Measurement and Computation of Streamflow: Volume 2. Computation of Discharge

By S. E. RANTZ *and others*

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Inch-pound	*Multiply by—*	*Metric*
acres	4.047×10^3	m^2 (square meters)
acre-ft (acre-feet)	1.233×10^3	m^3 (cubic meters)
acre-ft/yr (acre feet per year)	1.233×10^3	m^3/yr (cubic meters per year)
ft (feet)	3.048×10^{-1}	m (meters)
ft/hr (feet per hour)	3.048×10^{-1}	m/hr (meters per hour)
ft/s (feet per second)	3.048×10^{-1}	m/s (meters per second)
ft^3/s (cubic feet per second)	2.832×10^{-2}	m^3/s (cubic meters per second)
in (inches)	2.540×10	mm (millimeters)
lb (pounds)	4.536×10^{-1}	kg (kilograms)
mi (miles)	1.609	km (kilometers)
mi^2 (square miles)	2.590	km^2 (square kilometers)
oz (ounces)	2.835×10^{-2}	kg (kilograms)

MEASUREMENT AND COMPUTATION OF STREAMFLOW

VOLUME 2. COMPUTATION OF DISCHARGE

By S. E. RANTZ and others

CHAPTER 10.—DISCHARGE RATINGS USING SIMPLE STAGE-DISCHARGE RELATIONS

INTRODUCTION

Continuous records of discharge at gaging stations are computed by applying the discharge rating for the stream to records of stage. Discharge ratings may be simple or complex, depending on the number of variables needed to define the stage-discharge relation. This chapter is concerned with ratings in which the discharge can be related to stage alone. (The terms "rating," "rating curve," "stage rating," and "stage-discharge relation" are synonymous and are used here interchangeably.)

Discharge ratings for gaging stations are usually determined empirically by means of periodic measurements of discharge and stage. The discharge measurements are usually made by current meter. Measured discharge is then plotted against concurrent stage on graph paper to define the rating curve. At a new station many discharge measurements are needed to define the stage-discharge relation throughout the entire range of stage. Periodic measurements are needed thereafter to either confirm the permanence of the rating or to follow changes (shifts) in the rating. A minimum of 10 discharge measurements per year is recommended, unless it has been demonstrated that the stage-discharge relation is unvarying with time. In that event the frequency of measurements may be reduced. It is of prime importance that the stage-discharge relation be defined for flood conditions and for periods when the rating is subject to shifts as a result of ice formation (see section titled, "Effect of Ice Formation on Discharge Ratings") or as a result of the variable channel and control conditions discussed in the section titled, "Shifts in the Discharge Rating." It is essential that the stream-gaging program have sufficient flexibility to provide for the nonroutine scheduling of additional measurements of discharge at those times.

If the discharge measurements cover the entire range of stage experienced during a period of time when the stage-discharge relation is stable, there is little problem in defining the discharge rating for that

period. On the other hand, if, as is usually the case, discharge measurements are lacking to define the upper end of the rating, the defined lower part of the rating curve must be extrapolated to the highest stage experienced. Such extrapolations are always subject to error, but the error may be reduced if the analyst has a knowledge of the principles that govern the shape of rating curves. Much of the material in this chapter is directed toward a discussion of those principles, so that when the hydrographer is faced with the problem of extending the high-water end of a rating curve he can decide whether the extrapolation should be a straight line, or whether it should be concave upward or concave downward.

The problem of extrapolation can be circumvented, of course, if the unmeasured peak discharge is determined by use of the indirect methods discussed in chapter 9. In the absence of such peak-discharge determinations, some of the uncertainty in extrapolating the rating may be reduced by the use of one or more of several methods of estimating the discharge corresponding to high values of stage. Four such methods are discussed in the section titled "High-flow Extrapolation."

In the discussions that follow it was generally impractical to use both English and metric units, except where basic equations are given. Consequently English units are used throughout, unless otherwise noted.

STAGE-DISCHARGE CONTROLS

The subject of stage-discharge controls was discussed in detail in chapter 3, but a brief summary at this point is appropriate.

The relation of stage to discharge is usually controlled by a section or reach of channel downstream from the gage that is known as the station control. A section control may be natural or manmade; it may be a ledge of rock across the channel, a boulder-covered riffle, an overflow dam, or any other physical feature capable of maintaining a fairly stable relation between stage and discharge. Section controls are often effective only at low discharges and are completely submerged by channel control at medium and high discharges. Channel control consists of all the physical features of the channel that determine the stage of the river at a given point for a given rate of flow. These features include the size, slope, roughness, alinement, constrictions and expansions, and shape of the channel. The reach of channel that acts as the control may lengthen as the discharge increases, introducing new features that affect the stage-discharge relation.

Knowledge of the channel features that control the stage-discharge relation is important. The development of stage-discharge curves where more than one control is effective, and where the number of

measurements is limited, usually requires judgment in interpolating between measurements and in extrapolating beyond the highest measurements. That is particularly true where the controls are not permanent and the various discharge measurements are representative of changes in the positioning of segments of the stage-discharge curve.

GRAPHICAL PLOTTING OF RATING CURVES

Stage-discharge relations are usually developed from a graphical analysis of the discharge measurements plotted on either rectangular-coordinate or logarithmic plotting paper. In a preliminary step the discharge measurements available for analysis are tabulated and summarized on a form such as that shown in figure 139. Discharge is then plotted as the abscissa, corresponding gage height is plotted as the ordinate, and a curve or line is fitted by eye to the plotted points. The plotted points carry the identifying measurement numbers given in figure 139; the discharge measurements are numbered consecutively in chronological order so that time trends can be identified.

At recording-gage stations that use stilling wells, systematic and significantly large differences between inside (recorded) gage heights and outside gage heights often occur during periods of high stage, usually as a result of intake drawdown (see section in chapter 4 titled, "Stilling Wells"). For stations where such differences occur, both inside and outside gage heights for high-water discharge measurements are recorded on the form shown in figure 139, and in plotting the measurements for rating analysis, the outside gage readings are used first. The stage-discharge relation is drawn through the outside gage readings of the high-water discharge measurements and is extended to the stage of the outside high-water marks that are observed for each flood event. The stage-discharge relation is next transposed to correspond with the inside gage heights obtained from the stage-recorder at the times of discharge measurement and at flood peaks. It is this transposed stage-discharge relation that is used with recorded stages to compute the discharge.

The rationale behind the above procedure is as follows. The outside gage readings are used for developing the rating because the hydraulic principles on which the rating is based require the use of the true stage of the stream. The transposition of the rating to inside (recorded) stages is then made because the recorded stages will be used with the rating to determine discharge. The recorded stages are used for discharge determination because if differences exist between inside and outside gage readings, those differences will be known only for those times when the two gages are read concurrently. If the

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UNITED STATES DEPARTMENT OF THE INTERIOR
GEOLOGICAL SURVEY (WATER RESOURCES DIVISION)
DISCHARGE MEASUREMENT SUMMARY SHEET

Station No 11-4770

Discharge measurements of Eel River at Scotia, Calif. during the year ending Sept 30, 1966

No	Date	Made by—	Width	Area	Mean velocity	Gage height	Discharge	Rating: Shift adj	Rating: Percent diff	Method	Number meas sections	Gage height change	Time	Meas rated	Water temp °C	Outside gage	REMARKS
	1965		Feet	Sq ft	Fps	Feet	Cfs	Feet				Feet	Hr				
495	Aug. 31	Hammond McGuire	153	W 199	0.78	8.92	155			0.6	34	-.01	1.2	G	74	8.98	
496	Oct. 5	La Rue	155	W 148	0.82	8.79	121			.6	28	0	.5	G	61	8.87	Zero flow = 8.5 ± 0.05
497	Nov 2	do	154	W 158	0.85	8.88	135			.6	29	0	.5	G	60	8.89	Zero flow = 8.5 ± 0.05
498	30	La Rue Crumrine	454	2,030	2.47	12.52	5,010			.6, .2/.8	33	-.01	1.1	G	46	12.55	
499	1966 Jan. 3	La Rue	553	6,790	7.19	21.03	48,800			.2/.8	29	+0.52	1.75	G/F	46	21.05	
500	25	Hammond Crumrine	512	2350	1.73	12.74	4,050			.6, .2/.8	25	-.02	1.3	P	45	12.80	peak (Jan 5) 45.84
501	25	do	506	2270	1.71	12.73	3,880			.6, .2/.8	28	0	2	F	—	12.80	
502	31	La Rue	543	3960	4.18	16.08	16,500			.6, .2/.8	32	-.07	1.25	G	—	16.13	
503	Feb. 21	do	533	3000	2.70	13.85	8,100			.2/.8	36	-.03	1.4	G	48	13.86	
504	Mar. 31	do	2-Channels		—	13.59	7,420			.2/.8	37	.00	1.6	G	57	13.59	
505	Apr 28	do	454	1890	1.90	12.13	3,600			.6, .2/.8	36	-.01	1.3	G	58	12.18	
506	June 1	Palmer	434	1200	0.83	10.62	1,000			.6, .2/.8	40	0	1.7	G	62	10.64	
507	July 6	do	196	193	1.56	9.82	302			.6	39	0	.9	G	70	9.83	
508	Aug. 8	do	184	W 126	1.11	9.40	140			.6	38	0	.75	F/P	67	—	Zero flow = 9.05 ± 0.05
509	Sept. 12	Hammond	77	W 74.1	1.48	9.26	110			.6	31	0	.8	F	66	9.3 ± .02	Zero flow = 9.0 ± 0.0[illegible]
510	Oct. 4	Palmer	184	116	1.01	9.30	117			.6	34	0	.7	F	64	9.3 ± .01	

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FIGURE 139.—Example of form used for tabulating and summarizing current-meter discharge measurements.

outside gage heights were used with the rating to determine discharge, variable corrections, either known or assumed, would have to be applied to recorded gage heights to convert them to outside stages. We have digressed here to discuss differences between inside and outside gage heights, because in the discussions that follow no distinction between the two gages will be made.

The use of logarithmic plotting paper is usually preferred for graphical analysis of the rating because in the usual situation of compound controls, changes in the slope of the logarithmically plotted rating identify the range in stage for which the individual controls are effective. Furthermore, the portion of the rating curve that is applicable to any particular control may be linearized for rational extrapolation or interpolation. A discussion of the characteristics of logarithmic plotting follows.

The measured distance between any two ordinates or abscissas on logarithmic graph paper, whose values are printed or indicated on the sheet by the manufacturer of the paper, represents the difference between the logarithms of those values. Consequently, the measured distance is related to the ratio of the two values. Therefore, the distance between pairs of numbers such as 1 and 2, 2 and 4, 3 and 6, 5 and 10, are all equal because the ratios of the various pairs are identical. Thus the logarithmic scale of either the ordinates or the abscissas is maintained if all printed numbers on the scale are multiplied or divided by a constant. This property of the paper has practical value. For example, assume that the logarithmic plotting paper available has two cycles (fig. 140), and that ordinates ranging from 0.3 to 15.0 are to be plotted. If the printed scale of ordinates is used and the bottom line is called 0.1, the top line of the paper becomes 10.0, and values between 10.0 and 15.0 cannot be accommodated. However, the logarithmic scale will not be distorted if all values are multiplied by a constant. For this particular problem, 2 is the constant used in figure 140, and now the desired range of 0.3 to 15.0 can be accommodated. Examination of figure 140 shows that the change in scale has not changed the distance between any given pair of ordinates; the position of the ordinate scale has merely been transposed.

We turn now to a theoretical discussion of rating curves plotted on logarithmic graph paper. A rating curve, or a segment of a rating curve, that plots as a straight line of logarithmic paper has the equation,

$$Q = p(G - e)^N, \qquad (53)$$

where

Q is discharge;

$(G - e)$ is head or depth of water on the control—this value is indicated by the ordinate scale printed by the manufacturer or

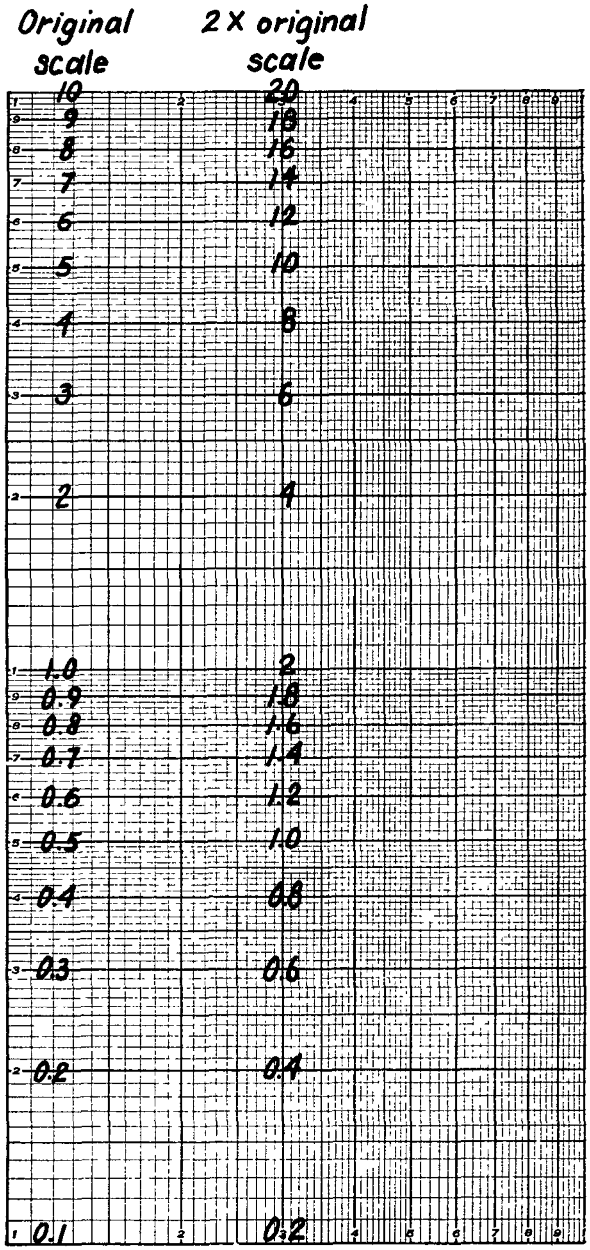


Figure 140.—Example showing how the logarithmic scale of graph paper may be transposed.

by the ordinate scale that has been transposed, as explained in the preceding paragraph;

G is gage height of the water surface;

e is gage height of zero flow for a section control of regular shape, or the gage height of effective zero flow for a channel control or a section control of irregular shape;

p is a constant that is numerically equal to the discharge when the head $(G - e)$ equals 1.0 ft or 1.0 m, depending on whether English or metric units are used; and

N is slope of the rating curve. (Slope in equation 53 is the ratio of the horizontal distance to the vertical distance. This unconventional way of measuring slope is necessary because the dependent variable Q is always plotted as the abscissa.)

We assume now that a segment of an established logarithmic rating is linear, and we examine the effect on the rating of changes to the control. If the width of the control increases, p increases and the new rating will be parallel to and to the right of the original rating. If the width of the control decreases, the opposite effect occurs; p decreases and the new rating will be parallel to and to the left of the original rating. If the control scours, e decreases and the depth $(G - e)$ for a given gage height increases; the new rating moves to the right and will no longer be a straight line but will be a curve that is concave downward. If the control becomes built up by deposition, e increases and the depth $(G - e)$ for a given gage height decreases; the new rating moves to the left and is no longer linear but is a curve that is concave upward.

When discharge measurements are originally plotted on logarithmic paper, no consideration is given to values of e. The gage height of each measurement is plotted using the ordinate scale provided by the manufacturer or, if necessary, an ordinate scale that has been transposed as illustrated in figure 140. We refer now to figure 141. The inside scale $(e = 0)$ is the scale printed by the paper manufacturer. Assume that the discharge measurements have been plotted to that scale and that they define the curvilinear relation between gage height (G) and discharge (Q) that is shown in the topmost curve. For the purpose of extrapolating the relation, a value of e is sought, which when applied to G, will result in a linear relation between $(G - e)$ and Q. If we are dealing with a section control of regular shape, the value of e will be known; it will be the gage height of the lowest point of the control (point of zero flow). If we are dealing with a channel control or section control of irregular shape, the value of e is the gage height of *effective* zero flow. The gage height of effective zero flow is not the gage height of some identifiable feature on the irregular section control or in the channel but is actually a mathematical constant

that is considered as a gage height to preserve the concept of a logarithmically linear head-discharge relation. Effective zero flow is usually determined by a method of successive approximations.

In successive trials, the ordinate scale in figure 141 is varied for e values of 1, 2 and 3 ft, each of which results in a different curve, but each new curve still represents the same rating as the top curve. For example, a discharge of 30 ft^3/s corresponds to a gage height (G) of 5.5 ft on all four curves. The true value of e is 2 ft, and thus the rating plots as a straight line if the ordinate scale numbers are increased by that value. In other words, while even on the new scale a discharge of 30 ft^3/s corresponds to a gage height (G) of 5.5 ft, the head or depth on the control for a discharge of 30 ft^3/s is $(G-e)$, or 3.5 ft; the linear rating marked $e = 2$ crosses the ordinate for 30 ft^3/s at 5.5 ft on the new scale and at 3.5 ft on the manufacturer's, or inside, scale. If values of e smaller than the true value of 2 ft are used, the rating curve will be concave upward, if values of e greater than 2 ft are used, the curve will be concave downward. The value of e to be used for a rating curve, or for a segment of a rating curve, can thus be determined by adding or subtracting trial values of e to the numbered scales on the logarithmic plotting paper until a value is found that results in a straight-line plot of the rating. It is important to note that if the logarithmic ordinate scale must be transposed by multiplication or division to accommodate the range of stage to be plotted, that transposition must be made before the ordinate scale is manipulated for values of e.

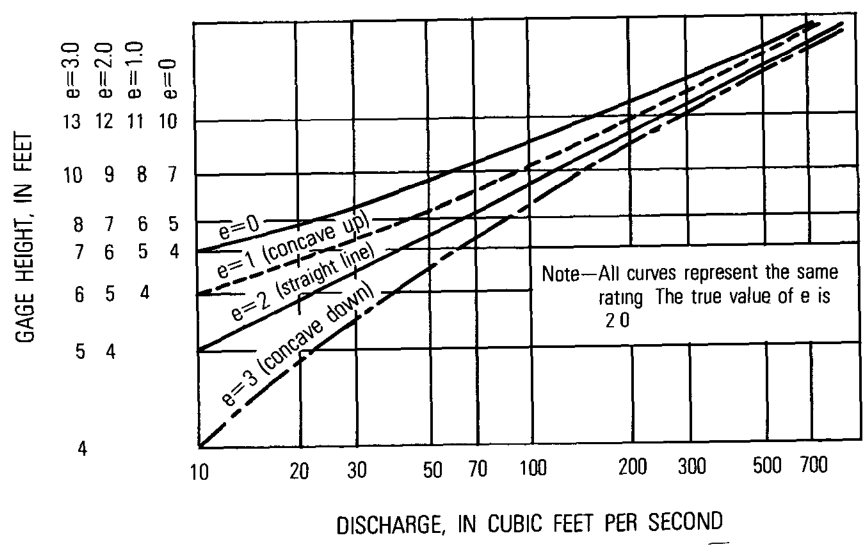


FIGURE 141.—Rating-curve shapes resulting from the use of differing values of effective zero flow.

A more direct solution for e, as described by Johnson (1952) is illustrated in figure 142. A plot of G versus Q has resulted in the solid-line curve which is to be linearized by subtracting a value of e from each value of G. The part of the rating between points 1 and 2 is chosen, and values of G_1, G_2, Q_1 and Q_2 are picked from the coordinate scales. A value of Q_3 is next computed, such that

$$Q^2_3 = Q_1 Q_2.$$

From the solid-line curve, the value of G_3 that corresponds to Q_3 is picked. In accordance with the properties of a straight line on logarithmic plotting paper,

$$(G_3 - e)^2 = (G_1 - e)(G_2 - e). \tag{54}$$

Expansion of terms in equation 54 leads to equation 55 which provides a direct solution for e.

$$e = \frac{G_1 G_2 - G_3^2}{G_1 + G_2 - 2G_3} \tag{55}$$

A logarithmic rating curve is seldom a straight line or a gentle curve for the entire range in stage. Even where a single cross section of the channel is the control for all stages, a sharp break in the

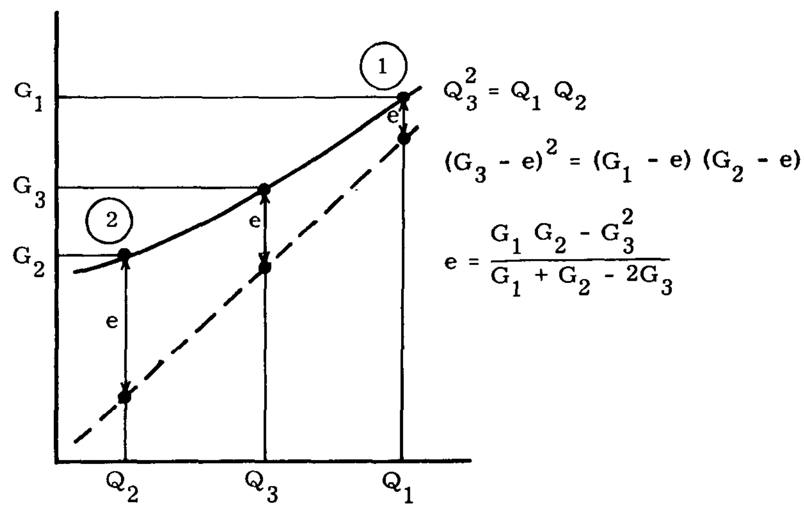


FIGURE 142.—Schematic representation of the linearization of a curve on logarithmic graph paper.

contour of the cross section, such as an overflow plain, will cause a break in the slope of the rating curve. Commonly, however, a break in slope is due to the low-water control being drowned out by a downstream section control becoming effective or by channel control becoming effective.

The use of rectangular-coordinate paper for rating analysis has certain advantages, particularly in the study of the pattern of shifts in the lower part of the rating. A change in the low-flow rating at many sites results from a change in the elevation of effective zero flow (e), which means a constant shift in gage height. A shift of that kind is more easily visualized on rectangular-coordinate paper because on that paper the shift curve is parallel to the original rating curve, the two curves being separated by a vertical distance equal to the change in the value of e. On logarithmic paper the two curves will be separated by a variable distance which decreases as stage increases. A further advantage of rectangular-coordinate paper is the fact that the point of zero flow can be plotted directly on rectangular-coordinate paper, thereby facilitating extrapolation of the low-water end of the rating curve. That cannot be done on logarithmic paper because zero values cannot be shown on that type of paper.

As a general rule logarithmic plotting should be used initially in developing the general shape of the rating. The final curve may be displayed on either type of graph paper and used as a base curve for the analysis of shifts. A combination of the two types of graph paper is frequently used with the lower part of the rating plotted on an inset of rectangular-coordinate paper or on a separate sheet of rectangular-coordinate paper.

SECTION CONTROLS

ARTIFICIAL CONTROLS

At this point we digress from the subject of logarithmic rating curves to discuss the ratings for artificial section controls. A knowledge of the rating characteristics of controls of standard shape is necessary for an understanding of the rating characteristics of natural controls, almost all of which have irregular shapes. On pages that follow we first discuss thin-plate weirs, then broad-crested weirs, and finally flumes.

Thin-plate weirs are generally used in small clear-flowing streams, particularly where high accuracy is desired and adequate maintenance can be provided, as in small research watersheds. Flumes are preferred for use in small streams and canals that carry sediment and debris, and in other situations where the head loss (backwater) associated with a thin-plate weir is unacceptable. Most types of flume may also be used under conditions of submergence, as opposed to free-flow

conditions, thereby permitting them to operate with even smaller head loss but with some loss of accuracy of the stage-discharge relation. The broad-crested weirs are commonly used in the larger streams.

TRANSFERABILITY OF LABORATORY RATINGS

Standard shapes or dimensions are commonly used in building artificial controls, and many of these standard structures have been rated in laboratory model studies (World Meteorological Organization, 1971). The transfer of a laboratory discharge rating to a structure in the field requires the existence, and maintenance, of similitude between laboratory model and prototype, not only with regard to the structure, but also with regard to the approach channel. For example, scour and (or) fill in the approach channel will change the head-discharge relation, as will algal growth on the control structure. Both the structure and the approach channel must be kept free from accumulations of debris, sediment, and vegetal growth. Flow conditions downstream from the structure are significant only to the extent that they control the tailwater elevation, which may influence the operation of structures designed for free-flow conditions.

Because of the likelihood of the existence or development of conditions that differ from those specified in a laboratory model study, the policy of the Geological Survey is to calibrate the prototype control in the field by discharge measurements for the entire range of stage that is experienced. (See section in chapter 3 titled, "Artificial Controls.") In-place calibration is sometimes dispensed with where the artificial control is a standard thin-plate weir having negligible velocity of approach.

THIN-PLATE WEIRS

The surface of the weir over which the water flows is the crest of the weir. A thin-plate weir has its crest beveled to a chisel edge and is always installed with the beveled face on the downstream side. The crest of a thin-plate weir is highly susceptible to damage from floating debris, and therefore such weirs are used as control structures almost solely in canals whose flow is free of floating debris. Thin-plate weirs are not satisfactory for use in canals carrying sediment-laden water because they trap sediment and thereby cause the gage pool to fill with sediment, sometimes to a level above the weir crest. The banks of the canal must also be high enough to accommodate the increase in stage (backwater) caused by the installation of the weir, the weir plate being an impedance to flow in the canal. The commonly used shapes for thin-plate weirs are rectangular, trapezoidal, and triangular or V-notch.

The information needed to compute the discharge over a thin-plate weir is as follows:

1. Static head (h), which is the difference in elevation between the weir crest and the water surface at the approach section; the approach section is located upstream from the weir face a distance equal to about $3h$ or more. (See section in chapter 2 titled, "Considerations in Specific Site Selection" for discussion of location of gage intakes.)
2. Length of crest of weir (b) if weir is rectangular or trapezoidal.
3. Width of channel in the plane of the weir face (B).
4. Angle of side slopes if weir is triangular or trapezoidal.
5. Average depth of streambed below elevation of weir crest (P). P is measured in the approach section.

RECTANGULAR THIN-PLATE WEIR

Flow over a rectangular thin-plate weir is illustrated in figure 143. The discharge equation for this type of weir is:

$$Q = Cbh^{3/2}, \tag{56}$$

where

Q = discharge,
C = discharge coefficient,
b = length of weir crest normal to flow, and
h = static or piezometric head on a weir, referred to the weir crest.

Information on discharge coefficients for rectangular thin-plate weirs is available from the investigations of Kindsvater and Carter (1959) and others, and is given in the previously cited WMO Technical Note No. 117 (1971). Those investigations show that the coefficient for free discharge is a function of certain dimensionless ratios which describe the geometry of the channel and the weir;

$$C = f\left(\frac{h}{P}, \frac{b}{B}, E\right), \tag{57}$$

where E is the slope of the weir face; the other variables are depicted in figure 143.

The relation between C, h/P and E for weirs with no side contraction ($b/B=1.0$) is shown in figure 144, where each of the four curves corresponds to a particular value of E. The coefficient is defined in the range of h/P from 0 to 5. The value of the coefficient becomes uncertain at high values of h/P. The greater the value of h/P, the greater the velocity of approach, and therefore the greater the coefficient. The coefficients in figure 144 are for use with English units, where all

linear measurements are expressed in feet and discharge is in cubic feet per second. If linear measurements are expressed in meters and discharge is in cubic meters per second, all values of C must be multiplied by the factor 0.552.

Side contractions reduce the effective length of the weir crest. That effect is accounted for by multiplying the value of C from figure 144 by a correction factor that is a function of b/B, h/P, and the degree of rounding of the upstream vertical edge of the weir-notch abutments. Rounding is a factor only in the situation where the horizontal weir crest is set between vertical abutments. For a rectangular thin-plate weir with sharp-edged entry, the correction factor is k_c; appropriate values of k_c are obtained from the curves in figure 145. For a

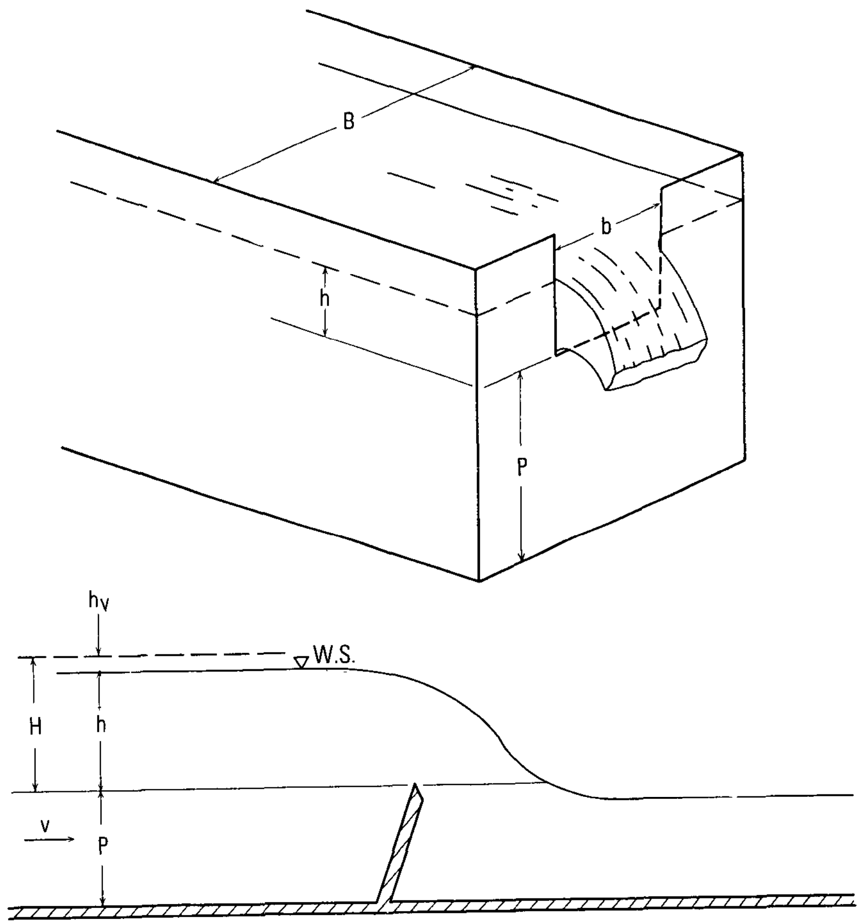


FIGURE 143.—Definition sketch of a rectangular thin-plate weir.

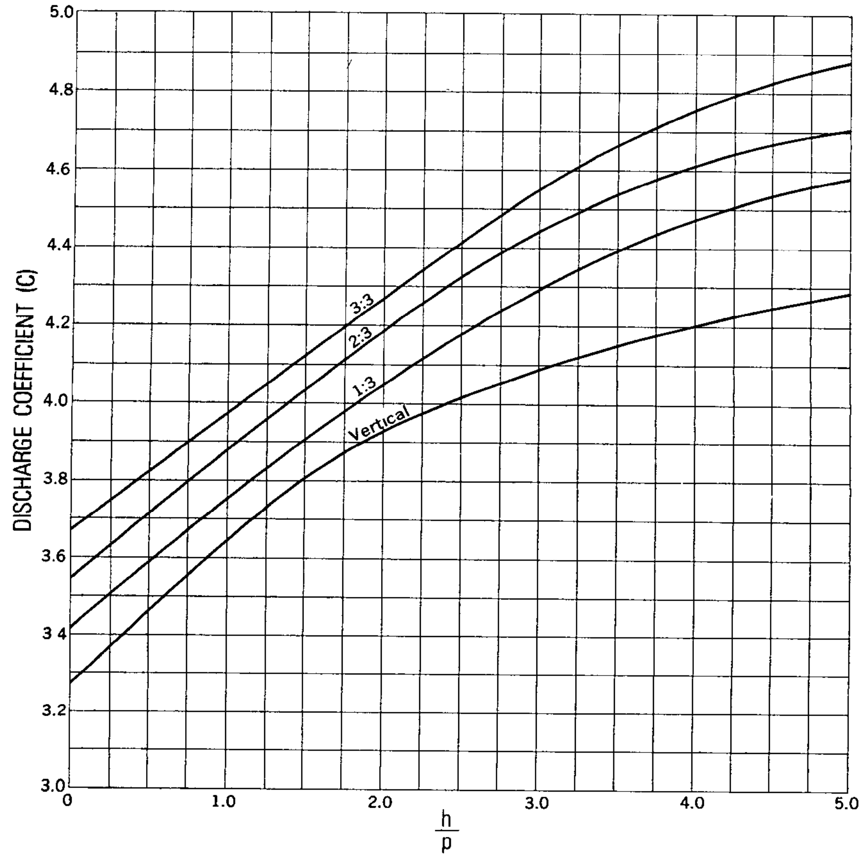


FIGURE 144.—Discharge coefficients for full-width, vertical and inclined, rectangular thin-plate weirs.

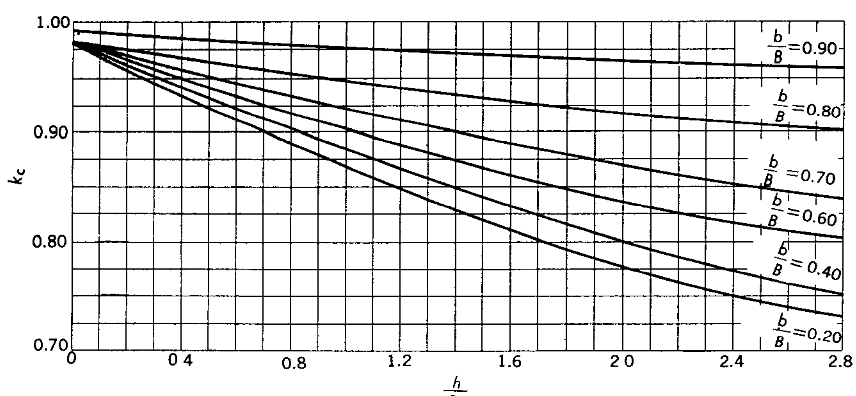


FIGURE 145.—Definition of adjustment factor, k_c, for contracted rectangular thin-plate weirs.

rectangular thin-plate weir having vertical abutments rounded with radius r, the correction factor, when $r/b \geqslant 0.12$, is assumed to equal $(1 + k_c)/2$, where again k_c is read from figure 145. For rounded abutments having a lesser value of r/b, the correction factor is obtained by interpolating between the appropriate k_c value from figure 145 and the value $(1 + k_c)/2$. In other words, for given values of b/B, h/P, and r/b between 0 and 0.12, we interpolate linearly between the value of k_c (corresponds to $r/b=0$) and the value of $(1 + k_c)/2$ (corresponds to $r/b = 0.12$).

We will now prepare the rating for a hypothetical rectangular thin-plate weir for the purpose of examining the implications of a logarithmic plot of the rating. The discharges corresponding to various stages will be computed by use of the theoretical equation for a rectangular weir, using figures 144 and 145 to obtain the constant in the equation. We will assume that the computed discharges represent the results of carefully made discharge measurements.

Assume that we have a rectangular thin-plate weir, with vertical face, and that discharge measurements (computations) have been made at heads ranging from 0.1 ft to 7.0 ft. The dimensions of the weir using the symbols in figure 142 are given in table 16. The data and computations are also shown in table 16 and should be self-explanatory. The weir constant is actually equal to $C(k_c)$. The stages and corresponding discharges are plotted on logarithmic graph paper and fitted with a curve by eye in figure 146.

Figure 146 shows that a tangent can be fitted to the plotted points at heads greater than 0.3 ft ($G=1.3$ ft). The intercept (p) of the tangent at $G - e=1.0$ ft is 67 ft³/s and the measured slope of the tangent is 1.55. (Note that the slope of the rating curve Q/h is the ratio of the horizontal distance to the vertical distance.) In accordance with equation 53, the equation of the tangent is therefore $Q=67h^{1.55}$ However, the equation for discharge over a rectangular weir is $Q=(Ck_cb)h^{1.50}$. Therefore (Ck_cb) must vary with stage, as we know it does, and $Ck_cb=67h^{0.05}$; the exponent 0.05 is obtained by subtracting the theoretical exponent 1.50 from the empirical exponent 1.55. Because b has a constant value of 20 ft, $Ck_c=3.35h^{0.05}$; the coefficient 3.35 is obtained by dividing the original coefficient (67) by the value of b (20 ft). We can extrapolate the tangent in figure 146 with some confidence. If we wish to determine the discharge from the curve for a gage height of 11 ft ($h=10$ ft), the extrapolated value of Q is 2,380 ft³/s; that is, if a value of 10 ft is substituted in the equation $Q=67h^{1.55}$, Q will equal 2,380 ft³/s. That value matches the true value computed on the bottom line of table 16 for a gage height of 11 ft.

TRAPEZOIDAL THIN-PLATE WEIR

Few experimental data are available for determining the discharge

TABLE 16.—*Computation of discharge rating for a hypothetical rectangular thin-plate weir*

[*Given* P=2.0 ft, b=20 ft, B=25 ft, b/B=0.8 Gage height of weir crest (e)=1.0 ft, h=Gage height (G) minus e]

G (ft)	$h=G-e$ (ft)	h/P	C (from fig 144)	k_t (from fig 145)	Ck_t	$Ck_t b$	$h^{3/2}$	$Q=Ck_t bh^{3/2}$ (ft^3/s)
1.0	0	----	----	----	----	----	----	0
1.1	0.1	0.05	3.29	0.980	3.224	64.48	0.0316	2.04
1.3	.3	.15	3.33	.977	3.253	65.06	.1643	10.7
1.5	.5	.25	3.37	.973	3.279	65.58	.3536	23.2
2.0	1.0	.50	3.46	.965	3.339	66.78	1.000	66.8
2.5	1.5	.75	3.56	.955	3.400	68.00	1.837	125
3.0	2.0	1.0	3.65	.947	3.456	69.12	2.828	195
4.0	3.0	1.5	3.81	.930	3.543	70.86	5.196	368
5.0	4.0	2.0	3.93	.920	3.616	72.32	8.000	579
6.0	5.0	2.5	4.02	.905	3.638	72.76	11.18	813
7.0	6.0	3.0	4.08	.900	3.672	73.44	14.70	1,080
Computation to check extrapolation obtained from figure 146								
11.0	10.0	5.0	4.28	.88	3.766	75.32	31.62	2,380

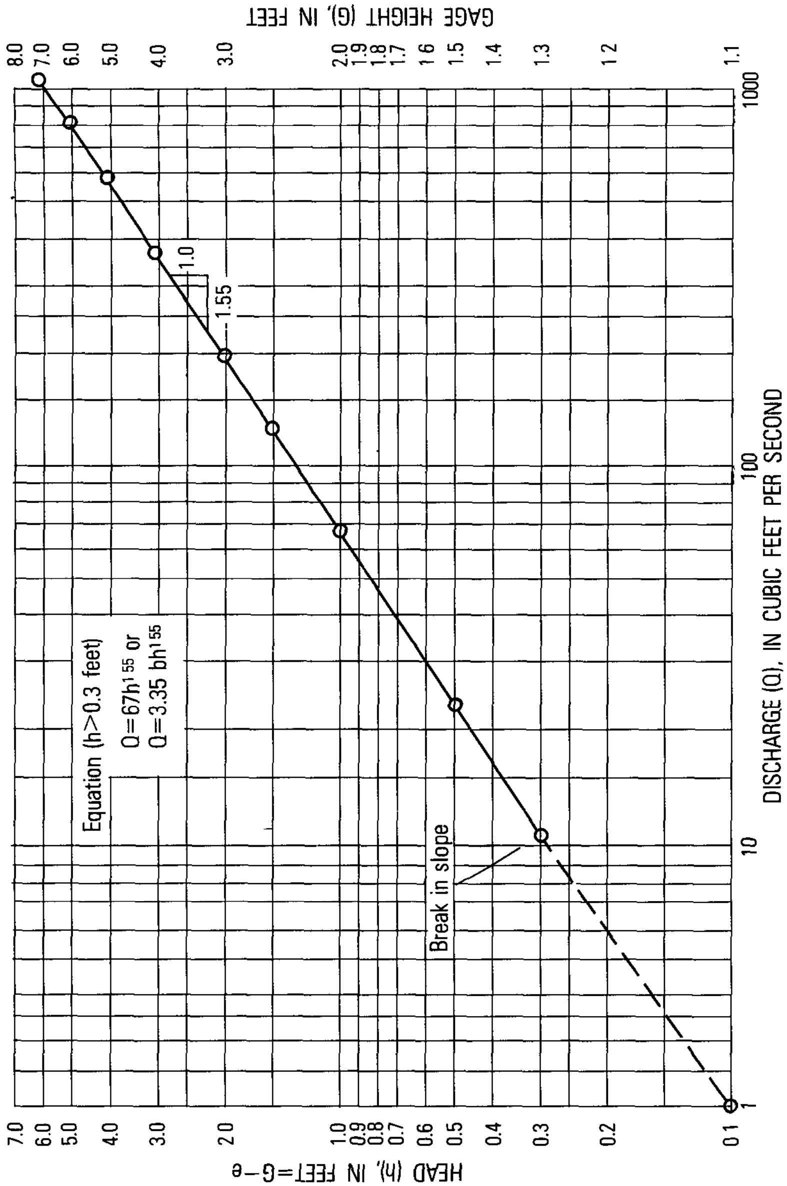


FIGURE 146.—Rating curve for hypothetical rectangular thin-plate weir.

coefficients of trapezoidal weirs (fig. 147). One exception is the vertical Cippoletti weir, which is a sharp-crested trapezoidal weir whose sides have a slope of 1 horizontal (x) to 4 vertical (y). The slope of the sides is approximately that required to obtain a discharge through the two triangular parts of the weir opening that equals the decrease in discharge resulting from end contractions. In other words, the Cippoletti weir acts as a rectangular thin-plate weir whose crest length is equal to b and whose contraction coefficient, k_c, is equal to 1.0. The dimension B (fig. 147) for a Cippoletti weir has little bearing on the discharge. The equation used to compute discharge is again

$$Q = Cbh^{3/2}, \tag{56}$$

and close approximations of values of C are obtained from figure 144. The head, h, and height of notch, P, are both measured in the approach section.

If we compute the discharge for a vertical thin-plate Cippoletti weir whose value of b is 20 ft and whose value of P is 2.0 ft, similar to the dimensions used in computing the hypothetical rating shown in table 16, the rating will approximate that obtained for the thin-plate rectangular weir of table 16. The only difference in discharge will be that attributable to the fact that the value of k_c is 1.00 for all values of head for the Cippoletti weir. A logarithmic plot of the rating (not shown here) indicates that the equation for all but the very small values of head is

$$Q = 69h^{1.58}, \text{ (English units)}$$

meaning that $C = 3.45h^{0.08}$.

For trapezoidal weirs other than Cippoletti weirs, the general empirical equation for discharge is

$$Q = Cb(h + h_r)^N, \tag{58}$$

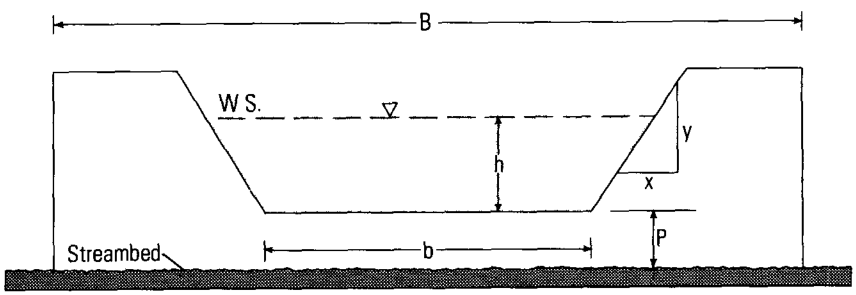


FIGURE 147.—Sketch of upstream face of a trapezoidal weir.

where h_v, the velocity head at the approach section, is equal to $V_a{}^2/2g$, V_a being the mean velocity in the approach section and g being the acceleration of gravity. The coefficient C and exponent N must be determined from current-meter discharge measurements that cover the entire range of stage that is experienced. If discharge measurements are not available for the highest stage experienced, a rating curve is obtained by plotting on logarithmic paper the head $(G-e)$ against discharge (Q) for the measurements that have been obtained, and then fitting a curve to the plotted points. The upper end of that curve should be a tangent, or possibly an extremely flat curve, that can be extrapolated to the highest stage experienced. Because the limiting shapes of a trapezoid are a rectangle at one extreme and a triangle at the other, the slope of the tangent will lie somewhere between 1.5, which is the theoretical slope for a rectangular weir, and 2.5 which is the theoretical slope for a triangular weir. The closer the shape is to a rectangle, the closer the slope will be to 1.5; the closer the shape is to a triangle, the closer the slope will be to 2.5.

The reader will note the difference in form between equations 56 and 58. Equation 56 uses static head (h), whereas equation 58 uses total head $(h+h_v)$. Velocity head is a factor in any discharge equation for a weir. In the more modern laboratory studies of weir discharge, the static-head term is used in the discharge equation, and velocity head, as indicated by a term h/P, is used directly as a variable in the determination of C. (See equation 57.) In older laboratory investigations, a more empirical approach for determining C was followed in that the total–head term was used in the discharge equation and the values of C that were determined do not vary directly with change in velocity head. Both forms of the weir-discharge equations will be found in this manual; the older type of equation is shown wherever it has not been superseded by later laboratory studies.

TRIANGULAR OR V-NOTCH THIN-PLATE WEIR

Triangular or V-notch thin-plate weirs (fig. 148) are installed at sites where low discharges occur; they are highly sensitive to low flows but have less capacity than rectangular or trapezoidal weirs. Because the area of the notch is invariably small compared to the cross-sectional area of the channel, water is pooled upstream from the weir and the approach velocity is necessarily low. The approach velocity head can usually be neglected in computing the discharge for a 90° V-notch weir ($\Theta=90°$ in fig. 148). Actually, for values of Θ equal to or less than 90°, it has been specified that velocity of approach is negligible if h/P is less than 0.4 and h/B is less than 0.2 (WMO Tech. Note 117, 1971). Whether or not the velocity head can be ignored in computing discharges for V-notch weirs having central angles greater

than 90° depends on the relative size of the areas occupied by the water in the notch and the water in the approach section. Virtually no experimental work has been done with triangular weirs having significant velocity of approach, and all equations discussed below are for installations where the velocity of approach can be neglected. Furthermore the equations are applicable only for thin-plate V-notch weirs whose faces are vertical.

Both the cross-sectional area of the flow in the notch and its velocity in the notch are functions of the head (h). Consequently, the general equation for a triangular thin-plate weir is $Q=Ch^x$, and the constants in that equation do not vary greatly from those in the following equations:

$$Q=2.5(\tan \Theta/2)h^{5/2},$$

where h is in feet and Q is in cubic feet per second; or

$$Q=1.38(\tan \Theta/2)h^{5/2},$$

where h is in meters and Q is in cubic meters per second.
The head is measured in the approach section, a distance about $3h$ upstream from the weir face. From an earlier discussion it is apparent that the above equations will plot as straight lines on logarithmic graph paper. The slope of the ratings will be 2.5, and the intercept, where $h=1$, will be either 2.5 tan $\Theta/2$ or 1.38 tan $\Theta/2$, depending on whether English or metric units are used.

V-notch weirs are most commonly built with a central angle of 90°. Much experimental work has been done with thin-plate 90°

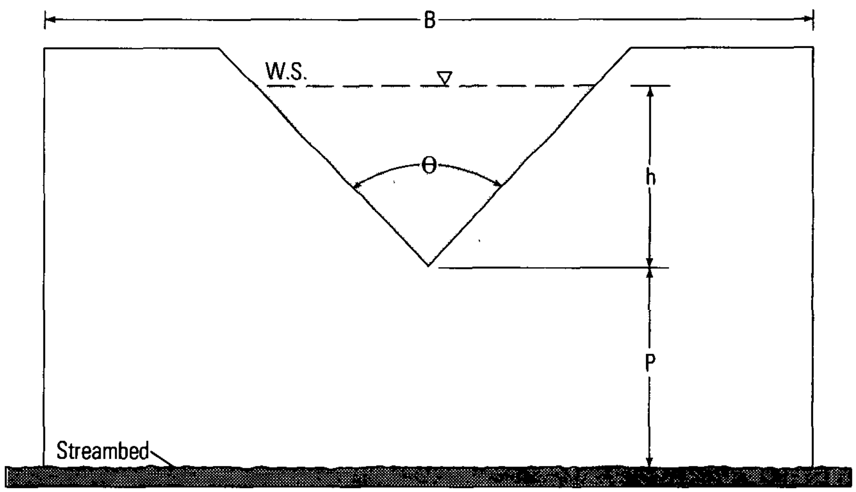


FIGURE 148.—Sketch of upstream face of a triangular or V-notch weir.

V-notches, and the discharge equation usually recommended in the U.S.A. is

$$Q=2.47h^{2.50} \text{ (English units).}$$

More precise values of the weir coefficient, which vary with h, are given for use with metric units in WMO Technical Note No. 117.

The only other central angle that is commonly used in the U.S.A. for V-notch weirs is 120°. The recommended discharge equation is

$$Q=4.35h^{2.50} \text{ (English units).}$$

SUBMERGED THIN-PLATE WEIRS

Submergence occurs at a weir when the elevation of the downstream water surface (tailwater) exceeds the elevation of the weir crest (fig. 149). The tailwater elevation is measured downstream from the turbulence that occurs in the immediate vicinity of the downstream face of the weir. The degree of submergence is expressed by the ratio h_t/h. For any given head h, submergence has the effect of reducing the discharge that would occur under the condition of free flow; the greater the submergence ratio h_t/h, the greater the reduction in discharge. Villemonte (1947) combined the results of his tests with those of several other investigators to produce the generalized relation shown in figure 150. Figure 150 is applicable to all shapes of vertical thin-plate weirs. In that figure, the abscissa is the submergence ratio raised to a power N, where N is the exponent in the free-flow discharge equation; for example, $N=1.5$ for a rectangular weir and $N=2.5$ for a triangular weir. The ordinate in figure 150 is the ratio of discharge under the submerged condition (Q_s) to free-flow discharge (Q). The relation shown in figure 150 agrees reasonably with the individual results obtained by the various investigators of submerged-weir discharge. However, if great accuracy is essential, it

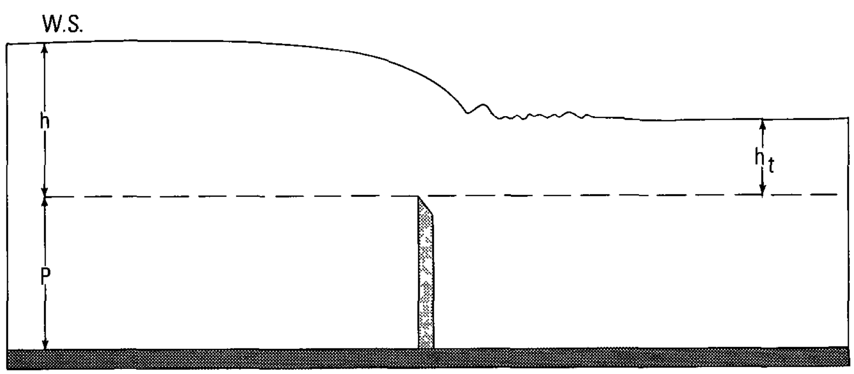


FIGURE 149.—Sketch showing submergence of a weir.

is recommended that the particular weir be calibrated in the field or in a laboratory under conditions similar to field conditions.

BROAD-CRESTED WEIRS

The term "broad-crested weir", as used here, refers to any weir that is not of the thin-plate type. The most common type of artificial control built in natural channels is the broad-crested weir. A structure of that type has the necessary strength and durability to withstand possible damage by floating debris. When installed in a stream channel that carries sediment-laden water, the weir is often built with a gently sloping upstream apron (slope: 1 vertical to 5 horizontal) so that there is no abrupt impedance to the flow and sediment is carried over the weir and not deposited in the gage pool. Because the

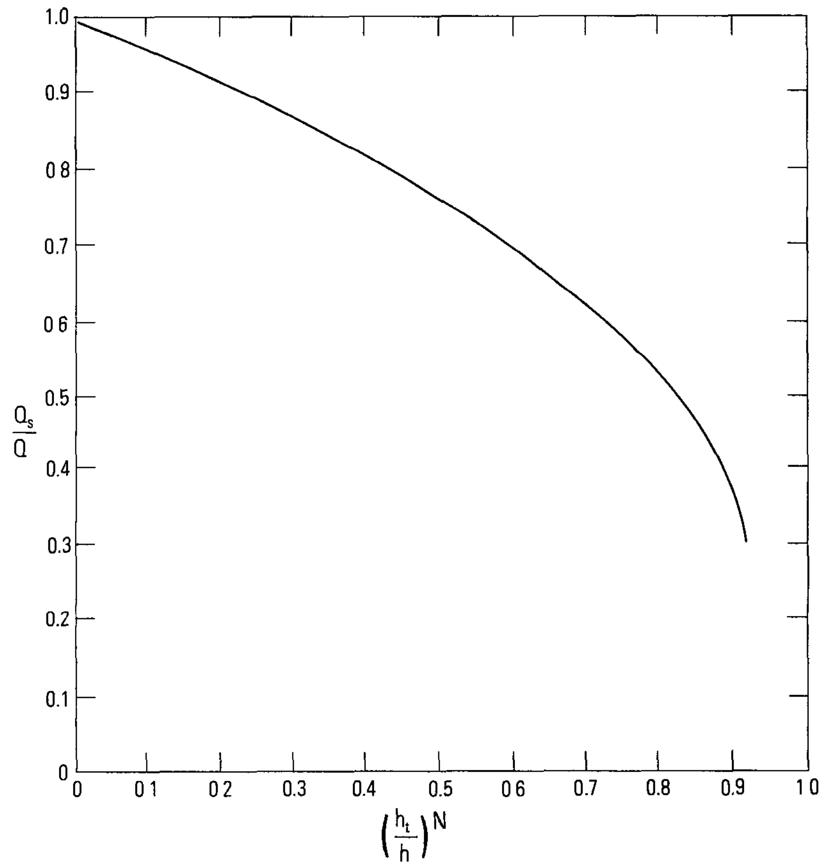


FIGURE 150.—Generalized relation of discharge ratio to submergence ratio for vertical thin-plate weirs. (After Villemonte, 1947.)

backwater caused by a high weir can aggravate flood problems along a stream, broad-crested weirs are usually built low to act as low-water controls and they become submerged at intermediate and high stages.

There are a myriad of crest shapes that can be used for broad-crested weirs, and there will be no attempt to describe the characteristics of the rating curve for each. Much of the material for such a discussion can be found in WMO Technical Note No. 117 (1971), in a report by Hulsing (1967), and in a textbook by King and Brater (1963). Instead, this section of the report will present a discussion of general principles as they apply to the definition of the discharge rating and will present the approximate ratings for broad-crested weirs commonly used as gaging-station controls in the U.S.A. The weirs are all intended to be field calibrated by current-meter discharge measurements.

Before proceeding to the discussion of broad-crested weirs commonly used in the U.S.A., it might be mentioned in passing that perhaps the most popular weir for use as a gaging-station control in Europe, and particularly in the United Kingdom, is the Crump weir (World Meteorological Organization, 1971). The Crump weir is triangular in cross section; the *upstream* face has a slope of 1 (vertical) to 2 (horizontal) and the *downstream* face has a slope of 1 (vertical) to 5 (horizontal). The crest, or apex of the triangular cross section, is usually horizontal over its entire length (b), but for greater sensitivity the crest may be given the shape of a flat Vee, the sides of which often have a slope of 1 (vertical) to 10 (horizontal). The basic equation for the Crump weir with horizontal crest is,

$$Q = Cb\,(h + h_v)^{3/2},$$

where C equals about 3.55 when English units are used and 1.96 when metric units are used.

FLAT-CRESTED RECTANGULAR WEIR

The simplest type of broad-crested weir is one that is rectangular in cross section and whose crest is horizontal over its entire length, b. The basic discharge equation for that weir is $Q = Cb(h + h_v)^{1.5}$, where h_v is the head attributable to velocity of approach. The coefficient C will increase with stage in the manner shown in figure 151, and h_v will also increase with stage as a result of the velocity of approach increasing with stage. (Figure 151 also shows the relation of C to stage for flat-crested weirs with sloping faces.) The rating curve for a flat-crested rectangular weir, when plotted on logarithmic graph paper, will be a straight line except for extremely low stages. The equation

of the line will be of the form $Q=p(G-e)^N$, where the slope of the line, N, will have a value greater than 1.5 because both the weir coefficient and the velocity head increase with stage.

Most flat-crested rectangular weirs are not sufficiently sensitive at low flows. To increase the low-flow sensitivity, the crest is often modified as shown in figure 152. Instead of the crest being horizontal over its entire length, b, the crest is given a gentle slope from one streambank to the other, or the crest is given the shape of an extremely flat Vee or catenary. As a result of this modification, the area of flow over the weir is triangular, or nearly so, at low flows and approximately rectangular at high flows. In other words, the length of weir crest that is utilized by the flow varies with stage until the stage rises high enough to flow over the entire length of the crest (b in fig. 152). In the general equation for the weir discharge, $Q=Cb(h+h_v)^{1.5}$, not only do C and h_v increase with stage, but length of weir crest, b, also increases with stage, as stated in the preceding sentence. Consequently if the weir rating plots as a straight line on logarithmic graph paper in accordance with equation, $Q=p(G-e)^N$, the slope of the line, N, will be considerably greater than 1.5, and invariably will be greater than 2.0.

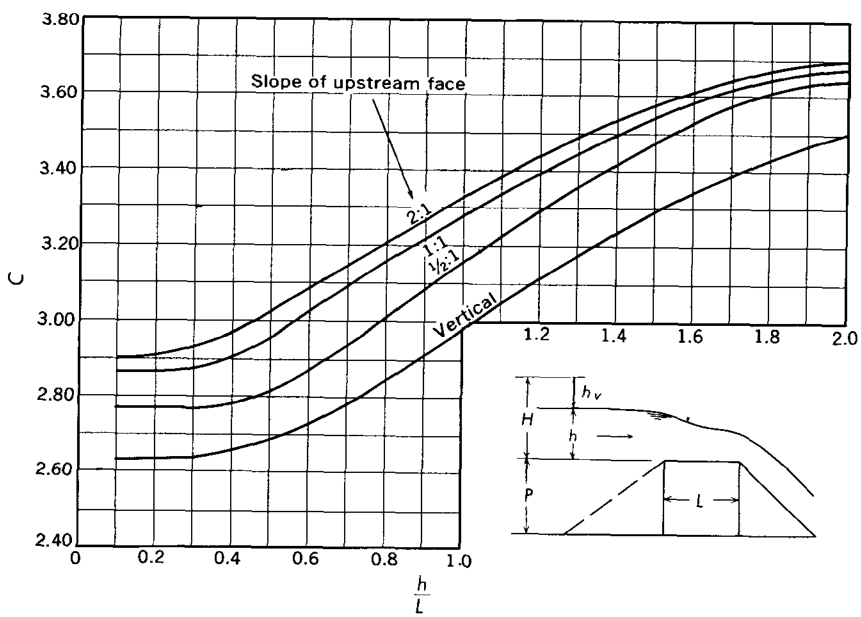


FIGURE 151.—Coefficients of discharge for full-width, broad-crested weirs with downstream slope ≤ 1:1 and various upstream slopes. (Slope is the ratio of horizontal to vertical distance.)

NOTCHED FLAT-CRESTED RECTANGULAR WEIR

Figure 153 shows the notched flat-crested rectangular weir that is the control for a gaging station on Great Trough Creek near Marklesburg, Pa.

Because there is a sharp break in the cross section at gage height 1.4 ft, a break occurs in the slope of the rating curve at that stage. The gage-height of zero flow for stages between 0.0 and 1.4 ft is 0.0 ft; for stages above 1.4 ft, the effective zero flow is at some gage height between 0.0 and 1.4 ft. If the low end of the rating is made a tangent, the gage height of zero flow (e) is 0.0 ft, and the slope of this tangent turns out to be 2.5, which, as now expected, is greater than the theoretical slope of 1.5. The upper part of this rating curve is concave upward because the value of e used (0.0 ft) is lower than the effective value of zero flow for high stages.

If the upper end of the rating is made a tangent, it is found that the value of e, or effective zero flow, must be increased to 0.6 foot. Because we have raised the value of e, the low-water end of the curve will be concave downward. The high-water tangent of the curve, principally because of increased rate of change of velocity of approach, will have a

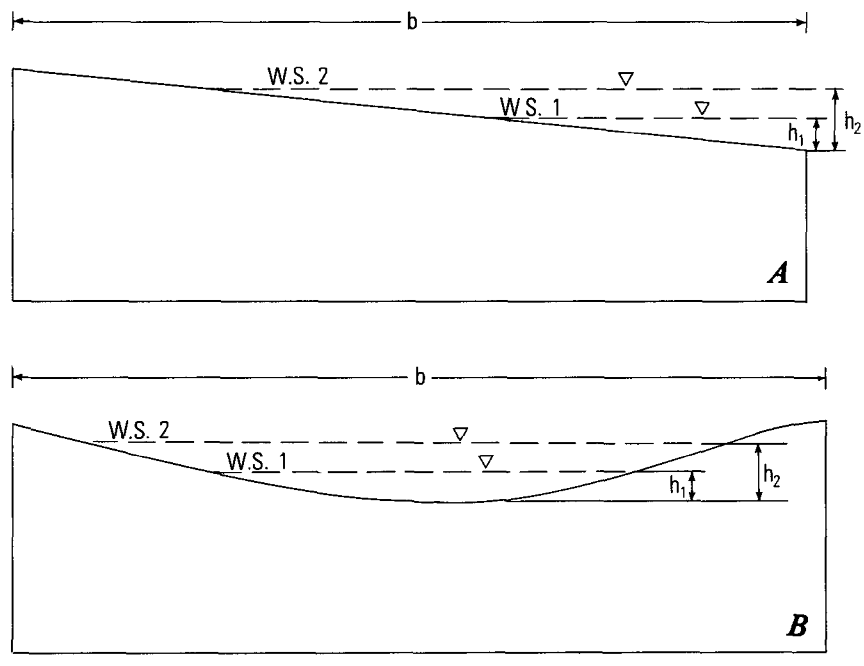


FIGURE 152.—Sketch of upstream face of flat-crested weir with (A) sloping crest and (B) catenary crest.

slope that is greater than that of the low-water tangent of the curve previously described; its slope is found to have a value of 3.0.

The low-water tangent for the notched control, which is defined by discharge measurements, warrants further discussion. Its slope of 2.5 is higher than one would normally expect for a simple flat-crested rectangular notch. One reason for the steep slope is the fact that the range of stage involved, 0.0 ft to 1.4 ft, is one in which the theoretical weir coefficient C increases very rapidly with stage. A more important reason is the geometric complexity of the notch which is not indicated in figure 153. At the downstream edge of the notch is a sharp-edged plate; its elevation is at 0.0 ft, but the sharp edge is about 0.1 ft higher than the concrete base of the notch. The details of the notch are not important to this discussion; they are mentioned here only to warn the reader not to expect a slope as great as 2.5 in the rating for a simple flat-crested rectangular notch. In fact, the sole purpose here of discussing the low-water tangent of the rating curve is to demonstrate the effect exerted on the curve by varying the applied values of e. The low-water end of a rating curve is usually well defined by discharge measurements, and if it is necessary to

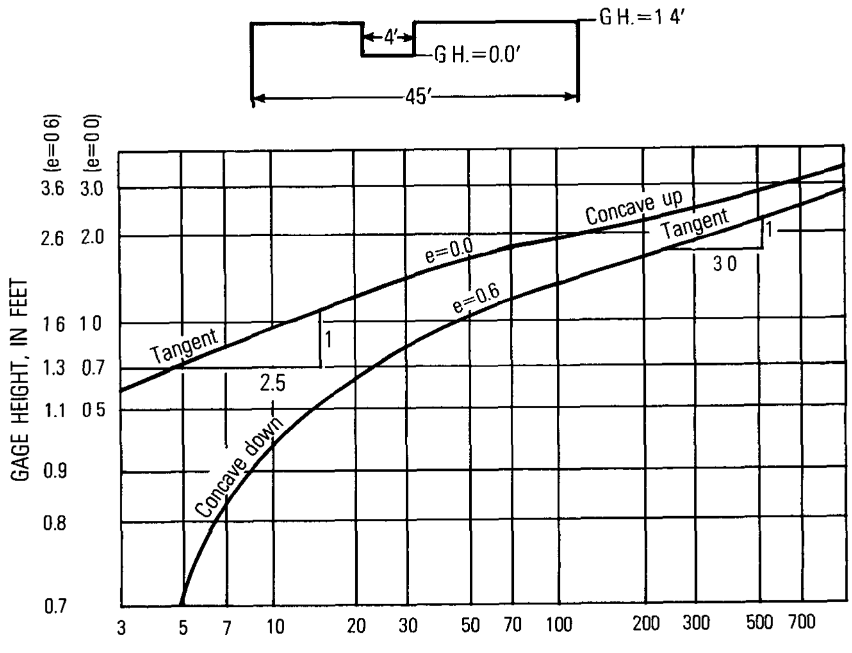


FIGURE 153.—Rating curve for a notched broad-crested control at Great Trough Creek near Marklesburg, Pa.

extrapolate the rating downward, it is best done by replotting the low-water end of the curve on rectangular-coordinate graph paper, and extrapolating the curve down to the point of zero discharge. (See section titled, "Low-Flow Extrapolation.")

TRENTON-TYPE CONTROL

The so-called Trenton-type control is a concrete weir that is frequently used in the U.S.A. The dimensions of the cross section of the crest are shown in figure 154. The crest may be constructed so as to be horizontal for its entire length across the stream, or for increased low-flow sensitivity the crest may be given the shape of an extremely flat Vee. For a horizontal crest, the equation of the stage-discharge relation, as obtained from a logarithmic plot of the discharge measurements, is commonly on the order of $Q=3.5bh^{1.65}$(English units). The precise values of the constants will vary with the height of the weir above the streambed, because that height affects the velocity of approach. The constants of the equation are greater than those for a flat-crested rectangular weir (see section titled, "Flat-crested Rectangular Weir") because the cross-sectional shape of the Trenton-type control is more efficient than a rectangle with regard to the flow of water.

When the Trenton-type control is built with its crest in the shape of a flat Vee, the exponent of h in the discharge equation is usually 2.5 or more, as expected for a triangular notch where velocity of approach

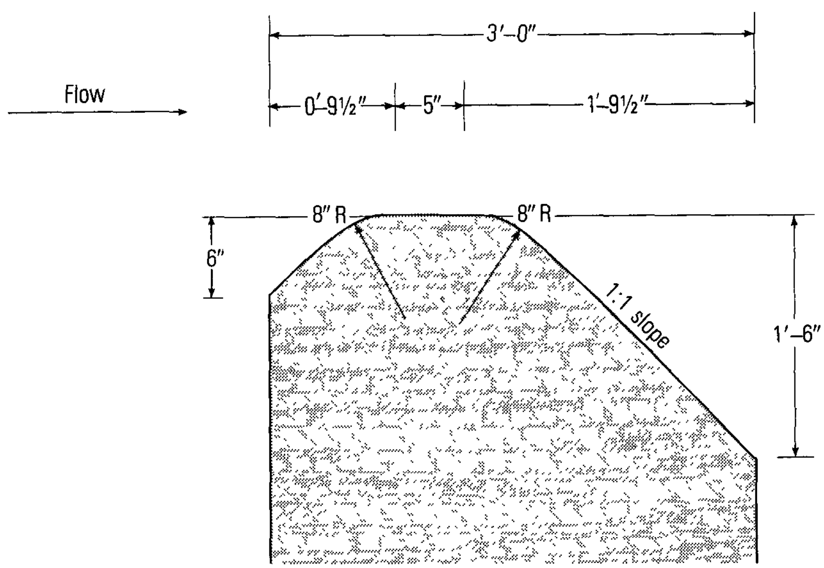


FIGURE 154.—Cross section of Trenton-type control.

is significant. Again, the precise values of the constants in the discharge equation are dependent on the geometry of the installation.

COLUMBUS-TYPE CONTROL

One of the most widely used controls in the U.S.A. is the Columbus-type control. This control is a concrete weir with a parabolic notch that is designed to give accurate measurement of a wide range of flows (fig. 155). The notch accommodates low flows; the main section, whose crest has a flat upward slope away from the notch, accommodates higher flows. The throat of the notch is convex along the axis of flow to permit the passage of debris. For stages above a head of 0.7 ft, which is the elevation of the top of the notch, the elevation of effective zero flow is 0.2 ft, and the equation of discharge is approximately,

$$Q=8.5(h-0.2)^{3.3} \text{ (English units)}$$

The precise values of the constants in the equation will vary with conditions for each installation. The shape of the crest above a stage of 0.7 ft is essentially a flat Vee for which the theoretical exponent of head is 2.5 in the discharge equation. However, the actual value of the exponent is greater than 2.5 principally because of the increase of velocity of approach with stage.

SUBMERGED BROAD-CRESTED WEIRS

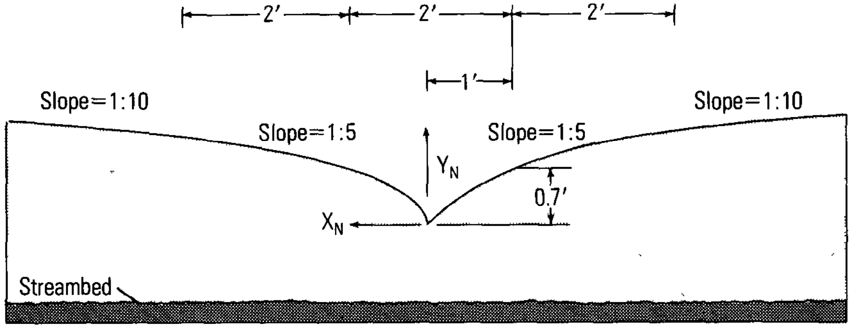
Weir submergence was defined earlier in the section titled, "Submerged Thin-Plate Weirs." As in the case of thin-plate weirs, for a given static head (h) the discharge decreases as the submergence ratio (h_t/h) increases. Little quantitative data are available to define the relation of discharge ratio to submergence ratio for the many types of broad-crested weir. However, it is known that for horizontal crests the submergence ratio must be appreciable before any significant reduction in discharge occurs. This threshold value of the submergence ratio at which the discharge is first affected ranges from about 0.65 to 0.85, depending on the cross-sectional shape of the weir crest.

FLUMES

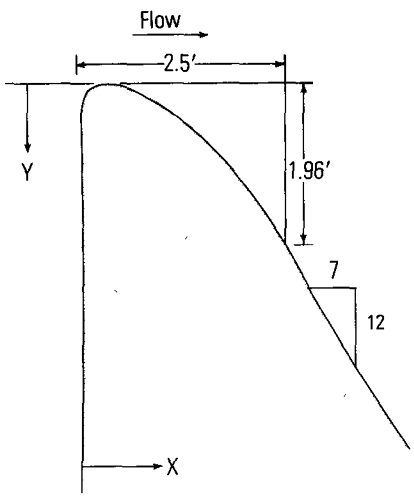
Flumes commonly utilize a contraction in channel width and free fall or a steepening of bed slope to produce critical or supercritical flow in the throat of the flume. The relation between stage measured at some standard cross section and discharge is thus a function only of the characteristics of the flume and can be determined, on an interim basis at least, prior to installation.

In the section in chapter 3 titled, "Artificial Controls," it was mentioned that flumes may be categorized with respect to the flow regime

that principally controls the measured stage; that is, a flume may be classed as either a critical-flow flume or a supercritical-flow flume. The most commonly used critical-flow flume is the Parshall flume, and it is the only one of that type that will be described here. The



Profile of weir crest and notch



Cross section of weir

Coordinates of notch profile, in feet

X_N	Y_N
0.00	0.00
.046	.1
.108	.2
.192	.3
.302	.4
.452	.5
.665	6
1.00	7

Coordinates of cross section of weir crest, in feet

X	Y
0.0	0.126
.1	.036
2	.007
.3	.000
.4	.007
.6	.060
8	.142
1.0	.257
1.2	.397
1.4	565
1.7	.870
2.0	1.22
2 5	1.96

FIGURE 155.—Dimensions of Columbus-type control.

supercritical-flow flume is less widely used, but fills a definite need. (See section in chapter 3 titled "Choice of an Artificial Control.") Of that type of flume, the trapezoidal supercritical-flow is preferred by the Geological Survey; it too will be described here.

PARSHALL FLUME

The principal feature of the Parshall flume is an approach reach having converging sidewalls and a level floor, the downstream end of which is a critical-depth cross section. Critical flow is established in the vicinity of that cross section by having a sharp downward break in the bed slope of the flume. In other words, the bed slope downstream from the level approach section is supercritical. The primary stage measurement is made in the approach reach at some standard distance upstream from the critical-depth cross section.

The general design of the Parshall flume is shown in figure 156. Table 17 gives the dimensions corresponding to the letters in figure 156 for various sizes of flumes. The flumes are designated by the width (W) of the throat. Flumes having throat widths from 3 in. to 8 ft have a rounded entrance whose floor slope is 25 percent. The smaller

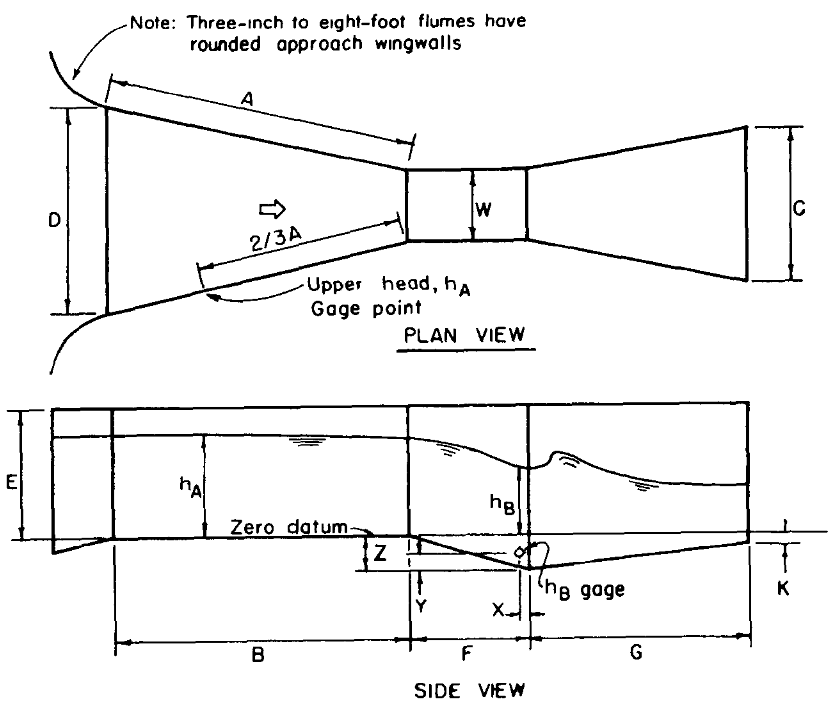


FIGURE 156.—Configuration and descriptive nomenclature for Parshall flumes.

TABLE 17.—*Dimensions and capacities of all sizes of standard Parshall flumes*

[For sizes 1 ft to 8 ft, $A = W/2 + 4$. For all sizes, h_A is located a distance of 2/3 A from crest, distance is converging wall length, not axial]

Size, Throat width W	Widths: Upstream end D	Widths: Downstream end C	Axial Lengths: Converging Section B	Axial Lengths: Throat Section F	Axial Lengths: Diverging Section G	Wall Depth in Converging Section E	Vertical distance below crest: Dip at Throat Z	Vertical distance below crest: Lower end of flume K	Converging wall length A	Gage Points: h_A, dist. upstream from crest	Gage Points: h_B X	Gage Points: h_B Y	Free Flow Capacities: Min.	Free Flow Capacities: Max.
inches	feet	feet	feet	feet	feet	feet	feet	feet	feet	feet	feet	feet	ft ^{3}/s	ft ^{3}/5s
1	0.549	0.305	1.17	0.250	0.67	0.5-0.75	0 094	0.062	1.19	0.79	0.026	0.042	0.005	0.15
2	.700	.443	1.33	.375	.83	0.50-0.83	.141	.073	1.36	.91	.052	.083	.01	.30
3	.849	.583	1.50	.500	1.00	1.00-2.00	.188	.083	1.53	1.02	.083	.125	.03	1.90
6	1.30	1.29	2.00	1.00	2.00	2.0	.375	.25	2.36	1.36	.167	.25	.05	3.90
9	1.88	1.25	2.83	1.00	1.50	2.5	.375	.25	2.88	1.93	.167	.25	.09	8.90
feet														
1.0	2.77	2.00	4.41	2.0	3.0	3.0	.75	.25	4.50	3.00	.167	.25	.11	16.1
1.5	3.36	2.50	4.66	2.0	3.0	3.0	.75	.25	4.75	3.17	.167	.25	.15	24.6
2.0	3.96	3.00	4.91	2.0	3.0	3.0	.75	.25	5.00	3.33	.167	.25	.42	33.1
3.0	5.16	4.00	5.40	2.0	3.0	3.0	.75	.25	5.50	3.67	.167	.25	.61	50.4
4.0	6.35	5.00	5.88	2.0	3.0	3.0	.75	.25	6.00	4.00	.167	.25	1.30	67.9
5.0	7.55	6.00	6.38	2.0	3.0	3.0	.75	.25	6.50	4.33	.167	.25	1.60	85.6
6.0	8.75	7.00	6.86	2.0	3.0	3.0	.75	.25	7.0	4.67	.167	.25	2.60	103.5
7.0	9.95	8.00	7.35	2.0	3.0	3.0	.75	.25	7.5	5.00	.167	.25	3.00	121.4
8.0	11.15	9.00	7.84	2.0	3.0	3.0	.75	.25	8.0	5.33	.167	.25	3.50	139.5
10	15.60	12.00	14.0	3.0	6.0	4.0	1.12	.50	9.0	6.00			6	300
12	18.40	14.67	16.0	3.0	8.0	5.0	1.12	.50	10.0	6.67			8	520
15	25.0	18.33	25.0	4.0	10.0	6.0	1.50	.75	11.5	7.67			8	900
20	30.0	24.00	25.0	6.0	12.0	7.0	2.25	1.00	14.0	9.33			10	1340
25	35.0	29.33	25.0	6.0	13.0	7.0	2.25	1.00	16.5	11.00			15	1660
30	40.4	34.67	26.0	6.0	14.0	7.0	2.25	1.00	19.0	12.67			15	1990
40	50.8	45.33	27.0	6.0	16.0	7.0	2.25	1.00	24.0	16.00			20.	2640
50	60.8	56.67	27.0	6.0	20.0	7.0	2.25	1.00	29.0	19.33			25	3280

Note: Flume sizes 3 inches through 8 feet have approach aprons rising at a 1:4 slope and the following entrance roundings: 3 through 9 inches, radius =1.33 feet; 1 through 3 feet, radius=1 67 feet, 4 through 8 feet, radius=2 00 feet

and larger flumes do not have that feature, but it is doubtful whether the performance of any of the flumes is significantly affected by the presence or absence of the entrance feature as long as approach conditions are satisfactory.

Parshall flumes have provision for stage measurements both in the approach reach and in the throat reach, but the downstream gage is required only when submerged-flow conditions exist. The datum for both gages is the level floor in the approach. The raised floor, length G in figure 156, in the downstream diverging reach is designed to reduce scour downstream and to produce more consistent stage-discharge relations under conditions of submergence. The percentage of submergence for Parshall flumes is computed by the formula,

$$\frac{h_B}{h_A} \times 100.$$

Where free-flow conditions exist for all flows, the downstream gage, h_B, may be omitted and the entire diverging reach may be dispensed with if desired. That simplification has been used in the design of small portable Parshall measuring flumes. (See section in chapter 8 titled, "Portable Parshall Flume.")

Tables 18 and 19 summarize the relation of discharge to stage at h_A under conditions of free flow (low stage at h_B) for flumes of the various sizes. Although the free-flow stage-discharge relations for the various flumes were derived experimentally, all relations can be expressed closely by the following equation, (Davis, 1963),

$$Y_0 + \frac{Q_0^2}{2Y_0^2\,(1 + 0.4X_0)^2} = 1.351\,Q_0^{\,0.645} \text{ (English units)} \tag{59}$$

in which

Y_0 = nondimensional depth, y_1/b
Q_0 = nondimensional discharge, $Q/g^{1/2}b^{5/2}$
X_0 = nondimensional distance, x/b
y_1 = depth at measuring section
b = channel width at throat
Q = discharge
g = acceleration of gravity
x = distance from throat crest to measuring section.

For flumes with throat widths no greater than 6 ft, the following simplified form of the above equation (Dodge, 1963) can be used:

$$Y_0 = 1.19Q_0^{\,0.645}\,X_0^{\,0.0494} \tag{60}$$

When the stage h_B is relatively high, the free-flow discharge corresponding to any given value of h_A is reduced. The percentage of submergence, or value of $[(h_B/h_A)\times 100]$, at which the free-flow discharge is first affected, varies with the size of flume. For flumes whose throat width is less than 1 ft, the submergence must exceed 50 percent before there is any backwater effect; for flumes with throat width from 1 to 8 ft, the threshold submergence is 70 percent; for flumes with throat width greater than 10 ft, the threshold submergence is 80 percent. Figure 157 shows the discharge ratings for Parshall flumes, from 2 inches to 9 inches, under both free-flow and submergence conditions. Figure 158 shows the correction in discharge, which is always negative, that is to be applied to free-flow discharges for various percentages of submergence and various values of h_A, for flumes having throat widths between 1 and 50 feet. The appropriate correction factor (k_s) for flume size is applied to the corrections read from the graphs. In other words,

$$Q_s = Q_f - k_s Q_c,$$

where

Q_s = discharge under submergence conditions,
Q_f = discharge under free-flow conditions, and
Q_c = discharge correction unadjusted for flume size.

The stage-discharge relations, both for free-flow and submergence conditions, given in the preceding tables and graphs, should be used only as guides or as preliminary ratings for Parshall flumes built in the field. Those installations should be field-calibrated because the structural differences that invariably occur between model and prototype flume usually cause the discharge rating for the field structure to differ from the experimental ratings given in this manual.

TABLE 18.—*Discharge table for Parshall flumes, sizes 2 inches to 9 inches, for free-flow conditions*

[Discharges for standard 3-inch Parshall flumes are slightly less than those for the modified 3-inch Parshall flume discussed in chapter 8; see table 14]

h_A (feet)	2 inches (ft^3/s)	3 inches (ft^3/s)	6 inches (ft^3/s)	9 inches (ft^3/s)
0.1	0.02	0.03	0.05	0.09
2	06	08	16	26
3	11	15	31	49
4	17	24	48	.76
5	24	34	69	1.06
6	31	45	92	1.40
7	40	57	1.17	1.78
8		70	1.45	2.18
9		84	1.74	2.61
1.0		89	2.06	3.07
1.1			2.40	3.55
1.2			2.75	4.06
1.3			3.12	4.59
1.4			3.51	5.14
1.5				5.71
1.6				6.31
1.7				6.92
1.8				7.54
1.9				8.20

TABLE 19. —*Discharge table for Parshall flumes, sizes 1 foot to 50 feet for free-flow conditions*

h_A (feet)	1 foot (ft^3/s)	1.5 feet (ft^3/s)	2 feet (ft^3/s)	3 feet (ft^3/s)	4 feet (ft^3/s)	5 feet (ft^3/s)	6 feet (ft^3/s)	7 feet (ft^3/s)	8 feet (ft^3/s)
0.10	0.11	0.15							
15	20	30	0.42	0.61					
20	35	51	66	97	1.26	1.55			
25	49	.71	93	1.37	1.80	2.22	2.63	3.02	3.46
30	.64	94	1.24	1.82	2.39	2.96	3.52	4.08	4.62
4	99	1.47	1.93	2.86	3.77	4.68	5.57	6.46	7.34
5	1.39	2.06	2.73	4.05	5.36	6.66	7.94	9.23	10.5
6	1.84	2.73	3.62	5.39	7.15	8.89	10.6	12.4	14.1
7	2.33	3.46	4.60	6.86	9.11	11.4	13.6	15.8	18.0
.8	2.85	4.26	5.66	8.46	11.3	14.0	16.8	19.6	22.4
9	3.41	5.10	6.80	10.2	13.6	16.9	20.3	23.7	27.0
1.0	4.00	6.00	8.00	12.0	16.0	20.0	24.0	28.0	32.0
1.2	5.28	7.94	10.6	16.0	21.3	26.7	32.1	37.5	42.9
1.4	6.68	10.1	13.5	20.3	27.2	34.1	41.1	48.0	55.0
1.6	8.18	12.4	16.6	25.1	33.6	42.2	50.8	59.4	68.1
1.8	9.79	14.8	19.9	30.1	40.5	50.8	61.3	71.8	82.3
2.0	11.5	17.4	23.4	35.5	47.8	60.1	72.5	84.9	97.5
2.2	13.3	20.2	27.2	41.3	55.5	69.9	84.4	98.9	113.6
2.4	15.2	23.0	31.1	47.3	63.7	80.3	97.0	113.7	130.7

h_A (feet)	10 feet (ft^3/s)	12 feet (ft^3/s)	15 feet (ft^3/s)	20 feet (ft^3/s)	25 feet (ft^3/s)	30 feet (ft^3/s)	40 feet (ft^3/s)	50 feet (ft^3/s)
0.30	5.75	6.75	8.4	11.1	13.8	16.5	21.8	27.3
04	9.05	10.85	13.3	17.7	21.8	26.1	34.6	43.2
0.5	13.0	15.4	19.1	25.1	31.2	37.2	49.5	61.8
0.6	17.4	20.6	25.5	33.7	41.8	50.0	66.2	82.6
0.7	22.2	26.2	32.7	43.1	53.4	64.0	84.8	105.5
0.8	27.5	32.7	40.4	53.4	66.3	79.2	105	131
0.9	33.3	39.4	48.9	64.3	80.1	95.5	127	158
1.0	39.4	46.8	57.9	76.3	94.8	113.2	150	187
1.2	52.7	62.6	77.3	102.0	127.0	152	201	250
1.4	67.4	80.1	99.0	130.5	162	194	257	320
1.6	83.5	99.1	122.8	162	201	240	318	396
1.8	100.9	119.8	148.0	195	243	290	384	479
2.0	119.4	141.8	175.3	232	287	343	454	567
2.2	139.0	165.0	204	269	334	400	530	660
2.4	159.9	189.8	235	310	384	459	609	758
2.6	181.7	215.7	267	352	437	522	692	864
3.0	228.4	271.2	335	442	549	656	870	1084
3.5	294	347	429	566	703	840	1113	1387
4.0	363	430	531	700	870	1040	1379	1717
4.5	437	518	641	846	1051	1255	1664	2073
5.0	517	614	759	1002	1244	1486	1970	2453
5.5			885	1166	1448	1730	2295	2860
6.0			1016	1340	1664	1988	2638	3285

Note: Available data indicates that extension of the above ratings to greater heads is reliable

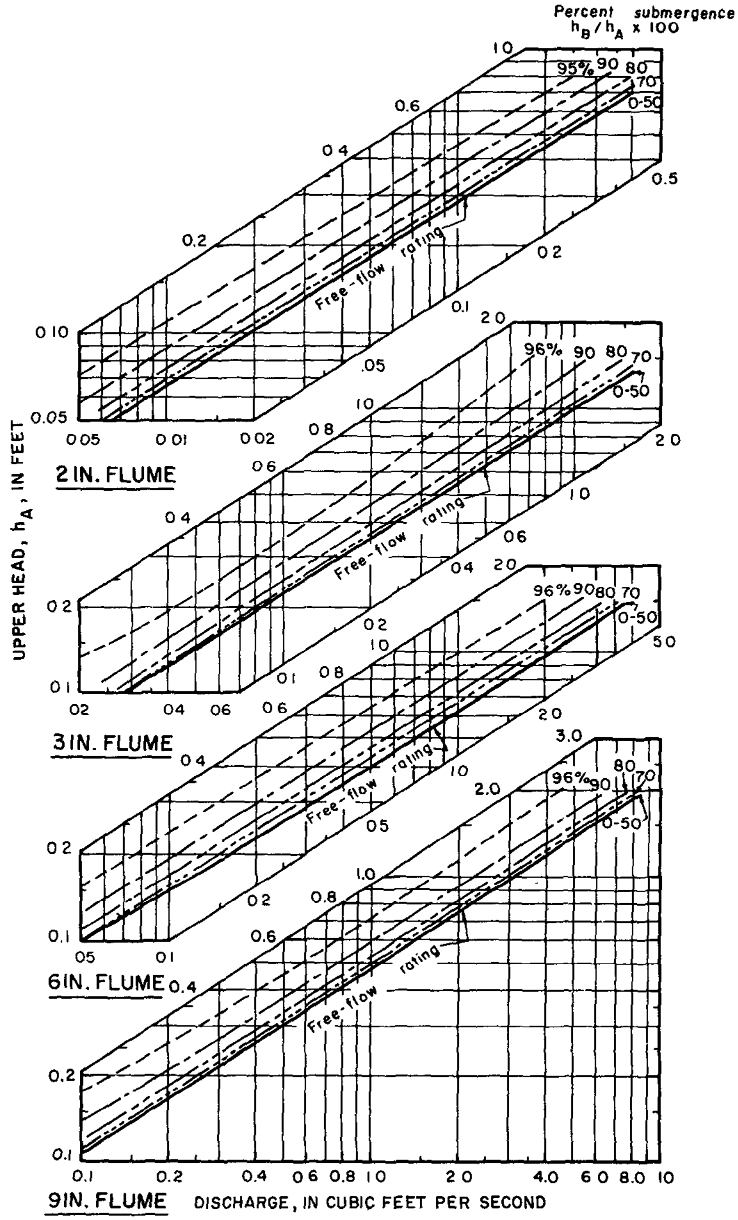


FIGURE 157.—Discharge ratings for "inch" Parshall flumes for both free-flow and submergence conditions.

TRAPEZOIDAL SUPERCRITICAL-FLOW FLUME

The principal feature of the trapezoidal supercritical-flow flume is a reach of flume (throat) whose bed has supercritical slope, upstream from which is a critical-depth cross section. The general design of the flume and the dimensions for the flumes of three throat widths that have been installed by the Geological Survey are shown in figure 159. The purpose of having the flume trapezoidal in cross section is to increase the sensitivity of the stage-discharge relation, particularly at low flows. Wide latitude exists with regard to the height (E) of the

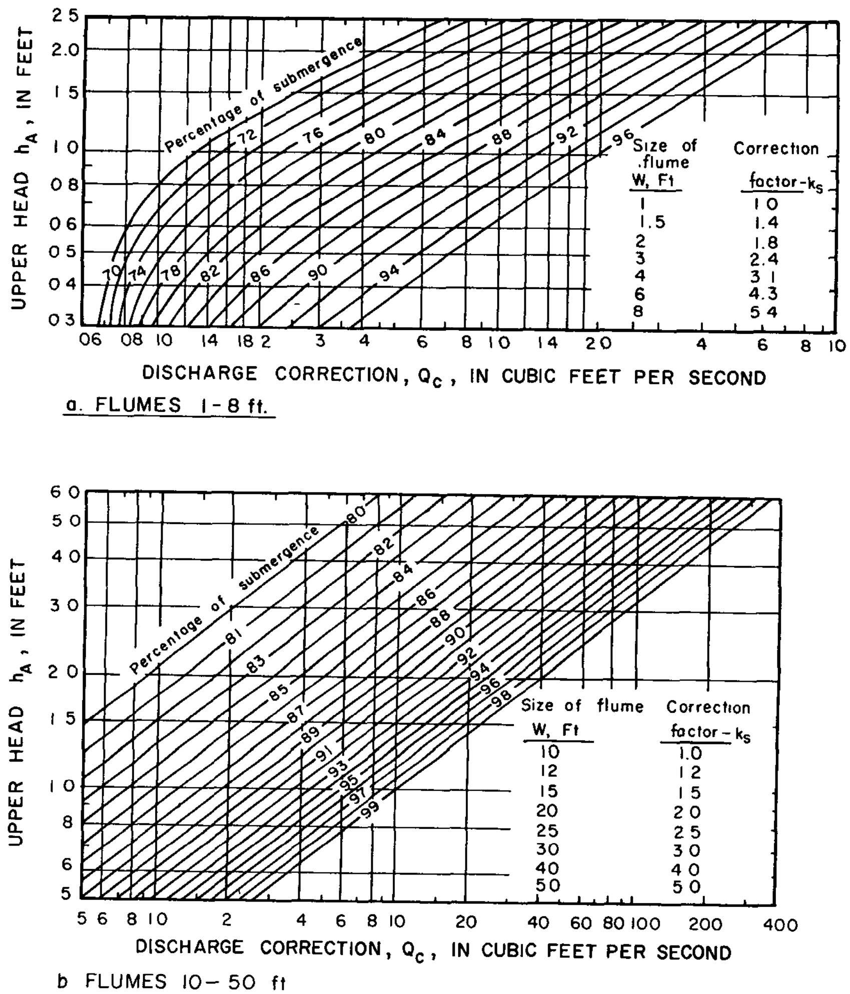


FIGURE 158.—Correction factors for submerged flow through 1- to 50-ft Parshall flumes.

sidewalls that can be used, and thus the range of discharge that can be accommodated by a supercritical-flow flume of any particular throat width is quite flexible. Stage (vertical depth of flow) is measured at a cross section at midlength of the throat reach, gage datum being the floor of the flume at the stage-measurement site. The measurement of stage must be precise because the stage-discharge relation for supercritical flow is extremely insensitive—a small change in stage corresponds to a large change in discharge.

Were it not for the severe width constriction at the downstream end of the converging reach, critical flow would occur at the break in floor slope at the downstream end of the approach reach and flow would be supercritical at all cross sections downstream from the approach reach. However for all but extremely low flows, the sharp constriction in width resulting from the use of a convergence angle (ϕ) of 21.8° (fig.159) causes backwater that extends upstream into the approach

Dimensions of trapezoidal supercritical-flow flume

Flume size, W_T feet	Width of approach reach, W_A feet	Angles		Lengths			Minimum capacity, ft³/s	Floor slopes	
		Sloping walls θ	Converging walls ϕ	Approach reach, L_A feet	Converging reach, L_C feet	Throat reach, L_T feet		Approach reach, percent	Converging and throat reaches, %
1	5	30°	21.8°	5.0	5.0	5.0	0.7	0	5
3	9	30°	21.8°	variable	7.5	10.0	2.0	0	5
8	14	30°	21.8°	variable	7.5	12.0	6.0	0	5

Note—Height of wall (E) is dependent on magnitude of maximum discharge to be gaged

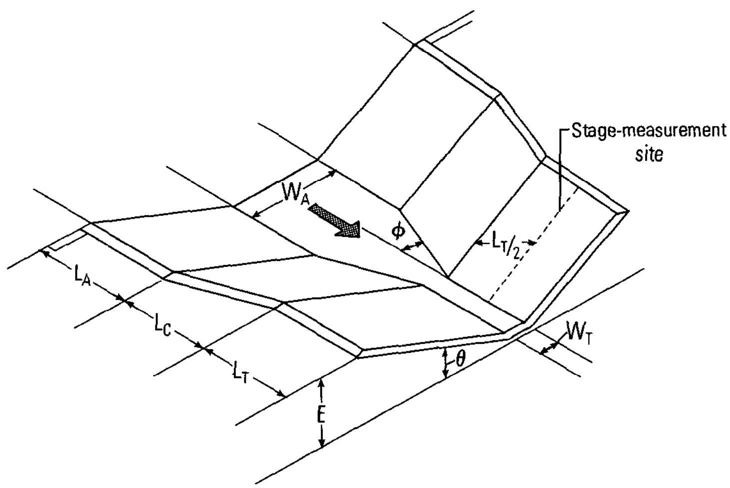


FIGURE 159.—Configuration and dimensions of trapezoidal supercritical-flow flumes of three throat widths.

reach. As a result critical depth occurs at the most constricted cross section in the converging reach, the flow being subcritical in the approach and converging reaches and supercritical in the throat reach. That is seen in figure 5 (chap. 3) which is a photograph of a 3-foot trapezoidal flume in Owl Creek in Wyoming. The purpose of the converging reach is to obtain an increased velocity at the critical-depth cross section and thereby reduce the likelihood of debris deposition at that cross section; such deposition could affect the stage-discharge relation in the throat of the flume.

The measured stage corresponding to any discharge is a function of the stage of critical depth at the head of the throat reach and the geometry of the throat reach upstream from the stage-measurement cross section. Consequently a theoretical rating for all but the smallest discharges can be computed by use of the Bernoulli or total-energy equation for the length of throat reach upstream from the stage-measurement site (fig. 160). By equating total energy at the critical-depth cross section (c) at the head of the throat reach to total energy at the stage-measurement cross section (m), we have,

$$\frac{V_c^{\,2}}{2g} + h_c + z_c = \frac{V_m^{\,2}}{2g} + h_m + z_m + h_f, \tag{61}$$

where

V is mean velocity,

g is acceleration of gravity,

h is vertical depth,

z is elevation of flume floor above any arbitrary datum plane, and

h_f is friction loss.

We make the assumption that the friction loss h_f in the short reach is negligible and may be ignored. Then by substituting, in equation 61 values from the two equations

$$Q = A_c V_c = A_m V_m \quad \text{and} \quad \Delta Z = Z_c - Z_m,$$

we obtain

$$\frac{Q^2}{2gA^2_c} + h_c + \Delta z = \frac{Q^2}{2gA^2_m} + h_m . \tag{62}$$

From the properties of critical-depth flow (Chow, 1959, p. 64), the critical-section factor (J) is computed by the formula

$$J = A_c \sqrt{\frac{A_c}{T_c}} \tag{63}$$

where A_c is the area and T_c is the top width at the critical-depth cross section. The discharge (Q) at the critical-depth cross section is

$$Q = J\sqrt{g} \tag{64}$$

By assuming a depth (h_c) at the critical-depth cross section, we can compute Q and A_c, and thus the values of all terms on the left side of equation 62 will be known for any chosen value of h_c. Because h_m is uniquely related to A_m, equation 62 can be solved by trial and error to obtain the depth (stage) at the measurement cross section corresponding to the value of Q that was computed earlier.

The entire procedure is repeated for other selected values of h_c to provide a discharge rating curve for the entire range of discharge. The value of h_c corresponding to the maximum discharge to be gaged represents the height to which the sidewalls of the throat section must be built to contain that discharge. An additional height of at least 0.5 ft should be added for freeboard to accommodate surge and wave action.

The computed discharge rating should be used only until the rating can be checked by current-meter discharge measurements. The sources of error in the computed rating are uncertainty as to the exact location of the critical-depth cross section for any given discharge and neglect of the small friction loss (h_f). However, the general shape of the discharge rating curve will have been defined by the computed values, and relatively few discharge measurements should be required for any needed modification of the rating.

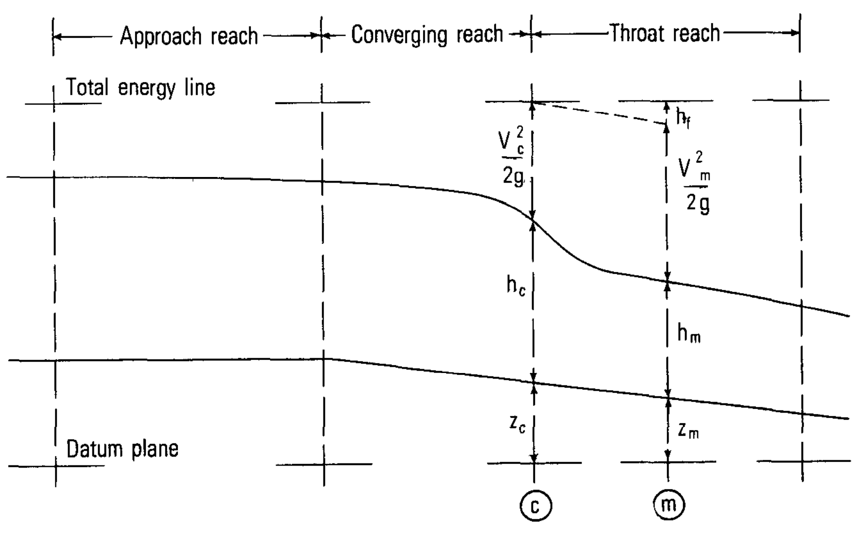


FIGURE 160.—Sketch illustrating use of the total-energy (Bernoulli) equation.

The total-energy equation (eq. 61) should also be applied to the converging reach to obtain the required height of the sidewalls at the upper end of the converging reach. The height plus an additional freeboard height of at least 0.5 ft should be used for the sidewalls in the approach reach. In applying equation 61 to the converging reach, the value of discharge used is the maximum discharge that is to be gaged, and the depth used at the lower end of the converging reach is the corresponding critical depth (h_c) that was computed earlier for the throat reach.

The solid-line curves on figures 161–163 are the theoretical discharge rating curves for the flumes of the three throat widths that have been field tested. The agreement between measured and theoretical discharges has generally been good except at extremely low stages. Nevertheless the theoretical curves should be considered as interim rating curves for newly built flumes until later measurements either corroborate the ratings or show the need for modification of the ratings. It is expected that the stage-discharge relation will not be affected by submergence, as long as submergence percentages do not exceed 80 percent. (Percentage of submergence for a given discharge is defined as the ratio, expressed as a percentage, of the stage in the natural channel immediately downstream from the throat reach to the stage at the stage-measurement site, both stages being referred to the floor elevation of the flume at the stage-measurement site.)

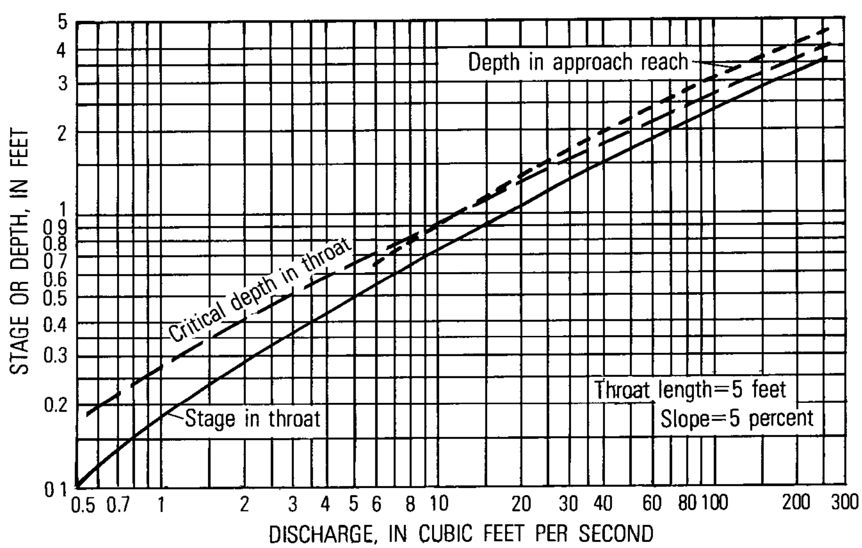


FIGURE 161.—Stage-discharge relation and significant depth-discharge relations for 1-ft trapezoidal supercritical-flow flume.

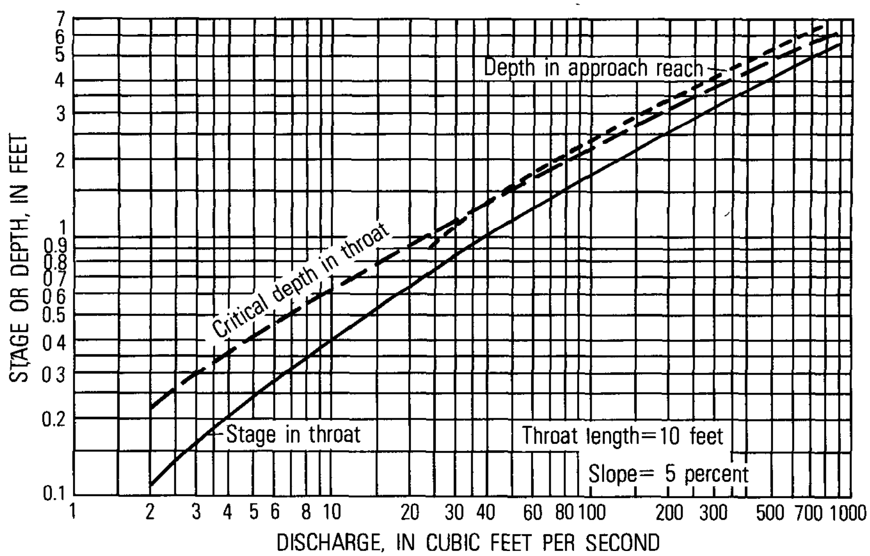


FIGURE 162.—Stage-discharge relation and significant depth-discharge relations for 3-ft trapezoidal supercritical-flow flume.

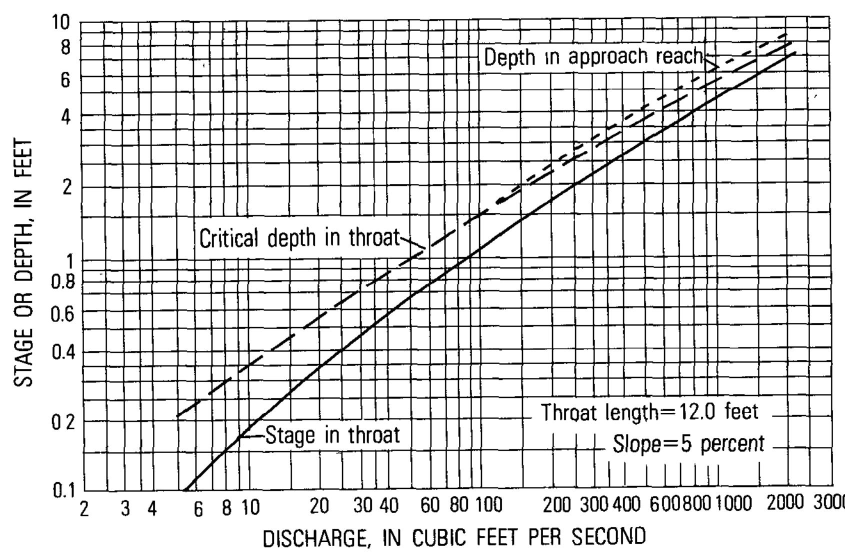


FIGURE 163.—Stage-discharge relation and significant depth-discharge relations for 8-ft trapezoidal supercritical-flow flume.

Also shown on each of the three rating-curve graphs are curves labeled "critical depth in throat" and "depth in approach reach." These curves are used to obtain the heights of sidewalls required to

contain the maximum discharge for which the flume is designed. For example, let us assume that an 8-ft flume is to be built to gage a range of discharges whose maximum value is 1200 ft^3/s. Figure 163 shows that the theoretical stage for that discharge is 5.2 ft; the height of the throat sidewall (critical depth) is shown to be 6.0 ft and the height of the approach sidewall is 6.8 ft. To those sidewall heights will be added at least 0.5 ft of freeboard, and the top of the sidewalls in the converging reach will be sloped uniformly to join the tops of the sidewalls of the approach and throat reaches.

Up to this point little has been said concerning the approach reach. Its sidewalls are extended upstream from the converging reach by means of rock fill or concrete to meet the natural channel banks. As long as the approach reach provides a smooth transition from the natural channel to the converging reach, its actual geometry will have no effect on the theoretical rating. The level floor of the approach reach will provide a site for current-meter measurements of discharge and will also induce the deposition of large debris, thereby helping to keep the more vital parts of the flume structure free of sediment deposition.

NATURAL SECTION CONTROLS

Natural section controls, listed in order of permanence, are usually a rock ledge outcrop across the channel, or a riffle composed of loose rock, cobbles, and gravel, or a gravel bar. Less commonly, the section control is a natural constriction in width of the channel, or is a sharp break in channel slope, as at the head of a cascade or brink of a falls.

Where the control is a rock outcrop, riffle, or gravel bar, the stage-discharge relation, when plotted on logarithmic paper, conforms to the general principles discussed for broad-crested artificial controls. If the natural control is essentially horizontal for the entire width of the control, the head on the control is the difference between the gage heights of the water surface and the crest of the control. The exponent (N) of the head in the equation of discharge,

$$Q = p(G-e)^N \tag{53}$$

will be greater than the theoretical value 1.5, primarily because of the increase in velocity of approach with stage. If the crest of the control has a roughly parabolic profile, as most natural controls have (greater depths on the control near midstream), the exponent N will be even larger because of the increase in width of the stream with stage, as well as the increase in velocity of approach with stage. The value of N will almost always exceed 2.0. If the control is irregularly

notched, as is often the case, the gage height of effective zero flow (*e*) for all but the lowest stages, will be somewhat greater than that for the lowest point in the notch. (The method of determining values of *e* was explained in the section titled, "Graphical Plotting of Rating Curves.")

The above principles are also roughly applicable to the discharge equations for an abrupt width contraction or an abrupt steepening of bed slope. The exponent *N* and the gage height of effective zero flow are influenced, as described above, by the transverse profile of the streambed at the control cross section.

An example of natural section control is treated in the following discussion.

COMPOUND SECTION CONTROLS

Where the control section is a local rise in the streambed, as at a rock outcrop, riffle, or gravel bar, that cross section is invariably a control only for low flows. The gaging station in that circumstance has a compound control, the high flows being subject to channel control. Occasionally there is a second outcrop or riffle, downstream from the low-water riffle, that acts as a section control for flows of intermediate magnitude. When the control for intermediate stages is effective it causes submergence of the low-water control. At high flows the section control for intermediate stages is in turn submerged when channel control becomes effective. An example of a compound control involving two section controls follows; an example of a compound control involving a section control that is submerged when channel control becomes effective is described in the section titled, "Compound Controls Involving Channel Control."

Figure 164 shows the rating for the compound section control at the gaging station on Muncy Creek near Sonestown, Pa. The control consists of two rock-ledge riffles, effective zero flow (e) for very low stages being at gage height 1.3 feet and for higher stages at gage height 1.2 feet. If the low end of the rating is made a tangent, it means that too large a value of *e* is used for the high end of the rating (1.3 ft vs 1.2 ft), and the high-water end of the curve becomes concave downward. Conversely, if the high end of the curve is made a tangent, the low-water end of the curve becomes concave upward. The high-water tangent of the curve has a greater value of exponent *N* than the low-water tangent of the other curve. This difference in the values of *N* reflects the effect of differences in the geometries of the two controls as well as the effect of increased rate of change of approach velocities at the higher stages. The slopes of the two tangents are 2.9 and 2.2, both values being greater than the theoretical slope of 1.5.

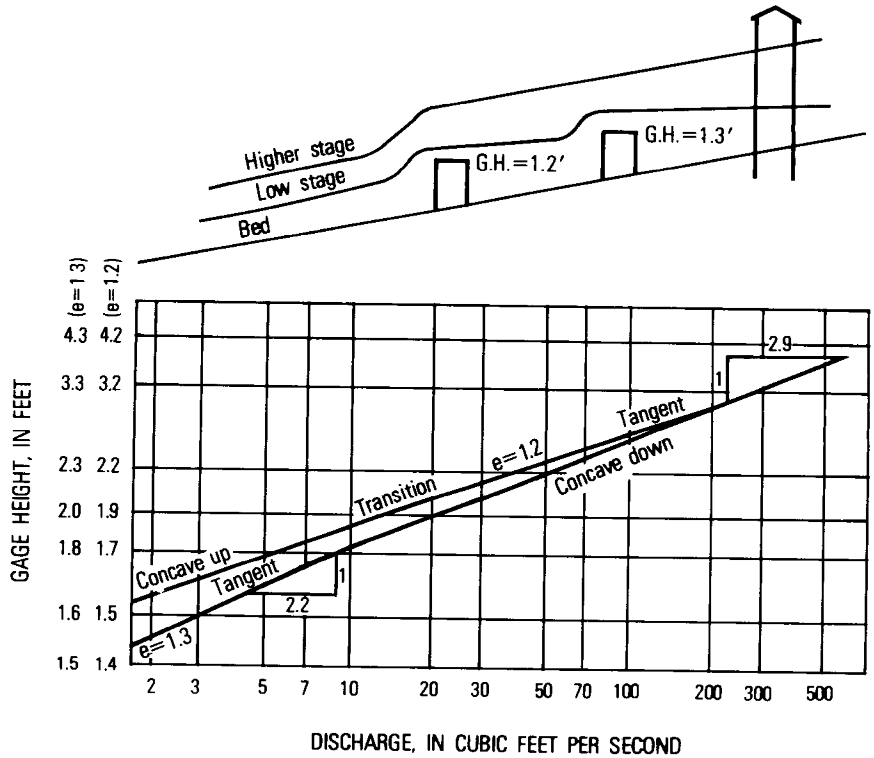


FIGURE 164.—Rating curve for a compound section control at Muncy Creek near Sonestown, Pa.

CHANNEL CONTROL

CHANNEL CONTROL FOR STABLE CHANNELS

The term "stable channels," as used in this report, is a relative term. Virtually all natural channels are subject to at least occasional change as a result of scour, deposition, or the growth of vegetation, but some alluvial channels, notably those whose bed and banks are composed of sand, have movable boundaries that change almost continuously, as do their stage-discharge relations. For the purpose of this manual, stable channels include all but sand channels. Sand channels are discussed in the section titled, "Sand-Channel Streams."

Almost all streams that are unregulated by man have channel control at the higher stages, and among those with stable channels, all but the largest rivers have section control at low stages. Because this section of the manual discusses only stable channels that have channel control for the entire range of stage experienced, the discussion is limited to the natural channels of extremely large rivers and

to artificial channels constructed without section controls. The artificial channels may be concrete-lined, partly lined or rip-rapped, or unlined. Streams that have compound controls involving channel control are discussed in the section titled, "Compound Controls Involving Channel Control."

The Manning discharge equation for the condition of channel control, as discussed in chapter 9, under the heading, "Slope-Area Method," is

$$Q = \frac{1.486}{n} AR^{2/3}S^{1/2} \text{ (English units)}, \tag{65}$$

or

$$Q = \frac{1}{n} AR^{2/3}S^{1/2} \text{ (metric units)} \tag{66}$$

In analyzing an artificial channel of regular shape, whose dimensions are fixed, flow at the gage is first assumed to be at uniform depth. Consequently, for any stage all dimensions on the right side of the equations are known except n. A value of n can be computed from a single discharge measurement, or an average value of n can be computed from a pair of discharge measurements, and thus a preliminary rating curve for the artificial channel can be computed for the entire range of stage from the results of a pair of discharge measurements. If subsequent discharge measurements depart from the computed rating curve, it is likely that the original assumption of flow at uniform depth was erroneous. That means that the energy slope, S, is not parallel to the bed slope, but varies with stage, and that the value of n, which was computed on the basis of bed slope, is also in error. The rating curve must be revised to fit the plotted discharge measurements, but the preliminary rating curve may be used as a guide in shaping the required extrapolation of the rating curve. The extrapolation should also be checked by application of the conveyance-slope method of rating extrapolation, which is described in the section titled, "Conveyance-Slope Method."

To understand the principles that underlie the stage-discharge relation for channel control in a natural channel of irregular shape we return to the Manning equation and make some simplifying assumptions in that equation. We assume, not unreasonably, that at the higher stages n is a constant and that the energy slope (S) tends to become constant. Furthermore, area (A) is approximately equal to depth (D) times width (W). We make the substitution for A in equation 65 or 66, and by expressing $S^{1/2}/n$ as a constant, C_1, we obtain

$$Q = C_1 \,(D)\,(W)R^{2/3}. \text{ (approx.)}$$

If the hydraulic radius (R) is considered equal to D, and W is considered a constant, the equation becomes

$$Q = CD^{1.67} = C(G-e)^{1.67} \text{ (approx.)} \tag{67}$$

However, unless the stream is exceptionally wide, R is appreciably smaller than D. This has the effect of reducing the exponent in the last equation although this reduction may be offset by an increase of S or W with discharge. Changes in roughness with stage will also affect the value of the exponent. The net result of all these factors is a discharge equation of the form

$$Q = C\,(G-e)^N$$

where N will commonly vary between 1.3 and 1.8 and practically never reach a value as high as 2.0.

An example of a discharge rating for channel control in a natural stream is given in the following section, where compound controls that involve channel control are discussed.

COMPOUND CONTROLS INVOLVING CHANNEL CONTROL

In the preceding section mention was made of the fact that compound control of the stage-discharge relation usually exists in natural channels, section control being effective for the lower stages and channel control being effective for the higher stages. An example of that situation is shown in figure 165, the rating curve for the Susquehanna River at Harrisburg, Pa. The low-water control is a low weir with zero flow at gage height 2.2 feet. At a stage of 3.9 feet this control starts to drown out and channel control becomes effective. If the low end of the rating is made a tangent, a value of e = 2.2 ft must be used. Because the value of e for the upper end of the rating is something less than 2.2 feet, the high end becomes concave downward. If the high end of the curve is made a tangent, the effective value of e is found to be 0.0 ft. This being too low a value of e for the lower end of the curve, the low end becomes concave upward.

If the rating for a section control (low end of the curve) is a tangent, the value of the exponent N is expected to be greater than 2.0. In this example, N = 2.3. If the rating for a channel control (high end of the curve) is a tangent, the value of N is expected to be less than 2.0, and probably between 1.3 and 1.8. In this example N = 1.3. Should overbank flow occur the rating curve will bend to the right.

It can be demonstrated, nonrigorously, that straight-line rating curves for section control almost always have a slope greater than 2.0 and that those for channel control have a slope less than 2.0. It has

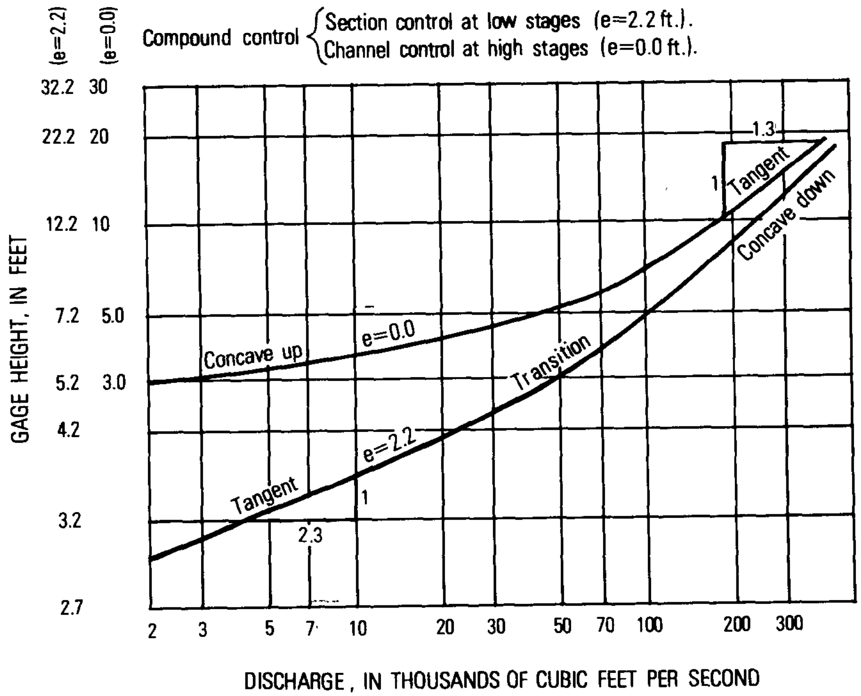


FIGURE 165.—Rating curve for a compound control at Susquehanna River at Harrisburg, Pa.

been shown that the equation for a straight-line rating on log paper is $Q = CH^N$, where N is the slope of the line. The first derivative of this equation is a measure of the change in discharge per tenth of a foot change in stage. The first derivative is:

$$\frac{dQ}{dH} = CNH^{N-1}.$$

Second differences are obtained by differentiating again. The second derivative is:

$$\frac{d^2Q}{dH^2} = CN\,(N-1)H^{N-2}.$$

Examination of the second derivative shows that second differences increase with stage when N is greater than 2.0 and decrease with stage when N is less than 2.0.

The hypothetical rating for a compound control is shown in table 20. This rating represents the condition of section control at the lower

TABLE 20.—*Hypothetical stage-discharge rating for a compound control*

[The higher values of discharge are rounded as normally used in a rating table; the precise values that required rounding are in parentheses]

Gage height (ft)	Discharge (ft^3/s)	Difference per tenth of a foot	Second difference
1.0	100		
		20	
1.1	120		1
		21	
1.2	141		1
		22	
1.3	163		2
		24	
1.4	187		2
		26	
1.5	213		3
		29	
1.6	242		3
		32	
1.7	274		4
		36	
1.8	310		3
		40 (39)	
1.9	350 (349)		2
		40 (41)	
2.0	390		2
		45 (43)	
2.1	435 (433)		2
		45 (45)	
2.2	480 (478)		2
		45 (47)	
2.3	525		1
		50 (48)	
2.4	575 (573)		1
		50 (49)	
2.5	625 (622)		1
		50 (50)	
2.6	675 (672)		1
		50 (51)	
2.7	725 (723)		1
		50 (52)	

stages and channel control at the higher stages. If two values of discharge are shown for an item in the rating table, the figure in parenthesis is the exact value and the figure without a parenthesis is the "rounded" value that normally would be used in the rating table. Experienced hydrographers will recognize the progression of discharge values in this table as being typical. Inspection of the second difference column shows the second differences to be increasing at the low-water end (section control, $N > 2$) and decreasing at the high-water end (channel control, $N < 2$). These are the results that one would predict from the discussion in the preceding paragraph.

EXTRAPOLATION OF RATING CURVES

Rating curves, more often than not, must be extrapolated beyond the range of measured discharges. The preceding material in this

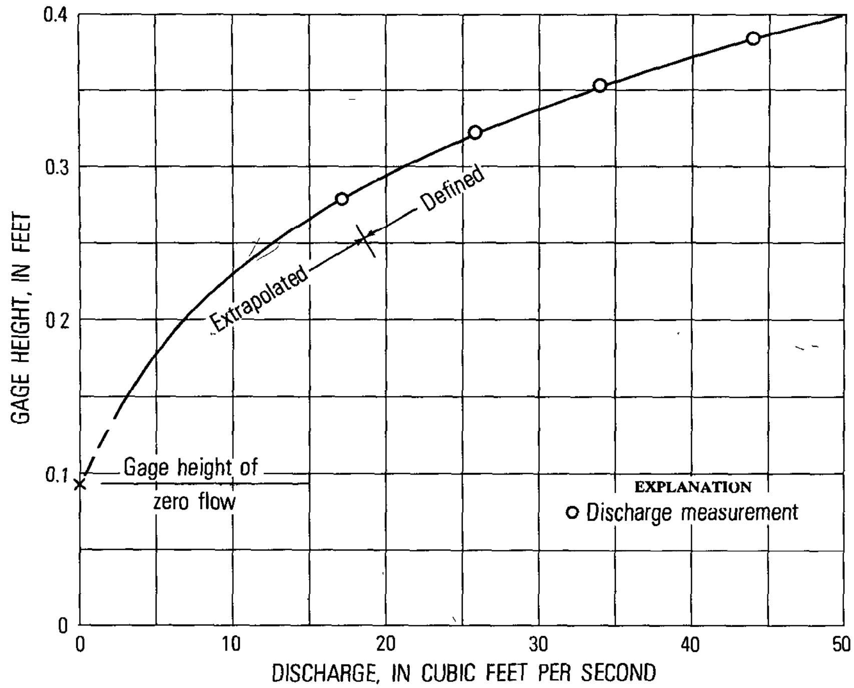


FIGURE 166.—Example of low-flow extrapolation on rectangular-coordinate graph paper.

chapter explained the principles governing the shape of logarithmic rating curves to guide the hydrographer in shaping the extrapolated segment of a rating. However, even with a knowledge of those principles, a large element of uncertainty exists in the extrapolation process. The purpose of this section of the manual is to describe methods of analysis that will reduce the degree of uncertainty.

LOW-FLOW EXTRAPOLATION

Low-flow extrapolation is best performed on rectangular-coordinate graph paper because the coordinates of zero flow can be plotted on such paper. (Zero discharge cannot be plotted on logarithmic graph paper.) An example of such an extrapolation is shown in figure 166, where the circled points represent discharge measurements plotted on the coordinate scales of gage height versus discharge. The rating in the example is defined by the measurements down to a gage height of 0.28 ft, but an extrapolation to a gage height of 0.14 ft is required. Field observation has shown the low point on the control (point of zero flow) to be at gage height 0.09 ft.

The method of extrapolation in figure 166 is self-evident. A curve

has been drawn between the plotted points at gage heights 0.09 ft and 0.28 ft to merge smoothly with the rating curve above 0.28 ft. There is no assurance that the extrapolation is precise—low-flow discharge measurements are required for that assurance—but the extrapolation shown is a reasonable one.

HIGH-FLOW EXTRAPOLATION

As mentioned in the "Introduction" of this chapter, the problem of high-flow extrapolation can be avoided if the unmeasured peak discharge for the rating is determined by the use of the indirect methods discussed in chapter 9. In the absence of such peak-discharge determinations, estimates of the discharges corresponding to high values of stage may be made by using one or more of the following four techniques:

1. conveyance-slope method,
2. areal comparison of peak-runoff rates,
3. step-backwater method, and
4. flood routing.

As a matter of fact, only as a last resort should the rating curve be extrapolated beyond a discharge value equal to twice the greatest measured discharge. If a greater extrapolation is required, the hydrologist should first try to define the upper end of the rating by use of one of the indirect peak-discharge determination methods of chapter 9. If for some reason, that course of action is not feasible, he should then use at least one of the four techniques listed above.

The knowledgeable reader of this manual may notice the absence from the above list of two techniques that used to be standard practice—the velocity-area method and the Q vs $Ad^{1/2}$ method. The Q vs $Ad^{1/2}$ method was superior to the velocity-area method and largely supplanted it; similarly, the conveyance-slope method, because of its superiority, has, in the last two decades, largely supplanted the Q vs $Ad^{1/2}$ method. Of the three somewhat similar methods, only the conveyance-slope method is described here, because a description of the two earlier methods (Corbett and others, 1943, p. 91–92) would have only academic, rather than practical, value.

CONVEYANCE-SLOPE METHOD

The conveyance-slope method is based on equations of steady flow, such as the Manning equation. In the Manning equation,

$$Q = KS^{1/2}. \tag{68}$$

The conveyance, K, equals $\frac{1.486}{n} AR^{2/3}$ when English units are used,

and $K=\frac{1}{n}AR^{2/3}$ when metric units are used. Values of A and R corresponding to any stage can be obtained from a field survey of the discharge-measurement cross section, and values of the coefficient n can be estimated in the field. Thus, the value of K, embodying all the elements that can be measured or estimated, can be computed for any given stage. (We shall soon see that errors in estimating n are usually not critical.) Values of gage height vs K, covering the complete range of stage up to the required peak gage height, are computed and plotted on rectangular graph paper. A smooth curve is fitted to the plotted points.

Values of slope, S, which is actually the energy gradient, are usually not available even for measured discharges. However, for the measured discharges, $S^{1/2}$ can be computed by dividing each measured discharge by its corresponding K value; S is then obtained by squaring the resulting value of $S^{1/2}$. Values of gage height vs S for the measured discharges are plotted on rectangular graph paper, a curve is fitted to the plotted points, and the curve is extrapolated to the required peak gage height. The extrapolation is guided by the knowledge that S tends to become constant at the higher stages. That constant slope is the "normal" slope, or slope of the streambed. If the upper end of the defined part of the curve of gage height vs S indicates that a constant or near-constant value of S has been attained, the extrapolation of the curve can be made with confidence. The discharge for any particular gage height will be obtained by multiplying the corresponding value of K from the K curve by the square root of the corresponding value of S from the S curve. We see that errors in estimating n will have minor effect because the resulting percentage error in computing K is compensated by a similar percentage error in the opposite direction in computing $S^{1/2}$. In other words, the constancy of S is unaffected, but if K is, say, 10 percent high, $S^{1/2}$ will be 10 percent low, and the two discrepancies are canceled when multiplication is performed. However, if the upper end of the defined part of the curve of gage height vs S has not reached the stage where S has a near-constant value, the extrapolation of the curve will be subject to uncertainty. In that situation the general slope of the streambed, as determined from a topographic map, provides a guide to the probable constant value of S that should be attained at high stages.

As mentioned in the preceding paragraph, the discharge for any particular gage height is obtained by the multiplication of appropriate values of K and $S^{1/2}$, and in that manner the upper end of the stage-discharge relation is constructed.

Figure 167 provides an example of the conveyance-slope method, as used for rating-curve extrapolation at the gaging station on Klamath River at Somes Bar, Calif. The conveyance curve is based on

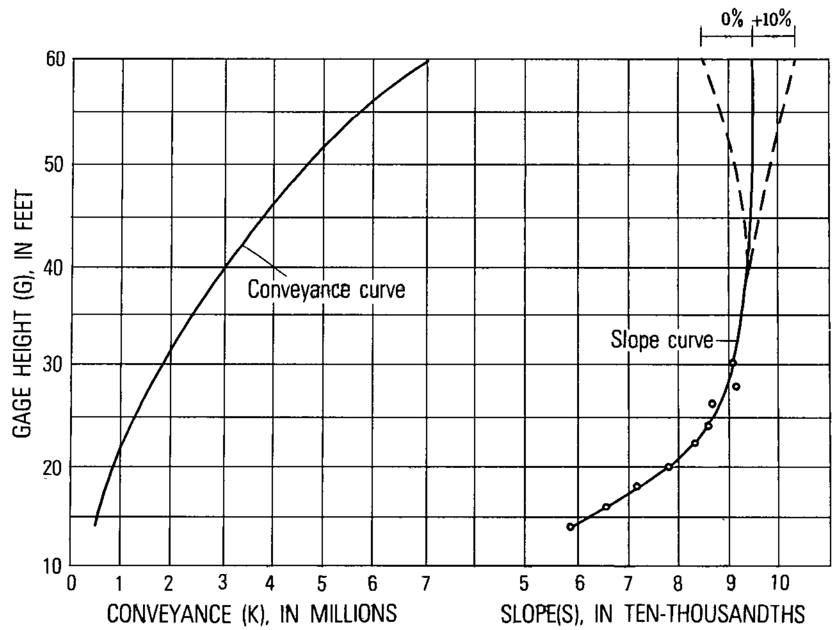


FIGURE 167.—High-flow extrapolation by use of conveyance-slope method—Klamath River at Somes Bar, Calif.

values of K computed from the geometry of the measurement cross section. The slope curve is defined to a gage height of 30 ft by discharge measurements (circled points), and extrapolated as the solid line to the peak gage height of 60 ft. It appears highly unlikely that the slope curve at a gage height of 60 ft will fall outside the limiting dashed curves shown in figure 167; in other words, it appears unlikely that the value of S at 60 ft (0.00095) is in error by more than ± 10 percent. If that is true, when the square root of S is computed and then used in a computation of peak discharge, the error for both $S^{1/2}$ and Q reduces to ± 5 percent. Although the attainment of so high an accuracy is highly improbable, the fact remains that one can place considerable confidence in the discharge computed for a gage height of 60 ft in this example. It should be mentioned here that the likelihood of a decrease in slope at high stages, as shown by the dashed curve on the left of the slope curve, is greatest when overbank flows occur.

In the above example conditions were ideal for application of the conveyance-slope method, and the example in figure 167 may therefore be misleading with regard to the general accuracy of the method. The conveyance-slope method assumes first that the geometry of the cross-section used for discharge measurements is fairly representa-

tive of that of a long reach of downstream channel. The need to meet this assumption immediately eliminates from consideration those gaging stations where discharge measurements are made at constricted cross sections, such as occur at many bridge- and cableway-measurement sections.

The conveyance-slope method also assumes that slope tends to become constant (uniform flow) at the higher stages. That is strictly true only for long, straight channels of uniform cross section, but natural channels that meet that description are virtually nonexistent. Consequently, the slope-stage relation may be anything but a vertical line at the upper stages. In the example in figure 167, a judgment decision, based on a knowledge of the channel characteristics, was made concerning the "probable" limiting positions of the stage-slope relation—the dashed lines on the graph—to give some idea of the "probable" error of the discharge computation. However, even given that knowledge of channel characteristics, if the two highest discharge measurements (two highest circles on the slope curve) had not been available it would have been impossible to position the upper end of the slope curve with any confidence. Fortunately there is a mitigating factor; an error of even as much as 40 percent in the value of slope at the upper end of the slope curve would give an error in discharge of either +18 percent ($\sqrt{1.40}-1.0=0.18$) or −23 percent ($1.0-\sqrt{0.60}=0.23$), depending on whether the estimate of slope was high or low.

In summary, the conveyance-slope method is a helpful adjunct in extrapolating rating curves, but its limitations must be understood so that it is not misused.

AREAL COMPARISON OF PEAK-RUNOFF RATES

When flood stages are produced over a large area by an intense general storm, the peak discharges can often be estimated, at gaging stations where they are lacking, from the known peak discharges at surrounding stations. Usually each known peak discharge is converted to peak discharge per unit of drainage area before making the analysis. In other words, peak discharge is expressed in terms of cubic feet per second per square mile or cubic meters per second per square kilometer.

If there has been relatively little difference in storm intensity over the area affected, peak discharge per unit area may be correlated with drainage area alone. If storm intensity has been variable, as in mountainous terrain, the correlation will require the use of some index of storm intensity as a third variable. Figure 168 illustrates a multiple correlation of that type where the independent variables

used were drainage area and maximum 24-hour basinwide precipitation during the storm of December 1964 in north coastal California.

The peak discharges estimated by the above method should be used only as a guide in extrapolating the rating curve at a gaging station. The basic principles underlying the extrapolation of logarithmic rating curves are not to be violated to accommodate peak-discharge values that are relatively gross estimates, but the estimated discharges should properly be given consideration in the extrapolation process.

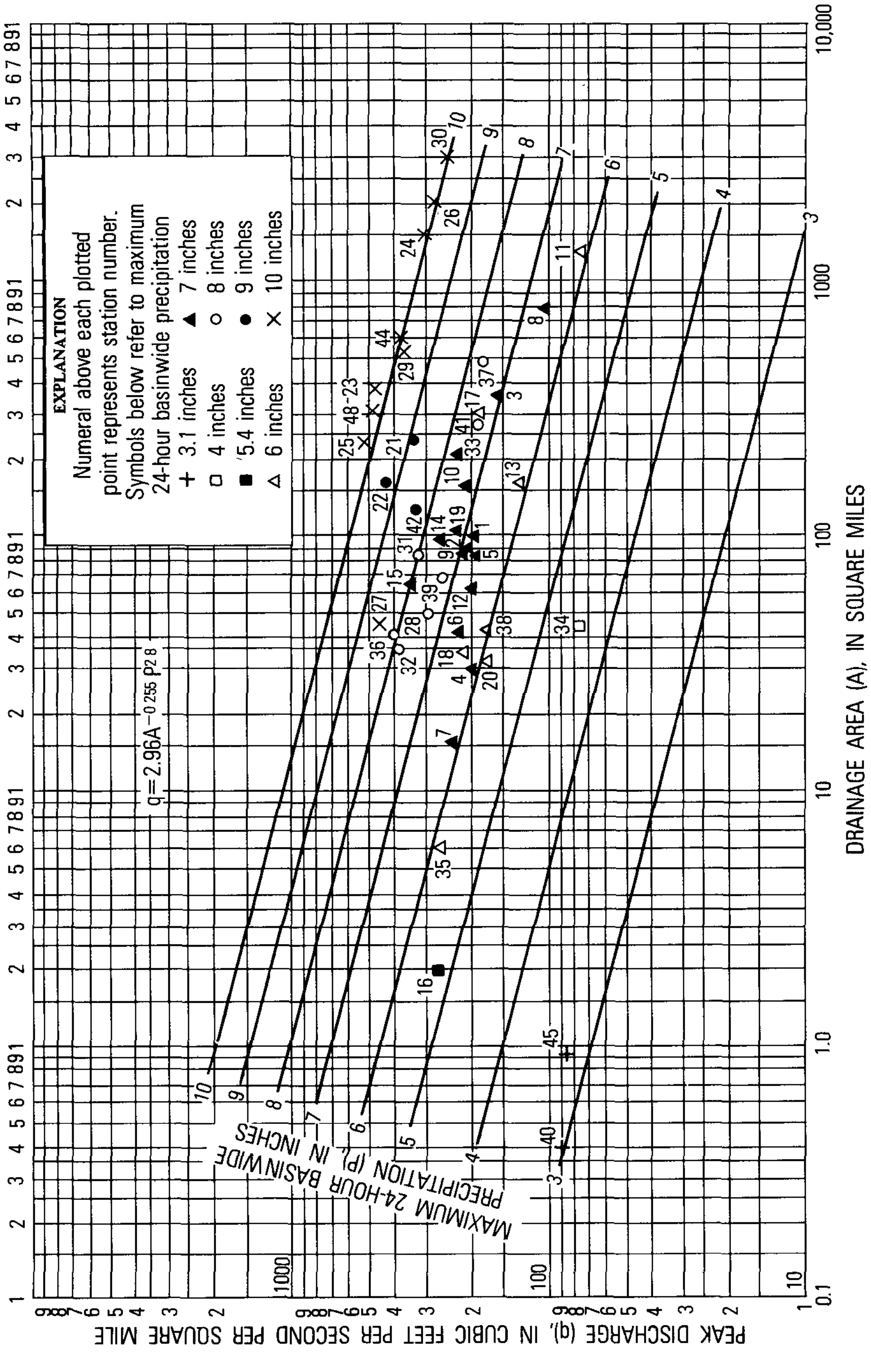
STEP-BACKWATER METHOD

The step-backwater method is a technique in which water-surface profiles for selected discharges are computed by successive approximations. The computations start at a cross section where the stage-discharge relation is known or assumed, and they proceed to the gage site whose rating requires extrapolation. If flow is in the subcritical regime, as it usually is in natural streams, the computations must proceed in the upstream direction; computations proceed in the downstream direction if flow is in the supercritical regime. In the discussion that follows, the usual situation of subcritical flow will be assumed.

Under conditions of subcritical flow, water-surface profiles converge upstream to a common profile. For example, the stage for a given discharge at a gated dam may have a wide range of values depending on the position of the gates. At a gaging station far enough upstream to be beyond the influence of the dam, the stage for that discharge will be unaffected by the gate operations. Consequently, when the water-surface profile is computed for a given discharge in the reach between the dam and the gaging station, the segment of the computed profile in the vicinity of the gage will be unaffected by the value of stage that exists at the dam. However, it will be necessary that the computations start at the dam and proceed upstream, subreach by subreach (in "steps"). It follows, therefore, that if an initial cross section for the computation of the water-surface profile is selected far enough downstream from the gage, the computed water-surface elevation at the gage, corresponding to any given discharge, will have a single value regardless of the stage selected for the initial site.

A guide for determining the required distance (L) between gaging station and initial section is found in the dimensionless graph in figure 169. The graph, (Bailey and Ray, 1966), has for its equation,

FIGURE 168.—Relation of peak discharge to drainage area and maximum 24-hour basinwide precipitation in north coastal California, December 1964.


EXPLANATION
Numeral above each plotted point represents station number.
Symbols below refer to maximum 24-hour basinwide precipitation
+ 3.1 inches
□ 4 inches
■ 5.4 inches
△ 6 inches
▲ 7 inches
○ 8 inches
● 9 inches
× 10 inches
q=2.96A^-0.255 P^2.8
MAXIMUM 24-HOUR BASINWIDE PRECIPITATION (P), IN INCHES
DRAINAGE AREA (A), IN SQUARE MILES
PEAK DISCHARGE (q), IN CUBIC FEET PER SECOND PER SQUARE MILE

$$\frac{LS_o}{\bar{d}} = 0.86-0.64\left(\frac{S_oC^2}{g}\right) \tag{69}$$

where

L is the distance required for convergence,
S_0 is bed slope,
$\bar{d}$ is mean depth for the smallest discharge to be considered,
g is the acceleration of gravity, and
C is the Chezy coefficient.

If a rated cross section is available downstream from the gage, that cross section would be used as the initial section, of course, and there would be no need to be concerned with the above computation of L.

After the initial site is selected, the next step is to divide the study reach, that is, the reach between the initial section and the gaging station, into subreaches. That is done by selecting cross sections where major breaks in the high-water profile would be expected to occur because of changes in channel geometry or roughness. Those cross sections are the end sections of the subreaches. The cross sections are surveyed and roughness coefficients are selected for each subreach. That completes the field work for the study.

The first step in the computations is to select a discharge, Q, for study, and obtain a stage at the initial section for use with that value of discharge. If the initial section is a rated cross section, that stage will be known. If the initial section is not a rated cross section, an estimated stage there is computed from the estimated mean depth ($\bar{d}$) for discharge Q; $\bar{d}$ in turn is estimated by cut-and-try computations from a variation of the Chezy equation,

$$\bar{d}=\frac{Q^2}{(CA)^2S_o} \tag{70}$$

where

C is the Chezy coefficient,
A is the cross-sectional area corresponding to $\bar{d}$, and
S_0 is the bed slope (or water-surface slope).

Step-backwater computations are then applied to the subreach farthest downstream. We have a known or estimated stage at the downstream cross section for the value of Q being considered; the object of the computations is to determine the stage at the upstream end of the subreach that is compatible with that value of Q. The computation for each subreach is based on a steady-flow equation, such as the Chezy or Manning equation, after the equation has been modified for nonuniformity in the subreach by use of the difference in velocity head at the end cross sections. (See section in chapter 9 titled,

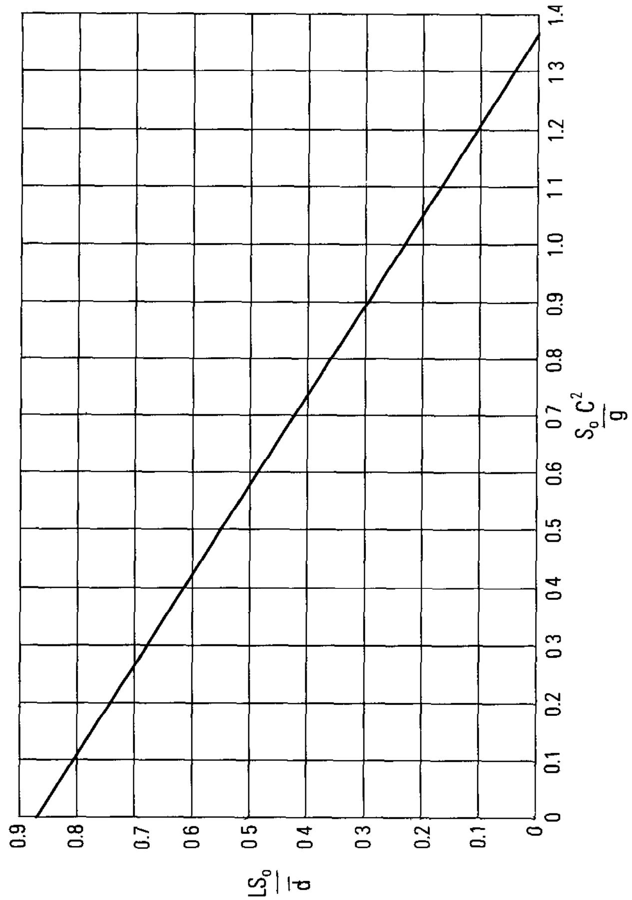


FIGURE 169.—Dimensionless relation for determining distance required for backwater profiles to converge.

"Slope-Area Method.") It will be recalled that the Chezy equation is related to the Manning equation by the formulas,

$$C = \frac{1.486}{n} R^{1/6} \text{ (English units)} \tag{71}$$

or

$$C = \frac{1}{n} R^{1/6} \text{ (metric units)} \tag{71a}$$

where n is the Manning roughness coefficient and R is the hydraulic radius.

By shifting terms in the modified Chezy equation, the following equation is obtained for the difference in water-surface elevation (Δh) between the upstream (subscript 1) and downstream (subscript 2) cross sections.

$$\Delta h = h_1 - h_2 = \frac{(\Delta L) V_1 V_2}{C^2 R_1^{1/2} R_2^{1/2}} + \frac{(\alpha_2 V_2^2 - \alpha_1 V_1^2)}{2g} (1+k), \tag{72}$$

where

h is stage;
ΔL is the length of the subreach;
V is average velocity in the cross section;
g is the acceleration of gravity;
k is a constant whose value is zero when $\alpha_2 V_2^2 > \alpha_1 V_1^2$; and whose value is 0.5 when $\alpha_2 V_2^2 < \alpha_1 V_1^2$; and
α is the velocity-head coefficient whose value is dependent on the velocity distribution in the cross section.

As for α, in many countries its value is assumed to be 1.1; in the U.S.A. its value is assumed to be 1.0 for cross sections of simple shape, but its value is computed for cross sections of complex shape that require subdivision. The equation used for that purpose is

$$\alpha = \frac{\Sigma\, (K_i^3 / a_i^2)}{K_T^3 / A_T^2}, \tag{73}$$

where the subscript i refers to the conveyance (K) or area (a) of the individual subsections, and the subscript T refers to the conveyance (K) or area (A) of the entire cross section. With regard to conveyance, K,

$$K_i = C a_i R_i^{1/2}, \text{ and } K_T = \Sigma K_i$$

We return to our computations for the downstream subreach. A trial value of stage for discharge Q is selected for the upstream cross

section, and values of A, V, and R are computed for the upstream and downstream cross sections. Those values are substituted in equations 72 and 73 and after solving for Δh, the computed value of Δh is compared with the difference between the trial value of stage at the upstream cross section and the known or assumed stage at the downstream cross section. Seldom will the two values agree after a single trial computation; if they do not agree, a second trial value of stage is selected for the upstream cross section. The computational procedure is repeated and the newly computed value of Δh is compared with its corresponding trial value. The computations are repeated as many times as are necessary to obtain agreement between the computed Δh and the difference between the trial stage at the upstream cross section and the known or assumed stage at the downstream cross section.

After a satisfactory value of stage has been determined for the upstream cross section, that cross section becomes the downstream cross section for the next subreach upstream. Computations similar to those described in the preceding paragraph are repeated for that subreach, and for each succeeding subreach, to provide a water-surface profile extending to the gaging station that is applicable to the discharge value (Q) being studied.

If the stage corresponding to discharge Q at the initial cross section was known, the stage computed for the gage is satisfactory. If the stage at the initial cross section was estimated from equation 70, it is necessary to repeat the above computations twice using other values of stage at the initial cross section for the same discharge Q. That is done to assure convergence of the water-surface profiles at the gage. The computations are repeated, first using an initial stage about 0.5 to 1.0 ft (0.15 to 0.30 m) higher than that originally used, and then using an initial stage about 0.5 to 1.0 ft lower than that originally used. All three sets of computations for discharge should result in almost identical values of stage at the gaging station for discharge Q. If they do not, the initial cross section for the step-backwater computation should be moved farther downstream, and all computations previously described must be repeated. If the three sets of computations give water-surface profiles that converge at a common stage at the gage, the entire procedure is repeated for other discharges until enough data are obtained to define the high-water rating for the gaging station.

From the preceding discussion it should be evident that the computations will be expedited if, in a preliminary step, the three relations of stage versus area (A), hydraulic radius (R), and conveyance (K), are computed for each cross section. Even then, the computations will be laborious and the use of a digital computer is therefore recommended.

The step-backwater method can be used to prepare a preliminary rating for a gaging station before a single discharge measurement is made. A smooth curve is fitted to the logarithmic plot of the discharge values that are studied. The preliminary rating can be revised, as necessary, when subsequent discharge measurements indicate the need for such revision. If the step-backwater method is used to define the high-water end of an existing rating curve, the discharge values investigated should include one or more of the highest discharges previously measured. By doing so, selected roughness coefficients can be verified, or can be modified so that step-backwater computations for the measured discharges provide stages at the gaging station that are in agreement with those observed. The computations for the high-water end of the rating can then be made with more confidence, in the knowledge that reasonable values of the roughness coefficients are being used. There will also be assurance of continuity between the defined lower part of the rating and the computed upper part.

FLOOD ROUTING

Flood-routing techniques may be used to test and improve the overall consistency of records of discharge during major floods in a river basin. The number of direct observations of discharge during such flood periods is generally limited by the short duration of the flood and the inaccessibility of certain stream sites. Through the use of flood-routing techniques, all observations of discharge and other hydrologic events in a river basin may be combined and used to evaluate the discharge hydrograph at a single site. The resulting discharge hydrograph can then be used with the stage hydrograph for that gage site to construct the stage-discharge relation for the site; or, if only a peak stage is available at the site, the peak stage may be used with the peak discharge computed for the hydrograph to provide the end point for a rating-curve extrapolation.

Flood-routing techniques, of which there are many, are based on the principle of the conservation of mass—inflow plus or minus change in storage equals outflow. It is beyond the scope of a stream-gaging manual to treat the subject of flood routing; it is discussed in most standard hydrology texts (for example, Linsley, Kohler, and Paulhus, 1949, p. 485–541).

SHIFTS IN THE DISCHARGE RATING

Shifts in the discharge rating reflect the fact that stage-discharge relations are not permanent but vary from time to time, either gradually or abruptly, because of changes in the physical features that form the control for the station. If a specific change in the rating

stabilizes to the extent of lasting for more than a month or two, a new rating curve is usually prepared for the period of time during which the new stage-discharge relation is effective. If the effective period of a specific rating change is of shorter duration, the original rating curve is usually kept in effect, but during that period shifts or adjustments are applied to the recorded stage, so that the "new" discharge corresponding to a recorded stage is equal to the discharge from the original rating that corresponds to the adjusted stage. For example, assume that vegetal growth on the control has shifted the rating curve to the left (minus shift), so that in a particular range of discharge, stages are 0.05 ft higher than they originally had been. To obtain the discharge corresponding to a recorded stage of, say, 1.30 ft, the original rating is entered with a stage of 1.25 ft (1.30−0.05) and the corresponding discharge is read. The period of time during which such stage adjustments are used is known as a period of shifting control.

Frequent discharge measurements should be made during a period of shifting control to define the stage-discharge relation, or magnitude(s) of shifts, during that period. However, even with infrequent discharge measurements the stage-discharge relation can be estimated during the period of shifting control if the few available measurements are supplemented with a knowledge of shifting-control behavior. This section of the report discusses such behavior. That part of the discussion that deals with channel-control shifts does not include alluvial channels, such as sand channels, whose boundaries change almost continuously; sand channels are discussed in the section titled, "Sand-Channel Streams."

The formation of ice in the stream and on section controls causes shifts in the discharge rating, but ice effect is not discussed here; it is discussed separately in the section titled. "Effect of Ice Formation on Discharge Ratings."

DETECTION OF SHIFTS IN THE RATING

Stage-discharge relations are usually subject to minor random fluctuations resulting from the dynamic force of moving water, and because it is virtually impossible to sort out those minor fluctuations, a rating curve that averages the measured discharges within close limits is considered adequate. Furthermore, it is recognized that discharge measurements are not error-free, and consequently an average curve drawn to fit a group of measurements is probably more accurate than any single measurement that is used to define the average curve. If a group of consecutive measurements subsequently plot to the right or left of the average rating curve, it is usually clearly evident that a shift in the rating has occurred. (An exception to that

statement occurs where the rating curve is poorly defined or undefined in the range of discharge covered by the subsequent measurements; in that circumstance the indication is that the original rating curve was in error and requires revision.) If, however, only one or two measurements depart significantly from a defined segment of the rating curve, there may be no unanimity of opinion on whether a shift in the rating has actually occurred, or whether the departure of the measurement (s) results from random error that is to be expected occasionally in measurements.

Two schools of thought exist with regard to identifying periods of shifting control. In the U.S.A. and many other countries, a pragmatic approach is taken that is based on certain guidelines and on the judgment of the analyst. In other countries, notably the United Kingdom, the approach used is based on statistical theory. (It is reiterated that the discussion that follows excludes the constantly shifting alluvial channels that are discussed in the section on "Sand-Channel Streams.")

In the U.S.A., if the random departure of a discharge measurement from a defined segment of the rating curve is within ±5 percent of the discharge value indicated by the rating, the measurement is considered to be a verification of the rating curve. If several consecutive measurements meet the 5-percent criterion, but they all plot on the same side of the defined segment of the rating curve, they may be considered to define a period of shifting control. It should be mentioned that when a discharge measurement is made, the measurement is computed before the hydrographer leaves the gaging station and the result is plotted on a rating curve that shows all previous discharge measurements. If the discharge measurement does not check a defined segment of the rating curve by 5 percent or less, or if the discharge measurement does not check the trend of departures shown by recent measurements, the hydrographer is normally expected to make a second discharge measurement to check his original measurement. However, at many stations the 5-percent criterion may be too stringent for low-flow measurements because of control insensitivity. At those installations departures in excess of 5 percent are generally acceptable if the indicated shift does not exceed 0.02 ft.

In making a check measurement, the possibility of systematic error is eliminated by changing the measurement conditions as much as possible. The meter and stopwatch are changed, or the stopwatch is checked against the movement of the second hand of a standard watch. If the measurements are being made from a bridge, boat, or cableway, the measurement verticals are changed by measuring at verticals between those originally used; if wading measurements are being made, a new measurement section is sought, or the meas-

urement verticals in the original section are changed. If the check measurement checks the original rating curve or current rating trend by 5 percent or less, the original discharge measurement will be given no consideration in the rating although it is still entered in the records. If the check measurement checks, by 5 percent or less, the original discharge measurement or the trend of that measurement if the stage has changed, the two measurements are considered to be reliable evidence of a new shift in the stage-discharge relation. If the check measurement fails to check anything that has gone before, a second check measurement is made and the most consistent two of the three measurements are used for rating analysis. The need for a second check measurement is a rarity, but it may possibly occur.

Thus, in the U.S.A., a single discharge measurement and its check measurement, even if unsupported by later measurements, may mark a period of shifting control. The engineer who analyzes the rating does have the responsibility of explaining the reason for the short-lived shift—it can often be explained as having started as a result of fill (or scour) on a preceding stream rise and as having ended as a result of scour (or fill) on the recession or on a following rise.

In the United Kingdom, the analysis of the rating starts in the usual way; the chronologically numbered discharge measurements are plotted on logarithmic graph paper and are fitted by eye with a smooth curve. Where compound controls exist, there may be one or more points of inflection in the curve. In the statistical analysis that follows, each segment of the rating curve between inflection points is treated separately. The standard deviation (S_D) of the plotted points, in percent, is computed for each segment, using the standard statistical equation,

$$S_D = \sqrt{\frac{\Sigma d^2}{N-1}}\ , \tag{74}$$

where

d is the departure of a discharge measurement from the rating curve, in percent, and

N is the number of measurements used to define the segment of the rating curve.

Use of the standard deviation (S_D) in detecting rating shifts is explained as follows in ISO Recommendation R 1100 (1969, p. 15). On the average, 19 out of 20 measurements should depart from the particular segment of the rating curve by no more than $2S_D$ percent. Any subsequent discharge measurement that departs by a much greater percentage—say, $3S_D$ percent—can be regarded as the result of faulty measurement, except in those cases where two or more consecutive measurements, either chronologically or over a range of stage, appear to be well on one side of the $\pm 2S_D$ limit. Where that occurs, a change

in the stage-discharge relation is required—either in the form of a reconstruction of the original relation using the additional discharge measurements, or in the form of a new stage-discharge relation because a shift in the control is indicated.

In the United Kingdom, additional statistical tests are given the rating to assure that : (1) the discharge measurements show no preponderance of either plus or minus departures from the rating curve; (2) the number of "runs" of successive plus or minus departures from the rating, examined in ascending order of stage, are neither excessively large nor excessively small, and (3) the average percentage departure of all measurements from the rating curve does not differ significantly from zero.

In the U.S.A. the above statistical approach is not favored for several reasons. First, it is felt that the limiting criteria of $2S_D$ percent will usually exceed the 5 percent criteria preferred in the U.S.A. Second, any statistical approach gives equal weight to all discharge measurements used in the analysis. In the U.S.A. hydrographers rate the probable accuracy of the measurements they make on the basis of measuring conditions at the time, without reference to how closely the measurements plot on the rating curve. The feeling in the U.S.A. is that more weight in the analysis should be given to measurements rated good to excellent than to measurements rated fair to poor. Third, while it is agreed that in general an average curve drawn to fit a group of measurements is probably more accurate than any single measurement that is used to define the average curve, it is also felt in the U.S.A. that any subsequent measurement that is verified by a check measurement is more accurate than the rating-curve value of discharge, particularly at a station that is historically known to have rating-curve shifts.

RATING SHIFTS FOR ARTIFICIAL CONTROLS

Weirs.—Artificial controls are not subject to scour and fill by high flows, but the streambed immediately upstream from the weir may be so affected. If scour occurs in the pool formed by the weir, the pool is deepened and velocity of approach decreases. The net result is a smaller discharge for a given stage than under pre-scour conditions; that is, the rating curve for the period of scour will shift to the left of the rating curve for pre-scour conditions. The converse occurs if the weir pool has been subjected to deposition or fill.

The effect of such scour and fill on the stage-discharge relation is usually relatively minor, and usually can be expressed by a parallel shift of most of the section-control portion of the rating curve that is plotted as a straight line on logarithmic graph paper. If only a single discharge measurement is available for defining the parallel shift

curve, the shift curve is drawn to pass through that measurement. If more than one discharge measurement is available, and there is no evidence of a progressive rating shift with time, the parallel shift curve is drawn to average the discharge measurements. If the discharge measurements indicate a progressive rating shift with time, shifts are prorated with time. However, what may appear to be a gradually progressive shift, may in fact be several discrete shifts caused by individual peak flows whose occurrences are not widely separated in time. The shift in stage to be applied to recorded gage heights during the period of shifting control is determined from the vertical spacing between the original rating curve and the shift curve.

The shift, if attributable to fill, is considered to start after the peak discharge of a stream rise that preceded the first of the variant discharge measurements. Shift adjustments are therefore started on the recession of that rise. The shift, if attributable to scour, is considered to start during the high stages of a stream rise that preceded the first of the variant discharge measurements. Because those high stages generally occur when the section control is "drowned out" by channel control, the shift in the section-control segment of the rating is again commonly first applied after the peak discharge of the rise. The shifts are ended on a stream rise that follows the last variant discharge measurement, using the general principle that scour in the gage pool usually occurs during high stages and fill usually occurs during the recession of a stream rise.

The parallel shift discussed in a preceding paragraph requires some elaboration. A parallel shift of the rating curve on logarithmic graph paper indicates that for all stages the discharge changes by a fixed percentage, and that the difference in stage between the two lines increases with stage. However, it is not quite true that the discharge changes by a fixed percentage when the weir pool has scoured or filled. At extremely low flows there will be no effect because velocity of approach is negligible; that section of the original rating has a break in slope (see fig. 146; $G=1.3$ ft), and the lower end of the parallel shift curve above the break in slope should be warped to join the extreme low-water curve. The effect of scour or fill on the percentage change in discharge increases rapidly with stage to a maximum value and then slowly decreases to a percent change that does not differ greatly from the maximum percentage. The parallel shift drawn through the available discharge measurement(s) will adequately fit those relatively large percentage changes in discharge at the higher stages; the warped section of the shift curve at the lower stages will adequately fit the rapidly increasing percentage change in discharge at those lower stages. Figure 170 illustrates the above discussion; the

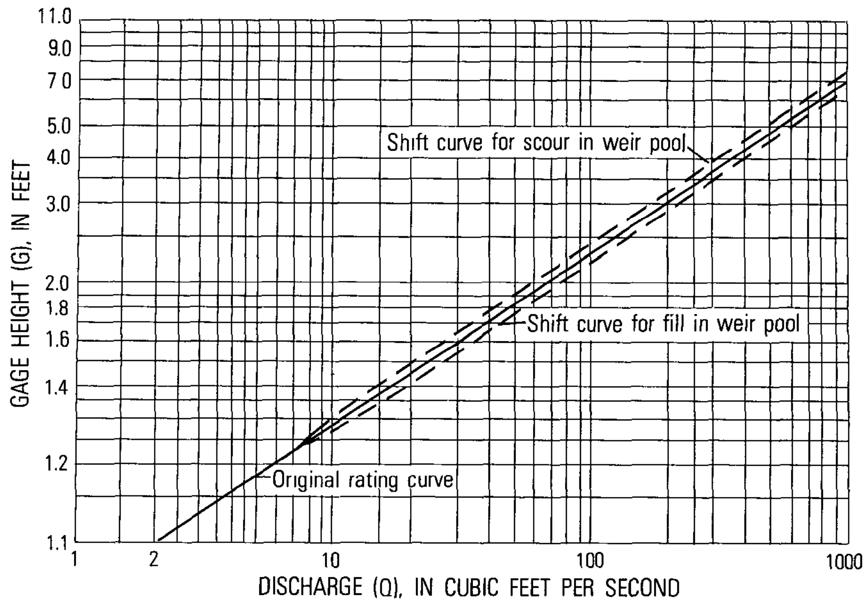


FIGURE 170.—Rating curve for hypothetical rectangular thin-plate weir, with shift curves for scour and fill in the weir pool.

original rating curve shown is a reproduction of that given in figure 146.

It has been mentioned frequently in this manual that section controls are usually submerged at high stages as a result of channel control becoming effective. The parallel shift curve described above should be extended to the stage where it either intersects the actual rating for channel control (in the case of scour in the weir pool) or can be warped into the rating for channel control (in the case of fill in the weir pool). If a shift has occurred simultaneously in the channel control (see section titled, "Rating Shifts for Channel Control"), the shift curves for the section-control and channel-control segments of the rating are drawn to form a continuous curve.

Up to now we have discussed changes in the velocity of approach that are caused only by scour and fill in the weir pool. The velocity of approach may also be affected by aquatic vegetation growing in the weir pool. Usually such an occurrence will reduce the velocity of approach by greatly increasing the friction loss, and the rating curve will shift to the left. However, the shift will not be abrupt, but will gradually increase as the growing season progresses. The aquatic growth in the pool may also encroach on the weir to the extent that the effective length (b) of the weir is reduced. The effect of a reduction in effective length of the weir is a parallel shift of the rating to the left

when plotted on logarithmic graph paper. At all stages the discharge will be reduced by a percentage that is equal to the percentage change in effective length of the weir. The shift will either decrease gradually as the vegetation dies in the dormant season, or the shift may terminate abruptly if the vegetation is washed out by a stream rise.

Moss or algal growth may sometimes attach itself to a weir crest and thereby reduce the head on the weir for any given gage height (G). The head will be reduced by a constant value that is equal to the thickness of the growth. In other words, in the equation, head$=G-e$, the value of e is increased by the thickness of the growth. The reduction in head causes the rating to shift to the left, it being displaced vertically by an amount equal to the thickness of the growth. If the shift rating is plotted on rectangular-coordinate graph paper, it will be parallel to the original rating. If the shift rating is plotted on logarithmic graph paper, it will be a curve that is concave upward and asymptotic to the original linear rating curve at the higher stages. The growth of algae or moss on the weir should be removed with a wire brush before it becomes heavy enough to affect the stage-discharge relation. The effect of the shift caused by the algal growth disappears during stages when channel control becomes effective.

Flumes.—Shifts in the stage-discharge relation for flumes are most commonly caused by changes in the approach section—either in the channel immediately upstream from the flume or in the contracting section of the flume upstream from the throat. In either event the change is caused by the deposition of rocks and cobbles that are too large to pass through the flume; the flume is self-cleaning with regard to sediment of smaller size. Manual removal of the large debris should restore the original discharge rating of the flume.

The deposition of rocks and debris upstream from the flume may divert most of the flow to the gage-side of the flume and the build-up of water at the gage will result in a shift of the discharge rating to the left. Conversely, if most of the flow is diverted to the side of the flume opposite the gage, the discharge rating will shift to the right. In the above situation, the shift curve is usually drawn parallel to the original rating curve on logarithmic graph paper in much the same manner as was described earlier for shifts resulting from scour and fill in the pool behind the weir.

If rocks and cobbles are deposited at the entrance to the throat of the flume, they will cause the discharge rating to shift to the left because the stage at the gage will be raised higher than normal for any given discharge. A similar backwater effect will result from the growth of algae at the entrance to the throat.

The backwater effect, or decrease in head for a given gage height

caused by deposition or algal growth at the entrance to the throat of the flume, has the effect of increasing the value of e in a linear logarithmic plot of the rating. The shift rating on logarithmic graph paper will be a curve that is concave upward and asymptotic to the original linear rating curve at the higher stages. The deposition of rocks and debris will be associated with a high-water event; the growth of algae will increase gradually with time.

Large rocks driven by high-velocity flow through the flume may erode the walls and floor of a concrete flume. The resulting increase in roughness and decrease in elevation of the concrete may cause shifts in the stage-discharge relation. The two effects tend to be compensating; an increase in roughness will shift the discharge rating to the left, and a decrease in elevation of the concrete surface will shift the discharge rating to the right. However, the latter effect usually predominates, particularly in supercritical-flow flumes.

RATING SHIFTS FOR NATURAL SECTION CONTROLS

The primary cause of changes in natural section controls is the high velocity associated with high discharge. Of those controls, a rock ledge outcrop will be unaffected by high velocities, but boulder, gravel, and sand-bar riffles are likely to shift, boulder riffles being the most resistant to movement and sand bars the least resistant. After a flood the riffles are often altered so drastically as to bear no resemblance to their pre-flood state, and a new stage-discharge relation must be defined. Minor stream rises usually move and sort the materials composing the riffle, and from the standpoint of the rating curve, the greatest effect is usually a change in the gage height of effective zero flow (e). The shift curve ideally should be defined by current-meter discharge measurements. However, if only one or two measurements are available for the purpose, they are examined and the gage-height shift that they indicate is applied to the section-control segment of the original rating curve. If the shift rating is plotted on rectangular paper, it will tend to be parallel to the original rating. The extreme low-water end of the curve can be extrapolated to the actual point of zero flow, as determined in the field when low-water discharge measurements are made. If the shift rating is plotted on logarithmic graph paper, it will be a curve that is either concave upward or downward, depending on whether the shift is to the left (increase in e) or the right (decrease in e). The shift curve will tend to be asymptotic to the linear rating at the higher stages of section control, but its precise slope in the range of stage where channel control is beginning to exert an effect, will depend on whether or not a shift has occurred in the channel-control segment of the rating curve.

(See section titled, "Rating Shifts for Channel Control.")

Vegetal growth in the approach channel of the control or on the control itself will affect the stage-discharge relation in the manner described on preceding pages, where rating shifts for weirs were discussed. Aquatic vegetation in the approach channel will affect the velocity of approach, and if the channel growth encroaches on the control, it may reduce the effective length of the control. Aquatic growth on the control itself will reduce the discharge corresponding to any given stage by reducing the head on the control and increasing the resistance to flow, and (or) by reducing the effective length of the control. The shifts associated with vegetal growth are cyclic and therefore change with time. The growth increases as the growing season progresses and declines during the dormant season, but shifts may terminate abruptly if the vegetation is washed out by a stream rise.

In temperate climates, accumulations of water-logged fallen leaves on section controls each autumn clog the interstices and raise the effective elevation of all section controls. The effect of an increase in the gage height of effective zero flow (e) is explained on a preceding page in the discussion of moss and algal growth on weirs. The build-up of water-logged leaves is progressive starting with the first killing frost (usually in October in the Northern Hemisphere) and reaching a maximum when the trees are bare of leaves. The first ensuing stream rise of any significance usually clears the control of fallen leaves.

Two other causes of backwater (increased gage height for a given discharge), unassociated with hydrologic events, also warrant discussion. Vacationers in the summer often use the gage pool for swimming, and they will often pile rocks on the control to create a deeper pool. This change in the height of the control manifests itself in the record of stage as an abrupt increase in gage height, usually during a rainless period, without any corresponding decline in stage that would be associated with the passage of a stream rise. The abrupt rise in stage fixes the time when the shift in the rating occurred; the magnitude of the change in stage is a measure of the change in the value of e. In some regions another cause of backwater is the construction of dams by beavers. These dams are built of boughs, logs, stones, and mud to create a pool that is part of the beavers' habitats. Again, the time of occurrence and the effect on the stage of the stream can be detected in the gage-height record which will show a gradual rise, usually over a period of a few days as the dam is being built, without the corresponding decline in stage that would be associated with a stream rise. The beaver dams usually remain in place until washed out by a high discharge.

RATING SHIFTS FOR CHANNEL CONTROL

As mentioned earlier, most natural streams have compound controls—section control for low-stages and channel control for high stages. The shifts in section control that were described on the preceding pages are commonly accompanied by shifts in channel control.

The most common cause of a shifting channel control, in a relatively stable channel, is scour or fill of the streambed caused by high-velocity flow. The scour usually occurs during a stream rise and fill usually occurs on the recession, but that statement is an oversimplification of the highly complex process of sediment transport. The degree of scour in a reach is dependent not only on the magnitude of the discharge and velocity, but also on the sediment load coming into the reach. On some streams it has been found that when scour is occurring in a pool at a meander bend there is simultaneous filling on the bar or riffle at the crossover, or point of inflection between successive meander bends; on other streams scour has been found to take place simultaneously through relatively long reaches of channel, both in pools and over bars. A further complication is the fact that the length of channel that is effective as a control is not constant, but increases with discharge.

From the preceding discussion it should be apparent that there is no really satisfactory substitute for discharge measurements in defining shifts in the channel-control segment of the rating; of particular importance are measurements made at or near the peak stage that occurs during periods of shifting control. However, in the usual situation a few (or less) measurements made at medium stages are the only ones available for analyzing channel-control shifts, and the shifts must be extrapolated to peak stages. The assumptions usually made in the rating analysis are those discussed below. The results are accepted unless they are shown to be invalid by a determination of peak discharge as described in chapter 9, or are shown to be invalid by use of one or more of the methods of rating-curve extrapolation as described in the section on "High-flow Extrapolation."

If a single predominantly large stream rise occurred shortly before the first measurement that indicated a shift, the shifts are assumed to have been caused solely by that rise. If more than one large stream rise occurred shortly before the first shift measurement, the shift curve may be prorated between rises. For example, if two rises of almost equal magnitude occurred just before the first shift measurement, and if the shift curve indicates a shift of 0.30 ft at a given stage, the shift to be used during the period between the two rises would be 0.15 ft at the given stage. It is often helpful to plot the shifts indicated by the discharge measurements against the observed stage of those measurements to obtain the trend of the shifts.

The pattern of scour and fill in the control channel determines whether the shift will increase with stage, decrease with stage, or be relatively constant at all stages. Figure 171 (graph *A*) illustrates a common situation where the shifts, either plus in the case of scour or minus in the case of fill, increase in absolute value as stage decreases. The highest value of the shift is assumed to be only slightly greater than the maximum value observed in order to avoid "overcorrecting" the original rating. Graph *B* of figure 171 shows the shift ratings corresponding to measurements nos. 1 and 2. The ratings have been plotted on rectangular-coordinate graph paper because the shifts are more easily visualized, at least by the inexperienced hydrographer, on that type of plotting paper. The stage-shift curve is usually plotted on rectangular-coordinate paper, but the rating curves are usually plotted on logarithmic graph paper. On logarithmic paper the shift curves in this example would converge more rapidly toward the original rating curve at high stages. The shift curves at low stages would be shaped to join smoothly with the shift curve for section control. The period for applying the shifts would be terminated on the stream rise following the last shift measurement; the original rating would be used on the recession from that rise.

In analyzing shifts there is no substitute for experience with a given stream because the shift pattern can often be interpreted logically in more than one way. For example, refer to the shift curve for channel fill in graph *B* of figure 171. Assume that measurements nos. 1 and 2 were made on a stream recession, and the measurement no. 1 was made a few days before measurement no. 2. Measurement no. 2 shows the effect of greater fill than measurement no. 1; fill usually occurs on a recession; therefore it is possible that the shifts should have been made to vary with time or to vary with time and stage, rather than with stage alone as shown in figure 171*A*. In the absence of additional knowledge the simplest interpretation is generally made, as was done here. Given more discharge measurements or a better knowledge of the behavior of the particular stream, a more accurate analysis can be made.

Figure 172 (graph *A*) illustrates a less common situation where the shifts increase as stage increases. Again the highest value of shift is assumed to be only slightly greater than the maximum value observed in order to prevent "overcorrecting" the original rating. Graph *B* of figure 172 shows the shift ratings corresponding to measurements nos. 1 and 2. The period for applying shifting-control corrections would be terminated on the stream rise following the last shift measurement; the original rating would be used on the rising limb of that rise. As in the case of figure 171, in the absence of additional knowledge, more than one interpretation can be given to

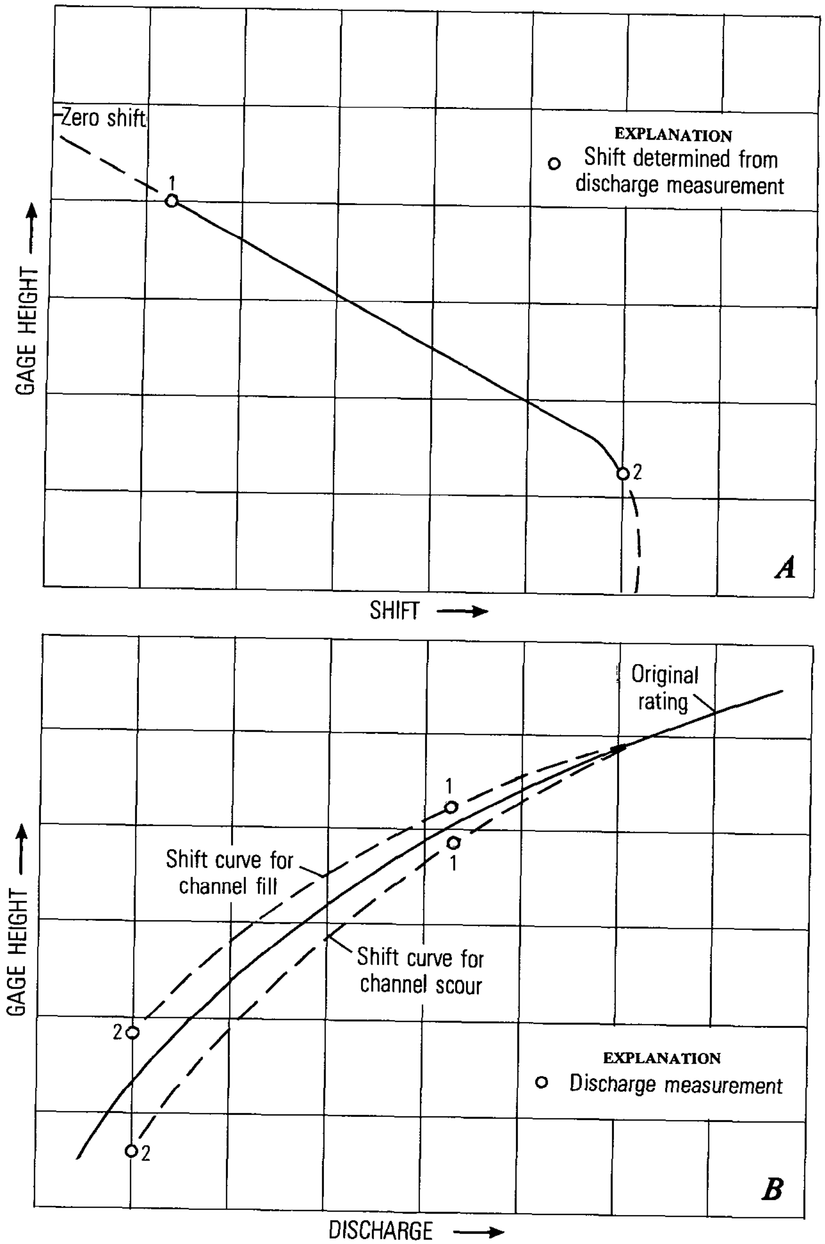


FIGURE 171.—First example of a stage-shift relation and the corresponding stage-discharge relation caused by scour or fill in the control channel.

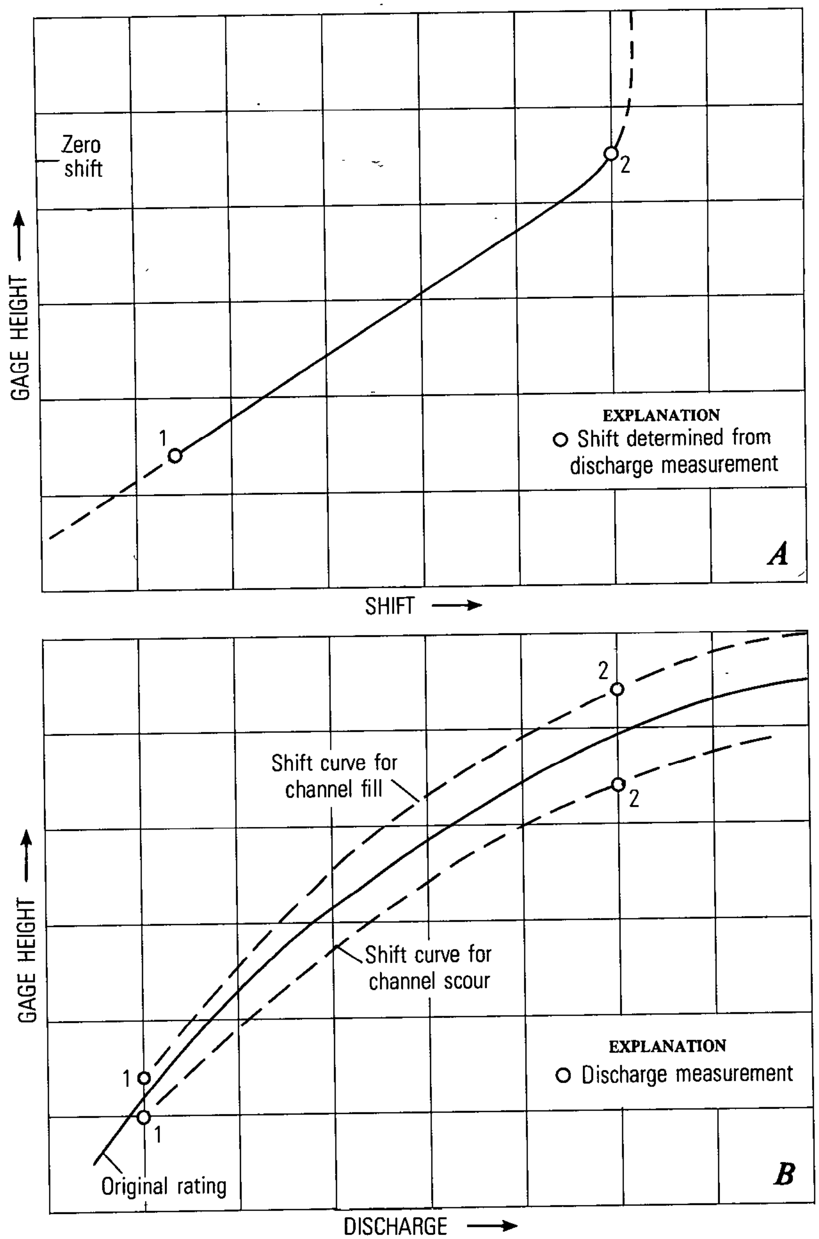


FIGURE 172.—Second example of a stage-shift relation and the corresponding stage-discharge relation caused by scour or fill in the control channel.

shifts shown by measurements nos. 1 and 2, depending on the relative times when the measurements were made and the fact that scour generally occurs on stream rises and fill generally occurs on stream recessions.

If there had been an additional major rise, one that occurred between the pairs of measurements shown in figures 171 and 172, other courses of action would be available. If the analyst had no additional data on which to base a judgment, he could assume that two separate shift events occurred, each attributable to the rise that preceded a discharge measurement. For each shift period, he could use a constant shift, equal to that shown by the discharge measurement made during that shift period. If, however, the analyst has had experience in the past with shifting control at the station caused by scour and fill in the control channel and if that experience had shown that shifts tend to vary with stage, another course of action would suggest itself. For each of the stage periods, the analyst could use a stage-shift relation of average shape that passed through the shift value shown by the appropriate discharge measurement. The above discussion would also apply to the situation of a single shift period and the availability of only a single discharge measurement made during that period. (It is assumed that the single discharge measurement would be accompanied by a check measurement to verify its accuracy, as discussed in the section on "Detection of Shifts in the Rating.")

If, during a period of shifting control, several measurements had been made but few of them could be fitted with a smooth shift curve, it would then be necessary to prorate the shifts with both time and stage, or possibly with time alone, based on the average shape of a stage-shift relation.

As mentioned earlier, scour in the control channel causes a plus shift because depth, and therefore discharge, is increased for a given gage height. Deposition or fill in the control channel causes a minus shift, because depth, and therefore discharge, is decreased for a given gage height. Thus the effect on the discharge of scour or fill in a channel control is opposite to that of scour and fill in a weir pool, which affects only the velocity of approach. Therefore, if a permanent weir is part of a compound control, scour in both the weir pool and in the channel control will cause a minus shift in the rating for section control and a plus shift in the rating for channel control. The converse is true when fill occurs in both the weir pool and the channel control. That situation is compatible with the stage-shift relation shown in figure 172, where a further decrease in stage would change the sign of the shifts. If the section control is a natural riffle, that riffle is likely to scour when the channel scours and fill when the channel fills, a situation that is compatible with the stage-shift relation shown in

figure 171. In any event, the shift curves for low stages of channel control should be shaped to join smoothly with the shift curves for high stages of section control where a compound control exists.

Up to now the discussion of channel-control shifts has been confined to shifts caused by streambed scour and deposition. Shifts may also be caused by changes in the width of the channel. Even in a relatively stable channel the width of the channel may be increased during intense floods by widespread bank-cutting, and in some areas (for example, north coastal California) channel widths may be constricted by widespread landslides that occur when steep streambanks are undercut. In meandering streams changes in channel width occur as point bars are built up by deposition and later eroded by flood flows. The effect of a change in channel width on the stage-discharge relation, unaccompanied by a change in streambed elevation, is to change the discharge, for a given gage height, by a fixed percentage. When the original rating curve for channel control is plotted linearly on logarithmic graph paper, in accordance with the equation,

$$Q = p(G-e)^N, \qquad (53)$$

the value of p increases with an increase in width and decreases with a decrease in width. The shift curve for a change in width alone will therefore plot on logarithmic graph paper as a straight line that is parallel to the original linear rating curve. Under those conditions a single discharge measurement is sufficient for constructing a shift curve for channel control.

When a change in channel width occurs concurrently with a change in streambed elevation, the effects of the two changes are compounded. The resulting shift curve is complex and requires at least several discharge measurements for its definition.

The growth of vegetation in a stream channel will affect the stage-discharge relation by reducing the discharge for a given gage height. The shift rating will therefore plot to the left (minus shift) of the original rating. The vegetation will increase the roughness coefficient of the channel and will tend to constrict the effective or unobstructed width of the channel. Both those factors reduce the value of p in equation 53, and if the changes in roughness coefficient and effective width are unvarying with stage, the shift curve will be parallel to, and to the left of, the original rating curve that has been plotted linearly on logarithmic graph paper. Usually, however the changes are not independent of stage. If the growth consists of aquatic weeds, the weeds will be overtopped and bent over by high water; if the growth consists of alders and willows, the backwater effect will be greater at higher stages when the tree crowns as well as when the tree trunks are submerged. The rating shift caused by channel vege-

tation is, of course, variable with time as the growth spreads and increases in size.

EFFECT OF ICE FORMATION ON DISCHARGE RATINGS

GENERAL

The formation of ice in stream channels or on section controls affects the stage-discharge relation by causing backwater that varies in effect with the quantity and nature of the ice, as well as with the discharge. Because of the variability of the backwater effect, discharge measurements should be made as frequently as is feasible when the stream is under ice cover, particularly during periods of freeze-up and thaw when flow is highly variable. (Procedures for making measurements under ice cover are described in the section in chapter 5 titled, "Current-Meter Measurements from Ice Cover.") In midwinter the frequency of measurements will depend on climate, accessibility, size of stream, winter runoff characteristics, and required accuracy of the discharge record. As a general rule, two measurements per month is the recommended frequency. At stations below powerplants that carry a variable load, it may be necessary to make two measurements during each winter visit—one at the high stage of the regulated flow and the other at the low stage. The backwater effects may be markedly different at the two stages. In very cold climates where winter ice-cover persists and winter discharge shows a relatively smooth recession, fewer winter measurements are needed than in a climate that promotes the alternate freezing and thawing of river ice.

Knowledge of the three types of ice formation—frazil, anchor, and surface ice—and their possible effects, is helpful in analyzing streamflow records for ice-affected periods. With regard to the type of stage recorder that is preferred for use at ice-affected stations, the graphic recorder, described under that heading in Chapter 4; is by far the best because the recorder graph generally provides dependable evidence of the presence and type of ice formation.

FRAZIL

Frazil is ice in the form of fine elongated needles, thin sheets, or cubical crystals, formed at the surface of turbulent water, as at riffles. The turbulence prevents the ice crystals from coalescing to form sheet ice. The crystals may form in sufficient numbers to give the water a milky appearance. When the crystals float into slower water they come together to coalesce into masses of floating slush. When the current carries slush ice under a sheet of downstream surface ice, the slush may become attached to the underside of the surface ice, thereby increasing the effective depth of the surface ice. Most of the slush that adheres to the surface ice does so near the upstream end of the ice sheet.

Frazil or floating slush has no effect on the stage-discharge relation, but it may interfere with the operation of a current meter. It is particularly troublesome to operators of hydroelectric plants; by adhering and building up on trash racks the ice may effectively reduce the flow to the turbines.

ANCHOR ICE

Anchor ice is an accumulation of spongy ice or slush adhering to the rocks of a streambed. In former years the theory was held that the ice resulted from loss of heat by longwave radiation from streambed to outer space, because anchor ice generally formed on clear cold nights on the streambeds of open reaches of river. This theory has been shown to be invalid, because all of the long-wave radiation that can be lost from the bed of a stream at 0°C would be absorbed in less than 1 cm of water. Anchor ice is now commonly believed to be either (1) frazil that turbulent currents have carried to the streambed where the ice adhered to the rocks, or (2) ice that formed as the result of supercooled water finding nucleating agents on the streambed on which to crystallize. The ice crystals first formed on the rocks act as a nucleating agents for the continued growth of the ice mass.

Regardless of how anchor ice forms, it cannot form or exist when the rocks are warmed by shortwave radiation from the sun which penetrates the water. When the morning sun strikes anchor ice that had formed the night before and the streambed is warmed by the incoming solar radiation, the anchor ice is released and floats to the surface, often carrying small stones that it has picked up from the bed. For the next few hours the stream will be full of floating slush released in a similar manner upstream.

Anchor ice on the streambed or on the section control may build up the bed and (or) control to the extent that a higher than normal stage results from a given discharge. The solid-line graph in figure 173 shows a typical effect of anchor ice on a water-stage recorder graph. The rise starts in late evening or early morning, many hours after the sun has set, when ice begins to adhere to the rocks and raise the water level. By 10 a.m. the sun has warmed the streambed sufficiently to release the ice and the stage starts to fall. The distinguishing feature of the "anchor-ice hump" is that the rise is slow compared to the fall, whereas an actual increase in streamflow would occur in the opposite sequence, or at least the rise would be as rapid as the fall.

The small rises in actual discharge in the late afternoon, shown by the short–dashed lines in figure 173, probably result from water being released from channel storage when anchor ice upstream goes out. There may also be some runoff from the melting of snow and ice during the warmer part of the day.

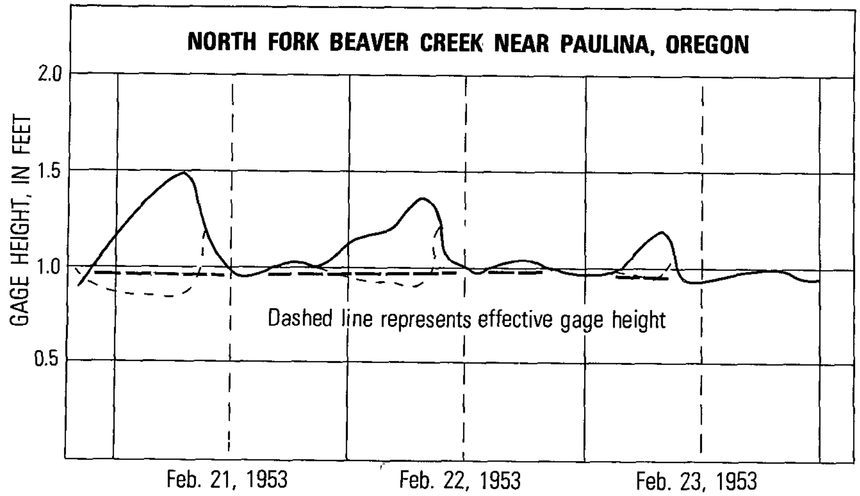


FIGURE 173.—Typical anchor-ice rises. (After Moore, 1957.)

SURFACE ICE

FORMATION OF ICE COVER

As the name implies, surface ice forms on the surface, first as a fringe of shore ice, which then, if the stream is not too turbulent, spreads to form a continuous ice cover spanning the stream from bank to bank. A description of the formation of surface ice follows.

With the onset of cold weather, the water in a stream is gradually cooled. Along the banks where the water is quiescent, temperature stratification occurs as in a lake. Because depths near the bank are usually very shallow, temperatures reach the freezing point more quickly there; ice crystals form and adhere to the banks, twigs, and projecting rocks, and a thin ice sheet forms. In the open part of the channel, temperature stratification is generally absent because of turbulent mixing, and the entire water body must reach 0°C before any freezing will occur. In the absence of nuclei or foreign material on which the ice crystals may form, there may be slight supercooling of the surface layer before any ice crystals are produced.

The ice sheet builds out from the shore as supercooled water, or water carrying ice crystals, impinges on the already-formed shore ice, and the transported or newly formed ice crystals adhere to the sheet. In the center of the stream, turbulence prevents coalescence of the ice crystals (frazil) that form. In the less turbulent areas, groups of crystals coalesce to form small pans of floating slush. These pans and (or) individual ice crystals are carried by the currents until they too impinge and adhere to existing ice sheets. In this manner an ice sheet finally forms across the entire stream. The ensuing increase in thick-

ness of the ice sheet occurs almost entirely at the interface of ice and water.

On a fairly wide stream there is no great buildup of pressure as a result of the ice cover because the ice is, to a large degree, in floatation. Ice is weak in tension. If the stage rises or if the ice thickens considerably, the increased upward force of the water causes tension cracks to appear at the banks. The ice floats up to a position in equilibrium with the water, and water fills the tension cracks and freezes. The result is again a solid sheet in equilibrium with the river. If the stage drops, the unsupported weight of the ice again causes tension cracks, especially at the banks, and the ice drops to an equilibrium position with respect to the water. Water again fills the tension cracks, freezes, and again a solid sheet of ice results.

On narrow streams the ice may be in floatation, bridged, or under pressure. If the stream is so narrow or the ice so thick that the ice can resist the tensile stress placed on it by changes in stage, the ice will not change position regardless of change in stage. At high stages the stream, in effect, will be flowing in a pressure conduit; at low stages the ice sheet will be bridged so that it makes no contact with the water. This is particularly true when there are large boulders in the stream to which the ice is frozen, thereby reducing the length of the unsupported free span.

EFFECT OF SURFACE ICE ON STREAM HYDRAULICS

Surface ice when in contact with the stream may, in effect, change streamflow from open-channel flow to closed-conduit flow. Frictional resistance is increased because a water-ice interface replaces the water-air interface, hydraulic radius is decreased because of the additional wetted perimeter of the ice, and the cross-sectional area is decreased to a degree by the thickness of the ice. The stage will therefore increase for a given discharge. Figure 174 shows the water-stage recorder graph for a gaging station as the formation of surface ice begins to cause backwater effect. In this example, daily mean discharge remained about the same as before the freezeup, although the discharge undoubtedly fluctuated somewhat during each day. It can be seen from figure 174 that surface ice can cause much uncertainty regarding the discharge because the stage-discharge relation becomes indeterminate. It is evident in figure 174 that backwater effect exists and is increasing, because the rise looks very unnatural, but the amount of backwater effect cannot be determined directly from the recorder chart.

Surface ice can also cause siphon action when it forms on a section control, but that effect is not very common. In figure 175 when water filled the entire space between control and ice, siphon action began

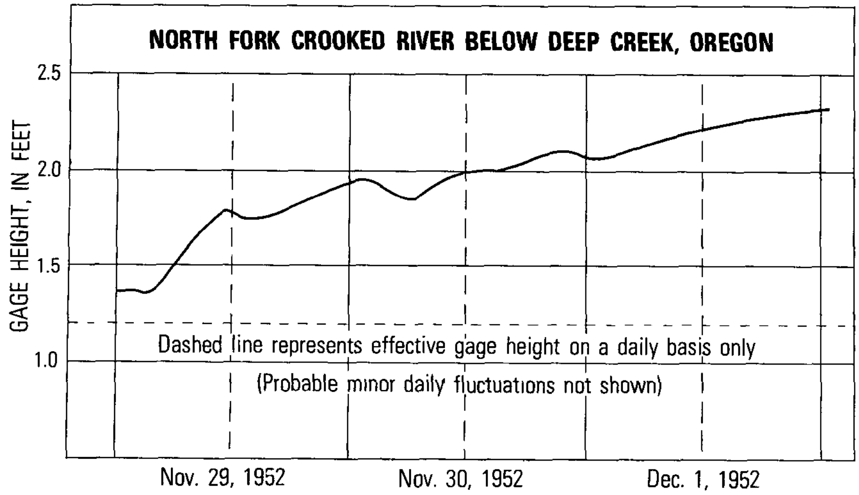


FIGURE 174.—Typical rise as complete ice cover forms. (After Moore, 1957.)

and water flowed over the control faster than it entered the gage pool. The gage pool was pulled down 0.3 ft below the point of zero flow before air entered the system and broke up the siphon action. Discharge ceased and then became a trickle while the inflow again filled the gage pool. When the entire space between control and ice was filled once more, siphon action began again. Siphon action is easily recognizable from the rapid fluctuations of the stage record. If the gaging station is visited at that time, the discharge measurement should be made far enough upstream from the gage pool to be beyond the effect of the fluctuating pool level.

If the section control is open and the gage is not too far removed from the control, there will probably be no backwater effect even though the entire pool is ice covered. The only effect of the ice cover will be to slow up the velocity of approach, and this effect will probably be minor. If the gage, however, is a considerable distance upstream from the riffle, surface ice on the pool may cause backwater as the covered reach of pool becomes a partial channel control.

Ice forming below an open-section control may jam and raise the water level sufficiently to introduce backwater effect at the control.

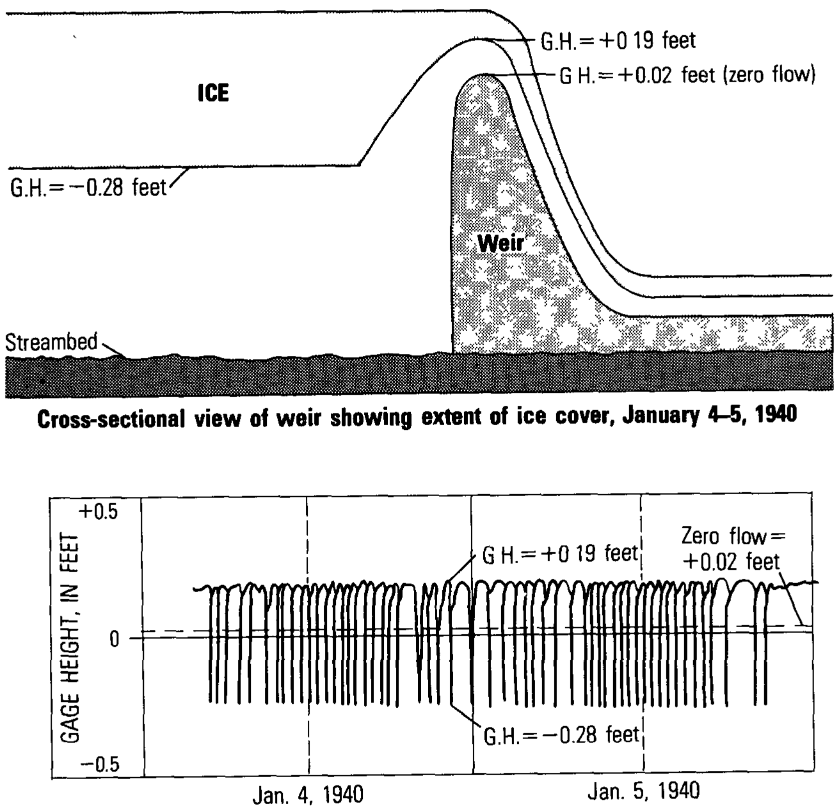
COMPUTATION OF DISCHARGE DURING PERIODS OF BACKWATER FROM ANCHOR ICE

Discharge measurements are usually not made when anchor ice is present for the following reasons. First, adjustment of the stage record for the effect of anchor ice can be made quickly and reliably. Second, a discharge measurement made at that time is of little help in

the analysis because discharge is highly variable with time as a result of water entering or leaving channel storage.

Anchor-ice rises are clearly recognizable on the recorder chart. In computing discharge for periods of anchor-ice effect, adjustments to gage-height are made directly on the gage-height graph. In figure 173 the long-dashed line connecting the low points of the "anchor-ice hump" is the effective gage height to use during the hours when the hump was recorded. Actually, the true effective gage height is shown by the short-dashed line. As the anchor ice builds up, the flow decreases faster than the normal recession shown by the long-dashed line, because some of the flow is going into storage as a result of the increased stage.

When the anchor ice goes out at about 9 or 10 a.m., a slug of water is released from storage and the true effective gage height rises. It can be seen however, that the areas formed by the short-dashed lines



Cross-sectional view of weir showing extent of ice cover, January 4–5, 1940

Gage height record for period January 4–5, 1940

FIGURE 175.—Effect of siphon action at artificial control in Sugar Run at Pymatuning, Pa., January 4–5, 1940.

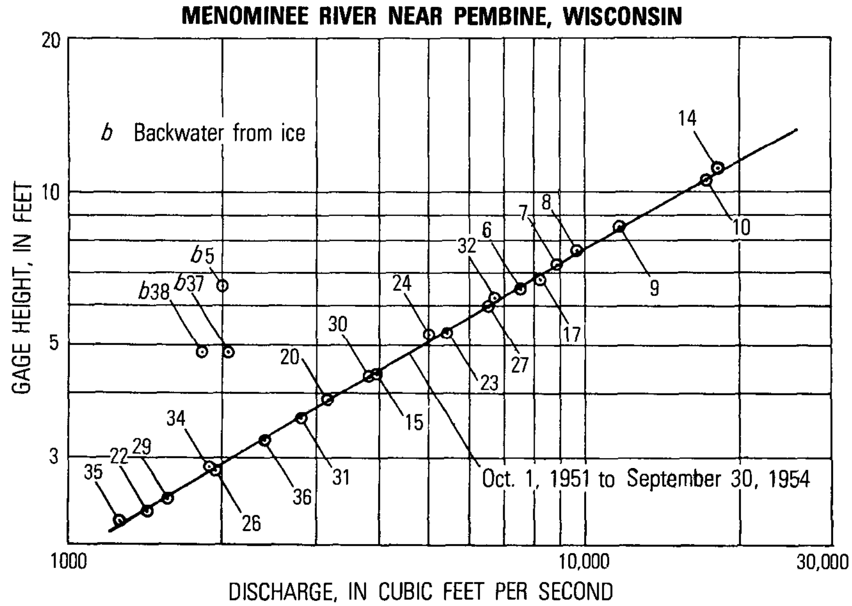


FIGURE 176.—Rating curve for Menominee River near Pembine, Wis. (After Moore, 1957.)

above and below the long-dashed line balance, and we would get identical daily mean values from use of either of the dashed lines. The rule then for obtaining effective gage height during anchor-ice periods is to cut off the hump with a straight line connecting the low points of the gage-height graph.

COMPUTATION OF DISCHARGE DURING PERIODS OF BACKWATER FROM SURFACE ICE

Figure 176 is an example of how discharge measurements (nos. 5, 37, 38), made during periods of ice effect, plot on a rating curve. Figure 174 is an example of a gage-height graph as complete ice cover forms. It is apparent from figure 174 that the backwater effect from surface ice cannot be determined directly from the recorder chart. The recorder chart is very helpful, however, in determining which periods during the winter are affected by ice. Complete notes describing ice conditions at the times the station was visited are also very valuable. Most important of all are discharge measurements made during ice-affected periods. A discharge measurement gives a definite point on a hydrograph plot of daily mean discharge versus date (fig. 177) through which the graph of estimated true daily discharge must pass. If little change in stage occurred during the day the discharge measurement was made, the measured discharge is considered to be the daily mean discharge. If a significant change in stage occurred that

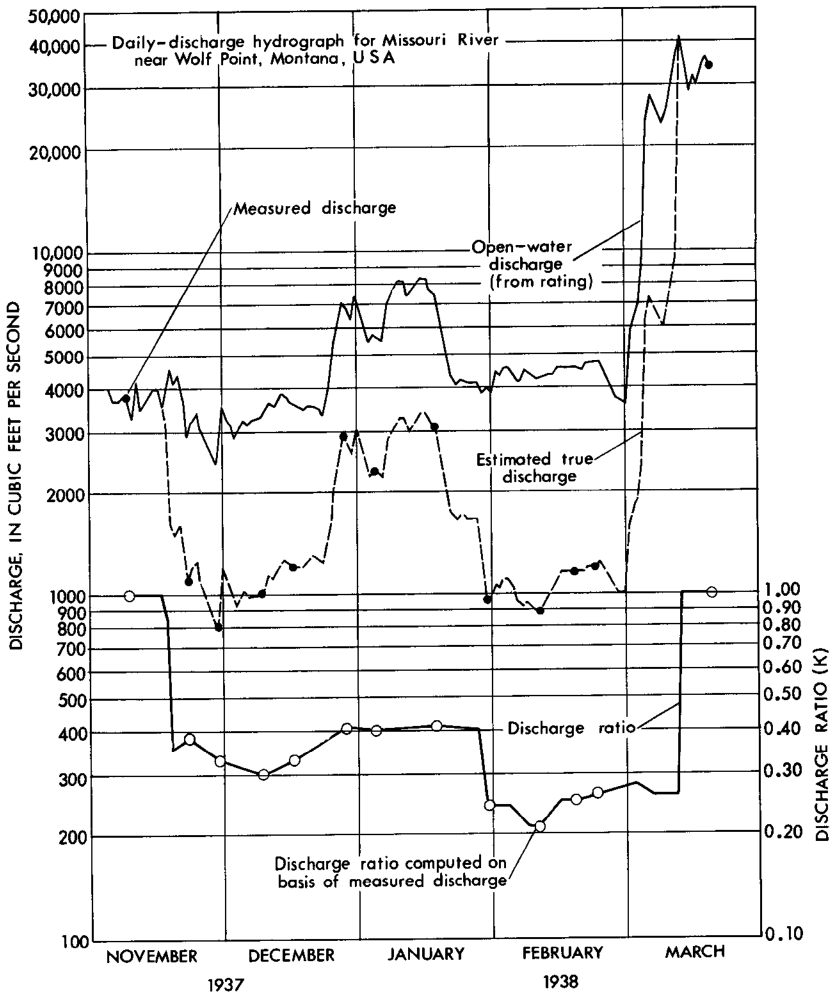


FIGURE 177.—Example of discharge-ratio method for correcting discharge record for ice effect.

day, the daily mean discharge (Q) is computed from the formula,

$$Q = Q_a \left(\frac{Q_m}{Q_r} \right), \tag{75}$$

where

Q_a is the discharge from the open-water (ice-free) rating curve corresponding to the daily mean gage height,

Q_m is the measured discharge and

Q_r is the discharge from the open-water rating curve corresponding to the gage height of the discharge measurement.

Three methods of correcting open-water discharge for ice effect are in use. (The term "open-water discharge", as used in this section of the manual, refers to the discharge for ice-free conditions obtained by applying the gage height record to the rating or shift curve that was in use immediately before the start of the ice-affected period.) The three methods are:

1. discharge-ratio method (sometimes known in the U.S.A. as the Lithuanian method),
2. shifting-control method or Stout method, and
3. hydrographic-and climatic-comparison method.

The reliability of each of the methods varies almost directly with the number of discharge measurements that were made during the ice-affected period that is being studied. Regardless of the method used, the corrected hydrograph of daily discharge, if possible, should be checked for consistency with other records. If the station being studied is on a stream that carries natural flow (flow not significantly affected by manmade development), its corrected record is compared with those for nearby streams that likewise carry natural flow. Particularly useful for that purpose are the hydrographs of streams that are unaffected by ice. If the station being studied is on a regulated stream, its corrected hydrograph is compared with the record of upstream reservoir releases or upstream hydroelectric generation, expressed either in units of discharge or in units of power output.

DISCHARGE-RATIO METHOD

In the discharge-ratio method which is used in many European countries, the open-water daily mean discharge is multiplied by a variable factor K to give the corrected discharge during periods of ice cover. A value of K is computed for each discharge measurement as the ratio of measured discharge (Q_m) to the open-water discharge (Q_r). Because K varies during the winter with time, as changes occur in the ice cover, the value of K for use on any given day is obtained by interpolation, on the basis of time, between K values computed for consecutive discharge measurements. Meteorological data are generally used to modify the simple interpolation between K values for consecutive discharge measurements; for example, during a period of extremely low temperatures the values of K indicated by simple interpolation would be reduced because the discharge usually decreases sharply at such times. The dates on which ice effect begins and ends are based on the observed or deduced beginning and end of ice cover.

An example of the discharge-ratio method is shown in figure 177. Note that discharge is plotted on a logarithmic scale. The upper daily hydrograph shows open-water discharges and the solid circles are

discharge measurements; the lower graph shows the K values obtained from discharge measurements (open circles) and the interpolation between those values; the middle graph is the hydrograph of estimated true daily discharges, obtained by multiplying concurrent values from the upper and lower graphs. The nonlinear interpolations for K values during the periods November 9–23, January 18 to February 19, and February 24 to March 20, were based on the observer's notes concerning ice conditions and on temperature and precipitation records (not shown in fig. 177).

SHIFTING-CONTROL METHOD

The shifting-control method, at one time the standard method used in the U.S.A., is seldom used here now, but it is still used in other countries. In the shifting-control method, recorded gage heights are reduced by a variable backwater value to obtain the effective daily gage heights. The effective gage heights are then applied to the open-water rating to obtain estimated true daily discharges. The backwater correction on days when discharge measurements are made is computed as the difference between the actual gage height and the effective gage height—effective gage height being the gage height from the open-water rating that corresponds to the measured discharge. The backwater correction for use on any given day is obtained by interpolation, on the basis of time, between the backwater corrections computed for consecutive discharge measurements. As in the discharge-ratio method, the interpolation is subject to modification on the basis of meteorological records, and the dates on which ice effect begins and ends are based on the observed or deduced beginning and end of ice cover.

An example of the shifting-control method is shown in figure 178. The method is applied to the same gaging station used in the example in figure 177. Note that a natural (not logarithmic) scale is used in figure 178. The upper daily hydrograph in figure 178 shows recorded gage heights and the solid circles are the effective gage heights for discharge measurements; the lower graph shows the backwater corrections obtained from discharge measurements (open circles) and the interpolation between those values; the middle graph is the hydrograph of effective gage height, obtained by subtracting values on the lower graph from concurrent values on the upper graph. The nonlinear interpolations for backwater corrections during various periods were based on the observer's notes concerning ice conditions and on temperature and precipitation records (not shown in fig. 178). As mentioned in the preceding paragraph, the effective gage heights (middle graph) are applied to the rating curve to obtain estimated true daily discharges.

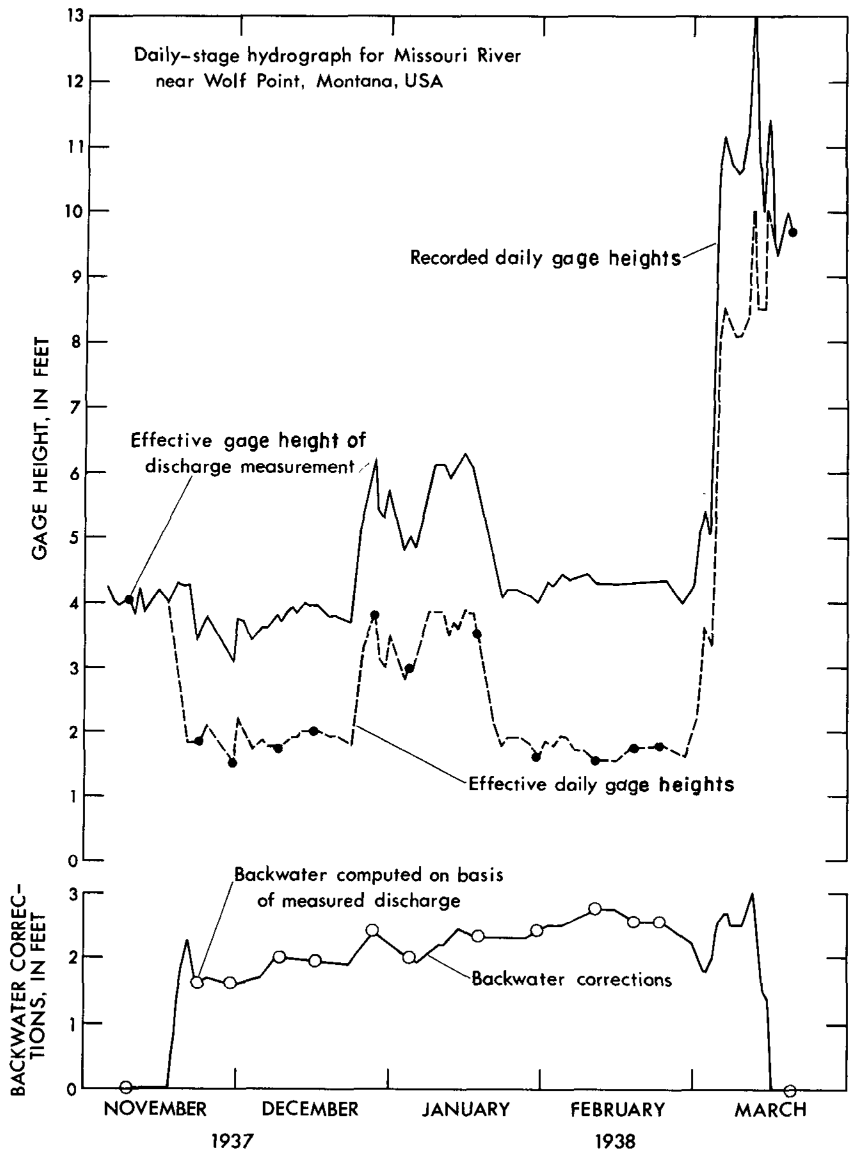


FIGURE 178.—Example of shifting-control method for adjusting stage record for ice effect.

HYDROGRAPHIC- AND CLIMATIC-COMPARISON METHOD

The method of hydrographic and climatic comparison has been favored in the U.S.A. for the last 30 years. The mechanics of the method differ from those of the discharge-ratio method, but both methods

basically correct the daily open-water discharge by a variable percentage.

The first step is to compute the station discharge record for the entire year as though there were no ice effect at any time. The daily hydrograph of open-water discharge and the discharge measurements are then plotted, using a logarithmic discharge scale, and notes concerning ice conditions are entered on the graph. At this point the hydrograph sheet resembles the upper graph in figure 177. If a measurement of ice-affected discharge is not representative of the daily mean discharge because of changing stage during the day, the daily mean discharge, as computed by equation 75, is also plotted. All is then in readiness for estimating the true daily discharge directly on the hydrograph sheet, and that is done on the basis of three comparisons:

1. comparison with records for nearby gaging stations,
2. comparison with weather records, and
3. comparison with the base-flow recession curve for the gaging station that is being studied.

Comparison with records for nearby gaging stations

Comparison with other discharge records is the most important basis for determining the probable discharge for periods between discharge measurements. Even though the record used for comparison may also have been corrected for ice effect, its use provides an additional independent set of basic data—another stage record and another set of current-meter measurements. Without a nearby record that compares well with the record being studied, the accuracy of the daily discharges estimated between the dates of discharge measurements may be greatly reduced. However, hydrographic comparisons are not infallible because the relation between the flow of two streams may vary significantly during the year; hence the importance of making many discharge measurements during ice-affected periods.

In making the hydrographic comparison, the nearby station with the most reliable winter streamflow record is selected for use as a reference station. The reliability of the reference station may have been established by the fact that its discharge is unaffected by ice or is affected by ice for only a relatively short period, or by the fact that many winter measurements have been made at the station and the true discharge between the dates of measurement can be estimated from weather records. (See discussion below on use of weather records.) A hydrograph of daily discharge, corrected for ice effect if necessary, is prepared for the reference station on a separate sheet of

graph paper, similar to that used for plotting the daily hydrograph for the station being studied.

A light table is used in comparing the two hydrographs. A light table is a glass-topped table that is illuminated by light from below the glass top, so that when one hydrograph is superposed on the other on the table top, both hydrographs can be viewed simultaneously. The hydrograph for the reference station is taped to the top of the light table. The hydrograph for the study station is then superposed on that of the reference station and positioned laterally so that the date lines of the two hydrographs coincide. The period preceding the first measurement (no. 1) that showed ice effect at the study station is the period first selected for consideration. The hydrograph for the study station is positioned vertically so that hydrographs for the two stations roughly coincide for the period immediately preceding the day or days when the start of ice effect is suspected. A comparison of the hydrographs and an inspection of the weather records should fix the date when ice effect started. That date will be preceded by a period of subfreezing weather, and on that date—usually a rainless day—the hydrograph for the study station will start a gradual rise not shown by the hydrograph for the reference station. For an appreciable period thereafter the hydrograph for the study station will remain above that of the reference station.

After the starting date (A) of ice-effect at the study station has been selected, the vertical position of the hydrograph for the study station is changed slightly, if necessary, to make the two hydrographs coincide on that date. If that positioning causes measurement no. 1 to fall directly on the hydrograph for the reference station, the hydrograph for the reference station between date A and measurement no. 1 is traced with dashed lines on the hydrograph sheet for the study station. The daily discharges indicated by the dashed lines are the estimated true discharges at the study station during the period between date A and measurement no. 1.

However, only rarely does measurement no. 1 coincide with the reference hydrograph when discharges at the two stations are made to coincide on date A; measurement no. 1 will usually lie above or below the hydrograph for the reference station. In that situation, as discharges from the reference hydrograph are being transferred to the sheet bearing the study hydrograph, the study sheet will in effect be moved up or down, as the case may be, so that when the transfer of discharge points reaches measurement no. 1, measurement no. 1 will coincide exactly with the reference hydrograph. If the temperature record shows no great fluctuation from day to day during the period between date A and measurement no. 1, the vertical displacement of the sheet bearing the study hydrograph will be made uniformly dur-

ing the transfer process. If the temperature record does fluctuate from day to day during the period, the vertical displacement will be made at a variable rate to reflect the fact that the ratio of true discharge to open-water discharge usually decreases during sharp drops in temperature; the ratio increases during sharp rises in temperature. In other words, the vertical distance between open-water discharge and true discharge increases on the study-hydrograph sheet during sharp drops in temperature; the vertical distance decreases during sharp rises in temperature. Observer's notes concerning major changes in the ice cover, particularly where complete cover is intermittent during the winter, are also very helpful in estimating the degree of ice effect.

After correcting the discharge between date *A* and measurement no. 1, the process is repeated for the period between discharge measurement no. 1 and the next successive discharge measurement (no. 2). The two hydrographs are made to coincide at measurement no. 1 and the transfer of discharge points to the study hydrograph proceeds to measurement no. 2. In that manner the open-water discharge for the study station is corrected until the date is reached when ice effect ceases.

Comparison with Weather Records

Records of air temperature and precipitation are a most valuable aid in making corrections for ice effect. The temperature record helps the engineer decide whether the precipitation is rain or snow—snow will have no immediate effect on the runoff. The temperature record also helps the engineer decide whether ice cover is forming, increasing, or dissipating. For stations for which there are no nearby discharge records for comparison and for which the recorder chart does not provide dependable clues to the fluctuation of discharge, it may be necessary to correct open-water discharges for ice effect almost solely on the basis of weather records and available measurements of discharge. Discharge usually follows closely the "ups-and-downs" of the air temperature record, and the discharge measurements help fix, within reasonable limits, the estimated rises and falls of the "true" discharge hydrograph. An exception to that statement is found in regions of extreme cold, such as the Arctic, that become blanketed with a heavy snow cover. The snow acts as an insulator for the underlying ground, and it then requires a prolonged change in temperature to significantly change the slow uniform recession of streamflow during the winter.

It should be mentioned here that a water-temperature recorder is a helpful adjunct to a gaging station. When the water temperature is above the freezing level, there is little likelihood of ice effect.

COMPARISON WITH BASE-FLOW RECESSION CURVES

During periods of subfreezing weather, virtually all the flow in a stream is base flow; that is, water that comes out of ground-water storage to sustain the flow of the stream during periods when there is no surface runoff. It will often be found that during cold ice-affected periods, the flow of the stream will be declining at a rate similar to the rate of recession shown by that stream during ice-free periods. Thus if we have a known discharge of say, 20 ft^3/s, on some day during the ice-affected period and we wish to estimate daily discharge during the next 10 days, all of which were free of rain or snowmelt, we look for an ice-free period elsewhere in the record for the study station when there was no surface runoff, and choose a day whose discharge is 20 ft^3/s. We then note the receding values of discharge for the following 10 days, and use those same discharges for the 10 days to be estimated. The ice-free period that is used for an index should preferably be in the nongrowing season because the use of water by vegetation affects the rate of base-flow recession.

It is possible that daily discharges estimated from the base-flow recession for a warmer period may be somewhat high because extremely cold weather reduces the rate at which water percolates through the ground, and because some of the water that does reach the stream may go into storage behind ice dams. Nevertheless a standard base-flow recession curve provides a valuable guide to the probable flow during recession periods when the stream is ice-covered. Because the discharge during periods of base flow originates as ground water, a record of the fluctuations of ground-water levels of wells in the area can be useful as an index for estimating the true discharge during those periods.

An example of the application of the hydrographic- and climatic-comparisons method is illustrated in figures 179 and 180. Figure 179 shows a portion of a plotted hydrograph of daily mean discharge for the gaging station on North Fork John Day River at Monument, Oreg. The solid line represents open-water discharge obtained by applying recorded gage heights to the rating curve, and the X on January 26 represents the open-water discharge corresponding to the gage height of discharge measurement C made on that date. The open-water discharge is almost 10 times as great as the measured discharge on January 26. The dashed line on figure 179 represents the estimated true daily discharge obtained by comparison with the hydrograph of daily mean discharge for John Day River at Service Creek, Oreg. and by comparison with the record of daily maximum and minimum temperature at Dayville, Oreg. The reference hydrograph and temperature record used for the comparison are shown in figure 180. Actually the precipitation record at Dayville was also

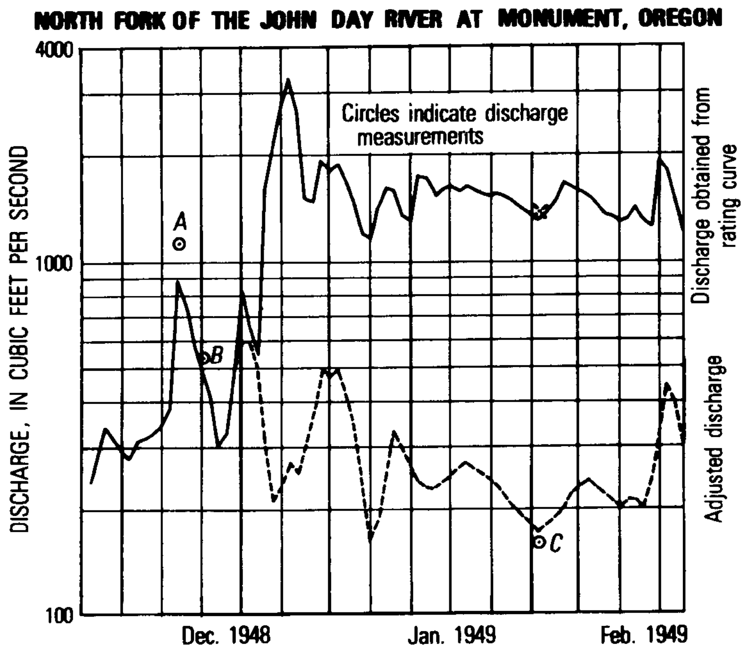


FIGURE 179.—Daily hydrographs for open-water discharge and for discharge corrected for ice effect. (After Moore, 1957.)

considered, but because all precipitation during the study period occurred as snow and therefore had no immediate effect on the runoff, the precipitation record is not shown in figure 180.

Also shown on figure 180 is the corrected hydrograph for the study station on North Fork John Day River at Monument; the hydrograph of open-water discharge at that station has been omitted to reduce clutter in the illustration. The discharge for the reference station on John Day River at Service Creek was unaffected by ice. The shapes of the two hydrographs are not identical, but useful comparison between the hydrographs for two stations does not require that their shapes be identical, as long as their discharge trends are similar. It can be seen on figure 180 that both hydrographs respond to the effect of air-temperature fluctuations during the winter period.

In applying the method of hydrographic and climatic comparison, the hydrograph of "true" daily discharge, plotted on a logarithmic scale, was displaced from the open-water hydrograph by a variable vertical distance. That means, in effect, that discharge ratios, variable with time, were applied to the open-water discharges, and there-

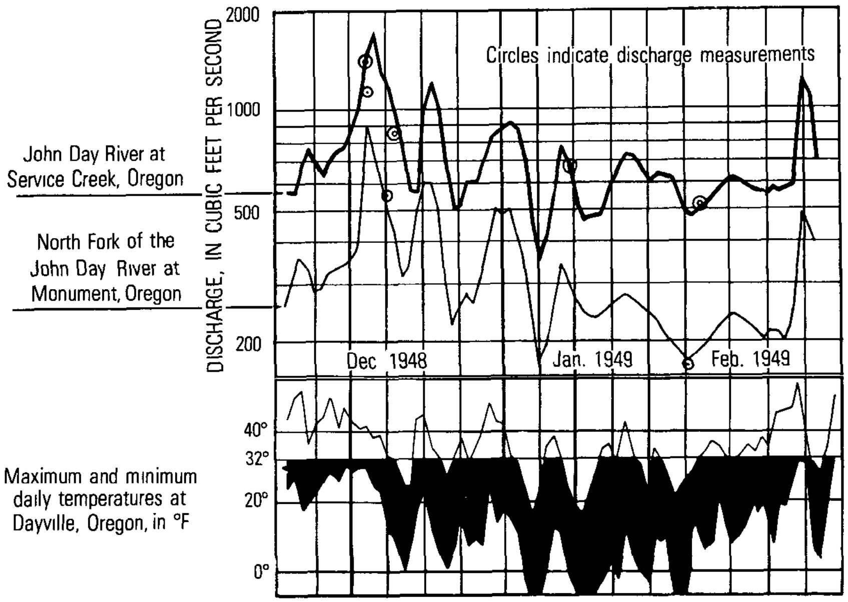


FIGURE 180.—Comparison of daily winter discharge at two gaging stations showing their response to air-temperature fluctuations. (After Moore, 1957.)

fore a basic similarity exists between the hydrographic-comparison method and the discharge-ratio method. It appears to the author that application of the hydrographic-comparison method would be greatly facilitated if the hydrograph of open-water discharge for the study station were first adjusted by the discharge-ratio method because application of that method is relatively simple. The adjusted hydrograph would then be refined by using it, rather than the open-water hydrograph, in the hydrographic-comparison method. It is much simpler to apply the hydrographic-comparison method for refining discharge estimates than it is to apply that method for making original discharge estimates.

SAND-CHANNEL STREAMS

In fixed channels, well-defined stage-discharge relations can usually be developed that show only minor shifting at low flow. In sand-channel streams, however, stage-discharge relations are continually changing with time because of scour and fill and because of changes in the configuration of the channel bed. These changes cause the shape and position of the stage-discharge relation to vary from time to time and from flood to flood, and it becomes very difficult to explain the apparent haphazard scatter of discharge measurements available to define the rating. Familiarity with the results of research

studies as reported by Colby (1960), Dawdy (1961), Simons and Richardson (1962), Beckman and Furness (1962), and Culbertson and Dawdy (1964) will greatly assist the analyst in defining the discharge rating.

For a stream with rigid boundaries, the best site for a stream-gaging station is upstream from a constriction because the constriction will provide a stable and sensitive control. An opposite effect occurs, however, at a constriction on a sand-channel stream; the rating will be unstable there because the constricted section will experience maximum streambed scour and fill. In fact, any contracting reach on a sand-channel stream is undesirable for use as a gaging-station site, and a straight uniform reach should be sought. Preferably both the gage and the cableway site for high-water discharge measurements should be located in a reach suitable for the determination of peak discharge by the slope-area method (chap. 9). This will permit the use of high-water current-meter measurements to verify computed peak discharges as well as develop the hydraulics of the stage-discharge relation. The fieldwork for a sand-channel stream should also include the collection of samples of bed materials at the stream-gaging site.

BED CONFIGURATION

On the basis of laboratory investigation, Simons and Richardson (1962) described the bed configuration of sand-channel streams as ripples, dunes, plane bed, standing waves, and antidunes. This sequence of bed configurations occurs with increasing discharge. When the dunes wash out, and the sand is rearranged to form a plane bed, there is a marked decrease in resistance to flow which may result in an abrupt discontinuity in the stage-discharge relation. The forms of bed roughness, as shown in figure 181 and described in table 21, are grouped according to the two separate conditions of depth-discharge relationship that are evident in a given channel. The sequence of configurations described in table 21 is developed by continually increasing discharge. The lower regime occurs with lower discharges; the upper regime with higher discharges; an unstable discontinuity in the depth-discharge relationship appears between these two more stable regimes.

The presence of fine sediment in the flow influences the configuration of the sand bed and thus the resistance to flow. It has been found by Simons and Richardson (1962, p. 4) that with concentrations on the order of 40,000 milligrams per liter of fine material, resistance to flow in the dune range is reduced as much as 40 percent. The effect is less pronounced in the upper regime, but fine sediment may change a standing-wave condition into a breaking antidune which will increase the resistance to flow. Thus the stage-discharge relation for a

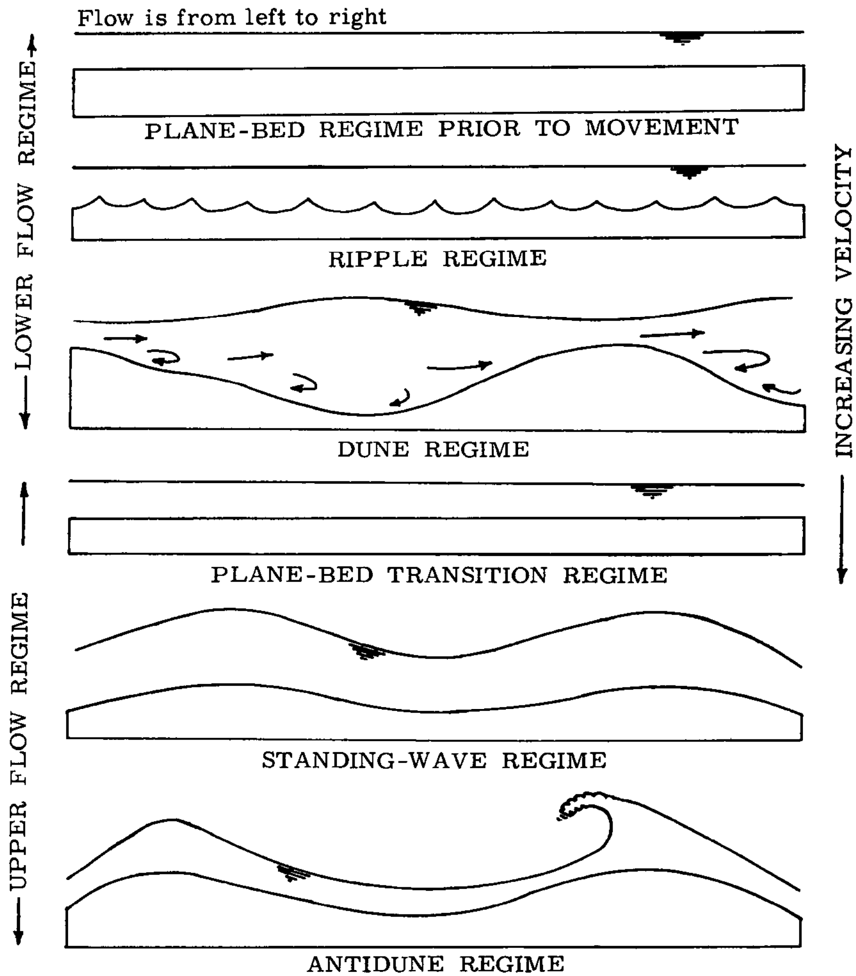


FIGURE 181.—Idealized diagram of bed and water-surface configuration of alluvial streams for various regimes of flow.

stream may vary with sediment concentration if the flow is heavily laden with fine sediment.

Changes in temperature can also alter the form of bed roughness, and, hence, the resistance to flow. Lowering the temperature increases the viscosity of the water and increases the mobility of the sand. If, for example, the form of bed roughness is in transition or nearly so, and if there is a reduction in the temperature of the water, the increased mobility of the sand may cause the dunes to wash out and the bed to become plane. This phenomenon is reversible.

Changes in bed forms do not occur instantaneously with increasing or decreasing discharge. The time lag between change in bed form

TABLE 21.—*Surface and bed descriptions for the various flow regimes*

Type of configuration	Description	
	Bed	Flow
Lower regime flow:		
Plane bed	Plane; no sediment movement.	Plane surface; little turbulence.
Ripples	Small uniform waves; no sediment movement.	Plane surface; little turbulence.
Dunes	Large, irregular, sawtoothed waves formed by sediment moving downstream; waves move slowly downstream.	Very turbulent; large boils.
Upper regime of flow:		
Plane bed	Dunes smoothed out to plane bed.	Plane surface; little turbulence.
Standing waves	Smooth sinusoidal waves in fixed position.	Standing sinusoidal waves in phase with bed waves; termed "sand waves."
Antidunes	Symmetrical sinusoidal waves progressing upstream and increasing in amplitude; suddenly collapse into suspension then gradually reform.	Symmetrical sand waves progressing upstream in phase with bed waves; amplitude increases until wave breaks, whole system collapses then gradually reforms.

and change in discharge may result in loop rating curves. For example, if bed configuration is initially dunes, the dunes will persist on rising stages to a discharge that is greater than the discharge at which the dunes will reform on falling stages. Thus at a given stage, the discharge may be greater when the stage is falling. Because the form of each loop curve depends on the initial condition of bed configuration and the rate of change of discharge, an infinite number of different loop curves, and even multiple-loop curves, may occur for a given reach of channel across the transition from dunes to plane bed. The stage-discharge relation within the transition band may be indeterminate. An example of a loop curve, typical of some channels, is shown in figure 182.

RELATION OF MEAN DEPTH TO DISCHARGE

A plot of stage against discharge in sand-channel streams often obscures any underlying hydraulic relationship because neither the bottom nor sides of these streams are fixed. Figure 183 shows as an extreme example the stage-discharge plot for Huerfano River near Undercliffe, Colo., for 1941 and 1942. The relation between stage and discharge is indeterminate. However, the underlying hydraulic relation may be revealed by a change in variables. The effect of variation in bottom elevation is eliminated by replacing stage by mean depth or hydraulic radius. The effect of variation in width is eliminated by

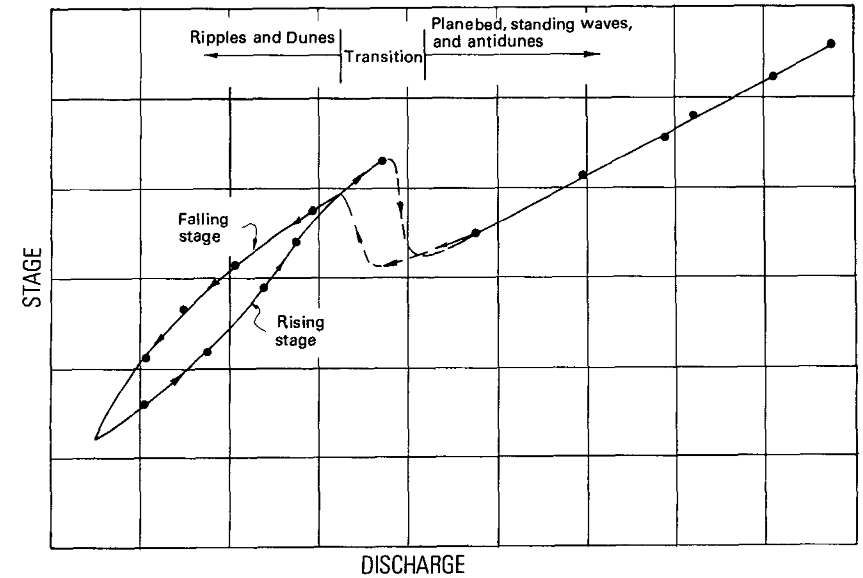


FIGURE 182.—Typical loop curve of stage versus discharge for a single flood event in a sand channel. (After Stepanich, Simons, and Richardson, 1964.)

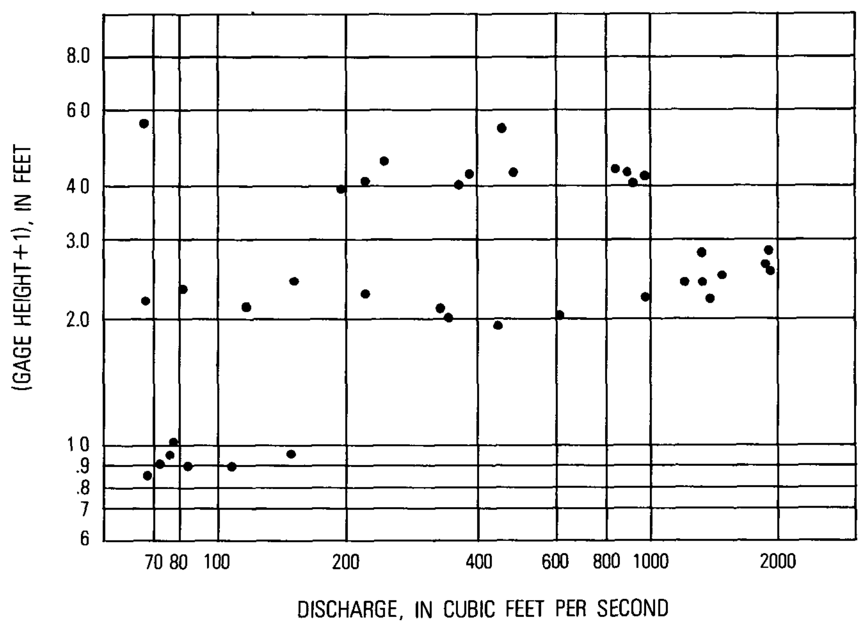


FIGURE 183.—Stage-discharge relation for Huerfano River near Undercliffe, Colo.

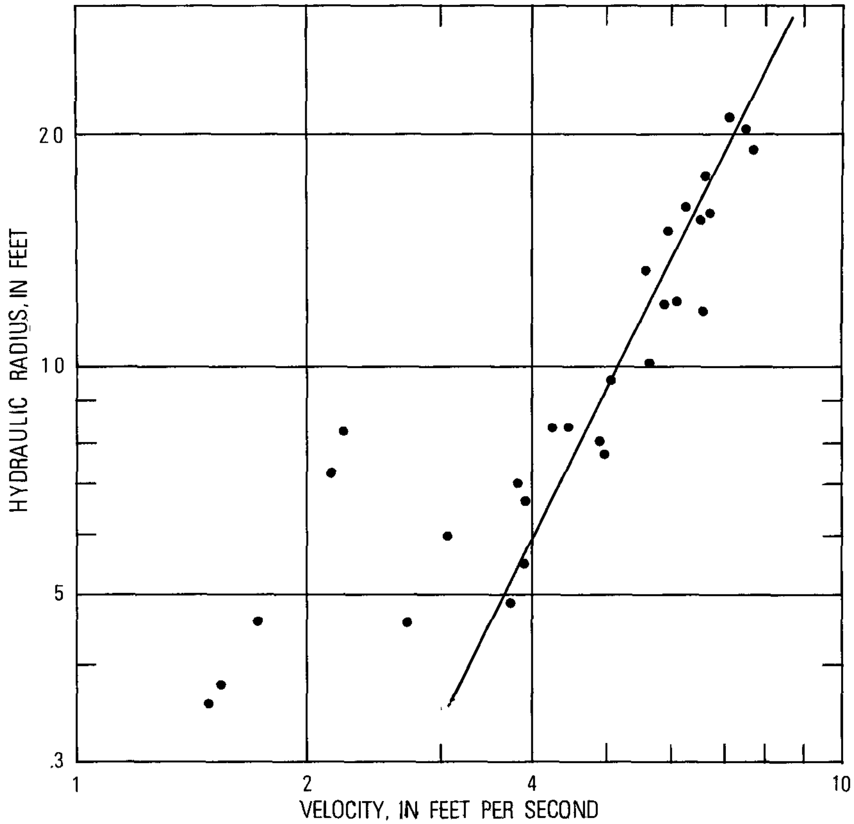


FIGURE 184.—Relation of velocity to hydraulic radius for Huerfano River near Undercliffe, Colo.

using mean velocity. Figure 184 shows most of the same measurements for Huerfano River that were plotted in figure 183, now replotted on the basis of velocity and hydraulic radius. Measurements for this stream with a hydraulic radius greater than one foot define a single curve with bed forms corresponding to the upper regime. Measurements in the transition range from dunes to plane bed scatter wildly as would be expected from the previous discussion.

The discontinuity in the depth-discharge relation is further illustrated in figure 185 which shows a plot of hydraulic radius against velocity for Rio Grande near Bernalillo, N. Mex. The measurements plotted on the left represent bed configurations of ripples and dunes and the curve on the right represents bed configurations of plane bed, standing waves, or antidunes.

According to Dawdy (1961), the curve representing the upper regime in a true sand-bed stream usually fits the following relation,

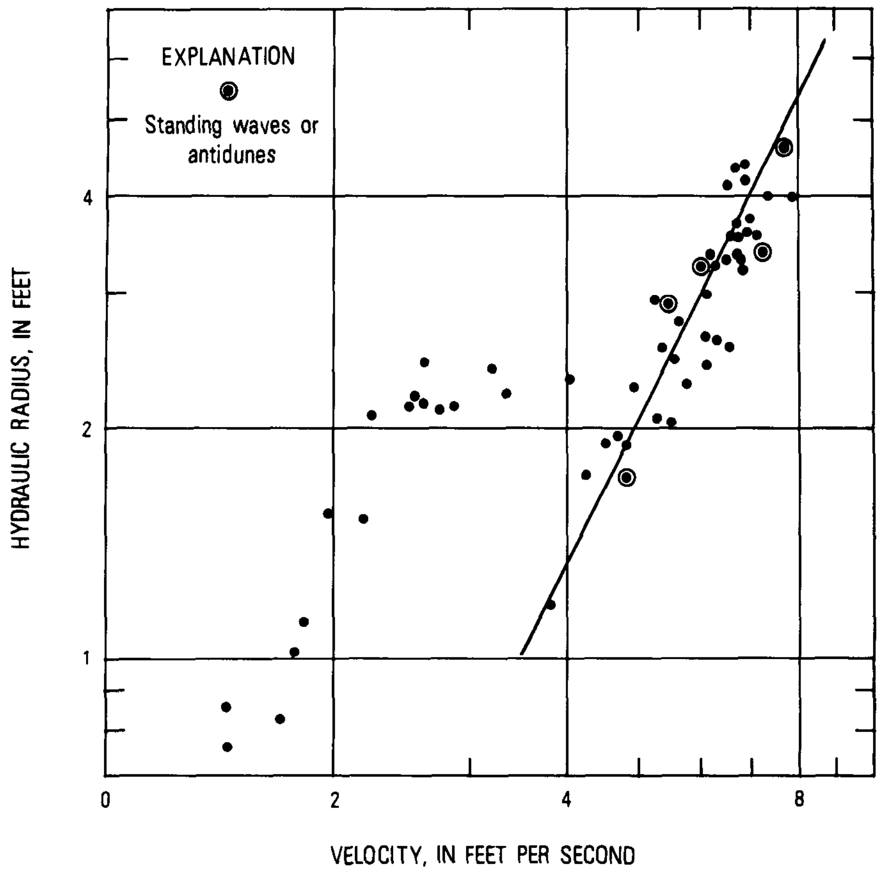


FIGURE 185.—Relation of velocity to hydraulic radius for Rio Grande near Bernalillo, N. Mex.

$$V = kR^{1/2},$$

where V is the mean velocity, k is a constant, and R is the hydraulic radius. He found this relation to be applicable for 26 of the 27 streams used in his study. More recent study has shown that the exponent of R ranges from ⅔, as in the Manning equation, to ½, the larger exponents being associated with the coarser grain sizes.

DEVELOPMENT OF DISCHARGE RATING

Plots of mean depth or hydraulic radius against mean velocity or discharge per foot of width are valuable in the analysis of stage-discharge relations. These plots clearly identify the regimes of bed configuration and assist in the identification of the conditions represented by individual discharge measurements. For example, only

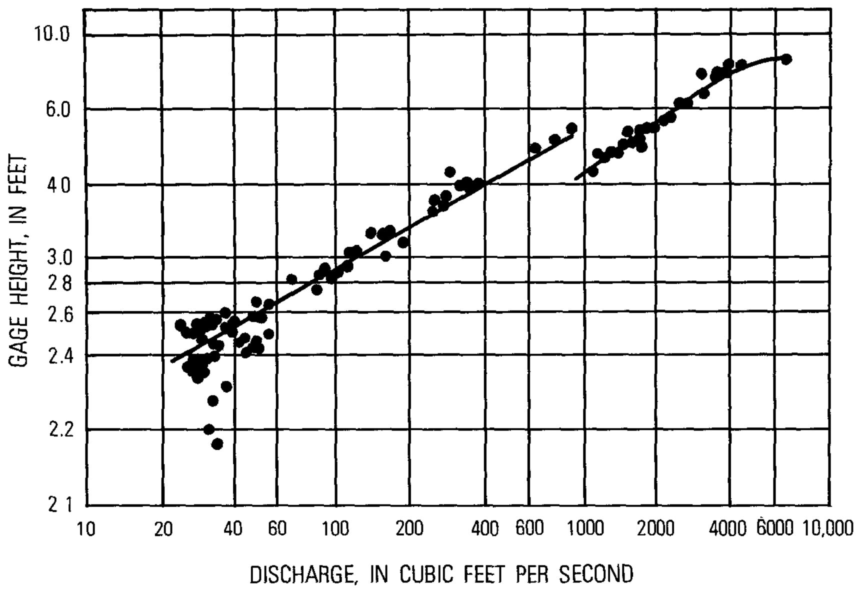


FIGURE 186.—Stage-discharge relation for station 34 on Pigeon Roost Creek, Miss. (After Colby, 1960.)

those measurements identified with the upper regime should be used to define the position and slope of the upper portion of the stage-discharge curve; similarly, only those measurements identified with the lower regime should be used to define the lower portion of the stage-discharge curve. Measurements made in the transition zone may be expected to scatter widely but do not necessarily represent shifts in more stable portions of the rating.

Plots of stage against mean depth and stage against width are also helpful in developing a mean stage-discharge relation and in analyzing the cause of shifts from the mean relation. In the upper regime the use of these plots in conjunction with the plot of velocity versus mean depth or hydraulic radius raised to the ½ to ⅔ power, depending on grain size, may be useful in establishing a reasonable slope to the upper part of the stage-discharge relation.

The stage-discharge relation developed by Colby (1960) for Pigeon Roost Creek, Miss., is shown in figure 186. This stream is about 75 ft wide, the banks are relatively stable, and the median size of the bed material is 0.4 mm. The mean elevation of the channel bed does not change appreciably with time or discharge. The discontinuity in the stage-discharge relation is very abrupt. Discharges from 900 to 1,800 ft^3/s may occur at a stage of 5.3 ft.

According to Colby (1960, p. 19, 20), stage-discharge relations may

be expected to have a discontinuity if the reach has all of the following characteristics:

1. a bed of uniform and readily shifting sediment that does not form distinct pools and riffles,
2. at some flows almost all of the stream-bed is covered with loose sand dunes,
3. at higher flows the bed of the stream is mostly plane or has antidunes,
4. the depth of flow at the point of discontinuity is sufficiently great so that changes in the stage-discharge relation at the discontinuity can be distinguished from changes caused by small local shifts of the channel bottom, and
5. the lateral distribution of depths and velocities is sufficiently uniform for the bed configuration to change across most of the streambed in a relatively short time.

The above conditions are very restrictive. Many streams with sand beds have well developed pools and riffles at the stage where the discontinuity might otherwise occur. Streams do not generally have uniform sediment sizes; many have large sorting coefficients. A few streams having suitable bed material may never show the discontinuity because dunes exist even at the highest flow rates. Others may have such high slopes that the lower regime cannot be defined by discharge measurements because of the shallow depths at which the discontinuity occurs. Winding streams seldom have uniform lateral distribution of velocity and depths. Some streams have such gradual or inconsistent transitions between dunes and plane bed that the discontinuity may be difficult, if not impossible, to define. Dunes may exist near the banks at the same time that a plane bed exists near the center of the stream. The transition in this case may occur so gradually with increasing stage that the discontinuity in rating is eliminated. However, at any station where dunes exist at low flows and a plane bed exists at higher flows, there is a major change in bed roughness. Knowledge of the bed forms that exist at each stage or discharge can be very helpful in developing the discharge rating.

EVIDENCES OF BED FORMS

Evidence of the bed forms that exist at a given time at a particular station can be obtained in several ways, a listing of which follows.

1. Visual observation of the water surface will reveal one of several conditions: large boils or eddies, which indicate dunes; a very smooth water surface, which indicates a plane bed; standing waves, which indicate smooth bed waves in phase with the surface waves; or breaking waves, which indicate antidunes. Visual observations of the water surface should be recorded on each discharge measurement.

2. Noting whether the sand in the bed is soft or firm. A soft bed often indicates lower regime conditions. The streambed during upper regime flow will usually be firm.

3. Measurements of bed elevations in a cross section will usually indicate the type of bed forms. A large variation in depth indicates dunes, and a small variation in depth a plane bed. The small variation in depths for a plane-bed (upper-regime) configuration should not be confused with small variations caused by ripples or by small dunes, both of which are definitely lower-regime configurations. A large variation in bed elevation at a particular point in the cross section during a series of discharge measurements indicates the movement of dunes.

4. The amount of surge on a recorder chart may also indicate the configuration of the channel bed. Medium surge may indicate dunes, little or no surge may indicate a plane bed, and violent surge may indicate standing waves or antidunes. The transition from plane bed to dunes during a discharge recession may cause a secondary hump on the gage-height trace if the transition occurs over a short time period.

5. Relations that define the occurrence of bed forms as a function of hydraulic radius (R), slope (S), mean velocity (V), and grain size (d), are useful in developing discharge ratings. A relation of that type, presented by Simons and Richardson (1962), is shown in figure 187. In that figure the dimension of R is feet; that of V is feet per second. Recent studies suggest that: the lower regime of bed forms will occur when the ratio,

$$\frac{V^4}{g^2 D^{1/2} d_{50}{}^{3/2}}, \tag{76}$$

is less than 1×10^3; the upper regime of bed forms will occur when the ratio is greater than 4×10^3; the bed will be in transition if the ratio is between those values. In the above ratio, V is the mean velocity in feet per second, g is the acceleration of gravity in feet per second per second, D is the mean depth in feet, and d_{50} is the median grain size of bed material in feet.

SHIFTING CONTROLS

The upper part of the stage-discharge relation is relatively stable if it represents the upper regime of bed forms. Rating shifts that occur in upper-regime flow can be analyzed in accordance with the methods or principles discussed in the section titled, "Rating Shifts for Channel Control." However, the shift ratings after minor stream rises will generally have a strong tendency to parallel the base rating when plotted on rectangular-coordinate graph paper; that is, the equation

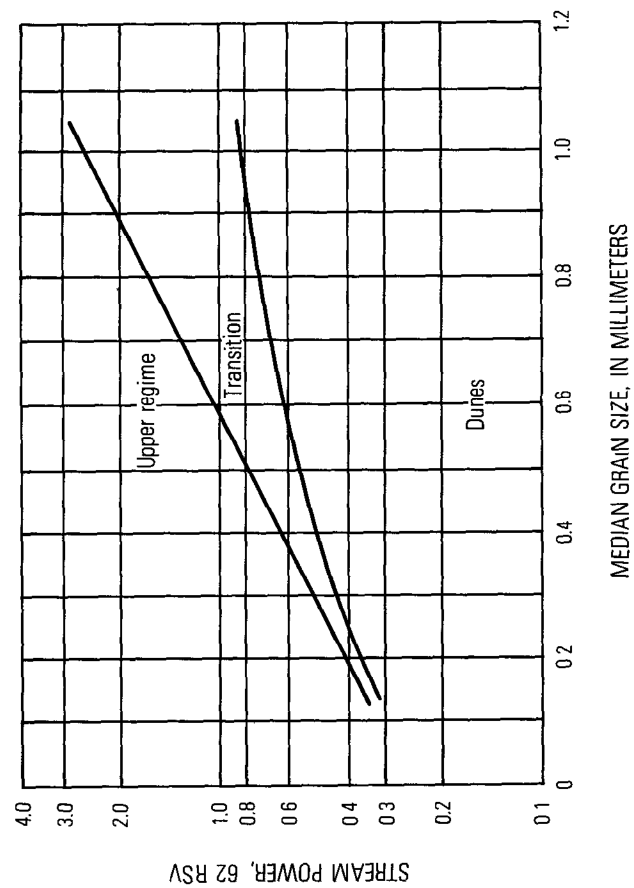


FIGURE 187.—Relation of stream power and median grain size to form of bed roughness.

for each shift curve will differ from that of the base rating by a change in the value of e in the basic equation,

$$Q=p(G-e)^N. \tag{53}$$

The shifts will change on stream rises and will often vary with time between rises. Major stream rises may also change the value of p in equation 53.

The lower part of the rating is usually in the dune regime and the stage-discharge relation varies almost randomly with time. Frequent discharge measurements are necessary to define the stage-discharge relation, and for some streams they are necessary to determine the variation of discharge with time in the absence of any usable relation between stage and discharge. In the U.S.A. a frequency of three discharge measurements per week is often recommended, but for some streams, even daily measurements barely suffice.

A mean curve for the lower regime is frequently used with shifts as defined by discharge measurements. In some instances the shift defined by a single discharge measurement represents only the temporary position of a dune moving over a partial section control. A series of discharge measurements made at short time intervals over the period of a day may define a pattern of shifts caused by dune movement. When discharge is constant but the stage fluctuates, the changing gage-height trace generally reflects dune movement.

Continuous definition of the stage-discharge relation in a sand channel stream at low flow is a very difficult problem. The installation of a control structure should be considered if at all feasible.

ARTIFICIAL CONTROLS FOR SAND CHANNELS

When conventional controls are installed in sand channels, they are seldom satisfactory, even those designed to be self-cleaning. The principal difficulty is that for such controls in a sand channel, discharge is dependent not only on water-surface elevation, but also on the bed elevation and flow regime upstream from the structure. A satisfactory control is one whose stage-discharge relation is unaffected by bed configuration. A few successful low-water controls have been designed for use in sand channels; one example is the weir designed for the gaging station on the Rio Grande conveyance channel near Bernardo, N. Mex. (Richardson and Harris, 1962). That structure will not be described here because generalizations concerning control shape are meaningless; each control structure must be individually designed for compatibility with channel and flow conditions that exist at the proposed site for the control. A laboratory model study involving a reach of channel is therefore needed for each

site investigated. Efforts continue to design low-water controls that are both relatively cheap and that have satisfactory operating characteristics when installed in sand channels (Stepanich and others, 1964).

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