model indicate a wide areal range in transmissivity from about 0.05 to 44 feet squared per second and in leakance from about $2.2 \times 10^{-9}$ to $6.0 \times 10^{-8}$ feet per second per foot. Along with parameter values, model statistics also were calculated, including the coefficient of correlation between calculated and observed head (0.996), the standard error of the estimates for head (40 feet), and the parameter coefficients of variation (about 10–40 percent). Additional boundary flux was added in some areas during calibration to achieve proper fit to ground-water flow directions. Model fit improved significantly when areas that violated model assumptions were removed. It also improved slightly when *y*-direction (northwest-southeast) transmissivity values were larger than *x*-direction (northeast-southwest) transmissivity values. The model was most sensitive to changes in recharge, and in some areas, to changes in transmissivity, particularly near the spring discharge area from Milner Dam to King Hill.


## INTRODUCTION

This report is one in a series resulting from the U.S. Geological Survey Snake River Plain RASA (Regional Aquifer-System Analysis) study that was initiated in October 1979. As stated by Lindholm (1981), the purposes of the study were to (1) refine knowledge of the regional ground-water flow system, (2) determine effects of conjunctive use of ground and surface water, and (3) describe solute chemistry. This report addresses the first of these objectives.

A two-dimensional, steady-state ground-water flow model was used to develop preliminary estimates of transmissivity, leakance, and boundary fluxes for the regional aquifer underlying the eastern Snake River Plain. Estimates of aquifer recharge and discharge during water year 1980 were made as a basis for calibrating the unknown parameters: transmissivity, leakance, and some boundary fluxes.

## Well-Numbering System

The well-numbering system (fig. 1) used by the U.S. Geological Survey in Idaho indicates the location of wells within the official rectangular subdivision of public lands, with reference to the Boise Base Line and Meridian. The first two segments of a number designate the township (north or south) and range (east or west). The third segment gives the section number, followed by three letters and a numeral, which indicate the ¼ section (160-acre tract), ¼-¼ section (40-acre tract), ¼-¼-¼ section (10-acre tract), and serial number of the well within the tract, respectively.

Quarter sections are designated by the letters A, B, C, and D in counterclockwise order from the northeast quarter of each section. Within quarter sections, 40-acre and 10-acre tracts are lettered in the same manner. For example, well 8S-19E-5DAB1 is in the SE¼NE¼NW¼ sec. 5, T. 8 S., R. 19 E., and is the first well inventoried in that tract.

## Location and Description of Study Area

The eastern Snake River Plain is part of the arcuate Snake River Plain that extends across southern Idaho into Oregon (fig. 2). The eastern plain is about 170 mi long, 60 mi wide, and 10,800 mi$^2$ in area. Altitudes range from about 2,500 ft above sea level at river level near King Hill to about 6,000 ft in the northeastern part of the plain. The surrounding mountains rise to 7,000-12,000 ft in altitude. The eastern plain is entirely within the Snake River drainage basin. Streams in several tributary intermontane valleys lose all flow to infiltration or evaporation after reaching the plain.

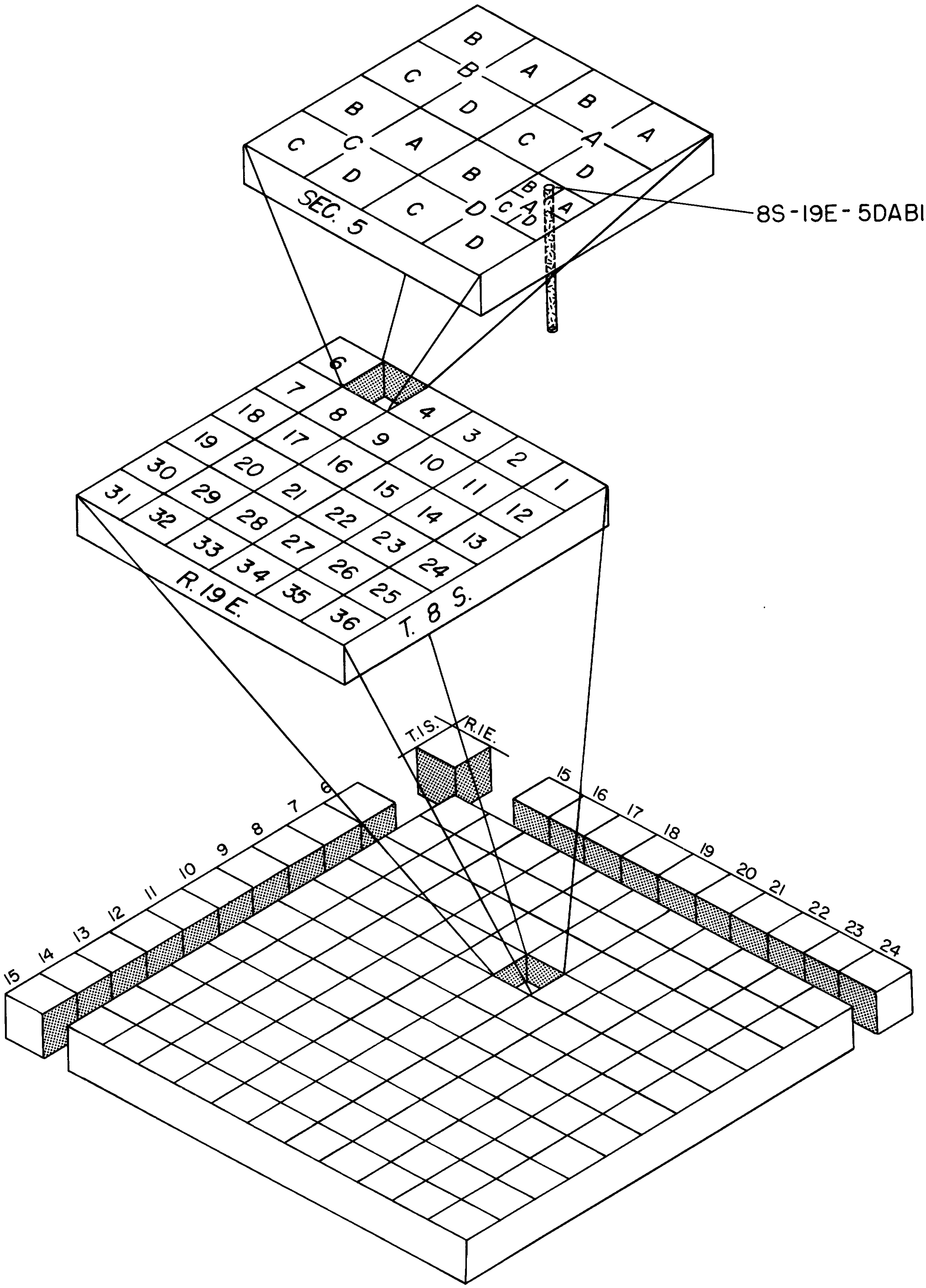


**Figure 1.** Well-numbering system.

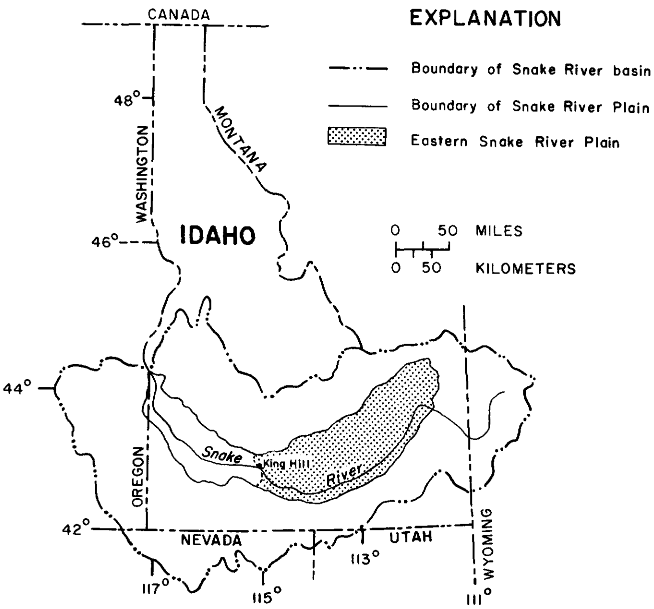


**Figure 2.** Location of study area.

The eastern Snake River Plain is underlain chiefly by basalt, which transmits large volumes of water and is a major regional aquifer in southern Idaho. Discharge from the aquifer is largely spring flow, which sustains a major part of streamflow in the Snake River. The basaltic aquifer is a major source of irrigation water. Most crops grown on the eastern plain are irrigated because annual precipitation over most of the area is only about 8–10 in.

## Previous Investigations

Numerous studies and reports have been made of the geology and ground-water resources of the eastern Snake River Plain. Notable early studies were those of Russell (1902) and Stearns and others (1938). A quantitative hydrologic study by Mundorff and others (1964) included estimates of transmissivity by use of a flow-net analysis. Electric analog model studies of the regional aquifer underlying the eastern plain were made by Skibitzke and da Costa (1962), Norvitch and others (1969), and Mantei (1974), and numerical model studies were carried out by deSonneville (1974), Newton (1978), and Wytzes (1980). Solute-transport modeling of radioactive wastes at the Idaho National Engineering Laboratory was done by Robertson (1974, 1977).

## Acknowledgments

The author wishes to thank Richard L. Cooley and Aldo V. Veechia of the U.S. Geological Survey for their suggestions and error analysis, which helped to improve this report.

# GEOLOGY OF THE EASTERN SNAKE RIVER PLAIN

The predominant rock type of the eastern Snake River Plain is Quaternary basalt (pl. 1*A*). Basalt, interbedded with terrestrial sediments, fills a structural basin bounded by faulting on the northwest and by downwarping and faulting on the southeast (Whitehead, 1984). The northeastern end of the plain is defined by silicic volcanic rocks (mainly rhyolite), which also are present southwest of the plain. Granitic rocks and pre-Cretaceous sedimentary and metamorphic rocks occur northwest of the plain. Adjacent and perpendicular to the axis of the plain are several intermontane valleys characterized by basin-and-range structure. Kuntz (1978) noted that volcanism on the eastern plain is localized along rift zones (pl. 1*B*). The rifts appear to be extensions of basin-and-range faults that are present in areas surrounding the plain. Kuntz (1978) indicated that faults are abundant owing to extension in the northeast-southwest direction along the axis of the eastern plain. In some places, this extension has caused open fissures at land surface.

Quaternary basalts were extruded from individual vents or series of vents, and individual flows are thin, averaging 20–25 ft in thickness. Aggregate basalt thickness may in places exceed several thousand feet, as shown in figures 3 and 4 (Whitehead, 1984). Individual flows are of variable areal extent, commonly 50–100 $mi^2$. Sediments interbedded with the basalt along the edges of the plain were deposited by the Snake River and tributary streams (figs. 3 and 4). In some areas, particularly in alluvial fans, sand and gravel predominate. In other areas, particularly where streams were dammed by basalt flows, silt and clay are the predominant sediments. Along some margins of the plain and possibly under the entire eastern plain, rhyolite underlies the basalt.

Soil cover is minimal over younger basalt and consists primarily of windblown material. Most agriculture is in areas where soils are developed on fluvial and lacustrine deposits.

# GROUND-WATER HYDROLOGY

Occurrence and movement of ground water in the regional aquifer underlying the eastern plain are dependent on both the geologic framework, which determines aquifer transmissivity and storage, and the recharge and discharge within that framework. Regionally, most water moves horizontally through basalt interflow zones, which are the broken and rubbly zones between lava flows. Locally, water moves vertically along joints and the interfingering edges of interflow zones.

The Quaternary basalt aquifer generally yields large quantities of water to wells. Where interflow zones include sediments and secondary minerals, transmissivity of the unit is decreased. Generally, older basalts yield less water than younger basalts as a result of secondary minerals filling

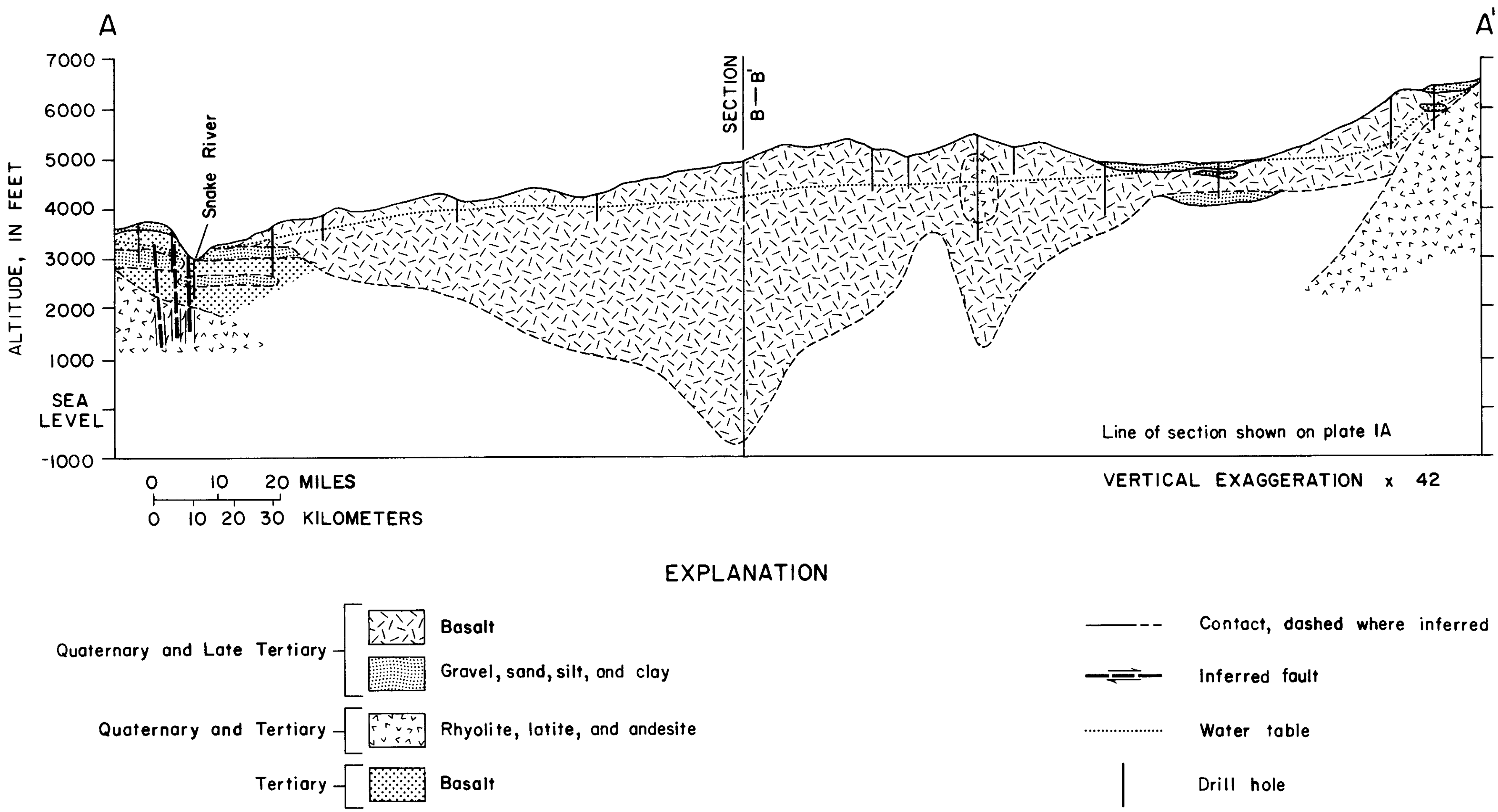


**Figure 3.** Hydrogeologic section A–A'.

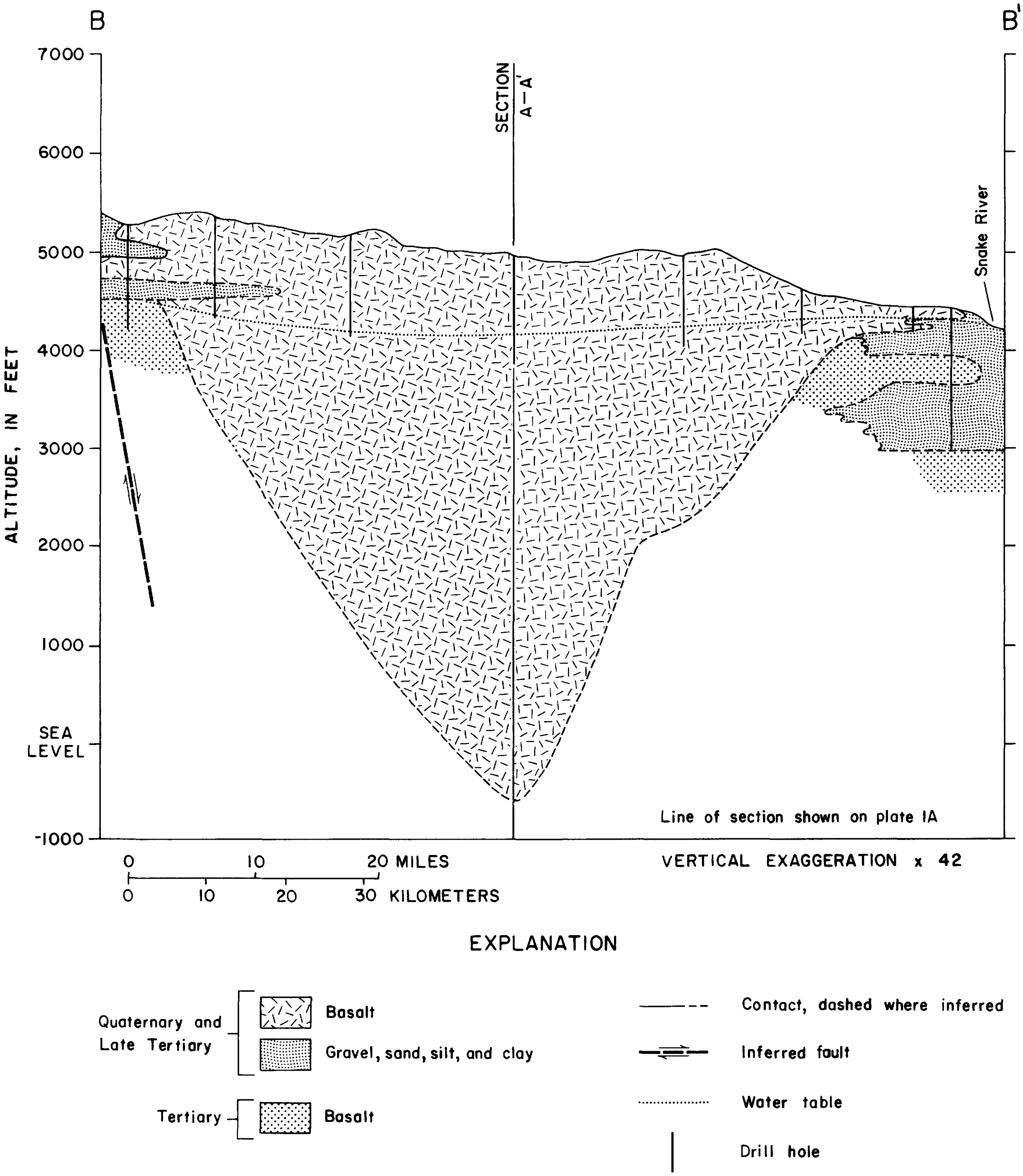


**Figure 4.** Hydrogeologic section B–B′.

vesicles, fractures, and interflow rubble zones. Aquifer thickness is largely unknown, but recent geophysical studies suggest that locally the Quaternary basalt aquifer may be several thousand feet thick (Whitehead, 1984). It is generally believed that the upper several hundred feet are the most transmissive. Along the margins of the plain, sand and gravel deposits several hundred feet thick transmit large volumes of water.

Recharge to the eastern Snake River Plain aquifer is from seepage of irrigation water, stream and canal leakage, tributary valley underflow, and direct precipitation. Aquifer discharge is largely spring flow to the Snake River and water pumped for irrigation. Major springs are near American Falls Reservoir and along the Snake River from Milner Dam to King Hill. Ground water generally moves from northeast to southwest, from areas of recharge to areas of discharge; movement generally is perpendicular to water-table contours (pl. 2).

A comparison of water levels for three different periods (1928–30, 1956–58, and March 1980; pl. 2) indicates that regional ground-water levels and the direction of flow have been relatively stable in the central part of the eastern Snake River Plain for at least the last 50 years. However, on large tracts of land in the eastern plain, water levels rose an average of 60 to 70 ft (Mundorff and others, 1964, p. 162) and ground-water discharge increased (fig. 5) soon after initiation of surface-water irrigation (about 1910). By 1928, most surface water for irrigation was appropriated; since that time (until 1980), the total amount of water diverted from Henrys Fork of the Snake River (hereafter referred to as Henrys Fork) and the Teton, Falls, Blackfoot, and Snake Rivers has been relatively stable, averaging about 8,600,000 acre-ft/yr (fig. 6). From 1945 to 1980, increased amounts of ground water were withdrawn for irrigation. The result has been a small but, in most areas, definite decline of ground-water levels and decrease in ground-water discharge (fig. 5). Hydrographs on plate 2 show that water-level declines in the past 30 years have been less than 10 ft in the eastern plain.

In several areas on the eastern plain, local shallow ground-water systems have developed in the alluvium. Some of the shallow systems are perched, usually in surface-water-irrigated areas where vertical flow of recharge water from irrigation is impeded by fine-grained sediments. In these areas, water levels in shallow wells may be higher than those in nearby deeper wells. Water levels in these shallow wells are representative of local ground-water conditions and not of the underlying regional flow system. For these reasons, water levels in shallow wells in several areas on the plain were not used to develop the regional water-table contours for March 1980 (Lindholm and others, 1983) shown on plate 2.

The regional aquifer underlying the eastern plain is nonhomogeneous and locally anisotropic. The complex interfingering of sedimentary and volcanic rocks results in aquifer nonhomogeneity along the margins of the plain. Basalts are also nonhomogeneous because hydraulic conductivity is greatest in randomly distributed and discontinuous rubbly interflow zones; however, the lateral anisotropy caused by these zones probably occurs only locally.

## Recharge

Sources of recharge to the regional aquifer system in the eastern Snake River Plain are seepage of irrigation water, stream and canal leakage, tributary valley underflow, and direct precipitation. Steady-state calculations of recharge and discharge were based on data for the 1980 water year (October 1979 to September 1980). Surface-water diversion and return-flow data for irrigation districts are presented in watermaster reports (Idaho Department of Water Resources, 1980; and Water Districts 37, 37M, 1980) or were calculated using U.S. Geological Survey records (1980) and data from other agencies (U.S. Bureau of Reclamation, written commun., 1981).

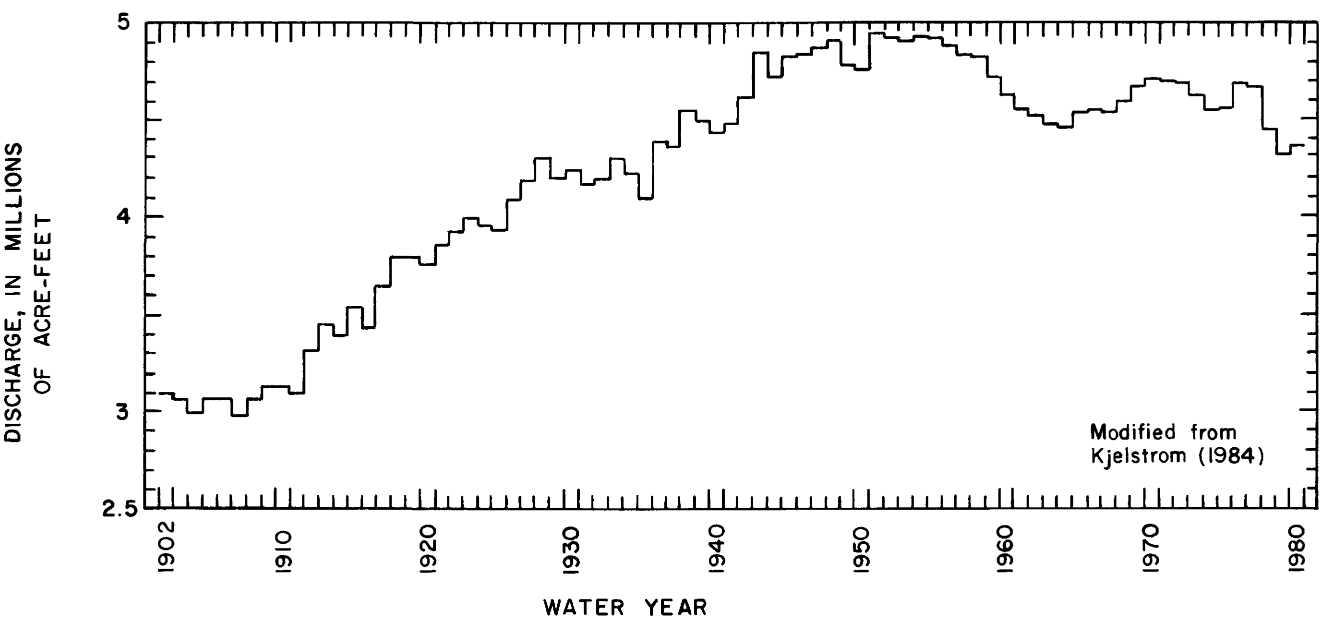


**Figure 5.** Mean annual north-side ground-water discharge to the Snake River between Milner and King Hill.

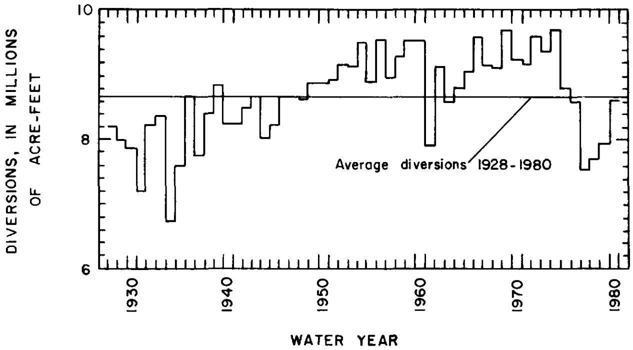


**Figure 6.** Total diversions for irrigation on the eastern Snake River Plain, water years 1928–80, from Henrys Fork and the Teton, Falls, Blackfoot, and Snake Rivers.

To simplify the determination of recharge from irrigation seepage, irrigation districts were grouped into areas similar to those used by Norvitch and others (1969), as shown on plate 1*B* and listed in appendix A.

Average recharge rates (table 1) for each surface-water-irrigated area were estimated by the following equation:

$$R = \frac{D - F}{A} - ET \qquad (1)$$

where

$R$ = recharge rate, in feet per year,
$D$ = irrigation diversions, in acre-feet per year,
$F$ = surface return flows, in acre-feet per year,
$A$ = area acreage, and
$ET$ = estimated evapotranspiration, in feet per year.

Irrigated acreages were estimated from a map of irrigated lands for 1979 (pl. 1*B*). Evapotranspiration rates represent crop requirements adjusted for growing-season precipitation (consumptive irrigation requirement). These rates were estimated by using an empirical formula developed by Jensen and Criddle (1952) on the bases of monthly temperature, length of growing season, monthly percentage of

**Table 1.** Estimated recharge rates for irrigated areas in 1980

| Area No. | Diversions minus surface returns [1] (acre-feet per year) | Total irrigated area (acres) | Estimated evapotranspiration (feet per year) | Recharge rate (feet per year) | Area included in model (acres) | Volume of recharge to modeled area (acre-feet per year) |
|---|---|---|---|---|---|---|
| 1 | 38,800 | 24,800 | 1.0 | 0.56 | 3,900 | 2,200 |
| 2 | 225,500 | 27,300 | 1.1 | 7.16 | 26,100 | 186,900 |
| 3 | 379,100 | 33,800 | 1.2 | 10.02 | 33,800 | 338,500 |
| 4 | 175,300 | 41,900 | 1.3 | 2.88 | 38,000 | 109,600 |
| 5 | 128,700 | 26,300 | 1.3 | 3.59 | 26,300 | 94,500 |
| 6 | 1,388,600 | 140,300 | 1.3 | 8.60 | 139,600 | 1,200,200 |
| 7 | 256,400 | 62,700 | 1.3 | 2.79 | 62,700 | 174,900 |
| 8 | 527,900 | 80,400 | 1.3 | 5.27 | 80,400 | 423,400 |
| 9 | 227,300 | 35,100 | 1.5 | 4.98 | 35,100 | 174,700 |
| 10 | 487,900 | 79,600 | 1.5 | 4.63 | 79,600 | 368,500 |
| 11 | 73,900 | 41,600 | 1.5 | .28 | 38,800 | 10,700 |
| 12 | 338,500 | 77,200 | 1.6 | 2.78 | 77,200 | 215,000 |
| 13 | 48,800 | 18,200 | 1.6 | 1.08 | 18,200 | 19,700 |
| 14 | 1,022,100 | 162,700 | 1.6 | 4.68 | 161,000 | 753,800 |
| 15 | 657,300 | 247,200 | 1.6 | 1.06 | 242,200 | 256,500 |
| 16 | 60,600 | 15,200 | 1.6 | 2.39 | 15,200 | 36,300 |
| 17 | 245,800 | 49,000 | 1.6 | 3.42 | 49,000 | 167,400 |
| 18 | 44,900 | 13,900 | 1.6 | 1.63 | 9,700 | 15,800 |
| 19 | 67,100 | 20,500 | 1.6 | 1.67 | 20,500 | 34,300 |
| 20 | 62,600 | 17,400 | 1.6 | 2.00 | 17,400 | 34,800 |
| 21 | 226,100 | 30,100 | 1.6 | 5.91 | 29,900 | 176,800 |
| 22 | 106,800 | 5,600 | 1.6 | 17.47 | 5,600 | 97,800 |
| 23 | 69,200 | 15,300 | 1.6 | 2.92 | 15,300 | 44,700 |
| 24 | 36,000 | 11,900 | 1.6 | 1.43 | 11,200 | 16,000 |
| 25 | 94,700 | 27,400 | 1.6 | 1.86 | 27,400 | 50,900 |
| 26 | 129,200 | 20,900 | 1.6 | 4.58 | 20,000 | 91,600 |
| TOTAL | 7,119,100 | 1,326,300 | | | 1,284,100 | 5,095,500 |

[1] Calculation of diversions minus returns discussed in appendix A.

annual daytime hours, precipitation, and crop type. Total volume of recharge to the eastern Snake River Plain regional aquifer system from surface-water irrigation during the 1980 water year was estimated to be about 5,100,000 acre-ft.

To estimate ground-water recharge from irrigation in the Henrys Fork–Teton River basin and the Big Wood–Little Wood River basin, river losses were included with the irrigated lands recharge rate because of unmeasured diversions and return flows. Most Snake River diversions and return flows were measured or estimated in water year 1980.

Snake River losses (about 880,000 acre-ft/yr) and gains (about 7,280,000 acre-ft/yr) in 1980, calculated by Kjelstrom (1984), are listed in table 2. Average losses to the ground-water system from other streams and canals (about 490,000 acre-ft/yr) over various periods of record, as determined by Kjelstrom (1984) and the U.S. Geological Survey (1980), are listed in table 3. Most canal losses were included in the determination of recharge rates for each irrigation area. However, the Milner-Gooding, Aberdeen-Springfield, and Reservation Canals lose water by seepage crossing nonirrigated lands before reaching points of delivery. These canals were treated separately in recharge calculations as distributed losses.

Average underflow from tributary valleys (about 1,230,000 acre-ft/yr) is listed in table 4 and was calculated by Kjelstrom (1984) using basin-yield equations. Recharge from precipitation was estimated by subdividing the eastern Snake River Plain into six areas (table 5 and pl. 1*C*) that differ in soil type and amount of average annual precipitation. Recharge rates were modified from those used by Mundorff and others (1964) and should be considered approximate. Total annual recharge to the eastern plain from direct precipitation is about 760,000 acre-ft.

**Table 2.** Snake River losses to and gains from ground water in water year 1980

| Reach | Loss (-) or gain (+) (cubic feet per second) | Loss (-) or gain (+) (acre-feet per year) |
|---|---|---|
| Heise to Lorenzo | -145 | -105,000 |
| Lorenzo to Lewisville | +289 | +208,900 |
| Lewisville to Shelley | -379 | -274,400 |
| Shelley to at Blackfoot | -204 | -147,800 |
| At Blackfoot to near Blackfoot | -270 | -195,600 |
| Near Blackfoot to Neeley | +2,635 | +1,907,900 |
| Neeley to Minidoka | +453 | +327,900 |
| Minidoka to Milner | -218 | -157,700 |
| Milner to Kimberly (north side) | +30 | +21,700 |
| Milner to Kimberly (south side) | +267 | +193,300 |
| Kimberly to Buhl (north side) | +1,115 | +807,200 |
| Kimberly to Buhl (south side) | +108 | +78,400 |
| Buhl to Hagerman (north side) | +3,405 | +2,465,400 |
| Buhl to Hagerman (south side) | +275 | +198,900 |
| Hagerman to King Hill | +1,472 | +1,065,700 |
| Total loss | -1,216 | -880,500 |
| Total gain | +10,049 | +7,275,300 |

**Table 3.** Tributary stream and canal losses to the ground-water system based on various periods of record

| Name | Loss (cubic feet per second) | Loss (acre-feet per year) | Source of data |
|---|---|---|---|
| Big Lost River | 95 | 69,000 | a |
| Little Lost River | 17 | 12,500 | a |
| Medicine Lodge Creek | 69 | 50,200 | a |
| Beaver Creek | 46 | 33,100 | a |
| Camas Creek | 125 | 90,300 | b |
| Clover Creek | 25 | 18,000 | a |
| Fish Creek | 21 | 14,900 | a |
| Aberdeen-Springfield Canal | 263c | 95,200 | a |
| Milner-Gooding Canal | 204c | 96,800 | a |
| Reservation Canal | 16c | 11,800 | a |
| Total | 679 | 491,800 | |

a--Kjelstrom (1984)
b--U.S. Geological Survey (1980)
c--Losses occur only during irrigation season

## Discharge

Ground water discharges from the regional aquifer system largely as seeps and springs along the Snake River from Blackfoot to Neeley and from Milner to King Hill (pl. 1). Ground-water discharge to the Snake River from Ferry Butte to American Falls Reservoir, and to the Portneuf River from Pocatello to the reservoir, was about 1,910,000 acre-ft in 1980 (Kjelstrom, 1984). Snake River gains in 1980 were 210,000 acre-ft in the Lorenzo to Lewisville reach and 330,000 acre-ft in the Neeley to Minidoka reach.

Springs along the Snake River from Milner to King Hill discharge from both the south and north sides of the river canyon. South-side springs discharged about 470,000 acre-ft

**Table 4.** Tributary valley underflow based on basin-yield equations

| Name | Underflow (cubic feet per second) | Underflow (acre-feet per year) |
|---|---|---|
| Big Bend area | 154 | 111,300 |
| Camas Creek | 208 | 150,700 |
| Beaver Creek | 82 | 59,200 |
| Medicine Lodge Creek | 12 | 8,400 |
| Deep Creek | 11 | 7,600 |
| Warm Springs Creek | 35 | 25,600 |
| Birch Creek | 97 | 70,000 |
| Little Lost River | 210 | 152,000 |
| Big Lost River | 418 | 302,600 |
| Little Wood River | 111 | 80,700 |
| Silver Creek | 52 | 38,000 |
| Salmon Falls Creek | 34 | 24,600 |
| Raft River | 113 | 82,000 |
| Rockland Valley | 70 | 51,000 |
| Portneuf River | 87 | 63,000 |
| Total | 1,694 | 1,226,700 |

**Table 5.** Estimated recharge from precipitation

| County or area | Soil cover[1] | Area (acres) | Annual precipitation (inches) | Recharge (inches) | Total volume of recharge (acre-feet per year) |
|---|---|---|---|---|---|
| Gooding, Jerome, Lincoln, Jefferson, Clark, Fremont | Thin soil cover (<40 in.), high infiltration-rate potential | 1,514,500 | 10 | 1 | 126,200 |
| Butte, Blaine, Minidoka | Recent lava flows, little soil cover | 338,800 | 10 | 3.5 | 98,800 |
| Central part of eastern Snake River Plain | Thick soil cover (>40 in.), low infiltration-rate potential | 4,244,500 | 8-10 | .3 | 106,100 |
| Blaine, Power, Bingham, Bonneville | Recent lava flows, little soil cover | 229,200 | 8 | 2.8 | 53,500 |
| Lands adjacent to the Snake River | Thin soil cover (<40 in.), high infiltration-rate potential | 747,300 | 10 | 2 | 124,500 |
| Northeastern part of eastern Snake River Plain, Jefferson, Fremont, Clark | Thin soil cover (<40 in.), high infiltration-rate potential | 508,100 | 16-20 | 6 | 254,100 |
| Total | | 7,582,400 | | | 763,200 |

[1] See plate 1 for distribution of generalized soil types.

and north-side springs discharged about 4,360,000 acre-ft in 1980 (Kjelstrom, 1984).

Ground-water pumpage for irrigation continues to increase. In 1980, ground water was used to irrigate about 930,000 acres. Based on crop evapotranspiration requirements, it was estimated that 1,640,000 acre-ft of water was pumped. Any excess pumpage beyond crop requirements was assumed to return to the ground-water system. The estimate of pumpage based on crop evapotranspiration requirements is similar to the estimate of 1,760,000 acre-ft made by Bigelow and others (1984) using data on electric power consumption.

## Water Budget

A 1980 water budget for the regional aquifer system underlying the eastern Snake River Plain is presented in table 6. A net loss in ground-water storage of about 130,000 acre-ft was estimated from water-level changes measured in 1980 and is shown on plate 1*D*. Storage coefficients used for the estimates are 0.05 for basalt, determined from pumping-test data (Mundorff and others, 1964), and 0.20 for sediments.

The most accurate estimates in the ground-water budget are Snake River gains and losses—errors of these estimates range from 3 to 10 percent. Estimates of recharge from surface-water irrigation are less accurate because the evapotranspiration values used in calculations are empirical estimates. Evapotranspiration, which is also an important component in the estimation of ground-water pumpage, is particularly difficult to estimate for large areas where climatic conditions and crop types vary. Estimates of recharge from tributary streams and tributary valley underflow vary in accuracy because flow is measured directly in some streams

**Table 6.** Aquifer budget for water year 1980

| Sources | Recharge (acre-feet) |
|---|---|
| Surface-water irrigation | 5,095,500 |
| Snake River loss | 880,500 |
| Tributary stream and canal losses | 491,800 |
| Tributary valley underflow | 1,226,700 |
| Precipitation | 763,200 |
| Total | 8,457,700 |

| Sources | Discharge (acre-feet) |
|---|---|
| Springs discharging to Snake River | 7,275,300 |
| Ground-water pumpage | 1,641,300 |
| Total | 8,916,600 |
| Change in storage | - 127,300 |

Recharge - discharge = change in storage + differences in estimates

$$\frac{\text{Differences in estimates}}{\text{Discharge}} = \frac{-331{,}600}{8{,}916{,}600} = -0.04$$

and estimated from basin-yield equations in others. The value for change in storage is approximate because it is based on estimates of the aquifer storage coefficient and on measurements made in widely scattered observation wells.

The least accurate estimate in the ground-water budget is recharge from direct precipitation. Although precipitation is measured at several sites, recharge from precipitation cannot be measured directly. Possible endpoints of the estimate are 0 and 100 percent recharge of the total direct precipitation. One hundred percent recharge from precipitation on the eastern plain would be about 6,000,000 acre-ft, or an average of 10.4 in./yr. This amount of recharge puts the ground-water budget far out of balance and is therefore unreasonable. The assumption of no recharge from precipitation causes an 11.5 percent residual in the 1980 budget. Mundorff and others (1964) estimated recharge from precipitation to be about 500,000 acre-ft. Given the difference in study areas (8,400 mi$^2$, from Mundorff and others, 1964; versus 10,800 mi$^2$, from the present study), the difference between previous and present estimates of recharge from direct precipitation is reasonable.

The overall budget error of estimate is from 10 to 20 percent, owing to compensating errors in calculations of evapotranspiration, basin yields, and recharge from precipitation. Within the context of these errors, the budget residual of 4 percent (table 6) was considered acceptable.

## GROUND-WATER FLOW MODEL

The flow model is based on the following partial differential equation that describes steady-state, two-dimensional ground-water flow (Cooley, 1977, 1979):

$$\frac{\partial}{\partial x}\left(T_{xx}\frac{\partial h}{\partial x}\right) + \frac{\partial}{\partial y}\left(T_{yy}\frac{\partial h}{\partial y}\right) + R(H-h) + W + \sum_{p=1}^{M} Q_p\delta(x-a_p)\delta(y-b_p) = 0 \quad (2)$$

where

$T_{xx}$, $T_{yy}$ = transmissivity in the $x$ and $y$ Cartesian coordinate directions, which are to be aligned with the principal transmissivity axes, in square feet per second,

$R$ = leakance (hydraulic conductivity divided by thickness) of streambed and spring confining bed, in feet per second per foot,

$H$ = head in stream, spring vent, or opposite side of confining bed, in feet,

$h$ = hydraulic head in the aquifer, in feet,

$W$ = areally distributed recharge and discharge, in feet per second,

$M$ = number of point-source sink terms, and

$Q_p$ = source or sink term (well) at point ($a_p$, $b_p$), as a volume per unit surface area per unit time, in feet per second.

The terms in equation (2) are time-averaged values, where variability of values such as seasonal pumpage is averaged over a given time period. Aquifer properties are distributed by zones in which a property is held constant; however, zonal boundaries of different properties need not be the same (fig. 7).

The numerical approximation and solution of equation (2) are based on a finite-difference, mesh-centered discretization (fig. 8). The mesh spacing is held constant ($\Delta x = \Delta y$) in this report.

The regression procedure used in solving for optimum model hydraulic parameters is a minimization of the sum of squared errors of head differences between model-simulated heads and observed heads; it is based on a modified Gauss-Newton method (Cooley, 1977). The sum of squares ($SQ$) criterion is defined as

$$SQ = \sum_{l=1}^{N} (WF)_l (h_l^{obs} - h_l^{c})^2 \quad (3)$$

where

$N$ = total number of observations,

$(WF)_l$ = weighting factor, an expression of the reliability of head observation, $l$,

$h_l^{obs}$ = observed head, and

$h_l^{c}$ = simulated head.

Regression parameters can be $T_{xx}$, $T_{yy}$, $R$, $W$, boundary flux, or constant head values. Zones may be combined to form a single regression parameter so that resultant values in each zone would be changed by the same amount during the regression procedure; that is, the ratio of the zonal values

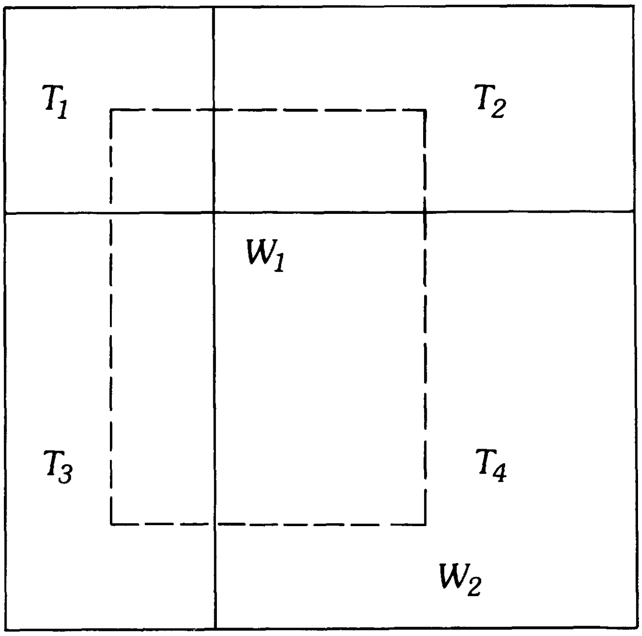


**Figure 7.** Example of aquifer zonation (*T* and *W* represent distributed aquifer parameter values; modified from Cooley, 1977).

would remain the same. Prior information about regression parameters may be incorporated into the model to improve calibrated estimates. This part of the technique is described by Cooley (1982) and requires an unbiased initial estimate of the parameter as well as the coefficient of variation (standard deviation divided by expected value) of the estimate.

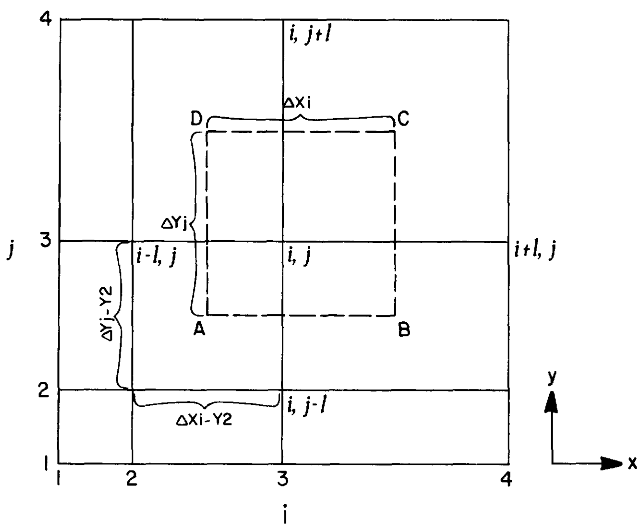


**Figure 8.** Finite-difference, mesh-centered grid notation (*A, B, C, D* represent blocks of distributed aquifer parameter volume around node *i, j*).

A computer program by R. L. Cooley (U.S. Geological Survey, written commun., 1981) that implements the least-squares procedure is described in appendix B; the program listing is appendix C. The program was modified by addition of mass-balance calculations and the use of interpolated simulated heads where measured heads were not at nodes. This was necessary because many blocks had several head observations and node points were not located at points of head observations. The following equation is used to calculate interpolated heads:

$$h^c = ax + by + cxy + d \tag{4}$$

where

$h^c$ = interpolated head,
$h_1, h_2, h_3, h_4$ = heads at block corners (fig. 9),
$a = h_2 - h_1$,
$b = h_3 - h_1$,
$c = h_4 + h_1 - h_2 - h_3$,
$d = h_1$,
$x$ = fractional distance from lower left corner of unit block along the $x$-axis (fig. 9),
and
$y$ = fractional distance from lower left corner of unit block along the $y$-axis (fig. 9).

This interpolation equation also was used to interpolate the arrays used in the least-squares estimation of parameters. Changes made using the interpolation equation are noted in comment statements throughout the program listing (appendix C).

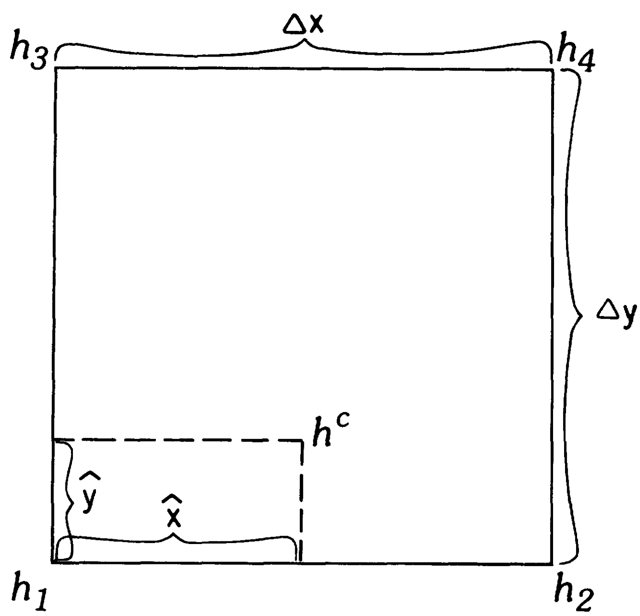


**Figure 9.** Notation for head interpolation ($x = \hat{x}/\Delta x$, $y = \hat{y}/\Delta y$).

## APPLICATION OF GROUND-WATER FLOW MODEL TO THE EASTERN SNAKE RIVER PLAIN REGIONAL AQUIFER SYSTEM

The approach taken in this study was to develop recharge estimates for the regional aquifer system as a starting point for calibrating estimates of transmissivity and other aquifer parameters. A similar approach was used by previous investigators (Mundorff and others, 1964; Norvitch and others, 1969; and Newton, 1978) because fluxes in the regional system, particularly spring discharges, are more accurately measured than aquifer transmissivities and leakances. Reported transmissivities calculated from aquifer tests are typically minimum values and represent only local conditions around a partially penetrating well.

Recharge rates based on 1980 water year data and distributed to each block were calculated using the following equation:

$$RB(i,j) = \frac{1}{AB(i,j)}\left[\left(\sum_{K=1}^{N} SW_K \cdot A_K\right)(i,j) + U(i,j) + SC(i,j) + P(i,j) + \Delta S(i,j) + GW(i,j)\right] \quad (5)$$

where

$RB(i,j)$ = recharge rate for block $(i,j)$, in feet per second,
$AB(i,j)$ = area of block $(i,j)$, in square feet,
$N(i,j)$ = number of irrigation areas in block $(i,j)$,
$SW_K$ = recharge rate for irrigation area $K$ (table 1), in feet per year,
$A_K(i,j)$ = total acreage for irrigation area $(K)$ in block $(i,j)$,
$U(i,j)$ = tributary valley underflow in block $(i,j)$ (table 4), in cubic feet per second,
$SC(i,j)$ = stream and canal losses in block $(i,j)$ (tables 2 and 3), in cubic feet per second,
$P(i,j)$ = recharge from precipitation in block $(i,j)$ (table 5), in cubic feet per second,
$\Delta S(i,j)$ = change in storage per unit time in block $(i,j)$, in cubic feet per second, and
$GW(i,j)$ = ground-water pumpage in block $(i,j)$, in cubic feet per second.

### Model Grid and Boundary Conditions

Model grid and boundary conditions are illustrated on plate 3. The grid was oriented in a northeast-southwest direction to minimize the number of inactive blocks, to make the *x*-direction parallel to the major direction of ground-water flow, and to make the *y*-direction parallel to rift zones crossing the plain (pl. 1*A*). Boundary conditions included constant flux and head-dependent river and spring blocks. Constant flux for each block was calculated using equation (5). Underflow from tributary valleys shown on plate 3 is listed in table 4. Where the amount of underflow was unknown, boundary flux was calculated as a separate regression parameter. Ground-water recharge calculated by equation (5) varied from block to block across the aquifer. Largest values were in areas of surface-water irrigation; smallest values were in areas where ground-water pumpage exceeded recharge, which resulted in a negative flux.

Ground-water discharge to the Snake River (largely as spring flow) was simulated as leakage across a confining bed. River or spring heads were held constant, but head losses were allowed between the ground-water system and these constant heads. The control on head loss—leakance—is the hydraulic conductivity of the confining bed divided by the bed thickness. This type of boundary condition allowed good control of discharge to the river and springs. Each of the gaining reaches of the Snake River listed in table 2, except the Lorenzo to Lewisville reach, was modeled using head-dependent discharge blocks (pl. 3). The Lorenzo to Lewisville reach was treated as a constant flux. In this reach, ground-water discharge during the irrigation season is primarily from the shallow alluvial system, which also recharges the deeper regional system. Head relations between the shallow system and the regional system do not indicate upward movement of water during the irrigation season, when the Lorenzo to Lewisville reach is gaining. Therefore, discharge to the river from the shallow alluvial system reduces the amount of recharge to the underlying regional system in this area.

### Model Assumptions

Several major assumptions were made in the use of this model. Ground-water flow in the aquifer was assumed to be laminar (Darcian flow). Although this may not be true locally, particularly near spring vents, head gradients across most of the eastern plain are relatively low (averaging about 17 ft/mi); therefore, ground-water flow is likely to be laminar. Isotropic permeability was initially assumed because rubbly interflow zones, the most transmissive part of basalt flows, appear to be random and discontinuous. However, large-scale fractures in the rift zones crossing the plain (pl. 1*A*) might cause anisotropic conditions over a broad area. Nonhomogeneity of aquifer properties (transmissivity and leakance) was accommodated by zoning the regional ground-water system into areas where a limited range of parameter values was expected. Model-calculated values then represented an average within each zone.

Ground-water flow was assumed to be generally horizontal (two dimensional). This assumption is valid for the central part of the plain, where there is little recharge or discharge to cause vertical flow; however, along the margins of the plain, vertical flow components do exist. Because much of the vertical flow is in sediments overlying the regional aquifer system, only heads from wells completed in the regional system were used in simulations. Therefore, calculated transmissivity values are representative of only the regional aquifer system.

In some parts of the eastern Snake River Plain, head changes significantly with depth. In the Mud Lake area, a complex interlayering of basalt and sediments causes variations in head within the regional ground-water system. Therefore, the assumption of two-dimensional flow is only an approximation of the flow system. Changing heads with depth also were observed in wells in Gooding and Jerome Counties near the major discharge area for the regional ground-water system. However, throughout most of the plain, regional ground-water flow is approximately horizontal.

Ground-water flow in the regional aquifer system was assumed to be near steady state during the 1980 water year. Water levels have been stable during the past 30 years, with rises and declines less than 10 ft across the plain. During water year 1980, storage changes in the regional system were small, as indicated by the small net change in storage (table 6) and the small head changes (pl. 1*D*). For the period 1912–80, annual average ground-water discharge to the Snake River from near Blackfoot to Neeley was stable, ranging from 2,400 to 2,700 $ft^3/s$. North-side ground-water discharge to the Snake River from Milner to King Hill from 1960 to 1980 also was fairly stable; the annual average ranged from 6,000 to 6,500 $ft^3/s$. The stability of water levels and discharge is due to the relatively constant recharge from surface-water diversions, as shown in figure 6. Diversions in 1980 were about the same as the average for the period 1928–80. Effects of transient flow in the shallow alluvial system were assumed to be negligible in the long-term, steady-state calculations of recharge based on 1980 water year data. It also was assumed that recharge moves from the shallow zones downward into the regional system within irrigated-area boundaries (pl. 1*B*).

## Model Calibration

Water levels measured in 824 wells in the spring of 1980 were used in the calibration procedure and were assigned a weighting factor $(WF)_l$ of 1.0, as there was little basis for differentiating among the measurements. Initial zonation of transmissivity was made on the basis of rock type (sediment, basalt) and water-level gradients.

Transmissivities and leakances initially were considered to be regression parameters in the calibration process. In early simulations, unrestrained leakance regression parameters were unstable, resulting in negative parameter values. When leakance and recharge were held constant and transmissivity was calibrated as a regression parameter, the model became stable. Leakance values were adjusted manually from simulation to simulation until model-calculated spring flow aproximated measured spring flow. Some spring-vent altitudes were varied within reasonable ranges to obtain a better comparison between measured and calculated water levels.

Calibration proceeded in a stepwise manner—results of the previous simulation were used as initial values for the next simulation. If an individual zone caused instability and poor simulation results, zones were combined to form a single regression parameter. Transmissivities were adjusted by modifying zone boundaries and allowing the program to compute an optimum fit to water levels. If, within a zone, major differences were noted between model-calculated and measured water levels, the zone was split or rearranged for further model simulations. Zones were added along margins of the plain where transmissivity is reduced because the aquifer is thin and sediments are interlayered with basalt (pl. 1*A*, figs. 3 and 4).

Boundary flux was added as a regression parameter in some areas where the flux was unknown to better simulate the direction of flow. This additional parameter resulted in better comparison of model-calculated discharge with measured aquifer discharge. In the final stages of calibration, transmissivity, leakance, recharge, and boundary flux were made regression parameters. Transmissivity and boundary flux were unrestrained, whereas recharge and leakance were used as prior-information parameters with various values of coefficients of variation.

A measure of overall goodness of fit of model simulations is the error variance ($s^2$) (Cooley, 1977):

$$s^2 = \frac{\sum_{l=1}^{J} (WF)_l (h_l^{obs} - h_l^c)^2}{J - K} \qquad (6)$$

where

$(WF)_l$ = weighting factor,
$h_l^{obs} - h_l^c$ = head residual, the difference between observed and calculated head,
$J$ = number of head observations, and
$K$ = number of regression parameters.

The standard error of estimate for head, defined as the square root of the error variance, was plotted against model-run number (fig. 10). The plot indicates a rapid initial decrease in standard error as more regression parameters were used. However, as more regression parameters were added, there were also more occurrences of nonconvergence and invalid parameter values (negative transmissivities). Figure 10 also indicates a diminishing return in model improvement (decreased standard error) as model runs and changes progressed.

Along with standard error of estimate for head, model-calculated spring discharge was also an important criterion of model fit. Calculated transmissivities were strongly affected by the distribution of flux within the model. Many of the final simulations involved adjustments of leakance to match calculated spring discharges to measured spring discharges.

Contours based on model-calculated heads (run 40) were compared with contours based on March 1980 water-level measurements (pl. 3). This comparison indicates that the flow model reasonably simulates both the major direction of ground-water flow and the magnitude of ground-water levels. The ratio of the standard error of estimate for

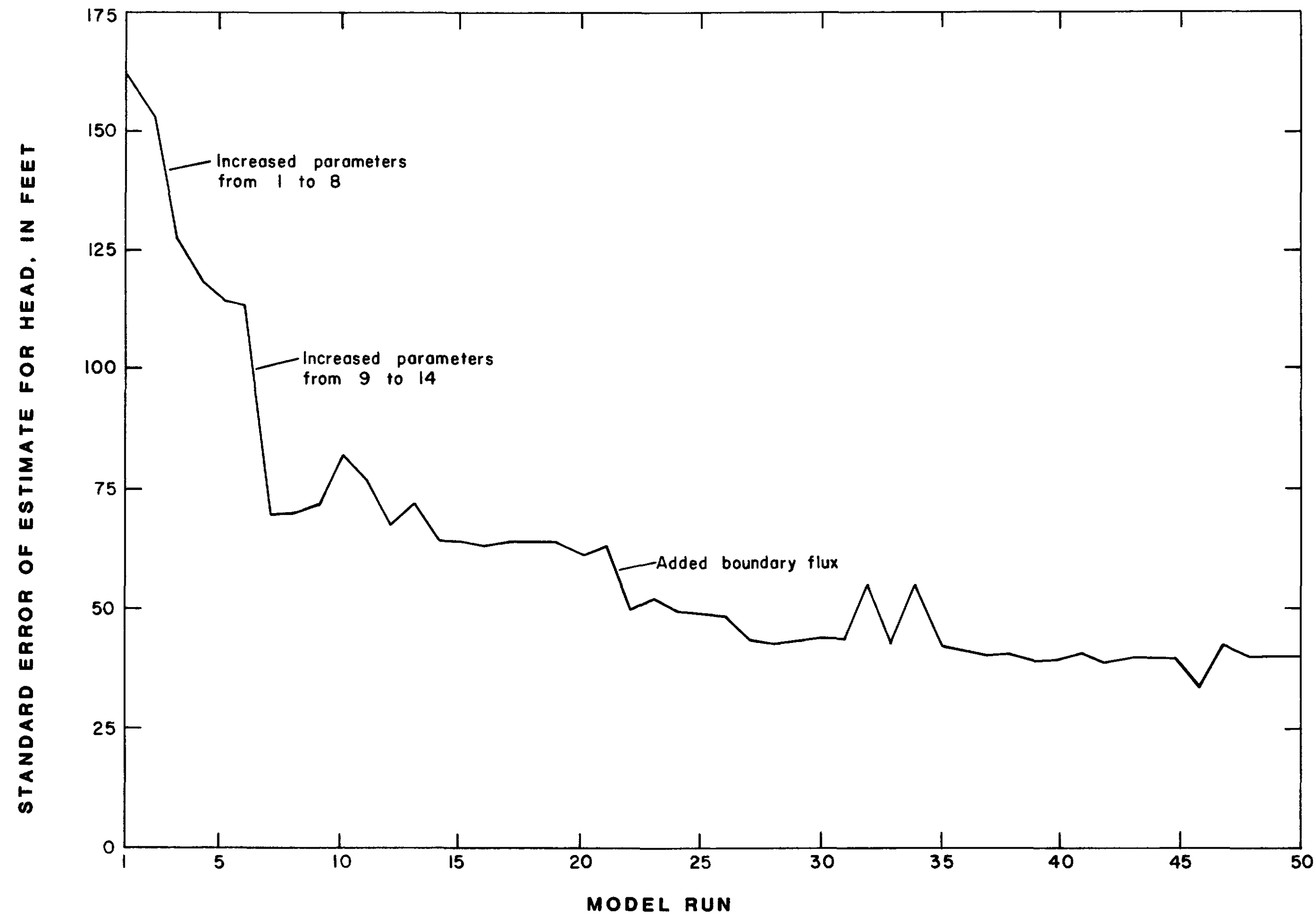


**Figure 10.** Model-run standard errors of estimates.

heads (run 40) to total head loss in the system is $s/\Delta h = 40.4/3{,}619 = 0.011$, which indicates that the errors are only a small part of the overall model response.

To statistically analyze results of the regression model, modeled head residuals were assumed to be random variables with zero mean, to have a constant variance, and to be uncorrelated (Cooley, 1979). It was also assumed that the set of head residuals had a multivariate normal distribution. These assumptions allow the use of statistical tests and measures involving the *F*- and *t*-distributions.

Modeled head residuals generally have a random distribution throughout the central part of the modeled area. However, in several places, particularly along the margins of the plain, large absolute values of head residuals are apparent. They appear where model assumptions were violated or where the model zones were inadequate to describe local variations in aquifer parameters. The steady-state flow assumption was violated in areas southwest of Burley (zones 19 and 20, pl. 3). Owing to continuing declines in water levels, the Idaho Department of Water Resources has declared a moratorium on further development in these areas. Although the water-level-change map (pl. 1*D*) indicates some recovery during water year 1980, these areas probably are not yet in steady-state conditions. Large head residuals also appear in the Mud Lake area (zone 12) owing to the large head changes with depth in this area.

Transmissivity changes in some areas were not simulated adequately owing to the block size used in this study. An example is the river reach from Kimberly to Buhl (zones 1, 2, and 3). In this reach, highly permeable rocks that fill ancestral Snake River canyons cause large transmissivity changes over short distances and changes in flow direction in the immediate vicinity of spring vents. Large changes in aquifer properties over short distances also occur in the Shoshone-Gooding (zones 2 and 18), Mud Lake (zone 12), and Camas Creek (zone 16) headwater areas. Changes in aquifer properties cause poor model fit and large absolute values of residuals in these areas.

## Comparison of Simulations

Regression parameters for seven different model simulations are presented in table 7. These simulations were used to test assumptions and various configurations of the model. There is no best simulation presented; instead, results are compared for indications of model improvement. Statistical

**Table 7.** Regression parameters for simulations 40, 42, 46–50

(Aquifer property and zone of calibration (pl. 2), where T = transmissivity, SL = leakance, QRE = recharge, and BF = boundary flux; the associated numbers are zone numbers)

| Regression parameter No. | Run 40 | Run 42 | Run 46 | Run 47 | Runs 48-50 |
|---|---|---|---|---|---|
| 1 | BF,1,19 | BF,1,19 | T,2,3 | BF,1,19 | BF,1,4,7,11, 13,14,16-20 |
| 2 | T,1,19 | T,x-direction | T,4 | T,1,19 | T,1,19 |
| 3 | T,2,3 | T,y-direction | T,5,10 | T,2,3 | T,2,3 |
| 4 | T,4 | | T,6 | T,4 | T,4 |
| 5 | T,5,10 | | T,7,8 | T,5,10 | T,5,10 |
| 6 | T,6 | | T,9 | T,6 | T,6 |
| 7 | T,7,8 | | T,11,17 | T,7,8 | T,7,8 |
| 8 | T,9 | | T,12 | T,9 | T,9 |
| 9 | T,11,17 | | T,13 | T,11,17 | T,11,17 |
| 10 | T,12 | | T,14,15 | T,12 | T,12 |
| 11 | T,13 | | T,16 | T,13 | T,13 |
| 12 | T,14,15 | | T,18 | T,14,15 | T,14,15 |
| 13 | T,16 | | T,20 | T,16 | T,16 |
| 14 | T,18 | | | T,18 | T,18 |
| 15 | T,20 | | | T,20 | T,20 |
| 16 | | | | | SL,7,17 |
| 17 | | | | | SL,1,2,3, 18,20 |
| 18 | | | | | QRE,2,3,4, 18,20 |
| 19 | | | | | QRE,11,13, 14,17 |
| 20 | | | | | QRE,1,5-10, 12,15,16,19 |

results for these simulations are presented in table 8, measured and model-calculated spring discharges are compared in table 9, calculated transmissivities are shown in table 10, and mass-balance calculations are presented in table 11.

Although discussion of individual simulations follows, the similarity of the results should be noted. In particular, the measured and model-calculated spring discharges show good agreement (table 9), which indicates that the mass-flux distribution was good in this set of simulations. The small variation of results obtained when zonation was held constant, as compared with the larger variation for earlier simulations where zonation was varied, implies that the underlying structure of the model zonation is the single most important influence on modeling results. The zones used in this set of simulations were developed using geologic and hydrologic information and represent a simplification of continuously varying geologic and hydrologic parameters. Within the constraints of the number of head observations, the stability of the model, and the accuracy of mass-flux estimates, the results from the model zonation presented here appear to be good representations of average aquifer conditions.

To investigate the significance of reduction in error variance from one model simulation to another, the approximate probability of occurrence of the ratio of the two error variances using the $F$-distribution was computed. The $F$-distribution assumes statistical independence of the two error variances used to form the ratio, whereas the two error variances used here are probably positively correlated (R. L. Cooley, U.S. Geological Survey, written commun., 1984). This correlation would cause the ratio to be consistently smaller than it would be if the variances were independent. Thus, the actual probability of occurrence is generally less than the probability computed using the $F$-distribution. Further limitations on the exact interpretation of the $F$-statistic are the assumptions of model linearity (the regression model is assumed to be linear in the parameters) and of normal distribution of head residuals.

In run 42, all of the $x$-direction transmissivities were calibrated as a combined regression parameter independent of the combined $y$-direction transmissivities to test for anisotropy in the regional aquifer. Thus, all the ratios between the $x$-direction transmissivities remain the same, as do the $y$-direction transmissivities; only the ratio between the $x$- and $y$-direction transmissivities changes. Transmissivity values determined from run 40 were used as initial values for run 42. The error variance for run 42 was reduced from that in

**Table 8.** Statistical results for simulations 40, 42, 46–50

| Statistic, or number of observations | Simulations 40 | 42 | 46 | 47 | 48 | 49 | 50 |
|---|---|---|---|---|---|---|---|
| Error variance, in feet squared | $1.63 \times 10^3$ | $1.54 \times 10^3$ | $1.16 \times 10^3$ | $1.82 \times 10^3$ | $1.63 \times 10^3$ | $1.62 \times 10^3$ | $1.62 \times 10^3$ |
| Standard error of estimate for heads, in feet | 40.4 | 39.2 | 34.1 | 42.70 | 40.32 | 40.29 | 40.25 |
| Sum of squared errors | $1.32 \times 10^6$ | $1.26 \times 10^6$ | $8.52 \times 10^5$ | $7.51 \times 10^5$ | $1.32 \times 10^6$ | $1.31 \times 10^6$ | $1.31 \times 10^6$ |
| Correlation between observed and calculated heads | 0.9964 | 0.9966 | 0.9976 | 0.9961 | 0.9964 | 0.9964 | 0.9965 |
| Mean head residual, in feet | -1.19 | -1.61 | -1.17 | -2.84 | -0.81 | -0.52 | -0.42 |
| Mean of absolute value of head residuals, in feet | 28.87 | 28.18 | 24.97 | 28.60 | 28.79 | 28.74 | 28.68 |
| Number of observations | 824 | 824 | [1]746 | [1]427 | 824 | 824 | 824 |

[1]Some data were removed; see discussions presented in text.

run 40 (table 8); the variance ratio was then computed to check the significance of the reduction:

$$\frac{\text{error variance for run 40 (1,630)}}{\text{error variance for run 42 (1,540)}} = 1.06$$

and Prob $[F(J-K,J-M)>1.06] \simeq 0.21$

where

$F(J-K,J-M)$ = value of the $F$-distribution with $J-K$ and $J-M$ degrees of freedom,
$J$ = number of head observations (824),
$K$ = number of regression parameters in run 40 (15), and
$M$ = number of regression parameters in run 42 (3).

Because the computed probability on the basis of the $F$-distribution is 0.21, the actual probability of occurrence of the value 1.06 is probably less than 0.21. Hence, the reduction of variance may be significant at about the 0.05 level, but further evidence is needed.

Run 42 results indicate an $x$-direction transmissivity to $y$-direction transmissivity ratio ($T_x/T_y$) of about 0.80 (table 10), which suggests that the medium is not highly anisotropic. However, the standard errors for $T_x$ and $T_y$ are small enough to suggest that $T_x$ and $T_y$ might be significantly different. From table 10, it can be seen that $T_x + 3 \cdot$ (standard error of $T_x$) $< T_y - 3 \cdot$ (standard error of $T_y$). Hence, with approximately 99 percent confidence, $T_x$ and $T_y$ are different (see Graybill, 1976, p. 360), even though they are not greatly

**Table 9.** Comparison of model-calculated and measured spring discharges

| Snake River reach/location | Measured discharge (cubic feet per second) | Model-calculated discharge (cubic feet per second) Run 40 | Run 42 | Run 46 | Run 47 | Run 48 | Run 49 | Run 50 | Model leakance, in foot per second per foot (standard error x $10^{-9}$) Runs 40,42,46,47 | Run 48 | Run 49 | Run 50 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Near Blackfoot to Neeley | 2,635 | 2,550 | 2,686 | 2,497 | 2,338 | 2,609 | 2,685 | 2,800 | 27.6 | 27.8 (1.4) | 28.7 (2.6) | 31.4 (5.0) |
| Neeley to Minidoka | 453 | 419 | 395 | 409 | 367 | 425 | 436 | 452 | 5.0 | 5.0 (0.2) | 5.2 (0.5) | 5.7 (0.9) |
| Milner to Kimberly | 297 | 318 | 242 | 160 | 279 | 306 | 292 | 271 | 6.0 | 5.8 (0.2) | 5.5 (0.4) | 5.1 (0.6) |
| Kimberly to Buhl | 1,223 | 1,232 | 1,170 | 1,055 | 1,219 | 1,202 | 1,160 | 1,080 | 6.0 | 5.8 (0.2) | 5.5 (0.4) | 5.1 (0.6) |
| Buhl to Hagerman | 3,680 | 4,081 | 4,049 | 3,773 | 4,326 | 4,122 | 4,054 | 3,750 | 60.0 | 57.8 (2.4) | 55.2 (4.4) | 51.0 (6.6) |
| Hagerman to King Hill | 1,472 | 1,210 | 1,290 | 1,166 | 1,269 | 1,223 | 1,207 | 1,130 | 2.2 | 2.1 (0.1) | 2.0 (0.1) | 1.9 (0.2) |
| Total | 9,760 | 9,810 | 9,832 | 9,060 | 9,798 | 9,887 | 9,834 | 9,483 | | | | |

**Table 10.** Transmissivity values for simulations 40, 42, 46–50

(T, transmissivity; x, x-direction; y, y-direction; ---, no data available; values in feet squared per second)

| Zone No. | Run 40 | | Run 42 | | | | Run 46 | | Run 47 | | Run 48 | | Run 49 | | Run 50 | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | T | Standard error | $T_x$ | Standard error | $T_y$ | Standard error | T | Standard error | T | Standard error | T | Standard error | T | Standard error | T | Standard error |
| 1 | 0.159 | 0.0064 | 0.151 | 0.0017 | 0.188 | 0.0055 | --- | --- | 0.152 | 0.00953 | 0.163 | 0.0105 | 0.163 | 0.0168 | 0.156 | 0.0272 |
| 2 | 11.7 | .577 | 11.1 | .124 | 13.8 | .408 | 11.0 | 0.466 | 12.7 | .840 | 11.9 | .765 | 11.7 | 1.05 | 10.7 | 1.56 |
| 3 | .064 | .0031 | .061 | .00068 | .075 | .0022 | .06 | .0025 | .0693 | .00458 | .0648 | .00417 | .0638 | .00574 | .0584 | .00852 |
| 4 | 9.11 | .676 | 8.64 | .097 | 10.7 | .317 | 12.7 | .96 | 10.2 | 1.15 | 8.95 | .785 | 8.48 | 1.02 | 7.27 | 1.39 |
| 5 | .516 | .059 | .489 | .0055 | .608 | .018 | .481 | .043 | .520 | .154 | .522 | .0658 | .524 | .0788 | .508 | .105 |
| 6 | 27.2 | 7.60 | 25.8 | .289 | 32.1 | .948 | 27.2 | 5.32 | 27.2 | 9.64 | 27.7 | 8.00 | 27.0 | 8.30 | 24.2 | 8.59 |
| 7 | 6.19 | .930 | 5.87 | .066 | 7.30 | .216 | 4.97 | .802 | 6.35 | 1.30 | 6.06 | .995 | 5.66 | 1.12 | 4.72 | 1.29 |
| 8 | 12.4 | 1.86 | 11.7 | .132 | 14.6 | .431 | 9.95 | 1.60 | 12.7 | 2.60 | 12.1 | 1.99 | 11.3 | 2.24 | 9.45 | 2.59 |
| 9 | .833 | .316 | .790 | .0089 | .982 | .029 | 1.40 | .376 | 1.01 | .425 | .829 | .317 | .796 | .310 | .693 | .291 |
| 10 | 1.03 | .119 | .978 | .011 | 1.22 | .036 | .963 | .086 | 1.04 | .308 | 1.04 | .132 | 1.05 | .158 | 1.02 | .210 |
| 11 | 43.9 | 2.42 | 41.6 | .467 | 51.8 | 1.53 | 41.0 | 2.06 | 40.7 | 3.10 | 44.3 | 3.08 | 43.7 | 4.07 | 41.0 | 5.91 |
| 12 | .672 | .092 | .638 | .0071 | .793 | .023 | .758 | .084 | .964 | .235 | .676 | .0952 | .669 | .101 | .632 | .117 |
| 13 | 7.92 | .822 | 7.50 | .084 | 9.33 | .276 | 8.17 | .719 | 6.37 | 1.06 | 7.94 | .886 | 7.79 | 1.01 | 7.34 | 1.28 |
| 14 | 9.15 | .974 | 8.67 | .097 | 10.8 | .319 | 9.39 | .85 | 9.28 | 1.90 | 9.21 | 1.03 | 9.10 | 1.13 | 8.60 | 1.37 |
| 15 | 13.2 | 1.41 | 12.5 | .141 | 15.6 | .460 | 13.6 | 1.23 | 13.4 | 2.75 | 13.3 | 1.48 | 13.2 | 1.63 | 12.4 | 1.98 |
| 16 | .147 | .0036 | .139 | .0016 | .173 | .0051 | .146 | .003 | 7.31 | 26.1 | .148 | .00799 | .148 | .0139 | .141 | .0234 |
| 17 | 43.9 | 2.42 | 41.6 | .467 | 51.8 | 1.53 | 41.0 | 2.06 | 40.7 | 3.10 | 44.3 | 3.08 | 43.7 | 4.07 | 41.0 | 5.91 |
| 18 | .170 | .009 | .162 | .0018 | .201 | .0059 | .177 | .0083 | .146 | .0101 | .175 | .0121 | .180 | .0168 | .184 | .027 |
| 19 | .050 | .002 | .047 | .00053 | .059 | .0017 | --- | --- | .0475 | .00298 | .0508 | .00329 | .0509 | .00523 | .0487 | .00849 |
| 20 | 12.8 | 1.23 | 12.2 | .136 | 15.1 | .447 | 8.02 | 1.05 | 12.2 | 1.78 | 12.8 | 1.33 | 12.4 | 1.51 | 11.4 | 1.93 |

**Table 11.** Mass-balance calculations[1] for simulations 40, 42, 46–50

(---, no data available; values in cubic feet per second)

| Run No. | Recharge: Value [2] | Recharge: Standard error | Spring discharge: Value | Spring discharge: Standard error | Additional boundary flux: Constant flux | Additional boundary flux: Value | Additional boundary flux: Standard error |
|---|---|---|---|---|---|---|---|
| 40 | 9,294 | --- | 9,810 | --- | 364 | 152 | 7.9 |
| 42 | 9,294 | --- | 9,832 | --- | 364 | 174 | 6.6 |
| 46 | 8,713 | --- | 9,060 | --- | 347 | --- | --- |
| 47 | 9,294 | --- | 9,798 | --- | 364 | 140 | 10.4 |
| 48 | 9,359 | 440 | 9,887 | 430 | --- | 528 | 37.5 |
| 49 | 9,306 | 840 | 9,834 | 774 | --- | 528 | 55.6 |
| 50 | 8,978 | 1,490 | 9,483 | 1,280 | --- | 505 | 87.7 |

[1] Mass-balance equation: recharge + boundary flux - spring discharge = 0
[2] Example of recharge calculation:

| | Acre-feet per year | |
|---|---|---|
| Surface-water irrigation | 5,095,500 | table 6 |
| Snake River loss | 880,500 | table 6 |
| Tributary-stream and canal losses | 491,800 | table 6 |
| Tributary-valley underflow | 1,226,700 | table 6 |
| Precipitation | 763,200 | table 6 |
| Ground-water pumpage | -1,641,300 | table 6 |
| Snake River gain (Lorenzo to Lewisville) | -208,900 | table 2 |
| Change in storage | 127,300 | table 6 |
| Total | 6,734,800, or 9,302 cubic feet per second | |

different numerically. This result is supported by geologic evidence shown in plate 1*A*. Fractures and faults in rift zones, parallel to the *y*-direction, may increase hydraulic conductivity along the trace of the fracture or fault, thereby increasing the *y*-direction transmissivity. However, the results of the statistical tests can only be interpreted as preliminary indications of anisotropy in the regional system, as there was little reduction in error variance, and calculated values of *x*-direction and *y*-direction transmissivities are similar.

In run 46, zones 1 and 19 and parts of zones 4 and 20, corresponding to the area south of the Snake River from Twin Falls to Burley, were removed from the system (table 7). Model run 40 had several large absolute values of head residuals within zones 1 and 19 that resulted from violation of model assumptions (two-dimensional and steady-state flow). By removing those zones from the model, the error variance was reduced greatly compared with run 40 (table 8):

$$\frac{1{,}630 \text{ (run 40)}}{1{,}160 \text{ (run 46)}} = 1.41$$

and Prob $[F(809{,}811) > 1.41] \simeq 2 \times 10^{-9}$.

Using a range of $\pm 2$ standard errors as an indication of roughly 95-percent confidence (similar to the comparison of $T_x$ and $T_y$ previously), values of transmissivity in zones 4 and 20 in run 46 are significantly different from those in run 40. Except for zone 4, all the standard errors for transmissivity are lower in run 46. This simulation indicates the importance of removing zones that violate model assumptions.

Run 47 is a test of the model, using 427 head observations measured in August 1980. Head differences occur in many wells between spring and late summer, owing to seasonal application of irrigation water and pumping (pl. 4). Water levels in pumping areas are typically lowest in August, whereas in areas of applied surface water, they are typically highest in late summer and early fall (pl. 4). Seasonal variations in many wells in irrigated areas are greater than the long-term trend for the period of record (pl. 4). The error variance for run 47 was slightly greater than for run 40 (table 8):

$$\frac{1{,}820 \text{ (run 47)}}{1{,}630 \text{ (run 40)}} = 1.12$$

and Prob $[F(412{,}809) > 1.12] \simeq 0.085$.

However, because two separate data sets were used for run 40 and run 47, the two variances used to compute the ratio may be nearly independent, and so the ratio may not be biased. Hence, for a level of significance of 0.05, the variance for run 47 may not be significantly different from the variance computed for run 40. Furthermore, transmissivity values determined in runs 40 and 47 (table 10) are within the $\pm 2$ standard-error range of each other. Comparison of the results based on run 47 with those of run 40 indicates both the usefulness of the steady-state approach in estimating aquifer properties and the stability of model results when using a different set of head observations.

In runs 48, 49, and 50, all model input, including transmissivity, leakance, recharge, and boundary flux, were used as regression parameters (table 7). Transmissivity and boundary flux were allowed to vary unrestrained, whereas recharge and leakance were restrained by available information. Recharge zones were grouped into major areas of agricultural lands (pl. 1*B*) and leakance into the two major spring areas, American Falls and Milner Dam to King Hill. The values used for leakance and recharge in runs 48, 49, and 50 were the initial values for run 40; the coefficients of variation for both leakance and recharge were set at 0.05, 0.10, and 0.20 for runs 48, 49, and 50, respectively.

The error variances and most other statistics given in table 8 for runs 48, 49, and 50 are essentially the same as for run 40. All parameter values—transmissivity (table 10), leakance (table 9), recharge (table 11), and boundary flux (table 11)—are within the ±2 standard-error range of values in run 40. Standard errors for all parameters increased from run 48 to run 50, owing to the increased uncertainty in the parameter values. These simulations indicate the stability of the model to increasing uncertainty in model recharge and leakance. Because of uncertainty in recharge and leakance values, the standard errors for all parameters are more realistic in simulations 48–50 than in run 40. Although the level of uncertainty is difficult to estimate, it is not likely to exceed 0.20. If uncertainty is 0.30, for example, calculated ground-water discharges are not within an acceptable error range, which is about 10 percent of measured spring discharges. Therefore, a reasonable range of parameters, within the model zones used in this study, is ±2 times the standard errors given in run 50 results.

The scaled sensitivity (*SW*) for regression parameter *m* is defined as:

$$SW = \frac{\partial h_l}{\partial a_m} \cdot a_m \qquad (7)$$

where

$h_l$ = head at location $l$, and

$a_m$ = regression parameter value.

Average scaled sensitivities for run 49 for each regression parameter in each model zone are presented in table 12.

**Table 12.** Scaled sensitivities for simulation 49

(T, transmissivity; SL, leakance, QRE, recharge; and BF, boundary flux)

| Regression parameter No. | Aquifer property and zone of calibration (pl. 3) | Average scaled sensitivity within each aquifer zone (pl. 3): 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | BF,1,4,7,11,13,14,16-20 | 54.31 | 5.707 | 2.599 | 10.01 | 10.30 | 8.478 | 6.108 | 6.377 | 6.383 | 7.103 |
| 2 | T,1,19 | -290.5 | .5782 | -2.466 | -.1447 | -.09394 | -.1178 | -.09421 | -.1046 | -.06723 | -.02794 |
| 3 | T,2,3 | -55.69 | -105.9 | -2.365 | -151.1 | -119.0 | -110.0 | -52.57 | -64.75 | -62.75 | -24.97 |
| 4 | T,4 | 15.84 | 45.64 | 12.78 | -14.90 | -84.69 | -79.36 | -36.66 | -45.22 | -45.31 | -17.98 |
| 5 | T,5,10 | .2226 | .4121 | .1264 | 1.327 | -112.1 | 2.153 | .6496 | .8604 | 2.189 | -83.50 |
| 6 | T,6 | 4.522 | 9.586 | 2.858 | 25.70 | 15.50 | 4.744 | -1.823 | -5.538 | -14.80 | -5.918 |
| 7 | T,7,8 | 12.96 | 24.48 | 7.471 | 73.27 | 76.95 | 82.57 | 42.61 | 51.97 | 36.97 | 3.082 |
| 8 | T,9 | 5.318 | 10.67 | 3.216 | 30.22 | 45.78 | 46.05 | 13.01 | 16.70 | 13.21 | -29.07 |
| 9 | T,11,17 | 1.201 | 2.237 | .6847 | 6.768 | 5.536 | 6.916 | 15.75 | 17.45 | -10.93 | -75.57 |
| 10 | T,12 | .02904 | .05757 | .01739 | .1648 | .2474 | .2368 | .1164 | .1558 | .6066 | -.1565 |
| 11 | T,13 | -.02698 | -.05352 | -.01617 | -.1532 | -.2306 | -.2203 | -.1072 | -.1435 | -.5673 | -16.40 |
| 12 | T,14,15 | -.005157 | -.01022 | -.003088 | -.02927 | -.04381 | -.04199 | -.02084 | -.02790 | -.1070 | -3.066 |
| 13 | T,16 | .000020 | .000040 | .000012 | .000114 | .000171 | .000163 | .0000791 | .000106 | .000422 | .01678 |
| 14 | T,18 | -.4857 | -.8245 | -.2926 | -3.004 | -16.84 | -2.987 | -1.172 | -1.495 | -1.749 | -.6918 |
| 15 | T,20 | 10.41 | 16.16 | 7.463 | -41.75 | -30.54 | -33.04 | -21.58 | -24.88 | -18.84 | -7.681 |
| 16 | SL,7,17 | -2.671 | -5.145 | -1.564 | -15.12 | -18.08 | -18.84 | -26.65 | -23.53 | -22.62 | -25.09 |
| 17 | SL,1,2,3,18,20 | -53.41 | -78.48 | -62.44 | -45.00 | -35.42 | -32.99 | -16.11 | -19.75 | -18.82 | -7.500 |
| 18 | QRE,2,3,4,18,20 | 25.05 | 52.49 | 17.27 | 65.74 | 53.82 | 49.30 | 26.79 | 31.93 | 28.18 | 11.32 |
| 19 | QRE,11,13,14,17 | 3.739 | 7.291 | 2.210 | 21.18 | 27.77 | 28.08 | 24.27 | 29.18 | 49.54 | 113.3 |
| 20 | QRE,1,5-10,12,15,16,19 | 269.2 | 15.07 | 12.45 | 36.80 | 181.1 | 49.04 | 27.50 | 30.81 | 59.49 | 162.8 |

| Regression parameter No. | Aquifer property and zone of calibration (pl. 3) | Average scaled sensitivity within each aquifer zone (pl. 3): 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | BF,1,4,7,11,13,14,16-20 | 4.834 | 12.47 | 9.525 | 19.03 | 17.02 | 24.95 | 2.602 | 55.54 | 272.6 | 11.36 |
| 2 | T,1,19 | -.01746 | -.01661 | -.01608 | -.1625 | -.01640 | -.01637 | -.01257 | .1864 | -164.8 | -.3708 |
| 3 | T,2,3 | -13.70 | -13.21 | -12.73 | -12.89 | -13.02 | -12.99 | -9.358 | -103.9 | -137.2 | -162.8 |
| 4 | T,4 | -9.762 | -9.427 | -9.079 | -9.191 | -9.291 | -9.269 | -6.639 | 7.659 | 31.14 | 40.89 |
| 5 | T,5,10 | -.4346 | -18.85 | -3.042 | -8.694 | -12.91 | -11.93 | -.1479 | -6.971 | .9661 | .9960 |
| 6 | T,6 | -2.999 | -2.968 | -2.835 | -2.878 | -2.916 | -2.907 | -1.837 | 15.34 | 15.72 | 17.69 |
| 7 | T,7,8 | -8.361 | -6.876 | -7.010 | -6.967 | -6.929 | -6.937 | -9.051 | 33.96 | 59.61 | 59.19 |
| 8 | T,9 | -9.781 | -10.70 | -9.897 | -10.15 | -10.38 | -10.33 | -3.296 | 16.92 | 21.62 | 22.59 |
| 9 | T,11,17 | -59.37 | -110.7 | -115.5 | -114.4 | -112.9 | -113.2 | -2.812 | 2.951 | 5.636 | 5.560 |
| 10 | T,12 | .8190 | -56.77 | -19.36 | -67.73 | -90.16 | -85.43 | -.05117 | .08912 | .1212 | .1253 |
| 11 | T,13 | -.7040 | -79.49 | -65.73 | -138.2 | -119.0 | -123.4 | .04638 | -.08291 | -.1126 | -.1164 |
| 12 | T,14,15 | -.1405 | -18.31 | .4551 | -84.95 | -70.99 | -117.0 | .009417 | -.01581 | -.02154 | -.02226 |
| 13 | T,16 | .000503 | .05498 | -.008902 | -.08296 | .04393 | -669.6 | -.0000339 | .0000615 | .000083 | .000086 |
| 14 | T,18 | -.3531 | -.3430 | -.3297 | -.3339 | -.3377 | -.3369 | -.2349 | -118.9 | -1.877 | -2.035 |
| 15 | T,20 | -4.546 | -4.349 | -4.202 | -4.249 | -4.291 | -4.282 | -3.206 | -4.213 | -58.88 | -42.30 |
| 16 | SL,7,17 | -26.03 | -25.81 | -25.78 | -25.79 | -25.80 | -25.80 | -26.59 | -7.446 | -11.82 | -11.92 |
| 17 | SL,1,2,3,18,20 | -4.135 | -3.986 | -3.842 | -3.888 | -3.930 | -3.921 | -2.831 | -65.54 | -49.15 | -54.13 |
| 18 | QRE,2,3,4,18,20 | 6.363 | 6.120 | 5.902 | 5.972 | 6.035 | 6.021 | 4.396 | 130.8 | 68.73 | 71.53 |
| 19 | QRE,11,13,14,17 | 86.93 | 222.3 | 190.6 | 323.0 | 294.3 | 311.9 | 41.33 | 10.85 | 16.13 | 16.45 |
| 20 | QRE,1,5-10,12,15,16,19 | 41.39 | 121.0 | 72.93 | 142.4 | 165.5 | 854.5 | 17.68 | 32.71 | -68.40 | 27.37 |

Absolute values of scaled sensitivity indicate the contribution of the regression parameter to model response (for example, calculated head). Large absolute values of scaled sensitivity indicate a large influence on calculated head when the parameter value is changed. Largest sensitivity values in each zone are footnoted in table 12. For example, model response in zone 1 is most sensitive to changes in transmissivity and recharge. Although the model generally is sensitive to changes in transmissivity, calculated heads in some zones are most sensitive to the transmissivity of adjacent zones. For example, the largest absolute value of scaled sensitivity in zones 4, 6, 7, 8, and 9 is related to transmissivity in zones 2 and 3 (table 12). Transmissivity and recharge are strong influences along the margins of the plain (zones 1, 5, 10, 16, and 18). Spring discharges are sensitive to leakance values; this information aided estimation of leakance. Constant flux, represented by the recharge parameter, was a strong influence everywhere except in the central and south-central parts of the plain (zones 6, 7, and 8), where most lands are undeveloped. Overall, the model was most sensitive to recharge and, in some areas, transmissivity.

Comparison and analysis of regression results are based on the assumption of linearity with respect to aquifer parameters. The technique developed by Cooley (1977, 1979) is nonlinear in transmissivity and leakance; therefore, statistical results are approximations. Cooley (1979) presented a measure of linearity ($Na$) derived by Beale (1960). This measure is based on the ratio between results of the nonlinear flow equation (2) and those predicted by use of the calculated sensitivities (linearized model). Models are considered definitely nonlinear if

$$Na > 1/F(K,J-K,\alpha) \qquad (8)$$

where

$F(K,J-K,\alpha)$ = the upper $\alpha$-percent point of the $F$-distribution with $K$ and $J-K$ degrees of freedom,

$K$ = number of regression parameters, and

$J$ = number of head observations.

If $Na$ is less than $0.01/F(K,J-K,\alpha)$, then the model is effectively linear. For run 40, $Na = 0.00978$, where $0.01/F(14,\ 810,\ 0.05) = 0.00585$, which indicates the model is nearly effectively linear, and the bias caused by nonlinearity is a minor component of the statistical results.

## Comparison With Previous Studies

Table 13 illustrates the comparison of transmissivity values obtained from run 40 with those obtained by Mundorff and others (1964), Norvitch and others (1969), and Newton (1978). Mundorff and others calculated transmissivity by using a flow-net analysis; Norvitch and others, by using analog modeling; and Newton, by using a digital numerical model. Transmissivity values from previous studies are the estimated averages in the zones used in the present study. Results of run 40 are similar to results of previous studies in most zones of the central plain area (zones 2, 4, 6, 8, 11–15, 17, and 20). Zone 9 in the central plain area has a much lower value in the present study, probably owing to the better head control now available in this area. Along the margins of the plain (zones 3, 5, 10, and 18), transmissivity values determined in the present study are also much lower than those determined in previous studies, owing to better head data.

**Table 13.** Comparison of transmissivity values with those of previous studies

(values in feet squared per second; --, no data available)

| Zone No. | Mundorff and others (1964, pl. 6) | Norvitch and others (1969, p. 37) | Newton (1978, table 16) | This study (run 40) |
|---|---|---|---|---|
| 1 | -- | -- | -- | 0.2 |
| 2 | 8 | 11 | 8 | 12 |
| 3 | 5 | 3 | 8 | .06 |
| 4 | 15 | 15 | 10 | 9 |
| 5 | 15 | 11 | 30 | .5 |
| 6 | 30 | 20 | 35 | 27 |
| 7 | -- | 3 | <.3 | 6 |
| 8 | 11 | 8 | 2 | 12 |
| 9 | 8 | 8 | 3 | .8 |
| 10 | >30 | 15 | 6 | 1 |
| 11 | 30 | 50 | 35 | 44 |
| 12 | 1 | <8 | 4 | .7 |
| 13 | 5 | 2 | 25 | 8 |
| 14 | 15 | 8 | 3 | 9 |
| 15 | >30 | 30 | 9 | 13 |
| 16 | -- | -- | -- | .1 |
| 17 | >30 | 15 | 10 | 44 |
| 18 | 8 | 3 | 10 | .2 |
| 19 | -- | -- | -- | .05 |
| 20 | 8 | 8 | 8 | 13 |

## SUMMARY AND CONCLUSIONS

Results of steady-state numerical simulations indicated a wide range in transmissivity and leakance values for the regional aquifer system underlying the eastern Snake River Plain. Model-calculated transmissivity values for the central part of the plain were similar to those reported in previous studies. In several locations, primarily along the margins, model fit was poor, owing primarily to a violation of one or more model assumptions. The regression model is approximately linear, and calculated standard errors of parameters may be used to calculate reasonable ranges of model parameters. Parameter values and ranges can be used as initial estimates for further modeling efforts. Using a numerical simulation and parameter-estimation technique for ground-water flow studies yields transmissivity values similar to those estimated by other methods. The advantage of the parameter estimation technique is that it has a sound statistical basis and yields values for a complex set of parameters other than transmissivity (for example, recharge rates and leakance).

Alternative models were tested to observe the effects on overall model fit. Model results improved when more zones were added in areas of poor model fit. Zonation is the single most important influence on modeling results. As the number of regression parameters increased, however, there was an increasing tendency for the model to not converge to a solution or to generate negative values of transmissivity and leakance. Model results also improved when boundary fluxes were increased in areas that initially were modeled as having small fluxes. There was a significant improvement in model results when zones 1 and 19 and parts of zones 4 and 20 (Twin Falls to Burley) were removed. Model fit in these zones was poor, owing to violation of model assumptions of two-dimensional, steady-state flow. Model analysis indicated that results improved slightly when the *y*-direction transmissivity was simulated as being greater than the *x*-direction transmissivity.

Use of a different set of head observations demonstrated the stability of model results and the usefulness of the steady-state approach. Although increased levels of uncertainty for recharge and leakance did not improve overall model fit, they did increase the standard errors of calculated parameters, making the standard errors more reasonable estimates of the parameter variability. The model was most sensitive to changes in recharge and, in some areas, transmissivity, particularly near the spring discharge area from Milner Dam to King Hill.

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## METRIC CONVERSION FACTORS

For the convenience of those who prefer to use International System (SI) units rather than inch-pound units, conversion factors for items used in this report are listed below.

| Multiply inch-pound unit | By | To obtain SI unit |
|---|---|---|
| acre | 4,047 | square meter ($m^2$) |
| acre-foot (acre-ft) | 1,233 | cubic meter ($m^3$) |
| cubic foot per second ($ft^3/s$) | 0.02832 | cubic meter per second ($m^3/s$) |
| foot (ft) | 0.3048 | meter (m) |
| foot squared per second ($ft^2/s$) | 0.0929 | meter squared per second ($m^2/s$) |
| inch (in.) | 25.40 | millimeter (mm) |
| mile (mi) | 1.609 | kilometer (km) |
| square mile ($mi^2$) | 2.590 | square kilometer ($km^2$) |

## APPENDIX A.—DIVERSION AND RETURN-FLOW DATA FOR WATER YEAR 1980

This appendix lists 1980 water year diversion and return-flow data and data sources for surface-water-irrigated areas on the eastern Snake River Plain. Areas shown in figure 8 include surface-water-irrigated lands where diversion records are available. Sources of data are the following:

a. Idaho Department of Water Resources (1980)
b. U.S. Geological Survey (1980)
c. U.S. Bureau of Reclamation, written commun., 1981
d. Water Districts 37, 37M (1980)
e. American Falls District No. 2, written commun., 1981
f. Wytzes (1980)
g. Kjelstrom (1984)
h. Idaho Department of Water Resources, written commun., 1981

The data-source identifier (a–h) is used as a prefix in the following tables for the irrigation areas.

*Irrigation Area 1.—Diversions from Falls River*

| Name | | Quantity (acre-ft) |
|---|---|---|
| Marysville Canal | a | 32,900 |
| Farmers Own Canal | a | 14,900 |
| Yellowstone Canal | a | 2,900 |
| Orme Canal | a | 800 |
| Squirrel Creek | a | 1,700 |
| Boom Creek | a | 800 |
| Conant Creek | a | 6,000 |
| Total | | 60,000 |
| Estimated surface-return flows | = | 21,200 |
| Diversions minus surface return | = | 38,800 |

*Irrigation Area 2.—Diversions from Henrys Fork, Falls River, and Teton River*

| Name | | Quantity (acre-ft) |
|---|---|---|
| Silkey | a | 5,000 |
| McBee | a | 500 |
| Stewart | a | 3,000 |
| Pioneer | a | 1,600 |
| Wilford | a | 52,200 |
| Salem Union | a | 60,600 |
| Farmers Friend | a | 33,500 |
| Twin Groves | a | 41,100 |
| Roxana | a | 4,400 |
| North Salem | a | 1,900 |
| Pincock Byington | a | 4,200 |
| Consolidated Farmers | a | 84,300 |
| Cross Cut | a | 39,700 |
| Pumps | a | 5,400 |
| Total | | 337,400 |
| Estimated surface-return flows | = | 111,900 |
| Diversions minus surface return | = | 225,500 |

*Irrigation Area 3.—Diversions from Henrys Fork*

| Name | | Quantity (acre-ft) |
|---|---|---|
| St. Anthony Union | a | 165,100 |
| Last Chance | a | 30,800 |
| Dewey | a | 5,100 |
| Independent | a | 90,700 |
| St. Anthony Union Feeder | a | 38,300 |
| Egin | a | 112,100 |
| Total | | 442,100 |
| Estimated surface-return flows | = | 63,000 |
| Diversions minus surface return | = | 379,100 |

*Irrigation Area 4.—Diversions from Falls River and Henrys Fork*

| Name | | Quantity (acre-ft) |
|---|---|---|
| Curr | a | 14,500 |
| Chester | a | 19,000 |
| Falls River | a | 55,300 |
| Enterprise | a | 20,300 |
| Teton Irrigation | a | 24,500 |
| Saurey-Somers | a | 4,600 |
| Island Ward | a | 7,500 |
| Teton Island Feeder | a | 92,300 |
| Pincock-Gardner | a | 1,300 |
| Rexburg City | a | 5,000 |
| Rexburg Irrigation | a | 52,400 |
| Woodmansee-Johnson | a | 5,400 |
| Siddoway | a | 1,200 |
| McCormick-Rowe | a | 400 |
| Bigler Slough | a | 800 |
| Pumps | a | 400 |
| Total | | 304,900 |
| Estimated surface-return flows | = | 129,600 |
| Diversions minus surface return | = | 175,300 |

Surface-return flows for irrigation areas 1–4 were estimated using data reported by Wytzes (1980) for the 1977 water year. Surface-return flows were adjusted for the 1980 water year by assuming that the total streamflow depletion for irrigation areas 1–4 was equal to the sum of the depletions within the areas, as expressed in the following equation:

$$\text{basin inflow} - \text{basin outflow} = \Sigma\ (\text{diversions minus surface returns}).$$

Therefore, if basin inflow, outflow, and diversions are known, the sum of all returns can be calculated. Knowing the total of all returns, returns reported by Wytzes (1980) were adjusted by a common multiplier to equal the estimated total. Basin inflows (in acre-ft) for water year 1980 were

| | | |
|---|---|---|
| Henrys Fork at Ashton | g | 1,102,400 |
| Falls River at Squirrel | g | 550,400 |
| Marysville Canal | g | 32,900 |
| Yellowstone Canal | g | 2,900 |
| Conant Creek | g | 61,900 |
| Teton River near St. Anthony | g | 559,300 |
| Moody Creek | g | 10,800 |
| Total | | 2,320,600 |

Basin outflows (in acre-ft) for water year 1980 were:

| | | |
|---|---|---|
| Henrys Fork near Rexburg | g | 1,491,900 |
| Rexburg Canal drain | g | 10,100 |
| Total | | 1,502,000 |

Total diversions for areas 1–4 were 1,144,400 acre-ft. Total returns (in acre-ft) for areas 1–4 were:

| Inflow | | Outflow | | Diversions | | Surface returns |
|---|---|---|---|---|---|---|
| 2,320,600 | − | 1,502,000 | − | 1,144,400 | = | −325,800 |

Surface-return flows (in acre-ft) estimated from data reported by Wytzes (1980) were

| | |
|---|---|
| Area 1 | 6,000 |
| Area 2 | 31,600 |
| Area 3 | 17,800 |
| Area 4 | 36,600 |
| Total | 92,000 |

The common multiplier is calculated as 325,800/92,000 = 3.54, and the estimated surface-return flows (in acre-ft) are

| | |
|---|---|
| Area 1 | 21,200 |
| Area 2 | 111,900 |
| Area 3 | 63,000 |
| Area 4 | 129,600 |

*Irrigation Area 5.—Right-bank diversions from the Snake River from Heise to Lorenzo*

| *Name* | | *Quantity (acre-ft)* |
|---|---|---|
| Hill-Pettinger | a | 900 |
| Nelson-Corey | a | 1,700 |
| Sunnydell | a | 47,400 |
| Lenroot | a | 41,000 |
| Reid | a | 58,500 |
| Texas + Liberty Park | a | 79,100 |
| Bannock Jim | a | 5,200 |
| Total | | 233,800 |
| *Surface-return flows:* | | |
| Texas Canal drain | g | 19,100 |
| Texas Slough | g | 77,200 |
| Bannock Jim Slough | g | 8,800 |
| Total | | 105,100 |
| Diversions minus surface return | | 128,700 |

*Irrigation Area 6.—Left-bank diversions from the Snake River from Heise to Lorenzo*

| *Name* | | *Quantity (acre-ft)* |
|---|---|---|
| Riley | g | 5,100 |
| Anderson | g | 93,400 |
| Eagle Rock | g | 135,400 |
| Farmers Friend | g | 112,900 |
| Enterprise | g | 56,500 |
| Dry Bed | g | 1,151,200 |
| Nelson | g | 700 |
| Mattson-Craig | g | 4,300 |
| Pumps | g | 700 |
| Willow Creek near Ririe | g | 73,500 |
| Total | | 1,633,700 |
| *Surface-return flows:* | | |
| Dry Bed | g | 174,500 |
| Spring Creek | g | 21,700 |
| Emigrant Creek | g | 1,400 |
| Drain | g | 700 |
| Anderson waste | g | 6,300 |
| Sand Creek | g | 6,700 |
| Little Sand Creek | g | 3,500 |
| Taylor | g | 10,600 |
| Henrys Creek | g | 11,100 |
| Willow Creek floodway | g | 8,600 |
| Total | | 245,100 |
| Diversions minus surface return | = | 1,388,600 |

*Irrigation Area 7.—Right-bank diversions from the Snake River from below Lorenzo to Shelley*

| *Name* | | *Quantity (acre-ft)* |
|---|---|---|
| Butte, Market Lake | a | 71,600 |
| Bear Trap | a | 6,000 |
| Osgood | a | 9,300 |
| Clements | a | 700 |
| Kennedy | a | 3,500 |
| Great Western | a | 126,300 |
| Porter | a | 80,800 |
| Woodville | a | 21,500 |
| McKay South | a | 600 |
| Total | | 320,300 |
| *Surface-return flows:* | | |
| Great Western waste | c | 400 |
| Great Western waste | c | 30,700 |
| Great Western waste | c | 25,600 |
| Butte, Market Lake return | c | 7,200 |
| Total | | 63,900 |
| Diversions minus surface return | = | 256,400 |

*Irrigation Area 8.—Left-bank diversions from the Snake River from Lewisville to Blackfoot*

| *Name* | | *Quantity (acre-ft)* |
|---|---|---|
| Idaho | a | 295,200 |
| Snake River valley | a | 198,000 |
| Blackfoot | a | 111,500 |
| Corbett | a | 47,500 |
| Nielson-Hansen | a | 2,600 |
| Sand Creek at Idaho Falls | c | 6,700 |
| Little Sand Creek at Ammon | c | 3,500 |
| Taylor | c | 10,600 |
| Henrys Creek | c | 11,100 |
| East Idaho Slough | c | 13,800 |
| Total | | 700,500 |
| *Surface-return flows:* | | |
| Cedar Point to Reservation Canal | c | 2,700 |
| Snake River valley waste to Reservation Canal (estimated) | | 20,000 |
| Sand Creek to Reservation Canal | c | 78,200 |
| Idaho Canal to Blackfoot River | c | 30,600 |
| Shull Lateral waste | c | 2,200 |
| End of East Idaho Slough into Blackfoot River | c | 25,500 |
| Corbett Slough waste to Snake River | c | 3,200 |
| Blackfoot Canal waste to Snake River | c | 10,200 |
| Total | | 172,600 |
| Diversions minus surface return | = | 527,900 |

*Irrigation Area 9.—Diversions from the Snake and Blackfoot Rivers*

| *Name* | | *Quantity (acre-ft)* |
|---|---|---|
| Little Indian Creek | c | 10,500 |
| Fort Hall Main | c | 178,900 |
| Fort Hall North | c | 70,200 |
| Total | | 259,600 |
| *Surface-return flows:* | | |
| End of Fort Hall North | c | 2,500 |
| End of Gibson | c | 2,100 |
| Teak Lateral to Ross Fork | c | 600 |
| Indian Lateral to Ross Fork | c | 700 |
| Ross Fork below Fort Hall Main | c | 3,600 |
| Tyhee waste to Ross Fork | c | 13,000 |
| Reider waste | c | 2,000 |
| Dubois Lateral waste | c | 800 |
| Tyhee Lateral waste | c | 2,000 |
| Church Lateral waste | c | 2,700 |
| End of Fort Hall Main | c | 2,300 |
| Total | | 32,300 |
| Diversions minus surface return | = | 227,300 |

*Irrigation Area 10.—Right-bank diversions from the Snake River below Shelley to Blackfoot*

| *Name* | | *Quantity (acre-ft)* |
|---|---|---|
| New Lava Side | a | 35,200 |
| Peoples | a | 109,000 |
| Aberdeen-Springfield | a | 312,000 |
| Riverside | a | 33,600 |
| Danskin | a | 58,800 |
| Trego | a | 17,700 |
| Wearyrick | a | 18,500 |
| Watson | a | 31,400 |
| Parsons | a | 14,500 |
| Total | | 630,700 |

*Surface-return flows:*

| | | |
|---|---|---|
| Riverside waste | c | 15,500 |
| Watson Slough waste | c | 9,400 |
| Peoples waste | c | 8,700 |
| Duncan waste | c | 5,200 |
| New Lava Side waste | c | 4,500 |
| Parsons waste | c | 1,900 |
| Crawford waste | c | 2,400 |
| Total | | 47,600 |

Diversions minus surface returns minus canal loss (Aberdeen-Springfield) = 583,100 − 95,200 ........ = 487,900

### *Irrigation Area 11.—Left-bank diversions from Portneuf River*

| Name | | Quantity (acre-ft) |
|---|---|---|
| Fort Hall Michaud | c | 30,600 |
| Falls Irrigation | c | 23,200 |
| Bannock Creek | g | 54,600 |
| Total | | 108,400 |
| *Surface-return flows:* | | |
| Bannock Creek | g | 34,500 |
| Diversions minus surface return | = | 73,900 |

### *Irrigation Area 12.—Right-bank diversion from the Snake River at Lake Walcott*

| | | Quantity (acre-ft) |
|---|---|---|
| Diversion | a | 385,900 |
| Surface return | g | 47,400 |
| Diversion minus surface return | = | 338,500 |

### *Irrigation Area 13.—Right-bank diversion from the Snake River at Lake Milner*

| | | Quantity (acre-ft) |
|---|---|---|
| Diversion | g | 50,500 |
| Surface return | g | 1,700 |
| Diversion minus surface return | = | 48,800 |

### *Irrigation Area 14.—Right-bank diversions from the Snake River at Lake Milner*

| Name | | Quantity (acre-ft) |
|---|---|---|
| North-side Twin Falls | a | 697,300 |
| North-side Crosscut-Gooding | g | 354,200 |
| North-side "A" Lateral | a | 18,100 |
| PA Lateral | a | 15,200 |
| Total | | 1,084,800 |
| Surface-return flows | g | 62,700 |
| Diversions minus surface return | = | 1,022,100 |

### *Irrigation Area 15.—Left-bank diversions from the Snake River at Lake Milner*

| Name | | Quantity (acre-ft) |
|---|---|---|
| South-side Twin Falls | a | 1,090,200 |
| Salmon Falls | b | 85,400 |
| Rock Creek | g | 25,000 |
| Dry Creek | g | 9,000 |
| Cedar Creek | g | 8,300 |
| Cottonwood, McMullen, Deep Creeks | g | 15,000 |
| Total | | 1,232,900 |
| Surface-return flows | g | 575,600 |
| Diversions minus surface return | = | 657,300 |

### *Irrigation Area 16.—Left-bank diversion from the Snake River at Lake Milner*

| | | Quantity (acre-ft) |
|---|---|---|
| Diversion | g | 61,100 |
| Surface return | g | 500 |
| Diversion minus surface return | = | 60,600 |

### *Irrigation Area 17.—Left-bank diversion from the Snake River at Lake Walcott*

| | | Quantity (acre-ft) |
|---|---|---|
| Diversion | a | 312,300 |
| Surface return | g | 66,500 |
| Diversion minus surface return | = | 245,800 |

### *Irrigation Area 18.—Goose Creek diversion from Goose Creek Reservoir*

| | | Quantity (acre-ft) |
|---|---|---|
| Diversion | b | 44,900 |
| Surface return | | 0 |
| Diversion minus surface return | = | 44,900 |

### *Irrigation Areas 19–26.—Milner-Gooding Canal, Big Wood and Little Wood Rivers*

Records of measured flows in irrigation areas 19–26 are in the Water Districts 37, 37M (1980) Watermaster report, the American Falls District No. 2 report, and the U.S. Geological Survey (1980) report. The approach used in these areas was to sum the inflow and outflow for each area and determine the difference. This approach includes river and canal losses and field seepage. The total consumed in the basin was compared with the total consumed in six of the eight subbasin areas.

| Name | | Quantity (acre-ft) |
|---|---|---|
| Inflow: | | |
| Big Wood below Magic Reservoir | b | 314,100 |
| Little Wood near Carey | b | 140,500 |
| Silver Creek at Sportsman Access | b | 114,100 |
| Milner-Gooding above Little Wood | b | 335,400 |
| X Canal | d | 101,100 |
| Total | | 1,005,200 |
| Outflow: | | |
| Big Wood near Gooding | b | 202,200 |
| Y Canal | d | 47,600 |
| X Canal | d | 22,200 |
| Dietrich Canal | d | 56,700 |
| Total | | 328,700 |
| Basin inflow minus basin outflow | = | 676,500 |

Total of subbasin consumption:

| Area | Inflow-Outflow (acre-ft) |
|---|---|
| 19 | 67,100 |
| 20 | 62,600 |
| 21 | 226,100 |
| 22 | 106,800 |
| 25 | 94,700 |
| 26 | 129,200 |
| Total | 686,500 |

$$\frac{686{,}500 - 676{,}500}{686{,}500} \times 100 = 1.5\text{-percent difference}$$

### *Irrigation Area 19.—South Gooding tract*

| Name | | Quantity (acre-ft) |
|---|---|---|
| Inflow: | | |
| Little Wood at Shoshone | d | 168,700 |
| X Canal | d | 101,100 |
| Big Wood River near Gooding No. 9 | d | 69,300 |
| Total | | 339,100 |
| Outflow: | | |
| Big Wood River near Gooding No. 21 | d | 202,200 |
| Y Canal | d | 47,600 |
| Z Canal | d | 22,200 |
| Total | | 272,000 |
| Inflow minus outflow | = | 67,100 |

*Irrigation Area 20.—North Gooding tract*

| Name | | Quantity (acre-ft) |
|---|---|---|
| Inflow: | | |
| Head of North Gooding Main | d | 62,600 |
| Outflow: | | 0 |
| Inflow minus outflow | = | 62,600 |

*Irrigation Area 21.—Shoshone tract*

| Name | | Quantity (acre-ft) |
|---|---|---|
| Inflow: | | |
| Big Wood River below Diversion No. 5 | d | 164,700 |
| Milner-Gooding Canal below Little Wood River | d | 193,300 |
| Total | | 358,000 |
| Outflow: | | |
| Head of North Gooding Main | d | 62,600 |
| Big Wood River near Gooding No. 9 | d | 69,300 |
| Total | | 131,900 |
| Inflow minus outflow | = | 226,100 |

*Irrigation Area 22.—Lower Little Wood River*

| Name | | Quantity (acre-ft) |
|---|---|---|
| Inflow: | | |
| Little Wood River near Richfield, nonirrigation season—estimated from historic records | g | 60,000 |
| Little Wood River near Richfield, irrigation season | d | 65,400 |
| JB Slough near Richfield | d | 40,300 |
| Marley Slough | d | 20,300 |
| Historic F-waste | h | 4,100 |
| Milner-Gooding Canal above Little Wood | d | 335,400 |
| Total | | 525,500 |
| Outflow: | | |
| Dietrich Canal No. 11 | d | 56,700 |
| Milner-Gooding Canal below Little Wood | d | 193,300 |
| Little Wood at Shoshone | d | 168,700 |
| Total | | 418,700 |
| Inflow minus outflow | = | 106,800 |

*Irrigation Area 23.—Dietrich tract*

| Name | | Quantity (acre-ft) |
|---|---|---|
| Inflow: | | |
| Head of Dietrich Canal | d | 56,700 |
| Milner-Gooding diversion | e | 16,600 |
| Total | | 73,300 |
| Outflow: | | |
| Historic F-waste | h | 4,100 |
| Inflow minus outflow | = | 69,200 |

*Irrigation Area 24.—Hunt tract*

| | | Quantity (acre-ft) |
|---|---|---|
| Inflow: | e | 36,000 |
| Outflow: | | 0 |
| Inflow minus outflow | = | 36,000 |

*Irrigation Area 25.—Richfield tract*

| Name | | Quantity (acre-ft) |
|---|---|---|
| Inflow: | | |
| Head of Richfield Canal | d | 159,300 |
| Outflow: | | |
| JB Slough near Richfield | d | 40,300 |
| Marley Slough | d | 20,300 |
| Sum of miscellaneous wastes | h | 4,000 |
| Total | | 64,600 |
| Inflow minus outflow | = | 94,700 |

*Irrigation Area 26.—Silver Creek, Upper Little Wood diversions*

| Name | | Quantity (acre-ft) |
|---|---|---|
| Inflow: | | |
| Silver Creek at Sportsman Access | b | 114,100 |
| Little Wood near Carey | b | 140,500 |
| Total | | 254,600 |
| Outflow: | | |
| Little Wood near Richfield, nonirrigation season—estimated from historic records | | 60,000 |
| Irrigation season | d | 65,400 |
| Total | | 125,400 |
| Inflow minus outflow | = | 129,200 |

# APPENDIX B.—PARAMETER-ESTIMATION PROGRAM DOCUMENTATION

## Introduction

This program documentation is based on materials authored by Steven P. Larson in March 1979, revised by James V. Tracy in September 1980, and distributed as instruction material for the course "Parameter-Estimation Techniques for Ground-Water Models," held September 15–26, 1980, at the U.S. Geological Survey's National Training Center in Denver, Colorado. Mr. Larson and Mr. Tracy are not responsible for errors or mistakes that may be present.

The program is designed to perform a nonlinear regression analysis to compute parameters associated with a finite-difference model of a steady-state, two-dimensional, ground-water flow system. The theory of the regression analysis is described by Cooley (1977), and the computer program follows his development explicitly.

The computer program is composed of a main program and seven subroutines. The main program controls input-output and performs all computations that cannot be accomplished more effectively with subroutines. The seven subroutines (D4SOLVE, COEF, LSTSQ, PRTOT, ORDER, ARRAY, ARRAYI) perform the following specialized tasks:

D4SOLVE—Solves the sets of linear algebraic equations resulting from the application of finite-difference methods by LDU factorization, assuming the equations are ordered in an alternating-diagonal fashion.

COEF—Computes coefficients necessary for the determination of transformed sensitivities and the flow equation.

LSTSQ—Computes the coefficients of the normal equations and solves the system of equations to determine the vector of parameter-change coefficients.

PRTOT—Prints matrices or vectors in a column configuration.

ORDER—Computes equation numbers at grid points corresponding to the alternating diagonal ordering scheme.

ARRAY—Loads and (or) prints one- and two-dimensional array variables.

ARRAYI—Loads and (or) prints one- and two-dimensional integer array variables.

The basic flow of the main program can be described as follows:

(A) Data input and variable initialization are accomplished.
(B) An initial solution corresponding to the initial parameter estimates is computed.
(C) In an iterative fashion, the following four steps are taken until the regression technique converges or until the number of iterations exceeds the maximum allowed.

(1) Compute residual of current estimate of head by invoking D4SOLVE subroutine.
(2) Compute transformed sensitivities using decomposed coefficient matrix and other coefficients computed by COEF subroutine.
(3) Invoke LSTSQ to form and solve normal equations.
(4) Update parameters using parameter-change coefficients generated by LSTSQ subroutine.

(D) Various statistics associated with the regression analysis are computed.

(E) Mass balances are computed.

## Aquifer Zonation and Variable Definition

The finite-difference grid is divided into zones that form the basic elements for the regression analysis. An aquifer property, such as transmissivity or leakance, at a particular node is computed as the product of the *zonal* value of the property times the *nodal* value of the property. Thus, if all nodal values for a property are given a value of unity, the zonal value becomes the value of the property for each node within that zone. Variation of a property within a zone is accomplished by assigning cell values that describe the relative variation within the zone. The zonal value then becomes a scalar for these relative values. Thus, the transmissivity of cell $(i,j)$ that is part of zone $k$ is $\text{TRAN}_{k\ (X \text{ or } Y)}\ T_{ij}$, where $\text{TRAN}_{k\ (X \text{ or } Y)}$ is the zonal value and $T_{ij}$ is the block value.

The zonal properties of one or more zones can be grouped to form a single parameter of the regression analysis. Computed changes in a regression parameter are applied equally to the zonal property of all zones that form that regression parameter. Nodal values of a property are unaffected by the regression procedure. The definitions of some of the more important variables in the computer program related to aquifer properties are as follows:

| *Variable name* | *Definition* |
|---|---|
| TRANX, TRANY, VLEAK, QDIST | Zonal values of transmissivity($X$ and $Y$), leakance, and distributed recharge. |
| IZN | An integer array that indicates the zone number of each block. |
| T, SL, QRE | Block values of transmissivity, leakance, and distributed recharge. |
| HR | Head on the boundary of the confining bed opposite the aquifer. |
| WELL | Block values of discharge (or recharge) from wells or other constant-rate source-sink phenomena. |

## Boundary Conditions and Boundary Parameters

Two types of boundary conditions may be used: specified flow and (or) specified head. Both may be considered as regression parameters. Nonzero specified-flow boundaries are imposed by assigning the appropriate value of the specified-flow rate to the nodal value of WELL. Specified-head boundaries are imposed by assigning a negative integer to variable IN, which corresponds to each node that is to be considered as specified head. The value of the known head at these nodes is entered via variable HO, which also describes nodal values of observed head.

Additional variables are used if specified-flow or specified-head boundary conditions are to be considered as regression parameters. Variables IBZN and QBND are used to indicate groups of nodes that form a specified-flow regression parameter and the value of the parameter. Thus, for a particular specified-flow regression parameter, the number of the parameter is entered into variable IBZN, which corresponds to the node or group of nodes that forms a specified-flow zone. The specified flow for each node within a zone is the product of QBND for that zone and a multiplier for the node. QBND is modified by the regression procedure; the multiplier is unaffected.

Specified-head boundary nodes that are to be considered regression parameters are defined in groups. Each group (variable IZ) is composed of a sequence of nodes (variables IH and JH). The specified head of the first and last nodes in the sequence are regression parameters (BH) or a single regression parameter. Adjustments to these nodes computed by the regression procedure are apportioned to other nodes in the sequence. The proportion is the ratio of A, the distance (along the sequence of nodes) from the end node to the node of interest, to B, the distance between the two end nodes. These factors are computed by the program for a given sequence.

## Prior Information on Regression Parameters

If estimates of the regression parameters are available from other sources (for example, aquifer tests), it may be desirable to introduce the information (parameter estimates and variability) into the regression analysis. The variability of the estimate is represented by a normalized standard error. Variable RK is used to store these values for aquifer parameters (TRANX, TRANY, VLEAK, and QDIST) and for specified-head and specified-flow boundary parameters (BH and QBND). The values are read directly into variable RK except for specified-head boundary parameters. These are read into variables STEHA and STEHB for each boundary segment, and appropriate elements of variable RK are set equal to these values.

The use of prior information requires an estimate of the error variance of the heads computed with the optimum parameters (variable EV). This estimate may be obtained from a solution that did not use prior information. If the estimate differs substantially from the value computed by the analysis using prior information, the problem should be re-solved using that computed value as the estimate of error variance.

## Solution-Only Mode

To facilitate the calculation of certain statistical measures, the program is capable of bypassing the regression analysis and computing only head distributions for various combinations of parameter values. This is accomplished by specifying the "solution-only" option (variable ISO) and providing the various combinations of parameter values for which solutions are desired. These solutions can be used to test the assumption of model "linearity" in the vicinity of the optimum parameter estimates.

## Using the Program

The computer code has been designed to be as machine independent as possible. Also, to minimize confusion, all arrays have been dimensioned explicitly. The following summarizes the minimum dimensions required for the program to operate properly for a specific problem. If

$N_o$ is the number of observed heads,
$N_g$ is the number of grid points ($N_x \times N_y$),

$N_x$ is the number of grid columns,
$N_y$ is the number of grid rows,
$N_e$ is the number of active nodes in the grid,
$N_z$ is the number of aquifer parameter zones in the model grid,
$N_p$ is the number of regression parameters for aquifer zones (excluding boundary parameters),
$N_q$ is the number of specified-flow boundary parameters,
$N_h$ is the number of specified-head boundary parameters,
$N_{qp}$ is the total number of nodes on boundary where flow is a parameter,
$N_{hp}$ is the total number of nodes on boundary where head is a parameter, and
$N_R$ is the total number of regression parameters, $N_p + N_q + N_h$,

then the array variables should be dimensioned as follows:

| *Variable name* | *Dimension*[1] |
|---|---|
| WELL, HR, W, HC, HO | $N_g$ |
| T, QRE, SL | $(N_x - 1)(N_y - 1)$ unchecked values |
| IZN | $N_g$ |
| IBZN | $N_{qp}$ |
| TRANX, TRANY, VLEAK, QDIST | $N_z$ |
| IPRM | 4, $N_z$ |
| QBND, BH | $N_q + N_h$ |
| DX | $N_x$ |
| DY, JPOSN$_y$ | $N_y$ |
| RK, P | $N_R$ |
| B | 4,$N_R$ |
| S | $N_R$, $N_e$ |
| A | $N_R$, $N_R$ |
| V | $N_e$ |
| IBPA, IBPB | $N_{hp} + N_{qp}$ |
| ILOC, JLOC | $N_g$ |
| PLA, PLB | $N_{hp}$ |
| QBF | $N_{qp}$ |
| IC, AU | 5, $N_e/2$ |
| AL | $N_y - 1$, $N_e/2$ |
| IN | $N_g$ |
| QBND | $N_q$ |
| BH | $N_h$ |
| HCA | 5, $N_o$ |

[1]These dimensions are approximate. The exact sizes required are calculated and printed in subroutine ORDER.

Note that array variables that have a single dimension (TRANX, TRANY, T, HO, and so forth) and that are "passed" to subroutines need only be dimensioned as "one" within the subroutine. (Only the initial address of an array is actually passed to a subroutine, and dimension sizes are required only for multidimensioned arrays.) This "unit" dimension need *not* be changed (in the subroutine) if the dimension of the variable is changed in the main program. Unit dimensions in subroutines cannot be used for multidimensioned arrays.

## Input Data

Cards 1–3—TITLE (Format, 20A4)

Card 4—Problem size parameters (Format, 16I5)

| *Card columns* | *Variable* | *Definition* |
|---|---|---|
| 1–5 | ID | Number of grid columns |
| 6–10 | JD | Number of grid rows |
| 11–15 | NZNS | Number of aquifer parameter zones |
| 16–20 | NWELS | Number of wells |
| 21–25 | NQBND | Number of specified-flow zones |
| 26–30 | NBQZ | Number of specified-flow boundary parameters |
| 31–35 | NBHZ | Number of specified-head boundary zones |
| 36–40 | NBHP | Number of specified-head boundary parameters |
| 41–45 | NPAR | Number of regression parameters associated with aquifer zones |
| 46–50 | NUM | Maximum number of iterations allowed for the regression analysis |
| 51–55 | IPRX | Additional print sensitivities and orthogonalize-sensitivities option. Code 1 to select |
| 56–60 | IRPF | Option for optimal bias parameter calculation. Code 1 to select |
| 61–65 | IPO | Additional printout option. Code 1 to select |
| 66–70 | ISO | Head solution-only option. Code 1 to select |
| 71–75 | NONMB | Number of nodes for mass-balance calculations; limit = 200 |

Card 5—Special regression parameters (Format, 8F10.0)

| *Card columns* | *Variable* | *Definition* |
|---|---|---|
| 1–10 | AP | Acceleration parameter for regression analysis (normally equal to 1.0) |
| 11–20 | AMP | Marquardt parameter. Code 0.0 if not used |
| 21–30 | RP | Ridge parameter for regression analysis (normally equal to 0.0) |
| 31–40 | RPF | Bias parameter for regression. Set to 0.0 |
| 41–50 | EV | Estimated error variance for problems using prior information. Code 0.0 if not used |

Array sets 1–9—Aquifer description arrays (Format, 2I5). Each array set (1–10) is prefaced by a single parameter card containing the following information:

| *Card columns* | *Variable* | *Definition* |
|---|---|---|
| 1–5 | NOZN | Number of rectangular input zones into which the variable has been subdivided |
| 6–10 | IPRN | Print option for full array. Set to 0 for print. Set to 1 for no output |

Each input zone (1, NOZN) of array sets (1–8) is prefaced by a parameter card providing the following information about that input zone:

(Format 4I5, F10.0, I5)

| *Card columns* | *Variable* | *Definition* |
|---|---|---|
| 1–5 | IB | Beginning column of the rectangular input zone |
| 6–10 | IE | Final column of the rectangular input zone |
| 11–15 | JB | Beginning row of the rectangular input zone |
| 16–20 | JE | Final row of the rectangular input zone |
| 21–30 | FACT | If the array set is uniform for the entire zone, FACT is the cell or nodal value that is assigned to each grid point. If the array set is not uniform, each cell or nodal value on the subsequent data cards is multiplied by FACT |
| 31–35 | IVAR | Code 0 if the array set is uniform. Code 1 if it is not uniform |

Each input zone (1, NOZN) of array sets (9–10) is prefaced by a parameter card providing the following information about that input zone:

(Format, 6I5)

| *Card columns* | *Variable* | *Definition* |
|---|---|---|
| 1–5 | IB | Beginning column of the rectangular input zone |
| 6–10 | IE | Final column of the rectangular input zone |

| | | |
|---|---|---|
| 11–15 | JB | Beginning row of the rectangular input zone |
| 16–20 | JE | Final row of the rectangular input zone |
| 21–25 | IFACT | If the array set is uniform for the entire zone, FACT is the cell or nodal value that is assigned to each grid point. If the array set is not uniform, each cell or nodal value on the subsequent data cards is not multiplied by IFACT |
| 26–30 | IVAR | Code 0 if the array set is uniform. Code 1 if it is not uniform |

The format for the data cards is variable and is given for each array set in the following list. Remember that data cards are not required if the array set is uniform for the entire zone (IVAR = 0). The following list gives the array sets in the order in which they must appear:

| *Array set number* | *Variable* | *Format* | *Definition* |
|---|---|---|---|
| 1 | DX(ID-1) | 8F10.0 | Distance between grid points in $x$ ($I$) direction |
| 2 | DY(JD-1) | 8F10.0 | Distance between grid points in $y$ ($J$) direction |
| 3, 4 | NHCA | I5 | Number of head observations |
| | HCA (4, NHCA) | 5F10.2 | HCA(1, $N$) $X$-location in nodal units<br>HCA(2, $N$) $Y$-location in nodal units<br>HCA(3, $N$) water level<br>HCA(4, $N$) weighting value<br>HCA(5, $N$) interpolated head, set to 0.0 |
| 5 | T(ID-1, JD-1) | 8F10.0 | Transmissivity |
| 6 | SL(ID-1, JD-1) | 8F10.0 | Leakance ($k'/m'$) of confining bed |
| 7 | HR(ID, JD) | 8F10.0 | Head on boundary of confining bed opposite the aquifer |
| 8 | QRE(ID-1, JD-1) | 8F10.0 | Recharge rate per unit area |
| 8b | (NLMBX, NLMBY) | (I5,5x,I5) | $X$, $Y$ locations of mass-balance nodes, NONMB cards |
| 9 | IZN(ID-1, JD-1) | 16I5 | Zone number of each cell. Each active grid point ($T \geq 0$) *must* have a zone number |

Card set 1—Zonal parameters (Format, I5, 4F10.0). Set will contain NZNS cards.

| *Card columns* | *Variable* | *Definition* |
|---|---|---|
| 1–5 | I | Zone number |
| 6–15 | TRANX(I) | Zonal $X$-transmissivity value for zone $I$ |
| 16–25 | TRANY(I) | Zonal $Y$-transmissivity value for zone $I$ |
| 26–35 | VLEAK(I) | Zonal leakance value for zone $I$ |
| 36–45 | QDIST(I) | Zonal distributed recharge value for zone $I$ |

Card set 2—Parameter numbers (Format, 16I5). Set will contain NZNS cards. Omit if NPAR = 0

| *Card columns* | *Variable* | *Definition* |
|---|---|---|
| 1–5 | I | Zone number |
| 6–10 | IPRM(1, I) | Parameter number for $X$-transmissivity in zone $I$. Code 0 if it is not a regression parameter |
| 11–15 | IPRM(2, I) | Parameter number for $Y$-transmissivity in zone $I$. Code 0 if it is not a regression parameter |
| 16–20 | IPRM(3, I) | Parameter number for leakance in zone $I$. Code 0 if it is not a regression parameter |
| 21–25 | IPRM(4, I) | Parameter number for distributed recharge in zone $I$. Code 0 if it is not a regression parameter |

Note: The cards in each set (1 or 2) may appear in any order with respect to zone number, but there must be NZNS cards.

Card set 3—Normalized parameter standard errors (Format, 8F10.0). Eight values per card; NPAR values. Use as many cards as required. Omit if NPAR = 0

| *Card columns* | *Variable* | *Definition* |
|---|---|---|
| 1–10 | RK(1) | Normalized standard errors (standard errors divided by initial parameter value) for each parameter. Enter values for parameters defined by IPRM (aquifer zone parameters) in increasing order. Code 0.0 if no prior information exists for the parameter |
| 11–20 | RK(2) | |
| | RK(NPAR) | |

Card set 4—Well rates (Format, 2I5, F10.0). Set will contain NWELS cards. Omit if NWELS = 0.

| *Card columns* | *Variable* | *Definition* |
|---|---|---|
| 1–5 | I | Column location of well |
| 6–10 | J | Row location of well |
| 11–20 | WELL(I, J) | Well rate, *negative* for withdrawal |

Each flow boundary segment (zone) is identified and quantified by the following card set 5, which gives the particular information regarding each zone.

Card set 5—Specified-flow boundary zone parameters (Format, 5I5, 3F10.0). Requires NQBND cards for zonal information. One card for each zone (1, NQBND); if NQBND = 0, omit the data set.

| *Card columns* | *Variable* | *Format* | *Definition* |
|---|---|---|---|
| 1–5 | IA | I5 | Column location of the A end of the segment (zone) |
| 6–10 | JA | I5 | Row location of the A end of the segment (zone) |
| 11–15 | IB | I5 | Column location of the B end of the segment (zone) |
| 16–20 | JB | I5 | Row location of the B end of the segment (zone) |
| 21–25 | IZ | I5 | Parameter number (set equal to zero if not a parameter) |
| 26–35 | QBND | F10.0 | Flow parameter value |
| 36–45 | RK | F10.0 | Normalized parameter standard error |
| 46–55 | QBM | F10.0 | Multiplier for flow parameter |

Array set 10—Specified-head boundary zonation

| *Array set number* | *Variable* | *Format* | *Definition* |
|---|---|---|---|
| 10 | IBZN(ID, JD) | 16I5 | To denote specified heads. Set equal to −1 |

Card set 6—Specified-head boundary parameter information. This set is composed of one card (6a) with descriptive information about the boundary segment or zone followed by a card or cards (6b) that describe the row and column locations of the nodes that form the segment. There are NBHZ groups of card set 6. If NBHZ = 0, all cards are omitted.

Card 6a—Segment description (Format, 4I5, 2F10.0)

| Card columns | Variable | Definition |
|---|---|---|
| 1–5 | IZ | Segment or zone number |
| 6–10 | NN | Number of nodes in segment |
| 11–15 | IBPA | Specified boundary parameter number for A end of segment |
| 16–20 | IBPB | Specified boundary parameter number for B end of segment |
| 21–30 | STEHA | Normalized standard error of head at A end of segment |
| 31–40 | STEHB | Normalized standard error of head at B end of segment |

Card(s) 6b—Nodes forming segment IZ (Format, 2I5, F10.0)

| Card columns | Variable | Definition |
|---|---|---|
| 1–5 | IH | Column location of node |
| 6–10 | JH | Row location of node |
| 11–20 | V | Estimated head at node IH, JH |

Note: For card(s) 6b, estimated heads (V) are required only at the ends of the segment (A and B). If estimated heads for the other nodes in the segment are set equal to zero, a linear interpolation (based on distance) is made from the endpoint values to determine the initial head estimate at these nodes. If nonzero values are entered, the initial head estimate is the nonzero value. Regression parameters (changes in head) are calculated only for the endpoints and are distributed linearly to zero from one endpoint to the other. The change in head at any other node point in the segment thus has two components: the percentage of the computed change at each of the two endpoints. If IBPA is the same as IBPB, the computed head change at both ends of the segment is the same, and in effect a single head change is applied to all nodes in the segment.

The following cards and card sets are required *only* if the solution-only option (ISO, card 4) is specified. Mass-balance calculations are not performed in the solution-only option.

Card 6—Additional solution specification (Format, I5)

| Card columns | Variable | Definition |
|---|---|---|
| 1–5 | N | Number of solutions required using alternative parameter sets. (Code 0 if a solution is desired only for the initial set of parameters) |

Card set 7—Zonal parameters for next parameter set (Format, I5, 4F10.0). Set contains NZNS cards.

| Card columns | Variable | Definition |
|---|---|---|
| 1–5 | I | Zone number |
| 6–15 | TRANX(I) | Zonal *X*-transmissivity parameter for zone *I* |
| 16–25 | TRANY(I) | Zonal *Y*-transmissivity parameter for zone *I* |
| 26–35 | VLEAK(I) | Zonal leakance parameter for zone *I* |
| 36–45 | QDIST(I) | Zonal distributed recharge parameter for zone *I* |

Note: The zones may appear in any order, but all zones must be defined.

Card set 8—Specified-flow boundary parameters for next parameter set (Format, I5, F10.0). Set will contain NBQZ cards. Omit if NBQZ = 0.

| Card columns | Variable | Definition |
|---|---|---|
| 1–5 | I | Specified-flow boundary zone number |
| 6–15 | QBND(I) | Specified-flow boundary parameter for zone *I* |

Note: The zones may appear in any order, but all zones must be defined if NBQZ is not zero.

Card set 9—Specified-head boundary parameters for next parameter set (Format, I5, F10.0). Set will contain NBHP cards. Omit if NBHZ = 0.

| Card columns | Variable | Definition |
|---|---|---|
| 1–5 | I | Specified-head boundary parameter number |
| 6–15 | BH(I) | *Fractional change* of specified-head boundary parameter *I*. The head computed for the first solution is the basis for computing the values for subsequent solutions |

Note: The parameters may appear in any order, but all parameters must be defined if NBHZ is not zero.

Card sets 7, 8, and 9 are repeated *N* times. The program computes and prints the solution corresponding to each set of parameters. This is useful in determining certain statistical measures about the regression analysis.

# APPENDIX C.--MODIFIED PARAMETER-ESTIMATION PROGRAM

```
C       FINITE DIFFERENCE PROGRAM FOR NONLINEAR REGRESSION SOLUTION
C       OF TWO-DIMENSIONAL, STEADY-STATE, GROUND-WATER FLOW PROBLEMS
C       BY R. L. COOLEY, USGS, DENVER, COLO.
C       MODIFIED BY S. GARABEDIAN TO USE NON-NODAL OBSERVATION HEADS
C
C
      PROGRAM MAIN(TAPE4,TAPE5,TAPE6,TAPE7,TAPE8,INPUT,OUTPUT)
      DIMENSION TITLE(20),DX(60),DY(30),T(1400),SL(1400),QRE(1400)
     1,WELL(1400),HR(1400),DUM(1400),HC(1400),TRANX(50),TRANY(50)
     2,VLEAK(50),QDIST(50),P(50),RK(50),QBND(10),QBF(150),BH(30),PLA(150
     3)
     4,PLB(150),S(20,1100),V(1400)
      DIMENSION A(20,20),AU(5,550),AL(30,500)
      DIMENSION HCA(5,1100)
      DIMENSION NLMA(400),NLMB(400),XLMB(400)
      DIMENSION JPOS(30),IZN(1400),IBZN(1400),IPRM(4,50),IBPA(150),IBPB(
     1150)
     2,ILOC(1400),JLOC(1400),IN(1400),IC(5,550)
      COMMON/INT/NIJ,NEQ,ICR,ICR1,IB1,LH1,ID,JD,IM,JM,NVAR,NQSD,NBPAR
     1,NVX2,NVX3,IPO,KOUNT,INDT,IBHZ
      COMMON/TNME/IIN,IOUT
      COMMON/FLT/AU,AL,AP,AMP,RP,RPF,YSQ
      EQUIVALENCE (ILOC(1),TITLE(1),AU(1,1)),(JLOC(1),AL(1,1),A(1,1))
C**DEFINE INPUT AND OUTPUT FILES, AND ARRAY DIMENSIONS
      IIN=7
      IOUT=6
      NVE=20
C**READ AND WRITE 3 TITLE CARDS
      WRITE(IOUT,804)
      DO 5 I=1,3
      READ(IIN,801) (TITLE(J),J=1,20)
    5 WRITE(IOUT,803) (TITLE(J),J=1,20)
C**READ JOB SPECIFICATION DATA
      READ(IIN,800) ID,JD,NZNS,NWELS,NQBND,NBQZ,NBHZ,NBHP,NPAR,NUM,IPRX
     1,IRPF,IPO,ISO,NONMB
      WRITE(IOUT,802) ID,JD,NZNS,NWELS,NQBND,NBQZ,NBHZ,NBHP,NPAR,NUM
     1,IPRX,IRPF,IPO,ISO
      READ(IIN,820) AP,AMP,RP,RPF,EV
      WRITE(IOUT,806) AP,AMP,RP,RPF,EV
C**READ INITIAL ARRAY DATA
      IM=ID-1
      JM=JD-1
      CALL ARRAY(DX,IM,1,1,0)
      CALL ARRAY(DY,JM,1,2,0)
      NIJ=ID*JD
      DO 14 I=1,NIJ
   14 HC(I)=0.0
C*MODIFICATION* READ IN THE LOCATION, VALUE AND WEIGHTING OF
C               OBSERVATION HEADS
      READ(IIN,12)NHCA
      DO 11 I=1,NHCA
      READ(IIN,13)(HCA(J,I),J=1,5)
      IX=INT(HCA(1,I))
      IY=INT(HCA(2,I))
      I1=(IY-1)*ID+IX
      HC(I1)=HCA(3,I)
      HC(I1+1)=HCA(3,I)
      HC(I1+ID)=HCA(3,I)
```

```
      HC(I1+ID+1)=HCA(3,I)
   11 CONTINUE
C*MODIFICATION*   RESET CONSTANT HEADS
      DO 15 I=1,NHCA
      IF(HCA(4,I).GE.0.0)GO TO 15
      IX=INT(HCA(1,I))
      IY=INT(HCA(2,I))
      I1=(IY-1)*ID=IX
      HC(I1)=HCA(3,I)
      HCA(4,I)=0.0
   15 CONTINUE
   12 FORMAT(3I5)
   13 FORMAT(F10.0,F10.0,3F10.0)
      CALL ARRAY(W,ID,JD,4,0)
      CALL ARRAY(T,IM,JM,5,0)
      CALL ARRAY(SL,IM,JM,6,0)
      CALL ARRAY(HR,ID,JD,7,0)
      CALL ARRAY(QRE,IM,JM,8,0)
C*MODIFICATION* READ IN NODAL LOCATIONS WHERE MASS BALANCE
C               CALCULATIONS ARE TO BE MADE
      IF(NONMB.EQ.0)GO TO 41
      DO 43 I=1,NONMB
   43 READ(IIN,44)NLMB(I*2-1),NLMB(I*2)
   44 FORMAT(I5,5X,I5)
      DO 42 I=1,NONMB
      NLMA(I)=NLMB(2*I-1)+(NLMB(2*I)-1)*ID
   42 XLMB(I)=0.0
   41 CONTINUE
C**READ GRID ZONATION
      CALL ARRAYI(IZN,IM,JM,1,0)
C**READ INITIAL ZONAL PARAMETERS
      WRITE(IOUT,810)
      DO 30 J=1,NZNS
      READ(IIN,812) I,TRANX(I),TRANY(I),VLEAK(I),QDIST(I)
      WRITE(IOUT,814) I,TRANX(I),TRANY(I),VLEAK(I),QDIST(I)
      IF(ISO.GT.0)WRITE(8,814)I,TRANX(I),TRANY(I),VLEAK(I),QDIST(I)
      DO 20 K=1,4
   20 IPRM(K,J)=0
   30 CONTINUE
C**READ AND PRINT PARAMETER #'S
      NBPAR=NBHP+NBQZ
      IF(NPAR.LE.0) GO TO 45
      WRITE(IOUT,816)
      DO 40 J=1,NZNS
      READ(IIN,800) I,(IPRM(K,I),K=1,4)
      WRITE(IOUT,818) I,(IPRM(K,I),K=1,4)
      DO 35 K=1,4
   35 IPRM(K,I)=IPRM(K,I)+NBPAR
   40 CONTINUE
C**   READ AND PRINT PARAMETER COEFFICIENTS OF VARIATION
      READ(IIN,820) (RK(K+NBPAR),K=1,NPAR)
      WRITE(IOUT,822)
      CALL PRTOT(RK(NBPAR+1),NPAR,1)
   45 JPOS(1)=0
      DO 50 J=2,JD
   50 JPOS(J)=JPOS(J-1)+ID
C**READ WELL DATA
      DO 55 N=1,NIJ
   55 WELL(N)=0.
```

```
      IF(NWELS.LE.0) GO TO 61
      WRITE(IOUT,824)
      DO 60 K=1,NWELS
      READ(IIN,826) I,J,WELL(I+JPOS(J))
      WRITE(IOUT,828) I,J,WELL(I+JPOS(J))
   60 CONTINUE
C** READ AND FORM ARRAYS FOR SPECIFIED POINT OR LINE FLUXES
   61 NQSD=0
      IF(NQBND.LE.0) GO TO 85
      WRITE(IOUT,830)
      N=0
      DO 80 J=1,NQBND
      READ(IIN,832) IA,JA,IB,JB,IZ,QB,STDER,QBM
      WRITE(IOUT,831) IA,JA,IB,JB,IZ,QB,STDER,QBM
      M=1
      K=IA-1
      IF(JA.EQ.JB) GO TO 62
      M=ID
      K=JA-1
   62 MA=IA+JPOS(JA)
      MB=IB+JPOS(JB)-M
      IF(MB.GE.MA) GO TO 64
      IF(IZ.LE.0) GO TO 63
      N=N+1
      IBPA(N)=MA
      IBPB(N)=MA
      QBF(N)=.5*QBM
      IBZN(N)=IZ
      GO TO 68
   63 WELL(MA)=QB*QBM
      GO TO 80
   64 QBM=.5*QBM
      IF(IZ.LE.0) GO TO 70
      DO 66 L=MA,MB,M
      N=N+1
      IBPA(N)=L
      IBPB(N)=L+M
      K=K+1
      TEMP=DX(K)
      IF(M.EQ.ID) TEMP=DY(K)
      QBF(N)=QBM*TEMP
   66 IBZN(N)=IZ
   68 QBND(IZ)=QB
      RK(IZ)=STDER
      GO TO 80
   70 TMP=QB*QBM
      DO 75 L=MA,MB,M
      K=K+1
      TEMP=DX(K)
      IF(M.EQ.ID) TEMP=DY(K)
      TEMP=TMP*TEMP
      WELL(L)=WELL(L)+TEMP
   75 WELL(L+M)=WELL(L+M)+TEMP
   80 CONTINUE
      NQSD=N
C**READ SPECIFIED BOUNDARY HEAD POSITIONS AS -1'S
   85 CALL ARRAYI(IN,ID,JD,2,0)
C**READ DATA AND FORM ARRAYS FOR SPECIFIED HEADS AND PARAMETERS
      IBHZ=0
```

```
      IF(NBHZ.LE.0) GO TO 110
      WRITE(IOUT,833)
      NBH=0
      DO 108 KK=1,NBHZ
      READ(IIN,834) IZ,NN,M,N,STEHA,STEHB
      WRITE(IOUT,836) IZ,NN,M,N,STEHA,STEHB
      DO 95 J=1,NN
       READ(IIN,826) ILOC(J),JLOC(J),V(J)
   95 WRITE(IOUT,840) ILOC(J),JLOC(J),V(J)
      IF(IZ.LE.0) GO TO 97
      BH(M)=V(1)
      RK(M+NBQZ)=STEHA
      BH(N)=V(NN)
      RK(N+NBQZ)=STEHB
      IZ=IZ+NQSD
      IBPA(IZ)=M+NBQZ
      IBPB(IZ)=N+NBQZ
   97 J=JLOC(1)
      K=ILOC(1)+JPOS(J)
      IF(IN(K).LT.-1) GO TO 100
      NBH=NBH+1
      IF(IZ.GT.0) IN(K)=-NBH-1
      IBZN(NBH+NQSD)=IZ
      PLA(NBH)=V(1)
      PLB(NBH)=0.
  100 IB(W(K).GT.0.) IBHZ=1
      IF(NN.LE.1) GO TO 108
      DIST=0.
      DO 102 KNT=2,NN
      J=JLOC(KNT)
      L=ILOC(KNT)+JPOS(J)
      IB(W(L).GT.0.) IBHZ=1
      NBH=NBH+1
      IF(IZ.GT.0) IN(L)=-NBH-1
      IBZN(NBH+NQSD)=IZ
      JM1=JLOC(KNT-1)
      IF(J.EQ.JM1) GO TO 101
      J=MIN0(J,JM1)
      DIST=DIST+DY(J)
      GO TO 102
  101 I=MIN0(ILOC(KNT),ILOC(KNT-1))
      DIST=DIST+DX(I)
  102 PLB(NBH)=DIST
      DO 106 KNT=2,NN
      J=JLOC(KNT)
      L=ILOC(KNT)+JPOS(J)
      N=-IN(L)-1
      TMP=PLB(N)/DIST
      TMPA=TMP*V(NN)
      TMPB=(1.-TMP)*V(1)
      TMPC=TMPA+TMPB
      IF(ABS(V(KNT)).LE.0.) GO TO 104
      TMP=V(KNT)/TMPC
      TMPA=TMPA*TMP
      TMPB=TMPB*TMP
      TMPC=V(KNT)
  104 PLA(N)=TMPB
      PLB(N)=TMPA
```

```
  106 HC(L)=TMPC
  108 HC(K)=V(1)
      IF(NBHP.LE.0) IBHZ=0
C**COMPARE T WITH IZN FOR CONFLICT
  110 IER=0
      N=0
      DO 115 J=1,JM
      DO 115 I=1,IM
      N=N+1
      IF(IZN(N).LE.0) GO TO 115
      IF(T(N).GT.0.) GO TO 115
      IER=1
      WRITE(IOUT,842) I,J
  115 CONTINUE
      IF(IER.LT.1) GO TO 120
      WRITE(IOUT,844)
      STOP
C**TRANSFER DOMAIN GEOMETRY TO IN(M)
  120 N=0
      DO 122 J=1,JM
      DO 122 I=1,IM
      N=N+1
      IF(IZN(N).LE.0) GO TO 122
      M=N+J
      IF(IN(M).GE.0) IN(M)=1
      IF(IN(M-1).GE.0) IN(M-1)=1
      IF(IN(M+ID-1).GE.0) IN(M+ID-1)=1
      IF(IN(M+ID).GE.0) IN(M+ID)=1
  122 CONTINUE
C**SET UP D4 ORDERING
      CALL ORDER(JPOS,IN,IC)
      NVAR=NPAR+NBPAR
      NVX2=NVAR+NVAR
      NVX3=NVX2+NVAR
C**COMPUTE AND COUNT PRIOR INFORMATION DATA
      NPRIR=0
      DO 125 I=1,NVAR
      P(I)=1.
      IF(RK(I).LE.0.) GO TO 125
      RK(I)=EV/(RK(I)*RK(I))
      NPRIR=NPRIR+1
  125 CONTINUE
C** ADJUST DX AND DY
      DO 130 I=1,IM
  130 DX(I)=.5*DX(I)
      DO 135 J=1,JM
  135 DY(J)=.5*DY(J)
C**COMPUTE INITIAL SOLUTION
      CALL COEF(DX,DY,T,SL,QRE,WELL,HR,HC,TRANX,TRANY,VLEAK,QDIST,QBND
     1,QBF,PLA,PLB,S,V,IZN,IBZN,IPRM,IBPA,IBPB,IN,IC,NVE)
      CALL D4SOLVE(HC,V,IN,IC,HCA,NHCA,ID,JD,ISO)
      WRITE(IOUT,846)
      CALL ARRAY(HC,ID,JD,0,1)
      IF(ISO.EQ.1) GO TO 640
C**COMPUTE INITIAL ERROR VARIANCE
      OBS=0.
      YSQ=0.
      DO 160 N=1,NHCA
C  CHECK LOCATION OF THE OBS HEADS, CORRECT WEIGHTHING IF OUTSIDE ACTIVE
```

```
C  AREA
      M4=0
      IX=INT(HCA(1,N))
      IY=INT(HCA(2,N))
      I1=(IY-1)*IM+IX
      RESX=HCA(1,N)-IX
      RESY=HCA(2,N)-IY
      I2=(IY-1)*ID+IX
      IF(RESX.EQ.0.0.AND.RESY.EQ.0.0)M4=1
      IF(M4.GT.0.AND.IN(I2).GT.0)GO TO 163
      IF(IZN(I1).LE.0)HCA(4,N)=0.0
  163 CONTINUE
      IF (HCA(4,N).LE.0.) GO TO 160
      OBS=OBS+1.
      TMP=HCA(3,N)-HCA(5,N)
      YSQ=YSQ+HCA(4,N)*TMP*TMP
      IF (W(N).LE.0.) GO TO 160
      OBS=OBS+1.
      TMP=HO(N)-HC(N)
      YSQ=YSQ+W(N)*TMP*TMP
  160 CONTINUE
      TEMP=NVAR-NPRIR
      VAR=YSQ/(OBS-TEMP)
      NTMP=OBS
      WRITE(IOUT,848) NTMP,NPRIR,YSQ,VAR
      DO 161 N=1,NHCA
  161 WRITE(IOUT,162)(HCA(I,N),I=1,5)
  162 FORMAT(5F10.2)
C**BEGIN ITERATIONS
      INDT=0
      ER=.01
      ERP=1000.
      DO 340 KNT=1,NUM
      KOUNT=KNT
C**COMPUTE HC(N)+DELTHC(N)
      CALL COEF(DX,DY,T,SL,QRE,WELL,HR,HC,TRANX,TRANY,VLEAK,QDIST,QBND
     1,QBF,PLA,PLB,S,V,IZN,IBZN,IPRM,IBPA,IBPB,IN,IC,NVE)
      CALL D4SOLVE(HC,V,IN,IC,HCA,NHCA,ID,JD,ISO)
C**SOLVE FOR SCALED SENSITIVITIES
      DO 260 K=1,NVAR
      DO 170 N=1,NEQ
  170 V(N)=S(K,N)
C**MODIFY R.H.S. UPPER HALF.
      DO 190 J=1,ICR1
      II=IC(1,J)
      DO 180 I=2,II
      LR=IC(I,J)
      V(LR)=V(LR)-AU(I,J)*V(J)
  180 CONTINUE
  190 V(J)=V(J)/AU(1,J)
C**MODIFY R.H.S. LOWER HALF.
      JJ=NEQ-ICR
      DO 210 J=1,JJ
      JR=J+ICR1
      LR=JR
      DO 200 I=2,IB1
      LR=LR+1
      IF (AL(I,J).NE.0.) V(LR)=V(LR)-AL(I,J)*V(JR)
```

```
  200 CONTINUE
  210 V(JR)=V(JR)/AL(1,J)
C**BACK SOLVE LOWER HALF
      V(NEQ)=V(NEQ)/AL(1,NEQ-ICR1)
      S(K,NEQ)=V(NEQ)
      DO 230 J=1,JJ
      KK=NEQ-J
      KL=KK-ICR1
      L=KK
      DO 220 I=2,IB1
      L=L+1
      IF (AL(I,KL).NE.0.) V(KK)=V(KK)-AL(I,KL)*V(L)
  220 CONTINUE
      S(K,KK)=V(KK)
  230 CONTINUE
C**BACK SOLVE UPPER HALF
      DO 250 J=1,ICR1
      KK=ICR-J
      II=IC(1,KK)
      DO 240 I=2,II
      L=IC(I,KK)
      V(KK)=V(KK)-AU(I,KK)*V(L)
  240 CONTINUE
      S(K,KK)=V(KK)
  250 CONTINUE
  260 CONTINUE
      IF(IPO.NE.1) GO TO 270
      WRITE(IOUT,850)
      N=0
      DO 265 J=1,JD
      DO 265 I=1,ID
      N=N+1
      L=IN(N)
      IF(L.LE.0) GO TO 265
      WRITE(IOUT,852) I,J,(S(K,L),K=1,NVAR)
  265 CONTINUE
C**CALL LEAST SQUARES
  270 CALL LSTSQ(HCA,HC,P,RK,PLA,PLB,S,V,IBZN,IBPA,IBPB,IN,NVE,NHCA)
      IF(INDT.GT.0) GO TO 521
      IF(IPO.EQ.1) WRITE(IOUT,854)
C**COMPUTE NEW SPECIFIED FLOW PARAMETERS
      IF(NBQZ.LE.0) GO TO 282
      DO 280 K=1,NBQZ
      QBND(K)=(V(K)+1.)*QBND(K)
      IF(IPO.EQ.1) WRITE(IOUT,856) K,QBND(K)
  280 CONTINUE
C**COMPUTE NEW SPECIFIED HEAD PARAMETERS
  282 IF(NBHP.LE.0) GO TO 290
      DO 284 J=1,NBHP
      BH(J)=(V(J+NBQZ)+1.)*BH(J)
      IF(IPO.EQ.1) WRITE(IOUT,856) J,BH(J)
  284 CONTINUE
      DO 286 J=1,NIJ
      N=IN(J)
      IF(N.GE.-1) GO TO 286
      N=-N-1
      M=IBZN(N+NQSD)
      K=IBPA(M)
```

```
      L=IBPB(M)
      PLA(N)=PLA(N)*(V(K)+1.)
      PLB(N)=PLB(N)*(V(L)+1.)
      HC(J)=PLA(N)+PLB(N)
  286 CONTINUE
C**UPDATE ZONAL PARAMETERS
  290 DO 300 K=1,NZNS
      L=IPRM(1,K)
      IF(L.GT.NBPAR) TRANX(K)=TRANX(K)*(V(L)+1.)
      L=IPRM(2,K)
      IF(L.GT.NBPAR) TRANY(K)=TRANY(K)*(V(L)+1.)
      L=IPRM(3,K)
      IF(L.GT.NBPAR) VLEAK(K)=VLEAK(K)*(V(L)+1.)
      L=IPRM(4,K)
      IF(L.GT.NBPAR) QDIST(K)=QDIST(K)*(V(L)+1.)
      IF(IPO.EQ.1) WRITE(IOUT,814) K,TRANX(K),TRANY(K),VLEAK(K),QDIST(K)
  300 CONTINUE
C**COMPUTE NEW HEADS AT GRID POINTS
      DO 320 N=1,NIJ
      L=IN(N)
      IF(L.LE.0) GO TO 320
      SUM=0.
      DO 310 K=1,NVAR
  310 SUM=SUM+V(K)*S(K,L)
      HC(N)=HC(N)+SUM
  320 CONTINUE
C*MODIFICATION* INTERPOLATE CALCULATED HEADS AT NODES TO
C               OBSERVATION HEAD LOCATIONS
      DO 371 N=1,NHCA
      IX=INT(HCA(1,N))
      IY=INT(HCA(2,N))
      I1=(IY-1)*ID+IX
      RESX=HCA(1,N)-IX
      RESY=HCA(2,N)-IY
      WA=HC(I1+1)-HC(I1)
      WB=HC(I1+ID)-HC(I1)
      WC=HC(I1+ID+1)+HC(I1)-HC(I1+1)-HC(I1+ID)
      WD=HC(I1)
  371 HCA(5,N)=WA*RESX+WB*RESY+WC*RESX*RESY+WD
C**CHECK FOR CONVERGENCE
      DO 330 K=1,NVAR
      IF(ABS(V(K)/AP).GT.ER) GO TO 335
  330 CONTINUE
      GO TO 350
C**  CALCULATE NEW SCALED PRIOR INFORMATION PARAMETERS
  335 IND=0
      DO 337 I=1,NVAR
      TEMP=V(I)+1.
      P(I)=P(I)/TEMP
      IF(ABS(P(I)).LT.ERP) GO TO 337
      WRITE(IOUT,858) I
      IND=1
  337 RK(I)=RK(I)*TEMP*TEMP
      IF(IND.GT.0) GO TO 360
  340 CONTINUE
      WRITE(IOUT,860) NUM
      GO TO 360
  350 WRITE(IOUT,862) KOUNT
```

```
C** COMPUTE SUM OF SQUARES AND CORRELATION COEFFICIENT
  360 SUMA=0.
C*MODIFICATION* BASED ON NON-NODAL HEAD OBSERVATIONS
      SUMB=0.
      SUMC=0.
      SUMD=0.
      SUM=0.
      DO 370 N=1,NHCA
      IF (HCA(4,N).LE.0.) GO TO 370
      TMP=HCA(4,N)**.5
      HCA(4,N)=TMP
      TEMP=TMP*HCA(3,N)
      TMP=TMP*HCA(5,N)
      SUMA=SUMA+TEMP
      SUMB=SUMB+TMP
      SUMC=SUMC+TEMP*TEMP
      SUMD=SUMD+TMP*TMP
      SUM=SUM+TEMP*TMP
  370 CONTINUE
      R=(OBS*SUM-SUMA*SUMB)/((OBS*SUMC-SUMA*SUMA)*(OBS*SUMD-SUMB*SUMB))
     1**.5
      TEMP=AP*(2.-AP)
      TMPA=2.*AP*RP/TEMP
      SUM=0.
      DO 380 I=1,NVAR
      TMP=P(I)-1.
  380 SUM=SUM+TEMP*V(I+NVAR)*(V(I+NVX2)-TMPA*V(I+NVX3)*(RPF*P(I)-1.))
     1-RK(I)*TMP*TMP
      YSQ=YSQ-SUM
C**COMPUTE SCALED VARIANCE-COVARIANCE MATRIX, ERROR VARIANCE, AND
C**OPTIMUM BIAS PARAMETER:
C**CORRECT A FOR MARQUARDT PARAMETER
      IF (NVAR.EQ.1) GO TO 435
      IF(AMP.LE.0) GO TO 386
      DO 384 I=1,NVAR
      A(I,I)=1.+RP
      DO 382 J=1,NVAR
  382 A(J,I)=A(I,J)
  384 CONTINUE
      AMP=-1.
      CALL LSTSQ(HCA,HC,P,RK,PLA,PLB,S,V,IBZN,IBPA,IBPB,IN,NVE,NHCA)
      IF(INDT.GT.0) GO TO 521
C**COMPUTE A-INVERSE
  386 A(NVAR,NVAR)=1./A(NVAR,NVAR)
      NM1=NVAR-1
      DO 430 K=1,NM1
      KP1=K+1
      DO 400 I=KP1,NVAR
      SUM=0.
      IM1=I-1
      DO 390 J=K,IM1
  390 SUM=SUM+A(I,J)*A(J,K)
      A(K,I)=-SUM
  400 A(I,K)=-SUM*A(I,I)
```

```
      SUM=A(K,J)
      DO 410 I=KP1,NVAR
  410 SUM=SUM+A(I,J)*A(K,I)
      A(K,J)=SUM
  420 A(J,K)=A(K,J)
  430 CONTINUE
      GO TO 440
  435 A(1,1)=1./(1.+RP)
  440 DO 450 J=1,NVAR
      V(J)=0.
      V(J+NVAR)=AP*V(J+NVAR)+V(J+NVX3)
      P(J)=V(J+NVX3)*P(J)
  450 A(J,NVAR)=A(NVAR,J)
C**COMPUTE TR(A-INVERSE**2) AND RPF
      TRACE=0.
      RPF=0.
      IF(RP.LE.0..AND.IRPF.LE.0) GO TO 459
      DO 457 N=1,NVAR
      DO 452 J=1,NVAR
  452 T(J)=A(J,N)
      SUMA=0.
      DO 456 J=N,NVAR
      SUM=0.
      DO 454 I=1,NVAR
  454 SUM=SUM+T(I)*A(I,J)
      SL(J)=SUM
      SUMA=SUMA+P(J)*SUM
      V(J)=V(J)+P(N)*SUM
  456 A(J,N)=A(J,N)-RP*SUM
      V(N)=V(N)+SUMA-SL(N)*P(N)
  457 TRACE=TRACE+SL(N)
      IF(IRPF.LE.0) GO TO 459
      SUM=0.
      DO 458 I=1,NVAR
      RPF=RPF+V(I)*V(I+NVAR)
  458 SUM=SUM+V(I)*P(I)
      RPF=RPF/SUM
C**COMPUTE ERROR VARIANCE AND COV(SCALED PARAMETERS)
  459 TEMP=NVAR-NPRIR
      VAR=YSQ/(OBS-TEMP+RP*RP*TRACE)
      SUM=0.
      DO 462 J=1,NVAR
      TMP=V(J+NVAR)-RPF*P(J)
      SUM=SUM+TMP*TMP
      TEMP=V(J+NVX3)
      DO 460 I=J,NVAR
      A(I,J)=VAR*A(I,J)/(V(I+NVX3)*TEMP)
  460 A(J,I)=A(I,J)
  462 V(J)=A(J,J)**.5
      TEMP=NVAR
      SUM=TEMP*VAR/SUM
C**PRINT ERROR VARIANCE, ESTIMATED SUM OF SQUARED ERRORS, CORRELATION
C**COEFFICIENT, AND OPTIMUM RIDGE PARAMETERS
      WRITE(IOUT,864) VAR,YSQ,R,SUM,RPF
C*********PRINT PARAMETERS AND STANDARD ERRORS
      IF(NBQZ.LE.0) GO TO 466
      WRITE(IOUT,866)
      DO 464 J=1,NBQZ
```

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      STDER=QBND(J)*V(J)
  464 WRITE(IOUT,856) J,QBND(J),STDER
  466 IF(NBHP.LE.0) GO TO 470
      WRITE(IOUT,868)
      DO 468 J=1,NBHP
      STDER=BH(J)*V(J+NBQZ)
  468 WRITE(IOUT,856) J,BH(J),STDER
  470 WRITE(IOUT,870)
      DO 480 J=1,NZNS
      WRITE(IOUT,814) J,TRANX(J),TRANY(J),VLEAK(J),QDIST(J)
  480 CONTINUE
      WRITE(IOUT,872)
      DO 490 J=1,NZNS
      K=IPRM(1,J)
      STERX=0.
      IF (K.GT.NBPAR) STERX=TRANX(J)*V(K)
      K=IPRM(2,J)
      STERY=0.
      IF (K.GT.NBPAR) STERY=TRANY(J)*V(K)
      K=IPRM(3,J)
      STERV=0.
      IF (K.GT.NBPAR) STERV=VLEAK(J)*V(K)
      K=IPRM(4,J)
      STERQ=0.
      IF (K.GT.NBPAR) STERQ=QDIST(J)*V(K)
      WRITE(IOUT,814) J,STERX,STERY,STERV,STERQ
  490 CONTINUE
C*MODIFICATION* MASS BALANCE CALCULATIONS
      SUMQRE=0.0
      SUMHED=0.0
      SUMLEK=0.0
      SUMQB=0.0
      SUMWELL=0.0
      N=0
      M=0
      MP1=0
C*MODIFICATION* CALCULATE WELL TOTALS
      DO 966 I=1,ID
      DO 966 J=1,JD
      MP1=MP1+1
      SUMWELL=SUMWELL+WELL(MP1)
  966 CONTINUE
      DO 900 J=1,JM
      DO 901 I=1,IM
      N=N+1
      M=M+1
      IPT=IZN(N)
      IF(IPT.LE.0)GO TO 901
      IXT1=0
      IXT2=0
      IXT3=0
      IXT4=0
      IF(NONMB.LE.0)GO TO 948
      DO 941 K=1,NONMB
      IF(M.EQ.NLMA(K))IXT1=K
      IF(M+1.EQ.NLMA(K))IXT2=K
      IF(M+ID.EQ.NLMA(K))IXT3=K
  941 IF(M+ID+1.EQ.NLMA(K))IXT4=K
```

```
  948 CONTINUE
      AREAN=DX(I)*DY(J)
      TP=TRANX(IPT)*T(N)
      TQ=TRANY(IPT)*T(N)
      SLP=VLEAK(IPT)*SL(N)*AREAN
      SUMQ=QRE(N)*QDIST(IPT)*AREAN
      IF(IN(M).LE.0)GO TO 930
      SUML=SLP*(HR(M)-HC(M))
      SUMQRE=SUMQRE+SUMQ
      SUMLEK=SUMLEK+SUML
      IF(IXT1.GT.0)XLMB(IXT1)=XLMB(IXT1)+SUML
  930 IF(IN(M+1).LE.0)GO TO 931
      SUML=SLP*(HR(M+1)-HC(M+1))
      SUMQRE=SUMQRE+SUMQ
      SUMLEK=SUMLEK+SUML
      IF(IXT2.GT.0)XLMB(IXT2)=XLMB(IXT2)+SUML
  931 IF(IN(M+ID).LE.0)GO TO 932
      SUML=SLP*(HR(M+ID)-HC(M+ID))
      SUMQRE=SUMQRE+SUMQ
      SUMLEK=SUMLEK+SUML
      IF(IXT3.GT.0)XLMB(IXT3)=XLMB(IXT3)+SUML
  932 IF(IN(M+ID+1).LE.0)GO TO 933
      SUML=SLP*(HR(M+ID+1)-HC(M+ID+1))
      SUMQRE=SUMQRE+SUMQ
      SUMLEK=SUMLEK+SUML
      IF(IXT4.GT.0)XLMB(IXT4)=XLMB(IXT4)+SUML
  933 CONTINUE
C     FIRST DIAGONAL
  910 IF(IN(M).GE.0)GO TO 902
      IF(IN(M+1).LE.0)GO TO 903
      SUMH=TP*DY(J)*(HC(M)-HC(M+1))/(DX(I)*2)
      SUMHED=SUMHED+SUMH
      IF(IXT1.GT.0)XLMB(IXT1)=XLMB(IXT1)+SUMH
  903 IF(IN(M+ID).LE.0)GO TO 902
      SUMH=TQ*DX(I)*(HC(M)-HC(M+ID))/(DY(J)*2)
      SUMHED=SUMHED+SUMH
      IF(IXT1.GT.0)XLMB(IXT1)=XLMB(IXT1)+SUMH
  902 IF(IN(M+ID+1).GE.0)GO TO 904
      IF(IN(M+1).LE.0)GO TO 905
      SUMH=TQ*DX(I)*(HC(M+ID+1)-HC(M+1))/(DY(J)*2)
      SUMHED=SUMHED+SUMH
      IF(IXT4.GT.0)XLMB(IXT4)=XLMB(IXT4)+SUMH
  905 IF(IN(M+ID).LE.0)GO TO 904
      SUMH=TP*DY(J)*(HC(M+ID+1)-HC(M+ID))/(DX(I)*2)
      SUMHED=SUMHED+SUMH
      IF(IXT4.GT.0)XLMB(IXT4)=XLMB(IXT4)+SUMH
C     SECOND DIAGONAL
  904 IF(IN(M+1).GE.0)GO TO 906
      IF(IN(M).LE.0)GO TO 907
      SUMH=TP*DY(J)*(HC(M+1)-HC(M))/(DX(I)*2)
      SUMHED=SUMHED+SUMH
      IF(IXT2.GT.0)XLMB(IXT2)=XLMB(IXT2)+SUMH
  907 IF(IN(M+ID+1).LE.0)GO TO 906
      SUMH=TQ*DX(I)*(HC(M+1)-HC(M+1+ID))/(DY(J)*2)
      SUMHED=SUMHED+SUMH
      IF(IXT2.GT.0)XLMB(IXT2)=XLMB(IXT2)+SUMH
  906 IF(IN(M+ID).GE.0)GO TO 908
      IF(IN(M).LE.0)GO TO 909
```

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      SUMH=TQ*DX(I)*(HC(M+ID)-HC(M))/(DY(J)*2)
      SUMHED=SUMHED+SUMH
       IF(IXT3.GT.0)XLMB(IXT3)=XLMB(IXT3)+SUMH
  909 IF(IN(M+1+ID).LE.0)GO TO 908
      SUMH=TP*DY(J)*(HC(M+ID)-HC(M+1+ID))/(DX(I)*2)
      SUMHED=SUMHED+SUMH
      IF(IXT3.GT.0)XLMB(IXT3)=XLMB(IXT3)+SUMH
  908 CONTINUE
  901 CONTINUE
      M=M+1
  900 CONTINUE
      IF(NQSD.LE.0)GO TO 944
      DO 946 K=1,NQSD
  946 SUMQB=SUMQB+QBND(IBZN(K))*QBF(K)*2
  944 CONTINUE
      TOTALS=SUMQRE+SUMHED+SUMLEK+SUMWELL+SUMQB
      WRITE(IOUT,999)TOTALS,SUMQRE,SUMHED,SUMLEK,SUMWELL,SUMQB
  999 FORMAT(1X,"MASS BALANCE",G10.4,5X,"SUMQRE",G10.4,5X,
     1"SUMHED",G10.4,5X,"SUMLEK",G10.4,5X,"SUMWELL",G10.4,"SUMQB",G10.4)
      WRITE(IOUT,997)
      IF(NONMB.LE.0)GO TO 947
      DO 995 I=1,NONMB
  995 WRITE(IOUT,996)NLMA(I),NLMB(I*2-1),NLMB(I*2),XLMB(I)
     1,HC(NLMA(I)),HR(NLMA(I))
  947 CONTINUE
  997 FORMAT(1X,"SEQUENCE NO.",5X,"COLUMN",5X,"ROW",6X,"LEAKAGE OR CONS
     1TANT HEAD FLUX",4X,"COMPUTED HEAD",2X,"FIXED HEAD")
  996 FORMAT(6X,I5,8X,I5,3X,I5,5X,G10.4,12X,G10.4,12X,G10.4,12X,G10.4
     1,19X,G10.4)
      WRITE(IOUT,961)
      CALL ARRAY(HC,ID,JD,0,1)
  961 FORMAT(1H0,1X,15HFINAL SOLUTION:)
C*********PRINT SCALED VARIANCE-COVARIANCE MATRIX
      WRITE(IOUT,874)
      CALL PRTOT(A,NVE,0)
C*********COMPUTE AND PRINT CORRELATION MATRIX FOR PARAMETERS
      DO 510 J=1,NVAR
      TEMP=V(J)
      DO 500 I=J,NVAR
      A(I,J)=A(I,J)/(V(I)*TEMP)
  500 A(J,I)=A(I,J)
  510 CONTINUE
      WRITE(IOUT,876)
      CALL PRTOT(A,NVE,0)
C*********COMPUTE AND PRINT RESIDUALS
C*MODIFICATION* BASED ON NON-NODAL HEAD OBSERVATIONS
      WRITE(IOUT,878)
      N=0
      RESMN=0.0
      RESABS=0.0
      XINTX=1.0
      YINTY=1.0
      CONIN=100.0
      NCITU=0
      DO 520 N=1,NHCA
      RES=HCA(4,N)*(HCA(5,N)-HCA(3,N))
      RESMN=RESNM+RES
      RESABS=RESABS+ABS(RES)
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      WRITE(IOUT,880) HCA(1,N),HCA(2,N),HCA(5,N),HCA(3,N),RES
      WRITE(4,958)HCA(1,N),HCA(2,N),RES
  520 CONTINUE
      TRESM=RESMN/NHCA
      TRESA=RESABS/NHCA
      WRITE(IOUT,943)TRESM,TRESA
  943 FORMAT(1X,"MEAN RESIDUAL",F10.2,2X,"ABSOLUTE MEAN RESIDUAL",F10.2)
      NDW=0
  958 FORMAT(3F10.2)
      DO 978 J=1,JD
      DO 978 I=1,ID
      NDW=NDW+1
      IF(IN(NDW).EQ.0)GO TO 978
      NCITU=NCITU+1
  978 CONTINUE
      WRITE(5,800)NCITU
      WRITE(5,957)XINTX,YINTY,CONIN
      N=0
      DO 956 J=1,JD
      DO 956 I=1,ID
      N=N+1
      IF(IN(N).EQ.0)GO TO 956
      XIX=FLOAT(I)
      YIY=FLOAT(J)
      WRITE(5,957)XIX,YIY,HC(N)
  956 CONTINUE
  957 FORMAT(2F5.1,F10.2)
C**PRINT SCALED SENSITIVITIES FOR EACH NODE
  521 IF(IPRX.LE.0.AND.KOUNT.LT.NUM) STOP
      WRITE(IOUT,882)
      DO 530 KK=1,NVAR
      WRITE(IOUT,884) KK
      DO 525 N=1,NIJ
      HC(N)=0.
      L=IN(N)
      IF(L.GT.0) GO TO 523
      IF(L.GE.-1) GO TO 525
      L=-L-1
      IZ=IBZN(L+NQSD)
      IF(IBPB(IZ).EQ.KK) HC(N)=PLB(L)
      IF(IBPA(IZ).EQ.KK) HC(N)=PLA(L)
      GO TO 525
  523 HC(N)=S(KK,L)
  525 CONTINUE
  530 CALL ARRAY(HC,ID,JD,0,1)
      IF(NVAR.LT.2) STOP
C**ORTHOGONALIZE SENSITIVITY MATRIX (S):
C**DEFINE I AND J POINTERS
C*MODIFICATION* BASED ON NON-NODAL HEAD OBSERVATIONS
      N=0
      DO 531 J=1,JD
      DO 531 I=1,ID
      N=N+1
      K=IN(N)
      IF(K.LE.0) GO TO 531
      ILOC(K)=I
      JLOC(K)=J
  531 CONTINUE
```

```
C**DEFINE DUMMY WEIGHT MATRIX
      DO 541 I=1,NIJ
  541 DUM(I)=0.0
      DO 542 I=1,NHCA
      IF(HCA(4,I).LT.1.E-10)GO TO 542
      IXI=INT(HCA(1,I)+.5)
      JYJ=INT(HCA(2,I)+.5)
      NUML=IXI+(JYJ-1)*ID
      IF(HCA(4,I).GT.DUM(NUML))DUM(NUML)=HCA(4,I)
  542 CONTINUE
C**COMPRESS S
      N=0
      DO 533 L=1,NEQ
      J=JLOC(L)
      K=ILOC(L)+JPOS(J)
      IF(DUM(K).LE.1.E-10) GO TO 533
      N=N+1
      ILOC(N)=ILOC(L)
      JLOC(N)=JLOC(L)
      DO 532 J=1,NVAR
  532 S(J,N)=S(J,L)*DUM(K)
  533 CONTINUE
      NOOB=N
      IF(IBHZ.LE.0) GO TO 536
      KK=0
      DO 535 J=1,JD
      DO 535 I=1,ID
      KK=KK+1
      NN=IN(KK)
      IF(NN.GE.-1.OR.DUM(KK).LE.1.E-10) GO TO 535
      N=N+1
      ILOC(N)=I
      JLOC(N)=J
      DO 534 K=1,NVAR
  534 S(K,N)=0.
      NN=-NN-1
      M=IBZN(NN+NQSD)
      K=IBPA(M)
      L=IBPB(M)
      S(L,N)=PLB(NN)*DUM(KK)
      S(K,N)=PLA(NN)*DUM(KK)
  535 CONTINUE
  536 IF(NPRIR.LE.0) GO TO 539
      DO 538 I=1,NVAR
      IF(RK(I).LT.1.E-10) GO TO 538
      N=N+1
      ILOC(N)=I
      JLOC(N)=0
      DO 537 J=1,NVAR
  537 S(J,N)=0.
      S(I,N)=RK(I)**.5
  538 CONTINUE
C**ORTHOGONALIZE S
  539 NTMP=NOOB+NPRIR
      DO 540 I=1,NTMP
  540 DUM(I)=S(1,I)
      DO 600 N=2,NVAR
      NM1=N-1
```

```
      SUM=0.
      DO 550 I=1,NTMP
      SUM=SUM+DUM(I)*DUM(I)
      S(NM1,I)=DUM(I)
  550 CONTINUE
      IF(SUM.LT.1.E-20) GO TO 610
      V(NM1)=1./SUM
      DO 570 J=1,NM1
      SUM=0.
      DO 560 K=1,NTMP
  560 SUM=SUM+V(J)*S(J,K)*S(N,K)
  570 T(J)=SUM
      DO 590 K=1,NTMP
      SUM=0.
      DO 580 I=1,NM1
  580 SUM=SUM+S(I,K)*T(I)
  590 DUM(K)=S(N,K)-SUM
  600 CONTINUE
C**PRINT ORTHOGONALIZED S
  610 WRITE(IOUT,886)
      K=1
      L=8
      DO 630 M=1,NVAR,8
      IF(L.GT.NVAR) L=NVAR
      WRITE(IOUT,888) (I,I=K,L)
      DO 620 J=1,NTMP
      S(NVAR,J)=DUM(J)
      WRITE(IOUT,890) ILOC(J),JLOC(J),(S(I,J),I=K,L)
  620 CONTINUE
      K=K+8
      L=L+8
  630 CONTINUE
      STOP
C**READ, PRINT, AND EXECUTE FOR ALTERNATE SOLUTIONS
  640 READ(IIN,800) N
      IF(N.LE.0) STOP
      DO 690 K=1,N
      WRITE(IOUT,892) K
      DO 650 L=1,NZNS
      READ(IIN,812) I,TRANX(I),TRANY(I),VLEAK(I),QDIST(I)
      WRITE(8,814)I,TRANX(I),TRANY(I),VLEAK(I),QDIST(I)
  650 WRITE(IOUT,814) I,TRANX(I),TRANY(I),VLEAK(I),QDIST(I)
      IF(NBQZ.LE.0) GO TO 665
      DO 660 L=1,NBQZ
  660 READ(IIN,812) I,QBND(I)
      WRITE(IOUT,894)
      CALL PRTOT(QBND,NBQZ,1)
  665 IF(NBHP.LE.0) GO TO 685
      DO 670 L=1,NBHP
  670 READ(IIN,812) I,BH(I)
      WRITE(IOUT,896)
      CALL PRTOT(BH,NBHP,1)
      DO 680 KK=1,NIJ
      NN=IN(KK)
      IF(NN.GE.-1) GO TO 680
      NN=-NN-1
      M=IBZN(NN+NQSD)
      L=IBPA(M)
```

```
      TMPA=BH(L)*PLA(NN)
      L=IBPB(M)
      TMPB=BH(L)*PLB(NN)
      HC(KK)=TMPA+TMPB
  680 CONTINUE
  685 CALL COEF(DX,DY,T,SL,QRE,WELL,HR,HC,TRANX,TRANY,VLEAK,QDIST,QBND
     1,QBF,PLA,PLB,S,V,IZN,IBZN,IPRM,IBPA,IBPB,IN,IC,NVE)
      CALL D4SOLVE(HC,V,IN,IC,HCA,NHCA,ID,JD,ISO)
      WRITE(IOUT,898) K
  690 CONTINUE
      STOP
C
  800 FORMAT (16I5)
  801 FORMAT (20A4)
  802 FORMAT(25H0NUMBER OF COLUMNS (ID) =,I5
     $/22H NUMBER OF ROWS (JD) =,I5
     $/39H NUMBER OF EQUIPARAMETER ZONES (NZNS) =,I5
     $/43H NUMBER OF POINT SOURCES OR SINKS (NWELS) =,I5
     $/42H NUMBER OF SPECIFIED FLOW ZONES (NQBND) = ,I5
     $/50H NUMBER OF SPECIFIED FLOW PARAMETER ZONES (NBQZ) =,I5
     $/40H NUMBER OF SPECIFIED HEAD ZONES (NBHZ) =,I5
     $/45H NUMBER OF SPECIFIED HEAD PARAMETERS (NBHP) =,I5
     $/44H NUMBER OF REGRESSION PARAMETERS OTHER THAN
     $/42H SPECIFIED HEAD OR SPECIFIED FLOW (NPAR) =,I5
     $/37H MAXIMUM NUMBER OF ITERATIONS (NUM) =,I5
     $/56H SENSITIVITY PRINT AND ORTHOGONALIZATION OPTION (IPRX) =,I5
     $/51H OPTION TO ESTIMATE OPTIMUM BIAS PARAMETER (IRPF) =,I5
     $/35H ADDITIONAL PRINTOUT OPTION (IPO) =,I5
     $/29H SOLUTION ONLY OPTION (ISO) =,I5)
  803 FORMAT (1H ,20A4)
  804 FORMAT (1H1)
  806 FORMAT (46H ACCELERATION PARAMETER FOR REGRESSION (AP) = ,G11.4
     $/44H MARQUARDT PARAMETER FOR REGRESSION (AMP) = ,G11.4
     $/39H RIDGE PARAMETER FOR REGRESSION (RP) = ,G11.4
     $/39H BIAS PARAMETER FOR REGRESSION (RPF) = ,G11.4
     $/33H ESTIMATED ERROR VARIANCE (EV) = ,G11.4)
  810 FORMAT (1H0,15X,26HINITIAL PARAMETERS BY ZONE/6H  ZONE,4X,5HTRANX
     1,8X,5HTRANY,8X,5HVLEAK,8X,5HQDIST)
  812 FORMAT (I5,4F10.0)
  814 FORMAT (1H ,I4,4(2X,E11.4))
  816 FORMAT (1H0,15X,17HPARAMETER NUMBERS/1H ,5X,4HZONE,4X,5HTRANX,3X
     1,5HTRANY,3X,5HVLEAK,3X,5HQDIST)
  818 FORMAT (1H ,8I8)
  820 FORMAT (8F10.0)
  822 FORMAT (1H0,16X,40HCOEFFICIENTS OF VARIATION FOR PARAMETERS
     1/1H ,3(2X,6HPARAM.,7X,5HCOEF.,4X)/1H ,3(4X,3HNO.,9X,4HVAR.,4X))
  824 FORMAT (1H0,4X,25HPOINT SOURCE OR SINK DATA/1H ,7X,1HI,7X,1HJ
     1,6X,4HRATE)
  826 FORMAT (2I5,F10.0)
  828 FORMAT (1H ,2I8,2X,E11.4)
  830 FORMAT (1H0,22X,27HINITIAL SPECIFIED FLOW DATA/1H ,6X,9HNODE NO.S
     1,7X,4HZONE,6X,4HFLOW,8X,5HCOEF./1H ,19H  IA   JA   IB   JB,4X
     2,3HNO.,3X,9HPARAMETER,7X,4HVAR.,6X,10HMULTIPLIER)
  831 FORMAT (1H ,4(1X,I3,1X),2X,I3,1X,3(2X,E11.4))
  832 FORMAT (5I5,3F10.0)
  833 FORMAT (1H0,17X,27HINITIAL SPECIFIED HEAD DATA)
  834 FORMAT(4I5,2F10.0)
  836 FORMAT (1H0,22H  NO. OF NODES IN ZONE,I4,3H = ,I3/1H ,15H  NO. PAR
```

```
     1. A = ,I3,17X,13HNO. PAR. B = ,I3/1H ,22H  COEF. VAR. PAR. A =
     2,E11.4,22H  COEF. PAR. PAR. B = ,E11.4/1H ,20X,22HINITIAL VALUES O
     3F HEAD/1H ,21X,1HI,5X,1HJ,7X,4HHEAD)
  840 FORMAT (1H ,19X,2(I3,3X),E11.4)
  842 FORMAT(9H0AT CELL ,2I5,15H, IZN>0 AND T=0)
  844 FORMAT(54H0PROGRAM ABORTED BECAUSE OF CONFLICT BETWEEN IZN AND T)
  846 FORMAT (1H0,1X,17HINITIAL SOLUTION:)
  848 FORMAT (23H0NO. OF OBSERVATIONS = ,I4/46H NO. OF PARAMETERS HAVING
     1 PRIOR INFORMATION = ,I4//56H ESTIMATED SUM OF SQUARED ERRORS FOR
     2INITIAL SOLUTION = ,E12.5/39H ERROR VARIANCE FOR INITIAL SOLUTION
     3= ,E12.5)
  850 FORMAT(1H0,5X,58HNODAL LOCATION AND SCALED SENSITIVITIES FOR EACH
     1PARAMETER)
  852 FORMAT(1H ,2(1X,I4),7(1X,G11.5)/1H ,(10X,7(1X,G11.5)))
  854 FORMAT(19H0UPDATED PARAMETERS)
  856 FORMAT (1H ,1X,I4,3X,G11.5,4X,G11.5)
  858 FORMAT (11H0PARAMETER ,I3,17H EFFECTIVELY ZERO)
  860 FORMAT (//32H0SOLUTION FAILED TO CONVERGE IN ,I3,11H ITERATIONS)
  862 FORMAT (//23H0SOLUTION CONVERGED IN ,I3,11H ITERATIONS)
  864 FORMAT (18H0ERROR VARIANCE = ,E12.5/35H ESTIMATED SUM OF SQUARED E
     1RRORS = ,E12.5/27H CORRELATION COEFFICIENT = ,F6.4/27H OPTIMUM RID
     2GE PARAMETER = ,E12.5/26H OPTIMUM BIAS PARAMETER = ,E12.5)
  866 FORMAT(1H0,1X,35HESTIMATED SPECIFIED FLOW PARAMETERS
     1/7H   ZONE,5X,4HQBND,9X,8HSTD. ER.)
  868 FORMAT (1H0,5X,25HESTIMATED SPECIFIED HEADS/1H ,6H  PAR./1H ,3X
     1,3HNO.,5X,4HHEAD,9X,8HSTD. ER.)
  870 FORMAT (1H0,14X,28HESTIMATED PARAMETERS BY ZONE/6H  ZONE,4X
     1,5HTRANX,8X,5HTRANY,8X,5HVLEAK,8X,5HQDIST)
  872 FORMAT (1H0,13X,33HESTIMATED STANDARD ERRORS BY ZONE/1H ,8X
     1,8HSTD. ER.,5X,8HSTD. ER.,5X,8HSTD. ER.,5X,8HSTD. ER./6H  ZONE,4X
     2,5HTRANX,8X,5HTRANY,8X,5HVLEAK,8X,5HQDIST)
  874 FORMAT (34H0SCALED VARIANCE-COVARIANCE MATRIX)
  876 FORMAT (34H0CORRELATION MATRIX FOR PARAMETERS)
  878 FORMAT (1H0,21X,18HTABLE OF RESIDUALS/1H ,5X,1HI,5X,1HJ,5X
     1,9HPREDICTED,10X,6HACTUAL,12X,8HWEIGHTED/1H ,19X,5HVALUE,12X
     2,5HVALUE,13X,8HRESIDUAL)
  880 FORMAT (1H ,1X,2(1X,FS.2),3(4X,G13.6))
  882 FORMAT (26H0SCALED SENSITIVITY ARRAYS)
  884 FORMAT(18H0PARAMETER NUMBER ,I5)
  886 FORMAT(1H0,5X,33HORTHOGONALIZED SENSITIVITY MATRIX
     1/1H ,45H NODAL LOCATION AND VALUES FOR EACH PARAMETER)
  888 FORMAT (1H ,14X,15HPARAMETER NOS.:/1H ,4X,1HI,4X,1HJ,5X,8(I3,9X))
  890 FORMAT (1H ,2(1X,I4),8(1X,G11.5))
  892 FORMAT (1H0,7X,39HPARAMETERS FOR ADDITIONAL SOLUTION NO. ,I3
     1/6H  ZONE,4X,5HTRANX,8X,5HTRANY,8X,5HVLEAK,8X,5HQDIST)
  894 FORMAT (1H0,27X,19HSPECIFIED FLOW DATA/1H ,3X,3(4HFLOW,5X
     1,9HSPECIFIED,6X)/1H ,3X,3(4HZONE,8X,4HFLOW,8X))
  896 FORMAT (1H0,9X,52HSPECIFIED HEAD PARAMETERS, FRACTIONAL CHANGE IN
     1HEAD/1H ,3(8H  PARAM.,4X,9HSPECIFIED,3X)/1H ,3(4X,3HNO.,8X,4HHEAD
     2,5X))
  898 FORMAT(13H0SOLUTION NO.,I5)
      END
      SUBROUTINE ARRAY(A,IND,JND,N,IT)
C**IF IT=0, SUBROUTINE FOR LOADING 1 AND 2 DIMENSIONAL ARRAYS
C**IF IT=1, SUBROUTINE FOR PRINTING 2 DIMENSIONAL ARRAYS
      DIMENSION A(IND,JND),NME(8)
```

```
      COMMON/TNME/IIN,IOUT
      DATA NME/4HDX   ,4HDY   ,4HHO   ,4HW    ,4HT    ,4HSL   ,4HHR   ,4HQRE /
      IF(IT.EQ.1) GO TO 55
      DO 5 J=1,JND
      DO 5 I=1,IND
    5 A(I,J)=0.
      WRITE(IOUT,105)
      READ(IIN,70) NOZN,IPRN
      DO 50 K=1,NOZN
      READ(IIN,70) IB,IE,JB,JE,FACT,IVAR
      WRITE(IOUT,80) K,IB,IE,JB,JE,NME(N),FACT
      IF (IVAR.EQ.1) GO TO 20
      DO 10 J=JB,JE
      DO 10 I=IB,IE
   10 A(I,J)=FACT
      GO TO 50
   20 DO 40 J=JB,JE
      READ(IIN,90) (A(I,J),I=IB,IE)
      DO 40 I=IB,IE
   40 A(I,J)=A(I,J)*FACT
   50 CONTINUE
      IF (IPRN.EQ.1) RETURN
      WRITE(IOUT,100) NME(N)
   55 DO 60 K=1,IND,10
      I10=K+9
      IF(I10.GT.IND) I10=IND
      WRITE(IOUT,110) (I,I=K,I10)
      WRITE(IOUT,105)
      DO 60 J=1,JND
      JR=JND-J+1
   60 WRITE(IOUT,120) JR,(A(I,JR),I=K,I10)
      RETURN
C
   70 FORMAT (4I5,F10.0,3I5)
   80 FORMAT (1H ,I3,2X,5HIB = ,I5,2X,5HIE = ,I5,2X,5HJB = ,I5,2X
     1,5HJE = ,I5,2X,A4,2H = ,G12.5)
   90 FORMAT (8F10.0)
  100 FORMAT (1H0,2X,A4,7H ARRAY:)
  105 FORMAT (1H )
  110 FORMAT (1H0,10X,10(I3,9X))
  120 FORMAT (1H ,1X,I3,1X,10(1X,G11.5))
      END
      SUBROUTINE ARRAYI(INT,IND,JND,N,IT)
C**IF IT=0, SUBROUTINE FOR LOADING 1 AND 2 DIMENSIONAL INTEGER ARRAYS
C**IF IT=1, SUBROUTINE FOR PRINTING 2 DIMENSIONAL INTEGER ARRAYS
      DIMENSION INT(IND,JND),NME(2)
      COMMON/TNME/IIN,IOUT
      DATA NME/4HIZN ,4HIBZN/
      IF(IT.EQ.1) GO TO 45
      DO 5 J=1,JND
      DO 5 I=1,IND
    5 INT(I,J)=0
      WRITE(IOUT,100)
      READ(IIN,60) NOZN,IPRN
      DO 40 K=1,NOZN
      READ(IIN,60) IB,IE,JB,JE,IFACT,IVAR
      WRITE(IOUT,70) K,IB,IE,JB,JE,NME(N),IFACT
      IF(IVAR.EQ.1) GO TO 20
```

```
      DO 10 J=JB,JE
      DO 10 I=IB,IE
   10 INT(I,J)=IFACT
      GO TO 40
   20 DO 30 J=JB,JE
      READ(IIN,60) (INT(I,J),I=IB,IE)
   30 CONTINUE
   40 CONTINUE
      IF(IPRN.EQ.1) RETURN
      WRITE(IOUT,80) NME(N)
   45 DO 50 K=1,IND,30
      I30=K+29
      IF(I30.GT.IND) I30=IND
      WRITE(IOUT,90) (I,I=K,I30)
      WRITE(IOUT,100)
      DO 50 J=1,JND
      JR=JND-J+1
   50 WRITE(IOUT,110) JR,(INT(I,JR),I=K,I30)
      RETURN
C
   60 FORMAT (16I5)
   70 FORMAT (1H ,I3,2X,5HIB = ,I5,2X,5HIE = ,I5,2X,5HJB = ,I5,2X
     1,5HJE = ,I5,2X,A4,2H = ,I5)
   80 FORMAT (1H0,2X,A4,7H ARRAY:)
   90 FORMAT (1H0,4X,30(1X,I3))
  100 FORMAT (1H )
  110 FORMAT (1H ,31(1X,I3))
      END
      SUBROUTINE ORDER(JPOS,IN,IC)
      DIMENSION JPOS(1),IN(1),IC(5,1)
      COMMON/INT/NIJ,NEQ,ICR,ICR1,IB1,LH1,ID,JD
      COMMON/TNME/IIN,IOUT
C*****COMPUTE EQUATION NUMBERS FOR D4 ORDERING:
      NXP=ID+JD-1
      K=0
C*****ORDER--LEFT TO RIGHT, BOTTOM TO TOP
      DO 20 I=1,NXP,2
      DO 20 J=1,JD
      IK=I-J+1
      IF(IK.LT.1.OR.IK.GT.ID) GO TO 20
      N=IK+JPOS(J)
      IF(IN(N).LE.0) GO TO 20
      K=K+1
      IN(N)=K
   20 CONTINUE
      ICR=K+1
      DO 30 I=2,NXP,2
      DO 30 J=1,JD
      IK=I-J+1
      IF(IK.LT.1.OR.IK.GT.ID) GO TO 30
      N=IK+JPOS(J)
      IF(IN(N).LE.0) GO TO 30
      K=K+1
      IN(N)=K
   30 CONTINUE
C*****COMPUTE BANDWIDTH AND DETERMINE CONNECTING EQUATION NUMBERS:
      MNO=9999
      MXO=0
```

```
      N=0
      DO 80 J=1,JD
      DO 80 I=1,ID
      N=N+1
      JR=IN(N)
      IF (JR.LE.0.OR.JR.GE.ICR) GO TO 80
      IU=1
C**   BELOW
      IF ((J-1).LT.1) GO TO 40
      IF (IN(N-ID).LE.0) GO TO 40
      IU=IU+1
      IC(IU,JR)=IN(N-ID)
      MM=IN(N-ID)-JR
      MXO=MAX0(MM,MXO)
      MNO=MIN0(MM,MNO)
C**   LEFT
   40 IF ((I-1).LT.1) GO TO 50
      IF (IN(N-1).LE.0) GO TO 50
      IU=IU+1
      IC(IU,JR)=IN(N-1)
      MM=IN(N-1)-JR
      MNO=MIN0(MM,MNO)
      MXO=MAX0(MM,MXO)
C**   RIGHT
   50 IF ((I+1).GT.ID) GO TO 60
      IF (IN(N+1).LE.0) GO TO 60
      IU=IU+1
      IC(IU,JR)=IN(N+1)
      MM=IN(N+1)-JR
      MXO=MAX0(MM,MXO)
      MNO=MIN0(MM,MNO)
C**   ABOVE
   60 IF ((J+1).GT.JD) GO TO 70
      IF (IN(N+ID).LE.0) GO TO 70
      IU=IU+1
      IC(IU,JR)=IN(N+ID)
      MM=IN(N+ID)-JR
      MXO=MAX0(MM,MXO)
      MNO=MIN0(MM,MNO)
   70 IC(1,JR)=IU
   80 CONTINUE
      NEQ=K
      ICR1=ICR-1
      IB1=MXO-MNO+1
      LH1=NEQ-ICR1
      WRITE(IOUT,90)
      WRITE(IOUT,100) ICR1,IB1,LH1,ICR1,NEQ
      RETURN
C
   90 FORMAT (51H0SOLUTION BY LDU FACTORIZATION ASSUMING D4 ORDERING)
  100 FORMAT (82H ****WARNING*****MINIMUM DIMENSIONS FOR ARRAYS USED BY
     1THIS METHOD ARE AS FOLLOWS:/1H ,4H AU:,8H     5 BY,I5/1H ,4H AL:
     2,I5,3H BY,I5/1H ,4H IC:,8H     5 BY,I5/1H ,4H  B:,I5)
      END
      SUBROUTINE COEF(DX,DY,T,SL,QRE,WELL,HR,HC,TRANX,TRANY,VLEAK,QDIST
     1,QBND,QBF,PLA,PLB,S,B,IZN,IBZN,IPRM,IBPA,IBPB,IN,IC,NVE)
      DIMENSION DX(1),DY(1),T(1),SL(1),QRE(1),WELL(1),HR(1),HC(1)
     1,TRANX(1),TRANY(1),VLEAK(1),QDIST(1),QBND(1),QBF(1),PLA(1),PLB(1)
```

```
     1,S(NVE,1),B(1)
      DIMENSION IZN(1),IBZN(1),IPRM(4,NVE),IBPA(1),IBPB(1),IN(1)
     1,IC(5,1)
      DIMENSION AU(5,550),AL(30,500)
      COMMON/INT/NIJ,NEQ,ICR,ICR1,IB1,LH1,ID,JD,IM,JM,NVAR,NQSD,NBPAR
      COMMON/FLT/AU,AL
C**INITIALIZE ARRAYS
      DO 10 J=1,ICR1
      DO 10 I=1,5
   10 AU(I,J)=0.
      DO 20 J=1,LH1
      DO 20 I=1,IB1
   20 AL(I,J)=0.
      DO 40 I=1,NIJ
      N=IN(I)
      IF(N.LE.0) GO TO 40
      DO 30 K=1,NVAR
   30 S(K,N)=0.
      B(N)=WELL(I)
   40 CONTINUE
C**CALCULATE -DF/DA*A AND B FOR SPECIFIED FLOW PARAMETERS
      IF(NQSD.LE.0) GO TO 52
      DO 50 I=1,NQSD
      IZ=IBZN(I)
      TMP=QBND(IZ)*QBF(I)
      INA=IBPA(I)
      L=IN(INA)
      IF(L.LE.0) GO TO 44
      S(IZ,L)=S(IZ,L)+TMP
      B(L)=B(L)+TMP
   44 INB=IBPB(I)
      L=IN(INB)
      IF(L.LE.0) GO TO 50
      S(IZ,L)=S(IZ,L)+TMP
      B(L)=B(L)+TMP
   50 CONTINUE
C**BEGIN MAIN LOOP
   52 N=0
      DO 150 J=1,JM
      DYN=DY(J)
      DO 150 I=1,IM
      N=N+1
      M=IZN(N)
      IF(M.LE.0) GO TO 150
      NB=N+J
      NA=NB-1
      NC=NB+ID
      ND=NA+ID
      INA=IN(NA)
      INB=IN(NB)
      INC=IN(NC)
      IND=IN(ND)
      TX=TRANX(M)*T(N)
      TY=TRANY(M)*T(N)
      DXN=DX(I)
      CX=TX*DYN/(DXN+DXN)
      CY=TY*DXN/(DYN+DYN)
      AREA=DXN*DYN
```

```
      VL=VLEAK(M)*SL(N)*AREA
      QRT=QDIST(M)*QRE(N)*AREA
      E=CX+CY+VL
C**CALCULATE AU, AL, B, AND -DF/DA*A FOR TRANSMISSIVITIES, LEAKANCES
C**AND SPECIFIED FLOWS
      L1=IPRM(1,M)
      L2=IPRM(2,M)
      L3=IPRM(3,M)
      L4=IPRM(4,M)
      K=-INA-1
      IF(K) 60,75,53
   53 IZ=IBZN(K+NQSD)
      I1=IBPA(IZ)
      I2=IBPB(IZ)
      IF(INB.LE.0) GO TO 55
      S(I1,INB)=S(I1,INB)+CX*PLA(K)
      S(I2,INB)=S(I2,INB)+CX*PLB(K)
   55 IF(IND.LE.0) GO TO 75
      S(I1,IND)=S(I1,IND)+CY*PLA(K)
      S(I2,IND)=S(I2,IND)+CY*PLB(K)
      GO TO 75
   60 CXT=CX*(HC(NB)-HC(NA))
      CYT=CY*(HC(ND)-HC(NA))
      VLT=VL*(HR(NA)-HC(NA))
      IF(L1.GT.NBPAR) S(L1,INA)=S(L1,INA)+CXT
      IF(L2.GT.NBPAR) S(L2,INA)=S(L2,INA)+CYT
      IF(L3.GT.NBPAR) S(L3,INA)=S(L3,INA)+VLT
      IF(L4.GT.NBPAR) S(L4,INA)=S(L4,INA)+QRT
      IF(INA.GE.ICR) GO TO 65
      AU(1,INA)=AU(1,INA)+E
      AU(4,INA)=AU(4,INA)-CX
      AU(5,INA)=AU(5,INA)-CY
      GO TO 70
   65 AL(1,INA-ICR1)=AL(1,INA-ICR1)+E
   70 B(INA)=B(INA)+QRT+VLT+CXT+CYT
   75 K=-INB-1
      IF(K) 85,100,77
   77 IZ=IBZN(K+NQSD)
      I1=IBPA(IZ)
      I2=IBPB(IZ)
      IF(INA.LE.0) GO TO 80
      S(I1,INA)=S(I1,INA)+CX*PLA(K)
      S(I2,INA)=S(I2,INA)+CX*PLB(K)
   80 IF(INC.LE.0) GO TO 100
      S(I1,INC)=S(I1,INC)+CY*PLA(K)
      S(I2,INC)=S(I2,INC)+CY*PLB(K)
      GO TO 100
   85 CXT=CX*(HC(NA)-HC(NB))
      CYT=CY*(HC(NC)-HC(NB))
      VLT=VL*(HR(NB)-HC(NB))
      IF(L1.GT.NBPAR) S(L1,INB)=S(L1,INB)+CXT
      IF(L2.GT.NBPAR) S(L2,INB)=S(L2,INB)+CYT
      IF(L3.GT.NBPAR) S(L3,INB)=S(L3,INB)+VLT
      IF(L4.GT.NBPAR) S(L4,INB)=S(L4,INB)+QRT
      IF(INB.GE.ICR) GO TO 90
      AU(1,INB)=AU(1,INB)+E
      AU(3,INB)=AU(3,INB)-CX
      AU(5,INB)=AU(5,INB)-CY
```

```
      GO TO 95
   90 AL(1,INB-ICR1)=AL(1,INB-ICR1)+E
   95 B(INB)=B(INB)+QRT+VLT+CXT+CYT
  100 K=-INC-1
      IF(K) 110,125,102
  102 IZ=IBZN(K+NQSD)
      I1=IBPA(IZ)
      I2=IBPB(IZ)
      IF(IND.LE.0) GO TO 105
      S(I1,IND)=S(I1,IND)+CX*PLA(K)
      S(I2,IND)=S(I2,IND)+CX*PLB(K)
  105 IF(INB.LE.0) GO TO 125
      S(I1,INB)=S(I1,INB)+CY*PLA(K)
      S(I2,INB)=S(I2,INB)+CY*PLB(K)
      GO TO 125
  110 CXT=CX*(HC(ND)-HC(NC))
      CYT=CY*(HC(NB)-HC(NC))
      VLT=VL*(HR(NC)-HC(NC))
      IF(L1.GT.NBPAR) S(L1,INC)=S(L1,INC)+CXT
      IF(L2.GT.NBPAR) S(L2,INC)=S(L2,INC)+CYT
      IF(L3.GT.NBPAR) S(L3,INC)=S(L3,INC)+VLT
      IF(L4.GT.NBPAR) S(L4,INC)=S(L4,INC)+QRT
      IF(INC.GE.ICR) GO TO 115
      AU(1,INC)=AU(1,INC)+E
      AU(2,INC)=AU(2,INC)-CY
      AU(3,INC)=AU(3,INC)-CX
      GO TO 120
  115 AL(1,INC-ICR1)=AL(1,INC-ICR1)+E
  120 B(INC)=B(INC)+QRT+VLT+CXT+CYT
  125 K=-IND-1
      IF(K) 135,150,127
  127 IZ=IBZN(K+NQSD)
      I1=IBPA(IZ)
      I2=IBPB(IZ)
      IF(INC.LE.0) GO TO 130
      S(I1,INC)=S(I1,INC)+CX*PLA(K)
      S(I2,INC)=S(I2,INC)+CX*PLB(K)
  130 IF(INA.LE.0) GO TO 150
      S(I1,INA)=S(I1,INA)+CY*PLA(K)
      S(I2,INA)=S(I2,INA)+CY*PLB(K)
      GO TO 150
  135 CXT=CX*(HC(NC)-HC(ND))
      CYT=CY*(HC(NA)-HC(ND))
      VLT=VL*(HR(ND)-HC(ND))
      IF(L1.GT.NBPAR) S(L1,IND)=S(L1,IND)+CXT
      IF(L2.GT.NBPAR) S(L2,IND)=S(L2,IND)+CYT
      IF(L3.GT.NBPAR) S(L3,IND)=S(L3,IND)+VLT
      IF(L4.GT.NBPAR) S(L4,IND)=S(L4,IND)+QRT
      IF(IND.GE.ICR) GO TO 140
      AU(1,IND)=AU(1,IND)+E
      AU(2,IND)=AU(2,IND)-CY
      AU(4,IND)=AU(4,IND)-CX
      GO TO 145
  140 AL(1,IND-ICR1)=AL(1,IND-ICR1)+E
  145 B(IND)=B(IND)+QRT+VLT+CXT+CYT
  150 CONTINUE
C**COMPRESS AU
      N=0
```

```
      DO 190 J=1,JD
      DO 190 I=1,ID
      N=N+1
      K=IN(N)
      IF(K.LE.0.OR.K.GE.ICR) GO TO 190
      IF(IC(1,K).EQ.5) GO TO 190
      IU=1
      IF((J-1).LT.1) GO TO 160
      IF(IN(N-ID).LE.0) GO TO 160
      IU=IU+1
      AU(IU,K)=AU(2,K)
  160 IF((I-1).LT.1) GO TO 170
      IF(IN(N-1).LE.0) GO TO 170
      IU=IU+1
      AU(IU,K)=AU(3,K)
  170 IF((I+1).GT.ID) GO TO 180
      IF(IN(N+1).LE.0) GO TO 180
      IU=IU+1
      AU(IU,K)=AU(4,K)
  180 IF((J+1).GT.JD) GO TO 190
      IF(IN(N+ID).LE.0) GO TO 190
      IU=IU+1
      AU(IU,K)=AU(5,K)
  190 CONTINUE
      RETURN
      END
      SUBROUTINE D4SOLVE(HC,B,IN,IC,HCA,NHCA,ID,JD,ISO)
      DIMENSION HCA(5,1100)
      DIMENSION HC(1),B(1)
      DIMENSION IN(1),IC(5,1)
      DIMENSION HC(1),B(1)
      DIMENSION IN(1),IC(5,1)
      DIMENSION AU(5,550),AL(30,500)
      COMMON/INT/NIJ,NEQ,ICR,ICR1,IB1
      COMMON/FLT/AU,AL
C*****ELIMINATE TO FILL AL

      DO 280 J=1,ICR1
      II=IC(1,J)
      DO 270 I=2,II
      LR=IC(I,J)
      L=LR-ICR1
      C=AU(I,J)/AU(1,J)
      DO 260 K=I,II
      KL=IC(K,J)-LR+1
      AL(KL,L)=AL(KL,L)-C*AU(K,J)
  260 CONTINUE
      AU(I,J)=C
      B(LR)=B(LR)-C*B(J)
  270 CONTINUE
  280 B(J)=B(J)/AU(1,J)
C*****ELIMINATE AL
      JJ=NEQ-ICR
      DO 310 J=1,JJ
      JR=J+ICR1
      L=J
      DO 300 I=2,IB1
      L=L+1
```

```
      IF (AL(I,J).EQ.0.) GO TO 300
      LR=L+ICR1
      C=AL(I,J)/AL(1,J)
      KL=0
      DO 290 K=I,IB1
      KL=KL+1
      IF (AL(K,J).NE.0.) AL(KL,L)=AL(KL,L)-C*AL(K,J)
  290 CONTINUE
      AL(I,J)=C
      B(LR)=B(LR)-C*B(JR)
  300 CONTINUE
  310 B(JR)=B(JR)/AL(1,J)
C*****BACK SOLVE--LOWER HALF
      B(NEQ)=B(NEQ)/AL(1,NEQ-ICR1)
      DO 330 J=1,JJ
      K=NEQ-J
      KL=K-ICR1
      L=K
      DO 320 I=2,IB1
      L=L+1
      IF (AL(I,KL).NE.0.) B(K)=B(K)-AL(I,KL)*B(L)
  320 CONTINUE
  330 CONTINUE
C*****BACK SOLVE--UPPER HALF
      DO 350 J=1,ICR1
      K=ICR-J
      II=IC(1,K)
      DO 340 I=2,II
      L=IC(I,K)
      B(K)=B(K)-AU(I,K)*B(L)
  340 CONTINUE
  350 CONTINUE
C*****COMPUTE HC + DELTHC
      DO 360 N=1,NIJ
      L=IN(N)
      IF(L.LE.0) GO TO 360
      HC(N)=HC(N)+B(L)
  360 CONTINUE
C*MODIFICATION* CALCULATE NEW HEADS AT OBSERVATION POINTS
      DO 370 N=1,NHCA
      IX=INT(HCA(1,N))
      IY=INT(HCA(2,N))
      I1=(IY-1)*ID+IX
      RESX=HCA(1,N)-IX
      RESY=HCA(2,N)-IY
      WA=HC(I1+1)-HC(I1)
      WB=HC(I1+ID)-HC(I1)
      WC=HC(I1+ID+1)+HC(I1)-HC(I1+1)-HC(I1+ID)
      WD=HC(I1)
  370 HCA(5,N)=WA*RESX+WB*RESY+WC*RESX*RESY+WD
      IF(ISO.GT.0)WRITE(8,900)(HCA(5,N),N=1,NHCA)
  900 FORMAT(8F10.2)
      RETURN
      END
      SUBROUTINE LSTSQ(HCA,HC,P,RK,PLA,PLB,S,B,IBZN,IBPA,IBPB,IN,NVE
     1,NHCA)
       DIMENSION HC(1),P(1),RK(1),PLA(1),PLB(1),S(NVE,1),B(4)
       DIMENSION HCA(5,1100),ST(60)
```

```
       DIMENSION IBZN(1),IBPA(1),IBPB(1),IN(1)
       DIMENSION AU(5,550),AL(30,500),C(20,20)
      COMMON/INT/NIJ,NEQ,ICR,ICR1,IB1,LH1,ID,JD,IM,JM,NVAR,NQSD,NBPAR
     1,NVX2,NVX3,IPO,KOUNT,INDT,IBHZ
      COMMON/TNME/IIN,IOUT
      COMMON/FLT/AU,AL,AP,AMP,RP,RPF,YSQ
      EQUIVALENCE (AL(1,1),C(1,1))
C**       CHECK FOR NONZERO MARQUARDT PARAMETER
      IF(AMP.LT.-.5) GO TO 105
C**       INITIALIZE
      DO 20 J=1,NVAR
      DO 10 I=1,NVAR
   10 C(I,J)=0.
   20 B(J+NVX2)=0.
      YSQ=0.
      NUMSUM=0.0
C**       FORM COEFFICIENT MATRIX AND RIGHT-HAND SIDE VECTOR
C*MODIFICATION* INTERPOLATE TO NON-NODAL HEAD OBSERVATION POINTS
      DO 70 N=1,NHCA
      IF(HCA(4,N).LE.0.0)GO TO 70
      IX=INT(HCA(1,N))
      IY=INT(HCA(2,N))
      RESX=HCA(1,N)-IX
      RESY=HCA(2,N)-IY
      I1=(IY-1)*ID+IX
      IF(RESX.NE.0.0.OR.RESY.NE.0.0)GO TO 15
      IF(IN(I1).LE.0)GO TO 70
      DO 17 I=1,NVAR
   17 ST(I)=S(I,IN(I1))
      GO TO 16
   15 CONTINUE
      K1=IN(I1)
      K2=IN(I1+1)
      K3=IN(I1+ID)
      K4=IN(I1+ID+1)
      IF(K1.LE.0)GO TO 70
      IF(K2.LE.0)GO TO 70
      IF(K3.LE.0)GO TO 70
      IF(K4.LE.0)GO TO 70
      DO 71 I=1,NVAR
      WA=S(I,K2)-S(I,K1)
      WB=S(I,K3)-S(I,K1)
      WC=S(I,K4)+S(I,K1)-S(I,K2)-S(I,K3)
      WD=S(I,K1)
   71 ST(I)=WA*RESX+WB*RESY+WC*RESX*RESY+WD
   16 CONTINUE
C*MODIFICATION* PRINT INTERPOLATED SENSITIVITIES TO TAPE B
      IF(IPO.EQ.-1)WRITE(8,900)(ST(I),I=1,NVAR)
  900 FORMAT(8F15.5)
      NUMSUM=NUMSUM+1
      TEMP=HCA(3,N)-HCA(5,N)
      DO 60 J=1,NVAR
      TMP=HCA(4,N)*ST(J)
      DO 50 I=J,NVAR
   50 C(I,J)=TMP*ST(I)+C(I,J)
   60 B(J+NVX2)=TMP*TEMP+B(J+NVX2)
```

```
      YSQ=YSQ+TEMP*TEMP*HCA(4,N)
   70 CONTINUE
      WRITE(IOUT,11)NHCA,NUMSUM
   11 FORMAT(5X,"NHCA",I5,5X,"NUMSUM",I5)
      IF(IBHZ.LE.0) GO TO 74
      DO 73 J=1,NIJ
      N=IN(J)
      IF(N.GE.-1.) GO TO 73
      DO 75 I=1,NHCA
      IX=INT(HCA(1,I))
      IY=INT(HCA(2,I))
      I1=(IY-1)*ID+IX
   75 IF(I1.EQ.J)MX=I
      N=-N-1
      M=IBZN(N+NQSD)
      K=IBPA(M)
      L=IBPB(M)
      TMPA=PLA(N)*HCA(4,MX)
      TMPB=PLB(N)*HCA(4,MX)
      C(K,K)=C(K,K)+PLA(N)*TMPA
      C(L,L)=C(L,L)+PLB(N)*TMPB
      TMP=TMPA*PLB(N)
      C(K,L)=C(K,L)+TMP
      C(L,K)=C(L,K)+TMP
      TMPC=HCA(3,MX)-HC(J)
      B(K+NVX2)=B(K+NVX2)+TMPC*TMPA
      B(L+NVX2)=B(L+NVX2)+TMPC*TMPB
      YSQ=YSQ+TMPC*TMPC*HCA(4,MX)
   73 CONTINUE
   74 IF (NVAR.EQ.1) GO TO 190
      DO 80 I=1,NVAR
      TEMP=C(I,I)+RK(I)
      IF(TEMP.GT.1.E-10) GO TO 78
      WRITE(IOUT,260) I
      INDT=1
      GO TO 80
   78 C(I,I)=TEMP**.5
   80 CONTINUE
      IF(INDT.GT.0) RETURN
      NM1=NVAR-1
      DO 100 J=1,NM1
      TEMP=C(J,J)
      JP1=J+1
      DO 90 I=JP1,NVAR
      C(I,J)=C(I,J)/(C(I,I)*TEMP)
   90 C(J,I)=C(I,J)
      B(J+NVX2)=(B(J+NVX2)+RK(J)*(P(J)-1.))/TEMP+RP*TEMP*(RPF*P(J)-1.)
      B(J)=B(J+NVX2)
      B(J+NVX3)=TEMP
  100 C(J,J)=1.+RP+AMP
      TEMP=C(NVAR,NVAR)
      B(NVX3)=(B(NVX3)+RK(NVAR)*(P(NVAR)-1.))/TEMP
     1+RP*TEMP*(RPF*P(NVAR)-1.)
      B(NVAR)=B(NVX3)
      B(NVAR+NVX3)=TEMP
      C(NVAR,NVAR)=1.+RP+AMP
      IF(IPO.NE.1) GO TO 105
      WRITE(IOUT,250)
```

```
      CALL PRTOT(C,NVE,0)
      WRITE(IOUT,230) (B(I+NVX2),I=1,NVAR)
C********SOLVE FOR B USING LDU FACTORIZATION:
C**        DECOMPOSITION AND FORWARD SUBSTITUTION
  105 DET=1.
      DO 140 K=1,NM1
      PIV=C(K,K)
      DET=DET*PIV
      IF(ABS(PIV).GT.1.E-10) GO TO 110
      WRITE(IOUT,210)
      INDT=1
      RETURN
  110 PIV=1./PIV
      KP1=K+1
      DO 130 J=KP1,NVAR
      TMP=C(J,K)*PIV
      DO 120 I=J,NVAR
  120 C(I,J)=C(I,J)-TMP*C(I,K)
  130 B(J)=B(J)-TMP*B(K)
      C(K,K)=PIV
  140 CONTINUE
      DET=DET*C(NVAR,NVAR)
      IF(ABS(C(NVAR,NVAR)).GT.1.E-10) GO TO 150
      WRITE(IOUT,210)
      INDT=1
      RETURN
  150 IF(AMP.LT.-.5) RETURN
C**        BACK SUBSTITUTION
      B(NVX2)=B(NVAR)/C(NVAR,NVAR)
      B(NVAR)=B(NVX2)/B(NVAR+NVX3)
      I=NVAR
  160 I=I-1
      IF (I.LE.0) GO TO 200
      IP1=I+1
      SUM=0.
      DO 170 J=IP1,NVAR
  170 SUM=SUM+C(J,I)*B(J+NVAR)
      B(I+NVAR)=(B(I)-SUM)*C(I,I)
      B(I)=B(I+NVAR)/B(I+NVX3)
      GO TO 160
C**        SOLUTION WHEN NVAR=1
  190 TEMP=C(1,1)+RK(1)
      IF(TEMP.GT.1.E-10) GO TO 195
      I=1
      WRITE(IOUT,260) I
      INDT=1
      RETURN
  195 B(4)=TEMP**.5
      B(3)=(B(3)+RK(1)*(P(1)-1.))/B(4)+RP*B(4)*(RPF*P(1)-1.)
      C(1,1)=1.+RP+AMP
      DET=C(1,1)
      B(2)=B(3)/C(1,1)
      B(1)=B(2)/B(4)
C**        ADJUST AND PRINT REGRESSION COEFFICIENTS
  200 WRITE(IOUT,220) KOUNT,YSQ,DET
      DO 202 J=1,NVAR
  202 B(J)=AP*B(J)
      WRITE(IOUT,230) (B(J),J=1,NVAR)
```

```
      RETURN
C
  210 FORMAT (43H0LEAST SQUARES COEFFICIENT MATRIX SINGULAR;/35H SOLUTIO
     1N FOR PARAMETERS NOT UNIQUE)
  220 FORMAT (1H0,14HITERATION NO. ,I3/1H ,6HYSQ = ,E12.5,2X
     1,9HDET(C) = ,E12.5/1H ,24HREGRESSION COEFFICIENTS:)
  230 FORMAT ((1H ,8(E12.5,2X)))
  250 FORMAT(49H0 SCALED LEAST SQUARES MATRIX AND GRADIENT VECTOR)
  260 FORMAT (29H0SENSITIVITIES FOR PARAMETER ,I4,17H EFFECTIVELY ZERO)
      END
      SUBROUTINE PRTOT(C,NO,IT)
C**IF IT=0, PRINT MATRICES DIVIDED VERTICALLY INTO BETWEEN ONE AND TEN PARTS
C**IF IT=1, PRINT VECTOR IN THREE COLUMNS
      DIMENSION C(1)
      COMMON/INT/NIJ,NEQ,ICR,ICR1,IB1,LH1,ID,JD,IM,JM,NUM
      COMMON/TNME/IIN,IOUT
      IF(IT.EQ.1) GO TO 25
      ITMP=(NUM-1)/10+1
      IB=1
      DO 20 IBK=1,ITMP
      INC=IBK*10
      IF(NUM.LT.INC) INC=NUM
      WRITE(IOUT,30) (I,I=IB,INC)
      WRITE(IOUT,50)
      K=-NO
      DO 10 J=1,NUM
      K=K+NO
   10 WRITE(IOUT,40) J,(C(I+K),I=IB,INC)
      WRITE(IOUT,60)
      IB=INC+1
   20 CONTINUE
      RETURN
   25 NR=NO/3
      IF((3*NR).NE.NO) NR=NR+1
      DO 26 K=1,NR
   26 WRITE(IOUT,80) (L,C(L),L=K,NO,NR)
      RETURN
C
   30 FORMAT (1H0,8X,I3,9(9X,I3))
   40 FORMAT (1H ,I3,10(1X,E11.4))
   50 FORMAT (1H )
   60 FORMAT (1H0)
   80 FORMAT(1H ,3X,3(I3,5X,E11.4,5X))
      END
```

764-204



GPO 687-049/45012