A Dynamical-Systems Approach for Computing Ice-Affected Streamflow

United States Geological Survey Water-Supply Paper 2473

Prepared in cooperation with the MICHIGAN DEPARTMENT OF NATURAL RESOURCES

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By DAVID J. HOLTSCHLAG

Prepared in cooperation with the
Michigan Department of Natural Resources

U.S. GEOLOGICAL SURVEY WATER-SUPPLY PAPER 2473

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UNITED STATES GOVERNMENT PRINTING OFFICE, WASHINGTON : 1996

For sale by the
U.S. Geological Survey
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Box 25286, Federal Center
Denver, CO 80225

Library of Congress Cataloging in Publication Data

A dynamic-systems approach for computing ice-affected streamflow / by David J. Holtschlag.
p. cm. — (U.S. Geological Survey water supply paper ; 2473)
"Prepared in cooperation with the Michigan Department of Natural Resources."
Includes bibliographical references.
Supt. of Docs. no.: I 19.13 ; 2473
1. Stream measurements. 2. Ice on rivers, lakes, etc.. I. Michigan. Dept. of Natural Resources. II. Title. III. Series.
GB1205.H65 1996
551.48'3—dc20 96–22075
CIP

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CONVERSION FACTORS

Multiply	By	To obtain
inch (in.)	25.4	millimeter
inch per year (in/yr)	25.4	millimeter per year
foot (ft)	0.3048	meter
square mile (mi^2)	2.590	square kilometer
cubic foot per second (ft^3/s)	0.02832	cubic meter per second

Temperature in degrees Fahrenheit (°F) can be converted to degrees Celsius (°C)
Temp °C = (temp °F–32)/1.8.

A Dynamical-Systems Approach for Computing Ice-Affected Streamflow

By David J. Holtschlag

Abstract

A dynamical-systems approach was developed and evaluated for computing ice-affected streamflow. The approach provides for dynamic simulation and parameter estimation of site-specific equations relating ice effects to routinely measured environmental variables. The form of the site-specific equations are user specified. Parameters are estimated by minimizing an objective function, which is also user specified. Error correction is used to prorate the error in the simulation estimates so that final estimates match daily mean discharge computed on days of direct streamflow measurements.

The accuracy of the dynamical-systems approach was evaluated by use of an extensive data set that provides detailed information on ice-affected streamflow for three streams in Iowa during the winter of 1987-88. The report expands on the work of previous investigators, who originally obtained the data and compared 11 analytical methods that are proposed or commonly used for estimating ice-affected streamflow.

Comparison indicates that the results from the dynamical-systems approach ranked higher than the results from 11 analytical methods previously investigated on the basis of accuracy and feasibility criteria. Given the modest data requirements, ease of computer application, and the reproducibility and objectivity of the method, the dynamical-systems approach has the potential to become an important method for computing streamflow records. Research on alternative forms of site-specific equations, choice of objective functions and optimization methods, simultaneous simulation of ice-effects in a network of streamflow-gaging stations, and the effect of time-step length will likely lead to further improvements in the dynamical-systems approach.

INTRODUCTION

The U.S. Geological Survey (USGS) operates hydrologic data-collection stations nationwide to provide the general public, the private sector, and all levels of government with water-resources information. Daily mean streamflow is a major component of this data-collection program. In 1993, daily mean streamflow was recorded by the USGS at 7,272 continuous-record gaging stations in the United States (Condes de la Torre, 1994).

The term "streamflow," as used by the USGS, is synonymous with discharge (volume of water per unit time) in a natural channel. Daily mean streamflow is computed on the basis of hourly or more frequent measurements of water-surface elevation (stage) and a rating curve that defines the relation between stage and discharge at a particular station. This relation is referred to as the stage-discharge rating or the open-water rating. This stage-discharge rating is affected by backwater from ice formation at about one-half of the USGS streamflow-gaging stations during part of the winter (Melcher and Walker, 1992, Appendix A2). The variable ice-backwater effect is a major source of uncertainty in the computation of streamflow records.

In a continuing effort to improve the quality of information provided by USGS streamflow-data network, Melcher and Walker (1992) analyzed 11 widely used or proposed analytical methods of estimating ice-affected streamflow. The methods were classified as analytical because they are based on systematic computation.

Purpose and Scope

The purpose of this report is to develop a new analytical method for estimating ice-affected streamflow that is based on a dynamical-systems approach. The accuracy of this method is compared with the accuracy of 11 analytical methods evaluated by Melcher and Walker (1992).

Previous Work

Melcher and Walker (1992) give a detailed description of methods evaluated for computing ice-affected streamflow. A brief description of the 11 analytical methods is provided in this report as an aid to the reader.

Prorated discharge.—The prorated-discharge method involves linear interpolation of daily discharge between periods of known discharge for estimation of discharge during ice-affected periods. Discharge is considered known on days when daily discharge can be reliably computed from the open-water rating or at times of direct measurements.

Discharge ratio.—In the discharge-ratio method, the open-water daily mean discharge is multiplied by a variable discharge ratio to give the corrected discharge during periods of ice cover. The discharge ratio is computed for each discharge measurement as the ratio of measured discharge to the open-water rated discharge. The discharge ratio varies during the winter with time as changes occur in the ice cover. The discharge ratio is obtained by interpolation, on the basis of time, between the discharge ratios computed for consecutive discharge measurements (Rantz and others, 1982, p. 368)

Backwater shift.—The backwater-shift method involves linear interpolation between backwater shifts defined by direct measurements of ice-affected streamflow. The backwater shift is defined as the difference between measured (ice-affected) gage height and gage height corresponding to the measured discharge on the open-water rating. The interpolated backwater shift is subtracted from the stage used in computing streamflow from the open-water rating.

Stage fall.—The stage-fall method involves use of an auxiliary gage height, measured upstream or downstream of the main gage, to adjust the open-water rating discharge for the water-surface fall over the stream reach (Carey, 1967). Implementation of the method requires an extensive set of direct streamflow measurements and corresponding auxiliary stage measurements to determine the relation between fall and discharge adjustment.

Adjusted rating curve. —The adjusted-rating-curve method is based on a modification of the open-water rating curve to compensate for increased slope of the rating curve due to additional flow resistance associated with the ice cover. Lavender (1984) derived an equation relating the slope of the ice-cover rating curve to the slope of the open-water rating curve from Manning's equation. To implement this method, the user determines the backwater shift and also determines the changes in ice roughness from vertical profiles obtained for each ice-affected measurement. The changes in shift and roughness are prorated between measurements and combined with stage data to compute streamflow.

Conductance correlation. —The conductance-correlation method is based on a statistical relation between streamflow and specific conductance. Specific conductance is generally inversely related to streamflow. To implement the method, the user needs one or more measurements of specific conductance per day.

Multiple regression. —The multiple-regression method is based on a statistical relation to predict the ice effect on the basis of other hydrologic or climatological variables. Explanatory variables commonly include gage height, rated discharge, concurrent daily streamflow at a correlated station, and maximum and minimum daily air temperature.

Index velocity.— The index-velocity method requires effective cross-sectional area of the channel and stream velocity to compute streamflow. Effective cross-sectional area is the portion of the cross-section that contains liquid. Implementation of this method requires continuous monitoring of stream velocity. Streamflow is computed as the product of mean velocity and effective cross-sectional area determined from the daily mean gage height.

Ice-adjustment factor.—In the ice-adjustment-factor method (Alger and Santeford, 1987), streamflow is computed by use of hydraulic-property and stage-discharge rating tables, interpolated values of ice-adjustment factors and float depths, and daily mean gage heights.

Pipe flow.—The pipe-flow method involves computation of streamflow under an ice cover as closed-conduit flow. The Darcy-Weisbach equation is used with a modified friction factor based on water-surface slope (Carey, 1967). The relation between the modified friction factor and the water-surface slope is estimated from direct measurements of streamflow and fall. Daily mean gage heights and once-daily auxiliary gage heights are used to compute daily estimates of streamflow.

Uniform flow.—The uniform-flow method involves Manning's equation for computation of streamflow during periods of ice effect. Bed and ice-cover roughness are determined from vertical velocity profiles and are prorated between measurements. Daily mean gage height and an estimate of the effective cross-sectional area are used to compute streamflow.

METHODS OF STUDY

The evaluation of the dynamical-systems approach is based on field data collected in Iowa and originally analyzed by Melcher and Walker (1992). The various ice conditions that typically occur in Iowa are representative of ice conditions that occur throughout much of the United States. Streams in the mid-latitude regions of the United States tend to have intermittent ice-affected periods during the winter. The intermittent periods are associated with alternating warm and cool periods that cause the channel ice to melt and refreeze.

Data used in comparing the various analytical methods with simulation modeling were collected at three test streamflow-gaging stations in Iowa (fig. 1) during the winter of 1987-88. The meteorological conditions in Iowa during the 1987-88 winter were near normal (Melcher and Walker, 1992). A description of the hydrologic and hydraulic conditions at each of the streamflow-gaging stations and the data-collection strategy follows.

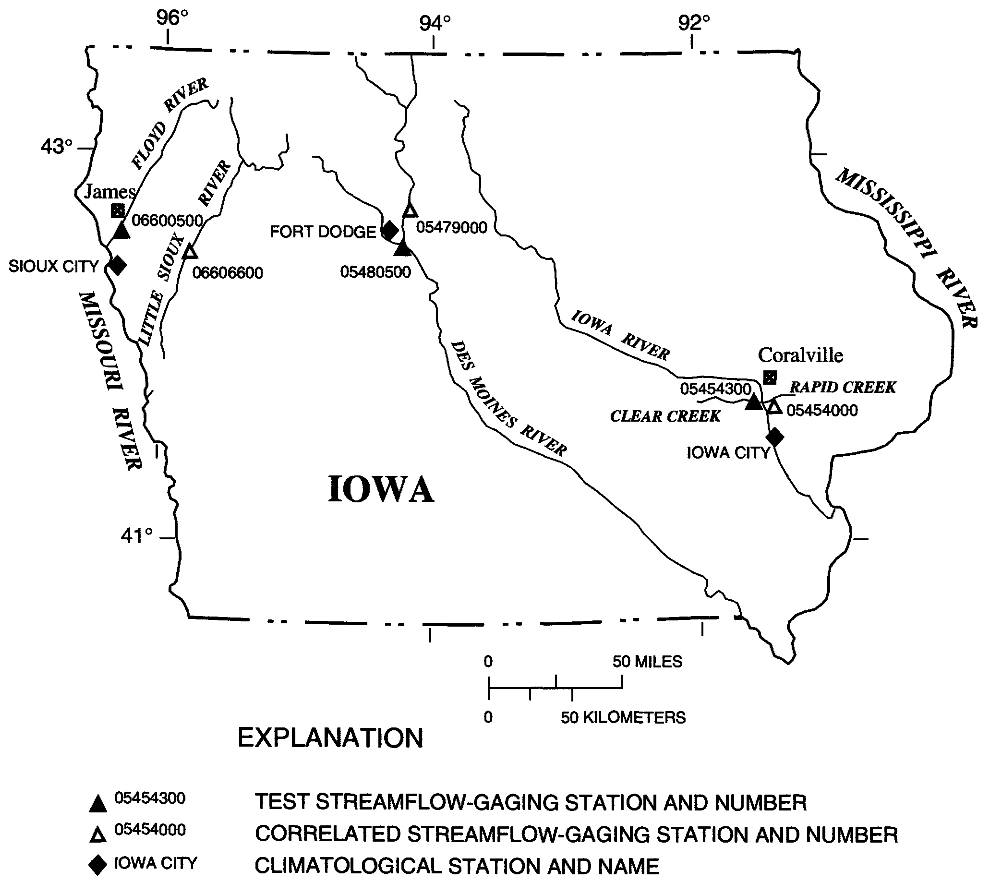

Figure 1. Location of selected streamflow-gaging and climatic stations in Iowa.

Sites of Data Collection

The streamflow-gaging station Clear Creek near Coralville, Iowa (USGS streamflow-gaging station 05454300), is on a small tributary of the Iowa River in the east-central part of Iowa. The region is humid (annual runoff is greater than 5 in.); average precipitation is 34.6 in/yr and average snowfall is 28.6 in/yr. The mean annual temperature is 50.2°F, and the mean January temperature is 19.8°F (E. May, Iowa State Climatologist, written commun., 1988). Clear Creek has a drainage area of 98.1 mi^2 at the streamflow-gaging station. During the winter, the channel width is about 45 ft and the mean depth is about 0.8 ft. The streambed is composed of sand and silt. Ancillary data needed for some of the methods of estimating ice-affected streamflow were obtained from the climatological station at Iowa City, Iowa, and the streamflow-gaging station Rapid Creek near Iowa City, Iowa (USGS streamflow-gaging station 05454000).

The streamflow-gaging station Des Moines River at Fort Dodge, Iowa (USGS streamflow-gaging station 05480500), is on the main stem of the Des Moines River in the north-central part of Iowa near Fort Dodge. The region is humid; average precipitation is 32.3 in/yr, and average snowfall is 39.5 in/yr. The mean annual temperature is 47.5°F, and the mean January temperature is 15.8°F (E. May, Iowa State Climatologist, written commun., 1988). Des Moines River has a drainage area of 4,190 mi^2 at the streamflow-gaging station. During the winter, the channel width is about 320 ft and the mean depth is about 1.5 ft. The streambed is composed of gravel and cobbles, and open-water stage-discharge relation at the station is stable. Ancillary data needed for some of the methods were obtained from the climatological station at Fort Dodge, Iowa, and the streamflow-gaging station East Fork Des Moines River at Dakota City, Iowa (USGS streamflow-gaging station 05479000).

The streamflow-gaging station Floyd River at James, Iowa (USGS streamflow-gaging station 06600500), is on a tributary to the Missouri River in the northwestern part of Iowa near Sioux City. The region is subhumid (annual runoff is 2 to 5 in.); average precipitation is 25.4 in/yr, and average snowfall is 31.6 in/yr. The mean annual temperature is 48.4°F and the mean January temperature is 16.2°F (E. May, Iowa State Climatologist, written commun., 1988). Floyd River has a drainage area of 882 mi^2 at the streamflow-gaging station. During the winter, the channel width is about 110 ft and the mean depth is about 0.7 ft. The streambed is composed of silt and fine sand. The stage-discharge relation at the station is subject to a moderate amount of shifting. Ancillary data needed to support some of the methods were obtained from the climatological station at Sioux City, Iowa, and the streamflow-gaging station Little Sioux River at Correctionville, Iowa (USGS streamflow-gaging station 06606600).

Hydrologic and Climatic Data

Hydrologic and climatic data were compiled to provide information needed for method application and to compare the accuracy of alternative methods. Data requirements for method application differed among methods (table 1). Those methods requiring only routinely collected information could be calibrated by use of historical data; thus, most of the data collected during the winter of 1987-88 could be used for independent verification of the accuracy of the method. Some of the special data required for other methods could be obtained only during the winter of 1987-88.

Table 1. Field-data requirements for application of selected methods for estimating ice-affected streamflow

[X indicates data required; --, data not needed]

Method	Field-data requirements: Routine				Field-data requirements: Special				
	Stage	Streamflow measurements	Air temperature	Correlated station	Auxiliary stage	Vertical velocity	Specific conductance	Point velocity	Floating ice depth
Unadjusted simulation	X	X	X	–	–	–	–	–	–
Error-corrected simulation	X	X	X	–	–	–	–	–	–
Prorated discharge	–	X	–	–	–	–	–	–	–
Discharge ratio	X	X	–	–	–	–	–	–	–
Backwater shift	X	X	–	–	–	–	–	–	–
Stage fall	X	X	–	–	X	–	–	–	–
Adjusted-rating curve	X	X	–	–	–	X	–	–	–
Conductance correlation	X	X	–	–	–	–	X	–	–
Multiple regression	X	X	X	X	–	–	–	–	–
Index velocity	X	X	–	–	–	–	–	–	–
Ice-adjustment factor	X	X	–	–	–	–	–	–	X
Pipe flow	X	X	–	–	X	–	–	–	–
Uniform flow	X	X	–	–	–	X	–	–	–

Routinely collected data include stage (at hourly or more frequent intervals), direct streamflow measurements at 4- to 6-week intervals, maximum and minimum daily air temperatures and precipitation at nearby climatological stations, and stage data from nearby streamflow-gaging stations. Rated discharge (the discharge indicated by the open-water stage-discharge rating and stage data) also is routinely computed.

Data specially collected during the winter of 1987-88 included vertical-velocity-profile data, continuous-point-velocity data, once-daily specific-conductance readings of the flowing water, and auxiliary measurements of stage needed to compute the slope or fall of the water surface. Some methods could not be evaluated for all three streamflow-gaging stations because the required special data were not available.

Extensive sets of direct streamflow measurements were obtained at 1- to 5-day intervals during the winter of 1987-88. These measurements were used by Melcher and Walker (1992) to compute a highly accurate estimate of daily mean streamflow referred to as the "baseline streamflow" (Qb_t). The baseline streamflow was used to measure the relative accuracy of the selected methods. Subsets of these measurements at 6-week intervals, corresponding to the routinely available streamflow measurements, were provided for application of all the methods.

A DYNAMICAL-SYSTEMS APPROACH FOR COMPUTING ICE-AFFECTED STREAMFLOW

A dynamical-systems approach was developed and evaluated for computing ice-affected streamflow. The approach provides for simulation and parameter estimation of site-specific equations relating ice effects to routinely measured environmental variables. The form of the site-specific equations and the objective function are user specified. Parameters are estimated by minimizing an objective function, which is also user specified. Error correction is used to adjust initial simulation estimates so that final simulation estimates match daily mean discharge computed on days of direct streamflow measurements.

Simulation and Error Correction

Simulation modeling provides a simplified description of dynamic, nonlinear process similar to the simplified description of static, linear process provided by regression modeling. In simulation modeling, an initial (unadjusted) estimate of the daily mean streamflow is computed by use of the identity

$$\tilde{Q}s_t = R_t \times Qr_t, \qquad (1)$$

where

$\tilde{Q}s_t$ is the unadjusted simulation estimate at time t;
R_t is the discharge ratio at time t, where the discharge ratio is the ratio of daily mean streamflow to the rated daily mean streamflow; and
Qr_t is the rated daily mean streamflow at time t, which indicates the daily mean discharge when the open-water rating is applicable.

The general form of the equation used to simulate changes in the discharge ratio can be written as

$$R_t + aR_{t-1} = bu_t + e_t\ , \qquad (2)$$

where

u_t is a column vector of explanatory variables at time t, which are selected on the basis of physical significance and site-specific conditions;
a and b are estimated parameters where a is a scalar and b is a row vector; and
e_t is the error component at time t.

To ensure consistency with the physical characteristics of this system, simulated values of the discharge ratio are limited to a maximum range of numbers between 0 and 1.

An iterative parameter estimation technique is used to minimize a user-specified objective function. After each simulation of the entire period of historical record, a multivariate optimization subroutine, which uses a direct-search polytope algorithm (IMSL, Inc., 1989, p. 831), is called from within the simulation program to compute parameter estimates. Iteration continues until either the least-squares objective function reaches a minimum or parameter estimates stabilize.

In this analysis, the objective function was the sum of squared differences between the logarithms of the unadjusted simulation estimates of daily mean streamflow, $\tilde{Q}s_t$, and the logarithms of daily mean streamflow computed for days of direct streamflow measurements Qm_t. A reliable value of daily mean streamflow on days of direct measurement can generally be computed on the basis of ice effects defined by the direct streamflow measurement together with hourly stage values and the open-water rating.

Error correction is used to prorate the error over time between direct streamflow measurements (the inter-measurement interval) for all streamflow measurements within the simulation time interval. The final simulation estimate is referred to as the error-corrected simulation estimate, Qs_t. In this analysis, the estimated error in the unadjusted simulation-model estimate is computed by linear interpolation as

$$\hat{e}_t = e_\alpha + \frac{(t - t_\alpha)\,(e_\omega - e_\alpha)}{t_\omega - t_\alpha}, \qquad (3)$$

where

$e_\alpha = Qm_{t_\alpha} - \tilde{Q}s_{t_\alpha}$,

$e_\omega = Qm_{t_\omega} - \tilde{Q}s_{t_\omega}$,

t_α is the time of the direct streamflow measurement at the start of the inter-measurement interval, and

t_ω is the time of the direct streamflow measurement at the end of the inter-measurement interval.

The error-corrected simulation estimate is computed as

$$Qs_t = \tilde{Q}s_t + \hat{e}_t. \qquad (4)$$

Site-specific forms for the simulation models developed for the three sites in Iowa are discussed in the following section.

Ice-Affected Streamflow at Selected Sites

Simulation models were developed for three streamflow-gaging stations in Iowa. All models used equation 1 to compute an unadjusted simulation estimate; however, the explanatory variables used to compute the discharge ratio (equation 2) varied among stations. Explanatory variables identified for possible inclusion in the models were selected from among environmental data routinely available at USGS gaging stations. In this analysis, explanatory variable were selected on the basis of site-specific conditions as indicated by standard statistical tests. Although this criteria was considered adequate for testing and evaluation of the dynamical-systems approach, a more consistent criteria for explanatory variable selection may be more appropriate for routine applications involving simulation of a network of gaging stations. Finally, an error estimate was computed by use of equation 3 and an error-corrected simulation estimate was computed by use of equation 4. A description of each of the simulation models follows.

Clear Creek near Coralville, Iowa

The form of the equation for simulating the discharge ratio for Clear Creek near Coralville, Iowa, is

$$R_t - a_1 R_{t-1} = b_0 + b_1 Al_{t-1} + b_2 \log \tilde{Q}s_{t|t-1} + b_3 J_{t-1} \qquad (5)$$

where

R_t and R_{t-1} are the simulated discharge ratios on day t and t-1, respectively, constrained to the interval 0.05 to 1.00;

Al_{t-1} is the minimum daily air temperature at time t-1;

$\tilde{Q}s_{t|t-1}$ is the unadjusted simulation estimate of daily mean streamflow at time t based on information available at time t-1. The estimate is computed as

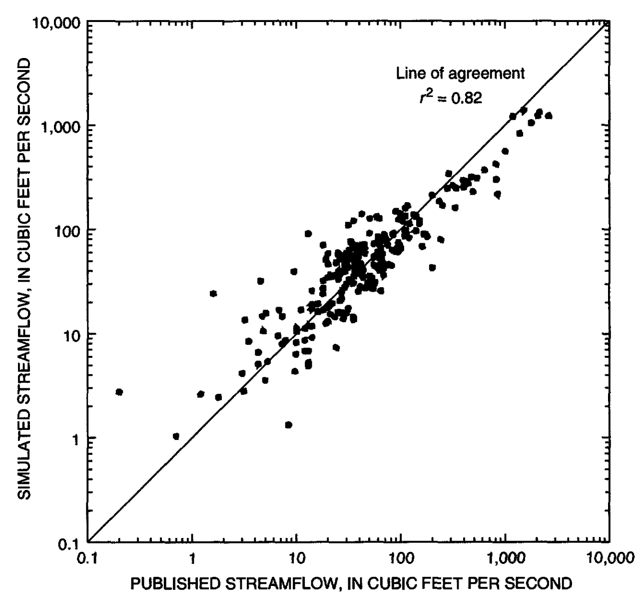

Figure 2. Relation between simulated and published daily streamflow data, Clear Creek near Coralville, Iowa, during the calibration period (winters of 1960-61 through 1986-87).

the rated discharge at day t times the computed discharge ratio for the previous day;

$J_{t\text{-}1}$ is the number of days since the beginning of the ice-affected period prior to time t; and

$a_1, b_0, b_1, b_2,$ and b_3 are parameters with estimated values of 0.9055, -0.01165, 0.0009423, 0.03338, and 0.0001434, respectively.

The parameters indicate that discharge ratios separated by 1 day are highly positively correlated and that higher air temperatures and higher discharge rates increase the discharge ratio. The last term in the equation adjusts estimates for an apparent trend within each ice-affected period.

The sample coefficient of determination (r^2) between published daily streamflow data and simulated daily streamflow data is 0.82 (fig. 2). The relation between simulated and published streamflows is based on 177 days when direct measurements defined the ice-backwater effects during the winters of 1960-61 through 1986-87. The residuals, formed from the differences between logarithms of simulated and published streamflows, are another measure of the accuracy of the simulation model. The mean residual, which describes the bias, is -0.0076. The standard deviation of the residuals is 0.2583. Streamflows during the winter of 1987-88 are shown in figure 3.

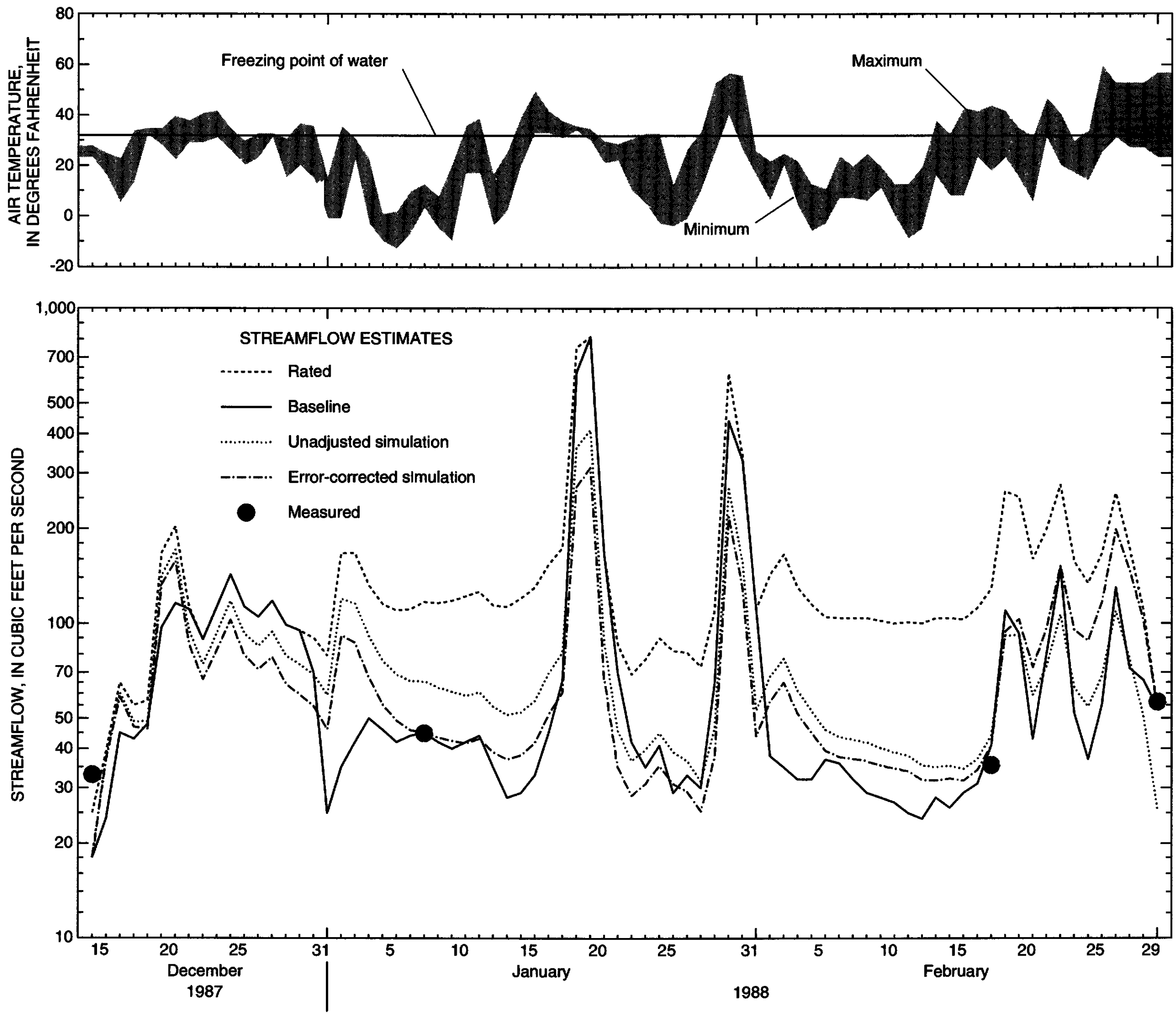

Figure 3. Daily streamflow at Clear Creek near Coralville, Iowa, during the winter of 1987-88 (December 15, 1987 through March 1, 1988).

Des Moines River at Fort Dodge, Iowa

The form of the equation for simulating the discharge ratio for Des Moines River at Fort Dodge, Iowa is

$$R_t - a_1 R_{t-1} = b_0 + b_1 Ah_{t-1} \quad , \qquad (6)$$

where

R_t and R_{t-1} are the discharge ratios at time t and t-1, respectively, constrained to the interval between 0.05 and 1.00;

Ah_{t-1} is the maximum daily air temperature at time t-1; and

a_1, b_0, and b_1 are parameters with estimated values of 0.9354, -0.01451, and 0.001780, respectively.

The parameters indicate that discharge ratios separated by 1 day are highly positively correlated and that higher air temperatures increase the discharge ratio.

The sample coefficient of determination (r^2) between published daily streamflow data and simulated daily streamflow data is 0.96 (fig. 4). The relation between simulated and published streamflows is based on 114 days when direct measurements defined the ice-backwater effects during the winters of 1961-62 through 1986-87. The mean residual, formed from the differences between logarithms of simulated and published streamflows, is -0.0017. The standard deviation of the residuals is 0.1277. Streamflows during the winter of 1987-88 are shown in figure 5.

Floyd River at James, Iowa

The form of the equation for simulating the discharge ratio for Floyd River at James, Iowa, is

$$R_t - a_1 R_{t-1} = b_0 + b_1 Aa_{t-1} \quad , \qquad (7)$$

where

R_t and R_{t-1} are the discharge ratios at time t and t-1, respectively, confined to the interval between 0.05 and 1.00;

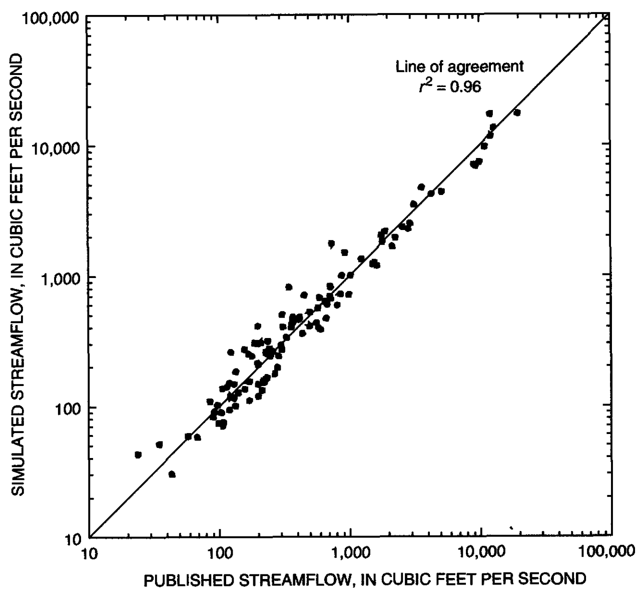

Figure 4. Relation between simulated and published daily streamflow data, Des Moines River at Fort Dodge, Iowa, during the calibration period (winters of 1961-62 through 1986-87).

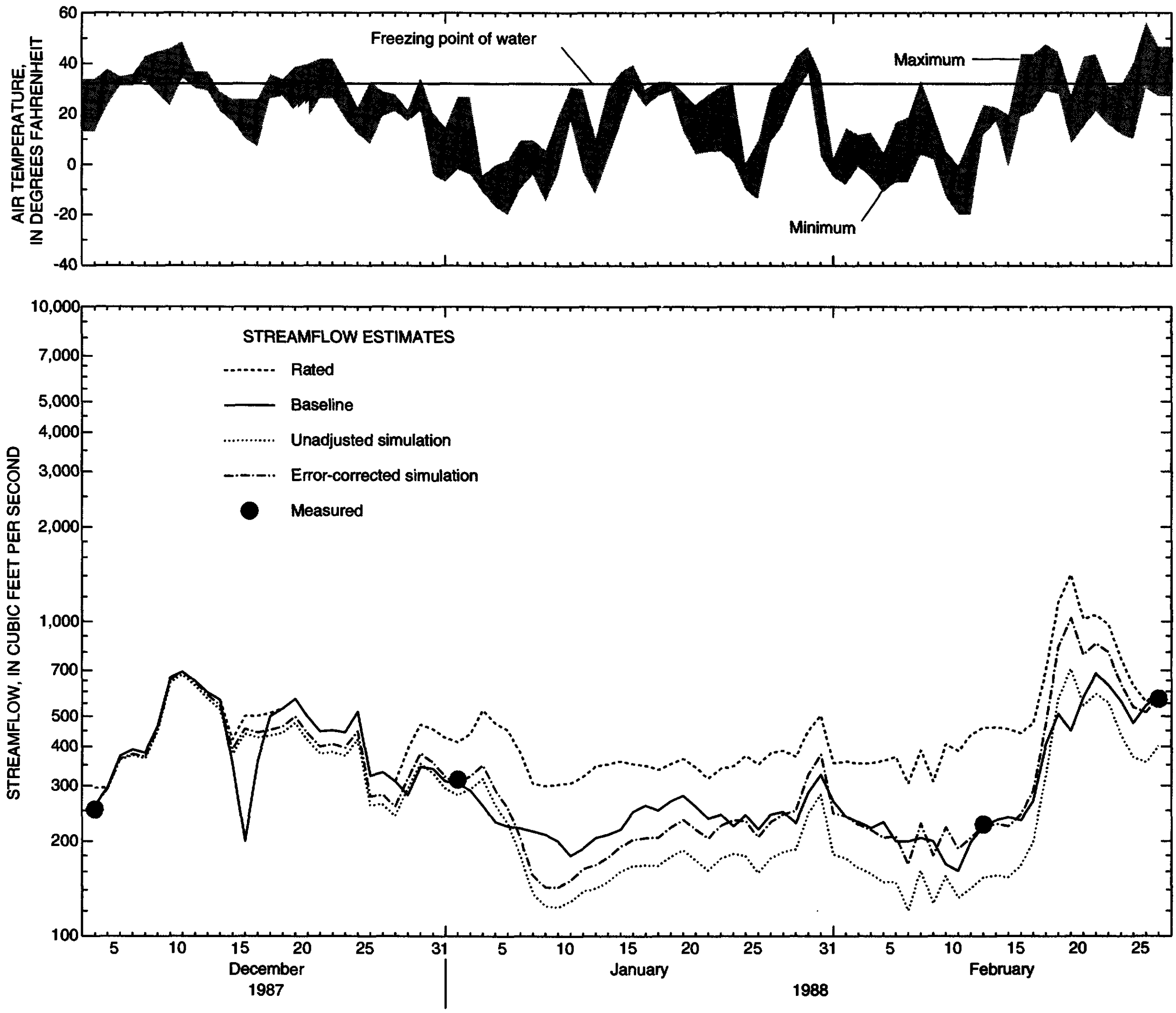

Figure 5. Daily streamflow at Des Moines River at Fort Dodge, Iowa, during the winter of 1987-88 (December 4, 1987 through February 27, 1988).

$Aa_{t\text{-}1}$ is the average of the maximum and minimum daily air temperatures at time t-1; and

a_1, b_0, and b_1 are parameters with estimated values of 0.8531, 0.006762, and 0.002956 respectively.

The parameters indicate that discharge ratios separated by 1 day are highly positively correlated and that higher air temperatures increase the discharge ratio.

The sample coefficient of determination (r^2) between published daily streamflow data and simulated daily streamflow data is 0.93 (fig. 6). The relation between simulated and published streamflows is based on 103 days when direct measurements defined the ice-backwater effects during the winters of 1961-62 through 1986-87. The mean residual, formed from the differences between logarithms of simulated and published streamflows, is 0.0005. The standard deviation of the residuals is 0.2404. Streamflows during the winter of 1987-88 are shown in figure 7.

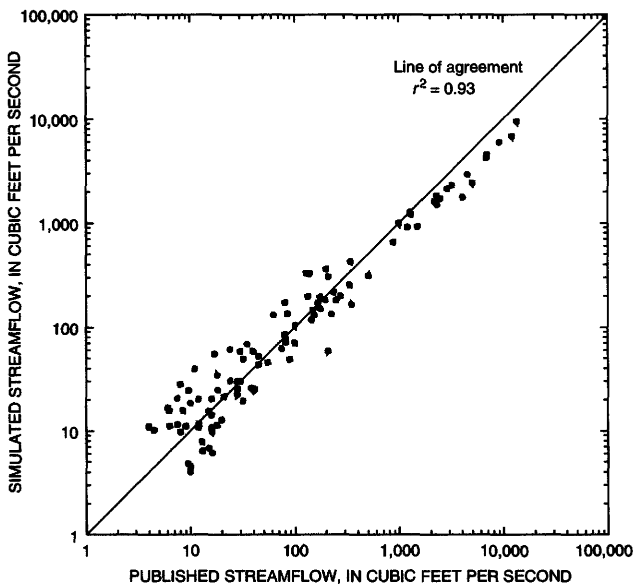

Figure 6. Relation between simulated and published daily streamflow data, Floyd River at James, Iowa, during the calibration period (winters of 1961-62 through 1986-87).

EVALUATION OF METHODS

Various criteria are available for evaluating alternative methods of estimating ice-affected streamflow. These include accuracy, cost, technical soundness, applicability to a range of ice conditions, ease of computer application, and feasibility within the existing data-collection network. In this section, the accuracies of the unadjusted and the error-corrected simulation estimates are compared to the accuracies of other analytical methods evaluated by Melcher and Walker (1992) on the basis of the relative daily error criterion.

The relative daily error is computed for each method as

$$\varepsilon_t = \frac{(\hat{Q}_t - Qb_t)}{Qb_t}, \qquad (8)$$

where

ε_t is the relative error at time t,

$\hat{Q}_t$ is the estimated streamflow at time t based on one of the selected methods, and

Qb_t is the baseline streamflow at time t.

The distribution of daily relative errors for each of the analytical methods is shown in figure 8 for the three selected streamflow-gaging stations. The periods during which the errors were evaluated within the ice-affected periods match those used by Melcher and Walker (1992). Among the three gaging stations, the relative errors tended to be higher at Clear Creek (drainage area, 98.1 mi^2) and at Floyd River (drainage area, 882 mi^2) than at Des Moines

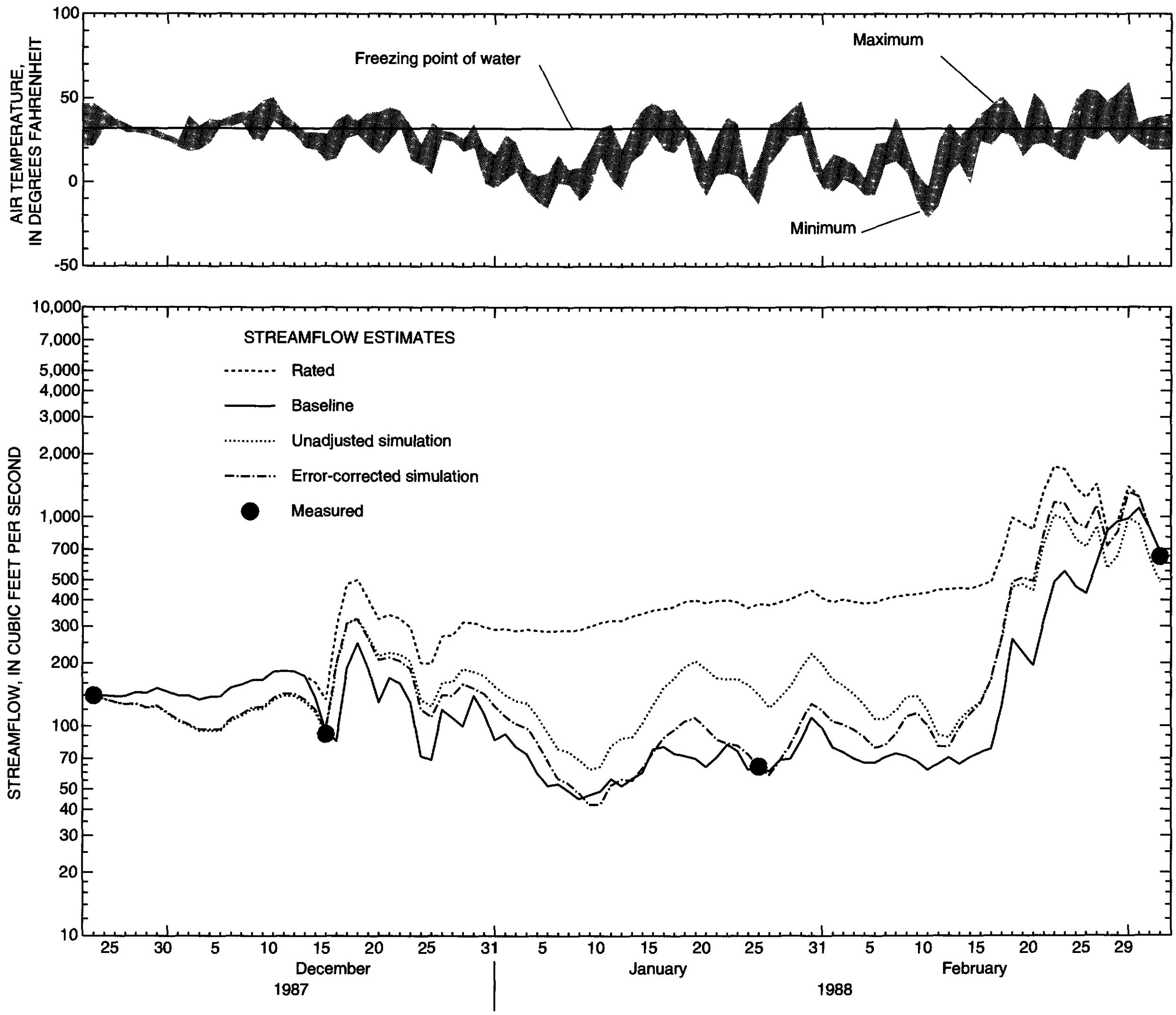

Figure 7. Daily streamflow at Floyd River at James, Iowa, during the winter of 1987-88 (November 24, 1987 through March 4, 1988).

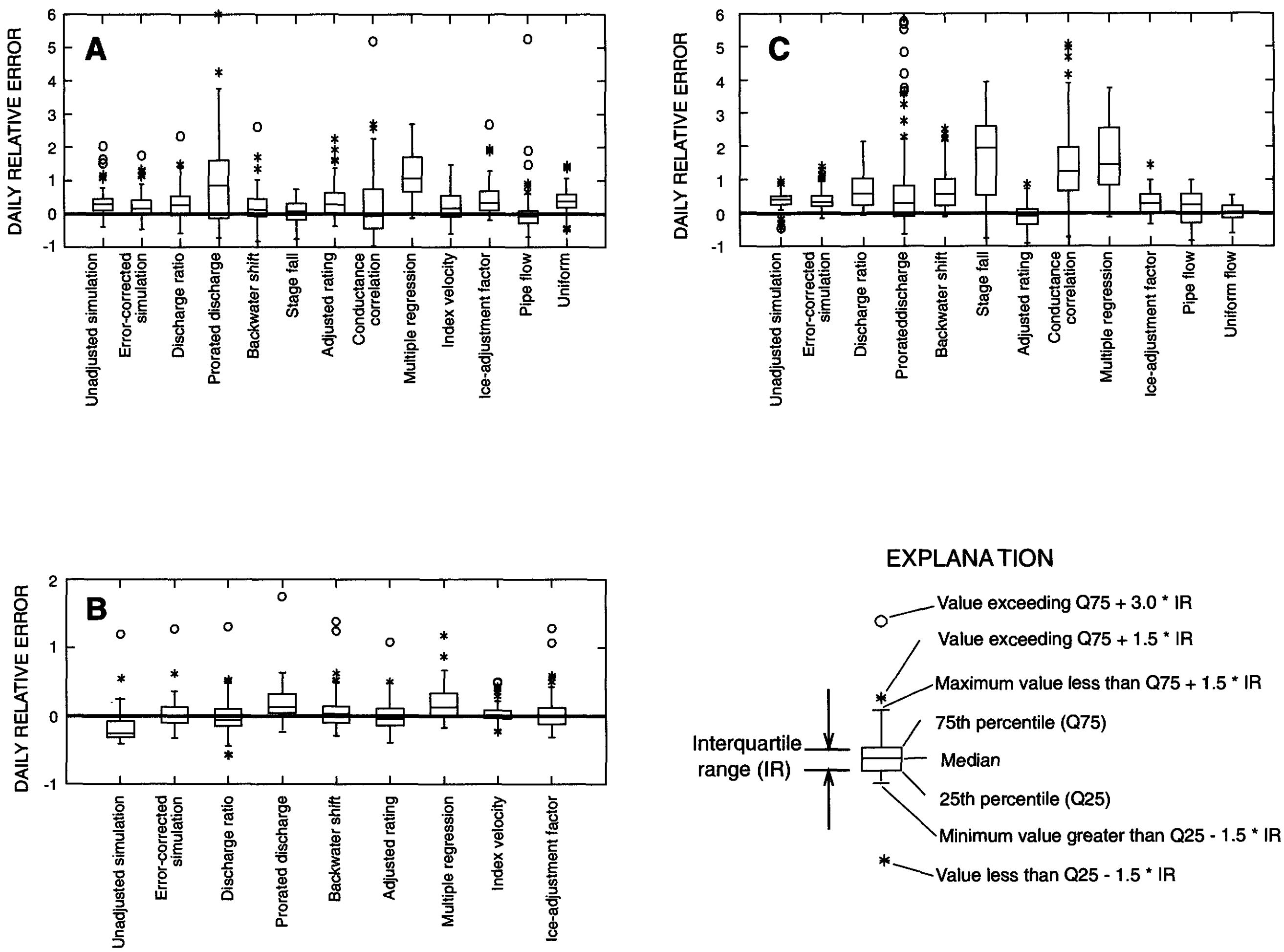

Figure 8. Summary of relative errors for analytical methods of estimating ice-affected streamflow as applied to data from selected streamflow-gaging stations. (*A*) Clear Creek near Coralville, Iowa, (*B*) Des Moines River at Fort Dodge, Iowa, and (*C*) Floyd River at James, Iowa.

River (drainage area, 4,190 mi^2). The smaller relative errors may be due to more stable ice conditions and less variable flow characteristics at the station with the largest drainage area. The greatest variation in the distribution of relative errors among methods was at Floyd River. This variation helps show the most accurate methods of estimating ice-affected streamflow at this site.

A numerical index of the relative accuracy and feasibility of the estimation methods was computed on the basis of the root-mean-square error (RMSE). The RMSE is computed as

$$\mathrm{RMSE} = \sqrt{(\bar{e})^2 + s_e^2}\,, \tag{9}$$

where

$\bar{e}$ is the estimated mean relative errors, which is a measure of bias, and

s_e^2 is the estimated variance of the relative errors, which is a measure of precision.

Thus, the RMSE is a single measure of the bias and uncertainty associated with estimation errors for each method.

The RMSE was computed for each method (table 2). Ranks were computed on the RMSE for each station. The method with the smallest RMSE was assigned a rank of 1 for that station; higher ranks were

Table 2. Ranks of analytical methods on the basis of accuracy and feasibility criteria

[RMSE, root-mean-square error; --, RMSE was not computed]

Method	Clear Creek		Des Moines River		Floyd River		Average rank	Rank of the average ranks
	RMSE	Rank	RMSE	Rank	RMSE	Rank		
Unadjusted simulation	0.598	6.0	0.296	6.0	0.880	6.0	6.00	6.0
Error-corrected simulation	.484	2.0	.279	4.0	.571	5.0	3.67	1.0
Prorated discharge	5.183	13.0	.333	9.0	2.226	11.0	11.00	12.0
Discharge ratio	.623	7.0	.271	3.0	.917	7.0	5.67	4.5
Backwater shift	.579	4.0	.300	7.5	1.062	8.0	6.50	8.0
Stage fall	.347	1.0	--	11.5	2.060	10.0	7.50	9.0
Adjusted-rating curve	.672	8.0	.230	2.0	.383	2.0	4.00	2.0
Conductance correlation	1.146	11.0	--	11.5	3.364	12.0	11.50	13.0
Multiple regression	1.312	12.0	.300	7.5	1.998	9.0	9.50	11.0
Index velocity	.582	5.0	.184	1.0	--	13.0	6.33	7.0
Ice-adjustment factor	.730	9.0	.283	5.0	.482	3.0	5.67	4.5
Pipe flow	.793	10.0	--	11.5	.517	4.0	8.50	10.0
Uniform flow	.554	3.0	--	11.5	.260	1.0	5.17	3.0

assigned to methods with larger RMSE's. Tied RMSE's were assigned an average rank. Methods that could not be used at a particular station because of limitations of data availability were assigned the highest rank. This convention penalizes methods whose special data requirements were not met. However, methods with special data requirements may have artificially small RMSE's because they were calibrated and verified on data from the same period and do not reflect year-to-year variations in ice conditions. The average rank was computed for each method across stations; a ranking of these averages is a measure of the accuracy and feasibility of each method for selected stations.

The results of the ranking indicate that the error-corrected simulation model ranks first among analytical methods for estimating ice-affected streamflow. The adjusted-rating-curve method and the uniform-flow method rank second and third, respectively; however, both of these analytical methods have special data requirements. Should these special data be generally available, it would be appropriate to specify an alternative form for the simulation-model equation and compare the results on the basis of the same supporting data.

The (unadjusted) simulation method ranked sixth, whereas the multiple-regression method ranked eleventh. Although the two methods have similar data requirements, the results indicate the significance of the dynamic component in the estimation of ice-affected streamflow. Inclusion of streamflow at a correlated station, comparable to the multiple-regression method, could lead to further improvements in the accuracy of the simulation-model estimate.

Additional research on alternative forms for the simulation-model equation, choice of objective functions and optimization methods, simultaneous simulation of ice-affected streamflow within a network of streamflow-gaging stations, and the effect of length of time step on numerical accuracy would lead to further improvements in the accuracy of the dynamical-systems approach.

SUMMARY AND CONCLUSIONS

Ice affects the stage-discharge relation for some part of the winter at more than half of the streamflow-gaging stations operated by the USGS. Ice-affected streamflow usually is estimated by subjective methods that are dependent on the judgment of a hydrographer and are not adaptable to automated data processing. Analytical methods, which are based on systematic computation, are needed to improve the reliability and reproducibility of ice-affected streamflow estimates and to improve the efficiency of processing streamflow data.

This report describes the development, application, and evaluation of a dynamical-systems approach for estimating ice-affected streamflow. The approach is based on simulation modeling and error correction. A simulation model is developed by use of a user-specified nonlinear dynamical equation that describes the characteristics of the ice-backwater effect. Historical streamflow data is used to estimate parameters for the equation. Classical statistical techniques can be used to determine the significance of all estimated parameters. Once developed, the simulation model is used to compute initial unadjusted estimates of ice-affected streamflow. Error correction is used to prorate the error in the unadjusted estimates so that final error-corrected estimates of ice-affected streamflow match daily mean streamflow computed on days of direct streamflow measurement.

The accuracy of the dynamical-systems approach was assessed on the basis of data originally collected and analyzed by Melcher and Walker (1992), who compared 11 analytical methods of estimating ice-affected streamflow. These methods were evaluated by applying the methods to data collected at three streamflow-gaging stations in Iowa during the winter of 1987-88. A baseline data set was compiled by collecting data needed for application of the 11 methods and making streamflow measurements at 1- to 5-day intervals at the three stations. The streamflow records for each method were compiled by use of data that is typical of a normal 6-week field schedule.

The results of the comparison indicate that the dynamical-systems approach ranks first on the basis of accuracy and feasibility criteria among the analytical methods evaluated for estimating ice-affected streamflow. This ranking is particularly significant because many of the other analytical methods had requirements for data that are not routinely collected. Given the modest and flexible data requirements, ease of computer application, and the reproducibility and objectivity of the method, the dynamical-systems approach for estimating ice-affected streamflow has the potential to become an important method for computing streamflow.

Additional research on alternative forms of site-specific equations, choice of objective functions and optimization methods, simultaneous simulation of ice-effects in a network of streamflow-gaging stations, and the effect of time-step length would likely lead to further improvements in the dynamical-systems approach.

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