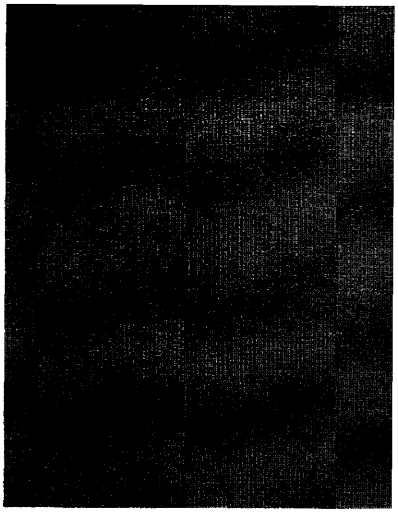
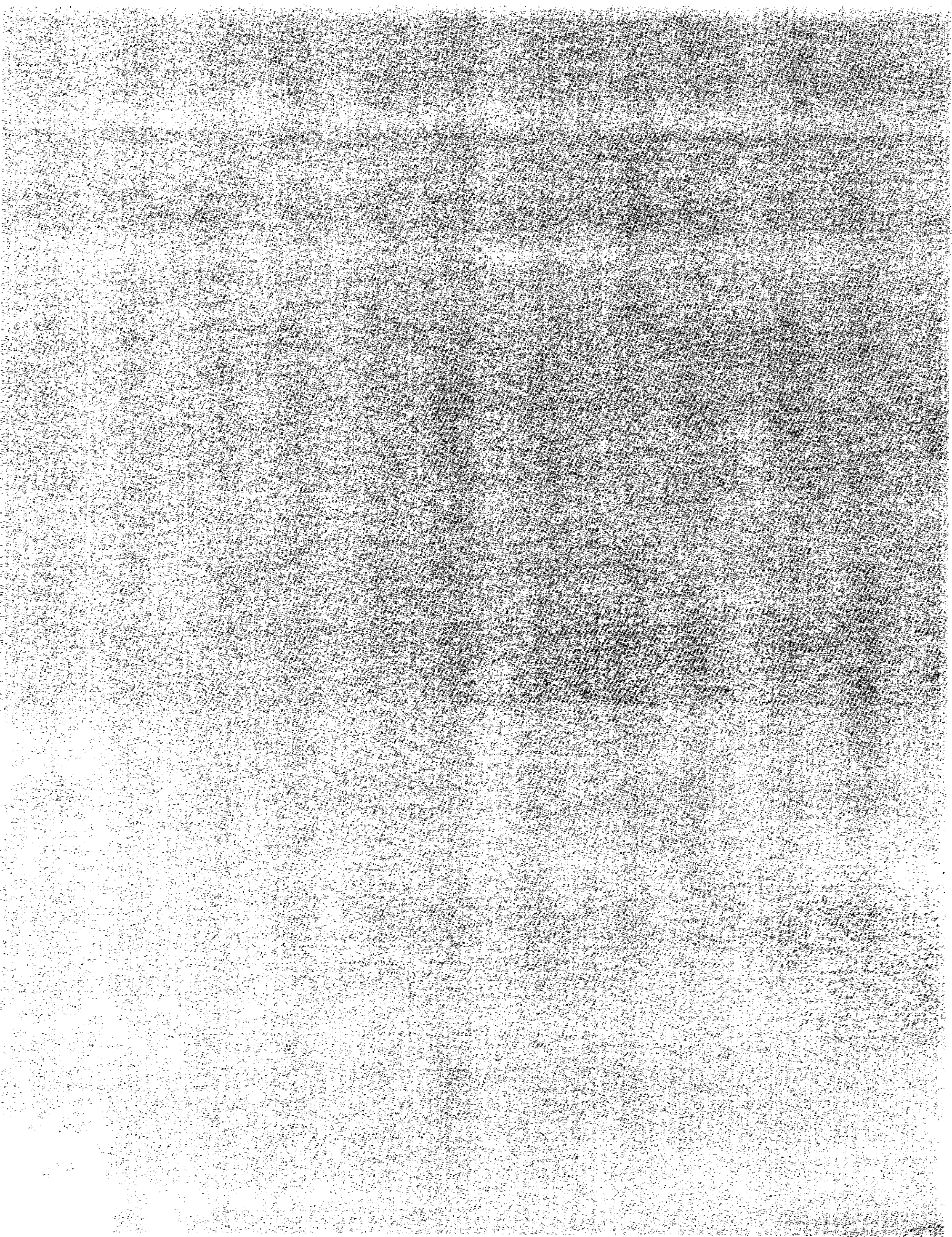
Using Geophysical Logs to Estimate Porosity, Water Resistivity, and Intrinsic Permeability





United States
Geological
Survey
Water-Supply
Paper 2321

U.S. DEPARTMENT OF THE INTERIOR
March 3, 1849

Using Geophysical Logs to Estimate Porosity, Water Resistivity, and Intrinsic Permeability

By DONALD G. JORGENSEN

U.S. GEOLOGICAL SURVEY WATER-SUPPLY PAPER 2321

DEPARTMENT OF THE INTERIOR
DONALD PAUL HODEL, Secretary

U.S. GEOLOGICAL SURVEY
Dallas L. Peck, Director

UNITED STATES GOVERNMENT PRINTING OFFICE: 1989

For sale by the
Books and Open-File Reports Section
U.S. Geological Survey
Federal Center
Box 25425
Denver, CO 80225

Library of Congress Cataloging-in-Publication Data

Jorgensen, Donald G.
Using geophysical logs to estimate porosity, water resistivity, and intrinsic permeability.

(U.S. Geological Survey Water-Supply Paper ; 2321)
Bibliography: p.
Supt. of Docs. no.: I 19.13:2321
1. Water, Underground—Methodology. 2. Geophysical well logging.
3. Porosity—Measurement. 4. Groundwater flow—Measurement.
5. Rocks—Permeability—Measurement.
I. Title. II. Series.
GB1001.72.G44J67 1988 622'.187 87-600025

CONTENTS

FIGURES

TABLES

NOMENCLATURE

A	Coefficient relating dissolved-solids concentration to sodium-chloride concentration.
API	Units defined by the American Petroleum Institute.
BHT	Bottom-hole temperature in a borehole, in units specified.
C_{ds}	Concentration of dissolved solids in units specified.
C_{NaCl}	Concentration of sodium chloride, in units specified.
***C*(Ŧ)**	Tortuosity function.
Dt	Total depth of borehole, in feet.
DS	Dissolved-solids concentration, in units specified.
F	Formation factor, dimensionless.
G	Specific gravity, dimensionless.
HR	Hydraulic radius, in units of length specified.
K	Constant at a specified absolute temperature relating to spontaneous potential, in millivolts.
K	Hydraulic conductivity, in feet per day.
L	Length of sample, in feet.
L_e	Effective length of flow path, in feet.
Ro	Resistivity of water-rock system, in ohm-meters.
Rm	Resistivity of mud, in ohm-meters.
Rmf	Resistivity of mud filtrate, in ohm-meters.
Rmfe	Resistivity of mud-filtrate equivalent, in ohm-meters.
Rw	Resistivity of water equivalent, in ohm-meters.
Rw_{77}	Resistivity of water at 77°F, in ohm-meters.
Rw_{75}	Resistivity of water at 75°F, in ohm-meters.
SC	Specific conductance, in microsiemens.
SF	Geometric shape factor, dimensionless.
S_s	Specific surface area of solids per unit volume of solids, in feet^{-1}.
S_p	Specific surface, in feet^{-1}.
SP	Spontaneous potential, in millivolts.
T_a	Absolute temperature, in degrees Rankine.
T_c	Temperature, in degrees Celsius.
T_f	Temperature, in degrees Fahrenheit.
Tf	Temperature of formation, in degrees Fahrenheit
Tma	Mean annual temperature, in degrees Fahrenheit.
Tmf	Temperature of mud filtrate, in degrees Fahrenheit.
Є	Kozeny coefficient relates to pore geometry, dimensionless.
Ᵽ	Porosity factor, dimensionless.
Ŧ	Tortuosity factor, dimensionless.
a	Empirical constant related to lithology, dimensionless.
Δd	Change of density in grams per cubic centimeter.
d	Density, in units specified of mass per unit volume.
d_b	Bulk density, which is density of matrix and fluid, in units specified of mass per unit volume.

d_f	Density of fluid, in units specified of mass per unit volume.
d_{ma}	Density of matrix, in units specified of mass per unit volume.
g	Acceleration due to gravity, in specified units of length per unit of time squared.
k	Intrinsic permeability, in millidarcies.
n	Porosity, dimensionless.
n_d	Apparent porosity from a density log, dimensionless.
n_n	Apparent porosity from a neutron log, dimensionless.
m	Cementation factor, dimensionless.
m^*	Factor related to the number of reductions of the pore-size openings, dimensionless.
p	Pressure, in Pascal.
r^2	Coefficient of determination, dimensionless.
μ	Dynamic viscosity, in units specified units of mass per length-time.

CONVERSION FACTORS

Factors to convert measurements in units other than the International System of Units (SI units) are presented. SI units are modernized metric units. The unit of time is s (second). The unit of mass is kg (kilogram). The unit of length is m (meter). The unit of force is N (newton) and is that force which gives a mass of 1 kilogram an acceleration of 1 meter per second per second. The unit of pressure or stress is Pa (pascal), which is 1 newton per square meter.

SI units may use the following prefixes:

tera	T	10^{12}
giga	G	10^{9}
mega	M	10^{6}
kilo	k	10^{3}
milli	m	10^{-3}
micro	μ	10^{-6}
nano	n	10^{-9}
pico	p	10^{-12}

The following conversions may be useful to hydrologists, geologists, and geophysicists:

acceleration due to gravity (g) = 9.807 m/s^2
centipoise (cp) = 1.000×10^{-3} Pa.s (pascal second)
degree Fahrenheit (°F) = (1.8 T_c + 32)
foot per day (ft/day) = 2.633×10^{4} m/s
foot per second·second (ft/s^2) = 3.048×10^{-1} m/s^2
foot (ft) = 3.048×10^{-1} m
gallon per day per square foot [(gal/d)/ft^2] = 4.716×10^{-7} m/s
grams per cubic centimeter (g/cm^3) = 1.000×10^{3} kg/m^3
inch (in.) = 2.540×10^{-2} m
kilogram per cubic centimeter (kg/cm^3) = 9.807×10^{4} Pa
millidarcy (mD) = 9.87×10^{-15} m^2
milligram per liter (mg/L) = 1.000×10^{-3} kg/m^3
millimho = millisiemen
parts per million (ppm) = milligrams per liter/relative density
percent salinity of sodium chloride = 10,000 ppm NaCl
pounds per square foot (lb/ft^2 or psf) = 4.788×10^{1} Pa
pounds per square foot (lb/in^2 or psi) = 6.895×10^{3} Pa
square foot (ft^2) = 9.290×10^{-2} m^2

Using Geophysical Logs to Estimate Porosity, Water Resistivity, and Intrinsic Permeability

By Donald G. Jorgensen

Abstract

Geophysical logs can be used to estimate porosity, formation-water resistivity, and intrinsic permeability for geohydrologic investigations, especially in areas where measurements of these geohydrologic properties are not available. The dual density and neutron porosity logs plus the gamma-ray log can be used to determine in-situ porosity and to qualitatively define lithology. Either a spontaneous-potential log or a resistivity log can be used to estimate relative resistivity of the formation water.

The spontaneous-potential and the cross-plot methods were tested for their usefulness as estimators of resistivity of the formation water. The spontaneous-potential method uses measurements of spontaneous potential and mud-filtrate resistivity to estimate the formation-water resistivity. The cross-plot method uses porosity values and observed resistivity of saturated rock to estimate the formation-water resistivity. Neither method was an accurate estimator. However, in areas of no data the methods can be used with caution.

A review of the literature of the basic relations for formation factor (F), porosity (n), and cementation factor (m) implies that the empirical Archie equation

$$F = n^{-m}$$

is applicable to carbonates, coarse-grained clastics, and uniformly fractured media. The relation of F, n, and m to intrinsic permeability (k) was also investigated. Merging the well-known Archie equation with the Kozeny equation establishes an equation for estimating intrinsic permeability. The resulting equation implies that intrinsic permeability is the function of a medium factor, ($\in/S_s{}^2$), and a porosity factor ₽, $[n^{m+2}/(1-n)^2]$. Where $\in$ is the Kozeny coefficient, S_s is the specific surface of grains per unit of volume of solids. Unfortunately, $\in$ and S_s values generally are impossible to determine from wireline geophysical logs.

A plot of the porosity factor as a function of intrinsic permeability defined the following empirical equation for k, in millidarcies:

$$k = 1.828 \times 10^5 \; (\text{₽}^{1.10}),$$

where

$$\text{₽} = (n^{m+2})/((1-n)^2).$$

This equation for k was based on a selected data base of 10 sets, which included in-place measurements of permeability, in-place measurements of porosity from two different types of porosity logs, and a measurement of bulk resistivity of the rock and water from a resistivity log. The equation has a coefficient of determination (r^2) of 0.90. The relation is applicable to permeable rocks in which surface conductance along grains is not dominant, such as most carbonate rocks, fractured rocks, and coarse clastics. Calculated permeability values for different lithologies using typical values of porosity and cementation factors compare well with typical permeability values of the different lithologies. Some data were available to support the equation, but considerably more data will be needed to better test the equation established for k.

INTRODUCTION

The problem of estimating rock, fluid, and hydraulic properties becomes more important as hydrologists are asked to solve problems related to ground-water flow in rock material about which little is known. For example, most studies of flow systems in deeply buried formations have little or no hydraulic data because those data are generally obtained from water wells, which are rare in deeply buried aquifers or aquifers containing saline water. Therefore, hydraulic property values of deep aquifers commonly are estimated using sparse and indirect data. Available analytical methods have been developed by petroleum engineers or geologists and are not widely known to hydrologists who study ground-water flow systems.

The purpose of this paper is to describe selected techniques of using borehole-geophysical logs for water-resources investigations. The procedure for each technique, the assumptions upon which it is based, and measured data for comparison and evaluation are presented for each technique. Special emphasis is given to techniques for estimating intrinsic permeability and water resistivity. Discussion of porosity and lithology is limited to those techniques directly related to the techniques used for determining permeability and water resistivity.

This paper is directed to hydrologists investigating water resources in aquifers for which hydraulic data are sparse, such as most deeply buried rock sections, and assumes a cursory knowledge of geophysical logging. Hydraulic property values are expressed in their customary units. Calculated estimates are compared with measured values allowing evaluation of the results. In addition, methods of estimating in-situ water-quality properties, such as dissolved-solids concentration, sodium-chloride concentration, density, and viscosity are discussed in the appendixes.

Water resistivity is a measure of the resistance of a unit volume of water to electric flow and is related to water chemistry and temperature. Water that has a low concentration of dissolved solids has a high electrical resistance, whereas water that has a high concentration of dissolved solids has a low resistance. Quantitative techniques are available for identifying water resistivity—a characteristic related to water chemistry. The relation of dissolved solids and common chemical constituents to water resistivity commonly is known or can be determined. (See appendixes B and C.)

Intrinsic permeability, usually called permeability, is a measure of the relative ease with which a medium (rocks) can transmit a liquid under a potential gradient; it is a property of the medium alone. The cross-plot method determines intrinsic permeability from borehole geophysical logs using the cementation factor.

Methods of estimating water resistivity and permeability in aquifers of a porous medium whose freshwater contains less than about 700 mg/L dissolved-solids (Huntley, 1986, p. 469) were described by Biella and others (1983), Jones and Buford (1951), Alger (1966), Pfannkuch (1969), Worthington (1976), and Urish (1981). The methods usually require detailed information about the water chemistry and about the nature of the porous medium. For example, to estimate permeability of a coarse-grained porous medium, the average grain size and the uniformity coefficient, as well as the surface conductance of the water molecules surrounding the grains must be known. Those data are seldom known.

ESTIMATING POROSITY AND LITHOLOGY

Because the techniques discussed rely on data about porosity, lithology, and rock and water resistivity from wireline-geophysical logs, some discussion of wireline geophysical logs with special reference to water is relevant. A method allowing a "quick" qualitative lithology interpretation is available that uses the same borehole-geophysical logs used to determine porosity and to estimate water resistivity and permeability.

A suitable geophysical log for determining porosity and lithology combines a compensated-neutron porosity log and a compensated formation-density log, especially if traces of both are printed on the same log chart as is shown in figure 1 (right side). The combination log is termed a "dual-porosity log" in this paper. Figure 1 also shows a gamma-ray trace and a caliper trace (both left side). The gamma-ray log supplements the dual-porosity log in lithologic interpretations. The dual-porosity log is especially powerful because not only are porosity values recorded, but also the position of the density trace with respect to the neutron trace generally indicates lithology. For example, if the density trace is to the right of the neutron trace on a dual-porosity log calibrated to limestone, either shale or dolostone or dolomite is implied. The gamma-ray trace defines the shale sequence; thus, shale and dolomite can be easily differentiated (fig. 2). If the density trace is close to the neutron trace, calcite, such as limestone, is indicated. If the density trace is to the left of the neutron trace, silica, such as sandstone and chert, or a gas effect is indicated (fig. 2). As an example, figure 1 shows the top of a sandstone that underlies a dolostone at 2,000 ft.

Quantitative techniques are available for identifying simple lithologies and for correcting for shaly conditions. Most of the techniques use neutron, density (gamma-gamma), dielectric, and sonic logs to define porosity for cross-plotting methods. These techniques are described in log interpretation texts and manuals; some uses of these techniques are presented by MacCary (1978).

Before the compensated-porosity logs were available, porosity was determined using uncompensated sonic, gamma-gamma, and neutron logs. Interpretations were difficult because of unknown lithologic differences and variations in hole size. MacCary (1983) discussed how to use uncompensated and (or) uncalibrated porosity or other geophysical logs in investigating carbonate aquifers. The resistivity versus porosity cross-plotting method can be used with these logs.

Porosity values determined from sonic logs normally are assumed to represent nonfracture porosity. Therefore, fracture porosity may be estimated if the porosity determined from sonic logs is subtracted from total porosity determined from compensated dual-porosity logs. However, porosity values from some sonic logs sometimes exceed total porosity values. Thus, the assumption that sonic-porosity values represent only nonfractured porosity cannot be made without additional information about conditions in the hole or about shaliness. Likewise, because the fracture porosity is usually small,

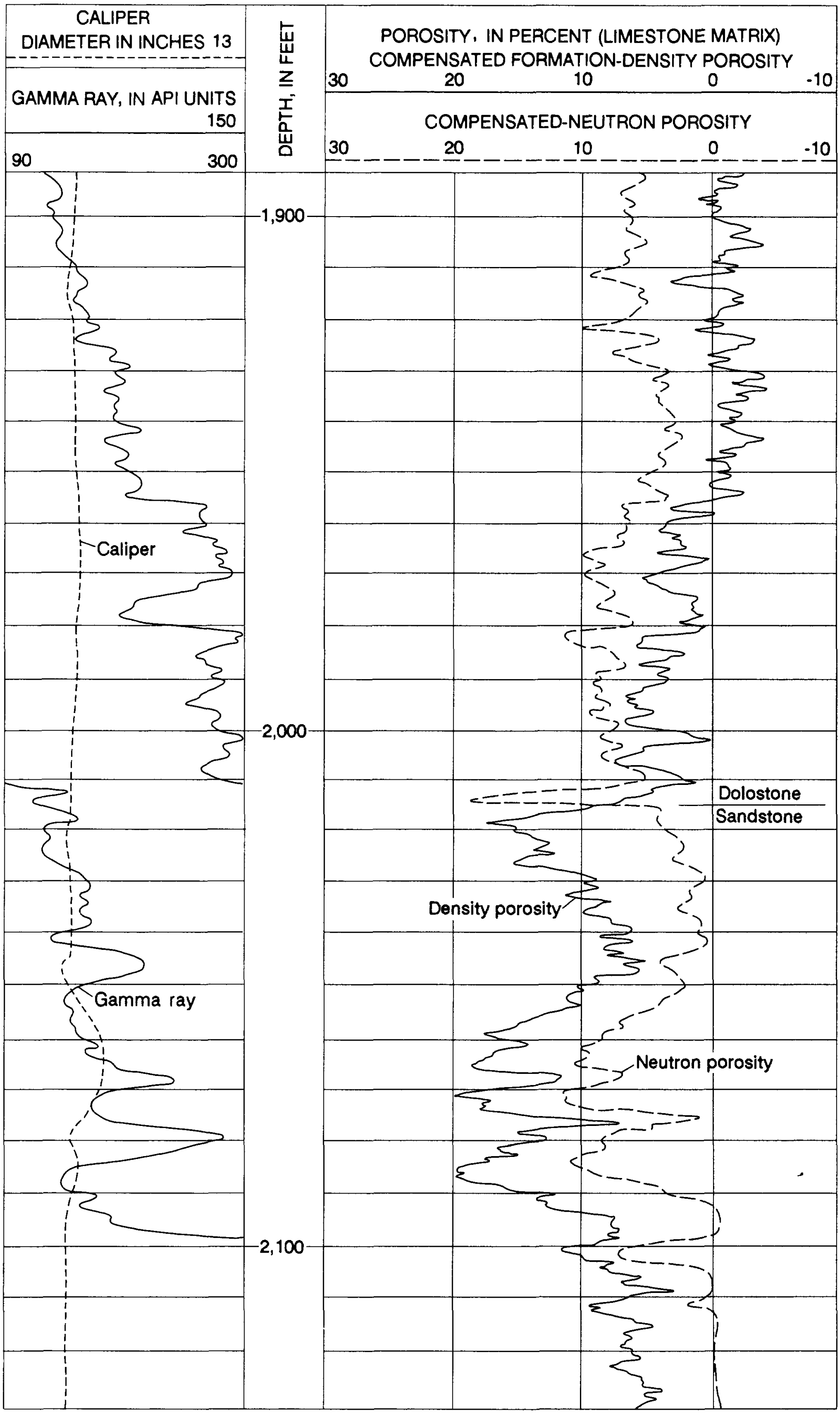


Figure 1. Dual-porosity, gamma-ray, and caliper logs.

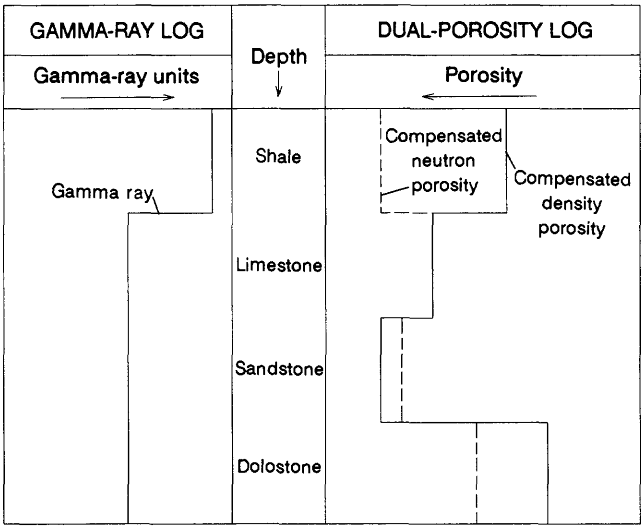


Figure 2. Idealized dual-porosity log calibrated to limestone matrix and gamma-ray log.

slight errors in either or both porosity logs can make fracture porosity impossible to detect.

ESTIMATING WATER RESISTIVITY

Resistivity and spontaneous-potential logs contain information useful in estimating water resistivity. Resistivity logs are the most widely used and most commonly available type of geophysical log. A resistivity log records resistance of the electrical current flow in rock with depth. There are many types of resistivity logs; most variations refer to the specific method of measuring, such as a lateral log, an induction log, or a conductivity log. A typical resistivity log is shown in figure 3.

The spontaneous-potential log records the spontaneous potential (*SP*) of the fluid-filled borehole (fig. 3). The *SP* measured is largely an electromotive potential between the mud filtrate, the water within the rock, or the adjacent saturated rock materials.

The combined *SP* log and resistivity log is common and is here called an electric log. A hypothetical electric log of a sandstone and shale sequence is shown on figure 4. Each deeper sandstone contains water of increased salinity. Two resistivity traces are shown—a deep and a shallow trace. Deep implies that the resistivity measured represents material at some distance from the well bore. In addition, deep resistivity measurements generally represent undisturbed formation water and rock. The shallow trace measures the resistivity of the material adjacent to the borehole. This shallow resistivity is most likely to be affected by the invading drilling fluid. The following methods can be used to estimate water resistivity.

Qualitative Methods

Two little-known but easy to apply methods for qualitatively estimating relative water resistivity use the resistivity log and the spontaneous-potential log. Figure 4 shows that within sandstone D, the resistivities, as recorded on both the shallow and deep traces, are equal. Therefore, the formation-water resistivity can be assumed to be equal to the mud filtrate as measured by the shallow trace. Thus, the formation-water resistivity is equal to the mud-filtrate resistivity. Mud-filtrate resistivity usually is measured, and its value can be found on the log heading.

Spontaneous potential is a function of the logarithm of the ratio of the ionic activity of the formation water to the mud filtrate. Therefore, for the SP deflection to be zero, as shown for sandstone D in figure 4, the ratio is 1 and the activites are equal. If resistivities and activities are assumed to be proportional, the usual assumption in interpreting SP logs, it follows that the resistivity of the water equals the resistivity of the mud filtrate, which is usually recorded in the log heading. This unique condition is useful in quickly estimating the water resistivity at one point and for qualitatively evaluating the relative water resistivity in overlying or underlying permeable rocks, assuming other conditions are equal.

Spontaneous-Potential Method

The two quantitative techniques typically used to estimate water resistivity are (1) the spontaneous-potential method, which is the most common, and (2) the cross-plot method. Although both methods are usually presented in most well-logging manuals and texts, their accuracy is not. In the present paper, estimated values are compared with measured values to evaluate the use and accuracy of the method.

The spontaneous-potential method can be used to estimate resistivity of sodium-chloride-type water. The method is widely known and described in nearly all texts and well-logging manuals, such as "Application of borehole geophysics to water-resources investigations" (Keys and MacCary, 1971), and is based on the equation:

$$SP = -K \log \frac{Rmf}{Rw} \quad ; \tag{1}$$

where *SP* is the spontaneous potential, in millivolts, at the in-situ (formation) temperature; *K*, in millivolts, is a constant porportional to its absolute temperature within the formation; *Rw* is resistivity of water equivalent, in ohm-meters, at in-situ temperature; and *Rmf* is the resistivity of mud filtrate, in ohm-meters, at in-situ temperature. This method of calculation requires spontaneous potential from an electric log and the mud-filtrate

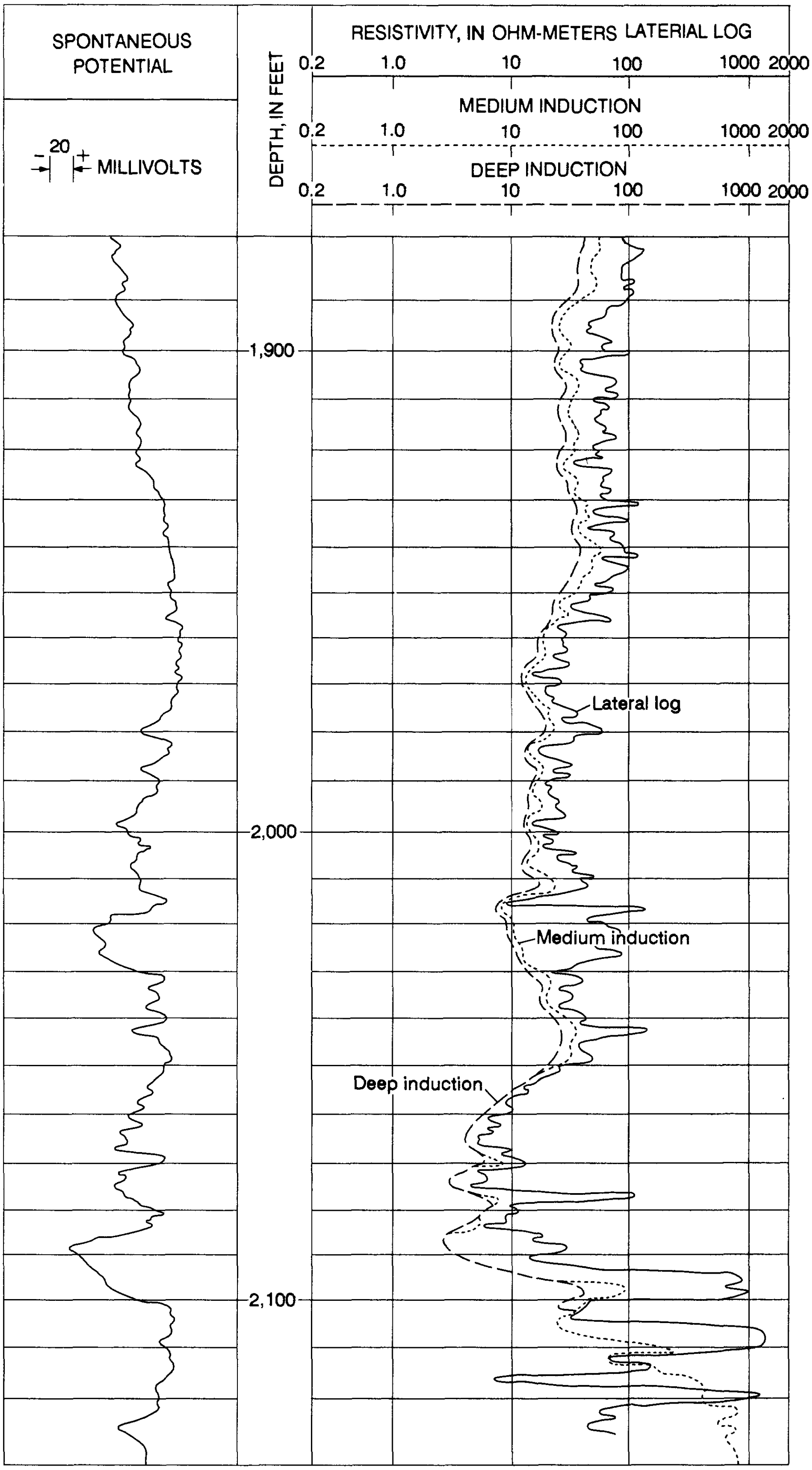


Figure 3. Electric log (spontaneous-potential and resistivity logs).

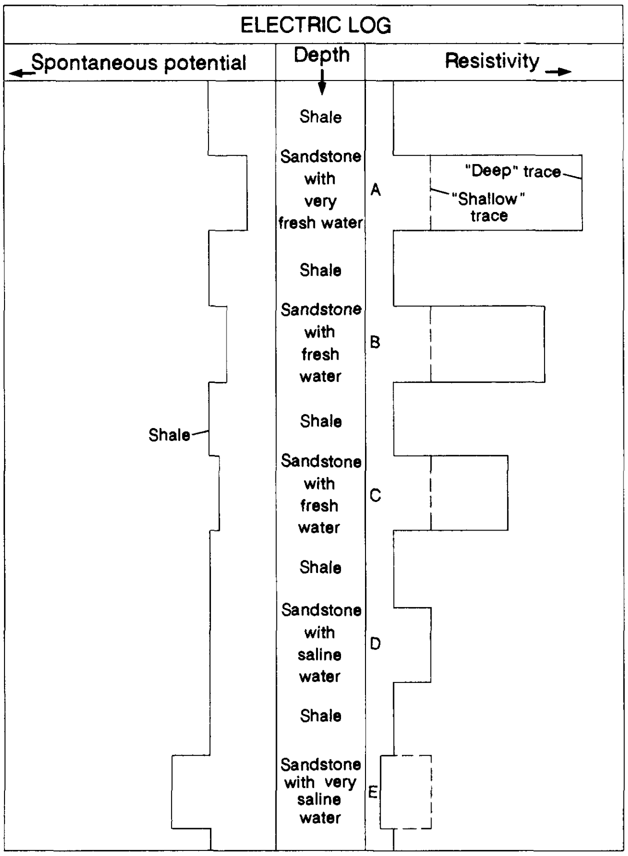


Figure 4. Idealized electric log of shale and sandstone sequence containing fresh and saline water. Letters identify sandstone units whose waters range in salinity from very fresh (A) to very saline (E).

resistance measurement, generally recorded on the log heading.

The spontaneous-potential method is commonly used because *SP* logs are readily available. (More electric logs are available than any other type of geophysical log.) This method of estimating water resistivity is useful in sand-shale sequences where good *SP* differences exist, but reportedly is not usable or works poorly in carbonate rocks (MacCary, 1980, p. 3). However, no terms in equation 1 refer to lithology; thus, the equation should be applicable to any permeable rock type.

An algorithm, similar to the one presented by Bateman and Konen (1977), for using *SP* to determine *Rw* is shown in figure 5. Mud-filtrate resistivity values (*Rmf*) at specific temperature (*Tmf*) and the in-situ temperature of the permeable material (*Tf*) at which the *SP* is measured are required. The *SP* value, in millivolts, is the signed (+ or -) difference between the potential of the aquifer material and the potential at the reference shale line (vertical line along which most shale or clay sequences plot). If the value of *Rmf* is not known, an *Rmf* can be estimated from the mud resistivity (*Rm*):

$$Rmf \cong 0.75\ Rm \quad . \qquad (2)$$

The in-situ (formation) temperature (*Tf*) is rarely known unless temperature was measured in the borehole after drilling had been completed. However, *Tf* may be estimated by assuming that the temperature between the mean annual temperature near the surface and the temperature at the bottom of the borehole (*BHT*) increases linearly with depth; mathematically, *Tf* can be shown as

$$Tf \cong Tma + \frac{(BHT\text{-}Tma)\ (Df)}{Dt} \quad ; \qquad (3a)$$

where *Df* is formation depth, and *Dt* is the total hole depth at which *BHT* was measured. A second and similar method of estimating *Tf* uses information on the geothermal gradient of the area:

$$Tf \cong Tma + (\text{geothermal gradient})\ (Df), \qquad (3b)$$

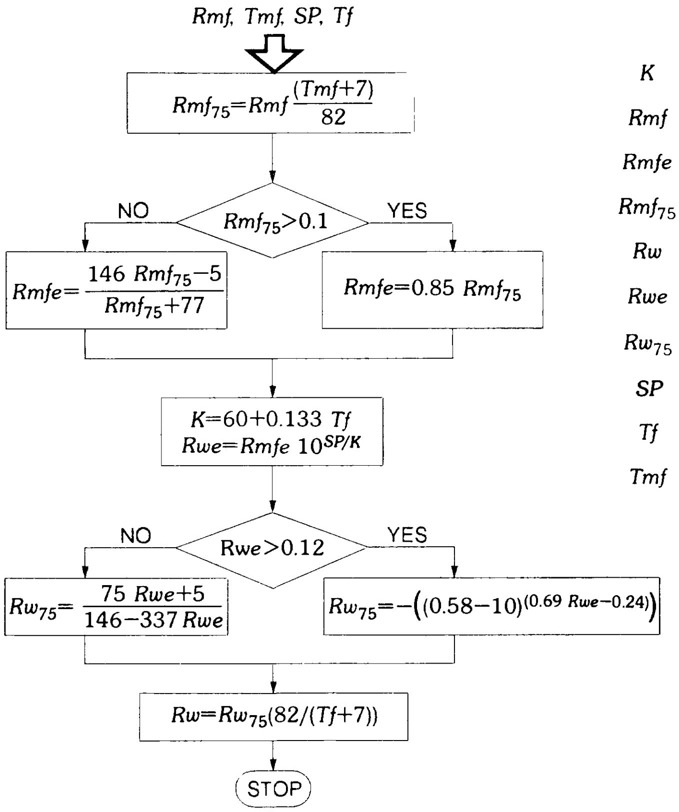
where geothermal gradient is in degrees per unit depth.

The procedure for the spontaneous-potential method to estimate the water resistivity is:

1. Determine *Rmf* and *Tmf*. Read values from the log heading. If *Rmf* is not available, estimate from *Rm* using equation 2.
2. Determine *SP* from the spontaneous-potential trace on the electric log.
3. Determine *Tf* from temperature log or estimate using equations 3a or 3b.
4. Determine formation-water resistivity (*Rw*) using the algorithm shown in figure 5.

The spontaneous-potential method was used to estimate *Rw* for 11 rock sequences in boreholes for which the formation-water resistivity had been measured. Results of comparing *Rw* estimated from *SP* method versus *Rw* measured from chemical analyses are listed in table 1 and are shown in figure 6. A least-squares analysis for a linear relation indicates a coefficient of determination (r^2) of 0.66. A coefficient of determination of zero indicates no correlation and a value of 1 indicates perfect correlation. The value of 0.66 indicates that a correlation exists. Examination of figure 6 indicates that a scatter of about 1 order of magnitude might be expected.

The coefficient of determination of 0.66 may be typical for the method if logs from the petroleum industry are used. The value of the coefficient might be interpreted as an inaccurate estimator. However, water resistivity in nature ranges from about 0.01 to more than 10—more than 3 orders of magnitude. Thus, if an accuracy of plus or minus one-half order of magnitude is acceptable, the method may be used with caution to estimate water resistivity in areas of no measured data.



EXPLANATION

K	Spontaneous potential constant at a specific temperature
Rmf	Resistivity of mud filtrate, in ohm-meters
$Rmfe$	Resistivity of mud filtrate equivalent, in ohm-meters
Rmf_{75}	Resistivity of mud at 75°F, in ohm-meters
Rw	Resistivity of water, in ohm-meters
Rwe	Resistivity of water equivalent, in ohm-meters
Rw_{75}	Resistivity of water at 75°F, in ohm-meters
SP	Spontaneous potential, in millivolts
Tf	Temperature of formation, in degrees Fahrenheit (°F)
Tmf	Temperature of mud filtrate, in degrees Fahrenheit (°F)

Figure 5. Flow chart of spontaneous-potential method of estimating resistivity of formation water.

The 11 rock sequences tested were mostly carbonates. No evidence was found to indicate that the method was better suited to sandstone than any other type of rock if the shale line for the *SP* curve could be established; however, only three sandstone sequences were used in the test.

The accuracy of the method depends on the accuracy of the spontaneous-potential measurement, which is difficult to measure accurately because spurious electromotive forces are inadvertently included in the measurement. Equation 1 is most applicable if the formation water is saline, sodium and chloride are the predominant ions, and the mud is fresh and contains no unusual additives.

Cross-Plot Method

The cross-plot method for determining water resistivity is discussed in MacCary (1978). It is also referred to as the "carbonate" method or the "Pickett" cross-plot method and is not as widely used as the spontaneous-potential method.

As the name implies, the method requires a cross plot of resistivity and porosity values of saturated material. These values are plotted on log-log graph paper and a line is fitted to the points. Ideally, the points define a straight line, and the intercept of the line projected to the 100-percent porosity value represents the water

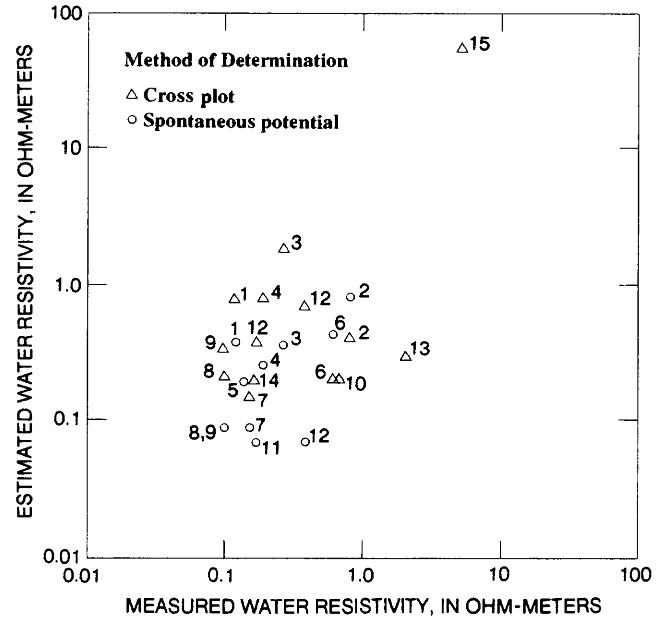


Figure 6. Measured and estimated resistivity of water. Numbers are those of boreholes listed in table 1.

Table 1. Water-resistivity data
[Leaders (---) indicate data not available]

Hole No.	Resistivity of water, in ohm-meters			Rock-section depth, lithology, and formation	Name and location of borehole
	Measured	Estimated			
		Spontaneous-potential method	Cross-plot method		
1	0.12	0.38	0.80	Depth 2,420-2,444 ft, vuggy dolostone, Jefferson City Dolomite.	DC & FA #1: SE1/4NW1/4NW1/4, sec. 13, T. 12 S., R. 17. E. Douglas County, Kansas.
2	.79	.86	.44	Depth 615-955 ft, limestone and shale, Pennsylvanian Lansing-Kansas City Groups.	Do.
3	.27	.36	1.9	Depth 2,240-2,335 ft, cherty dolostone, Ordovician Viola Limestone.	Do.
4	.19	.26	.80	Depth 2,398-2,508 ft, dolostone and sandstone, Ordovician and Cambrian Simpson and Arbuckle Groups.	Do.
5	.14	.20	---	Depth 2,934-3,985 ft, calcareous sandstone and granite, Cambrian Lamotte Sandstone and Precambrian rock.	Do.
6	.61	.48	.20	Depth 3,343-3,665 ft, sandy dolostone and cherty dolostone, Simpson and Arbuckle Groups.	Geis #1: SW1/4SW1/4SW1/4, sec. 32, T. 13 S., R. 2 W., Saline County, Kansas
7	.15	.09	.15	Depth 1,980-2,200 ft, limestone, Lansing and Kansas City Groups.	Do.
8	.10	.09	.22	Depth 2,616-2,804 ft, limestone, Mississippian Warsaw, Keokuk, Burlington, and Fern Glen Limestones.	Do.
9	.10	.09	.35	Depth 2,970-3,070 ft, dolostones, Devonian and Silurian Hunton Group.	Do.
10	.67	---	.20	Depth 3,384-3,498, dolostone, slightly cherty, Arbuckle Group.	Do.
11	.17	.07	.38	Depth 1,760-2,179 ft, dolostone and sandstone, Arbuckle Group and Lamotte Sandstone.	Watson #1: SE1/4SW1/4SE1/4, sec. 18, T. 18 S., R. 23 E. Miami County, Kansas.
12	.38	.07	.70	Depth 1,245-1735 ft, limestone and dolostone, Hunton and Arbuckle Groups.	Do.
13	2.09	---	3	Depth 900-1,500 ft, dolostone and sandy dolostone, Ordovician Cotter, Jefferson City, and Gasconade, Dolomites, and Roubidoux Formation.	Ordinance #1: Center SW1/4, sec. 22, T. 31 S., R. 20 E., Labette County, Kansas.
14	.16	---	.20	Depth 1,501-1,816 ft, dolostones with vuggy zones, Ordovician and Cambrian Gasconade, Eminence, and Bonneterre Dolomites.	Do.
15	5.31	---	56	Depth about 2,600 ft, dolostone, Mississippian Mission Canyon Limestone.	Madison #1: NE1/4SE1/4, sec. 15, T. 57 N., R. 65 W., Crook County, Wyoming.

resistivity (*Rw*). This hypothesis was tested using data from two dolostone cores (DC & FA #1 and Geis #1) from Douglas and Saline Counties, Kansas. The cores were saturated with water of known resistivity. The results are shown in figure 7 and are encouraging because the projection of a line, which fitted most of the data from the Geis #1 core, intercepted the 100-percent porosity line close to the measured Rw value. However, DC & FA #1 core results show a few data points falling below the straight line, probably because the *Ro* values for porosity values less than 3 percent may be affected by surface conductance along the grains. (*Ro* is the combined resistance of water and saturated rock.)

Porosity and resistivity values for the cross plot can be obtained from geophysical logs. Homogeneous lithology, constant water resistivity, and 100-percent water saturation are assumed. Resistivity values from logging devices that "look deep" into a formation are preferable. Suitable logs might be a deep-induction log (as shown in fig. 3), a long-lateral log, a deep-conductivity log, and so forth. (Conductivity is the inverse of resistivity, generally recorded in millimhos or micromhos per meter.)

The most accurate porosity values are obtained from a dual-porosity log, such as the log shown on figure 1. However, other porosity logs, such as sonic, neutron, density, or dielectric, could also be used. Porosity determined from a resistivity log cannot be used.

The procedure for determining water resistivity using a dual-porosity log (including a gamma-ray trace), and an electric log (including a resistivity trace) is as follows:

1. Identify the lithology on the dual-porosity log.
2. Select a rock sequence of uniform lithology to analyze.
3. Record both neutron and density porosity values for the selected points. Be certain to include the maximum and minimum porosities as well as values within the range. About 10 points typically are needed to define the line.
4. Calculate porosity values using a graph similar to that shown in figure 8. (See log-interpretation manuals for more details and for the appropriate graphs for the tool used.)

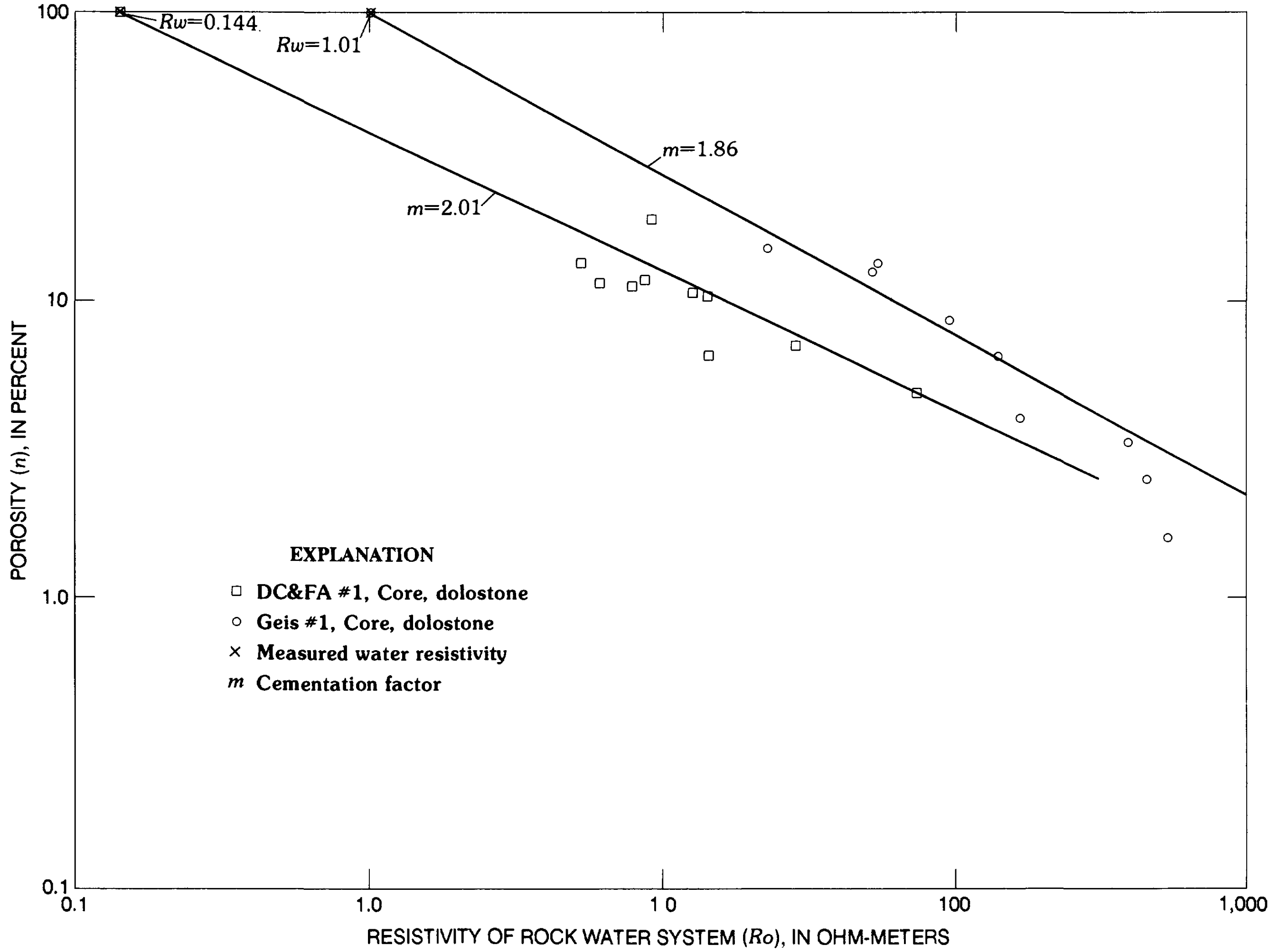


Figure 7. Cross plot of resistivity and porosity measured on dolostone cores.

5. Record resistivity values from the deep resistivity trace of the same points selected in step 3.

6. Plot the resistivity and porosity values as shown in figure 9 using log-log graph paper.

7. Fit a straight line to the data points. During the straight-line fitting, give less weight to the lower porosity values because their resistivities may be affected by conductance along the grain. The resistivity indicated by this intercept is the water-formation resistivity. Project the straight line to 100-percent porosity.

When using the cross-plot method, points defining a straight line to the degree of desired accuracy cannot always be selected. Logs with expanded depth scales are easier to use in selecting better porosity and resistivity value sets because the same point on all logs can be located more accurately. An example of the method is shown in figure 9 using data from the logs shown in figures 1 and 3.

If the rock section is not 100-percent saturated with water, the rock-water system resistivity (*Ro*) values obtained from the log will be larger than *Ro* values of the aquifer material if it had been 100-percent saturated. The values would plot to the right of the line defining *n* versus *Ro* for a 100-percent water-saturated section. The unusually large resistivity values may indicate an unsaturated zone, hydrocarbons, or gases.

Because the porosity and resistivity logarithms define a straight line, standard least-squares techniques can be applied to determine the standard estimate of error. Accordingly, the standard estimate of error for *Rw* can also be defined. Note that the estimated *Rw* is for formation conditions. The accuracy of the method was tested by estimating *Rw* for 15 rock sections for which *Rw* had been measured (fig. 6). A least-squares analysis gave a coefficient of determination of 0.88, a value that may be typical of estimates based on available logs. Figure 6 shows variations or scatter of about 1 order of magnitude in a range of more than 3 orders of magnitude that can be expected in nature. Thus, based on the results shown in figure 6, the method did not accurately estimate

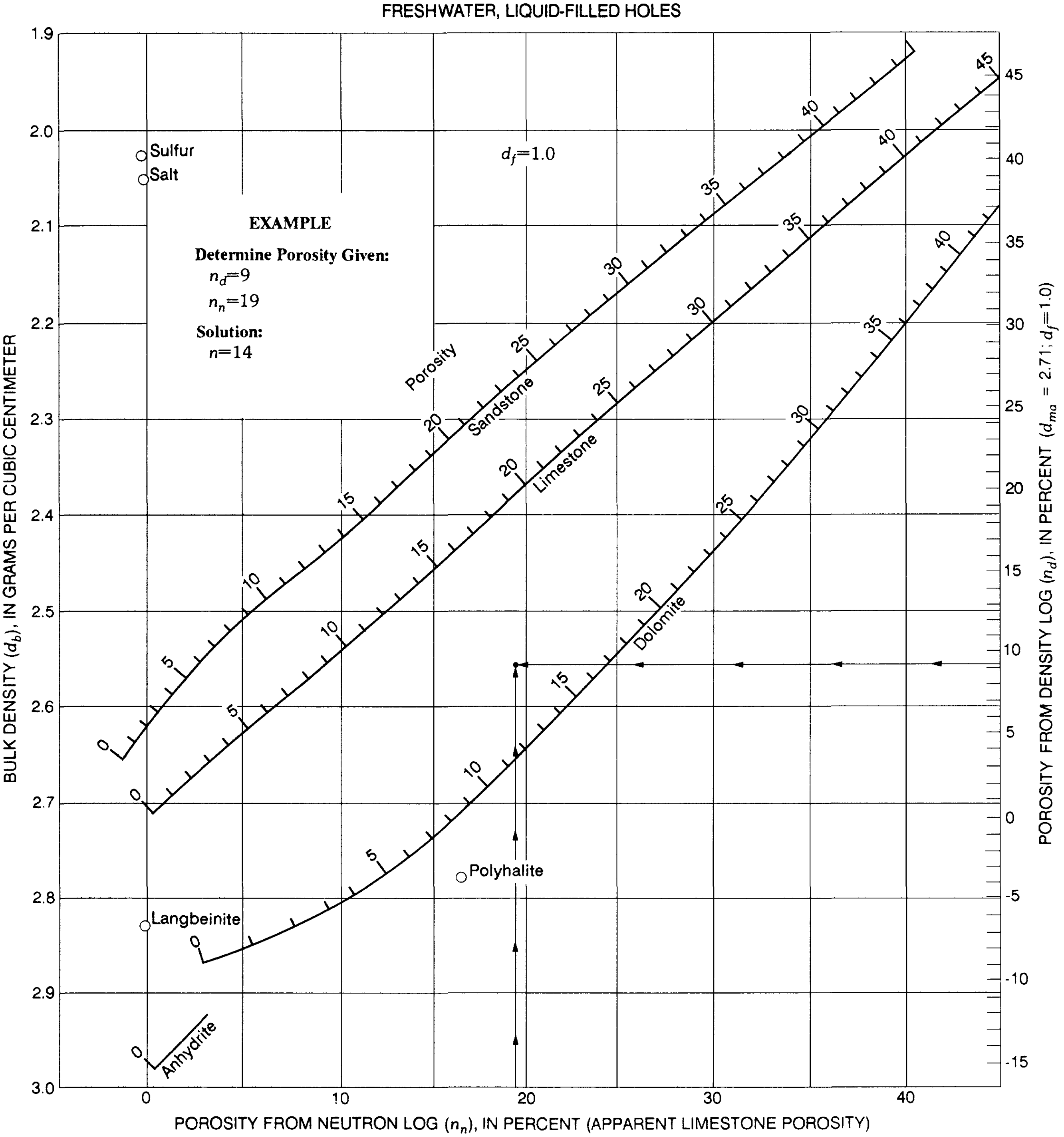
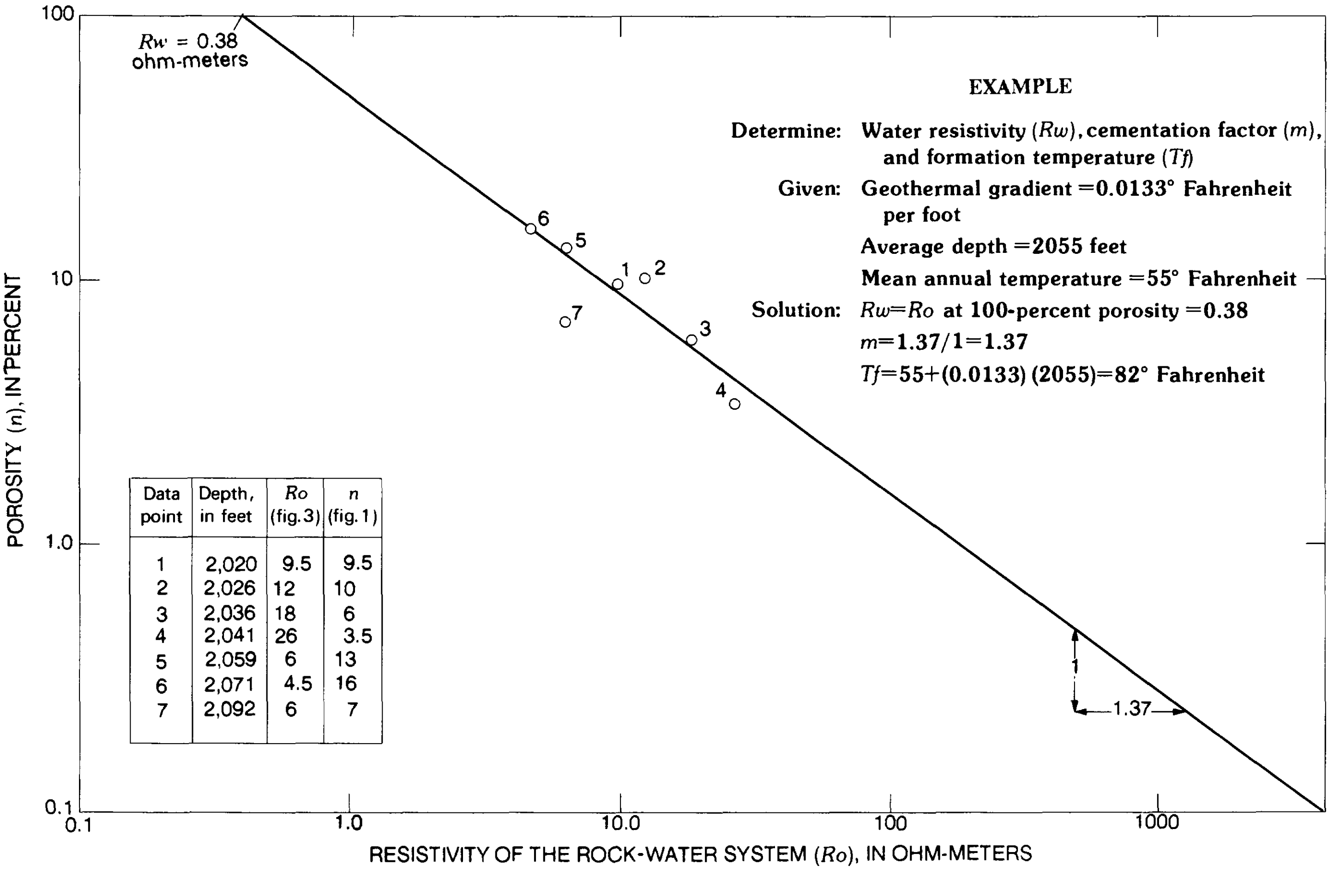


Figure 8. Porosity and lithology from formation-density log and compensated-neutron log (from Schlumberger Well Surveying Corp., 1979). Data are from figures 1 and 3. Arrows show how to solve the example problem.

Rw; however, the method can be used to estimate formation-water resistivity in areas of no data if an estimated accuracy of plus or minus one-half order of magnitude is acceptable. The method is applicable irrespective of whether the water is saline or fresh.

The accuracy of the method generally is proportional to the range of porosity measured within the section of interest. The wider the range, the more accurately the line can be defined. The recorded measurement can be read more accurately from the log (trace) if the scale is expanded. Also, the accuracy of the measurement is of concern. The consistency of resistivity values as measured by



Data point	Depth, in feet	Ro (fig. 3)	n (fig. 1)
1	2,020	9.5	9.5
2	2,026	12	10
3	2,036	18	6
4	2,041	26	3.5
5	2,059	6	13
6	2,071	4.5	16
7	2,092	6	7

EXAMPLE

Determine: Water resistivity (*Rw*), cementation factor (*m*), and formation temperature (*Tf*)

Given: Geothermal gradient =0.0133° Fahrenheit per foot

Average depth =2055 feet

Mean annual temperature =55° Fahrenheit

Solution: *Rw*=*Ro* at 100-percent porosity =0.38

m=1.37/1=1.37

Tf=55+(0.0133) (2055)=82° Fahrenheit

Figure 9. Cross plot of geophysical-log values of *Ro* and *n*. See table 1 for description of boreholes.

the induction tools, lateral-resistivity tools, and conductivity tools are generally unknown to the user. For example, tools may be adjusted in the field to produce a "sharp" trace irrespective of the ohm-meter scales used on the logs.

The ***Rw*** value may be used to estimate water chemistry if the relation has been established. For example, the sodium-chloride content can be estimated for many saline waters if the in-situ temperature of the water is available or can be estimated. Turcan (1966) used resistivity logs to determine ***Rw***, which he then correlated with chloride or dissolved-solids concentration.

Dissolved-solids concentration can be estimated from specific conductance (conductance at 75 °F), usually expressed in microsiemens or micromhos per centimeter, if the relationship between specific conductance and dissolved solids is known. A method of estimating dissolved-solids concentration in water from ***Rw*** is given in appendix A.

ESTIMATING PERMEABILITY

Background and Research

In the deep subsurface and (or) areas where the media (rocks) contain saline water, few hydraulic properties of rocks, such as intrinsic permeability or hydraulic conductivity, have been measured. Of the few measurements available, most are of intrinsic permeability and most have been made on rocks in the search for oil and gas. Intrinsic permeability (*k*) is a measure of the relative ease with which a medium can transmit a liquid under potential gradient, and it is a property of the medium alone. Hydraulic conductivity (*K*) is a measure of the ease of the flow of water at a specific viscosity through a rock. Hydrologists studying shallow ground water generally use the term "hydraulic conductivity," whereas petroleum engineers and geologists use "intrinsic permeability." The relation between the two properties is

$$K = \frac{k\,d\,g}{\mu}\ , \qquad (4)$$

where K is hydraulic conductivity,

k is intrinsic permeability,

d is the density of the fluid,

g is the acceleration due to gravity, and

μ is the dynamic viscosity of the fluid.

Both density and viscosity are functions of temperature, salinity, dissolved gases, and, to a lesser degree, pressure.

Water viscosity and density are functions of temperature and salinity. Density, viscosity, and temperature relations are shown in figures 10, 11, and 12. The viscosity-to-temperature relation for various sodium-chloride solutions shown in figure 11 can be approximated by an equation derived by Weiss (1982), which is given in appendix D. Methods of estimating density from dissolved-solids concentration are given in appendix C.

Most intrinsic-permeability values are determined from testing cores in the laboratory or from a drill-stem test. Most drill-stem tests are conducted on petroleum reservoirs, and the results, even if accurate, must be used with care because they may represent hydrocarbon reservoirs rather than aquifers.

Intrinsic-permeability values determined from core tests usually do not completely represent conditions in the rock because any method of collecting cores disturbs the rock. Also, recovery of unconsolidated material is difficult and rarely successful. Laboratory tests should be run under conditions duplicating those in the subsurface. Determining *k* values of fractured rock from cores is extremely difficult because fractured core pieces are nearly impossible to arrange exactly as they were positioned in the subsurface. Also, the scale problem must be considered. Does a small volume of core represent the large volume of rock and fluid being considered? The scale problem arises when determining aquifer permeability that may be considered homogeneous over a thickness of tens or hundreds of feet, but that is extremely variable over a short distance, such as 1 inch. For example, the intrinsic permeability values for sample 10 listed in table 2 ranged from 4,890 mD to 0.01 mD. Obviously, an averaging technique, such as the geometric mean or a thickness-weighted mean, is required to determine the effective permeability of the core. Thus, even if an undisturbed fractured core could be obtained, how many point samples would be necessary to define the effective intrinsic permeability of the aquifer? From the discussion, obviously the *k* value determined from a core or a drill-stem test must be carefully evaluated before assuming the value is representative of a large rock mass.

The concept of relating formation factor to intrinsic permeability is appealing and has been investigated by many. Bear (1972, p. 113-117) related formation factor (F) to a tortuosity factor by

$$F = C(Ŧ)\, n^{-m^*} \quad , \qquad (5)$$

where $C(Ŧ)$ is some tortuosity function, n is porosity, and m^* is a function of the number of reductions in pore size openings. The function $C(Ŧ)$ may be 1 or less because the tortuosity factor is 1 or less. The tortuosity factor is defined as:

$$Ŧ = (L/L_e)^2 \quad , \qquad (6)$$

where L is sample length, and L_e is effective electrical-flow path length. Note that the tortuosity factor of equation 6 differs from the tortuosity defined by Winsauer and others (1952, p. 255). Equation 5 indicates that formation factor depends on pore size and its reduction, and tortuosity. Equation 5 may be useful, but a procedure to apply data from geophysical logs to equations 5 and 6 has not yet been fully developed.

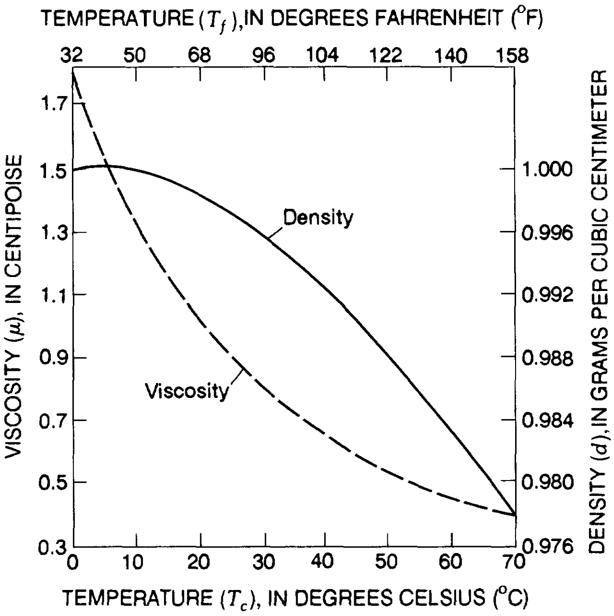


Figure 10. Viscosity, density, and temperature of freshwater (data from Weast, 1984).

The relations among the resistivity recorded on a geophysical log, water resistivity, and rock-material resistivity is not entirely straightforward. Archie (1942) assumed that the rock material was nonconductive and derived empirical equations to define observed resistivities in terms of reservoir properties. The most generalized form of the Archie equation (Archie, 1942, p. 56) is

$$F = n^{-m} \quad , \qquad (7)$$

where F is the formation factor (dimensionless),
n is porosity (dimensionless), and
m is the cementation factor (dimensionless).

The relation of formation factor to pressure and temperature is not completely known. In reference to temperature effect, Somerton (1982, fig. 12, p. 188) showed that the logarithm of the ratio of the formation factor at a specific temperature to formation factor at a specified reference temperature for the Berea Sandstone varied nonlinearly with temperature change.

For increasing pressure, Helander and Campbell (1966, p. 1) reported that formation factor changes can

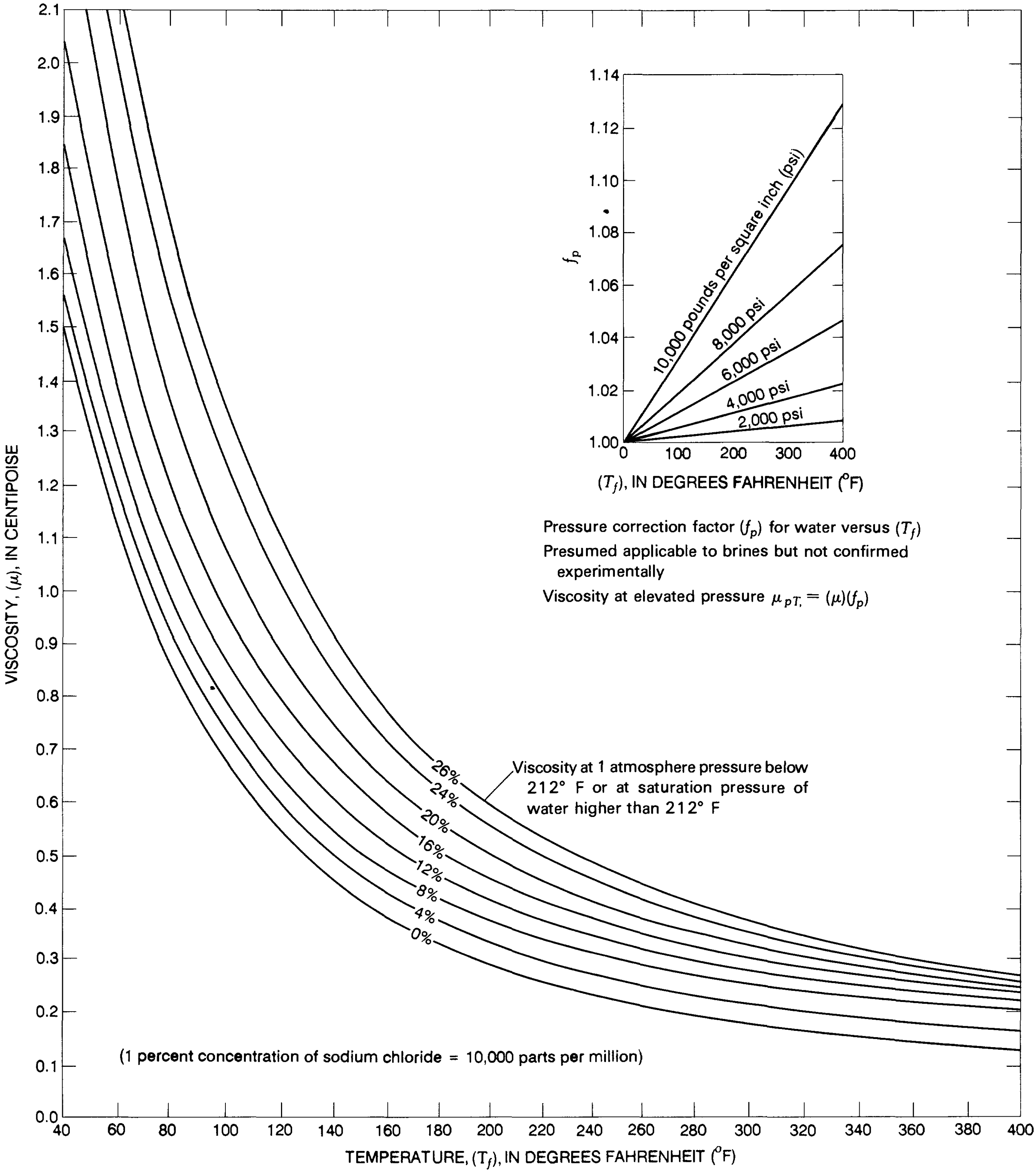


Figure 11. Water viscosity at various temperatures and percentages (%) of salinity (modified from Matthews and Russell, 1967).

be attributed to the following: (1) The increase in length of the mean free path for current (increased tortuosity) results from increased constriction as pores close, (2) the amount of constriction is largely due to the closing of the smallest pores, and (3) the effect of the double layer is increased by the reduced area of the pores as porosity decreases.

The cementation factor (*m*) is a function of

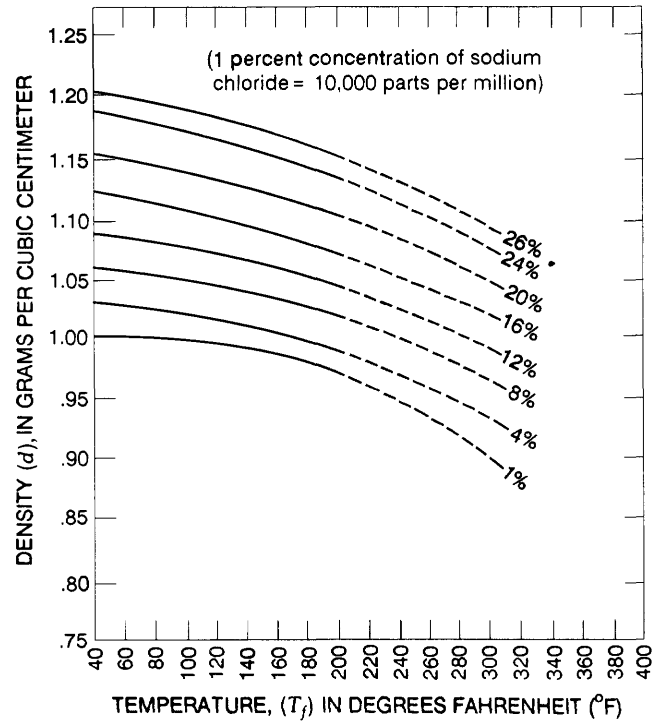


Figure 12. Water density at various temperatures and percentages (%) of salinity (data from Arps, 1953).

tortuosity and pore geometry. Tortuosity is the ratio of the fluid path length to the sample length. Aquilera (1976) studied the effect of fractured rock on the formation and cementation factor, and used a double-porosity model to define *m*. The model implies that *m* will approach 1 for a rock mass in which all porosity is the result of fractures (that is, there is no interconnected primary porosity). Because the length of the flow path in a fractured medium is much shorter than the length of flow path in a porous medium, the tortuosity of the fractured medium is small; the cementation factor is also small and approaches 1. The relation of porosity (*n*), formation factor (*F*), and the tortuosity factor (Ŧ) is:

$$F = \frac{1}{Ŧ n} \quad . \tag{8}$$

Archie further defined

$$F = Ro/Rw \quad , \tag{9}$$

where *Ro* is resistivity observed (log resistivity), and *Rw* is the formation-water resistivity. (In the present paper, *Ro* is assumed to be the bulk water and rock resistivity unaffected by fluid invasion, sometimes called true resistivity or *Rt*.)

Table 2. Porosity and permeability data

[*k*, intrinsic permeability in millidarcies (mD); *n*, porosity, dimensionless; *m*, cementation factor, dimensionless; *Rw*, resistivity of water, in ohm-meters; *F*, formation factor, dimensionless; leaders (---) indicate data not available]

Hole No.	Average *k*	Average *n*	Average *m*	Average *Rw*	Average *F*	Rock-section depth, lithology remarks, and formation	Name and location of borehole (source)
1	6.45	0.105	2.01	0.144	---	Depth 2,420-2,443 ft, vuggy dolostone, maximum *k*=17 mD, minimum *k*=0.1 mD, core, Jefferson City Dolomite.	DCL & FA #1: SE1/4NW1/4NW1/4, sec. 13, T. 12 S., R. 17 E., Douglas County, Kansas (Gogel, 1981).
2	4.9	.13	2.37	---	---	Depth 1,980-2,200 ft, oolitic limestone, drill-stem test, Lansing and Kansas City Groups.	Geis #1: SW1/4SW1/4SW1/4, sec. 32, T. 13 S., R. 2 W., Saline County, Kansas (Gogel, 1981).
3	4.74	.074	1.86	1.01	---	Depth 3,482-3,493 ft, dolostone, maximum *k*=29 mD, minimum *k*=0.02 mD, core, Roubidoux Formation.	Do.
4	4.54	.092	2.15	---	---	Depth 2,934-2,985 ft, calcareous sandstone and granite, drill-stem test No. 1, Lamotte Sandstone and Precambrian rock.	DCL & FA #1: SE1/4NW1/4NW1/4, sec. 13, R. 17 E., Douglas County, Kansas (Gogel, 1981).
5	101	.181	1.78	---	---	Depth 2,616-2,804 ft, cherty limestone and dolostone, drill-stem test, Warsaw, Keokuk, and Burlington Limestones.	Geis #1: SW1/4SW1/4SW1/4, sec. 32, T. 13 S., R. 2 W., Saline County, Kansas (Gogel, 1981).
6	300	.125	1.09	---	---	Depth 2,944-3,046 ft, porous dolostone, drill-stem test, Hunton Group.	Do.
7	1,147	.22	1.10	14.1	5.29	Depth 210-610 ft, limestone, aquifer test on well tapping the Floridian aquifer.	GF 18: sec. 20, T. 7 S., R. 10 W. Gulf County, Florida (Kwader, 1982).
8	171	.10	1.26	5	---	Depth 2,500-2,760 ft, dolomite, breccia, some intensely fractured, some very dense, flow-dye test, Mission Canyon Limestone (maximum *k*=789 mD, minimum *k*=0.4, from core test).	Madison #1: NE1/4SE1/4, sec. 15, T. 57 N., R. 65 W., Crook County, Wyoming (Blankennagel and others, 1977).
9	155	.136	1.49	---	---	Depth 2,760-3,030 ft, limestone and dolostone with some thin anhydrite and interbedded shale, flow-dye test (maximum *k*=320 mD; minumum *k*=0.1 mD).	Do.
10	360	.149	1.49	---	---	Depth 3,100-3,400 ft, dolostone, some breccia texture, some crystalline, some vuggy, some dense; flow-dye test (from core test, maximum *k*=4,890 mD, minimum *k*=0.1 mD).	Do.

Combining equations 7 and 9 yields:

$$Rw = Ro\ n^{m} \quad . \qquad (10)$$

If Rw is constant, equation 10 will yield a straight line with the slope of $-m$ on a log-log plot of n versus Ro as shown in figures 7 and 9.

Sethi (1979) comprehensively reviewed the work of many researchers in defining formation-factor relations. The paper includes data originally presented by Winsauer and others (1952) in their resistivity study of brine-saturated sands and pore geometry. Raiga-Clemenceau (1977) reviewed the derivation and accuracy of the common forms of the modified Archie equation:

$$F = a\ n^{-m} \quad , \qquad (11)$$

where a is an empirically determined constant probably related to lithology. Some authors term "a" the cementation factor and "m" the cementation exponent (Dewan, 1983, p. 19). The two terms are covariant because they are not independent variables. Two variations of equation 11 are

$$F = 0.62\ n^{-2.15} \quad \text{, and} \qquad (12)$$

$$F = 1.0\ n^{-2} \quad . \qquad (13)$$

Equation 12 is sometimes called the Humble equation and is used for clastics. Equation 13 is sometimes called the carbonate equation. Raiga-Clemenceau (1977) noted that both equations are empirical and concluded that intrinsic permeability might be used to better define F. Accordingly, he chose to define F by setting a equal to 1, which is the Archie equation, and making m a function of k. The equation yielded a formation factor with less error than the formation factor estimated by the Humble equation. In conclusion, the Archie equation (eq. 7) may be as appropriate for clastics as it is for carbonates, especially considering the empirical nature of the equations.

The following discussion relates formation factor, cementation factors, and tortuosity factor to intrinsic permeability. Regression analysis is one technique for defining intrinsic permeability in terms of formation factor. Carothers (1968) derived two equations for intrinsic permeability (in millidarcies):

$$k = 4.0 \times 10^{8}/F^{3.65} \qquad (14)$$

for limestone, and

$$k = 7.0 \times 10^{8}/F^{4.5} \qquad (15)$$

for sandstone. Ogbe and Bassiouni (1978, p. 10) used a similar approach.

Croft (1971) successfully correlated the ratio Ro/Rw with intrinsic permeability. However, the correlation requires information about water chemistry, which is not available from geophysical logs. The method works well locally where the change in Ro is primarily due to a change in porosity. MacCary (1984) pointed out that in thick hydrologic units, neither Rw nor Ro can be considered constant.

Stephens and Lin (1978) derived an equation for intrinsic permeability that includes a geometric shape factor (SF) of the pore and the hydraulic radius (HR), which is defined as the ratio between area of cross section of the pore and its wetted perimeter. Their equation is

$$k = \frac{HR^{2}}{SF\ n^{0.5} F^{1.5}} \quad . \qquad (16)$$

SF is 2 for circular pores and 3 for narrow cracks. Stephens and Lin reported good correlation between calculated values of intrinsic permeability and the measured permeability values presented by Brace (1977).

If equation 7 is substituted into equation 16,

$$k = (HR^{2}/SF) \cdot n^{(1.5m-0.5)} \quad . \qquad (17)$$

Equation 17 implies that intrinsic permeability is a function of the medium, the first set of terms, and a term that includes porosity and cementation factor.

The Kozeny equation is a common starting point for relating intrinsic permeability and resistivity of porous media. This equation as stated by Bear (1972, p. 166) and Herdan (1960, p. 196) is

$$k = \in \frac{\mp\, n^{3}}{S_{p}^{\,2}} \quad . \qquad (18)$$

where $\in$ is the Kozeny coefficient, and S_p is the specific surface, which is defined as the total interstitial surface area of the pores per unit volume of the medium.

The relation between specific surface area of the solids per unit volume of solids (S_s) to specific surface (S_p) is

$$S_s = \frac{S_p}{(1-n)} \quad . \qquad (19)$$

Substituting equations 8 and 19 into equation 18 yields:

$$k = \frac{\in}{F\ S_s^{2}} \cdot \frac{n^{2}}{(1-n)^{2}} \quad . \qquad (20)$$

Equation 20 would be especially useful if F and Rw were independently known, but generally they are not. If equation 7 is substituted into equation 20:

$$k = \frac{\mathcal{C}}{S_s^2} \cdot \frac{n^{m+2}}{(1-n)^2} \quad . \tag{21}$$

Equation 21 indicates that permeability is a function of a medium factor (first set of terms right of the equal sign) and a porosity factor ($\mathcal{P}$) (the last set of terms). Specifically, the porosity factor is

$$\mathcal{P} = \frac{n^{m+2}}{(1-n)^2} \quad . \tag{22}$$

The numerator of $\mathcal{P}$ indicates that as the cementation factor (m) decreases, the porosity factor and the intrinsic permeability increase. Similarly, the denominator indicates that as the porosity increases, the porosity factor and permeability increase. All values for the porosity factor can be evaluated from information from geophysical logs. The factor $\mathcal{C}$ can be evaluated; however, efforts to evaluate S_s from geophysical logs are not straightforward. Nevertheless, equation 21 does imply that any relation for permeability should include a medium factor and a porosity factor.

A search was made to find data sets of intrinsic permeability, porosity, and cementation factor for unconsolidated, consolidated, and fractured rock of various lithologies. The literature reports numerous values for cementation factor, core permeability, and empirical constant (a), but little is said about how the properties were measured or what equation was used to determine the factors. Because several different procedures are commonly used, finding reported data to compare is difficult. The approach used here was that of Raiga-Clemenceau (1977); that is, equation 7 was considered an appropriate form of the Archie equation to use. The cementation factor was calculated from equation 10 or it was determined from the Ro versus porosity cross plot. Special preference was given to in-situ permeability and porosity measurements from dual-porosity logs. Only a few sites were found that met the requirements (table 2). Nearly all data in table 2 are for carbonate rocks. The data from table 2 are plotted in figure 13, which shows porosity factor versus intrinsic permeability. Because of the wide range in values, a log-log plot was chosen.

A least-squares line was used to fit data as shown on figure 13, and the regression in equation of the line is

$$k = 1.828 \times 10^5 \, (\mathcal{P}^{1.10}) \quad . \tag{23}$$

The correlation of determination (r^2) is 0.90, which is a good correlation. With the limited data available, it cannot be ascertained if the exponent 1.10 is significantly different from 1 if the factor 1.828×10^5 were reduced accordingly. Equation 23 does not include the medium factor as defined in equation 21. Medium factors for many different samples have been plotted; no single line fits the data. Variations should be expected because the basic implied assumption that the flow of electricity in an aquifer is in all ways analogous to the flow of water is not completely true. Permeability is a function of porosity, surface area, and tortuosity. Electrical conductivity is a function of rock conductivity, ion mobility, temperature, pressure, surface area, charge on the surface area, and the conductivity of the double layer surrounding the grains. The straight line shown in figure 13 might be assumed to define an approximate relation applicable to coarse-grained clastics (such as sandstone and siltstone, most porous carbonates, and uniformly fractured rocks) because surface conductance along the pore walls is not dominant. The relations in equation 23 are shown in figure 14, which relates porosity to permeability for various cementation factors. The Humble sandstone reference samples reported by Winsauer and others (1952) whose permeability exceeds 1,000 mD (not shown on figure 13) correlate well with estimates using equation 23. Equation 23 was tested against data collated by Brace (1977) as listed in table 3. The correlation between calculated and measured permeabilities is good except if the hydraulic radius, in micrometers, was less than 1. Again, this comparison shows the importance of including a geometric factor, such as S_s, S_p, or SF, to better estimate intrinsic permeability of a medium with large surface areas.

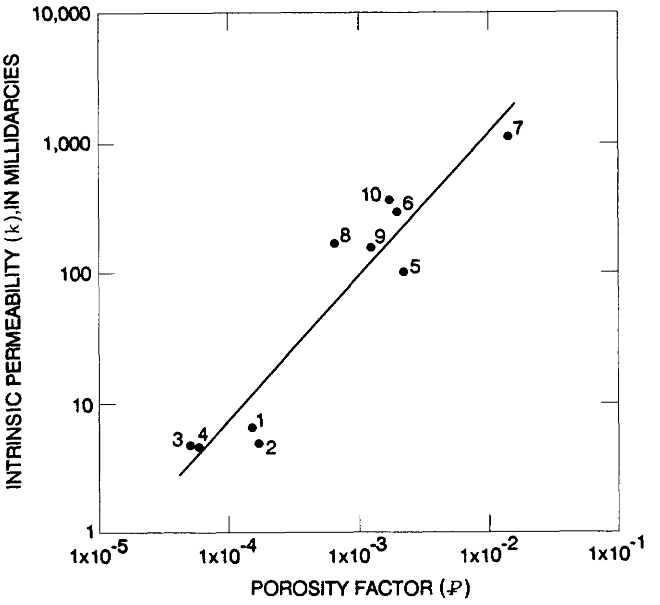


Figure 13. Relation between porosity factor and intrinsic permeability.

To better test the applicability of equation 23 for other regions and for other lithologies of rock sections listed in table 2, typical porosities and cementation factors (table 4) were input into equation 23 to calculate

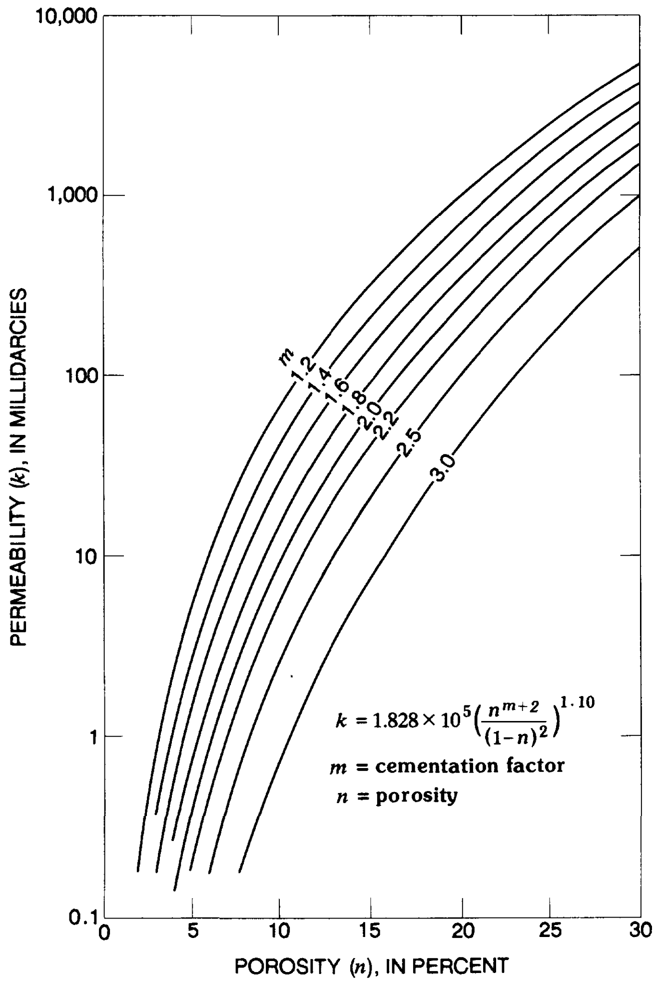


Figure 14. Permeability and porosity.

permeabilites of other lithologies. The comparison between typical permeabilities and calculated permeabilities is good. Considering that the range of typical permeabilities in nature is about 12 orders of magnitude (from 10^{-4} to 10^8 mD as listed in table 5), the accuracy and usefulness of equation 23 are rather surprising because it allows use of data from borehole-geophysical logs to estimate intrinsic permeability with an accuracy of less than one-half an order of magnitude (fig. 3). The equation, based on available data, successfully estimates across a range of about 10^{-3} to 10^3.

Cross-Plot Method

Equation 23 implies that intrinsic permeability can be estimated if the cementation factor and porosity are known. The value of m can be found from a resistivity-to-porosity cross plot, such as that shown in figure 9. The inverse of the slope of the cross plot is the cementation factor, *m*. The value of *m* can be scaled directly from the resistivity-to-porosity cross plot as follows. The slope of the cross plot (*m*) is

$$m = \text{(horizontal distance)/(vertical distance)}.$$

The cross-plot method of estimating intrinsic permeability is as follows:

1-7. The first seven steps are the same as used in determining resistivity of water (*Rw*) using a resistivity-to-porosity cross plot.

8. Determine *m* from cross plot.

9. Calculate the thickness-weighted mean porosity of the section being tested.

10. Calculate *k* using equation 23.

The cross-plot method assumes that lithology is constant except for variations in porosity. The method also assumes that *Rw* in the section is constant.

Table 3. Permeability from formation factor and pore shape
[μm, micrometer]

Rock	Porosity (n)	Formation factor (F)	Cementation factor (m)	Hydraulic radius (HR), in μm	Intrinsic permeability (k)[1], in millidarcies		Reference
					Calculated	Observed	
Western granite:							
50 megapascals---	0.01	3,100	1.746	0.1	1.1×10^{-3}	6.3×10^{-5}	Brace and others (1968).
0.4 gigapascals--	.01	16,300	2.106	.1	1.7×10^{-4}	4×10^{-6}	Do.
Sherman Granite----	.03	880	1.933	6	.05	0.10	Do.
Berea Sandstone----	.22	11.7	1.624	7	750	890	Wyllie and Rose (1950).
Pyrex 1------------	.37	4.4	1.490	.38	11,000	8.1	Do.
Pyrex 2------------	.29	6.2	1.474	12	3,400	3,900	Do.
Alundum------------	.24	11.5	1.711	2.2	990	63	Do.
Nichols Buff------- sandstone.	.20	12.5	1.569	4.2	538	230	Do.
Eocene sandstone---	.22	13	1.694	4.4	670	340	Wyllie and Spangler (1952).
Pennsylvanian------ sandstone 1.	.16	20	1.635	4.1	180	120	Do.
Pennsylvanian------ sandstone 2.	.21	13	1.644	5.5	590	520	Do.

[1] Calculated $k = 1.828 \times 10^5 (P^{1.10})$, where $P = (n^{m+2})/(1-n)^2$.

Table 4. Typical cementation factors, porosities, and permeabilities and calculated permeabilities for various lithologies
[mD, millidarcies]

Lithology	Commonly observed: Cementation factor (*m*)	Commonly observed: Porosity (*n*)	Commonly observed: Intrinsic permeability of (*k*), in mD	Calculated range of *k*, in mD[1]	Average *k*, in mD: Commonly observed	Average *k*, in mD: Calculated
Sand, fine to medium	[2]1.3-1.4	[4]15-30	[4]1,000-3,500	220-5,100	2,300	2,600
Sandstone, slightly cemented	[2]1.4-1.5	10-30	[6]10-5,000	30-4,400	[6]2,500	2,200
Sandstone, well cemented	[3]1.5-2.0	[5]13-18	[5]1-200	30-380	100	200
Limestone, crystalline	[2]1.5-1.8	[5]8-18	[5]1-300	6-380	150	190
Limestone, dolomitic	[2]1.8-2.0	[5]8-18	[5]1-300	3-220	150	110
Dolostone, sucrosic	[2]2.0-2.2	[5]7-25	[5]2-470	1-770	240	390
Dolostone, crystalline	[2]2.2-2.4	[5]8-18	[5]1-300	1-100	150	50
Rocks, dense, fractured	[6]1.1-1.8	2-20	[7]0.001-1,000	0.015-870	500	440

[1] Calculated using typical *m*, typical *n*, and $k=1{,}828\times10^{5}\,P^{1.10}$, where $P=n^{(m+2)}/(1-n)^{2}$.

[2] Kwader (1985).

[3] Coates and Dumanoir (1973).

[4] Jorgensen (1980).

[5] Core Laboratories Inc. (1983).

[6] This study.

[7] Brace (1980).

Table 5. Typical range of permeabilities of various consolidated and unconsolidated rocks

INTRINSIC PERMEABILITY (*k*)

Square meter	10^{-19}			10^{-16}				10^{-12}			10^{-9}		10^{-7}
Square foot	10^{-18}			10^{-15}				10^{-11}			10^{-8}		10^{-6}
Millidarcy	10^{-4}	10^{-3}	10^{-2}	10^{-1}	1	10^{1}	10^{2}	10^{3}	10^{4}	10^{5}	10^{6}	10^{7}	10^{8}

	10^{-4} to 10^{-1} mD	10^{-1} to 10^{3} mD	10^{3} to 10^{6} mD	10^{6} to 10^{8} mD
Unconsolidated rocks	Unweathered clay	Very fine sand; silt;mixtures of sand, silt and clay; glacial till; stratified clay	"Clean" sand; mixtures of clean sand and gravel	"Clean" well-sorted gravel or cobble
Consolidated rocks	Shale deeper than 250 feet; evaporites; limestone deeper than 15,000 feet	Sandstone; dolostone; limestone shallower than 15,000 feet; shale shallower than 250 feet	Extensively fractured rock; weathered limestone or dolostone	Rocks with larger than pore-size openings
Permeability characteristic	Very slightly permeable	Slightly permeable	Permeable	Very permeable

Gallons per day per square foot	10^{-5}	10^{-4}	10^{-3}	10^{-2}	10^{-1}	1	10	10^{2}	10^{3}	10^{4}	10^{5}	10^{6}
Foot per day	10^{-6}	10^{-5}	10^{-4}	10^{-3}	10^{-2}	10^{-1}	1	10	10^{2}	10^{3}	10^{4}	10^{5}
Foot per second	10^{-11}	10^{-10}	10^{-9}	10^{-8}	10^{-7}	10^{-6}	10^{-5}	10^{-4}	10^{-3}	10^{-2}	10^{-1}	1

HYDRAULIC CONDUCTIVITY (*k*), AT 1.005 CENTIPOISE

The data may not always closely define the resistivity-porosity curve. Resistivity and porosity logs with expanded depth scales are easiest to use and generally yield the most accurate data. If *Rw* is known from other sources and is plotted at 100-percent porosity, the line and its slope can be easily and accurately defined. *Rw* might be available from chemical analyses of a water sample. The value of *m* can sometimes be estimated if the type of porosity and lithology are known (Asquith, 1985).

Porosity values from other types of porosity logs, such as density, neutron, sonic, and dielectric, can also be used in the cross plot. Logs not in porosity units also can be used in cross plots to determine *m* (Aquilera, 1976, p. 767; Pickett, 1973). For example, a sonic log, which records transit time, can be used. A plot on log-log paper of the difference between the transit time from the log minus transit time of the matrix versus rock-water resistivity (*Ro*) will define a straight line with the slope of $-m$. Density logs recording bulk density can be used similarly as a plot on log-log paper of grain density minus bulk density versus *Ro* will define a straight line with a slope of $-m$. Porosity values from an epithermal-neutron log versus *Ro* when plotted on log-log paper also will define a straight line with the slope of $-m$. Values, in either counts per second or API units, from thermal neutron logs can be plotted on a linear scale versus values of *Ro* on a log scale. The slope of the line defined will be $-m/D$, where D is a function of borehole size and scale function. Thus, ideally, even uncalibrated logs can be used to determine *m*.

The resistivity-to-porosity cross-plot method gives average values within the logged section because the logging tools have a "radius of investigation" and the values measured are a volumetric average for the material within their radius of investigation. The radius of investigation for a "resistivity-logging tool" might be 10 ft or more. A dielectric logging tool measures to about 2 in. depth beyond the borehole wall. The radius of investigation for a "radioactive-logging tool" might be 6 in. to 1 ft. Recorders for radioactive tools commonly average emissions over a time period, which results in additional averaging. Also, the cross-plot method requires several sets of readings at different depths to define the empirical linear relationship; thus "averaging," which is needed to determine effective intrinsic permeability of the entire thickness of a formation, is inherent in the procedure.

The accuracy of the procedure can be checked and improved if the procedure can be calibrated to local aquifer conditions. For example, the results can be checked against permeability values from an aquifer test or a drill-stem test.

SUMMARY

Geophysical logs can be used in geohydrologic studies to estimate porosity, water resistivity, and intrinsic permeability, especially in areas where few hydrologic data are available.

Dual density and neutron porosity logs can be used in conjunction with gamma-ray logs to determine in-situ porosity and to qualitatively identify lithology. Either a spontaneous-potential log or a resistivity log can be used to define relative water resistivity in a rock section.

The spontaneous-potential and cross-plot methods of estimating water resisitivity were tested. The spontaneous-potential method uses spontaneous-potential log measurements and mud-filtrate resistivity to estimate water resistivity. The cross-plot method uses the relation between porosity and the observed resistivity of the saturated rock to estimate water resistivity in the rock. Estimates of water resistivity were compared with measured values. A coefficient of determination of 0.66 for the spontaneous-potential method and 0.88 for the cross-plot method were determined. Plots of the estimated values relative to the measured values show variations of about 1 order of magnitude. The methods are not accurate estimators, but can be used with caution to estimate water resistivity if no measurements are available.

The relations among resistivity measured on a geophysical log, formation-water resistivity, rock-matrix resistivity, and degree of cementation have been investigated by many, such as Archie (1942) and Winsauer and others (1952). Several empirical relations have been proposed and are described in the literature. Raiga-Clemenceau (1977) investigated the relations among cementation factor, permeability, and formation factor and questioned the validity of many of the empirical formation factors that have been proposed since Archie's original work. Raiga-Clemenceau empirically defined the cementation factor as a function of permeability.

By merging the well-known Archie equation with the Kozeny equation, an equation for the intrinsic permeability is derived as shown:

$$k = \frac{\epsilon}{S_s^2} \cdot \frac{n^{m+2}}{(1-n)^2} \quad . \tag{21}$$

The equation implies that permeability is a function of the medium, represented by the first set of terms, and a porosity factor (P) represented by the last set of terms. Porosity values can be determined from logs, such as neutron, density, sonic, and dielectric. A cross plot of observed resistivity to porosity defines the cementation factor. However, the Kozeney coefficient and surface area are not easily determined from borehole-geophysical logs.

The following linear regression equation, established on the basis of 10 carefully selected data sets, describes the relationship between the porosity factor and intrinsic permeability plot.

$$k = 1.828 \times 10^5 (P^{1.10}). \qquad (23)$$

The regression equation has a coefficient of determination of 0.90 and applies to rocks in which surface conductance along grains is not dominant, such as fractured rock, coarse-grained clastics, and most carbonates. Commonly observed porosities and cementation factors for different lithologies were used in the equation to calculate permeabilities. The calculated permeabilities compared well with typical permeabilities for the different lithologies, indicating the general usefulness of the equation. However, additional data, which include in-situ permeability measurements, are needed to better evaluate the relations.

REFERENCES CITED

Alger, R.P., 1966, Interpretation of electric logs in fresh water in unconsolidated formations: Society of Professional Well Log Analysts, 7th Annual Logging Symposium, Tulsa, Oklahoma, May 1966, Transactions, p. CC1-225.

Aquilera, R., 1976, Analysis of naturally fractured reservoirs from conventional well logs: Journal of Petroleum Technology, p. 764-772.

Archie, G.E., 1942, The electrical resistivity log as an aid in determining some reservoir characteristics: American Institute of Mining and Metallurgical Engineers Transactions, v. 146, p. 54-62.

Arps, J.J., 1953, Technical note 195: Journal of Petroleum Technology, sec. 1, p. 18.

Asquith, G.B., 1985, Handbook of log evaluation techniques for carbonate reservoirs: American Association of Petroleum Geologists Methods in Exploration 5, 47 p.

Bateman, R.M., and Konen, C.E., 1977, The log analyst and the programmable pocket calculator: Log Analyst (September-October), v. 18, no. 5, p. 3-10.

Bear, Jacob, 1972, Dynamics of fluids in porous media: New York, American Elsevier, 764 p.

Biella, Giancarlo, Lozei, Alfredo, and Tabacco, Ignazio, 1983, Experimental study of some hydrogeophysical properties of unconsolidated porous media: Ground Water, v. 21, no. 6, p. 741-751.

Blannkenagel, R.K., Miller, W.R., Brown, D.L., and Cushing, E.M., 1977, Report on preliminary data for Madison Limestone test well No. 1, NE, SE, sec. 15, T. 57 N., R. 65 W., Crook County, Wyoming: U.S. Geological Survey Open-File Report 77-164, 97 p.

Brace, W.F., 1977, Permeability from resistivity and pore shape: Journal of Geophysical Research, v. 82, no. 23, p. 3343-3349.

______1980, Permeability of crystalline and argillaceous rocks: International Journal of Rock Mechanics, Mineral Sciences, and Geomechanics Abstracts, v. 17, p. 241-251,

Brace, W.F., Walsh, J.B., and Framgos, W.T., 1968, Permeability of granite under high pressure: Journal of Geophysical Research, v. 73, no. 6, p. 225-2236.

Carothers, J.E., 1968, A statistical study of the formation factor relation: Log Analyst, September-October, p. 13-20.

Coates, G.R., and Dumanoir, J.L., 1973, A new approach to log-derived permeability: Society of Professional Well Log Analysts, 14th Annual Logging Symposium, Transactions, p. 1-28.

Core Laboratories Inc., 1983, Variations of permeability with rock types, *in* Dewan, J.T., Modern open-hole log interpretations: Tulsa, Okla., Penn Well Publishing, figs. 7-8, 361 p.

Croft, M.G., 1971, A method of calculating permeability from electric logs: U.S. Geological Survey Professional Paper 750-B, p. 265-269.

Desai, K.P., and Moore, E.J., 1969, Equivalent NaCl solutions from ionic concentrations: Log Analyst, v. 10, no. 3, p. 12.

Dewan, J.T., 1983, Essentials of modern open-hole log interpretation: Tulsa, Oklahoma, Penn Well Publishing, 361 p.

Gogel, Tony, 1981, Preliminary data from Arbuckle test wells, Miami, Douglas, Saline, and Labette Counties, Kansas: U.S. Geological Survey Open-File Report 81-1112, 155 p.

Helander, D.P., and Campbell, J.M., 1966, The effect of pore configuration, pressure, and temperature on rock resistivity: Society of Professional Well Log Analysts, 7th Annual Logging Symposium, Transactions, 29 p.

Herdan, George, 1960, Small particle statistics: London, Butterworth Scientific Publications, 418 p.

Huntley, David, 1986, Relation between permeability and electrical resistivity in granular aquifers: Ground Water, v. 24, no. 4, p. 466-474.

Jones, P.H., and Buford, T.B., 1951, Electric logging applied to ground-water exploration: Geophysics, v. 16, no. 1, p. 115-139.

Jorgensen, D.G., 1980, Relationships between basic soil-engineering equations and basic ground-water flow equations: U.S. Geological Survey Water-Supply Paper 2064, 40 p.

Keys, S.W., and MacCary, L.M., 1971, Application of borehole geophysics to water-resources investigations: U.S. Geological Survey Techniques of Water-Resources Investigations, book 2, chap. E1, 123 p.

Kwader, Thomas, 1982, Interpretation of borehole geophysical logs in shallow carbonate environments and their application to ground-water resources investigations: Tallahassee, Florida State University Ph.D. dissertation, 186 p.

______1985, Estimating aquifer permeability from formation resistivity factor: Ground Water, v. 23, no. 6, p. 762-766.

MacCary, L.M., 1978, Interpretation of well logs in a carbonate aquifer: U.S. Geological Survey Water-Resources Investigations Report 78-88, 30 p.

______1980, Use of geophysical logs to estimate water-quality trends in carbonate aquifers: U.S. Geological Survey Water-Resources Investigations Report 80-57, 23 p.

______1983, Geophysical logging in carbonate aquifers: Ground Water, v. 21, p. 334-342.

______1984, Relation of formation factor to depth of burial along the Texas Gulf Coast, *in* Surface and borehole geophysical methods in ground-water investigations: U.S. Environmental Protection Agency and National Water Well Association (Feb. 6-9, 1984), p. 722-741.

Matthews, C.S., and Russell, D.C., 1967, Pressure build up and flow tests in wells: Dallas, Texas, Society of Petroleum Engineers of American Institute of Mining, Metallurgical, and Petroleum Engineers Monograph 1, 158 p.

Meyer, C.A., McIntosh, R.B., Silvestri, G.J., and Spencer, R.C., 1968, 1967 steam tables (2d ed.): New York, American Society of Mechanical Engineers, 328 p.

Miller, R.W., 1977, A Galerkin finite-element analysis of steady-state flow and heat transport in the shallow hydrothermal system in the east Mesa area, Imperial Valley California: U.S. Geological Survey Journal of Research, v. 5, no. 4, p. 497-508.

Ogbe, David, and Bassiouni, Zaki, 1978, Estimating of aquifer permeabilities from electric well logs: Log Analyst, September-October, p. 21-27.

Pfannkuch, H.O., 1969, On the correlation of electrical conductivity properties of porous systems with viscous flow tranport coefficients: Haifa, First International Symposium of the Fundamentals of Transport Phenomena in Porous Media, International Association of Hydraulic Research, p. 42-54.

Phillips, S.L., Igbene, A., Fair, J.A., and Ozebek, H., 1981, A technical databook for geothermal energy utilization: Lawrence Berkley Laboratory, University of California LBL-12810, 46 p.

Pickett, G.R., 1973, Pattern recognition as a means of formation evaluation: Society of Professional Well Log Analysts, 14th Annual Logging Symposium, Transactions, p. A1-A21.

Raiga-Clemenceau, J., 1977, The cementation exponent in the formation factor-porosity relation—The effect of permeability: Society of Professional Well Log Analysts, 18th Annual Logging Symposium, Transactions, 13 p.

Schlumberger Limited, 1972, Log interpretation. Volume 1—Principles: New York, 112 p.

Schlumberger Well Surveying Corporation, 1979, Log interpretation charts: Houston, Texas, 97 p.

Sethi, D.K., 1979, Some consideration about formation resistivity factor-porosity relations: Society of Professional Well Log Analysts, 20th Annual Logging Symposium, Transactions, 11 p.

Somerton, W.H., 1982, Porous rock-fluid systems at elevated temperatures and pressures: Geological Society of America Special Paper 189, p. 183-197.

Stephens, D.R., and Lin, W., 1978, Relationships of permeability, porosity, and formation factor: University of California Research Letter 80856, 9 p.

Turcan, A.N., Jr., 1966, Calculation of water quality from electrical logs—Theory and practice: Louisiana Geological Survey Water Resources Pamphlet 19, 23 p.

Urish, D.W., 1981, Electrical resistivity-hydraulic conductivity relationships in glacial outwash aquifers: Water Resources Research, v. 17, no. 5, p. 1401-1408.

Weast, R.C., ed., 1984, CRC Handbook of chemistry and physics: Boca Raton, Florida, CRC Press, p. F-11, F-38.

Weiss, Emanuel, 1982, A computer program for calculating relative transmissivity input arrays to aid model calibration: U.S. Geological Survey Open-File Report 82-447, 18 p.

Winsauer, W.O., Shearin, H.M., Masson, P.H., and Williams, M., 1952, Resistivity of brine-saturated sands in relation to pore geometries: American Association of Petroleum Geologists Bulletin, v. 36, no. 2, p. 253-277.

Worthington, P.F., 1976, Hydrogeophysical equivalence of water salinity, porosity, and matrix conduction in arenaceous aquifers: Ground Water, p. 224-232.

Wyllie, M.R.J., and Rose, W.D., 1950, Application of the Kozeny equation to consolidated porous media: Nature, no. 165, p. 972.

Wyllie, M.R.J., and Spangler, M.B., 1952, Application of electrical resistivity measurements to the problem of fluid flow in porous media: American Association of Petroleum Geologists Bulletin, v. 36, no. 2, p. 359-403.

APPENDIX A

Estimating Dissolved-Solids Concentration

Dissolved-solids concentration can be estimated from either resisitivity or specific conductance measurements. Specific conductance is a measure of the conductance of electrical current through a fluid and is usually expressed in microsiemens or micromhos. A specific-conductance measurement is referenced to a specified temperature, usually 25 °C (77 °F).

Specific conductance (SC), which is the resistivity reciprocal at 77 °F, can be calculated from the equation:

$$SC = \frac{1\times 10^4}{Rw_{77}} \quad , \tag{A1}$$

where Rw_{77} is the water resistivity at 77 °F, in ohm-meters.

Resistivity of water at any temperature, T_x, in degrees Fahrenheit, can be converted to Rw_{77} by:

$$Rw_{77} = Rw_x\,(T_x+7)/84 \quad . \tag{A2}$$

The relation between dissolved-solids concentration, in parts per million, and specific conductance, in microsiemens is

$$DS \cong (P)\,(SC) \tag{A3}$$

where P is a factor to be determined for each water, but which typically is about 0.67 for many ground waters if specific conductance is in microsiemens.

Dissolved-solids concentration can also be estimated from the sodium-chloride concentration. The empirical relation between sodium-chloride concentration (C_{NaCl}) and dissolved-solids concentration (DS) is:

$$DS \cong (A)\,(C_{\mathrm{NaCl}}) \quad , \tag{A4}$$

where DS and C_{NaCl} are both in the same units. The coefficient A should be determined for each water type; however, the value of A generally used is about 1.04 for many natural waters.

APPENDIX B

Estimating Sodium-Chloride Concentration

Many useful interpretations of geophysical-log data are based on relations for sodium-chloride solutions. Relations among water resistivity, temperature, and dissolved-solids concentration are shown on figure B1. Most of the curves on figure B1 can be approximated by the equation:

$$C_{\mathrm{NaCl,ppm}} \cong (2500/Rw_{75})^{1.125} \quad , \tag{B1}$$

where $C_{\mathrm{NaCl,ppm}}$ is sodium-chloride concentration, in parts per million, and Rw_{75} is the resistivity, in ohm-meters, of the solution at 75 °F. 75 °F is the usual reference temperature in geophysical logging.

Values of resistivity, in ohm-meters, at any temperature, T_x, in degrees Fahrenheit, can be converted to resistivity at 75 °F by the equation:

$$Rw_{75} = (Rw_x)\,(T_x+7)/(82) \quad . \tag{B2}$$

Sodium-chloride concentration, in milligrams per liter, can be converted to concentration in parts per million:

$$C_{\mathrm{NaCl,ppm}} = (C_{\mathrm{NaCl,mg/L}})/(G) \quad , \tag{B3}$$

where G is specific gravity and is the solution density divided by pure-water density.

Concentration of a sodium-chloride solution in milligrams per liter ($C_{\mathrm{NaCl,mg/L}}$) can be converted to estimated concentrations in part per million ($C_{\mathrm{NaCl,ppm}}$) by the following equation (Robert Leonard, U.S. Geological Survey, written commun., 1984):

$$C_{\mathrm{NaCl,ppm}} \cong (1 - 6.7\times 10^{-7})\; C_{\mathrm{NaCl,mg/L}} \quad . \tag{B4}$$

Investigators, such as Turcan (1966) and Desai and Moore (1969), list multipliers that allow resistivity of equivalent sodium-chloride solutions to be estimated for solutions that contain ions other than Na and Cl.

APPENDIX C

Estimating Density

Density is a function of temperature, pressure, and salinity (fig. 12). Miller (1977) reported a relation derived by J.W. Mercer (U.S. Geological Survey, written commun., 1975) to estimate density (d) of freshwater, in kg/m^3 (kilograms per cubic meter) as a function of temperature and pressure:

$$\begin{aligned} d_{T,p} = {} & 2.0743\times 10^{3} + 5.56172\times 10^{-9}p \\ & -4.26895\times 10^{-5}(1/T_a) \\ & -2.35494\times 10^{-3}\;T_a^{2} \\ & +4.95912\times 10^{7}(1/T_a^{2}) \quad ; \end{aligned} \tag{C1}$$

where p is pressure, in Pa (pascals); and T_a is absolute temperature.

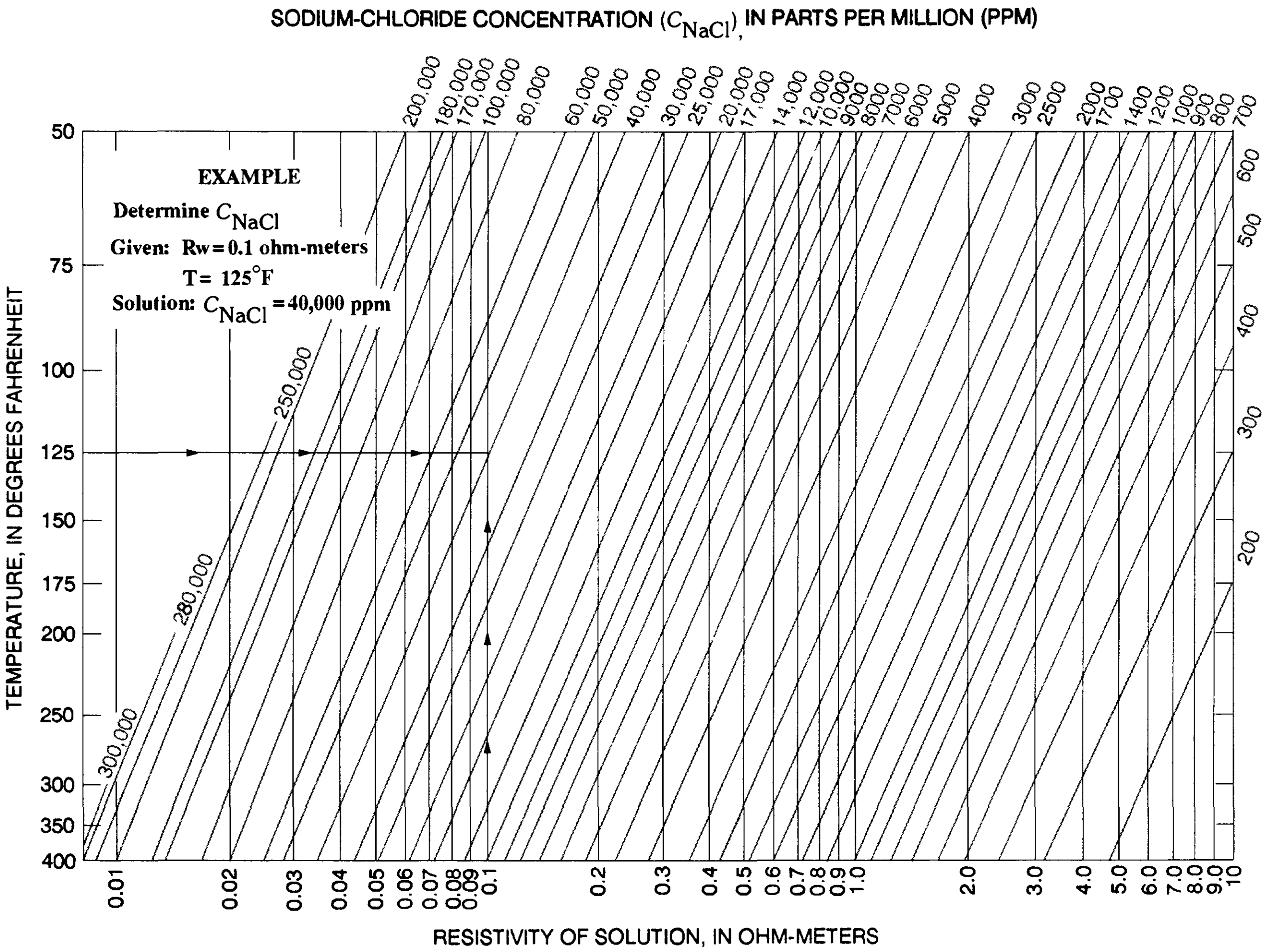


Figure B1. Resistivity of water as a function of salinity and temperature (modified from Schlumberger, 1972). Arrows show how to solve the example problem.

T_a can be calculated by

$$T_a = T_c + 273.15 \quad , \qquad \text{(C2)}$$

Equation C1 is based on a least-squares regression of data by Meyer and others (1968) for a range of 0° to 350°C and 1.0×10^5 to 1.75×10^7 P_a.

Curves shown by Schlumberger Limited (1972, fig. 8-5, p. 47) indicate that density (d) is affected by the sodium-chloride concentration:

$$\Delta d \cong 7.3\times10^{-4}\ C_{\mathrm{NaCl,ppm}} \quad , \qquad \text{(C3)}$$

where Δd is change of density in grams per cubic centimeter and $C_{\mathrm{NaCl,ppm}}$ is sodium-chloride concentration in parts per million. Thus, an estimate of density as a function of pressure, temperature, and salinity might be made by the use of both equations C1 and C3:

$$d \cong d_{\mathrm{T,p}} + \Delta d \qquad \text{(C4)}$$

where d and Δd are in the same units.

In borehole geophysical logging applications, many useful interpretations are based on the properties of sodium-chloride solutions. Phillips and others (1981) derived a regression equation of aqueous sodium-chloride solutions:

$$d = A + Bx + Cx^2 + Dx^3 \qquad \text{(C5)}$$

and

$$x = C_1 e^{(a1\ mol)} + C_2 e^{(a2\ T_c)} + C_3 e^{(a3p)};$$

where d is density, in grams per cubic centimeter;
T_c is temperature, in degrees Celsius;
p is pressure, in mega pascals (MPa);
mol is molal concentration of NaCl;
C_1 = -9.9595; C_2 = 7.0845; C_3 = 3.9093;
$a1$ = 0.004539 ($a1$ = -0.00500 for vapor saturated values);
$a2$ = -0.0001638; $a3$ = 0.00002551; A = -3.033405; B = 10.128163; C = -8.7507567; D = 2.663107;

and

e = the exponential constant.

Phillips and others (1981) reported that the experimental data were reproduced to a maximum deviation of ± 2 percent over the entire range of concentration, for temperatures of 0°–350°C, and for pressure as high as 50 MPa.

An empirical relation derived by Robert Leonard (U.S. Geological Survey, written commun., 1984) to estimate specific gravity, which is solution density divided by density of pure water, is

$$G \cong 1/(1 - 6.7\times10^{-7} C_{\mathrm{NaCl,ppm}}) \quad , \qquad \text{(C6)}$$

where G is specific gravity and $C_{\mathrm{NaCl,ppm}}$ is sodium-chloride concentration in parts per million.

APPENDIX D

Estimating Viscosity

An equation to estimate dynamic viscosity of water, (μ), was derived by Emanuel Weiss (1982, p. 4). The equation approximates the curves of Matthews and Russell (1967) shown in figure 11. The approximate equation is:

$$\mu \cong [(38.3/T_f^{0.5}) - (14.6/T_f^{0.25}) + 1.48]$$

$$[1 + (C_{ds,\mathrm{mg/L}}/300)] \ . \qquad \text{(D1)}$$

Where μ is dynamic viscosity, in centipoise;

T_f is temperature, in degrees Fahrenheit; and

$C_{ds,\mathrm{mg/L}}$ is dissolved-solids concentration, in milligrams per liter.

Weiss (1982) reported that the largest error is at 55°F, at which temperature the value of the calculated μ from equation D1 is inaccurate by nearly 10 percent.

Miller (1977) reported on the work of Meyer and others (1968) to estimate viscosity. If viscosity is a function of temperature alone, viscosity, in pascal-seconds, can be estimated by:

$$\mu \cong (10^{-7})\,(241.4\times10^{x}) \quad . \qquad \text{(D2)}$$

The exponent x is:

$$x = (247.8)/(T_c + 133.15) \quad ,$$

where T_c is temperature in degrees Celsius. The equation is reported valid for liquid water along the saturation line from 0–300 degrees Celsius.

Phillips and others (1981) reported a correlation for sodium-chloride solutions using more than 1,500 selected experimental values. The model was originally developed for colloidal suspensions and solutions of nonelectrolytes; however, the equation has been used extensively for electrolytes, such as NaCl, after suitable modification:

$$\mu/\mu_w = 1 + a\,mol + b\,mol^2 + c\,mol^3 + d\,T_c(1 - e^{i\,mol}) \quad , \qquad \text{(D3)}$$

where μ is viscosity of sodium-chloride solution, in centipoise;

μ_w is viscosity of water, in centipoise;

$a = 0.0816$; $b = 0.0122$;

$c = 0.000128$; $d = 0.000629$;

$i = -0.7$; T_c is temperature, in degrees Celsius;

mol = molal concentration, in gram-moles NaCl per kilogram of water;

and e is the exponential constant.

Phillips and others (1981) reported that the equation reproduces data to an average of better than ±2 percent over the range 10 to 350°C, 0.1 to 50 MPa, and 0–5 *mol.*

☆U.S. GOVERNMENT PRINTING OFFICE: 1989-673-047/86048