CHAPTER 11—DISCHARGE RATINGS USING SLOPE AS A PARAMETER

GENERAL CONSIDERATIONS

If variable backwater or highly unsteady flow exists at a gaging station, the energy slope is variable at a given stage and the discharge rating cannot be defined by stage alone.

Variable backwater is most commonly caused by variable stage at a downstream confluence for a given discharge upstream or by the manipulation of gates at a downstream dam. The discharge under those conditions is a function of both stage and slope of the energy gradient. If the rate of change of stage is sufficiently great, the acceleration head must also be considered, but this chapter deals only with situations where the acceleration head has insignificant effect and can be neglected.

The unsteady-flow situation treated in this chapter is that of a natural flood wave, in which the flow maintains a stable wave profile as it moves down the channel. That type of wave is known as a uniformly progressive wave, and it often produces a loop rating at the gaging station; that is, for a given stage the discharge is greater when the stream is rising than it is when the stream is falling. The difference between the two discharges is significant only when the flow is highly unsteady. The term "highly unsteady", when associated only with the property of producing loop ratings, is a relative term, because channel slope is of equal importance in determining whether or not loop ratings will occur. A flood wave in a steep mountain channel will have a simple stage-discharge relation; that same flood wave in a flat valley channel may have a loop rating. The sections of this chapter that deal with unsteady flow are concerned only with loop ratings whose definition requires the use of slope, as well as stage, in a relation with discharge.

When a new gaging station is established, the need for a slope parameter in the rating can often be anticipated from the rating procedures used for existing stations nearby in a similar hydrologic and hydraulic environment. At other times the need for a slope parameter is not as evident. However, a plot of a series of discharge measurements made at medium and high stages will indicate the type of rating required for the station and will dictate whether or not an auxiliary gage is necessary to continuously measure water-surface slope.

If a pair of gages is needed, the locations of the base and auxiliary gage are based on the characteristics of the slope reach. The length of the reach should be such that ordinary errors that occur in the determination of gage heights at stage stations will cause no more than minor error in computing the fall in the reach. A fall of about 0.5 ft (0.15 m) is desirable but satisfactory records can often be obtained in reaches where the minimum fall is considerably less than 0.5 ft. Channel slope in the reach should be as uniform as possible. The reach should be as far upstream from the source of backwater as is practicable, and inflow between the two gages should be negligible. If possible, reaches with frequent or appreciable overbank flow should be avoided, as should reaches with sharp bends or unstable channel conditions. If the reach includes a natural control for low stages, the upstream (base) gage should be located just upstream from that control so that a simple stage-discharge relation will apply at low stages. Rarely will a slope reach be found that has all of the above attributes, but they should be considered in making a selection from the reaches that are available for slope measurement.

THEORETICAL CONSIDERATIONS

Variable slopes that affect flow in open channels are caused by variable backwater, by changing discharge, or by variable backwater in conjunction with changing discharge. The pair of differential equations given below provides a general solution to both gradually varied and unsteady flow.

$$\frac{Q^2}{K^2} = -\frac{\partial H}{\partial x} - \frac{1}{g}\frac{\partial V}{\partial t} \tag{77}$$

$$\frac{\partial Q}{\partial x} = -B\frac{\partial h}{\partial t} \tag{77a}$$

In the equations Q is the discharge, K is the conveyance of the cross section, H is the total energy head, x the distance along the channel, g the acceleration of gravity, V the mean velocity, t the time, B the top width of the channel, and h is the water-surface elevation. A solution to these equations in uniform channels may be obtained by approximate step methods after the conveyance term has been evaluated by discharge measurements.

In those practical problems of determining flow in open channels that require application of equation 77 the increment of slope due to the acceleration head $\frac{1}{g}\frac{\partial V}{\partial t}$ is, in general, so small with respect to the other two terms that its effect may be neglected. Thus, in equation 77 the terms that remain in addition to discharge (Q), are conveyance (K) which is a function of stage, and energy gradient ($\partial H/\partial x$) which is related to water-surface slope. At those sites where tidal action or variation in power production cause the acceleration head to be large, approximate methods of integration of equations 77 and 77a are used in conjunction with an electronic computer. Those methods are described briefly in chapter 13 of this manual.

The discussion of stage-fall-discharge ratings presented in the present chapter draws heavily on previously published reports. The three primary references used are Corbett and others (1945), Eisenlohr (1964), and Mitchell (1954).

VARIABLE SLOPE CAUSED BY VARIABLE BACKWATER

The stage at a gaging station for a given discharge, under the usual subcritical flow conditions, is influenced by downstream control elements. A brief discussion of those elements is now in order.

Previous discussions of controls in this manual have dealt primarily with such elements as natural riffles, weirs and dams, flumes, and the physical properties of the stream channel. It had also been explained that a control may act independently for some range of stage or it may act in concert with one or more other controls. However, it had also been mentioned in appropriate places in this manual that the stage at downstream stream confluences may affect the stage-discharge relation at a gaging station. Where that occurs, the confluent stream must be classed as a control element that acts in concert (partial control) with the control(s) in the gaged stream. Furthermore, when a confluent stream acts as a control element, it usually does so as a variable element. That is, the stage at the gaging station will no longer be related solely to the discharge of the main stream, but will also vary with variation of the discharge in the confluent stream.

At gaging stations on tide-affected streams, the tide itself must be considered as a variable control element because of its effect on the stage-discharge relation at the gaging station. As mentioned earlier tide-affected stage-discharge relations are treated in chapter 13.

A less clear-cut situation with regard to control elements exists in many streams in southeastern United States. These streams have extremely wide flood plains that are crossed in places by highway embankments whose bridge openings locally constrict the flow severely. At high flow if water occupies the flood plain, the stage-discharge relation at the bridge is affected; for a given discharge through the bridge the corresponding stage will vary, depending on whether streamflow is entering the overbank areas as on a rising stage, or whether water is returning to the main channel from the overbank areas as on a falling stage. In that situation the overbank flow itself is acting as a variable control element in concert with the "more conventional" and more stable control elements, such the geometry of the bridge opening and the geometry and roughness of the downstream main channel and overbank areas. The streamflow that is entering the overbank areas acts, in effect, as an extremely wide downstream distributary; the overbank flow that is returning to the stream acts, in effect, as an extremely wide downstream tributary. The streams usually have extremely flat gradients and the rating may possibly be complicated by the effect of changing discharge on streams of flat slope. However, as explained in the section titled, "Variable Slope Caused by a Combination of Variable Backwater and Changing Discharge," streams affected by both variable backwater and changing discharge are treated as though they were affected by variable backwater alone.

The control elements that affect the stage-discharge relation for a stream have now been identified and their descriptions have been amplified for the discussion of backwater that follows. At any given discharge the effect on the stage at the gaging station that is attributable to the operative control element(s) is known as backwater. As long as the control elements are unvarying, the backwater for a given discharge is unvarying, and the discharge is a function of stage only; the slope of the water surface at that stage is also unvarying. If some of the control elements are variable—for example, movable gates at a downstream dam or the varying stage at a downstream stream confluence—for any given discharge the stage at the station and the slope are likewise variable. In a preceeding discussion titled "Theoretical Considerations," it was demonstrated that for the above variable conditions, discharge can be related to stage and slope. Because the slope between two fixed points is measured by the fall between those points, it is more convenient to express discharge as a function of stage and fall.

Stage-fall-discharge ratings are usually determined empirically for observations of (1) discharge, (2) stage at the base gage, which is usually the upstream gage, and (3) the fall of the water surface between the base gage and an auxiliary gage. The general procedure used in developing the ratings is as follows:

1. A base relation between stage and discharge for uniform flow or for a fixed backwater condition is developed from the observations. The discharge from that relation is termed Q_r.
2. The corresponding relation between stages and the falls for conditions of uniform flow or fixed backwater is developed. Those falls are termed rating falls, F_r. Figure 188 shows schematically three forms the stage-fall relation may have.
3. The ratios of discharges Q_m, measured under conditions of variable backwater, to Q_r, are correlated with the ratios of the measured falls F_m to the rating falls F_r. Thus,

$$\frac{Q_m}{Q_r} = f\left(\frac{F_m}{F_r}\right). \tag{78}$$

The form of the relation depends primarily on the channel features that control the stage-discharge relation. The relation commonly takes the form,

$$\frac{Q_m}{Q_r} = \left(\frac{F_m}{F_r}\right)^N, \tag{78a}$$

where N varies from 0.4 to 0.6, the theoretical value of N being 0.5. Generally speaking, the stage-fall-discharge rating can be extrapolated with more confidence when the data are such that they fit equation 78a best when an N value of 0.5 is used.

The fall between the base and auxiliary gage sites, as determined from recorded stages at the two gages, may not provide a true representation of the slope of the water surface between the two sites. That situation may result from the channel and gaging conditions that are described below.

First, the water surface in any reach affected by backwater is not a plane surface between points in the reach, as sinuosity of the channel

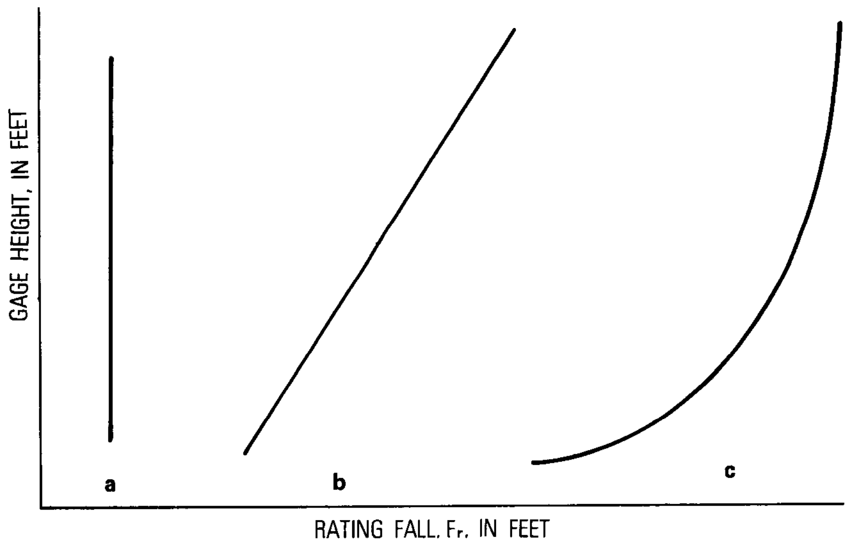

FIGURE 188.—Schematic representation of typical stage-fall relations. Curve (a), rating fall constant; curve (b), rating fall a linear function of stage; curve (c), rating fall a curvilinear function of stage.

will produce variations in the height of the water surface, both across and along the reach; variations in channel cross section and the effects of backwater also tend to produce curvature of the water surface. The slope determined from observed differences in stages is that of a chord connecting the water-surface elevations at points at the ends of a reach. It may not represent the slope of the water surface at either end of the reach but may be parallel to a line that is tangent to the water surface at some point in the reach.

Second, no reach of a natural stream selected for the determination of slope is completely uniform. The area of the cross section may vary considerably from point to point in the reach, but more important is the effect that shoals, riffles, rapids, or bends in the stream channel within the reach may have on the slope of the water surface, as well as on the energy gradient.

Third, the positions of the gages at the ends of the reach with respect to the physical features of the channel may have a material effect on the recorded gage heights and hence on the indicated slope. For example, if one gage is on the inside of a rather sharp bend and the other on the outside of a similar bend, the slope computed from records of stages at those gages may be widely different from the average slope of the water surface. Also, if differing drawdown effects exist at the intakes of the two gages, the two stage records obtained may not provide a true index of the water-surface slope.

Fourth, both gages may not be set to exactly the same datum, the difference in datum possibly being a large percentage of the total fall if the fall is small. The slope determined from gages not set to the same datum would not indicate the true water-surface slope because the computed slope would include the quantity y/L, where y is the difference in datum and L is the length of the reach.

Because of those conditions, theoretical relations between stage, fall, and discharge cannot be directly applied, and the relations must be empirically defined by discharge measurements made throughout the range of backwater conditions. Thus, the "best" value of the exponent of F_m/F_r in equation 78a will often be found to be in the range from 0.4 to 0.6, rather than having the theoretical value of 0.5; or, it may even be necessary to depart from a pure exponential curve in order to fit the plotted points satisfactorily. At other times the substitution of a term, $F+y$, for F values in equation 78a will improve the discharge relation. The use of a constant, y, whose best value is determined by trial computations, compensates in part for the inaccuracies in the value of F that were discussed above.

It is convenient to classify stage-fall-discharge ratings according to the types of relation that may be developed between stage and rating fall. The two types are:

1. *Rating fall constant.*—This type of relation (curve *a* in fig. 188) may be developed for channels that tend to be uniform in nature and for which the water-surface profile between gages does not have appreciable curvature.

2. *Rating fall a function of stage.*—This type of relation (curves *b* and *c* in fig. 188) may be developed if any of the following conditions exist:

   a. appreciable curvature occurs in the water-surface profile between gages;
   b. the reach is nonuniform;
   c. a submerged section control exists in the reach between gages, but the control does not become completely drowned by channel control even at high discharges; and
   d. a combination of some of the conditions listed above.

It is not uncommon for variable backwater to be effective for only part of the time. That follows from the two general principles that apply to backwater effect. The first states that for a given stage at the variable control element, backwater decreases at the base gage as discharge increases. For example, in a long gage reach of fairly steep slope, a given stage at the variable control element may cause significant backwater at the base gage when the discharge in the gaged stream is low but cause no backwater during periods when the discharge is high. The second principle states that for a given discharge, backwater decreases at the base gage as stage decreases at the variable control element. For example, at a given discharge in the gaged stream a high stage at the variable control element may cause significant backwater at the base gage, but a low stage at the variable control element may cause no backwater.

Other basic principles and detailed procedures used in defining stage-fall-discharge ratings are discussed on the pages that follow. The discussions are arranged in accordance with the preceding classification of stage-rating fall relations. A knowledge of the hydraulic principles applicable to a given slope reach is essential as a guide to the empirical analysis of the data.

RATING FALL CONSTANT

GENERAL DISCUSSION OF RATING PRINCIPLES

In uniform channels the water-surface profile is parallel to the bed; the slope, and therefore the fall, is the same for all discharges. The rating fall, F_r, for the condition of no variable backwater (uniform-flow conditions) would be the same at any stage. The stage-discharge relation with no backwater could be described by the Chezy equation,

$$Q_0 = CA_0\sqrt{R_0 S_0},$$

where the subscripts denote uniform flow; or by the equation,

$$Q_r = CA\sqrt{RF_r/L,} \tag{79}$$

where the subscripts denote the base rating conditions.

If variable backwater is imposed on the reach by a downstream tributary, the measured fall, F_m, and measured discharge, Q_m, would be less at a given stage than indicated by the uniform-flow rating. If the slope or fall as measured truly represents the slope at the base gage, those measurements would define, as shown in figure 189, a family of stage-discharge curves, each for a constant but different value of fall. The relation of each curve in the family to the curve for base rating conditions according to equation 79, is expressed by the equation,

$$\frac{Q}{Q_r} = \sqrt{\frac{F}{F_r}} \tag{80}$$

The discharge under variable backwater conditions may be computed as the product of (*a*) the discharge Q_r from the base rating and (*b*) the

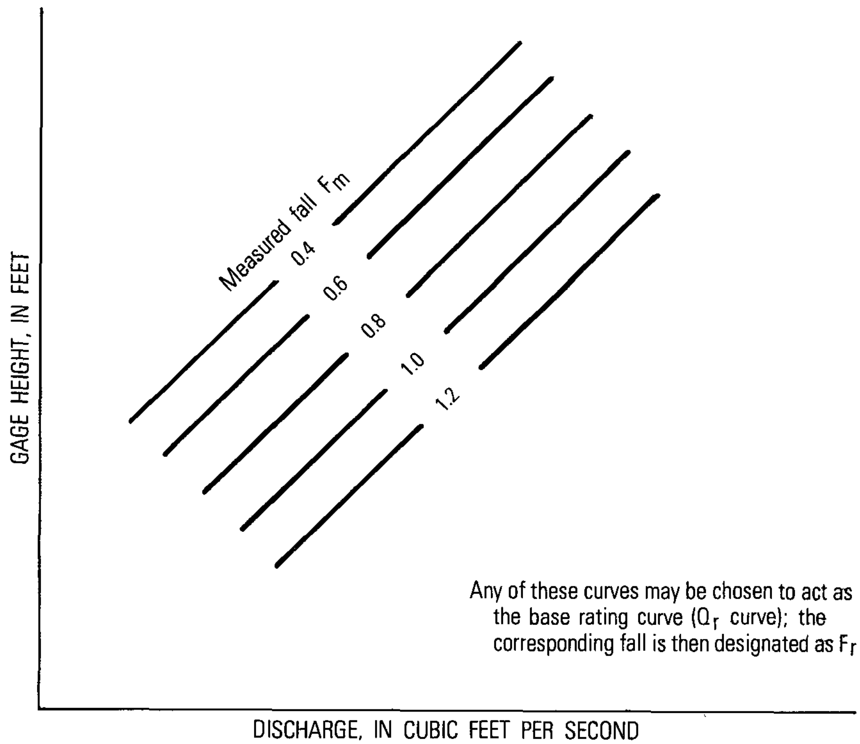

FIGURE 189.—Schematic representation of family of stage-discharge curves, each for a constant but different value of fall.

square root of the ratio of the measured fall to the constant-value rating fall.

A constant rating fall may also exist at sites where the base rating is controlled by a dam downstream from the reach in which fall is measured. If the curvature in the backwater profile is not significant, and if the channel is uniform, the water-surface profile will approximately parallel the channel-bed profile at all discharges. For example, the curve in figure 189 for a constant fall of 1.2 ft may be taken to represent the base stage-discharge relation for a fixed or stable control element. The curve for lesser falls that might result from variable submergence of the dam, are theoretically related to this base curve by the square root of the fall ratios, as described above. Quite commonly a constant value of 1.0 ft is used for F_r in equation 80. That special case of the constant rating-fall method, usually referred to as the unit-fall method, simplifies the computations because equation 80 then reduces to

$$Q_r = Q/\sqrt{F}. \tag{81}$$

A constant rating fall is not the usual case encountered in natural streams. However, if discharge measurements cover the entire range of flow conditions and if such measurements conform to a constant rating fall, there is no need to use a more complicated technique. If profile curvature and velocity-head increments are truly negligible, the relation between the discharge ratio and fall ratio should resolve into a single curve; otherwise the relation may be a family of curves with stage as a third variable.

PROCEDURE FOR ESTABLISHING THE RATING

The general procedure used in establishing a stage-fall-discharge rating with constant rating fall is outlined as follows:

1. Plot all discharge measurements using stages at the base gage as ordinates and discharges as abscissas, and note the measured fall (F_m) beside each plotted point. If the information on this plot indicates a family of curves, each corresponding to a constant value of fall (fig. 189), the use of a constant rating fall should be investigated.

2. The most satisfactory type of constant-fall rating, from the standpoint of high-water extrapolation, is one whose discharge ratio-fall ratio relation is a pure parabolic relation, as in equation 80, with the exponent equal to, or nearly equal to, 0.5. If such a relation fits the measured discharges, the results are unaffected by whatever value of constant fall (F_r) is used. For convenience, unit fall is used, as in equation 81.

3. For each discharge measurement (Q_m), compute Q_r by use of the equation $Q_r = Q_m/(F_m)^{0.5}$.

4. Plot values of gage height versus Q_r for each discharge measurement and fit a curve to the plotted points to obtain the Q_r discharges from the Q_r rating curve.

5. Compute and tabulate the percentage departures of the plotted Q_r discharges from the Q_r rating curve.

6. Repeat steps 3–5, using exponents of F_m other than, but close to, 0.5. Try exponents equal to 0.40, 0.45, 0.55, and 0.60.

7. Compare the five Q_r rating curves and select the curve that best fits the plotted points used to define it. In steps 8 and 9 that follow, the discharges from that "best" rating curve will be referred to as Q_{rd}, and the corresponding exponent of F_m will be referred to as d.

8. If the plotted discharges closely fit the Q_{rd} rating curve, that curve and the relation of (Q_m/Q_{rd}) to F_m are accepted for use.

9. If the plotted discharges do not closely fit the Q_{rd} rating curve repeat steps 3–5, using the exponent d but substituting the term (F_m+y) for F_m. Several values of y, a small quantity that may be either positive or negative, are tried to obtain a Q_r rating curve that closely fits the plotted discharge.

10. Compare the various Q_r rating curves obtained from step 9 and select the curve that best fits the plotted points used to define it. If the plotted discharges closely fit that Q_r rating curve, that rating curve and the corresponding relation of (Q_m/Q_r) to (F_m+y) are accepted for use. If the fit is not considered to be sufficiently close, the use of a pure parabolic relation, such as equation 81, is abandoned and the strictly empirical approach described in the following steps is used.

11. From the family of stage-discharge curves discussed in step 1, select one as the base Q_r curve and use the constant fall for this curve as F_r.

12. Compute the ratios Q_m/Q_r and F_m/F_r, plot the discharge ratios as ordinates and the fall ratios as abscissas, and draw an average curve through the plotted points that passes through the point whose coordinates are 1.0, 1.0.

13. Adjust each measured discharge by dividing it by the discharge ratio corresponding to the fall ratio on the above curve. Plot these computed values of Q_r against stage, and draw an average curve (Q_r curve) through the plotted points.

14. Repeat steps 11–13 using alternative constant values of F_r until the best relation between stage, fall, and discharge is established.

15. If the best relation derived from the application of steps 11–14 is still unsatisfactory, use the more flexible method described in the section titled, "Rating Fall a Function of Stage."

EXAMPLE OF RATING PROCEDURE

The stage-fall-discharge rating for Tennessee River at Guntersville, Ala. is presented in figure 190 as an example of a rating with constant rating fall. The upper gage is a water-stage recorder installed in a well attached to a pier of a highway bridge. The lower gage is a water-stage recorder installed on the right bank 43,700 ft below the upper gage and 3,300 ft above Guntersville Dam. The channel conditions in this reach are reasonably uniform. Variable backwater is caused by the operations at Guntersville Dam.

A satisfactory relation between stage, fall, and discharge could not be established for the upper (base) gage by use of the procedures for a pure parabolic fall-ratio curve that are described in steps 1–10. The empirical approach described in steps 11–14 was therefore used. The best rating was obtained by using a value of F_r equal to 1.5 ft. The fall-ratio curve in figure 190 approximately fits equation 80 for all fall ratios no greater than 1.0; for fall ratios greater than 1.0 the curve is flatter than a parabola defined by equation 80.

To plot, on the Q_r rating curve, a subsequent discharge measurement (Q_m) having a fall F_m, the fall ratio, F_m/F_r or $F_m/1.5$, is first computed. The fall-ratio curve is then entered with the computed fall ratio, and the discharge ratio, Q_m/Q_r, is read. Q_m is then divided by that value of the discharge ratio to give the value of Q_r to be plotted.

The method of obtaining the discharge corresponding to a given gage height and a given fall (F_m) is explained in the section titled, "Determination of Discharge from Relations for Variable Backwater."

RATING FALL A FUNCTION OF STAGE

GENERAL DISCUSSION OF RATING PRINCIPLES

Where variable backwater is a factor in the discharge rating, it will generally be found that fall is a function of stage. The average relation between fall and discharge may be linear, or fall may be a complex function of stage. Rating principles are best discussed by reference to examples.

The right-hand graph in figure 191 for the Columbia River at The Dalles, Oreg., is an example of a linear relation between stage and fall. The stage-discharge relation at the base gage is affected by reservoir operations at Bonneville Dam, more than 80 miles downstream. The auxiliary gage is located at Hood River bridge, 19 miles downstream from the base gage. Within the range of measured discharges, fall increases linearly with stage.

A much more complex stage-fall relation is shown in the right-hand graph in figure 192 for the Ohio River at Metropolis, Ill. At the downstream (auxiliary) gage, the stage-discharge relation is affected only at the lower stages by a constriction, the backwater from which causes fall to decrease with stage in the slope reach. At the higher

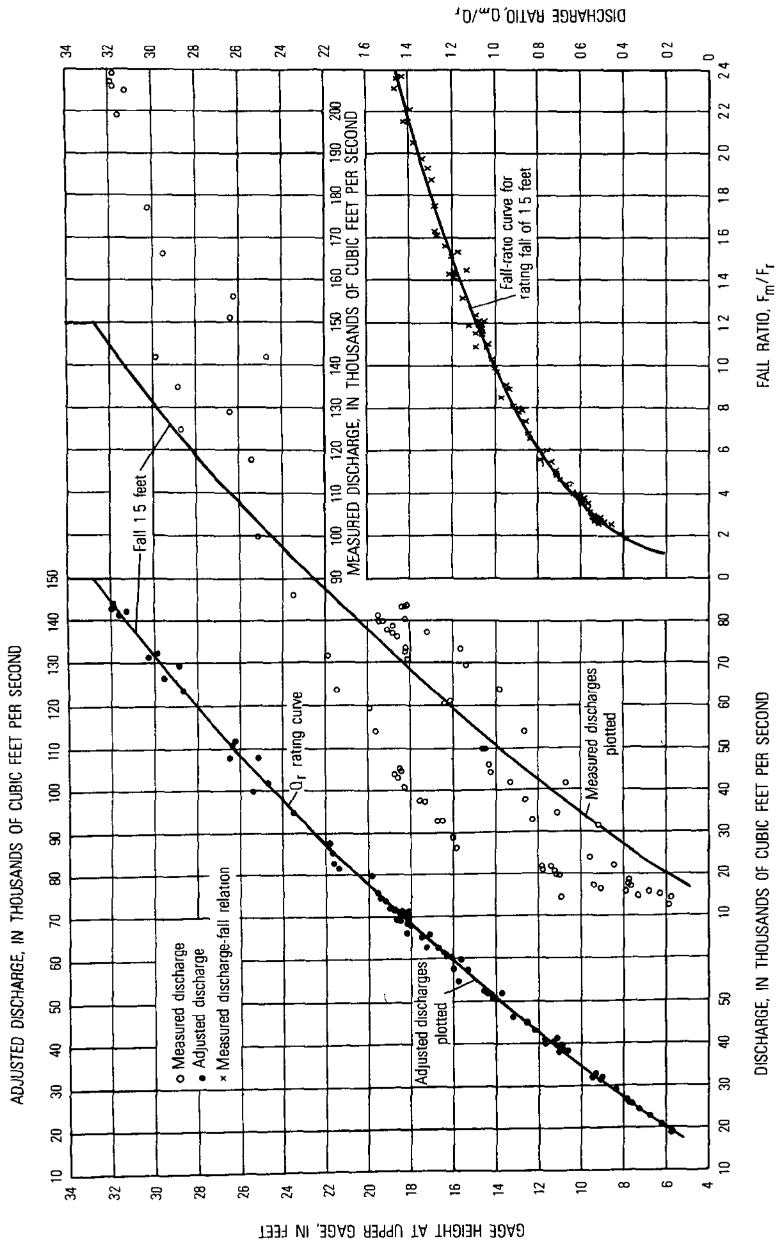

FIGURE 190.—Stage-fall-discharge relations for Tennessee River at Guntersville, Ala.

stages the constriction has little effect and fall increases with stage.

Another example of a complex stage-fall relation is shown in the right-hand graph in figure 193 for Kelly Bayou near Hosston, La. The base gage for this rating is about 2.7 miles upstream from the mouth of Kelly Bayou. The auxiliary gage is on Black Bayou, 4.2 miles downstream from the base gage. At low stages, fall increases with stage; at medium and high stages the backwater effect from Black Bayou is more pronounced and fall tends to assume a constant value.

Where a section control exists just downstream from the base gage, it is necessary to identify those situations when backwater effect is absent at the base gage. Obviously there will be no backwater when the tailwater at the section control is below the crest of the control. Most artificial controls are broad-crested, and submergence is generally effective only when tailwater rises to a height above the crest that is equal to or greater than 0.7 times the head on the control. Looked at another way, submergence is effective only when the fall between the upstream and downstream stages is equal to or less than 0.3 times the head on the control. Thus a straight line of initial submergence may be drawn on the curve of stage versus fall; the line passes through the coordinates representing the elevation of the control crest and zero fall, with a slope of 3 ft of stage per foot of fall.

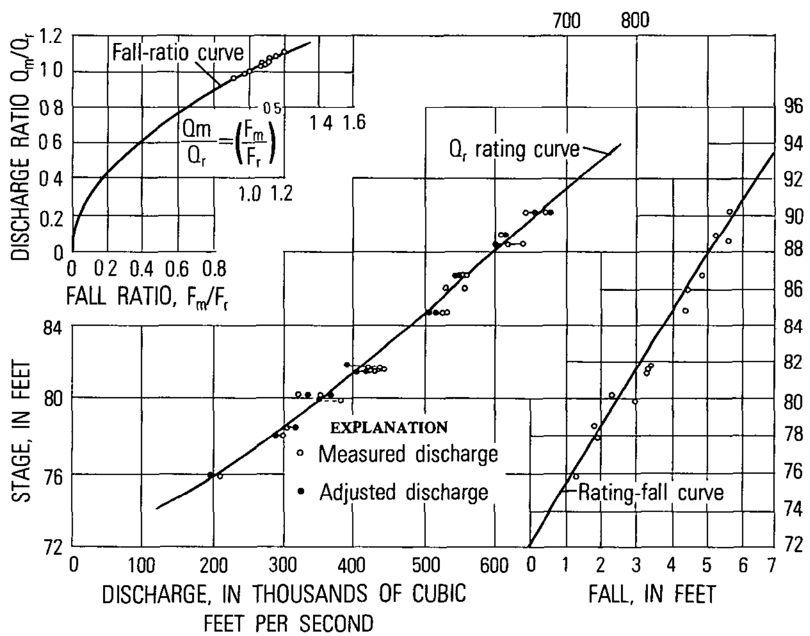

FIGURE 191.—Stage-fall-discharge relations for Columbia River at The Dalles, Oreg.

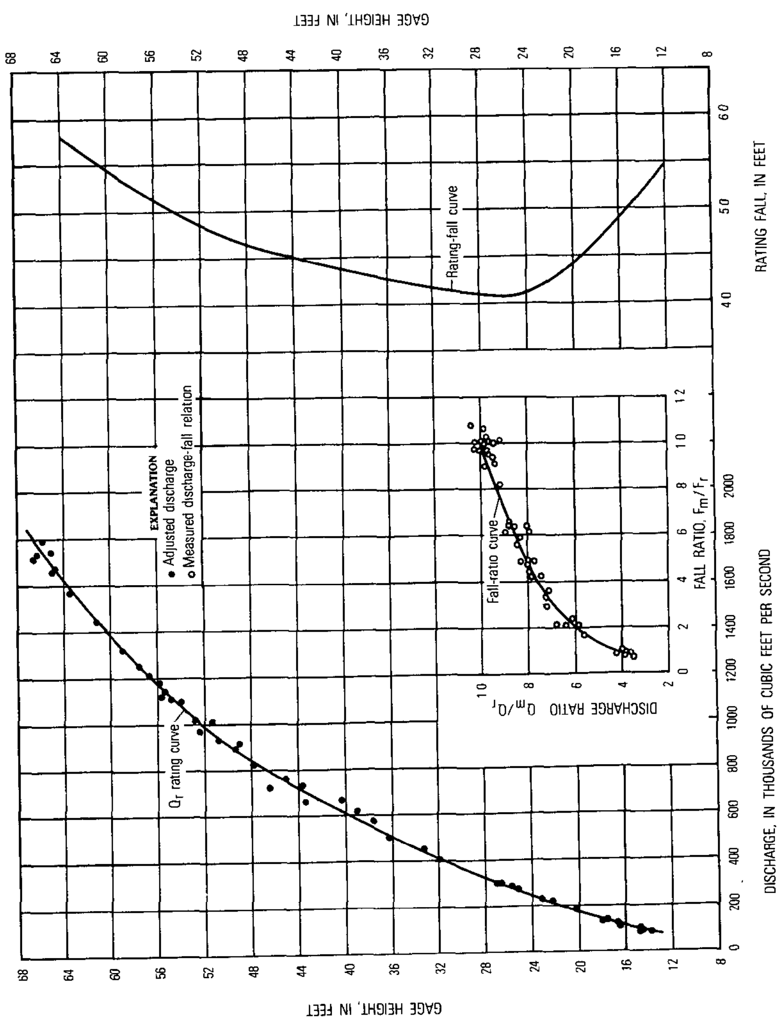

FIGURE 192.—Stage-fall-discharge relations for Ohio River at Metropolis, Ill.

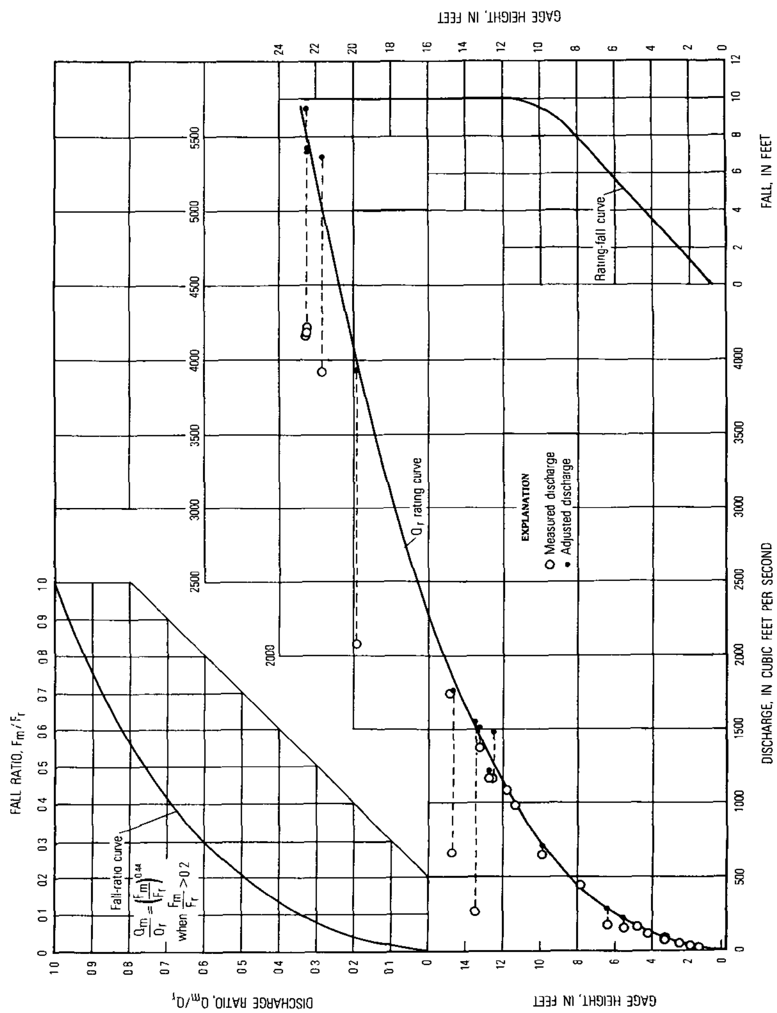

FIGURE 193.—Stage-fall-discharge relations for Kelly Bayou near Hosston, La.

The precise position and slope of the line will depend on the location of the downstream auxiliary gage with respect to the section control. If the auxiliary gage is immediately downstream from the control, the line of initial submergence will have the position and slope stated above. If the auxiliary gage is far downstream from the control, the line on the stage-fall graph will intersect the elevation of the control crest at a value of fall greater than zero, and the slope of the line will depend on the hydraulic features of the station; field observation will be necessary to define the graph coordinates of the line of initial submergence. All observed or recorded values of fall that lie below the line of initial submergence indicate free-fall discharge (discharge unaffected by the tailwater elevation); all observed or recorded values of fall that lie above the line of initial submergence indicate discharge affected by variable backwater. Furthermore, if the auxiliary (tailwater) gage is close to the control, the fall-ratio curve for discharges affected by backwater should closely fit the theoretical equation,

$$(Q_m/Q_r) = (F_m/F_r)^{0.5}.$$

If the auxiliary (tailwater) gage is distant from the control, the fall-ratio curve will depart from the theoretical equation.

The right-hand graph in figure 194 shows the stage-fall relation for Colusa Weir near Colusa, Calif. The base gage for the station is a short distance upstream from an ungated weir which acts as a section control, and the auxiliary gage is a short distance downstream from the control. There is no pool immediately upstream from Colusa Weir, the streambed being at the elevation of the weir crest; there is a drop of about 2 ft immediately downstream from the weir. The line of initial submergence shown crossing the lower part of the stage-fall relation has the theoretical position and slope discussed above. Colusa Weir is at the downstream end of a large natural detention basin along the left bank of the Sacramento River, and water that passes over the weir immediately enters the river. Because the river stage rises faster than the stage of the detention pool, fall decreases with stage at the base gage, as shown by the rating-fall curve.

The right-hand graph in figure 195 is a plot of stage versus fall for the Kootenay River at Grohman, B.C., Canada. The base gage for this station is on the west arm of Kootenay Lake about 2 miles upstream from Grohman Narrows. Downstream from the narrows is the forebay of the Corra Linn powerplant, and in the forebay is the auxiliary gage, about 8 miles downstream from the base gage. Grohman Narrows is the control for the base gage, but operations of Corra Linn Dam cause variable submergence of the control when the stage of the

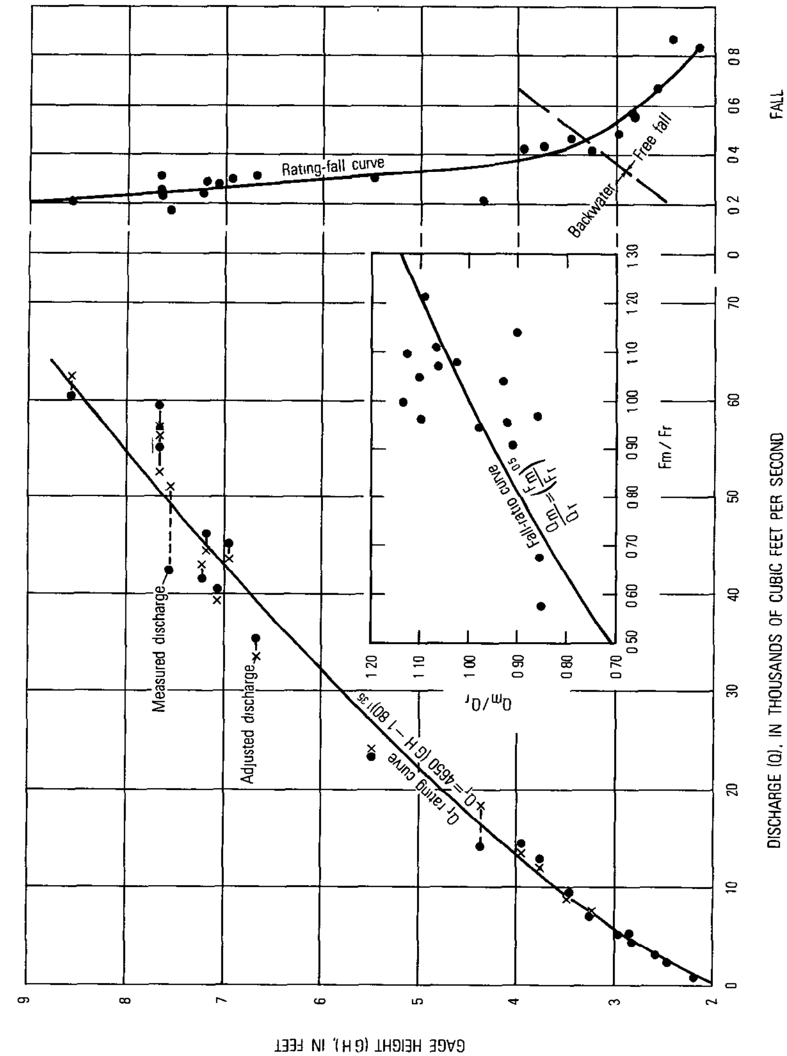

FIGURE 194.—Stage-fall-discharge relations for Colusa Weir near Colusa, Calif

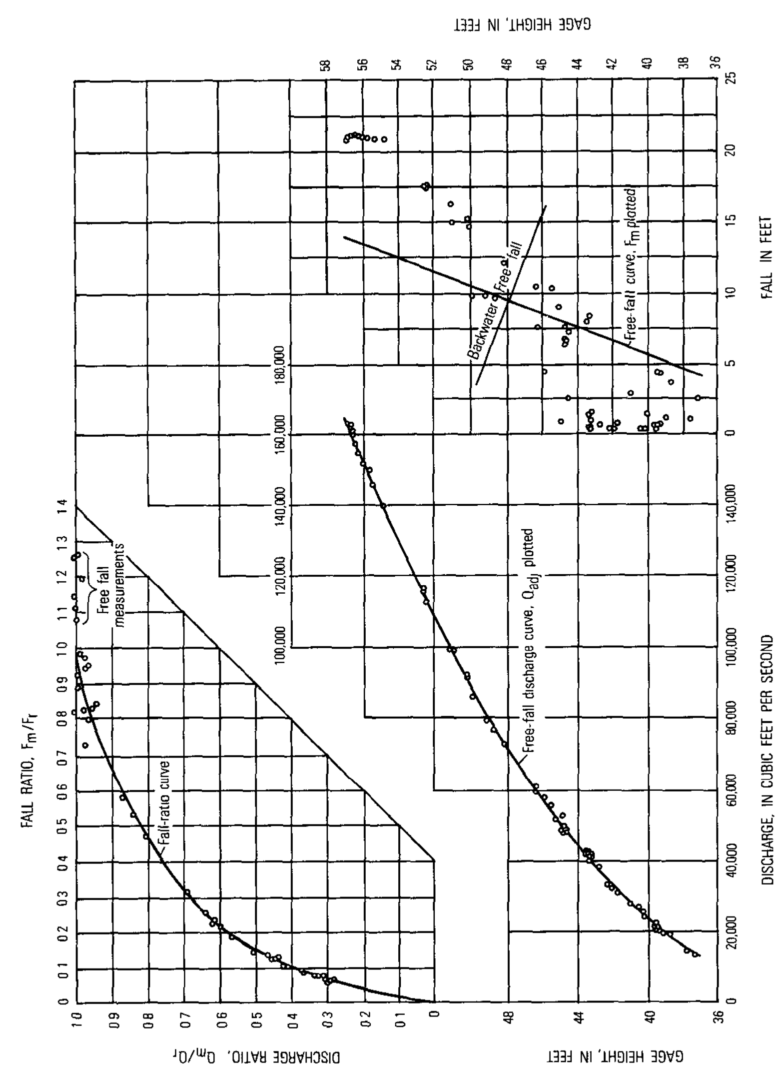

FIGURE 195.—Stage-fall-discharge relations for Kootenay River at Grohman, British Columbia, Canada. Measurements with falls less than 0.4 feet not plotted.

forebay is sufficiently high. The line of initial submergence, shown as the free-fall curve in figure 195, was determined from observation and discharge measurements. Discharge measurements whose values of fall plot below, or to the right of, the free-fall curve are unaffected by backwater and those discharges are therefore independent of fall. Discharge measurements whose values of fall plot above, or to the left of, the free-fall curve are affected by variable backwater. For those measurements the graph shows no apparent relation between stage and fall, and the free-fall curve (line of initial submergence) was used as the rating-fall curve for the measurements affected by variable backwater.

The rating for a gaging station whose base gage has no section control is analyzed in a manner similar to that previously described in the section on "Rating Fall Constant—Procedure for Establishing the Rating," the principal difference being that instead of using a constant value of rating fall, the rating fall for any stage is obtained from the rating-fall curve. The rating for a gaging station whose base gage has a section control is analyzed in two separate steps. The free-fall part of the rating (no variable backwater) is analyzed as explained in chapter 10, where simple stage-discharge relations are discussed. That part of the rating that is affected by variable backwater is analyzed as though no section control existed. It is not necessary to use the free-fall rating curve as the basis for establishing that part of the rating that is affected by variable backwater although that course of action is commonly followed.

Summary.—In view of the many different and complex situations that exist in natural channels, it is difficult to give general guidelines for establishing stage-fall-discharge relations. The analyst should make every effort to acquaint himself with the physical characteristics of the channel and the source of variable backwater. The best position of the relation curves that comprise the discharge rating must be determined by trial and error. The complexity of those relations determines, to a large degree, the number of discharge measurements necessary to define the discharge rating. Although the methods are empirical, experience has shown that there may be found a stage-discharge relation (the Q_r curve) which, taken in conjunction with its associated stage-fall relation (the rating-fall curve), will give close approximation to the true discharge under all possible combinations of stage and fall, by the application of a single-curve relation, Q_m/Q_r versus F_m/F_r. It is desirable, but not always possible, to have that relation take the theoretical form,

$$Q_m/Q_r = (F_m/F_r)^{0.5} \tag{80}$$

PROCEDURE FOR ESTABLISHING THE RATING

The general procedure used in establishing a stage-fall-discharge rating with variable fall is outlined as follows:

1. Plot all discharge measurements using stages at the base gage as ordinates and discharges (Q_m) as abscissas, and note the measured fall (F_m) beside each plotted point.
2. On another graph plot the measured fall (F_m) for each discharge measurement against stage at the base gage, using stage as the ordinate.
3. If the base gage has a section control, determine the position of the line of initial submergence on the plot of stage versus measured fall. Its position is based on discharge measurements known to have been made under conditions of free fall. Those measurements, plotted against stage on logarithmic graph paper, are fitted with a free-fall rating curve which is extrapolated in accordance with the principles discussed in chapter 10. The remaining measurements are added to the logarithmic rating plot; those measurements that plot to the left of the extrapolation are considered to be affected by backwater. That knowledge, along with a knowledge of the probable degree of submergence required to cause backwater effect, enables the analyst to fix the position of the line of initial submergence. Only those measurements that plot above, or to the left of, the line of initial submergence are used in the analysis of the rating for variable backwater that is discussed in the steps that follow.
4. Fit a curve, Q_r rating curve, to the stage-discharge plot in step 1, and another curve, F_r, or rating-fall curve, to the stage-fall plot in step 2.
5. From the curves in step 4 obtain values of Q_r and F_r corresponding to the stage of each discharge measurement.
6. Compute the ratios Q_m/Q_r and F_m/F_r for each discharge measurement.
7. Plot Q_m/Q_r as ordinate against F_m/F_r as abscissa, and on that graph draw the curve $Q_m/Q_r=(F_m/F_r)^{0.5}$.
8. On the basis of the scatter of the plotted points about the curve in step 7, adjust the Q_r and F_r curves (step 4) to obtain revised values of Q_r and F_r (step 5), such that the new ratios of Q_m/Q_r and F_m/F_r fit the curve in step 7 as closely as possible. The adjustments to the Q_r and F_r curve should not be so drastic that the adjusted curves are no longer smooth curves.
9. Repeat steps 4–8, using exponents of (F_m/F_r) other than, but close to 0.5. Try exponents equal to 0.40, 0.45, 0.55, and 0.60.
10. Compare the five plots of Q_m/Q_r versus F_m/F_r and select the one which shows the best fit between curve and plotted points. (The ratio of plotted values of Q_m/Q_r to curve values of Q_m/Q_r is identical with

the ratio of measured discharge to discharge obtained from the stage-fall-discharge relations.) In steps 11 and 12 that follow, the exponent of that best fall-ratio curve will be referred to as d.

11. If the plotted ratios closely fit the curve $(Q_m/Q_r) = (F_m/F_r)^d$, that curve and the corresponding Q_r and F_r curves are accepted for use.

12. If the plotted ratios do not closely fit the curve $(Q_m/Q_r) = (F_m/F_r)^d$, repeat steps 4–8, using the exponent d but substituting the terms $(F_m + y)$ for F_m and $(F_r + y)$ for F_r. Several values of y, a small quantity that may be either positive or negative, are tried to obtain a close fit between plotted points and the curve $(Q_m/Q_r) = \left[(F_m+y)/(F_r+y)\right]^d$.

13. Compare the various plots of the fall-ratio graph obtained from step 12 and select the one showing the best fit between curve and plotted points. If the fit is satisfactory, that curve and the corresponding Q_r and F_r curves are selected for use. If the fit is not considered to be sufficiently close, the use of a pure parabolic relation, such as

$$Q_m/Q_r = (F_m/F_r)^d \tag{82}$$

or

$$Q_m/Q_r = \left[(F_m+y)/(F_r+y)\right]^d \tag{83}$$

is abandoned and the strictly empirical approach described in the following steps is used.

14. Select one of the trial Q_r and F_r curves, such as were constructed in step 4, along with the corresponding values of Q_r, F_r, Q_m/Q_r, and F_m/F_r, such as were obtained in steps 5 and 6.

15. Plot the discharge ratios as ordinates and the fall ratios as abscissas, and draw an average curve through the plotted points that passes through the point whose coordinates are (1.0, 1.0).

16. On the basis of the scatter of the plotted points about the curve in step 15, adjust the Q_r and F_r curves (step 14), as well as the fall-ratio curve. Again, the reminder that the adjusted curves must remain smooth curves.

17. Repeat steps 14–16, using other trial curves of Q_r, F_r, and fall ratio versus discharge ratio, until the best relation is established between stage, fall, and discharge; in other words, until a close fit is obtained between plotted points and the fall-ratio curve.

18. After having obtained acceptable Q_r, F_r, and fall-ratio curves, plot adjusted values of the discharge measurements on the Q_r rating curve. The adjusted values are computed as follows: Given a measured discharge (Q_m) and a measured fall (F_m). Enter the F_r curve (stage-fall relation) with the gage height of the discharge measurement and read F_r. Next, compute the fall ratio, F_m/F_r, and enter

the fall-ratio curve to obtain the discharge ratio, Q_m/Q_r. Obtain the value Q_r to be plotted by dividing Q_m by (Q_m/Q_r).

The method of obtaining the discharge corresponding to a given gage height and a given fall (F_m) will be explained in the section titled, "Determination of Discharge from Relations for Variable Backwater."

EXAMPLES OF RATING PROCEDURE

Figures 191–195 are examples of stage-fall-discharge relations for slope stations where fall is a function of stage.

Figure 191 for a Columbia River station shows that excellent results were achieved in the range of discharge that was measured. The linear trend of fall increasing with stage is clearly evident, and the fall-ratio curve not only is represented by the theoretical equation 80, but is closely fitted by the plotted points. Where the rating-fall curve (stage versus fall) is so well defined, the first estimate of the Q_r curve is usually made by the use of equation 80, in which Q would represent the measured discharges. The computed Q_r values for the discharge measurements would then be plotted against stage, and a curve fitted to the plotted points would represent the first trial Q_r curve.

Figure 192 for an Ohio River station is an extremely complex example, as can be seen from the shape of the rating-fall curve. It is not surprising that the fall-ratio curve could not be expressed by a simple parabolic equation such as equation 82 or 83.

Figure 193 for a station on Kelly Bayou shows that there is relatively minor effect from variable backwater at low stages. At medium and high stages, the variable stage of Black Bayou causes variable backwater at the base gage. The rating-fall used during high-water periods has the constant value of 10.0 ft. The fall-ratio curve, for values of F_m/F_r greater than 0.2, has the equation

$$Q_m/Q_r = (F_m/F_r)^{0.44}.$$

Because the exponent 0.44 does not differ greatly from its theoretical value of 0.5, the Q_r rating curve can be extrapolated with some confidence.

Figure 194 for Colusa Weir is an example of the stage-fall-discharge relation for a station whose base gage has a section control. There is no variable backwater at low flow, as shown by the 6 discharge measurements that plot below the line of initial submergence on the graph of stage versus fall. The remaining 16 discharge measurements show the effect of variable backwater. While the fit of adjusted measured discharges to the Q_r rating curve is not completely satisfactory, there is some satisfaction to be derived from the facts that the equation of the fall-ratio curve is theoretically correct and the fall-ratio curve balances the plotted points.

Figure 195 for a station on the Kootenay River is an example of the stage-fall-discharge relation for a station whose base gage has a control that is unsubmerged at high stages. Of the 59 discharge measurements shown, 23 were made under free-fall conditions; they plot below, or to the right of, the line of initial submergence on the graph of stage versus fall. The remaining 36 discharge measurements are affected by variable backwater and were used in the stage-fall-discharge analysis. Because the line of initial submergence was used as F_r in the analysis, the value of F_m for any measurement affected by backwater is less than F_r. Consequently the fall-ratio curve was fitted empirically to the plotted points and is not expressed by a simple parabolic equation such as equation 82 or 83.

DETERMINATION OF DISCHARGE FROM RELATIONS FOR VARIABLE BACKWATER

After the three necessary graphical relations are available—stage versus rating fall (F_r), stage versus rating discharge (Q_r), and Q_m/Q_r versus F_m/F_r—the graphs are converted to tables. The determination of discharge (Q_m) corresponding to a given stage and a given fall (F_m) proceeds as follows:

1) From the stage-fall table determine the rating fall, F_r, for the known stage.
2) Compute the ratio F_m/F_r.
3) From the table of discharge ratios, (Q_m/Q_r) and fall ratios (F_m/F_r), determine the value of the ratio Q_m/Q_r.
4) From the stage-discharge table, determine the rating discharge, Q_r, for the known stage.
5) Compute Q_m by multiplying the ratio Q_m/Q_r by the value of Q_r.

Much emphasis has been placed on obtaining a purely parabolic function, such as equation 82 or 83, for the relation between fall ratio and discharge ratio. Such a relation not only permits the analyst to extrapolate the Q_r curve with more confidence, but it also expedites the computation of discharge. For example equation 82 may be transposed to

$$Q_m = \left(\frac{Q_r}{F_r^{\;N}}\right)\left(F_m^{\;N}\right). \qquad (82a)$$

Two tables can be prepared, one giving the values of the quantity ($Q_r/F_r^{\;N}$) corresponding to stage, and the other giving values of ($F_m^{\;N}$) corresponding to values of F_m. The discharge is then computed as the

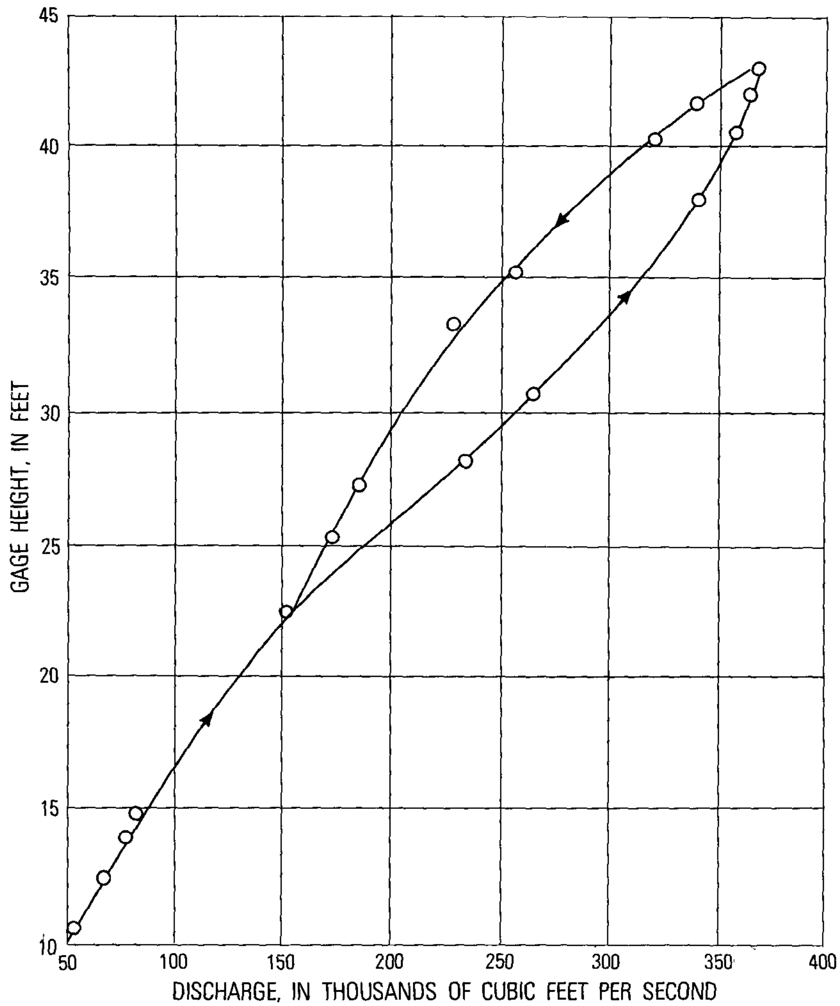

FIGURE 196.—Stage-discharge loop for the Ohio River at Wheeling, W. Va., during the flood of March 14–27, 1905.

product of the two values picked from the tables. Equation 83 may be transposed in a similar way.

VARIABLE SLOPE CAUSED BY CHANGING DISCHARGE

THEORETICAL CONSIDERATIONS

Where channel control is effective, the effect of changing discharge on a graph of the stage-discharge relation is such as to produce a loop curve (fig. 196), on which the discharge for a given stage is greater when the stream is rising than it is when the stream is falling. In other words, given a simple stage-discharge relation for steady flow—that is, a rating that averages all discharge measurements—it will be found that the measurements made on a rising stage plot to the right of the curve and those made on a falling stage plot to the left. The discharge measurements for individual flood waves will commonly describe individual loops in the rating. The departure of measurements from the rating curve for steady flow is of significant magnitude only if the slope of the stream is relatively flat and the rate of change of discharge is rapid. For gaging stations where this scatter of discharge measurements does occur, the discharge rating must be developed by the application of adjustment factors that relate steady flow to unsteady flow. (Unsteady flow refers to discharge at a site that changes appreciably with time, as in the passage of a flood wave.)

The relation between the discharges for steady and unsteady conditions at the same stage can be derived from the general equations for unsteady flow (Rouse, 1950). A simplified equation shown below may also be derived by neglecting all terms representing change of velocity head or acceleration.

$$\frac{Q_m}{Q_c} = \sqrt{1 + \frac{1}{S_c\, v_w}\,\frac{dh}{dt}} \tag{84}$$

where Q_m is the discharge for unsteady flow, Q_c and S_c are the discharge and energy slope for steady flow at the same stage, v_w is the wave velocity, and dh/dt is the rate of change of stage with respect to time (dh is positive for rising stages).

Because equation 84 is basic to the methods commonly used for adjusting discharge ratings for the effect of changing discharge, it is appropriate to elaborate on its derivation. The ratio of the magnitudes of two discharges that occur at a given stage is equal to the ratio of the square roots of their energy slopes. That principle can be expressed in the following basic equation, which is similar to equation 80 that was used in preceding sections of the manual.

$$\frac{Q_m}{Q_c} = \frac{\sqrt{S_m}}{\sqrt{S_c}} \tag{85}$$

where S_m is the energy slope for unsteady flow at the time of Q_m; the remaining terms are defined above for equation 84.

During changing discharge, the slope of the water surface increases or decreases by an increment of slope (ΔS), where

$$\Delta S = \frac{1}{v_w}\frac{dh}{dt} . \tag{86}$$

If we assume that the increment of slope by which the energy gradient changes is likewise equal to ΔS, then

$$S_m = S_c + \Delta S = S_c + \frac{1}{v_w}\frac{dh}{dt} . \tag{87}$$

By combining equations 85 and 87,

$$\frac{Q_m}{Q_c} = \left(\frac{S_c + \frac{1}{v_w}\frac{dh}{dt}}{S_c}\right)^{1/2} , \tag{88}$$

or

$$\frac{Q_m}{Q_c} = \left(1 + \frac{1}{S_c\, v_w}\frac{dh}{dt}\right)^{1/2} . \tag{84}$$

The wave velocity v_w in the above equations may be evaluated by the Seddon principle (Seddon, J. E., 1900).

$$v_w = \frac{1}{B}\frac{dQ}{dh} ,$$

where B is the width of the channel at the water surface, and dQ/dh is the slope of the stage-discharge curve for constant-flow conditions. From examination of formulas for mean velocity (V_m) in open channels, the ratio of wave velocity to mean velocity may be shown to vary as follows,

	Ratio v_w/V_m	
Channel Type	*Manning*	*Chezy*
Triangular	1.33	1.25
Wide rectangular	1.67	1.50
Wide parabolic	1.44	1.33

Experience seems to indicate that the most probable value of the ratio in natural channels is 1.3.

Equation 84 explains why the effect of changing discharge is significant only on flat streams during rapid changes in discharge; that combination is necessary to make the right-hand side of the equation differ significantly from unity. During rapid changes in discharge, absolute values of dh/dt are large. On flat streams both energy slope (S_e) and wave velocity (v_w) are small. The combination of a large value of dh/dt and small values of S_e and v_w gives the right-hand side of the equation a value that is significantly larger than unity during a rising stage (dh/dt is positive) and significantly smaller than unity during a falling stage (dh/dt is negative).

METHODS OF RATING ADJUSTMENT FOR CHANGING DISCHARGE

The two methods used to adjust discharge for the effect of changing slope attributable to changing discharge are the Boyer method and the Wiggins method. Both methods are based on equation 84. The knowledgeable reader of this manual may notice that the Jones and Lewis methods are not included among the techniques for adjusting discharge. Those two methods have been supplanted by the somewhat similar Boyer method and therefore are not described here. For a description of the Jones and Lewis methods the interested reader is referred to the manual by Corbett (1943, p. 159–165).

BOYER METHOD

The Boyer method provides a solution of equation 84 without the necessity for individual evaluation of v_w and S_e. The method requires numerous discharge measurements made under the conditions of rising and falling stage. Measured discharge (Q_m) is plotted against stage in the usual manner, and beside each plotted point is noted the value of dh/dt for the measurement. For convenience dh/dt is expressed in feet or meters per hour and the algebraic sign of dh/dt is included in the notation—plus for a rising stage and minus for a falling stage. A trial Q_c rating curve, representing the steady-flow condition where dh/dt equals zero, is fitted to the plotted discharge measurements, its position being influenced by the values of dh/dt noted for the plotted points. Values of Q_c from the curve corresponding to the stage of each discharge measurement, are used in equation 84, along with the measured discharge (Q_m) and observed change in stage (dh/dt), to compute corresponding values of the adjustment factor, $1/S_c v_w$. The computed values of $1/S_c v_w$ are then plotted against stage and a smooth curve is fitted to the plotted points. If the plotted values of $1/S_c v_w$ scatter widely about the curve, the Q_c curve is modified to produce some new values of $1/S_c v_w$ that can be better fitted by a smooth curve. The modifications of the curves of Q_c and $1/S_c v_w$ should not be so drastic that the modified curves are no longer smooth curves, nor should the modified shape of the Q_c rating curve violate the principles underlying rating curves, as discussed in chap-

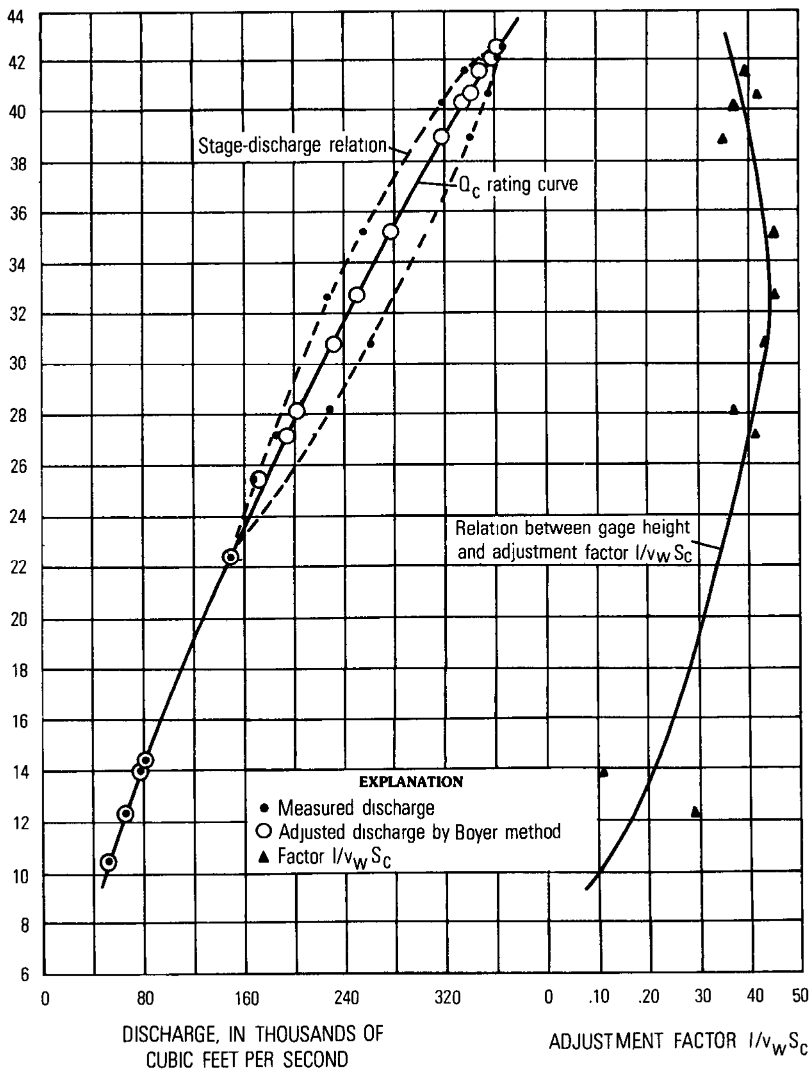

FIGURE 197.—Adjustment of discharge measurements for changing discharge, Ohio River at Wheeling, W. Va., during the period March 14–27, 1905.

ter 10. Construction of the two curves completes the rating analysis. Figure 197 is an example of such an analysis.

To adjust the value of subsequent discharge measurements for plotting on the Q_c rating curve, the adjustment-factor curve is first entered with the stage of the measurement to obtain the appropriate value of the factor, $1/S_c v_w$. Next, the observed value of dh/dt is used with that factor to compute the term, $\left(1+\frac{1}{S_c v_w}\ \frac{dh}{dt}\right)^{0.5}$. That term is then divided into the measured discharge (Q_m) to obtain the required value of Q_c.

To determine discharge from the Q_c rating curve and adjustment-factor curve, during a period when the stage and rate of change of stage are known, the procedure described above is used to obtain the value of the term $\left(1+\frac{1}{S_c v_w}\ \frac{dh}{dt}\right)^{1/2}$. That term is then multiplied by Q_c, which is obtained by entering the Q_c rating curve with the known stage. The product is the required discharge (Q_m).

WIGGINS METHOD

The Wiggins method is convenient for adjusting measured discharge (Q_m) for the effect of changing discharge to obtain the corresponding steady-flow discharge (Q_c). However, the reverse procedure of computing discharge for unsteady flow (Q_m) from the steady-flow discharge rating is rather complicated. Consequently, the Wiggins method is used only for those stations where only occasional adjustment of measured discharge at high stages is required. If the discharge is affected by changing stage on numerous days each year, the more accurate Boyer method of discharge adjustment should be used. Unlike the Boyer method, application of the Wiggins method does not require numerous discharge measurements that have been made under conditions of both rising and falling stage.

The discharge measurement adjusted by the Wiggins method are used to define the steady-flow rating, and that rating is used directly with the gage-height record to obtain daily values of discharge. That course of action is justifiable for those streams whose discharge is affected by changing discharge on only a few days each year. For that type of stream, it will generally be found that the discharge adjustment is less than 10 percent. On the affected days, the discharge obtained from the steady flow rating will be underestimated by a small percentage when the discharge is rising rapidly, and overestimated by a small percentage when the discharge is falling rapidly. The discrepancies are compensating, and if only few days are involved, the streamflow record is not significantly impaired. The advantage of applying the adjustment to discharge measurements made under unsteady-flow conditions is that the scatter of discharge measurements on the rating curve is reduced, and the rating curve can therefore be more precisely defined.

Application of the Wiggins method has been simplified by the preparation of diagrams that eliminate much of the computational labor. Figures 198*A*-*D* are used to determine the value of the energy slope

(S_m) at the time of the discharge measurement (Q_m), for combinations of values of mean velocity (V_m) and hydraulic radius (R). The Manning equation was used in preparing the graphs, and each of the four sheets is applicable for a particular value of Mannings n, as shown in the following tabulation:

Figure 198A—n=0.025 Smooth bed and banks.
198B—n=0.035 Fairly smooth.
198C—n=0.050 Rough.
198D—n=0.080 Very rough.

Figure 199 is used to determine the increment of energy slope $\left(\frac{1}{v_w}\frac{dh}{dt}\right)$ attributable to changing discharge, for combinations of values of flood-wave velocity (v_w) and rate of change of stage (dh/dt). Flood-wave velocity is assumed to equal $1.3V_m$.

Figures 200*A* and *B* are used to determine the factor to apply to the measured discharge (Q_m) to obtain the steady-flow discharge (Q_c). The factor, which is equal to

$$\left[\frac{S_m-\left(\frac{1}{\bar{v}_w}\right)\left(\frac{dh}{dt}\right)}{S_m}\right]^{0.5},$$

is given for combinations of values of S_m from figure 198 and of $\left(\frac{1}{v_w}\frac{dh}{dt}\right)$ from figure 199. (Note that the factor differs from that given in equation 88, because S_m is used here as the base slope, rather than S_c as in equation 88.) Figure 200*A* is used for rising stages and figure 200*B* is used for falling stages.

An example of the use of the Wiggins diagrams follows.

Given: a discharge measurement with the following data for a stream with fairly smooth bed (n=0.035);

Q_m=23,000 ft³/s
Area=53,900 ft²
Width= 2,700 ft
V_m=4.27 ft/s
Change in stage=0.87 ft in 1.5 hours (rising)

Compute adjusted discharge to be plotted on rating curve.

First compute: $R = \frac{\text{Area}}{\text{Width}} = \frac{53{,}900}{2{,}700} = 20$ ft

$v_w = 1.3\ V_m = 1.3 \times 4.27 = 5.55$ ft/s

$\frac{dh}{dt}$ =change in stage per hour $= \frac{0.87}{1.5} = 0.58$ ft/hr

Then: (a) Enter figure 198*B* with $V_m = 4.27$ and $R = 20$ and read $S_m = 0.00018$

(b) Enter figure 199 with $\frac{dh}{dt} = 0.58$ and $v_w = 5.55$ and read slope increment $(\frac{1}{v_w}\frac{dh}{dt}) = 0.000029$

(c) Enter figure 200*A* (rising stage) with $S_m = 0.00018$ and slope increment = 0.000029 and read factor = 0.915.

Adjusted discharge = 0.915 × 230,000 = 210,000 ft³/s.

Because the stage was rising, the unadjusted discharge would plot to the right of the rating curve. The computed adjustment moves the measurement to the left.

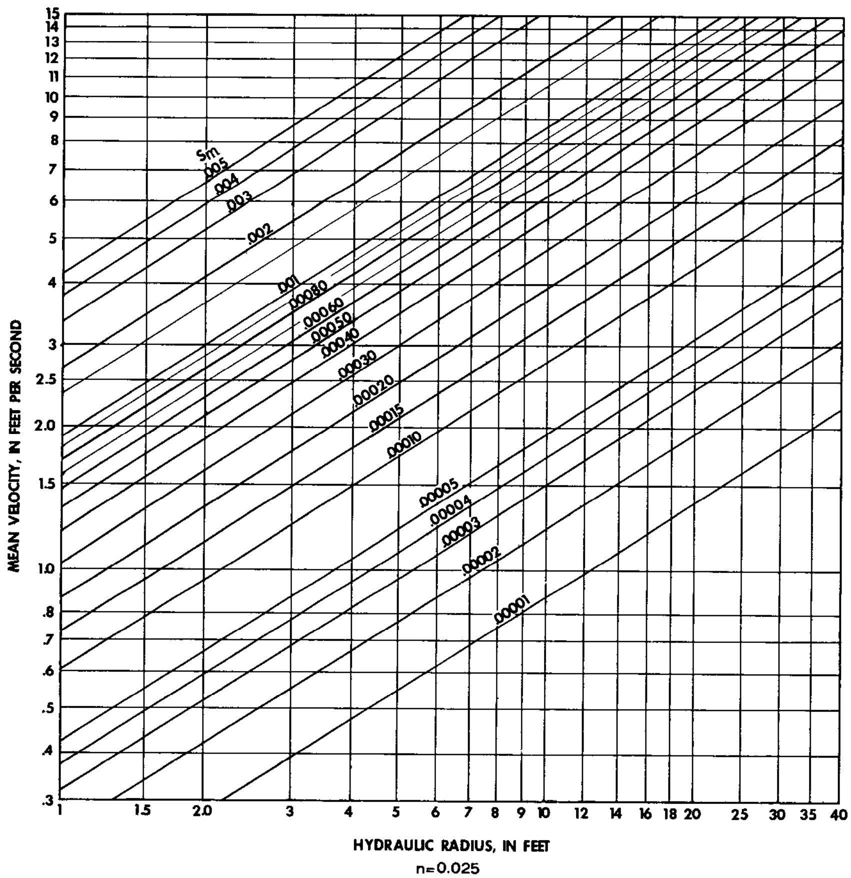

FIGURE 198*A*.—Diagram for solution of the Manning equation to determine S_m. Smooth bed and banks (n=0.025).

Both the measured (Q_m) and adjusted (Q_c) discharges are entered in the list of discharge measurements and both are plotted on the rating curve. Suitable symbols are used, however, to differentiate between the measured and adjusted discharges.

VARIABLE SLOPE CAUSED BY A COMBINATION OF VARIABLE BACKWATER AND CHANGING DISCHARGE

Where the rating for a gaging station is affected by a combination of variable backwater and changing discharge, the rating should be analyzed as though it were affected by variable backwater only, using the fall-rating methods described in the section titled, "Rating Fall a Function of Stage." The basic equation for variable-backwater adjustments (eq. 80) and that for changing-discharge adjustments (eq.

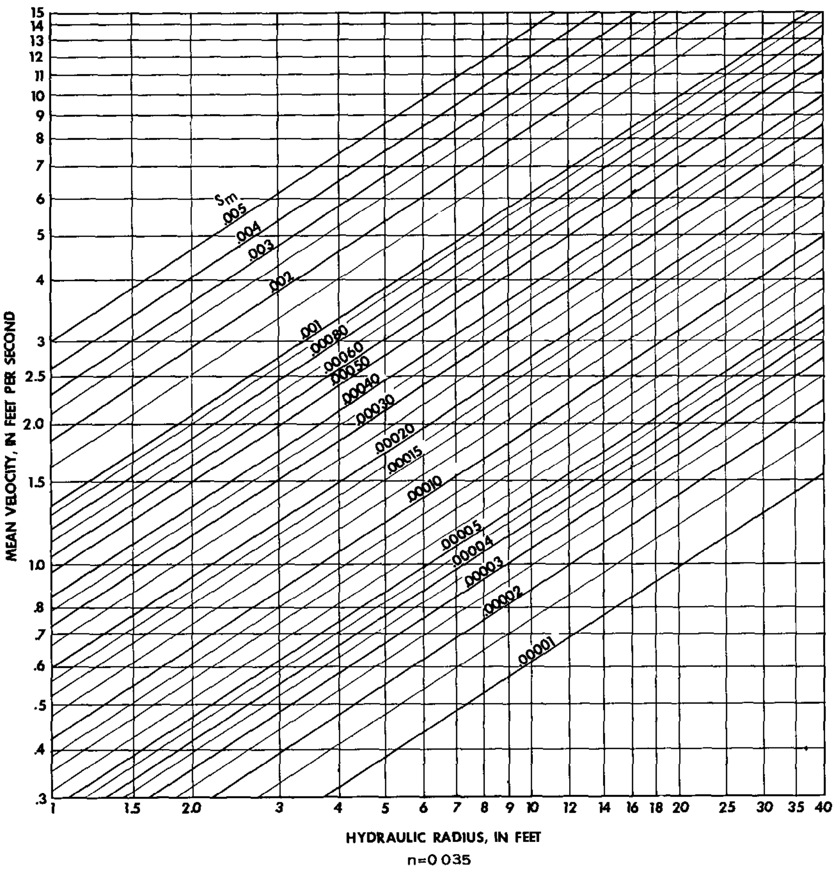

FIGURE 198*B*.—Diagram for solution of the Manning equation to determine S_m. Fairly smooth bed (n = 0.035).

85) are similar, but only the fall-rating methods are versatile enough to handle the combined effect of the two factors.

SHIFTS IN DISCHARGE RATINGS WHERE SLOPE IS A FACTOR

Changes in channel geometry (scour or fill) and (or) changes in flow conditions (vegetal growth) will cause shifts in the discharge rating where slope is a factor, just as they cause shifts in simple stage-discharge relations. When discharge measurements indicate a shift in the rating for a slope station, the shifts should be applied to the Q_r rating curve if the station is affected by variable backwater, or to the Q_c rating curve if the station is affected by changing discharge. Extrapolation of the shift curves should be performed in accordance with the principles discussed in chapter 10 for shifts in simple stage-

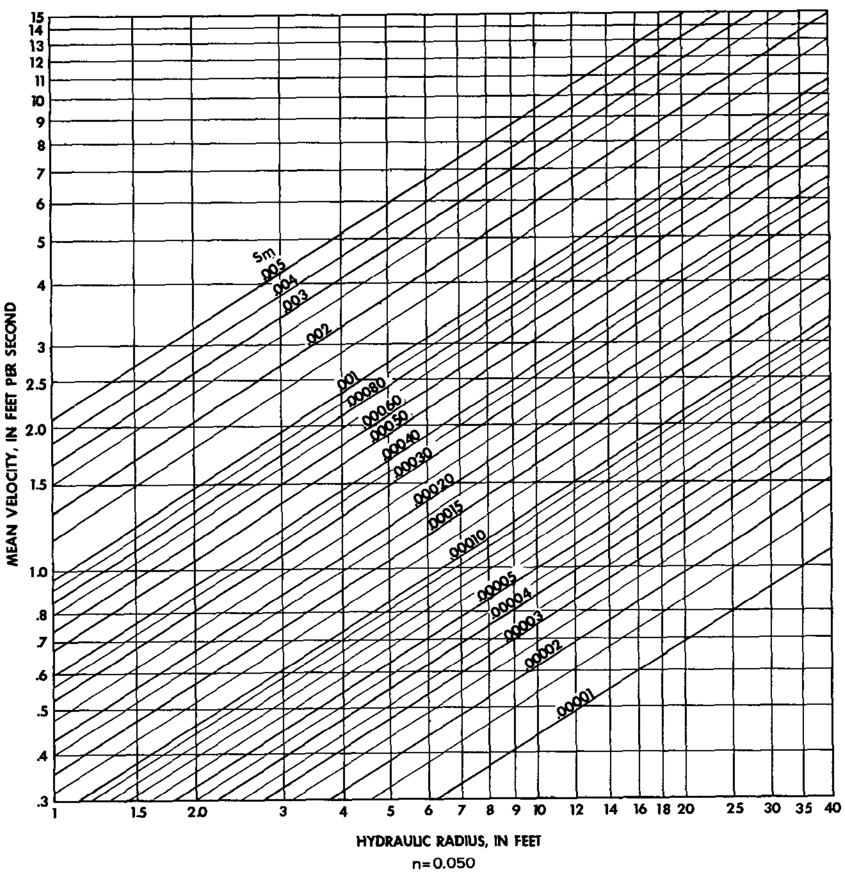

FIGURE 198*C*.—Diagram for solution of the Manning equation to determine S_m Rough bed (n=0.050).

discharge relations. (See section in chapter 10 titled, "Shifts in the Discharge Rating.")

A SUGGESTED NEW APPROACH FOR COMPUTING DISCHARGE RECORDS FOR SLOPE STATIONS

Now that the use of electronic computers has become commonplace, it appears that a fresh approach might be tried with regard to computing streamflow records for gaging stations equipped with a stage-recorder at each end of a slope reach. Instead of using the various graphical empiricisms that were described in this chapter, a computer program could be written to compute discharge for the reach by the Manning equation or by some similar equation for open-channel flow. (It is assumed that acceleration head can be neglected.) Dis-

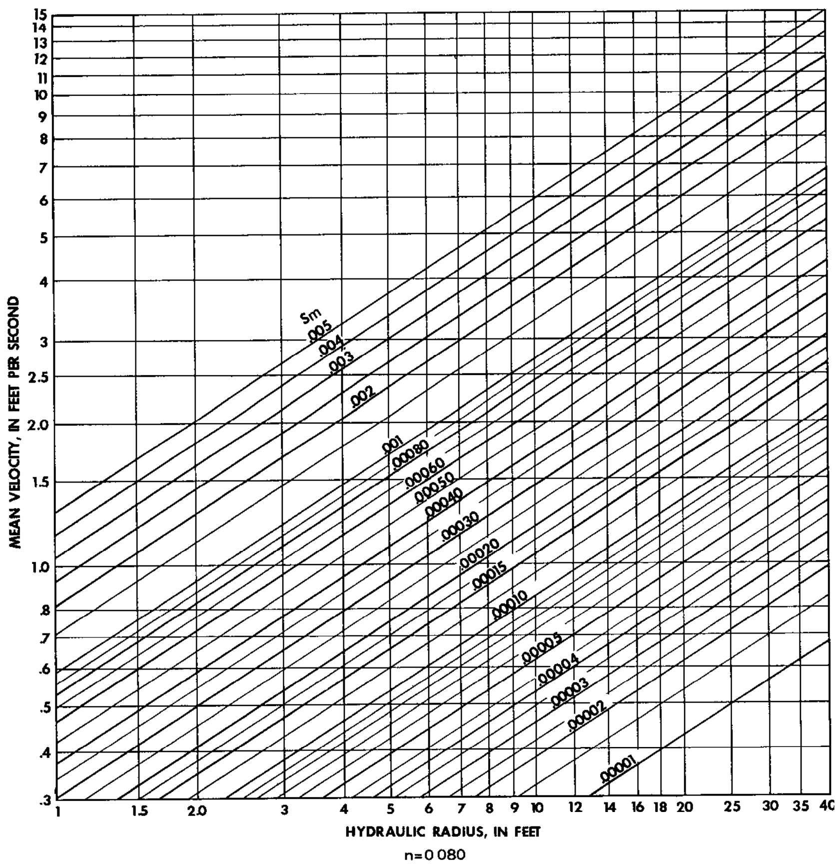

FIGURE 198*D*.—Diagram for solution of the Manning equation to determine S_m. Very rough bed (n=0.080).

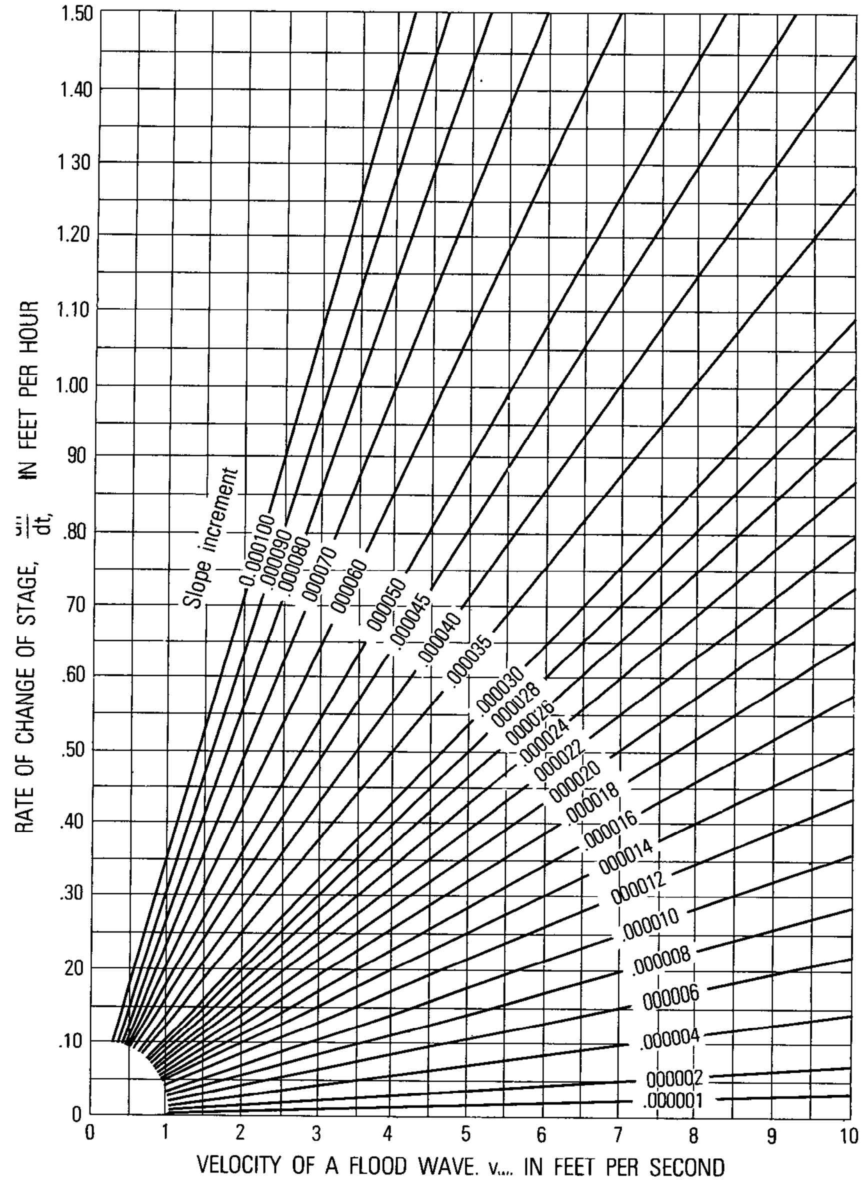

FIGURE 199.—Diagram for determining slope increment resulting from changing discharge.

charge measurements would be made solely for the purpose of determining the Manning roughness coefficient (n) from the measured discharge, thereby obtaining the only unkown factor needed to compute the conveyance (K) at each end of the slope reach.

The value of n computed from a discharge measurement usually would not represent the true value of the roughness coefficent but would actually be a "catchall" value that included the effect of error in the computed value of the energy slope in the reach. The computed values of n would likely vary with stage.

The discharge computations would proceed along the following lines. The basic form of the Manning equation is

$$Q = KS^{1/2} \tag{89}$$

where

Q is discharge;
K is conveyance, which is equal to $\frac{1.49}{n} AR^{2/3}$ (A is cross-sectional area and R is hydraulic radius); and
S is the energy gradient.

Equation 89 can be expanded to

$$Q = K_2 \sqrt{\frac{F}{\frac{K_2}{K_1} L + \frac{K_2^2}{2gA_2^2}\left[-\alpha_1\left(\frac{A_2}{A_1}\right)^2(1-k)+\alpha_2(1-k)\right]}} \tag{90}$$

where
F is fall in the reach,
L is length of reach,
g is the acceleration of gravity,
α is the velocity-head coefficient whose value is dependent on the velocity distribution in the cross section,
k is a coefficient of energy loss whose value is considered to be zero for contracting reaches and 0.5 for expanding reaches;
subscript 1 refers to the upstream cross section, and
subscript 2 refers to the downstream cross section.

For the cross section at each end of the slope reach, relations would be prepared between stage and each of the following three elements: K, A, and α. A computer program would be written to solve equation 90. Then, given the stage at each end of the reach, the computer would compute F, A, K, α, and finally, Q.

For those slope stations where the change in velocity head in the reach is so minor an item that it can be neglected, the conventional constant-fall method (see section titled, "Rating Fall Constant") could be continued in use; computer computation would be optional.

It is emphasized that the above method of computing discharge records is as yet untried, but it is suggested that it be tested.

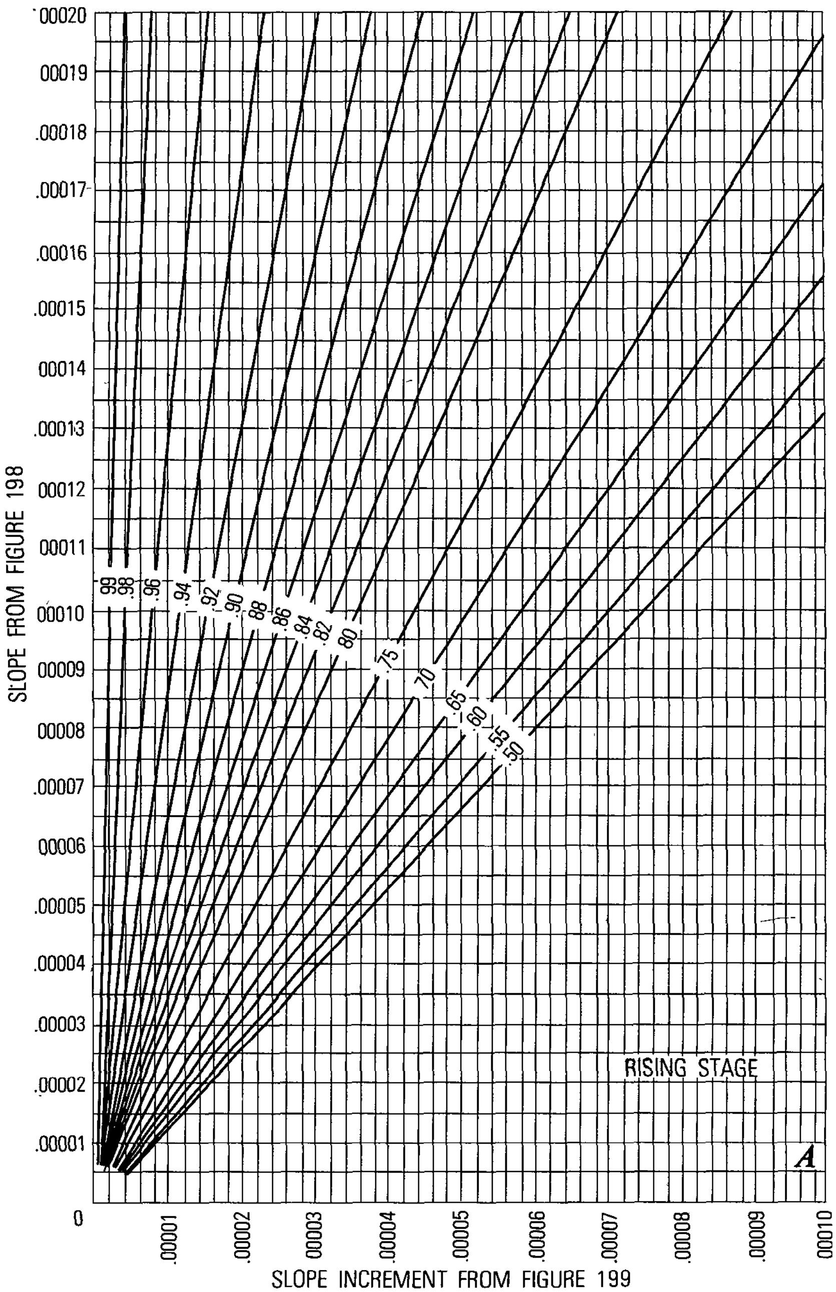

FIGURE 200*A*.—Diagram for determining factor to apply to measured discharge—rising stage.

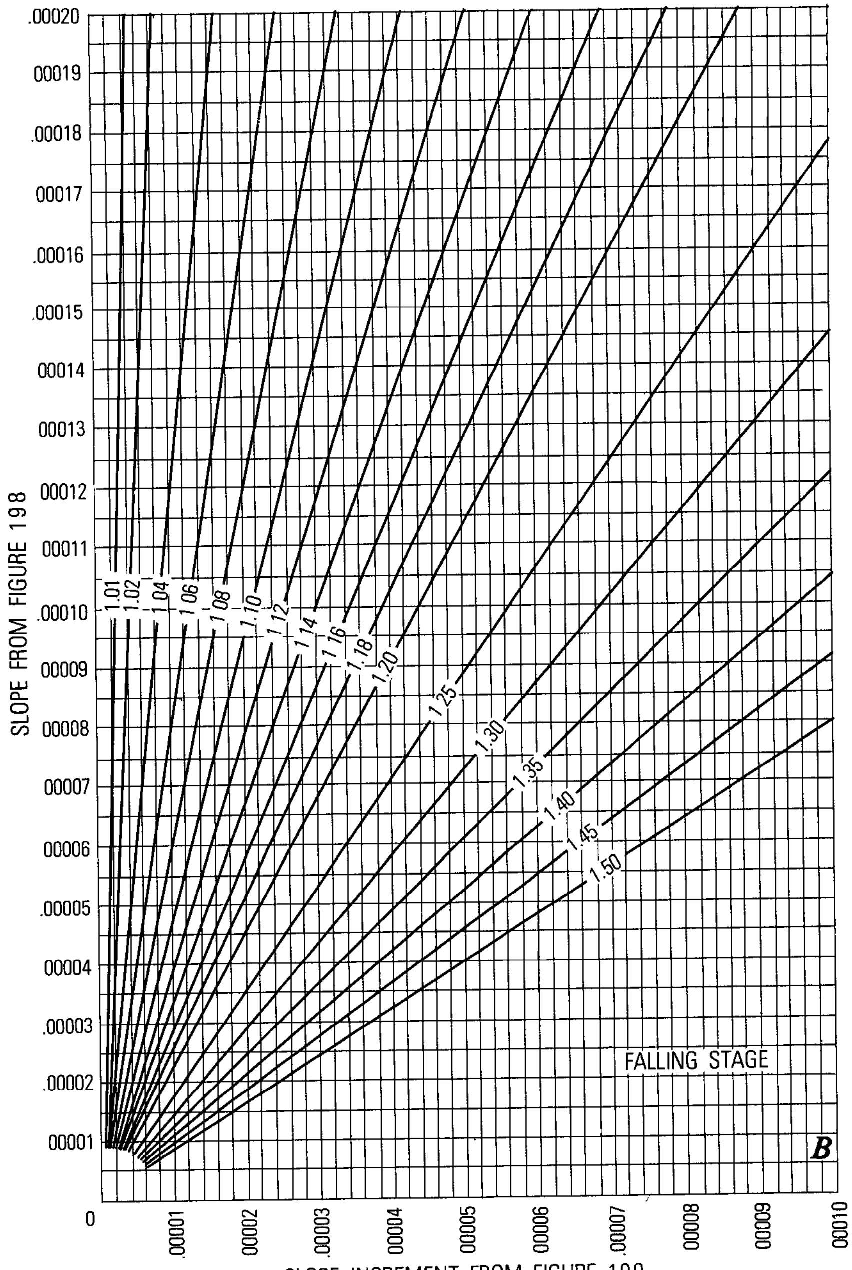

FIGURE 200*B*.—Diagram for determining factor to apply to measured discharge—falling stage.

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