CHAPTER 13—DISCHARGE RATINGS FOR TIDAL STREAMS

GENERAL

A discharge rating can be obtained for a tidal or tide-affected stream if a velocity index is used as a parameter in the rating along with stage. If the stream is quite small, a deflection meter is a satisfactory device for obtaining a continuous record of velocity; the electromagnetic method of stream-gaging is still in the experimental stage. For the larger streams the acoustic velocity meter is a satisfactory device for obtaining an index of velocity. (The instrumentation and methodologies involved in the use of deflection meters, electromagnetic velocity meters, and acoustic velocity meters were described in Chapter 12.) In the absence of such devices, there are two general approaches for obtaining a continuous discharge record—the theoretical approach involving evaluation of the equations of unsteady flow for a tide-affected reach of channel, and the empirical approach involving empirical relations whose effectiveness generally varies inversely with the degree of importance of the acceleration head (see section in chapter 11, titled "Theoretical Considerations"). The theoretical approach is much preferred.

Either approach requires a recording stage gage at each end of a long reach of channel. The two gages must be synchronized so that simultaneous stages at the two sites can be obtained from the stage records. Either approach requires discharge measurements for calibrating the discharge model; the moving-boat method of measuring discharge (chap. 6) is recommended for the larger streams.

EVALUATION OF UNSTEADY-FLOW EQUATIONS

It is beyond the scope of this manual to treat in detail the various methods of evaluating the equations of unsteady flow. Basic to all methods is the solution, by approximate step procedures, of the following pair of differential equations:

$$\frac{Q^2}{K^2} = -\frac{\partial H}{\partial x} - \frac{1}{g}\frac{\partial V}{\partial t} \qquad (122)$$

$$\frac{\partial Q}{\partial x} = -B\frac{\partial h}{\partial t}, \qquad (122a)$$

where Q is the discharge, K is the conveyance of the cross section, H is the total energy head, x the distance along the channel, g the acceleration of gravity, V the mean velocity, t the time, B the top width of the channel, and h is the water-surface elevation. Solution of the equations requires the use of a digital computer.

The four methods of equation evaluation that will be briefly discussed are:

1. power series,
2. method of characteristics,
3. implicit method, and
4. Fourier series.

For any of the four methods, it is necessary that a series of current-meter discharge measurements be made over several tidal cycles at either end of the reach. The measurements are needed to compute the resistance and conveyance properties of the reach and to serve as checks on the computed discharges. The series of measurements should include one series made at a time of low freshwater discharge and large tidal range, and another made when the freshwater discharge is high. These two series of measurements may be sufficient if the channel is stable with regard to scour and fill, but for an unstable alluvial channel, it is necessary that several additional series of discharge measurements be made. For meaningful results it is necessary that the channel either be stable (unchanging) or if subject to change, the channel changes must occur in the same way for each change in discharge during rises and subsequent recessions.

During a series of discharge measurements over a tidal cycle, enough measurements must be made to define the discharge hydrograph to an accuracy that will permit momentary discharges to be determined at 15-minute intervals for the duration of the cycle. The discharge measurements are commonly made at about hourly intervals and cover a few hours more than the duration of the tidal cycle.

The exact procedure used to measure the discharge in a tidal reach will vary with size of channel and flow conditions. For a small tide-affected stream, one or two field crews measuring continuously across the stream may be adequate for obtaining the data for an accurate definition of the discharge hydrograph. For a large tide-affected river, several measuring crews may be required, and it may be necessary to compute the hydrograph in the manner described for flash floods on small streams (see section in chapter 5 titled, "Measurement Procedures During Rapidly Changing Stage—Case B. Small Streams"). In that method a stage-mean velocity relation is first determined for each measurement vertical, and total stream discharge is then computed for selected stages. The preferred method of measuring discharge in a large tide-affected stream is, of course, the moving-boat method (chap. 6).

One of the basic assumptions of the methods to be presented is that the water in the tidal reach is substantially of homogeneous density, thereby eliminating the possibility of density currents. Therefore, consideration should be limited to those portions of tidal reaches that are affected by the propagation of long, low-amplitude, translatory waves but in which salinity intrusion does not cause a saline wedge to form or cause distinct stratification of flow.

An important consideration in the choice of a reach is the availability of a 110-volt power supply at both ends to drive the digital stage recorders synchronously. Although standby DC units, equipped with DC to AC inverters, are available to take over immediately in the event of a power failure, standby units should be used for periods no longer than absolutely necessary; the use of standby units for long periods of time invariably results in loss of synchronization. Any loss of synchronization between the two clocks, even fractions of a minute, may cause significant error in the computation of discharge. Recent (1976) tests indicate that battery-operated electronic clocks may have the required accuracy and reliability, and if so, the availability of a 110-volt power supply will no longer be a requirement.

POWER SERIES

In solving the equations of unsteady flow by use of a power series—commonly a Taylor series—finite differences are used. Velocity and stage and all orders of their derivatives are continuous functions with respect to x and t. The value of Δx, or length of reach to be used, is generally 3 to 7 miles—there is a theoretical maximum length that depends on tidal wave length (hours) and mean channel depth. The value of Δt to be used is usually 15 minutes.

In the power-series method, tidal flow is considered to be one-dimensional unsteady flow in a prismatic channel. However, a natural reach of channel usually differs greatly from an idealized tidal reach that has an unvarying prismatic cross-section and a constant bottom slope. Consequently, it is necessary to determine a mean cross-section that is representative of the reach and whose dimensions are variable with stage. From such a representation, the geometric parameters required for the discharge computations can be obtained. The number of cross sections to be surveyed in the field in order to compute a representative mean cross-section depends on the length and degree of uniformity of the tidal reach. Usually ten or more cross sections, somewhat evenly spaced in the reach, are required.

Local inflow or outflow (diversions) to or from the tidal reach is considered in the solution, but the quantity of such flow must be small in comparison with the flow in the main channel. The local inflow or outflow is often assumed to be constant throughout a complete tidal cycle and is considered to enter or leave the main channel uniformly along the entire length of reach; thus large concentrations of flow at a single point cannot be accommodated by the method.

A mathematical description of the mechanics of computing discharge in tidal reaches, by the use of a power series for solving the equations of unsteady flow, is beyond the scope of this manual. For such information the reader is referred to papers by Baltzer and Shen (1961, 1964) and by Davidian (1964).

METHOD OF CHARACTERISTICS

The method of characteristics is well adapted to the solution of partial differential equations in two variables, such as equations 122 and 122a. (All terms in those equations are related to velocity and (or) stage.) In the method, the basic partial differential equations are first transformed to characteristic equations and then to corresponding difference equations.

As in the power-series method, tidal flow is considered to be one-dimensional unsteady flow in a prismatic channel. In the finite-difference solution of the equations, computations are made at equal time intervals Δt (usually 15 minutes, or less) and at equal increments of distance x along the channel. The selected value of Δx must meet the criterion,

$$\Delta x \geqslant \Delta t \, (V + \sqrt{gd}), \tag{123}$$

where

- V is mean velocity of flow at the starting cross section of increment Δx,
- g is the acceleration of gravity, and
- d is depth.

Both velocity and stage, as well as discharge, which is the product of velocity and area, can be obtained explicitly for each new step of Δt. Unlike the power-series method which provides the desired information for only one variable at a time—for example, the discharge at one end of the reach—the method of characteristics provides the desired information simultaneously for any selected points in the reach that lie at multiples of Δx from the end of the reach. However, it is possible to change the values of Δx and Δt during the computation, if that is desired, as long as the relation of Δt to Δx meets the criterion given in equation 123.

A single representative cross section is normally used for the entire reach, as in the power-series method. However, a multiple reach may also be used (Lai, 1967a). A long reach is divided into several subreaches, each with its own individual geometry and roughness coefficient. The basic method of characteristics is applied to each subreach, and additional boundary conditions are imposed at each junction between subreaches. Furthermore, the method of characteristics will accommodate the entrance of a large gaged tributary into the reach; the boundary conditions are then more involved, but there is no complication in principle.

The mathematical details of the method of characteristics are described by Lai (1965a, 1967a), Lister (1960), and Stoker (1953).

IMPLICIT METHOD

The implicit method is yet another finite-difference procedure for solving the basic partial differential equations for flows of homogeneous density in tidal reaches. The implicit method has one advantage over the method of characteristics in that the choice of Δt is less restricted; the stability of the solution is not limited by the criterion of equation 123.

The mathematical details of the implicit method are described by Lai (1965b, 1967b, 1968) and by O'Brien, Hyman, and Kaplan (1951).

FOURIER SERIES

The equations of unsteady flow have also been solved by a method of harmonics in which a Fourier series is used. The distinctive characteristic of a Fourier series is the periodicity of the trigonometric terms of which it is composed. The Fourier series lends itself well to the expression of periodic phenomena that are represented by linear differential functions; the equations of unsteady flow, however, are quasi-linear hyperbolic differential functions. Consequently, it is necessary to linearize the equation system when using a Fourier series. That distorts the equation system. An even more significant consideration is whether or not tidal flow can be described as a truly periodic phenomenon. The long translatory wave motion introduced into a tidal reach by the astronomical tide is periodic, but its periodicity is disturbed when the natural upland flow of a river system is superimposed on the tidal wave motion or when storm surges from the ocean occur.

Despite these drawbacks, Fourier series evaluation techniques for determining flow in tidal reaches have been developed (Dronkers, 1947, p. 127–137; Dronkers and Schönfeld, 1955, p.11–24; Schonfeld, 1951, p. 70–87 and 143–152). However this type of solution is the least suitable of the four methods that have been briefly described here for solving the differential equations of unsteady flow in tidal reaches.

EMPIRICAL METHODS

Four empirical methods of rating tidal reaches have been in use, all but one of which were developed before the use of digital computers became commonplace. Those techniques are:

1. method of cubatures,
2. rating-fall method,
3. tide-correction method, and
4. coaxial graphical-correlation method.

All the above methods have their shortcomings which are discussed, where appropriate, in the sections of the manual that follow.

METHOD OF CUBATURES

One of the oldest methods of computing discharge in tidal estuaries is the method of cubatures (Pillsbury, 1956). The method, still in use, is based on the equation of the conservation of mass;

Outflow at the study station = inflow ± change in storage.

The inflow term in the above equation is the freshwater discharge measured at a gaging station at or upstream from the head of tide—that is, a gaging station having a simple stage-discharge relation. The storage term refers to volume of water in the reach between the inflow gaging station and the study station on the estuary. Intermediate stage gages are usually needed for evaluating the storage term. The gages are spaced at such distances that no significant error is introduced in the computations by considering the water surfaces between gages as planes. That requirement ordinarily is met by stations some miles apart but suitably placed with regard to marked changes in the cross section of the waterway. The differences in the tidal ranges on the opposite shores of a wide estuary may usually be disregarded, but it may be necessary to establish tidal stations on any long tidal tributaries of the main waterway. For convenience in the computations, the tides at all stations should be reduced to the same horizontal datum, preferably taken low enough to make all stages positive.

If existing surveys do not afford reliable data on the areas of the water surfaces between the selected tidal stations, a survey to establish these surfaces is required. Usually such surface areas may be taken as increasing uniformly from low water to high water, but if there are any considerable tide flats that are exposed at the lower tidal stages, the area at the stage at which such flats are covered also should be found.

Freshwater inflow to the reach from tributary streams is estimated if the tributary flow is relatively small. If the tributary streams are large, they are gaged upstream from the head of tide to provide a continuous record of freshwater inflow, just as is done with the principal inflow stream.

A sample computation is shown in figure 222 for a 5.8 mile reach of the Delaware River between Trenton and Fieldsboro, N.J. This is the second reach in the estuary; the first reach extends upstream from

Time	Tidal stage			Mean stage during time interval	Water-surface area (A) at mean stage	Change in stage (Δy) during time interval	Change in storage			Outflow Q
	Upper station	Lower station	Mean (y)				In study reach $\Delta Q = A(\Delta y/\Delta t)$	In upstream reach	Total (upstream plus study reaches)	
(hr)	(ft)	(ft)	(ft)	(ft)	(1000's of ft^2)	(ft)	(ft^3/sec)	(ft^3/sec)	(ft^3/sec)	(ft^3/sec)
1	2	3	4	5	6	7	8	9	10	11
0500	2.78	2.52	2.65							
				2.48	39,703	-0.35	-7,720	-500	-8,220	-20,420
0530	2.43	2.16	2.30							
				2.12	38,880	-.35	-7,560	-490	-8,050	-20,250
0600	2.09	1.81	1.95							
				1.80	38,206	-.31	-6,580	-430	-7,010	-19,210
0630	1.79	1.50	1.64							
				1.52	37,656	-.25	-5,230	-410	-5,640	-17,340
0700	1.51	1.27	1.39							
				1.59	37,800	.40	8,400	20	8,420	-3,780
0730	1.52	2.06	1.79							
				2.52	39,686	1.47	32,410	2,840	35,250	23,050
0800	3.48	3.04	3.26							
				3.77	42,247	1.02	23,940	1,350	25,290	13,090
0830	4.41	4.16	4.78							
				4.66	44,053	.76	18,600	1,090	19,690	7,490
0900	5.16	4.93	5.04							
				5.36	45,478	.64	16,170	920	17,090	4,890
0930	5.79	5.58	5.68							
				5.96	46,702	.55	14,270	770	15,040	2,840
1000	6.32	6.14	6.23							
				6.48	47,755	.49	13,000	690	13,690	1,490
1030	6.80	6.64	6.72							

FIGURE 222.—Sample computation of tide-affected discharge by method of cubatures, using 30-minute time intervals. (After Pillsbury, 1956.)

Trenton to the Delaware River gaging station that is upstream from head of tide. Total freshwater inflow to the second reach is −12,200 ft^3/s, flow in the downstream (ebb) direction being considered negative in the computations. That inflow consists of 12,000 ft^3/s for the Delaware River mainstream and 200 ft^3/s for tributary inflow. The time interval (Δt) used in the computations is 30 minutes, or 1,800 seconds. The computations in the table in figure 222 are largely self-explanatory. The figures in column 9 were obtained from similar computations (not shown) for the reach upstream from the study reach. Column 10 is the sum of columns 8 and 9; column 11, the outflow from the study reach, is the sum of the total storage change (column 10) and the total freshwater inflow (−12,200 ft^3/s).

Figure 223 is the discharge hydrograph obtained by first plotting the outflow histogram (values from column 11 of fig. 222), and then drawing a smooth hydrograph to give balance between areas above and below the horizontal bars of the histogram.

The method of cubatures is not only cumbersome, but the discharge figures obtained are only rough approximations of the true values

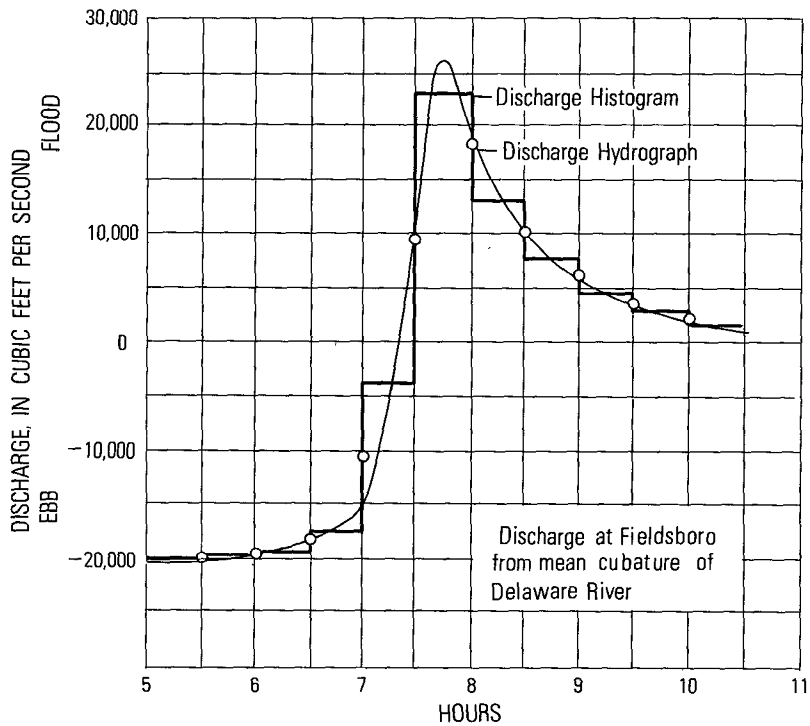

FIGURE 223.—Discharge hydrograph obtained for sample problem by method of cubatures. (After Pillsbury, 1956.)

because of the large errors inherent in computing the storage component of the continuity equation. When the method is used, the results, although approximate, should be checked for consistency—total computed outflow should approximate total inflow over some long-term period whose net change in storage is negligible.

RATING-FALL METHOD

Stage-fall-discharge relations have been used successfully for rating tide-affected streams where acceleration head is a minor factor. The rating-fall method that is discussed in detail in the section in chapter 11 titled, "Variable Slope Caused by Variable Backwater," is used for that purpose. Acceleration head is often a minor factor where the slope reach is located at the upper end of an estuary near the head of tide. Consequently, it is usually only at or near such locations that the rating-fall method can be used successfully.

TIDE-CORRECTION METHOD

The tide-correction method assumes that a direct proportionality exists between the cyclic range in stage observed at any two points within a tidal reach. On the basis of that assumption, a relation of mean discharge for a tidal cycle to mean stage for a tidal cycle is developed for the base-gage site. In calibrating that relation, the mean discharge for a tidal cycle, obtained by averaging several individual measurements made 1–2 hours apart throughout the cycle, is plotted against adjusted mean stage at the base gage. The adjustment applied to the mean stage at the base gage is determined from the difference, at the secondary gage, between observed mean stage and the stage that is presumed to exist under conditions of least tide fluctuation. That difference (D) is multiplied by the ratio of the stage range at the base gage to the stage range at the secondary gage; the product is the stage adjustment required at the base gage. In practice, the secondary stage observations are frequently made at a nearby ocean inlet. Mean sea level is assumed to represent the condition of least tidal fluctuation, and therefore, if all gages have their datums set to mean sea level, D is always equal to the mean stage for a tidal cycle at the secondary gage. Essentially the tide-correction method attempts to approximate the stage that would occur for a particular steady-flow discharge under a fixed backwater condition. An example of the tide-correction method (Parker and others, 1955) follows.

At Hialeah, Fla., the base gage is on the Miami Canal, 7.6 miles upstream from the ocean. A tide gage on the ocean is used as the secondary gage. Both gages have their datum at mean sea level. On a given date the following tidal-cycle data were obtained from the stage gages:

	Base gage	*Secondary gage*
Mean stage (ft)	2.64	1.61 = D
Maximum stage (ft)	3 18	2.74
Minimum stage (ft)	2.10	.48
Stage range (ft)	1.08	2.26

$$\text{Stage correction (base gage)} = D\left(\frac{\text{stage range at base gage}}{\text{stage range at secondary gage}}\right)$$

$$= 1.61\left(\frac{1.08}{2.26}\right)$$

$$= 0.77 \text{ ft}$$

The stage correction is always negative and therefore the stage at the base gage, to be applied to the mean discharge for the tidal cycle on that date, is

$$\text{Gage height} = 2.64-0.77 = 1.87 \text{ ft.}$$

The mean-cycle discharge, as determined from 20 sets of discharge measurements, was plotted against the actual mean-cycle gage height and also against the tide-corrected gage height, as indicated on figure 224. The rating curve shows the relation between the tide-corrected gage height and the mean tide-cycle discharge for the upper gage. The discharge, when plotted against actual mean-cycle gage height, shows a considerable scatter of the plotted points, but the discharge plotted against tide-corrected gage height shows a very

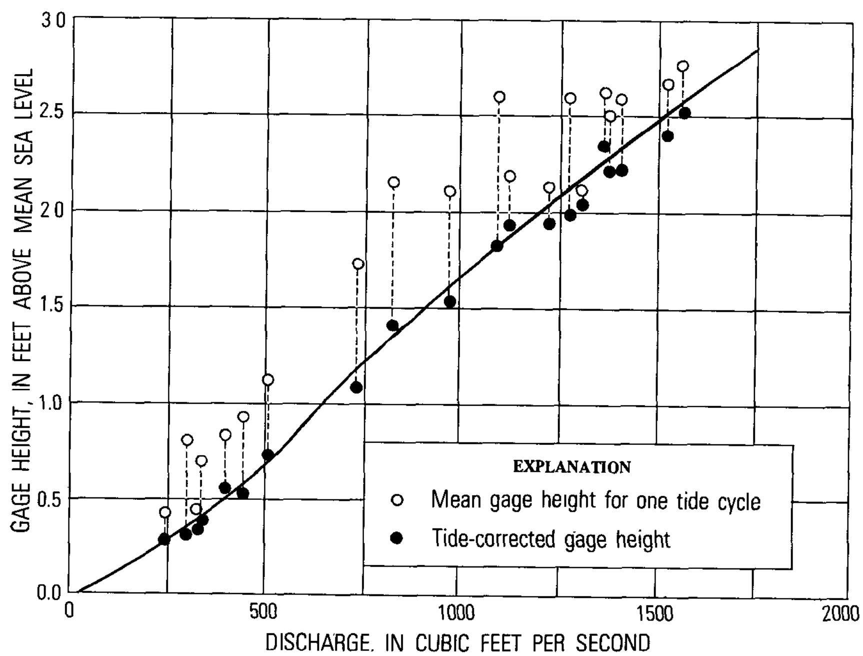

FIGURE 224.—Graph of relation between tide-corrected gage height and discharge for Miami Canal at Water Plant, Hialeah, Fla.

close agreement for the numerous measurements. The shape of the rating curve is characteristic of that for a stream having a large initial cross-sectional area at the point of zero flow.

The tide-correction method of rating a tide-affected stream may be used where reverse flows occur during a part of each tide cycle because the mean discharge for the cycle is the value used in the computation. It is also applicable to a reach of tidal waterway, on which both observation stations are upstream from the mouth of the waterway. Mean-cycle discharge obtained from the rating curve can be plotted against mean-cycle time on a hydrograph sheet, and after connecting the points by straight lines, the daily mean discharges can be determined.

The tide-correction method has been satisfactory, though cumbersome, for computing the daily discharge of tide-affected canals in Florida, but efforts to adopt the method for use elsewhere in the U.S.A. have generally been unsuccessful.

COAXIAL RATING-CURVE METHOD

The coaxial method of graphical correlation to determine discharge in a tidal reach was developed to fill the need for a simple method of making reasonably accurate "on-the-spot" determinations of streamflow. A method of this kind is required, for example, in the operation of a sewage plant discharging its effluent into a tide-affected stream. The method that was developed fills this need in that readings from a pair of stage gages can be used to determine momentary discharge directly from a set of rating curves.

The coaxial method is best described by an example. Coaxial rating curves were developed for the Sacramento River at Sacramento, Calif., on the basis of 302 discharge measurements made during the years 1957–60 (Rantz, 1963). Actually only 52 of the measurements were used to develop the curves; the remaining 250 discharge measurements were used to test the rating curves and refine them slightly. Measured discharges ranged from 4,060 ft^3/s (115 m^3/s) to 19,300 ft^3/s (547 m^3/s).

The streamflow-measurement section is at the site of the stage recorder in the city of Sacramento; the auxiliary stage recorder is 10.8 mi (17.4 km) downstream near the town of Freeport. Local inflow into the 10.8-mile reach of channel is negligible. The reach itself is located far enough upstream on the Sacramento River estuary so that no reversal of flow occurs. When upland flow (streamflow) into the estuary is less than about 30,000 ft^3/s (850 m^3/s), however, the discharge is affected by tidal action, and the flow in the reach is unsteady. The relative magnitude of the tidal effect in the reach increases with decrease in the upland flow and with increase in the range of elevation between high and low tides. The stages at Sacramento and

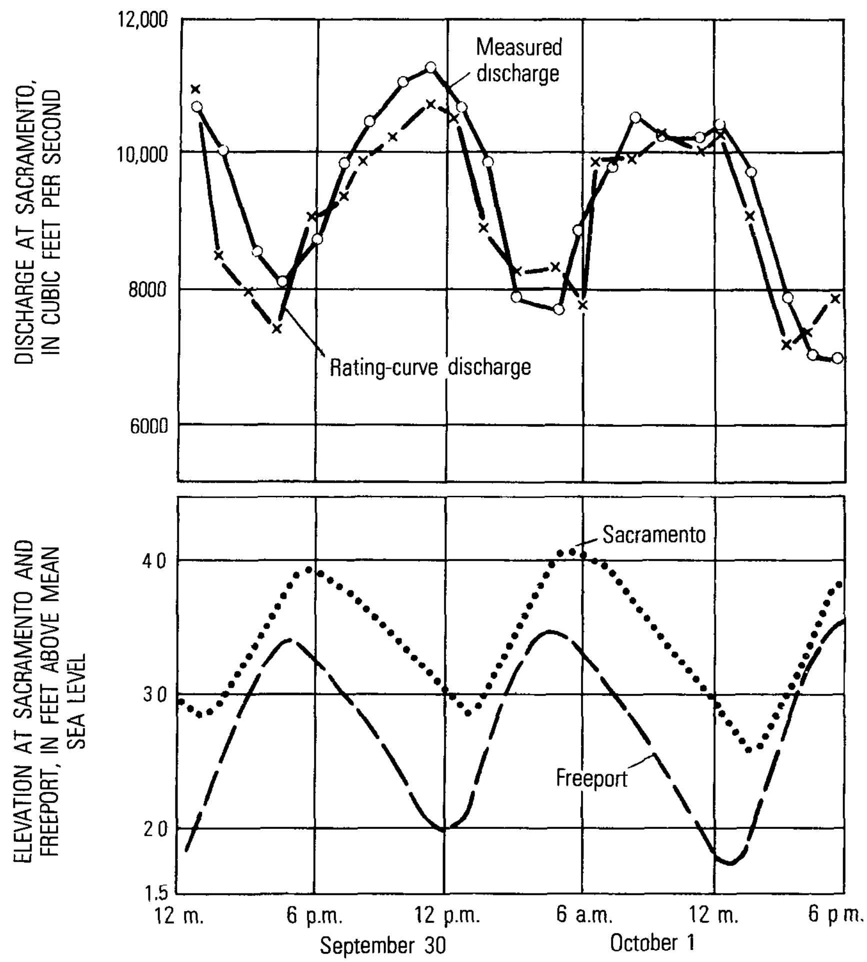

FIGURE 225.—Stage and discharge of the Sacramento River at Sacramento, Calif., Sept. 30 to Oct. 1, 1959.

Freeport during a 36-hour period and the fluctuation of discharge at Sacramento illustrate a typical low-flow condition (fig. 225). The upland flow above Sacramento was 9,300 ft/s (263 m^3/s). As a result of tidal effect, the discharge at Sacramento varied from 6,800 ft^3/s (193 m^3/s) to 11,300 ft^3/s (320 m^3/s).

The differential equations of unsteady flow were used to devise a graphical technique for determining discharge. The following parameters serve as indices of the terms that appear in these differential equations:

Dependent variable.—Measured discharge at Sacramento.

Independent variables.—(1) Stage at Sacramento, (2) fall in the

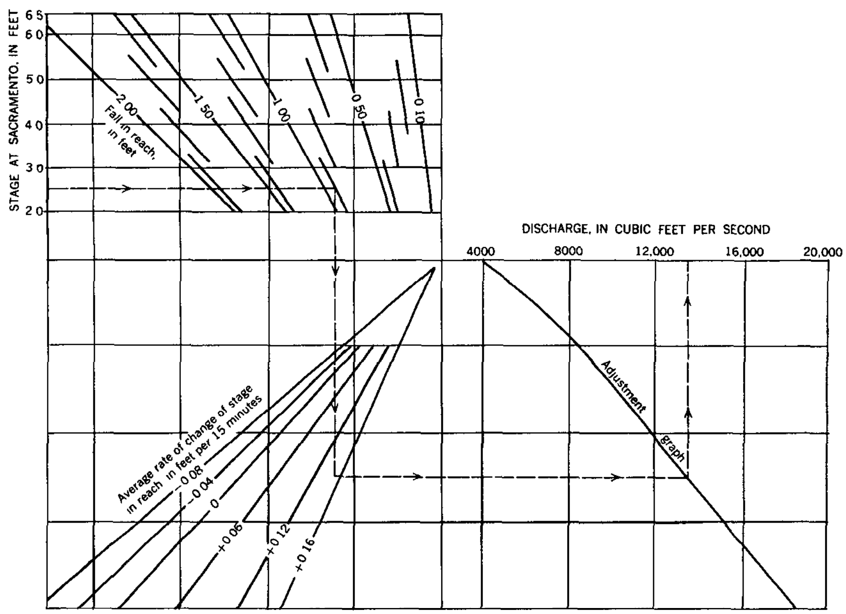

FIGURE 226.—Coaxial rating curves for the Sacramento River at Sacramento, Calif. (Dashed lines and arrows illustrate use of the curves.)

reach between Sacramento and Freeport, and (3) the algebraic average of the change in stage observed at Sacramento and Freeport during a 15-minute interval.

Because of the differential form of the equations of unsteady flow, there is no statistical model on which to base the relationship of these variables. A further complication arises from the fact that joint functions are involved, for interrelations among the independent variables affect the flow at Sacramento. The versatile statistical technique known as the coaxial method of graphical multiple correlation (Linsley and others, 1949, p. 650–655) was adopted for developing the rating curves for the Sacramento River.

The coaxial graphical correlation that was the end product of this study is shown in figure 226. In the interest of simplicity, only a few lines are shown in each family of curves. To use the graph, first, the curves in the upper left-hand group are entered with the stage at Sacramento and the fall in the reach; next, the curves in the lower left-hand group are entered with the average rate of change of stage in the reach; finally, the adjustment graph to the right is entered and the discharge is read. The adjustment graph was added to the correlation to introduce a necessary curvilinearity to the relationship. This curve may also serve another purpose—if the relation should change, as a result of channel dredging, for example, only the adjustment graph need be revised, thereby eliminating the laborious task of revising the two families of curves.

Of the 305 discharge measurements, 268, or 89 percent of them, checked the rating within ±10%; 286 measurements, or 95 percent of the measurements, checked the rating within ±15%. The rating was weakest during periods of low upland flow, but for only a few hours of any day during those periods. Those hours coincided with times when the acceleration head in the equations of unsteady flow was of relatively major importance.

The coaxial rating-curve method fulfills its purpose of being useful for making "on-the-spot" estimates of tidal flow at Sacramento, but the method is too cumbersome for use in computing a continuous record of discharge for a gaging station. Solution of the theoretical equations of unsteady flow (see section on "Evaluation of Unsteady-Flow Equations") is much better for the latter purpose.

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